1 Operatoren

Operatoren aus Chiral Effective Theory heißen R,

$$
\begin{aligned}
R^{(6)}_{1q} &= (\bar{\chi}\gamma_\mu\chi)(\bar{q}\gamma^\mu q) \\
R^{(6)}_{2q} &= (\bar{\chi}\gamma_\mu\gamma_5\chi)(\bar{q}\gamma^\mu q) \\
R^{(6)}_{3q} &= (\bar{\chi}\gamma_\mu\chi)(\bar{q}\gamma^\mu\gamma_5 q) \\
R^{(6)}_{4q} &= (\bar{\chi}\gamma_\mu\gamma_5\chi)(\bar{q}\gamma^\mu\gamma_5 q)
\end{aligned}
$$

die anderen Q

$$
\begin{aligned}
Q^{(6)}_{1ij} &= (\bar{\chi}\gamma_\mu\tilde{\tau}^a\chi)(\bar{Q}^i_L\gamma^\mu\tau^a Q^j_L) \\
Q^{(6)}_{2ij} &= (\bar{\chi}\gamma_\mu\chi)(\bar{Q}^i_L\gamma^\mu Q^j_L) \\
Q^{(6)}_{3ij} &= (\bar{\chi}\gamma_\mu\chi)(\bar{u}^i_R\gamma^\mu u^j_R) \\
Q^{(6)}_{4ij} &= (\bar{\chi}\gamma_\mu\chi)(\bar{d}^i_R\gamma^\mu d^j_R) \\
Q^{(6)}_{5ij} &= (\bar{\chi}\gamma_\mu\gamma_5\tilde{\tau}^a\chi)(\bar{Q}^i_L\gamma^\mu\tau^a Q^j_L) \\
Q^{(6)}_{6ij} &= (\bar{\chi}\gamma_\mu\gamma_5\chi)(\bar{Q}^i_L\gamma^\mu Q^j_L) \\
Q^{(6)}_{7ij} &= (\bar{\chi}\gamma_\mu\gamma_5\chi)(\bar{u}^i_R\gamma^\mu u^j_R) \\
Q^{(6)}_{8ij} &= (\bar{\chi}\gamma_\mu\gamma_5\chi)(\bar{d}^i_R\gamma^\mu d^j_R)
\end{aligned}
$$

Das heißt es werden 12 (3 Flavour x 4 Operatoren) Koeffizienten auf 72 (8 Operatoren x 3x3 Möglichkeiten) Koeffizienten aufgeblasen, wobei sich die 72 auf 42 reduzieren durch die Annahmen *nur u,d,s* und *keine Mischterme*.

2 Kürzen der $Q^{(6)}$: Nur u,d,s und diagonale Terme

$Q^{(6)}_{1ij}, Q^{(6)}_{2ij}, Q^{(6)}_{5ij}, Q^{(6)}_{6ij}$ können zusammen ausgerechnet werden, mit $q_L^{id} = V_{id}d_L + V_{is}s_L + V_{ib}b_L$:

$$\begin{aligned}
Q_L^i A Q_L^j &= \begin{pmatrix} \bar{q}_L^{iu} \\ \bar{q}_L^{id} \end{pmatrix} A \begin{pmatrix} q_L^{ju} \\ q_L^{jd} \end{pmatrix} \\
&= \bar{q}_L^{iu} A q_L^{ju} + \bar{q}_L^{id} A q_L^{jd} \\
&= \bar{q}_L^{iu} A q_L^{ju} + (V_{id}^* \bar{d}_L + V_{is}^* \bar{s}_L + V_{ib}^* \bar{b}_L) A (V_{jd} d_L + V_{js} s_L + V_{jb} b_L) \\
\tilde{Q}_1 = \tilde{Q}_5 &\approx \bar{q}_L^{iu} \gamma^\mu \frac{1}{2} q_L^{ju} \delta_{3a} + (V_{id}^* \bar{d}_L + V_{is}^* \bar{s}_L + V_{ib}^* \bar{b}_L) \gamma^\mu \frac{-1}{2} (V_{jd} d_L + V_{js} s_L + V_{jb} b_L) \delta_{3a} \\
&\approx \bar{u}_L \gamma^\mu \frac{1}{2} u_L \delta_{ij} \delta_{iu} \delta_{3a} - \frac{1}{2} (V_{id}^* V_{jd} \bar{d}_L \gamma^\mu d_L + V_{is}^* V_{js} \bar{s}_L \gamma^\mu s_L) \delta_{3a} \\
\tilde{Q}_2 = \tilde{Q}_6 &\approx \bar{q}_L^{iu} \gamma_\mu q_L^{ju} \delta_{ij} + V_{id}^* V_{jd} \bar{d}_L \gamma_\mu d_L + V_{is}^* V_{js} \bar{s}_L \gamma_\mu s_L \\
&= \bar{u}_L \gamma_\mu u_L \delta_{iu} \delta_{ij} + V_{id}^* V_{jd} \bar{d}_L \gamma_\mu d_L + V_{is}^* V_{js} \bar{s}_L \gamma_\mu s_L \\
\tilde{Q}_3 = \tilde{Q}_7 &= \bar{q}_R^{iu} \gamma^\mu q_R^{ju} \\
&\approx \bar{q}_R^{iu} \gamma^\mu q_R^{ju} \delta_{ij} \\
&= \bar{u}_R \gamma^\mu u_R \delta_{ij} \delta_{iu} \\
\tilde{Q}_4 = \tilde{Q}_8 &= \bar{q}_R^{id} \gamma^\mu q_R^{jd} \\
&\approx \bar{d}_R \gamma^\mu d_R \delta_{ij} \delta_{id} + \bar{s}_R \gamma^\mu s_R \delta_{ij} \delta_{is}
\end{aligned}$$

Das heißt, dass einige der Koeffizienten automatisch 0 sind. Nämlich alle

$$\begin{array}{lll}
C_{3ij} & \text{außer} & C_{311} \\
C_{4ij} & \text{außer} & C_{411}, C_{422} \\
C_{7ij} & \text{außer} & C_{711} \\
C_{8ij} & \text{außer} & C_{811}, C_{822} \quad .
\end{array}$$

Für den Vorfaktor gilt

$$(\bar{\chi} A \tilde{\tau}^a \chi) = \tau_0 (\bar{\chi}_0 A \chi_0)$$

mit $\tau_0 = 0$, falls a nicht der richtige Index ist.

3 Umschreiben der $Q^{(6)}$ von q_L, q_R in q

$$
\begin{aligned}
\tilde{Q}_1^{(6)} = \tilde{Q}_5^{(6)} &= \bar{u}\gamma^\mu \frac{1-\gamma_5}{4} u\delta_{ij}\delta_{iu}\delta_{3a} - V_{id}^* V_{jd}\bar{d}\gamma^\mu \frac{1-\gamma_5}{4} d\delta_{3a} - V_{is}^* V_{js}\bar{s}\gamma^\mu \frac{1-\gamma_5}{4} s\delta_{3a} \\
&= \frac{1}{4}(\bar{u}\gamma^\mu u\delta_{ij}\delta_{iu}\delta_{3a} - V_{id}^* V_{jd}\bar{d}\gamma^\mu d\delta_{3a} - V_{is}^* V_{js}\bar{s}\gamma^\mu s\delta_{3a}) - \frac{1}{4}(\bar{u}\gamma^\mu\gamma_5 u\delta_{ij}\delta_{iu}\delta_{3a} - V_{id}^* V_{jd}\bar{d}\gamma^\mu\gamma_5 d\delta_{3a} - V_{is}^* V_{js}\bar{s}\gamma^\mu\gamma_5 s\delta_{3a}) \\
\tilde{Q}_2^{(6)} = \tilde{Q}_6^{(6)} &= \bar{u}\gamma^\mu \frac{1-\gamma_5}{2} u\delta_{iu}\delta_{ij} + V_{id}^* V_{jd}\bar{d}\gamma^\mu \frac{1-\gamma_5}{2} d + V_{is}^* V_{js}\bar{s}\gamma^\mu \frac{1-\gamma_5}{2} s \\
&= \frac{1}{2}(\bar{u}\gamma^\mu u\delta_{iu}\delta_{ij} - \bar{u}\gamma^\mu\gamma_5 u\delta_{iu}\delta_{ij}) + \frac{1}{2}(V_{id}^* V_{jd}\bar{d}\gamma^\mu d - V_{id}^* V_{jd}\bar{d}\gamma^\mu\gamma_5 d + V_{is}^* V_{js}\bar{s}\gamma^\mu s - V_{is}^* V_{js}\bar{s}\gamma^\mu\gamma_5 s) \\
\tilde{Q}_3^{(6)} = \tilde{Q}_7^{(6)} &= \bar{u}_R\gamma^\mu u_R\delta_{ij}\delta_{iu} \\
&= \bar{u}\gamma^\mu \frac{1+\gamma_5}{2} u\delta_{ij}\delta_{iu} \\
&= \frac{1}{2}(\bar{u}\gamma^\mu u\delta_{ij}\delta_{iu} + \bar{u}\gamma^\mu\gamma_5 u\delta_{ij}\delta_{iu}) \\
\tilde{Q}_4^{(6)} = \tilde{Q}_8^{(6)} &= \bar{d}_R\gamma^\mu d_R\delta_{ij}\delta_{id} + \bar{s}_R\gamma^\mu s_R\delta_{ij}\delta_{is} \\
&= \bar{d}\gamma^\mu \frac{1+\gamma_5}{2} d\delta_{ij}\delta_{id} + \bar{s}\gamma^\mu \frac{1+\gamma_5}{2} s\delta_{ij}\delta_{is} \\
&= \frac{1}{2}(\bar{d}\gamma^\mu d\delta_{ij}\delta_{id} + \bar{d}\gamma^\mu\gamma_5 d\delta_{ij}\delta_{id} + \bar{s}\gamma^\mu s\delta_{ij}\delta_{is} + \bar{s}\gamma^\mu\gamma_5 s\delta_{ij}\delta_{is})
\end{aligned}
$$

4 Umschreiben der $Q^{(6)}$ in $R^{(6)}$

$$
\begin{aligned}
Q^{(6)}_{1ij} &= (\bar{\chi}_0\gamma_\mu\chi_0)\delta_{3a}\tau_0\frac{1}{4}(\bar{u}\gamma^\mu u\delta_{ij}\delta_{iu} - V^*_{id}V_{jd}\bar{d}\gamma^\mu d - V^*_{is}V_{js}\bar{s}\gamma^\mu s) - (\bar{\chi}_0\gamma_\mu\chi_0)\tau_0\frac{1}{4}(\bar{u}\gamma^\mu\gamma_5 u\delta_{ij}\delta_{iu} - V^*_{id}V_{jd}\bar{d}\gamma^\mu\gamma_5 d - V^*_{is}V_{js}\bar{s}\gamma^\mu\gamma_5 s) \\
&= \frac{\delta_{3a}\tau_0}{4}(R^{(6)}_{1u}\delta_{ij}\delta_{iu} - V^*_{id}V_{jd}R^{(6)}_{1d} - V^*_{is}V_{js}R^{(6)}_{1s}) - \frac{\delta_{3a}\tau_0}{4}(R^{(6)}_{3u}\delta_{ij}\delta_{iu} - V^*_{id}V_{jd}R^{(6)}_{3d} - V^*_{is}V_{js}R^{(6)}_{3s}) \\
Q^{(6)}_{2ij} &= (\bar{\chi}\gamma_\mu\chi)\frac{1}{2}(\bar{u}\gamma_\mu u\delta_{iu}\delta_{ij} - \bar{u}\gamma_\mu\gamma_5 u\delta_{iu}\delta_{ij}) + (\bar{\chi}\gamma_\mu\chi)\frac{1}{2}(V^*_{id}V_{jd}\bar{d}\gamma_\mu d - V^*_{id}V_{jd}\bar{d}\gamma_\mu\gamma_5 d + V^*_{is}V_{js}\bar{s}\gamma_\mu s - V^*_{is}V_{js}\bar{s}\gamma_\mu\gamma_5 s) \\
&= \frac{1}{2}(R^{(6)}_{1u}\delta_{iu}\delta_{ij} - R^{(6)}_{3u}\delta_{iu}\delta_{ij}) + \frac{1}{2}(V^*_{id}V_{jd}R^{(6)}_{1d} - V^*_{id}V_{jd}R^{(6)}_{3d} + V^*_{is}V_{js}R^{(6)}_{1s} - V^*_{is}V_{js}R^{(6)}_{3s}) \\
Q^{(6)}_{3ij} &= (\bar{\chi}\gamma_\mu\chi)\frac{1}{2}(\bar{u}\gamma^\mu u\delta_{ij}\delta_{iu} + \bar{u}\gamma^\mu\gamma_5 u\delta_{ij}\delta_{iu}) \\
&= \frac{1}{2}(R^{(6)}_{1u}\delta_{ij}\delta_{iu} + R^{(6)}_{3u}\delta_{ij}\delta_{iu}) \\
Q^{(6)}_{4ij} &= (\bar{\chi}\gamma_\mu\chi)\frac{1}{2}(\bar{d}\gamma^\mu d\delta_{ij}\delta_{id} + \bar{d}\gamma^\mu\gamma_5 d\delta_{ij}\delta_{id} + \bar{s}\gamma^\mu s\delta_{ij}\delta_{is} + \bar{s}\gamma^\mu\gamma_5 s\delta_{ij}\delta_{is}) \\
&= \frac{1}{2}(R^{(6)}_{1d}\delta_{ij}\delta_{id} + R^{(6)}_{3d}\delta_{ij}\delta_{id} + R^{(6)}_{1s}\delta_{ij}\delta_{is} + R^{(6)}_{3s}\delta_{ij}\delta_{is}) \\
Q^{(6)}_{5ij} &= (\bar{\chi}_0\gamma_\mu\chi_0)\delta_{3a}\tau_0\frac{1}{4}(\bar{u}\gamma^\mu u\delta_{ij}\delta_{iu} - V^*_{id}V_{jd}\bar{d}\gamma^\mu d - V^*_{is}V_{js}\bar{s}\gamma^\mu s) - (\bar{\chi}_0\gamma_\mu\chi_0)\delta_{3a}\tau_0\frac{1}{4}(\bar{u}\gamma^\mu\gamma_5 u\delta_{ij}\delta_{iu} - V^*_{id}V_{jd}\bar{d}\gamma^\mu\gamma_5 d - V^*_{is}V_{js}\bar{s}\gamma^\mu\gamma_5 s) \\
&= \frac{\delta_{3a}\tau_0}{4}(R^{(6)}_{2u}\delta_{ij}\delta_{iu} - V^*_{id}V_{jd}R^{(6)}_{2d} - V^*_{is}V_{js}R^{(6)}_{2s}) - \frac{\delta_{3a}\tau_0}{4}(R^{(6)}_{4u}\delta_{ij}\delta_{iu} - V^*_{id}V_{jd}R^{(6)}_{4d} - V^*_{is}V_{js}R^{(6)}_{4s}) \\
Q^{(6)}_{6ij} &= (\bar{\chi}\gamma_\mu\chi)\frac{1}{2}(\bar{u}\gamma_\mu u\delta_{iu}\delta_{ij} - \bar{u}\gamma_\mu\gamma_5 u\delta_{iu}\delta_{ij}) + (\bar{\chi}\gamma_\mu\chi)\frac{1}{2}(V^*_{id}V_{jd}\bar{d}\gamma_\mu d - V^*_{id}V_{jd}\bar{d}\gamma_\mu\gamma_5 d + V^*_{is}V_{js}\bar{s}\gamma_\mu s - V^*_{is}V_{js}\bar{s}\gamma_\mu\gamma_5 s) \\
&= \frac{1}{2}(R^{(6)}_{2u}\delta_{iu}\delta_{ij} - R^{(6)}_{4u}\delta_{iu}\delta_{ij}) + \frac{1}{2}(V^*_{id}V_{jd}R^{(6)}_{2d} - V^*_{id}V_{jd}R^{(6)}_{4d} + V^*_{is}V_{js}R^{(6)}_{2s} - V^*_{is}V_{js}R^{(6)}_{4s}) \\
Q^{(6)}_{7ij} &= (\bar{\chi}\gamma_\mu\gamma_5\chi)\frac{1}{2}(\bar{u}\gamma^\mu u\delta_{ij}\delta_{iu} + \bar{u}\gamma^\mu\gamma_5 u\delta_{ij}\delta_{iu}) \\
&= \frac{1}{2}(R^{(6)}_{2u}\delta_{ij}\delta_{iu} + R^{(6)}_{4u}\delta_{ij}\delta_{iu}) \\
Q^{(6)}_{8ij} &= (\bar{\chi}\gamma_\mu\gamma_5\chi)\frac{1}{2}(\bar{d}\gamma^\mu d\delta_{ij}\delta_{id} + \bar{d}\gamma^\mu\gamma_5 d\delta_{ij}\delta_{id} + \bar{s}\gamma^\mu s\delta_{ij}\delta_{is} + \bar{s}\gamma^\mu\gamma_5 s\delta_{ij}\delta_{is}) \\
&= \frac{1}{2}(R^{(6)}_{2d}\delta_{ij}\delta_{id} + R^{(6)}_{4d}\delta_{ij}\delta_{id} + R^{(6)}_{2s}\delta_{ij}\delta_{is} + R^{(6)}_{4s}\delta_{ij}\delta_{is})
\end{aligned}
$$

5 Ausrechnen der Koeffizienten

$$\mathcal{L}_Q^{(6)} = \sum_{k,i,j} C_{kij} Q_{kij}^{(6)} \stackrel{!}{=} \sum_{l,m} D_{lm} R_{lm}^{(6)} = \mathcal{L}_R^{(6)}$$

$$\begin{aligned}
\mathcal{L}_Q^{(6)} &= \sum_{k,i,j} C_{kij} Q_{kij}^{(6)} \\
&= \sum_{i,j} C_{1ij}(\frac{\delta_{3a}\tau_0}{4}(R_{1u}^{(6)}\delta_{ij}\delta_{iu} - V_{id}^* V_{jd} R_{1d}^{(6)} - V_{is}^* V_{js} R_{1s}^{(6)}) - \frac{\delta_{3a}\tau_0}{4}(R_{3u}^{(6)}\delta_{ij}\delta_{iu} - V_{id}^* V_{jd} R_{3d}^{(6)} - V_{is}^* V_{js} R_{3s}^{(6)})) \\
&+ C_{2ij}(\frac{1}{2}(R_{1u}^{(6)}\delta_{iu}\delta_{ij} - R_{3u}^{(6)}\delta_{iu}\delta_{ij}) + \frac{1}{2}(V_{id}^* V_{jd} R_{1d}^{(6)} - V_{id}^* V_{jd} R_{3d}^{(6)} + V_{is}^* V_{js} R_{1s}^{(6)} - V_{is}^* V_{js} R_{3s}^{(6)})) \\
&+ C_{3ij}(\frac{1}{2}(R_{1u}^{(6)}\delta_{ij}\delta_{iu} + R_{3u}^{(6)}\delta_{ij}\delta_{iu})) \\
&+ C_{4ij}(\frac{1}{2}(R_{1d}^{(6)}\delta_{ij}\delta_{id} + R_{3d}^{(6)}\delta_{ij}\delta_{id} + R_{1s}^{(6)}\delta_{ij}\delta_{is} + R_{3s}^{(6)}\delta_{ij}\delta_{is})) \\
&+ C_{5ij}(\frac{\delta_{3a}\tau_0}{4}(R_{2u}^{(6)}\delta_{ij}\delta_{iu} - V_{id}^* V_{jd} R_{2d}^{(6)} - V_{is}^* V_{js} R_{2s}^{(6)}) - \frac{\delta_{3a}\tau_0}{4}(R_{4u}^{(6)}\delta_{ij}\delta_{iu} - V_{id}^* V_{jd} R_{4d}^{(6)} - V_{is}^* V_{js} R_{4s}^{(6)})) \\
&+ C_{6ij}(\frac{1}{2}(R_{2u}^{(6)}\delta_{iu}\delta_{ij} - R_{4u}^{(6)}\delta_{iu}\delta_{ij}) + \frac{1}{2}(V_{id}^* V_{jd} R_{2d}^{(6)} - V_{id}^* V_{jd} R_{4d}^{(6)} + V_{is}^* V_{js} R_{2s}^{(6)} - V_{is}^* V_{js} R_{4s}^{(6)})) \\
&+ C_{7ij}(\frac{1}{2}(R_{2u}^{(6)}\delta_{ij}\delta_{iu} + R_{4u}^{(6)}\delta_{ij}\delta_{iu})) \\
&+ C_{8ij}(\frac{1}{2}(R_{2d}^{(6)}\delta_{ij}\delta_{id} + R_{4d}^{(6)}\delta_{ij}\delta_{id} + R_{2s}^{(6)}\delta_{ij}\delta_{is} + R_{4s}^{(6)}\delta_{ij}\delta_{is}))
\end{aligned}$$

$$
\begin{aligned}
&= \sum_{i,j} \\
&+ R^{(6)}_{1u} \frac{\delta_{ij}\delta_{iu}}{2} \left(C_{1ij} \frac{\delta_{3a}\tau_0}{2} + C_{2ij} + C_{3ij} \right) \\
&+ R^{(6)}_{1d} \frac{1}{2} \left(-V^*_{id} V_{jd} C_{1ij} \frac{\delta_{3a}\tau_0}{2} + V^*_{id} V_{jd} C_{2ij} + \delta_{ij}\delta_{id} C_{4ij} \right) \\
&+ R^{(6)}_{1s} \frac{1}{2} \left(-V^*_{is} V_{js} C_{1ij} \frac{\delta_{3a}\tau_0}{2} + V^*_{is} V_{js} C_{2ij} + \delta_{ij}\delta_{is} C_{4ij} \right) \\
&+ R^{(6)}_{2u} \frac{\delta_{ij}\delta_{iu}}{2} \left(\frac{\delta_{3a}\tau_0}{2} C_{5ij} + C_{6ij} + C_{7ij} \right) \\
&+ R^{(6)}_{2d} \frac{1}{2} \left(-V^*_{id} V_{jd} \frac{\delta_{3a}\tau_0}{2} C_{5ij} + C_{6ij} V^*_{id} V_{jd} + \delta_{ij}\delta_{id} C_{8ij} \right) \\
&+ R^{(6)}_{2s} \frac{1}{2} \left(-V^*_{is} V_{js} \frac{\delta_{3a}\tau_0}{2} C_{5ij} + C_{6ij} V^*_{is} V_{js} + \delta_{ij}\delta_{is} C_{8ij} \right) \\
&+ R^{(6)}_{3u} \frac{\delta_{ij}\delta_{iu}}{2} \left(-C_{1ij} \frac{\delta_{3a}\tau_0}{2} - C_{2ij} + C_{3ij} \right) \\
&+ R^{(6)}_{3d} \frac{1}{2} \left(C_{1ij} \frac{\delta_{3a}\tau_0}{2} V^*_{id} V_{jd} - V^*_{id} V_{jd} C_{2ij} + \delta_{ij}\delta_{id} C_{4ij} \right) \\
&+ R^{(6)}_{3s} \frac{1}{2} \left(C_{1ij} \frac{\delta_{3a}\tau_0}{2} V^*_{is} V_{js} - V^*_{is} V_{js} C_{2ij} + \delta_{ij}\delta_{is} C_{4ij} \right) \\
&+ R^{(6)}_{4u} \frac{\delta_{ij}\delta_{iu}}{2} \left(-\frac{\delta_{3a}\tau_0}{2} C_{5ij} - C_{6ij} + C_{7ij} \right) \\
&+ R^{(6)}_{4d} \frac{1}{2} \left(\frac{\delta_{3a}\tau_0}{2} C_{5ij} V^*_{id} V_{jd} - C_{6ij} V^*_{id} V_{jd} + \delta_{ij}\delta_{id} C_{8ij} \right) \\
&+ R^{(6)}_{4s} \frac{1}{2} \left(\frac{\delta_{3a}\tau_0}{2} C_{5ij} V^*_{is} V_{js} - C_{6ij} V^*_{is} V_{js} + \delta_{ij}\delta_{is} C_{8ij} \right)
\end{aligned}
$$