technische universität
dortmund

Arbeit zur Erlangung des akademischen Grades
Bachelor of Science

Flavour Mixing Effects in the Direct Detection of Dark Matter

Anja Beck
geboren in Kempten (Allgäu)

2017

Lehrstuhl für Theoretische Physik IV
Fakultät Physik
Technische Universität Dortmund

Erstgutachter:	Jun.-Prof. Dr. Joachim Brod
Zweitgutachter:	Prof. Dr. Heinrich Päs
Abgabedatum:	17. Juli 2017

Abstract

Quark flavour mixing often is neglected because of its little effects. But since dark matter only interacts weakly with baryonic matter, these tiny effects might be of importance for direct detection. This thesis examines the effects of flavour mixing on dark matter direct detection in one specific example. After explaining the theoretical origin of the flavour mixing, we therefore expand an existing mathematical framework for direct detection with the CKM mixing matrix. We then present a new interaction proposed to explain anomalies in the decay $B \to Kl\bar{l}$, which also makes predictions about the direct detection of dark matter. Finally we compare the new cross sections with the flavour mixing cross sections and see that, in this case, flavour mixing was rightfully neglected.

Contents

1 Introduction

Observing the movement of stars and planets is one of the earliest scientific acts of mankind, e. g. Babylonians analysed the orbit of the Venus around 1500 B.C. [**History**]. Throughout all human civilisations night sky has fascinated people and is probably continuing to do so, because still little is known about the particles that make up the universe. Scientists only understand about 20% of the universe's matter. They call the remaining 80% dark matter, due to its lack of luminosity. [**DM**]

Astronomers in the 1930's found first indications of its existence. One of them was F. Zwicky, a Swiss astronomer who was interested in the Coma cluster. Through measurements of the Doppler shifts in galactic spectra, he obtained the velocity distribution and the kinetic energy of the galaxies in the cluster. Zwicky assumed that there were only gravitational interactions that could be described with Newtonian gravity. The virial theorem then gives the relation $\langle T \rangle = -1/2 \langle U \rangle$ between the average kinetic energy $\langle T \rangle$ and the average potential energy $\langle U \rangle$. Using this equation, Zwicky calculated the Cluster mass to be $M_{\text{Coma}} \approx 4.5 \cdot 10^{-13} M_{\text{sun}}$. Surprisingly this was 50 times the mass obtained by luminosity measurements. Later, a small ratio of the missing mass could be accounted to intracluster gas, but a huge discrepancy remained and was attributed to dark matter. [**DM**]

However, even today, after decades of intensive dark matter research, physicists do not know what dark matter particles are and how they interact. To shed light on dark matter, there currently are various detection experiments around the globe, which can be classified in two categories: indirect and direct detection. Indirect detection experiments try to measure the secondary products of dark matter annihilation, whereas direct detection experiments aim at detecting the dark matter particles. As shown in figure **??** this means scattering of a dark matter particle off a nucleus.

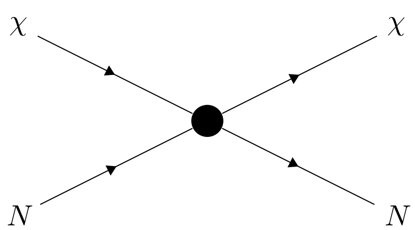


Figure 1.1: Direct detection, respectively scattering of a dark matter particle χ off a nucleus N.

Nuclei consist of protons and neutrons, which contain quarks, hence quark flavour mixing might have an effect on direct detection. In this thesis we want to examine whether flavour mixing was rightfully neglected in one particular example. Therefore we first expand an existing formalism for dark matter direct detection with the CKM mixing matrix. Secondly, we present a new interaction proposed to explain anomalies in the decay $B \to K l \bar{l}$ that also makes predictions about the direct detection of dark matter. And finally we compare these predictions with the direct detection cross sections that respect flavour mixing.

2 The Flavour Mixing Mechanism

The origins of flavour mixing go back to the 1960s, when the Italian physicist Cabibbo resolved anomalies in data of weak interactions by proposing a flavour mixing of left-handed down-type quarks. Later in 1973, Kobayaski and Maskawa extended this idea to three quark generations to explain CP violation [**Griffiths**]. On a mathematical level, quark flavour mixing arises from the fact that the fermion mass eigenstates do not necessarily coincide with the flavour eigenstates. In the course of this chapter we first derive how the Higgs mechanism gives mass to particles and second take a look at fermion masses and why this leads to flavour mixing. The following calculations are based on the outlines in [**Peskin**] and [**Tevatron**].

2.1 The Higgs Mechanism

We consider a complex scalar field Φ that interacts with itself through a potential

$$V(\Phi) = -\mu^2 \Phi^\dagger \Phi + \frac{\lambda}{2}(\Phi^\dagger \Phi)^2 \ , \quad \mu^2 > 0 \ . \tag{2.1}$$

If $\lambda > 0$, this potential has two two minima, which occur at

$$\langle \Phi \rangle = \pm \sqrt{\frac{\mu^2}{\lambda}} \ . \tag{2.2}$$

These are the vacuum expectation values of Φ.

We now introduce a symmetry, for example $SU(2)$, coupled to Φ to show how breaking the symmetry leads to massive particles. In order to couple to $SU(2)$, Φ has to be a doublet (Φ_1, Φ_2) with covariant derivative

$$D_\mu \Phi = (\partial_\mu - ig \sum_{a=1}^{3} A_\mu^a \tau^a)\Phi \ , \tag{2.3}$$

where τ^a are the generators of the $SU(2)$. Since the condition **??** still holds for the absolute value of Φ, there now is an infinite number of vacuum expectation values, arranged in a circle. We are free to choose one and make the simple choice

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \ , \qquad v = \sqrt{\frac{2\mu^2}{\lambda}} \ . \tag{2.4}$$

The kinetic energy of Φ is then

$$\begin{aligned}(D_\mu\Phi)^2 =& \frac{1}{2}(\partial_\mu v)(\partial^\mu v) \\ & -ig(\partial^\mu \begin{pmatrix}0 & v\end{pmatrix})\left(\sum_{a=1}^{3} A_\mu^a \tau^a \begin{pmatrix}0\\ v\end{pmatrix}\right) \\ & -\frac{1}{2}g^2 \begin{pmatrix}0 & v\end{pmatrix} \sum_{a,b=1}^{3} \tau^a\tau^b \begin{pmatrix}0\\ v\end{pmatrix} A_\mu^a A^{b\mu} \ .\end{aligned} \tag{2.5}$$

Using the anticommutator $\{\tau^a, \tau^b\} = {}^{\delta_{ab}}/2$, we can simplify the last expression in **??** to

$$-\frac{1}{2}g^2 \begin{pmatrix}0 & v\end{pmatrix} \sum_{a,b=1}^{3} \tau^a\tau^b \begin{pmatrix}0\\ v\end{pmatrix} A_\mu^a A^{b\mu} = -\frac{g^2v^2}{8}\sum_{a=1}^{3} A_\mu^a A^{a\mu} \ , \tag{2.6}$$

which is a mass term $\mathcal{L}_m = -\frac{1}{2}m_A^2 A_\mu A^\mu$ that assigns the mass $m_A = \frac{gv}{2}$ to all three gauge bosons that are needed in a $SU(2)$ symmetry. By expanding the system with an additional $U(1)$ symmetry, the kinetic energy would again provide three gauge boson masses, leaving the fourth gauge boson massless. The massive bosons can be identified as $W^\pm, Z^0$ and the massless as the photon.

The scalar field Φ that gives mass to the gauge bosons of a $SU(2) \times U(1)$ symmetry is usually called Higgs boson. Obtaining particle mass terms in the kinetic energy of the Higgs, is, unsurprisingly, referred to as the Higgs mechanism.

2.2 Fermion Masses and Flavour Mixing

Hereinafter we describe how the standard model fermions get their masses and how this leads to quark flavour mixing. But beforehand, we introduce the notation we use in this and the following chapters. The leptons and quark chiral particle multiplets are

$$E_R = (e_R, \mu_R, \tau_R) \ , \qquad Y_E = -2 \ ; \tag{2.7}$$

$$L_L = \left(\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L , \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L , \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L \right) \ , \qquad Y_L = -1 \ ; \tag{2.8}$$

$$U_R = (u_R, c_R, t_R) \ , \qquad Y_U = \frac{4}{3} \ ; \tag{2.9}$$

$$D_R = (d_R, s_R, b_R) \ , \qquad Y_D = -\frac{2}{3} \ ; \tag{2.10}$$

$$Q_L = \left(\begin{pmatrix} u \\ d \end{pmatrix}_L , \begin{pmatrix} c \\ s \end{pmatrix}_L , \begin{pmatrix} t \\ b \end{pmatrix}_L \right) \ , \qquad Y_Q = \frac{1}{3} \ , \tag{2.11}$$

where the hypercharge Y is given. It is related to the electric charge Q and the third component of the weak isospin I_3 through the Gell-Mann-Nishijima formula [**Griffiths**]

$$Q = \frac{Y}{2} + I_3 \ . \tag{2.12}$$

The right-handed particles are singlets under $SU(2)$ and therefore $I_3^{(r.h.)} = 0$. We later also use the left-handed components $E_L = (e_L, \mu_L, \tau_L)$, $D_L = (d_L, s_L, b_L)$, and $U_L = (u_L, c_L, t_L)$.

The electroweak interaction lagrangian for the standard model fermions is

$$\begin{aligned} \mathcal{L}^{(\mathrm{int})} =& \bar{E}_R \gamma^\mu (i\partial_\mu - g_1 Y_E B_\mu) E_R + \bar{L}_L \gamma^\mu (i\partial_\mu - g_1 Y_L B_\mu - g_2 \sum_{i=1}^{3} A_\mu^i \tau^i) L_L \\ &+ \bar{D}_R \gamma^\mu (i\partial_\mu - g_1 Y_D B_\mu) D_R + \bar{U}_R \gamma^\mu (i\partial_\mu - g_1 Y_U B_\mu) U_R \\ &+ \bar{Q}_L \gamma^\mu (i\partial_\mu - g_1 Y_Q B_\mu - g_2 \sum_{i=1}^{3} A_\mu^i \tau^i) Q_L \ , \end{aligned} \tag{2.13}$$

where B_μ, A_μ^i are the gauge bosons corresponding to $U(1)_Y \times SU(2)$. The coupling constants are g_1 and g_2, and the τ^i are again the $SU(2)$ generators. This lagrangian describes massless particles. In order to get a fermion mass term, one has to couple the left- and right-handed part of a particle. Since a direct coupling between a $SU(2)$ singlet and a $SU(2)$ doublet violates gauge invariance, a connecting field is necessary. To preserve invariance under Lorentz, $U(1)_Y$, and $SU(2)$ transformations this field must have spin 0, hypercharge $Y = 1/2$, and be a doublet. We identify this field with Φ from the previous chapter and write down the mass terms for the fermions

$$\mathcal{L}^{(\mathrm{mass})} = - \left[\bar{L}_L \Phi \lambda^e E_R + \bar{Q}_L \Phi \lambda^d D_R + \bar{Q}_L i\sigma^2 \Phi^\dagger \lambda^u U_R + \mathrm{h.c.} \right] \ , \tag{2.14}$$

with complex matrix coupling constants $\lambda^e, \lambda^d, \lambda^u$. Replacing Φ with its vacuum expectation value (**??**) gives

$$\mathcal{L}^{(\text{mass})} = -\frac{v}{\sqrt{2}} \left[\bar{E}_L \lambda^e E_R + \bar{D}_L \lambda^d D_R + \bar{U}_L \lambda^u U_R + \text{h.c.} \right] \ . \tag{2.15}$$

The interaction lagrangian $\mathcal{L}^{(\text{int})}$ (see (**??**)) is invariant under unitary transformations

$$E_L \to S_e E_L \ , \qquad E_R \to R_e E_R \ , \tag{2.16}$$
$$U_L \to S_u U_L \ , \qquad U_R \to R_u U_R \ , \tag{2.17}$$
$$D_L \to S_d D_L \ , \qquad D_R \to R_d D_R \ . \tag{2.18}$$

Thus, we can diagonalize the interactions in (**??**), using these transformations. The diagonal lepton coupling is $\tilde{\lambda}^e = S_e \lambda^e R_e^\dagger$ and parametrizes the lepton masses

$$m_e = \frac{v}{\sqrt{2}} \tilde{\lambda}^e_{11} \ , \quad m_\mu = \frac{v}{\sqrt{2}} \tilde{\lambda}^e_{22} \ , \quad m_\tau = \frac{v}{\sqrt{2}} \tilde{\lambda}^e_{33} \ . \tag{2.19}$$

The diagonal coupling for up-type quarks is $\tilde{\lambda}^u = S_u \lambda^u R_u^\dagger$, giving the corresponding masses

$$m_u = \frac{v}{\sqrt{2}} \tilde{\lambda}^u_{11} \ , \quad m_c = \frac{v}{\sqrt{2}} \tilde{\lambda}^u_{22} \ , \quad m_t = \frac{v}{\sqrt{2}} \tilde{\lambda}^u_{33} \ . \tag{2.20}$$

The transformed coupling of the down-type quarks is $\tilde{\lambda}^d = S_d \lambda^d R_d^\dagger$, leading to the down-type masses

$$m_d = \frac{v}{\sqrt{2}} \tilde{\lambda}^d_{11} \ , \quad m_s = \frac{v}{\sqrt{2}} \tilde{\lambda}^d_{22} \ , \quad m_b = \frac{v}{\sqrt{2}} \tilde{\lambda}^d_{33} \ . \tag{2.21}$$

The transformed particle multiplets are now mass eigenstates. But when looking at couplings of up- and down-type quarks, e.g. the current

$$\bar{U}_L \gamma^\mu D_L \ , \tag{2.22}$$

the unitary transformations change the interaction to

$$\bar{U}_L \gamma^\mu S_u^\dagger S_d D_L \ , \tag{2.23}$$

where we identify the CKM matrix $V = S_u^\dagger S_d$. Because S_u, S_d are already determined by the diagonalizations above, the CKM matrix is not equal to the identity matrix in general. In fact, it alters left-handed currents measurably. The most recent data by the Particle Data Group determines the parameters of the Wolfenstein parametrization

$$V = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} \tag{2.24}$$

to be (see [**PDG**])

$$\begin{aligned} \lambda &= 0.224\,96 \pm 0.000\,48 \ , & A &= 0.823 \pm 0.013 \ , \\ \rho &= 0.141 \pm 0.019 \ , & \eta &= 0.349 \pm 0.012 \ .^{1} \end{aligned} \tag{2.25}$$

[1] The Particle Data Group actually suggests two different sets of parameters obtained by two different methods. We choose one arbitrarily, because they only differ in the second decimal position, which does not make any difference in our calculations later.

3 Introduction of the Flavour Mixing into an Existing Formalism

In this chapter we include the flavour mixing into an existing formalism. We use the model described in [**ChiralEFT**]. It provides a framework to calculate cross sections for the direct detection of dark matter. The model bases on a set of dimension-five, -six, and -seven operators. We restrict our calculations to the dimension-six operators, which are

$$\begin{aligned} R_{1,q} &= (\bar{\chi}_0\gamma_\mu\chi_0)(\bar{q}\gamma^\mu q)\ , & R_{3,q} &= (\bar{\chi}_0\gamma_\mu\chi_0)(\bar{q}\gamma^\mu\gamma_5 q)\ , \\ R_{2,q} &= (\bar{\chi}_0\gamma_\mu\gamma_5\chi_0)(\bar{q}\gamma^\mu q)\ , & R_{4,q} &= (\bar{\chi}_0\gamma_\mu\gamma_5\chi_0)(\bar{q}\gamma^\mu\gamma_5 q)\ . \end{aligned} \tag{3.1}$$

Here $q = (u, d, c, s, t, b)$ is a quark and χ_0 is the component of the weak dark matter mutliplet that has no electric charge.

Since the CKM mixing only applies to the left-handed down-type quarks, we need to rewrite these operators in terms of the left- and righthanded particle functions to include the CKM matrix. These chiral operators are

$$\begin{aligned} Q_{1ij} &= (\bar{\chi}\gamma_\mu\tilde{\tau}^a\chi)(\bar{Q}^i_L\gamma^\mu\tau^a Q^j_L)\ , & Q_{5ij} &= (\bar{\chi}\gamma_\mu\gamma_5\tilde{\tau}^a\chi)(\bar{Q}^i_L\gamma^\mu\tau^a Q^j_L)\ , \\ Q_{2ij} &= (\bar{\chi}\gamma_\mu\chi)(\bar{Q}^i_L\gamma^\mu Q^j_L)\ , & Q_{6ij} &= (\bar{\chi}\gamma_\mu\gamma_5\chi)(\bar{Q}^i_L\gamma^\mu Q^j_L)\ , \\ Q_{3ij} &= (\bar{\chi}\gamma_\mu\chi)(\bar{U}^i_R\gamma^\mu U^j_R)\ , & Q_{7ij} &= (\bar{\chi}\gamma_\mu\gamma_5\chi)(\bar{U}^i_R\gamma^\mu U^j_R)\ , \\ Q_{4ij} &= (\bar{\chi}\gamma_\mu\chi)(\bar{D}^i_R\gamma^\mu D^j_R)\ , & Q_{8ij} &= (\bar{\chi}\gamma_\mu\gamma_5\chi)(\bar{D}^i_R\gamma^\mu D^j_R)\ . \end{aligned} \tag{3.2}$$

The operators $\tilde{\tau}^a, \tau^a$ are the generators of the $SU(2)$ in the corresponding spin-representation. The left-handed quarks come in doublets, therefore the generators are the Pauli matrices σ_a:

$$\tau^a = \frac{\sigma_a}{2}\ .$$

Regarding the dark matter, we do not know the size of the multiplet and thus use the general spin-representation

$$\begin{aligned} (\tilde{\tau}^1 \pm i\tilde{\tau}^2)_{\sigma'\sigma} &= \delta_{\sigma'\sigma\pm1}\sqrt{(j\mp\sigma)(j\pm\sigma+1)}\ , \\ \tilde{\tau}^3_{\sigma'\sigma} &= \sigma\delta_{\sigma'\sigma}\ , \end{aligned}$$

where j is the spin value and $-j \leq \sigma, \sigma' \leq j$. However, we only keep the electrically uncharged component of the multiplet, χ_0. **To do** (**??**)

In the following sections, we go through the steps of including the CKM matrix in the formalism:

1. Inclusion of the Flavour Mixing by replacing the pure left-handed down-type quarks with the mixed quarks.
2. Rewriting the chiral particle functions in terms of the unchiral particle functions and projection operators.
3. Writing down the entire interaction lagrangians in terms of the operators in (**??**) and (**??**) separately, and then comparing the coefficients.

To do (**??**)

3.1 Including the Flavour Mixing

The inclusion of the CKM matrix only affects the chiral operators with left-handed quarks: $Q_{1ij}, Q_{2ij}, Q_{5ij}, Q_{6ij}$. Since the dark matter part of the interaction remains unchanged, we only look at the quark part of the interaction, which becomes

$$
\begin{aligned}
\bar{Q}^i_L \gamma^\mu \tau^a Q^j_L &= \begin{pmatrix} \bar{u}^i_L \\ \bar{d}^i_L \end{pmatrix} \gamma^\mu \begin{pmatrix} u^j_L \\ d^j_L \end{pmatrix} = \frac{1}{2}\bar{u}^i_L \gamma^\mu \sigma_a d^j_L + \frac{1}{2}\bar{d}^i_L \gamma^\mu \sigma_a d^j_L \\
&= \frac{1}{2}\bar{u}^i_L \gamma^\mu \sigma_a u^j_L + \frac{1}{2}(V^*_{id}\bar{d}_L + V^*_{is}\bar{s}_L + V^*_{ib}\bar{b}_L)\gamma^\mu \sigma_a (V_{jd} d_L + V_{js} s_L + V_{jb} b_L)
\end{aligned}
$$

for Q_{1ij}, Q_{5ij}, respectively

$$
\begin{aligned}
\bar{Q}^i_L \gamma^\mu Q^j_L &= \begin{pmatrix} \bar{u}^i_L \\ \bar{d}^i_L \end{pmatrix} \gamma^\mu \begin{pmatrix} u^j_L \\ d^j_L \end{pmatrix} = \bar{u}^i_L \gamma^\mu d^j_L + \bar{d}^i_L \gamma^\mu d^j_L \\
&= \bar{u}^i_L \gamma^\mu u^j_L + (V^*_{id}\bar{d}_L + V^*_{is}\bar{s}_L + V^*_{ib}\bar{b}_L)\gamma^\mu (V_{jd} d_L + V_{js} s_L + V_{jb} b_L)
\end{aligned}
$$

for Q_{2ij}, Q_{6ij}. For simplicity we only keep the light quarks u, d, s and neglect mixed terms. So the whole set of quark interactions is

$$
\begin{aligned}
\bar{Q}^i_L \gamma^\mu \tau^a Q^j_L &\approx \frac{1}{2}\bar{u}_L \gamma^\mu u_L \delta_{ij}\delta_{iu}\delta_{a3} - \frac{1}{2}\delta_{a3}(V^*_{id}V_{jd}\bar{d}_L\gamma^\mu d_L + V^*_{is}V_{js}\bar{s}_L\gamma^\mu s_L) \ , \\
\bar{Q}^i_L \gamma^\mu Q^j_L &\approx \bar{u}_L \gamma^\mu u_L \delta_{ij}\delta_{iu} + V^*_{id}V_{jd}\bar{d}_L\gamma^\mu d_L + V^*_{is}V_{js}\bar{s}_L\gamma^\mu s_L \ , \\
\bar{u}^i_R \gamma^\mu u^j_R &\approx \bar{u}_R \gamma^\mu u_R \delta_{ij}\delta_{iu} \ , \\
\bar{d}^i_R \gamma^\mu d^j_R &\approx \bar{d}_R \gamma^\mu d_R \delta_{ij}\delta_{id} + \bar{s}_R \gamma^\mu s_R \delta_{ij}\delta_{is} \ .
\end{aligned} \tag{3.3}
$$

Note that, by only keeping diagonal interactions, we abolish the expressions including τ^1, τ^2, respectively $\tilde{\tau}^1, \tilde{\tau}^2$. Therefore the dark matter part of the interactions Q_{1ij}, Q_{5ij} becomes

$$\bar{\chi}\gamma_\mu\tilde{\tau}^3\chi = \sigma^0\bar{\chi}_0\gamma_\mu\chi_0 \ , \tag{3.4}$$
$$\bar{\chi}\gamma_\mu\gamma_5\tilde{\tau}^3\chi = \sigma^0\bar{\chi}_0\gamma_\mu\gamma_5\chi_0 \ , \tag{3.5}$$

because remember: we only want the electrically uncharged component of χ.

3.2 Replacing Chiral Particle Functions

The next step is rewriting the chiral particle functions in terms of the unchiral particle functions using the projection operators P_L, P_R. This leads to

$$\begin{aligned}
\bar{Q}^i_L\gamma^\mu\tau^a Q^j_L =& \frac{1}{4}(\bar{u}\gamma^\mu u\delta_{ij}\delta_{iu}\delta_{3a} - V^*_{id}V_{jd}\bar{d}\gamma^\mu d\delta_{3a} - V^*_{is}V_{js}\bar{s}\gamma^\mu s\delta_{3a}) \\
&-\frac{1}{4}(\bar{u}\gamma^\mu\gamma_5 u\delta_{ij}\delta_{iu}\delta_{3a} - V^*_{id}V_{jd}\bar{d}\gamma^\mu\gamma_5 d\delta_{3a} - V^*_{is}V_{js}\bar{s}\gamma^\mu\gamma_5 s\delta_{3a}) \\
\bar{Q}^i_L\gamma^\mu Q^j_L =& \frac{1}{2}(\bar{u}\gamma^\mu u\delta_{iu}\delta_{ij} + V^*_{id}V_{jd}\bar{d}\gamma^\mu d + V^*_{is}V_{js}\bar{s}\gamma^\mu s) \\
&-\frac{1}{2}(\bar{u}\gamma^\mu\gamma_5 u\delta_{iu}\delta_{ij} + V^*_{id}V_{jd}\bar{d}\gamma^\mu\gamma_5 d + V^*_{is}V_{js}\bar{s}\gamma^\mu\gamma_5 s) \\
\bar{u}^i_R\gamma^\mu u^j_R =& \frac{1}{2}(\bar{u}\gamma^\mu u\delta_{ij}\delta_{iu} + \bar{u}\gamma^\mu\gamma_5 u\delta_{ij}\delta_{iu}) \\
\bar{d}^i_R\gamma^\mu d^j_R =& \frac{1}{2}(\bar{d}\gamma^\mu d\delta_{ij}\delta_{id} + \bar{d}\gamma^\mu\gamma_5 d\delta_{ij}\delta_{id} + \bar{s}\gamma^\mu s\delta_{ij}\delta_{is} + \bar{s}\gamma^\mu\gamma_5 s\delta_{ij}\delta_{is}) \ .
\end{aligned} \tag{3.6}$$

At this point we can express the chiral operators in terms of the original operators from (**??**):

$$\begin{aligned}
Q_{1ij} =& \frac{\delta_{3a}\tau_0}{4}(R_{1u}\delta_{ij}\delta_{iu} - V^*_{id}V_{jd}R_{1d} - V^*_{is}V_{js}R_{1s}) \\
&-\frac{\delta_{3a}\tau_0}{4}(R_{3u}\delta_{ij}\delta_{iu} - V^*_{id}V_{jd}R_{3d} - V^*_{is}V_{js}R_{3s}) \\
Q_{2ij} =& \frac{1}{2}(R_{1u}\delta_{iu}\delta_{ij} + V^*_{id}V_{jd}R_{1d} + V^*_{is}V_{js}R_{1s}) \\
&-\frac{1}{2}(R_{3u}\delta_{iu}\delta_{ij} + V^*_{id}V_{jd}R_{3d} + V^*_{is}V_{js}R_{3s}) \\
Q_{3ij} =& \frac{1}{2}(R_{1u}\delta_{ij}\delta_{iu} + R_{3u}\delta_{ij}\delta_{iu}) \\
Q_{4ij} =& \frac{1}{2}(R_{1d}\delta_{ij}\delta_{id} + R_{3d}\delta_{ij}\delta_{id} + R_{1s}\delta_{ij}\delta_{is} + R_{3s}\delta_{ij}\delta_{is}) \ .
\end{aligned} \tag{3.7}$$

The operators $Q_{5ij} - Q_{8ij}$ can be obtained from $Q_{1ij} - Q_{4ij}$ by replacing $R_{1q} \leftrightarrow R_{2q}$ and $R_{3q} \leftrightarrow R_{4q}$.

3.3 Comparing Coefficients

To do (??) Our final step is expressing the coefficients $K_{l,q}$ of the original operators $R_{l,q}$ in (**??**) in terms of the coefficients C_{lij} of the chiral operators Q_{lij} in (**??**). To get there we look at the overall interaction. The interaction cannot depend on the choice of particle representation (chiral or non-chiral), therefore

$$\sum_{l,q} K_{l,q} R_{l,q} \stackrel{!}{=} \sum_{l,i,j} C_{lij} Q_{lij} \; . \tag{3.8}$$

By putting the interactions (**??**) into the right side of the equation and rearranging the expression in terms of the $R_{l,q}$, we conclude that $K_{l,q}$ must be equal to the terms in front of $R_{l,q}$ on the right side. We get the dependencies

$$\begin{aligned}
K_{1,u} &= \sum_{i,j} \frac{\delta_{ij}\delta_{iu}}{2}\left(C_{1ij}\frac{\delta_{3a}\tau_0}{2} + C_{2ij} + C_{3ij}\right)\\
K_{1,d} &= \sum_{i,j} \frac{1}{2}\left(-V^*_{id}V_{jd}C_{1ij}\frac{\delta_{3a}\tau_0}{2} + V^*_{id}V_{jd}C_{2ij} + \delta_{ij}\delta_{id}C_{4ij}\right)\\
K_{1,s} &= \sum_{i,j} \frac{1}{2}\left(-V^*_{is}V_{js}C_{1ij}\frac{\delta_{3a}\tau_0}{2} + V^*_{is}V_{js}C_{2ij} + \delta_{ij}\delta_{is}C_{4ij}\right)\\
K_{2,u} &= \sum_{i,j} \frac{\delta_{ij}\delta_{iu}}{2}\left(\frac{\delta_{3a}\tau_0}{2}C_{5ij} + C_{6ij} + C_{7ij}\right)\\
K_{2,d} &= \sum_{i,j} \frac{1}{2}\left(-V^*_{id}V_{jd}\frac{\delta_{3a}\tau_0}{2}C_{5ij} + C_{6ij}V^*_{id}V_{jd} + \delta_{ij}\delta_{id}C_{8ij}\right)\\
K_{2,s} &= \sum_{i,j} \frac{1}{2}\left(-V^*_{is}V_{js}\frac{\delta_{3a}\tau_0}{2}C_{5ij} + C_{6ij}V^*_{is}V_{js} + \delta_{ij}\delta_{is}C_{8ij}\right)\\
K_{3,u} &= \sum_{i,j} \frac{\delta_{ij}\delta_{iu}}{2}\left(-C_{1ij}\frac{\delta_{3a}\tau_0}{2} - C_{2ij} + C_{3ij}\right)\\
K_{3,d} &= \sum_{i,j} \frac{1}{2}\left(C_{1ij}\frac{\delta_{3a}\tau_0}{2}V^*_{id}V_{jd} - V^*_{id}V_{jd}C_{2ij} + \delta_{ij}\delta_{id}C_{4ij}\right)\\
K_{3,s} &= \sum_{i,j} \frac{1}{2}\left(C_{1ij}\frac{\delta_{3a}\tau_0}{2}V^*_{is}V_{js} - V^*_{is}V_{js}C_{2ij} + \delta_{ij}\delta_{is}C_{4ij}\right)\\
K_{4,u} &= \sum_{i,j} \frac{\delta_{ij}\delta_{iu}}{2}\left(-\frac{\delta_{3a}\tau_0}{2}C_{5ij} - C_{6ij} + C_{7ij}\right)\\
K_{4,d} &= \sum_{i,j} \frac{1}{2}\left(\frac{\delta_{3a}\tau_0}{2}C_{5ij}V^*_{id}V_{jd} - C_{6ij}V^*_{id}V_{jd} + \delta_{ij}\delta_{id}C_{8ij}\right)\\
K_{4,s} &= \sum_{i,j} \frac{1}{2}\left(\frac{\delta_{3a}\tau_0}{2}C_{5ij}V^*_{is}V_{js} - C_{6ij}V^*_{is}V_{js} + \delta_{ij}\delta_{is}C_{8ij}\right) \;. \qquad (3.9)
\end{aligned}$$

To have a hermitian interaction the coefficients need to fulfil the relation $C_{lij} = C^*_{lji}$.

4 The $L_\mu - L_\tau$ Model

In this chapter we present an extension to the standard model proposed in [**InColour**]. The authors originally aimed at explaining anomalies in the decay $B \to K l \bar{l}$, but also obtained predictions for the direct detection of dark matter in the succeeding publication [**Z**]. We will later compare their results with the formalism in the previous chapter that includes the CKM mixing.

4.1 The New Interaction

The extension to the standard model in [**InColour**] is a new $U(1)'$ gauge group. The related vector-boson is called Z', and it couples to the muon and tau lepton families, and a new set of vector-like quarks U, D, Q. The standard model quarks indirectly couple to the Z' as well, since they mix with the new quarks through a Yukawa coupling:

$$\begin{aligned}\mathcal{L}^{(\mathrm{mix})} =& \Phi' \bar{\tilde{D}}_R (Y_{Qb} b_L + Y_{Qs} s_L + Y_{Qd} d_L) \\ &+ \Phi' \bar{\tilde{U}}_R (Y_{Qt} t_L + Y_{Qc} c_L + Y_{Qu} u_L) \\ &+ \Phi'^\dagger \bar{U}_L (Y_{Ut} t_R + Y_{Uc} c_R + Y_{Uu} u_R) \\ &+ \Phi'^\dagger \bar{D}_L (Y_{Db} b_R + Y_{Ds} s_R + Y_{Dd} d_R) + \text{h.c.} \ ,\end{aligned} \tag{4.1}$$

where $\tilde{Q}_R = (\tilde{U}_R, \tilde{D}_R)$, $Q_L = (U_L, D_L)$ are the weak doublets, Y_{ij} are the coupling constants, and Φ' is a Higgs-like field that gives mass to the Z'.

In [**Z**] also a coupling to a dark matter fermion χ is established. The full interaction lagrangian is:

$$\begin{aligned}\mathcal{L}^{(\mathrm{int})}_{Z'} =& g' Z'_\alpha \times q_l \left(\bar{L}^2_L \gamma^\alpha L^2_L - \bar{L}^3_L \gamma^\alpha L^3_L + \bar{\mu}_R \gamma^\alpha \mu_R - \bar{\tau}_R \gamma^\alpha \tau_R \right) \\ &+ g' Z'_\alpha \times v_\Phi^2 \sum_{i,j}^{3} \left(-\frac{Y_{Di} Y^*_{Dj}}{2m_D^2} \bar{D}^i_R \gamma^\alpha D^j_R - \frac{Y_{Ui} Y^*_{Uj}}{2m_U^2} \bar{U}^i_R \gamma^\alpha U^j_R + \frac{Y_{Qi} Y^*_{Qj}}{2m_Q^2} \bar{Q}^i_L \gamma^\alpha Q^j_L \right) \\ &+ g' Z'_\alpha \times q_\chi (\bar{\chi} \gamma^\alpha \chi) \ ,\end{aligned} \tag{4.2}$$

where q_l, q_χ are the $U(1)'$ charge of the leptons and the dark matter particle, $m_{U,D,Q}$ are the masses of the new quarks, and $v_{\Phi'}$ is the vacuum expectation value of Φ'.

To do (??)

4.2 Restrictions on the Parameter Space

In [**Z**] Altmannshofer et. al. discuss restrictions on the parameter space by looking at the B decay mentioned above and discussing dark matter relic density and direct detection. They find that experimental data from $B \to K l \bar{l}$ limits the ratio of the Z' mass and the coupling g' to

$$540\,\mathrm{GeV} \lessapprox \frac{m_{Z'}}{g'} \lessapprox 4.9\,\mathrm{TeV}\ , \tag{4.3}$$

with $m_{Z'} \gtrapprox 10\,\mathrm{GeV}$.

Regarding the dark matter relic density, they conclude that only

$$m_{Z'} \approx 2 m_\chi \tag{4.4}$$

leads to correct results. Since they neglect flavour mixing in the nucleus, direct detection has to occur through the loop diagram in figure **??**. The corresponding cross section at zero momentum transfer is

$$\sigma_{0,\mathrm{loop}} = \frac{\mu_{A\chi}^2}{A^2 \pi} \left(\frac{\alpha_{em} Z}{3\pi} \frac{g'^2 q_\chi q_l}{m_{Z'}^2} \log \left(\frac{m_\mu^2}{m_\tau^2} \right) \right)^2\ , \tag{4.5}$$

where $\mu_{A\chi}$ is the reduced mass of the nucleus and the dark matter particle χ and A, Z are the nucleon and proton numbers.

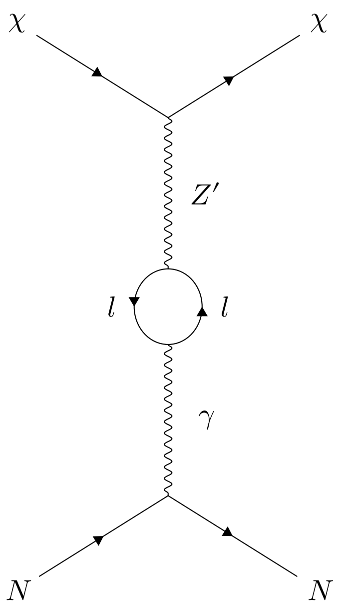


Figure 4.1: Direct detection loop diagram

When discussing limits to the parameter space they distinguish two cases. For $q_l = q_\chi = 1$, experimental data favours the parameter region

$$\begin{aligned} 10\,\mathrm{GeV} \lessapprox m_{Z'} \lessapprox 46\,\mathrm{GeV} \\ 2 \cdot 10^{-3} \lessapprox g' \lessapprox 10^{-2} \ , \end{aligned} \tag{4.6}$$

leaving possible dark matter masses in the range $(5 - 23)$GeV. For $q_l = 1, q_\chi = 1/6$ no further restriction of the parameters can be found.

To do (**??**)

5 Comparison

We now assume that only b, s, μ, τ and dark matter particles couple to Z', and dark matter couples exclusively to Z'. Taking into account flavour mixing, a tree level interaction of dark matter particles and nucleons is then possible. We take a look at this option in the first section. Afterwards we compare the direct detection cross section of the tree level interaction with the one of the loop interaction presented in **??**, respectively [**Z**]. **To do** (**??**)

5.1 Tree Level Interaction Cross Section

By considering flavour mixing, a dark matter particle can interact with the strange and bottom parts of a down quark in a nucleus through the tree level diagram shown in figure **??**. In terms of the operators in (**??**), the diagram corresponds to Q_{2bs}, Q_{2sb}. The related coefficients are

$$C_{2bs} = C^*_{2sb} = q_\chi \frac{Y_{Qb} Y^*_{Qs}}{2m_Q^2} \ , \tag{5.1}$$

for which [**InColour**] gives the approximation

$$\mathrm{Re}\,(C_{2bs}) \approx 8 \cdot 10^{-10}\,\mathrm{GeV}^{-2} \ . \tag{5.2}$$

This leads to the non-chiral coefficients (see (**??**))

$$\begin{aligned}
K_{1,d} &= +\mathrm{Re}(V^*_{cd} V_{td} C_{2sb}) \ , \\
K_{1,s} &= +\mathrm{Re}(V^*_{cs} V_{ts} C_{2sb}) \ , \\
K_{3,d} &= -\mathrm{Re}(V^*_{cd} V_{td} C_{2sb}) \ , \\
K_{3,s} &= -\mathrm{Re}(V^*_{cs} V_{ts} C_{2sb}) \ ,
\end{aligned} \tag{5.3}$$

and all other $K_{l,q}$ vanish. These are the coefficients of the spin-dependent interaction $R_{3,q} = (\bar\chi \gamma^\mu \chi)(\bar q \gamma_\mu \gamma_5 q)$ and the spin-independent interaction $R_{1,q} = (\bar\chi \gamma^\mu \chi)(\bar q \gamma_\mu q)$. We neglect the former one because it is various orders of magnitude smaller than the spin-independent cross section. However, there is no interaction with strange quarks as they are only available as sea quarks. Thus, we are left with the spin-independent vector interaction

$$\mathcal{L} = K_{1,d}(\bar{\chi}\gamma^{\mu}\chi)(\bar{d}\gamma_{\mu}d) \ . \tag{5.4}$$

As explained in [**Supersymmetric**], this operator counts the number of down quarks. So the nucleon cross section is

$$\sigma^{\mathrm{SI}}_{0,\mathrm{tree}} = \frac{\mu^2_{A\chi}}{A^2\pi} \left|ZC_p + (A-Z)C_n\right|^2 \ , \tag{5.5}$$

where $C_p = K_{1,d}$ and $C_n = 2K_{1,d}$ are the proton and neutron coefficients.

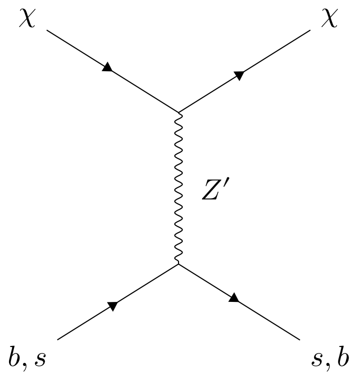


Figure 5.1: Tree level diagram for direct detection considering flavour mixing.

5.2 Results

We now compare the nucleon cross sections for the loop interaction $\sigma_{0,\mathrm{loop}}$ (see equation (**??**) and figure **??**) and the tree level interaction $\sigma_{0,\mathrm{tree}}$ (see equation (**??**) and figure **??**). We choose Xenon, which is used in the LUX and XENON100 experiments, as detection material. The most abundant isotope is Xenon-129, therefore $Z = 54, A = 129$ [**DD**]. For the CKM matrix elements we use the values given in (**??**).

In the following we show various plots, in which we follow [**Z**] with the distinction of the cases $q_\chi = 1$ and $q_\chi = 1/6$. Remember that for the former the dark matter mass cannot be much larger than 23 GeV, but for the latter no restriction is made. Therefore we choose different m_χ-axis scales in the plots. The shaded green area always corresponds to $\sigma_{0,\mathrm{loop}}$, whereas the lines visualize $\sigma_{0,\mathrm{tree}}$.

First, we consider the bound (??) for $\sigma_{0,\text{loop}}$. Figure ?? shows $\sigma_{0,\text{loop}}$ (shaded green) within this bound and $\sigma_{0,\text{tree}}$ with various choices for $C_{2bs} = C$ that are in accordance with (??). But even for quiet large values of C_{2bs}, the tree level cross section cannot reach the loop cross section. This is a strong hint that Altmannshofer et. al. correctly neglected the CKM mixing in their predictions for dark matter direct detection.

Since there are no restrictions concerning the imaginary part of C_{2bs}, we enlarged $\text{Im}(C_{2bs})$ until we reached $\sigma_{0,\text{loop}}$ in figure ??. But this was only achieved by imaginary parts that are two orders of magnitude larger than the real part of C_{2bs}, which we consider unreasonable. **To do** (??)

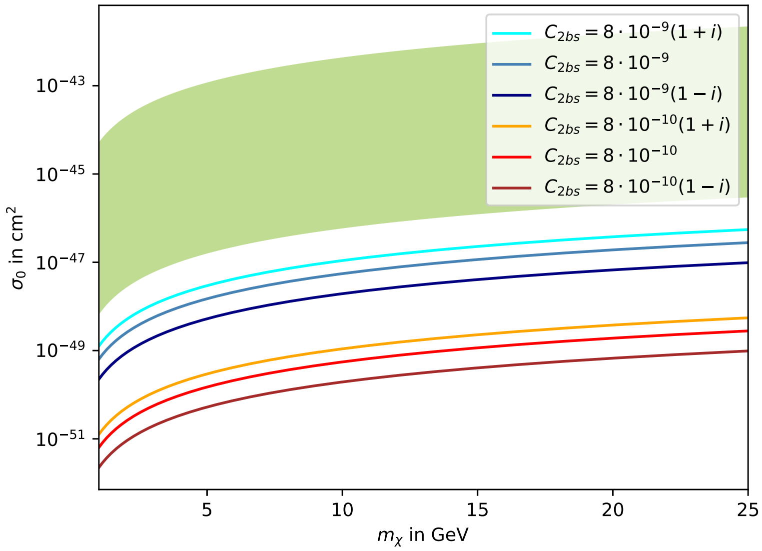
Second, we concentrate on the relic density condition in (??). Therefore we set $m_{Z'} = 2m_\chi$ but allow a 30 % tolerance. For $q_\chi = 1$, we set g' to the lower bound in (??): $g' = 2 \cdot 10^{-3}$, and for $q_\chi = 1/6$, we guess $g' = 10^{-2}$. Figure ?? shows the corresponding loop cross section again as shaded green area. Regarding the tree level cross section we choose C_{2bs} in strong accordance with (??). We find that there are regions for m_χ, where the tree level cross section reaches the loop cross section. The shaded lines show the lower bound of these regions. At the lower end of the green area $m_{Z'} = 2m_\chi \cdot 1.3$. The tables ?? and ?? show the obtained lower bounds (l.b.) for the dark matter mass, the resulting values for the Z' mass, and the ratio of the Z' mass and the coupling g'.

Table 5.1: $q_\chi = 1$. Bounds for the dark matter and Z' masses (in GeV) and ratio $m_{Z'}/g'$ (in GeV).

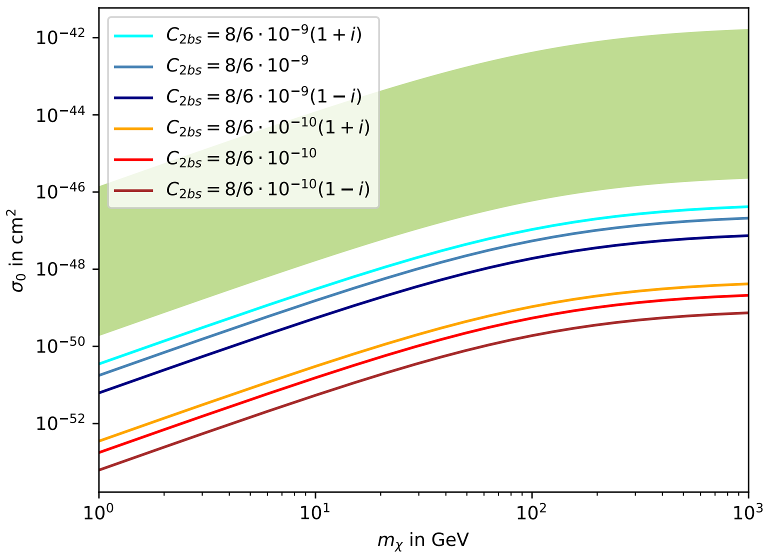
C_{2bs} in GeV^{-2}	l. b. for m_χ	$m_{Z'}$	$m_{Z'}/g'$
$8 \cdot 10^{-10}$	22	57	$29 \cdot 10^3$
$8 \cdot 10^{-10}(1+i)$	18	48	$24 \cdot 10^3$
$3 \cdot 10^{-9}$	11	29	$15 \cdot 10^3$

Table 5.2: $q_\chi = 1/6$. Bounds for the dark matter and Z' masses (in GeV) and ratio $m_{Z'}/g'$ (in GeV).

C_{2bs} in GeV^{-2}	l. b. for m_χ	$m_{Z'}$	$m_{Z'}/g'$
$1/68 \cdot 10^{-10}$	108	281	$28 \cdot 10^3$
$1/68 \cdot 10^{-10}(1+i)$	91	237	$24 \cdot 10^3$
$1/63 \cdot 10^{-9}$	56	146	$15 \cdot 10^3$

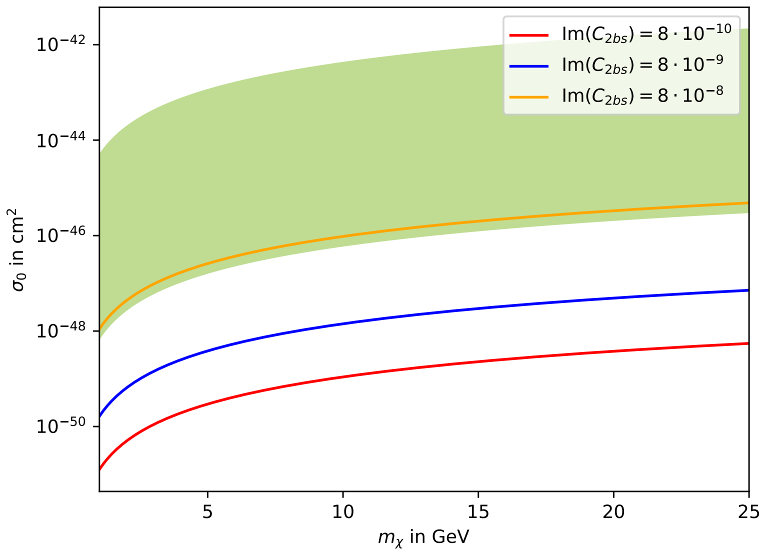


(a) $q_l = q_\chi = 1$

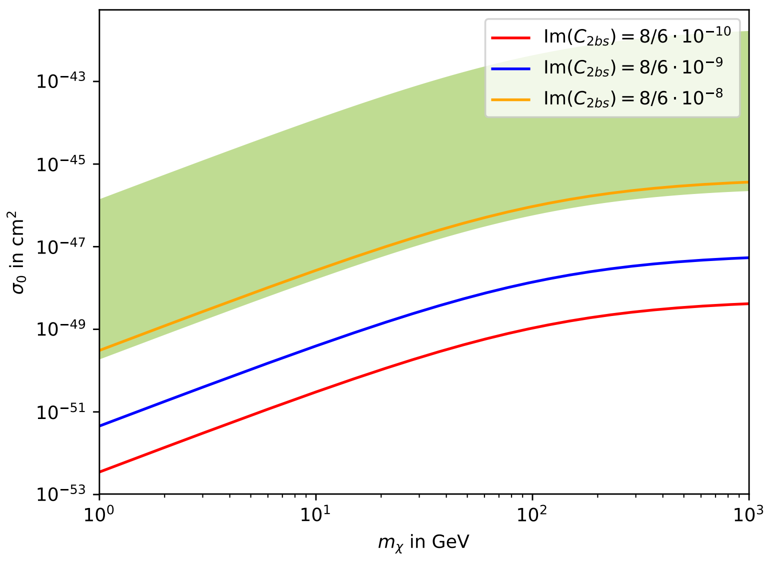


(b) $q_l = 1, q_\chi = {}^1\!/_6$

Figure 5.2: The shaded green area represents $\sigma_{0,\text{loop}}$ with the bounds in (??). The coloured lines show $\sigma_{0,\text{tree}}$ for different values of C_{2bs} (in GeV^{-2}).

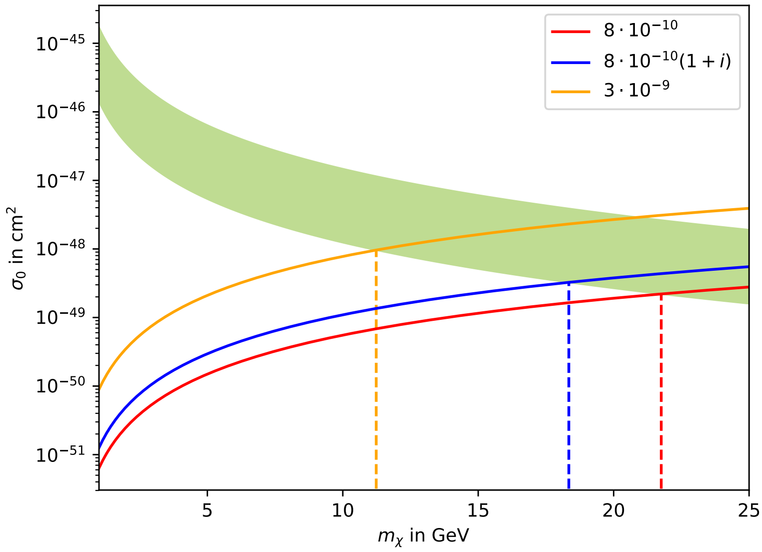


(a) $q_l = q_\chi = 1$

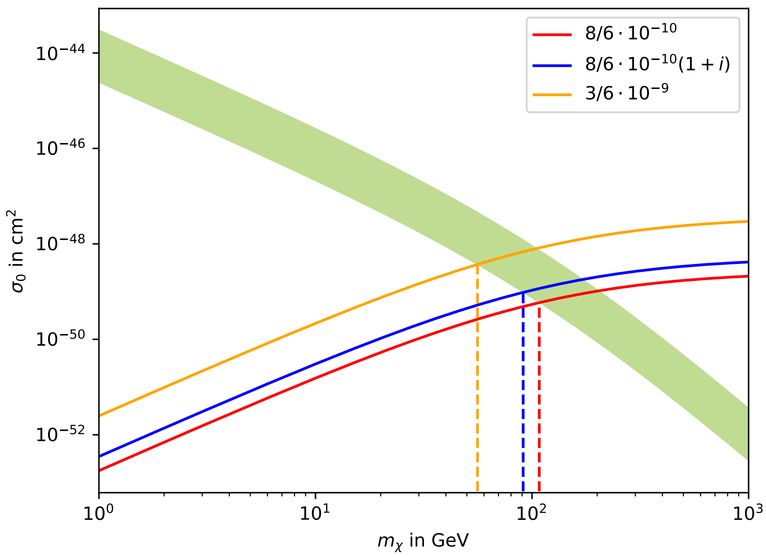


(b) $q_l = 1, q_\chi = 1/6$

Figure 5.3: Like **??**, but the real part of C_{2bs} is fixed at $8 \cdot 10^{-10}\,\mathrm{GeV}^{-2}$ and only $\mathrm{Im}(C_{2bs})$ is varied.



(a) $q_l = q_\chi = 1, g' = 2 \cdot 10^{-3}$



(b) $q_l = 1, q_\chi = 1/6, g' = 10^{-2}$

Figure 5.4: The shaded green area represents the loop cross section $\sigma_{0,\text{loop}}$ at fixed coupling constant g' and $m_{Z'} = 2m_\chi$ with a $\pm 30\,\%$ tolerance.

To do...

- ☐ 1 (p. **??**): Warum behält man eigentlich nur die ungeladene Komponente?
- ☐ 2 (p. **??**): Der letzte Punkt ist schlecht formuliert.
- ☐ 3 (p. **??**): Dieser Text ist nicht schön.
- ☐ 4 (p. **??**): Kann man tatsächlich hier die Ups drehen und dann später mit gedrehten Downs rechnen?
- ☐ 5 (p. **??**): Evtl. kann man den Ursprung der Grenzen noch näher erläutern.
- ☐ 6 (p. **??**): Der Text ist nicht so gut.
- ☐ 7 (p. **??**): Warum ist das eigentlich blöd den Imaginärteil so hoch zu machen.

Eidesstattliche Versicherung

Ich versichere hiermit an Eides statt, dass ich die vorliegende Abschlussarbeit mit dem Titel “Flavour Mixing Effects in the Direct Detection of Dark Matter” selbstständig und ohne unzulässige fremde Hilfe erbracht habe. Ich habe keine anderen als die angegebenen Quellen und Hilfsmittel benutzt, sowie wörtliche und sinngemäße Zitate kenntlich gemacht. Die Arbeit hat in gleicher oder ähnlicher Form noch keiner Prüfungsbehörde vorgelegen.

Ort, Datum	Unterschrift

Belehrung

Wer vorsätzlich gegen eine die Täuschung über Prüfungsleistungen betreffende Regelung einer Hochschulprüfungsordnung verstößt, handelt ordnungswidrig. Die Ordnungswidrigkeit kann mit einer Geldbuße von bis zu 50 000 € geahndet werden. Zuständige Verwaltungsbehörde für die Verfolgung und Ahndung von Ordnungswidrigkeiten ist der Kanzler/die Kanzlerin der Technischen Universität Dortmund. Im Falle eines mehrfachen oder sonstigen schwerwiegenden Täuschungsversuches kann der Prüfling zudem exmatrikuliert werden (§ 63 Abs. 5 Hochschulgesetz –HG–).

Die Abgabe einer falschen Versicherung an Eides statt wird mit Freiheitsstrafe bis zu 3 Jahren oder mit Geldstrafe bestraft.

Die Technische Universität Dortmund wird ggf. elektronische Vergleichswerkzeuge (wie z. B. die Software “turnitin”) zur Überprüfung von Ordnungswidrigkeiten in Prüfungsverfahren nutzen.

Die oben stehende Belehrung habe ich zur Kenntnis genommen.

Ort, Datum	Unterschrift