technische universität
dortmund

Arbeit zur Erlangung des akademischen Grades
Bachelor of Science

Flavour Mixing Effects in the Direct Detection of Dark Matter

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2017

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Abgabedatum:	17. Juli 2017

Abstract

Contents

1 Introduction

We will take a look at the effects of the flavour mixing mechanism on the direct detection of dark matter. Therefore we expand an existing formalism for dark matter direct detection with the CKM matrix. Afterwards we present a new interaction proposed to explain anomalies in the decay $B \to K l \bar{l}$ that also makes predictions about the direct detection of dark matter. Finally, we compare these predictions with the direct detection cross sections that respect flavour mixing.

Direct detection means detecting dark matter directly through interaction with a nucleus, in contrary to indirect detection, which means measuring secondary products of dark matter annihilation or dark matter decay.

To do (1)

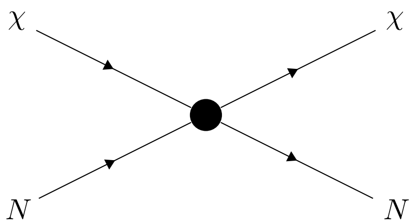

Figure 1.1: Direct detection

2 The Flavour Mixing Mechanism

[**Peskin**] [**Tevatron**] The origins of the flavour mixing go back to the 1960s, when Italian physicist Cabibbo resolved anomalies in weak decay data by proposing a flavour mixing of lefthanded down-type quarks. Later in 1973, Kobayaski and Maskawa extended this idea to three quark generation to explain CP violation. On a theoretical level the flavour mixing arises from the fact that the fermion mass eigenstates do not necessarily equal the flavour eigenstates. In the course of this chapter we introduce the Glashow-Weinberg-Salam Model of electroweak interaction and derive how the fermions gain their masses and why this leads to flavour mixing.

2.1 The Glashow-Weinberg-Salam Model

The Glashow-Weinberg-Salam model is a gauge theory with gauge group $SU(2) \times U(1)$. Therefore we need four gauge bosons, A^1, A^2, A^3 for $SU(2)$ and B for $U(1)$. The covariant derivative of a field Ψ that couples to all of the gauge bosons is then

$$D_\mu \Psi = \left(\partial_\mu - ig \sum_{a=1}^{3} A_\mu^a \tau^a - i Y_\Psi g' B_\mu \right) \Psi \,, \tag{2.1}$$

where $\tau^a = \sigma^a/2$ with the Pauli matrices σ^a, g, g' are the coupling constants, and Y_Ψ is the charge under $U(1)$.

2.1.1 The Higgs Mechanism and the Gauge Bosons of the Electroweak Unification

We consider a complex scalar field Φ that interacts with itself through a potential

$$V(\Phi) = -\mu^2 \Phi^\dagger \Phi + \frac{\lambda}{2} (\Phi^\dagger \Phi)^2 \,, \quad \mu^2 > 0 \,. \tag{2.2}$$

If $\lambda > 0$, the minimum of this potential occurs at

$$\langle \Phi \rangle = \Phi_0 = \sqrt{\frac{\mu^2}{\lambda}} \,. \tag{2.3}$$

This is the vacuum expectation value of Φ.

$SU(2)$

We add a $SU(2)$ gauge field coupled to Φ, so Φ is a doublet (Φ_1, Φ_2) with covariant derivative

$$D_\mu \Phi = (\partial_\mu - ig \sum_{a=1}^{3} A^a_\mu \tau^a) \Phi \ , \tag{2.4}$$

where τ^a are the generators of $SU(2)$. In this case there is an infinite number of vacuum expection values for Φ arranged in a circle. We are free to choose one, and we make the simple choice

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \ . \tag{2.5}$$

Note that by choosing one vacuum value, we break the symmetry.

The kinetic energy of Φ is

$$\begin{aligned} (D_\mu \Phi)^2 =& \frac{1}{2} (\partial_\mu v)(\partial^\mu v) \\ & - ig (\partial^\mu \begin{pmatrix} 0 & v \end{pmatrix}) \left(\sum_{a=1}^{3} A^a_\mu \tau^a \begin{pmatrix} 0 \\ v \end{pmatrix} \right) \\ & - \frac{1}{2} g^2 \begin{pmatrix} 0 & v \end{pmatrix} \sum_{a,b=1}^{3} \tau^a \tau^b \begin{pmatrix} 0 \\ v \end{pmatrix} A^a_\mu A^{b\mu} \ . \end{aligned} \tag{2.6}$$

Using the relation $\{\tau^a, \tau^b\} = 1/2 \cdot \delta_{ab}$, we can simplify the last expression to get

$$-\frac{1}{2} g^2 \begin{pmatrix} 0 & v \end{pmatrix} \sum_{a,b=1}^{3} \tau^a \tau^b \begin{pmatrix} 0 \\ v \end{pmatrix} A^a_\mu A^{b\mu} = -\frac{g^2 v^2}{8} \sum_{a=1}^{3} A^a_\mu A^{a\mu} \ , \tag{2.7}$$

which is a mass term $\mathcal{L}_m = -\frac{1}{2} m_A^2 A_\mu A^\mu$ that assigns the mass $m_A = \frac{gv}{2}$ to all three gauge bosons.

$SU(2) \times U(1)$

We expand the system with an additional $U(1)$ symmetry with gauge boson B. The field Φ has a charge Y_Φ under $U(1)$. The new covariant derivative is

$$D_\mu \Phi = \left(\partial_\mu - ig \sum_{a=1}^{3} A^a_\mu \tau^2 - i Y_\Phi g' B_\mu \right) \Phi \ . \tag{2.8}$$

Again, we examine the kinetic term

$$
\begin{aligned}
(D_\mu \Phi)^2 =& \frac{1}{2}(\partial_\mu v)(\partial^\mu v) \\
&- \frac{1}{2} i \left(\partial^\mu \begin{pmatrix} 0 & v \end{pmatrix}\right) \left(g \sum_{a=1}^{3} A_\mu^a \tau^2 + Y_\Phi g' B_\mu \right) \begin{pmatrix} 0 \\ v \end{pmatrix} \\
&- \frac{1}{2} \begin{pmatrix} 0 & v \end{pmatrix} \left(g^2 \sum_{a,b=1}^{3} A_\mu^a A^{b\mu} \tau^a \tau^b + 2 g g' Y_\Phi \sum_{a=1}^{3} A_\mu^a \tau^a B^\mu + Y_\Phi^2 g'^2 B_\mu B^\mu \right) \begin{pmatrix} 0 \\ v \end{pmatrix}
\end{aligned}
\tag{2.9}
$$

Using $\{\tau^a, \tau^b\} = 1/2 \cdot \delta_{ab}$ and replacing $\tau^a = \sigma^a/2$, we find for the last term

$$
\begin{aligned}
\mathcal{L}^{(\text{mass})} &= -\frac{v^2}{2} \left(g^2 \frac{1}{4} \sum_{a=1}^{3} A_\mu^a A^{a\mu} + Y_\Phi^2 g'^2 B_\mu B^\mu - g g' Y_\Phi B^\mu A_\mu^3 \right) \\
&= -\frac{1}{2}\frac{v^2}{4} \left(g^2 A_\mu^1 A^{1\mu} + g^2 A_\mu^2 A^{2\mu} + (g A_\mu^3 - 2 g' Y_\Phi B_\mu)^2 \right) \\
&= -\frac{1}{2}\frac{v^2}{4} \left(g^2 2 W^+ W^- + (g^2 + 4 g'^2 Y_\Phi^2) Z_0^2 \right) \ .
\end{aligned}
\tag{2.10}
$$

Here we identified the known vector bosons

$$
\begin{aligned}
W_\mu^\pm &= \frac{1}{\sqrt{2}}(A_\mu^1 \mp i A_\mu^2) \ , & m_W &= \frac{v}{2} g \\
Z_\mu^0 &= \frac{1}{\sqrt{g^2 + 4 g'^2 Y_\Phi^2}} (g A_\mu^3 - 2 g' Y_\Phi B_\mu) \ , & m_Z &= \frac{v}{2}\sqrt{g^2 + 4 g'^2 Y_\Phi^2} \ .
\end{aligned}
\tag{2.11}
$$

Since we are in a $SU(2) \times U(1)$ symmetry, there has to be a fourth gauge boson. As we have just derived, it is massless. We renounce giving an elaborate explanation for this, but we want to give a motivation. Therefore we need to remember that the masses arise through choosing a vacuum expectation value for Φ and thereby breaking the $SU(2)$ symmetry. Looking at the gauge transformation of Φ

$$
\Phi \to e^{i \sum_{a=1}^{3} \alpha^a \tau^a} e^{i \beta Y_\Phi} \Phi \ ,
\tag{2.12}
$$

we find that the choice $\alpha^1 = \alpha^2 = 0$, $\alpha^3 = 2\beta Y_\Phi$ leaves the vacuum expectation value unchanged. Thus, parts of the symmetry are conserved and keep one gauge boson from acquiring mass. The fourth gauge boson is the photon, and it is orthogonal to Z_μ^0:

$$
A_\mu = \frac{1}{\sqrt{g^2 + 2 g'^2 Y_\Phi^2}} (2 g' Y_\Phi A_\mu^3 + g B_\mu) \ .
\tag{2.13}
$$

2.2 Fermion Masses and Flavour Mixing

The electroweak interaction lagrangian for the standard model fermions is

$$\begin{aligned}\mathcal{L}^{(\mathrm{int})} =&\bar{E}_R(i\partial\!\!\!/ - g_1 Y_E B\!\!\!/)E_R + \bar{D}_R(i\partial\!\!\!/ - g_1 Y_D B\!\!\!/)D_R + \bar{U}_R(i\partial\!\!\!/ - g_1 Y_U B\!\!\!/)U_R \\ &+\bar{L}_L(i\partial\!\!\!/ - g_1 Y_L B\!\!\!/ - g_2 A\!\!\!/)L_L + \bar{Q}_L(i\partial\!\!\!/ - g_1 Y_Q B\!\!\!/ - g_2 A\!\!\!/)Q_L \ ,\end{aligned} \tag{2.14}$$

where Y is the charge under $U(1)_Y$, or hypercharge. It is related to the electric charge and the third component of the weak isospin through the Gell-Mann–Nishijima formula: $Y = 2(Q - I_3)$. We already split the particle functions in their chiral components: $\psi = \psi_L + \psi_R$. For the leptons this means

$$E_R = (e_R, \mu_R, \tau_R) \ , \qquad Y_E = -1 \ ; \tag{2.15}$$

$$L_L = \left(\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L , \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L , \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L \right) \ , \qquad Y_L = -\frac{1}{2} \ ; \tag{2.16}$$

and accordingly for the quarks

$$U_R = (u_R, c_R, t_R) \ , \qquad Y_U = \frac{2}{3} \ ; \tag{2.17}$$

$$D_R = (d_R, s_R, b_R) \ , \qquad Y_D = -\frac{1}{3} \ ; \tag{2.18}$$

$$Q_L = \left(\begin{pmatrix} u \\ d \end{pmatrix}_L , \begin{pmatrix} c \\ s \end{pmatrix}_L , \begin{pmatrix} t \\ b \end{pmatrix}_L \right) \ , \qquad Y_Q = \frac{1}{6} \ . \tag{2.19}$$

The righthanded particles are singlets under $SU(2)$ and therefore $I_3 = 0$.

The lagrangian in (2.14) describes massless particles. A fermion mass term couples the lefthanded and righthanded part of a particle. Since direct coupling between for example e_R and $(\nu_e, e)_L$ would violate gauge invariance, a connecting field. To preserve invariance under Lorentz, $U(1)_Y$, and $SU(2)$ transformations this field must have spin 0, hypercharge $Y = 1/2$, and be a doublet. We identify this field with Φ from the previous chapter and write the mass terms for the fermions

$$\mathcal{L}^{(\mathrm{mass})} = -\left[\lambda^e \bar{L}_L \Phi E_R + \lambda^d \bar{Q}_L \Phi D_R + \lambda^u \bar{Q}_L i\sigma^2 \Phi^\dagger U_R + \mathrm{h.c.}\right] \ , \tag{2.20}$$

with complex matrix coupling constants $\lambda^e, \lambda^d, \lambda^u$. Replacing Φ with its vacuum expectation value gives

$$\mathcal{L}^{(\mathrm{mass})} = -\frac{v}{\sqrt{2}}\left[\lambda^e \bar{E}_L E_R + \lambda^d \bar{D}_L D_R + \lambda^u \bar{U}_L U_R + \mathrm{h.c.}\right] \ , \tag{2.21}$$

where $E_L = (e_L, \mu_L, \tau_L)$, $D_L = (d_L, s_L, b_L)$, and $U_L = (u_L, c_L, t_L)$.

The interaction lagrangian (2.14) is invariant under unitary transformations

$$L_L \to R_l L_L \qquad E_R \to U_e E_R \ , \tag{2.22}$$
$$Q_L \to R_q Q_L \qquad U_R \to U_u U_R \ , \tag{2.23}$$
$$D_R \to U_d D_R \ . \tag{2.24}$$

Thus, we can diagonalize the interactions in (2.21). The diagonal lepton coupling is $\tilde{\lambda}^e = U_l \lambda^e U_e$ and parametrizes the lepton masses

$$m_e = \frac{v}{\sqrt{2}} \tilde{\lambda}^e_{11} \ , \quad m_\mu = \frac{v}{\sqrt{2}} \tilde{\lambda}^e_{22} \ , \quad m_\tau = \frac{v}{\sqrt{2}} \tilde{\lambda}^e_{33} \ . \tag{2.25}$$

$$\mathcal{L}_l^{(\text{mass})} = -\frac{v}{\sqrt{2}} \tilde{\lambda}^e \bar{E}_L E_R + \text{h.c.} \ , \tag{2.26}$$

and we identify the lepton masses

$$m^i = \frac{v \tilde{\lambda}^e_{ii}}{\sqrt{2}} \ . \tag{2.27}$$

The new diagonal coupling matrices $\tilde{\lambda}$ are

$$\tilde{\lambda}^e = U_l \lambda^e U_e^\dagger \ , \tag{2.28}$$
$$\tilde{\lambda}^u = U_l \lambda^e U_e^\dagger \ , \tag{2.29}$$
$$\tilde{\lambda}^d = U_l \lambda^e U_e^\dagger \tag{2.30}$$

2.3 Peskin & Schroeder

Erklären:

- Wieso braucht man ein neues Feld/ Symmetriebrechung?
- Warum muss das die Eigenschaften xy haben?
- Wie und wieso koppelt das an die Fermionen?
- Wie ergibt sich aus der most general gauge-invariant coupling die CKM matrix?

3 Introduction of the Flavour Mixing into an Existing Formalism

In this chapter we include the flavour mixing into an existing formalism. We use the model described in [**ChiralEFT**]. It provides a framework to calculate cross sections for the direct detection of dark matter. The model bases on a set of dimension-five, -six, and -seven operators. We restrict our calculations to the dimension-six operators, which are

$$\begin{aligned} R_{1,q} &= (\bar{\chi}\gamma_\mu\chi)(\bar{q}\gamma^\mu q) & R_{3,q} &= (\bar{\chi}\gamma_\mu\chi)(\bar{q}\gamma^\mu\gamma_5 q) \\ R_{2,q} &= (\bar{\chi}\gamma_\mu\gamma_5\chi)(\bar{q}\gamma^\mu q) & R_{4,q} &= (\bar{\chi}\gamma_\mu\gamma_5\chi)(\bar{q}\gamma^\mu\gamma_5 q)\ . \end{aligned} \tag{3.1}$$

Since the CKM mixing only applies to the lefthanded down-type quarks, we need to rewrite these operators in terms of the left- and righthanded particle functions to include the CKM matrix. These chiral operators are

$$\begin{aligned} Q_{1ij} &= (\bar{\chi}\gamma_\mu\tilde{\tau}^a\chi)(\bar{Q}^i_L\gamma^\mu\tau^a Q^j_L) & Q_{5ij} &= (\bar{\chi}\gamma_\mu\gamma_5\tilde{\tau}^a\chi)(\bar{Q}^i_L\gamma^\mu\tau^a Q^j_L) \\ Q_{2ij} &= (\bar{\chi}\gamma_\mu\chi)(\bar{Q}^i_L\gamma^\mu Q^j_L) & Q_{6ij} &= (\bar{\chi}\gamma_\mu\gamma_5\chi)(\bar{Q}^i_L\gamma^\mu Q^j_L) \\ Q_{3ij} &= (\bar{\chi}\gamma_\mu\chi)(\bar{u}^i_R\gamma^\mu u^j_R) & Q_{7ij} &= (\bar{\chi}\gamma_\mu\gamma_5\chi)(\bar{u}^i_R\gamma^\mu u^j_R) \\ Q_{4ij} &= (\bar{\chi}\gamma_\mu\chi)(\bar{d}^i_R\gamma^\mu d^j_R) & Q_{8ij} &= (\bar{\chi}\gamma_\mu\gamma_5\chi)(\bar{d}^i_R\gamma^\mu d^j_R)\ , \end{aligned} \tag{3.2}$$

here $Q^i_L = (u^i_L, d^i_L)$ is the isospin doublet of the i^{th} quark generation. The operators $\tilde{\tau}^a, \tau^a$ are the generators of the SU(2) in the corresponding spin-representation. So for the quarks, we use

$$\tau^a = \frac{\sigma_a}{2}\ , \tag{3.3}$$

where σ_a are the pauli matrices. Regarding the dark matter particle, we decide to only keep the terms with $\tilde{\tau}^a = \tilde{\tau}^3$ and get

$$\tilde{\tau}^3\chi = \tau_0\chi\ , \tag{3.4}$$

with the weak isospin value τ_0 of the dark matter particle.

Including the CKM matrix in the fromalism follows these steps:

1. Replace the pure lefthanded down-type quarks with the mixed quarks.

2. Rewrite the chiral particle functions in terms of the normal particle functions and projection operators.

3. Write down the entire interaction lagrangian in terms of (3.1) and (3.2) and compare the coefficients.

3.1 Including the Flavour Mixing

The inclusion of the CKM matrix only affects the chiral operators $Q_{1ij}, Q_{2ij}, Q_{5ij}, Q_{6ij}$. The quark part of the interaction with flavour mixing is therefore

$$\begin{aligned}\bar{Q}^i_L\gamma^\mu Q^j_L &= \begin{pmatrix}\bar{u}^i_L\\ \bar{d}^i_L\end{pmatrix}\gamma^\mu\begin{pmatrix}u^j_L\\ d^j_L\end{pmatrix} = \bar{u}^i_L\gamma^\mu d^j_L + \bar{d}^i_L\gamma^\mu d^j_L\\ &= \bar{u}^i_L\gamma^\mu u^j_L + (V^*_{id}\bar{d}_L + V^*_{is}\bar{s}_L + V^*_{ib}\bar{b}_L)\gamma^\mu(V_{jd}d_L + V_{js}s_L + V_{jb}b_L)\end{aligned}$$

for Q_{2ij}, Q_{6ij}, respectively

$$\begin{aligned}\bar{Q}^i_L\gamma^\mu\tau^a Q^j_L &= \begin{pmatrix}\bar{u}^i_L\\ \bar{d}^i_L\end{pmatrix}\gamma^\mu\begin{pmatrix}u^j_L\\ d^j_L\end{pmatrix} = \frac{1}{2}\bar{u}^i_L\gamma^\mu\sigma_a d^j_L + \frac{1}{2}\bar{d}^i_L\gamma^\mu\sigma_a d^j_L\\ &= \frac{1}{2}\bar{u}^i_L\gamma^\mu\sigma_a u^j_L + \frac{1}{2}(V^*_{id}\bar{d}_L + V^*_{is}\bar{s}_L + V^*_{ib}\bar{b}_L)\gamma^\mu\sigma_a(V_{jd}d_L + V_{js}s_L + V_{jb}b_L)\end{aligned}$$

for Q_{1ij}, Q_{5ij}. For simplicity we only keep the light quarks u, d, s and neglect mixed terms. So the whole set of quark interactions is

$$\begin{aligned}\bar{Q}^i_L\gamma^\mu\tau^a Q^j_L &\approx \frac{1}{2}\bar{u}_L\gamma^\mu u_L\delta_{ij}\delta_{iu}\delta_{a3} - \frac{1}{2}\delta_{a3}(V^*_{id}V_{jd}\bar{d}_L\gamma^\mu d_L + V^*_{is}V_{js}\bar{s}_L\gamma^\mu s_L)\\ \bar{Q}^i_L\gamma^\mu Q^j_L &\approx \bar{u}_L\gamma^\mu u_L\delta_{ij}\delta_{iu} + V^*_{id}V_{jd}\bar{d}_L\gamma^\mu d_L + V^*_{is}V_{js}\bar{s}_L\gamma^\mu s_L\\ \bar{u}^i_R\gamma^\mu u^j_R &\approx \bar{u}_R\gamma^\mu u_R\delta_{ij}\delta_{iu}\\ \bar{d}^i_R\gamma^\mu d^j_R &\approx \bar{d}_R\gamma^\mu d_R\delta_{ij}\delta_{id} + \bar{s}_R\gamma^\mu s_R\delta_{ij}\delta_{is}\ .\end{aligned} \tag{3.5}$$

3.2 Replacing Chiral Particle Functions

The next step is rewriting the chiral particle functions in terms of the normal particle functions using the projection operators P_L, P_R. This leads us to

$$
\begin{aligned}
\bar{Q}^i_L \gamma^\mu \tau^a Q^j_L =& \frac{1}{4}(\bar{u}\gamma^\mu u \delta_{ij}\delta_{iu}\delta_{3a} - V^*_{id}V_{jd}\bar{d}\gamma^\mu d\delta_{3a} - V^*_{is}V_{js}\bar{s}\gamma^\mu s\delta_{3a}) \\
&-\frac{1}{4}(\bar{u}\gamma^\mu\gamma_5 u\delta_{ij}\delta_{iu}\delta_{3a} - V^*_{id}V_{jd}\bar{d}\gamma^\mu\gamma_5 d\delta_{3a} - V^*_{is}V_{js}\bar{s}\gamma^\mu\gamma_5 s\delta_{3a}) \\
\bar{Q}^i_L\gamma^\mu Q^j_L =& \frac{1}{2}(\bar{u}\gamma^\mu u\delta_{iu}\delta_{ij} + V^*_{id}V_{jd}\bar{d}\gamma^\mu d + V^*_{is}V_{js}\bar{s}\gamma^\mu s) \\
&-\frac{1}{2}(\bar{u}\gamma^\mu\gamma_5 u\delta_{iu}\delta_{ij} + V^*_{id}V_{jd}\bar{d}\gamma^\mu\gamma_5 d + V^*_{is}V_{js}\bar{s}\gamma^\mu\gamma_5 s) \\
\bar{u}^i_R\gamma^\mu u^j_R =& \frac{1}{2}(\bar{u}\gamma^\mu u\delta_{ij}\delta_{iu} + \bar{u}\gamma^\mu\gamma_5 u\delta_{ij}\delta_{iu}) \\
\bar{d}^i_R\gamma^\mu d^j_R =& \frac{1}{2}(\bar{d}\gamma^\mu d\delta_{ij}\delta_{id} + \bar{d}\gamma^\mu\gamma_5 d\delta_{ij}\delta_{id} + \bar{s}\gamma^\mu s\delta_{ij}\delta_{is} + \bar{s}\gamma^\mu\gamma_5 s\delta_{ij}\delta_{is}) \ .
\end{aligned}
\tag{3.6}
$$

At this point we can express the chiral operators in terms of the original operators from (3.1):

$$
\begin{aligned}
Q_{1ij} =& \frac{\delta_{3a}\tau_0}{4}(R_{1u}\delta_{ij}\delta_{iu} - V^*_{id}V_{jd}R_{1d} - V^*_{is}V_{js}R_{1s}) \\
&-\frac{\delta_{3a}\tau_0}{4}(R_{3u}\delta_{ij}\delta_{iu} - V^*_{id}V_{jd}R_{3d} - V^*_{is}V_{js}R_{3s}) \\
Q_{2ij} =& \frac{1}{2}(R_{1u}\delta_{iu}\delta_{ij} + V^*_{id}V_{jd}R_{1d} + V^*_{is}V_{js}R_{1s}) \\
&-\frac{1}{2}(R_{3u}\delta_{iu}\delta_{ij} + V^*_{id}V_{jd}R_{3d} + V^*_{is}V_{js}R_{3s}) \\
Q_{3ij} =& \frac{1}{2}(R_{1u}\delta_{ij}\delta_{iu} + R_{3u}\delta_{ij}\delta_{iu}) \\
Q_{4ij} =& \frac{1}{2}(R_{1d}\delta_{ij}\delta_{id} + R_{3d}\delta_{ij}\delta_{id} + R_{1s}\delta_{ij}\delta_{is} + R_{3s}\delta_{ij}\delta_{is}) \ .
\end{aligned}
\tag{3.7}
$$

The operators $Q_{5ij} - Q_{8ij}$ can be obtained from $Q_{1ij} - Q_{4ij}$ by replacing $R_{1q} \leftrightarrow R_{2q}$ and $R_{3q} \leftrightarrow R_{4q}$.

3.3 Comparing Coefficients

Our final step is expressing the coefficients $K_{l,q}$ of the original operators $R_{l,q}$ in (3.1) in terms of the coefficients C_{lij} of the chiral operators Q_{lij} in (3.2). To get there we look at the overall interaction. The interaction cannot depend on the representation of particle functions we choose, either normal or chiral, therefore

$$\sum_{l,q} K_{l,q} R_{l,q} \stackrel{!}{=} \sum_{l,i,j} C_{lij} Q_{lij} \ . \tag{3.8}$$

By putting the interactions (3.7) into the right side of the equation and rearranging the expression in terms of the $R_{l,q}$, we conclude that $K_{l,q}$ must be equal to the terms in front of $R_{l,q}$ on the right side. We get the dependencies

$$\begin{aligned}
K_{1,u} &= \sum_{i,j} \frac{\delta_{ij}\delta_{iu}}{2} \left(C_{1ij} \frac{\delta_{3a}\tau_0}{2} + C_{2ij} + C_{3ij} \right) \\
K_{1,d} &= \sum_{i,j} \frac{1}{2} \left(-V_{id}^* V_{jd} C_{1ij} \frac{\delta_{3a}\tau_0}{2} + V_{id}^* V_{jd} C_{2ij} + \delta_{ij}\delta_{id} C_{4ij} \right) \\
K_{1,s} &= \sum_{i,j} \frac{1}{2} \left(-V_{is}^* V_{js} C_{1ij} \frac{\delta_{3a}\tau_0}{2} + V_{is}^* V_{js} C_{2ij} + \delta_{ij}\delta_{is} C_{4ij} \right) \\
K_{2,u} &= \sum_{i,j} \frac{\delta_{ij}\delta_{iu}}{2} \left(\frac{\delta_{3a}\tau_0}{2} C_{5ij} + C_{6ij} + C_{7ij} \right) \\
K_{2,d} &= \sum_{i,j} \frac{1}{2} \left(-V_{id}^* V_{jd} \frac{\delta_{3a}\tau_0}{2} C_{5ij} + C_{6ij} V_{id}^* V_{jd} + \delta_{ij}\delta_{id} C_{8ij} \right) \\
K_{2,s} &= \sum_{i,j} \frac{1}{2} \left(-V_{is}^* V_{js} \frac{\delta_{3a}\tau_0}{2} C_{5ij} + C_{6ij} V_{is}^* V_{js} + \delta_{ij}\delta_{is} C_{8ij} \right) \\
K_{3,u} &= \sum_{i,j} \frac{\delta_{ij}\delta_{iu}}{2} \left(-C_{1ij} \frac{\delta_{3a}\tau_0}{2} - C_{2ij} + C_{3ij} \right) \\
K_{3,d} &= \sum_{i,j} \frac{1}{2} \left(C_{1ij} \frac{\delta_{3a}\tau_0}{2} V_{id}^* V_{jd} - V_{id}^* V_{jd} C_{2ij} + \delta_{ij}\delta_{id} C_{4ij} \right) \\
K_{3,s} &= \sum_{i,j} \frac{1}{2} \left(C_{1ij} \frac{\delta_{3a}\tau_0}{2} V_{is}^* V_{js} - V_{is}^* V_{js} C_{2ij} + \delta_{ij}\delta_{is} C_{4ij} \right) \\
K_{4,u} &= \sum_{i,j} \frac{\delta_{ij}\delta_{iu}}{2} \left(-\frac{\delta_{3a}\tau_0}{2} C_{5ij} - C_{6ij} + C_{7ij} \right) \\
K_{4,d} &= \sum_{i,j} \frac{1}{2} \left(\frac{\delta_{3a}\tau_0}{2} C_{5ij} V_{id}^* V_{jd} - C_{6ij} V_{id}^* V_{jd} + \delta_{ij}\delta_{id} C_{8ij} \right) \\
K_{4,s} &= \sum_{i,j} \frac{1}{2} \left(\frac{\delta_{3a}\tau_0}{2} C_{5ij} V_{is}^* V_{js} - C_{6ij} V_{is}^* V_{js} + \delta_{ij}\delta_{is} C_{8ij} \right) \ .
\end{aligned} \tag{3.9}$$

To have a hermitian interaction the coefficients need to fulfil the relation $C_{lij} = C_{lji}^*$.

4 The $L_\mu - L_\tau$ Model

In this chapter we present an extension to the standard model proposed in [**InColour**]. The authors originally aimed at explaining anomalies in the decay $B \to K l \bar{l}$, but also obtained predictions for the direct detection of dark matter in the succeeding publication [**Z**]. We will later compare their results with the formalism in the previous chapter including the CKM mixing.

4.1 The New Interaction

The extension to the standard model in [**InColour**] is a new $U(1)'$ gauge group. The related vector-boson is called Z', and it couples to the muon and tau lepton families, and a new set of vector-like quarks U, D, Q. The standard model quarks indirectly couple to the Z' as well, since they mix with the new quarks through a Yukawa coupling:

$$\mathcal{L}^{(\mathrm{mix})} \; . \tag{4.1}$$

In [**Z**] also a coupling to a dark matter fermion χ is established. Before present the entire interaction, we want to mention that the Z' and the new quarks get their masses through a new higgs-like field Φ. Its vacuum expectation value is $\langle \Phi \rangle = {}^{v_\Phi}/\sqrt{2}$, and it connects the coupling strength g' to the Z' mass: $m_{Z'} = v_\Phi g'$. The full interaction lagrangian is:

$$\begin{aligned}
\mathcal{L}^{(\mathrm{int})}_{Z'} =& g' Z'_\alpha \times q_l \left(\bar{l}_2 \gamma^\alpha l_2 - \bar{l}_3 \gamma^\alpha l_3 + \bar{\mu}_R \gamma^\alpha \mu_R - \bar{\tau}_R \gamma^\alpha \tau_R \right) \\
&+ g' Z'_\alpha \times v_\Phi^2 \left(- \frac{Y_{Di} Y^*_{Dj}}{2 m_D^2} \bar{d}^i_R \gamma^\alpha d^j_R - \frac{Y_{Ui} Y^*_{Uj}}{2 m_U^2} \bar{u}^i_R \gamma^\alpha u^j_R + \bar{u}^i_L \gamma^\alpha u^j_L + \bar{d}^i_L \gamma^\alpha d^j_L \right) \\
&+ g' Z'_\alpha \times q_\chi (\bar{\chi} \gamma^\alpha \chi) \; ,
\end{aligned} \tag{4.2}$$

where q_l, q_χ are the $U(1)'$ charge of the leptons and the dark matter particle, l_2, l_3 are the electroweak lefthanded lepton doublets, u^i, d^i are the standard model quarks, and $m_{U,D,Q}$ are the masses of the new quarks.

To do (2) **To do** (3) **To do** (4)

4.2 Restrictions to the Parameter Space

In [**Z**] Altmannshofer et. al. discuss restrictions for the parameter space by looking at the B decay mentioned above and discussing dark matter relic density and direct detection. They find that experimental data from $B \to K l \bar{l}$ limits the ratio of the Z' mass and the coupling g' to

$$540\,\mathrm{GeV} \lessapprox \frac{m_{Z'}}{g'} \lessapprox 4{,}9\,\mathrm{TeV}\ , \tag{4.3}$$

with $m_{Z'} \gtrapprox 10\,\mathrm{GeV}$.

Regarding the dark matter relic density, they conclude that only

$$m_{Z'} \approx 2 m_\chi \tag{4.4}$$

leads to correct results. Since they neglect flavour mixing in the nucleus, direct detection has to occur through the loop diagram in figure 4.1. The corresponding cross section at zero momentum transfer is (see [**Z**])

$$\sigma_{0,\mathrm{Loop}} = \frac{\mu_{A\chi}^2}{A^2 \pi} \left(\frac{\alpha_{em} Z}{3\pi} \, \frac{g'^2 q_\chi q_l}{m_{Z'}^2} \log\left(\frac{m_\mu^2}{m_\tau^2}\right)\right)^2\ , \tag{4.5}$$

where $\mu_{A\chi}$ is the reduced mass of the nucleus and the dark matter particle χ and A, Z are the nucleon and proton numbers.

When discussing limits to the parameter space they distinguish two cases. For $q_l = q_\chi = 1$, experimental data favours the parameter region

$$\begin{aligned} 10\,\mathrm{GeV} \lessapprox m_\chi \lessapprox 46\,\mathrm{GeV} \\ 2 \cdot 10^{-3} \lessapprox g' \lessapprox 10^{-2}\ , \end{aligned} \tag{4.6}$$

leaving possible dark matter masses in the range $(5-23)$GeV. For $q_l = 1, q_\chi = {}^1\!/_6$ no further restriction of the parameters can be found.

To do (5)

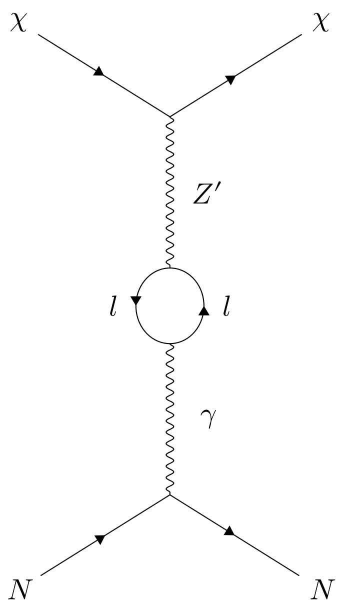

Figure 4.1: Direct detection loop diagram

5 Comparison

5.1 Cross Section

We are only interested in the interactions Q_{2bs}, therefore all coefficients C_{lij} vanish except $C_{2sb} = C^*_{2bs}$. In terms of the coefficients in (3.9) this means

$$\begin{aligned}
K_{1,d} &= +\mathrm{Re}(V^*_{cd} V_{td} C_{2sb}) \\
K_{1,s} &= +\mathrm{Re}(V^*_{cs} V_{ts} C_{2sb}) \\
K_{3,d} &= -\mathrm{Re}(V^*_{cd} V_{td} C_{2sb}) \\
K_{3,s} &= -\mathrm{Re}(V^*_{cs} V_{ts} C_{2sb}) \ ,
\end{aligned} \tag{5.1}$$

and all the other $K_{l,q}$ are zero.
We are interested in spin-independent cross sections, therefore we only use the interactions $(\bar{\chi}\gamma^\mu\chi)(\bar{q}\gamma_\mu q)$. Protons do not contain strange quarks, so the lagrangian only consists of one interaction:

$$\mathcal{L} = K_{1,d}(\bar{\chi}\gamma^\mu\chi)(\bar{d}\gamma_\mu d) + \text{h.c.} \ . \tag{5.2}$$

Since this operator only counts the number of down quarks in the nucleus, the matrix element is

$$M = Z \cdot 2K_{1,d} + (A - Z) \cdot K_{1,d} \ . \tag{5.3}$$

$$\sigma^{\mathrm{SI}}_{0,\mathrm{tree}} = \frac{\mu^2_{A\chi}}{A^2\pi} \left| Z C_p + (A - Z) C_n \right|^2 \ . \tag{5.4}$$

$$= \frac{\mu^2_{A\chi}}{A^2\pi} K^2_{1,d} \times \mathcal{O}(10^2) \tag{5.5}$$

In case of the spin-dependent cross section we have

$$\sigma_{0,\text{tree}}^{\text{SD}} = \frac{\mu_{A\chi}^2}{A^2\pi} 32\Lambda^2 J(J+1) 64 G_F^2 \tag{5.6}$$

$$= \frac{\mu_{A\chi}^2}{A^2\pi} 32 \left(\frac{K_{3,d}}{J} \frac{\Delta d^{(n)}}{\sqrt{2} G_F} \frac{\mu}{3.826} \right)^2 J(J+1) 64 G_F^2 \tag{5.7}$$

$$= \frac{\mu_{A\chi}^2}{A^2\pi} 2^{10} K_{3,d}^2 (\Delta d^{(n)})^2 \left(\frac{\mu}{3.826} \right)^2 \frac{J(J+1)}{J^2} \tag{5.8}$$

$$= \frac{\mu_{A\chi}^2}{A^2\pi} K_{3,d}^2 \times \mathcal{O}(10^3 10^{-2} 10^{-2}) \tag{5.9}$$

$$\begin{aligned} &\langle N | K_{1,d} \bar{d} \gamma^\mu d | N \rangle + \langle N | K_{1,u} \bar{u} \gamma^\mu u | N \rangle \\ =& Z \left(2K_{1,u} + K_{1,d} \right) (\bar{p} \gamma^\mu p) + (A - Z) \left(K_{1,u} + 2K_{1,d} \right) (\bar{n} \gamma^\mu n) \end{aligned}$$

$$\begin{aligned} &\langle N | K_{3,d} \bar{d} \gamma^\mu \gamma_5 d | N \rangle + \langle N | K_{3,u} \bar{u} \gamma^\mu \gamma_5 u | N \rangle \\ =& Z (K_{3,d} 2 s^\mu \Delta d^{(p)} + K_{3,u} 2 s^\mu \Delta u^{(p)}) (\bar{p} \gamma^\mu \gamma_5 p) \\ +& (A - Z)(K_{3,d} 2 s^\mu \Delta d^{(n)} + K_{3,u} 2 s^\mu \Delta u^{(n)}) (\bar{n} \gamma^\mu \gamma_5 n) \end{aligned}$$

To do (6)

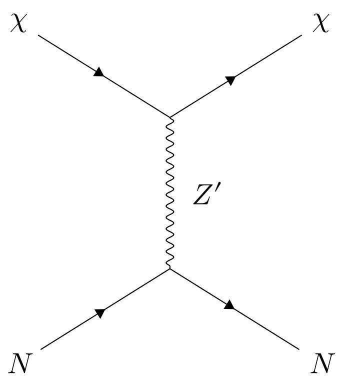

Figure 5.1: Direct detection tree level diagram

6 Conclusion

To do...

☐ 1 (p. 1): Irgendwo muss noch kurz auf dunkle Materie eingegangen werden. Als Einleitung wären daher auch Detection Experimente gut. Evtl. hier direct vs indirect erklären.

☐ 2 (p. 11): Was genau sind die neuen Quarks? Gibt es davon nur 3?

☐ 3 (p. 11): Erkläre die Mischung der neuen und SM Quarks etwas ausführlicher.

☐ 4 (p. 11): Kann man tatsächlich hier die Ups drehen und dann später mit gedrehten Downs rechnen? Die entsprechenden Konstanten dann noch einfügen.

☐ 5 (p. 12): Evtl. kann man den Ursprung der Grenzen noch näher erläutern.

☐ 6 (p. 15): Wieso ist eigentlich $Q_L^2 \gamma_\mu Q_L^3 = s_L \gamma_\mu b_L$?

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Ich versichere hiermit an Eides statt, dass ich die vorliegende Abschlussarbeit mit dem Titel "Flavour Mixing Effects in the Direct Detection of Dark Matter" selbstständig und ohne unzulässige fremde Hilfe erbracht habe. Ich habe keine anderen als die angegebenen Quellen und Hilfsmittel benutzt, sowie wörtliche und sinngemäße Zitate kenntlich gemacht. Die Arbeit hat in gleicher oder ähnlicher Form noch keiner Prüfungsbehörde vorgelegen.

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