technische universität
dortmund

Arbeit zur Erlangung des akademischen Grades
Bachelor of Science

Flavour Mixing Effects in the Direct Detection of Dark Matter

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2017

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Abgabedatum:	17. Juli 2017

Abstract

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The origins of the flavour mixing go back to the 1960s, when Italian physicist Cabibbo resolved anomalies in data of weak interactions by proposing a flavour mixing of lefthanded down-type quarks. Later in 1973, Kobayaski and Maskawa extended this idea to three quark generations to explain CP violation [1]. On a mathematical level the flavour mixing arises from the fact that the fermion mass eigenstates do not necessarily equal the flavour eigenstates. In the course of this chapter we first derive how the Higgs mechanism gives mass to particles and second take a look at fermion masses and why this leads to flavour mixing. The following calculations are based on the outlines in [2, Chapter 20] and [3, Chapter 1.2.1].

2.1 The Higgs Mechanism

We consider a complex scalar field Φ that interacts with itself through a potential

$$V(\Phi) = -\mu^2 \Phi^\dagger \Phi + \frac{\lambda}{2}(\Phi^\dagger \Phi)^2 \ , \quad \mu^2 > 0 \ . \tag{2.1}$$

If $\lambda > 0$, there are to minima of this potential. They occur at

$$\langle \Phi \rangle = \pm\sqrt{\frac{\mu^2}{\lambda}} \ . \tag{2.2}$$

This is the vacuum expectation value of Φ.

We add a $SU(2)$ gauge field coupled to Φ, so Φ is a doublet (Φ_1, Φ_2) with covariant derivative

$$D_\mu \Phi = (\partial_\mu - ig \sum_{a=1}^{3} A_\mu^a \tau^a)\Phi \ , \tag{2.3}$$

where τ^a are the generators of $SU(2)$. In this case there is an infinite number of vacuum expection values for Φ arranged in a circle. We are free to choose one and make the simple choice

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \ , \qquad v = \sqrt{\frac{2\mu^2}{\lambda}} \ . \tag{2.4}$$

Note that by choosing one vacuum value, we break the symmetry.

The kinetic energy of Φ is

$$\begin{aligned}(D_\mu \Phi)^2 =& \frac{1}{2}(\partial_\mu v)(\partial^\mu v) \\ & -ig(\partial^\mu \begin{pmatrix}0 & v\end{pmatrix})\left(\sum_{a=1}^{3} A_\mu^a \tau^a \begin{pmatrix}0 \\ v\end{pmatrix}\right) \\ & -\frac{1}{2}g^2 \begin{pmatrix}0 & v\end{pmatrix} \sum_{a,b=1}^{3} \tau^a \tau^b \begin{pmatrix}0 \\ v\end{pmatrix} A_\mu^a A^{b\mu} \ . \end{aligned} \tag{2.5}$$

Using the relation $\{\tau^a, \tau^b\} = {}^1\!/_2 \cdot \delta_{ab}$, we can simplify the last expression to get

$$-\frac{1}{2}g^2 \begin{pmatrix}0 & v\end{pmatrix} \sum_{a,b=1}^{3} \tau^a \tau^b \begin{pmatrix}0 \\ v\end{pmatrix} A_\mu^a A^{b\mu} = -\frac{g^2 v^2}{8} \sum_{a=1}^{3} A_\mu^a A^{a\mu} \ , \tag{2.6}$$

which is a mass term $\mathcal{L}_m = -\frac{1}{2} m_A^2 A_\mu A^\mu$ that assigns the mass $m_A = \frac{gv}{2}$ to all three gauge bosons. By expanding the system with an additional $U(1)$ symmetry, the kinetic energy would again provide three gauge boson masses, leaving the fourth gauge boson massless. The massive bosons can be identified as $W^\pm, Z^0$ and the massless as the photon.

The scalar field Φ is usually called Higgs boson. Obtaining particle mass terms in the kinetic energy of the Higgs, is, unsurprisingly, referred to as the Higgs mechanism.

2.2 Fermion Masses and Flavour Mixing

Before we discuss fermion mass terms, we introduce the notation we use in this and the following chapters. The leptons and quark chiral particle functions are

$$E_R = (e_R, \mu_R, \tau_R) \ , \qquad Y_E = -1 \ ; \tag{2.7}$$

$$L_L = \left(\begin{pmatrix}\nu_e \\ e\end{pmatrix}_L , \begin{pmatrix}\nu_\mu \\ \mu\end{pmatrix}_L , \begin{pmatrix}\nu_\tau \\ \tau\end{pmatrix}_L \right) \ , \qquad Y_L = -1 \ ; \tag{2.8}$$

$$U_R = (u_R, c_R, t_R) \ , \qquad Y_U = \frac{2}{3} \ ; \tag{2.9}$$

$$D_R = (d_R, s_R, b_R) \ , \qquad Y_D = -\frac{1}{3} \ ; \tag{2.10}$$

$$Q_L = \left(\begin{pmatrix}u \\ d\end{pmatrix}_L , \begin{pmatrix}c \\ s\end{pmatrix}_L , \begin{pmatrix}t \\ b\end{pmatrix}_L \right) \ , \qquad Y_Q = \frac{1}{3} \ , \tag{2.11}$$

where the hypercharge Y is given. It is related to the electric charge Q and the third component of the weak isospin I_3 through the Gell-Mann-Nishijima formula $Y = 2(Q - I_3)$. The righthanded particles are singlets under $SU(2)$ and therefore $I_3^{(r.h.)} = 0$. We also use the lefthanded components $E_L = (e_L, \mu_L, \tau_L)$, $D_L = (d_L, s_L, b_L)$, and $U_L = (u_L, c_L, t_L)$.

The electroweak interaction lagrangian for the standard model fermions is

$$\begin{aligned}\mathcal{L}^{(\text{int})} =& \bar{E}_R\gamma^\mu(i\partial_\mu - g_1 Y_E B_\mu)E_R + \bar{L}_L\gamma^\mu(i\partial_\mu - g_1 Y_L B_\mu - g_2\sum_{i=1}^{3} A_\mu^i\tau^i)L_L \\ &+\bar{D}_R\gamma^\mu(i\partial_\mu - g_1 Y_D B_\mu)D_R + \bar{U}_R\gamma^\mu(i\partial_\mu - g_1 Y_U B_\mu)U_R \\ &+\bar{Q}_L\gamma^\mu(i\partial_\mu - g_1 Y_Q B_\mu - g_2\sum_{i=1}^{3} A_\mu^i\tau^i)Q_L \ ,\end{aligned} \tag{2.12}$$

where B_μ, A_μ^i are the gauge bosons corresponding to $U(1)_Y \times SU(2)$. The coupling constants are g_1, g_2, and τ^i are again the generators of $SU(2)$. This lagrangian describes massless particles. In order to get a fermion mass term, one has to couple the lefthanded and righthanded part of a particle. Since direct coupling between a $SU(2)$ singlet and a $SU(2)$ doublet violates gauge invariance, a connecting field is necessary. To preserve invariance under Lorentz, $U(1)_Y$, and $SU(2)$ transformations this field must have spin 0, hypercharge $Y = 1/2$, and be a doublet. We identify this field with Φ from the previous chapter and write down the mass terms for the fermions

$$\mathcal{L}^{(\text{mass})} = -\left[\bar{L}_L\Phi\lambda^e E_R + \bar{Q}_L\Phi\lambda^d D_R + \bar{Q}_L i\sigma^2\Phi^\dagger\lambda^u U_R + \text{h.c.}\right] \ , \tag{2.13}$$

with complex matrix coupling constants $\lambda^e, \lambda^d, \lambda^u$. Replacing Φ with its vacuum expectation value (2.4) gives

$$\mathcal{L}^{(\text{mass})} = -\frac{v}{\sqrt{2}}\left[\bar{E}_L\lambda^e E_R + \bar{D}_L\lambda^d D_R + \bar{U}_L\lambda^u U_R + \text{h.c.}\right] \ . \tag{2.14}$$

The interaction lagrangian (2.12) is invariant under unitary transformations

$$E_L \to S_e E_L \qquad E_R \to R_e E_R \ , \tag{2.15}$$
$$U_L \to S_u U_L \qquad U_R \to R_u U_R \ , \tag{2.16}$$
$$D_L \to S_d D_L \qquad D_R \to R_d D_R \ . \tag{2.17}$$

Thus, we can diagonalize the interactions in (2.14). The diagonal lepton coupling is $\tilde{\lambda}^e = S_e\lambda^e R_e^\dagger$ and parametrizes the lepton masses

$$m_e = \frac{v}{\sqrt{2}}\tilde{\lambda}^e_{11} \ , \quad m_\mu = \frac{v}{\sqrt{2}}\tilde{\lambda}^e_{22} \ , \quad m_\tau = \frac{v}{\sqrt{2}}\tilde{\lambda}^e_{33} \ . \tag{2.18}$$

The diagonal coupling for up-type quarks is $\tilde{\lambda}^u = S_u \lambda^u R_u^\dagger$, giving the corresponding masses

$$m_u = \frac{v}{\sqrt{2}} \tilde{\lambda}^u_{11} \ , \quad m_c = \frac{v}{\sqrt{2}} \tilde{\lambda}^u_{22} \ , \quad m_t = \frac{v}{\sqrt{2}} \tilde{\lambda}^u_{33} \ . \tag{2.19}$$

The transformed coupling of the down-type quarks is $\tilde{\lambda}^d = S_d \lambda^d R_d^\dagger$, leading to the down-type masses

$$m_d = \frac{v}{\sqrt{2}} \tilde{\lambda}^d_{11} \ , \quad m_s = \frac{v}{\sqrt{2}} \tilde{\lambda}^d_{22} \ , \quad m_b = \frac{v}{\sqrt{2}} \tilde{\lambda}^d_{33} \ . \tag{2.20}$$

The transformed particle functions are now mass eigenstates. But when looking at couplings of up- and down-type quarks, e.g. the current

$$\bar{U}_L \gamma^\mu D_L \ , \tag{2.21}$$

the unitary transformations change the interaction to

$$\bar{U}_L \gamma^\mu S_u^\dagger S_d D_L \ , \tag{2.22}$$

where we identify the CKM matrix $V = S_u^\dagger S_d$.

3 Introduction of the Flavour Mixing into an Existing Formalism

In this chapter we include the flavour mixing into an existing formalism. We use the model described in [4]. It provides a framework to calculate cross sections for the direct detection of dark matter. The model bases on a set of dimension-five, -six, and -seven operators. We restrict our calculations to the dimension-six operators, which are

$$\begin{aligned} R_{1,q} &= (\bar{\chi}\gamma_\mu\chi)(\bar{q}\gamma^\mu q)\ , & R_{3,q} &= (\bar{\chi}\gamma_\mu\chi)(\bar{q}\gamma^\mu\gamma_5 q)\ , \\ R_{2,q} &= (\bar{\chi}\gamma_\mu\gamma_5\chi)(\bar{q}\gamma^\mu q)\ , & R_{4,q} &= (\bar{\chi}\gamma_\mu\gamma_5\chi)(\bar{q}\gamma^\mu\gamma_5 q)\ , \end{aligned} \tag{3.1}$$

with the dark matter particle χ and a quark $q = (u, d, c, s, t, b)$.

Since the CKM mixing only applies to the lefthanded down-type quarks, we need to rewrite these operators in terms of the left- and righthanded particle functions to include the CKM matrix. These chiral operators are

$$\begin{aligned} Q_{1ij} &= (\bar{\chi}\gamma_\mu\tilde{\tau}^a\chi)(\bar{Q}^i_L\gamma^\mu\tau^a Q^j_L) & Q_{5ij} &= (\bar{\chi}\gamma_\mu\gamma_5\tilde{\tau}^a\chi)(\bar{Q}^i_L\gamma^\mu\tau^a Q^j_L) \\ Q_{2ij} &= (\bar{\chi}\gamma_\mu\chi)(\bar{Q}^i_L\gamma^\mu Q^j_L) & Q_{6ij} &= (\bar{\chi}\gamma_\mu\gamma_5\chi)(\bar{Q}^i_L\gamma^\mu Q^j_L) \\ Q_{3ij} &= (\bar{\chi}\gamma_\mu\chi)(\bar{U}^i_R\gamma^\mu U^j_R) & Q_{7ij} &= (\bar{\chi}\gamma_\mu\gamma_5\chi)(\bar{U}^i_R\gamma^\mu U^j_R) \\ Q_{4ij} &= (\bar{\chi}\gamma_\mu\chi)(\bar{D}^i_R\gamma^\mu D^j_R) & Q_{8ij} &= (\bar{\chi}\gamma_\mu\gamma_5\chi)(\bar{D}^i_R\gamma^\mu D^j_R)\ . \end{aligned} \tag{3.2}$$

The operators $\tilde{\tau}^a, \tau^a$ are the generators of the $SU(2)$ in the corresponding spin-representation. For the quarks, we use

$$\tau^a = \frac{\sigma_a}{2}\ , \tag{3.3}$$

where σ_a are the pauli matrices. For simplicity reasons we are only interested in interactions with only one kind of quark and therefore abolish $a = 1, 2$. Regarding the dark matter, we therby obtain

$$\tilde{\tau}^3\chi = \tau_0\chi\ , \tag{3.4}$$

with the weak isospin value τ_0 of the dark matter particle.

In the following subchapters, we go through the steps of including the CKM matrix in the formalism:

1. Replacing the pure lefthanded down-type quarks with the mixed quarks.

2. Rewriting the chiral particle functions in terms of the normal particle functions and projection operators.

3. Writing down the entire interaction lagrangian in terms of (3.1) and (3.2) and comparing the coefficients.

3.1 Including the Flavour Mixing

The inclusion of the CKM matrix only affects the chiral operators $Q_{1ij}, Q_{2ij}, Q_{5ij}, Q_{6ij}$. The quark part of the interaction with flavour mixing is therefore

$$\begin{aligned}\bar{Q}^i_L \gamma^\mu Q^j_L &= \begin{pmatrix} \bar{u}^i_L \\ \bar{d}^i_L \end{pmatrix} \gamma^\mu \begin{pmatrix} u^j_L \\ d^j_L \end{pmatrix} = \bar{u}^i_L \gamma^\mu d^j_L + \bar{d}^i_L \gamma^\mu d^j_L \\ &= \bar{u}^i_L \gamma^\mu u^j_L + (V^*_{id}\bar{d}_L + V^*_{is}\bar{s}_L + V^*_{ib}\bar{b}_L)\gamma^\mu (V_{jd} d_L + V_{js} s_L + V_{jb} b_L)\end{aligned}$$

for Q_{2ij}, Q_{6ij}, respectively

$$\begin{aligned}\bar{Q}^i_L \gamma^\mu \tau^a Q^j_L &= \begin{pmatrix} \bar{u}^i_L \\ \bar{d}^i_L \end{pmatrix} \gamma^\mu \begin{pmatrix} u^j_L \\ d^j_L \end{pmatrix} = \frac{1}{2}\bar{u}^i_L \gamma^\mu \sigma_a d^j_L + \frac{1}{2}\bar{d}^i_L \gamma^\mu \sigma_a d^j_L \\ &= \frac{1}{2}\bar{u}^i_L \gamma^\mu \sigma_a u^j_L + \frac{1}{2}(V^*_{id}\bar{d}_L + V^*_{is}\bar{s}_L + V^*_{ib}\bar{b}_L)\gamma^\mu \sigma_a (V_{jd} d_L + V_{js} s_L + V_{jb} b_L)\end{aligned}$$

for Q_{1ij}, Q_{5ij}. For simplicity we only keep the light quarks u, d, s and neglect mixed terms. So the whole set of quark interactions is

$$\begin{aligned}\bar{Q}^i_L \gamma^\mu \tau^a Q^j_L &\approx \frac{1}{2}\bar{u}_L \gamma^\mu u_L \delta_{ij}\delta_{iu}\delta_{a3} - \frac{1}{2}\delta_{a3}(V^*_{id}V_{jd}\bar{d}_L\gamma^\mu d_L + V^*_{is}V_{js}\bar{s}_L\gamma^\mu s_L) \\ \bar{Q}^i_L \gamma^\mu Q^j_L &\approx \bar{u}_L \gamma^\mu u_L \delta_{ij}\delta_{iu} + V^*_{id}V_{jd}\bar{d}_L\gamma^\mu d_L + V^*_{is}V_{js}\bar{s}_L\gamma^\mu s_L \\ \bar{u}^i_R \gamma^\mu u^j_R &\approx \bar{u}_R \gamma^\mu u_R \delta_{ij}\delta_{iu} \\ \bar{d}^i_R \gamma^\mu d^j_R &\approx \bar{d}_R \gamma^\mu d_R \delta_{ij}\delta_{id} + \bar{s}_R \gamma^\mu s_R \delta_{ij}\delta_{is}\ .\end{aligned} \tag{3.5}$$

3.2 Replacing Chiral Particle Functions

The next step is rewriting the chiral particle functions in terms of the normal particle functions using the projection operators P_L, P_R. This leads us to

$$
\begin{aligned}
\bar{Q}^i_L\gamma^\mu\tau^a Q^j_L =&\frac{1}{4}(\bar{u}\gamma^\mu u\delta_{ij}\delta_{iu}\delta_{3a} - V^*_{id}V_{jd}\bar{d}\gamma^\mu d\delta_{3a} - V^*_{is}V_{js}\bar{s}\gamma^\mu s\delta_{3a})\\
&-\frac{1}{4}(\bar{u}\gamma^\mu\gamma_5 u\delta_{ij}\delta_{iu}\delta_{3a} - V^*_{id}V_{jd}\bar{d}\gamma^\mu\gamma_5 d\delta_{3a} - V^*_{is}V_{js}\bar{s}\gamma^\mu\gamma_5 s\delta_{3a})\\
\bar{Q}^i_L\gamma^\mu Q^j_L =&\frac{1}{2}(\bar{u}\gamma^\mu u\delta_{iu}\delta_{ij} + V^*_{id}V_{jd}\bar{d}\gamma^\mu d + V^*_{is}V_{js}\bar{s}\gamma^\mu s)\\
&-\frac{1}{2}(\bar{u}\gamma^\mu\gamma_5 u\delta_{iu}\delta_{ij} + V^*_{id}V_{jd}\bar{d}\gamma^\mu\gamma_5 d + V^*_{is}V_{js}\bar{s}\gamma^\mu\gamma_5 s)\\
\bar{u}^i_R\gamma^\mu u^j_R =&\frac{1}{2}(\bar{u}\gamma^\mu u\delta_{ij}\delta_{iu} + \bar{u}\gamma^\mu\gamma_5 u\delta_{ij}\delta_{iu})\\
\bar{d}^i_R\gamma^\mu d^j_R =&\frac{1}{2}(\bar{d}\gamma^\mu d\delta_{ij}\delta_{id} + \bar{d}\gamma^\mu\gamma_5 d\delta_{ij}\delta_{id} + \bar{s}\gamma^\mu s\delta_{ij}\delta_{is} + \bar{s}\gamma^\mu\gamma_5 s\delta_{ij}\delta_{is})\ .
\end{aligned}
\tag{3.6}
$$

At this point we can express the chiral operators in terms of the original operators from (3.1):

$$
\begin{aligned}
Q_{1ij} =&\frac{\delta_{3a}\tau_0}{4}(R_{1u}\delta_{ij}\delta_{iu} - V^*_{id}V_{jd}R_{1d} - V^*_{is}V_{js}R_{1s})\\
&-\frac{\delta_{3a}\tau_0}{4}(R_{3u}\delta_{ij}\delta_{iu} - V^*_{id}V_{jd}R_{3d} - V^*_{is}V_{js}R_{3s})\\
Q_{2ij} =&\frac{1}{2}(R_{1u}\delta_{iu}\delta_{ij} + V^*_{id}V_{jd}R_{1d} + V^*_{is}V_{js}R_{1s})\\
&-\frac{1}{2}(R_{3u}\delta_{iu}\delta_{ij} + V^*_{id}V_{jd}R_{3d} + V^*_{is}V_{js}R_{3s})\\
Q_{3ij} =&\frac{1}{2}(R_{1u}\delta_{ij}\delta_{iu} + R_{3u}\delta_{ij}\delta_{iu})\\
Q_{4ij} =&\frac{1}{2}(R_{1d}\delta_{ij}\delta_{id} + R_{3d}\delta_{ij}\delta_{id} + R_{1s}\delta_{ij}\delta_{is} + R_{3s}\delta_{ij}\delta_{is})\ .
\end{aligned}
\tag{3.7}
$$

The operators $Q_{5ij} - Q_{8ij}$ can be obtained from $Q_{1ij} - Q_{4ij}$ by replacing $R_{1q} \leftrightarrow R_{2q}$ and $R_{3q} \leftrightarrow R_{4q}$.

3.3 Comparing Coefficients

Our final step is expressing the coefficients $K_{l,q}$ of the original operators $R_{l,q}$ in (3.1) in terms of the coefficients C_{lij} of the chiral operators Q_{lij} in (3.2). To get there we look at the overall interaction. The interaction cannot depend on the representation of particle functions we choose, either normal or chiral, therefore

$$\sum_{l,q} K_{l,q} R_{l,q} \stackrel{!}{=} \sum_{l,i,j} C_{lij} Q_{lij} \; . \tag{3.8}$$

By putting the interactions (3.7) into the right side of the equation and rearranging the expression in terms of the $R_{l,q}$, we conclude that $K_{l,q}$ must be equal to the terms in front of $R_{l,q}$ on the right side. We get the dependencies

$$\begin{aligned}
K_{1,u} &= \sum_{i,j} \frac{\delta_{ij}\delta_{iu}}{2} \left(C_{1ij} \frac{\delta_{3a}\tau_0}{2} + C_{2ij} + C_{3ij} \right) \\
K_{1,d} &= \sum_{i,j} \frac{1}{2} \left(-V^*_{id} V_{jd} C_{1ij} \frac{\delta_{3a}\tau_0}{2} + V^*_{id} V_{jd} C_{2ij} + \delta_{ij}\delta_{id} C_{4ij} \right) \\
K_{1,s} &= \sum_{i,j} \frac{1}{2} \left(-V^*_{is} V_{js} C_{1ij} \frac{\delta_{3a}\tau_0}{2} + V^*_{is} V_{js} C_{2ij} + \delta_{ij}\delta_{is} C_{4ij} \right) \\
K_{2,u} &= \sum_{i,j} \frac{\delta_{ij}\delta_{iu}}{2} \left(\frac{\delta_{3a}\tau_0}{2} C_{5ij} + C_{6ij} + C_{7ij} \right) \\
K_{2,d} &= \sum_{i,j} \frac{1}{2} \left(-V^*_{id} V_{jd} \frac{\delta_{3a}\tau_0}{2} C_{5ij} + C_{6ij} V^*_{id} V_{jd} + \delta_{ij}\delta_{id} C_{8ij} \right) \\
K_{2,s} &= \sum_{i,j} \frac{1}{2} \left(-V^*_{is} V_{js} \frac{\delta_{3a}\tau_0}{2} C_{5ij} + C_{6ij} V^*_{is} V_{js} + \delta_{ij}\delta_{is} C_{8ij} \right) \\
K_{3,u} &= \sum_{i,j} \frac{\delta_{ij}\delta_{iu}}{2} \left(-C_{1ij} \frac{\delta_{3a}\tau_0}{2} - C_{2ij} + C_{3ij} \right) \\
K_{3,d} &= \sum_{i,j} \frac{1}{2} \left(C_{1ij} \frac{\delta_{3a}\tau_0}{2} V^*_{id} V_{jd} - V^*_{id} V_{jd} C_{2ij} + \delta_{ij}\delta_{id} C_{4ij} \right) \\
K_{3,s} &= \sum_{i,j} \frac{1}{2} \left(C_{1ij} \frac{\delta_{3a}\tau_0}{2} V^*_{is} V_{js} - V^*_{is} V_{js} C_{2ij} + \delta_{ij}\delta_{is} C_{4ij} \right) \\
K_{4,u} &= \sum_{i,j} \frac{\delta_{ij}\delta_{iu}}{2} \left(-\frac{\delta_{3a}\tau_0}{2} C_{5ij} - C_{6ij} + C_{7ij} \right) \\
K_{4,d} &= \sum_{i,j} \frac{1}{2} \left(\frac{\delta_{3a}\tau_0}{2} C_{5ij} V^*_{id} V_{jd} - C_{6ij} V^*_{id} V_{jd} + \delta_{ij}\delta_{id} C_{8ij} \right) \\
K_{4,s} &= \sum_{i,j} \frac{1}{2} \left(\frac{\delta_{3a}\tau_0}{2} C_{5ij} V^*_{is} V_{js} - C_{6ij} V^*_{is} V_{js} + \delta_{ij}\delta_{is} C_{8ij} \right) \; .
\end{aligned} \tag{3.9}$$

To have a hermitian interaction the coefficients need to fulfil the relation $C_{lij} = C^*_{lji}$.

4 The $L_\mu - L_\tau$ Model

In this chapter we present an extension to the standard model proposed in [5]. The authors originally aimed at explaining anomalies in the decay $B \to K l \bar{l}$, but also obtained predictions for the direct detection of dark matter in the succeeding publication [6]. We will later compare their results with the formalism in the previous chapter that includes the CKM mixing.

4.1 The New Interaction

The extension to the standard model in [5] is a new $U(1)'$ gauge group. The related vector-boson is called Z', and it couples to the muon and tau lepton families, and a new set of vector-like quarks U, D, Q. The standard model quarks indirectly couple to the Z' as well, since they mix with the new quarks through a Yukawa coupling:

$$\begin{aligned}\mathcal{L}^{(\mathrm{mix})} =& \Phi \bar{\tilde{D}}_R (Y_{Qb} b_L + Y_{Qs} s_L + Y_{Qd} d_L) \\ &+ \Phi \bar{\tilde{U}}_R (Y_{Qt} t_L + Y_{Qc} c_L + Y_{Qu} u_L) \\ &+ \Phi^\dagger \bar{U}_L (Y_{Ut} t_R + Y_{Uc} c_R + Y_{Uu} u_R) \\ &+ \Phi^\dagger \bar{D}_L (Y_{Db} b_R + Y_{Ds} s_R + Y_{Dd} d_R) + \text{h.c.} \ ,\end{aligned} \tag{4.1}$$

where $\tilde{Q}_R = (\tilde{U}_R, \tilde{D}_R)$, $Q_L = (U_L, D_L)$ are the weak doublets and Y_{ij} are the coupling constants.

In [6] also a coupling to a dark matter fermion χ is established. The full interaction lagrangian is:

$$\begin{aligned}\mathcal{L}^{(\mathrm{int})}_{Z'} =& g' Z'_\alpha \times q_l \left(\bar{L}^2_L \gamma^\alpha L^2_L - \bar{L}^3_L \gamma^\alpha L^3_L + \bar{\mu}_R \gamma^\alpha \mu_R - \bar{\tau}_R \gamma^\alpha \tau_R \right) \\ &+ g' Z'_\alpha \times v_\Phi^2 \sum_{i,j}^{3} \left(-\frac{Y_{Di} Y^*_{Dj}}{2 m_D^2} \bar{D}^i_R \gamma^\alpha D^j_R - \frac{Y_{Ui} Y^*_{Uj}}{2 m_U^2} \bar{U}^i_R \gamma^\alpha U^j_R + \frac{Y_{Qi} Y^*_{Qj}}{2 m_Q^2} \bar{Q}^i_L \gamma^\alpha Q^j_L \right) \\ &+ g' Z'_\alpha \times q_\chi (\bar{\chi} \gamma^\alpha \chi) \ ,\end{aligned} \tag{4.2}$$

where q_l, q_χ are the $U(1)'$ charge of the leptons and the dark matter particle, $m_{U,D,Q}$ are the masses of the new quarks, and v_Φ is the vacuum expectation value of a new Higgs-like field that gives mass to the Z'.

4.2 Restrictions to the Parameter Space

In [6] Altmannshofer et. al. discuss restrictions for the parameter space by looking at the B decay mentioned above and discussing dark matter relic density and direct detection. They find that experimental data from $B \to K l \bar{l}$ limits the ratio of the Z' mass and the coupling g' to

$$540\,\mathrm{GeV} \lessapprox \frac{m_{Z'}}{g'} \lessapprox 4.9\,\mathrm{TeV} \ , \tag{4.3}$$

with $m_{Z'} \gtrapprox 10\,\mathrm{GeV}$.

Regarding the dark matter relic density, they conclude that only

$$m_{Z'} \approx 2 m_\chi \tag{4.4}$$

leads to correct results. Since they neglect flavour mixing in the nucleus, direct detection has to occur through the loop diagram in figure 4.1. The corresponding cross section at zero momentum transfer is

$$\sigma_{0,\mathrm{loop}} = \frac{\mu_{A\chi}^2}{A^2 \pi} \left(\frac{\alpha_{em} Z}{3\pi} \, \frac{g'^2 q_\chi q_l}{m_{Z'}^2} \log\left(\frac{m_\mu^2}{m_\tau^2} \right) \right)^2 \ , \tag{4.5}$$

where $\mu_{A\chi}$ is the reduced mass of the nucleus and the dark matter particle χ and A, Z are the nucleon and proton numbers.

When discussing limits to the parameter space they distinguish two cases. For $q_l = q_\chi = 1$, experimental data favours the parameter region

$$\begin{aligned} 10\,\mathrm{GeV} \lessapprox m_{Z'} &\lessapprox 46\,\mathrm{GeV} \\ 2 \cdot 10^{-3} \lessapprox g' &\lessapprox 10^{-2} \ , \end{aligned} \tag{4.6}$$

leaving possible dark matter masses in the range $(5-23)$GeV. For $q_l = 1, q_\chi = 1/6$ no further restriction of the parameters can be found.

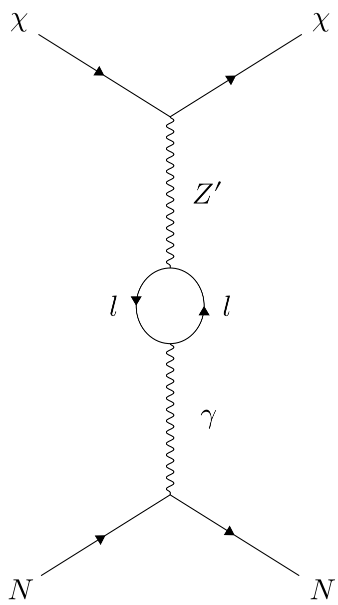


Figure 4.1: Direct detection loop diagram

5 Comparison

We now assume that only b, s, μ, τ and dark matter particles couple to Z', and dark matter couples exclusively to Z'. Considering flavour mixing, a tree level interaction of dark matter particles and nucleons is then possible. We take a look at this option in the first subchapter. Afterwards we compare the direct detection cross section of the tree level interaction with the one of the loop interaction presented in 4.2, respectively [6].

5.1 Tree Level Interaction Cross Section

By considering flavour mixing, a dark matter particle can interact with the strange and bottom parts of a down quark in a nucleus through the tree level diagram shown in figure 5.1. In terms of the operators in (3.2), the diagram corresponds to Q_{2bs}, Q_{2sb}. The related coefficients are

$$C_{2bs} = C^*_{2sb} = q_\chi \frac{Y_{Qb} Y^*_{Qs}}{2m_Q^2} \ , \tag{5.1}$$

for which [5] gives the approximation

$$\mathrm{Re}\,(C_{2bs}) \approx 8 \cdot 10^{-10}\,\mathrm{GeV}^{-2} \ . \tag{5.2}$$

This leads to the non-chiral coefficients (3.9)

$$\begin{aligned} K_{1,d} &= +\mathrm{Re}(V^*_{cd} V_{td} C_{2sb}) \\ K_{1,s} &= +\mathrm{Re}(V^*_{cs} V_{ts} C_{2sb}) \\ K_{3,d} &= -\mathrm{Re}(V^*_{cd} V_{td} C_{2sb}) \\ K_{3,s} &= -\mathrm{Re}(V^*_{cs} V_{ts} C_{2sb}) \ , \end{aligned} \tag{5.3}$$

and all other $K_{l,q}$ vanish. These are the coefficients of the spin-dependent interaction $R_{3,q} = (\bar{\chi}\gamma^\mu \chi)(\bar{q}\gamma_\mu \gamma_5 q)$ and the spin-independent interaction $R_{1,q} = (\bar{\chi}\gamma^\mu \chi)(\bar{q}\gamma_\mu q)$. We neglect the former one because it is various orders of magnitude smaller than the spin-independent cross section. Since nucleons consist of up and down quarks, we abolish the interaction with strange quarks as they are only available as sea quarks. We are left with the spin-independent vector interaction

$$\mathcal{L} = K_{1,d}(\bar{\chi}\gamma^{\mu}\chi)(\bar{d}\gamma_{\mu}d) \ . \tag{5.4}$$

As is explained in [7, Chapter 7], this operator basically counts the number of down quarks in the nucleus. So the nucleon cross section is

$$\sigma^{\mathrm{SI}}_{0,\mathrm{tree}} = \frac{\mu^2_{A\chi}}{A^2\pi}\left|ZC_p + (A-Z)C_n\right|^2 \ , \tag{5.5}$$

where $C_p = K_{1,d}$ and $C_n = 2K_{1,d}$ are the proton and neutron coefficient.

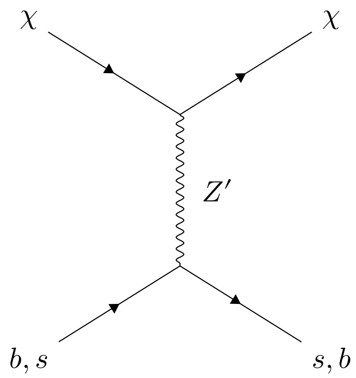


Figure 5.1: Tree level diagram for direct detection considering flavour mixing.

5.2 Results

We now compare the nucleon cross sections for the loop interaction $\sigma_{0,\mathrm{loop}}$ (see (4.5) and figure 4.1) and the tree level interaction $\sigma_{0,\mathrm{tree}}$ (see (5.5) and figure 5.1).

In the following we show various plots, in which we follow [6] with the distinction of the cases $q_\chi = 1$ and $q_\chi = 1/6$. Remember that for the former the dark matter mass cannot be much larger than 23 GeV, but for the latter no restriction is made. Therefore we choose different m_χ-axis scales in the plots. The shaded green area always corresponds to $\sigma_{0,\mathrm{loop}}$, whereas the lines display $\sigma_{0,\mathrm{tree}}$.

First we consider the bound (4.3) for $\sigma_{0,\mathrm{loop}}$. Figure 5.2 shows $\sigma_{0,\mathrm{loop}}$ (shaded green) within the bound and $\sigma_{0,\mathrm{tree}}$ with various choices for $C_{2bs} = C$ that are in accordance with (5.2). But even for quiet large values of C_{2bs}, the tree level cross section cannot reach the loop cross section. This is a strong hint that Altmannshofer et. al. correctly neglected the CKM mixing in their predictions for dark matter direct detection.

Since there are no restrictions concerning the imaginary part of C_{2bs}, we enlarged $\mathrm{Im}(C_{2bs})$ until we reached $\sigma_{0,\mathrm{loop}}$ in figure 5.3. But this was only achieved by imaginary parts that are two orders of magnitude larger than the real part of C_{2bs}, which we consider unreasonable.

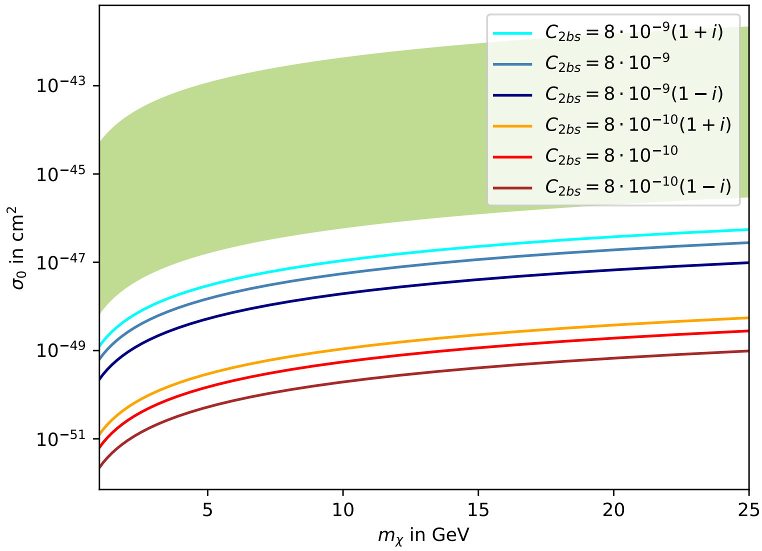
Second, we concentrate on the relic density condition in (4.4). Therefore we set $m_{Z'} = 2m_\chi$ but allow a 30 % tolerance. For $q_\chi = 1$, we set g' to the lower bound in (4.6): $g' = 2 \cdot 10^{-3}$, and for $q_\chi = 1/6$, we guess $g' = 10^{-2}$. Figure 5.4 shows the corresponding loop cross section again as shaded green area. Regarding the tree level cross section we choose C_{2bs} in strong accordance with (5.2). We find that there are regions for m_χ, where the tree level cross section reaches the loop cross section. The shaded lines show the lower bound of these regions. At the lower end of the green area $m_{Z'} = 2m_\chi \cdot 1.3$. The tables 5.1 and 5.2 show the obtained lower bounds (l.b.) for the dark matter mass, the resulting values for the Z' mass, and the ratio of the Z' mass and the coupling g'.

Table 5.1: $q_\chi = 1$. Bounds for the dark matter and Z' masses (in GeV) and ratio $m_{Z'}/g'$ (in GeV).

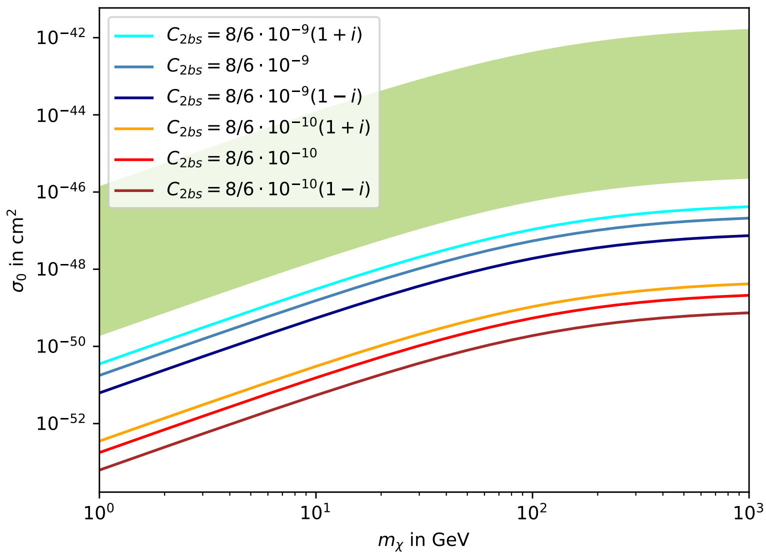
C_{2bs} in GeV^{-2}	l. b. for m_χ	$m_{Z'}$	$m_{Z'}/g'$
$8 \cdot 10^{-10}$	22	57	$29 \cdot 10^3$
$8 \cdot 10^{-10}(1+i)$	18	48	$24 \cdot 10^3$
$3 \cdot 10^{-9}$	11	29	$15 \cdot 10^3$

Table 5.2: $q_\chi = 1/6$. Bounds for the dark matter and Z' masses (in GeV) and ratio $m_{Z'}/g'$ (in GeV).

C_{2bs} in GeV^{-2}	l. b. for m_χ	$m_{Z'}$	$m_{Z'}/g'$
$1/68 \cdot 10^{-10}$	108	281	$28 \cdot 10^3$
$1/68 \cdot 10^{-10}(1+i)$	91	237	$24 \cdot 10^3$
$1/63 \cdot 10^{-9}$	56	146	$15 \cdot 10^3$

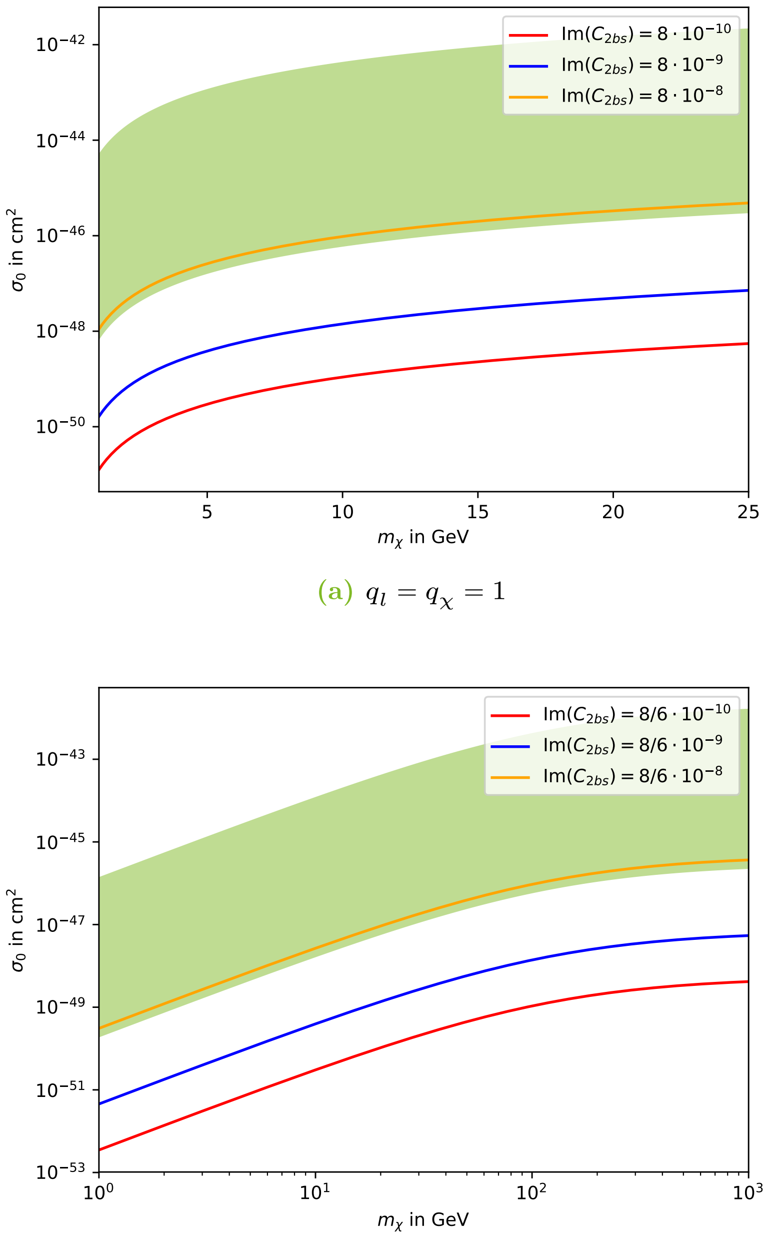


(a) $q_l = q_\chi = 1$



(b) $q_l = 1, q_\chi = 1/6$

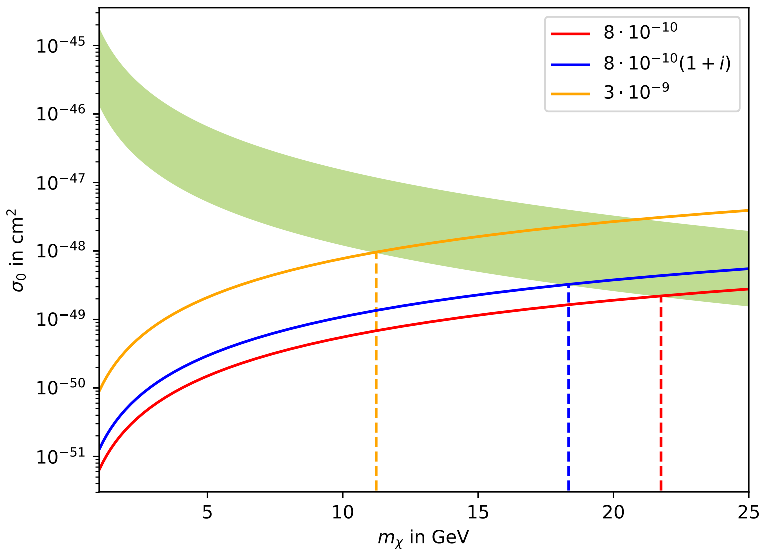
Figure 5.2: The shaded green area represents $\sigma_{0,\text{loop}}$ with the bounds in (4.3). The coloured lines show $\sigma_{0,\text{tree}}$ for different values of C_{2bs} (in GeV^{-2}).



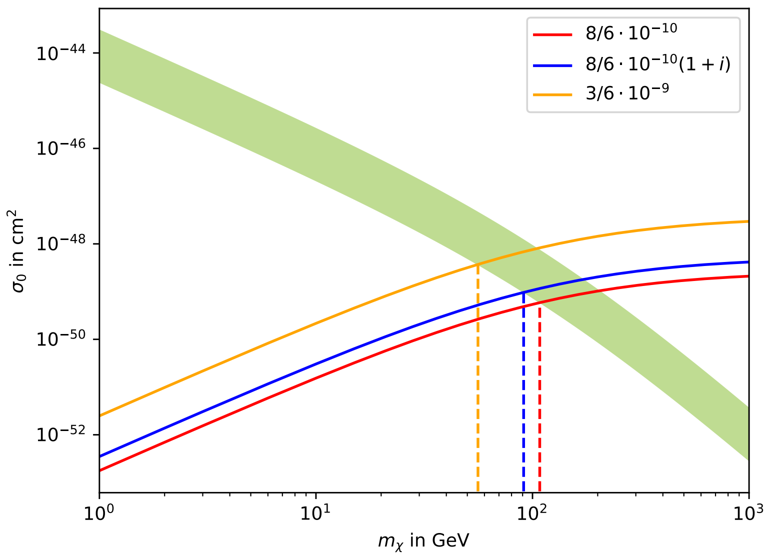
(a) $q_l = q_\chi = 1$

(b) $q_l = 1, q_\chi = 1/6$

Figure 5.3: Like 5.2, but the real part of C_{2bs} is fixed at $8 \cdot 10^{-10}\,\mathrm{GeV}^{-2}$ and only $\mathrm{Im}(C_{2bs})$ is varied.



(a) $q_l = q_\chi = 1, g' = 2 \cdot 10^{-3}$



(b) $q_l = 1, q_\chi = 1/6, g' = 10^{-2}$

Figure 5.4: The shaded green area represents the loop cross section $\sigma_{0,\text{loop}}$ at fixed coupling constant g' and $m_{Z'} = 2m_\chi$ with a $\pm 30\,\%$ tolerance.

6 Conclusion

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