# BioCro Equations

## Canopy Radiation

$$\delta = -23.5 \cdot \cos\left(\frac{360(D_j + 10)}{365}\right) \tag{1}$$

$$\cos(\theta) = \sin(\Omega)\sin(\delta) + \cos(\Omega)\cos(\delta)\cos(15 \cdot (t - t_{sn})) \tag{2}$$

$$I_{dir} = I_s \alpha^{\frac{(P/P_o)}{\cos(\theta)}} \tag{3}$$

$$I_{diff} = 0.5 \cdot I_s \cdot (1 - \alpha^{(P/P_o)/\cos(\theta)})\cos(\theta) \tag{4}$$

$$\frac{1}{2}\cos((15 \cdot t_{len}) = -\tan(\Omega)\tan(\delta) \tag{5}$$

$$t_{\text{len}} = \frac{2cos^{-1}(-\tan(\Omega)\tan(\delta))}{15} \tag{6}$$

$$t_{\text{down}} = 12 - t_{\text{len}}/2 \tag{7}$$

$$t_{\text{up}} = 12 + t_{\text{len}}/2 \tag{8}$$

what is the relevance of equation 5? Steve H's thesis contains the original, equation 6

## Weather Downscaling

$$T_{\text{mean}} = \frac{1}{2}(T_{\text{max}} + T_{\text{min}}) \tag{9}$$

$$T_{\text{range}} = T_{\text{max}} - T_{\text{min}} \tag{10}$$

$$T_{\text{excursion}} = \sin\left(2\pi\frac{h_r - 10}{24}\right) \tag{11}$$

$$T_{\text{air}} = T_{\text{mean}} + T_{\text{range}} \cdot T_{\text{excursion}} \tag{12}$$

## Canopy Radiation

combine or use clearly distinguished titles for different sections on canopy radiation; energy balance, etc

$$q = \frac{n_r}{n} \tag{13}$$

$$N_{\text{eff}} = \frac{\frac{(1-q)}{q}}{C_{ov}^2} \tag{14}$$

$$r^{\sim} = \frac{m_r}{n} \tag{15}$$

$$h = \frac{r^{\sim}}{q} \tag{16}$$

## C4 Photosynthesis test

From Collatz 1992 Coupled Photosynthesis-Stomata1 Conductance Model for Leaves of C4 Plants. Aust. J. Plant Physiol. 19 519-538

$$V_{\max} = \frac{V_{\max_0} Q_{10}^{\frac{T_{\text{leaf}}-25}{10}}}{(1+\exp(0.3(T_{\text{lower}} - T_{\text{leaf}})) \, (1+\exp(0.3(T_{\text{leaf}} - T_{\text{upper}}))} \tag{17}$$

$$R_d = \frac{R_0 Q_{10}^{\frac{T_{\text{leaf}}-25}{10}}}{1+\exp(1.3(T_{\text{leaf}} - 55))} \tag{18}$$

$$k_t = k Q_{10}^{\frac{T_{\text{leaf}}-25}{10}} \tag{19}$$

$$c_i = c_a - \frac{1.6 A_n P}{g_s} \tag{20}$$

$$A_{\text{net}} = A_{\text{gross}} - R_d \tag{21}$$

$$M = \min\left[\frac{(V_{\max}+\alpha_{\text{slope}} I_{\text{abs}}) \pm \sqrt{(V_{\max}+\alpha_{\text{slope}} I_{\text{abs}})^2 - 4(V_{\max}\alpha_{\text{slope}} I_{\text{abs}})\theta_{\text{curve}}}}{2\theta_{\text{curve}}}\right] \tag{22}$$

$$A_{\text{gross}} = \min\left[\frac{\left(M + k_t \cdot \frac{c_i}{P}\right) \pm \sqrt{\left(M + k_t \cdot \frac{c_i}{P}\right)^2 - \left(4 \cdot M \cdot k_t \cdot \frac{c_i}{P} \cdot \beta\right)}}{2 \cdot \beta}\right] \tag{23}$$

## C3 Photosynthesis

From Appendix 2 in Bernacchi et al 2003 Plant, Cell and Environment 26, 14191430 doi: 10.1046/j.0016-8025.2003.01050.x

$$A = (1 - \Gamma^*/c_i) \tag{24}$$

$$w_c = \frac{V_{c\max} c_i}{c_i + K_c(1 + O_a/K_0)} \tag{25}$$

$$w_j = \frac{J c_i}{4.5 c_i + 10.5 \Gamma^*} \tag{26}$$

$$\Gamma^* = exp(19.02 - 37.83/(R(T_{\text{leaf}} + 273.15))) \tag{27}$$

$$K_c = exp(38.05 - 36.38/R(T_{\text{leaf}} + 273.15)) \tag{28}$$

$$K_0 = exp(20.30 - 36.38/R(T_{\text{leaf}} + 273.15)) \tag{29}$$

$$V_{c,\max} = V_{c,\max @25C} \cdot \exp(26.35 - 65.33/R(T_{\text{leaf}} + 273.15)) \tag{30}$$

$$J = \frac{Q_2 + J_{\max,\text{T}} - \sqrt{(Q_2 + J_{\max,\text{T}})^2 - 4\Theta_{PSII} Q_2 J_{\max,T}}}{2\Theta_{\text{PSII}}} \tag{31}$$

$$J_{\max,T} = J_{\max @25C} \exp(17.57 - 43.54/(R(T_{\text{leaf}} + 273.15))) \tag{32}$$

$$\Theta_{\text{PSII}} = 0.76 + 0.018 T_{\text{leaf}} - 3.7 \cdot 10^{-4} T_{\text{leaf}}^2 \tag{33}$$

$$Q_2 = Q \cdot k \cdot \Phi_{\text{PSII,max}} \cdot \beta_\Phi \tag{34}$$

$$\Phi_{\text{PSII,max}} = 0.352 + 0.022 T_{\text{leaf}} - 3.4 \cdot 10^{-4} T_{\text{leaf}}^2 \tag{35}$$

From Appendix 1, Equations 7-9 in Long 1991 Plant, Cell and Environment 14, 729-739. doi:10.1111/j.1365-3040.1991.tb01439.x

$$c_i = 0.7c_a \left( \frac{1.6740 - 6.1294 \cdot 10^{-2} T_{\text{leaf}} + 1.1688 \cdot 10^{-3} T_{\text{leaf}}^2 - 8.8741 \cdot 10^{-6} T_{\text{leaf}}^3}{0.73547} \right) \tag{36}$$

$$c_i = 0.7c_a @25°C \tag{37}$$

$$O_i = 210 \left( \frac{4.7000 \cdot 10^{-2} - 1.3087 \cdot 10^{-3} T_{\text{leaf}} + 2.5603 \cdot 10^{-5} T_{\text{leaf}}^2 - 2.1441 \cdot 10^{-7} T_{\text{leaf}}^3}{2.6934 \cdot 10^{-2}} \right) \tag{38}$$

$$O_i = O_a @25°C \tag{39}$$

$$\phi = \frac{A_{\text{I=50}} - A_{\text{I=25}}}{25f} \tag{40}$$

is there a reason not to divide by the denominator when it is constant?

## Water Stress

$$h_s = \frac{e_l - \rho_{va}}{e_l} \cdot 100 \tag{41}$$

$$g_s = g_0 + g_1 \cdot A_{\text{gross}} \cdot \frac{h_s}{c_a} \tag{42}$$

$$\text{Four options for water stress model:} \tag{43}$$

$$g_{\text{ws, linear}} = \frac{W_s - W_p}{F_c - W_p} \tag{44}$$

$$g_{\text{ws, logistic}} = \frac{1}{1 + \exp\left(\frac{\frac{1}{2}(F_c - W_p) - W_s)}{\phi_i}\right)} \tag{45}$$

$$g_{\text{ws, exponential}} = \frac{1 - \exp\left(\frac{F_c - W_s}{F_c - W_p} + \frac{W_p}{1 - W_p}\right)}{0.631206} \tag{46}$$

$$g_{\text{ws, none}} = 1 \tag{47}$$

$$\text{Calculate } g_s \text{ and } A_n \text{ under water stress:} \tag{48}$$

$$g_s^{\text{water stress}} = g_{\text{ws},*} g_s \tag{49}$$

$$A_n^{\text{water stress}} = g_{\text{ws},*} A_n \tag{50}$$

should there be only one equation for Anet(?) is either correct? The first seems strange in that it implies water limited Anet equals Anet times humidity

## Canopy Energy Balance

$$J_a = 2 \cdot I_{\text{abs}} \cdot \left( \frac{1 - r - \tau}{1 - \tau} \right) \cdot \ell \tag{51}$$

$$L_b = (2.126 \cdot 10^{-5} + 1.48 \cdot 10^{-7} \cdot T_{\text{air}})/0.004 \cdot \sqrt{L_w/u_{\text{layer}}} \tag{52}$$

$$u_a = \frac{u \cdot 0.41}{log((u-d)/z_o)} \tag{53}$$

I can not reconcile units

$$g_a = \frac{(u_a^2/u_{\text{layer}}) \cdot L_b}{(u_a^2/u_{\text{layer}}) + L_b} \tag{54}$$

I can not reconcile units

$$\rho_v' = 610.78 \cdot \exp(\left(17.269 \cdot \frac{T_a}{T_a + 237.3}\right)) \tag{55}$$

$$\Delta\rho_{va} = \rho_v' \cdot \left(1 - \frac{h_s}{100}\right) \tag{56}$$

$$\gamma = \frac{\rho \cdot c_p}{\lambda} \tag{57}$$

$$s = 18 \cdot (2501 - 2.373 \cdot T_a) \cdot \left( \frac{\rho_v'}{8.314 \cdot (T_a + 273)^2} \right) \tag{58}$$

$$R_{lc} = 4\sigma \cdot (273 + T_{\text{air}})^3 \cdot \Delta T \tag{59}$$

$$\Phi_N = J_a - R_{lc} \tag{60}$$

$$\Delta T = T_{\text{leaf}} - T_{\text{air}} = \frac{\Phi_n \left(\frac{1}{g_a} + \frac{1}{g_c}\right)}{\lambda \left[ s + \gamma \left(1 + \frac{g_a}{g_c}\right) \right]} - \frac{\lambda \Delta\rho_{va}}{\lambda \left[ s + \gamma \left(1 + \frac{g_a}{g_c}\right) \right]} \tag{61}$$

should thermal conductivity be in this equation?

$$E = \frac{s \cdot \Phi_N + \lambda \cdot g_a \cdot \Delta\rho_{va}}{\lambda \cdot [s + \lambda \cdot (1 + g_a/g_c)]} \tag{62}$$

$$\mathbf{E_c} = \sum_{\text{layer}=1}^{\text{N}} (\mathbf{E}_{\text{sun}} \cdot l_{\text{sun}}) + (\mathbf{E}_{\text{shade}} \cdot l_{\text{shade}}) \tag{63}$$

$$\mathbf{E}_{\text{tot}} = \sum_{\text{day}=1}^{365} \sum_{\text{hr}=1}^{24} \mathbf{E_c} \tag{64}$$

## Sun / Shade Canopy

$$k = \frac{\sqrt{\chi^2 + \tan^2(\theta)}}{\chi + 1.744 \cdot [\chi + 1.183]^{-0.733}} \tag{65}$$

$$F_{\text{sun}} = \frac{1 - \exp[-k \cdot F_{\text{canopy}}]}{k} \tag{66}$$

$$F_{\text{shade}} = F_{\text{canopy}} - F_{\text{sun}} \tag{67}$$

$$I_{\text{sun}} = k \cdot I_{beam} + I_{\text{diff}} + I_{\text{scat}} \tag{68}$$

$$I_{\text{beam}} = I_{\text{dir}} \cos(\theta) \tag{69}$$

$$I_{\text{shade}} = I_{\text{diff}} + I_{\text{scat}} \tag{70}$$

$$I_{\text{diff}} = I_{\text{od}} \exp(-k_d F_{\text{canopy}}) \tag{71}$$

$$I_{\text{scat}} = I_{\text{beam}} \exp(-k\sqrt{\alpha_{\text{scat}}} F_{\text{canopy}}) - I_{\text{beam}} \exp(-k F_{\text{canopy}}) \tag{72}$$

(73)

## Total Canopy Assimilation

$$A_c = (A_{c,\text{sun}} \cdot F_{\text{sun}}) + (A_{c,\text{shade}} \cdot F_{\text{shade}}) \tag{74}$$

$$F_{\text{sun}} = \sum_{\text{layer}=1}^{\text{N}} l_{\text{sun}};\ l_{\text{sun}} = \frac{1 - e^{(-k \cdot F_{\text{sun}})}}{k} \tag{75}$$

$$F_{\text{shade}} = \sum_{\text{layer}=1}^{\text{N}} \ell_{\text{shade}};\ \ell_{\text{shade}} = F_{\text{sun}} - \ell_{\text{sun}} \tag{76}$$

$$F_{\text{canopy}} = F_{\text{sun}} + F_{\text{shade}} \tag{77}$$

$$A_c = \sum_{\text{layer}=1}^{\text{N}} (A_{c,\text{sun}} \cdot F_{\text{sun}}) + (A_{c,\text{shade}} \cdot F_{\text{shade}}) \tag{78}$$

$$A_{c,\text{tot}} = \sum_{\text{day}=1}^{365} \sum_{\text{hr}=1}^{24} A_c \tag{79}$$

$$g_c = \sum_{\text{layer}=1}^{\text{N}} (g_{s,\text{sun}} \cdot l_{\text{sun}}) + (g_{s,\text{shade}} \cdot l_{\text{shade}}) \tag{80}$$

$$g_{c,\text{tot}} = \sum_{\text{day}=1}^{365} \sum_{\text{hr}=1}^{24} g_c \tag{81}$$

is $\ell_{\text{sun}} \equiv l_{\text{sun}}$?

## Allocation

$$A_{\text{storage}} = |\min(0, \omega_{\text{storage}} \cdot k_{\text{storage}})| \tag{82}$$

$$A_{\text{total}} = A_{\text{leaf}} + A_{\text{stem}} + A_{\text{root}} + A_{\text{storage}} \tag{83}$$

$$\omega_{\text{leaf}} = \omega_{\text{leaf}} + (A_{\text{total}} \cdot k_{\text{leaf}}) \tag{84}$$

$$\omega_{\text{stem}} = \omega_{\text{stem}} + (A_{\text{total}} \cdot k_{\text{stem}}) \tag{85}$$

$$\omega_{\text{stroot}} = \omega_{\text{storage}} + (A_{\text{total}} \cdot k_{\text{storage}}) \tag{86}$$

$$\omega_{\text{root}} = \omega_{\text{root}} + (A_{\text{total}} \cdot k_{\text{root}}) \tag{87}$$

$$\Psi_{\text{adl}} < \Psi_{\text{pt}} \tag{88}$$

$$k_{\text{leaf}} = k_{\text{leaf}} \cdot k_{\text{mod}} \tag{89}$$

$$k_{\text{stem}} = k_{\text{stem}} \cdot k_{\text{mod}} \tag{90}$$

$$k_{\text{storage}} = k_{\text{storage}} \cdot k_{\text{mod}} \tag{91}$$

$$k_{\text{mod}} = (\Psi_{\text{adl}} - \Psi_{\text{pt}}) \cdot \Psi_g; 0 \leq k_{\text{mod}} \leq 1 \tag{92}$$

$$\Delta F_{\text{canopy}} = \frac{\omega_{\text{leaf}}}{Sp_{\text{leaf}}} \tag{93}$$

$$\Delta L_{\text{stem}} = \frac{\omega_{\text{stem}}}{Sp_{\text{stem}}} \tag{94}$$

$$\Delta L_{\text{sroot}} = \frac{\omega_{\text{sroot}}}{Sp_{\text{sroot}}} \tag{95}$$

$$\Delta L_{\text{storage}} = \frac{\omega_{\text{storage}}}{Sp_{\text{storage}}} \tag{96}$$

$$\text{Stem}_{\text{coppice}} = 0.95 - \omega_{\text{stem}} \tag{97}$$

should restrictions on values of $k$ in equations 82 and 92 be moved to the parameter definitions?

would it make sense to subscript values of $\omega$ with $t$, $t+1$ when updating them to avoid confusion?

$\Delta t$ is the timescale for updating biomass.; need to define $\Delta T$ for both daily and hourly

## Soil Evaporation

$$E_{\text{soil}} = \sum \frac{(\Psi_{\text{si}} - g \cdot z_i - \Psi_x)}{R_{\text{si}} + R_{\text{ri}}} \tag{98}$$

$$R_{\text{ri}} = R_r \cdot \frac{\sum L_i}{L_i} \tag{99}$$

$$\Psi_x = \sum \frac{(\Psi_{\text{si}} - q_w \cdot z_i)}{R_{\text{si}} + R_{\text{ri}}} \Big/ \sum \frac{1}{R_{\text{si}} + R_{\text{ri}}} \tag{100}$$

$$\Psi_L = \Psi_x - E \cdot R_L \tag{101}$$

$$E_d = \begin{cases} E_p, & \theta^* \geq \theta_1 \\ E_p\left(\frac{\theta - \theta_2}{\theta_1 - \theta_2}\right), & \theta_2 < \theta^* < \theta_1 \\ 0, & \theta^* \leq \theta_2 \end{cases} \tag{102}$$

$$\theta_{i+1} = \theta_i - \frac{E_i \cdot \theta_i}{\rho_w \cdot d_s} \tag{103}$$

$$g_{a,\text{soil}} = \frac{(2.126 \cdot 10^{-5}) + (1.48 \cdot 10^{-7}) \cdot T_{\text{soil}}}{\left(0.004 \cdot \sqrt{\frac{S_{size}}{u_{\text{soil}}}}\right)} \tag{104}$$

$$R_{lc,\text{soil}} = ((4\sigma) \cdot (273 + T_{\text{soil}})^3 \cdot \Delta T) \tag{105}$$

$$J_{a,\text{soil}} = 2 \cdot I_{\text{soil}} \cdot \left(\frac{1 - S_r - S_\tau}{1 - S_\tau}\right) \tag{106}$$

$$\Phi_{N,soil} = J_{a,soil} - R_{lc,soil} \tag{107}$$

$$E_{\text{soil}} = \frac{s \cdot \Phi_{N,soil} + \lambda \cdot g_{a,soil} \cdot \Delta\rho_{va}}{\lambda \cdot [s + \gamma]} \tag{108}$$

is “soil” subscript correct?

## Respiration

$$R_{\text{total}} = aA_n + b_{\text{stem}}\Delta\omega_{\text{stem}} + b_{\text{root}}\Delta\omega_{\text{root}} + b_{\text{storage}}\Delta\omega_{\text{storage}} \tag{109}$$

## Soil Energy Balance

$$HS_{\text{soil}} = HO_{\text{soil}} \cdot exp\left[\frac{h_{\text{soil}}}{46.97 \cdot (T_{\text{soil}} + 273.16)}\right] \tag{110}$$

$$HO_{\text{soil}} = 1.323 \cdot exp\left[\frac{17.27 \cdot T_{\text{soil}}}{273.3 + T_{\text{soil}}}\right] \Big/ T_{\text{soil}} + 273.16 \tag{111}$$

$$G_{\text{soil}} = -\lambda_{\text{soil}} \frac{\delta T}{\delta x} \tag{112}$$

$$G_{\text{soil}} = -\lambda_{\text{soil}} \cdot \left[\frac{T_2 - T_{\text{soil}}}{\Delta z}\right] + (T_{\text{soil}} - T_l) \cdot C \cdot \frac{\Delta z}{(2 \cdot \Delta t)} \tag{113}$$

should denominator in equation 112 be $\delta z$?

what is $t_l$?

$C$ in from equation 113 is undefined - is this the specific heat of soil?

# Definition of Terms

| Term | Units | Definition | Value | |
|---|---|---|---|---|
| $A_{\text{gross}}$ | $\mu mol\, mol^{-1}$ | Gross rate of $CO_2$ uptake per unit leaf area | - | |
| $A_{\text{net}}$ | $\mu mol\, mol^{-1}$ | Net rate of $CO_2$ uptake per unit leaf area | - | |
| $A_{net,\text{water stress}}$ | $\mu mol\, mol^{-1}$ | $A_{\text{net}}$ under water stress | | |
| $A_c$ | $\mu mol\, mol^{-1}$ | Net canopy rate of $CO_2$ uptake per unit ground area | - | |
| $A_{c,\text{tot}}$ | $g\, m^{-2}\, yr^{-1}$ | $A_c$ integrated over the course of a year | - | |
| $A_{c,\text{sun}}$ | $mol\, mol^{-1}$ | Net rate of $CO_2$ uptake per unit area sunlit leaves | - | |
| $A_{c,\text{shade}}$ | $mol\, m^{-2}\, s^{-1}$ | Net rate of $CO_2$ uptake per unit area shaded leaves | - | |
| $A$ | $\mu mol\, mol^{-1}$ | Predicted rate of $CO_2$ uptake | - | |
| $c_a$ | $\mu mol\, mol^{-1}$ | Atmospheric $CO_2$ concentration | 378 | |
| $C$ | $\text{J}^\circ\text{C}^{-1}\text{m}^-3$ | volumetric heat capacity | | |
| $a$ | Dimensionless | Coefficient for growth respiration | 0.2 | |
| $\alpha$ | dimensionless | Atmospheric transmittance | 0.85 | |
| $\alpha_{\text{slope}}$ | mol $\text{mol}^{-1}$ | The quantum yield of $CO_2$ uptake determined by the initial slope of the response of A versus $\text{I}_{abs}$ | 0.04 | |
| $b_{\text{leaf}}$ | Dimensionless | Coefficient for maintenance respiration for leaf | 0.03 | |
| $b_{\text{stem}}$ | Dimensionless | Coefficient for maintenance respiration for stem | 0.015 | |
| $b_{\text{root}}$ | Dimensionless | Coefficient for maintenance respiration for root | 0.01 | |
| $\beta$ | | $C_4$ curvature parameter | 0.93 | |
| $\beta_\Phi$ | % | Fraction of absorbed quanta reaching PSII | | |
| $c_i$ | $\mu mol\, mol^{-1}$ | Intercellular concentration of $O_2$ in air corrected for solubility relative to 25°C | | |
| $c_p$ | $J\, kg^{-1}\, K{-}1$ | Specific heat capacity of dry air | 1010 | |
| $C_{ov}$ | Dimensionless | Coefficient of Variation for probability of rain in each month | - | |
| $d_s$ | m | Soil depth | - | |
| $D_j$ | d | day of year | - | |
| $D_{\text{start}}$ | d | Day of year on which the sinusoidal temperature function is assumed to start | 45 | |
| $d$ | dimensionless | Zero plane displacement | 0.77 | |
| $\delta$ | degrees | Solar declination | - | |
| $e_l$ | kPa | Saturated water VPD in the leaf | - | is “saturated VPD” an oxymoron? |
| $E$ | $J\, mol^{-1}$ | Activation energy | $R_d$ = 66405 $V_{\max}$ = 6800 | |
| $E_d$ | | | | undefined from equation 102 |
| $E_i$ | | | | undefined from equation 103 |
| $E_c$ | $mmol\, m^{-2}\, s^{-1}$ | Instantaneous canopy evapo/transpiration rate | - | |
| $E_d$ | $g\, m^{-2}\, s^{-1}$ | Potential soil evaporation | - | |
| $E_l$ | $mmol\, m^{-2}\, s^{-1}$ | Evapo/transpiration rate at sunlit/shaded leaves in a canopy layer | - | |
| $E_p$ | $g\, m^{-2}\, s^{-1}$ | Actual soil evaporation | - | |
| $E_{R_d}$ | J $\text{mol}^{-1}$ | Activation energy of $\text{R}_d$ | - | |
| $E_{\text{tot}}$ | $mmol\, m^{-2}\, yr^{-1}$ | $\text{E}_c$ integrated over the course of a year | - | |
| $E_{V_{\max}}$ | J $\text{mol}^{-1}$ | Activation energy of $\text{V}_{\text{cmax}}$ | - | |
| $f$ | | fraction of light not absorbed by photosynthesis | 0.23 | |
| $f_{s,l}$ | | fraction of sunlit leaves at depth $l$ ($l$ is cumulative leaf area index from top) | | |
| $F_c$ | $m^3 m^{-3}$ | Field Capacity | | |
| $F_{\text{canopy}}$ | $m^2\, m^{-2}$ | Cumulative canopy leaf area index from top at depth | 9 | |
| $F_{\text{shade}}$ | $m^2\, m^{-2}$ | Canopy shaded leaf area index | - | |
| $F_{\text{sun}}$ | $m^2\, m^{-2}$ | Canopy sunlit leaf area index | - | |
| $F_{\text{sum}}$ | $m^2\, m^{-2}$ | Summed leaf area index from top of canopy to layer considered in calculation | - | |
| $G_{\text{soil}}$ | $W\, m^{-2}$ | Soil heat flux | - | |
| $g$ | $m\, s^{-2}$ | Gravitional constant | 9.8 | |

**Table 1 – continued from previous page**

| Term | Units | Definition | Value | |
|---|---|---|---|---|
| $g_a$ | $mmol\,m^{-2}\,s^{-1}$ | Leaf boundary layer conductance | - | |
| $g_c$ | $mmol\,m^{-2}\,s^{-1}$ | Canopy conductance of $CO_2$ | - | |
| $g_{\text{c, root}}$ | | | | undefined |
| $g_s$ | $mmol\,m^{-2}\,s^{-1}$ | Leaf stomatal conductance | - | |
| $g_0$ | dimensionless | Stomatal slope factor | 3 | |
| $g_1$ | dimensionless | Stomatal intercept factor | 0.08 | |
| $g_{s,\text{sun}}$ | $mmol\,m^{-2}\,s^{-1}$ | The sum of stomatal conductance of sunlit leaves | - | |
| $g_{s,\text{shade}}$ | $mmol\,m^{-2}\,s^{-1}$ | The sum of stomatal conductance of shaded leaves | - | |
| $g_{\text{ws}}$ | dimensionless | Species-specific water stress sensitivity factor | | |
| $g_{\text{ws*}}$ | dimensionless | water stress stomatal conductance factor; see equations 43 | | |
| $\gamma$ | Pa $K^{-1}$ | psychrometer constant | - | |
| $\Gamma*$ | $\mu$mol $\text{mol}^{-1}$ | $CO_2$ compensation point in the absence of dark respiration | | |
| $h_r$ | h | Hour of day | - | |
| $h_s$ | % | Relative humidity | - | |
| $h_{\text{canopy}}$ | m | Height of canopy | 5 | |
| $h_{ms}$ | m | Wind speed measurement height | 2 | |
| $h_{\text{layer}}$ | m | Height of canopy layer above ground | - | |
| I | $\mu mol\,m^{-2}\,s^{-1}$ | Photon flux | - | |
| $h$ | $mmday^{-1}$ | The amount of water received on a given rainy day | - | |
| $h_{\text{soil}}$ | m | Water pressure head | - | |
| $HO_{\text{soil}}$ | $kg\,m^{-3}$ | Saturated humidity of the air at the soil surface | - | |
| $HS_{\text{soil}}$ | $Kg\,m^{-3}$ | Humidity of the air at the soil surface | - | |
| $I_{\text{abs}}$ | $\mu mol\,m^{-2}\,s^{-1}$ | Photon flux absorbed by either sunlit or shaded leaves within a canopy layer | - | |
| $I_{\text{dir}}$ | $\mu mol\,m^{-2}\,s^{-1}$ | Photon flux in direct solar beam | - | |
| $I_{\text{diff}}$ | $\mu mol\,m^{-2}\,s^{-1}$ | Photon flux in diffuse radiation | - | |
| $I_{\text{total}}$ | $\mu mol\,m^{-2}\,s^{-1}$ | Total photon flux incident on canopy | - | |
| $I_s$ | $\mu mol\,m^{-2}\,s^{-1}$ | Solar constant, photon flux in a plane perpedicular to the solar beam above the atmosphere | 2600 | |
| $I_d$ | | | | undefined |
| $I_{\ell,d}$ | | | | undefined |
| $I_{\text{short}}$ | $\mu mol\,m^{-2}\,s^{-1}$ | Short wave radiation component of incident light | - | |
| $I_{\text{beam}}$ | | flux density of beam radiation on horizontal surface at top of canopy | | |
| $I_{\text{od}}$ | | flux density of diffuse radiation on horizontal surface at top of canopy | | |
| $I_{\text{soil}}$ | $\mu mol\,m^{-2}\,s^{-1}$ | Solar radiation incident upon soil surface | - | |
| $I_{\text{soil}}$ | $W\,m^{-2}$ | Solar radiation on soil | - | which units for $I_{\text{soil}}$ are correct? |
| $I_{\text{sun}}$ | $\mu mol\,m^{-2}\,s^{-1}$ | Mean I for leaves which receive direct solar radiation, i.e. are sunlit | - | |
| $I_{\text{shade}}$ | $\mu mol\,m^{-2}\,s^{-1}$ | Mean I for leaves shaded from direct solar radiation | - | |
| $I_{\text{scat}}$ | $\mu mol\,m^{-2}s^{-1}$ | Direct beam radiation scattered by surfaces within the canopy | - | |
| $J_{\text{a}}$ | $\mu mol\,m^{-2}\,s^{-1}$ | Total solar radiation absorbed by either sunlit or shaded leaves within a canopy layer | - | |
| $J_{\text{a, soil}}$ | $\mu mol\,m^{-2}\,s^{-1}$ | Total solar radiation absorbed by soil | - | |
| $k$ | dimensionless | Foliar absorption coefficient ($\alpha_\ell$ in Bernacchi 2003) | - | |
| $k_d$ | dimensionless | extinction coefficient for diffuse light | | |
| $K_c$ | $\mu mol\,mol^{-1}$ | Michaelis Menton constant for the carboxylation of RuBISCO | 460 | |
| $K_{CO_2}$ | $mol\,m^{-2}\,s^{-1}$ | Initial slope of photosynthetic $CO_2$ response | 0.7 | |
| $Kt$ | | $C_4$ slope factor | - | |
| $K_o$ | $mmol\,mol^{-1}$ | Michaelis Menton constant for oxygenation of RuBISCO | 330 | |
| $k_{\text{slope}}$ | Dimensionless | Initial slope of photosynthetic light response | 0.04 | |
| LN | $g\,m^{-2}$ | Leaf nitrogen concentration | - | |

**Table 1 – continued from previous page**

| Term | Units | Definition | Value | |
|---|---|---|---|---|
| $k_{\text{leaf}}$ | Dimensionless | Partitioning coefficient for leaf | - | |
| $k_{\text{stem}}$ | Dimensionless | Partitioning coefficient for stem | - | |
| $k_{\text{sroot}}$ | Dimensionless | Partitioning coefficient for storage root | - | |
| $k_t$ | | temperature-dependent pseudo-first order rate constant with respect to $P_i$ (Collatz 1992) | | |
| $k_{\text{froot}}$ | Dimensionless | Partitioning coefficient for fine root | - | |
| $k_{\text{stroot}}$ | Dimensionless | Partitioning coefficient for structural root | - | |
| $\ell$ | | | | undefined from Ja: equation 51 |
| $\ell_{\text{sun}}$ | | | | undefined from equation **??** |
| $l_{\text{sun}}$ | | | | undefined from equation 75 |
| $L_i$ | $cm\,cm^{-3}$ | Root density of ith zone | - | |
| $L_w$ | m | Leaf width in the direction of the wind | 0.04 | |
| $\Delta L_{\text{stem}}$ | | | | undefined |
| $\Delta L_{\text{sroot}}$ | | | | undefined |
| $\lambda$ | MJ/Kg | Latent heat of vapourisation | - | |
| $\lambda_{\text{soil}}$ | $W/(m^{\circ}C)$ | Thermal conductivity for the soil surface | - | |
| $M$ | | | 22 | what is$M$? |
| $m_r$ | mm month$^{-1}$ | monthly precipitation rate | | |
| $N_{\text{eff}}$ | days/mo | effective length of rainy period | | check units with equation 14 |
| $n$ | day | The number of days in a month | 29, 30, or 31 | |
| $nr$ | day | The number of rainy days in a month | - | |
| $O_a$ | $mmol\,mol^{-1}$ | Atmospheric $O_2$ concentration | 210 | is this corected to 25C like $O_i$? |
| $O_i$ | $mmol\,m^{-2}\,s^{-1}$ | Intercellular concentration of $O_2$ in air corrected for solubility relative to 25°C | - | |
| $\omega_{\text{leaf}}$ | g | Leaf biomass | - | |
| $\omega_{\text{stem}}$ | g | Stem biomass | - | |
| $\omega_{\text{sroot}}$ | g | Biomass of storage root | - | |
| $\omega_{\text{froot}}$ | g | Biomass of fine root | - | |
| $\omega_{\text{stroot}}$ | g | Biomass of structural root | - | |
| $\omega_{\text{storage}}$ | g | Biomass of storage | - | |
| $\Omega$ | degrees | Latitude | - | |
| $P$ | kPa | Atmospheric pressure | | |
| $P_o$ | kPa | Standard atmospheric pressure at sea level | 101.324 | |
| $P_s$ | kPa | Leaf surface partial pressure of $CO_2$ | - | |
| $\Psi_{\text{g}}$ | | | | undefined |
| $\Psi_l$ | MPa | Leaf water potential | - | |
| $\Psi_{\text{L}}$ | | | | undefined |
| $\Psi_t$ | MPa | Threshold leaf water potential for decreasing gs | - | |
| $\Phi_{\text{N}}$ | W m$^{-2}$ | Net radiation | - | |
| $\Phi_{N,soil}$ | W m$^{-2}$ | Net radiation at soil surface | - | |
| $\phi_i$ | | coefficient which controls spread of logistic function used to calculate water stress factor in 43 | | |
| $\Psi_{\text{adl}}$ | MPa | Average daily plant water potential | - | |
| $\Psi_{\text{pt}}$ | MPa | Threshold water potential | - | |
| $\Psi_{\text{si}}$ | MPa | Soil water potential of the ith layer | - | |
| $\Psi_x$ | MPa | xylem water potential | - | |
| $q$ | Dimensionless | The probability that there is no rainfall | - | during a month? |
| $q_w$ | kg s$^{-1}$ | Flux of water | - | |
| $Q_{10}$ | dimensionless | Is the proportional rise in a parameter for a 10°C increase in temperature | 2 | |
| $r$ | dimensionless | Leaf reflection coefficient for total solar radiation | 0.2 | |
| $r^{\sim}$ | mm day$^{-1}$ | Mean daily rainfall in each month | - | |
| $R$ | $J\,k^{-1}\,mol^{-1}$ | Real gas constant | 8.314 | |
| $R_L$ | $m^3\,kg^{-1}\,s^{-1}$ | Leaf resistance | - | |
| $R_{\text{si}}$ | $m^3 kg^{-1} s^{-1}$ | Soil resistance of the ith zone | - | |
| $R_{\text{ri}}$ | $M^3 kg^{-1} s^{-1}$ | root resistance of the ith zone | - | |
| $R_o$ | $mol\,m^{-2}\,s^{-1}$ | Dark respiration rate at $25^{\circ}C$ | 3 | |

**Table 1 – continued from previous page**

| Term | Units | Definition | Value | |
|---|---|---|---|---|
| $R_d$ | $mol\,m^{-2}\,s^{-1}$ | Dark respiration at a given temperature | - | |
| $R_{\text{lc}}$ | $mol\,m^{-2}\,s^{-1}$ | Longwave radiation | - | |
| $R_{lc,\text{soil}}$ | $mol\,m^{-2}\,s^{-1}$ | Soil longwave radiation | - | |
| $\rho_w$ | $kg\,m^{-3}$ | Density of water | 1000 | |
| $\rho_a$ | kPa | vapor pressure deficit in air | | is this distinct from $\Delta\rho_{\text{va}}$? |
| $\rho_a'$ | | | | undefined from equation 56 |
| $s$ | kPa $\text{K}^{-1}$ | Slope of saturated water vapor pressure change with respect to temperature (look up table) | - | also defined by equation 58; is one correct? |
| $s_p$ | dimensionless | Spectral imbalance | - | |
| $S_{\text{size}}$ | m | Average size of soil particles | 0.04 | |
| $S_r$ | Dimensionless | Soil reflectance | 0.2 | |
| $S_\tau$ | Dimensionless | Soil transmission | 0.01 | |
| $Sp_{\text{leaf}}$ | gram $\text{m}^{-2}$ | Specific leaf area | 50 | |
| $Sp_{\text{stem}}$ | gram $\text{m}^{-1}$ | Specific stem elongation factor | 60 | |
| $Sp_{\text{froot}}$ | gram $\text{m}^{-1}$ | Specific fine root elongation factor | 10 | |
| $Sp_{\text{stroot}}$ | gram $\text{m}^{-1}$ | Specific structural root elongation factor | 60 | |
| $\sigma$ | $\text{Wm}^{-2}\text{K}^{-4}$ | Stefan-Boltzmann constant | 5.67 $10^{-8}$ | |
| $t$ | h | Time of day | - | |
| $t_{\text{up}}$ | h | time of dawn | | |
| $t_{\text{down}}$ | h | time of dusk | | |
| $t_{\text{len}}$ | h | day length | - | is this a constant, 24? |
| $t_{\text{sn}}$ | h | Time of solar noon | 12 | |
| $T_{\text{leaf}}$ | °C | Leaf temperature | - | |
| $T_{\text{air}}$ | °C | Ambient air temperature | - | |
| $T_{\text{mean}}$ | °C | Daily mean $T_{\text{air}}$ | | |
| $T_{\text{range}}$ | °C | $\frac{T_{\text{air}}-T_{\text{mean}}}{T_{\text{range}}}$ | | |
| $T_{\text{excursion}}$ | fraction | Difference between current $T_{\text{mean}}$ | | |
| $T_{\text{soil}}$ | °C | Soil surface temperature | - | |
| $T_{\text{lower}}$ | °C | Lower T limitation on photosynthesis | | |
| $T_{\text{upper}}$ | °C | Upper T limitation on photosynthesis | | |
| $T_1$ | °C | Annual mean air temperature | 18 | |
| $T_2$ | °C | Annual range in air temperature | 2 | |
| $T_3$ | °C | Average daily range in air temperature | 7 | |
| $T_4$ | °C | Maximum daily range in air temperature | 7 | |
| $\Delta T$ | °C | Temperature difference between (leaf or soil) and air | | |
| $\tau$ | Dimensionless | Leaf transmittance coefficient | | |
| $\Theta_{\text{curve}}$ | dimensionless | Curvature parameter | - | |
| $\Theta^*$ | $kgm^{-3}$ | Actual volumetric water content | - | |
| $\Theta_1$ | $kg\,m^{-3}$ | The volumetric water content for maximizing Evaporation | | |
| $\Theta_2$ | $kg\,m^{-3}$ | The volumetric water content for wilting point | - | |
| $\Theta_i$ | $kg\,m^{-3}$ | The volumetric water content of the ith day | - | |
| $\Theta$ | degrees | Solar zenith angle | - | |
| $u$ | m $\text{s}^{-1}$ | Measured wind speed at known height (2m) | 2 | |
| $u_{\text{layer}}$ | m $\text{s}^{-1}$ | Wind speed in a given canopy layer | - | |
| $u_{\text{soil}}$ | m $\text{s}^{-1}$ | Wind speed at soil surface | - | |
| $v$ | | Saturated water vapour concentration | - | |
| $V_{\text{max}}$ | $mol\,m^{-2}\,s^{-1}$ | Maximum rubP saturated rate of carboxylation at a given temperature | - | |
| $V_{\text{max}_0}$ | $mol\,m^{-2}\,s^{-1}$ | Maximum rubP saturated rate of carboxylation at a given temperature | - | |
| $V_{c,\text{max}_0}$ | $mol\,m^{-2}\,s^{-1}$ | Maximum rubP saturated rate of carboxylation at $25°C$ | 39 | |
| VPD | kPa | Leaf-air water vapour pressure deficit | - | |
| $V_T$ | $mol\,m^{-2}\,s^{-1}$ | $V_{\text{max}}$ at current $T$ | | |
| $w_c$ | $mol\,m^{-2}\,s^{-1}$ | RuBISCO limited rate of photosynthesis | | units? |

**Table 1 – continued from previous page**

| Term | Units | Definition | Value |
|---|---|---|---|
| $w_c$ | $mol\, m^{-2}\, s^{-1}$ | RuBP limited rate of photosynthesis | |
| $W_p$ | $m^3 m^{-3}$ | Wilting point | |
| $W_s$ | $m^3 m^{-3}$ | Soil water content | |
| $z_o$ | m | Roughness length | 0.234 |
| $\chi$ | dimensionless | The ratio of horizontal:vertical projected area of leaves in the canopy segment | 1 |
| slope | $mol\, m^{-1}$ | Initial slope of photosynthetic $CO_2$ response | 0.7 |
| curve | dimensionless | Curvature parameter | 0.83 |
| $Z$ | m | Thickness of a soil layer | - |

units?