# KALMAN FILTER

It has two steps, prediction and updation.

***PREDICTION***:

It takes previous state of the model and predicts the current state using the following equation:

X(k)=A*X(k-1) + B*U

Where A is the state transition matrix and B is the control input.

U represents the acceleration.

State vector X is of the form [[x],[y],[vx],[vy]] so A and B are chosen such that the new state is predicted according to kinematic equations..

x(k) =x(k-1)+ vx*dt + 0.5*dt^2*ax

y(k) =y(k-1)+ yx*dt + 0.5*dt^2*ay

vx(k)=vx(k-1) + ax*dt

vy(k)=vy(k-1) + ay*dt

from these equations, we get

```
A=[[1 , 0 , dt , 0] ,
   [0 , 1 ,  0 , dt],
   [0 ,  0 ,  1  , 0],
   [0 , 0  , 0 ,  1]]
B=[[0.5dt^2   ,    0      ],
   [    0       , 0.5dt^2],
   [    dt       ,     0      ],
   [    0         ,     dt    ]]
```

B*U term comes out be zero as

Thus the prediction equation become X(k)=A*X(k-1)

The Kalman filter also predicts the covariance of the state estimation, which represents the uncertainty in the predicted state. It is given by the following equation:

$P_k^- = A*P_{k-1}*A^T + Q$

Where $P_k^-$ is the predicted state covariance matrix of the current state ,

$P_{k-1}$ is the state covariance matrix of the previous state and Q is the process noise covariance matrix.

Q=[[var(x), 0, SD(x)*SD(vx), 0 ],
[0 , var(y), 0, SD(y)*SD(vy)],
[SD(x)*SD(vx),0,var(vx),0 ],
[0, SD(y)*SD(vy),0,var(vy)]]

Q matrix is taken such that it takes into account the independency of (x,vx) And (y,vy) pair and dependency of x and vx respectively and similarly y and vy.

Q=[[var(x), 0, covariance(x,vx), 0 ],
[0 , var(y), 0,covariance(y,vy)],
[covariance(x,vx),0,var(vx),0 ],
[0 covariance(y,vy),0,var(vy)]]

Var=variance

sD=standard deviance

***UPDATION:***

Kalman gain is calculated as follows:

$K_k = P_k^- * H^T * S^{-1}$

Where

$S = H * P_k^- * H^T + R$

H=transformation matrix=[[1 0 0 0],[0 1 0 0]]

R=measurement noise covariance=[[var(x),0 ],[ 0,var(y)]]

R matrix is taken as a diagonal matrix because the measurement(x and y coordinates )are assumed to be independent.So we take the individual variances of x and y coordinates.

Var(x) and var(y) are the variances of the x and y gps coordinates given in the question respectively.

Updated estimate with measurement $z_k$:

$X_k = X_k^- + K_k(z_k - H * X_k^-)$

Where the term $(z_k - H * X_k^-)$ is the difference between gps coordinates and predicted coordinates.

Updated error covariance:

$P_k = (I - K_k * H)P_k^-$