Unique Paper Code	**:**	**32377911**
Name of the Paper	**:**	**Financial Statistics (DSE)**
Name of the Course	**:**	**B.Sc. (H) Statistics under CBCS**
Semester	**:**	**VI**
Duration	**:**	**3 hours**
Maximum Marks	**:**	**75 Marks**

Instructions for candidates:

1. Attempt ***five*** questions in all.
   Section **A** is compulsory.
   Choose **two** questions from each of the sections **B and C.**
2. Use of non programmable calculator is permitted.

Section A

Attempt any five parts. All parts carry equal marks. (5X5=25 marks)

1(a) As per an agreement, a company is required to make annual payments of \$7,500 for the maintenance of the building it occupies. These payments are due on January1, 2021, January 1, 2022 and January1, 2023. If the company wishes to cover these payments by investing a single sum in its bank account that pays 7.5% *p.a.* compounded annually, what sum must be invested on January1, 2018?

(b) If a bank offers sweep-account interest at a nominal rate of 6.5% *p.a.* compounded continuously, what is the effective interest rate per year?

(c) The cash flows of a 3-year project are:

- an initial outlay of £25,000
- regular expenditure of £2,000 *p.a.* during the first 2 years (assumed to be payable continuously)
- an income of £50,000 during the third year (assumed to be payable continuously)

Calculate the IRR of this project.

(d) Consider a strategy, a long call option with an exercise price of ₹ 100, another long call option with an exercise price of ₹110 on the same stock and with the same maturity. Obtain the payoff for this strategy considering different stock prices? When will this strategy be preferred?

(e) Let $X_t = \sum_{k=1}^{t} Z_k$ be a general random walk for $t = 1, 2, 3...$; $X_0 = 0$ and $Z_1, Z_2, ...$ are *i.i.d* random variables with $Var(Z_i) = 1;\ i = 1, 2, ...$ Calculate $Corr(X_s, X_t);\ s, t = 1, 2, ...$

(f) Consider a forward contract on a non-dividend paying share for which the stock price process is an Itô-process. If *'r'* is the risk free rate of interest, then find the SDE for the contract F_T maturing at time *T*.

(g) Obtain 'delta' of a European call option.

(h) Let W_t be a standard Wiener process. Define $X_t = e^{\lambda W_t - \frac{1}{2}\lambda^2 t}$ where λ is a constant. Show that X_t is a martingale with respect to F_t, the filtration associated with W_t.

Section B

2(a) A zero-coupon bond having face value 'F' pays the bondholder the amount 'F' when the bond matures. Assuming a continuously compounded interest rate of r%, find the present value 'P' of a zero-coupon bond with face value that matures at the end of 't' years. Calculate 'P' when $F = 100$, $r = 8\%$ and $t = 10$ years.

(b) Suppose that a person deposits money in a bank that pays interest at a nominal rate of 10% per year. How long will it take for his money to (i) double (ii) triple; and (iii) quadruple, if the interest is compounded continuously? $\left(6\frac{1}{2}, 6\right)$

3(a) The correlation (ρ) between two assets 'A' and 'B' is 0.1. The expected returns and standard deviations of returns are given in the following table:

Asset	Return (%)	Standard deviation (%)
A	10	15
B	18	30

(i) Find the proportions α and (1-α) of 'A' and 'B' respectively that defines a portfolio consisting of them such that it has minimum variance.

(ii) Find the expected return and the standard deviation of this portfolio.

(b) Consider a long forward contract to buy a stock which has a price of S_t at time t. Let K be the delivery price and T be the maturity date. Further Let $V\left(t, S_t\right)$ denote the value of the long forward contract at time t and $\tau = T - t$ is the time to maturity. Assume that 'r' is the interest rates during the time to maturity and is constant. If the stock pays dividends at discrete time points during the time to maturity or involves any costs whose current time t discounted total value is equal to D_t, then show that

$$V(t, S_t) = V_{K,T}(t, S_t) = S_t - D_t - Ke^{-r\tau}$$

$\left(6\frac{1}{2}, 6\right)$

4(a) Show that the price of a call option (American or European) is a convex function of the delivery price.

(b) Show that for a generalized binomial process with starting value X_0 and large t, the distribution of the random walk X_t can be approximated by a normal distribution.

$$\left(6\frac{1}{2}, 6\right)$$

Section C

5(a) Define a Wiener process. Let W_t be a standard Wiener process. Show that the process defined as $X_t = \frac{1}{\sqrt{c}} W_{ct};\ \ c > 0$ is also a Wiener process.

(b) Consider the process $dS_t = \mu dt + \sigma dW_t$ where W_t is a standard Wiener process. Let $Y_t = g(S_t)$ be a process given by $Y_t = \log(S_t)$. Find the SDE associated with Y_t, the mean and the variance of Y_t. $\left(6, 6\frac{1}{2}\right)$

6(a) Let $\{X_t; t \geq 0\}$ be an Itô-process given by $dX_t = \mu(t, X_t)dt + \sigma(t, X_t)dW_t$. Further let $Y_t = g(X_t)$ is a differentiable function of X_t. Then show that the SDE for $\{Y_t; t \geq 0\}$ is given by

$$dY_t = \left(\frac{dg(X_t)}{dX}\mu(t, X_t) + \frac{1}{2}\frac{d^2 g(X_t)}{dX^2}\sigma^2(t, X_t)\right)dt + \frac{dg(X_t)}{dX}\sigma(t, X_t)dW_t$$

(b) For a derivative based on a stock whose price process is an Itô-process, in usual notations, find a relationship between F, Δ, Γ and Θ $\left(6\frac{1}{2}, 6\right)$

7(a) Assume a call option with exercise price $K = 8$ at $T = 2$, which is to be priced at $t = 0$. The current stock price is 10 which is expected to increase or decrease by 20% in the first period. For the second period, the stock price is expected to increase or decrease by 10%. Assuming the risk free rate of interest to be 0, find the value of the call at $t = 0$.

(b) Let the current price of a stock is 50. If the risk free rate is 10% and volatility associated with the stock price movement is 20%, find the price of a put in 3 months if the exercise price is 47. Use put call parity to determine the price of a call on this stock. $\left(6\frac{1}{2}, 6\right)$

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<u>Instructions for candidates:</u>

1. Attempt *<u>five</u>* questions in all.
   Section A is compulsory.
   Choose **two** questions from each of the sections **B and C.**
2. Use of non programmable calculator is permitted.

Section A

Attempt any five parts. All parts carry equal marks. (5X5=25 marks)

1(a) Define (i) Nominal interest rate; and (ii) Effective interest rate. Find the corresponding effective rates for nominal rates: (a) 3% compounded monthly (b) 18% compounded quarterly

(b) The cash flows of a 3-year project are:

- an initial outlay of £25,000
- an income of £10,000 *p.a.* during the first year (assumed to be payable continuously)
- an income of £12,000 *p.a.* each during the next two year (assumed to be payable continuously)
- regular expenditure of £2,000 *p.a.* during the first 2 years (assumed to be payable continuously)
- a decommissioning expense of £25,000 at the end of the 3th year.

Calculate the IRR of this project.

(c) Explain the terms Arbitrage and Arbitrageurs with the help of example.

(d) Consider a strategy: Buy one stock, one long European put with delivery price *K*, one short European call with delivery price *K*. Calculate the payoff of the strategy and explain the risk in this strategy.

(e) A company's stock price is S_0 = ₹ 110 today. It will either rise or fall by 20% after one period. The risk free interest rate for one period is 10%. Calculate the risk neutral probability that makes expected return of the asset equal to risk free rate.

(f) Consider a forward contract on a non-dividend paying share for which the stock price process is an Itô-process. If the risk free rate of interest is 0%, then find the SDE for the contract F_T maturing at time *T*.

(g) Obtain 'delta' of a European put option.

(h) Let W_t be a standard Wiener process. Define $X_t = W_t^2 - t$. Show that X_t is a martingale with respect to F_t, the filtration associated with W_t.

Section B

2(a) An 8% bond with 18 years to maturity has a yield of 9% What is the price of this bond? Another bond with the same price, face value and maturity is there in the market but is a zero coupon bond. What would be the yield of this second bond?

(b) Consider two five- year bonds: one has a coupon of 9% and sells for $101.00; and the other has a coupon of 7% and sells for $93.20. Find the price of a 5-year zero-coupon bond.

(c) A person is required to make annual payments of $6,700 upkeep of his garden. These payments are due on 1st January 2019, 1st January 2020 and 1st January 2021. The person wishes to cover these payments by investing a single sum in a bank account that pays 4.5% p.a. compound. What sum must he invest on 1st January 2018? $\left(4,4,4\frac{1}{2}\right)$

3(a) Two stocks are available. The corresponding expected rates of return are $\overline{r}_1$ and $\overline{r}_2$. The respective variances and covariance are, σ_1^2, σ_2^2 and σ_{12} .Construct a portfolio consisting of the two stocks so that it has the minimum variance in the class of all portfolios consisting of these two assets. What is the mean rate of return of this portfolio? What would be the impact of addition of a risk-free asset to this portfolio?

(b)(i) Consider a short call option with delivery price K and option price C_0 in time t=0. Plot the corresponding payoff and profit functions.

(ii) Define the strategy called "straddle" and draw the corresponding payoff table. $\left(6\frac{1}{2},6\right)$

4(a) Show that the price of an American or European put option is the convex function of the delivery price.

(b) Let $\{X_t; t \geq 0\}$ denote a geometric random walk. Prove that the process $\{Y_t = \log(X_t); t \geq 0\}$ is a binomial process.

$\left(6\frac{1}{2},6\right)$

Section C

5(a) Define a Wiener process. Let W_t be a standard Wiener process. Show that the process defined as $X_t = W_{T+t} - W_t;\ T > 0$ is also a Wiener process.

(b) Let W_t be a standard Wiener process. Derive the value of $\int_0^t W_s^2 dW_s$.

$\left(6\frac{1}{2}, 6\right)$

6(a) Let $\{X_t; t \geq 0\}$ be an Itô-process given by $dX_t = \mu(t, X_t)dt + \sigma(t, X_t)dW_t$. Further let $Y_t = g(t, X_t)$ is a differentiable function of t and X_t . Then show that the SDE for $\{Y_t; t \geq 0\}$ is given by

$$dY_t = \left(\frac{\partial g(t, X_t)}{\partial X}\mu(t, X_t) + \frac{1}{2}\frac{\partial^2 g(t, X_t)}{\partial X^2}\sigma^2(t, X_t) + \frac{\partial g(t, X_t)}{\partial t}\right)dt + \frac{\partial g(t, X_t)}{\partial X}\sigma(t, X_t)dW_t$$

(b) For a call based on a stock whose price process is an Itô-process, show that, in usual notations,

$$rC = \Theta + rS\Delta + \frac{1}{2}\sigma^2 S^2 \Gamma$$

$\left(6\frac{1}{2}, 6\right)$

7(a) Assume a put option with exercise price K = 8 at T = 2, which is to be priced at t = 0. The current stock price is 10 which is expected to increase or decrease by 20% in the first period. For the second period, the stock price is expected to increase or decrease by 10%. Assuming the risk free rate of interest to be 0, find the value of the put at t = 0.

(b) Let the current price of a stock is 50. If the risk free rate is 10% and volatility associated with the stock price movement is 20%, find the price of a call in 3 months if the exercise price is 47. Use put call parity to determine the price of a put on this stock.

$\left(6\frac{1}{2}, 6\right)$

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   Section A is compulsory.
   Choose **two** questions from each of the sections **B and C.**
2. Use of non programmable calculator is permitted.

Section A

Attempt any five parts. All parts carry equal marks. (5X5=25 marks)

1(a) Describe the process of hedging associated with investments, with the help of example.

(b) If we put money into an account that pays interest at rate 'r' compounded annually, how many years does it take for the funds to double? What will be the time required if the compounding is continuous?

(c) An investor borrows money at an effective rate of interest of 10% *p.a* to invest in a 3-year project. The cash flows for the projectare:

- an initial outlay of £25,000
- an income of £20,000 *p.a.* during all the three year (assumed to be payable continuously)
- regular expenditure of £2,000 *p.a.* during the first 2 years (assumed to be payable continuously)
- a decommissioning expense of £25,000 at the end of the 3th year.

Calculate the NPV of this project.

(d) Consider a long forward contract on a 5 year bond currently trading at a price of ₹ 900. The delivery price is ₹910, the time to maturity of the forward contract is 1 year. The coupon payment of the bond of ₹ 60 occurs after 6 and 12 months. The continuously compounded annual interest rate for 6 and 12 months are 9% and 10% respectively. Obtain the value of forward contract.

(e) One of the most popular types of spread is a bull spread. A bull call-price spread can be made by buying a call option with a certain strike price and selling a call option on the same stock with higher exercise price. Both call options have same maturity date. Consider a European call with a delivery price of K_1 and second European call with delivery price of K_2. Draw the payoff for this strategy.

(f) Consider a forward contract on a non-dividend paying share for which the stock price process is an Itô-process. If 'r' is the risk free rate of interest, and $F_t = S_0 e^{r(T-t)}; 0 \le t < T$ is the price of a forward, then find the SDE of the contract maturing at time T.

(g) Obtain 'theta' of a European call option.

(h) Let W_t be a standard Wiener process. Define $X_t = W_t^3 - 3tW_t$. Show that X_t is a martingale with respect to F_t, the filtration associated with W_t.

Section B

2(a) Define 'Nominal interest rate' and 'Effective interest rate'. Find the corresponding effective rates for nominal rates : (a) 3% compounded quarterly (b) 18% compounded semi-annually.

(b) A person is required to make annual payments of \$8,200 for upkeep of his garden. These payments are due on 1st January 2019, 1st January 2020 and 1st January 2021. The person wishes to cover these payments by investing a single sum in a bank account that pays 5.5% pa compound. What sum must he invested on 1st January 2018?

(c) If you receive 5% interest compounded yearly, approximately how many years will it take for your money to quadruple? What is it if this interest were only 4%? $\left(4, 4\frac{1}{2}, 4\right)$

3(a) Three stocks are available. The corresponding expected rates of return are $\bar{r}_1$, $\bar{r}_2$ and $\bar{r}_3$. The respective variances are, σ_1^2, σ_2^2 and σ_3^2 and covariance are $\sigma_{ij} (i, j = 1, 2, 3)$ .Construct a portfolio consisting of the three stocks so that it has the minimum variance in the class of all portfolios consisting of these assets.

(b) Show that the values of a European call and a put option on a stock that pays a dividend yield with a time *t* discounted total value of D_t during the time to maturity $\tau = T - t$; and which have the same maturity date *T* and the same delivery price *K* are related as

$$C(t, S_t) = P(t, S_t) + S_t - D_t - Ke^{-r\tau}$$

where *r* denotes continuous interest rate; and $C(t, S_t)$ and $P(t, S_t)$ denote the time value of a call and a put with maturity date *T* respectively. $\left(6, 6\frac{1}{2}\right)$

4(a) Consider a long forward contract to buy a stock which has a price of S_t at time *t*. Let *K* be the delivery price and *T* be the maturity date. Further let $V(t, S_t)$ denote the value of the long forward contract at time *t* as a function of the current price $S_t = s$. The time to maturity is $\tau = T - t$. Assume that the interest rate is constant during the time to maturity. The stock does not pay any dividend and does not involve any costs during the time to maturity *T*. Then show that

$$V(t, S_t) = V_{K,T}(t, S_t) = S_t - Ke^{-r\tau} .$$

Hence obtain the forward price.

(b) If interest rates are constant during contract period, then show that the forward and the future prices are equal. $\left(6\frac{1}{2}, 6\right)$

Section C

5(a) Define a Wiener process. Let W_t be a standard Wiener process. Then the process defined as $X_t = \begin{cases} W_t; & t \le T \\ 2W_T - W_t; & t > T \end{cases}$ is also a Wiener process.

(b) Let W_t be a standard Wiener process. Derive the value of $E\left(\int_0^t W_s^3 dW_s\right)$.

$\left(6\frac{1}{2}, 6\right)$

6(a) Let $\{X_t; t \ge 0\}$ be an Itô-process given by $dX_t = \mu(t, X_t)dt + \sigma(t, X_t)dW_t$. Further let $Y_t = g(X_t)$ is a differentiable function of X_t. Then obtain the SDE for $\{Y_t; t \ge 0\}$

(b) For a put based on a stock whose price process is an Itô-process, show that, in usual notations,

$$rP = \Theta + rS\Delta + \frac{1}{2}\sigma^2 S^2 \Gamma$$

$\left(6\frac{1}{2}, 6\right)$

7(a) Assume a call option with exercise price K = 8 at T = 2, which is to be priced at t = 0. The current stock price is 10 which is expected to increase by 10% or decrease by 20% in the first period. For the second period, the stock price is expected to increase by 20% or decrease by 10%. Assuming the risk free rate of interest to be 0, find the value of the call at t = 0.

(b) Let the current price of a stock is 50. If the risk free rate is 10% and volatility associated with the stock price movement is 20%, find the price of a call in 3 months if the exercise price is 53. Use put call parity to determine the price of a put on this stock.

$\left(6\frac{1}{2}, 6\right)$

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2. Use of non programmable calculator is permitted.

Section A

Attempt any five parts. All parts carry equal marks. (5X5=25 marks)

1(a) Explain 'The Comparison Principle' with an example in the context of financial markets.

(b) If the amount 'P' is borrowed for 't' years at a nominal interest rate of 'r' per year compounded continuously, then show that the amount owed at time 't ' is 'Pe^{rt}'.

(c) An investor borrows money at an effective rate of interest of 10% *p.a.* to invest in a 5-year project. The cash flows for the project are:

- an initial outlay of £25,000
- an income of £20,000 *p.a.* each during the fourth and fifth years (assumed to be payable continuously)
- a decommissioning expense of £25,000 at the end of the 3th year.

Calculate the NPV of this project.

(d) Define 'intrinsic value' of a call and put option at time t. Under what conditions is an option said to be 'in-the-money', 'at-the-money'; and 'out-of-money'?

(e) Define the strategy called 'straddle' and draw the corresponding payoff table.

(f) Consider a forward contract on a non-dividend paying share for which the stock price process is an Itô-process. If 'r' is the risk free rate of interest, and $F_t = S_0 e^{r\tau}; 0 \le \tau, t < T$ is the price of a forward, then find the SDE of the forward contract maturing at time T.

(g) Obtain 'theta' of a European put option.

(h) Let W_t be a standard Wiener process. Define $X_t = W_t + 4t$. Check if X_t is a martingale with respect to F_t, the filtration associated with W_t.

Section B

2(a) Consider two possible sequences of end-of-year returns:

{20, 20, 20, 15, 10, 5} and {10, 10, 15, 20, 20, 20}.

Which sequence is preferable if the interest rate, compounded annually, were:

(a) 3%; (b) 5%; (c) 10%?

(b) A perpetual annuity or perpetuity entitles its holder to be paid the constant amount at the end of each of an infinite sequence of years. If the interest rate is compounded yearly, then what is the present value of such a cash flow sequence? If a person is to receive book grant of \$1,000 per year, at 8% interest, what is the present value of such a cash flow stream?

(c) If a bank offers sweep-account interest at a nominal rate of 6.5% *p.a.* compounded continuously, what is the effective interest rate per year? $\left(5, 5, 2\frac{1}{2}\right)$

3(a) Three assets are available for construction of a portfolio: two stocks with corresponding expected returns $\bar{r}_1$ and $\bar{r}_2$; the respective variances σ_1^2 and σ_2^2 and covariance σ_{12} and the third asset being a risk-free security with return $\bar{r}_3$ and variance σ_3^2.

(i) What should be the value of σ_3^2 so that the asset is a 'risk free' asset?

(ii) Construct a portfolio consisting of the three assets so that it has the minimum variance in the class of all portfolios consisting of these assets.

(b) An investor has a capital of ₹ 10,500 at his disposal to buy stocks whose current price is ₹100. Further put options on the same stock with a delivery price of *K*= ₹100 and a time to maturity of 1 year are quoted at a market price of ₹5 per contract. Consider two alternative investment plans:

Portfolio A : Buying 105 stocks

Portfolio B: Buying 100 stocks for 10,000 and buying 100 put options for 500.

Discuss the effect of portfolio insurance on portfolio value and return. $\left(6\frac{1}{2}, 6\right)$

4(a) For two European calls with the same maturity date T and delivery prices K_1 and K_2 such that $K_1 \le K_2$, show that at time $t \le T$

$$0 \le C_{K_1,T}\left(t, S_t\right) - C_{K_2,T}\left(t, S_t\right) \le \left(K_2 - K_1\right)e^{-r\tau}$$

where $\tau = T - t$ is the time to maturity and *r* denotes the risk-free rate of interest.

(b) Consider the following Binary one period model :
A stock with price S_0 ; and a European call option on the stock with strike price *K*. Determine price C_0 of the call option, the call being valued one period before expiration under the hedge strategy. $\left(6\frac{1}{2},6\right)$

Section C

5(a) Define a Wiener process. Let W_t be a standard Wiener process. Then the process defined as $X_t = tW_{\frac{1}{t}};\ \ t > 0$ is also a Wiener process.

(b) Let W_t be a standard Wiener process. Derive the value of $E\left(W_s^4\right)$.
$\left(6\frac{1}{2},6\right)$

6(a) Let $\{X_t; t \geq 0\}$ be an Itô-process given by $dX_t = \mu(t, X_t)dt + \sigma(t, X_t)dW_t$. Further let $Y_t = g(t, X_t)$ is a differentiable function of *t* and X_t. Then obtain the SDE for $\{Y_t; t \geq 0\}$.

(b) For a derivative based on a stock whose price process is an Itô-process, show that, in usual notations,

$$rF = \Theta + rS\Delta + \frac{1}{2}\sigma^2 S^2 \Gamma$$ $\left(6\frac{1}{2},6\right)$

7(a) Assume a put option with exercise price *K* = 8 at *T* = 2, which is to be priced at *t* = 0. The current stock price is 10 which is expected to increase by 10% or decrease by 20% in the first period. For the second period, the stock price is expected to increase by 20%or decrease by 10%. Assuming the risk free rate of interest to be 0, find the value of the call at *t* = 0.

(b) Let the current price of a stock is 50. If the risk free rate is 10% and volatility associated with the stock price movement is 20%, find the price of a put in 3 months if the exercise price is 53. Use put call parity to determine the price of a call on this stock.
$\left(6\frac{1}{2},6\right)$