Convex Optimization: SMAT430C
Practice Problem Set-2

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1. From Boyd's Book, Chapter 2 (Convex Set), Exercises: 2.23, 2.24(a), 2.28, 2.29, 2.30, 2.31 (a, b, d, e).

2. Let $f : \mathbb{R}^n \to \mathbb{R}^m$ and $g : \mathbb{R}^m \to \mathbb{R}^k$ be two maps such that f and g preserve convexity, i.e., if S is a convex subset of **dom** f (or **dom** g), then $f(S)$ (or $g(S)$) is convex set. Show that $gof : \mathbb{R}^n \to \mathbb{R}^k$ preserves convexity.

3. Express linear-fractional function as the composition of a perspective function with an affine function and hence show that the image of a convex set S under a linear fractional function is convex.

4. Let $S = \{2, 3, 4, 5, 9, 11\}$. Then find maximal element(s), mamimum element, minimal element(s) and minimal element for the relation $\preceq$:

   (a) $a \preceq b \Leftrightarrow a \leq b$

   (b) $a \preceq b \Leftrightarrow a|b$ (i.e., a divides b).

5. Let K be a proper cone with the generalized inequality $\preceq$. For any $a, b \in K$, define $a \preceq b \Leftrightarrow b - a \in K$. Show that

   (a) If $a \preceq b$ and $c \preceq d$, then $a + c \preceq b + d$.

   (b) If $a \preceq b$ and $b \preceq a$, then $a = b$.

   (c) If $a \preceq b$, $b \preceq c$, then $a \preceq c$.

   (d) $a \preceq a \; \forall \, a \in K$

   (e) If $a \preceq b$ and $\alpha \geq 0$, then $\alpha a \preceq \alpha b$

6. Let $T : \mathbb{R}^n \to \mathbb{R}$ be a linear functional which is bounded below. Show that $T = \mathbf{0}$ (i.e. T is zero transformation).

7. Write the converse of the separating hyperplane theorem. Is it true in general? Justify your answer.

8. Define strict separation of two sets. Can we always separate two sets strictly? Justify your answer.

9. Find $\text{Sup}\{a^T x : ||x||_2 \leq \epsilon\}$.

10. Suppose C and D are convex sets, with C open, and there exists an affine function f that is nonpositive on C and nonnegative on D. Then show that $C \cap D = \emptyset$.

Strict separation theorem: Let C and D be two closed convex sets in $\mathbb{R}^n$ with at least one of them is bounded. If $C \cap D = \emptyset$, then $\exists \; a \neq 0$ and b s.t. $a^T x < b \; \forall \; x \in C$ and $a^T x > b \; \forall \; x \in D$.