Relational Algebra and SQL

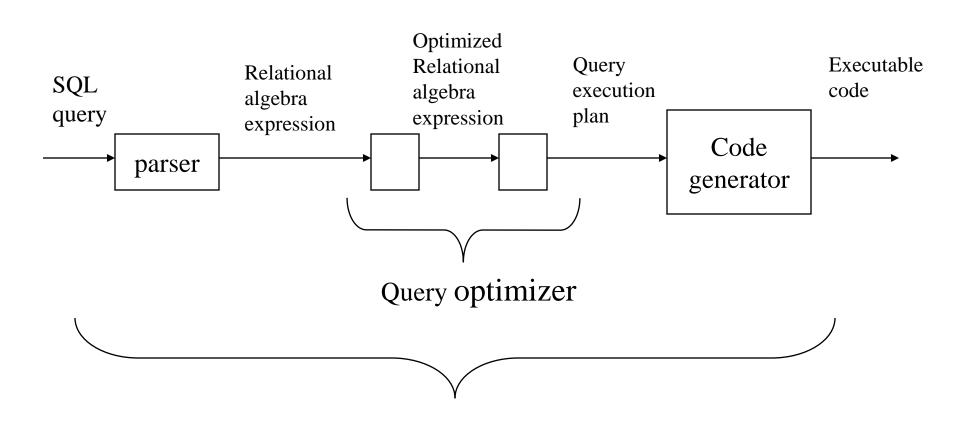
What is an Algebra?

- A language based on operators and a domain of values
- Operators map values taken from the domain into other domain values
- Hence, an expression involving operators and arguments produces a value in the domain
- When the domain is a set of all relations (and the operators are as described later), we get the *relational algebra*
- We refer to the expression as a *query* and the value produced as the *query result*

Relational Algebra

- *Domain*: set of relations
- *Basic operators*: select, project, union, set difference, Cartesian product
- Derived operators: set intersection, division, join
- *Procedural*: Relational expression specifies query by describing an algorithm (the sequence in which operators are applied) for determining the result of an expression

Relational Algebra in a DBMS



DBMS

Select Operator

• Produce table containing subset of rows of argument table satisfying condition

$$\sigma_{condition}$$
 relation

• Example:

Person

$$\sigma_{Hobby=\text{`stamps'}}$$
(Person)

Id	Name	Address	Hobby
1123	John	123 Main	stamps
1123	John	123 Main	coins
5556	Mary	7 Lake Dr	hiking
9876	Bart	5 Pine St	stamps
			<u>-</u>

Id	Name	Address	Hobby
1123	John	123 Main	stamps
9876	Bart	5 Pine St	stamps

Selection Condition

- Operators: $\langle, \leq, \geq, \rangle, =, \neq$
- Simple selection condition:
 - <attribute> operator <constant>
 - <attribute> operator <attribute>
- <condition> AND <condition>
- <condition> OR <condition>
- NOT < condition>

Selection Condition - Examples

- $\sigma_{Id>3000 \text{ Or } Hobby=\text{'hiking'}}$ (Person)
- $\sigma_{Id>3000 \text{ AND } Id < 3999}$ (Person)
- $\sigma_{\text{NOT}(Hobby='hiking')}$ (Person)
- $\sigma_{Hobby \neq 'hiking'}$ (Person)

Project Operator

 Produces table containing subset of columns of argument table

$$\Pi_{attribute\ list}(relation)$$

• Example:

Person

Id	Name	Address	Hobby
1123	John	123 Main	stamps
1123	John	123 Main	coins
5556	Mary	7 Lake Dr	hiking
9876	Bart	5 Pine St	stamps

 $\Pi_{Name, Hobby}(Person)$

Name	Hobby
John	stamps
John	coins
Mary	hiking
Bart	stamps

Project Operator

• Example:

Person

Id	Name	Address	Hobby
1123	John	123 Main	stamps
		123 Main	-
5556	Mary	7 Lake Dr	hiking
9876	Bart	5 Pine St	stamps

$\Pi_{Name,Address}(Person)$

Name	Address
John	123 Main
Mary	7 Lake Dr
Bart	5 Pine St

Result is a table (no duplicates)

Expressions

 $\Pi_{Id, Name} (\sigma_{Hobby='stamps'}) \circ (Person)$

Id	Name	Address	Hobby
1123	John	123 Main	stamps
1123	John	123 Main	coins
5556	Mary	7 Lake Dr	hiking
9876	Bart	5 Pine St	stamps

Id	Name
1123	John
9876	Bart

Result

Person

Set Operators

- Relation is a set of tuples => set operations should apply
- Result of combining two relations with a set operator is a relation => all its elements must be tuples having same structure
- Hence, scope of set operations limited to union compatible relations

Union Compatible Relations

- Two relations are union compatible if
 - Both have same number of columns
 - Names of attributes are the same in both
 - Attributes with the same name in both relations have the same domain
- Union compatible relations can be combined using *union*, *intersection*, and *set difference*

Example

Tables:

Person (SSN, Name, Address, Hobby)

Professor (Id, Name, Office, Phone)

are not union compatible. However

 Π_{Name} (Person) and Π_{Name} (Professor) are union compatible and

 Π_{Name} (Person) - Π_{Name} (Professor) makes sense.

Cartesian Product

- If R and S are two relations, $R \times S$ is the set of all concatenated tuples $\langle x, y \rangle$, where x is a tuple in R and y is a tuple in S
 - (R and S need not be union compatible)
- $R \times S$ is expensive to compute:
 - Factor of two in the size of each row
 - Quadratic in the number of rows

a b	c d	a b c d
x1 x2	y1 y2	x1 x2 y1 y2
x3 x4	y3 y4	x1 x2 y3 y4
		x3 x4 y1 y2
R	S	x1 x2 y1 y2 x1 x2 y3 y4 x3 x4 y1 y2 x3 x4 y3 y4
		$R \times S$

Renaming

- Result of expression evaluation is a relation
- Attributes of relation must have distinct names.
 This is not guaranteed with Cartesian product
 - e.g., suppose in previous example a and c have the same name
- Renaming operator tidies this up. To assign the names $A_1, A_2, ... A_n$ to the attributes of the n column relation produced by expression expr use $expr [A_1, A_2, ... A_n]$

Example

Transcript (StudId, CrsCode, Semester, Grade)

Teaching (ProfId, CrsCode, Semester)

 $\Pi_{StudId, CrsCode}$ (Transcript)[StudId, CrsCode1] \times $\Pi_{ProfId, CrsCode}$ (Teaching) [ProfId, Crscode2]

This is a relation with 4 attributes: StudId, CrsCode1, ProfId, CrsCode2

Derived Operation: Join

The expression:

$$\sigma_{join\text{-}condition'}(R \times S)$$

where join-condition' is a conjunction of terms:

$$A_i$$
 oper B_i

in which A_i is an attribute of R, B_i is an attribute of S, and oper is one of =, <, >, $\ge \ne$, \le , is referred to as the (theta) join of R and S and denoted:

$$R \bowtie_{join\text{-}condition} S$$

Where *join-condition* and *join-condition* are (roughly) the same ...

Join and Renaming

• **Problem**: *R* and *S* might have attributes with the same name – in which case the Cartesian product is not defined

• Solution:

- Rename attributes prior to forming the product and use new names in join-condition'.
- Common attribute names are qualified with relation names in the result of the join

Theta Join – Example

Output the names of all employees that earn more than their managers.

 $\Pi_{\text{Employee.}Name}$ (Employee \bowtie Manger Manger Manager)

The join yields a table with attributes:

Employee. Name, Employee. Id, Employee. Salary, MngrId Manager. Name, Manager. Id, Manager. Salary

Equijoin Join - Example

Equijoin: Join condition is a conjunction of equalities.

$$\Pi_{Name,CrsCode}(Student) \bowtie_{Id=StudId} \sigma_{Grade='A'}(Transcript))$$

Student

Id	Name	Addr	Status
111	John	• • • •	• • • •
222	Mary	• • • •	• • • •
333	Bill	• • • •	• • • •
444	Joe	• • • •	• • • •

Transcript

StudId	CrsCode	Sem	Grade
111	CSE305	S 00	В
222	CSE306	S 99	A
333	CSE304	F99	A

Mary CSE306 Bill CSE304 The equijoin is used very frequently since it combines related data in different relations.

Natural Join

- Special case of equijoin:
 - join condition equates all and only those attributes with the same name (condition doesn't have to be explicitly stated)
 - duplicate columns eliminated from the result

Transcript (StudId, CrsCode, Sem, Grade)
Teaching (ProfId, CrsCode, Sem)

```
Transcript Teaching =

\pi_{StudId, Transcript.CrsCode, Transcript.Sem, Grade, ProfId}

Transcript CrsCode = CrsCo
```

Natural Join (con't)

• More generally:

$$R \bowtie S = \pi_{attr-list} (\sigma_{join-cond} (R \times S))$$

where

 $attr-list = attributes (R) \cup attributes (S)$ (duplicates are eliminated) and join-cond has the form:

$$A_1 = A_1 \text{AND} \dots \text{AND} A_n = A_n$$

where

$$\{A_1 \dots A_n\} = attributes(R) \cap attributes(S)$$

Natural Join Example

• List all Id's of students who took at least two different courses:

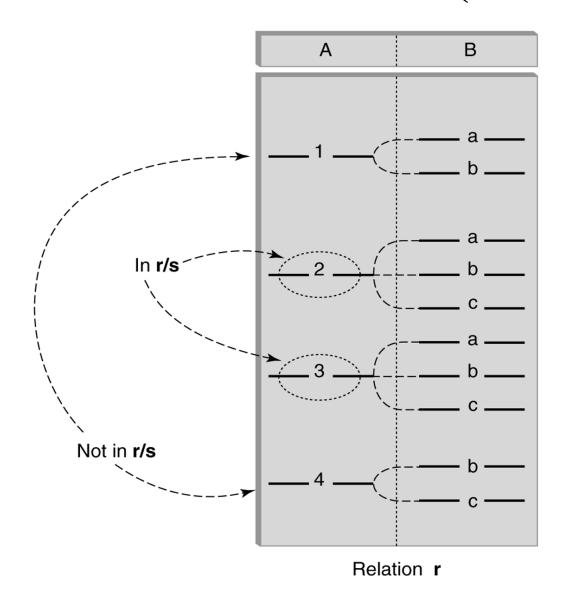
```
\Pi_{StudId} ( \sigma_{CrsCode \neq CrsCode2} (
Transcript \bowtie
Transcript [StudId, CrsCode2, Sem2, Grade2]))
```

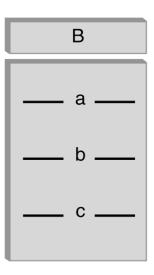
(don't join on *CrsCode*, *Sem*, and *Grade* attributes)

Division

- Goal: Produce the tuples in one relation, r, that match *all* tuples in another relation, s
 - $-r(A_1, ...A_n, B_1, ...B_m)$
 - $-s(B_1...B_m)$
 - -r/s, with attributes A_1 , ... A_n , is the set of all tuples < a > such that for every tuple < b > in s, < a, b > is in r
- Can be expressed in terms of projection, set difference, and cross-product

Division (con't)





Relation s

Division - Example

- List the Ids of students who have passed *all* courses that were taught in spring 2000
- <u>Numerator</u>: *StudId* and *CrsCode* for every course passed by every student
 - $-\pi_{StudId, CrsCode}(\sigma_{Grade \neq 'F'} (Transcript))$
- <u>Denominator</u>: *CrsCode* of all courses taught in spring 2000
 - $-\Pi_{CrsCode} (\sigma_{Semester='S2000'}, (Teaching))$
- Result is numerator/denominator

Schema for Student Registration System

Student (Id, Name, Addr, Status)
Professor (Id, Name, DeptId)
Course (DeptId, CrsCode, CrsName, Descr)
Transcript (StudId, CrsCode, Semester, Grade)
Teaching (ProfId, CrsCode, Semester)
Department (DeptId, Name)

Query Sublanguage of SQL

SELECT C.*CrsName*FROM Course C
WHERE C.*DeptId* = 'CS'

- Tuple variable C ranges over rows of Course.
- Evaluation strategy:
 - FROM clause produces Cartesian product of listed tables
 - WHERE clause assigns rows to C in sequence and produces table containing only rows satisfying condition
 - SELECT clause retains listed columns
- Equivalent to: $\pi_{CrsName} \sigma_{DeptId='CS'}$ (Course)

Join Queries

SELECT C.*CrsName*FROM Course C, Teaching T
WHERE C.*CrsCode*=T.*CrsCode* AND T.*Sem*='S2000'

- List CS courses taught in S2000
- Tuple variables clarify meaning.
- Join condition "C. CrsCode=T. CrsCode"
 - eliminates garbage
- Selection condition "T.Sem='S2000'"
 - eliminates irrelevant rows
- Equivalent (using natural join) to:

```
\pi_{CrsName}(Course \bowtie \sigma_{Sem=`S2000}, (Teaching))
\pi_{CrsName}(\sigma_{Sem=`S2000}, (Course \bowtie Teaching))
```

Correspondence Between SQL and Relational Algebra

```
SELECT C.CrsName
FROM Course C, Teaching T
WHERE C.CrsCode=T.CrsCode AND T.Sem='S2000'
```

Also equivalent to:

```
\pi_{CrsName} \ \sigma_{C\_CrsCode=T\_CrsCode\ AND\ Sem=`S2000'}
(Course [C_CrsCode, DeptId, CrsName, Desc] \times Teaching [ProfId, T_CrsCode, Sem])
```

This is the simple evaluation algorithm for SELECT. Relational algebra expressions are procedural. Which of the two equivalent expressions is more easily evaluated?

Self-join Queries

Find Ids of all professors who taught at least two courses in the same semester:

```
SELECT T1.ProfId
FROM Teaching T1, Teaching T2
WHERE T1.ProfId = T2.ProfId
AND T1.Semester = T2.Semester
AND T1.CrsCode <> T2.CrsCode
```

Tuple variables essential in this query!

Equivalent to:

```
\pi_{ProfId}(\sigma_{T1.CrsCode \neq T2.CrsCode}(\text{Teaching}[ProfId, T1.CrsCode, Sem])) Teaching[ProfId, T2.CrsCode, Sem]))
```

Duplicates

- Duplicate rows not allowed in a relation
- However, duplicate elimination from query result is costly and not automatically done; it must be explicitly requested:

```
SELECT DISTINCT .....
FROM .....
```

Use of Expressions

Equality and comparison operators apply to strings (based on lexical ordering)

WHERE S.Name < 'P'

Concatenate operator applies to strings

WHERE S.*Name* || '--' || S.*Address* =

Expressions can also be used in SELECT clause:

SELECT S.Name || '--' || S.Address AS NmAdd FROM Student S

Set Operators

- SQL provides UNION, EXCEPT (set difference), and INTERSECT for union compatible tables
- Example: Find all professors in the CS Department and all professors that have taught CS courses

```
(SELECT P.Name
FROM Professor P, Teaching T
WHERE P.Id=T.ProfId AND T.CrsCode LIKE 'CS%')
UNION
(SELECT P.Name
FROM Professor P
WHERE P.DeptId = 'CS')
```

Nested Queries

List all courses that were not taught in S2000

```
SELECT C.CrsName
FROM Course C
WHERE C.CrsCode NOT IN
(SELECT T.CrsCode ---subquery
FROM Teaching T
WHERE T.Sem = 'S2000')
```

Evaluation strategy: subquery evaluated once to produces set of courses taught in S2000. Each row (as C) tested against this set.

Correlated Nested Queries

Output a row <prof, dept> if prof has taught a course in dept.

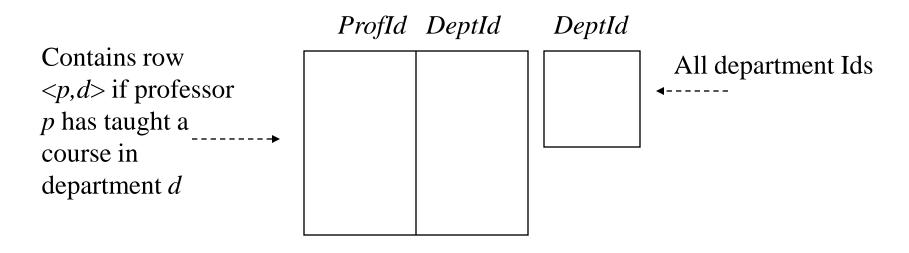
```
SELECT P.Name, D.Name --outer query
FROM Professor P, Department D
WHERE P.Id IN
-- set of all ProfId's who have taught a course in D.DeptId
(SELECT T.ProfId --subquery
FROM Teaching T, Course C
WHERE T.CrsCode=C.CrsCode AND
C.DeptId=D.DeptId --correlation
)
```

Correlated Nested Queries (con't)

- Tuple variables T and C are *local* to subquery
- Tuple variables P and D are *global* to subquery
- Correlation: subquery uses a global variable, D
- The value of D. DeptId parameterizes an evaluation of the subquery
- Subquery must (at least) be re-evaluated for each distinct value of D. DeptId
- Correlated queries can be expensive to evaluate

Division

- Query type: Find the subset of items in one set that are related to all items in another set
- Example: Find professors who have taught courses in *all* departments
 - Why does this involve division?



Division

- Strategy for implementing division in SQL:
 - Find set, A, of all departments in which a particular professor, p, has taught a course
 - Find set, B, of all departments
 - Output p if $A \supseteq B$, or, equivalently, if B-A is empty

Division – SQL Solution

```
SELECT P.Id
FROM Professor P
WHERE NOT EXISTS
  (SELECT D.DeptId -- set B of all dept Ids
   FROM Department D
      EXCEPT
  SELECT C.DeptId
                          -- set A of dept Ids of depts in
                          -- which P has taught a course
   FROM Teaching T, Course C
   WHERE T.ProfId=P.Id -- global variable
        AND T.CrsCode=C.CrsCode)
```

Aggregates

- Functions that operate on sets:
 - COUNT, SUM, AVG, MAX, MIN
- Produce numbers (not tables)
- Not part of relational algebra

SELECT COUNT(*)
FROM Professor P

SELECT MAX (Salary) FROM Employee E

Aggregates

Count the number of courses taught in S2

SELECT COUNT (T.CrsCode) FROM Teaching T WHERE T.Semester = 'S2'

But if multiple sections of same course are taught, use:

SELECT COUNT (DISTINCT T.CrsCode)
FROM Teaching T
WHERE T.Semester = 'S2'

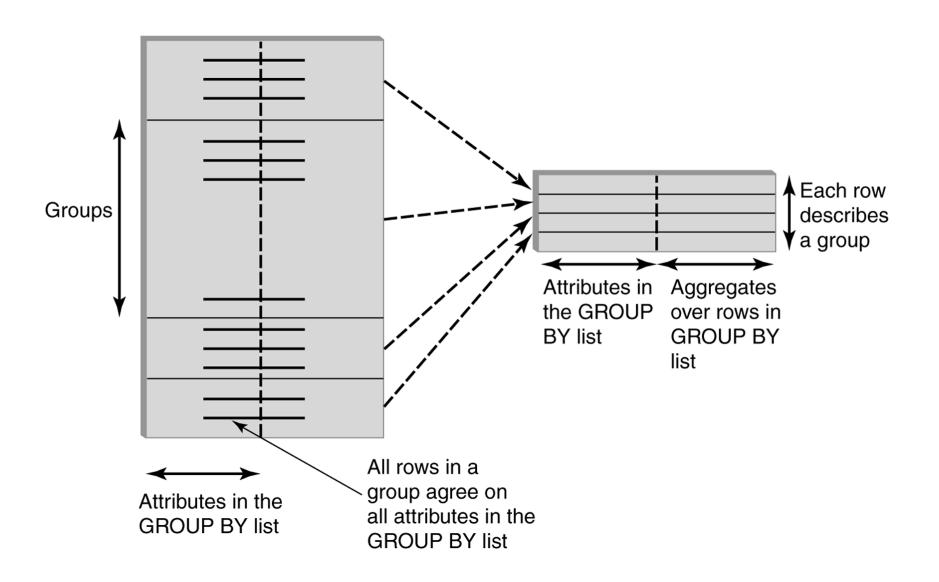
Aggregates: Proper and Improper Usage

SELECT COUNT (T. CrsCode), T. ProfId
..... --makes no sense (in the absence of
GROUP BY clause)

SELECT COUNT (*), AVG (T. Grade) --but this is OK

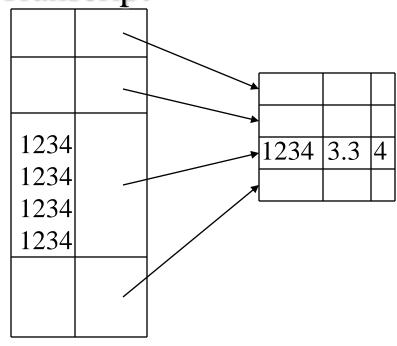
WHERE T.*Grade* > COUNT (SELECT -aggregate *cannot* be applied to result of SELECT statement

GROUP BY



GROUP BY - Example





Attributes:

- -student's Id
- -avg grade
- -number of courses

SELECT T. StudId, AVG(T. Grade), COUNT (*) FROM Transcript T GROUP BY T. StudId

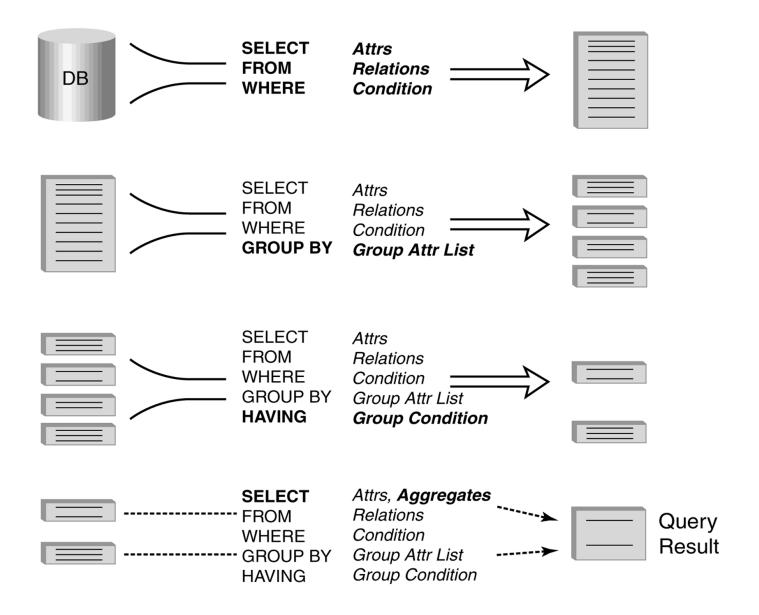
HAVING Clause

- Eliminates unwanted groups (analogous to WHERE clause)
- HAVING condition constructed from attributes of GROUP BY list and aggregates of attributes not in list

SELECT T.StudId, AVG(T.Grade) AS CumGpa,
COUNT (*) AS NumCrs
FROM Transcript T

WHERE T. CrsCode LIKE 'CS%'
GROUP BY T. StudId
HAVING AVG (T. Grade) > 3.5

Evaluation of GroupBy with Having



Example

 Output the name and address of all seniors on the Dean's List

```
SELECT S.Name, S.Address
FROM Student S, Transcript T
WHERE S.StudId = T.StudId AND S.Status = 'senior'
```

GROUP BY
$$< \frac{S.StudId}{S.Name}, S.Address$$
 -- right

HAVING AVG (T.Grade) > 3.5 AND SUM (T.Credit) > 90

ORDER BY Clause

Causes rows to be output in a specified order

SELECT T.StudId, COUNT (*) AS NumCrs, AVG(T.Grade) AS CumGpa FROM Transcript T WHERE T.CrsCode LIKE 'CS%' GROUP BY T.StudId HAVING AVG (T.Grade) > 3.5 ORDER BY DESC CumGpa, ASC StudId

Query Evaluation Strategy

- 1 Evaluate FROM: produces Cartesian product, A, of tables in FROM list
- 2 Evaluate WHERE: produces table, B, consisting of rows of A that satisfy WHERE condition
- 3 Evaluate GROUP BY: partitions B into groups that agree on attribute values in GROUP BY list
- 4 Evaluate HAVING: eliminates groups in B that do not satisfy HAVING condition
- 5 Evaluate SELECT: produces table C containing a row for each group. Attributes in SELECT list limited to those in GROUP BY list and aggregates over group
- 6 Evaluate ORDER BY: orders rows of C

Views

- Used as a relation, but rows are not physically stored.
 - The contents of a view is *computed* when it is used within an SQL statement
- View is the result of a SELECT statement over other views and base relations
- When used in an SQL statement, the view definition is substituted for the view name in the statement
 - SELECT statement can be nested in FROM clause

View - Example

CREATE VIEW CumGpa (StudId, Cum) AS SELECT T.StudId, AVG (T.Grade) FROM Transcript T GROUP BY T.StudId

SELECT S.Name, C.Cum FROM CumGpa C, Student S WHERE C.StudId = S.StudId AND C.Cum > 3.5

View Benefits

- Access Control: Users not granted access to base tables. Instead they are granted access to the view of the database appropriate to their needs.
 - External schema is composed of views.
 - View allows owner to provide SELECT access to a subset of columns (analogous to providing UPDATE and INSERT access to a subset of columns)

Views - Limiting Visibility

CREATE VIEW PartOfTranscript (StudId, CrsCode, Semester) AS
SELECT T. StudId, T.CrsCode, T.Semester -- limit columns
FROM Transcript T
WHERE T.Semester = 'S2000' -- limit rows

GRANT SELECT ON PartOfTranscript TO joe

This is analogous to:

GRANT UPDATE (Grade) ON Transcript TO joe

View Benefits (con't)

- *Customization*: Users need not see full complexity of database. View creates the illusion of a simpler database customized to the needs of a particular category of users
- A view is *similar in many ways to a subroutine* in standard programming
 - Can be used in multiple queries

Nulls

- Conditions: x op y (where op is <, >, <>, =, etc.) has value unknown (U) when either x or y is null
 - WHERE T.cost > T.price
- *Arithmetic expression*: x *op* y (where *op* is +, -, *, etc.) has value NULL if x or y is null
 - WHERE (T. price/T.cost) > 2
- *Aggregates*: COUNT counts nulls like any other value; other aggregates ignore nulls

SELECT COUNT (T.*CrsCode*), AVG (T.*Grade*) FROM Transcript T WHERE T.*StudId* = '1234'

Nulls (con't)

• WHERE clause uses a *three-valued logic* to filter rows. Portion of truth table:

<i>C1</i>	<i>C</i> 2	<i>C1</i> AND <i>C2</i>	<i>C1</i> OR <i>C2</i>
T	U	U	T
F	U	F	U
U	U	U	U

- Rows are discarded if WHERE condition is *false* or *unknown*
- Ex: WHERE T. CrsCode = 'CS305' AND T. Grade > 2.5

Modifying Tables - Insert

- Inserting a single row into a table
 - Attribute list can be omitted if it is the same as in CREATE TABLE (but do not omit it)
 - NULL and DEFAULT values can be specified

INSERT INTO Transcript(StudId, CrsCode, Semester, Grade) VALUES (12345, 'CSE305', 'S2000', NULL)

Bulk Insertion

Insert the rows output by a SELECT

```
CREATE TABLE DeansList (

StudId INTEGER,

Credits INTEGER,

CumGpa FLOAT,

PRIMARY KEY StudId )
```

```
INSERT INTO DeansList (StudId, Credits, CumGpa)
SELECT T.StudId, 3 * COUNT (*), AVG(T.Grade)
FROM Transcript T
GROUP BY T.StudId
HAVING AVG (T.Grade) > 3.5 AND COUNT(*) > 30
```

Modifying Tables - Delete

- Similar to SELECT except:
 - No project list in DELETE clause
 - No Cartesian product in FROM clause (only 1 table name)
 - Rows satisfying WHERE clause (general form, including subqueries, allowed) are deleted instead of output

DELETE FROM Transcript T
WHERE T. Grade IS NULL AND
T. Semester <> 'S2000'

Modifying Data - Update

- Updates rows in a single table
- All rows satisfying WHERE clause (general form, including subqueries, allowed) are updated

Updating Views

- Question: Since views look like tables to users, can they be updated?
- Answer: Yes a view update changes the underlying base table to produce the requested change to the view

CREATE VIEW CsReg (StudId, CrsCode, Semester) AS SELECT T.StudId, T. CrsCode, T.Semester FROM Transcript T
WHERE T.CrsCode LIKE 'CS%' AND T.Semester='S2000'

Updating Views - Problem 1

INSERT INTO CsReg (StudId, CrsCode, Semester) VALUES (1111, 'CSE305', 'S2000')

- **Question**: What value should be placed in attributes of underlying table that have been projected out (e.g., *Grade*)?
- **Answer**: NULL (assuming null allowed in the missing attribute) or DEFAULT

Updating Views - Problem 2

INSERT INTO CsReg (StudId, CrsCode, Semester) VALUES (1111, 'ECO105', 'S2000')

- **Problem**: New tuple not in view
- **Solution**: Allow insertion (assuming the WITH CHECK OPTION clause has not been appended to the CREATE VIEW statement)

Updating Views - Problem 3

• Update to the view might not *uniquely* specify the change to the base table(s) that results in the desired modification of the view

CREATE VIEW ProfDept (*PrName*, *DeName*) AS SELECT P.*Name*, D.*Name*FROM Professor P, Department D
WHERE P.*DeptId* = D.*DeptId*

Updating Views - Problem 3 (con't)

- Tuple <Smith, CS> can be deleted from ProfDept by:
 - Deleting row for Smith from Professor (but this is inappropriate if he is still at the University)
 - Deleting row for CS from Department (not what is intended)
 - Updating row for Smith in Professor by setting
 DeptId to null (seems like a good idea)

Updating Views - Restrictions

- Updatable views are restricted to those in which
 - No Cartesian product in FROM clause
 - no aggregates, GROUP BY, HAVING

– ...

```
For example, if we allowed:

CREATE VIEW AvgSalary (DeptId, Avg_Sal) AS

SELECT E.DeptId, AVG(E.Salary)

FROM Employee E

GROUP BY E.DeptId

then how do we handle:

UPDATE AvgSalary

SET Avg_Sal = 1.1 * Avg_Sal
```