



# Relational Algebra

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# Database Query Languages

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- Use "Campus" schema
- Given a database, ask questions, get data as answers
  - Ex: Get all students with GPA > 3.7 who applied to Berkeley and Stanford and nowhere else
  - Ex: Get all humanities departments at campuses in Florida with < 1000 applicants
  - Ex: Get the campus with highest average accept rate over the last five years
- Some questions are easy to pose, some are not
- Some questions are easy for DBMS to answer, some are not.
- "Query language" also used to update the database



# Relational Query Languages

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- Formal: relational algebra, relational calculus, Datalog
- Actual: SQL, Quel, Query-by-Example (QBE)
- In **ALL** languages, a query is executed over a set of relations, get single relation as the result



# Relational Algebra

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- Notation for describing queries in the relational model
- Relational model has concrete set of “standard” operations
- Operations are not “Turing Complete”
  - Not a defect, helps with query processing and optimization
  - FYI, a language is Turing Complete if it is powerful enough to implement any Turing machine. It's widely believed that Turing machines can do any calculation that can be performed by a modern computer program
- Start by introducing operations of relational algebra, SQL next
- Algebra applies to sets of tuples, i.e., relations
  - Commercial DBMS use different notation of relations which are multisets



# Relational Algebra

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- Construct new relations from old ones
  - Set of operators
  - Relations are operands
- Build progressively more complex expressions by applying operators to relations or to relational algebra expressions (which are relations as well)
- Query is an expression of relational algebra
  - First concrete example of a query language
- Four broad classes of operations
  - Set operations, selection operations, operations that combine data from two relations, rename operation



# Sample Relational Schema

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Movie (Title, Year, length, filmType,  
studioName, producerC#)

StarsIn (MovieTitle, MovieYear, StarName)

MovieStar (Name, address, gender, birthdate)

MovieExec (name, address, Cert#, netWorth)

Studio (Name, address, presC#)



# Basics

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- Operations of traditional relational algebra fall into four broad classes:
  1. Set operations
  2. Operations that remove parts of a relation
  3. Operations that combine tuples of two relations
  4. Renaming



# Set Operations

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Union (binary, commutative, associative)

- $R \cup S$

Intersection (binary, commutative, associative)

- $R \cap S$

Set Difference (binary)

- $R - S$

- Set of elements in R but not in S

- $R - S \neq S - R$  !!

- $R(A_1, A_2, \dots, A_n), S(B_1, B_2, \dots, B_n)$  must be union compatible

- R and S are of the same degree

- for each  $i$ ,  $\text{dom}(A_i) = \text{dom}(B_i)$

- Columns of R and S must be ordered so that order of attributes is same for both relations





# Example

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*R*

<i>name</i>	<i>address</i>	<i>gender</i>	<i>birthdate</i>
Carrie Fisher	123 Maple St., Hollywood	F	9/9/99
Mark Hamil	456 Oak Rd., Brentwood	M	8/8/88

*S*

<i>name</i>	<i>address</i>	<i>gender</i>	<i>birthdate</i>
Carrie Fisher	123 Maple St., Hollywood	F	9/9/99
Harrison Ford	789 Palm Dr., Beverly Hills	M	7/7/77



# Sample Operations

$R \cap S$	<i>name</i>	<i>address</i>	<i>gender</i>	<i>birthdate</i>
	Carrie Fisher	123 Maple St., Hollywood	F	9/9/99

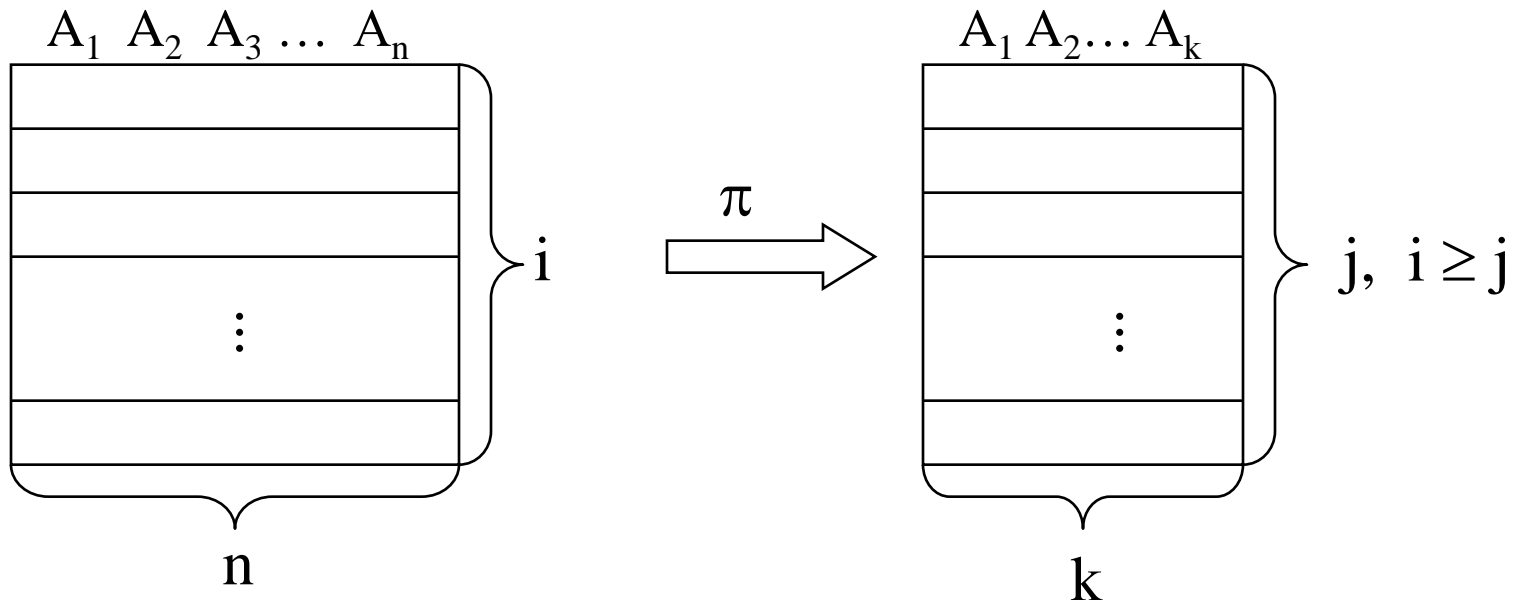
$R \cup S$	<i>name</i>	<i>address</i>	<i>gender</i>	<i>birthdate</i>
	Carrie Fisher	123 Maple St., Hollywood	F	9/9/99
	Harrison Ford	789 Palm Dr., Beverly Hills	M	7/7/77
	Mark Hamil	456 Oak Rd., Brentwood	M	8/8/88

$R - S$	<i>name</i>	<i>address</i>	<i>gender</i>	<i>birthdate</i>
	Mark Hamil	456 Oak Rd., Brentwood	M	8/8/88

# Relational Operator: Project

Project (unary)

- $\pi_{\langle \text{attr list} \rangle} (R)$
- $\langle \text{attr list} \rangle$  is a list of attributes (columns) from R only
- Ex:  $\pi_{\text{title, year, length}} (\text{Movie})$  “horizontal restriction”





# Project

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- PROJECT can produce many tuples with same value
- Relational algebra semantics says remove duplicates
- SQL does not -- one difference between formal and actual query languages



# Relational Operator: Select

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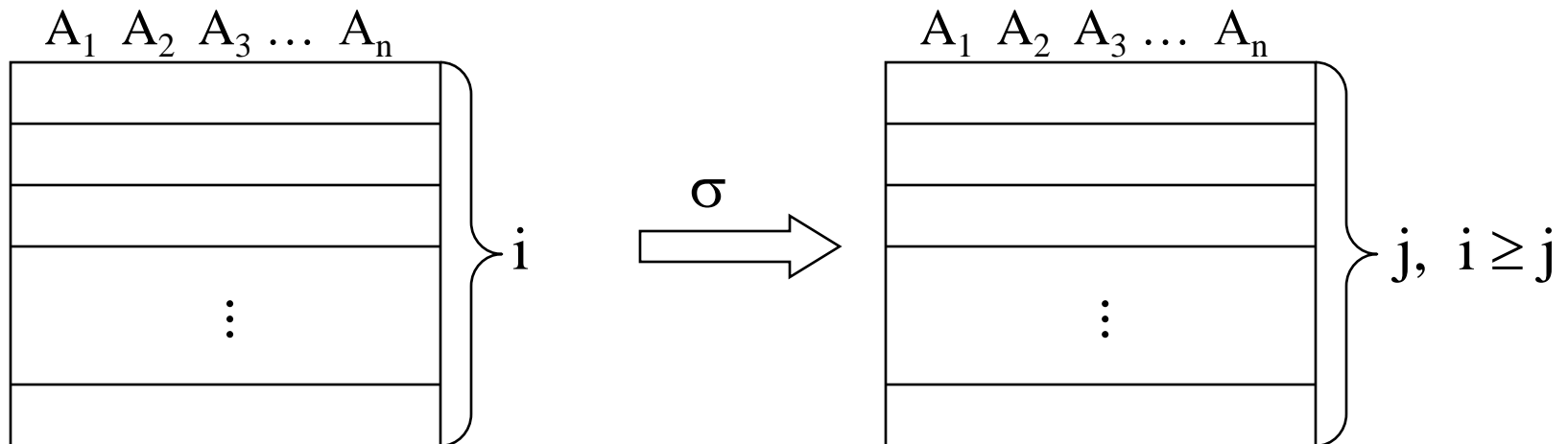
Select **or** Restrict (unary, commutative)

- $\sigma_{\langle \text{predicate} \rangle} (R)$
- $\langle \text{predicate} \rangle$  is a conditional expression of the type that we are familiar with from conventional programming languages
  - $\langle \text{attribute} \rangle \langle \text{op} \rangle \langle \text{attribute} \rangle$
  - $\langle \text{attribute} \rangle \langle \text{op} \rangle \langle \text{constant} \rangle$
  - attribute in  $R$
  - $\text{op} \in \{=, \neq, <, >, \leq, \dots, \text{AND}, \text{OR}\}$
- Ex:  $\sigma_{\text{length} \geq 100} (\text{Movie})$       vertical restriction"

# Pictorially

Movie	<i>title</i>	<i>year</i>	<i>length</i>	<i>filmType</i>
	Star Wars	1977	124	color
	Mighty Ducks	1991	104	color
	Wayne's World	1992	95	color

result set



# of selected tuples is referred to as the selectivity of the condition



# Cartesian Product

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Cartesian Product (binary, commutative, associative)

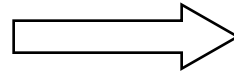
- $R \times S$
- Sets of all pairs that can be formed by choosing the first element of the pair to be any element of  $R$ , the second any element of  $S$
- Relation schema is union of schemas for  $R$  and  $S$
- Resulting schema may be ambiguous
  - Use  $R.A$  or  $S.A$  to disambiguate an attribute that occurs in both schemas

# Example

R		
	A	B
	1	2
	3	4

X

S			
	B	C	D
	2	5	6
	4	7	8
	9	10	11



A	R.B	S.B	C	D
1	2	2	5	6
1	2	4	7	8
1	2	9	10	11
3	4	2	5	6
3	4	4	7	8
3	4	9	10	11





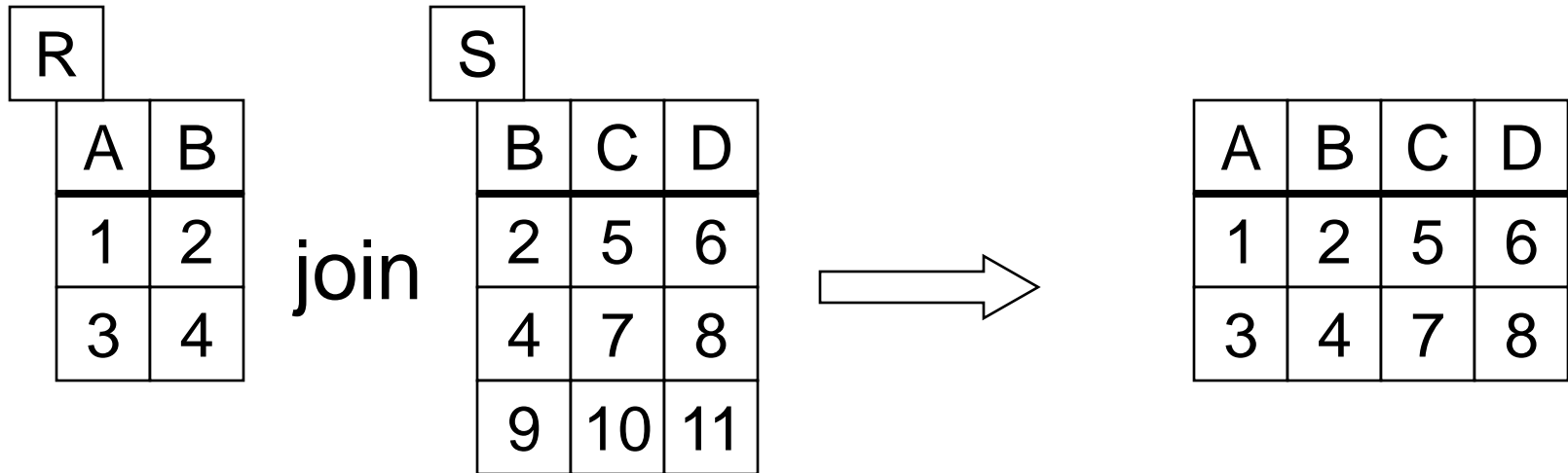
# Join Operations

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## Natural Join (binary)

- $R \text{ join } S$
- Match only those tuples from  $R$  and  $S$  that agree in whatever attributes are common to the schemas of  $R$  and  $S$ 
  - If  $r$  and  $s$  from  $r(R)$  and  $s(S)$  are successfully paired, result is called a *joined tuple*
- This join operation is the same we used in earlier section to recombine relations that had been projected onto two subsets of their attributes (e.g., as a result of a BCNF decomposition)

# Example



- Resulting schema has attributes from R, either R or S (i.e., joining attribute(s)), and S
- Tuples that fail to pair with any tuple of the other relation are called *dangling tuples*



# Join Operations

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## Theta Join (binary)

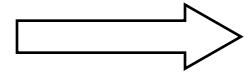
- $R \text{ join}_C S$ , where  $C$  is an arbitrary join condition
- Step 1: take the product of  $R$  and  $S$
- Step 2: Select from the product only those tuples that satisfy condition  $C$
- As with the product operation, the schema for the result is the union of the schemas of  $R$  and  $S$

# Example

U			
	A	B	C
	1	2	3
	6	7	8
	9	7	8

join<sub>A<D AND U.B≠V.B</sub>

V			
	B	C	D
	2	3	4
	2	3	5
	7	8	10



A	U.B	U.C	V.B	V.C	D
1	2	3	7	8	10



# Final Word on Join

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- DBMS often implements theta-join as basic operation
- Use of term "join" in implementation circles usually refers to theta-join or sometimes to cross-product