Quiz 2

Problem 1. A Dynamic Set

A dynamic set Q contains items, where each item x has an associated key *key*$[x]$. The dynamic set Q supports the following operations:

- $\text{INSERT}(x, Q)$: Insert item x into Q.
- $x \leftarrow \text{EXTRACT-OLDEST}(Q)$: Remove and return the oldest item $x \in Q$. (The oldest item is the one inserted least recently.)
- $x \leftarrow \text{FIND-MAX}(Q)$: Return (but do not remove) the item $x \in Q$ for which *key*$[x]$ is maximal.

Design a data structure for Q that can perform any sequence of n operations efficiently.

Problem 2. Great Minds Need Coffee

It is a beautiful autumn morning, and Professor Indyk has decided to walk from his apartment to MIT where he will give a lecture for 6.046. To prepare for this trip, the professor has gone online and downloaded a map of the n roads and m intersections in the Boston area. For each road e connecting two intersections, the map lists the length $w[e]$ of the road in meters. (Recall that in the Boston area, an arbitrarily large number of roads can meet at a single intersection, e.g., Davis Square.)

Like many great theorists, Professor Indyk cannot walk more than 1000 meters without sitting down and assuaging his caffeine addiction. (This is especially true before 9:30 A.M.) Fortunately, the Moonbucks Coffee Company has recently opened a large number of coffee shops throughout the Boston area. Let $B = \{b1, b2, \ldots, bk\}$ be the set of k intersections hosting Moonbucks coffee shops.

Thus, Professor Indyk is looking for a route from his apartment (located at one given intersection) to the Stata Center (located at another given intersection) in which he never travels more than 1000 meters without passing through an intersection with a coffee shop. Help the professor get to lecture on time by designing an efficient algorithm to find the shortest acceptable route from his apartment to the Stata Center.

Problem 3. Radio 107.9 FM

The Eccentric Motors (EM) corporation is about to roll out their 2005 year car model when they discover that many of their automobiles have faulty radio tuners that cannot access the highest frequencies in the FM spectrum. In particular, these defective radios cannot receive radio stations broadcast at 107.9 FM. Replacing the defective radios would delay the roll out of the 2005 model, which would cost the company millions of dollars in lost sales. Fortunately, not every city has a 107.9 FM station. Therefore, EM has decided to send the defective cars only to those dealerships that are not within range of any 107.9 FM station.

Eccentric Motors has given you a list of their n dealerships and has asked you to locate the dealerships that are out of range of all 107.9 FM stations. The list is organized so that dealership i is $D_x[i]$ miles east and $D_y[i]$ miles north of St. Louis, Missouri. (Negative values of $D_x[i]$ and $D_y[i]$ denote miles west and south, respectively.) You have contacted the FCC, which has given you the location of the k radio towers that broadcast at 107.9 FM. Tower j is located $T_x[j]$ miles east and $T_y[j]$ miles north of St. Louis, Missouri, and it broadcasts at a signal strength that allows the signal to be received within a radius of $T_r[j]$ miles from the tower. FCC regulations guarantee that no point is able to receive signals from two different 107.9 FM broadcast towers.

Design an efficient algorithm to locate the EM dealerships that are out of range of all 107.9 FM broadcast towers.

Problem 4. Isolating Kryptonite

Professor Luthor has obtained a meteorite fragment that contains high levels of a useful Kryptonite isotope. The fragment is a perfect cube *n* centimeters on each side. The professor would like to distribute this fragment to m secret laboratories for study in the hopes that the Kryptonite isotope can be synthesized in large quantities. The Kryptonite is not evenly distributed throughout the meteorite, however. The density of the Kryptonite at coordinate (i, j, k) is given by $D(i, j, k)$, that is, $D(i, j, k)$ gives the amount of isotope in a cubic centimeter with origin (i, j, k).

Professor Luthor has obtained access to a machine that will cut across the meteorite fragment at a given position, thereby dividing it into exactly two pieces. The machine only cuts at right angles (parallel to the x-, y-, or z-axis) and on centimeter boundaries. Thus, each cut results in two rectangular parallelepipeds with integral dimensions.

Design an efficient algorithm for dividing the meteorite into m pieces so as to maximize the minimum amount of Kryptonite contained in any piece. (Partial credit will be given for solving the analogous two-dimensional problem.)

Clarification: The machine can only be used to cut a single piece into two smaller pieces. That is, after cutting the meteorite into two pieces, you must separate the pieces; you can then perform different cuts on each piece. Also, you cannot glue pieces back together.

Problem 5. Finding Coolmail Users

Macrohard Corporation has decided to start a free email service known as Coolmail. Coolmail users choose their favorite k-digit number x as their user ID and get the email address x`@coolmail.com`. For instance, with $k = 9$ a user might chose 314159265 as her user ID and be given the address `314159265@coolmail.com`. Since many people find it hard to remember k-digit numbers, Coolmail provides the following helpful service. Whenever an email is sent to an address y`@coolmail.com`, where y is not a valid user ID, Coolmail finds a nearest valid user ID x, that is, an x that minimizes $|x - y|$, and sends the reply, "Sorry, but y`@coolmail.com` is not a valid address. Did you mean to type x`@coolmail.com`?"

It has come to the attention of the opportunistic Professor Ralsky that many Coolmail users are paying too much interest on their mortgages. He would therefore like to send every Coolmail user a message informing him or her that "You can ref1nance your m0rtgage at the amazing rate of only 3.49%!!" Unfortunately, the professor does not know the email addresses of all Coolmail users. Let $0 \leq x_1 < \cdots < x_n < 10^k$ be the user ID's of the n Coolmail users, where n is unknown to the professor. He has hired you as an MIT student intern to design an efficient algorithm to compute, with the help of Coolmail's automated reply service, the email addresses of all the Coolmail users. Minimize the worst-case number of emails required by your algorithm.