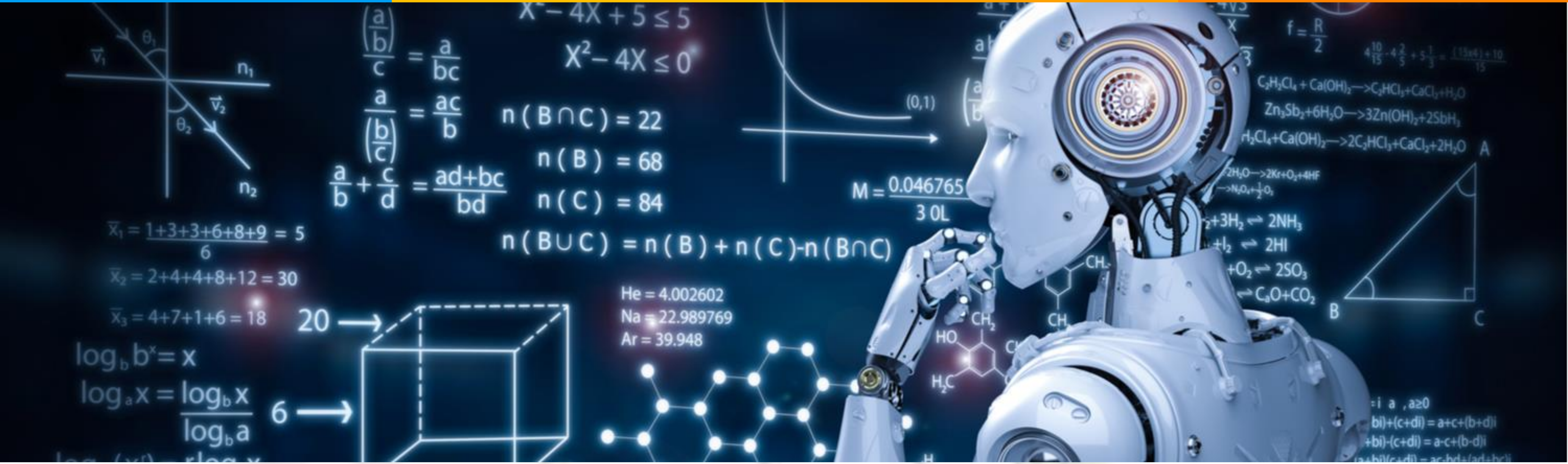
Duration: 60 min | Srikanth Shenoy | AML 5101 | M.E. AI & ML



Lecture 04 – Dot product and applications

Recap

- Geometric & algebraic meaning of a n-vector
- Formula for magnitude of a vector
- Vector is always column vector for convenience
- Scalar multiplication, vector addition, & subtraction
- Averaged vector is centroid vector
- Subtraction gives difference vector.
- Magnitude of difference vector is distance between vectors
- Word count vector
- Dot product – projection link
- Leads to single number for similarity in n-dimensions

Mandatory Homework for today

- Read:
  - To read entire chapter 1 from Stephen Boyd
    - Except linear combination
- Solve:
  - Work out a few problems at end of Chapter 1
- Incentive for solving the chapter end problems:
  - Your exam problems will be of this type

Approach

- Keep slide deck handy while reading book chapter
- Cursory look at minor topics skipped in lecture
  - E.g. Properties of vector addition, subtraction
  - Prove $(a+b)^T(c+d) = a^Tc + a^Td + b^Tc + b^Td$
- Understand the formulas intuitively
- Focus on examples in book and slide deck
  - Your exam problems will be like examples and chapter end problems

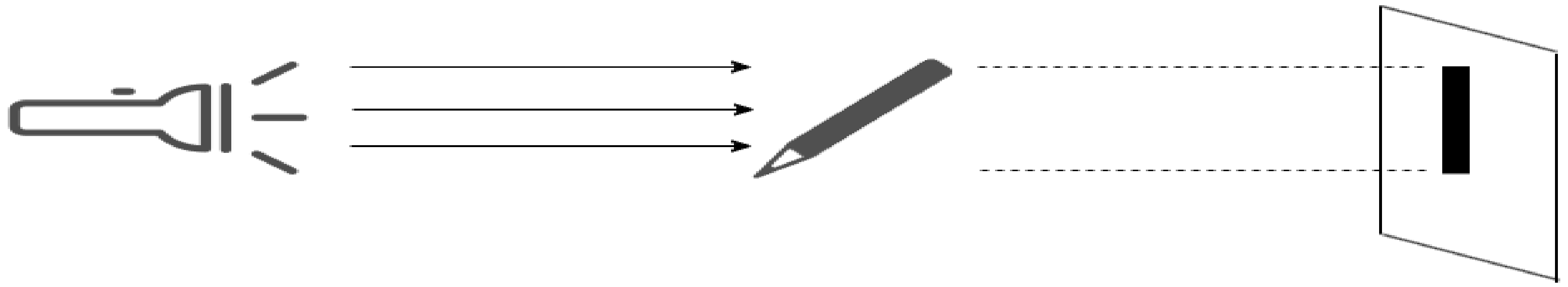
1. Dot product intuition & examples

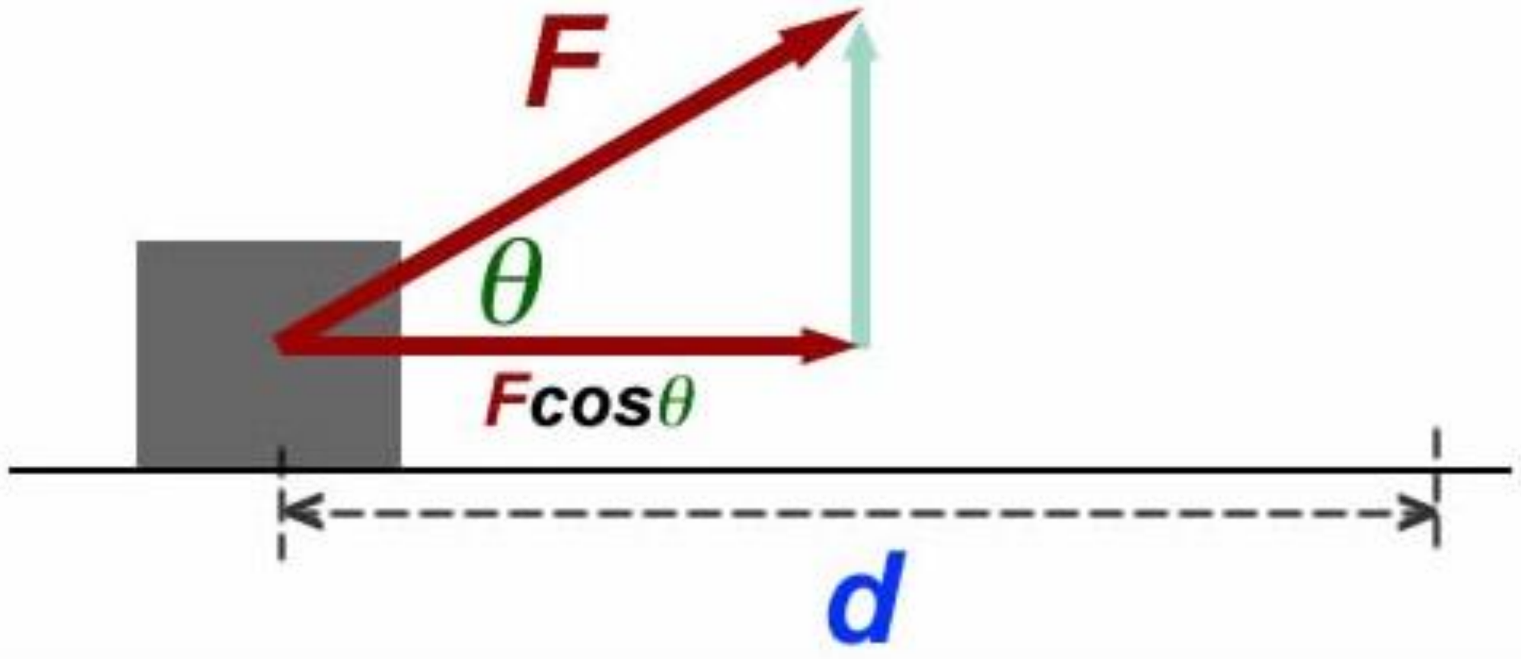
Dot Product (Inner product) definition

- Dot product of two vectors a and b

$$a = \begin{bmatrix} a_1 \\ a_2 \\ .. \\ a_n \end{bmatrix} \qquad b = \begin{bmatrix} b_1 \\ b_2 \\ .. \\ b_n \end{bmatrix}$$

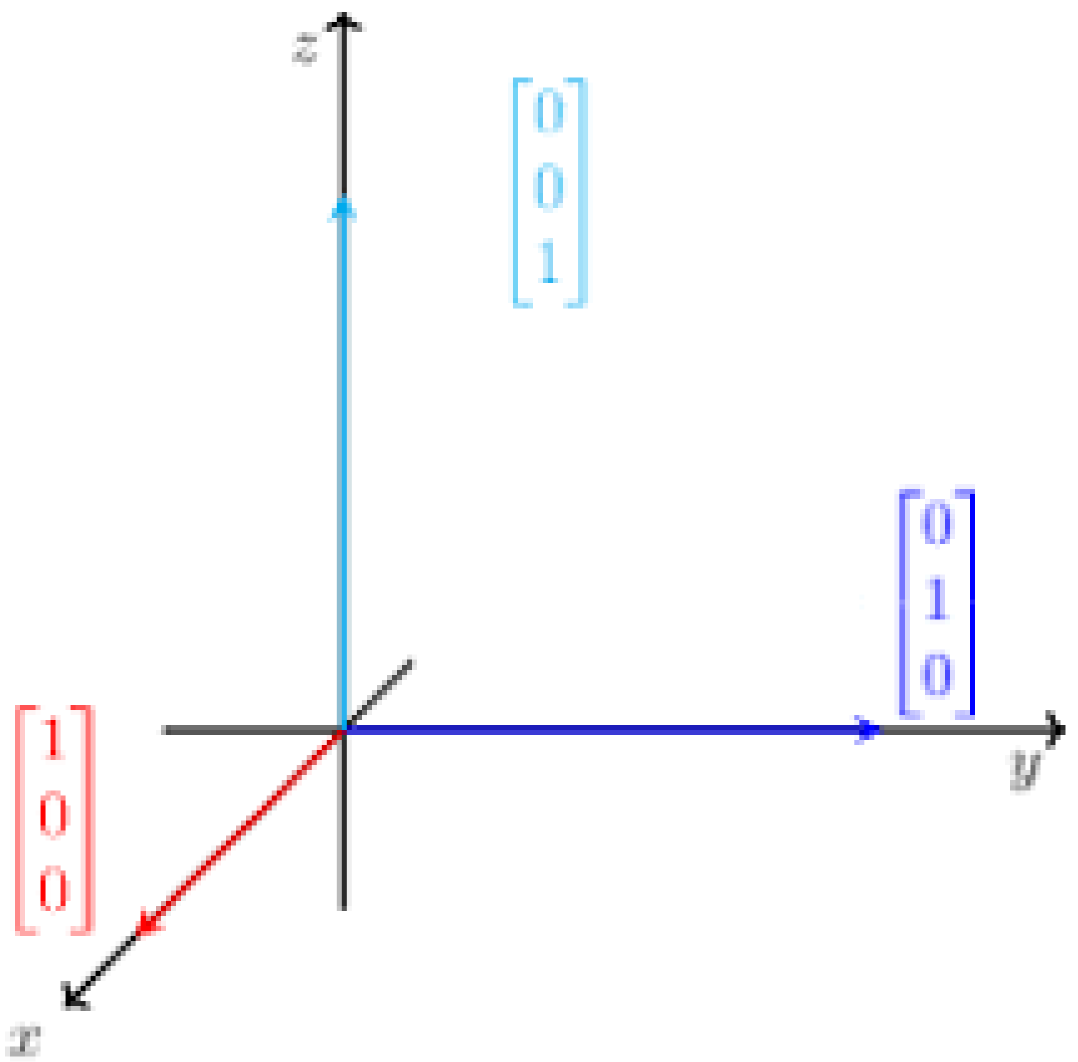
$$a^T b = a_1 b_1 + a_2 b_2 + \ldots + a_n b_n$$





- Two interpretations:
- Measures the projection of one vector on another
- Single number measure of 2-vector similarity in higher dimension

Standard unit vectors



$$e_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad e_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad e_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$e_{ij} = \begin{cases} 1, & \text{if } i = j \\ 0, & \text{if } i \neq j \end{cases}$$

Some dot product examples

$$a = \begin{bmatrix} a_1 \\ a_2 \\ .. \\ a_n \end{bmatrix} \qquad a^T a = \quad a_1 * a_1 + a_2 * a_2 + \ldots + a_n * a_n$$

$$= a_1^2 + a_2^2 + \ldots + a_n^2$$

$$= \|a\|^2 \qquad \Longrightarrow \|a\| = \sqrt{a^T a}$$

$$e_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad e_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad e_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad e_i^T a = a_i$$

- Picks out the ith entry of vector a

$$e_1^T a = \ 1 * a_1 + 0 * a_2 + 0 * a_3 = a_1$$

Some dot product examples (contd.)

$$a = \begin{bmatrix} a_1 \\ a_2 \\ .. \\ a_n \end{bmatrix} \qquad \mathbf{1} = \begin{bmatrix} 1 \\ 1 \\ .. \\ 1 \end{bmatrix}$$

$$\begin{aligned} \mathbf{1}^T a = \quad & 1 * a_1 + 1 * a_2 + \ldots + 1 * a_n \\ & = a_1 + a_2 + \ldots + a_n \end{aligned}$$

- Sum of vector entries

$$a = \begin{bmatrix} a_1 \\ a_2 \\ .. \\ a_n \end{bmatrix} \qquad \mathbf{1} = \begin{bmatrix} 1 \\ 1 \\ .. \\ 1 \end{bmatrix}$$

$$\frac{1}{n}\mathbf{1}^T a = \quad \frac{1}{n}(a_1 + a_2 + \ldots + a_n)$$

- Average of vector entries

Dot product toy applications

- Given Price & quantity vectors

$$p = \begin{bmatrix} p_1 \\ p_2 \\ .. \\ p_n \end{bmatrix} \quad q = \begin{bmatrix} q_1 \\ q_2 \\ .. \\ q_n \end{bmatrix} \quad p^T q = \quad p_1 * q_1 + p_2 * q_2 + ... + p_n * q_n$$

- Total cost of goods

- Evaluating a polynomial x=b

$$p(x) = a_1 * x + a_2 * x^2 + ... + a_n * x^n \quad a = \begin{bmatrix} a_1 \\ a_2 \\ .. \\ a_n \end{bmatrix} \quad c = \begin{bmatrix} b \\ b^2 \\ .. \\ b^n \end{bmatrix}$$

$$p(x)\Big|_{(x=b)} = a^T c$$

1. Dot product usage

Real world example snippets of applying dot product

- **Reminder: Your exam problems are of this type**
- Some examples worked out on the board today
  - Make notes
- Expected value of discrete probability distribution
- Hypothesis function of Linear regression & prediction
- Matrix vector multiplication
- Neural network single unit in a layer
- Anomaly detection in log file messages using co-occurrence vectors


QUESTIONS

Literature Survey