Implementation of LMS FIR based Adaptive Filter for System Identification

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***Abstract*—In this project, an LMS FIR based adaptive filter for system identification is discussed. The purpose of the project is to identify an unknown system by utilizing the nature of adaptive filters. Furthermore, characteristics of the adaptive filter is analyzed to determine optimum solution for the system. By executing the tasks 01, 02 and 03, significants of number of filter coefficients and step size are demonstrated.**

***Index Terms*—Adaptive Filter, FIR Filter, LMS Algorithm, Learning Curve, System Identification**

I. Introduction

In signal processing, filters are used for removing unwanted features from signals. They are one of the most significant part of signal processing that has applications in many fields such as telecommunication, control system, electronics, image processing etc. An adaptive filter is a type of filter which uses the current and past values of input and desired signals to determine the relationship between them. Over time, the adaptive filter coefficients are converged to an optimum value. By utilizing an appropriate adaptation algorithm an FIR based adaptive algorithm can be used for identifying an unknown system. Many approaches has been discussed in the literature [1] [2] [3] [5] to realize such a system. One of the well known system for system identification is the LMS FIR based adaptive filter. This system uses Least Mean Square (LMS) algorithm as the adaptation algorithm to update the filter coefficients. As a result, the filter approximates the unknown system by reducing the Mean Square Error (MSE).

II. Implementation of LMS FIR adaptive filter

A. Adaptive Filter

Adaptive filters are the filters which uses the current and past signal values of input and desired signal to determine the relationship between the input and desired signal [4]. During the learning process, the filter coefficients converges to the optimum over time. Figure 1 shows the block diagram of an adaptive filter. It consist of a discrete input signal $x[k]$, output signal $y[k]$, desired signal $d[k]$ and an error signal $e[k]$. For a given time k the adaptive filter output $y[k]$ should give the desired signal $d[k]$. The adaptation algorithm uses the error signal $e[k]$ in order to adjust the filter coefficients. The purpose of adaptation algorithm is to minimize the Mean Square Error (MSE). Considering the Wiener-Filter in a stationary environment, which assumes the statistics of signals does not change overtime, filter coefficients can be directly calculated from the auto-correlation (R) and cross-correlation (p) functions of the input and the output signals. As a result, Wiener-Filter are optimal that, the MSE of the error signal is minimum. In practical applications, the statistics of the signals are not known, in which, adaptive filters are more adequate.

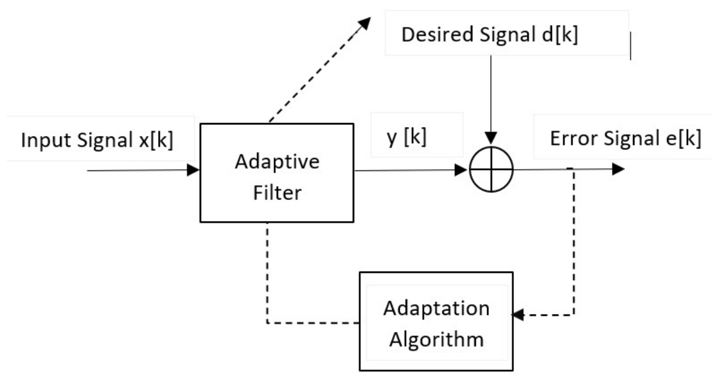

Fig. 1: Block Diagram of an Adaptive Filter

Based on the application, adaptive filters are classified mainly into four classes, such as: System Identification, Inverse Modeling, Prediction and Elimination of Disturbance. In case of system identification, an unknown system is provided whose output is the desired signal which needs to be identified. Here, the input signal is provided to both unknown system and adaptive filter. In a real system, the desired signal can also contain noise signal $n[k]$. In this case, the adaptive filter output is approximated to the desired signal, so that, the filter coefficient of adaptive filter become approximated to the filter coefficient of unknown system.

B. FIR based Adaptive Filter

The transfer function of an FIR filter with coefficients a_n and b_n can be represented as in Equation 1.

$$y[k] = -\sum_{n=1}^{N} a_n y[k-n] + \sum_{m=0}^{M} b_m y[k-m] \tag{1}$$

Since the FIR filer has finite impulse response given as $h[0], h[1], h[2], ..., h[M]$, we can state the direct relationship between impulse response and filter coefficients as: $b_i = h[i]]$, ($a_i = 0$, except: $a0 = 1$) [4]. The filter coefficients b_i of the FIR adaptive filter can be represented as the weights w_i of an adaptive filter. For discrete-time k, the filter weights changes with respect to time, due to adaptation. In a similar way, the output of an FIR adaptive filter can be represented as the convolution of the input signal $x[k]$ and the filter coefficients $w_i[k]$. This convolution can be represented as a formula in Equation 2.

$$y[k] = X^T[k]W[k] = W^T[k]X[k] \tag{2}$$

where $X[k]$ and $W[k]$ are given by:

$$\begin{aligned} X[k] &= [x[k], x[k1], ..., x[k-N+1]]^T, \\ W[k] &= [w_1[k], w_2[k], ..., w_N[k]]^T \end{aligned} \tag{3}$$

The difference between the filter output $y[k]$ from the desired signal $d[k]$ gives the error signal, which can defined as in Equation 4.

$$e[k] = d[k] - y[k] \tag{4}$$

For a random signal, the expected value of the squared error can be taken as the optimality measure, and so, mean square error (MSE) is given by Equation 5.

$$\begin{aligned} E\{e^2[k]\} = E\{d^2[k]\} + W^T E\{X[k]X^T[k]\}W \\ -2E\{d[k]X^T[k]\}W \end{aligned} \tag{5}$$

By taking auto-correlation $R = E\{X[k]X^T[k]\}$, cross-correlation $p = E\{d[k]X^T[k]$ and by considering $E\{e^2[k]\} = \sigma_d^2$, it can be seen that the MSE depends only on the weighting vector W. Hence an error function can be introduced as in Equation 6.

$$J(W) = E\{e^2[k]\} = \sigma_d^2 + W^T R W - 2p^T W \tag{6}$$

From the equation we can see that the error function is a quadratic function that depends on the filter coefficient vector W.

C. LMS Algorithm

Least Mean Square or LMS algorithms are a class of adaptive filter algorithms to determine the filer coefficients to provide least mean square of the error signal [6]. It belongs to the class of gradient search algorithms that the filter adaptation dependents only on the current error. They are best suitable for linear adaptive FIR filters. When considering LMS algorithm, the auto-correlation (R) and cross-correlation (p) values are not available. Here, the input signal $x[k]$ and desired signal $d[k]$ are only known. The error function for LMS algorithm can be shown as in Equation 7.

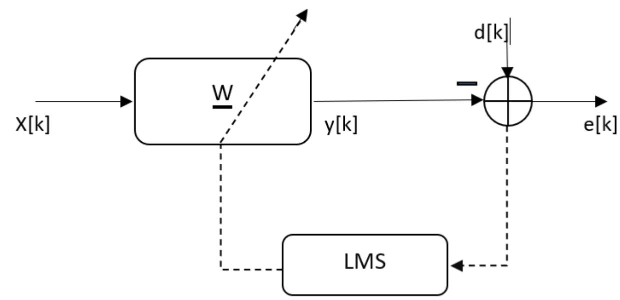

Fig. 2: Block Diagram for LMS Algorithm

$$J(W) = E\{e^2[k]\} = \lim_{M\to\infty} \sum_{k=0}^{M-1} e^2[k] \tag{7}$$

The averaging length M is normally finite in practical scenarios, and so, the result gives only an estimation of error function. By simplifying the above equation we can see that:

$$W[k+1] = W[k] + \mu e[k]X[k], \mu > 0 \tag{8}$$

Where μ is a positive constant known as step size.

For an LMS filter, it can be observed that the squared error e^2 (learning curve) decreases with time k. The learning curve drops or converges to e-th fraction after a number of iterations known as convergence time. It can be observed that the convergence speed depends on the step size μ and conditioning of input signal. Hence, LMS algorithm is considered comparatively a slower algorithm.

D. Task 01 - LMS Algorithm

The purpose of task 01 is to define an FIR based adaptive filter using LMS algorithm. Figure 2 represents the block diagram of such an adaptive filter. As shown in figure, $x[k]$ represents the discrete time input signal with sampling period T, $y[k]$ is the output signal of the adaptive filter, $d[k]$ is the desired signal and $e[k]$ is the error signal that fed in to the proposed adaptive algorithm. Here, the input signal $x[k]$ and desired signal $d[k]$ are taken as column vectors with values 2 and 1 respectively. For a filter vector containing single filter coefficient different outputs are generated using three different values of step size (μ).

Figure 3 represents the output containing $x[k]$, $[k]$ and $d[k]$ for step size 0.01. Here output signal adapts the desired output even before the k value become 200. Where as in Figure 4 the adaptation takes place very quickly with a higher step size of value 0.1. For step size 0.5 as shown in Figure 5, the filter does not gives a desired output.

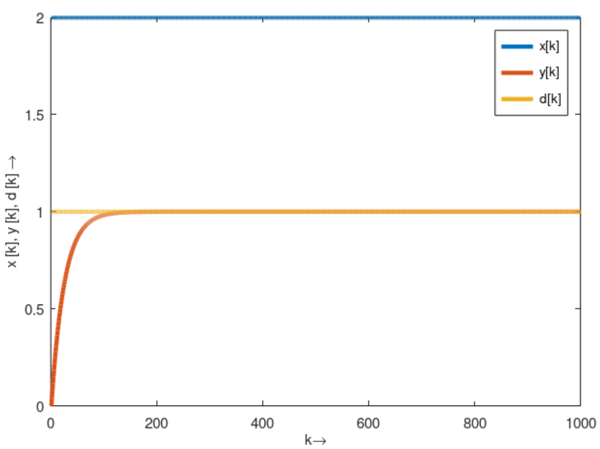

Fig. 3: x(k), y(k), d(k) for N = 1 and μ = 0.01

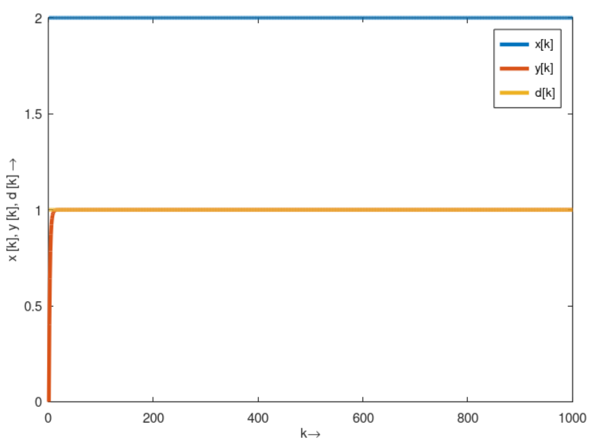

Fig. 4: x(k), y(k), d(k) for N = 1 and μ = 0.1

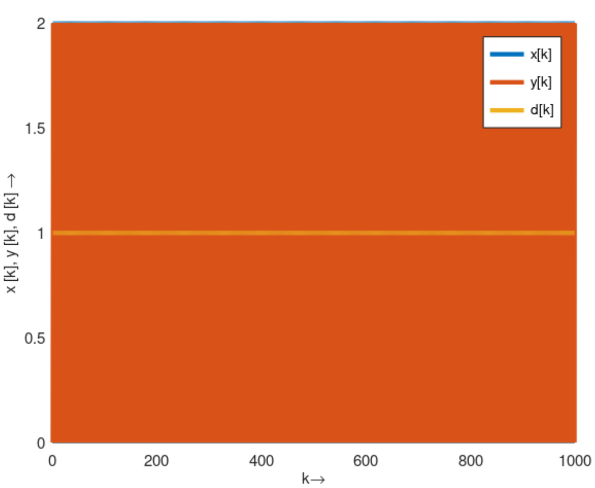

Fig. 5: x(k), y(k), d(k) for N = 1 and μ = 0.5

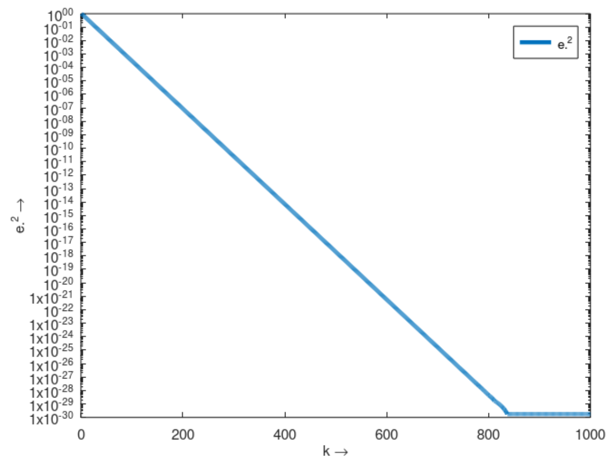

Fig. 6: $e^2[k]$ for N = 1 and μ = 0.01

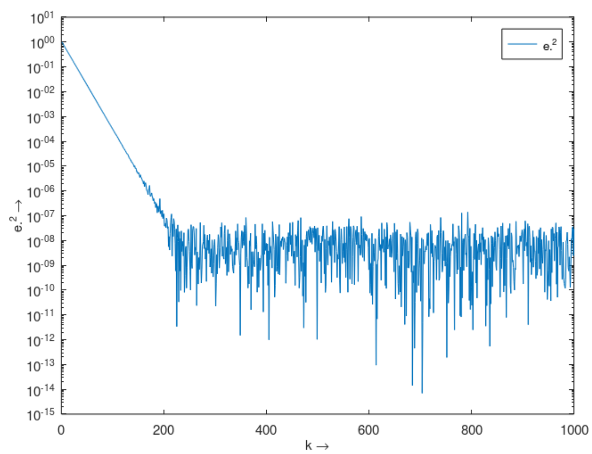

Fig. 7: $e^2[k]$ for N = 1 and μ = 0.01 with noise

E. Task 02 - Learning Curve

In task 02, the adaptation of filter coefficient is considered along with the behavior of error with respect to time. In order to examine the behavior of error, the input signal $x[k]$, desired signal $d[k]$, number of coefficients N and step size μ has been taken as of task 01. The variation of learning curve $e^2[k]$ has been plotted as shown in Figure 6 for $N = 1$ and $\mu = 0.01$. From the figure, it is clear that the adaptive filter is able to reduce $e^2[k]$ up to 10^{-30} only after a k value of 800. Here, the filter coefficient is converged to a value of 0.5.

$$d = d + 1e - 4 * randn(length(d), 1) \tag{9}$$

Considering the case of adding a random Gaussian noise signal with variance $\sigma_n^2 = 10^{-4}$ along with the desired signal as shown in Equation 9, the variation in the learning curve is shown in Figure 7. The learning curve is shown to reduce to a value near 10^{-8} around a k value of 200. Compared to the previous case, there are some fluctuation in the value of $e^2[k]$ due to presence of noise in desired signal. Although the filter coefficient seems to converges to a value of 0.5.

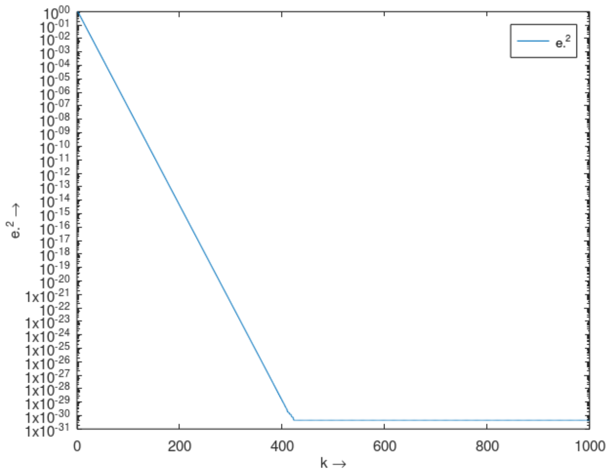

Fig. 8: $e^2[k]$ for N =2 and μ = 0.01

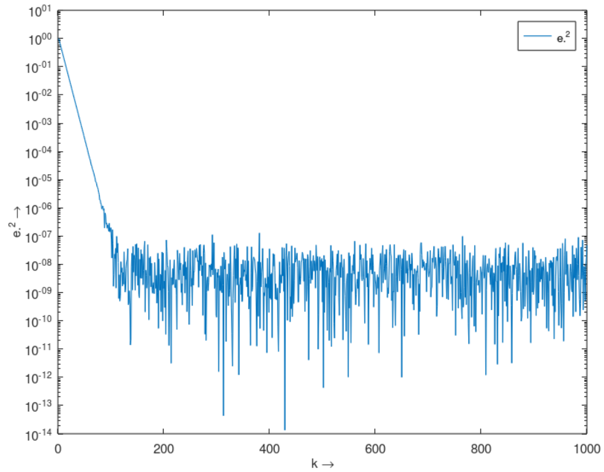

Fig. 10: $e^2[k]$ for N =2 and μ = 0.01 with noise

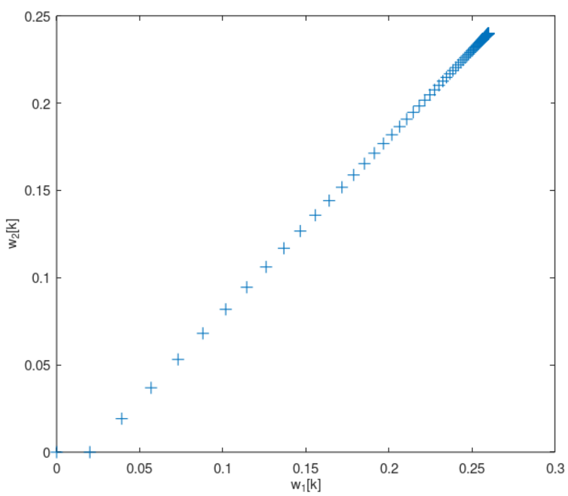

Fig. 9: w_1 and w_2 for N =2 and μ = 0.01

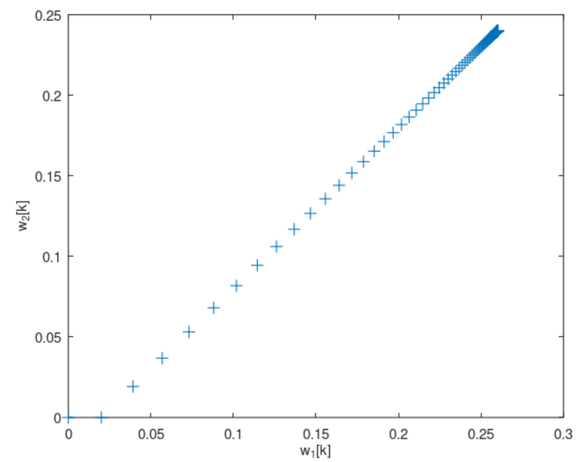

Fig. 11: w_1 and w_2 for N =2 and μ = 0.01 with noise

The above procedure is repeated for increasing the number of coefficient N to 2. When there are no noise introduced to the desired signal, the learning curve converges to a value around 10^{-31} near 400. The two filter coefficient vectors are plotted in Figure 9. The values of filter coefficients decreased and converged to a value of 0.26 and 0.24.

When a noise is added to the desired signal, the learning curve behavior is examined for $N = 2$. Figure 10 shows the behavior of learning curve with respect to k. From the figure it can be seen that $e^2[k]$ converges to a value of around 10^{-8}. As seen before there exist fluctuations in the value of $e^2[k]$ due to the presence of noise. Figure 11 shows the plot of two filter coefficient in presence of noise. The values of filter coefficients are converged to values 0.26 and 0.24 as before.

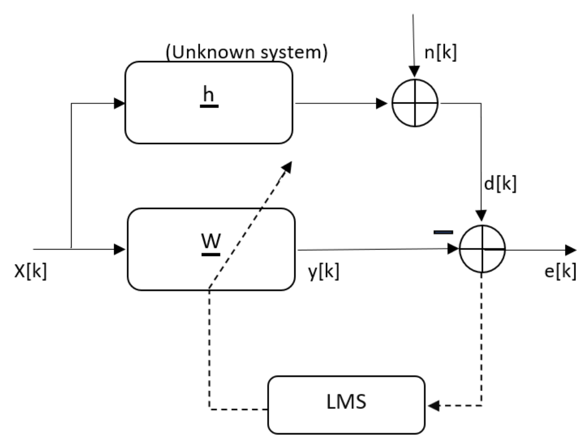

Fig. 12: Block diagram for system identification

F. Task 03 - System Identification

In this task, the system identification using LMS FIR based adaptive filter is considered. The block diagram indicating the system is shown in Figure 12. The purpose of this task is to approximate an unknown system described by FIR filter coefficient vector $h = [h_1, h_2, ...h_L]$ of length L. New value for input signal $x[k]$ and desired signal $d[k]$ are provided in the file 13_$task3_x_d.mat$. By applying the LMS FIR function used in task 02 the procedure is carried out to obtain optimal value for number of filter coefficients N and step size μ. Using different values of N it is found that the best result is provided for N value of 4. Also, by applying different values of step sizes μ, the best result seems to obtained for μ value 0.04. Figure 13 shows the plot of input signal $x[k]$, desired signal d[k] and error signal $y[k]$.

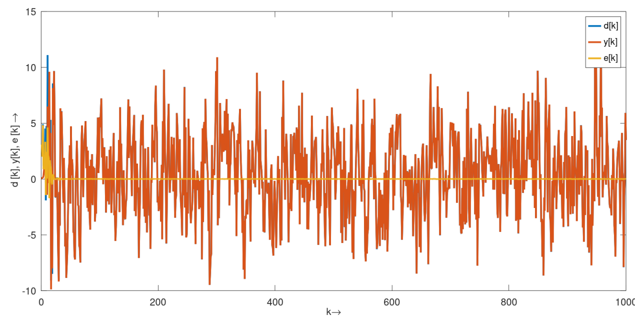

Fig. 13: x(k), y(k), d(k) for N = 4

Figures 14 and 15 shows the comparison of $e^2[k]$ for N = 2 and 4, in which N = 4 gives better result. Figure 16 shows the behavior of learning curve for N value 4 and μ values 0.1, 0.01 and 0.04. Even though there are some fluctuations, the $e^2[k]$ seems to be approximately converged to around 10^{-4}.

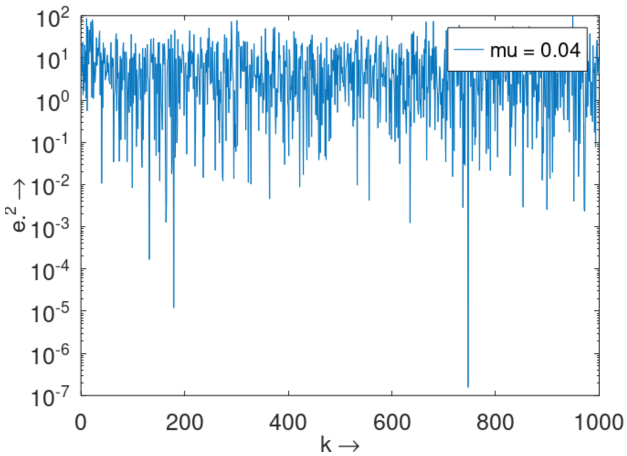

Fig. 14: $e^2[k]$ for N = 2

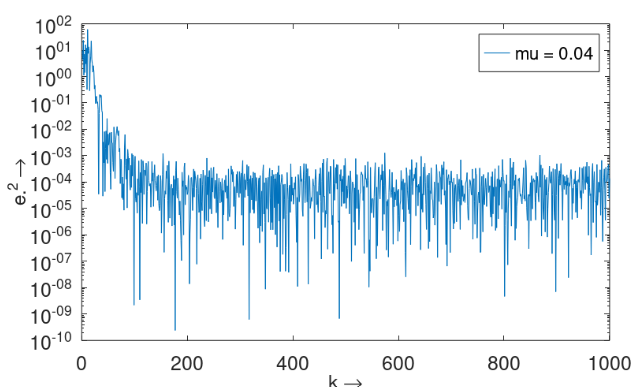

Fig. 15: $e^2[k]$ for N = 4

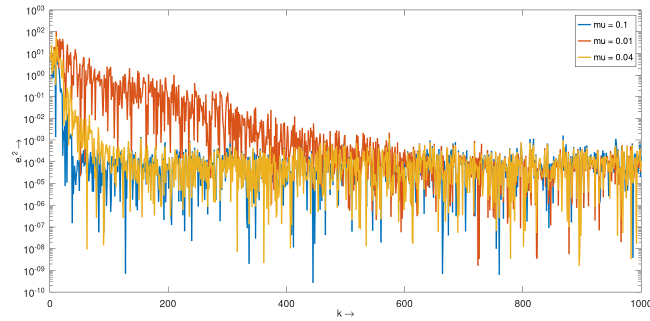

Fig. 16: $e^2[k]$ for N = 4 and μ = 0.1 / 0.01 / 0.04

1.9974	1.9970	1.9974	1.9967	1.9975	1.9977	1.9978
1.0016	1.0009	1.0013	1.0019	1.0027	1.0029	1.0025
2.9978	2.9982	2.9989	2.9995	2.9987	2.9990	2.9986
-0.0018	-0.0014	-0.0019	-0.0007	-0.0014	-0.0016	-0.0020

Fig. 17: Filter Coefficient for N = 4 and μ = 0.04

Based on the μ value, $e^2[k]$ converges faster when $\mu = 0.1$ but produces relatively more excess error. The converges is slower with $\mu = 0.01$ and better with $\mu = 0.04$. Finally, the filter coefficients produced for $N = 4$ and $\mu = 0.04$ are displayed as shown in Figure 17, where the final values are 1.9978, 1.0025, 2.9986 and -0.0020 which are approximately 2, 1, 3 and 0 respectively.

III. Conclusion

In this project, an LMS FIR based adaptive filter for system identification is successfully implemented. The filter is able to approximate the unknown system up to an extent. The analysis showed that variation in number of filter coefficient and step size has influence on correctly identifying the system. With increase in the number of filter coefficients, the minimum mean square error (MMSE) decreases while the excess error increases. There for an optimal value of N needs to be selected in order to achieve the best system. In a similar way, step size μ needs to selected carefully as the convergence speed of LMS algorithm increases with increase in step size but causes large excess error.

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