

Syllabus

Unit-I: Introduction to Artificial Intelligence.

- Introduction
- History
- Intelligent Systems
- Foundations of AI
- Applications
- Tic-Tac-Toe game playing
- Development of AI languages
- Current Trends in AI

Problem Solving - State-Space Search and Control Strategies.

- Introduction
- General problem Solving
- characteristics of problem.

Unit-II: Search Strategies.

- Exhaustive Search
- Heuristic Search Techniques
- Iterative Deepening a*
- Constraint Satisfaction
- Two-player Games.

Unit-III: Logic Concepts

- Introduction
- Propositional Calculus
- Propositional logic
- Natural Deduction System.

- Axiomatic System.
- Semantic Tableau System in Propositional logic
- Resolution Refutation in Propositional logic
- Predicate Logic

part-II knowledge Representation.

- Introduction
- Approaches to knowledge Representation.
- knowledge Representation using semantic network.
- Extended Semantic networks for knowledge Representation.
- knowledge Representation using Frames.

Unit-IV: Advanced knowledge Representation Techniques

- Introduction
- Conceptual Dependency Theory
- Script Structure.

Unit-IV-II Expert System and Applications.

- Introduction. phases in building expert system.
- Expert System Vs Traditional Systems.
- Rule based expert Systems
- Black Board Systems.
- Truth Maintenance Systems
- List of shells and tools.

Unit-11 Uncertainty Measure - Probability Theory

Introduction

Probability Theory

Bayesian Belief networks

Certainty factor Theory

Dempster-Shafer Theory.

Unit-I Introduction to Artificial Intelligence

Introduction:-

- * AI Foundation was laid with the development of BOOLEAN THEORY.
- * Boolean theory operates on the principle that all statements are either "True" or "False".
- * Building Blocks of Computing: Every computing system, at its core, operates on principles of Boolean logic. This is because computer h/w is built on binary states(0 and 1).
- * Logic Programming in AI: Boolean logic forms the foundation for many AI algorithms, particularly in the field of logic programming. Programs can make decisions based on logical rules, which are often expressed in terms of Boolean operations.

For example, an AI might have a rule like "IF condition1 AND condition2 are True, THEN perform action1". Here, "condition1 AND condition2" is a Boolean expression.

So, in essence, Boolean logic is fundamental to both the hardware that AI systems run on; and the software algorithms that drive their decision-making processes.

- * The goal of AI has always been to make machines smart, like trying to get a robot to think (or) make decisions like a human.
- * There are two main ways people try to create AI:-

Bottom-up approach: It's like building a mini-brain with electronic parts. Imagine creating a toy brain and hoping it will think like us.

Top-down approach:-

Here, we don't built anything physically. Instead, we write computer programs to mimic our thinking. It's like creating a virtual brain inside the computer.

Intelligent Systems:-

challenges of AI:

- Difficult to understand.
- Constantly changing
- AI deals with problem which AI don't understand
- AI makes mistakes (like Human Being).
- Don't give perfect answers every time [try to give a good solution].

Two views on AI goals:-

1. To copy the human brain behaviour [Acting]
2. Doing things logically [Thinking].

Testing Techniques

Turing Test:-

Simple Test introduced by Alan Turing. It will test whether the system is behaving like a human (or) not is called.

ELIZA

A Computer program which has a ~~test~~ based conversation with users.



ELIZA Communication

<u>ELIZA</u>	<u>USER</u>
Hello I am ELIZA	Can you help me?
Don't you believe that I can help you?	Yes

I See

How long have you been
feeling depressed?

This is quite interesting.

Have you asked anyone
else?

Are you saying no just
to be negative?

why no?

Say, do you have any
Psychological problems?

Tell me more.

I feel depressed

since last month

what is interesting in this?

No

No I did not ask anyone.

I do not believe other
people.

I do not know

Components of AI program

1) knowledge Base

A Memory stores like rules
facts

characteristics:-

1. voluminous

2. Incomplete Imprecise

3. Dynamic.

2, Control Strategy - Decision Making

(Heuristics/Thumrules)

3, Inference Mechanism - thought processing

(Inference rules)

Foundations of AI:-

1. Mathematics

2. Neuroscience - study brain & functions

3. Control Theory - Change their behaviour based on situations. \rightarrow decision making.

4. Linguistics. - The study of language & how to communicate.

Applications:-

1. Business

2. Engineering

3. Education

4. Medicine

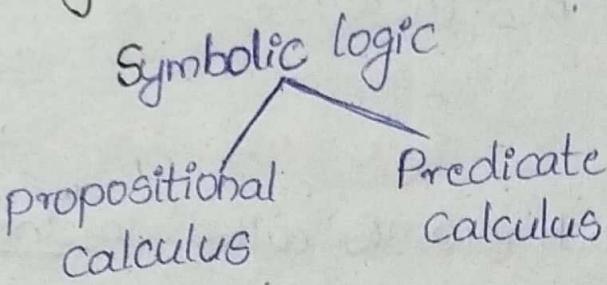
5. Manufacturing

6. Fraud Detection

7. Object Identification

8. Space Shuttle Scheduling

9. Information Retrieval

Logic ConceptsWell formed formula:-

A well-formed formula is defined as a symbol (or) string of symbols generated by the formal grammar of a formal language.

1. Smallest unit is considered to be well-formed formula (WFF). Ex:- P, Q, R, -
2. If α is a well-formed formula, then $\neg\alpha$ is also WFF.
3. If α and β are well-formed formulas then $(\alpha \vee \beta)$, $(\alpha \wedge \beta)$, $(\alpha \rightarrow \beta)$, $(\alpha \leftrightarrow \beta)$ implies equivalence.
4. A propositional expression is called a WFF, if and only if it satisfies the above properties.

Equivalent Formulas:-

Two formulas α and β are said to logically equivalent ($\alpha \cong \beta$). if and only if truth values are same for all possible assignments of logical constants (T/F).

Equivalence Laws:-

$$\text{Absorption law: } \begin{array}{c|c} A \vee (A \wedge B) \cong A & A \vee (\neg A \wedge B) \cong A \vee B \\ A \wedge (A \vee B) \cong A & A \wedge (\neg A \vee B) \cong A \wedge B \end{array}$$

Excluded Middle law:- $A \vee \neg A \cong T$ (True)

Contradiction law:- $A \wedge \neg A \cong F$ (False)

Commonly used equivalence relations:-

- | | |
|--------------------------|--|
| 1. $AVF \equiv A$ | 5. $A \rightarrow B \equiv \neg A \vee B$ |
| 2. $AVT \equiv T$ | 6. $(A \leftrightarrow B) \equiv (A \rightarrow B) \wedge (B \rightarrow A) \equiv (A \wedge B) \vee (\neg A \wedge \neg B)$ |
| 3. $A \wedge T \equiv A$ | 7. |
| 4. $A \wedge F \equiv F$ | |

Other

* Truth values on each row of is called an "Interpretation" for formula.

Valid = Tautology.

Stmt formula/group of Stmt Formula \rightarrow Satisfiable/Unsatisfiable.

* A formula ' α ' is said to be valid, if it is a tautology.

* A formula ' α ' is said to be satisfiable, if there exist at least one interpretation for which ' α ' is true.

* A formula ' α ' is said to be unsatisfiable, if the value of ' α ' is false under all interpretations.

1) Show that "If it is humid then it will rain and since it is humid today it will rain" is a valid argument.

Ans: Let, A: It is humid and B: It will rain.

$$\alpha: [(A \rightarrow B) \wedge A] \rightarrow B$$

$$A \quad B \quad A \rightarrow B \quad (A \rightarrow B) \wedge A \quad [(A \rightarrow B) \wedge A] \rightarrow B$$

T	T	T	T	T	Tautology
T	F	F	F	F	
F	T	T	F	T	
F	F	T	F	T	

Methods to check the condition is valid (or) not.

1. Natural Deduction System.

2. Axiomatic System

3. Semantic tableau Method

4. Resolution repetition method.

① ~~for~~ ~~for~~ 2) Axiomatic System:-

3 axioms and 1 deduction rule

\rightarrow (\sim), \rightarrow (condition). These two connectives are used.

Axiom:1 $\alpha \rightarrow (B \rightarrow \alpha)$ where α & B are WFF

Axiom:2 $[\alpha \rightarrow (B \rightarrow \alpha)] \rightarrow [(\alpha \rightarrow B) \rightarrow (\alpha \rightarrow \alpha)]$

Axiom:3 $(\neg \alpha \rightarrow \neg B) \rightarrow (B \rightarrow \alpha)$

Axiom \rightarrow means assumed to be true but there is no proof.

1 Deduction rule: Moden's Ponens rule i.e.,

In this,

$\alpha \rightarrow B$ and α :- Hypothesis

B :- Conclusion | consequent

$\underbrace{P, P \rightarrow \alpha \leftarrow \alpha}_{\text{antecedents}}$
 α :- Hypothesis

* Let $\Sigma = \{\alpha_1, \alpha_2, \dots, \alpha_n\}$ be a set of hypothesis then, α is said to be a deductive consequence of Σ , if either α is an axiom (or) a hypothesis - is (or) is derived from α_j where $1 \leq j \leq n$. using Moden's Ponen Rule.

* It is represented as $\{\alpha_1, \alpha_2, \dots, \alpha_n\} \vdash \alpha$ (or)

$\Sigma \vdash \alpha$
↳ inference

* If α is empty then the above representation becomes $\vdash \alpha$, which means α is derived only from axioms and no hypothesis are involved and it is said to be theorem.

Example:- *

① Validate the inference $\{A \rightarrow B, B \rightarrow C\} \vdash (A \rightarrow C)$ using axiomatic system.

Solution:- Description | Formula | Comments.

Theorem | $\{A \rightarrow B, B \rightarrow C\} \vdash (A \rightarrow C)$ | To be proved

Hypothesis 1 | $A \rightarrow B$ | 1

Hypothesis 2 | $B \rightarrow C$ | 2

Instance of axiom 1 | $(B \rightarrow C) \rightarrow [(A \rightarrow (B \rightarrow C))]$ | 3

$\alpha \rightarrow (B \rightarrow \alpha)$

Here
 $\alpha = (B \rightarrow C)$

$B = A$

Modus ponens Rule
(2, 3)

\downarrow
 $\alpha, \alpha \rightarrow B = B$

$A \rightarrow (B \rightarrow C)$ | 4

Instance of axiom 2.

$(\alpha \rightarrow (B \rightarrow \alpha)) \rightarrow [(\alpha \rightarrow B) \rightarrow (\alpha \rightarrow \alpha)]$

$[A \rightarrow (B \rightarrow C)] \rightarrow [(A \rightarrow B) \rightarrow (A \rightarrow C)]$ | 5

Modens Ponens Rule
(4,5)

$[(A \rightarrow B) \rightarrow (A \rightarrow C)]$

6

Moden's ponens Rule
(1,6)

$A \rightarrow C$

Proved

* Validate the inference $\{\neg A\} \text{ Infers } A \rightarrow B$.

sd:- Description Formula Comments
Theorem - $\{\neg A\} \vdash (A \rightarrow B)$ - To be Proved

Hypothesis - $\neg A$

Instance of axiom 1 - $\neg A \rightarrow (\neg B \rightarrow \neg A)$ - 2

\downarrow
 $\alpha \rightarrow (B \rightarrow \alpha)$

Here $\alpha = \neg A$
Let $B = \neg B$

Modens ponens rule (1)(2) - $\neg B \rightarrow \neg A$ - 3

Instance of axiom 3 - $(\neg B \rightarrow \neg A) \rightarrow$ - 4
 $(\alpha \rightarrow (B \rightarrow \alpha)) \rightarrow [(\alpha \rightarrow B) \rightarrow (\alpha \rightarrow \alpha)]$
 $(A \rightarrow B) \rightarrow (A \rightarrow A)$

Moden's ponens Rule - $A \rightarrow B$ - Proved

Deduction Theorem

* Given that Σ is a set of hypothesis and $\alpha \& \beta$ are well formed formulas.

* If β is proved from $\Sigma \cup \{\alpha\}$ then according to deduction theorem $(\alpha \rightarrow \beta)$ is proved from Σ .

* We can write $\underline{\Sigma \cup \{\alpha\} \vdash \beta}$ implies $\underline{\Sigma \vdash (\alpha \rightarrow \beta)}$.

Converse of Deduction Theorem.

* Given $\underline{\Sigma \vdash (\alpha \rightarrow \beta)}$ then $\underline{\Sigma \cup \{\alpha\} \vdash \beta}$.

3) Prove the theorem $\text{infer}(\neg A) \stackrel{(\vdash)}{\rightarrow} (A \rightarrow B)$ $(\Sigma = \emptyset)$

Sol:- Given, $\vdash (\neg A) \rightarrow (A \rightarrow B)$

By applying deduction theorem,

if $\Sigma \cup \{\alpha\} \vdash \beta$ then $\Sigma \vdash (\alpha \rightarrow \beta)$

Here, $\Sigma = \emptyset$, $\alpha = \neg A$, $\beta = (A \rightarrow B)$

Then

$\emptyset \cup \{(\neg A)\} \vdash (A \rightarrow B)$

$\{(\neg A)\} \vdash (A \rightarrow B)$

already proved in ②

Hence By DT, $\vdash (\neg A) \rightarrow (A \rightarrow B)$ is proved.

Semantic Tableau Method:-

(meaning) (Graphical Representation)

* It is a binary tree which is constructed by using Semantic Tableau Rules with a formula as a root.

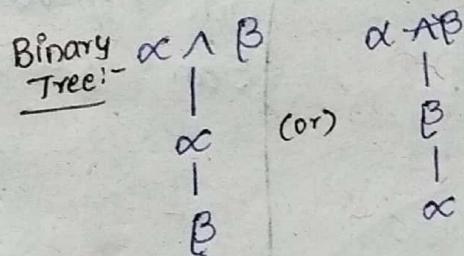
Semantic Tableau Rules for $\alpha \wedge \beta$:-

Rule No.

Tableau Tree It is a BT.

Rule 1

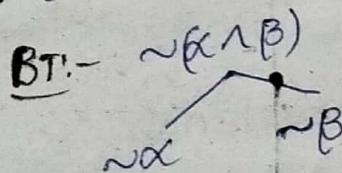
$\alpha \wedge \beta$ is true if both α and β are true.



Rule 2

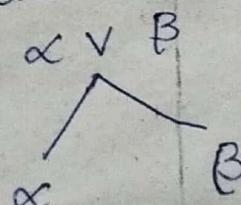
$\sim(\alpha \wedge \beta)$ is true if either $\sim\alpha$ is true or $\sim\beta$ is true.

$$\sim(\alpha \wedge \beta) \Leftrightarrow \sim\alpha \vee \sim\beta.$$



Rule 3

$\alpha \vee \beta$ is true if either α or β is true.



Explanation

A tableau for the formula $(\alpha \wedge \beta)$ is constructed by adding both α and β to the same path (branch).

A tableau for a formula $\sim(\alpha \wedge \beta)$ is constructed by adding two new paths:-
One containing $\sim\alpha$ and other containing $\sim\beta$.

Rule 4

$\sim(\alpha \vee \beta)$ is true
if both $\sim\alpha$ and
 $\sim\beta$ are true.

$$\sim(\alpha \vee \beta) \Leftrightarrow \sim\alpha \wedge \sim\beta$$

$$\sim(\alpha \vee \beta)$$

$$\mid$$

$$\sim\alpha$$

$$\mid$$

$$\sim\beta$$

Rule 5

$\sim(\sim\alpha)$ is true then
 α is true.

$$\sim(\sim\alpha)$$

$$\mid$$

$$\alpha$$

A tableau for $\sim(\sim\alpha)$
is constructed by
adding α on the
same path;

Rule 6

$\alpha \rightarrow \beta$ is true $\sim\alpha \vee \beta$
is true.

$$\alpha \rightarrow \beta$$

$$\swarrow \quad \searrow$$

$$\sim\alpha$$

$$\beta$$

Rule 7

$\sim(\alpha \rightarrow \beta)$ is true
then $\alpha \wedge \sim\beta$ is
true

$$\sim(\alpha \rightarrow \beta)$$

$$\mid$$

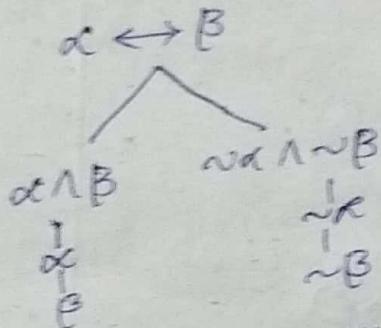
$$\alpha$$

$$\mid$$

$$\sim\beta$$

Rule 8

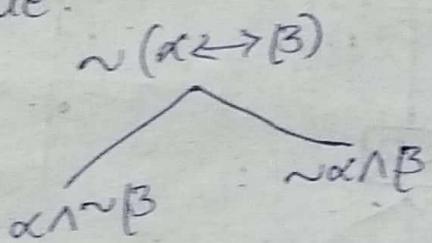
$\alpha \leftrightarrow \beta$ is true then
either $(\alpha \wedge \beta) \vee (\neg \alpha \wedge \neg \beta)$
is true.



$$\begin{aligned} \alpha \leftrightarrow \beta &= (\alpha \rightarrow \beta) \wedge (\beta \rightarrow \alpha) \\ &= (\alpha \wedge \beta) \vee (\neg \alpha \wedge \neg \beta) \end{aligned}$$

Rule 9

$\neg(\alpha \leftrightarrow \beta)$ is true then
 $(\alpha \wedge \neg \beta) \vee (\neg \alpha \wedge \beta)$ is
true.



$$\begin{aligned} \neg(\alpha \leftrightarrow \beta) &\Leftrightarrow \\ \neg[(\alpha \rightarrow \beta) \wedge (\beta \rightarrow \alpha)] &\Leftrightarrow \neg[(\neg \alpha \vee \beta) \wedge (\neg \beta \vee \alpha)] \\ &\Leftrightarrow (\alpha \wedge \neg \beta) \vee (\neg \alpha \wedge \beta) \end{aligned}$$

Ques:-
1) Explain the construction of semantic tableau
for the formula $(A \wedge \neg B) \wedge (\neg B \rightarrow C)$

Ans:- The thumb rule to construct a semantic tableau is to apply non-branching rules before branching rules.

Description

Formula

line number

Tableau root

$$(A \wedge \sim B) \wedge (\sim B \rightarrow C)$$

1

Rule 1 (1)

$$(A \wedge \sim B)$$

2

Rule 1 (2)

$$(\sim B \rightarrow C)$$

3

Rule 6 (3)

$$A$$

4

$$\sim B$$

5

Rule 5 (6)

$$\sim(\sim B) \quad C$$

6

$$B$$

7

$$X \{B, \sim B\}$$

* X - (closed) if there is complementary elements like $B, \sim B$ in a path.

* \checkmark - (open) no complement elements

* $A, \sim B, C$ are the atoms in opened path.
Prove that Truth values of $A, \sim B, C$ are

free: $\{A=T; \sim B=T; C=T\}$

$\{A=T; B=F; C=T\}$ Model.

- * Paths in a tableau tree extend from root to the leaf nodes.
- * There are 2 paths in the tree. starting from the root node to leaf nodes ending at B and C.
- * It is observed that first path from root to B becomes closed. Because of the presence of complementary atoms B and $\sim B$. while the other path remains open.
- * The model of the formula is obtained by assigning 'T' to all atomic formulas appearing on the open path.

$$\therefore \{ A=T ; B=F ; C=T \} \quad \text{Model}$$

\hookrightarrow is a model under which the given formula is true.

Model:-

For a given formula α ; a model is the interpretation under which the formula is true, we assign 'T' to all atomic formulas appearing on the open path.

Note-1:- Path is said to be contradictory (or) closed, if complementary atoms appear on the same path.

Note-2:- If all paths are closed, it is contradictory tableau, such a formula α is inconsistent (or) unsatisfiable.

Note-3: - A formula ' α ' is said to be consistent (or) satisfiable, if the tableau with root ' α ' is not a contradictory tableau, that means there is atleast one open path.

Note-4: - If we obtained a contradictory tableau with root $\sim\alpha$. we say that, the formula ' α ' is tableau provable.

Note-5: - If ' α ' is tableau provable then it is a valid formula (Tautology).

Note-6: - Let ' S ' be a set of formulae.

Formula ' α ' is said to be tableau provable from ' S '. if there is a contradictory tableau from S with $\sim\alpha$ as root.

★ ^{Ques} 2) Show that $S = \{\sim(A \vee B), (C \rightarrow B), (A \vee C)\}$ is unsatisfiable / inconsistent.

Sl:- Description	Formula	Line number
Tableau Root	$\sim(A \vee B) \wedge (C \rightarrow B) \wedge (A \vee C)$	1
Rule 1 (1)	$\sim(A \vee B)$	2
	$(C \rightarrow B)$	3
	$(A \vee C)$	4

Rule 4(2)

$(A \vee C)$

$\sim A$

1

$\sim B$

4

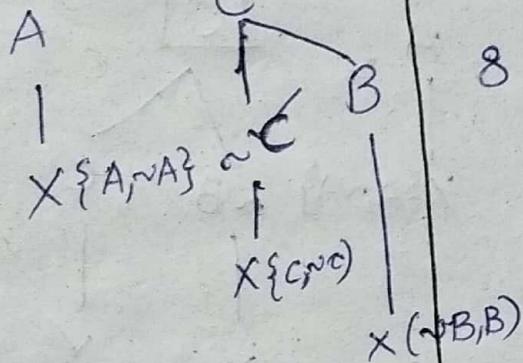
5

6

7

8

Rule 3(4)



Rule 6(3)

\therefore The set of formula $S = \{\sim(A \vee B), C \rightarrow B\}, \{A \vee C\}$ is inconsistent / unsatisfiable.

Since, all paths are closed, the obtained tableau is contradictory.

\therefore The set S is unsatisfiable / consistent.

*^{up} 3) show that $S = \{\sim(A \vee B), (B \rightarrow C), (A \vee C)\}$ is inconsistent / satisfiable.

Set:- Description Formula Linenumber

Tableau root

$\{\sim(A \vee B) \wedge (B \rightarrow C) \wedge (A \vee C)\}$

rule 1(1)

$\sim(A \vee B)$

1

$\sim(A \vee B)$

2

$(B \rightarrow C)$

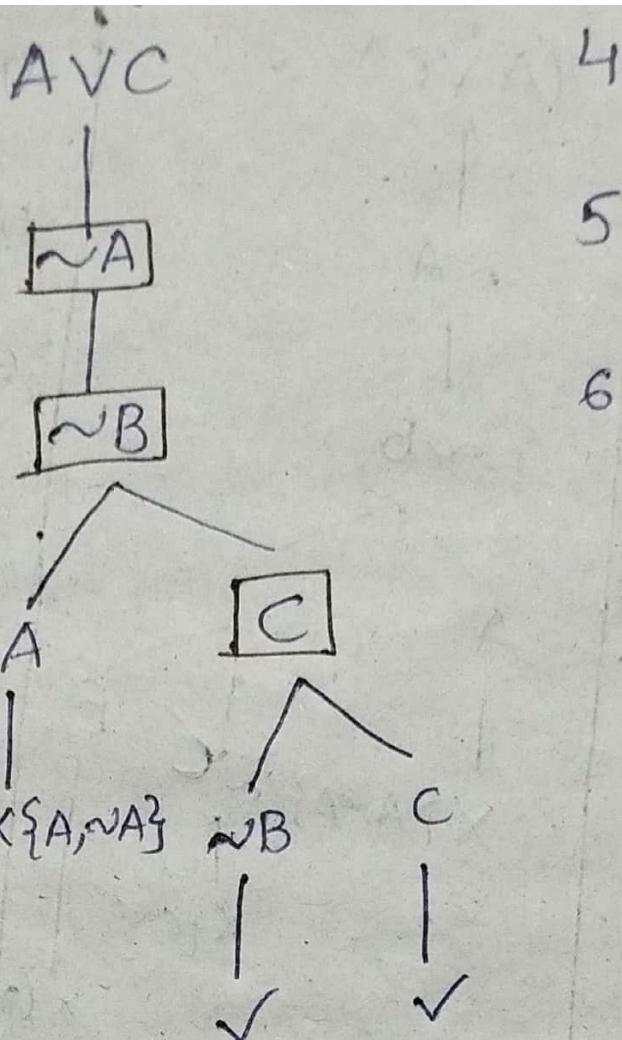
3

$(B \rightarrow C)$

4

$A \vee C$

5



Rule 6(2)

Rule 3(4)

Rule 6(3)

~~Model~~ :-

Since, there is atleast one open path the tableau is not contradictory and therefore, the set S is satisfiable/consistent.

The Model for the formula $\sim(A \vee B) \wedge (B \rightarrow C) \wedge (A \vee C)$ is $\{A=F, B=F, C=T\}$ (or) $\{A=T, B=T, C=T\}$

Show that B is a logical consequence of $S = \{A \rightarrow B, A\}$.

<u>Sol:- Description</u>	<u>Formula</u>	<u>line number</u>
Tableau root	$\sim B$	1
Premise 1	$A \rightarrow B$	2
Premise 2	A	3
Rule 6(2)	$\begin{array}{c} \sim A & B \\ & \\ x\{\sim A\} & x\{B, \sim B\} \end{array}$	4

From the above tableau, 'B' is tableau provable from S.
 $\therefore B$ is a logical consequence of S.

5. Show that $\alpha : B \vee \neg(A \rightarrow B) \vee \neg A$ is valid.

In order to show that α is valid, we have to show that α is tableau ~~provable~~ \leftarrow i.e., tableau tree with $\neg\alpha$ is contradictory.

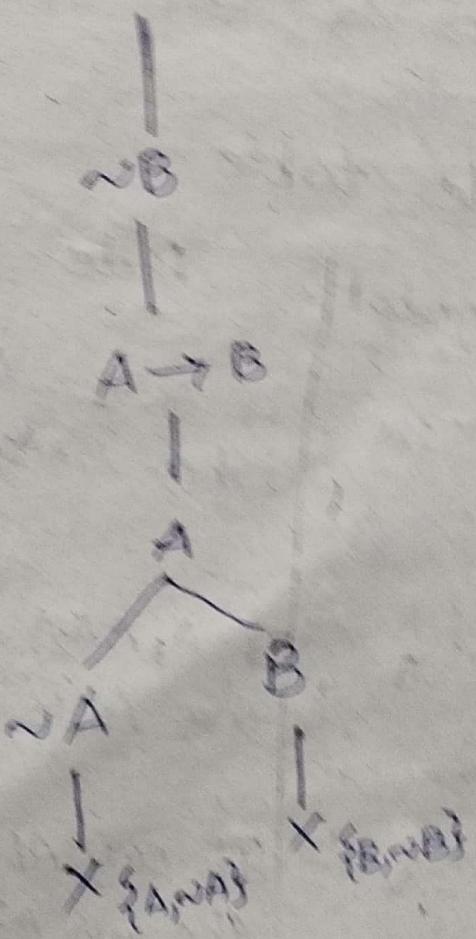
Set- Description

Tableau

$\neg(B \vee \neg(A \rightarrow B) \vee \neg A)$

Rule 4(1)

Rule 6(3)



5. Show that $\alpha : B \vee \sim(A \rightarrow B) \vee \sim A$ is valid.

Sol:- Description Formula Linenumber

Tableau

$\sim(B \vee \sim(A \rightarrow B) \vee \sim A)$

Rule 4(i)

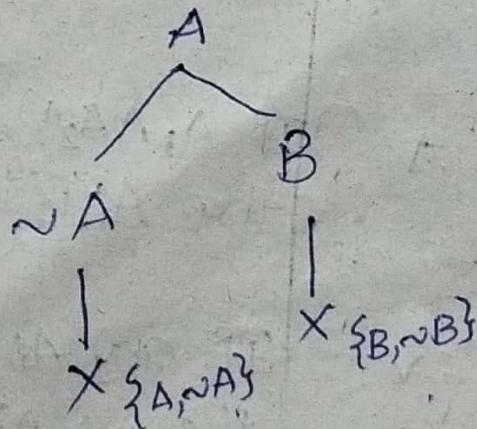
$\sim B$

2

$A \rightarrow B$

3

Rule 6(3)



* Unit-1 Current Trends in AI

1. Neural Networks & Deep learning - Patterns, multiple layers of analysis.
2. Transfer learning - Pre trained model eg:- cars, trucks.
3. Natural language Processing - Machine will understand and produce human language.
4. Reinforcement Learning - System learns "trial & error" approach
5. Explainable AI (XAI) - AI decision making

Eliminating

Introducing

Eliminating

Introducing

Eliminating

Introducing

Eliminating

* Unit-1 Part-2

character

Solved

1. Types

NDS rule table

<u>Rule Name</u>	<u>Symbol</u>	<u>Rule</u>	<u>Description</u>
Introducing \wedge	$I:\wedge$	If A_1, A_2, \dots, A_n then $A_1 \wedge A_2 \wedge \dots \wedge A_n$	If A_1, \dots, A_n are true then their conjunction is also true.
Eliminating \wedge	$E:\wedge$	If $A_1 \wedge A_2 \wedge \dots \wedge A_n$ then $A_i (1 \leq i \leq n)$	If $A_n - A_i$ is true then any A_i is true.
Introducing \vee	$I:\vee$	If any $A_i (1 \leq i \leq n)$, then $A_1 \vee A_2 \vee \dots \vee A_n$	If any A_i is true then disjunction of

* Unit-1 Current Trends in AI

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 - ↓
 - transparent
 - &
 - understandable to humans.
- U-2P Explain significance of NDS with Example?

* Natural Deduction System:-

It is based on a set of deductive inference rules. Let A_1, A_2, \dots, A_k where $1 \leq k \leq n$ are a set of atoms and α_j where $1 \leq j \leq m$ and β are well-formed formulae, the inference rules may be stated as follows:-

NDS rule table

<u>Rule Name</u>	<u>Symbol</u>	<u>Rule</u>	<u>Description</u>
Introducing \wedge	I: \wedge	If A_1, A_2, \dots, A_n then $A_1 \wedge A_2 \wedge \dots \wedge A_n$	If A_1, \dots, A_n are true then their conjunction is also true.
Eliminating \wedge	E: \wedge	If $A_1 \wedge A_2 \wedge \dots \wedge A_n$ then $A_i (1 \leq i \leq n)$	If $A_1 \wedge \dots \wedge A_n$ is true then any A_i is true.
Introducing \vee	I: \vee	If any $A_i (1 \leq i \leq n)$, then $A_1 \vee A_2 \vee \dots \vee A_n$	If any A_i is true then disjunction of A_1, A_2, \dots, A_n is true.

Eliminating \vee

E: \vee

If $A_1 \vee A_2 \vee \dots \vee A_n$,
 $A_1 \rightarrow A, \dots, A_n \rightarrow A$
then A .

If $A_1 \vee \dots \vee A_n$,
 $A_1 \rightarrow A, \dots, A_n \rightarrow A$
are true then
 A has to be
true.

Introducing \rightarrow

I: \rightarrow

If from $\alpha_1, \dots, \alpha_n$
infer β is proved,
then $(\alpha_1 \wedge \alpha_2 \wedge \dots \wedge \alpha_n) \rightarrow \beta$
 $\rightarrow \beta$ is proved.

Eliminating \rightarrow

E: \rightarrow

If $A_1 \rightarrow A, A_1$ then A

Introducing \leftrightarrow

I: \leftrightarrow

If $A_1 \rightarrow A_2, A_2 \rightarrow A_1$, then
 $A_1 \leftrightarrow A_2$.

Eliminating \leftrightarrow

E: \leftrightarrow

If $A_1 \leftrightarrow A_2$ then
 $A_1 \rightarrow A_2, A_2 \rightarrow A_1$

Introducing \sim

I: \sim

If from A infer
 $A_1 \wedge \sim A_1$ is prove
-ed then $\sim A$ is
proved.

Eliminating \sim

E: \sim

If from $\sim A$
infer $A_1 \wedge \sim A_1$
is proved then
 A is proved.

* Unit 1
Part 2 Topics

Characteristics of problems that can be
solved by various AI techniques.

1. Types of problem - Ignorable (Theorem proving-
lemmas - simple control
strategies).

— Recoverable (water Jug,
Back tracking, push
down stack).

— Irrecoverable (chess, card
controlled process-
ing)

Eliminating \vee

EV

IF $A_1 \vee A_2 \vee \dots \vee A_n$,
 $A_1 \rightarrow A, \dots, A_n \rightarrow A$
then A

IF $A_1 \vee \dots \vee A_n$,
 $A_1 \rightarrow A, \dots, A_n \rightarrow A$
are true then
 A has to be
true.

Introducing \rightarrow

I: \rightarrow

IF From A_1, \dots, A_n
infer B is proved,
then $(A_1 \wedge A_2 \wedge \dots \wedge A_n) \rightarrow B$
 $\rightarrow B$ is proved

Eliminating \rightarrow

E: \rightarrow

IF $A_1 \rightarrow A, A_1$ then A

Introducing \leftrightarrow

I: \leftrightarrow

IF $A_1 \rightarrow A_2, A_2 \rightarrow A_1$, then
 $A_1 \leftrightarrow A_2$

Eliminating \leftrightarrow

E: \leftrightarrow

IF $A_1 \leftrightarrow A_2$ then
 $A_1 \rightarrow A_2, A_2 \rightarrow A_1$

Introducing \sim

I: \sim

IF from A infer
 $A_1 \wedge \sim A_1$, PB prove/
-ed then $\sim A_1$ is
proved.

Eliminating \sim

E: \sim

IF from $\sim A$
infer $A_1 \wedge \sim A_1$,
PB proved then
 A is proved

* Unit 1
part 2 topics

characteristics of problems that can be
solved by various AI techniques.

1. Types of problem - Irresolvable (Theorem proving-
lemmas = simple control
strategies).

— Recoverable (water jug,
back tracking, push
down stack).

— Irrecoverable (chess, card
controlled process
= n)

Eliminating \vee

E: \vee

If $A_1 \vee A_2 \vee \dots \vee A_n$,
 $A_1 \rightarrow A, \dots, A_n \rightarrow A$
then A .

If $A_1 \vee \dots \vee A_n$,
 $A_1 \rightarrow A, \dots, A_n \rightarrow A$
are true then
 A has to be
true.

Introducing \rightarrow

I: \rightarrow

If from $\alpha_1, \dots, \alpha_n$
infer B is proved.
then $(\alpha_1 \wedge \alpha_2 \wedge \dots \wedge \alpha_n) \rightarrow B$
is proved.

Eliminating \rightarrow

E: \rightarrow

If $A_1 \rightarrow A, A_1$ then A

Introducing \leftrightarrow

I: \leftrightarrow

If $A_1 \rightarrow A_2, A_2 \rightarrow A_1$, then
 $A_1 \leftrightarrow A_2$.

Eliminating \leftrightarrow

E: \leftrightarrow

If $A_1 \leftrightarrow A_2$ then
 $A_1 \rightarrow A_2, A_2 \rightarrow A_1$

Introducing \sim

I: \sim

If from A infer
 $A_1 \wedge \sim A_1$ is prove
-ed then $\sim A$ is
proved.

Eliminating \sim

E: \sim

If from $\sim A$
infer $A_1 \wedge \sim A_1$
is proved then
 A is proved.

* Unit-1
part-2: Topics
Characteristics of problems that can be
solved by various AI techniques.

1. Types of problem - Ignorable (Theorem proving-
lemmas - simple control
strategies).

— Recoverable (water jug,
back tracking, push
down stack).

— Irrecoverable (chess, card
controlled process
-ng)

2. Decomposability of a problem

3. Role of knowledge - facts & rules -
problem solving -
state space

4. Consistency of knowledge Base - It is daytime/
It is night time -

inconsistent any path

5. Requirements of Solutions ~~Absolute (exact ans)~~

Relative (approximate ans)
(or)

Relatively good answer.

Travel Sales Person
problem

Best path
problems

multiple/Any
path prblm.

* Example for NDS:-

* Show that $A \wedge (B \vee C)$ is deduced from $A \wedge B$.

SL:-	Description	Formula	Comments
	Theorem	from $A \wedge B$ infer $A \wedge (B \vee C)$	To be proved
	Hypothesis	$A \wedge B$	1
	E: $\wedge(1)$	A	2
	E: $\wedge(1)$	B	3
	I: $\vee(3)$	$B \vee C$	4
	I: $\wedge(2)(4)$	$A \wedge (B \vee C)$	Proved.

* Deduction Theorem:-

* To prove a formula $(\alpha_1 \wedge \alpha_2 \wedge \dots \wedge \alpha_n) \rightarrow B$..
 It is sufficient to prove a theorem from $\alpha_1, \alpha_2, \dots, \alpha_n$ infer B . (a) $\alpha_1, \alpha_2, \dots, \alpha_n \vdash B$ (or)
 $\{\alpha_1, \alpha_2, \dots, \alpha_n\} \Rightarrow B$.

* Conversely, If $(\alpha_1 \wedge \alpha_2 \wedge \dots \wedge \alpha_n) \rightarrow B$ is proved then the theorem from $\alpha_1, \alpha_2, \dots, \alpha_n + B$ is assumed to be proved.

* Show that infer $[A \xrightarrow{\alpha_1} B] \wedge [B \xrightarrow{\alpha_2} C] \rightarrow (A \rightarrow C)$

→

Set	Description	Formula	Comments
	Theorem	from $((A \rightarrow B) \wedge (B \rightarrow C))$ infer $(A \rightarrow C)$	To be proved
	Hypothesis 1	$A \rightarrow B$	1
	Hypothesis 2	$B \rightarrow C$	2
	Sub-theorem	from A. infer C	3
	Hypothesis	A	3.1
	Elimination/E.	B	3.2
	E: $\rightarrow (1, 3.1)$	C	3.3
	E: $\rightarrow (2, 3.2)$	$A \rightarrow C$	proved.
	I: $\rightarrow (3)$		

* Problem Statement:- The theorem infer $[(A \rightarrow B) \wedge (B \rightarrow C)] \rightarrow (A \rightarrow C)$ is reduced to the theorem from $(A \rightarrow B), (B \rightarrow C)$ infer $(A \rightarrow C)$ using Deduction theorem.

* Further, to prove $(A \rightarrow C)$. we will have to prove a sub-theorem from $A \vdash C$ / $A \text{ infer } C$.

The proof of the theorem is as

follows:-

Development of AI languages

knowledge Representation Schemes

Pattern Recognition

Flexible Search

Program

} AI
program
language
Data

prolog - logic Programming language

- Inorder to predicate logic

LISP - Functional language

- Lambda calculus (Formal System of mathematical logic).

pop-2 - Stack Based language

- high flexible (POP II - embedded in AI environment).

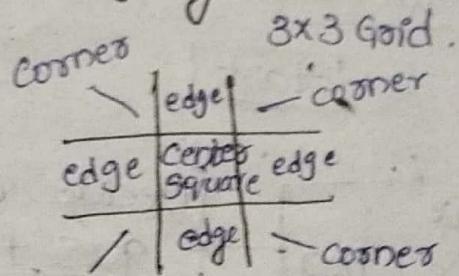
ML - Machine Learning.

TIC-TAC-TOE Game Playing.

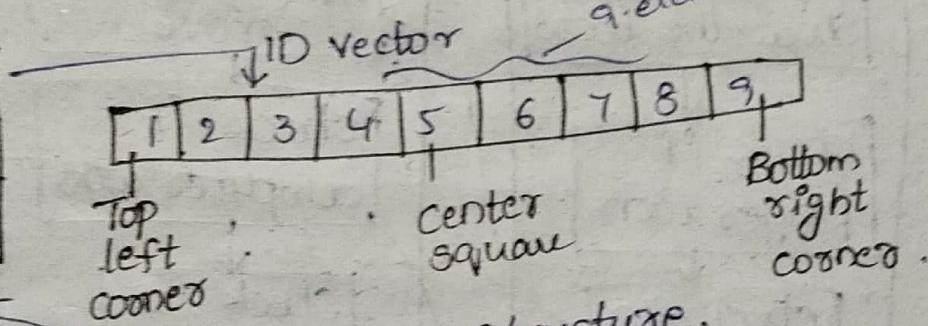
Human Computer
(1) x, 0-(2)

↓
first person / player

choose 'x' (Mask) corner



1	2	3
4	5	6
7	8	9



Approach 1: Board is a Data Structure.

0	0	0
0	0	0
0	0	0

Blank initially

* 'x' represented with '0'

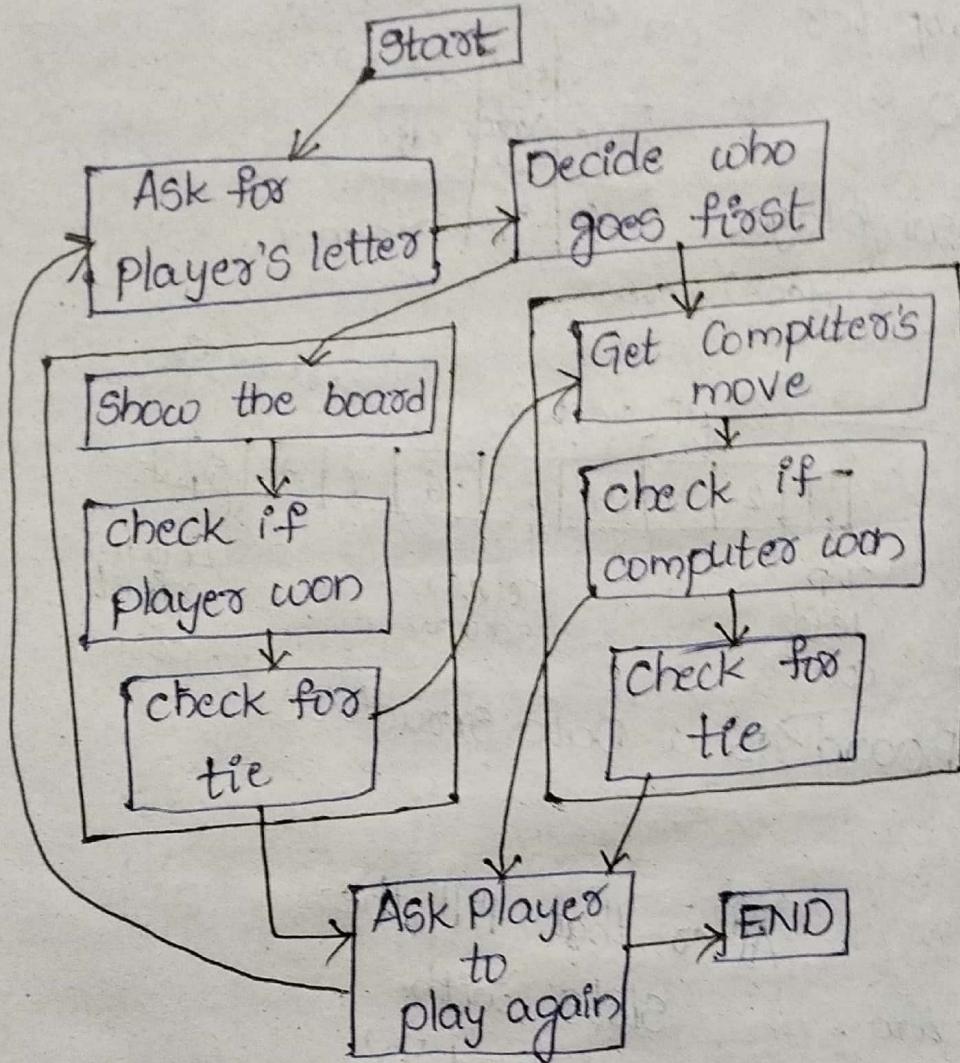
亲 0

After that fill the
etc

slots with
'x' and 'o'

Approach 2:- Move Table DS → used by Computer System not by HB
 3rd element vector. Ternary number - 0, 1, 2
 Base (3)

Flowchart



Advantages:-

* The program is very efficient in time.

Disadvantages:-

- * Lot of space is required to store the move table.
- * Lot of work to specify all entries in the move table.
- * Difficult to extend.
- * Highly error prone. as a data is voluminous and not at all intelligent.
- * The program is not at all intelligent.

II-Approach *

Data Structures

Board: 9 element vector, 2 → blank, 3 → x, 5 → 0

Turn: Integer → move to be played.

1 → 1: ... 9 → last move

Algorithm: 3 Sub Procedures

1. Make2: Return 5 → Center Square blank
otherwise, return any blank non-corner
square → (2, 4, 6 or 8).

2. Poss win(P): Returns 0, if P cannot move on
next move. otherwise, returns no. of square
that constitutes the winning move.

- checks each row, column and diagonal, one
at a time.

- Product is 18 ($3 \times 3 \times 2$), X can win, If it
is 50 ($5 \times 5 \times 2$), O can win.

3. Go(n): Makes a move in square "n"
sets Board to 3 if turn is odd and
to 5 if turn is even.

Strategy of each turn:-

Turn=1 Go(1)

Turn(2): Bound[5] = blank, Go(5) else Go(1)

Turn=3: If Board[9] is blank, Go(9) else Go(3).

Turn=4: If Posswin(x) is not zero, then Go(Posswin(x)), else Go(Make2);

Turn=5: If posswin(x) is not zero, then Go(Posswin(x)), else if posswin(o) is not zero, then Go(Posswin(o)), else if posswin(o) is ~~not zero~~, then Board[7] is blank, Go(7) else Go(3);

Turn=6: If posswin(o) is not zero, then Go(Posswin(x)), else if posswin(x) is not zero, then Go(Posswin(x)), else Go(Make2);

Turn=7: - If posswin(x) is not 0, then Go(Posswin(x)), else if posswin(o) is not 0, then Go(Posswin(o)), else go anywhere that is blank.

Turn=8: - If posswin(o) is not zero, Go(Posswin(o)) else if posswin(x) is not 0, then Go(Posswin(x)) else go anywhere that is blank,

Turn=9: is same as turn=7,

Turn=3: If Board[9] is blank, Go(9) else Go(3).

Turn=4: If Posswin(x) is not zero, then Go(Posswin(x)), else Go(Make2);

Turn=5: If posswin(x) is not zero, then Go(Posswin(x)), else if posswin(o) is not zero, then Go(Posswin(o)), else if posswin(o) is not zero, then Board[7] is blank, Go(7) else Go(3);

Turn=6: If posswin(o) is not zero, then Go(Posswin(x)), else if posswin(x) is not zero, then Go(Posswin(x)), else Go(Make2);

Turn=7: - If posswin(x) is not 0, then Go(Posswin(x)), else if posswin(o) is not 0, then Go(Posswin(o)), else go anywhere that is blank.

Turn=8: - If posswin(o) is not zero, Go(Posswin(o)), else if posswin(x) is not 0, then Go(Posswin(x)) else go anywhere that is blank,

Turn=9: is same as turn=7,

2nd approach (Tic-Tac-Toe)

Advantages:-

- * program is efficient in terms of space.
- * It is easy to understand the strategy.

Disadvantages:-

- * It is not efficient in terms of tie.
- * It is not possible to generalize the programmers knowledge.

III Approach

Data Structures

Board: Magic square of order 3

List: Block list of each player. is maintained.

Strategy :-

- Each pair of blocks a player owns is considered.
- Difference D between 15 and the sum of the two blocks is computed.
 - If $D \leq 0$ or $D > 9$, then these two blocks are not collinear and so can be ignored; otherwise if the block representing the difference is blank, then player can make in that block.
 - This strategy will produce a possible for the player.

Construction of Magic Square :-

3x3

8	1	6
3	5	7
4	9	2

$$\frac{n(n^2+1)}{2} = 15$$

State-Space

Set of all possible states from start state to goal state.

Production System - It is a formalism that helps AI programs to do search process more conveniently in state-space problem.

* It consists of start and goals state of the problem along with one or more databases consisting of suitable & necessary information for the particular task.

* It consists of no. of production Rules in which each production rule has left side that determines applicability of the rule and a right-side that describes the action to be performed if the rule is applied.

* It consists of Control strategy that specifies sequence in which the rules are applied when several rules match at once.

* Water Jug Problem

problem statement:- we have 2 Jugs, a five(5-gallon) jug and the other 3-gallon(3-g) jug with low measuring marks on them.

- * There is endless supply of water through the tap.
- * Our task is to get 4-gallon(4-g) of water in 5-gallon(5-g) Jug.

Solution:-

State-space for this problem can be described as the set of ordered pairs of integers (x, y) such that x represents the number of gallons of water in 5-gallons(5-g) jug and y for 3-g) Jug.

Start state is $(0, 0)$

goal state is $(4, n)$

For any Value $N \leq 3$ (0-3)

* Production Rules for Water Jug problem.

Rule no.	Left of Rule	Right of Rule	Description
1.	$(x, y \mid x < 5)$	$(5, y)$	Full 5-g
2.	$(x, y \mid x > 0)$	$(0, y)$	Empty 5-g jug
3.	$(x, y \mid y < 3)$	$(x, 3)$	Fill 3-g jug
4.	$(x, y \mid y > 0)$	$(x, 0)$	Empty 3-g jug
5.	$(x, y \mid x + y \leq 5 \wedge y > 0)$	$(x+y, 0)$	Empty 3-g into 5-g jug
6.	$(x, y \mid x + y \leq 3 \wedge x > 0)$	$(0, x+y)$	Empty 5-g into 3-g jug
7.	$(x, y \mid x + y \geq 5 \wedge y > 0)$	$(5, y - (5-x))$	Pour water from 3-g jug into 5-g jug until 5-g jug is full
8.	$(x, y \mid x + y \geq 3 \wedge x > 0)$	$(x - (3-y), 3)$	Pour water from 5-g jug into 3-g jug until 3-g jug is full

5 3
X Y

(0, 0)

↓

(5, 0) — R(1)

↓

(2, 3) — R(8)

↓

(2, 0) — R(4)

↓

(0, 2) — R(6)

↓

(5, 2) — R(1)

↓

(4, 3) — R(8)

Solution:-

F-SL(4, N)

Rule applied	5-giug	3-giug	Step No
Start State	0	0	1
	5	0	2
	2	3	3
	0	0	4
	2	2	5
	4	3	6
goal state	4	—	—

* Missionaries and Cannibals Problem

problem Statement:-

Three ~~Missionaries~~ ^{Missionaries} and 3 ^{Cannibals} wants to cross a river.

There is a boat on their side of the river, that can be used by either 1 (or) 2 persons.

How should they use this boat to cross the river in such a way that cannibals never out no. missionaries on either side of the river?

If the cannibals ever out numbers the missionaries (on either bank). Then the missionaries will be eaten.

How can they all cross over without

anyone being eaten.

Solution:-

State-Space for this problem can be described as the set of ordered pairs of left and right banks of the river as (L, R) where each bank is represented as a list $[nM, mC, B]$.

Here, n is the no. of missionaries 'M'
 m is the no. of cannibals 'C'.
and B represents the Boats

$\boxed{([nM, mC, OB], [nM, mC, !OB])}$

1. Start-State:- $([3M, 3C, !OB], [0M, 0C, OB])$

• $!OB$ means when boat is present at OB means it is absent.

2. Any State:- $([n_1M, m_1C, -], [n_2M, m_2C, -])$

With Constraints at any state as,

$$n_1 (\neq 0) \geq m_1 ; n_2 (\neq 0) \geq m_2 ;$$

$$n_1 + n_2 = 3 ; m_1 + m_2 = 3$$

Boat can be on either side

3. Goal States:- $([0M, 0C, OB], [3M, 3C, !OB])$.

Production Rules for missionaries cannibals problem.

Rule No	Left side of Rule	Right side of Rule
	Rules for boat going from left bank to right bank of river.	
L ₁	$[(n, M, m, C, IB)], [(n_2 M, m_2 C, OB)] \rightarrow [(n_1-2)M, m_1C, OB]$	$[(n_2+2)M, m_2C, IB]$
L ₂	"	$\rightarrow [(n_1-1)M, (m_1-1)C, OB],$ $[(n_2+1)M, (m_2+1)C, IB]$
L ₃	"	$\rightarrow [(n_1 M, (m_1-2)C, OB],$ $[(n_2 M, (m_2+2)C, IB)]$
L ₄	"	$\rightarrow [(n_1 M, (m_1-1)C, OB],$ $[(n_2 M, (m_2+1)C, IB)]$

Rules for boat going from R to L.

R ₁	$[(n, M, m, C, OB)],$ $[(n_2 M, m_2 C, IB)]$	$\rightarrow [(n_1+2)M, m_2 C, IB]$ $[(n_2-2)M, m_2 C, OB]$
----------------	---	--

Solution:-

L		R	
M	C	M	C
2	2	1	1
3	2	0	1
3	0	0	3
3	1	0	2
1	1	2	2
2	2	1	1
0	2	3	1
0	3	3	0
0	1	3	2
0	2	3	1
0	0	3	3

goal state

Unit-II

Cryptarithmetic

$$\begin{array}{r} 1 \ 2 \\ + 9 \ 4 \\ \hline 106 \end{array}$$

0-9
can be
replaced
with
A-Z

$$\begin{array}{r} \downarrow \\ K \ D \\ + S \ N \\ \hline KZ0 \end{array}$$

Ex:- $C_2 C_1$ $T O$

$+ G O$

OUT

↓
has to be '1' (msb)

letter digit

$T \rightarrow 2$

$O \rightarrow 1$

$G \rightarrow 8$

$U \rightarrow 0$

$\rightarrow C_2 = 0 = 1$

$O + O = T$

$\Rightarrow 1 + 1 = 2$

$T = ?$

C_2 C_1
 $\boxed{T2}$

$\boxed{01}$

$+ \boxed{G8}$

$\boxed{01}$

No carry
so, C_1 is '0'

$C_1 + T + G = U + 10$

$\Rightarrow 0 + 2 + G = U + 10$

$\Rightarrow G = U + 8$

Let U

Then,

$$\begin{aligned} G &= U + 8 \\ &= 8 \end{aligned}$$

1 00 62

* SEND [0-9]

+ MORE

MONEY

c_4

$c_3=0$
9

$c_2=1$

5

$c_1=1$

6

7

+

1

0

8

5

1

0

6

5

2

$$1) M=1 \quad (\because C_4=1 \text{ & } 10 \leq S+M \leq 19)$$

$$2) C_3+S+M=0+10 \quad \text{Let } C_3=0. \quad \boxed{0=0}$$

$$\text{Then } S+M \geq 10$$

i.e., It is possible only when $\boxed{S=9}$

3) $C_2+E+0=N \Rightarrow$ either $E=N$ (or) $E+1=N$
 depending on the value of $C_2 (=0 \text{ or } 1)$.
 $E \neq N \quad \therefore E+1=N$, only when $C_2=1$.

$$\text{Let, } \boxed{E=5}$$

$$\text{Then } N=E+1=5+1=6$$

$$\therefore \boxed{N=6}$$

$$4) C_1+N+R=E+10.$$

Let $C_1=0$. Then $6+R=15 \Rightarrow R=9$ (conflict with S).

$$\text{So, } C_1 \neq 0. \quad \text{Let } \boxed{C_1=1}$$

$$\text{Then } 1+6+R=15 \Rightarrow \boxed{R=8}$$

$$5) D+5=Y+10 \Rightarrow D=Y+5 \Rightarrow D \geq 5$$

i.e., D can hold 5, 6, 7, 8 or 9.

5, 6, 8 & 9 are already assigned.

$$\text{Let, } \boxed{D=7}. \quad \text{Then } Y=D-5$$

$$Y=7-5$$

$$\boxed{Y=2}$$

* CROSS
ROADS
+
DANGER

C_5	C_4	C_3	C_2	C_1	
C	9	R	O	S^2	S^2
$+$	R	O	A	D	S^2
D	A	N	G	E^3	R^4

1) $D=1$ ($\because C_5=1$ & $10 \leq C_4 + R \leq 19$)

2) $S+R=2S=R$ ($\because R$ is even)

i.e., R can be $0, 2, 4, 6$ (or) 8 .

Let $R=4$ Then $S=2$

3) $C_4 + C + R = A + 10$. Let $C_4 = 0$.

Then $C+4=A+10 \Rightarrow C=A+6$

$\Rightarrow C \geq 6$. C can be $6, 7, 8$ (or) 9

Let, $C=9$.

Then $A=3$ which has a conflict

with $E=3$.

Our assumption that $R=4$ is wrong.

Backtracking and assume

$$R=6 \quad \text{Then} \quad S=3$$

$$C_5=1 \quad C_4=0 \quad C_3=0 \quad C_2=0 \quad C_1=0$$

$$\begin{array}{cccccc} & & & & 3 & 3 \\ & 9 & 6 & 2 & 3 & \\ \end{array}$$

$$\begin{array}{cccccc} & & & 1 & 3 \\ & 6 & 2 & 5 & & \\ \end{array}$$

$$\begin{array}{cccccc} & & 7 & 4 & 6 \\ \hline 1 & 5 & 8 & & & \end{array}$$

$$4) C_4 + C + 6 = A + 10. \quad \text{Let} \quad C_4 = 0$$

$$\text{Then} \quad C + 6 = A + 10. \Rightarrow C = A + 4$$

$\Rightarrow C \geq 4$. C can hold $4, 5, 6, 7, 8$ or 9 .

$$\text{Let } C=9. \text{ Then } A = C - 4$$

$$= 9 - 4$$

$$A = 5$$

$$5) C_3 + R + 0 = N. \quad \text{Let} \quad C_3 = 0$$

$$\text{Then,} \quad 6 + 0 = N \quad \therefore N \leq 9$$

$$\Rightarrow 0 + 6 \leq 9 \Rightarrow 0 \leq 3$$

The possible combinations for $N \leq 9$

are $(6,0)$, $(6,1)$, $(6,2)$ and $(6,3)$. The possible

combinations are $(6,2)$

$$O=2$$

$$N=8$$

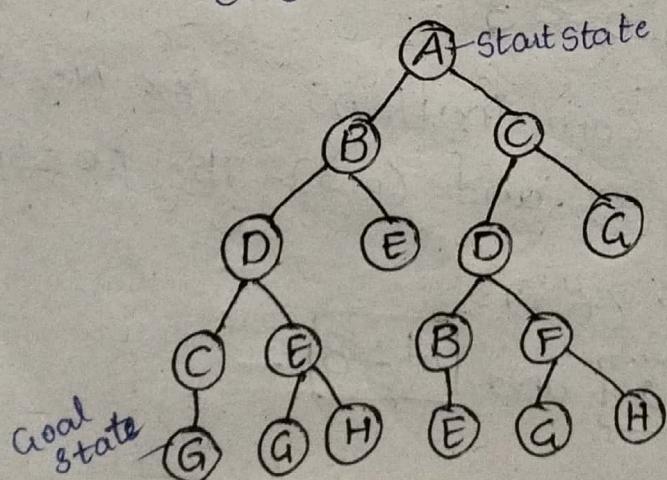
Exhaustive Search (Uninformed Search)

- * Breadth First Search
- * Depth First Search.

Breadth First Search:- (BFS)

Algorithm:

- 1) Create a variable called NODE-LIST and set it to the initial state.
- 2) Until a goal state is formed, (i) NODE-LIST is empty:
 - a) Remove the first element from NODE-LIST and call it E. If NODE-LIST is empty, then quit.
 - b) For element E, do the following:
 - i) Apply the rule to generate a new state. (get the successors of E).
 - ii) If the new state is a goal state, quit and return this state.
 - iii) Otherwise, add the new state to the end of NODE-LIST



1) NODE_LIST: A

2) E:A , NODE_LIST: B,C

3) E:B , NODE_LIST: C,D,E.

4) E:C , NODE_LIST: D,E, D,G

* T_1 , the new state that is generated is a goal state. Return G and quit.

* The algorithm returns the path $A \rightarrow C \rightarrow G$ by following the parent pointers of the node corresponding to G.

Advantages:-

* It is one of the simplest search strategies

* It is Complete.

If there is a solution, BFS is guaranteed to find it.

* If there are multiple solutions, the minimal solution can be formed.

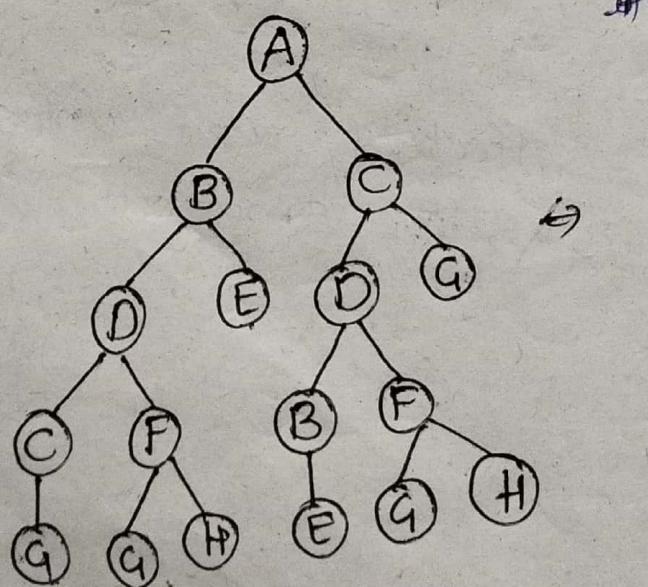
Disadvantages:-

It cannot be effectively used unless the search space is quite small.

Depth First Search (DFS):-

Algorithm:-

- 1) Create a variable called NODE-LIST and set it to the initial state.
- 2) If the initial state is a goal state, quit and return SUCCESS.
- 3) Otherwise, do the following until SUCCESS (or) FAILURE is signaled:
 - a) Generate a successor, E, of the initial state. If there are no more successors, signal failure.
 - b) Call depth first search with E as initial state.
 - c) If success is returned, signal success. Otherwise, continue in this loop.



1) NODE_LIST: A

2) E:A A is not the goal state.

* NODE_LIST: BC

3) E:B B is not the goal state.

* NODE_LIST: DEC

4) E:D, D is not the goal state

NODE_LIST: CFEC

5) E:C, C is not goal state

NODE_LIST: GFEC

6) E:G, G is a goal state

Return SUCCESS.

Algorithm will return the path

A - B - D - C - G.

Advantages:-

* It requires less memory since, only the nodes on current path are stored.

* It may find a solution without examining much of the search space at all.

Disadvantages:-

* It may find a sub-optimal solution.

* It is incomplete.

i.e., means a solution may not be found even if it exists.

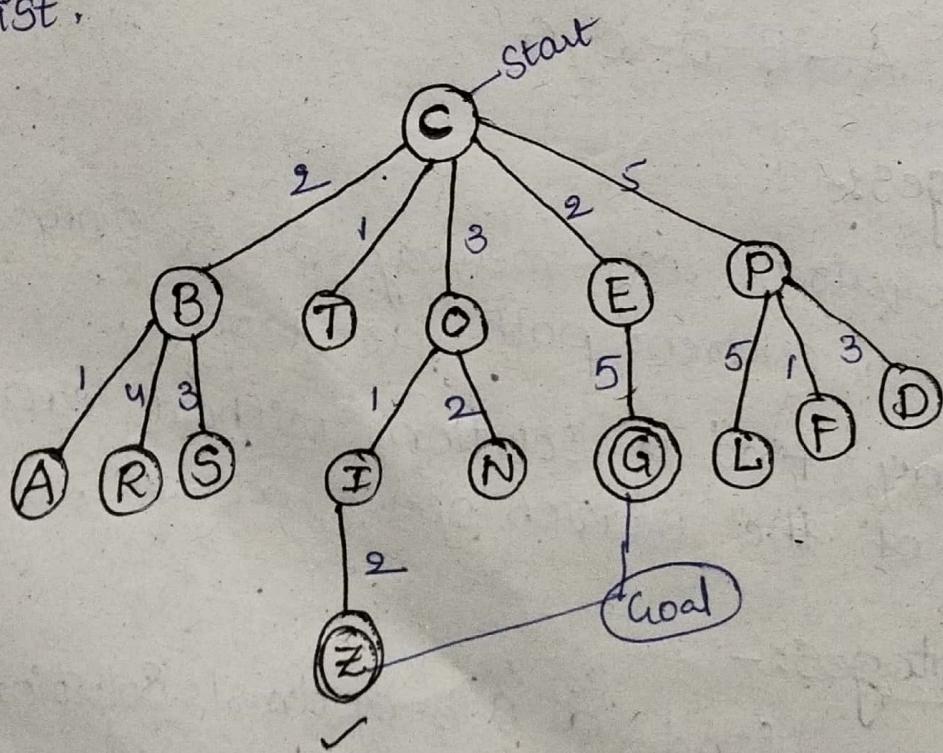
Branch and Bound Search (Uniform Cost Search)

Algorithm:-

- 1) Insert root node into the priority queue.
- 2) Remove the element with highest priority.
(Element with least cumulative cost.)
- 3) If the removed node is the goal node,
 - print total cost and stop. the algorithm.

else

- Enqueue all the children of the current node to the priority queue, with the cumulative cost from the root as priority and the current node to the visited list.



$\Theta: C(0)$

$\Theta: B(2), T(1), O(3), E(2), P(5)$

\Downarrow

~~$\Theta: T(1), B(2), E(2), O(3), P(5)$~~

$\Theta: A(3), R(6), S(5), E(2), O(3), P(5)$

\Downarrow

~~$\Theta: E(2), A(3), O(3), P(5), S(5), R(6)$~~

$\Theta: G(7), A(3), O(3), P(5), S(5), R(6)$

\Downarrow

~~$\Theta: A(3), O(3), P(5), S(5), R(6), G(7)$~~

$\Theta: I(4), N(5), S(5), P(5), R(6), G(7)$

~~$\Theta: I(4), N(5), S(5), P(5), R(6), G(7)$~~

$\Theta: Z(6), N(5), S(5), P(5), R(6), G(7)$

\Downarrow

~~$\Theta: N(5), S(5), P(5), R(6), Z(6), G(7)$~~

~~$\Theta: N(5), S(5), P(5), R(6), Z(6), G(7)$~~

(no child).

$\Theta: L(10), F(6), D(8), P(5), R(6), Z(6), G(7)$

\Downarrow

~~$\Theta: P(5), R(6), Z(6), L(10), F(6), D(8), G(7)$~~

$\Theta: G(7), D(8), L(10)$

$\therefore Z$ is the goal node with cumulative cost of 6. The path is $C-O-I-Z$.