

Assignment 2

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vector

Abstract—This document contains the solution to find the vertex of parabola

Download all python codes from

<https://github.com/Anjalibagade/EE5600/tree/master/Assignment2>

and latex codes from

<https://github.com/Anjalibagade/EE5600/Assignment2>

Problem

Vector-2, Example-4, Question-5

Sketch the loci of the following equation

$$y^2 = x \quad (0.0.1)$$

Solution: Given equation is

$$y^2 = x \quad (0.0.2)$$

We can write above equation as

$$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (0.0.3)$$

where,

$$\mathbf{V} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} -\frac{1}{2} \\ 0 \end{pmatrix}, f = 0 \quad (0.0.4)$$

$$|V| = 0 \quad (0.0.5)$$

From above equation we can say that the curve is parabola.

The vertex of parabola can be given as \mathbf{c}

$$\begin{pmatrix} \mathbf{u}^T + \eta \mathbf{p}_1^T \\ V \end{pmatrix} \mathbf{c} = \begin{pmatrix} -f \\ \eta \mathbf{p}_1 - \mathbf{u} \end{pmatrix} \quad (0.0.6)$$

where

$$\mathbf{p}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \eta = \mathbf{p}_1^T \mathbf{u} \quad (0.0.7)$$

$$\eta = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} -\frac{1}{2} \\ 0 \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} \end{pmatrix} \quad (0.0.8)$$

From 0.0.6 we can find out few values given below

$$(\mathbf{u}^T + \eta \mathbf{p}_1^T) = \begin{pmatrix} -\frac{1}{2} & 0 \end{pmatrix} + \begin{pmatrix} -\frac{1}{2} \end{pmatrix} \begin{pmatrix} 1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 \end{pmatrix} \quad (0.0.9)$$

$$(\eta \mathbf{p}_1 - \mathbf{u}) = \begin{pmatrix} -\frac{1}{2} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} - \begin{pmatrix} -\frac{1}{2} \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (0.0.10)$$

Substituting all the values in equation 0.0.6

$$\begin{pmatrix} -1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{c} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad (0.0.11)$$

$$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{c} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (0.0.12)$$

$$\mathbf{c} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (0.0.13)$$

The eccentricity of parabola is given by,

$$e = \sqrt{1 - \frac{\lambda_1}{\lambda_2}} \quad (0.0.14)$$

$$\lambda_1 = 0, \lambda_2 = 1 \quad (0.0.15)$$

$$e = 1 \quad (0.0.16)$$

Directrix of parabola is given by,

$$c = \frac{\|\mathbf{u}\|^2 - \lambda_2 f}{2e^2 \mathbf{u}^T \mathbf{n}} \quad (0.0.17)$$

Substituting all the values in above equation we get,

$$c = \frac{-1}{4} \quad (0.0.18)$$

Foci of the parabola is given by,

$$\mathbf{F} = \frac{ce^2 \mathbf{n} - \mathbf{u}}{\lambda_2} \quad (0.0.19)$$

Substituting all the values in above equation

$$\mathbf{F} = \begin{pmatrix} \frac{1}{4} \\ 0 \end{pmatrix} \quad (0.0.20)$$

Plot of parabola obtained from Python code is shown below.

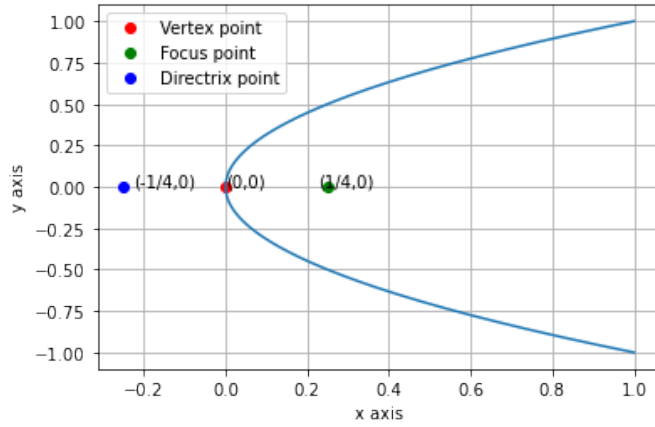


Fig. 0: Parabola with vertex c(0,0)