

- Algorithm: It is a combination of sequence of finite steps to solve a particular problem. ①

Properties of Algorithm:

- output should be generated after finite time.

- There should be at least one input.
- It's independent from programming language.

Difference between Algorithm and Program:

Algorithm

- Written at Design stage
- Need domain expert
- Written in any language
- H/W or OS independent
- Can be Analyze

Program

- Written at implementation stage
- Need programmer
- Written in programming language
- H/W or OS dependent
- Can be tested

Pseudo code: It's a description of an algorithm that is more structured than usual prose but less formal than a programming language.

- Pseudo code is our preferred notation for describing algorithms.

Measuring the running time of an algorithm:

App 1: Experimental study: Write a program that implements the algorithm.

App 2: Frequency count Method:



Prob 1:

```
main()
{
  x = y + z; → 1
}
```

$O(1)$

Prob 3:

```
main()
{
  x = y + z; → 1
  for (i = 1; i <= n; i++)
  {
    x = y + z; → n
  }
  for (i = 1; i <= n; i++)
  {
    for (j = 1; j <= n; j++)
    {
      x = y + z; →  $n^2$ 
    }
  }
}
```

$\frac{n^2 + n + 1}{O(n^2)}$

Prob 2:

```
main()
{
  x = y + z; → 1
  for (i = 1; i <= n; i++)
  {
    x = y + z; → n
  }
}
```

$\frac{n+1}{O(n)}$

Prob 4:

```
main()
{
  while (n > 1)
  {
    n = n - 10 →  $\frac{n}{10}$ 
  }
}
```

$\Rightarrow O(n)$

Prob 5:

```
main()
{
  i = 0
  while (i <= n)
  {
    i = i + 5 →  $\frac{n}{5}$ 
  }
}
```

$\Rightarrow O(n)$

Prob 6:

```
main()
{
  while (n > 1)
  {
    n =  $\frac{n}{2}$ ;
  }
}
```

$n$	$\vdots$	$k = \log_2 n$
$\frac{n}{2}$	$\vdots$	$O(\log n)$
$\frac{n}{2^2}$	$\frac{n}{2^k} = 1$	



Prob 7:

```
main ()
{
  i = 1
  while (i <= n)
  {
    i = 2 * i;
  }
}
```

$$\begin{aligned}
 &i = 1 \\
 &2 * 1 = 2 \\
 &2^2 \\
 &\vdots \\
 &2^k = n \\
 &k = \log_2 n \\
 &O(\log_2 n)
 \end{aligned}$$

Prob 8:

(3)

```
main ()
{
  while (n > 2)
  {
    n = n1/2;
  }
}
```

$$\begin{aligned}
 &n \\
 &/ \\
 &n^{1/2} \\
 &/ \\
 &n^{1/4} \\
 &\vdots \\
 &n^{1/2^k}
 \end{aligned}$$

$$\begin{aligned}
 n^{\frac{1}{2^k}} &= 2 \\
 \frac{1}{2^k} \log_2 n &= \log_2 2 \\
 \log_2 n &= 2^k \\
 k &= \log_2 \log_2 n
 \end{aligned}$$

Prob 9:

```
main ()
{
  while (n > 23)
  {
    n = n1/255;
  }
}
```

$$\begin{aligned}
 &n \\
 &/ \\
 &n^{1/255} \\
 &/ \\
 &n^{1/255^2} \\
 &\vdots \\
 &n^{1/255^k}
 \end{aligned}$$

$$\begin{aligned}
 \frac{1}{255^k} \log_{23} n &= \log_{23} 23 \\
 k &= \log_{255} \log_{23} n \\
 &O(\log_{255} \log_{23} n)
 \end{aligned}$$



Prob 10:

main()  
{ while (n > 15)  
{  
  n = n<sup>1/5</sup>  
}

}  
  n  
  |  
  n<sup>1/5</sup>  
  |  
  n<sup>1/5<sup>2</sup></sup>  
  |  
  n<sup>1/5<sup>3</sup></sup>  
  |  
  n<sup>1/5<sup>k</sup></sup>

$$n^{1/5^k} = 15$$

$$\frac{1}{5^k} \log_{15} n = \log_{15} 15$$

$$\log_{15} n = 5^k$$

$$k = \log_5 \log_{15} n$$

$$\boxed{O(\log_5 \log_{15} n)}$$

Prob 11:

main()  
{ i = 2  
  while (i < n)  
  { i = i<sup>2</sup>  
  }

$$2^k = \log_2 n$$

$$k = \log_2 \log_2 n$$

$$O(\log_2 \log_2 n)$$

}  
  2  
  |  
  2<sup>2</sup>  
  |  
  (2<sup>2</sup>)<sup>2</sup>

$$(2^2)^k = n$$

Prob 12:

main()  
{ i = 3  
  while (i < n)  
  { i = i<sup>3</sup>

}  
  3  
  |  
  3<sup>2</sup>  
  |  
  (3<sup>2</sup>)<sup>2</sup>

$$(3^2)^k = n$$

$$2^k = \log_3 n$$

$$k = \log_2 \log_3 n$$

$$\boxed{O(\log_2 \log_3 n)}$$