

B-Trees

- B-trees are balanced search trees designed to work well on disks or other direct access secondary storage devices.
- B-Trees are better at minimizing disks I/O operations.
- Database systems use B-Trees and its variants for indexing.

→ A B-tree T is a rooted tree (whose root is $T.root$) having following properties:

1. Every node x has the following attributes:
 - a. $x.n$, the number of keys currently stored in node x .
 - b. The $x.n$ keys themselves, $x.key_1, x.key_2, \dots, x.key_n$, stored in non-decreasing, so that $x.key_1 \leq x.key_2 \leq \dots \leq x.key_n$.
 - c. $x.leaf$, a Boolean value that is true if x is a leaf and false if x is an internal node.
2. Each internal node x also contains $x.n+1$ pointers $x.c_1, x.c_2, \dots, x.c_{n+1}$ to its children. Leaf nodes have no children, and so their c_i attributes are undefined.
3. The key $x.key_i$ separates the ranges of keys stored in each subtree, if k_i is any key stored in the subtree with root $x.c_i$ then,

$$k_i \leq x.key_1 \leq x.key_2 \leq \dots \leq x.key_n \leq k_{n+1}$$

4. All leaves have the same depth, which is the tree's height h .

5. Nodes have lower and upper bounds, on the number of keys, they can contain.

These bounds are fixed integers $t \geq 2$ is called minimum degree of B-tree:

a. Every node other than the root, must have at least $(t-1)$ keys.

Every internal node other than the root thus has at least t children. If the tree is non-empty, the root must have at least one key.

b. Every node must contain at most $(2t-1)$ keys.

Therefore an internal node may have at most $2t$ children. A node is full if it has $(2t-1)$ keys.

B-Trees - Creation

Array = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15

Degree $t = 2$

Soln: Min Keys = $(t-1) = 1$
Max Keys = $(2t-1) = 3$

Step-1 Insert-1

1		
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Step-2 Insert-2

1	2	
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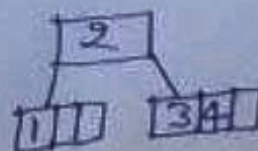
Step-3 Insert-3

1	2	3
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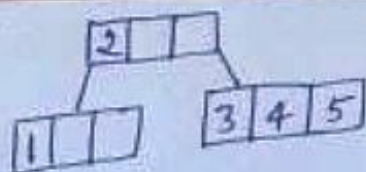
Step-4 Insert-4

1	2	3
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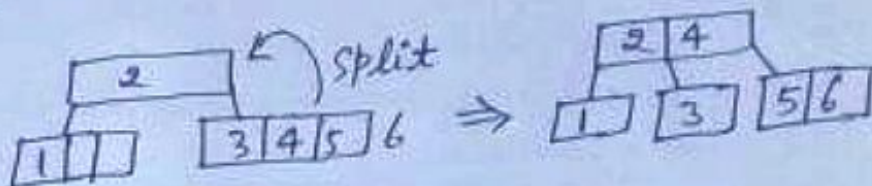
 4 \Rightarrow Split



Step-5 Insert-5



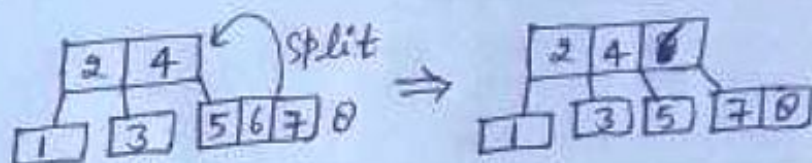
Step-6 Insert-6



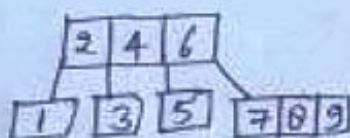
Step-7 Insert-7



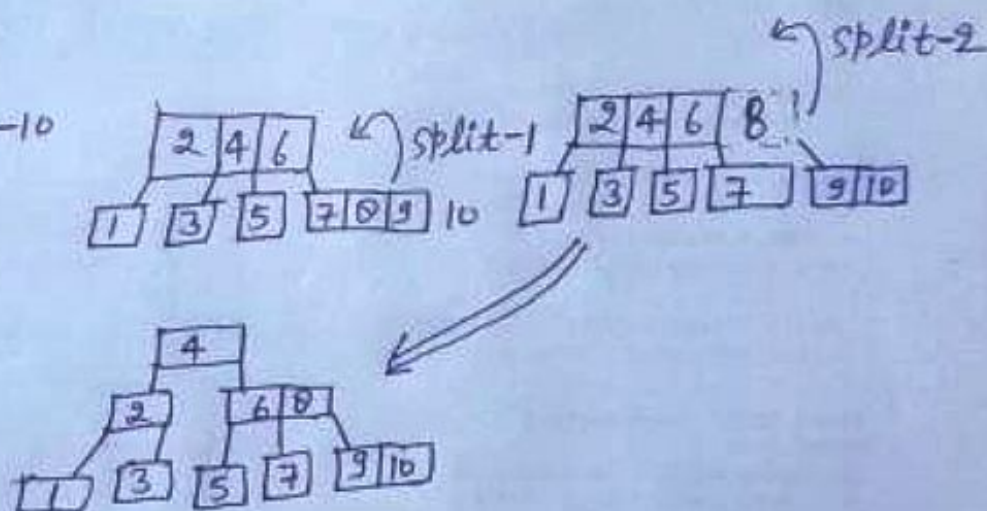
Step-8 Insert-8



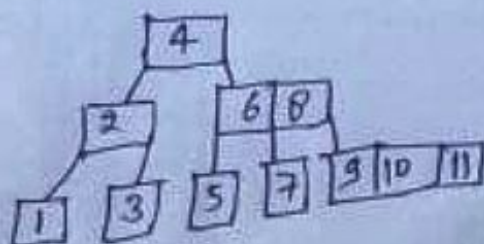
Step-9 Insert-9



Step-10 Insert-10

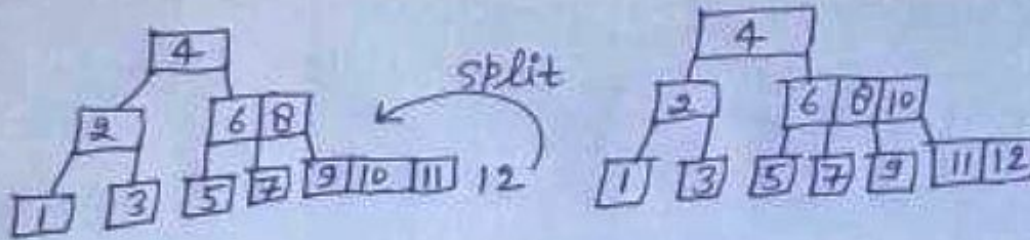


Step-11 Insert-11



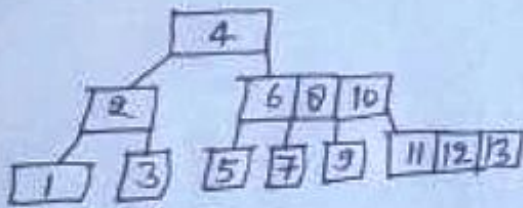
Step-12

Insert -12



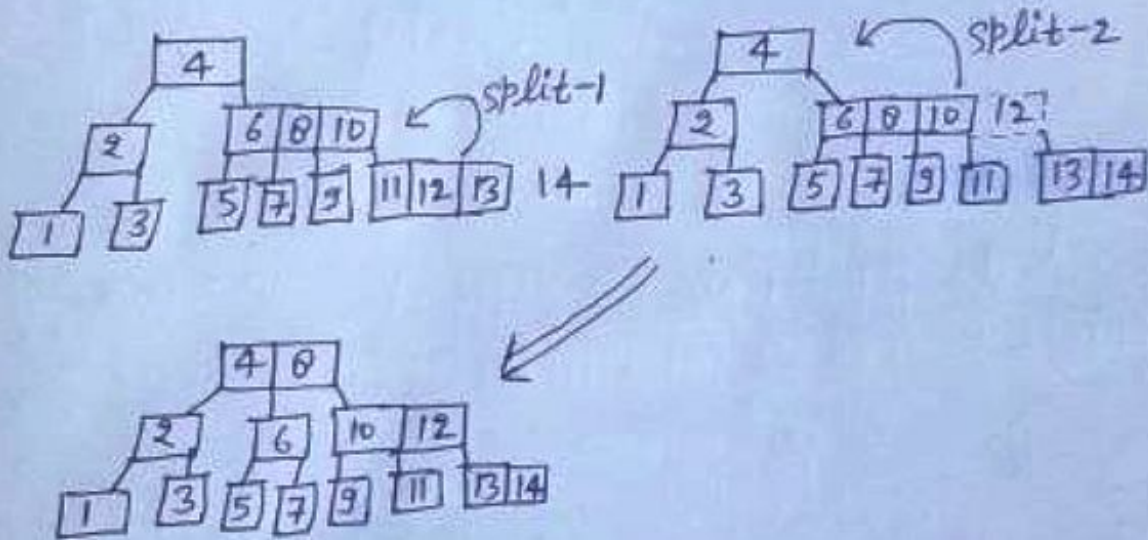
Step-13

Insert -13



Step-14

Insert -14



Step-15

Insert -15

