Decision Tree Classifier

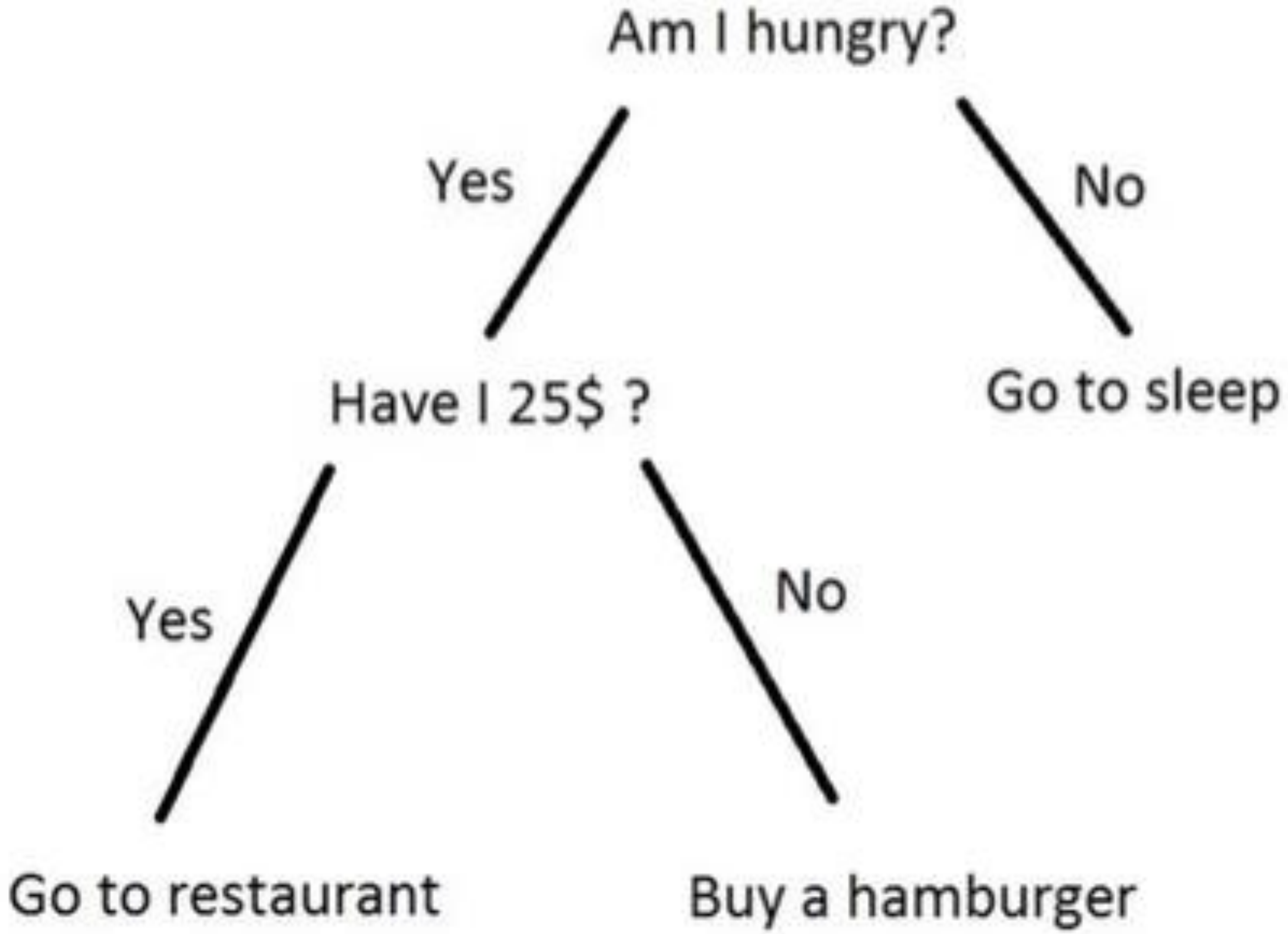
BCSE 0105 MACHINE LEARNING

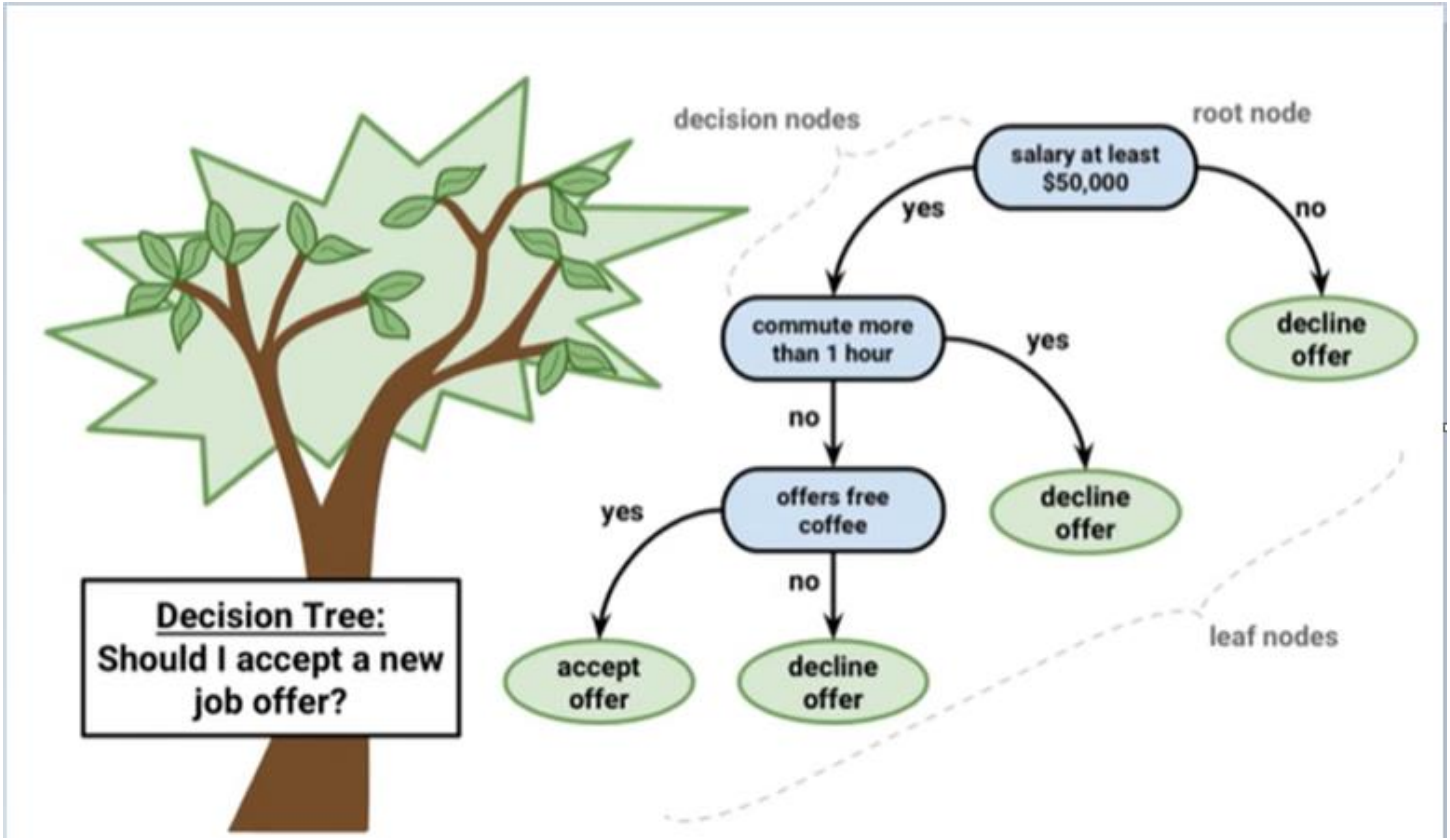
Introduction

"A **decision tree** is a graphical representation of all the possible solutions to a decision based on certain conditions"

Decision Tree

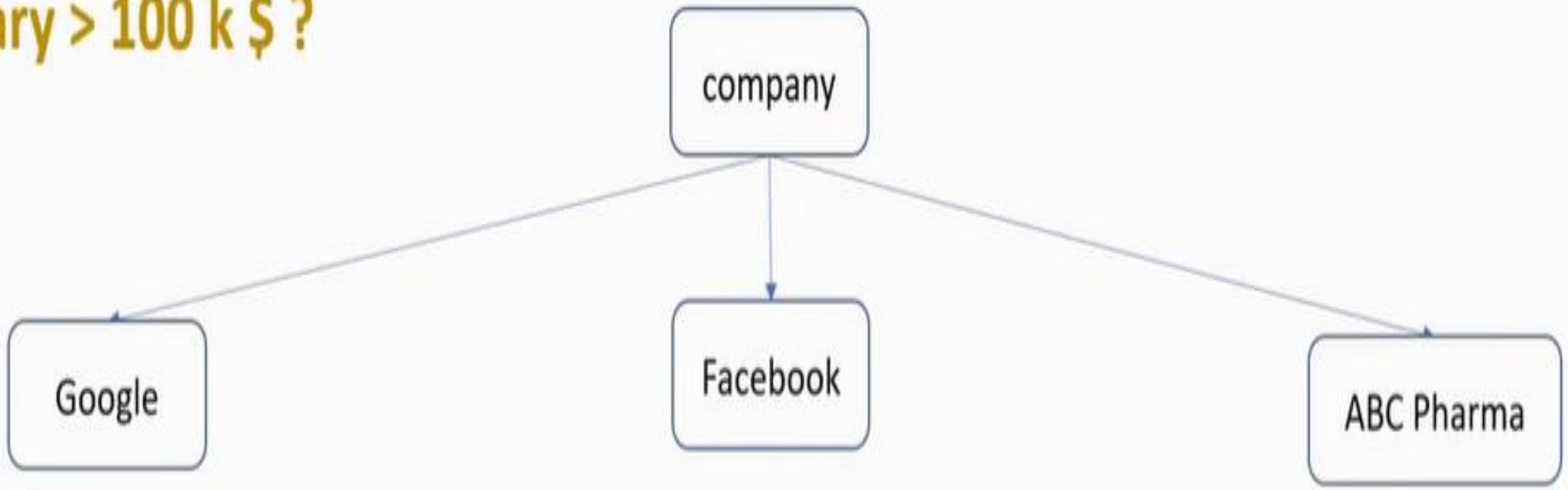
- Graphical representation of all the possible solutions to a decision
- Decisions are based on some conditions
- Decision made can be easily explained


Am I hungry?
Yes
No
Have I 25$?
Go to sleep
Yes
No
Go to restaurant
Buy a hamburger


decision nodes
root node
salary at least $50,000
yes
no
commute more than 1 hour
yes
decline offer
no
offers free coffee
decline offer
yes
no
accept offer
decline offer
leaf nodes
Decision Tree:
Should I accept a new job offer?

Company	Job	Degree	Salary_more_then_100k
google	sales executive	bachelors	0
google	sales executive	masters	0
google	business manager	bachelors	1
google	business manager	masters	1
google	computer programmer	bachelors	0
google	computer programmer	masters	1
abc pharma	sales executive	masters	0
abc pharma	computer programmer	bachelors	0
abc pharma	business manager	bachelors	0
abc pharma	business manager	masters	1
facebook	sales executive	bachelors	1
facebook	sales executive	masters	1
facebook	business manager	bachelors	1
facebook	business manager	masters	1
facebook	computer programmer	bachelors	1
facebook	computer programmer	masters	1

Salary > 100 k $?



google	sales executive	bachelors
google	sales executive	masters
google	business manager	bachelors
google	business manager	masters
google	computer programmer	bachelors
google	computer programmer	masters

?

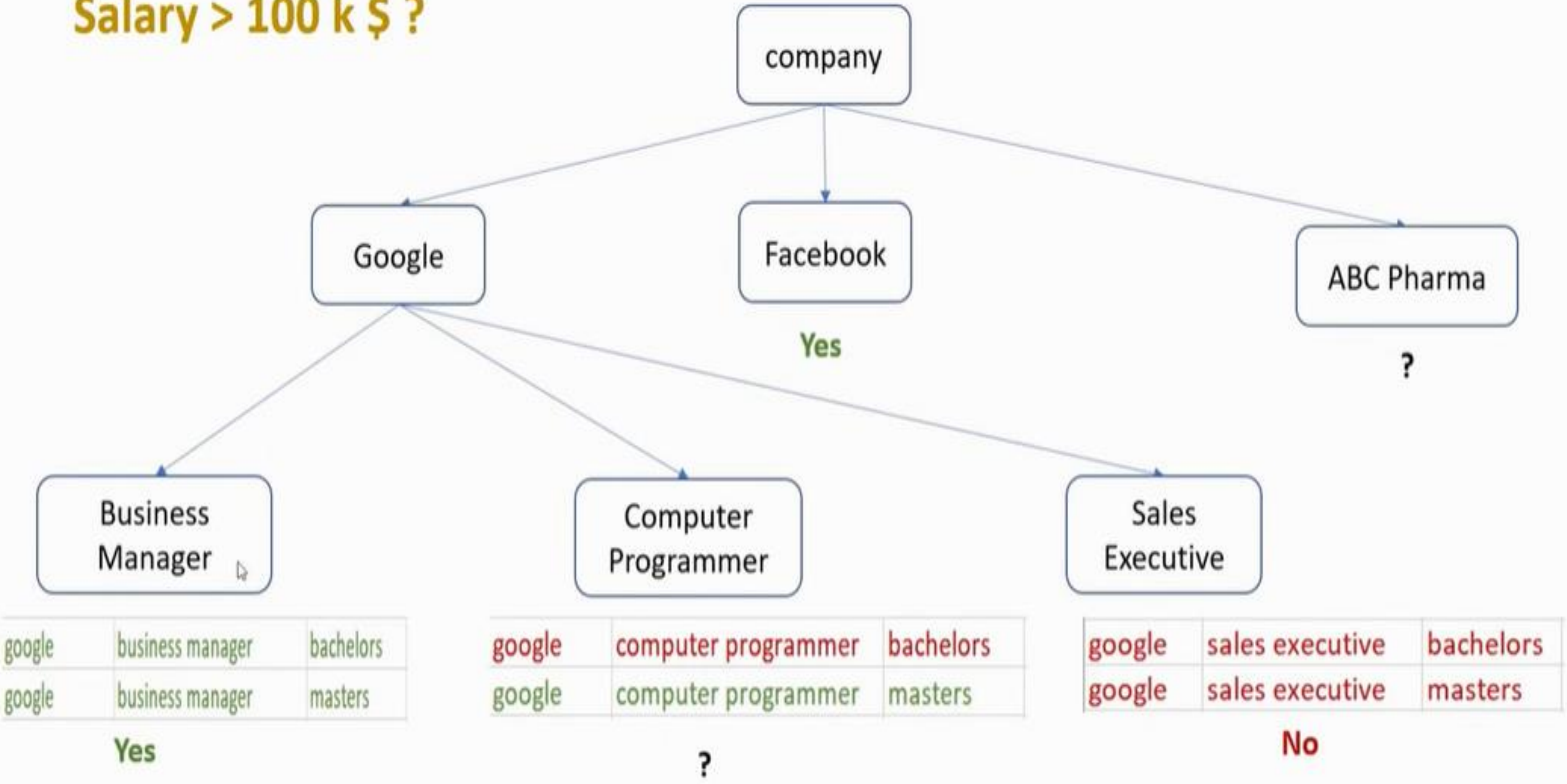
facebook	sales executive	bachelors
facebook	sales executive	masters
facebook	business manager	bachelors
facebook	business manager	masters
facebook	computer programmer	bachelors
facebook	computer programmer	masters

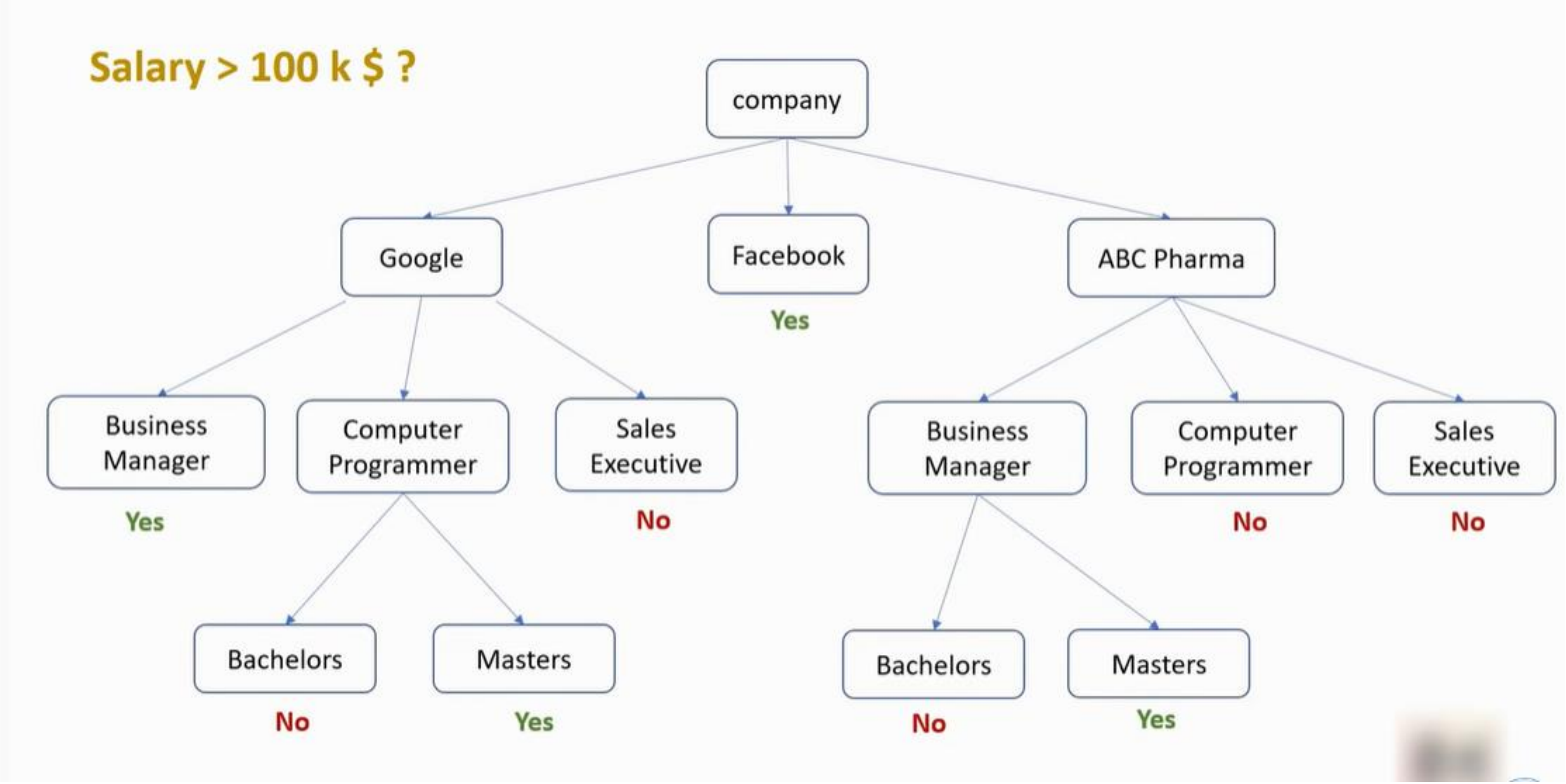
Yes

abc pharma	sales executive	masters
abc pharma	computer programmer	bachelors
abc pharma	business manager	bachelors
abc pharma	business manager	masters

?

Salary > 100 k $?

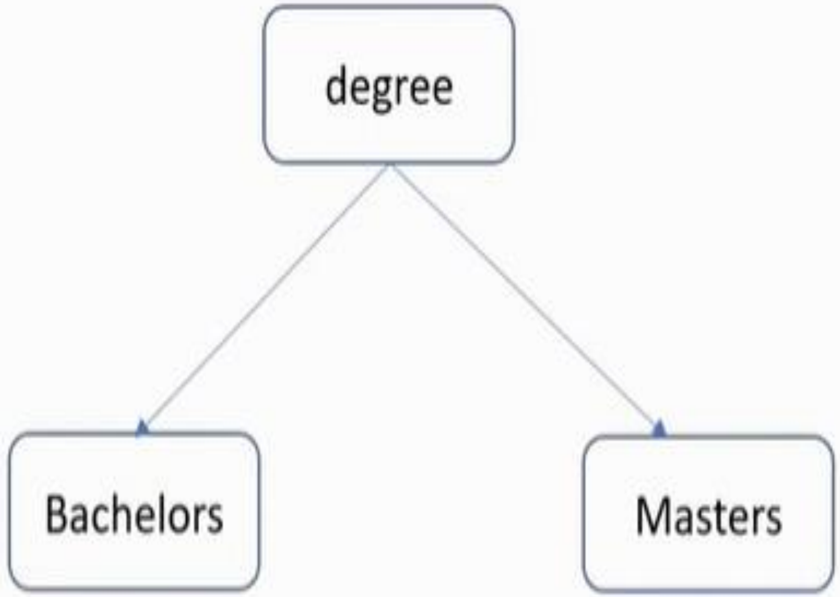

company
Google
Facebook
ABC Pharma
Yes
?
Business Manager
Computer Programmer
Sales Executive
google business manager bachelors
google business manager masters
Yes
google computer programmer bachelors
google computer programmer masters
?
google sales executive bachelors
google sales executive masters
No


Salary > 100 k $?
company
Google
Facebook
ABC Pharma
Yes
Business Manager
Computer Programmer
Sales Executive
Business Manager
Computer Programmer
Sales Executive
Yes
No
No
No
Bachelors
Masters
Bachelors
Masters
No
Yes
No
Yes

- It matters in which order you have split the tree

- Which feature to be chosen first will effect the performance of the algorithm

Alternatively



google	sales executive	bachelors
google	business manager	bachelors
google	computer programmer	bachelors
abc pharma	computer programmer	bachelors
abc pharma	business manager	bachelors
facebook	sales executive	bachelors
facebook	business manager	bachelors
facebook	computer programmer	bachelors

google	sales executive	masters
google	business manager	masters
google	computer programmer	masters
abc pharma	sales executive	masters
abc pharma	business manager	masters
facebook	sales executive	masters
facebook	business manager	masters
facebook	computer programmer	masters

Classification Problem:

Whether John will play Tennis or not?

Day	Outlook	Temperature	Humidity	Wind	Play Tennis
1	Sunny	Hot	High	Weak	No
2	Sunny	Hot	High	Strong	No
3	Overcast	Hot	High	Weak	Yes
4	Rain	Mild	High	Weak	Yes
5	Rain	Cool	Normal	Weak	Yes
6	Rain	Cool	Normal	Strong	No
7	Overcast	Cool	Normal	Strong	Yes
8	Sunny	Mild	High	Weak	No
9	Sunny	Cool	Normal	Weak	Yes
10	Rain	Mild	Normal	Weak	Yes
11	Sunny	Mild	Normal	Strong	Yes
12	Overcast	Mild	High	Strong	Yes
13	Overcast	Hot	Normal	Weak	Yes
14	Rain	Mild	High	Strong	No

To make a decision tree:
Need to select a feature as root node

Which one among them should you pick first?

Answer: Determine the attribute that best classifies the training data

But How do we choose the best attribute?

Or

How does a tree decide where to split?

How Does A Tree Decide Where To Split?

- Entropy (to measure the impurity)
- Information Gain (decision tree split)
- Gini Index/Gini Impurity

Algorithms to construct Decision Tree

- ID3 Algorithm (using entropy and Information Gain)
- CART Algorithm (using Gini Index/Gini Impurity)

Entropy

- Entropy can be defined as **a measure of the purity of the split** or measure of impurity of a node.
- Entropy always lies between 0 to 1.
- A node having multiple classes is impure whereas a node having only one class is pure.

$$\textbf{Entropy(s)} = -P(yes)\log_2 P(yes) - P(no)\log_2 P(no)$$

Where,

- S is the total sample space,
- P(yes) is probability of yes

Equal yes and no

$E(S) = -P(Yes) \log_2 P(Yes)$

When P(Yes) =P(No) = 0.5 ie YES + NO = Total Sample(S)

$E(S) = 0.5 \ \log_2 0.5 - 0.5 \ \log_2 0.5$

$E(S) = 0.5(\log_2 0.5 - \ \log_2 0.5)$

$E(S) = 1$

All yes or All no

$E(S) = -P(Yes)\log_2 P(Yes)$

When P(Yes) = 1 ie YES = Total Sample(S)

$E(S) = 1\ \log_2 1$

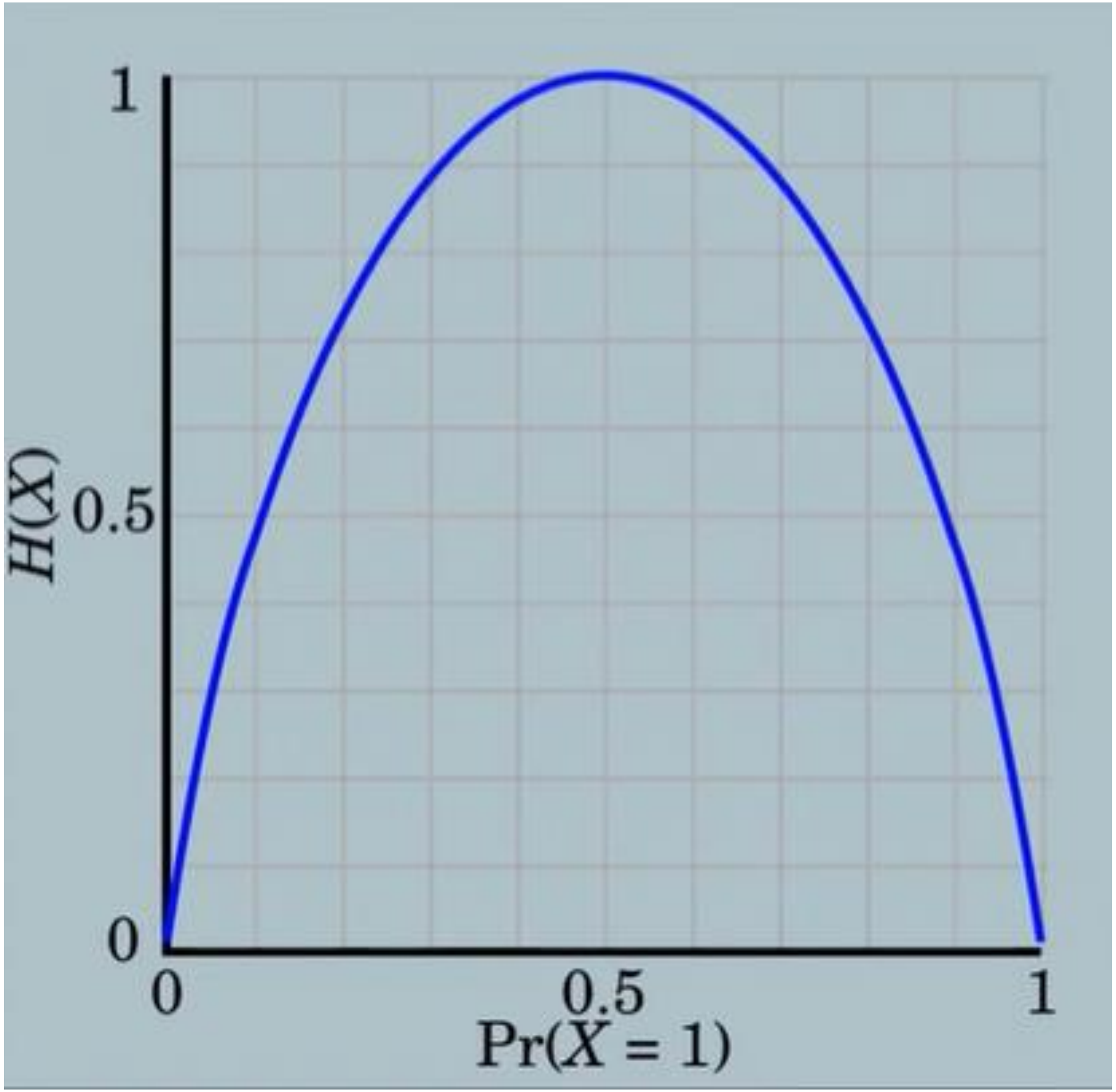
$E(S) = 0$

$E(S) = -P(No)\log_2 P(No)$

When P(No) = 1 ie No = Total Sample(S)

$E(S) = 1\ \log_2 1$

$E(S) = 0$


1
0.5
0
H(X)
0
0.5
1
Pr(X = 1)

Day	Outlook	Play Tennis
1	Sunny	No
2	Sunny	No
3	Overcast	Yes
4	Rain	Yes
5	Rain	Yes
6	Rain	No
7	Overcast	Yes
8	Sunny	No
9	Sunny	Yes
10	Rain	Yes
11	Sunny	Yes
12	Overcast	Yes
13	Overcast	Yes
14	Rain	No

Entropy of dataset

Out of 14 instances we have 9 YES and 5 NO

So we have the formula,

$$E(S) = -P(Yes)\log_2 P(Yes) - P(No)\log_2 P(No)$$

$$E(S) = -(9/14)^* \log_2 9/14 - (5/14)^* \log_2 5/14$$

$$E(S) = 0.41+0.53 = 0.94$$

Entropy of attribute
E(Outlook)

$$E(A) = \sum \frac{|S_v|}{|S|} Entropy(S_v)$$

Where

A is attribute (eg. Outlook)

v is values of attribute A (eg. sunny, overcast, rainy)

$\frac{|S_v|}{|S|}$ is probability of v i.e. P(v)

Eg. E(Outlook)

$$E\ (Outlook) = P(Sunny) * E(Sunny) + P(Overcast) * E(Overcast) + P(Rainy) * E(Rainy)$$

$$= P(Sunny) * E(3,2) + P(Overcast) * E(4,0) + P(Rainy) * E(2,3)$$

$$= \ 5/14 \times 0.971 + 4/14 \times 0 + 5/14 \times 0.971$$

$$= \ 0.693$$

Information Gain

- Information gain or IG is a statistical property that measures how well a given attribute separates the training examples according to their target classification.
- Constructing a decision tree is all about finding an attribute that returns the highest information gain and the smallest entropy.
- Information gain computes the difference between entropy before split and average entropy after split of the dataset based on given attribute values.
- ID3 (Iterative Dichotomiser) decision tree algorithm uses information gain to split a node.

Information Gain

$$Gain(S, A) = Entropy(S) - \sum \frac{|S_v|}{|S|} Entropy(S_v),$$

where

S is the given set or sample space

Or, Gain(S,A)=E(S)-E(A)

Eg.

Gain(S, Outlook)=E(S)-E(Outlook)= $0.94 - 0.693 = 0.247$

Let’s Build Our Decision Tree

Construct a decision tree of the given training set using ID3 algorithm

Day	Outlook	Temperature	Humidity	Wind	Play Tennis
1	Sunny	Hot	High	Weak	No
2	Sunny	Hot	High	Strong	No
3	Overcast	Hot	High	Weak	Yes
4	Rain	Mild	High	Weak	Yes
5	Rain	Cool	Normal	Weak	Yes
6	Rain	Cool	Normal	Strong	No
7	Overcast	Cool	Normal	Strong	Yes
8	Sunny	Mild	High	Weak	No
9	Sunny	Cool	Normal	Weak	Yes
10	Rain	Mild	Normal	Weak	Yes
11	Sunny	Mild	Normal	Strong	Yes
12	Overcast	Mild	High	Strong	Yes
13	Overcast	Hot	Normal	Weak	Yes
14	Rain	Mild	High	Strong	No

Step 1: Compute the entropy for the Data set

Out of 14 instances we have 9 YES and 5 NO

So we have the formula,

$$E(S) = -P(Yes)\log_2 P(Yes) - P(No)\log_2 P(No)$$

$$E(S) = -(9/14)^* \log_2 9/14 - (5/14)^* \log_2 5/14$$

$$E(S) = 0.41+0.53 = 0.94$$

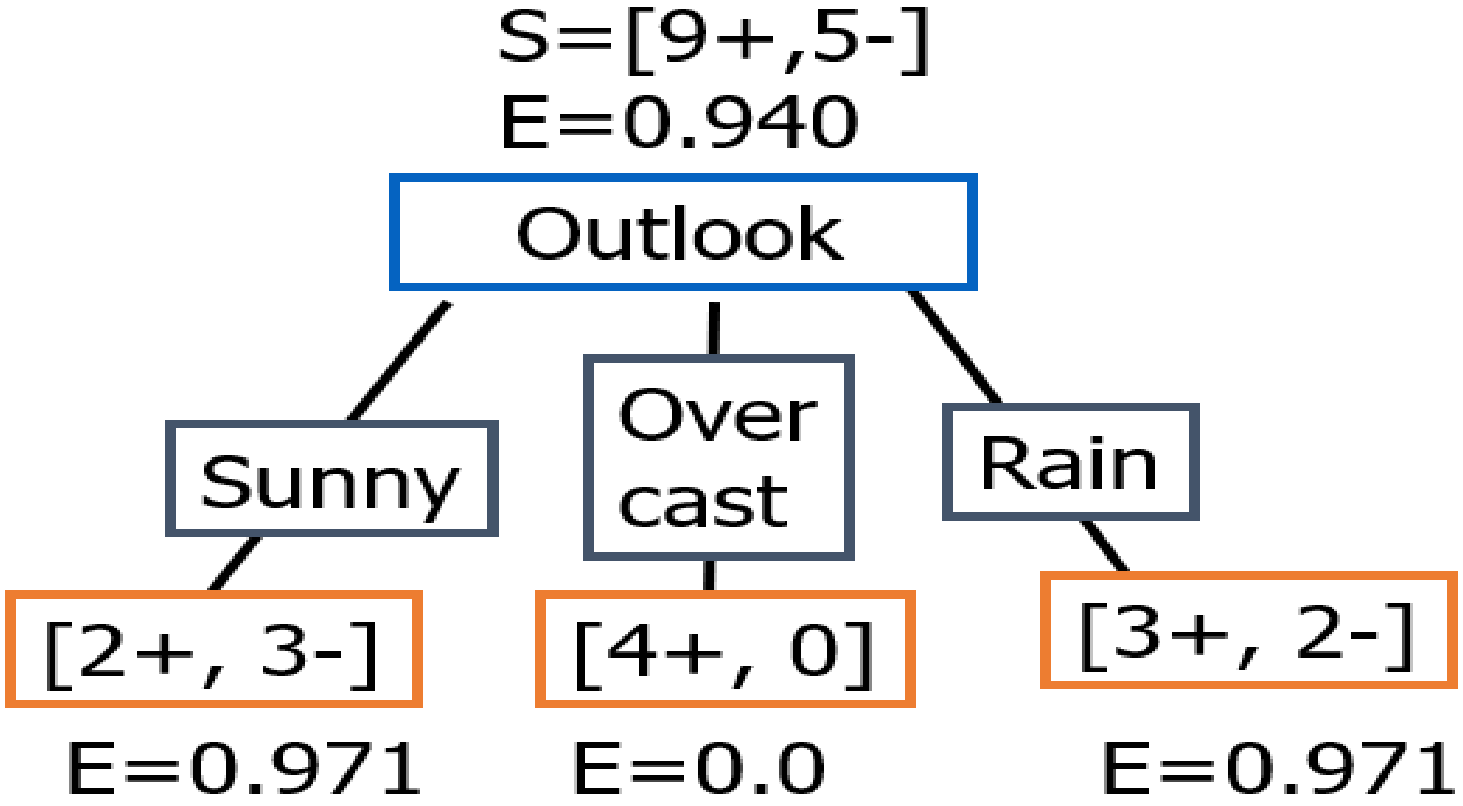
Which Node To Select As Root Node?

Outlook ?
Humidity ?
Temperature ?
Wind ?

First iteration

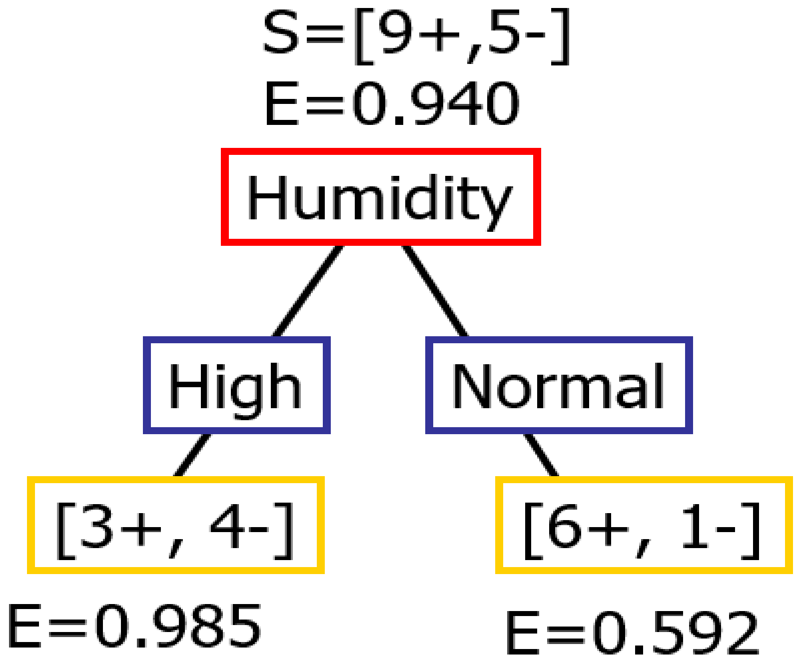
- At first iteration, we need to know which is best attribute to be chosen as top root in our decision tree.
- To do that, ID3 will find the best attribute which has maximum information gain.
- The information gain for each attribute:
- attributes = [Outlook, Humidity, Wind, Temperature]
- G(S, Outlook) = 0.247
- G(S, Humidity) = ?
- G(S, Wind) = ?
- G(S, Temperature) = ?

Information Gain of Outlook

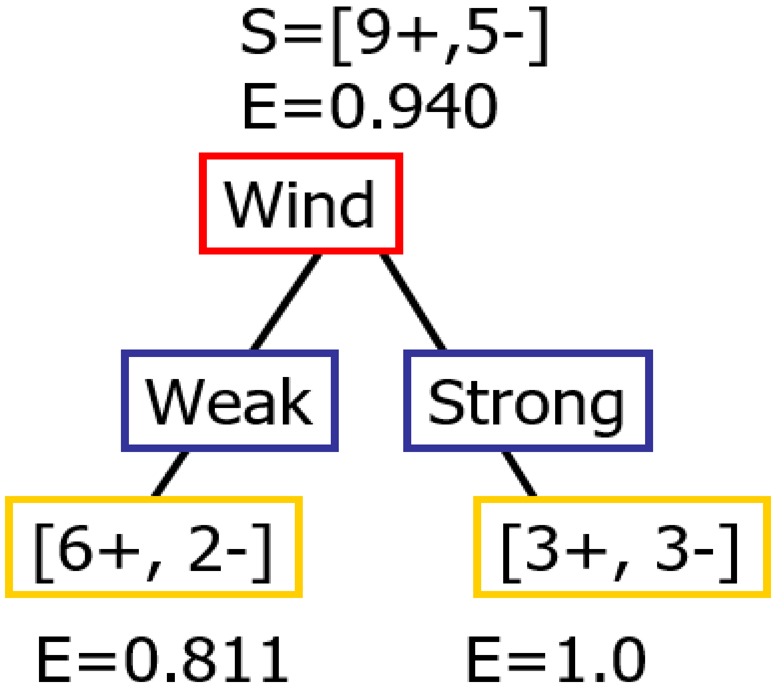


Gain(S,Outlook)
=0.940-(5/14)*0.971
-(4/14)*0.0 – (5/14)*0.0971
=0.247

Information Gain of Humidity, windy



Gain(S,Humidity)
=0.940-(7/14)*0.985
– (7/14)*0.592
=0.151



Gain(S,Wind)
=0.940-(8/14)*0.811
– (6/14)*1.0
=0.048

Humidity provides greater info. gain than Wind, w.r.t target classification.

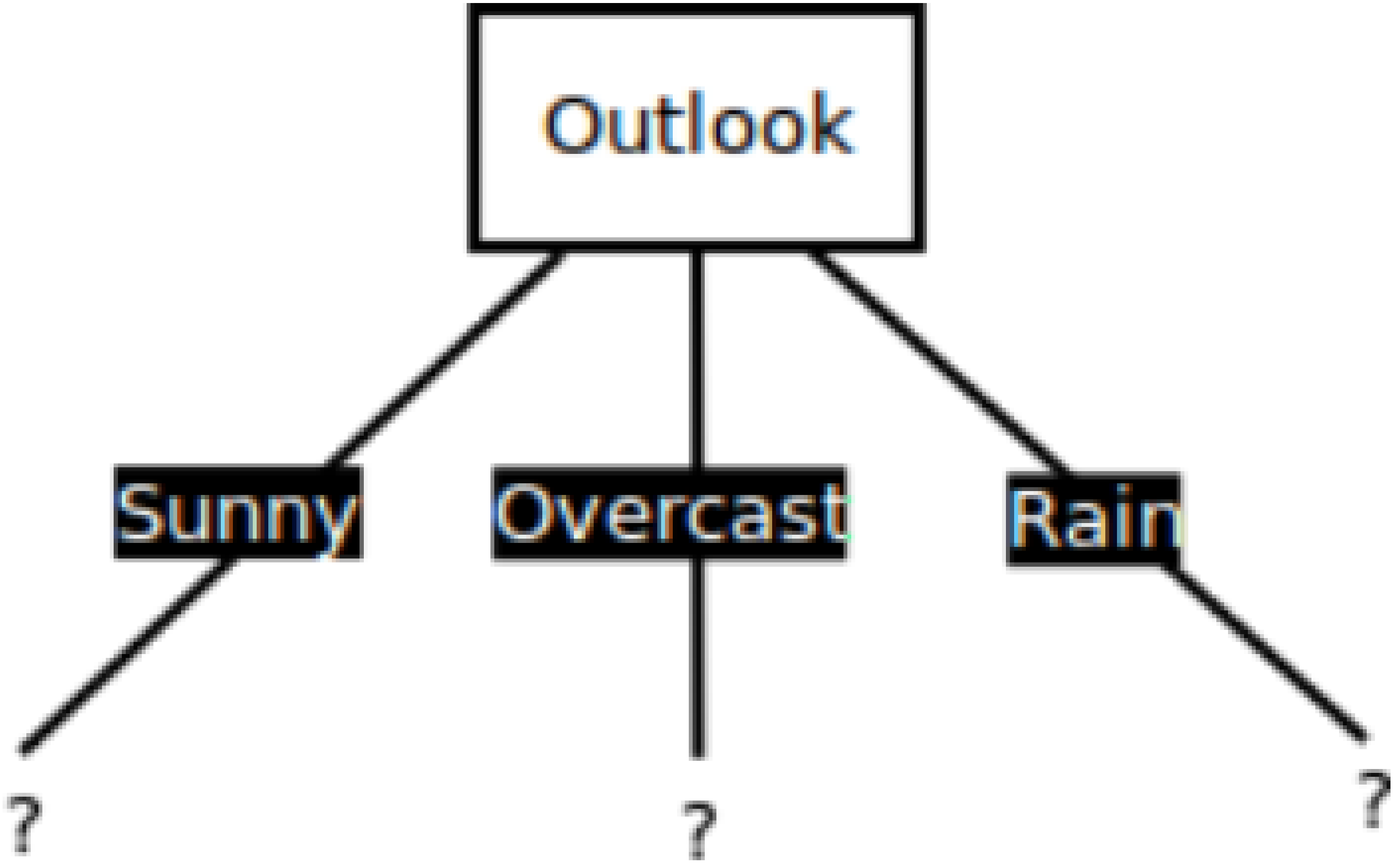
Similarly, we can find the Information Gain of Temperature

First iteration

- At first iteration, we need to know which is best attribute to be chosen as top root in our decision tree.
- To do that, ID3 will find the best attribute which is has maximum information gain.
- The information gain for each attribute:
- Gain(S,Outlook) =0.247
- Gain(S,Humidity) =0.151
- Gain(S,Wind) =0.048
- Gain(S,Temperature) =0.029

First iteration

- So, based on the information gains calculated
- we choose attribute Outlook as root node which has three branches Sunny, Rain, and Overcast.
- Our decision tree will look like an image below



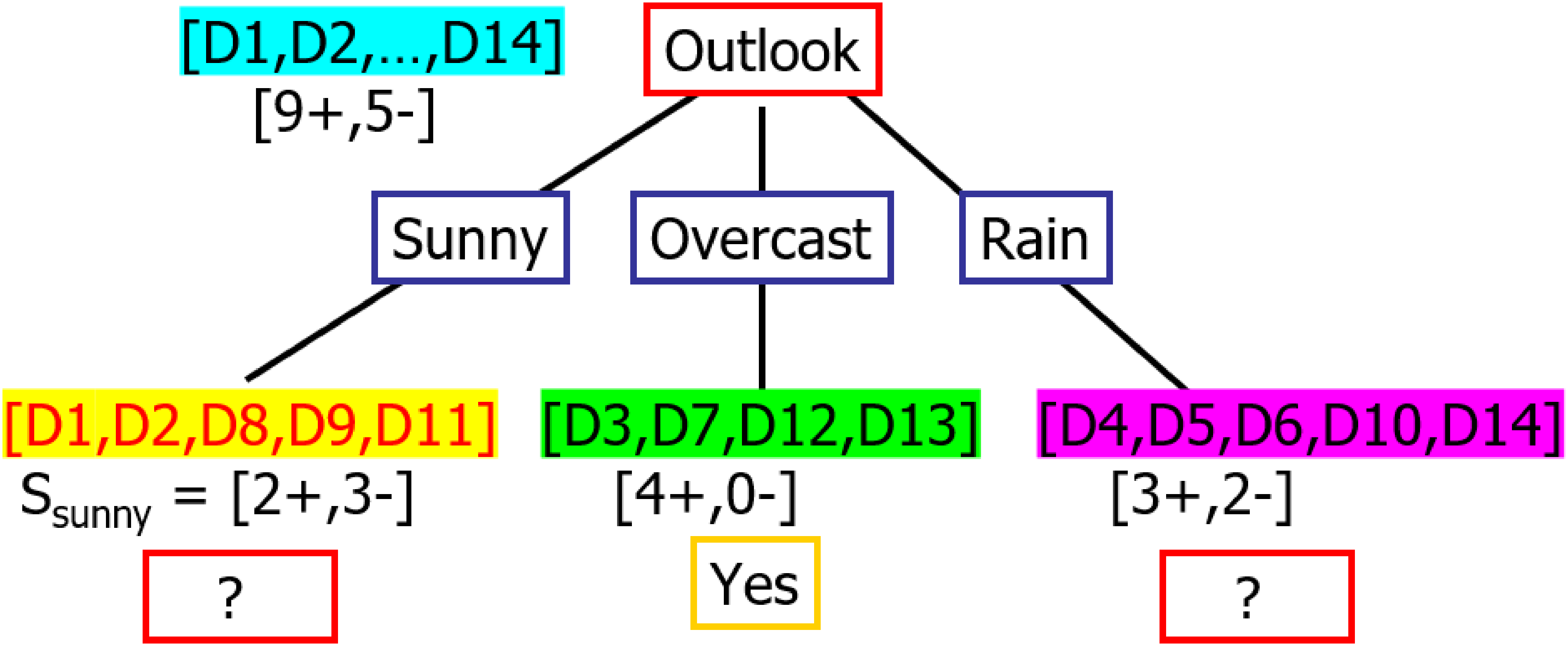
Initial decision tree

Second iteration

- Now we remove Outlook from attribute list.
- We want examine which is the best attribute for branch of Sunny.
- Remember, that new S is rows containing values of Sunny.

Now, attributes left are [Humidity, Wind, Temperature]

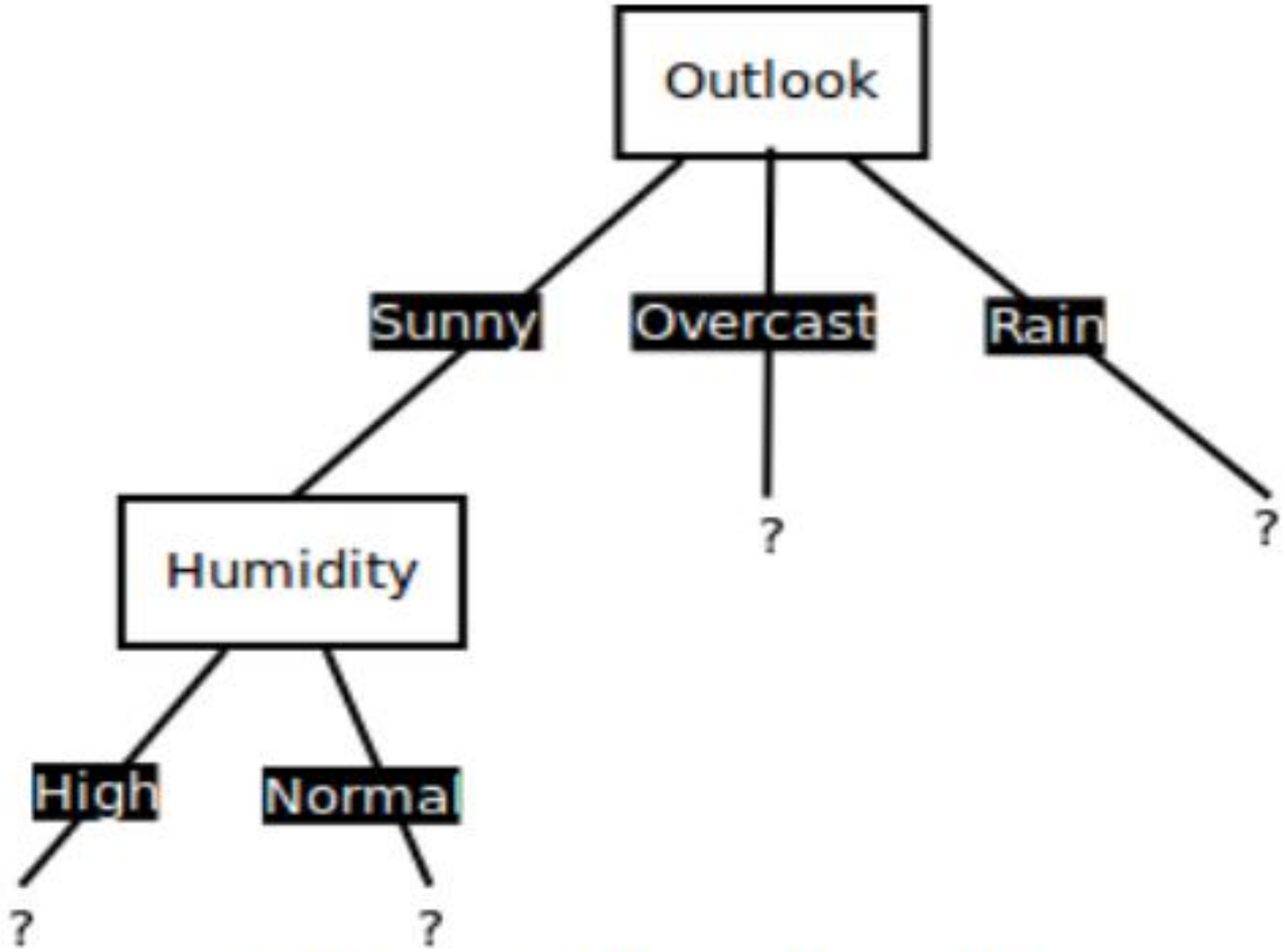
ID3 Algorithm



Gain(S_{sunny}, Humidity)=0.970-(3/5)0.0 – 2/5(0.0) = 0.970
Gain(S_{sunny}, Temperature)=0.970-(2/5)0.0 –2/5(1.0)-(1/5)0.0 = 0.570
Gain(S_{sunny}, Wind)=0.970 -(2/5)1.0 – 3/5(0.918) = 0.019

Second iteration

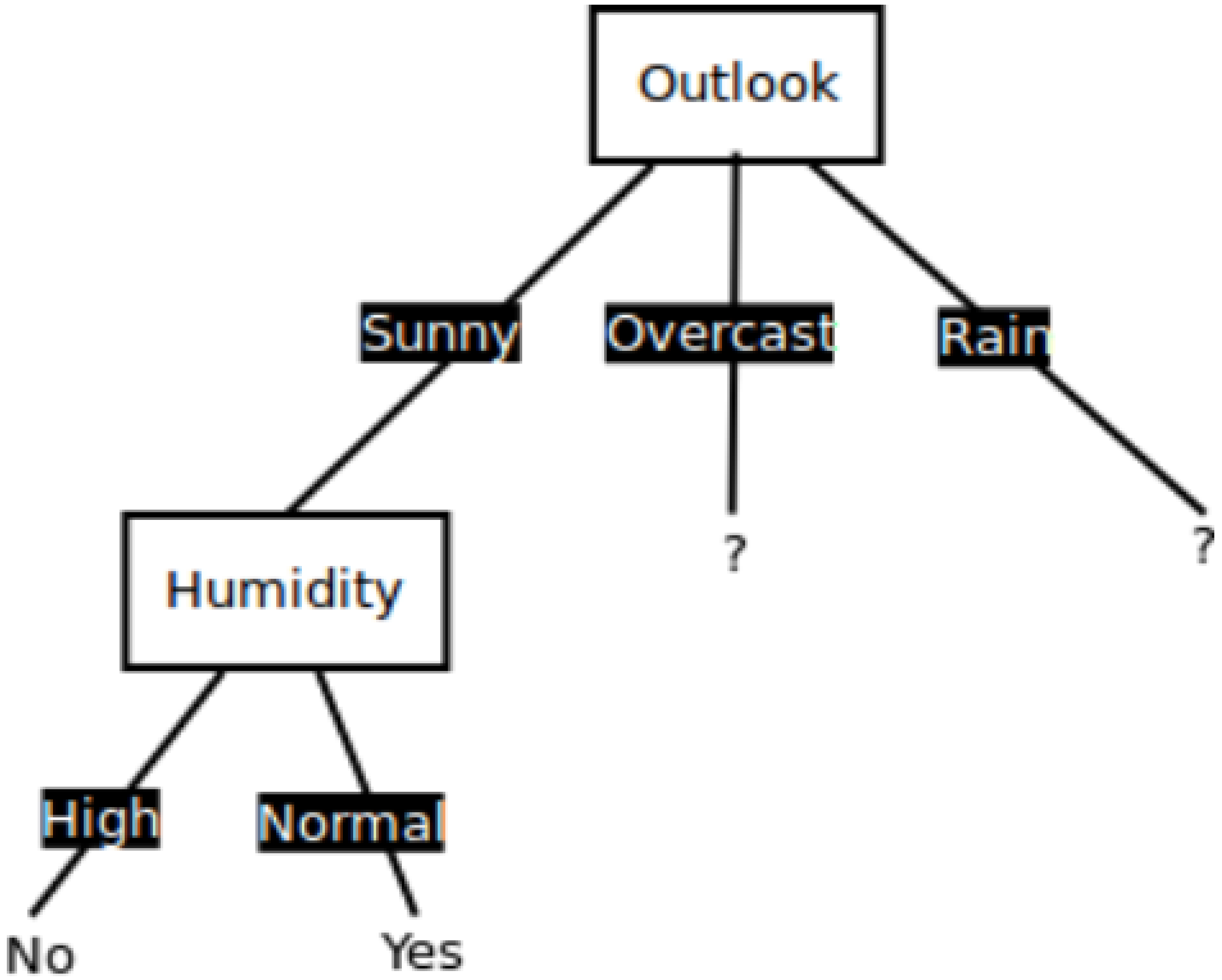
- So, based on the information gains calculated above,
- we choose attribute Humidity as attribute in branch Sunny.
- Our decision tree will look like an image below.



Add new attribute Humidity

Third iteration

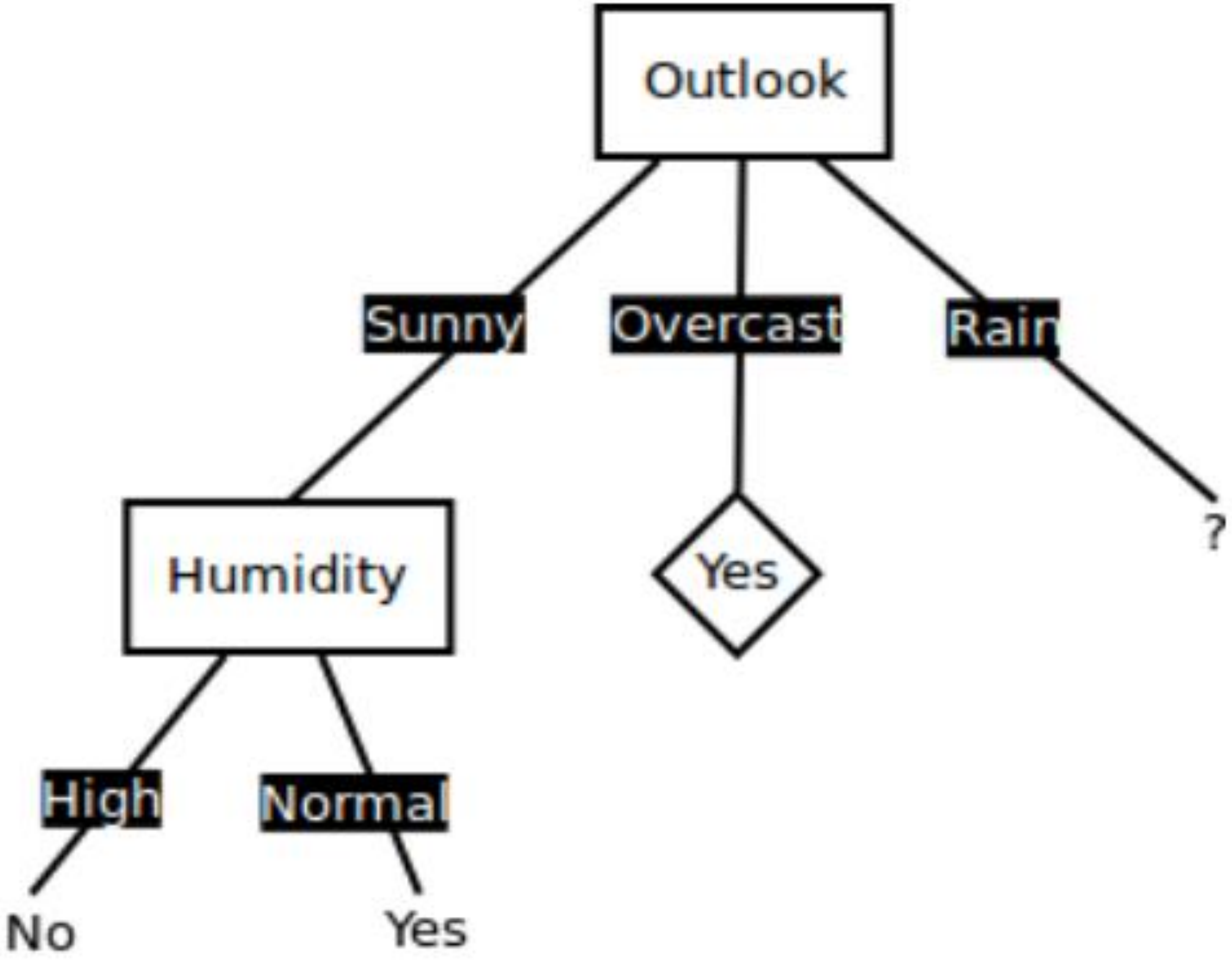
- Now, we remove Humidity from attribute list.
- Attributes left= [Wind, Temperature]
- Next node is an attribute Humidity which has two possible values {High, Normal}.
- A branch High dominated by single label which is No, caused this branch ended with a leaf contains label No. Same case with branch Normal which ended with a leaf contains label Yes.



Humidity's Leafs

Fourth iteration

All rows contain value Overcast are dominated by single label Yes, so branch of Overcast ended with a leaf contains label Yes.



Overcast's leaf

Fifth iteration

- We want examine which the best attribute for branch of Rain.

- Remember, that new S is rows containing values of Rain.

- Attributes left= [Wind , Temperature]

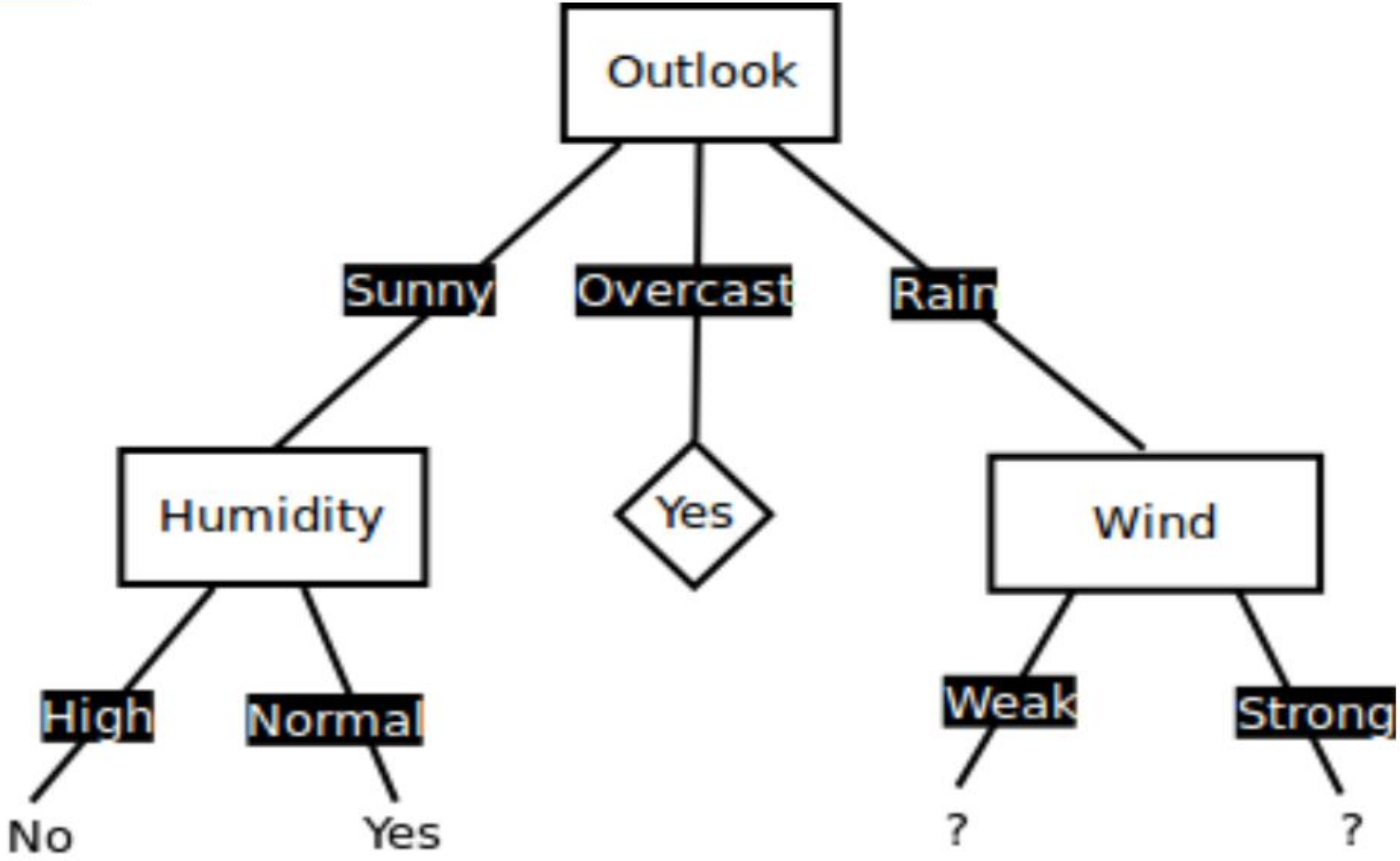
- $G(S_{rain}, Wind) = .970 - (2/5) * 0 - (3/5) * 0$

  $= .970$
- $G(S_{rain}, Temperature) = .970 - (0/5) * 0 - (3/5) * .918 - (2/5) * 1.0$

  $= .019$

Fifth iteration

- So, based on the information gains calculated above,
- we choose attribute Wind as attribute in branch of Rain.
- Our decision tree will look like an image below.

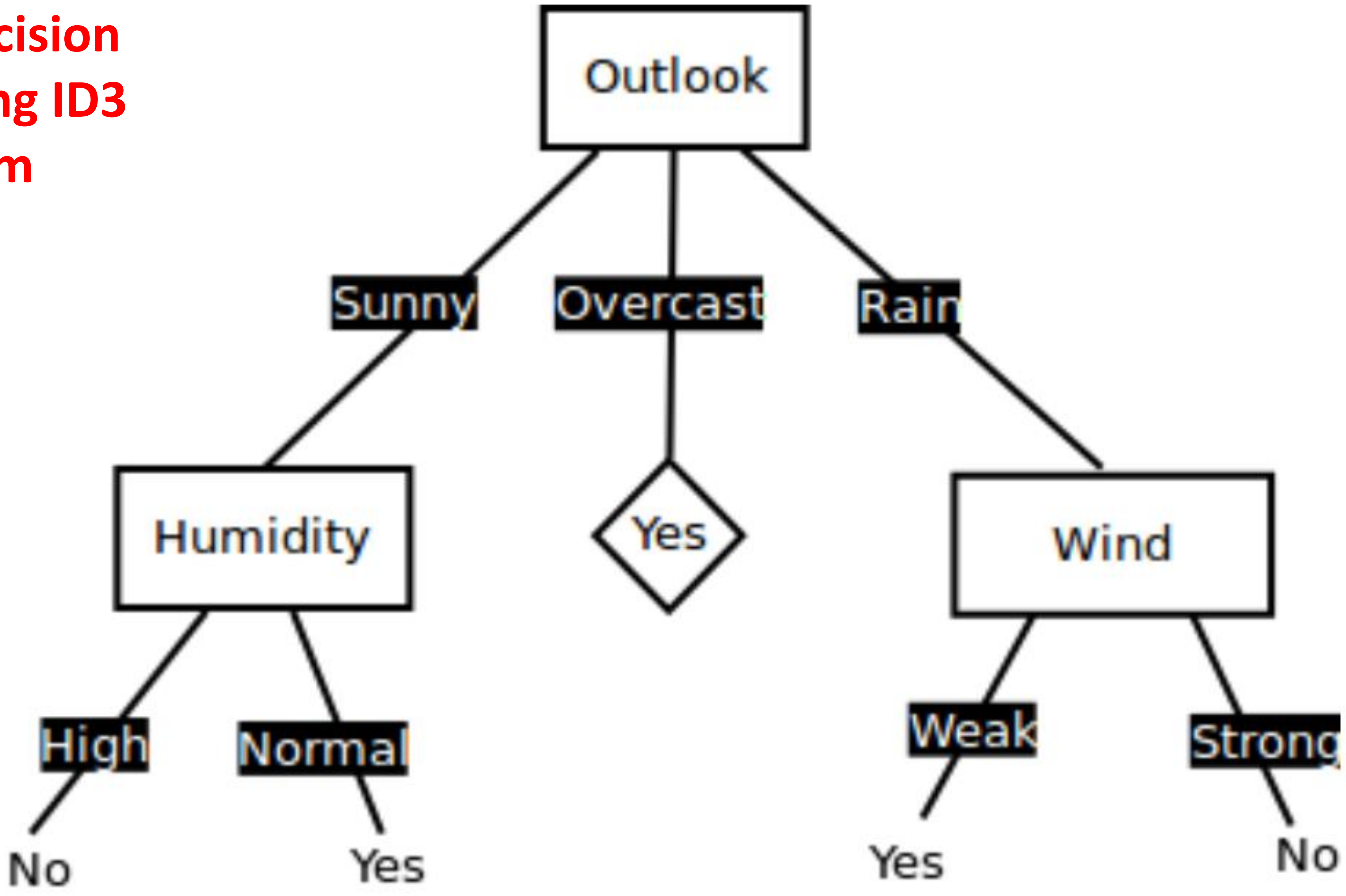


Add new attribute Wind

Sixth iteration

- For next iteration, we remove attribute Wind from attribute list.
- Next node is an attribute Wind which has two possible values {Weak, Strong}.
- A branch Strong dominated by single label which is No, caused this branch ended with a leaf contains label No.
- Same case with branch Weak which ended with a leaf contains label Yes.

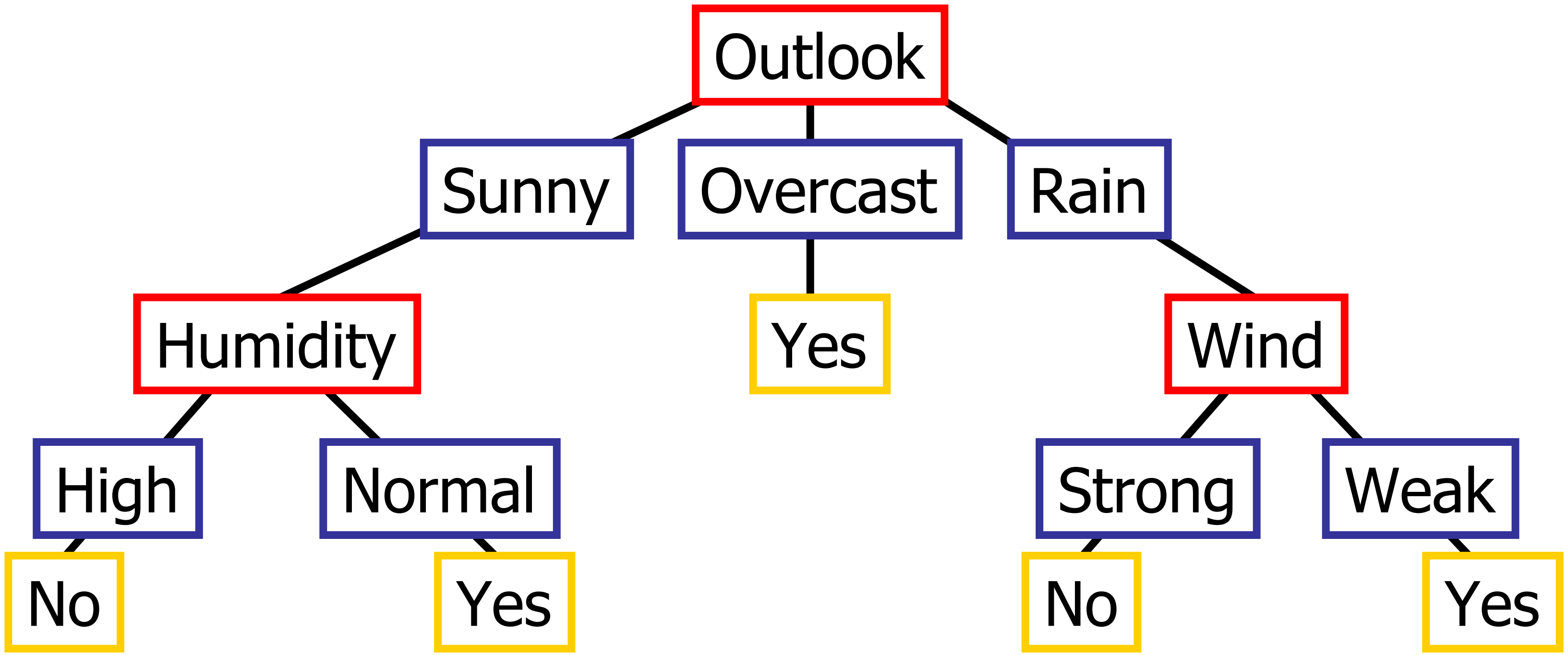
Final decision tree using ID3 algorithm



Termination condition

- Since all branches in our decision tree ended with leaves.
- We stop here.
- We prune attribute Temperature from our decision tree.

Converting a Tree to Rules



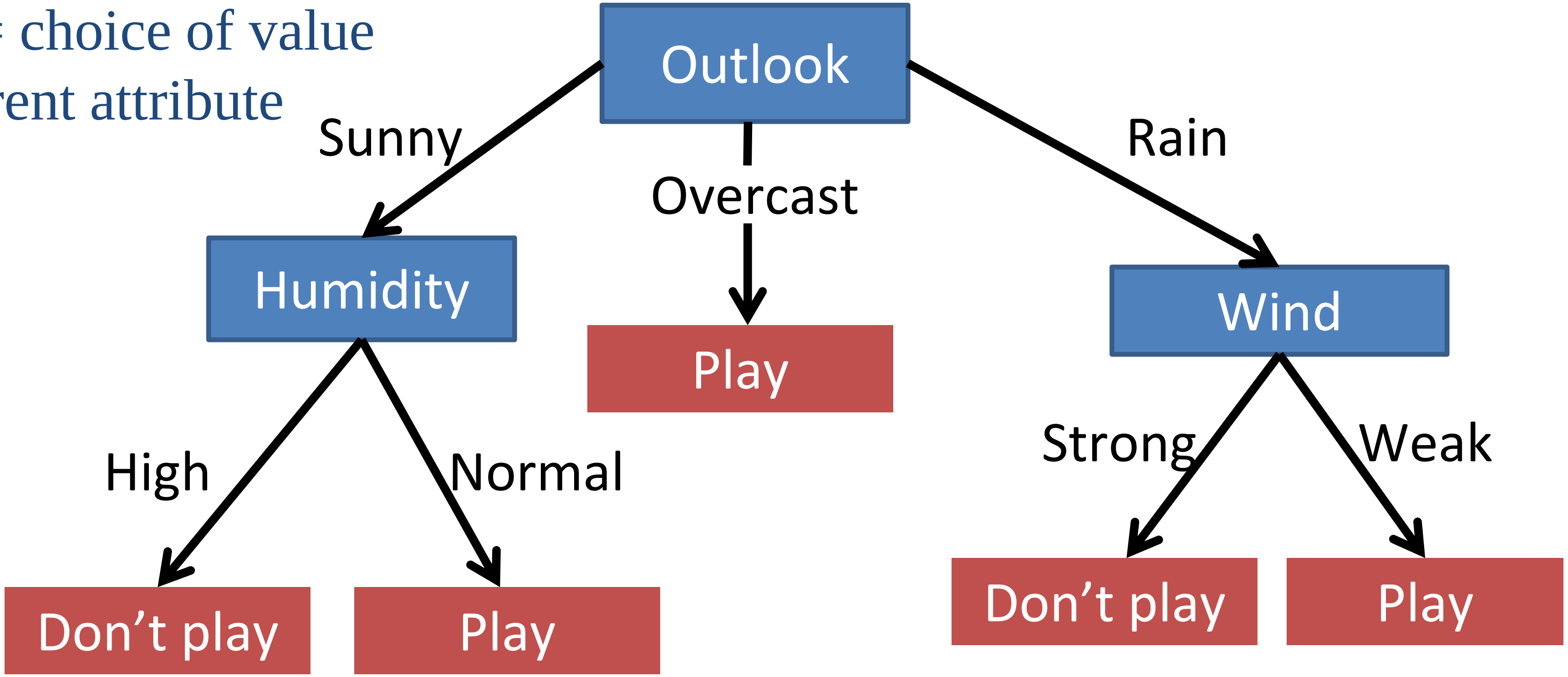
R_1: If (Outlook=Sunny) $\wedge$ (Humidity=High) Then PlayTennis=No
R_2: If (Outlook=Sunny) $\wedge$ (Humidity=Normal) Then PlayTennis=Yes
R_3: If (Outlook=Overcast) Then PlayTennis=Yes
R_4: If (Outlook=Rain) $\wedge$ (Wind=Strong) Then PlayTennis=No
R_5: If (Outlook=Rain) $\wedge$ (Wind=Weak) Then PlayTennis=Yes

Decision Tree Representation

Good day for tennis?

Leaves = classification

Arcs = choice of value for parent attribute



Decision tree is equivalent to logic in disjunctive normal form

$\text{Play} \Leftrightarrow (\text{Sunny} \wedge \text{Normal}) \vee \text{Overcast} \vee (\text{Rain} \wedge \text{Weak})$

Construct a decision tree of the given training set using CART (Classification and Regression Tree) Algorithm

Gini Index/Gini Impurity

$$\text{Gini Index}(\text{Attribute} = \text{Value}) = 1 - \sum_{i=1}^{N} (P_i)^2$$

$$\text{Ginni Index}(\text{Attribute}) = \sum_{v = \text{Values}} P_v * GI(v)$$

Compute the Gini index of Outlook

Outlook

Outlook	Yes	No	Number of instances
Sunny	2	3	5
Overcast	4	0	4
Rain	3	2	5

$$\text{Gini (Outlook = Sunny)} = 1 - (2/5)^2 - (3/5)^2$$
$$= 1 - 0.16 - 0.36 = 0.48$$

$$\text{Gini (Outlook = Overcast)} = 1 - (4/4)^2 - (0/4)^2 = 0$$

$$\text{Gini (Outlook = Rain)} = 1 - (3/5)^2 - (2/5)^2$$
$$= 1 - 0.36 - 0.16 = 0.48$$

Now Calculate weighted Sum of Gini Indexes

$$\text{Gini (Outlook)} = (5/14) * 0.48 + \left(\frac{4}{14}\right) * 0 + \left(\frac{5}{14}\right) * 0.48$$

$$= 0.171 + 0 + 0.171 = 0.342$$

Compute the Gini index of Temperature

Temperature

Temperature	Yes	No	Number of Inst^n
Hot	2	2	4
Cool	3	1	4
Mild	4	2	6

$$\text{Gini}(\text{Temp} = \text{Hot}) = 1 - (2/4)^2 - (2/4)^2 = 0.5$$

$$\text{Gini}(\text{Temp} = \text{Cool}) = 1 - (3/4)^2 - (1/4)^2 = 0.375$$

$$\text{Gini}(\text{Temp} = \text{mild}) = 1 - (4/6)^2 - (2/6)^2 = 0.445$$

Calculate Weighted sum of Gini Indixes

$$\text{Gini}(\text{Temperature}) = (4/14) * 0.5 + (4/14) * 0.375 + (6/14) * 0.445$$

$$= 0.142 + 0.107 + 0.190 = 0.439$$

Compute the Gini index of Wind and Humidity

Wind

Wind	Yes	No	Number of Instances
Weak	6	2	8
Strong	3	3	6

$Gini(wind = Weak) = 1 - (6/8)^2 - (2/8)^2 = 0.375$

$Gini(wind = Strong) = 1 - (3/6)^2 - (3/6)^2 = 0.5$

$Gini(Wind) = \left(\frac{8}{14}\right) * 0.375 + \left(\frac{6}{14}\right) * 0.5 = 0.428$

Humidity

Humidity	Yes	No	Number of Instances
High	3	4	7
Normal	6	1	7

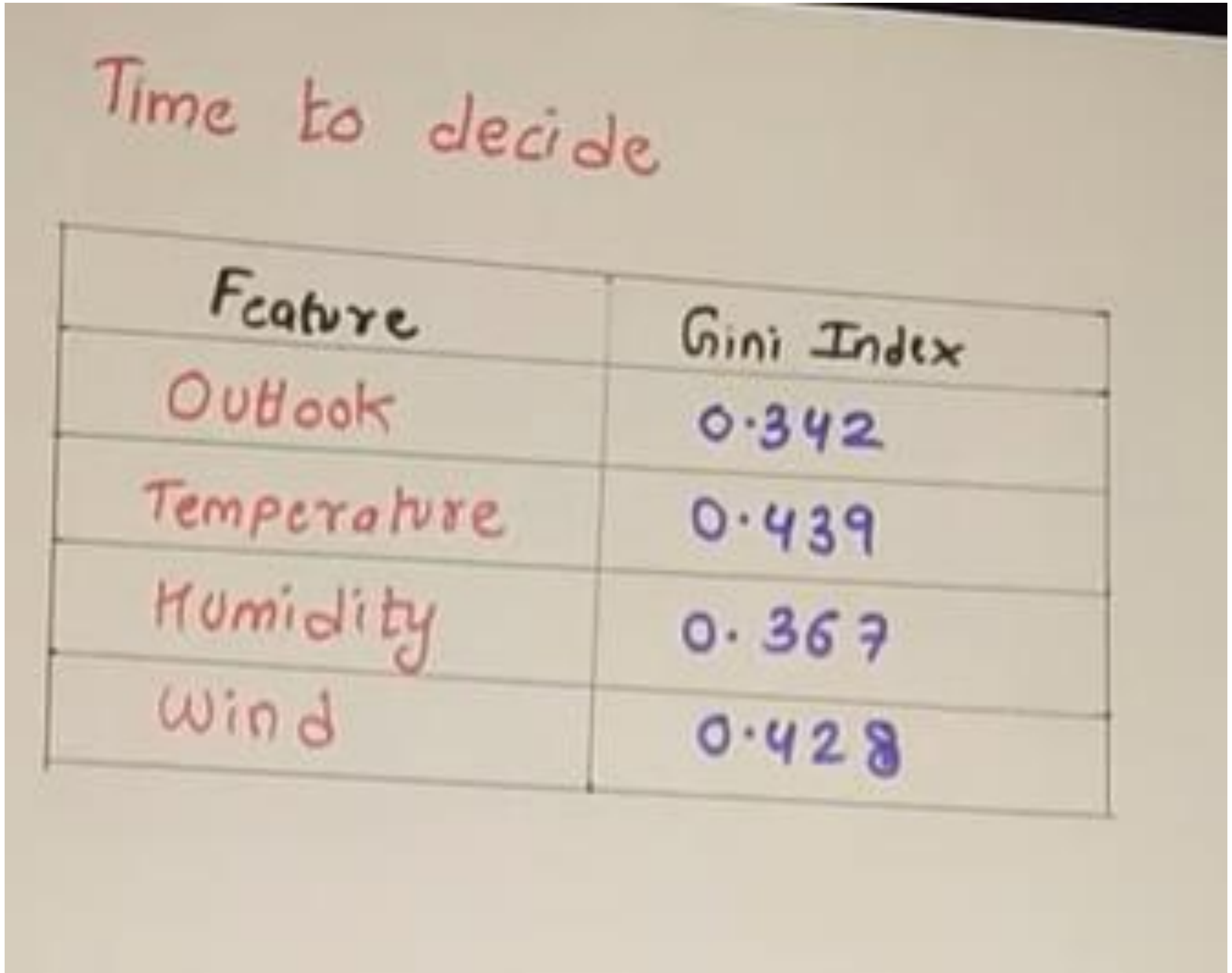
$Gini(Humidity = High) = 1 - (3/7)^2 - (4/7)^2 = 0.489$

$Gini(Humidity = Normal) = 1 - (6/7)^2 - (1/7)^2 = 0.244$

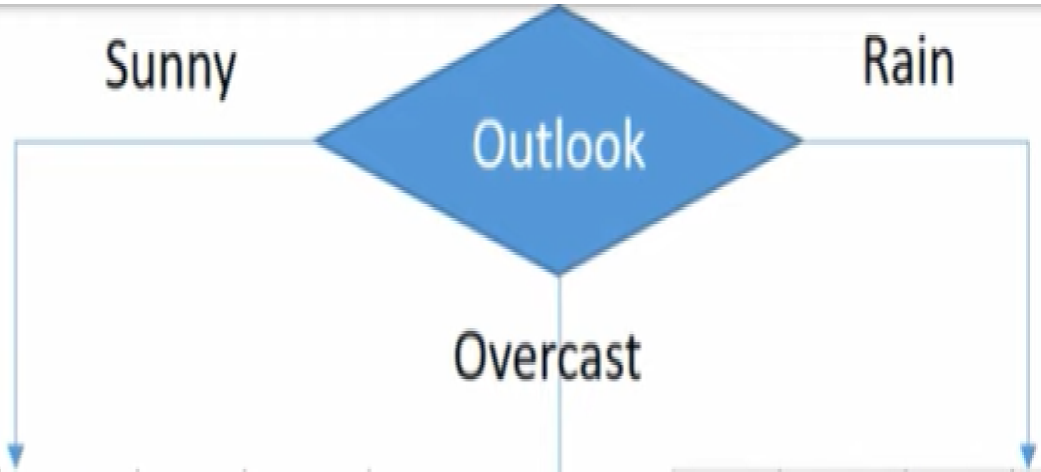
$Gini(Humidity) = \left(\frac{7}{14}\right) * 0.489 + \left(\frac{7}{14}\right) * 0.244$

$= 0.367$

- Attribute with lowest value of Gini Index is considered as Root node
- Here, Outlook has lowest Gini Index.


Time to decide

Feature	Gini Index
Outlook	0.342
Temperature	0.439
Humidity	0.367
Wind	0.428



Day	Outlook	Temp.	Humidity	Wind	Decision
1	Sunny	Hot	High	Weak	No
2	Sunny	Hot	High	Strong	No
8	Sunny	Mild	High	Weak	No
9	Sunny	Cool	Normal	Weak	Yes
11	Sunny	Mild	Normal	Strong	Yes

Day	Outlook	Temp.	Humidity	Wind	Decision
4	Rain	Mild	High	Weak	Yes
5	Rain	Cool	Normal	Weak	Yes
6	Rain	Cool	Normal	Strong	No
10	Rain	Mild	Normal	Weak	Yes
14	Rain	Mild	High	Strong	No

Day	Outlook	Temp.	Humidity	Wind	Decision
3	Overcast	Hot	High	Weak	Yes
7	Overcast	Cool	Normal	Strong	Yes
12	Overcast	Mild	High	Strong	Yes
13	Overcast	Hot	Normal	Weak	Yes

Focus on the sub dataset for sunny outlook. We need to find the Gini Index scores for temperature, humidity and wind features respectively.

Day	Outlook	Temp.	Humidity	Wind	Decision
1	Sunny	Hot	High	Weak	No
2	Sunny	Hot	High	Strong	No
8	Sunny	Mild	High	Weak	No
9	Sunny	Cool	Normal	Weak	Yes
11	Sunny	Mild	Normal	Strong	Yes

Gini of temperature for sunny outlook

Temperature	Yes	No	Number of instances
Hot	0	2	2
Cool	1	0	1
Mild	1	1	2

Gini(Outlook=Sunny and Temp.=Hot) = $1 - (0/2)^2 - (2/2)^2 = 0$

Gini(Outlook=Sunny and Temp.=Cool) = $1 - (1/1)^2 - (0/1)^2 = 0$

Gini(Outlook=Sunny and Temp.=Mild) = $1 - (1/2)^2 - (1/2)^2 = 1 - 0.25 - 0.25 = 0.5$

Gini(Outlook=Sunny and Temp.) = $(2/5)*0 + (1/5)*0 + (2/5)*0.5 = 0.2$

Gini of humidity for sunny outlook

Humidity	Yes	No	Number of instances
High	0	3	3
Normal	2	0	2

Gini(Outlook=Sunny and Humidity=High) = $1 - (0/3)^2 - (3/3)^2 = 0$
Gini(Outlook=Sunny and Humidity=Normal) = $1 - (2/2)^2 - (0/2)^2 = 0$
Gini(Outlook=Sunny and Humidity) = $(3/5)*0 + (2/5)*0 = 0$

Gini of wind for sunny outlook

Wind	Yes	No	Number of instances
Weak	1	2	3
Strong	1	1	2

Gini(Outlook=Sunny and Wind=Weak) = $1 – (1/3)^2 – (2/3)^2 = 0.266$

Gini(Outlook=Sunny and Wind=Strong) = $1- (1/2)^2 – (1/2)^2 = 0.2$

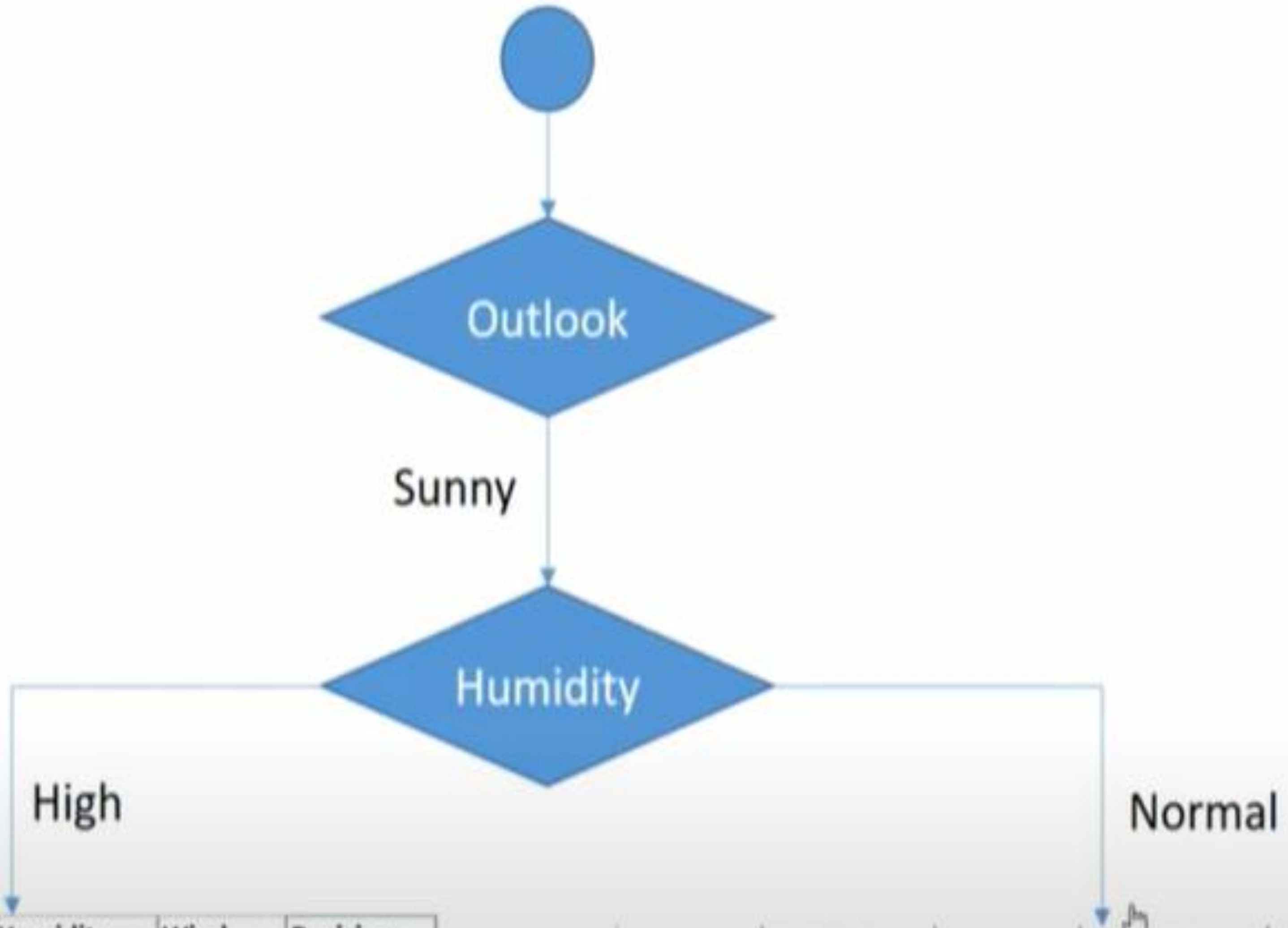
Gini(Outlook=Sunny and Wind) = (3/5)*0.266 + (2/5)*0.2 = 0.466

Decision for sunny outlook

We've calculated Gini Index scores for feature when outlook is sunny. The winner is humidity because it has the lowest value.

Feature	Gini index
Temperature	0.2
Humidity	0
Wind	0.466

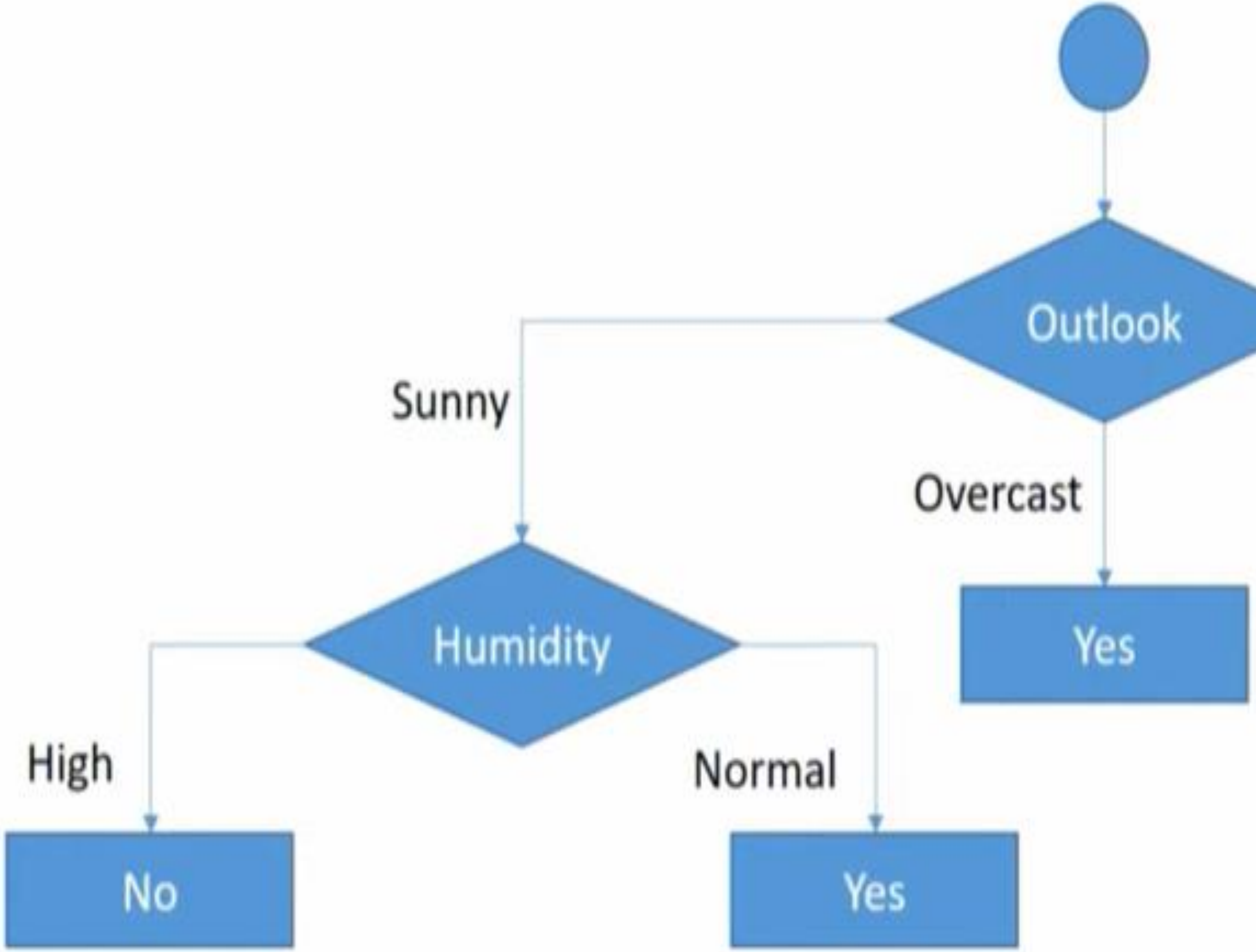
We'll put humidity check at the extension of sunny outlook.



Day	Outlook	Temp.	Humidity	Wind	Decision
1	Sunny	Hot	High	Weak	No
2	Sunny	Hot	High	Strong	No
8	Sunny	Mild	High	Weak	No

Day	Outlook	Temp.	Humidity	Wind	Decision
9	Sunny	Cool	Normal	Weak	Yes
11	Sunny	Mild	Normal	Strong	Yes

Sub datasets for high and normal humidity



Day	Outlook	Temp.	Humidity	Wind	Decision
4	Rain	Mild	High	Weak	Yes
5	Rain	Cool	Normal	Weak	Yes
6	Rain	Cool	Normal	Strong	No
10	Rain	Mild	Normal	Weak	Yes
14	Rain	Mild	High	Strong	No

Decisions for high and normal humidity

Now, we need to focus on rain outlook.

Rain outlook

Day	Outlook	Temp.	Humidity	Wind	Decision
4	Rain	Mild	High	Weak	Yes
5	Rain	Cool	Normal	Weak	Yes
6	Rain	Cool	Normal	Strong	No
10	Rain	Mild	Normal	Weak	Yes
14	Rain	Mild	High	Strong	No

We'll calculate Gini index scores for temperature, humidity and wind features when outlook is rain.

Gini of temperature for rain outlook

Temperature	Yes	No	Number of instances
Cool	1	1	2
Mild	2	1	3

Gini(Outlook=Rain and Temp.=Cool) = $1 - (1/2)^2 - (1/2)^2 = 0.5$

Gini(Outlook=Rain and Temp.=Mild) = $1 - (2/3)^2 - (1/3)^2 = 0.444$

Gini(Outlook=Rain and Temp.) = $(2/5)*0.5 + (3/5)*0.444 = 0.466$

Gini of humidity for rain outlook

Humidity	Yes	No	Number of instances
High	1	1	2
Normal	2	1	3

Gini(Outlook=Rain and Humidity=High) = $1 - (1/2)^2 - (1/2)^2 = 0.5$

Gini(Outlook=Rain and Humidity=Normal) = $1 - (2/3)^2 - (1/3)^2 = 0.444$

Gini(Outlook=Rain and Humidity) = $(2/5)*0.5 + (3/5)*0.444 = 0.466$

Gini of wind for rain outlook

Wind	Yes	No	Number of instances
Weak	3	0	3
Strong	0	2	2

Gini(Outlook=Rain and Wind=Weak) = $1 - (3/3)^2 - (0/3)^2 = 0$

Gini(Outlook=Rain and Wind=Strong) = $1 - (0/2)^2 - (2/2)^2 = 0$

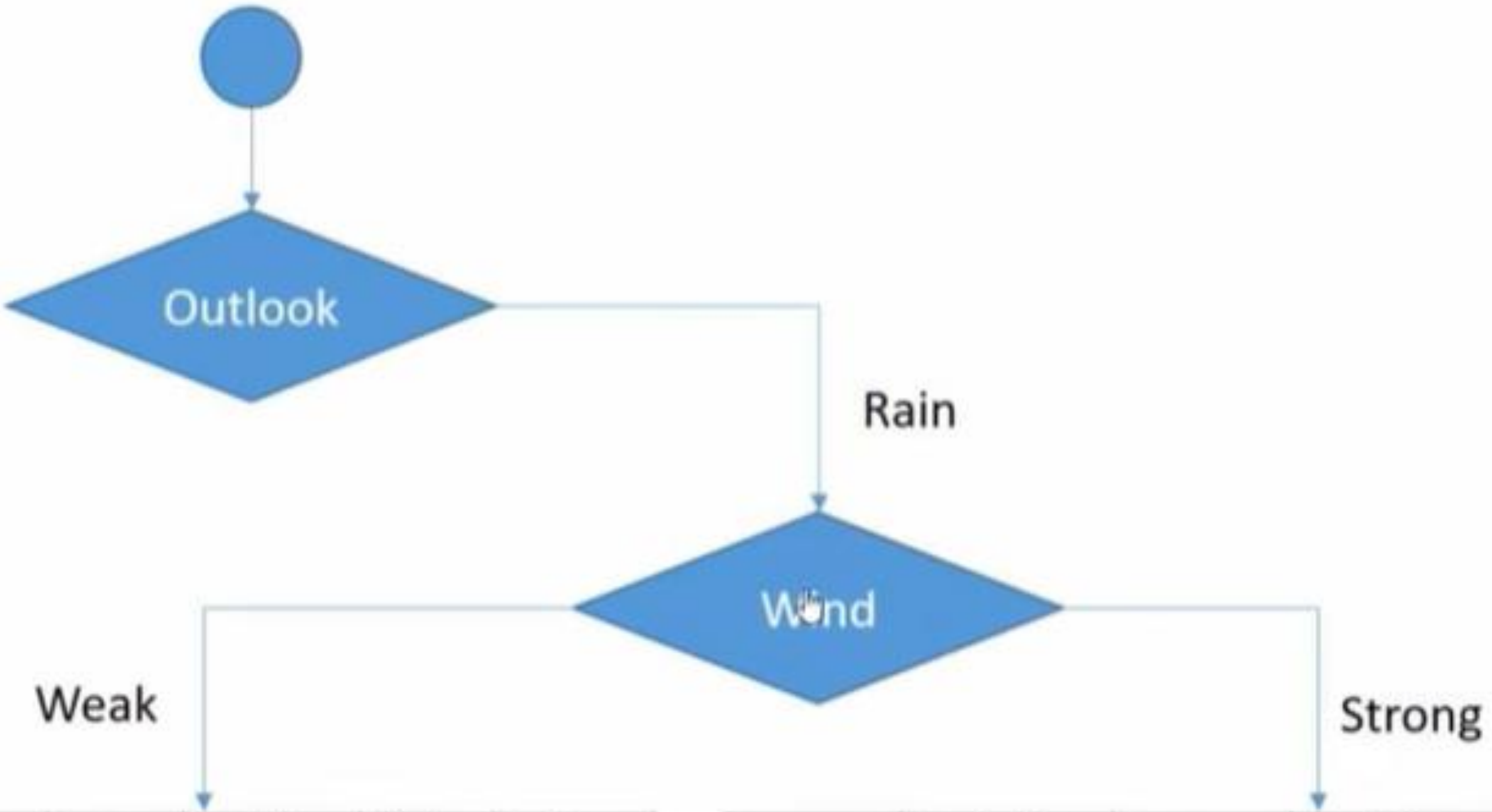
Gini(Outlook=Rain and Wind) = $(3/5)*0 + (2/5)*0 = 0$

Decision for rain outlook

The winner is wind feature for rain outlook because it has the minimum Gini Index score in features.

Feature	Gini index
Temperature	0.466
Humidity	0.466
Wind	0

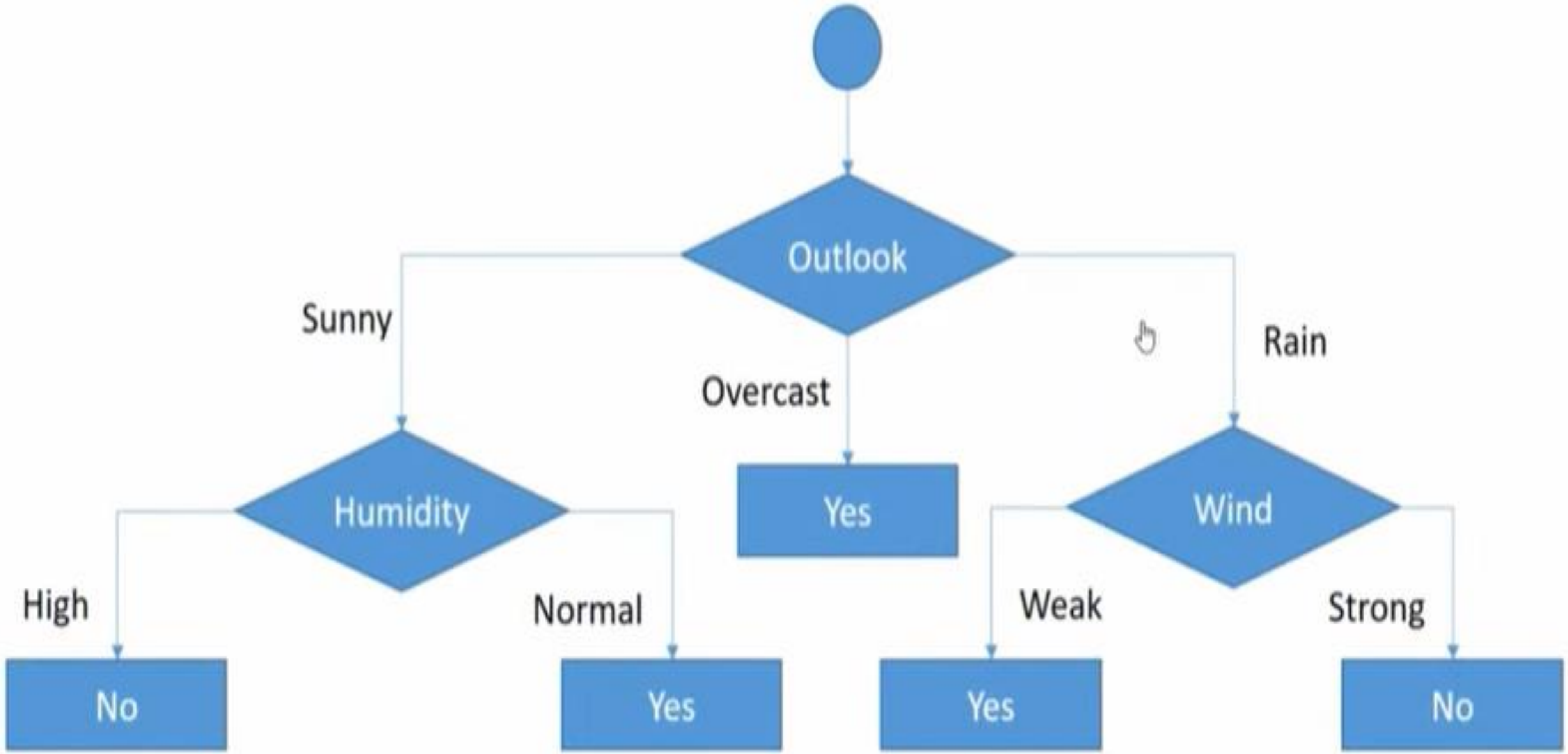
Put the wind feature for rain outlook branch and monitor the new sub data sets.



Day	Outlook	Temp.	Humidity	Wind	Decision
4	Rain	Mild	High	Weak	Yes
5	Rain	Cool	Normal	Weak	Yes
10	Rain	Mild	Normal	Weak	Yes

Day	Outlook	Temp.	Humidity	Wind	Decision
6	Rain	Cool	Normal	Strong	No
14	Rain	Mild	High	Strong	No

Sub data sets for weak and strong wind and rain outlook



Final form of the decision tree built by CART algorithm.