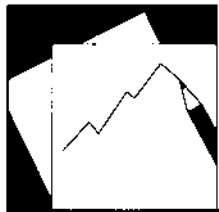


# Working Paper

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## Empirical Modeling of Contagion: A Review of Methodologies

*Mardi Dungey, Renée Fry,  
Brenda González-Hermosillo,  
and Vance Martin*

## **IMF Working Paper**

IMF Institute

### **Empirical Modeling of Contagion: A Review of Methodologies**

Prepared by Mardi Dungey, Renée Fry, Brenda González-Hermosillo, and Vance Martin<sup>1</sup>

Authorized for distribution by Sunil Sharma

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#### **Abstract**

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The existing literature suggests a number of alternative methods to test for the presence of contagion during financial market crises. This paper reviews those methods and shows how they are related in a unified framework. A number of extensions are also suggested that allow for multivariate testing, endogeneity issues, and structural breaks.

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## I. INTRODUCTION

There is now a reasonably large body of empirical work testing for the existence of contagion during financial crises. A range of different methodologies are in use, making it difficult to assess the evidence for and against contagion, and particularly its significance in transmitting crises between countries.<sup>2</sup>

The origins of current empirical studies of contagion stem from Sharpe (1964) and Grubel and Fadner (1971), and more recently from King and Wadhwani (1990); Engle, Ito, and Lin (1990); and Bekaert and Hodrick (1992). Many of the methods proposed in these papers are adapted in some form to the current empirical literature on measuring contagion.

The aim of the present paper is to provide a unifying framework to highlight the key similarities and differences among the various approaches. The proposed framework is based on a latent factor structure, which forms the basis of the models of Dungey and Martin (2001); Corsetti, Pericoli, and Sbracia (2001); and Bekaert, Harvey, and Ng (2003). This framework is used to compare directly the correlation analysis approach popularized in this literature by Forbes and Rigobon (2002); the vector autoregression (VAR) approach of Favero and Giavazzi (2002); the probability models of Eichengreen, Rose, and Wyplosz (1995, 1996); and the co-exceedance approach of Bae, Karolyi, and Stulz (2003).

An important outcome of this paper is that differences in the definitions used to define and test for contagion are minor and under certain conditions are even equivalent. In particular, all papers are interpreted as working from the same model, with the differences stemming from the amount of information used in the data to detect contagion. Interpreting the approaches in this way provides a natural ordering of models across the information spectrum with some models representing full information methods and others representing partial information methods.

The paper proceeds as follows. In Section II a framework drawn from basic relationships between asset returns is used to model returns in a non-crisis environment. This framework is augmented in Section III to give a model that includes an avenue for contagion during a crisis. The relationship between this model and the correlation tests for contagion are examined in Section IV, which includes a generalization of the Forbes and Rigobon bivariate test to a multivariate environment. The remaining non linear tests are examined in Section V, and additional methods are canvassed in Section VI. Each of the tests is shown to be a test of the significance of a slope dummy. Section VII concludes.

## II. A MODEL OF INTERDEPENDENCE

Before developing a model of contagion, a model of interdependence of asset markets

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<sup>2</sup>While the current paper focuses on empirical models of contagion, the literature on financial crises and contagion also covers theoretical and policy issues. These additional issues are reviewed in Flood and Marion (1998), with more recent papers given by Allen and Gale (2000); Calvo and Mendoza (2000); Kyle and Xiong (2001); and Kodres and Pritsker (2002).

during non-crisis periods is specified as a latent factor model of asset returns. The model has its origins in the factor models in finance based on Arbitrage Pricing Theory, for example, whereby asset returns are determined by a set of common factors and a set of idiosyncratic factors representing non-diversifiable risk (Sharpe, 1964; Solnik, 1974). Similar latent factor models of contagion are used by Dungey and Martin (2001); Dungey, Fry, González-Hermosillo, and Martin (2002a); Forbes and Rigobon (2002); and Bekaert, Harvey, and Ng (2003).

To simplify the analysis, the number of assets considered is three. Extending the model to  $N$  assets is straightforward, with an example given below. Let the returns of three asset markets during a non-crisis period be defined as

$$\{x_{1,t}, x_{2,t}, x_{3,t}\}. \quad (1)$$

All returns are assumed to have zero means. The returns could be on currencies, or national equity markets, or a combination of currency and equity returns in a particular country or across countries. The following trivariate factor model is assumed to summarize the dynamics of the three processes during a period of tranquility:

$$x_{i,t} = \lambda_i w_t + \delta_i u_{i,t}, \quad i = 1, 2, 3. \quad (2)$$

The variable  $w_t$  represents common shocks that impact upon all asset returns with loadings  $\lambda_i$ . These shocks could represent financial shocks arising from changes to the risk aversion of international investors, or changes in world endowments (Mahieu and Schotman, 1994; Rigobon, 2003b). In general,  $w_t$  represents market fundamentals that determine the average level of asset returns across international markets during “normal” (that is tranquil) times. This variable is commonly referred to as a world factor, which may or may not be observed. For simplicity, the world factor is assumed to be a latent stochastic process with zero mean and unit variance:

$$w_t \sim (0, 1). \quad (3)$$

The properties of this factor are extended below to capture richer dynamics, including both autocorrelation and time-varying volatility. The terms  $u_{i,t}$  in equation (2) are idiosyncratic factors that are unique to a specific asset market. The contribution of idiosyncratic shocks to the volatility of asset markets is determined by the loadings  $\delta_i > 0$ . These factors are also assumed to be stochastic processes with zero mean and unit variance:

$$u_{i,t} \sim (0, 1), \quad \forall i. \quad (4)$$

To complete the specification of the model, all factors are assumed to be independent

$$E[u_{i,t} u_{j,t}] = 0, \quad \forall i \neq j \quad (5)$$

$$E[u_{i,t} w_t] = 0, \quad \forall i. \quad (6)$$

To highlight the interrelationships among the three asset returns in equation (2) during a non-crisis period, the covariances are given by

$$E[x_{i,t} x_{j,t}] = \lambda_i \lambda_j, \quad \forall i \neq j, \quad (7)$$

while the variances are

$$E[x_{i,t}^2] = \lambda_i^2 + \delta_i^2 \quad \forall i. \quad (8)$$

Expression (7) shows that any dependence between asset returns is solely the result of the influence of common shocks, arising from  $w_t$ , that simultaneously impact upon all markets. Setting

$$\lambda_1 = \lambda_2 = \lambda_3 = 0, \quad (9)$$

results in independent asset markets with all movements determined by the idiosyncratic shocks,  $u_{i,t}$ .<sup>3</sup> The identifying assumption used by Mahieu and Schotman (1994) in a similar problem is to set  $\lambda_i \lambda_j$  to a constant value,  $L$ , for all  $i \neq j$ .

### III. UNANTICIPATED-SHOCK MODELS OF CONTAGION

The definition of the term contagion varies widely across the literature. In this paper contagion is represented by the transmission of unanticipated local shocks to another country or market. This definition is consistent with Masson (1998, 1999a,b), who divides shocks to asset markets as either common, spillovers that result from some identifiable channel, local or contagion, and, as shown below, that of other approaches, such as Forbes and Rigobon (2002), where contagion is represented by an increase in correlation during periods of crises.

The first model discussed is based on the factor structure developed by Dungey, Fry, González-Hermosillo and Martin (2002a,b) among others, where contagion is defined as the effects of unanticipated shocks across asset markets during a period of crisis. To distinguish between asset returns in non-crisis and crisis periods,  $y_{i,t}$  represents the return during the crisis period and  $x_{i,t}$  the return during the non-crisis period. Consider the case of contagion from country 1 to country 2. The factor model in equation (2) is now augmented as follows:

$$\begin{aligned} y_{1,t} &= \lambda_1 w_t + \delta_1 u_{1,t} \\ y_{2,t} &= \lambda_2 w_t + \delta_2 u_{2,t} + \gamma u_{1,t} \\ y_{3,t} &= \lambda_3 w_t + \delta_3 u_{3,t}, \end{aligned} \tag{10}$$

where the  $x_{i,t}$  are replaced by  $y_{i,t}$  to signify demeaned asset returns during the crisis period. The expression for  $y_{2,t}$  now contains a contagious transmission channel as represented by unanticipated local shocks from the asset market in country 1, with its impact measured by the parameter  $\gamma$ . The fundamental aim of all empirical models of contagion is to test the statistical significance of the parameter  $\gamma$ .<sup>4</sup>

#### A. Bivariate Testing

Bivariate tests of contagion focus on changes in the volatility of pairs of asset returns. From equation (10), the covariance between the asset returns of countries 1 and 2 during the crisis period is

$$E[y_{1,t} y_{2,t}] = \lambda_1 \lambda_2 + \gamma \delta_1. \tag{11}$$

Comparing this expression with the covariance for the pre-crisis period in equation (7) shows that the change in the covariance between the two periods is

$$E[y_{1,t} y_{2,t}] - E[x_{1,t} x_{2,t}] = \gamma \delta_1. \tag{12}$$

---

<sup>3</sup>Of course, just two of the restrictions in equation (7) are sufficient for independence of asset markets.

<sup>4</sup>An important assumption underlying the factor model in equation (10) is that the common shock and idiosyncratic shocks have the same impact during the crisis period as they do during the non-crisis period. This assumption is discussed in Section III.C.

If  $\gamma > 0$ , there is an increase in the covariance of asset returns during the crisis period because  $\delta_1 > 0$  by assumption. This is usually the situation observed in the data. However, it is possible for  $\gamma < 0$ , in which case there is a reduction in the covariance. Both situations are valid because both represent evidence of contagion via the impact of unanticipated shocks in factor model (10). Hence a test of contagion is given by testing the restriction

$$\gamma = 0 \quad (13)$$

in the factor model (10). This is the approach adopted by Dungey, Fry, González-Hermosillo, and Martin (2002a,b; 2003); and Dungey and Martin (2004).

An alternative way to construct a test of contagion is to use the volatility expression for  $y_{2,t}$ , which is given by

$$E[y_{2,t}^2] = \lambda_2^2 + \delta_2^2 + \gamma^2. \quad (14)$$

Since the change in volatility over the two periods is solely attributed to the presence of contagion

$$E[y_{2,t}^2] - E[x_{2,t}^2] = \gamma^2, \quad (15)$$

the contagion test based on restriction (13) can be interpreted as a test of whether there is an increase in volatility. The expression (14) suggests that a useful decomposition of the volatility of  $y_{2,t}$  is to decompose the effects of shocks into global, idiosyncratic, and contagion respectively as follows:

$$\frac{\lambda_i^2}{\lambda_2^2 + \delta_2^2 + \gamma^2}, \frac{\delta_i^2}{\lambda_2^2 + \delta_2^2 + \gamma^2}, \frac{\gamma^2}{\lambda_2^2 + \delta_2^2 + \gamma^2}. \quad (16)$$

This decomposition provides a descriptive measure of the relative strength of contagion in contributing to volatility. As before, the strength of contagion is determined by the parameter  $\gamma$ , which can be tested formally.

## B. Multivariate Testing

The test for contagion presented so far is a test for contagion from country 1 to country 2. However, it is possible to test for contagion in many directions, provided that there are sufficient moment conditions to identify the unknown parameters. For example, factor model (10) can be extended as

$$\begin{aligned} y_{1,t} &= \lambda_1 w_t + \delta_1 u_{1,t} + \gamma_{1,2} u_{2,t} + \gamma_{1,3} u_{3,t} \\ y_{2,t} &= \lambda_2 w_t + \delta_2 u_{2,t} + \gamma_{2,1} u_{1,t} + \gamma_{2,3} u_{3,t} \\ y_{3,t} &= \lambda_3 w_t + \delta_3 u_{3,t} + \gamma_{3,1} u_{1,t} + \gamma_{3,2} u_{2,t}, \end{aligned} \quad (17)$$

or more succinctly,

$$y_{i,t} = \lambda_i w_t + \delta_i u_{i,t} + \sum_{j=1, j \neq i}^3 \gamma_{i,j} u_{j,t}. \quad (18)$$

In this case there are 6 parameters,  $\gamma_{i,j}$ , controlling the strength of contagion across all asset markets. This model, by itself, is unidentified because there are 12 unknown parameters. However, by combining the empirical moments of the variance-covariance matrix of the three asset returns during the crisis period, 6 moments, with the empirical moments from the variance-covariance matrix of the pre-crisis period in equation (2), another 6 moments, gives 12 empirical moments in



total, which can be used to identify the 12 unknown parameters.

A joint test of contagion using the factor model in equations (2) and (17) can be achieved by comparing the objective function from the unconstrained model,  $q_u$ , with the value obtained from estimating the constrained model,  $q_c$ , whereby the contagion parameters are set to zero. Since the unconstrained model is just identified,  $q_u = 0$ , in which case the test is simply a test that under the null hypothesis of no contagion,

$$H_0 : q_c = 0, \quad (19)$$

which is distributed asymptotically as  $\chi^2$  with six degrees of freedom under the null. As before, the test of contagion can be interpreted as testing for changes in both variances and covariances.

### C. Structural Breaks

The model given by equations (2) and (18) is based on the assumption that the increase in volatility during the crisis period is solely generated by contagion; that is,  $\gamma_{i,j} \neq 0, \forall i, j$ . However, another scenario is that there is a general increase in volatility without any contagion, denoted as increased interdependence by Forbes and Rigobon (2002). This would arise if either the world loadings ( $\lambda_i$ ) change, or idiosyncratic loadings ( $\delta_i$ ) change, or a combination of the two. The first would be representative of a general increase in volatility across all asset markets brought about, for example, by an increase in the risk aversion of international investors. The second would arise from increases in the shocks of (some) individual asset markets that are entirely specific to those markets and thus independent of other asset markets.

To allow for structural breaks in the underlying relationships, the number of contagious linkages that can be entertained needs to be restricted. In the case where changes in the idiosyncratic shocks are allowed across the sample periods in all  $N$  asset markets, equation (18) becomes

$$y_{i,t} = \lambda_i w_t + \delta_{y,i} u_{i,t} + \sum_{j=1, j \neq i}^3 \gamma_{i,j} u_{j,t}, \quad (20)$$

where  $\delta_{y,i} \neq \delta_i$  are the idiosyncratic parameters during the crisis period. Bekaert, Harvey, and Ng (2003) adopt a different strategy for modeling structural breaks by specifying time-varying factor loadings.

The number of world and idiosyncratic parameters now increases to  $3N$ . Because the model is still block-recursive, there are just  $N(N+1)/2$  empirical moments from the crisis period available to identify the contagion parameters ( $\gamma_{i,j}$ ) and the structural break parameters ( $\delta_{y,i}$ ). This means that there are  $N(N+1)/2 - N = N(N-1)/2$  excess moments to identify contagion channels.

Extending the model to allow for structural breaks in both global and idiosyncratic factors in all  $N$  asset markets increases the number of world and idiosyncratic parameters to  $4N$ , now yielding  $N(N+1)/2 - 2N = N(N-3)/2$  excess moments to identify contagion channels in the crisis period. For a trivariate model ( $N=3$ ) that allows for all potential structural breaks in world and idiosyncratic factors, no contagion channels can be tested because the model is just

identified. Extending the model to  $N = 4$  assets allows for  $N(N - 3)/2 = 2$  potential contagion channels. Further extending the model to  $N = 6$  assets means that the number of contagion channels that can be tested increases to  $N(N - 3)/2 = 9$ .<sup>5</sup>

#### D. Using Only Crisis Data

Identification of the unknown parameters in the factor model framework discussed above is based on using information on both non-crisis and crisis periods. However, there may be a problem for certain asset markets in using non-crisis data to obtain empirical moments to identify unknown parameters, such as in the move from fixed to floating exchange rate regimes during the East Asian currency crisis. However, it is possible to identify the model using only crisis-period data, provided that the number of asset returns exceeds three and a limited number of contagious links are entertained. For example, for  $N = 4$  asset returns, there are 10 unique empirical moments from the variance-covariance matrix using crisis data. Specifying the factor model in (2) for  $N = 4$  assets means that there are four world parameters and four idiosyncratic parameters. This suggests that two contagious links can be specified and identified.

#### E. Autoregressive and Heteroscedastic Dynamics

The factor model can be easily extended to include a range of dynamics. Four broad avenues are possible. The first consists of including lagged values of the returns in the system. In Section VI these are interpreted as spillover effects. When the number of assets being studied is large, this approach can give rise to a large number of unknown parameters, thereby making estimation difficult. The second approach is to capture the dynamics through lags in the world factor  $w_t$ . This provides a more parsimonious representation of the system's dynamics as a result of a set of across-equation restrictions that arise naturally out of the factor structure. A third approach is to specify autoregressive representations for the idiosyncratic factors  $u_{i,t}$ . The specification of dynamics on all of the factors yields a state-space representation that can be estimated using a Kalman filter.

A fourth approach for specifying dynamics, which is potentially more important in models of asset returns than the specification of dynamics in the mean, is the specification of dynamics in the variance. This is especially true in models of contagion because increases in volatility are symptomatic of crises. A common way to capture this phenomenon is to include a GARCH (generalized autoregressive conditional heteroscedastic) structure on the factors.<sup>6</sup> This approach is used by Dungey, Fry, González-Hermosillo, and Martin (2002a, 2003); Bekaert, Harvey, and Ng (2003); and Dungey and Martin (2004). In the case where there is a single factor, a suitable specification is

$$w_t = e_t, \quad (21)$$

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<sup>5</sup>In general, an allowance for both contagion and structural breaks results in identification problems if the number of structural breaks entertained is unrestricted. Without the imposition of restrictions on the model, identification issues immediately arise.

<sup>6</sup>Chernov, Gallant, Ghysels, and Tauchen (2003) provide a recent investigation of the dynamics of asset markets.

where

$$e_t \sim N(0, h_t), \quad (22)$$

with conditional volatility  $h_t$ , given by a GARCH factor structure (Diebold and Nerlove, 1989; Dungey, Martin, and Pagan, 2000):

$$h_t = (1 - \alpha - \beta) + \alpha e_{t-1}^2 + \beta h_{t-1}. \quad (23)$$

The choice of the normalization  $(1 - \alpha - \beta)$  constrains the unconditional volatility to equal unity and is adopted for identification. For example, using factor model (10) augmented by equations (21)-(23), total (conditional) volatility of  $y_{2,t}$ , the asset return in the crisis period, is now given by

$$\begin{aligned} E_{t-1} [y_{2,t}^2] &= E_{t-1} [(\lambda_2 w_t + \delta_2 u_{2,t} + \gamma u_{1,t})^2] \\ &= \lambda_2^2 h_t + \delta_2^2 + \gamma^2, \end{aligned}$$

where the assumption of independence in equations (5) and (6) is utilized. The conditional covariance between  $y_{1,t}$  and  $y_{2,t}$  during the crisis period, for example, is

$$\begin{aligned} E_{t-1} [y_{1,t} y_{2,t}] &= E_{t-1} [(\lambda_1 w_t + \delta_1 u_{1,t})(\lambda_2 w_t + \delta_2 u_{2,t} + \gamma u_{1,t})] \\ &= \lambda_1 \lambda_2 h_t + \gamma \delta_1. \end{aligned}$$

Both the conditional variance and covariance during the crisis period are affected by the presence of contagion ( $\gamma \neq 0$ ). In particular, contagion has the effect of causing a structural shift during the crisis period in the conditional covariance by  $\gamma \delta_1$ , and in the conditional variance by  $\gamma^2$ .

The inclusion of a GARCH world factor into an  $N$  factor model of asset returns provides a parsimonious multivariate GARCH model. This model, when combined with a model of contagion, can capture changes in the variance and covariance structures of asset returns during financial crises. The parsimony of the factor GARCH model specification contrasts with multivariate GARCH models based on the BEKK specification (Engle and Kroner, 1995) which require a large number of parameters for even moderate size models.<sup>7</sup>

#### IV. CORRELATION AND COVARIANCE ANALYSIS

Forbes and Rigobon (2002) define contagion as the increase in correlation between two variables during a crisis period. In performing their test, the correlation between the two asset returns during the crisis period is adjusted to overcome the problem that correlations are a positive function of volatility. Because crisis periods are typically characterized by an increase in volatility, a test based on the (conditional) correlation is biased upward resulting in evidence of spurious contagion (Forbes and Rigobon, 2002; Boyer, Gibson, and Loretan, 1999; Loretan and English, 2000).<sup>8</sup>

A feature of the correlation applications is that they are based on pair-wise comparisons and thus do not consider potential multivariate analogues of the test. To overcome this problem, a

<sup>7</sup>Problems in estimating multivariate GARCH models are noted by Malliaroupulos (1997), although research on this problem proceeds apace.

<sup>8</sup>Butler and Joaquin (2002) conduct the same test across bull and bear markets, although they do not specifically use the terminology of contagion.

multivariate approach is proposed below that is based on simple regression equations augmented by dummy variables. This extension stems from the insight that the adjustments to the correlation coefficients proposed by Forbes and Rigobon can be placed conveniently within a bivariate regression framework where the underlying variables are scaled appropriately.

### A. Bivariate Testing

To demonstrate the Forbes and Rigobon (2002) approach, consider testing for contagion from country 1 to country 2. The correlation between the asset returns of the two asset markets is  $\rho_y$  during the crisis (high-volatility) period and  $\rho_x$  in the pre-crisis (low-volatility) period.<sup>9</sup> If there is an increase in volatility in the asset return of country 1,  $\sigma_{y,1}^2 > \sigma_{x,1}^2$ , without there being any change to the fundamental relationship between the asset returns in the two markets, then  $\rho_y > \rho_x$ , giving the false appearance of contagion. To adjust for this bias, Forbes and Rigobon show that the adjusted (unconditional) correlation is given by<sup>10</sup>

$$\nu_y = \frac{\rho_y}{\sqrt{1 + \left( \frac{\sigma_{y,1}^2 - \sigma_{x,1}^2}{\sigma_{x,1}^2} \right) (1 - \rho_y^2)}}. \quad (24)$$

This is the unconditional correlation ( $\nu_y$ ), which is the conditional correlation ( $\rho_y$ ) scaled by a non linear function of the percentage change in volatility in the asset return of the source country ( $(\sigma_{y,1}^2 - \sigma_{x,1}^2) / \sigma_{x,1}^2$ ), country 1 in this case, over the high and low-volatility periods. This adjustment allows for a level shift in the volatility of asset 1, whereby  $\nu_y = \rho_x$  if there is no fundamental change in the relationship between the two asset markets.

To test that there is a significant change in correlation, the null hypothesis is

$$H_0 : \nu_y = \rho_x, \quad (25)$$

against the alternative hypothesis of

$$H_1 : \nu_y > \rho_x. \quad (26)$$

A  $t$ -statistic for testing these hypotheses is given by

$$FR_1 = \frac{\hat{\nu}_y - \hat{\rho}_x}{\sqrt{\frac{1}{T_y} + \frac{1}{T_x}}}, \quad (27)$$

where the  $\hat{\cdot}$  signifies the sample estimator, and  $T_y$  and  $T_x$  are the respective sample sizes of the high-volatility and low-volatility periods. The standard error in equation (27) derives from the asymptotic distribution of the estimated correlation coefficient. To improve the finite sample properties of the test statistic, Forbes and Rigobon (2002) suggest using the Fisher

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<sup>9</sup>Forbes and Rigobon (2002) in their empirical application, compare the crisis period correlation with the correlation calculated over the total sample period (low volatility period). That is,  $x$  is replaced by  $z = (x; y)$ . This alternative formulation is also discussed below.

<sup>10</sup>See also Boyer, Gibson, and Loretan, 1999; and Loretan and English, 2000. Other approaches using correlation analysis are Karolyi and Stultz, 1996, and Longin and Solnik, 1995.

transformation:<sup>11</sup>

$$FR_2 = \frac{\frac{1}{2} \ln \left( \frac{1+\hat{\nu}_y}{1-\hat{\nu}_y} \right) - \frac{1}{2} \ln \left( \frac{1+\hat{\rho}_x}{1-\hat{\rho}_x} \right)}{\sqrt{\frac{1}{T_y-3} + \frac{1}{T_x-3}}}. \quad (28)$$

## B. Alternative Formulation

In implementing the correlation test in equation (28), equation (24) shows that the conditional correlation needs to be scaled initially by a nonlinear function of the change in volatility in the asset return of the source country, country 1 in this case, over the pertinent sample periods. Another way to implement the Forbes and Rigobon test of contagion is to scale the asset returns and perform the contagion test within a regression framework.<sup>12</sup> Continuing with the example of testing for contagion from the asset market of country 1 to the asset market of country 2, consider scaling the asset returns during the pre-crisis period by their respective standard deviations. First define the following pre-crisis regression equation in terms of the scaled asset returns

$$\left( \frac{x_{2,t}}{\sigma_{x,2}} \right) = \alpha_1 \left( \frac{x_{1,t}}{\sigma_{x,1}} \right) + \eta_{x,t}, \quad (29)$$

where  $\eta_{x,t}$  is a disturbance term and  $\alpha_1$  is a regression parameter, and as before all variable are transformed to have zero means. The pre-crisis slope regression parameter is related to the pre-crisis correlation coefficient as  $\alpha_1 = \rho_x$ . For the crisis returns the regression equation is given as follows, where the scaling of asset returns is still by the respective standard deviations from the pre-crisis periods

$$\left( \frac{y_{2,t}}{\sigma_{x,2}} \right) = \beta_1 \left( \frac{y_{1,t}}{\sigma_{x,1}} \right) + \eta_{y,t}, \quad (30)$$

where  $\eta_{y,t}$  is a disturbance term and  $\beta_0$  and  $\beta_1$  are regression parameters. The crisis regression slope parameter  $\beta_1 = \nu_y$ , which is the Forbes-Rigobon adjusted correlation coefficient given in equation (24).

This alternative formulation suggests that another way to implement the Forbes-Rigobon adjusted correlation is to estimate equations (29) and (30) by ordinary least squares (OLS) and test the equality of the regression slope parameters. This test is equivalent to a Chow test for a structural break of the regression slope. Implementation of the test can be based on the following

<sup>11</sup>This transformation is valid for small values of the correlation coefficients,  $\rho_x$  and  $\nu_y$ .

Further refinements are discussed in Kendall and Stuart (1969, Vol.I, p.391). For the case of independence,  $\rho_x = \nu_y = 0$ , an exact expression for the variance of the transformed correlation coefficient is available. An illustration of these problems for the Forbes and Rigobon method is given in Dungey and Zhumabekova (2001).

<sup>12</sup>Corsetti, Pericoli, and Sbracia (2001) extend the Forbes and Rigobon framework to a model equivalent to the factor structure given in (10). Their approach requires evaluating quantities given by the ratio of the contribution of idiosyncratic and common factors to volatility,  $\delta_i^2/\lambda_i^2$  for example. These quantities can be estimated directly using the framework discussed in Section III.

pooled regression equation over the entire sample:

$$\left( \frac{z_{2,t}}{\sigma_{x,2}} \right) = \beta_1 \left( \frac{z_{1,t}}{\sigma_{x,1}} \right) + \gamma_1 \left( \frac{z_{1,t}}{\sigma_{x,1}} \right) d_t + \eta_t, \quad (31)$$

where

$$z_i = \left( x_{i,1}, x_{i,2}, \dots, x_{i,T_x} \quad y_{i,1}, y_{i,2}, \dots, y_{i,T_y} \right)', \quad i = 1, 2, \quad (32)$$

represents the  $(T_x + T_y) \times 1$  scaled pooled data set by stacking the pre-crisis and crisis scaled data;  $d_t$  is a slope dummy variable defined as

$$d_t = \begin{cases} 1 & : t > T_x \\ 0 & : \text{otherwise} \end{cases}; \quad (33)$$

and  $\eta_t$  is a disturbance term. The parameter

$$\gamma_1 = \beta_1 - \alpha_1 \quad (34)$$

in equation (31) captures the effect of contagion. It represents the additional contribution of information on asset returns in country 2 to the pre-crisis regression. If there is no change in the relationship, the dummy variable provides no new additional information during the crisis period, resulting in  $\gamma_1 = 0$ . Thus, the Forbes and Rigobon contagion test can be implemented by estimating equation (31) by OLS and performing a one-sided  $t$ -test of

$$H_0 : \gamma_1 = 0, \quad (35)$$

in (31), which is equivalent to testing  $\alpha_1 = \beta_1$  in equations (29) and (30).<sup>13</sup> Of course, the test statistic to perform the contagion test is invariant to scaling transformations of the regressors, such as the use of  $\sigma_{x,1}$  and  $\sigma_{x,2}$  to standardize  $z_t$ . This suggests that an even more direct way to test for contagion is to implement a standard test of parameter constancy in a regression framework simply based on  $z_t$ , the unscaled data.<sup>14</sup>

There is one difference between the regression approach to correlation testing for contagion based on equation (31) and the Forbes and Rigobon approach: the standard errors used in the test statistics are different in small samples. The latter approach is based on the small sample asymptotic adjustment given in equation (28), while the former is based, in general, on the usual least-squares standard errors.

### C. Relationship with Unanticipated-Shock Models

Interpreting the Forbes-Rigobon contagion test as a Chow test provides an important link connecting this approach with the unanticipated-shock model discussed in the previous section.

<sup>13</sup>Caporale, Cipollini, and Spagnolo (2002) conduct a test of contagion based on a slope dummy, but do not identify the connection of the test with the Forbes and Rigobon (2002) correlation approach.

<sup>14</sup>To implement the form of the Forbes and Rigobon (2002) version of the correlation test within the regression framework in equation (31), the pre-crisis data are now replaced by the total sample data. That is, the low volatility period is defined as the total sample period and not the pre-crisis period. This requires redefining the pertinent variables as  $z_t = (x, y, y, )$  and the slope dummy as  $d = (0_{T_x}, 1_{T_y}, 1_{T_y})$ , and scaling the variables in  $z_t$  by the respective standard deviations obtained from the total sample period  $(x_t, y_t)$ .

To highlight this link, let the dynamics of the processes be represented by the first two expressions of the contagion model in (10)

$$y_{1,t} = \lambda_1 w_t + \delta_1 u_{1,t} \quad (36)$$

$$y_{2,t} = \lambda_2 w_t + \delta_2 u_{2,t} + \gamma u_{1,t}, \quad (37)$$

where as before, contagion from the asset market in country 1 to country 2 is controlled by the parameter  $\gamma$ . Combining these expressions to substitute out  $u_{1,t}$  from the equation for  $y_{2,t}$  gives

$$y_{2,t} = \left( \frac{\lambda_2 \delta_1 - \lambda_1 \gamma}{\delta_1} \right) w_t + \frac{\gamma}{\delta_1} y_{1,t} + \delta_2 u_{2,t}. \quad (38)$$

The corresponding asset equation in the pre-crisis period is then by definition ( $\gamma = 0$ , and changing  $y_{i,t}$  to  $x_{i,t}$ )

$$x_{2,t} = \left( \frac{\lambda_2 \delta_1 - \lambda_1 \gamma}{\delta_1} \right) w_t + \delta_2 u_{2,t}, \quad (39)$$

where  $x_{i,t}$  is the asset return in the pre-crisis period. Equations (38) and (39) are of the same form as equation (31), provided that the common factor is taken as  $w_t = z_{1,t}$ , which is the stacked vector of asset returns in country 1 across pre-crisis and crisis returns. In this scenario the unconditional shock model and the Forbes-Rigobon approach are equivalent with the test of contagion based on  $\gamma = 0$ , which amounts to testing the additional explanatory power of the asset returns in country 1 to explain movements in the asset returns in country 2 over and above the factors that govern movements in asset markets during non-crisis periods.

In practice, Forbes and Rigobon (2002) identify the world factor  $w_t$  using a number of observed variables, including U.S. interest rates. These variables are initially extracted from the asset returns data by regressing the returns on the chosen set of world factors and using the residuals from these regressions in the contagion tests given in equations (24)-(28). In conducting the contagion tests, the analysis is performed in pairs with the source country changing depending on the hypothesis being tested. This testing strategy is highlighted in (38) and (39) where the source country is country 1. An implication of the approach though is that it requires switching the exogeneity status of the variables, an issue that is discussed further below.

#### D. Multivariate Testing

The regression framework developed above for implementing the Forbes and Rigobon test suggests that a multivariate analogue can be easily constructed as follows.<sup>15</sup> In the case of three

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<sup>15</sup>As will become apparent, issues of endogeneity immediately arise. These issues are discussed below.

asset returns, the pre-crisis period equations are

$$\begin{aligned}\left(\frac{x_{1,t}}{\sigma_{x,1}}\right) &= \alpha_{1,2} \left(\frac{x_{2,t}}{\sigma_{x,2}}\right) + \alpha_{1,3} \left(\frac{x_{3,t}}{\sigma_{x,3}}\right) + \eta_{x,1,t} \\ \left(\frac{x_{2,t}}{\sigma_{x,2}}\right) &= \alpha_{2,1} \left(\frac{x_{1,t}}{\sigma_{x,1}}\right) + \alpha_{2,3} \left(\frac{x_{3,t}}{\sigma_{x,3}}\right) + \eta_{x,2,t} \\ \left(\frac{x_{3,t}}{\sigma_{x,3}}\right) &= \alpha_{3,1} \left(\frac{x_{1,t}}{\sigma_{x,1}}\right) + \alpha_{3,2} \left(\frac{x_{2,t}}{\sigma_{x,2}}\right) + \eta_{x,3,t},\end{aligned}\tag{40}$$

while the crisis equations are specified as

$$\begin{aligned}\left(\frac{y_{1,t}}{\sigma_{x,1}}\right) &= \beta_{1,2} \left(\frac{y_{2,t}}{\sigma_{x,2}}\right) + \beta_{1,3} \left(\frac{y_{3,t}}{\sigma_{x,3}}\right) + \eta_{y,1,t} \\ \left(\frac{y_{2,t}}{\sigma_{x,2}}\right) &= \beta_{2,1} \left(\frac{y_{1,t}}{\sigma_{x,1}}\right) + \beta_{2,3} \left(\frac{y_{3,t}}{\sigma_{x,3}}\right) + \eta_{y,2,t} \\ \left(\frac{y_{3,t}}{\sigma_{x,3}}\right) &= \beta_{3,1} \left(\frac{y_{1,t}}{\sigma_{x,1}}\right) + \beta_{3,2} \left(\frac{y_{2,t}}{\sigma_{x,2}}\right) + \eta_{y,3,t}.\end{aligned}\tag{41}$$

A joint test of contagion is given by

$$\alpha_{i,j} = \beta_{i,j}, \quad \forall i \neq j,\tag{42}$$

which represents six restrictions. A convenient way to implement the multivariate version of the Forbes and Rigobon test is to adopt the strategy of (31) and write the model as a three-equation system augmented by a set of slope dummy variables to capture the impact of contagion on asset returns

$$\begin{aligned}\left(\frac{z_{1,t}}{\sigma_{x,1}}\right) &= \alpha_{1,2} \left(\frac{z_{2,t}}{\sigma_{x,2}}\right) + \alpha_{1,3} \left(\frac{z_{3,t}}{\sigma_{x,3}}\right) + \gamma_{1,2} \left(\frac{z_{2,t}}{\sigma_{x,2}}\right) d_t + \gamma_{1,3} \left(\frac{z_{3,t}}{\sigma_{x,3}}\right) d_t + \eta_{1,t} \\ \left(\frac{z_{2,t}}{\sigma_{x,2}}\right) &= \alpha_{2,1} \left(\frac{z_{1,t}}{\sigma_{x,1}}\right) + \alpha_{2,3} \left(\frac{z_{3,t}}{\sigma_{x,3}}\right) + \gamma_{2,1} \left(\frac{z_{1,t}}{\sigma_{x,1}}\right) d_t + \gamma_{2,3} \left(\frac{z_{3,t}}{\sigma_{x,3}}\right) d_t + \eta_{2,t} \\ \left(\frac{z_{3,t}}{\sigma_{x,3}}\right) &= \alpha_{3,1} \left(\frac{z_{1,t}}{\sigma_{x,1}}\right) + \alpha_{3,2} \left(\frac{z_{2,t}}{\sigma_{x,2}}\right) + \gamma_{3,1} \left(\frac{z_{1,t}}{\sigma_{x,1}}\right) d_t + \gamma_{3,2} \left(\frac{z_{2,t}}{\sigma_{x,2}}\right) d_t + \eta_{3,t},\end{aligned}\tag{43}$$

where the  $z_i$  pooled asset returns are as defined in equation (32);  $\eta_i$  are disturbance terms;  $d_t$  is the dummy variable defined in equation (33); and  $\gamma_{i,j} = \beta_{i,j} - \alpha_{i,j}$ , are the parameters that control the strength of contagion.

The multivariate contagion test is based on testing the null hypothesis

$$H_0 : \gamma_{i,j} = 0, \quad \forall i \neq j.\tag{44}$$

Implementation of the test can be performed by using standard multivariate test statistics, including likelihood ratio, Wald and Lagrange multiplier tests.

Rigobon (2003b) suggests an alternative multivariate test of contagion. This test is referred to as the determinant of the change in the covariance matrix (DCC) because it is based on comparing the covariance matrices across two samples and then taking the determinant to express



the statistic as a scalar. The DCC statistic is formally defined as

$$DCC = \frac{|\hat{\Omega}_y - \hat{\Omega}_x|}{\hat{\sigma}_{DCC}}, \quad (45)$$

where  $\hat{\Omega}_y$  and  $\hat{\Omega}_x$  are the estimated covariance matrices of asset returns in the crisis and pre-crisis periods respectively, and  $\hat{\sigma}_{DCC}$  is an estimate of the pertinent standard error of the statistic. Under the null hypothesis there is no change in the covariance structure of asset returns across sample periods, resulting in a value of  $DCC = 0$ . If contagion increases volatility during the crisis period, then  $DCC > 0$ , resulting in a rejection of the null hypothesis of no contagion.

The DCC test represents a test of parameter stability and thus provides an alternative to a Chow test. However, given the relationship between Chow and contagion tests discussed above, this implies that potentially the DCC test is also a test of contagion. To highlight this point, consider the following bivariate factor model based on the first two equations in (2) and (10). The pre-crisis and crisis covariance matrices are respectively

$$\Omega_x = \begin{bmatrix} \lambda_1^2 + \delta_1^2 & \lambda_1 \lambda_2 \\ \lambda_1 \lambda_2 & \lambda_2^2 + \delta_2^2 \end{bmatrix}, \quad \Omega_y = \begin{bmatrix} \lambda_1^2 + \delta_1^2 & \lambda_1 \lambda_2 + \gamma \delta_1 \\ \lambda_1 \lambda_2 + \gamma \delta_1 & \lambda_2^2 + \delta_2^2 + \gamma^2 \end{bmatrix}.$$

The numerator of the DCC statistic in this case is

$$|\hat{\Omega}_y - \hat{\Omega}_x| = \begin{vmatrix} 0 & \hat{\gamma} \hat{\delta}_1 \\ \hat{\gamma} \hat{\delta}_1 & \hat{\gamma}^2 \end{vmatrix} = -\hat{\gamma}^2 \hat{\delta}_1^2,$$

where the  $\hat{\cdot}$  signifies a parameter estimator. Under the null hypothesis  $DCC = 0$ , which is achieved when  $\gamma = 0$ , a result that is equivalent to the tests of contagion already discussed.

In implementing the DCC test, the covariance matrices employed tend to be conditional covariance matrices if dynamics arising from lagged variables and other exogenous variables are controlled for. The simplest approach is to estimate a VAR for the total period,  $T_x + T_y$ , and base the covariances on the VAR residuals. This is the approach adopted in the empirical application of Rigobon (2003b). The advantage of working with VAR residuals, as compared with structural residuals, is that the VAR represents an unconstrained reduced form, thereby circumventing problems of simultaneity bias. These issues are now discussed.

## E. Endogeneity Issues

The potential simultaneity biases arising from the presence of endogenous variables are more evident when the Forbes and Rigobon test is cast in a linear regression framework. Forbes and Rigobon perform the correlation test in pairs of countries under the assumption that contagion spreads from one country to another with the source country being exogenous. The test is then performed in the reverse direction with the implicit assumption of exogeneity on the two asset returns reversed. Performing the two tests in this way is inappropriate because it clearly ignores the simultaneity bias problem.<sup>16</sup>

<sup>16</sup>Forbes and Rigobon recognize this problem and do not test for contagion in both directions, being very clear about their exogeneity assumptions.

Forbes and Rigobon (2002) show, using a Monte Carlo analysis, that the size of the simultaneity bias is unlikely to be severe if the size of the correlations between asset returns are relatively small. Interestingly, Rigobon (2003b) notes that the volatility adjustment in performing the test in equation (24) is incorrect in the presence of simultaneity bias. However, as noted above, the Forbes and Rigobon adjustment acts as a scaling parameter which has no affect on the properties of the test statistic in a linear regression framework. The problem of simultaneity bias is the same whether the endogenous explanatory variables are scaled or not.

To perform the Forbes and Rigobon contagion test while correcting for simultaneity bias, equations (40) and (41) need to be estimated initially using a simultaneous equations estimator, and the tests of contagion performed on the simultaneous equation estimates of  $\gamma_{i,j}$  in (43). To demonstrate some of the issues, the bivariate model in equations (29) and (30) is expanded to allow for structural breaks in the idiosyncratic loadings. The bivariate versions of the model without intercepts during the pre-crisis and crisis periods are respectively (where for clarity,  $\varrho_{i,t} = \eta_{x,i,t}$  and  $\xi_{i,t} = \eta_{y,i,t}$  in comparison with equations (29) and (30))

$$\left( \frac{x_{1,t}}{\sigma_{x,1}} \right) = \alpha_1 \left( \frac{x_{2,t}}{\sigma_{x,2}} \right) + \varrho_{1,t} \quad (46)$$

$$\left( \frac{x_{2,t}}{\sigma_{x,2}} \right) = \alpha_2 \left( \frac{x_{1,t}}{\sigma_{x,1}} \right) + \varrho_{2,t}, \quad (47)$$

where  $\varrho_{i,t}$  are independent and identically distributed (*iid*) with zero means and variances  $E[\sigma_{\varrho,i}^2]$ , and

$$\left( \frac{y_{1,t}}{\sigma_{x,1}} \right) = \beta_1 \left( \frac{y_{2,t}}{\sigma_{x,2}} \right) + \xi_{1,t} \quad (48)$$

$$\left( \frac{y_{2,t}}{\sigma_{x,2}} \right) = \beta_2 \left( \frac{y_{1,t}}{\sigma_{x,1}} \right) + \xi_{2,t}, \quad (49)$$

where  $\xi_{i,t}$  are *iid* with zero means and variances  $E[\sigma_{\xi,i}^2]$ . The respective reduced forms are

$$\left( \frac{x_{1,t}}{\sigma_{x,1}} \right) = \frac{1}{1 - \alpha_1 \alpha_2} (\varrho_{1,t} + \alpha_1 \varrho_{2,t}) \quad (50)$$

$$\left( \frac{x_{2,t}}{\sigma_{x,2}} \right) = \frac{1}{1 - \alpha_1 \alpha_2} (\varrho_{2,t} + \alpha_2 \varrho_{1,t})$$

for the pre-crisis period and

$$\left( \frac{y_{1,t}}{\sigma_{x,1}} \right) = \frac{1}{1 - \beta_1 \beta_2} (\xi_{1,t} + \beta_1 \xi_{2,t}) \quad (51)$$

$$\left( \frac{y_{2,t}}{\sigma_{x,2}} \right) = \frac{1}{1 - \beta_1 \beta_2} (\xi_{2,t} + \beta_2 \xi_{1,t})$$

for the crisis period. For the two subperiods the variance-covariance matrices are

$$\Omega_x = \frac{1}{(1 - \alpha_1 \alpha_2)^2} \begin{bmatrix} \sigma_{\varrho,1}^2 + \alpha_1^2 \sigma_{\varrho,2}^2 & \alpha_1 \sigma_{\varrho,2}^2 \\ \alpha_1 \sigma_{\varrho,2}^2 & \sigma_{\varrho,2}^2 + \alpha_2^2 \sigma_{\varrho,1}^2 \end{bmatrix} \quad (52)$$

$$\Omega_y = \frac{1}{(1 - \beta_1 \beta_2)^2} \begin{bmatrix} \sigma_{\xi,1}^2 + \beta_1^2 \sigma_{\xi,2}^2 & \beta_1 \sigma_{\xi,2}^2 \\ \beta_1 \sigma_{\xi,2}^2 & \sigma_{\xi,2}^2 + \beta_2^2 \sigma_{\xi,1}^2 \end{bmatrix}. \quad (53)$$

The model at present is underidentified because there is a total of just six unique moments across the two samples, to identify the eight unknown parameters

$$\{\alpha_1, \alpha_2, \beta_1, \beta_2, \sigma_{\varepsilon,1}^2, \sigma_{\varepsilon,2}^2, \sigma_{\xi,1}^2, \sigma_{\xi,2}^2\}.$$

In a study of the relationship between Mexican and Argentinian bonds, Rigobon (2003a) identifies the model by setting  $\alpha_1 = \beta_1$  and  $\alpha_2 = \beta_2$ . However, from equation (34), this implies that there is no contagion, just a structural break in the idiosyncratic variances. An alternative approach to identification that is more informative in the context of testing for contagion is to not allow for a structural break and set  $\sigma_{x,1}^2 = \sigma_{y,1}^2$ , and  $\sigma_{x,2}^2 = \sigma_{y,2}^2$ . Now there are six equations to identify the six unknowns. A test of contagion is given by a test of the over-identifying restrictions under the null hypothesis of no contagion. The observational equivalence between the two identification strategies has already been noted above in the discussion of the factor model. However, if the idiosyncratic variances are changing over the sample, the contagion test is undersized (Toyoda and Ohtani, 1986). One solution is to expand the number of asset markets investigated. For example, increasing the number of assets to  $N = 3$  results in a just identified model because there are 12 unknown parameters  $\{\alpha_1, \alpha_2, \alpha_3, \beta_1, \beta_2, \beta_3, \sigma_{\varepsilon,1}^2, \sigma_{\varepsilon,2}^2, \sigma_{\varepsilon,3}^2, \sigma_{\xi,1}^2, \sigma_{\xi,2}^2, \sigma_{\xi,3}^2\}$ , but there are now 12 moments because there are six unique moments from each of the variance-covariance matrices from the two subperiods.

Another solution to the identification problem is to expand the model to include own lagged variables, for example. In this case, the reduced-form expressions in (51) represent VARs over the two subperiods. Corresponding to each sample period, in each VAR there are four parameters associated with the lagged variables which are used to identify the four structural parameters. Favero and Giavazzi (2002) do this within a related context in testing for contagion; see Section V.A below.

Rigobon (2002) also suggests using instrumental variables to obtain consistent parameter estimates, with the instruments defined as

$$s_i = \left( -x_{i,1}, -x_{i,2}, \dots, -x_{i,T_x} \quad y_{i,1}, y_{i,2}, \dots, y_{i,T_y} \right)', \quad i = 1, 2.$$

This choice of instruments is an extension of the early suggestions of Wald (1940) and Durbin (1954). For example, Wald defined the instrument set as a dummy variable, with 1 signifying observations above the median and  $-1$  signifying observations below the median. In the case of contagion and modeling financial crises, observations above (below) the median can be expected to correspond to crisis (pre-crisis) observations. This suggests that the Rigobon instrument is likely to be more efficient than the Wald choice because it uses more information.<sup>17</sup> Rigobon then proceeds to estimate pooled equations as in (43), but with  $\gamma_{i,j} = 0$ . But this is not a test of contagion, since  $\alpha_i = \beta_i$  is imposed and not tested. Not surprisingly, the IV estimator of the structural parameters in this case is equivalent to the matching moment estimator using equations (52) and (53) subject to the restrictions  $\alpha_1 = \beta_1$ , and  $\alpha_2 = \beta_2$ .

<sup>17</sup>A similar issue arises in the Eichengreen, Rose, and Wyplosz (1995, 1996) framework in testing for contagion. This approach is discussed below in Section V.B.

## V. MODELS OF ASYMMETRIES AND NONLINEARITIES

A number of papers have concentrated on modeling contagion through a range of asymmetrical adjustments. The motivation of these approaches is that the transmission processes across asset markets may be nonlinearly different during periods of extreme returns than during “normal” times. In these models contagion arises when significant relationships across asset markets are detected during periods of extreme movements. The underlying differences in the proposed approaches lie in the ways that extreme observations are modeled. Four models of asymmetries are now outlined. In discussing these models, the model represented by equations (36)-(39) is used.

### A. Outliers

Favero and Giavazzi (2002) use a VAR to control for the interdependence between asset returns, and use the heteroscedasticity and nonnormalities of the residuals from that VAR to identify unexpected shocks that may be transmitted across countries and hence considered contagion. The methodology first estimates a simple VAR and considers the distribution of the residuals. Residuals that contribute to nonnormality and heteroscedasticity in the data are identified with a set of dummies associated with “unusual” residuals for each country, indicating crisis observations. The test for contagion is then given as the significance of those dummies in explaining the returns for the alternate assets in a structural model. That is, the test for contagion is the significance of an unexpected shock in country  $i$  on the returns for country  $j$ , a definition completely consistent with that given in the model of equations (36)-(38).

To highlight the properties of the Favero and Giavazzi (2002) testing framework, consider testing for contagion from  $y_{1,t}$  to  $y_{2,t}$ . Define the dummy variable corresponding to a single outlier in  $y_{1,t}$ , as

$$d_{1,t} = \begin{cases} 1 : & |u_{1,t}| > 3\delta_i^2 \\ 0 : & \text{otherwise} \end{cases} \quad (54)$$

The Favero and Giavazzi test amounts to replacing  $u_{1,t}$  in equation (37) by  $d_{1,t}$

$$y_{2,t} = \lambda_2 w_t + \delta_2 u_{2,t} + \gamma d_{1,t}, \quad (55)$$

and performing a  $t$ -test of  $\gamma = 0$ . In practice, a separate dummy variable is defined for each outlier with the contagion test corresponding to a joint test that all parameters associated with the dummy variables are zero. To demonstrate their approach more formally, consider the following  $N$ -variate first-order VAR model:

$$z_t = \Phi z_{t-1} + v_t, \quad (56)$$

where  $z_t$  are the pooled asset returns across the two sample periods as defined in equation (32),  $\Phi$  contains the  $(N \times N)$  VAR parameters, and  $v_t$  are the reduced-form disturbances with zero means and constant covariance matrix with variances given by  $E[v_i^2] = \sigma_i^2$ . The dummy variables are defined as

$$d_{i,k,t} = \begin{cases} 1 : & |v_{i,t}| > 3\sigma_i^2 \\ 0 : & \text{otherwise} \end{cases}, \quad (57)$$

where one (unique) dummy variable is defined for each observation that is an outlier. That is,  $k = 1$  for the first outlier,  $k = 2$  for the second outlier, etc. These dummy variables are then included in a structural model that is effectively equation (43) without any scaling of the variables,

augmented by the inclusion of own lagged variables to achieve identification. For a bivariate model where there is just one outlier in each returns series, the structural equations are

$$\begin{aligned} z_{1,t} &= \alpha_{1,2}z_{2,t} + \theta_1 z_{1,t-1} + \gamma_{1,1}d_{1,1,t} + \gamma_{1,2}d_{2,1,t} + \eta_{1,t} \\ z_{2,t} &= \alpha_{2,1}z_{1,t} + \theta_2 z_{2,t-1} + \gamma_{2,1}d_{1,1,t} + \gamma_{2,2}d_{2,1,t} + \eta_{2,t}, \end{aligned} \quad (58)$$

where  $\theta_1$  and  $\theta_2$  are the parameters on own lags and  $\eta_{i,t}$  are the structural disturbances. A joint test of contagion is given by testing the significance of the shock in asset returns in the second (first) country on asset returns in the first (second) country:

$$H_0 : \gamma_{1,2} = \gamma_{2,1} = 0.$$

This test can also be conducted individually to identify significant contagious linkages between countries 1 and 2.

The Favero and Giavazzi (2002) approach is very similar to the Forbes and Rigobon (2002) correlation test because both tests are based on testing the significance of dummy variables in an augmented model. In both cases the dummy variables are slope dummies: this is especially clear if the dummy variable in equation (57) is defined to equal the observation itself.<sup>18</sup> The similarities between the two testing frameworks are made more transparent by defining a crisis period to be where the dummy variable is non-zero. Observations when the dummy variables are not defined by default correspond to pre-crisis periods. Since the dummy variables are defined for a single observation, the parameter estimates of  $\{\alpha_{1,2}, \theta_1, \alpha_{2,1}, \theta_2\}$  in equation (58) can be computed simply by using pre-crisis data.

There are, however, two differences between the Forbes and Rigobon (2002) and Favero and Giavazzi (2002) approaches. Forbes and Rigobon identify a crisis period as a period of higher volatility using a single dummy that has a non-zero value during the entire crisis period. The Favero and Giavazzi test identifies potentially many (short-lived) crisis periods associated with extreme returns. Second, the Favero and Giavazzi test assigns a different parameter to each dummy variable, whereas the Forbes and Rigobon approach is based on a single parameter to represent contagion between two countries. This last property reflects that the two test procedures both represent Chow tests: the Forbes and Rigobon approach is based on the covariance version of the Chow test, while the Favero and Giavazzi test is based on the predictive version of the Chow test.

In implementing the Favero and Giavazzi (2002) test, the structural model needs to be estimated using a simultaneous equation estimator to correct for simultaneity bias. This correction appears to be adopted by Favero and Giavazzi. However, an important assumption underlying this framework is that asset returns exhibit autocorrelation. If this assumption is not valid, then the choice of instruments based on lagged returns will not be valid, and identification of the structural model via the inclusion of own lags will no longer be appropriate. Even if there is some autocorrelation in asset returns, it is not likely to be strong, resulting in weak instruments with the moments of the sampling distribution of the test statistics possibly not existing.

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<sup>18</sup>Defining the dummy variable this way changes the point estimates when estimating the structural model in equation (58), but not the value of the test statistic.

## B. Probability Models: Dichotomous Classifications

Eichengreen, Rose, and Wyplosz (1995, 1996) and related studies by Kaminsky and Reinhart (2000) consider the transmission of contagion in currency markets across exchange rate regimes. To accommodate all of the possible avenues for pressure on an exchange rate they construct an Exchange Market Pressure Index (EMP).<sup>19</sup> The EMP index is then used to construct a binary CRISIS index that indicates whether or not a particular currency is experiencing extreme pressure.

The Eichengreen, Rose, and Wyplosz (1995, 1996) framework for testing contagion can be interpreted as a variant of the dummy variable approach of Favero and Giavazzi (2002). However, unlike the Favero and Giavazzi approach, which requires constructing dummy variables for the explanatory variables, in the Eichengreen, Rose, and Wyplosz approach a dummy variable is also constructed for the dependent variable. To highlight the Eichengreen, Rose, and Wyplosz approach, rewrite equation (38) by replacing the asset returns,  $y_{1,t}$  and  $y_{2,t}$ , by dummy variables

$$d_{2,t} = \left( \frac{\lambda_2 \delta_1 - \lambda_1 \gamma}{\delta_1} \right) w_t + \frac{\gamma}{\delta_1} d_{1,t} + \delta_2 u_{2,t}, \quad (60)$$

where the dummy variables are defined in terms of large negative asset returns as<sup>20</sup>

$$d_{1,t} = \begin{cases} 1 : & y_{1,t} > f(EMP_{1,t}) \\ 0 : & \text{otherwise} \end{cases} \quad (61)$$

$$d_{2,t} = \begin{cases} 1 : & y_{2,t} > f(EMP_{2,t}) \\ 0 : & \text{otherwise} \end{cases} . \quad (62)$$

Because the dependent variable is now binary, the model is respecified as a probit model

$$y_{2,t}^* = \left( \frac{\lambda_2 \delta_1 - \lambda_1 \gamma}{\delta_1} \right) w_t + \frac{\gamma}{\delta_1} d_{1,t} + \delta_2 u_{2,t} \quad (63)$$

$$d_t = \begin{cases} 1 : & y_{2,t}^* < f(EMP_{2,t}) \\ 0 : & \text{otherwise} \end{cases} , \quad (64)$$

where  $y_{2,t}^*$  is a latent normal random variable. The test of contagion, estimated by the probit model in this instance, is again the significance of the parameter  $\gamma$  in equation (63). Thus the Eichengreen, Rose, and Wyplosz (1995, 1996) approach can be viewed as focusing on the change

<sup>19</sup>The threshold indicator  $EMP_{i,t}$  represents the Exchange Market Pressure Index corresponding to the  $i$ th asset return at time  $t$ , which is computed as a linear combination of the change in exchange rates, interest differentials, and changes in levels of reserve assets for country  $i$  with respect to some numeraire country, 0,

$$EMP_{i,t} = a\Delta e_{i,t} + b(r_{i,t} - r_{0,t}) + c(\Delta R_{i,t} - \Delta R_{0,t}), \quad (59)$$

where  $e_i$  is the log of the bilateral exchange rate,  $r_i$  is the short-term interest rate and  $R_i$  is the stock of reserve assets. The weights,  $a$ ,  $b$  and  $c$ , are given by the inverse of the variance of the individual component series over the sample period. Kaminsky and Reinhart (2000) adopt a different weighting scheme whereby the weight on interest rates is zero.

<sup>20</sup>The CRISIS indicator in Eichengreen et al is constructed for the cases  $y_{i,t} > \mu_{EMP} + 1.5\sigma_{EMP}$ , that is a linear combination of the mean and standard deviation of the calculated EMP index. Other relative weightings on the two components were trialed in their paper.

in the strength of the correlation during crisis periods. Since the dummy variable is defined for large (negative) movements, these correlations are based on the large returns compared with all returns, as in the Forbes and Rigobon (2002) approach.

An important part of the Eichengreen et al (1995, 1996) approach is that it requires choosing the threshold value of the EMP index for classifying asset returns into crisis and non-crisis periods. As with the threshold value in equation (57) adopted by Favero and Giavazzi (2002), the empirical results are contingent on the choice of the threshold value. In both the Eichengreen, Rose, and Wyplosz and Favero and Giavazzi approaches, this choice is based on sample estimates of the data, resulting in potentially non-unique classifications of the data for different sample periods.<sup>21</sup>

The construction of binary dummies in equations (61)-(64) in general amounts to a loss of sample information, resulting in inefficient parameter estimates and a loss of power in testing for contagion. A more direct approach that does not result in any loss of sample information is to estimate equation (38) by least squares and perform a test of contagion by undertaking a  $t$ -test of  $\gamma$ . In fact, the probit model of equations (63)-(64) delivers consistent estimates of the same unknown parameters given in (38), but these estimates are inefficient as a result of the loss of sample information in constructing the dummy variables.

One of the attractions of the Eichengreen, Rose, and Wyplosz (1995, 1996) approach is that it generates probability estimates ( $P_t$ ) of the spread of financial crises across countries. The change in the probability of a crisis is given by evaluating the probability at the two values of the dummy variable

$$\frac{\partial P_t}{\partial d_{1,t}} = \Phi(z_1) - \Phi(z_0),$$

where  $z_1 = \left(\frac{\lambda_2\delta_1 - \lambda_1\gamma}{\delta_1}\right) w_t + \frac{\gamma}{\delta_1}$  and  $z_0 = \left(\frac{\lambda_2\delta_1 - \lambda_1\gamma}{\delta_1}\right) w_t$ . However, if the attraction of computing probability estimates of financial crises is the guide to model choice, such estimates can also be recovered by using equation (38). Adopting the assumption that  $u_{2,t}$  in equation (38) is normally distributed, the probability of a crisis in  $y_{2,t}$  is simply given by

$$P(y_{2,t} < y^*) = \Phi(z),$$

where  $\Phi(z)$  is the cumulative normal density evaluated at

$$z = \left(y^* - \left(\frac{\lambda_2\delta_1 - \lambda_1\gamma}{\delta_1}\right) w_t + \frac{\gamma}{\delta_1} y_{1,t}\right) / \delta_2,$$

and  $y^*$  is a preassigned threshold variable.

Kaminsky and Reinhart (2000) find that the calculation of unconditional probabilities does not provide particularly useful information for predicting crises based on data across regions.

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<sup>21</sup>Both Eichengreen, Rose, and Wyplosz and Kaminsky and Reinhart use some matching of their CRISIS index constructed using these thresholds to market events to validate the threshold choice.

### C. Probability Models: Polychotomous Classifications

Bae, Karolyi, and Stulz (2003) concentrate explicitly on the tails of the distribution of asset returns by identifying the exceedances of individual returns and co-exceedances across asset returns. The exceedance at time  $t$  is simply the difference between a large (negative) asset return and some preassigned threshold value ( $THRESH$ ). For two asset markets, the exceedances corresponding to large negative asset returns are<sup>22</sup>

$$E_{1,t} = \begin{cases} 1 : & y_{1,t} < THRESH \\ 0 : & \text{otherwise} \end{cases} \quad (65)$$

$$E_{2,t} = \begin{cases} 1 : & y_{2,t} < THRESH \\ 0 : & \text{otherwise} \end{cases} \quad (66)$$

Comparing equations (65) and (66) with the Eichengreen, Rose, and Wyplosz (1995, 1996) dummy variables in equations (61) and (62), shows that the exchange rate market pressure indexes  $EMP_{i,t}$  is equivalent to the threshold index used to identify exceedances. Bae, Karolyi and Stulz (2003) choose  $THRESH = -5$  percent, while Baur and Schulze (2002) extend this to consider a number of different thresholds endogenously.<sup>23</sup> A co-exceedance occurs at a point in time when  $E_{1,t}$  and  $E_{2,t}$  both exceed the threshold level

$$E_{1,t}, E_{2,t} = 1.$$

That is, both asset markets experience a large fall in asset values at the same point in time. For  $N$  asset markets, categorizing asset returns into co-exceedances yields a polychotomous variables that gives the number of co-exceedances occurring at each point in time. Bae, Karolyi and Stulz (2003) use a multinomial logit model to analyze the co-exceedances

$$P_{j,t} = \frac{\exp(\beta_j x_{j,t})}{\sum_{k=0}^N \exp(\beta_k x_{k,t})}, \quad j = 0, 1, 2, \dots, N, \quad (67)$$

where  $P_{j,t}$  is the probability that there are  $j$  co-exceedances occurring at time  $t$ , and  $x_{k,t}$  represent a set of explanatory variables used to explain asset returns and hence co-exceedances. The model is normalized by setting  $\beta_0 = 0$ , which corresponds to the case of no exceedances (i.e. no outliers). This is a natural extension of the Eichengreen, Rose, and Wyplosz (1995, 1996) framework, which uses a probit model because it is based on a binary classification of asset returns.<sup>24</sup>

In a similar vein Kaminsky and Reinhart (2002) consider periods of turmoil to be due to extreme events, and also examine the 5th and 95th percentiles of the distribution and a multinomial logit framework. In their examination of the transmission of shocks between countries they distinguish between “weak form globalization” and “strong form globalization”. The latter

<sup>22</sup>In extreme value theory, the exceedances are usually defined as  $y_{1,t} - THRESH$  when the threshold is reached. Butler and Joaquin (2002) find that the number of co-exceedances in the observed data exceeds that which could be generated from an underlying normal, GARCH or student- $t$  distribution, consistent with their results on the correlation coefficients.

<sup>23</sup>Bae, Karolyi, and Stultz (2003) also identify exceedances corresponding to large positive returns, in which case  $THRESH = 5$  percent.

<sup>24</sup>Eichengreen, Rose, and Wyplosz (1995) in studying the transmission of crises also use a multinomial logit model.



is somewhat similar to Bae, Karolyi, and Stulz's (2003) concept of co-exceedances involving contemporaneous extreme returns events in both countries. Weak form globalization, however, considers the effect of a large shock in one country on another country, without imposing that the effects of the shock must also result in a large shock on the second country.

#### D. One-Sided Asymmetries

Butler and Joaquin (2002) report that the change in correlations in asset returns between tranquil periods and crisis periods differs depending on the direction of the crisis. In the case of positive shocks (bull markets) the difference in the correlations is broadly consistent with a model drawn from a normal distribution. However, for the crisis caused by negative shocks (bear markets) the rise in correlation between asset returns far exceeds that associated with a normal distribution. There seem to be nonlinearities in the behavior of extreme shocks as proposed in the co-exceedance literature. To distinguish between positive and negative shocks, Dungey, Fry, and Martin (2003) expand equation (37) as

$$y_{2,t} = \lambda_2 w_t + \delta_2 u_{2,t} + \gamma_+ u_{1,t} d_t + \gamma_- u_{1,t} (1 - d_t), \quad (68)$$

where

$$d_t = \begin{cases} 1 : & u_{1,t} > 0 \\ 0 : & \text{otherwise} \end{cases} . \quad (69)$$

A test of symmetry is given by

$$H_0 : \gamma_+ = \gamma_-.$$

This is also the approach of Baig and Goldfajn (2000) (see also Ellis and Lewis, 2000 and Kaminsky and Schmukler, 1999) in modeling equity returns in Brazil and Russia. Dummy variables are constructed that represent good and bad news in both Brazil and Russia (see Baig and Goldfajn, 2000, for a list of dates) as follows

$$\begin{aligned} d_{bad,t} &= \begin{cases} 1 : & \text{bad news in country 1} \\ 0 : & \text{no news} \end{cases} \\ d_{good,t} &= \begin{cases} 1 : & \text{good news in country 1} \\ 0 : & \text{no news} \end{cases} . \end{aligned}$$

The model is then given by

$$y_{2,t} = \lambda_2 w_t + \delta_2 u_{2,t} + \gamma_+ d_{bad,t} + \gamma_- d_{good,t},$$

with a test of contagion from country 1 to country 2 given by testing

$$H_0 : \gamma_+ = \gamma_- = 0.$$

The studies mentioned so far concentrate on the asymmetric transmission of shocks in the level of the shocks. However, it is also possible that it is the asymmetric volatility of the shocks that matters – this is explored in Bekaert, Harvey, and Ng (2003), who incorporate asymmetric GARCH in the idiosyncratic or country-specific shock of the latent factor model.

## VI. ADDITIONAL METHODS

### A. Principal Components

Principal components provide an alternative way to identify factors; examples include Calvo and Reinhart (1995) and Kaminsky and Reinhart (2001). The principal components are based on an eigen decomposition of either the variance-covariance matrix or the correlation matrix, with the principal components computed as the eigenvectors associated with the largest eigenvalues. Thus, each computed principal component represents a weighted average of individual asset returns.

Principal component analysis is based on the assumption of a constant variance-covariance matrix. However, this assumption is unlikely to be appropriate when using high-frequency asset returns data, especially estimated over a sample containing financial crises where volatilities may change over time. One solution is to use a dynamic factor approach (Mody and Taylor, 2003), while a more general approach is to use the extended factor model discussed in Section II.

### B. Multiple Equilibria

An important feature of theoretical models of contagion is that they yield multiple equilibria (Dornbusch, Park, and Claessens, 2000). This suggests that the underlying distribution is multimodal in general, where the modes correspond to stable equilibria and the antimodes correspond to the unstable equilibria. In the case of two stable equilibria, these properties can be captured by a mixture distribution

$$f(y_{i,t}) = \phi f_1(y_{i,t}) + (1 - \phi) f_2(y_{i,t}), \quad (70)$$

where  $0 < \phi < 1$  is a parameter that weights the individual densities  $f_i()$  with means corresponding to the stable equilibria, to form the overall density. Jeanne and Masson (2000) adopt this strategy by employing Hamilton's Markovian switching model (Hamilton, 1994), which is equivalent to equation (70) with a time-varying weighting parameter,  $\phi_t$ , based on a Markovian updating formula; see also Masson (1999c) for a discussion of the approach.<sup>25</sup>

Peseran and Pick (2003) show that the class of models that incorporate binary variables can generate multiple equilibria. They consider a model that is equivalent to the Favero and Giavazzi (2002) which uses equation (55) in the case of one outlier. In the case of many outliers, the two classes of models are equivalent when the parameters associated with each set of dummy variables are equal. For a bivariate model of asset returns  $y_{1,t}$  and  $y_{2,t}$ , the system is given by

$$\begin{aligned} y_{1,t} &= \lambda_1 w_t + \delta_1 u_{1,t} + \gamma_1 d_{2,t} \\ y_{2,t} &= \lambda_2 w_t + \delta_2 u_{2,t} + \gamma_2 d_{1,t}, \end{aligned} \quad (71)$$

where  $d_{i,t}$  are threshold dummy variables given by

$$d_{i,t} = \begin{cases} 1 & : y_{i,t} > c_i \\ 0 & : \text{otherwise} \end{cases} . \quad (72)$$

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<sup>25</sup>Lim and Martin (1999) use another approach based on a generalised normal distribution to capture multimodality.

As with the contagion models discussed above, the parameters  $\gamma_i$  control the strength of contagion with no contagion occurring when  $\gamma_i = 0$ .

### C. Spillovers

A number of papers undertake measurements of contagion where contagion is not identified as the effects of foreign unexpected shocks on a domestic asset return. In particular, transmissions through an identified channel such as fundamental variables or financial streams are more consistent with the concept of spillovers in the terminology of Masson (1999a,b,c). For example, Glick and Rose (1999) and Dasgupta (2001) consider contagion as the spread of a crisis from its origin by any means, and the examples they give of competitive devaluation and trade links are easily incorporated into current concepts of anticipated linkages, and therefore are more consistent with notions of spillovers than contagion. In a similar vein van Rijikghem and Weder (2001) consider financial flows. A further example is Lowell, Neu, and Tong (1998); where contagion is assessed by examining the effects of lagged values of foreign asset returns on domestic asset returns. Trade links are often associated with the location of countries. A strong working hypothesis in the crisis literature is that crises seem to have strong regional features. This has been investigated using gravity models in Kaminsky and Reinhart (2002) and Bayoumi and others (2003), who make the case that geographical relationships matter, and Dasgupta (2001), who finds that regions are less important.

However, Kiyotaki and Moore (2002) present the case of known ex-ante linkages through balance sheets, but argue if these are sufficiently complex they may not be fully anticipated, but behave as contagion. A related channel of contagion is information flows and investor preferences. Empirical work on this stream of research is limited to calibration and simulation experiments, due to the obvious lack of data. However, the ideas include the search costs of obtaining information and the role of increasing global integration across asset markets. Calvo and Mendoza (2000) and Chue (2002) provide simulations of theoretical models that show how contagion can spread through information flows and investor preferences. Rigobon (2002), in an application of the correlation tests discussed in Section IV to Latin American markets, shows that the upgrading of Mexican debt ratings dramatically changed the characteristics of the Mexican markets to move it away from its previous common regional association with other Latin American countries.

### D. Multiple Classes of Assets

The majority of the existing literature on contagion considers transmissions across geographical borders for a particular asset market, although one important exception to this is the relatively large literature discussing joint banking and currency crises, such as Kaminsky and Reinhart (1999) and Bordo and Eichengreen (1999). There have been a number of applications of similar techniques to the same data period and different asset markets across country borders; such as Baig and Goldfajn (1999) and Ellis and Lewis (2000), which look at currency, equity, and bond markets in a correlation and VAR framework. Dungey, Fry, González-Hermosillo, and Martin (2002a,b) look at contagion in developing market bonds during the Russian and LTCM crisis, and then at the equity markets for the same period in Dungey, Fry, González-Hermosillo, and Martin (2003). None of these papers formally models the linkages between the different asset

classes during the crisis. Kaminsky and Reinhart (2001) consider coincidence in dates of the greatest movements and of shocks in univariate conditional volatility estimates, somewhat similar in idea to the co-exceedances of Bae, Karolyi, and Stulz (2003).

There is a rich empirical finance literature modeling the interactions of several markets jointly. For example, Bekaert and Hodrick (1992) consider the common and idiosyncratic factors determining equity and currency markets in a modeling framework related to that used in Dungey, Fry, González-Hermosillo, and Martin (2002b). In the context of modeling contagion, McKibbin and Martin (1998) emphasize the importance of studying the linkages across markets of different classes of assets as they argue that the East Asian crisis originated in the equity markets and then transmitted to currency markets. However, there is very little empirical literature that identifies and tests these additional linkages. Some exceptions are Granger, Huang, and Yang (2000); Kaminsky and Reinhart (2001); Fang and Miller (2002); and Dungey and Martin (2001). Granger, Huang, and Yang conduct Granger causality tests across pairs of assets and borders and find that evidence of dual causality. Kaminsky and Reinhart find a relatively large component of variance in overnight interest rates, stock returns, exchange rate changes, and bond spreads can be attributed to an initial principal component across a wide sample of countries in 1997-1999. Fang and Miller (2002) use a bivariate GARCH model to examine the effects of currency depreciation on equity market returns in East Asia, and Dungey and Martin find evidence of contagion from equity to currency markets in the East Asian crisis.

Another approach for modeling the strength of the interactions between markets is to determine if financial assets are priced using the same stochastic discount factor. If markets are indeed integrated, the stochastic discount factor imposes a set of no-arbitrage constraints on the model in the form of cross-equation restrictions, which are consistent with all financial assets and across all sample periods. Flood and Rose (2003) adopt this approach; although their application focuses on individual equities during financial crises, it could be expanded to include a broader range of assets. As with the previous models discussed, the results are conditioned on a number of factors: Flood and Rose choose observable factors.

Some of the difficulties in modeling transmission across financial assets include controlling for different time zone issues, data frequency, and volatility structures across both country and asset types. This could potentially increase the complexity of the modeling problem and result in issues of dimension; these issues are discussed in an earlier version of this paper.<sup>26</sup>

## VII. CONCLUSIONS

This paper has reviewed the literature on tests for the presence and characteristics of contagion in financial markets. Using an overarching framework of a latent factor model, similar to that proposed in the finance literature, the different test methodologies are shown to be related. In essence each method is shown to be a test on a particular parameter regarding the transmission of a shock from one country or market to another—the parameter

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<sup>26</sup>The earlier version is available at <http://rspas.anu.edu.au/economics/staff/dungey/>. The present paper contains the technical material from the previous version.

$\gamma$  in the exposition of this paper.

The five tests of contagion specifically considered in the paper were, first, the latent factor framework developed by the current authors, and similar to that of Corsetti, Pericoli, and Sbracia (2001) and Bekaert, Harvey, and Ng (2003) in which testing for contagion is a test on the parameter  $\gamma$ . Second, the relatively popular correlation approach associated with Forbes and Rigobon (2002) was also shown to be a test on the parameter  $\gamma$  within the latent factor framework. Third, the test of Favero and Giavazzi (2002), based on using a VAR to identify unusual events and then testing the potential transmission of these events in a structural framework, was transformed to be a reorganization of the latent factor model and the contagion test consistent with a test on the parameter  $\gamma$ . Fourth, the probability-based framework of Eichengreen, Rose, and Wyplosz (1995, 1996), which tests for contagion as a non-zero probability of associating foreign crises with a domestic crisis, was shown to have a similar form in the latent factor model to the previous test, but with alternate conditioning information. Finally, the extreme returns test of Bae, Karolyi, and Stulz (2003) is a further refinement of the Eichengreen, Rose and Wyplosz framework, and hence can be similarly cast in a latent factor model and expressed as a test on the parameter  $\gamma$ .

While the paper has drawn together many of the existing empirical methods to identify contagion, some questions remain to be addressed. In a companion paper to this one, Dungey, Fry, González-Hermosillo, and Martin (2004) look at issues raised by time zones, data frequency, missing observations, and endogenous definitions of crisis periods. In addition, issues associated with the practical implementation of the tests described here are discussed. Many other areas remain to be explored; for example, the small sample properties of the tests outlined here, and their performance in detecting contagion across a range of financial crises.

## References

- Allen, F. and Gale, D. (2000), "Financial Contagion," *Journal of Political Economy*, Vol.108, pp.1-33.
- Bae, K.H, Karolyi, G.A. and Stulz, R.M. (2003), "A New Approach to Measuring Financial Contagion," *Review of Financial Studies*, forthcoming.
- Baig, T. and Goldfajn, I. (1999), "Financial Market Contagion in the Asian Crisis," *IMF Staff Papers*, 46(2), pp. 167–195.
- (2000), "The Russian Default and the Contagion to Brazil," IMF Working Paper 00/160 (Washington: International Monetary Fund).
- Baur, D. and Schulze, D. (2002), "Coexceedances in Financial Markets - A Quantile Regression Analysis of Contagion," unpublished, University of Tuebingen.
- Bayoumi, T., Fazio, G., Kumar, M., and MacDonald, R. (2003), "Fatal Attraction: A New Measure of Contagion," IMF Working Paper 03/80 (Washington: International Monetary Fund).
- Bekaert, G., Harvey, C.R. and Ng, A. (2003), "Market Integration and Contagion," *Journal of Business*, forthcoming.
- Bekaert, G. and Hodrick, R. (1992) "Characterizing Predictable Components in Excess Returns on Equity and Foreign Exchange Markets," *Journal of Finance*, Vol. 47, pp. 467–509.
- Bordo, M. and Eichengreen, B. (1999), "Is Our Current International Environment Unusually Crisis Prone?" in Gruen, D. (ed) *Capital Flows and the International Financial System*, Reserve Bank of Australia, Sydney, pp. 18–74.
- Boyer, B.H., Gibson, M.S. and Loretan, M. (1999), "Pitfalls in Tests for Changes in Correlations," Federal Reserve Board International Finance Division, Working Paper 597R.
- Butler, K.C. and Joaquin, D.C. (2002), "Are the Gains from International Portfolio Diversification Exaggerated? The Influence of Downside Risk in Bear Markets?" *Journal of International Money and Finance*, Vol. 21, pp. 981–1,011.
- Calvo, S. and Mendoza, E. (2000), "Rational Contagion and the Globalization of Securities Markets," *Journal of International Economics*, Vol. 51, pp. 79–113.
- Calvo, S. and Reinhart, C. (1995), "Capital Flows to Latin America: Is there Evidence of Contagion Effects?" World Bank and International Monetary Fund, unpublished.
- Caporale, G.M., Cipollini, A. and Spagnolo, N. (2002), "Testing for Contagion: A Conditional Correlation Analysis," South Bank University Centre for Monetary and Financial Economics, Discussion Paper #01-2002.
- Chernov, M.E., Gallant, R.A., Ghysels, E. and Tauchen, G. (2003), "Alternative Models for Stock Price Dynamics," *Journal of Econometrics*, forthcoming.
- Chue, T.K. (2002), "Time Varying Risk Preferences and Emerging Market Covariances," *Journal of International Money and Finance*, Vol. 21, pp. 1,053–1,072.
- Corsetti, G., Pericoli, M. and Sbraccia, M. (2001), "Correlation Analysis of Financial Contagion: What one Should Know Before Running a Test," unpublished, University of Rome III.

- (2002), “Some Contagion, Some Interdependence: More Pitfalls in Testing for Contagion,” unpublished, University of Rome III.
- Dasgupta, A. (2000), “Regionality Revisited: An Examination of the Direction of Spread of Currency Crises,” unpublished, London School of Economics.
- Diebold, F.X. and Nerlove, M. (1989), “The Dynamics of Exchange Rate Volatility: A Multivariate Latent-Factor ARCH Model,” *Journal of Applied Econometrics*, 4, 1-22.
- Dornbusch, R., Park, Y.C. and Claessens, S. (2000), “Contagion: Understanding How it Spreads,” *The World Bank Research Observer*, Vol.15(2), pp. 177–97.
- Dungey, M. and Martin, V.L. (2001), “Contagion Across Financial Markets: An Empirical Assessment,” New York Stock Exchange Conference Paper, February 16–17, 2001, Hawaii.
- (2004), “A Multifactor Model of Exchange Rates With Unanticipated Shocks: Measuring Contagion in the East Asian Currency Crisis,” forthcoming, *Journal of Emerging Markets Finance*.
- and Pagan, A.R. (2000), “A Multivariate Latent Factor Decomposition of International Bond Yield Spreads,” *Journal of Applied Econometrics*, Vol. 15, pp. 697–715.
- Dungey, M., Fry, R.A., and Martin, V.L. (2003), “Equity Transmission Mechanisms from Asia to Australia: Interdependence or Contagion?” *Australian Journal of Management*, 28(2), pp. 157–182.
- Dungey, M., Fry, R.A., González-Hermosillo, B. and Martin, V.L. (2002a), “International Contagion Effects from the Russian Crisis and the LTCM Near-Collapse,” IMF Working Paper 02/74 (Washington: International Monetary Fund)..
- (2002b), “The Transmission of Contagion in Developed and Developing International Bond Markets,” in Committee on the Global Financial System (ed), *Risk Measurement and Systemic Risk*, Proceedings of the Third Joint Central Bank Research Conference, pp. 61–74 .
- (2003), “Unanticipated Shocks and Systemic Influences: The Impact of Contagion of Contagion in Global Equity Markets in 1998,” IMF Working Paper 03/84 (Washington: International Monetary Fund).
- (2004), “A Comparison of Alternative Tests of Contagion with Applications,” unpublished, Australian National University.
- Dungey, M. and Zhumabekova, D. (2001), “Testing for Contagion Using Correlations: Some Words of Caution,” Federal Reserve Bank of San Francisco, Pacific Basin Working Paper PB01-09.
- Durbin, J.M. (1954), “Errors in Variables,” *Review of the International Statistical Institute*, Vol. 22, pp. 23–32.

- Eichengreen, B., Rose, A.K. and Wyplosz, C. (1995), "Exchange Market Mayhem: The Antecedents and Aftermath of Speculative Attacks," *Economic Policy*, Vol. 21, pp. 249–312.
- (1996), "Contagious Currency Crises," NBER Working Paper 5681.
- Ellis, L. and Lewis, E. (2000), "The Response of Financial Markets in Australia and New Zealand to News about the Asian Crisis," BIS Conference on International Financial Markets and the Implications for Monetary and Financial Stability, Basle, 25-26 October, 1999, Vol. 8.
- Engle R.F., Ito, T. and Lin, W. (1990), "Meteor Showers or Heat Waves? Heteroscedastic Intra-Daily Volatility in the Foreign Exchange Market," *Econometrica*, Vol. 58, pp. 525-542.
- Engle, R.F. and Kroner, K.F. (1995), "Multivariate Simultaneous Generalized ARCH," *Econometric Theory*, Vol. 11(1), pp. 22–50.
- Fang, W. and Miller, S. (2002), "Dynamic Effects of Currency Depreciation on Stock Market Returns During the Asian Financial Crisis," unpublished, Feng Chai University, Taiwan.
- Favero, C.A. and Giavazzi, F. (2002), "Is the International Propagation of Financial Shocks Non Linear? Evidence from the ERM," *Journal of International Economics*, Vol. 57(1), pp. 231–246.
- Flood, R. and Marion, N. (1998), "Perspectives on the Recent Currency Crisis Literature," NBER Working Paper #6380.
- Flood, R. and Rose, A.K. (2003), "Equity Integration in Times of Crisis," unpublished.
- Forbes, K. and Rigobon, R. (2002), "No Contagion, Only Interdependence: Measuring Stock Market Co-movements," *Journal of Finance*, Vol. 57 (5), pp. 2,223–2,261.
- Glick, R. and Rose, A.K. (1999), "Contagion and Trade: Why are Currency Crises Regional?" *Journal of International Money and Finance*, Vol. 18(4), pp. 603–17.
- Granger, C., Huang, B. and Yang, C. (2000), "Bivariate Causality Between Stock Prices and Exchange Rates in Asian Countries," *The Quarterly Review of Economics and Finance*, Vol. 40, pp. 337–54.
- Grubel, H.G. and Fadner R. (1971), "The Interdependence of International Equity Markets," *Journal of Finance*, Vol. 26, pp. 89–94.
- Hamilton, J.D. (1994), *Time Series Analysis*, Princeton New Jersey, Princeton University Press.
- Jeanne, O. and Masson, P. (2000), "Currency Crises, Sunspots and Markov-Switching Regimes," *Journal of International Economics*, Vol. 50(2), pp. 327–50.
- Kaminsky, G.L. and Schmukler, S.L. (1999), "What Triggers Market Jitters? A Chronicle of the Asian Crisis," *Journal of International Money and Finance*, Vol. 18, pp. 537–60.
- Kaminsky, G.L. and Reinhart, C.M. (1999), "The Twin Crises: The Causes of Banking and Balance of Payments Problems," *American Economic Review*, Vol. 89(3), pp. 473–500.
- (2000), "On Crises, Contagion and Confusion," *Journal of International Economics*, Vol. 51(1), pp. 145–68.



- (2001), “Financial Markets in Times of Stress,” NBER Working Paper #8569.
- (2002), “The Center and the Periphery: Tales of Financial Turmoil,” GWU, unpublished.
- Karoyli, A. and Stulz, R. (1996), “Why do Markets Move Together? An Investigation of U.S. - Japan Stock Return Co-movements,” *Journal of Finance*, Vol. 51 (3), pp. 951–86.
- Kendall, M. and Stuart, A. (1969), *The Advanced Theory of Statistics*, Charles Griffin and Co., London.
- King, M. and Wadhwani, S. (1990), “Transmission of Volatility Between Stock Markets,” *Review of Financial Studies*, Vol. 3 (1), pp. 5–33.
- Kiyotaki, N. and Moore, J. (2002), “Balance-Sheet Contagion,” *American Economic Review Papers and Proceedings*, Vol. 92 (2), pp. 46–50.
- Kodres, L.E. and Pritsker, M. (2002), “A Rational Expectations Model of Financial Contagion,” *Journal of Finance*, Vol. 57 (2), pp. 768–99.
- Kyle, A. and Xiong W. (2001), “Contagion as a Wealth Effect,” *Journal of Finance*, Vol. 56(4), pp. 1,401–1,440.
- Lim, G.C. and Martin, V.L. (1999), “Forecasting Large Changes in Exchange Rates,” *Economic Forecasting Techniques*, eds. Abelson. P. and Joyeaux, R., Allen and Unwin.
- Longin, F. and Solnik, B. (1995), “Is the Correlation in International Equity Returns Constant: 1960-1990?” *Journal of International Money and Finance*, Vol. 14 (1) pp. 3–26.
- Loretan, M., and English, W. (2000), “Evaluating ‘Correlation Breakdowns’ During Periods of Market Volatility,” Board of Governors of the Federal Reserve System, International Finance Discussion Paper No. 658.
- Lowell, J., Neu, C.R. and Tong, D. (1998), “Financial Crises and Contagion in Emerging Market Countries,” Monograph, RAND.
- McKibbin, W. and Martin, W. (1998), “The East Asian Crisis: Investigating Causes and Policy Responses,” Australian National University Working Paper in Trade and Development, 98/6.
- Mahieu, R. and Schotman, P. (1994), “Neglected Common Factors in Exchange Rate Volatility,” *Journal of Empirical Finance*, Vol. 1, pp. 279–311.
- Malliaroupulos, D. (1997), “A Multivariate GARCH Model of Risk Premia in Foreign Exchange Markets,” *Economic Modelling*, Vol. 14, pp. 61–79.
- Masson, P. (1999a), “Contagion: Macroeconomic Models with Multiple Equilibria,” *Journal of International Money and Finance*, Vol. 18, pp. 587–602.
- (1999b), “Contagion: Monsoonal Effects, Spillovers, and Jumps Between Multiple Equilibria,” in Agenor, P.R., Miller, M., Vines, D. and Weber, A. (eds), *The Asian Financial Crisis: Causes, Contagion and Consequences*, Cambridge University Press, Cambridge, UK.

- (1999c), “Multiple Equilibria, Contagion and the Emerging Market Crises,” IMF Working Paper 99/164 (Washington: International Monetary Fund).
- Mody, A. and Taylor, M.P. (2003), “Common Vulnerabilities,” University of Warwick, unpublished.
- Pericoli, M. and Sbracia, M. (2001), “A Primer on Financial Contagion,” Banca D’Italia Temi di Discussione #407.
- Pesaran, H. and Pick, A. (2003), “Econometric Issues in the Analysis of Contagion,” unpublished, University of Cambridge.
- Rigobon, R. (2002), “The Curse of Non-Investment Grade Countries,” *Journal of Development Economics*, Vol. 69, pp. 423–49.
- (2003a), “Identification Through heteroscedasticity,” *Review of Economics and Statistics*, Vol. 85, forthcoming.
- (2003b), “On the Measurement of the International Propagation of Shocks: Is the Transmission Stable?” *Journal of International Economics*, forthcoming.
- Sharpe, W. (1964), “Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk,” *Journal of Finance*, Vol. 19, pp. 425–42.
- Solnik, B.H. (1974), “An Equilibrium Model of the International Capital Market,” *Journal of Economic Theory*, Vol. 8, pp. 500–24.
- Toyoda, T. and Ohani, K. (1986), “Testing Equality Between Sets of Coefficients after a Preliminary Test for Equality of Disturbance Variances in Two Linear Regressions,” *Journal of Econometrics*, Vol. 31, pp. 67–80.
- Wald, A. (1940), “The Fitting of Straight Lines if Both Variables are Subject to Error,” *Annals of Mathematical Statistics*, Vol. 11, pp. 284–300.
- Van Rijikghem, C.V. and Weder, B. (2001), “Sources of Contagion: Is it Finance or Trade?” *Journal of International Economics*, Vol. 54, pp. 293–300.