A comparison of multivariate GARCH models with respect to Value at Risk

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ABSTRACT

Since the introduction univariate GARCH models number of available models have grown rapidly and has been extended to the multivariate area. This paper compares three different multivariate GARCH models and they are evaluated using out of sample Value at Risk of different portfolios. Sector portfolios are used with different market capitalization. The models compared are the DCC,CCC and the GO-Garch model. The forecast horizon is 1-day, 5-day and 10-day ahead forecast of the estimated VaR limit. The DCC performs best with regards to both conditional anc unconditional violations of the VaR estimates.
Keywords: multivariate GARCH, Value at Risk, forecasting, conditional correlation.

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1 Introduction

Modelling volatility has been an important research area since the introduction of autoregressive conditional heteroskedasticity model (ARCH) by Engle (1982). Bollerslev (1986) later extend this research to the Generalized autoregressive conditional heteroskedasticity (GARCH) models to remedy the heteroskedasticity properties that financial time series usually exhibits. GARCH models are used to model risk in financial time series, in particular the measure Value at risk (VaR). However most of this research is limited to the univariate approach. Since a portfolio usually consists of multiple assets or even multiple different type of assets understanding the volatility and co-volatility of these assets are of major interest. Multivariate GARCH (MGARCH) can help remedy these issues. This is because MGARCH models the covariance matrix of the multiple time series. Since the introduction of MGARCH models, the number of available models have grown rapidly. There exits a large quantity of previous research regarding MGARCH models for instance Silvennoinen and Teräsvirta (2008) have written an extensive overview of existing MGARCH models where they also discuss estimation issues of these models, Bauwens, Laurent and Rombourts (2006) have written an similar review.

Many of the papers regarding volatility forecasting and VaR focuses on the univariate approach. While univariate approaches can measure interesting relationships they tend to ignore the covariance/co-volatility between financial assets. In asset management this may lead an investor to overestimate or underestimate the risk of an certain portfolio. With regards to a portfolio of assets it is not possible to use individual VaR estimates for the assets within the portfolio due to the fact that VaR is not additive, the VaR of a portfolio can be either larger or smaller than the individual VaR estimates of the assets in the portfolio due to co-volatility. In a paper by Santos and Nogales (2012) the authors compare a number of multivariate models and univariate models and find that multivariate models outperform the univariate volatility models with respect to VaR.

The purpose of this thesis is to investigate different formulations of MGARCH models and evaluating them through VaR. Three different MGARCH models is used to forecast the VaR. The models used is the dynamic conditional correlation (DCC) model, the conditional constant correlation model and the Generalized Ortogonal-Garch(GO-Garch) model.

2 Background

2.1 VaR

VaR was introduced by Riskmetrics^{TM}/JP Morgan in 1994. Since the introduction of VaR its use has become widely accepted and is now regulated in the Basel II accord. VaR is an estimate of how much an invest might lose over a specified holding period and a given confidence level. Usually the confidence level is set at 0.99 or 0.95.

There exists a variaty of methods to calculate VaR, Riskmetrics^{TM} proposed historical simulation or exponential weighted moving average (EMWA) but other methods are available as well. Multivariate GARCH models can be used to forecast out of sample VaR measures. For a 1-day forecast horizon the VaR under normality conditions of a portfolio of assets can be expressed as

$$VaR_{\alpha} = \boldsymbol{w}'\boldsymbol{\mu}_{t+1|t} + z_{\alpha}\sqrt{\boldsymbol{w}'\boldsymbol{\Sigma}_{t+1|t}\boldsymbol{w}} \tag{1}$$

where $\boldsymbol{\mu}_{t+1|t}$,$\boldsymbol{\Sigma}_{t+1|t}$ are the 1-day ahead forecast of $\boldsymbol{\mu}_t$ and $\boldsymbol{\Sigma}_t$, $\boldsymbol{w}$ is the weights of the assets in the portfolio and z_{α} is the left quantile $\alpha\%$ of the normal distribution. Other distributions can be used as well for instance students t (Bauwens, Laurent,2005).

2.2 Modelling volatility

Volatility is defined to be the standard deviation of the price of the underlying asset. In order to model volatility, the error term is often assumed to be $\epsilon_t = \sigma^2 z_t$ where ϵ_t is the residuals of the time series, z_t is assumed to be iid white noise and σ^2 is the variable of interest.

Engle (1982) derived the autoregressive conditional heteroskedasticity model(ARCH) in order to model the British inflation. An ARCH(q) model models the conditional variance as a function of previous squared error terms.

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 \tag{2}$$

Where ϵ_{t-i}^2 is the squared error terms. For the process to be stationary, the requirements lie upon the values of the α terms where $\sum_{i=1}^{q} \alpha_i < 1$ and $\alpha_0 > 0$.

An extension of the ARCH models is the Generalized autoregressive conditional heteroskedasticity (GARCH) derived by Bollerslev (1986). Which remedy's the heteroskedasticity issue that financial time series often exhibits as well as volatility clustering.

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \epsilon_{t-1}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-i}^2 \tag{3}$$

Equation 3 shows an GARCH(p,q) model. In order for this process to be stationary it needs to fulfill the following requirements must be fulfilled, $\alpha_0 > 0$, $\alpha_i > 0$, $\beta_j > 0$ as well as $\sum(\alpha_i + \beta_j) < 1$.

2.3 Volatility clustering and leverage effects

It is widely known that financial returns often exhibits volatility clustering meaning that for shorter periods the volatility may spike and then be low for other periods (Tsay, 2005). This can cause problems for volatility modelling in the sense that the model needs to quickly adjust to structural change in volatility. One other common feature is that the distribution of the returns usually shows excess kurtosis in other words the series usually are leptokurtic. This means that the distribution exhibits fatter tails and more peaked compared to a normal distribution.

3 Theory

3.1 Multivariate GARCH

Multivariate Generalized autoregressive conditional heteroskedasticity (MGARCH) is an extension of GARCH. Where we consider a vector of returns $\boldsymbol{y_t} = (y_{1t,}y_{2t}...y_{Nt})$, where,

$$\boldsymbol{y_t} = \boldsymbol{\mu_t} + \boldsymbol{\epsilon_t} \tag{4}$$

where $\boldsymbol{\mu_t}$ is the conditional mean vector given the information set and

$$\boldsymbol{\epsilon_t} = \boldsymbol{H_t^{1/2} z_t} \tag{5}$$

where $\boldsymbol{H_t}$ is the covariance matrix of $\boldsymbol{y_t}$ and $\boldsymbol{H_t}^{1/2}$ is obtained using cholesky decomposition and $\boldsymbol{z_t}$ is a random vector with expected two first moments to be $E[\boldsymbol{z_t}] = 0$ and $V[\boldsymbol{z_t}] = \boldsymbol{I_N}$.

There exists a number of different MGARCH models and they can be divided in to different subgroups:

- Models of conditional covariance matrices: Models of conditional covariance matrices were the early Multivariate approaches, for instance the VEC model of Bollerslev, Engle and Wooldridge (1988)and the BEKK model of Baba, Engle Kraft and Kroner(1993) . One major drawback of these models are the large numbers of parameters required in order to create a model.

- Models of correlations and conditional correlations: For instance the Constant Conditional Correlation(CCC) model and the Dynamic Conditional Correlation(DCC) model. The variance and the correlation matrix modelled then used to estimate the conditional covariance matrix.

- Factor Models: Factor models assumes that the returns are driven by some heteroskedastic unobservable factors. Contrary to the previous mentioned models this approach benefits from dimensionality reduction. (Silvennoinen, Teräsvirta, 2008))

- Semi parametric and non parametric models: Semi parametric models and non parametric models has the advantage of not imposed an distribution or structure on the data, so there is no possible misspecification issues. (Silvennoinen and Teräsvirta, 2008)

3.1.1 CCC model

The CCC-model of Bollerslev (1990) decomposes the covariance matrix into standard deviations and correlations.

$$\boldsymbol{H}_t = \boldsymbol{D}_t \boldsymbol{R}_t \boldsymbol{D}_t \tag{6}$$

Where $\boldsymbol{R}_t$ is the conditional correlation matrix and $\boldsymbol{D_t}$ is a diagonal matrix only consisting of the standard deviations of the assets, which are typically modelled by univariate GARCH models. Such that

$$\boldsymbol{D}_t = \begin{bmatrix} \sqrt{h_{1,t}} & & \\ & \ddots & \\ & & \sqrt{h_{j,t}} \end{bmatrix} \tag{7}$$

Where each $h_{,i,t}$ is a univariate GARCH process. This model assumes that the correlations between assets are not time varying. So $\boldsymbol{R_t} = \boldsymbol{R}$ for all t. Then the off diagonal elements are given by

$$H_{t,j} = h_i^{1/2} h_j^{1/2} \rho_{i,j},\, \forall\, i \neq j \tag{8}$$

3.1.2 DCC GARCH

An extension of the CCC-model is the DCC-model of Engle and Sheppard (2001) that in contrary to the CCC-model does not assume that the correlations between the series are constant, so the model can account for possible time varying co-volatility. The estimation process is similar to the CCC model the first step is to model the diagonal with an univariate GARCH model the second step is to estimate the covariance of the returns.

Let the conditional correlation matrix $\boldsymbol{R}_t$ be estimated using the proxy variable $\boldsymbol{Q}$

$$\boldsymbol{R}_t = diag\, \boldsymbol{Q}_t^{-1/2} \boldsymbol{Q}_t diag\, \boldsymbol{Q}_t^{-1/2} \tag{9}$$

The proxy variable $\boldsymbol{Q}$ is in turn estimated by

$$\boldsymbol{Q_t} = (1 - \delta - \gamma)\bar{\boldsymbol{Q}} + \delta \boldsymbol{z_t} \boldsymbol{z_t}' + \gamma \boldsymbol{Q_{t-1}} \tag{10}$$

$$\tag{11}$$

Where δ γ are non negative scalars and $\bar{\boldsymbol{Q}}$ is the unconditional matrix of the errors z_t. (Ghalanos, 2019)

3.1.3 GO-Garch

Contrary to the BEKK and DCC model which models the covariance and the variances and correlations the Generalized Orthogonal (GO) Garch assumes that ϵ_t is generated by unobserved heteroskedastic factors. The underlying assumption is that the observed process y_t is generated by n-vector of linear combination of conditionally uncorrelated factors. (van der Weide, 2002)

Let r_t be the return of the asset at time t, with conditional mean μ_t given the information set.

$$\boldsymbol{r}_t = \boldsymbol{\mu}_t + \boldsymbol{\epsilon}_t, t = 1, ..., T \tag{12}$$

where $\boldsymbol{\epsilon}_t$ can be decomposed such that

$$\epsilon_t = \boldsymbol{A}\boldsymbol{f}_t \tag{13}$$

where $\boldsymbol{A}$ is a constant over time and invertible matrix and $\boldsymbol{f}_t$ is a vector of the unobserved factors. $\boldsymbol{A}$ can in turn be decomposed into

$$\boldsymbol{A} = \boldsymbol{\Sigma}^{1/2}\boldsymbol{U} \tag{14}$$

Where $\boldsymbol{\Sigma}_t$ is the square root of the unconditional covariance matrix and U is an orthogonal matrix. Then $\boldsymbol{f}_t$ can be specified to be

$$\boldsymbol{f}_t = \boldsymbol{H}_t\boldsymbol{z}_t \tag{15}$$

Where $\boldsymbol{H}_t$ is an diagonal matrix consisting the conditional variances of the factors and can be modelled by univariate garch processes. $\boldsymbol{z}_t$ is a random variable with $E[z_t] = 0$ and unit variance. Then the modelled can be described as

$$\boldsymbol{r}_t = \boldsymbol{m}_t + \boldsymbol{A}\boldsymbol{H}_t^{1/2}\boldsymbol{z}_t \tag{16}$$

and the conditional covariance matrix is given by

$$\boldsymbol{\Sigma}_t = \boldsymbol{A}\boldsymbol{H}_t\boldsymbol{A}' \tag{17}$$

The estimation procedure of van der Wiede (2002) was maximum likelihood which restricted the number of assets. The matrix U is estimated following the approach proposed by Broda and Paolella (2009) . (Ghalanos, 2019)

4 Method

4.1 Forecasting and Model selection

The estimation is done in the program R using the package, rmgarch in combination with rugarch and CCgarch. The Akaike Information Criterion is used to select the order of the models.

Rolling window is used for the forecasting where by the model is estimated using a sample size of T observations from the time period $1, 2...T$ and the out of sample VaR is calculated for $T+1$ and for the next time period the sample size remains the same but the time period changes to $2, 3...T + 1$ and so forth for the whole evaluation period. The model is re estimated every day and the VaR is calculated. The VaR forecasts that will be evaluated is the 1-day, 5-day and 10-day ahead forecasts.

4.2 Evaluating VaR forecasts

The VaR forecasts are a sequence of a binary outcome, yes or no if the portfolio breaches the estimated VaR limit. For a given significance level of the VaR the expected number of exceedances should equal the significance level if the estimated VaR is unbiased.

$$\alpha = f, \quad f = \frac{x}{N} \tag{18}$$

Where f is the failure rate and x is the number of values that exceed the estimated VaR limit and N is the number of observations.

Kupiec (1995) derived a test in order to test for the unconditional coverage of the VaR.

$$H_0 : f = \alpha \tag{19}$$

$$H_1 : f \neq \alpha \tag{20}$$

Where f is the failure rate and is estimated by $\hat{f}$ which in turn is estimated by $\hat{f} = \frac{x}{N}$, where x is the number of VaR exceedances and N is the number of observations. The test statistic is

$$LR_k = 2ln[\hat{f}^N(1 - \hat{f})^{T-N}] - 2ln[\alpha^N(1 - \alpha)^{T-N}] \tag{21}$$

the test statistic which is asymptotically $\chi^2(1)$ where T is the sample size and N is the number of breaches of VaR. (Bauwens, Hafner and Laurent, 2012) One drawback of the Kupiec test is that it is not possible to test for dependence of violations of VaR, rendering it not able to determine if the model captures changes to the conditional variance.

Christoffersen (1998) derived a test in order to test for independence among VaR violations. The null hypothesis is that $H_0 : \pi_{01} = \pi_{11} = \alpha$ and the test statistic is the following:

$$LR_c = 2ln[(1-\hat{\pi}_{01})^{n_{00}}\hat{\pi}_{01}^{n_{01}}(1-\hat{\pi}_{11})^{n_{10}}\hat{\pi}_{11}^{n_{11}}] - 2ln[\alpha^N(1-\alpha)^{T-N}] \tag{22}$$

where $\pi_{i,j} = P(I_t = j|I_{t-1} = i)$ and n_{ij} is the number of violations of i followed by j. $\hat{\pi}_{01}$ is estimated by $n_{01}/(n_{00} + n_{01})$ and $\hat{\pi}_{11}$ is estimated by $n_{11}/(n_{10} + n_{11})$. The test statistic is asymptotically $\chi^2(2)$ distributed under the null hypothesis. (Bauwens, Hafner and Laurent, 2012)

Both the tests are conducted on the $\alpha = 0.05$ significance level.

5 Data

This paper will focus on sector portfolios from three different sectors and three different market caps. The portfolios contains five stocks each and are equally weighted, and are listed in the tables below.

Table 1: Large Cap Portfolios

Finance	**Technology**	**Energy**
AMEX	HP inc	Emerson Electric
Blackrock	L3 Technologies	Haliburton
Citigroup	Oracle Corp	Apache Corp
Morgan Stanley	Motorola solutions Inc	Cummins Inc
Moodys	United Rentals	Valero Energy

Table 2: Mid Cap Portfolios

Finance	**Technology**	**Energy**
Associated Banc-Corp	ASGN Inc	Brunswick Corp
Eaton Vance Corp	Juniper Networks Inc	Chesapeake Energy Corp
First Horizon National Corp	Manpower Group	CNX Resources Corp
MGIC Investment Corp	Robert Half Int. Corp	Murphy Oil Corp
Radian Group Inc	Xerox Corp	World Fuel Services Corp

Table 3: Small Cap Portfolios

Finance	**Technology**	**Energy**
Berkshire Hills Bancorp Inc	3D Systems	BP Prudhoe Royalty Trust
Cittizens Inc	Kadant Inc	Newpark Resources Inc
CPB Inc	SPX Corp	Oceaneering Int. Inc
FBL Fin Corp	Tennent Company	Superior Energy Services
Oppenheimer Holdings Inc	Unisys Corp	Unit Corp

The data ranges from 2000-01-01 to 2019-01-01, where the period ranging from 2001-01-01 up until 2014-12-30 will be used for estimation of the models and the period ranging from 2015-01-01 to 2019-01-01 is used for evaluation of the models. The evaluation period consists of 1006 trading days and the total number of trading days equals 4779.

Table 4: Descriptive statistics of large cap companies

	mean	sd	skew	kurt		mean	sd	skew	kurt		mean	sd	skew	kurt
AXP	0.00	0.02	-0.05	10.34	HPQ	-0.00	0.02	-0.34	8.62	EMR	0.00	0.02	-0.08	6.84
BLK	0.00	0.02	0.08	7.50	LLL	0.00	0.02	1.10	26.94	HAL	0.00	0.03	-1.71	39.22
C	-0.00	0.03	-0.54	42.01	ORCL	0.00	0.02	-0.01	8.51	APA	0.00	0.02	-0.10	4.78
MS	-0.00	0.03	1.27	47.98	MSI	-0.00	0.03	-0.51	11.29	CMI	0.00	0.03	0.00	6.93
MCO	0.00	0.02	-0.26	7.83	URI	0.00	0.03	-0.76	10.42	VLO	0.00	0.03	-0.51	5.78

Table 5: Descriptive statistics of mid cap companies

	mean	sd	skew	kurt		mean	sd	skew	kurt		mean	sd	skew	kurt
ASB	0.00	0.02	-0.23	12.25	ASGN	0.00	0.04	-0.59	16.70	BC	0.01	0.70	-0.69	11.04
EV	0.00	0.02	0.20	11.73	JNPR	-0.00	0.04	0.24	8.57	CHK	-0.00	0.58	-0.87	12.17
FHN	-0.00	0.03	-1.07	33.06	MAN	0.00	0.02	-0.04	6.43	CNX	0.00	0.94	-1.39	22.69
MTG	-0.00	0.05	-1.96	60.63	RHI	0.00	0.02	0.57	11.08	MUR	0.00	0.94	-0.43	8.16
RDN	-0.00	0.05	0.63	18.55	XRX	-0.00	0.03	-0.41	18.97	INT	0.00	0.61	-1.37	23.57

Table 6: Descriptive statistics of small cap companies

	mean	sd	skew	kurt		mean	sd	skew	kurt		mean	sd	skew	kurt
CPB	-0.00	0.52	-0.45	10.68	DDD	0.00	0.70	-1.60	35.75	BPT	0.00	1.31	-1.28	15.71
CIA	0.00	0.24	0.22	4.43	KAI	0.01	0.79	1.38	34.63	NR	0.00	0.24	0.03	5.08
CPF	-0.06	7.84	0.41	11.38	SPXC	0.00	0.43	-0.25	9.87	OII	0.00	0.72	-0.22	5.25
FFG	0.01	0.73	0.02	7.01	TNC	0.01	0.83	-0.17	5.71	SPN	-0.00	0.65	-0.34	8.50
OPY	0.00	0.59	0.34	19.12	UIS	-0.06	3.08	-3.82	137.87	UNT	0.00	0.95	-0.41	3.62

The tables above shows descriptive statistics of the different portfolios. All portfolios show excess kurtosis and skewness. Table 25 in appendix 1 shows the results of Jarque bera test of normality for the individual series, the test is rejected for all of the different time series.

Table 1 in Appendix A shows the realized return for the different portfolios. All portfolios shows a spike in volatility during the financial crises of 2008. The small cap portfolios especially the finance and tech portfolio shows a decrease in volatility after the crises of 2008. All portfolios show volatility clustering.

6 Results

The results presented below focuses on one sector with a summary of all sectors due to the fact that the results hardly differ between sectors. Table A shows the AIC values of the different models. For all models the order $(1, 1)$ shows the lowest AIC value and is henceforth used for the out of sample VaR estimates forecasting.

6.1 1-day VaR

Table 7: 1 % VaR,Large Cap finance

	Expected exceed	Actual exceed	Kupiec test	P-value	Christoffersen	P-value
DCC	10.00	1.00	13.59	0.00	13.59	0.00
CCC	10.00	14.00	1.39	0.24	1.78	0.41
GO-Garch	10.00	13.00	0.79	0.37	7.59	0.02

Table 8: 5 % VaR,Large Cap finance

	Expected exceed	Actual exceed	Kupiec test	P-value	Christoffersen	P-value
DCC	50.00	39.00	2.89	0.09	8.55	0.01
CCC	50.00	66.00	4.72	0.03	11.22	0.00
GO-Garch	50.00	46.00	0.40	0.53	5.90	0.05

The table above shows the expected amount of exceedances given the significance level and the actual amount of exceedances given the estimated VaR limit. The table also shows the kupiec- and the Christoffersen test for the estimated VaR of the Large cap finance portfolio. The DCC-model underestimate the number of VaR breaches and shows significant results for both both the Kupiec test and the Christoffersen. While the CCC-model and the GO-Garch both rejects the Kupeic test but the GO-Garch is the only one that shows adequate results for both test. For the 5% VaR the DCC-model is the only one that shows adequate results for both tests.

Table 9: 1 % VaR, Mid Cap finance

	Expected exceed	Actual exceed	Kupiec test	P-value	Christoffersen	P-value
DCC	10.00	3.00	6.91	0.01	6.93	0.03
CCC	10.00	15.00	2.13	0.14	3.65	0.16
GO-Garch	10.00	6.00	1.93	0.16	7.00	0.03

Table 10: 5 % VaR, Mid Cap finance

	Expected exceed	Actual exceed	Kupiec test	P-value	Christoffersen	P-value
DCC	50.00	44.00	0.87	0.35	2.78	0.25
CCC	50.00	66.00	4.72	0.03	7.65	0.02
GO-Garch	50.00	34.00	6.24	0.01	8.51	0.01

For the mid cap finance portfolio, DCC also underestimates number of acutal breaches when VaR is set to the 1% significance level. The DCC model also shows significant Kupeic and Christoffersen results. while the GO-Garch model is the only model that shows adequate results for both tests.

Table 11: 1 % VaR, Small Cap finance

	Expected exceed	Actual exceed	Kupiec test	P-value	Christoffersen	P-value
DCC	10.00	7.00	1.05	0.31	1.15	0.56
CCC	10.00	8.00	0.46	0.50	0.59	0.75
GO-Garch	10.00	8.00	0.46	0.50	0.59	0.75

Table 12: 5 % VaR, Small Cap finance

	Expected exceed	Actual exceed	Kupiec test	P-value	Christoffersen	P-value
DCC	50.00	50.00	0.00	0.97	0.11	0.94
CCC	50.00	47.00	0.23	0.63	1.57	0.46
GO-Garch	50.00	27.00	13.57	0.00	15.20	0.00

For the 1% VaR the all models reject the Kupeic test but the breaches does are not independent according to the Christoffersen test. As seen in table 12 the DCC and the CCC-model rejects the Kupiec test but does not show a significant results for the Christoffersen test

6.2 5-day VaR

Table 13: 1 % VaR, Large Cap finance

	Expected exceed	Actual exceed	Kupiec test	P-value	Christoffersen	P-value
DCC	10.00	17.00	4.01	0.05	19.44	0.00
CCC	10.00	23.00	12.33	0.00	18.39	0.00
GO-Garch	10.00	15.00	2.13	0.14	3.65	0.16

Table 14: 5 % VaR, Large Cap finance

	Expected exceed	Actual exceed	Kupiec test	P-value	Christoffersen	P-value
DCC	50.00	46.00	0.40	0.53	8.59	0.01
CCC	50.00	71.00	8.00	0.00	16.55	0.00
GO-Garch	50.00	44.00	0.87	0.35	10.09	0.01

The 5-day forecast of VaR shows that the GO-garch has the lowest amounts of VaR breaches as well as the DCC-model. The DCC and CCC-model also rejects the Christoffersen test. For the 5% VaR the DCC-model shows the best results.

Table 15: 1 % VaR, Mid Cap finance

	Expected exceed	Actual exceed	Kupiec test	P-value	Christoffersen	P-value
DCC	10.00	21.00	9.15	0.00	12.34	0.00
CCC	10.00	23.00	12.33	0.00	12.68	0.00
GO-Garch	10.00	9.00	0.12	0.73	3.51	0.17

Table 16: 5 % VaR, Mid Cap finance

	Expected exceed	Actual exceed	Kupiec test	P-value	Christoffersen	P-value
DCC	50.00	52.00	0.06	0.81	5.62	0.06
CCC	50.00	73.00	9.52	0.00	19.33	0.00
GO-Garch	50.00	37.00	4.06	0.04	10.57	0.01

The mid cap finance portfolio show similar results, the GO-garch outperforms the other models for actual breaches, both the DCC and CCC-model show insignificant test results for the Kupiec test but also shows significant results for the Christoffersen test. For the 5% VaR the CCC-model and GO-garch model show significant Kupiec results aswell as Christoffersen test results.

Table 17: 1 % VaR, Small Cap finance

	Expected exceed	Actual exceed	Kupiec test	P-value	Christoffersen	P-value
DCC	10.00	16.00	3.00	0.08	4.32	0.12
CCC	10.00	13.00	0.79	0.37	2.81	0.25
GO-Garch	10.00	11.00	0.09	0.77	0.33	0.85

Table 18: 5 % VaR, Small Cap finance

	Expected exceed	Actual exceed	Kupiec test	P-value	Christoffersen	P-value
DCC	50.00	59.00	1.50	0.22	2.19	0.33
CCC	50.00	47.00	0.23	0.63	1.57	0.46
GO-Garch	50.00	29.00	11.13	0.00	12.39	0.00

For the small cap finance portfolios all models shows insignificant Kupiec and Christoffersen test results. The GO-Garch model has the fewest amount of VaR breaches. For the 5% VaR the CCC and the DCC-model shows insignificant Kupeic and Christoffersen test results. GO-garch has the fewest amount of breaches of 29.

6.3 10-day VaR

Table 19: 1 % VaR, Large Cap finance

	Expected exceed	Actual exceed	Kupiec test	P-value	Christoffersen	P-value
DCC	10.00	17.00	4.01	0.05	19.44	0.00
CCC	10.00	23.00	12.33	0.00	18.39	0.00
GO-Garch	10.00	15.00	2.13	0.14	3.65	0.16

Table 20: 5 % VaR, Large Cap finance

	Expected exceed	Actual exceed	Kupiec test	P-value	Christoffersen	P-value
DCC	50.00	46.00	0.40	0.53	8.59	0.01
CCC	50.00	71.00	8.00	0.00	16.55	0.00
GO-Garch	50.00	44.00	0.87	0.35	10.09	0.01

For the 10 day VaR estimates the DCC and the GO-Garch shows insignificant Kupeic test results, the DCC model also shows significant Christoffersen test results, but the GO-Garch has fewest amount of breaches. For the 5% VaR the DCC and the GO-garch models produced similar results with 46 and 44 breaches respectively and produces insignificant Kupiec and significant Christoffersen test results.

Table 21: 1 % VaR, Mid Cap finance

	Expected exceed	Actual exceed	Kupiec test	P-value	Christoffersen	P-value
DCC	10.00	21.00	9.15	0.00	12.34	0.00
CCC	10.00	25.00	15.86	0.00	16.07	0.00
GO-Garch	10.00	10.00	0.00	0.98	2.98	0.22

Table 22: 5 % VaR, Mid Cap finance

	Expected exceed	Actual exceed	Kupiec test	P-value	Christoffersen	P-value
DCC	50.00	51.00	0.01	0.92	11.44	0.00
CCC	50.00	69.00	6.59	0.01	16.14	0.00
GO-Garch	50.00	32.00	8.00	0.00	13.58	0.00

For the mid cap portfolios the GO-Garch accurately estimates 10 violations of VaR but the Christoffersen test is not significant. The DCC- and CCC-model overestimates the number of violations but shows significant results for the Christoffersen test. For the 5% VaR the DCC-model shows insignificant Kupiec test results, however all model shows insignificant Christoffersen test results.

Table 23: 1 % VaR, Small Cap finance

	Expected exceed	Actual exceed	Kupiec test	P-value	Christoffersen	P-value
DCC	10.00	18.00	5.13	0.02	6.09	0.05
CCC	10.00	15.00	2.13	0.14	3.65	0.16
GO-Garch	10.00	11.00	0.09	0.77	0.33	0.85

Table 24: 5 % VaR, Small Cap finance

	Expected exceed	Actual exceed	Kupiec test	P-value	Christoffersen	P-value
DCC	50.00	55.00	0.45	0.50	0.78	0.68
CCC	50.00	50.00	0.00	0.97	0.11	0.95
GO-Garch	50.00	29.00	11.13	0.00	12.39	0.00

For the small cap portfolios the results is simliar, GO-Garch shows 11 VaR violations as well as a insignificant test results for the Kupeiec test. However all models shows insignificant Christoffersen test results. For the 5% VaR the Go-Garch performs poorly with regards to the Kupiec test however it shows significant Christoffersen test. The DCC- and the CCC model shows opposite results.

The results is similar for the the other portfolios. For the 1% VaR the DCC-model and the GO-Garch outperforms the CCC-model when it comes to the Kupeic test but the DCC and CCC-model performs best with regards to the Christoffersen test. The same applies to the 5% VaR estimates, the CCC-model shows good results with regards to the Christoffersen test but the DCC and GO-Garch show better results with regards to the Kupiec test. For the 5-day and 10-day ahead VaR estimates the results are simliar, the DCC-model and GO-Garch show good results with regards to the Kupiec test but the DCC and CCC-model perform better with regards to the Christoffersen test.

7 Discussion and conclusion

The aim of this paper has been to examine the out of sample predictive power of MGARCH models with respect to VaR estimates. The DCC-model and GO-garch provides more unbiased results compared to the CCC-model indicating that time-varying volatility should be taken in to account when calculating VaR estimates. The GO-Garch model does not perform as good as the DCC and CCC-model with regards to the independence of VaR violations. For the 5% VaR the results are simliar, for unconditional coverage the DCC-model and GO-Garch provide good results but the same is not true for the independence of the VaR violations. The CCC-model and DCC-model shows good results with regards to the independence of VaR violations. The results are similar for the longer forecasting horizon. Moreover the CCC-model tends to underestimate the changes in volatility indicating that the assumption of constant correlation of the series is false. From a risk management standpoint the DCC-model and GO-Garch gives better estimates since it in the most cases show fewer breaches of the VaR limit.

To summarize the aim of this paper has been to compare different formulations of MGARCH models and evaluate them using VaR, all models show reasonable results but the DCC-model shows the best results of the models tested despite significance level and forecasting horizon.

8 Further studies

One interesting comparison available for further work is the comparison of MGARCH and multivariate stochastic volatility models and the implications of higher frequency data for instance intra daily data. Due to the distributional assumptions of financial returns non parametric and semi parametric models might provide better results and is available for further work.

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A Appendix A

Table 25: Jarque Bera test

Stock	χ^2	P-value	Stock	χ^2	P-value
AXP	21324.21	0.00	XRX	71858.80	0.00
BLK	11208.26	0.00	BC	24671.02	0.00
MUR	13414.92	0.00	CHK	30093.08	0.00
INT	112246.56	0.00	CNX	104159.74	0.00
C	351944.31	0.00	CPB	22913.22	0.00
MS	459919.06	0.00	CIA	3956.51	0.00
MCO	12287.14	0.00	CPF	25957.50	0.00
HPQ	14900.93	0.00	FFG	9787.95	0.00
LLL	145579.42	0.00	OPY	72945.29	0.00
ORCL	14438.54	0.00	DDD	256759.03	0.00
MSI	25617.35	0.00	KAI	240490.46	0.00
URI	22103.74	0.00	SPXC	19449.42	0.00
EMR	9338.71	0.00	TNC	6513.61	0.00
HAL	308852.88	0.00	UIS	3799049.80	0.00
APA	4571.68	0.00	BPT	50512.97	0.00
CMI	9568.87	0.00	NR	5149.75	0.00
VLO	6859.58	0.00	OII	5539.99	0.00
ASB	29926.44	0.00	SPN	14496.62	0.00
EV	27441.60	0.00	UNT	2744.98	0.00
FHN	218703.35	0.00	MTG	735495.61	0.00
RDN	68889.54	0.00	ASGN	55848.49	0.00
JNPR	14681.53	0.00	MAN	8253.72	0.00
RHI	24714.22	0.00			

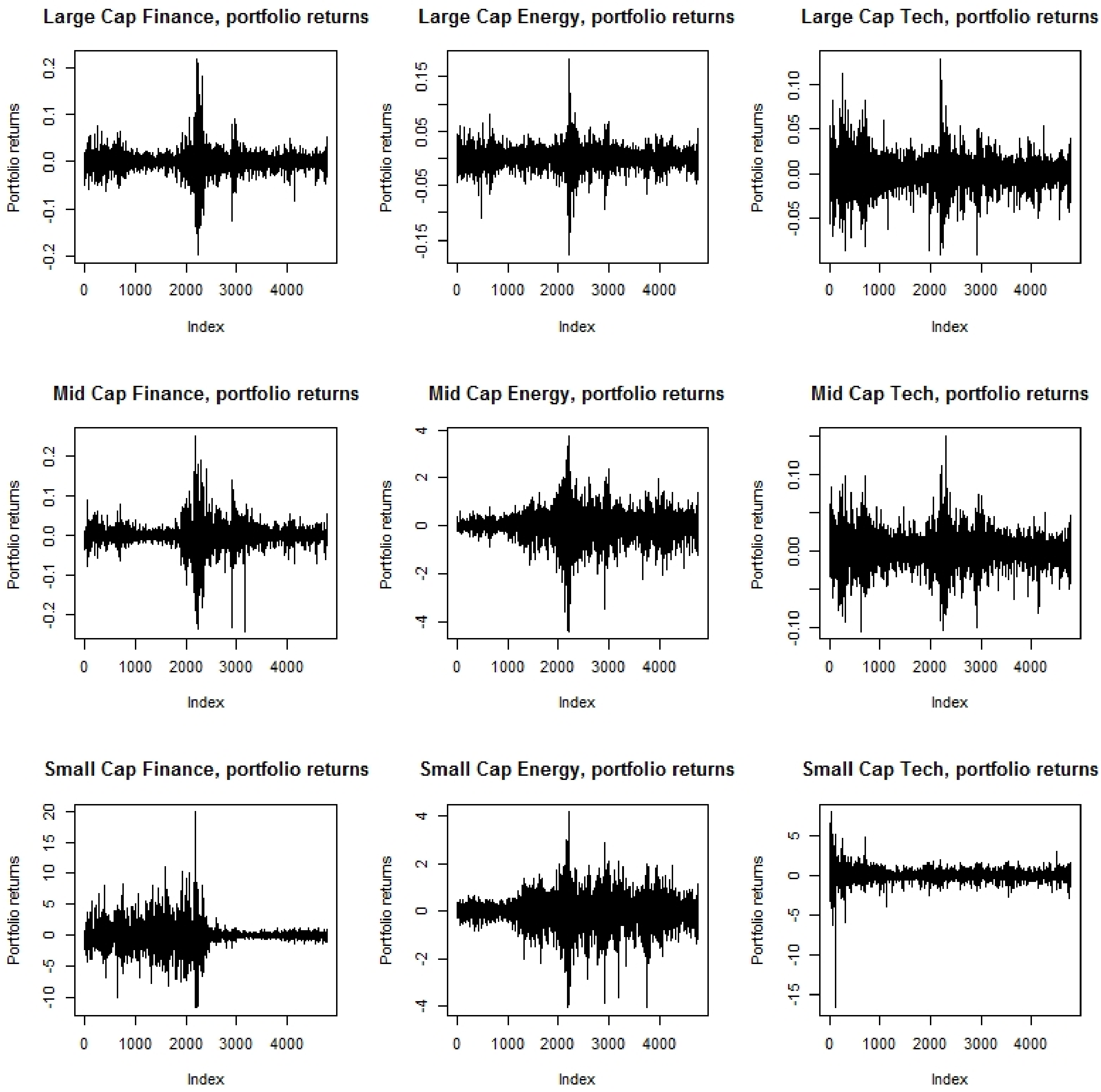

Figure 1: Portfolio returns.

Table 26: AIC values of Finance portfolios

Large Cap				Mid Cap				Small Cap			
Order	DCC	CCC	GO-Garch	Order	DCC	CCC	GO-Garch	Order	DCC	CCC	GO-Garch
1,1	-27,4*	-26,7*	-27,1*	1,1	-26,1*	-25,9*	-26.0*	1,1	7,9*	8,1*	9,0*
1,2	-27,2	-26,2	-26,6	1,2	-26,2	-25,7	-25.2	1,2	8,0	8,5	9,6
2,1	-27,2	-26,2	-27,0	2,1	-26,2	-25,7	-26,0	2,1	8,0	8,5	9,1
2,2	-27,3	-25,9	-26,6	2,2	-26,2	-24,9	-25.2	2,2	8,0	8,8	9,5

A.1 1-day VaR

Table 27: Large Cap Energy

	Expected exceed	Actual exceed	Kupiec test	P-value	Christoffersen	P-value
DCC	10.00	2.00	9.72	0.00	9.73	0.01
CCC	10.00	15.00	2.13	0.14	7.78	0.02
GO-Garch	10.00	1.00	13.59	0.00	13.59	0.00
	Expected exceed	Actual exceed	Kupiec test	P-value	Christoffersen	P-value
DCC	50.00	46.00	0.40	0.53	3.66	0.16
CCC	50.00	88.00	24.55	0.00	28.31	0.00
GO-Garch	50.00	38.00	3.45	0.06	12.68	0.00

Table 28: Large Cap Tech

	Expected exceed	Actual exceed	Kupiec test	P-value	Christoffersen	P-value
DCC	10.00	6.00	1.93	0.16	2.01	0.37
CCC	10.00	8.00	0.46	0.50	0.59	0.75
GO-Garch	10.00	8.00	0.46	0.50	4.32	0.12
	Expected exceed	Actual exceed	Kupiec test	P-value	Christoffersen	P-value
DCC	50.00	54.00	0.28	0.60	1.70	0.43
CCC	50.00	34.00	6.24	0.01	14.20	0.00
GO-Garch	50.00	43.00	1.17	0.28	7.92	0.02

Table 29: Mid Cap Energy

	Expected exceed	Actual exceed	Kupiec test	P-value	Christoffersen	P-value
DCC	10.00	12.00	0.36	0.55	0.65	0.72
CCC	10.00	3.00	6.91	0.01	6.93	0.03
GO-Garch	10.00	6.00	1.93	0.16	2.01	0.37
	Expected exceed	Actual exceed	Kupiec test	P-value	Christoffersen	P-value
DCC	50.00	61.00	2.25	0.13	2.72	0.26
CCC	50.00	24.00	17.80	0.00	18.07	0.00
GO-Garch	50.00	53.00	0.15	0.70	0.67	0.72

Table 30: Mid Cap Tech

	Expected exceed	Actual exceed	Kupiec test	P-value	Christoffersen	P-value
DCC	10.00	7.00	1.05	0.31	5.47	0.07
CCC	10.00	4.00	4.78	0.03	11.62	0.00
GO-Garch	10.00	6.00	1.93	0.16	7.00	0.03
	Expected exceed	Actual exceed	Kupiec test	P-value	Christoffersen	P-value
DCC	50.00	59.00	1.50	0.22	3.27	0.20
CCC	50.00	34.00	6.24	0.01	8.51	0.01
GO-Garch	50.00	52.00	0.06	0.81	5.62	0.06

Table 31: Small Cap Energy

	Expected exceed	Actual exceed	Kupiec test	P-value	Christoffersen	P-value
DCC	10.00	1.00	13.58	0.00	13.58	0.00
CCC	10.00	0.00	20.22	0.00	20.22	0.00
GO-Garch	10.00	2.00	9.72	0.00	9.73	0.00
	Expected exceed	Actual exceed	Kupiec test	P-value	Christoffersen	P-value
DCC	50.00	56.00	0.66	0.42	7.27	0.03
CCC	50.00	11.00	46.75	0.00	47.00	0.00
GO-Garch	50.00	46.00	0.40	0.53	4.81	0.09

Table 32: Small Cap Tech

	Expected exceed	Actual exceed	Kupiec test	P-value	Christoffersen	P-value
DCC	10.00	9.00	0.12	0.73	0.28	0.87
CCC	10.00	50.00	82.09	0.00	97.46	0.00
GO-Garch	10.00	5.00	3.15	0.08	3.20	0.20
	Expected exceed	Actual exceed	Kupiec test	P-value	Christoffersen	P-value
DCC	50.00	66.00	4.72	0.03	4.75	0.09
CCC	50.00	122.00	78.31	0.00	79.75	0.00
GO-Garch	50.00	44.00	0.87	0.35	0.87	0.65

A.2 5-day Var

Table 33: Large Cap Tech

	Expected exceed	Actual exceed	Kupiec test	P-value	Christoffersen	P-value
DCC	10.00	11.00	0.09	0.77	8.26	0.02
CCC	10.00	11.00	0.09	0.77	2.71	0.26
GO-Garch	10.00	12.00	0.36	0.55	2.66	0.27
	Expected exceed	Actual exceed	Kupiec test	P-value	Christoffersen	P-value
DCC	50.00	58.00	1.19	0.28	8.93	0.01
CCC	50.00	42.00	1.52	0.22	6.05	0.05
GO-Garch	50.00	46.00	0.40	0.53	8.59	0.01

Table 34: Mid Cap Energy

	Expected exceed	Actual exceed	Kupiec test	P-value	Christoffersen	P-value
DCC	10.00	17.00	4.01	0.05	5.14	0.08
CCC	10.00	5.00	3.15	0.08	3.20	0.20
GO-Garch	10.00	9.00	0.12	0.73	3.51	0.17
	Expected exceed	Actual exceed	Kupiec test	P-value	Christoffersen	P-value
DCC	50.00	65.00	4.16	0.04	7.36	0.03
CCC	50.00	27.00	13.57	0.00	15.20	0.00
GO-Garch	50.00	57.00	0.90	0.34	4.74	0.09

Table 35: Mid Cap Tech

	Expected exceed	Actual exceed	Kupiec test	P-value	Christoffersen	P-value
DCC	10.00	13.00	0.79	0.37	7.59	0.02
CCC	10.00	7.00	1.05	0.31	5.47	0.07
GO-Garch	10.00	10.00	0.00	0.98	2.98	0.22
	Expected exceed	Actual exceed	Kupiec test	P-value	Christoffersen	P-value
DCC	50.00	65.00	4.16	0.04	9.07	0.01
CCC	50.00	38.00	3.45	0.06	6.89	0.03
GO-Garch	50.00	56.00	0.66	0.42	6.92	0.03

Table 36: Small Cap Energy

	Expected exceed	Actual exceed	Kupiec test	P-value	Christoffersen	P-value
DCC	10.00	9.00	0.12	0.73	0.28	0.87
CCC	10.00	1.00	13.59	0.00	13.59	0.00
GO-Garch	10.00	3.00	6.91	0.01	6.93	0.03
	Expected exceed	Actual exceed	Kupiec test	P-value	Christoffersen	P-value
DCC	50.00	56.00	0.66	0.42	1.17	0.56
CCC	50.00	11.00	46.75	0.00	47.00	0.00
GO-Garch	50.00	47.00	0.23	0.63	1.12	0.57

Table 37: Small Cap Tech

	Expected exceed	Actual exceed	Kupiec test	P-value	Christoffersen	P-value
DCC	10.00	15.00	2.13	0.14	7.78	0.02
CCC	10.00	47.00	72.42	0.00	80.12	0.00
GO-Garch	10.00	10.00	0.00	0.98	2.98	0.22
	Expected exceed	Actual exceed	Kupiec test	P-value	Christoffersen	P-value
DCC	50.00	72.00	8.74	0.00	11.55	0.00
CCC	50.00	126.00	86.17	0.00	86.56	0.00
GO-Garch	50.00	56.00	0.66	0.42	4.81	0.09

A.3 10-day Var

Table 38: Large Cap Energy

	Expected exceed	Actual exceed	Kupiec test	P-value	Christoffersen	P-value
DCC	10.00	12.00	0.36	0.55	22.13	0.00
CCC	10.00	23.00	12.33	0.00	27.83	0.00
GO-Garch	10.00	8.00	0.46	0.50	4.32	0.12
	Expected exceed	Actual exceed	Kupiec test	P-value	Christoffersen	P-value
DCC	50.00	56.00	0.66	0.42	18.48	0.00
CCC	50.00	94.00	32.18	0.00	38.25	0.00
GO-Garch	50.00	47.00	0.23	0.63	10.91	0.00

Table 39: Large Cap Tech

	Expected exceed	Actual exceed	Kupiec test	P-value	Christoffersen	P-value
DCC	10.00	13.00	0.79	0.37	7.59	0.02
CCC	10.00	12.00	0.36	0.55	7.80	0.02
GO-Garch	10.00	12.00	0.36	0.55	2.66	0.27
	Expected exceed	Actual exceed	Kupiec test	P-value	Christoffersen	P-value
DCC	50.00	58.00	1.19	0.28	3.15	0.21
CCC	50.00	42.00	1.52	0.22	8.73	0.01
GO-Garch	50.00	45.00	0.61	0.44	6.51	0.04

Table 40: Mid Cap Energy

	Expected exceed	Actual exceed	Kupiec test	P-value	Christoffersen	P-value
DCC	10.00	19.00	6.36	0.01	10.25	0.01
CCC	10.00	6.00	1.93	0.16	2.01	0.37
GO-Garch	10.00	9.00	0.12	0.73	3.51	0.17
	Expected exceed	Actual exceed	Kupiec test	P-value	Christoffersen	P-value
DCC	50.00	62.00	2.68	0.10	5.15	0.08
CCC	50.00	29.00	11.13	0.00	12.39	0.00
GO-Garch	50.00	55.00	0.45	0.50	3.11	0.21

Table 41: Mid Cap Tech

	Expected exceed	Actual exceed	Kupiec test	P-value	Christoffersen	P-value
DCC	10.00	14.00	1.39	0.24	7.59	0.02
CCC	10.00	6.00	1.93	0.16	7.00	0.03
GO-Garch	10.00	10.00	0.00	0.98	2.98	0.22

	Expected exceed	Actual exceed	Kupiec test	P-value	Christoffersen	P-value
DCC	50.00	61.00	2.25	0.13	6.66	0.04
CCC	50.00	39.00	2.89	0.09	8.55	0.01
GO-Garch	50.00	54.00	0.28	0.60	5.11	0.08

Table 42: Small Cap Energy

	Expected exceed	Actual exceed	Kupiec test	P-value	Christoffersen	P-value
DCC	10.00	7.00	1.05	0.31	1.15	0.56
CCC	10.00	2.00	9.72	0.00	9.73	0.01
GO-Garch	10.00	3.00	6.91	0.01	6.93	0.03

	Expected exceed	Actual exceed	Kupiec test	P-value	Christoffersen	P-value
DCC	50.00	55.00	0.45	0.50	0.78	0.68
CCC	50.00	12.00	43.72	0.00	44.01	0.00
GO-Garch	50.00	46.00	0.40	0.53	0.40	0.82

Table 43: Small Cap Tech

	Expected exceed	Actual exceed	Kupiec test	P-value	Christoffersen	P-value
DCC	10.00	18.00	5.13	0.02	9.41	0.01
CCC	10.00	51.00	85.40	0.00	93.92	0.00
GO-Garch	10.00	12.00	0.36	0.55	2.66	0.27

	Expected exceed	Actual exceed	Kupiec test	P-value	Christoffersen	P-value
DCC	50.00	69.00	6.59	0.01	6.95	0.03
CCC	50.00	127.00	88.18	0.00	88.86	0.00
GO-Garch	50.00	56.00	0.66	0.42	3.07	0.22