

MATHEMATICS

SECTION A

January 27, 2024

1. Find the integrating factor of the differential equation $x \frac{dy}{dx} - 2y = 2x^2$.
2. Find $\frac{dy}{dx}$, if $xy^2 - x^2 = 4$.
3. If A is a square matrix of order 3 with $|A| = 4$, then write the value of $|-2A|$.
4. If a line has the direction ratios $-18, 12, -4$, then what are its direction cosines?
5. Find the cartesian equation of the line which passes through the point $(-2, 4, -5)$ and is parallel to the line $\frac{x+3}{3} = \frac{4-y}{5} = \frac{z+8}{6}$.
6. Let $*$ be a binary operation on $R - \{-1\}$ defined by $a * b = \frac{a}{b+1}$, for all $a, b \in R - \{-1\}$. show that $*$ is neither commutative nor associative in $R - \{-1\}$.
7. If $A = \begin{pmatrix} -3 & 6 \\ -2 & 4 \end{pmatrix}$, then show that $A^3 = A$.
8. Form the differential equation representing the family of curves $y^2 = m(a^2 - x^2)$ by eliminating the arbitrary constants ' m ' and ' a '.
9. Find:

$$\int \frac{\sin x - \cos x}{\sqrt{1 + \sin 2x}} dx, 0 < x < \frac{\pi}{2}$$

10. Find:

$$\int \frac{\sin(x-a)}{\sin(x+a)} dx$$

11. Find:

$$\int (\log x)^2 dx$$

12. Mother, father and son line up at random for a family photo. If A and B are two events given by A = son on one end, B = Father in the middle, find $P\left(\frac{B}{A}\right)$.
13. Let X be a random variable which assumes values x_1, x_2, x_3, x_4 such that

$$2P(X = x_1) = 3P(X = x_2) = P(X = x_3) = 5P(X = x_4).$$

Find the probability distribution of X.

14. A coin is tossed 5 times. Find the probability of getting
 - (i) at least 4 heads, and
 - (ii) at most 4 heads.

15. Find a unit vector perpendicular to both the vectors \vec{a} and \vec{b} , where $\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$.
16. Show that the vectors $\hat{i} - 2\hat{j} + 3\hat{k}$, $-2\hat{i} + 3\hat{j} - 4\hat{k}$ and $\hat{i} - 3\hat{j} + 5\hat{k}$ are coplanar.
17. Show that the relation R on the set Z of all integers, given by $R = \{(a, b) : 2 \text{ divides } (a - b)\}$ is an equivalence relation.
18. If $f(x) = \frac{4x+3}{6x-4}$, $x \neq \frac{2}{3}$, show that $f \circ f(x) = x$ for all $x \neq \frac{2}{3}$. Also, find the inverse of f .
19. If $\sin y = x \sin(a + y)$, prove that

$$\frac{dy}{dx} = \frac{\sin^2(a + y)}{\sin a}$$

20. If $(\sin x)^y = x + y$, find $\frac{dy}{dx}$.
21. If $\sin^{-1}\left(\frac{3}{x}\right) + \sin^{-1}\left(\frac{4}{x}\right) = \frac{\pi}{2}$, then find the value of x .
22. Using properties of determinants, prove that

$$\begin{vmatrix} a^2 + 1 & ab & ac \\ ab & b^2 + 1 & bc \\ ac & bc & c^2 + 1 \end{vmatrix} = 1 + a^2 + b^2 + c^2$$

23. If $y = (\cot^{-1} x)^2$, show that $(x^2 + 1)^2 \frac{d^2y}{dx^2} + 2x(x^2 + 1) \frac{dy}{dx} = 2$.
24. Find the equations of the tangent and normal to the curve $y = \frac{x-7}{(x-2)(x-3)}$ at the point where it cuts the x-axis.
25. Find the local maxima and local minima, if any, of the following function. Also find the local maximum and the local minimum values, as the case may be :

$$f(x) = \sin x + \frac{1}{2} \cos 2x, \quad 0 \leq x \leq \frac{\pi}{2}$$

26. If $A = \begin{pmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$, find A^2 and show that $A^2 = A^{-1}$.

27. Using matrix method, solve the following system of equations:

$$\begin{aligned} 2x - 3y + 5z &= 13 \\ 3x + 2y - 4z &= -2 \\ x + y - 2z &= -2 \end{aligned}$$