

# MATHEMATICS

## SECTION A

January 30, 2024

### 1 Matrix

1. If  $(A)$  is a square matrix of order 3 with  $|A| = 4$ , then write the value of  $|-2A|$ .
2. If  $A = \begin{pmatrix} -3 & 6 \\ -2 & 4 \end{pmatrix}$ , then show that  $A^3 = A$ .
3. Using properties of determinants, prove that

$$\begin{vmatrix} a^2 + 1 & ab & ac \\ ab & b^2 + 1 & bc \\ ac & bc & c^2 + 1 \end{vmatrix} = 1 + a^2 + b^2 + c^2$$

4. If  $A = \begin{pmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$ , find  $A^2$  and show that  $A^2 = A^{-1}$ .

5. Using matrix method, solve the following system of equations:

$$2x - 3y + 5z = 13$$

$$3x + 2y - 4z = -2$$

$$x + y - 2z = -2$$

### 2 Differentiation

6. Find the integrating factor of the differential equation  $x \frac{dy}{dx} - 2y = 2x^2$ .
7. Find  $\frac{dy}{dx}$ , if  $xy^2 - x^2 = 4$ .
8. Form the differential equation representing the family of curves  $y^2 = m(a^2 - x^2)$  by eliminating the arbitrary constants ' $m$ ' and ' $a$ '.
9. If  $\sin y = x \sin(a + y)$ , prove that

$$\frac{dy}{dx} = \frac{\sin^2(a + y)}{\sin a}$$

10. If  $(\sin x)^y = x + y$ , find  $\frac{dy}{dx}$ .
11. If  $y = (\cot^{-1} x)^2$ , show that  $(x^2 + 1)^2 \frac{d^2y}{dx^2} + 2x(x^2 + 1) \frac{dy}{dx} = 2$ .

### 3 Integration

12. Find:

$$\int \frac{\sin x - \cos x}{\sqrt{1 + \sin 2x}} dx, 0 < x < \frac{\pi}{2}$$

13. Find:

$$\int \frac{\sin(x-a)}{\sin(x+a)} dx$$

14. Find:

$$\int (\log x)^2 dx$$

### 4 Probability

15. Mother, father and son line up at random for a family photo. If A and B are two events given by A = son on one end, B = Father in the middle, find  $P(B | A)$ .

16. Let  $X$  be a random variable which assumes values  $x_1, x_2, x_3, x_4$  such that

$$2P(X = x_1) = 3P(X = x_2) = P(X = x_3) = 5P(X = x_4).$$

Find the probability distribution of  $X$ .

17. A coin is tossed 5 times. Find the probability of getting

- (i) at least 4 heads, and
- (ii) at most 4 heads.

### 5 Vectors

18. If a line has the direction ratios  $-18, 12, -4$ , then what are its direction cosines?

19. Find the cartesian equation of the line which passes through the point  $(-2, 4, -5)$  and is parallel to the line  $\frac{x+3}{3} = \frac{4-y}{5} = \frac{z+8}{6}$ .

20. Find a unit vector perpendicular to both the vectors  $\vec{a}$  and  $\vec{b}$ , where  $\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$  and  $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$ .

21. Show that the vectors  $\hat{i} - 2\hat{j} + 3\hat{k}$ ,  $-2\hat{i} + 3\hat{j} - 4\hat{k}$  and  $\hat{i} - 3\hat{j} + 5\hat{k}$  are coplanar.

### 6 Functions

22. Let  $*$  be a binary operation on  $\mathbf{R} - -1$  defined by  $a * b = \frac{a}{b+1}$ , for all  $a, b \in \mathbf{R} - -1$ . show that  $*$  is neither commutative nor associative in  $\mathbf{R} - -1$ .

23. Show that the relation  $R$  on the set  $\mathbf{Z}$  of all integers, given by  $R = \{(a, b) : 2 \text{ divides } (a - b)\}$  is an equivalence relation.

24. If  $f(x) = \frac{4x+3}{6x-4}$ ,  $x \neq \frac{2}{3}$ , show that  $f \circ f(x) = x$  for all  $x \neq \frac{2}{3}$ . Also, find the inverse of  $f$ .

25. Find the local maxima and local minima, if any, of the following function. Also find the local maximum and the local minimum values, as the case may be :

$$f(x) = \sin x + \frac{1}{2} \cos 2x, 0 \leq x \leq \frac{\pi}{2}$$

## 7 Algebra

26. If  $\sin^{-1}\left(\frac{3}{x}\right) + \sin^{-1}\left(\frac{4}{x}\right) = \frac{\pi}{2}$ , then find the value of  $x$ .

## 8 Intersection of Conics

27. Find the equations of the tangent and normal to the curve  $y = \frac{x-7}{(x-2)(x-3)}$  at the point where it cuts the x-axis.