Assignment - II

DAA (CSA 0666)

Name : Anjana Iyer. B

Reg no : 192324078

sub /sub
code : Design and analysis of algorithm (CSA 0666).

Q1). If $t_1(n) \in O(g_1(n))$ and $t_2(n) \in O(g_2(n))$ then $t_1(n)+t_2(n) \in O(\max\{g_1(n), g_2(n)\})$. Prove the assertions.

$$f_1(n) \le c_1 g_1(n) \quad \text{for all } n \ge n_0$$

$$f_2(n) \le c_2 g_2(n) \quad \text{for all } n \ge n_0$$

Adding

$$f_1(n) + f_2(n) \le c_1 g_1(n) + c_2 g_2(n)$$

Since

$$\max\{g_1(n), g_2(n)\} \ge g_1(n)$$

$$\max\{g_1(n), g_2(n)\} \ge g_2(n)$$

$$f_1(n) + f_2(n) \le c_1 \max\{g_1(n), g_2(n)\} + c_2 \max\{g_1(n), g_2(n)\}$$

$$\le (c_1 + c_2) \max\{g_1(n), g_2(n)\}$$

Let $c = c_1 + c_2$

$$f_1(n) + f_2(n) \le c \max\{g_1(n), g_2(n)\} \quad \text{for all } n \ge n_0$$

$$\therefore f_1(n) + f_2(n) = O(\max\{g_1(n), g_2(n)\}).$$

Q2).

$$T(n) = 2T(n/2) + 1$$

$a=2$
$b=2$ $\quad k=1,\ P=1$

$$\log_a b = \log_2 2 = 1$$

$$\log_a b = k$$

$P \ge -1 \quad \Theta(n^k \log_n^{P+1})$

$$\Theta(n^1 \log_n^2)$$

$$\Rightarrow \Theta(n \log n)$$

Q3). $T(n) = \begin{cases} 2T(n/2) + 1 & \text{if } n > 1 \\ 1 & \text{otherwise} \end{cases}$

$T(n) = aT(n/b) + f(n)$

if $f(n) = O(n \log_b a - \epsilon)$

then $T(n) = \theta(n \log_a b)$

if $f(n) = \theta(n \log_b a \log n k)$

then $T(n) = \theta(n \log_b a \log n^{k+1})$

if $f(n) = \Omega(n \log_b a + \epsilon)$

then $T(n) = \theta(f(n))$

Q4). $T(n) = \begin{cases} 2T(n-1) & \text{if } n > 0 \\ 1 & \text{otherwise} \end{cases}$

$T(n) = 2T(n-1)$

$T(n-1) = 2[2T(n-2)]$

$= [2^2 T(n-2)]$

$T(n) = 2^2 [2T(n-3)]$

$= 2^3 T(n-3)$

$T(n) = 2^k T(n-k)$

$n-k = 0 \; ; n = k$

$T(0) = 1$

$$\boxed{T(n) = O(2^n)}$$

5). Big O notation : ST $f(n)=n^2+3n+5$ is $O(n^2)$

$f(n)=n^2+3n+5$

$f(n) \leq c.n^2$

for all $n>n_0$

$f(n)=n^2+3n+5$

$=n^2+3n+5$

$f(n)=n^2+3n+5 \leq c.n^2$

$n^2+3n+5 \leq c.n^2$

$3n+5 \leq c.n^2$

$3n+5 \leq c.n^2$

$\therefore$ when n is close to 0, $3n+5 \leq c.n^2$ can be (-ve)

6) Big omega notation : Prove that $g(n)=n^3+2n^2+4n$ is $\Omega(n^3)$

$g(n)=n^3+2n^2+4n$

$g(n) > c.n^3$

$g(n)=n^3+2n^2+4n$

$=n^3(n+2)+4n$

$g(n)$ & n^3

$g(n)=n^2(n+2)+4n \geq c.n^3$

$n^2(n+2)+4n-c.n^3 \geq 0$

$n^2(n+2)+4n-cn^3 \geq 0$

$\therefore$ This inequality is not always true.

when n is close to $0 \cdot n^2(n+2)+4n-c \cdot n^3$ can be (-ve)

$$\boxed{\therefore g(n) \neq \Omega(n^3)}$$

Q7). Big Theta notation : Determine whether $h(n)=4n^2+3n$ is $\Theta(n^2)$ or not.

$$h(n)=4n^2+3n$$

$$h(n) \geq c \cdot n^2$$

$$h(n)=4n^2+3n$$

$$=n^2\left(4+\frac{3}{n}\right)$$

$$h(n)=n^2\left(4+\frac{3}{n}\right) \geq c \cdot n^2$$

$$\cancel{n^2}\left(4+\frac{3}{n}\right) \geq c \cdot \cancel{n^2}$$

$$4+\frac{3}{n} \geq c.$$

This inequality to hold for all n, we need

$4+3/n \geq c$ for all n.

Inequality is not always true. when n is close

$0 \cdot 4+3/n$ can be less than c.

We can't find a constant c such that

$$h(n) \geq c \cdot n^2$$

$$\boxed{h(n) \neq \Theta(n^2)}$$

Let $f(n) = n^3 - 2n^2 + n$ and $g(n) = n^2$. Show whether $f(n) = \Omega(g(n))$ is true or false and justify your answer

$f(n) = n^3 - 2n^2 + n$

$g(n) = n^2$

$f(n) \geq c \cdot g(n)$

$f(n) = n^3 - 2n^2 + n$

$= n^2(n-2) + n$

$= n^2 (n - 2 + 1/n)$

compare $f(n)$ & $g(n)$

$f(n) = n^2 (n - 2 + 1/n) \geq c \cdot n^2$

$n^2 (n - 2 + 1/n) \geq c \cdot n^2$

$n^2 (n - 2 + 1/n) + c \cdot n^2 \geq 0$

$n^2 (n - 2 + 1/n + c) \geq 0.$

$\therefore n - 2 + \frac{1}{n} + c \geq 0.$

This inequality is not always true

$\therefore f(n) \neq \Omega(g(n)).$

Q9). Determine whether $h(n) = n\log n + n$ in $\Theta(n\log n)$ prove a rigorous proof your conclusion

$h(n) = n\log n + n$

$c_1 \cdot n\log n \leq n\log n + n \leq c_2 \cdot n\log n$

upper bond:

$n \log n + n \leq c_2 . n\log n$

$n\log n + n \leq n \log n + n\log n = 2n \log n$

$c_2 = 2$

$n \log n + n \leq 2n \log n$

lower bond:

$c_1 . n\log n \leq n\log n + n$

$c_1 . n\log n \leq n\log n + n$

devide b.s by n

$c_1 . \log n \leq \log n + 1$

$\frac{1}{2} \log n \leq \log n$

$\boxed{\therefore c_1 . n\log n \leq n\log n + n \leq c_2 . n\log n}$

$\therefore h(n) = n\log n + n \in \theta (n\log n)$

Q10) Solve the following recurrence relations and find the order of growth for solutions

$T(n) = 4T(n/2) + n^2 \quad T(1) = 1$

By master theorem

$a = 4 \quad k = 2 \quad p = 0$

$b = 2 \quad f(n) = n^2$

$\log_b a = \log_2 4 = \log_2 2^2 = \log_2 2 = 2$

$\therefore \log_b a = k$

$p > -1$, so,

$= \theta(n^k \log_n^{p+1})$

$= \theta(n^2 \log_n^{1})$

$= \theta(n^2 \log n)$

$= T(n)$

Q11) Given an array of {4, -2, 5, 3, 10, -3, 2, 8 -3, 6, 7, -4, 1, 9, -1, 0, -6, -8, 11, -9} integers, find maximum and minimum product that can be obtained by multiplying 2 integers from array.

Maximum product:

2 Largest no's = 11, 10

2 smallest (-ve no's) = -9, -8.

Product:

$11 \times 10 = 110$

$-9 \times -8 = 72$

Max product = 110

Minimum product:

$11 \times -9 = -99$

$10 \times -9 = -90$

$\therefore$ Min product = -99.

Q12) Demonstrate binary search method to search key = 23, from the array arr[] = [2, 5, 8, 12, 16, 23, 38, 56, 72, 91].

arr [ ] = [2, 5, 8, 12, 16, 23, 38, 56, 72, 91]

Key = 23

0	1	2	3	4	5	6	7	8	9
2	5	8	12	16	23	38	56	72	91

$mid = \frac{l+h}{2} = \frac{0+9}{2} = 4.5 \approx 5$

0	1	2	3	4	5	6	7	8	9
2	5	8	12	16	$\boxed{23}$	38	56	72	91
					mid				

arr [mid] = 23

arr[mid] = Key

23 = 23

∴ Key is found.

Q13) Apply merge sort and order the list of elements data d = (45, 67, -12, 5, 22, 30, 50, 20). Set up a recurrence relation for the number of key comparisons made by mergesort.

d = (45, 67, -12, 5, 22, 30, 50, 20)

$m = \frac{0+7}{2} = 4$

0	1	2	3	4	5	6	7
45	67	-12	5	22	30	50	20

$m = \frac{0+4}{2} = 2$

$m = \frac{0+2}{2} = 1$

$$\begin{array}{cccccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 45 & 67 & -12 & 5 & 22 & 30 & 50 & 20 \end{array}$$

45 | 67 | -12 | 5 | 22 | 30 | 50 | 20

| 45 | 67 | | -12 | 5 | | 22 | | 30 | 50 | | 20 |

| -12 | 5 | 45 | 67 | | 22 | | 20 | 30 | 50 |

| -12 | 5 | 20 | 22 | 30 | 45 | 50 | 67 |

sorted

RR

$T(n) = 2T(n/2) + c(n)$

$a = 2, \quad K = 1$

$b = 2 \quad p = 1.$

$\log_b a = \log_2 2 = 1$

$\log_b a = K$

$\Theta(n^K \log_n^{p+1})$

$\Theta(n^1 \log_n^2)$

$\boxed{\therefore \Theta(n \log n)}$

Q15). Find the index of the target value 10 using binary search from the following list of elements

[2, 4, 6, 8, 10, 12, 14, 16, 18, 20]
0 1 2 3 4 5 6 7 8 9

$M = \frac{l+h}{2} = \frac{0+9}{2} = 4.5 \simeq 5$ (or) 4

0	1	2	3	4	5	6	7	8	9
2	4	6	8	$\boxed{10}$	12	14	16	18	20

target = 10

a [mid] = target

∴ 10 = 10

Target found

Q17). Sort the array 64, 34, 25, 12, 22, 11, 90 using bubble sort what is time complexity of bubble sort in best, worst average cases.

64 34 25 12 22 11 90
i j

34 64 25 12 22 11 90.
i j

34 25 64 12 22 11 90
i j

34 25 12 64 22 11 90
i j

34 25 12 22 64 11 90
i j

34 25 [illegible] 11 64 90

It-2

34 25 12 22 11 64 90
i j

25 34 12 22 11 64 90
i j

25 12 34 22 11 64 90
i j

25 12 22 34 11 64 90
i j

25 12 22 11 34 64 90.
i j

It-3

12 25 22 11 34 64 90
i j

12 22 25 11 34 64 90
i j

12 22 11 25 34 64 90
i j

It-4

12 11 22 25 34 64 90.
i j

11 12 22 25 34 64 90.

Q15). Sort the array 64, 25, 12, 22, 11 using selection sort. What is time complexity of selection in the best, worst average cases.

0	1	2	3	4
64	25	12	22	11
start				min

11	25	12	22	64
	start	min		

11	12	25	22	64
		start	min	

11	12	22	25	64

sorted.

Time complexity:

Best case : $O(n^2)$

Avg case : $O(n^2)$

worst case : $O(n^2)$