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1) $V_1 = \{2, -3, 5\}$

$V_2 = \{6, 2, 1\}$

By Euclidean distance formula :

$$d(i,j) = \sqrt{(x_{i1} - x_{j1})^2 + (x_{i2} - x_{j2})^2 \ldots (x_{in} - x_{jn})^2}$$

$V_1 = (2, -3, 5)$

$x_{i1} = 2$

$x_{i2} = -3$

$x_{i3} = 5$

$V_2 = (6, 2, 1)$

$x_{j1} = 6$

$x_{j2} = 2$

$x_{j3} = 1$

$\Rightarrow \sqrt{(2-6)^2 + (-3-2)^2 + (5-1)^2}$

$\Rightarrow \sqrt{(-4)^2 + (-5)^2 + (4)^2}$

$\Rightarrow \sqrt{16 + 25 + 16}$

$\Rightarrow \sqrt{57}$

$\Rightarrow \underline{7.549}$

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2) $x = \{6, -8, 0\}$

In Vector Form,

$x = \sqrt{6\hat{i} + -8\hat{j} + \hat{k}}$

$|x| = \sqrt{(6)^2 + (-8)^2 + 0}$

$= \sqrt{36 + 64}$

$= \sqrt{100}$

$\Rightarrow 10$

$x = \frac{6\hat{i}}{10} + \frac{-8\hat{j}}{10} + \frac{\hat{k}}{10}$

$= \frac{\cancel{6}^{3}}{\cancel{10}_{5}} + \frac{-\cancel{8}^{4}}{\cancel{10}_{5}} + 0$

$x \Rightarrow \left\{\frac{3}{5}, \frac{-4}{5}, 0\right\}$

3)

$$A = \begin{bmatrix} 3 & 4 & 2 \\ 2 & 1 & 5 \\ 6 & 0 & -1 \end{bmatrix}$$

$d(A) = 3(-1-0) - 4(-2-30) + 2(0-6)$

$= 3(-1) - 4(-32) + 2(-6)$

$\Rightarrow -3 + 128 + (-12)$

$\Rightarrow \underline{\underline{113}}$

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4)

$$B = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$$

$$B^{-1} = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$= \frac{1}{\det(B)} \times adj(B)$$

$$= \frac{1}{6-(-1)} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$= \frac{1}{7} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

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$$\Rightarrow \begin{bmatrix} \frac{3}{7} & \frac{1}{7} \\ \frac{-1}{7} & \frac{2}{7} \end{bmatrix}$$

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5) $V_1 = \{1, 2, -1\}$

$V_2 = \{3, -6, 2\}$

$$V_1 \cdot V_2 = (1 \times 3) + (2 \times -6) + (-1 \times 2)$$

$$= 3 + (-12) + (-2)$$

$$= -11$$

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6) $V_1 = \{1, 2\}$

$V_2 = \{3, 4\}$

$$V_1 \cdot V_2 = (1 \times 3) + (2 \times 4)$$

$$= 3 + 8$$

$$= 11$$

$$|V_1| = \sqrt{(1)^2 + (2)^2}$$

$$= \sqrt{1 + 4}$$

$$= \sqrt{5}$$

$$= 2.236$$

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$|V_2| = \sqrt{3^2 + 4^2}$

$= \sqrt{9 + 16}$

$= \sqrt{25}$

$= 5$

To find angle

$\cos\theta = \frac{a \cdot b}{|a| \cdot |b|}$

$= \frac{11}{2.236 \times 5}$

$= \frac{11}{11.18}$

$\cos\theta = 0.983$

$\theta = \cos^{-1}(0.983)$

$= 10.57$

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7) $C = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

equation = $|A - \lambda I| = 0$.

$$\left[\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right] = 0$$

$$= \left[\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\right] = 0$$

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$$\begin{bmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{bmatrix} = 0$$

$(2-\lambda)(2-\lambda) - 1 = 0.$

$(4 - 2\lambda - 2\lambda + \lambda^2) - 1 = 0.$

$4 - 4\lambda + \lambda^2 - 1 = 0.$

$\lambda^2 - 4\lambda + 3 = 0.$

$(\lambda - 3)(\lambda - 1) = 0.$

$\lambda = 3$ and 1

eigen Values $= 3, 1$

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8) probability of red ball

$P(\text{red ball}) = 4/9$

Probability of blue ball

$P(\text{blue ball}) = 3/9$

Probability of green ball

$P(\text{green ball}) = 2/9$

$P(\text{red or blue}) = \frac{\text{fav outcome}}{\text{Total outcome}}$

$= \frac{P(\text{red}) + P(\text{blue})}{9}$

$= \frac{4+3}{9}$

$= 7/9$

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q) Probability of having disease

$P(\text{disease}) = 0.01\ (1\%)$

Probability of no disease

$P(\text{no disease}) = 0.99\ (99\%)$

$P(\text{Total +ve} \mid \text{disease}) = 0.95\ (95\%)$

$P(\text{Test -ve} \mid \text{no disease}) = 1 - 0.95$

$= 0.05$

By Bayes theorem,

$$P(\text{Disease} \mid \text{Test positive}) = \frac{P(\text{Test } P \mid D) \cdot P(\text{Disease})}{P(\text{Test } P)} \quad — ①$$

$P(\text{Test } P) = P(\text{Test Positive} \mid \text{Disease}) \cdot P(\text{Disease}) + P(\text{Test} \mid D') \cdot P(\text{no }D)$

$= (0.95 \times 0.01) + (0.05 \times 0.99)$

$= 0.059$

Substituting the Value in — ①.

$$P(\text{Disease} \mid \text{Test positive}) = \frac{0.95 \times 0.01}{0.059}$$

$$= \frac{0.0095}{0.059}$$

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$\Rightarrow 0.161$

10) Probability of student pass math.

$P(\text{pass math}) = 0.7$

Probability of Student pass physics

$P(\text{pass physics}) = 0.5$

Probability of Student pass maths & physics

$P(\text{pass maths \& physics}) = 0.3$

Conditional probability (~~[illegible]~~)

$$= \frac{P(\text{maths} \cap \text{physics})}{P(\text{math})}$$

$$= \frac{0.3}{0.7}$$

$$= 0.428$$

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11)

$$H(x) = -\Sigma P(x) \log P(x)$$

$P(H) = 0.8$

$P(T) = 0.2$

$$H(x) = -[P(H)\log_2 P(H) + P(T)\log_2 P(T)]$$

$$= -[0.8 \log_2(0.8) + 0.2 \log_2(0.2)]$$

$$= -[0.8 \times (-0.32) + 0.2 \times (-2.321)]$$

$$= -[-0.2568 + -0.4642]$$

$$= \underline{\underline{0.721}}$$

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12) $P(1) = 0.1$

$P(2) = 0.2$

$P(3) = 0.3$

$P(4) = 0.4$

Expected Value $= E(x) = \sum_{i=1}^{4} (x_i)\, P(x_i)$

$$E(x) = (1 \times 0.1) + (2 \times 0.2) + (3 \times 0.3) + (4 \times 0.4)$$

$$= 0.1 + 0.4 + 0.9 + 1.6$$

$E(x) = 3//$

13)

Probability of total outcomes = 36

Two dice = 8

Probability possible.

(3,6), (4,5), (4,6), (5,4), (5,5), (5,6), (6,3), (6,4)
(6,5), (6,6)

Total 10 favourable outcome.

$$P(\text{sum} > 8) = \frac{\text{favourable outcome}}{\text{Total outcomes}}$$

$$= \frac{\cancel{10}\ 5}{\cancel{36}\ 18} = 5/18$$

$$= 0.27$$

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14) To find,

probability of getting 7 heads = ?.

Binomial co-efficient $\binom{n}{k} = \frac{n!}{k!\,(n-k)!}$

n = no of trials = 10
k = no of successful trials = 7

$$\binom{10}{7} = \frac{10!}{7!\,(10-7)!}$$

$$= \frac{10 \times 9 \times 8 \times \cancel{7} \times \cancel{6} \times \cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times \cancel{1}}{\cancel{7} \times \cancel{6} \times \cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times \cancel{1}\ (3)!}$$

$$\Rightarrow \frac{10 \times 9 \times 8}{3 \times 2 \times 1} = \frac{720}{6}$$

$$\Rightarrow \underline{120}$$

P of head = $P(x=6)$

P of tail = $P(x=4)$

P (headers) $= 0.6$

P (tail) $= 0.4$

$$P(x=7) = 120 \times (0.6)^7 \times (0.4)^3$$

$$= 120 \times 0.027 \times 0.064$$

$$= \underline{0.207}$$

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15)

Vector ϕ : $2i + 4j$

$= i + 2j$

$$A = \begin{bmatrix} 2 & 4 \\ 1 & 2 \end{bmatrix}$$

$$\det(A) = (2 \times 2) - (1 \times 4)$$

$$= 4 - 4 = 0 //$$

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det(A) = 0, So the given Vector are linearly independent.

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16)

$a = (1, -2, 3)$

$b = (4, 0, -1)$

$a \cdot b = (1 \times 4) + (-2 \times 0) + (3 \times -1)$

$= 4 + 0 - 3$

$= 1/1$

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17)

Total no of cards = 52

P of getting ace = $4/52$

P of getting heart = $\frac{13}{52}$

P of getting ace or heart = $\frac{1}{52}$

Conditional Prob. = $A \cap B$

$= \frac{4+13-1}{52}$

$= \frac{\cancel{16}\,4}{\cancel{52}\,13} = 4/13$

$= \underline{\underline{0.307}}$

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18)

if the dot product of the vector is zero, then the vectors are said to be orthogonal.

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19) P of getting rain = 0.3

P of umbrella = 0.6

P (rain and umbrella) = 0.2

$$P(\text{rain} \mid \text{umbrella}) = \frac{P(\text{Rain and umbrella})}{P(\text{umbrella})}$$

$$= \frac{0.2}{0.6}$$

$$= \underline{\underline{0.333}}$$

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20)

$$C = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$$

By equation,

$$[A - \lambda I] = 0$$

$$\Rightarrow \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0.$$

$$= \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 2-\lambda & 0 \\ 0 & 3-\lambda \end{bmatrix} = 0$$

$$= (2-\lambda)(3-\lambda) = 0$$

$$= 6 - 2\lambda - 3\lambda + \lambda^2 = 0$$

$$6 - 5\lambda + \lambda^2 = 0$$

$$= (\lambda-2)(\lambda-3) = 0.$$

$$\lambda = 2, 3$$