A Solow-Malthus Approach to Modeling the Pre-Industrial Economy

November 29, 2019

1 The Solow Model

In the standard Solow growth model, total output and income Y is:

1. $Y = (\kappa)^{\theta} EL$

where E is the efficiency of labor, L is the labor force, κ is the capital-output ratio $\kappa = K/L$, and the parameter θ is a transform of the more usual Cobb-Douglas constant-returns-to-scale α , with $\theta = \alpha/(1-\alpha)$. Along this model's steady-state growth path:

1. $Y^* = (\kappa^*)^{\theta} EL$

with κ equal to the quotient of the savings-investment share s and the investment requirements $n + g + \delta$, which are the sum of the labor-force growth rate n, the efficiency-of-labor growth rate g, and the depreciation rate δ : κ. With κ taken as the state variable, for constant s, n, g and δ, this model economy converges exponentially to its steady state according to:

1. $\frac{d\kappa}{dt} = -(1-\alpha)(n+g+\delta)(\kappa - \kappa^*)$

2 Malthusianism: Resource Scarcity, Population, and the Efficiency of Labor

Now let's make the model Malthusian:

1. Let's make efficiency-of-labor growth g a function of the rate h at which economically useful ideas are generated, but also of the rate of population and labor force growth n, because a higher population makes resources per capita scarce, as determined by an effect-of-resource scarcity parameter γ:

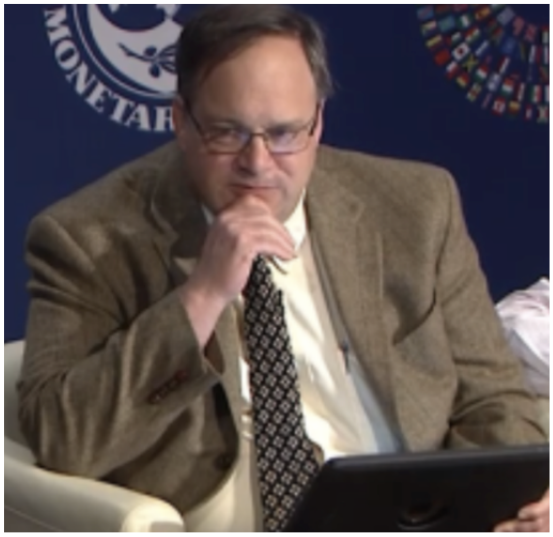
    1. $\frac{1}{E}\frac{dE}{dt} = \frac{d\ln(E)}{dt} = g = h - \frac{n}{\gamma}$

1. Let's make the rate of growth of the population and labor force depend on the level of prosperity $y = Y/L$; on the "subsistence" standard of living for necessities y^{sub}; and also on the fraction $1/\phi$ of production that is devoted to necessities, not conveniences and luxuries, and thus enters into reproductive and survival fitness:

    1. $\frac{1}{L}\frac{dL}{dt} = \frac{d\ln(L)}{dt} = n = \beta\left(\frac{y}{\phi y^{sub}} - 1\right)$

At high levels of income per worker, we get the "demographic transition": population growth then does not speed up but slows down as prsoperity increases, because women learn to read, acquire social power, and begin to use artificial means of birth control for family planning. But neglect that for now.

3 Catch Our Breath—Further Notes:



https://tinyurl.com/20190119a-delong

- Weblog Support https://github.com/braddelong/LS2019/blob/master/2019-09-06-210a-ancient-intro.ipynb
- nbViewer https://nbviewer.jupyter.org/github/braddelong/LS2019/blob/master/2019-09-06-210a-ancient-intro.ipynb