Analytical Questions-1

Name:- S.Neda Anjum
Reg no:- 192311071
Sub code:- CSA0672

Q) Solve the following recurrence relations:

a) $x(n) = x(n-1)+5$ for $n>1$ $x(1)=0$

Given

$$x(n) = x(n-1)+5$$
$$x(1) = 0$$

Sub: $n=2$

$$x(2) = x(2-1)+5$$
$$= x(1)+5 \quad (\text{from } \because x(1)=0)$$
$$= 0+5 \rightarrow (1)$$

Sub: $n=3$

$$x(3) = x(3-1)+5$$
$$= x(2)+5$$
$$= 5+5 \quad (\text{from } (1))$$
$$x(3) = 10 \rightarrow (2)$$

Sub $n=4$

$$x(4) = x(4-1)+5$$
$$= x(3)+5$$
$$= 10+5$$
$$= 15$$

The general for the given equation is $x(n) = x(1)+(n-1)$

In the given equation $d=5$ and $x(1)=0$

$$x(n) = 0+5(n-1)$$
$$x(n) = 5(n-1)$$

$\boxed{x(n) = 5(n-1)}$ is the recurrence relation.

b) $x(n) = 3x(n-1)$ for $n>1$, $x(1)=4$

Given

$$x(n) = 3x(n-1)$$
$$x(1) = 4$$

Sub $n=2$

$x(2) = \cancel{3x(n-1)}$ $\quad x(2) = 3x(n-1)$

$= \cancel{3(2-1)}$ $\quad = 3x(2-1)$

$= \cancel{3(1)}$ $\quad = 3x(1)$

$=$ $\quad = 3 \times 4$

$= 12$

Sub $n=3$

$x(3) = 3x(3-1)$

$= 3x(2)$

$= 3(12)$

$= 36$

Sub $n=4$

$x(4) = 3x(4-1)$

$= 3x(3)$

$= 3(36)$

$= 108$

The general form of the given eq^n is $x(n) = 3^{n-1} \cdot x(1)$

$$\boxed{x(n) = 3^{n-1} \cdot 4}$$

$\therefore x(n) = 3^{n-1} \cdot 4$ is the recurrence relation

c) $x(n) = x(n/2) + n$ for $n > 1$ $x(1) = 1$ (solve for $n = 2k$).

Given, $x(n) = x(n/2) + n$

$x(1) = 1$; $n = 2k$

$x(2k) = x\left(\frac{\cancel{2}k}{\cancel{2}}\right) + 2k$

$x(2k) = xk + \cancel{x}$

Sub $k=1$

$x(2 \cdot 1) = x(1) + 2$

$= \cancel{2} 1 + 2$

$x(2) = 3$

Sub $k=2$

$x(2 \cdot 2) = x(2) + 2 \cdot (2)$

$= x(2) + 4$

$x(4) = 3 + 4$

$= 7$

Sub $k=3$

$x(2 \cdot 3) = x(3) + 2(3)$

$= \cancel{x(3)+}$

$x(3) = x(1.5) + 3$.

The general equation for given expression is

$$\boxed{x(2k) = x(k) + 2k}$$

d) $x(n$

x

d) $x(n) = x(n/3)+1$ for $n>1$ $x(1)=1$ (solve for $n=3k$).

$x(n) = x(n/3)+1$

$n = 3k$

$x(3k) = x\left(\frac{3k}{3}\right)+1$

$x(3k) = x(k)+1$

Sub $\underline{k=1}$

$x(3\cdot1) = x(1)+1$

$= 1+1$

$= 2$

Sub $\underline{k=2}$

$x(3\cdot2) = x(2)+1$

$x(2) = x(2/3)+1$

The general equation for $\boxed{x(3k) = 1+\log_3(k)}$

2) Evaluate the following recurrences completely

(i) $T(n) = T(n/2)+1$, where $n=2k$ for all $k \geq 0$.

Given $n = 2k$; i.e $k = \log n$

$T(2k) = T\left(\frac{2k}{k}\right)+1$

$T(2k) = T(k)+1$

$T(2\cdot k) = T(k/2)+2$ (if k is even)

$T(2\cdot k) = T(k-1/2)+2$ (if k is odd)

$T(2\cdot k) = T(1)+k)$

$\Rightarrow$ Recurrences $\Rightarrow$ $\boxed{T(n) = O(\log n)}$

(ii) $T(n) = T(n/3)+T(2n/3)+cn$, where 'c' is a constant and 'n' is the input size.

$T(n) = aT(n/b) + f(n)$

$a=2,\ b=3,\ f(n)=cn$

Using Masters theorem:

$f(n) = O(n^c)$

where $c < \log_b^a$

$T(n) = \Theta(n \log_b^a)$

$f(n) = \Theta(n \log_b^a)$

Then $T(n) = \Theta(n^{\log_b^a} \log n)$

$f(n) = \Omega(n^c)$

where $c > \log_b^a$, $aT(\frac{n}{b}) \le k f(n)$

for $k < 1$

$T(n) = \Theta(f(n))$

find $\log_b^a \Rightarrow \log_b^a = \log_3^2$

$f(n) = cn = n \log_b^a$

Recurrence relation $\Rightarrow T(n) = \Theta(n)$

3) consider the following recursion algorithm

Min1[A[0.......n-1]]

if n=1 return A[0]

Else temp = Min1[A[0.....n_2]]

if temp <= A[n-1] return temp.

Else

Return A[n-1]

a) What does this algorithm compute?

→ The algorithm computes the minimum element in an array A of size n using a recursive approach.

⇒ If the array has only one element (n== 1), it returns that single element as the minimum

⇒ Recursive case:

* If the array has more than one element (n>1) the function makes a recursive call to find the min element in subarray consisting of the first n-1 elements

* The result of this recursive call ("temp") is then compared to the last element of the current array segment ("A[n-1]").

* The function returns the smaller of these two values.

b) Setup a recurrence relation for the algorithms basic operation count and solve it

```
Min1(A[0....n-1])
  if n=1
     return A[0]
     Else
        temp = Min1[A[0...n-2]) — n-1
  if temp <= A[n-1]
     return temp
  Else
     Return A[n-1]
```

T(n) = No. of basic operations.

if n=1 then T(1) = 0

" $T(n) = T(n-1) + 1$ " is the recurrence relation

$T(1) = 0$

$T(2) = T(2-1) + 1$

$= T(1) + 1$

$= 0 + 1$

$T(2) = 1$

$T(3) = T(3-1) + 1$

$= T(2) + 1$

$= 1 + 1$

$= 2$

$T(4) = T(4-1) + 1$

$= T(3) + 1$

$= 2 + 1$

$= 3$

$T(n) = n - 1$

Time complexity $= O(n)$

4) Analyse the order of growth

(i) $F(n) = 2n^2 + 5$ & $g(n) = 7n$. Use the $\Omega(g(n))$ notat

$f(n) = 2n^2 + 5$

$g(n) = 7n$

if $n = 1 \Rightarrow f(n) = 2(1)^2 + 5$ $\quad g(n) = 7(1)$

$= 7$ $\quad = 7$

$n = 2 \Rightarrow f(n) = 2(2)^2 + 5$ $\quad g(n) = 7(2)$

$= 13$ $\quad = 14$

$n = 3 \Rightarrow f(n) = 2(3)^2 + 5$ $\quad g(n) = 7(3)$

$= 23$ $\quad = 21$

$n = 4 \Rightarrow f(n) = 2(4)^2 + 5$ $\quad g(n) = 7(4)$

$= 2(16) + 5$ $\quad = 28$

$= 37$

$f(n) \geq g(n) \cdot c$ condition satisfies at $n = 1$ onwards

So the $\Omega(7n)$ is the recurrence relation

Time complexity is $\Omega(n)$