Name:- S.Neda Anjum
Reg no:- 192311071
Subcode:- CSA0672

1) calculate the no. of ways to achieve a sum of 15 when rolling four six-sided dice. provide a detailed step-by-step solution.

No. of solutions.

$x_1+x_2+x_3+x_4=15$

This becomes

$y_1+1+y_2+1+y_3+1+y_4+1=15$

$y_1+y_2+y_3+y_4+4=15$

$y_1+y_2+y_3+y_4=15-4$

$y_1+y_2+y_3+y_4=11$

using Inclusion & Exculsion principle.

"stars and bars" $\binom{n+k-1}{k-1}$

$\binom{11+4-1}{4-1}=\binom{14}{3}$

$\binom{14}{3}=\frac{14\times13\times\overset{\cancel{6}\,2}{\cancel{12}}}{\cancel{3}\times\cancel{2}\times1}=364$

Set $y_1' = y_1-6$ then

$y_1'+y_2+y_3+y_4=5$

$\binom{5+4-1}{4-1}=\binom{8}{3}$

$\cancel{8}$ $\binom{8}{3}=\frac{8\times7\times\overset{\cancel{3}}{\cancel{6}}}{\cancel{3}\times\cancel{2}\times1}=56$

since any of four variables $=4\times56=224$.

set $y_1'=y_1-6$ and $y_2'=y_2-6$ then

$y_1'+y_2'+y_3+y_4 \cancel{=} -1$ $\boxed{y_1'+y_2'+y_3+y_4=-1}$

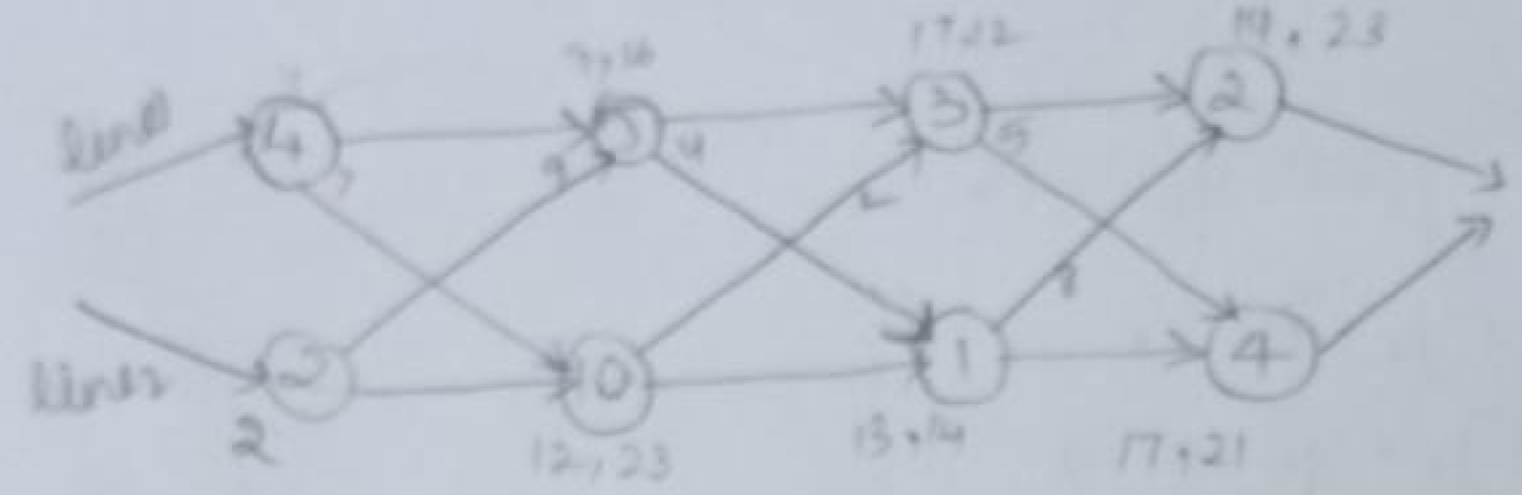
$y_1-6+y_2-6+y_3+y_4$

The no. of valid solutions are: $364-224=140$

③ Two assembly lines have station times as follows line1 [4,5,3,2] line2 [7,4,5] line2 : [2,10,1,4] Transfer times b/w lines are line1 to line2 [7,4,5] from line2 to line1 : [9,2,8] . calculate the minimum time to assemble a product

Given lines are

line1 : [4,5,3,2], line2 : [2,10,1,4]



	1	2	3	4
F1[J]	4	9	12	14
F2[J]	2	12	13	17

	1	2	3	4
l1	1	1 ←	1 ←	1 ←
l2	2	2	2	2

3) Given keys {10, 20, 30, 40} with access probabilities {0.1, 0.2, 0.4, 0.3} respectively. construct the optimal binary search tree. Calculate the total cost of the tree

Given keys {10, 20, 30, 40}

probabilities {0.1, 0.2, 0.4, 0.3}

j-i=0
1-1=0
2-2=0
3-3=0
4-4=0

j-i=1
1-0=1
2-1=1
3-2=1
4-3=1

j-i=2
2-0=2 (0,2)
3-1=2 (1,3)
4-2=2 (2,4)

j-i=3
3-0=3
4-1=3

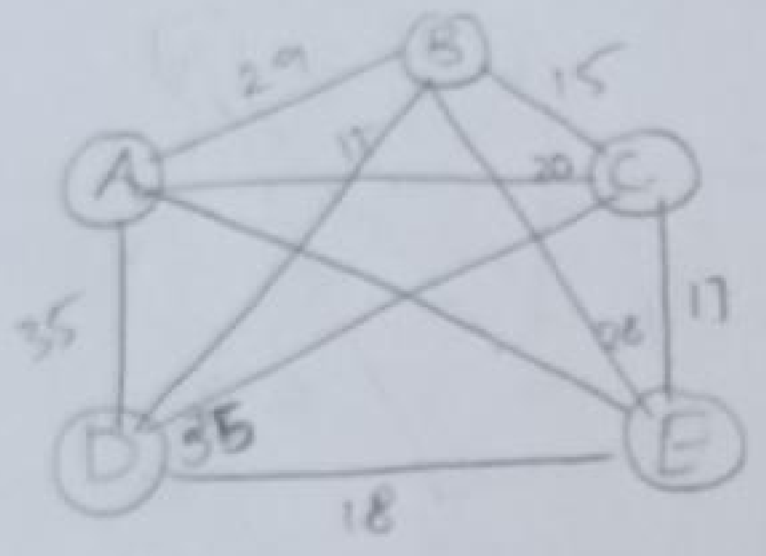
j-i=4
4-0=4

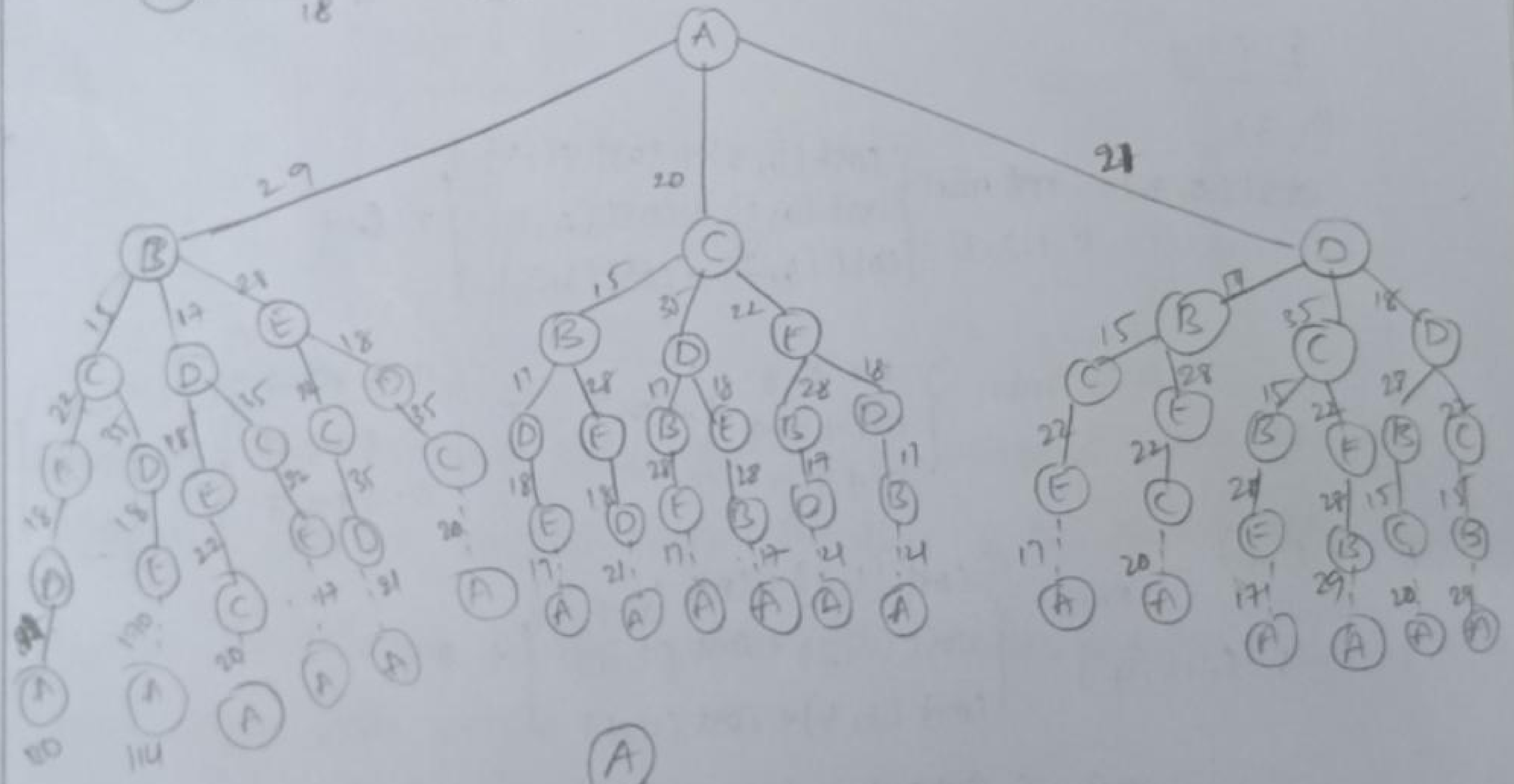
i\j	0	1	2	3	4
0	0	0.1	0.4 [2]	1.1 [3]	1.7 [3]
1		0	0.2	0.8 [3]	1.0 [2]
2			0	0.4	1.0 [3]
3				0	0.3
4					0

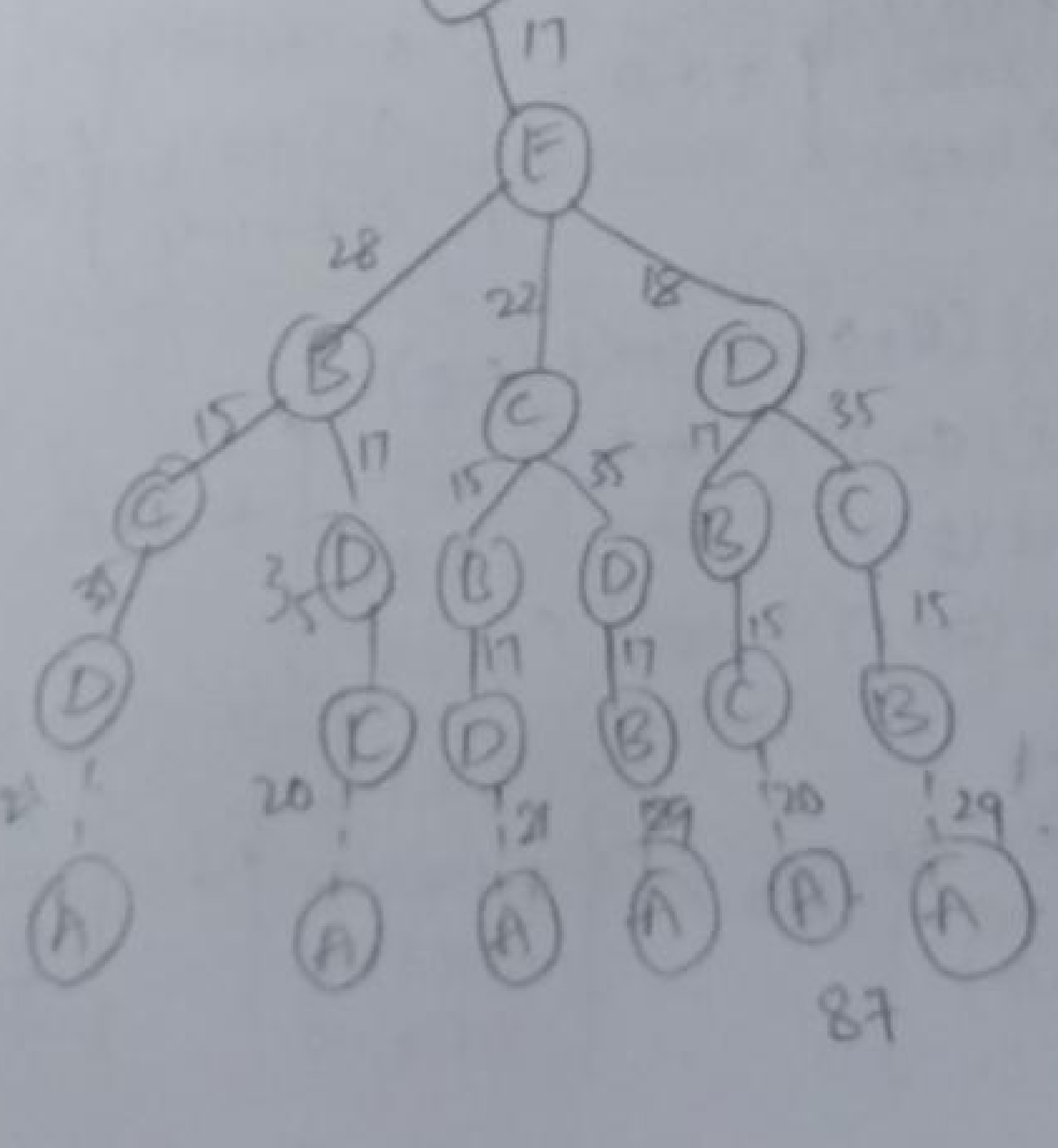
(4) Solve the TSP for the following 5-city distance matrix using dynamic programming.

A : [0, 29, 20, 21, 17]
B : [29, 0, 15, 17, 28]
C : [20, 15, 0, 35, 22]
D : [21, 17, 35, 0, 18]
E : [17, 28, 22, 18, 0].

$$\begin{array}{c|ccccc} & A & B & C & D & E \\ A & 0 & 29 & 20 & 21 & 17 \\ B & 29 & 0 & 15 & 17 & 28 \\ C & 20 & 15 & 0 & 35 & 22 \\ D & 21 & 17 & 35 & 0 & 18 \\ E & 17 & 28 & 22 & 18 & 0 \end{array}$$





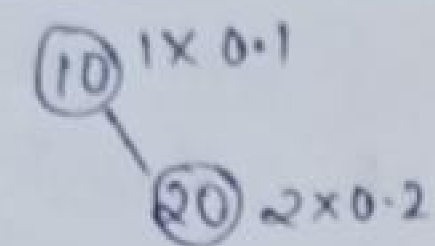


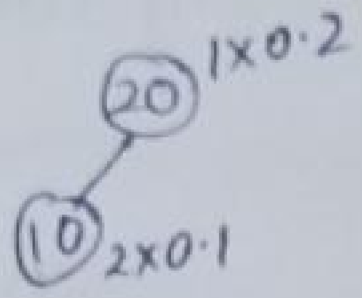
minimum cost

A — E — D — B — C — A

87

cost (0,2) ②

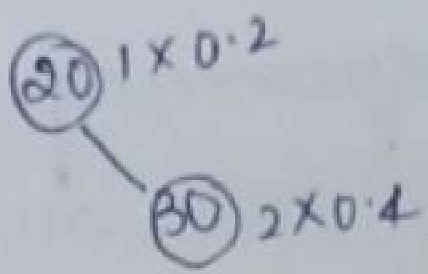


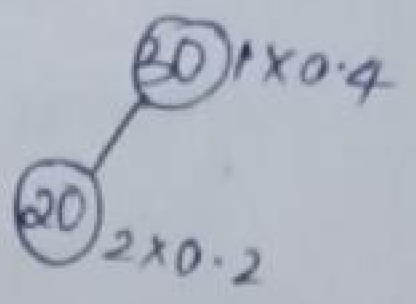


$0.1+0.4$
$=0.5$

$0.2+0.2$
$\underbrace{0.4}_{\text{min}}$

cost (1,3)

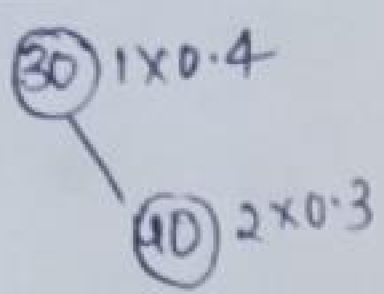


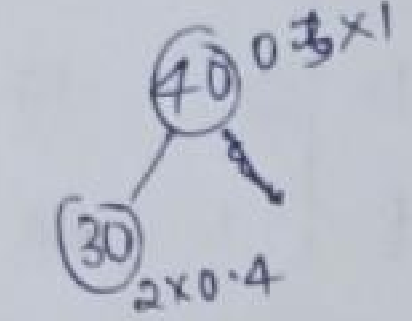


$0.2+0.8$
1.0

$0.4+0.4$
$\underbrace{0.8}_{\text{min}}$

cost(2,4)





$0.4+0.6$
$\underbrace{1.0}_{\text{min}}$

$0.8+0.3$
1.1

j - i = 3

(0, 3)

$$cost(0,3) = \min_{K=1,2,3} \begin{cases} cost(0,0)+cost(1,3) \\ cost(0,1)+cost(2,3) \\ cost(0,2)+cost(3,3) \end{cases} + 0.7$$

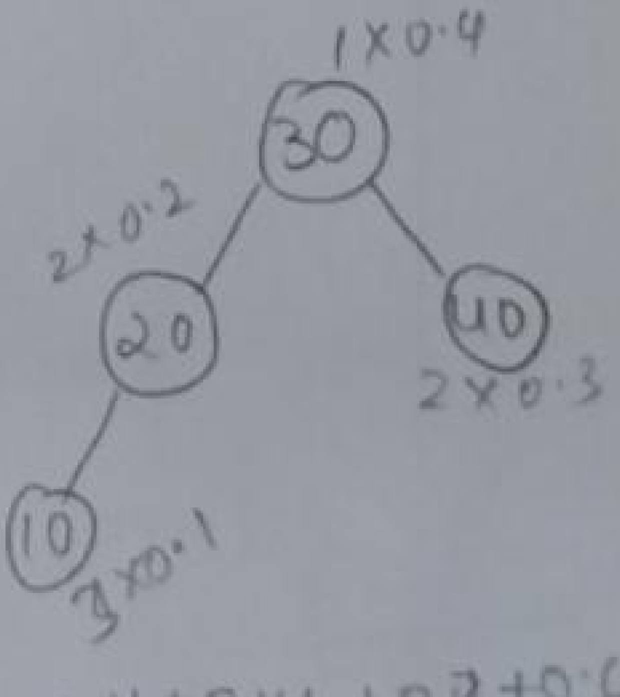
$$= \min \begin{cases} 0+0.8 \\ 0.1+0.4 \\ 0.4+0 \end{cases} + 0.7 = \begin{cases} 0.8+0.7 \\ 0.5+0.7 \\ 0.4+0.7 \end{cases} = \begin{cases} 1.5 \\ 1.2 \\ \underline{1.1} \end{cases}$$

$$(1,4) = \min_{K=(2,3,4)} \begin{cases} cost(1,1)+cost(2,4) \\ cost(1,2)+cost(3,4) \\ cost(1,3)+cost(4,4) \end{cases} + 0.9$$

$$= \min \begin{cases} 0+0.1 \\ 0.2+0.3 \\ 0.8+0 \end{cases} + 0.9 = \begin{cases} 0.1+0.9 \\ 0.5+0.9 \\ 0.8+0.9 \end{cases} = \begin{cases} \underline{1.0} \\ 1.4 \\ 1.7 \end{cases}$$

$$(0,4) = \min_{K=1,2,3,4} \begin{cases} cost(0,0)+cost(1,4) \\ cost(0,1)+cost(2,4) \\ cost(0,2)+cost(3,4) \\ cost(0,3)+cost(4,4) \end{cases} + 1.0$$

$$= \min \begin{cases} \text{[illegible]} \\ 0+1.0 \\ 0.1+1.0 \\ 0.4+0.3 \\ 1.1+0 \end{cases} + 1.0 = \begin{cases} 1.0+1.0 \\ 0.1+1.0 \\ 0.7+1.0 \\ 1.1+1.0 \end{cases} = \begin{cases} 2.0 \\ 2.1 \\ \underline{1.7} \\ 2.1 \end{cases}$$



$0.4+0.4+0.3+0.6$
1.7

⑤ you have a knapsack with a capacity 50 units. There are 4 items with the following weights & values

item1: wei = 10, Val = 60
item2: wei = 20, val = 100
item3: wei = 30, val = 120
item4: wei = 40, val = 200

item	weight	value
1	10	60
2	20	100
3	30	120
4	40	200

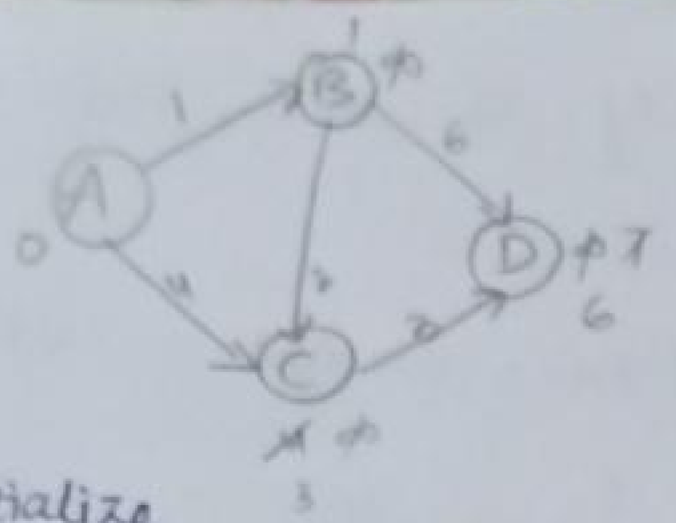
w/v	0	10	20	30	40	50
0	0	0	0	0	0	0
1	0	60	60	60	60	60
2	0	60	100	160	160	160
3	0	60	100	120	180	220
4	0	60	100	120	200	260

⑥ Determine the maximum value that can be obtained using the 0/1 knapsack problem approach show the steps & the final sol^n.

⑥ Given the following directed graph with vertices, A, B, C, D, A, B, C, D, A, B, C, D and edges with weights:

A → BC | rightarrow BA → B with weight 1
A → CA | right arrow CA → C with weight 4
B → CB | rightarrow CB → C with weight 2
B → DB | " DB → C " " 6
C → DC | " DC → D " " 3

Use the Bellman-ford algorithm to find the shortest path from vertex AAA to all other vertices show the steps & the final distances



A→B — 1
A→C — 4
B→C — 2
B→D — 6
C→D — 3

Initialize

V	A	B	C	D
d	0	∞	∞	∞
P	-	-	-	-

①

V	A	B	C	D
d	0	1	4	∞
P	-	A	A	B

②

V	A	B	C	D
d	0	1	3	7
P	-	A	B	B

④

V	A	B	C	P
d	0	1	3	6
P	-	A	B	C

path	shortest dist	Shortest path
A-B	1	A-B
A-C	3	A-B-C
A-D	6	A-B-C-D

o/p A→B→C→D

⑦ Determine the probability of rolling five dice such that the sum is exactly 20. Include a combinational approach to arrive at the solution.

$6^n = 6^5 = 7776$

$x_1 + x_2 + x_3 + x_4 + x_5 = 20$ where $1 \le x_i \le 6$

$y_i = x_i - 1$ for $i = 1, 2, 3, 4, 5$.

$y_1 + 1 + y_2 + 1 + y_3 + 1 + y_4 + 1 + y_5 + 1 = 20$

$y_1 + y_2 + y_3 + y_4 + y_5 + 5 = 20$

$y_1 + y_2 + y_3 + y_4 + y_5 = 15$

where $0 \le y_i \le 5$

By "stars & bars"

$\binom{15+5-1}{5-1} = \binom{19}{4}$

$$\binom{19}{4} = \frac{19\times18\times17\times16}{4\times3\times2\times1} = 3876$$

If $y_1 \geq 6$, let $y_1' = y_1 - 6$

$$y_1' + y_2 + y_3 + y_4 + y_5 = 9$$

$$\binom{9+5-1}{5-1} = \binom{13}{4} \Rightarrow \frac{13\times\overset{3}{\cancel{12}}\times11\times\overset{5}{\cancel{10}}}{\cancel{4}\times\cancel{3}\times\cancel{2}\times1} = 715$$

$$5\times713 = 3575$$

If two variables $y_i, y_j \geq 6$ let $y_1' = y_1 - 6$ & $y_2' = y_2 - 6$

$$y_1' + y_2' + y_3 + y_4 + y_5 = 3$$

$$\binom{3+5-1}{5-1} = \binom{7}{4}$$

$$\binom{7}{4} = \frac{7\times\overset{2}{\cancel{6}}\times5\times\cancel{4}}{\cancel{4}\times\cancel{3}\times\cancel{2}\times1} = 35$$

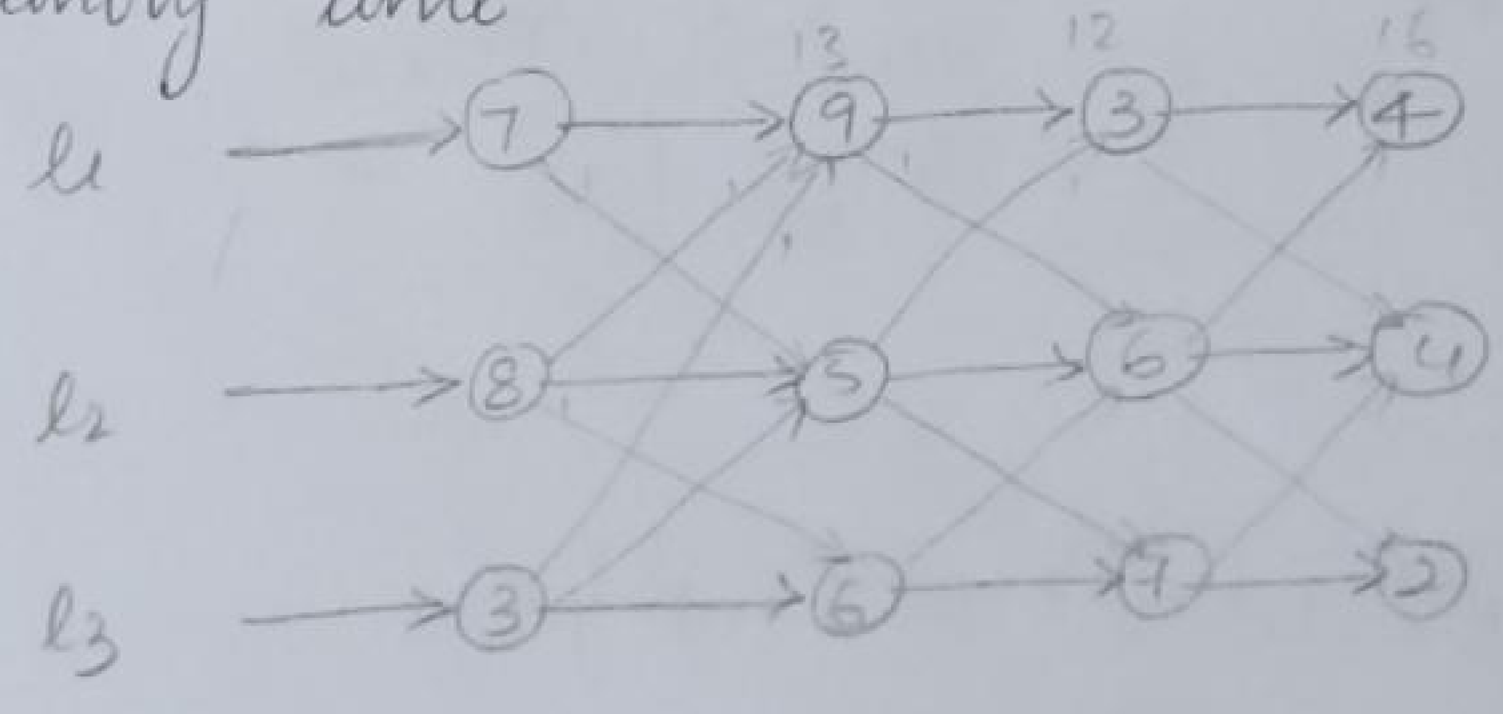
$$\binom{5}{2}\times35 = 10\times35 = 350$$

using inclusion-Exculsion principle

$$3876 - 3575 + 350 = 651$$

$$\frac{651}{7776} = \frac{651}{7776} \approx 0.0837.$$

⑧ For three assembly lines with station times line 1: [7, 9, 3, 4], line 2 [8, 5, 6, 4], line 3: [3, 6, 7, 2] and transfer time b/w lines given; determine the optimal Scheduling and the total minimum assembly time



$f_1[j] = \min\{f_1(j-1)+a_{1j}), (f_2(j-1)+(t_2, j-1)+a_{1j}), \{f_3(j-1)+(t_3, j-1)+a_{1j})\}\}$

$= \min\{(7+9), (8+1+9), (3+1+9)\} = 13.$

	1	2	3	4
$f_1[i]$	7	13	12	16
$f_2[i]$	8	9	15	17
$f_3[j]$	3	9	16	15

	1	2	3	4
$L_1[j]$	1	3	1	1
$L_2[i]$	2	3	2	1
$L_3[j]$	3	3	3	1

⑨ Consider keys {15, 25, 35, 45, 55} | {0.05, 0.15, 0.4, 0.25, 0.15} determine the structure of the optimal binary search tree & compute the Expected cost

Given {15, 25, 35, 45, 55}

{0.05, 0.15, 0.4, 0.25, 0.15}

	0	1	2	3	4	5
0	0	0.05	0.25[2]	0.85[3]	1.35[3]	1.80[3]
1		0	0.15	0.70[3]	1.20[3]	1.80[4]
2			0	0.4	0.90[3]	1.35[3,4]
3				0	0.25	0.55[4]
4					0	0.15
5						0

$j-i=0$

$1-1=0$

$2-2=0$

$3-3=0$

$4-4=0$

$j-i=1$

$1-0=1$

$2-1=1$

$3-2=1$

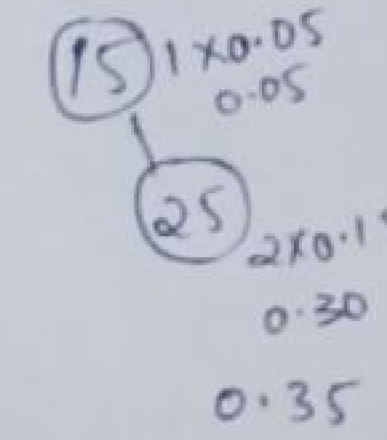
$4-3=1$

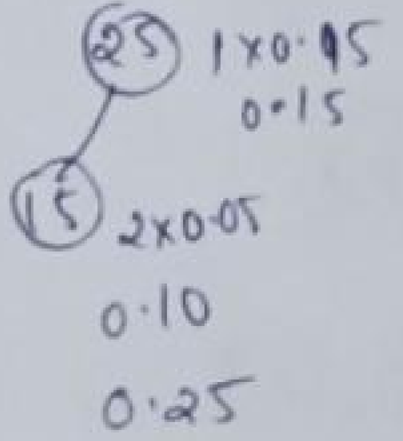
$j-i=2$

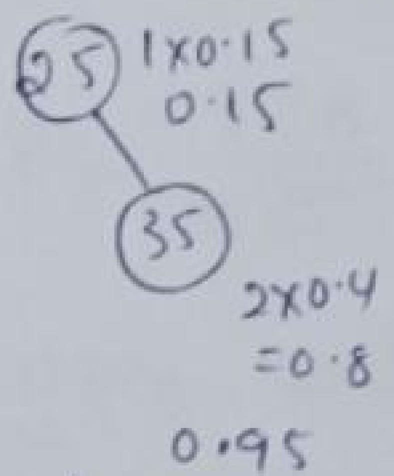
$2-0=2$ (0, 2)

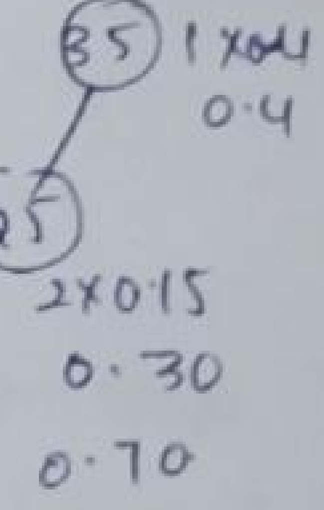
$3-1=2$ (1, 3)

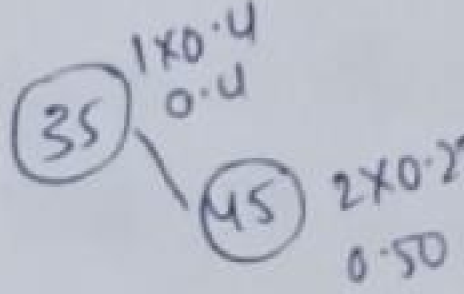
$4-2=2$ (2, 4)

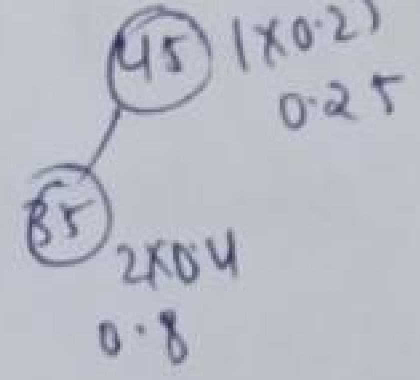


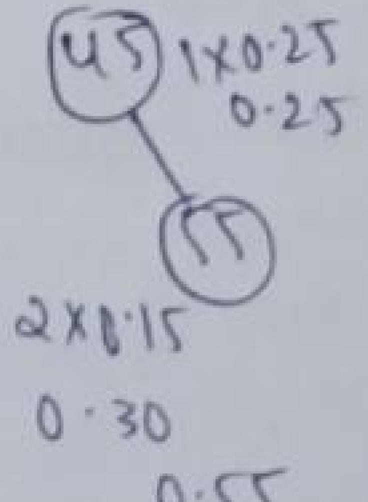


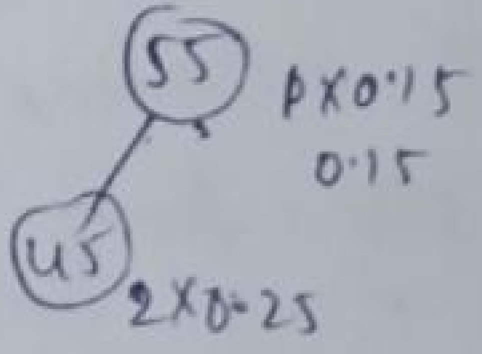












$j-i=3$

$3-0=3$

$4-1=3$

$5-2=3$

$cost(i,j) = \min\{cost(i,k-1) + cost(k,j)\} + w_i$

$$\text{cost}(0,3) = \underset{k=1,2,3}{\min} \left\{ \begin{matrix} \text{cost}(0,0) + \text{cost}(1,3) \\ \text{cost}(0,1) + \text{cost}(2,3) \\ \text{cost}(0,3) + \text{cost}(3,3) \end{matrix} \right\} + 0.6$$

$$\min \left\{ \begin{matrix} 0 + 0.70 \\ 0.05 + 0.4 \\ 0.25 + 0 \end{matrix} \right\} + 0.6$$

$$\min \left\{ \begin{matrix} 1.30 \\ 1.05 \\ 0.85 \end{matrix} \right\} = \underline{0.85}$$

$$\text{cost}(1,4) = \underset{K=2,3,4}{\min} \left\{ \begin{matrix} \text{cost}(1,1) + \text{cost}(2,4) \\ \text{cost}(1,2) + \text{cost}(3,4) \\ \text{cost}(1,3) + \text{cost}(4,4) \end{matrix} \right\} + 0.8$$

$$\min \left\{ \begin{matrix} 1.70 \\ 1.20 \\ 1.50 \end{matrix} \right\} + 1.20$$

$$\text{cost}(2,5) = \min \left\{ \begin{matrix} \text{cost}(2,2) + \text{cost}(3,5) \\ \text{cost}(2,3) + \text{cost}(4,5) \\ \text{cost}(2,4) + \text{cost}(5,5) \end{matrix} \right\} + 0.8$$

$$= \min \left\{ \begin{matrix} 1.35 \\ 1.35 \\ 1.70 \end{matrix} \right\} = 1.35$$

$$\text{cost}(0,4) = \overset{c}{\min} \left\{ \begin{matrix} \text{cost}(0,0) + \text{cost}(1,4) \\ \text{cost}(0,1) + \text{cost}(2,4) \\ \text{cost}(0,2) + \text{cost}(3,4) \\ \text{cost}(0,3 + \text{cost}(4,4) \end{matrix} \right\} + 0.85$$

$$= \left\{ \begin{matrix} 2.05 \\ 1.80 \\ 1.35 \\ 1.7 \end{matrix} \right\} = 1.35$$

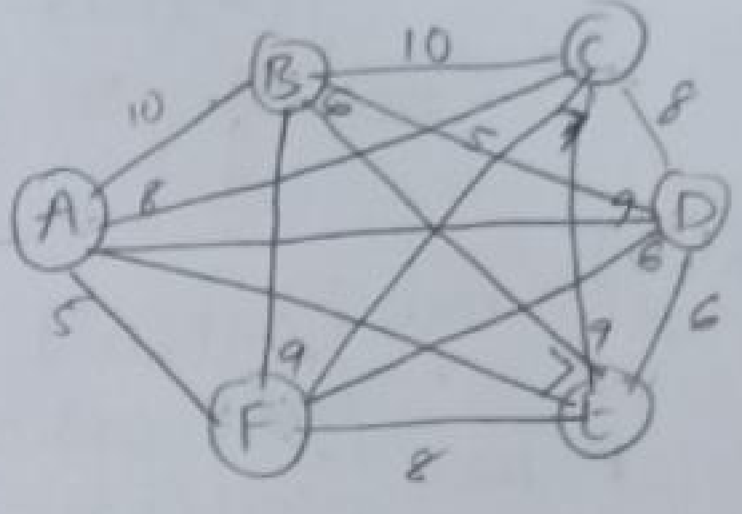
$$\text{cost}(1,5) = \underset{k=2,3,4,5}{\min} \left\{ \begin{matrix} \text{cost}(1,1) + \text{cost}(2,5) \\ \text{cost}(1,2) + \text{cost}(3,5) \\ \text{cost}(1,3) + \text{cost}(4,5) \\ \text{cost}(1,4) + \text{cost}(5,5) \end{matrix} \right\} + 0.95$$

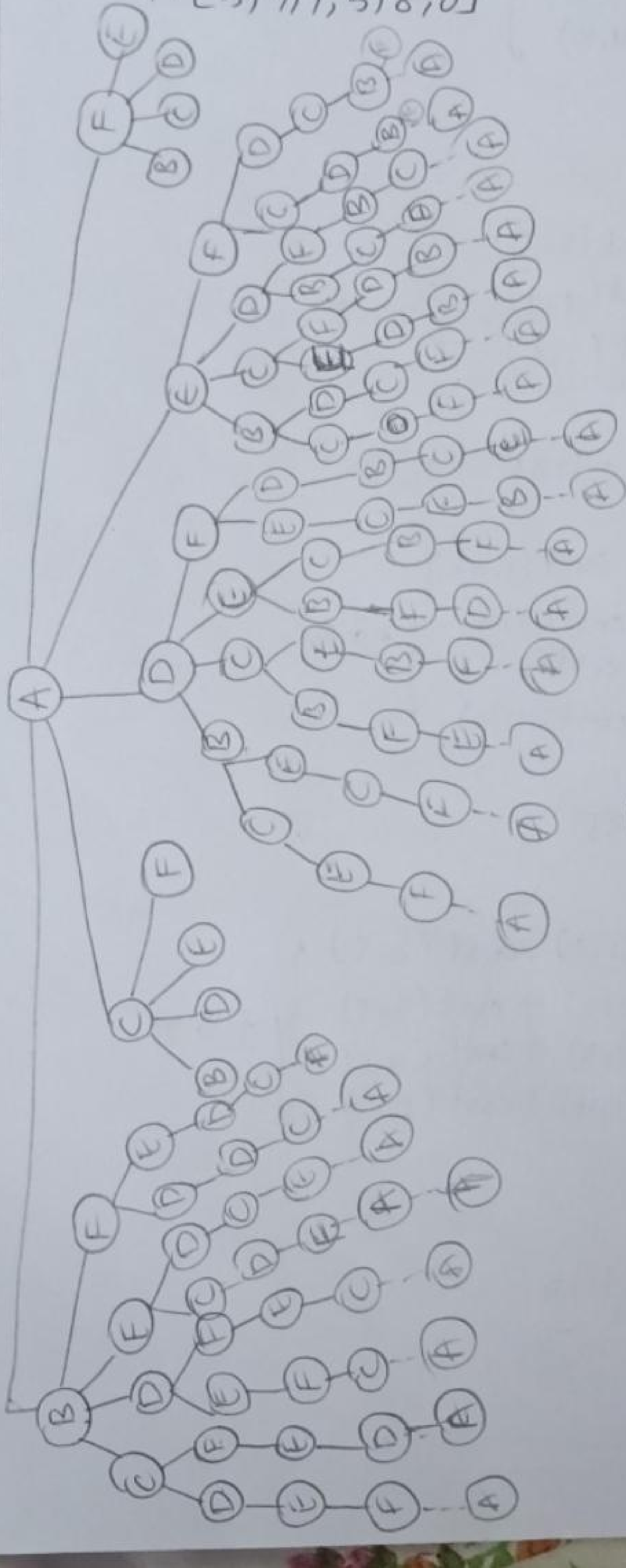
$$\left\{ \begin{matrix} 2.30 \\ 2.45 \\ 1.80 \\ 2.15 \end{matrix} \right\} = \underline{1.80}$$

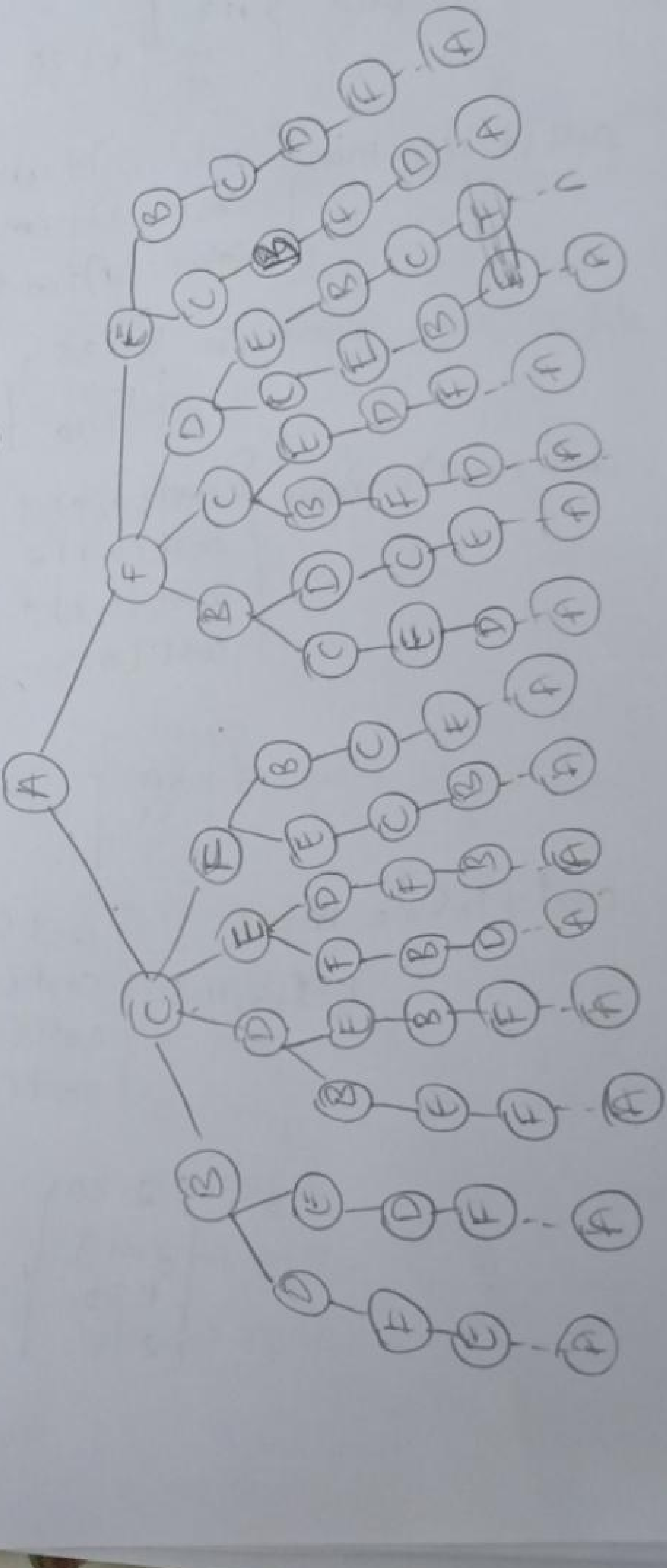
olve the fractional knapsack - ..

⑩ Given a distance matrix for 6 cities, find the shortest path using the nearest neighbours heuristic

	A	B	C	D	E	F
A:	0	10	8	9	7	5
B:	10	0	10	5	6	9
C:	8	10	0	8	9	7
D:	9	5	8	0	6	5
E:	7	6	9	6	0	8
F:	5	9	7	5	8	0







⑪ Solve the fractional knapsack problem for a knapsack with a capacity of 60 units & the following items

I_1 w=20 v=100

I_2 w=30 v=120

I_3 w=10 v=60.

Calculate the maximum value that can be achieved & describe the fractions of items taken.

w / v	0	10	20	30	40	50	60
0	0	0	0	0	0	0	0
1	0	0	100	100	100	100	100
2	0	0	100	120	120	220	220
3	0	10	100	120	180	220	280

Item	w	val
1	20	100
2	30	120
3	10	60

⑫ Consider a directed graph with 5 vertices v_1, v_2, v_3, v_4, v_5 $V1, v_2, v_3, v_4, v_5$ v_1, v_2, v_3, v_4, v_5 & the following edges with weights

$v_1 \to v_2 v_1$ | rightarrow $v_2 v_1 \to v_2$ with w- 3

$v_1 \to v_3 v_1$ w-8

$v_2 \to v_3 v_2$ w-2

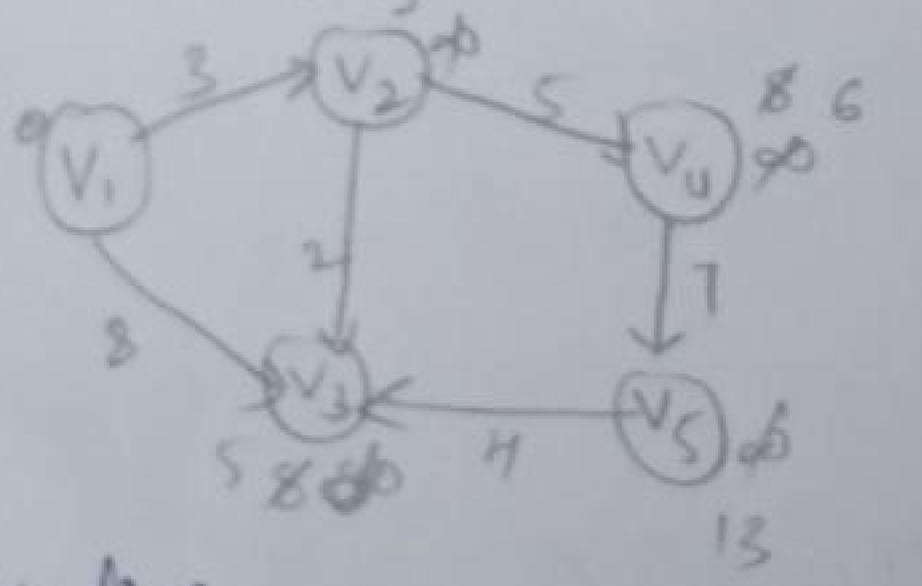
$v_2 \to v_4 v_2$ w-5

$v_3 \to v_4 v_3$ w-1

$v_4 \to v_5 v_4$ w-7

$v_5 \to v_3 v_{5_3}$ w-4

Apply Bellman ford algorithm.



$v_1 \to v_2$ 3

$v_1 \to v_3$ -8

$v_2 \to v_3$ 2

$v_3 \to v_4$ 1

$v_2 \to v_4$ 5

$v_4 \to v_5$ 7

$v_5 \to v_3$ 4

Initialise

v	v_1	v_2	v_3	v_4	v_5
d	0	∞	∞	∞	∞
P	-	-	-	-	-

①

v	v_1	v_2	v_3	v_4	v_5
d	0	3	8	∞	∞
P	-	v_1	v_1	v_2	v_4

②

v	v_1	v_2	v_3	v_4	v_5
d	0	3	5	8	∞
P	-	v_1	v_2	v_2	v_4

③

V	v_1	v_2	v_3	v_4	v_5
d	0	3	5	6	5
P	-	v_1	v_2	v_3	v_4

④

V	v_1	v_2	v_3	v_4	v_5
d	0	3	5	6	13
P	-	v_1	v_2	v_3	v_4

path	distance	path.
v_1-v_2	3	v_1-v_2
v_1-v_3	5	$v_1-v_2-v_3$
v_1-v_4	6	$v_1-v_2 v_3-v_4$
v_1-v_5	13	$v_1-v_2-v_3-v_4-v_5$

o/p $v_1 \to v_2 \to v_3 \to v_4 \to v_5$

⑬ Given two eight-sided dice, compute the number of ways to acheive a sum of 10. then Extend this to three dice & find the new no. of ways to get the same sum.

we need to count the pairs (x,y) such that

$x+y=10$ where $1 \leq x ; y \leq 8$

$(x,y) = (2,8)$

$(x,y) = (3,7)$

$(x,y) = (4,6)$

$(x,y) = (5,5)$

$(x,y) = (6,4)$

$(x,y) = (7,3)$

$(x,y) = (8,2)$

1. $x=1$

$y+2=9$: $(1,1,8),(1,2,7),(1,3,6),(1,4,5),(1,5,4),(1,6,3)$

2. $x=2$

$y+2=8$: $(2,1,7),(2,2,6),(2,3,5),(2,4,4),(2,5,3),(2,6,2),$
$(2,7,1)$

3. $x=3$

$y+2=7$: $(3,1,6),(3,2,5),(3,3,4),(3,4,3),(3,5,2),(3,6,1)$

4. x = 4 :
y+z = 6 (4,1,5), (4,2,4), (4,3,3), (4,4,2), (4,5,1)

5. x = 5
y+z = 5 (5,1,4), (5,2,3), (5,3,2), (5,4,1)..

6. x = 6
y+z = 4 (6,1,3), (6,2,2), (6,3,1)

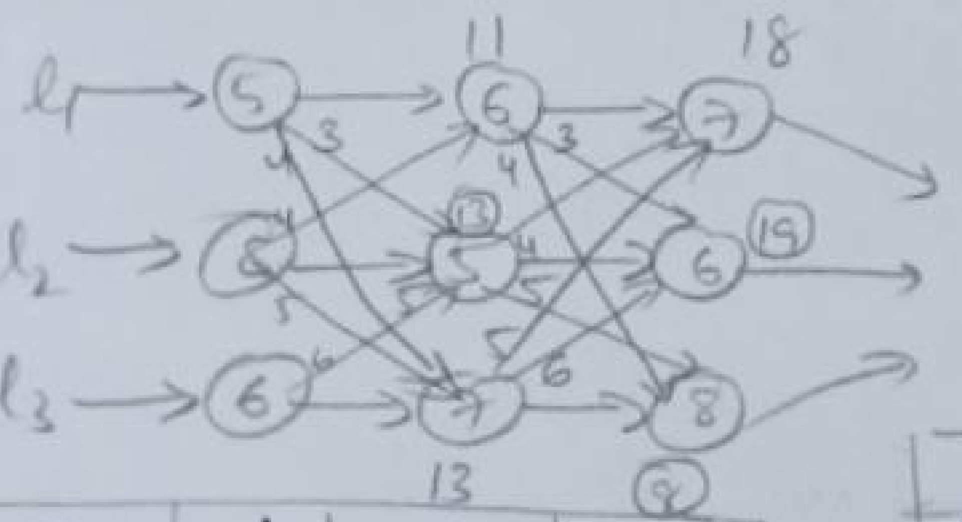
7. x = 7
y+z = 3 : (7,1,2), (7,2,1)

8. x = 8
y+z = 8 : (8,1,1).

Sum = 8+7+6+5+4+3+2+1 = 36

The no. of ways to a sum of 10 = 36.

14) Given Statione times for l_1 [5,6,9], l_2 [8,5,6] & l_3 [6,7,8] & transfer lines b/w lines [3,4], [4,5] & [5,6], calculate the minimum time required to complete the product assembly.



	1	2	3
F1[j]	5	11	18
F2[j]	8	13	19
F3[j]	6	13	21

	1	2	3
$l_1[j]$	1 ←	1 ←	1 ←
$l_2[j]$	2	2	2
$l_3[j]$	3	3	3

$F_1[j] = \min\{(f_1(j-1)+a_1j), (f_2(j-1)+(t_2, j-1)+a_1j), (f_3(j-1)+(t_3, j-1+a_1j)\}$

min {11, 18, 17} = 11

⑮ Given keys {05, 15, 25, 35, 45, 55} with access probabilities {0.1, 0.05, 0.2, 0.25, 0.3, 0.1} use dp to find OBST. show the steps to your calculation & the resulting cost

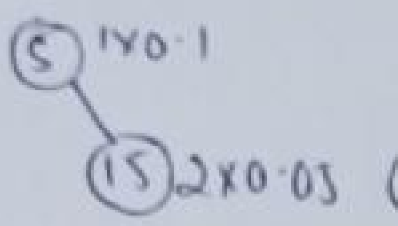
Given keys $\{\overset{1}{5},\overset{2}{15},\overset{3}{25},\overset{4}{35},\overset{5}{45},\overset{6}{55}\}$

$\{0.1, 0.05\ 0.2, 0.25, 0.3, 0.1\}$

j - i = 1	j - i = 2	j - i = 3	j - i = 4
1-0=1	2-0=2	3-0=3	4-0=4
2-1=1	3-1=2	4-1=3	5-1=4
3-2=1	4-2=2	5-2=3	6-2=4
4-3-1	5-3=2	6-3=3	
5-4=1	6-4=2		
6-5=1			

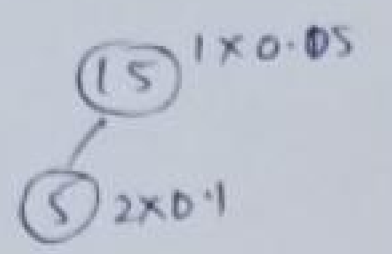
– 1.15 |

	0	1	2	3	4	5	6
0	0	0.1	0.2$^{[1]}$	0.55$^{[3]}$	1.05$^{[3]}$	1.75$^{[3]}$	2.05$^{[4]}$
1		0	0.05	0.3$^{[2]}$	0.8$^{[3]}$	1.4$^{[4]}$	1.7$^{[4]}$
2			0	0.2	0.65$^{[4]}$	1.25$^{[4]}$	1.55$^{[4]}$
3				0	0.25	0.8$^{[5]}$	1$^{[5]}$
4					0	0.8	0.5$^{[5]}$
5						0	0.1
6							0



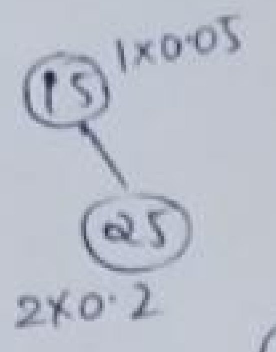
0.1+0.1

0.2



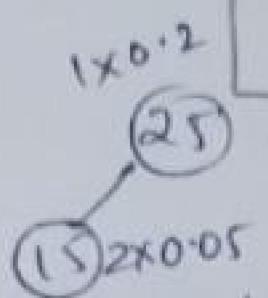
0.2+0.05

0.25



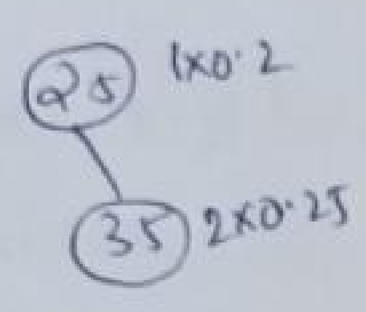
0.4+0.05

0.45

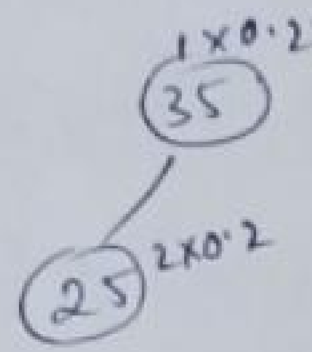


0.2+0.1

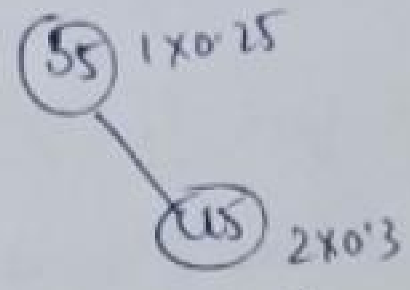
0.3



0.5

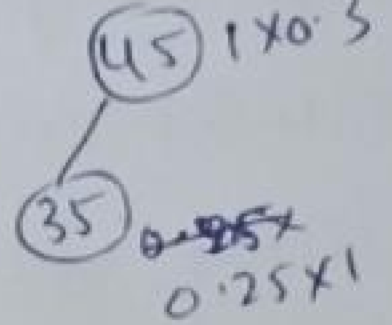


0.65

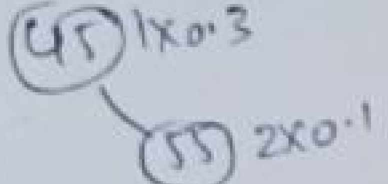


0.25+0.6

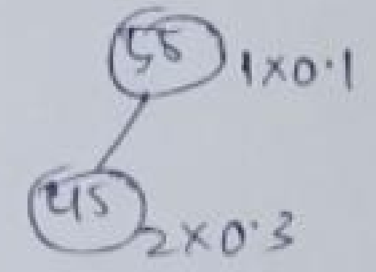
0.85



0.8



0.5



0.7

$$cost(i,j) = min\{cost(i,k-1) + cost(k,j)\} + wi$$

$$cost(0,3) = min\begin{Bmatrix} cost(0,0) + cost(1,3) \\ cost(0,1) + cost(2,3) \\ cost(0,2) + cost(3,3) \end{Bmatrix} + 0.35 = \begin{Bmatrix} 0+0.3 \\ 0.1+0.65 \\ 0.2+0 \end{Bmatrix} + 0.35$$

$$= \begin{Bmatrix} 0.65 \\ 1.1 \\ 0.55 \end{Bmatrix} = 0.55$$

$$cost(1,4) = min\begin{Bmatrix} cost(1,1) + cost(2,4) \\ cost(1,2) + cost(3,4) \\ cost(1,3) + cost(4,4) \end{Bmatrix} + 0.5$$

$$= \begin{Bmatrix} 0+0.65 \\ 0.05+0.25 \\ 0.3+0 \end{Bmatrix} + 0.5 = \begin{Bmatrix} 1.15 \\ 0.8 \\ 0.8 \end{Bmatrix}$$

$$cost(2,5) = min\begin{Bmatrix} cost(2,2) + cost(3,5) \\ cost(2,3) + cost(4,5) \\ cost(2,4) + cost(5,5) \end{Bmatrix} + 0.75 = \begin{Bmatrix} 0+0.8 \\ 0.2+0.3 \\ 0.65+0 \end{Bmatrix} + 0.75$$

$$= \begin{Bmatrix} 1.55 \\ 1.25 \\ 1.4 \end{Bmatrix} = 1.25$$

$$cost(3,\bar{6}) = \min \begin{cases} cost(3,3)+cost(4,6) \\ cost(3,4)+cost(5,6) \\ cost(3,5)+cost(6,6) \end{cases} + 0.65 = \begin{cases} 1.15 \\ 1 \\ 1.45 \end{cases} = 1$$

$$cost(0,4) = \begin{cases} 0+0.8 \\ 0.1+0.65 \\ 0.2+0.25 \\ 0.55+0 \end{cases} + 0.6 = \begin{cases} 1.4 \\ 1.35 \\ 1.05 \\ 1.15 \end{cases} = 1.05$$

$$cost(1,5) = \min \begin{cases} cost(1,1)+cost(2,5) \\ cost(1,2)+cost(3,5) \\ cost(1,3)+cost(4,5) \\ cost(1,4)+cost(5,5) \end{cases} + 0.8 = \begin{cases} 2.05 \\ 1.65 \\ 1.4 \\ 1.6 \end{cases} = 1.4$$

$\underline{j-i=5}$

5-0=5 (0,5)

6-1=5 (1,6)

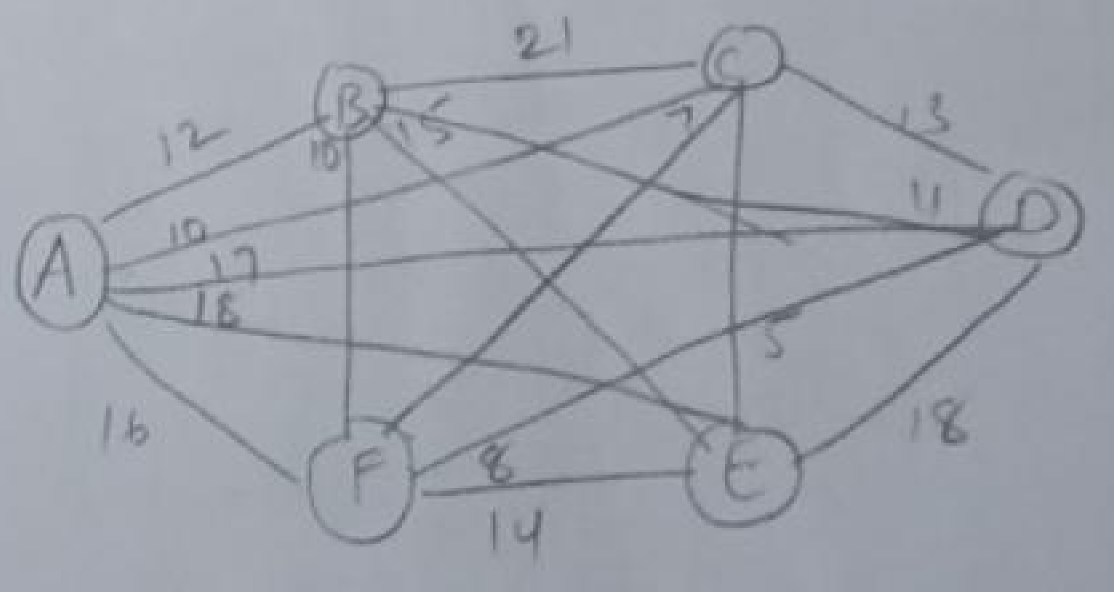
$$cost(0,5) = \min \begin{cases} cost(0,0)+cost(1,5) \\ cost(0,1)+cost(2,5) \\ cost(0,2)+cost(3,5) \\ cost(0,3)+cost(4,5) \\ cost(0,4)+cost(5,5) \end{cases} + 0.9 = \begin{cases} 2.3 \\ 2.25 \\ 1.9 \\ 1.75 \\ 1.95 \end{cases} = 1.75$$

$$cost(0,6) = \min \begin{cases} cost(0,0)+cost(1,6) \\ cost(0,1)+cost(2,6) \\ cost(0,2)+cost(3,6) \\ cost(0,3)+cost(4,6) \\ cost(0,4)+cost(5,6) \\ cost(0,5)+cost(6,6) \end{cases} + 1$$

$$= \begin{cases} 2.7 \\ 2.65 \\ 2.2 \\ 2.05 \\ 2.15 \\ 2.75 \end{cases} = 2.05$$

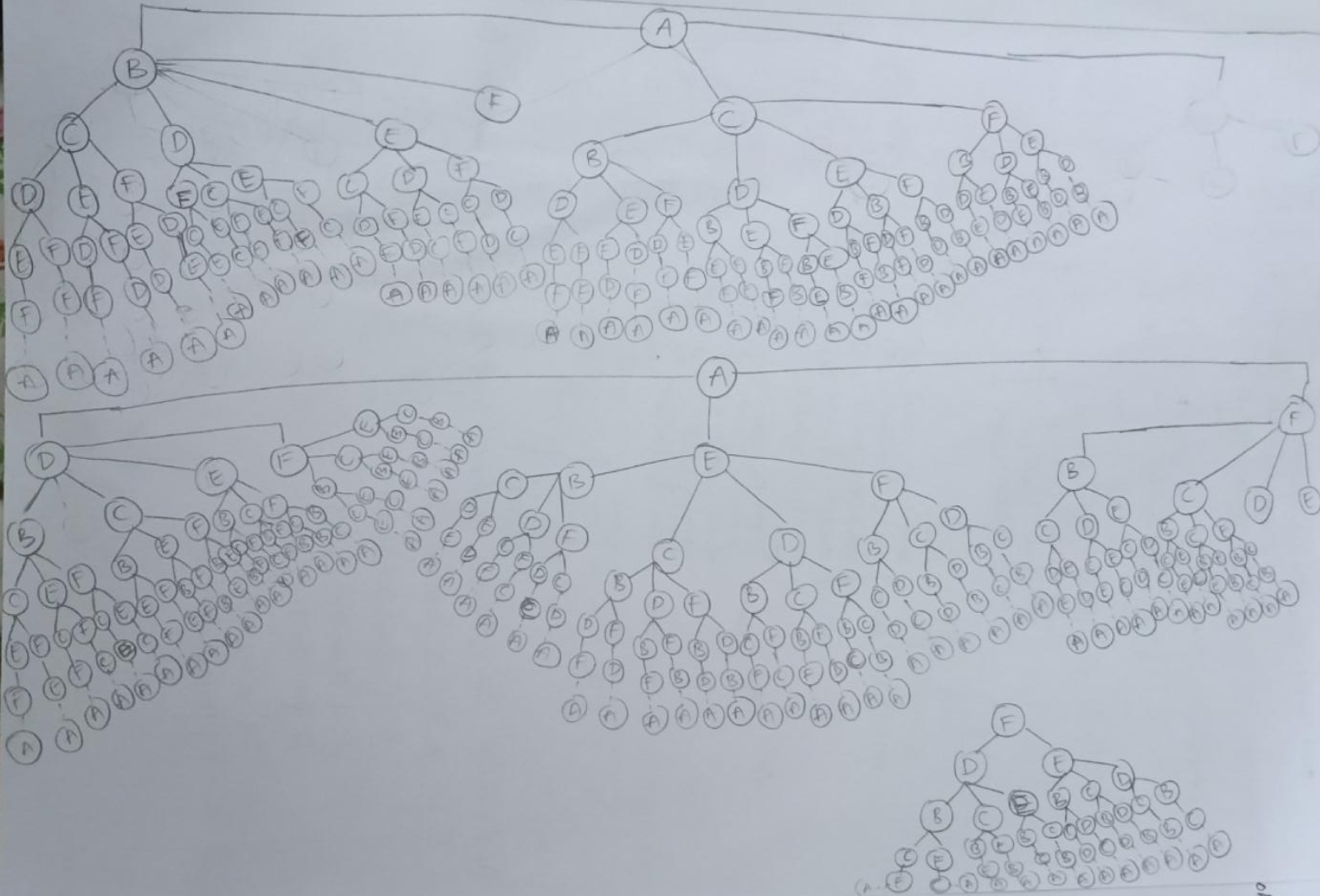
⑯ Extend the following distance matrix to 7 cities & solve the TSP

A: [0,12,10,19,08,16]
B: [12,0,21,11,15,10]
C: [10,21,0,13,5,7]
D: [19,11,13,0,18,8]
E: [8,15,5,18,0,14]
F: [16,10,7,8,14,0]



o|p = A-B-C-D-E-F-83//

I_1 : $w = 25$, $v = 80$



oblem

⑰ Given a knapsack capacity of 70 units & the following items

I_1 : w = 25 , v=80
I_2 : w = 35 , V=90
I_3 : w = 45, v=120
I_4 w = 30, v=70 use dp to solve 0/1 knapsack problem

w/V	0	25	35	45	30	70
0	0	0	0	0	0	0
1	0	80	80	80	80	80
2	0	80	90	90	90	90
3	0	80	90	120	120	90
4	0	80	90	80	120	150

Item	w	val
1	25	80
2	35	90
3	45	120
4	30	70

⑱ For a graph A→BA, W-1
A→CA, w-4
B→CB, w-3
B→DB, w-2
B→EB, w-2
D→BD, w-1
D→CD, w-5
E→DE, w-3 use Bellman ford algorithm.

10/64

V	A	B	C	D	E
d	0	∞	∞	∞	∞
P	-	-	-	-	-

①

V	A	B	C	D	E
d	0	1	4	∞	∞
P	-	A	A	-	-

②

V	A	B	C	D	E
d	0	-1	4	30	1
P	-	A	A	-	B

③

V	A	B	C	D	E
d	0	-1	4	3	1
P	-	A	A	E	B

④

V	A	B	C	D	E
d	0	-1	4	3	1
P	-	A	A	E	B

path	distance	shortest path
A	0	A
B	-1	A→B
C	4	A→C
D	3	A→E→D
E	7	A→B→C

Knapsack 0|1 50 units

(19) I_1, w = 10, v = 50

I_2, w = 20, v = 70

I_3, w = 30, v = 90

I_4, w = 25, v = 60

I_5 w = 15, v = 40.

w/v	0	10	20	30	25	15	50
0	0	0	0	0	0	0	0
1	0	50	50	50	50	50	50
2	0	50	70	70	70	70	70
3	0	50	70	90	90	90	160
4	0	50	70	90	90	90	160
5	0	50	70	90	90	90	160

(20) using 5 cities

A: [0, 14, 4, 10, 20]

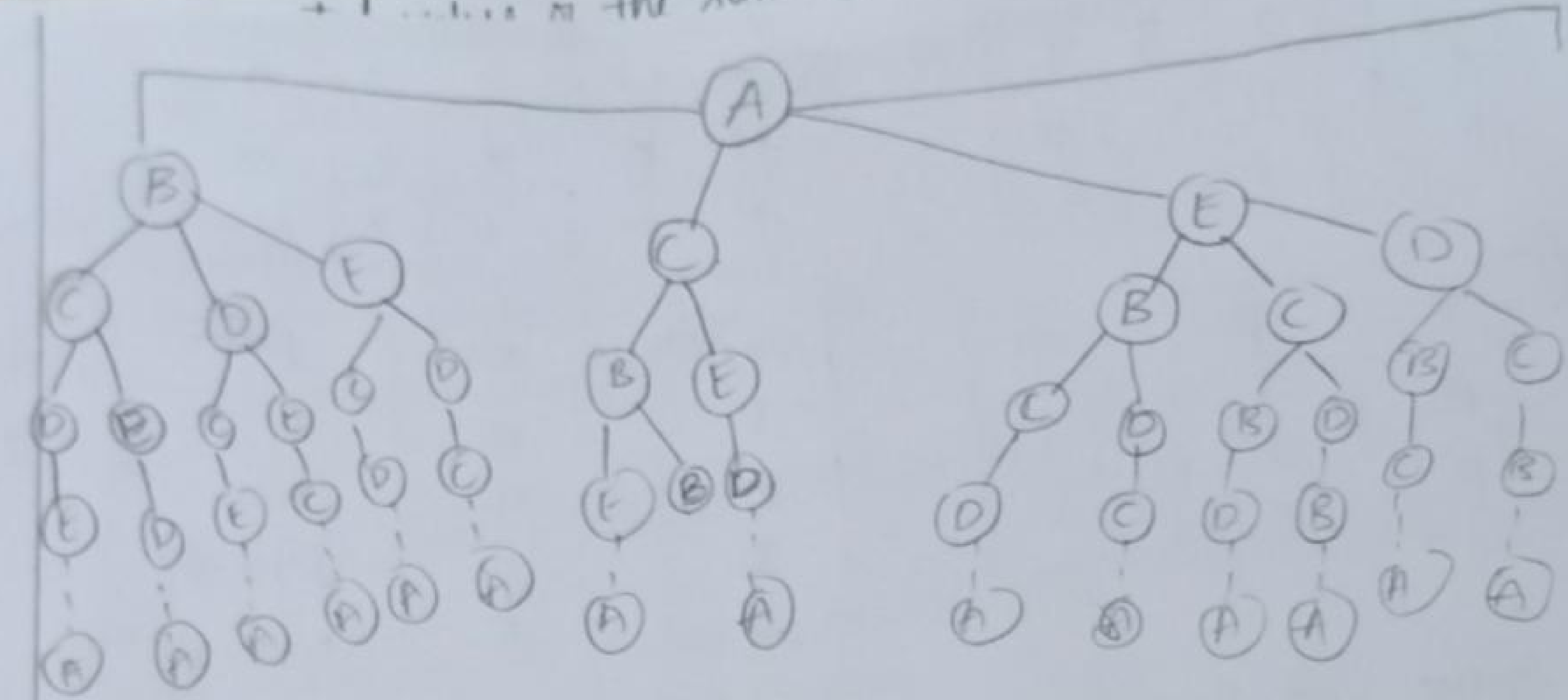
B: [14, 0, 7, 8, 7]

C: [4, 7, 0, 12, 6]

D: [10, 8, 12, 0, 15]

E: [20, 7, 6, 15, 0].

→ I ...... n the sum of the



(21) Bellman ford.

1→21, w=4
1→31, w=5
2→32, w=-2
3→~~1~~43, w=3
4→24, w=-10

1→2 -4
1→3 - 5
2→3 - 3
4→2 = -10

V	1	2	3	4
d	0	∞	∞	∞
P	-	-	-	-

the Expected values of the sum of reasoning

①

V	1	2	3	4
d	0	4	5	∞
P	–	1	1	–

②

V	1	2	3	4
d	0	4	2	∞
P	–	1	2	–

③

V	1	2	3	4
d	0	4	2	5
P	–	1	2	3

vertex	Dist	path
1	0	1
2	4	1→2
3	2	1→2→3
4	5	1→2→3→4

Solve TSP '4' cities

six-sided dice. Determine the no. of ways

㉒ Find the Expected value of the sum of the outcomes when rolling three four sided dice show your calculations & reasoning ⑪

→ sum − 3

way (1+1+1)

→ sum

$4 = \frac{3}{64}$ (1+1+2, 1+2+1, 2+1+1)

→ $5 = \frac{6}{64}$ (1+1+3, 1+2+2, 1+3+1, 2+1+2, 2+2+1, 3+1+1)

→ 6 = 10/64 (1+2+3, 1+3+2, 2+1+3, 2+2+2, 2+3+1, 3+1+2, 3+2+1, 1+4+1, 2+2+2, 2+3+1)

→ 12/64 = 7 (1+3+3, 2+2+3, 2+3+2, 3+1+3, 3+2+2, 3+3+1, 1+4+2, 2+3+2, 2+4+1, 3+2+2, 3+3+1, 4+1+2)

→ 8 = 12/64 → 11 = 3/64

→ 9 = 10/64 → 12 = 1/64

→ 10 = 6/64

→ 11 = 3/64

Σ (sum * probability)

$$= \left(3\times 1/64\right) + \left(4\times\frac{3}{64}\right) + \left(5\times\frac{6}{64}\right) + \left(6\times\frac{10}{64}\right) + \left(7\times\frac{12}{64}\right) + \left(8\times\frac{12}{64}\right) + \left(9\times\frac{10}{64}\right)$$

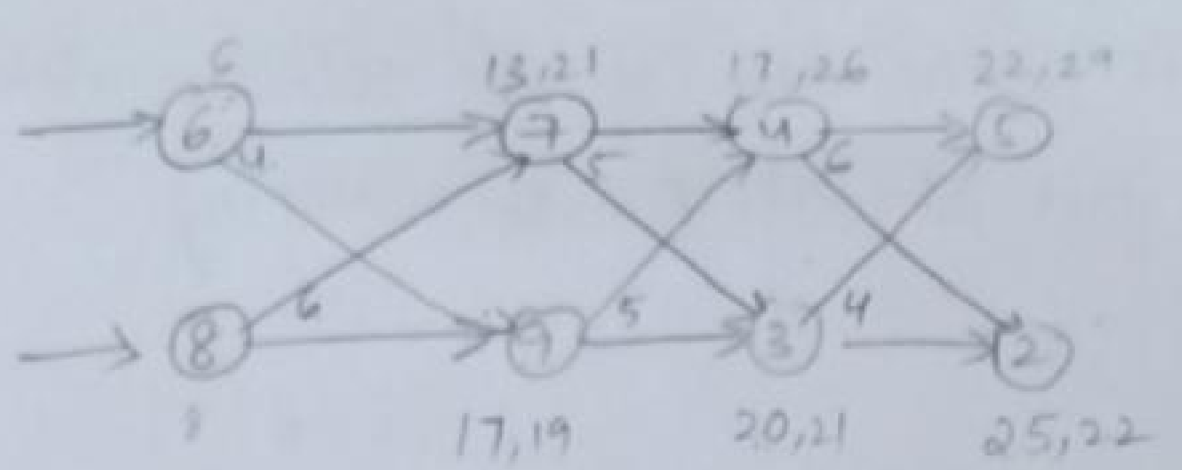
$$+ \left(10\times\frac{6}{64}\right) + \left(11\times\frac{3}{64}\right) + \left(12\times\frac{1}{64}\right)$$

$$= \frac{3}{64} + \frac{12}{64} + \frac{30}{64} + \frac{60}{64} + \frac{84}{64} + \frac{96}{64} + \frac{90}{64} + \frac{60}{64} + \frac{33}{64} + \frac{12}{64}$$

$$= \frac{480}{64} = \underline{\underline{7.5}}$$

The sum of the outcomes when rolling three four-sided dice is 7.5

㉓ Calculate the minimum time for product assembly for two assembly lines where line1 = [6,7,4,5] & l_2: [8,9,3,2] with transfer times b/w lines [4,5,6] from l_1 to l_2 & [6,5,4] from l_2 to l_1



	1	2	3	4
$f_1[j]$	6	13	17	22
$f_1[j]$	8	17	20	22

	1	2	3	4
l_1	1	1	1	1
l_2	2	.2	2	2

㉔ Keys {10, 20, 30} have probabilities {0.2, 0.5, 0.3}. construct the optimal binary search tree & calculate the total search cost. Additionally compare this with a suboptimal BST structure & analyse the difference in cost

	0	1	2	3
0	0	0.2	$0.7^{[1]}$	$1.1^{[3]}$
1		0	0.5	$1.7^{[2]}$
2			0	0.3
3				0

Given keys $\{\overset{1}{10}, \overset{2}{20}, \overset{3}{30}\}$

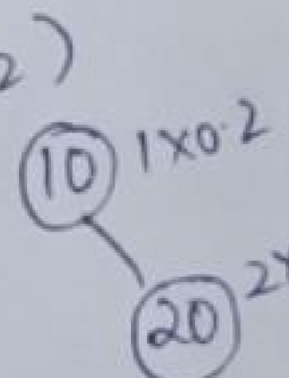
probabilities {0.2, 0.5, 0.3}

j-i=0	j-i=1	j-i=2
1-1=0	1-0=1	2-0=(0,2) 2
2-2=0	2-1=1	3-1=2 (1,3).
3-3=0	3-2=1	

j-i=3

3-0=3 (0,3)

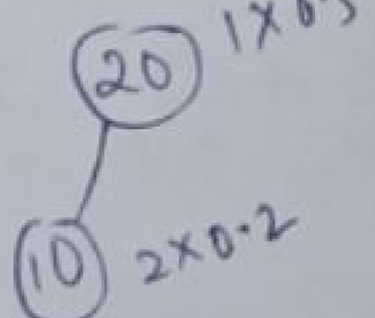
(0,2)



0.2+0.5

0.7

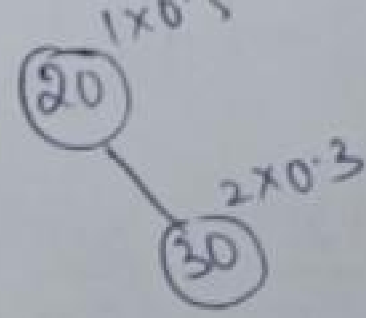
min



0.5+0.4

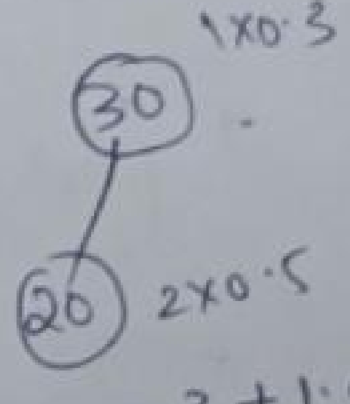
0.9

(1,3)



0.5+0.6

1.1



0.3+1.0

1.3

$$\text{cost}(i,j) = \min\{\text{cost}(i,k-1) + \text{cost}(k,j)\} + w_i$$

$$\underset{k=1,2,3}{\text{cost}(0,3)} = \min\begin{cases}\text{cost}(0,0) + \text{cost}(1,3)\\ \text{cost}(0,1) + \text{cost}(2,3)\\ \text{cost}(0,2) + \text{cost}(3,3)\end{cases} + 1.0 \quad = \min\begin{cases}0+1.1\\ 0.2+0.3\\ 0.1+0\end{cases} + 1.0$$

$$= \begin{cases}2.1\\ 1.5\\ \underline{1.1}\end{cases}$$

(25) Roll six-six-sided dice. Determine the no. of ways to get sum of ② 18, ensuring that atleast one ~~dice~~ die shows a 6.

The no. of ways to get sum of 18.

Total no. of possibilities. 46656

→ 18 (1 die will shows 18; which is not possible with 6-sided dice)

→ 17+1 (1 die shows 17)

→ 16+2 (1 die will shows 16)

→ 16+1+1 (1 die will shows 16, but not possible with a 6-sided die and 2 dice show 1)

The total ways to get a sum 18 is 34

The no. of ways to get a sum of 18 without any 6s

Then we should roll only 1, 2, 3, 4 & 5. then there are 10 partitions.

$\boxed{\text{Total ways} - \text{ways without 6s}}$

34 − 10

24

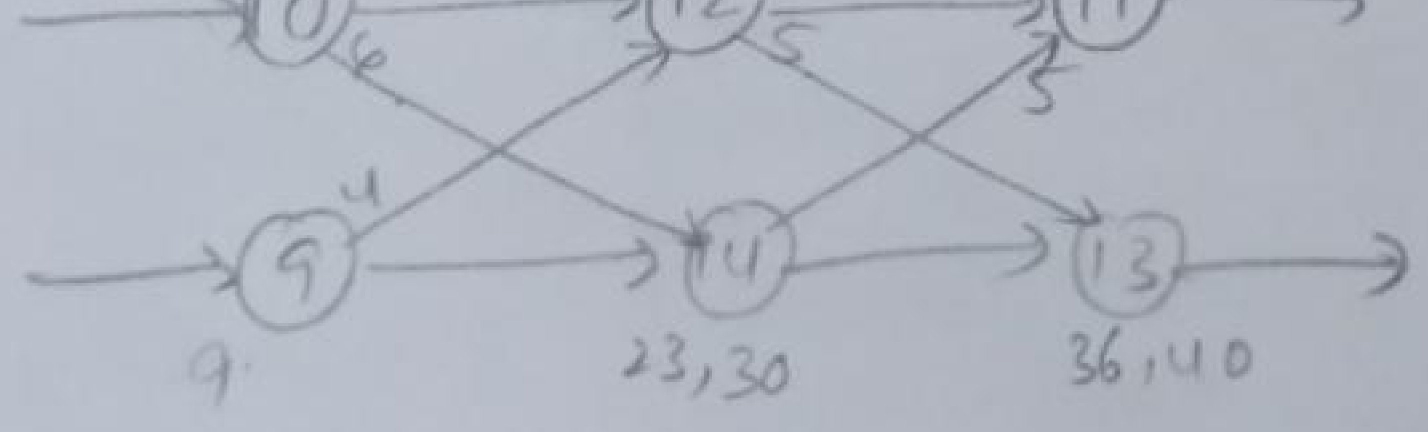
∴ there are 24 way to get sum of 18.

(26) Given line 1 [10, 12, 11], line 2 [9, 14, 13] & transfer times b/w lines [6, 5], calculate the minimum assembly time, considering a reduction in one of the transfer times by 2 units. Determine the optimal schedule.

Given line 1 [10, 12, 11] and l_2 [9, 14, 13]

time b/w lines [6, 5]

reduction in one of the transfer times by 2 units [4,



j-i=3	j-i=4	j-i=5
	4-0=(0,4)	5-0=5 (0,5)

	1	2	3
f1[1]	10	22	33
f1[3]	9	23	36

	1	2	3
l1	1	1	1
l2	2	2	2

㉗ For keys {8, 12, 16, 20, 24} with access probabilities {0.2, 0.05, 0.4, 0.25, 0.1} determine the optimal binary search tree using the dynamic programming approach. compute the total cost & discuss how changing one of the probabilities affects the tree structure & cost.

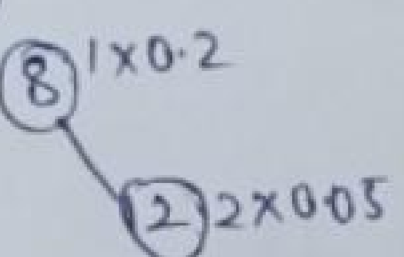
Given Keys $\{\overset{1}{8}, \overset{2}{12}, \overset{3}{16}, \overset{4}{20}, \overset{5}{24}\}$

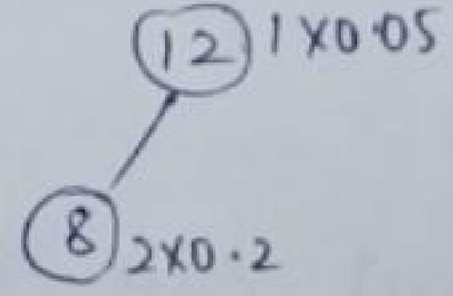
probabilities {0.2, 0.05, 0.4, 0.25, 0.1}

	0	1	2	3	4	5
0	0	0.2	0.21 [1]	0.86 [3]	1.36 [3]	1.9 [4]
1		0	0.05	0.41 [3]	1 [3]	1.3 [3]
2			0	0.4	0.9 [3]	1.2 [3]
3				0	0.25	0.45 [4]
4					0	0.1
5						0

j-i=0	j-i=1	j-i=2
1-1=0	1-0=1	2-0=2 (0,2)
2-2=0	2-1=1	3-1=2 (1,3)
3-3=0	3-2=1	4-2=2 (2,4)
4-4=0	4-3=1	5-3=2 (3,5)
5-5=0	5-4=1	

(0,2)



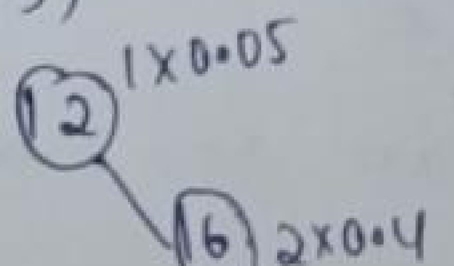


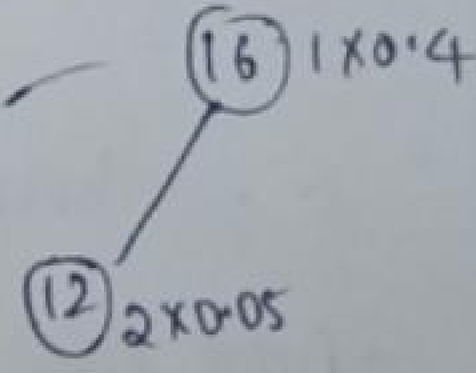
0.2 + 0.01   0.05 + 0.4

0.21   0.45

min

(1,3)



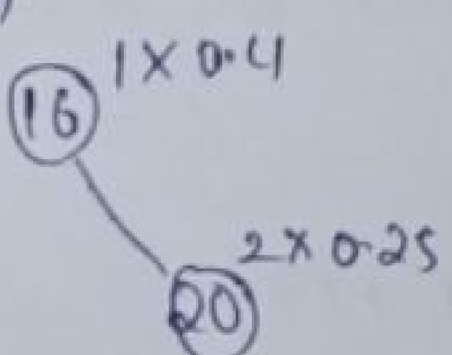


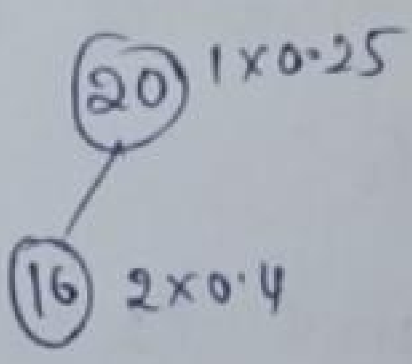
0.05 + 0.8   0.4 + 0.01

0.85   0.41

min

(2,4)



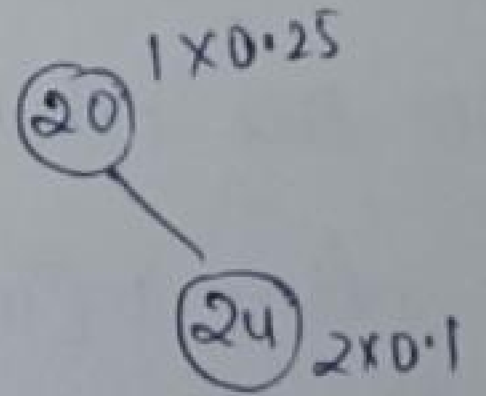


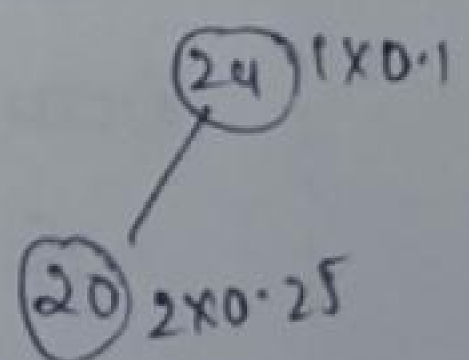
0.4 + 0.5   0.25 + 0.8

0.9   1.05

min

(3,5)





0.25 + 0.2   0.1 + 0.5

0.45   0.6

min

j-i=3	j-1=4	j-1=5
3-0=3 (0,3)	4-0=(0,4)	5-0=5 (0,5)
4-1=3 (1,4)	5-1=(1,5) 4	
5-2=3 (2,5)		

$$\text{cost}(1,j) = \min\{\text{cost}(i, k-1) + \text{cost}(k,j)\} + w_i$$

$$\underset{k=1,2,3}{\text{cost}(0,3) = \min} \begin{cases} \text{cost}(0,0) + \text{cost}(1,3) \\ \text{cost}(0,1) + \text{cost}(2,3) \\ \text{cost}(0,2) + \text{cost}(3,3) \end{cases} + 0.65$$

$$= \min \begin{cases} 0+0.41 \\ 0.2+0.4 \\ 0.21+0 \end{cases} + 0.65 = \begin{cases} 1.06 \\ 1.25 \\ \underline{0.86} \end{cases} = 0.86$$

$$\underset{k=2,3,4}{\text{cost}(1,4) = \min} \begin{cases} \text{cost}(1,1) + \text{cost}(2,4) \\ \text{cost}(1,2) + \text{cost}(3,4) \\ \text{cost}(1,3) + \text{cost}(4,4) \end{cases} + 0.7$$

$$= \min \begin{cases} 0+0.9 \\ 0.05+0.25 \\ 0.41+0 \end{cases} + 0.7 = \begin{cases} 1.6 \\ \underline{1} \\ 1.11 \end{cases} = 1$$

$$\underset{k=3,4,5}{\text{cost}(2,5) = \min} \begin{cases} \text{cost}(2,2) + \text{cost}(3,5) \\ \text{cost}(2,3) + \text{cost}(4,5) \\ \text{cost}(2,4) + \text{cost}(5,5) \end{cases} + 0.75$$

$$= \min \begin{cases} 0+0.45 \\ 0.4+0.1 \\ 0.9+0 \end{cases} + 0.75 = \begin{cases} \underline{1.2} \\ 1.25 \\ 1.65 \end{cases} = 1.2$$

$$\underset{k=1,2,3,4}{\text{cost}(0,4) = \min} \begin{cases} \text{cost}(0,0) + \text{cost}(1,4) \\ \text{cost}(0,1) + \text{cost}(2,4) \\ \text{cost}(0,2) + \text{cost}(3,4) \\ \text{cost}(0,3) + \text{cost}(4,4) \end{cases} + 0.9$$

$$\min \begin{cases} 0+1 \\ 0.2+0.9 \\ 0.21+0.25 \\ 0.86+0 \end{cases} + 0.9 = \begin{cases} 1.9 \\ 2 \\ \underline{1.36} \\ 1.76 \end{cases} = 1.36$$

$$\underset{k=2,3,4,5}{\text{cost}(1,5) = \min} \begin{cases} \text{cost}(1,1) + \text{cost}(2,5) \\ \text{cost}(1,2) + \text{cost}(3,5) \\ \text{cost}(1,3) + \text{cost}(4,5) \\ \text{cost}(1,4) + \text{cost}(5,5) \end{cases} + 0.8 =$$

$$\min\begin{cases}\cancel{0 + 0.9} \\ \cancel{0.05 + 0.25} \\ \cancel{0.44} .\end{cases} \quad \min\begin{cases}0 + 1.2 \\ 0.05 + 0.45 \\ 0.41 + 0.1 \\ 1 + 0\end{cases} + 0.8 = \min\begin{cases}2 \\ 1.3 \\ 1.31 \\ 1.8\end{cases} = 1.3$$

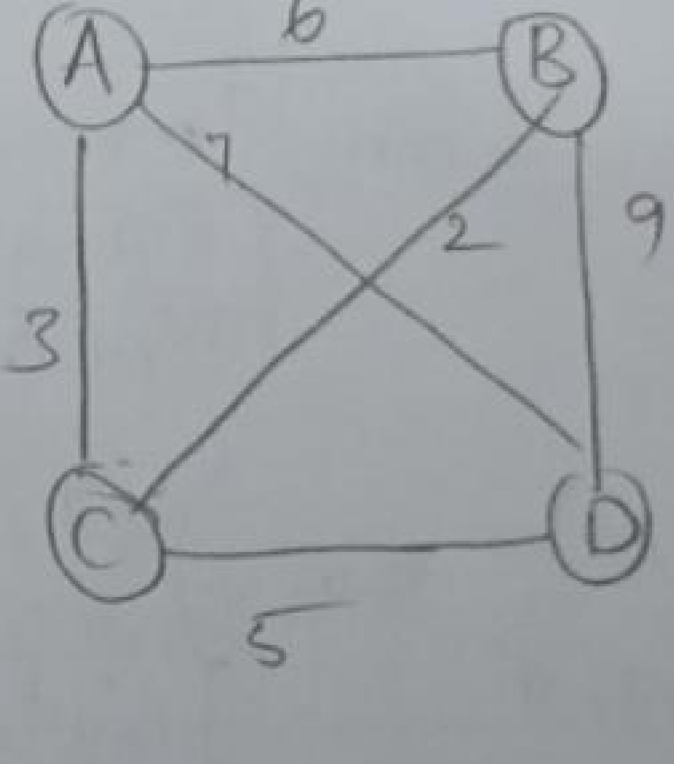
$$\text{cost}(0,5) = \min_{k=1,2,3,4,5} \begin{cases}\text{cost}(0,0) + \text{cost}(1,5) \\ \text{cost}(0,1) + \text{cost}(2,5) \\ \text{cost}(0,2) + \text{cost}(3,5) \\ \text{cost}(0,3) + \text{cost}(4,5) \\ \text{cost}(0,4) + \text{cost}(5,5)\end{cases} + 1 = \begin{cases}0 + 1.3 \\ 0.2 + 1.2 \\ 0.21 + 0.45 \\ 0.86 + 0.1 \\ 1.36 + 0\end{cases} + 1$$

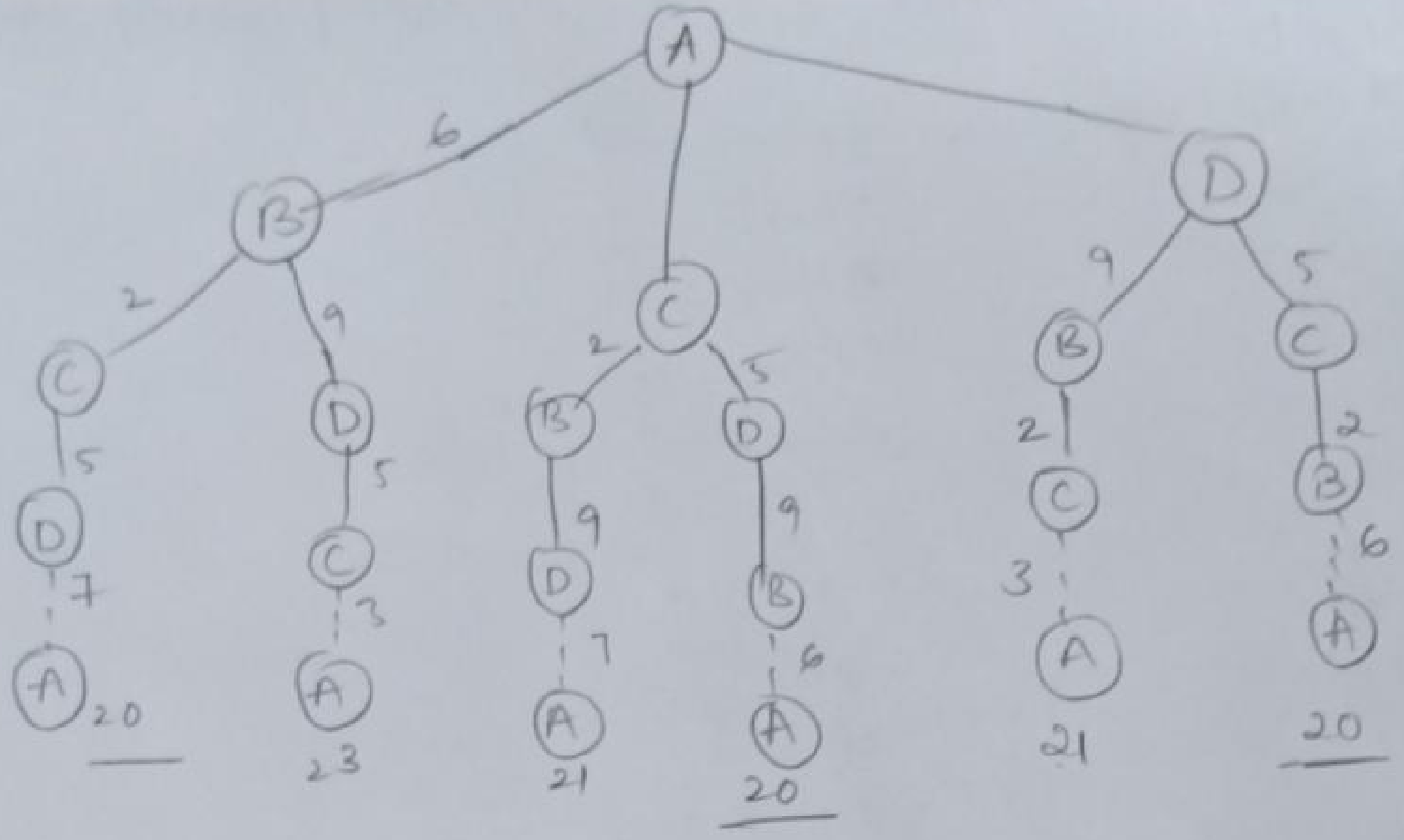
$$= \begin{cases}2.3 \\ 2.4 \\ 2.41 \\ 1.9 \\ 2.36\end{cases} = 1.9$$

㉘ Solve the TSP for 4 cities using simulated annealing with the following distance matrix.

A [0, 6, 3, 7]
B [6, 0, 2, 9]
C [3, 2, 0, 5]
D [7, 9, 5, 0]

$$\begin{array}{c|cccc} & A & B & C & D \\ \hline A & 0 & 6 & 3 & 7 \\ B & 6 & 0 & 2 & 9 \\ C & 3 & 2 & 0 & 5 \\ D & 7 & 9 & 5 & 0 \end{array}$$





A-B-C-D-A-20

A-D-C-B-A-20

(29) You have a knapsack with capacity of 50 units that are 4 items with following weights & values.

I_1 w=10 V=60

I_2 w=20 V=100

I_3 w=30 V=120

I_4 w=40 V=200

Determine the max value by using knapsack

W \ V	0	10	20	30	40	50
0	0	0	0	0	0	0
1	0	60	60	60	60	60
2	0	60	100	160	160	160
3	0	60	100	120	180	180
4	0	60	100	120	200	260

260 is maximum v

㉚. For a graph with vertices A, B, C, D, E, F & the following edges & weights

A → BA) rightarrow BA → B - wt - 6
A → DA / " DA → D - 7
B → CB / " ~~C~~B → C - 5
B → EB / " EB → E - 4
B → DB / " DB → D - 8
C → BC / " BC → B - 2
D → CD / " CD → C - 3
D → ED / " ED → E - 9
E → FE / " FE → F - 7
F → CF / · CF → C - 2

use the Bellman ford algorithm starting from Vertex A

V	A	B	C	D	E	F
d	0	∞	∞	∞	∞	∞
p	-	-	-	-	-	-

①

V	A	B	C	D	E	F
d	0	6	4	7	2	9
p	-	A	D	A	B	E

②

V	A	B	C	D	E	F
d	0	2	4	7	2	9
p	-	C	D	A	B	E

③

V	A	B	C	D	E
d	0	2	4	7	2
p	-	C	D	A	B

④

V	A	B	C	D	E	F
d	0	2	4	7	2	9
P	-	C	D	A	B	E

⑤

V	A	B	C	D	E	F
d	0	2	4	7	2	9
P	-	C	D	A	B	C

Vertex	Distance	path
A	0	A
B	2	A-D-C-B
C	4	A-D-C
D	7	A-D
E	2	A-D-C-B-E
F	9	A-D-C-B-E-F