DSO530 Statistical Learning Methods

Lecture 3b: Classification II

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What is the best classifier?

- It depends on what is your objective function
- Suppose your goal is to find h that minimizes $P(h(X) \neq Y)$
- What is the best classifier if you knew the joint distribution of (X, Y)?
- intuitively, how should we think about "know the joint distributin of (X, Y)"?
- Let's take for granted that the `best` classifier is $1(\eta(x) > 1/2)$, where $\eta(\cdot)$ is the so called regression function.
- This classifier is the so-called `Bayes classifier`
- Recall the regression function: $\eta(x) = E(Y|X = x)$.
- In the binary classification scenario, $E(Y|X = x) = P(Y = 1|X = x)$.

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Are we done now?

Since some statisticians have found this `best` classifier, why don't we just use it and save all the trouble to learn classification methods?

We cannot use the Bayes classifier

- Knowing the distribution of (X, Y) is impossible.
- We only know some instances sampled from the distribution.
- So we have to estimate $\eta(x)$ based on the sample
- Where does the logistic regression model come into the picture?

- The logistic regression model is a parametric model for $\eta(x)$ or $P(Y = 1|X = x)$
- But why do we often want to impose such a restrictive form for η?

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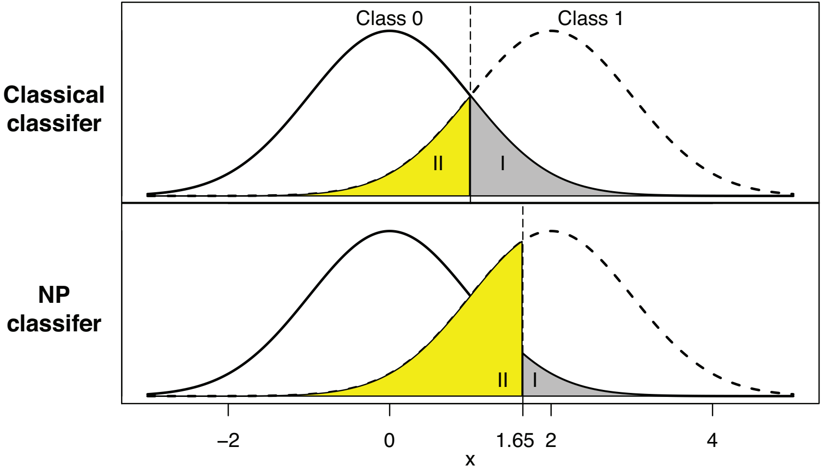
A typical question

- Q: Getting infinite observations, can I achieve perfect classification?
- A: Typically, no!
- Why: look at the Bayes classifier. Focus on 1st row: class 0: N(0, 1); class 1: N(2, 1); balanced classes.



Type I vs. type II error

- Modify the Bayes classifier
- You can move the decision threshold to the left or to the right
- How will type I and type II error change?
- type I error definition: $P(h(X) \neq Y | Y = 0)$
- type I error definition: $P(h(X) \neq Y | Y = 1)$
- a takeaway message: we can change the decision threshold so that we rebalance the trade-off between type I and type II errors

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Connection to reality

- Suppose 0 codes disease status and 1 codes normal
- Then type I error is the false negative rate and type II error is the false positive rate
- In the above, we showed that even if you can have the entire instances in the world, you still likely cannot achieve 0% false negative rate and false positive rate.
- Given the current training data and machine learning model, one can push down one kind of error at the expense of the other.
- How can I lower both type I error and type II error at the same time in practice? (1) a better model. (2) enlarge the sample size (3) get more powerful features that can separate the two classes better.
- We should note that the first two solutions have a limit.

A question for you to ponder

If one gives you a classifier that has false negative rate of 50% and false positive rate of 60%, is it acceptable?