DSO530 Week1 Technical Notes: Review of Probability I

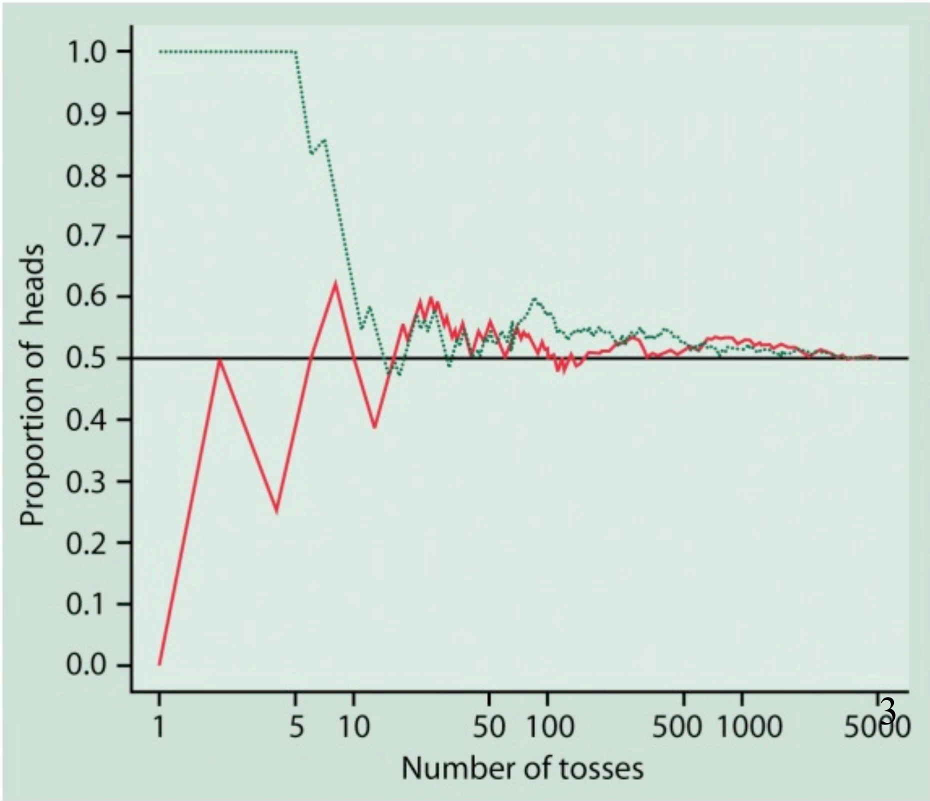
Statistics and Probability

- *Statistics* enables us to make decisions/inference under uncertainty.
- By using *Probability*, we can make numerical statements about uncertainty.
- By uncertainty, we mean *randomness (defined on the next slide)*.

Randomness

- Randomness $\neq$ complete chaos!
- A phenomenon is **random** if individual outcomes are uncertain but outcomes have a regular pattern in a large number of repetitions

e.g. tossing a fair coin



Probability Models

Any process that results in an *outcome* is an *experiment*.

An experiment may have more than one possible outcome.

S = **sample space** = set of *all* possible outcomes.

E.g. Experiment: toss a coin once;

Outcomes: H, T;

$S = \{H,T\}$

Probability Models (cont)

An **event** is a collection of *some* outcomes

E.g. A = (get exactly one head in 3 tosses)

$= \{\text{HTT, THT, TTH}\}$

Each event is assigned a **probability**, *i.e.*, a number between 0 and 1.

If A is an event, $\boldsymbol{P(A)}$ denotes the probability of A.

Equally-likely Case

When all possible outcomes are equally likely,

$$P(A) = \frac{\#\,\text{outcomes in } A}{\#\,\text{outcomes in } S}$$

E.g. tossing a coin once, with $S = \{H, T\}$.

If $A = \{H\}$, then

$$P(A) = \frac{1}{2}$$

Equally-likely Case

E.g. roll two dice. What is the probability of getting a total of at least 11?

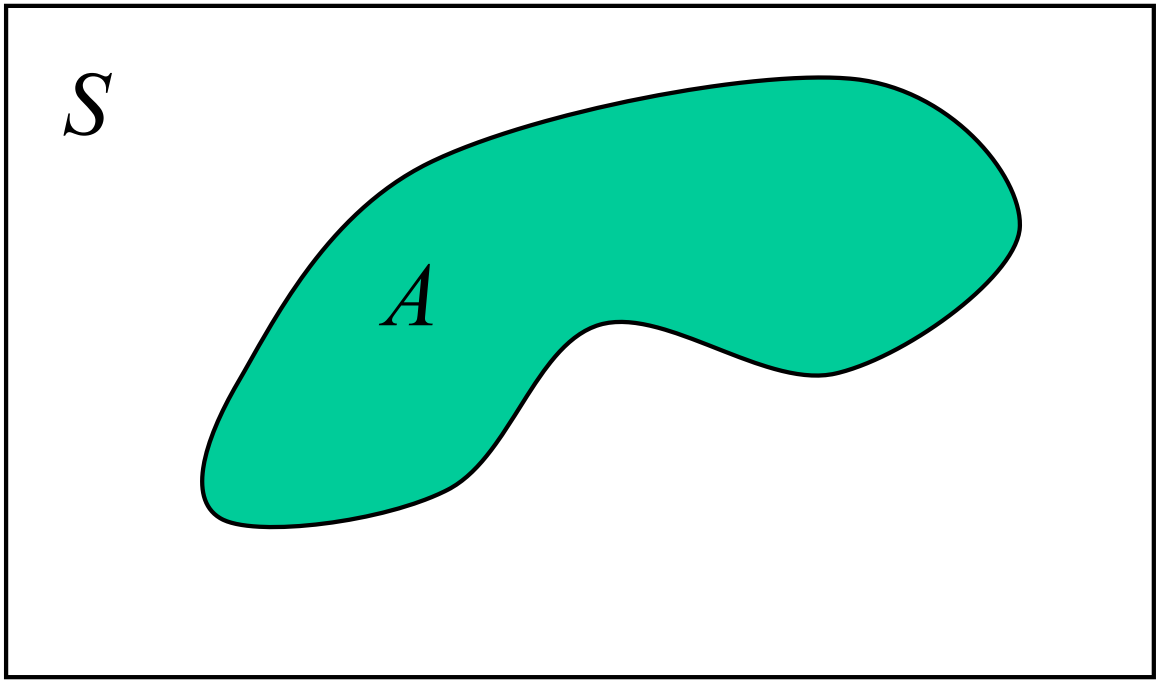
Here is the sample space S

36 outcomes, equally likely, each with 1/36 probability

(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

P{total at least 11} = 3/36 = 0.083

A useful picture/example of probability



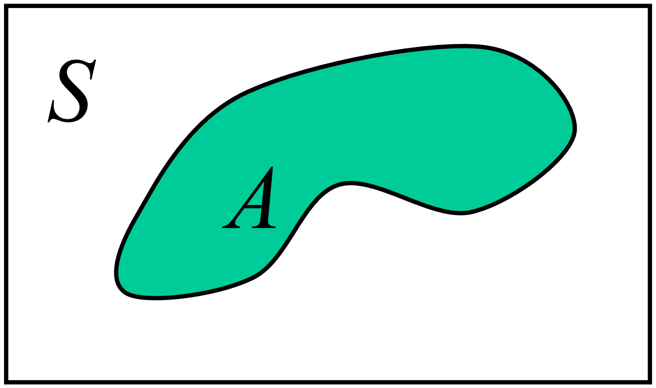
Venn diagram

S is the sample space, A is an event

You're driving and it's about to start raining. Think of S as your windshield. Event A corresponds to statement {the first drop to hit the windshield will hit the set A}.

A useful picture/example

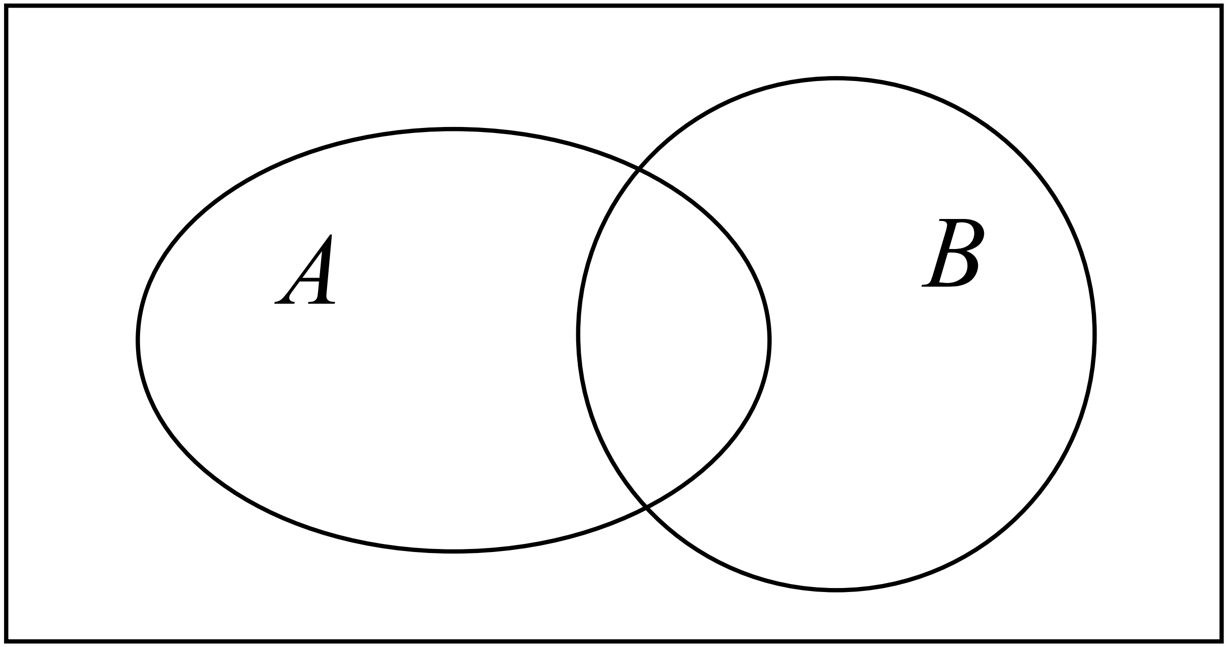
A simple probability measure to model this:



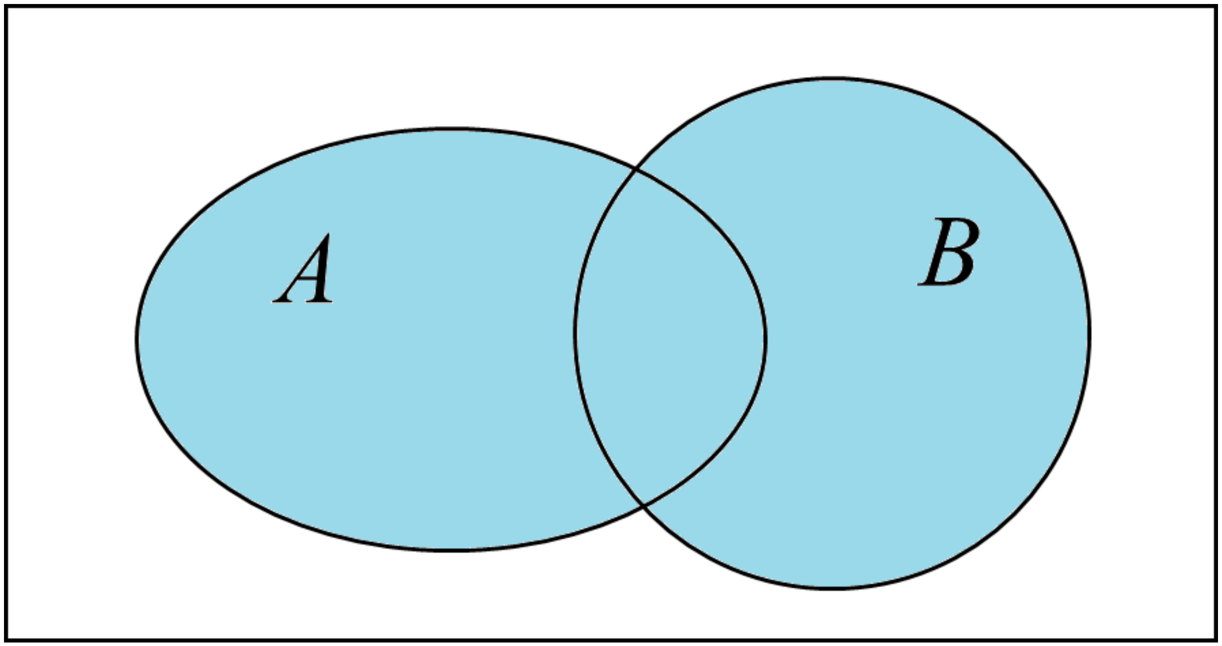
$$P(A) = \frac{\text{area of } A}{\text{area of } S}$$

Note that $0 \leq P(A) \leq 1$ and $P(S) = 1$

New events from old



New events from old

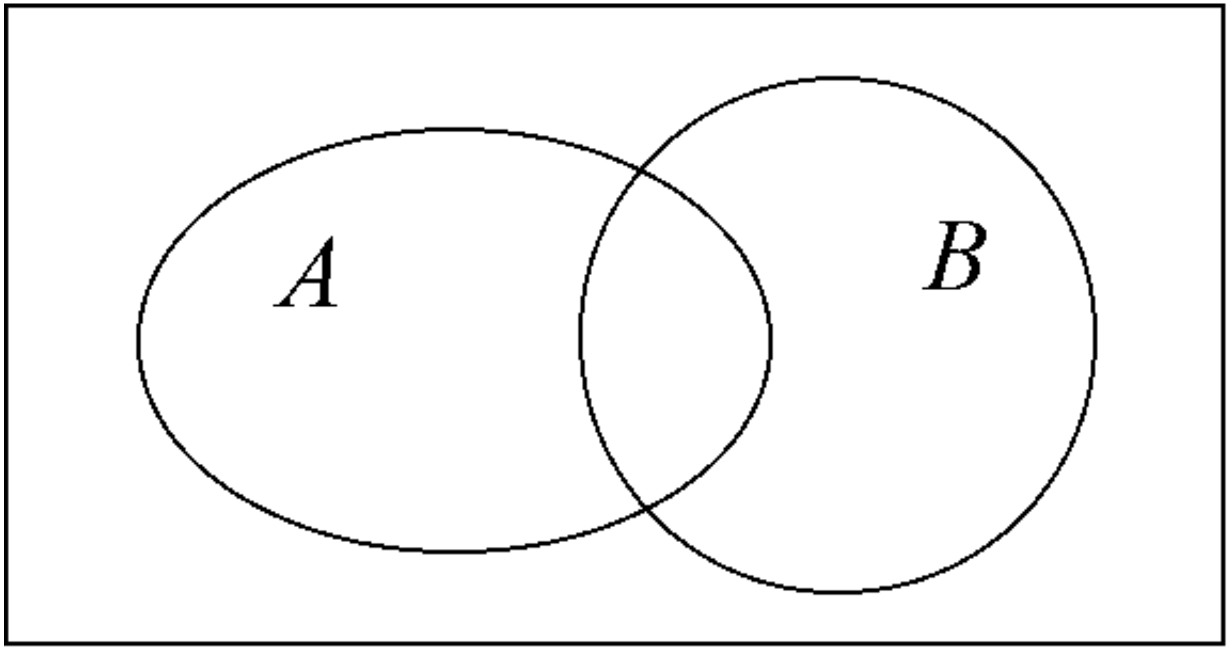


What should we call this?

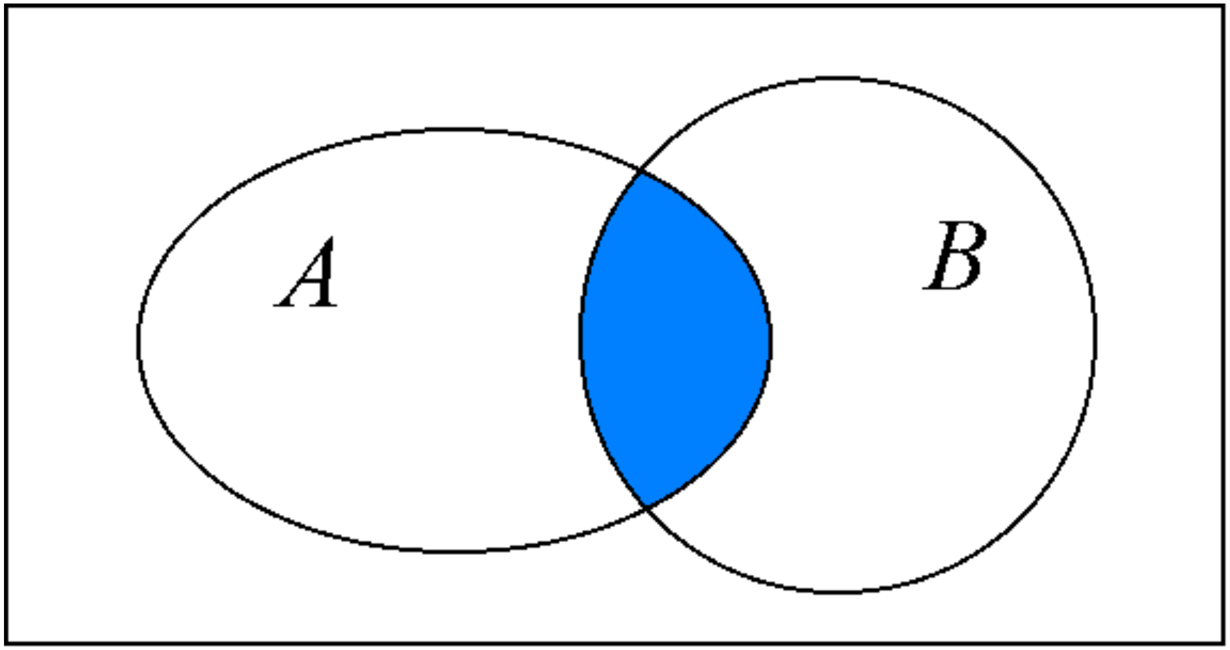
A and B ?

A or B ?

So what's (*A* and *B*) ?

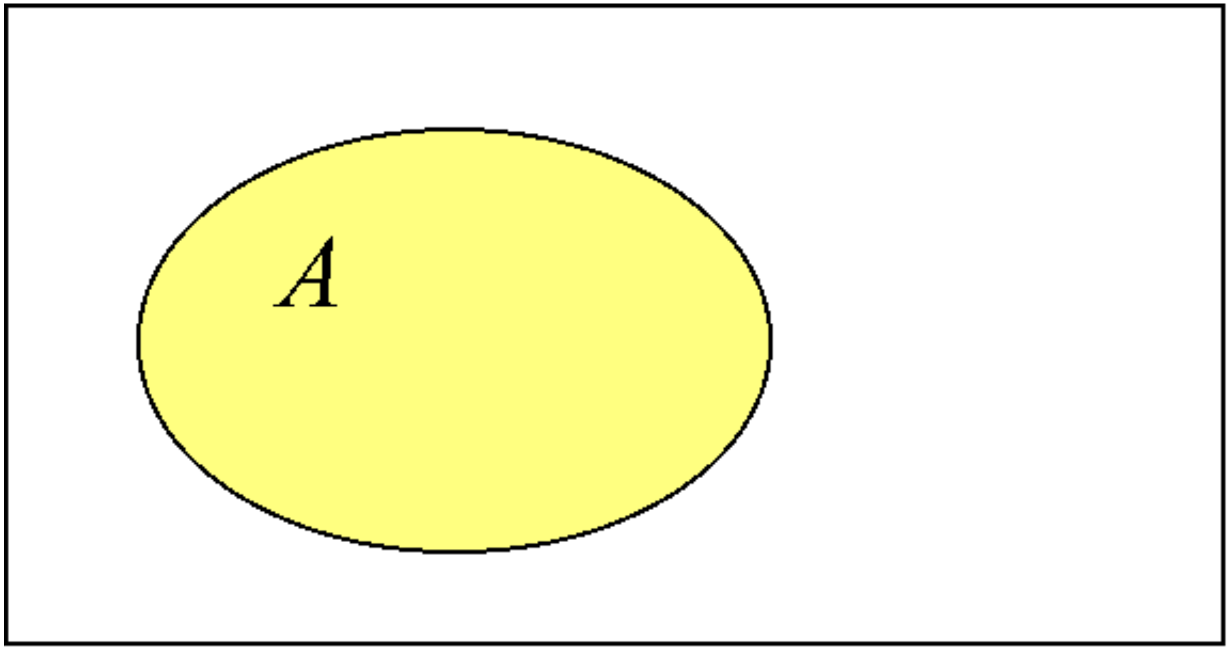


So what's (*A* and *B*) ?



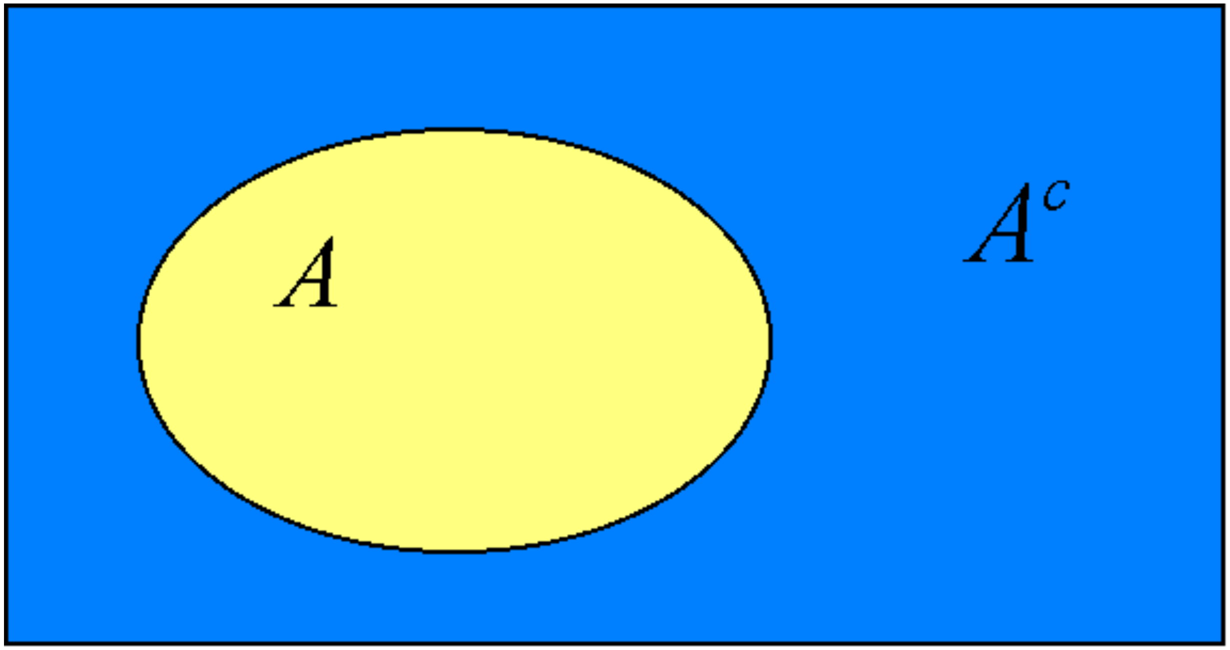
(raindrop falls in *A*) *and* (raindrop falls in *B*)

Complement of A?



$$\text{"complement of } A\text{"} = \text{"not } A\text{"} = A^c$$

Complement of A



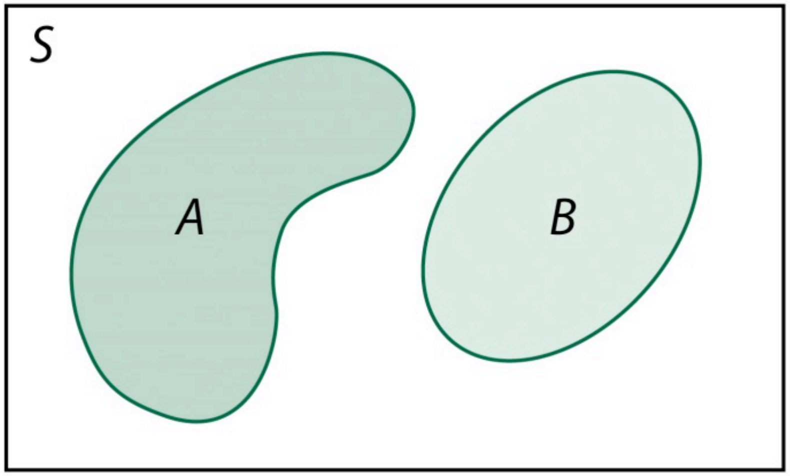
$$\text{"complement of } A\text{"} = \text{"not } A\text{"} = A^c$$

Rules of Probability

- For every event A,
  $P(A) \geq 0$ and $P(A) \leq 1$.

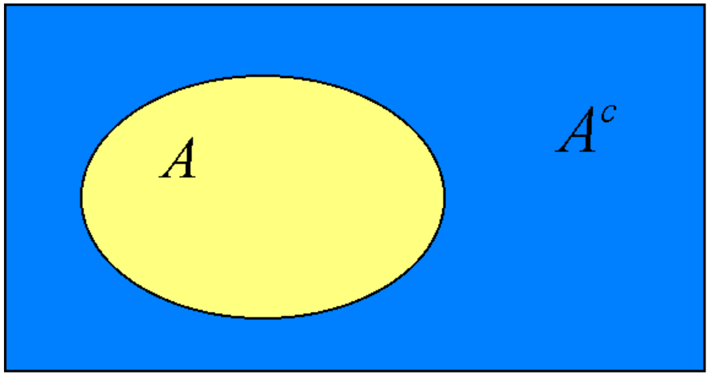
- $P(S) = 1$, where S is the sample space.

- If events A and B are **disjoint**,
  $P(A \text{ or } B) = P(A) + P(B)$.



Example: Complement rule

$$P(A^c) = 1 - P(A)$$



Why?

$$(A \text{ or } A^c) = S$$

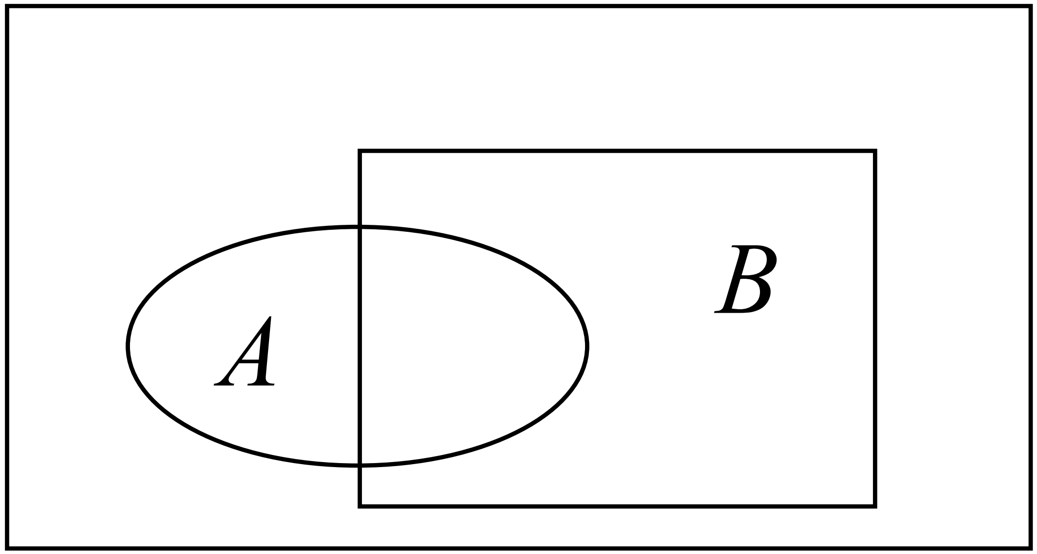
So $P(A \text{ or } A^c) = P(S) = 1$

But A and A^c are disjoint.

So $P(A \text{ or } A^c) = P(A) + P(A^c)$

So $P(A) + P(A^c) = 1.$

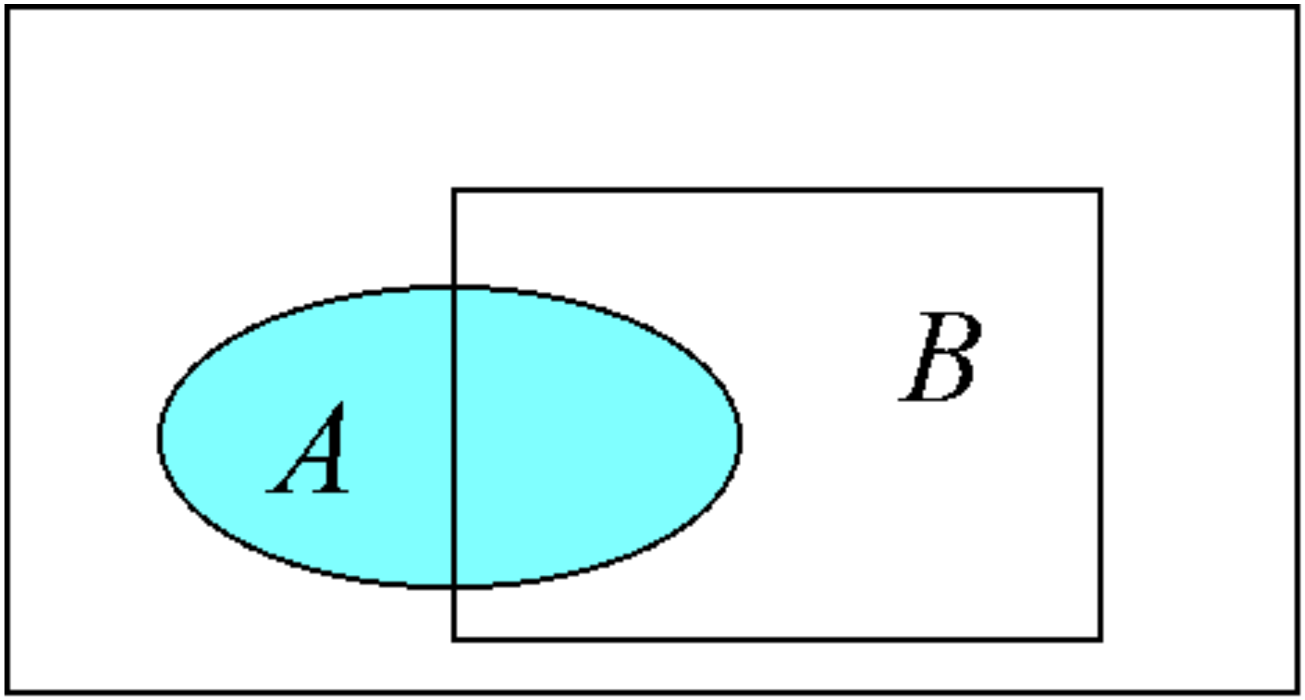
Conditional Probability $P(\mathrm{B} \mid \mathrm{A})$



Idea of $P(B|A)$: *Given* that A occurs, what is the probability that B also occurs?

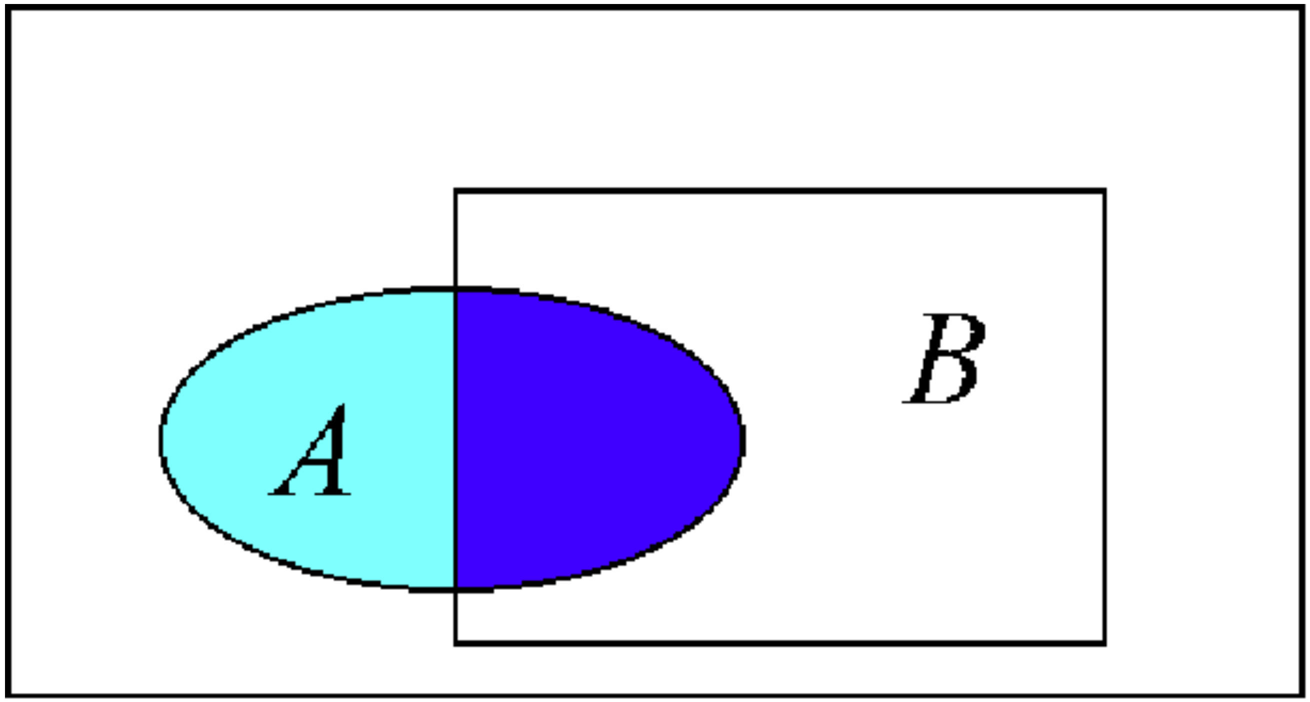
Question: By eyeball, what is $P(B|A)$?

Definition of $P(B \mid A)$



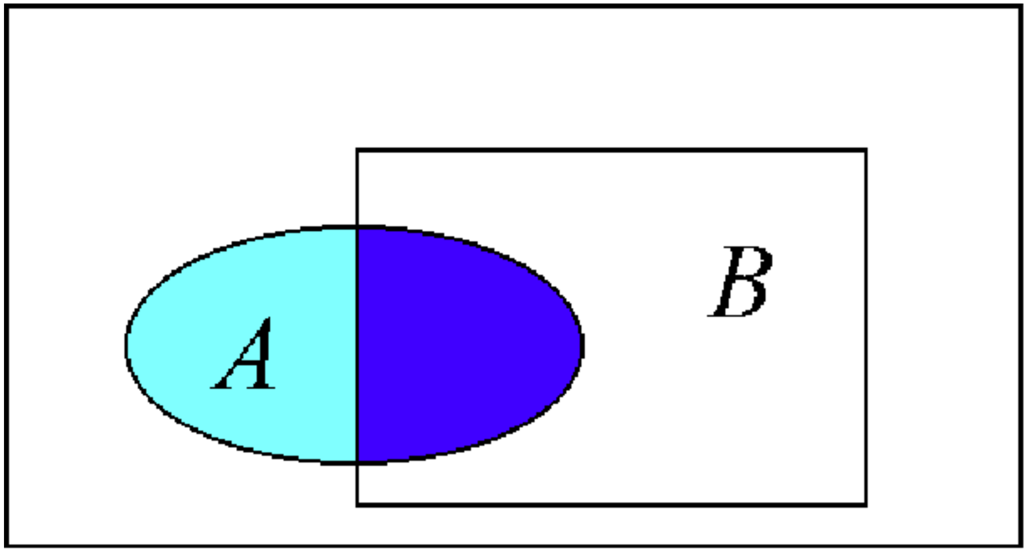
Given that the raindrop fell in A, we restrict our attention to the set A. The drop is equally likely to fall anywhere within A.

Definition of $P(\text{B} \mid \text{A})$



Given A, the event B occurs when the drop falls in the dark blue region, i.e., the event (A and B).

Definition of P(B | A)



$$P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)}$$

Independence

Two events A and *B* are **independent** if knowing one event's occurrence does not change the probability of the other event.

i.e. $P(B \mid A) = P(B)$

i.e. $\dfrac{P(A \text{ and } B)}{P(A)} = P(B)$

i.e. $P(A \text{ and } B) = P(A)P(B)$

E.g. Experiment: two tosses of a coin

A=(get H in the first toss)

B=(get H in the second toss)

Multiplication Rule

$$P(A \text{ and } B) = P(A)P(B)$$

the **multiplication rule for independent events**