13-Handout-DSO570

February 18, 2019

1 Session 13: Simulation Modeling II

In this session, we analyze a simulation case that puts everything we learned in the course so far together, including probability, algorithmic thinking and Python programming.

2 Case 10: Pricing Two Substitutable Products by Simulation

2.1 Part I: Simulating Customer Valuations

A firm sells two styles of headphones, which we refer to as model 0 and model 1. Based on a clustering analysis using historic data, the firm estimates that customers will come from three segments (A, B or C), and the valuation (maximum willingness to pay) of customers for the two products can be modelled as normally distributed according to the following parameters.

Segment	μ_0	σ_0	μ_1	σ_1	Proportion
A	30	30	70	30	0.1
B	80	20	20	10	0.3
C	-10	20	-10	20	0.6

As in the above table, a randomly chosen customer will be from segment A with 10% probability, segment B with 30% probability and segment C with 60% probability. Segment A customers have high valuations for model 1, while segment B customers have high valuations for model 0. Segment C customers, which make up the majority, do not on average value either products.

Generate a pandas DataFrame called "values" representing the simulated valuations of 10,000 randomly chosen customers. Each row represents a customer. There are three columns:

- **segment**: The segment of the customer, being "A", "B" or "C".
- **product_0**: The customer's maximum willingness to pay for Model 0.
- **product_1**: The customer's maximum willingness to pay for Model 1.

```
In [1]: import pandas as pd
        from scipy.stats import norm
        import numpy as np
        np.random.seed(0)
        data=[]
        dist0={'A':norm(30,30),'B':norm(80,20),'C':norm(-10,20)}
        dist1={'A':norm(70,30),'B':norm(20,10),'C':norm(-10,20)}
```

```
for i in range(10000):
    segment=np.random.choice(['A','B','C'],p=[.1,.3,.6])
    data.append([segment,dist0[segment].rvs(),dist1[segment].rvs()])
values=pd.DataFrame(data,columns=['segment','product_0','product_1'])
```

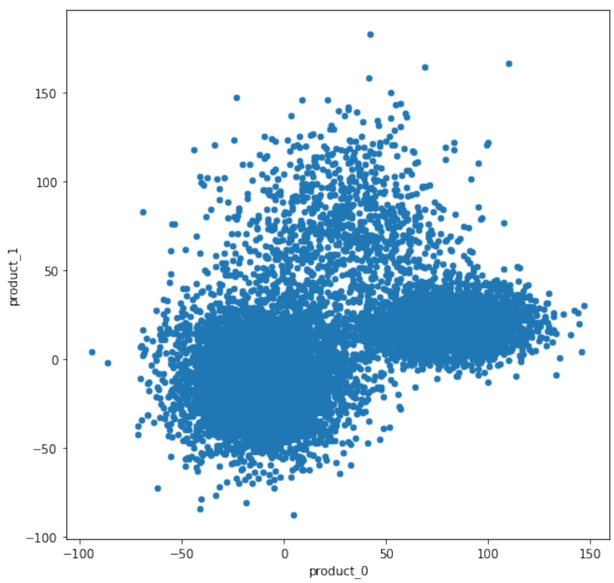
Once you have completed this part, you should be able to run the following code and obtain similar outputs.

```
In [2]: values.head()
```

```
Out[2]:   segment  product_0  product_1
        0       C   4.831835  21.058274
        1       C  27.351160 -29.545558
        2       C  15.322371 -20.117531
        3       C   5.220755  -7.566500
        4       A  44.529365  87.374214
```

```
In [23]: values.plot(x='product_0',y='product_1',kind='scatter',figsize=(8,8))
```

```
Out[23]: <matplotlib.axes._subplots.AxesSubplot at 0x7f4337d99f28>
```



2.2 Part II: Analysis and Optimization

The following code is a modification of the solution to case 9, using Pandas vectorized functions instead of for loops for improved performance.

```
In [4]: import numpy as np
        def demand(df,priceVector):
            diff=df[['product_0','product_1']]-priceVector
            demand0=((diff['product_0']>=diff['product_1'])&(diff['product_0']>=0)).sum()
            demand1=((diff['product_0']<diff['product_1'])&(diff['product_1']>=0)).sum()
            return demand0,demand1
        demand(values,[30,50])
```

```
Out[4]: (3314, 652)
```

Write a function called "tabluate" which takes as input a DataFrame in the format of the "values" DataFrame from Part I and outputs a DataFrame with the following columns:

- **Price_0**: The price for Model 0.
- **Price_1**: The price for Model 1.
- **Demand_0**: The simulated demand for Model 0 under the above prices.
- **Demand_1**: The simulated demand for Model 1 under the above prices.
- **Revenue**: The total revenue from the two products.

The rows of the DataFrame corresponds to every combination of Price_0 and Price_1 with values from `range(0,200,5)`, which is equivalent to the list `[0,5,10,...,195]`.

```
In [6]: result=tabulate(values)
        result.shape

Out[6]: (1600, 5)

In [7]: result.head()

Out[7]:    Price_0  Price_1  Demand_0  Demand_1  Revenue
        0        0        0      4622      2431        0
        1        0        5      4744      1996     9980
        2        0       10      4838      1605    16050
        3        0       15      4925      1284    19260
        4        0       20      4989      1049    20980
```

Using the "result" DataFrame, you can obtain the best revenue found using a number of ways, as below.

```
In [8]: result['Revenue'].max()

Out[8]: 194215

In [9]: result['Revenue'].idxmax()

Out[9]: 491

In [10]: result.iloc[491,:]

Out[10]: Price_0         60
         Price_1         55
         Demand_0      2587
         Demand_1       709
         Revenue     194215
         Name: 491, dtype: int64

In [11]: result.sort_values(by='Revenue',ascending=False).head(1)

Out[11]:      Price_0  Price_1  Demand_0  Demand_1  Revenue
         491       60       55      2587       709   194215
```

2.3 Part III: Obtaining Additional Insights

2.3.1 A. Value of price discrimination

Suppose that the company can observe which segment each customer belongs to, and charge separate prices to each segment. What would be the optimal prices for each segment and what would be the additional revenue from this flexibility?

(**Hint:** Filter the "values" DataFrame by whether the segment is A, B or C, and use the tabulate function to obtain DataFrames "resultA", "resultB", "resultC", which are analogous to the "result" DataFrame from above but are computed using valuations from one segment at a time.)

```
In [13]: # Best Pricing for Segment A
```

Out[13]:

	Price_0	Price_1	Demand_0	Demand_1	Revenue
451	55	55	103	692	43725

```
In [14]: # Best Pricing for Segment B
```

Out[14]:

	Price_0	Price_1	Demand_0	Demand_1	Revenue
486	60	30	2465	98	150840

```
In [15]: # Best Pricing for Segment C
```

Out[15]:

	Price_0	Price_1	Demand_0	Demand_1	Revenue
123	15	15	580	603	17745

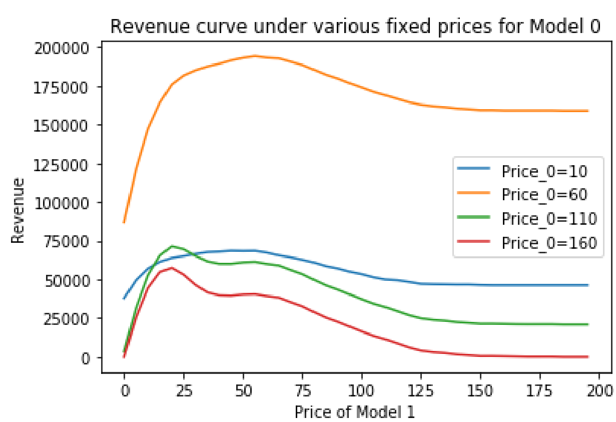
```
In [16]:
```

```
Potential benefit of price differentiation is about $18095.
```

2.3.2 B. Fixed Prices for One Product

Suppose that the price for Model 0 is fixed and the firm can only alter the price for Model 1, plot the total Revenue as a function of the price for Model 1, when the price for Model 0 is 10, 60, 110, and 160. (Note: you do not have to plot them on the same figure as below.)

```
In [17]:
```

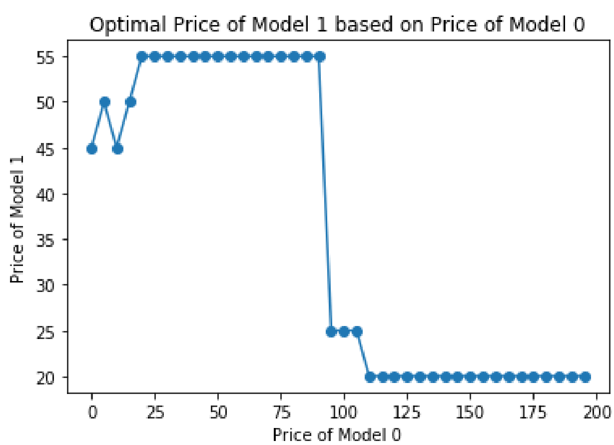


An alternative analysis based on more advanced Pandas functionality is as follows.

In [18]:
```
def bestPrice1(df):
    return df.sort_values(by='Revenue',ascending=False)['Price_1'].iloc[0]
optPrice1=result.groupby('Price_0').apply(bestPrice1)
optPrice1.head()
```

Out[18]:
```
Price_0
0      45
5      50
10     45
15     50
20     55
dtype: int64
```

In [19]:
```
import matplotlib.pyplot as plt
optPrice1.plot(marker='o')
plt.xlabel('Price of Model 0')
plt.ylabel('Price of Model 1')
plt.title('Optimal Price of Model 1 based on Price of Model 0')
plt.show()
```

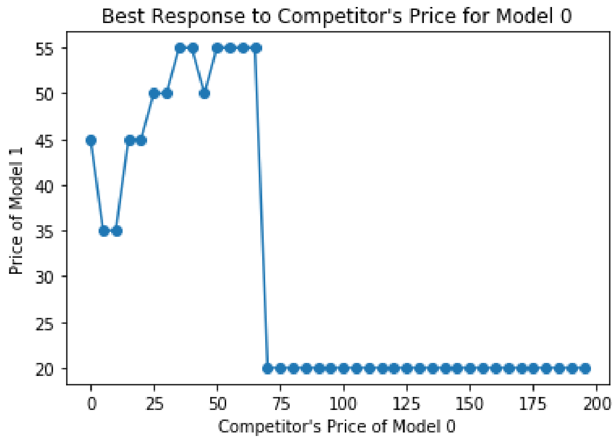


2.3.3 (Optional) C. Competitive Pricing

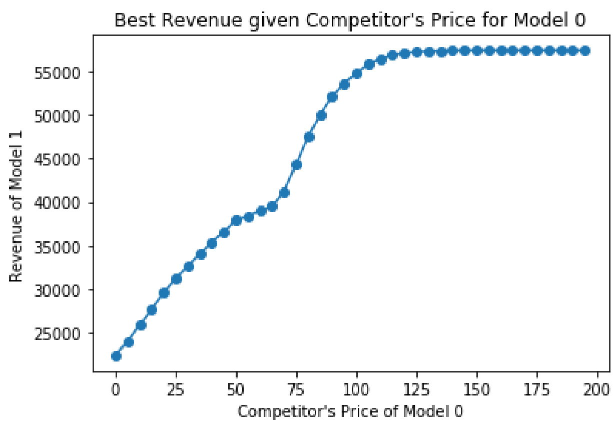
Suppose now that Model 0 is sold by a competitor, and only revenue from Model 1 counts. Modify the above code to display the optimal price for Model 1 given the competitor's pricing for Model 0. Moreover, plot the optimal attainable revenue given the competitor's pricing.

(**Hint**: You can create a new column in the "result" DataFrame corresponding to the revenue for Model 1 only, and sort by that revenue instead of by total revenue in the above code.)

In [21]:


Best Response to Competitor's Price for Model 0
Price of Model 1
Competitor's Price of Model 0
55
50
45
40
35
30
25
20
0
25
50
75
100
125
150
175
200

In [22]:


Best Revenue given Competitor's Price for Model 0
Revenue of Model 1
Competitor's Price of Model 0
55000
50000
45000
40000
35000
30000
25000
0
25
50
75
100
125
150
175
200