From Concrete Formulation to General Code (Single-Item Transportation)

1. Concrete LP Formulation

There are 2 production plants, A and B, with capacities 20 and 15 respectively. There are 3 demand centers, 1, 2, 3, with demand of 10 each. The cost of transporting each unit of good from plant A to the three centers are 3, 7, and 5 respectively. The unit transportation cost from plant B to the centers are 5, 3, and 3 respectively.

The following LP minimizes total transportation cost subject to satisfying demand at all three centers and not exceeding the capacity of each plant.

Decision variables: x_{A1} is the amount to be shipped from plant A to region 1, x_{A2} is from plant B to region 2, etc.

$$
\begin{aligned}
\text{minimize:} \quad & 3x_{A1} + 7x_{A2} + 5x_{A3} + 5x_{B1} + 3x_{B2} + 3x_{B3} \\
\text{subject to:} \quad & \\
\text{(Capacity A)} \quad & x_{A1} + x_{A2} + x_{A3} \leq 20 \\
\text{(Capacity B)} \quad & x_{B1} + x_{B2} + x_{B3} \leq 15 \\
\text{(Demand 1)} \quad & x_{A1} + x_{B1} \geq 10 \\
\text{(Demand 2)} \quad & x_{A2} + x_{B2} \geq 10 \\
\text{(Demand 3)} \quad & x_{A3} + x_{B3} \geq 10 \\
\text{(Non-negativity)} \quad & x_{ij} \geq 0 \quad \text{for all } i \in \{A, B\}, j \in \{1, 2, 3\}
\end{aligned}
$$

2. Using Variable Names

Let $I = \{A, B\}$ be the set of plants. Let $J = \{1, 2, 3\}$ be the set of demand centers. For every plant $i \in I$ and demand center $j \in J$, let c_{ij} be the unit transportation cost from plant i to demand center j. Let q_i be the capacity at plant i. Let d_j be the demand requirement at center j. let x_{ij} denote the amount transported from plant i to plant j. The above LP becomes,

$$
\begin{aligned}
\text{minimize:} \quad & c_{A1}x_{A1} + c_{A2}x_{A2} + c_{A3}x_{A3} + c_{B1}x_{B1} + c_{B2}x_{B2} + c_{B3}x_{B3} \\
\text{subject to:} \quad & \\
\text{(Capacity A)} \quad & x_{A1} + x_{A2} + x_{A3} \leq q_A \\
\text{(Capacity B)} \quad & x_{B1} + x_{B2} + x_{B3} \leq q_B \\
\text{(Demand 1)} \quad & x_{A1} + x_{B1} \geq d_1 \\
\text{(Demand 2)} \quad & x_{A2} + x_{B2} \geq d_2 \\
\text{(Demand 3)} \quad & x_{A3} + x_{B3} \geq d_3 \\
\text{(Non-negativity)} \quad & x_{ij} \geq 0 \quad \text{for all } i \in I, j \in J
\end{aligned}
$$

3. Using Summation Notation

The previous LP can be written as follows in summation notation.

$$
\begin{array}{lll}
\text{minimize:} & \displaystyle\sum_{i \in I} \sum_{j \in J} c_{ij} x_{ij} & \\
\text{subject to:} & & \\
\text{(Capacity)} & \displaystyle\sum_{j \in J} x_{ij} \leq q_i & \text{for each plant } i \in I. \\
\text{(Demand)} & \displaystyle\sum_{i \in I} x_{ij} \geq d_j & \text{for each demand center } j \in J. \\
\text{(Non-negativity)} & x_{ij} \geq 0 & \text{for all } i \in I, j \in J.
\end{array}
$$

4. Implementing in Python

[13]:
```python
# Explicitly constructing a simple production planning LP
import gurobipy as grb
import pandas as pd

# Data
I=['A','B'] # plants
J=[1,2,3] # demand centers
q=pd.Series([20,15],index=I) # capacities for each plant
d=pd.Series([10,10,10],index=J) # demand from each center
c=pd.DataFrame([[3,7,5],[5,3,3]],index=I,columns=J) # unit transportation costs

# Build model
mod=grb.Model()
x={}
for i in I:
    for j in J:
        x[i,j]=mod.addVar(lb=0,name='x[{0},{1}]'.format(i,j))
mod.setObjective(sum(c.loc[i,j]*x[i,j] for i in I for j in J),sense=grb.GRB.MINIMIZE)
cap={}
for i in I:
    cap[i]=mod.addConstr(sum(x[i,j] for j in J)<=q.loc[i],name='Capacity {0}'.format(i))
for j in J:
    mod.addConstr(sum(x[i,j] for i in I)>=d.loc[j],name='Demand {0}'.format(j))

# Solve and save output to Excel
mod.setParam('OutputFlag',False)
mod.optimize()

print('Optimal Objective:', mod.ObjVal)        # Value of optimal solution
outTable=[]
for i in cap:
    outTable.append([i,cap[i].PI])
outDf=pd.DataFrame(outTable,columns=['Plant','Shadow Price'])
outDf.to_excel('output.xlsx')
outDf
```

```
Optimal Objective: 100.0
```

	Plant	Shadow Price
0	A	0.0
1	B	-2.0