

## Homework 2 (Due before class on 1/23)

### Learning Objectives Assessed:

- Identify types of uncertainty that can be appropriately modeled as random variables. (Model)
- Apply basic concepts in probability to solve problems. (Analyze)
- Apply a decision tree to model sequential decision making and find the optimal solution. (Model and Analyze)

1. For each of the following uncertainties, decide whether or not you would model it as a random variable. If so, describe briefly how will you go about estimating the underlying probabilities. If not, describe why it would not be appropriate and what might be a better alternative to cope with the uncertainty? (There may not be a definitive "right answer." This question assesses how you think. What's important is the arguments you make.)

- a) The Dow Jones Industrial Average exactly one month from today.
- b) Whether or not there will be a nuclear holocaust in the next five years.
- c) The outcome of the next presidential election.
- d) (Suppose you are part of the airport security defending against terrorist attacks, and whatever you decide might be leaked to the terrorists.) Where in the airport will the next attack will take place.

2. **(DMD Exercise 2.1)** A four-sided die is engraved with the numbers 1 through 4 on its four different sides. Suppose that when rolled, each side (and hence each number) has an equal probability of being the bottom face when it lands. We roll two such dice. Let  $X$  be the sum of the numbers on the bottom faces of the two dice.

- a) What is the probability that  $X$  is at least five?
- b) How does your answer to (a) change if you are told that the bottom face of the first die has the number 3 on it?
- c) How does your answer to (a) change if you are told that the bottom face of one of the dice has the number 3 on it?

3. **(DMD Exercise 2.2)** We toss a coin three times. Let the outcome of this experiment be the sequence of heads (H) and tails (T) resulting from the three tosses.

- a) Enumerate all of the possible outcomes of this experiment.
- b) What is the probability of the outcome "HHT"?
- c) What is the probability of the event "The first two tosses resulted in heads"?
- d) What is the probability of the event "There were two heads in a row among the three tosses"?

4. **(DMD Exercise 2.4)** An oil company is drilling for oil at three promising sites. According to geological tests, the probabilities of finding oil at these three sites are 0.70, 0.85, and 0.80, respectively. The presence of oil at any one of the sites is presumed to be independent of the presence of oil at any of the other sites.

- a) What is the probability of finding oil at all three of the sites?
- b) What is the probability of not finding oil at any of the three sites?

**5. (DMD Exercise 2.8)** On a television game show, there are three boxes. Inside one of the boxes there is a check for 10,000 dollars. If you pick the box that contains the check, you keep the money. Suppose you pick one of the boxes at random. The host of the game opens one of the other boxes and reveals that it is empty. The host then offers you the chance to change your pick. Should you change your pick? If so, why? If not, why not?

**6. (DMD Exercise 2.10)** A hardware store has received two shipments of halogen lamps. The first shipment contains 100 lamps, 4% of which are defective. The second shipment contains 50 lamps, 6% of which are defective. Suppose that Emanuel picks a lamp (at random) off of the shelf and purchases it, and he later discovers that the lamp he purchased is defective. Is the defective lamp more likely to come from the first shipment or from the second shipment?

**7. (DMD Exercise 2.11)** It is estimated that one third of the population in a given county is infected with the tuberculosis (TB) bacteria. The human body is usually able to successfully fight the TB bacteria and so prevent the onset of the TB disease. Consequently, a person infected with the TB bacteria has only a 10% chance of development the TB disease over his/her lifetime.

- a) Suppose that we choose a person at random from all of the people in the county. What is the probability that this person has the TB disease?
- b) Suppose that among those people who have died in the county in the last year, we perform an autopsy on one of the bodies chosen at random, and we find that this person did not have the TB disease. What is the probability that this person had been infected with the TB bacteria?

**8. (DMD Section 1.6, "Buying a House" case)** This problem begins with the text "Debbie and George Calvert are thinking of making an offer to purchase a house..."

**9. (DMD Exercise 1.4)** This problem begins with the text "Anders and Michael were classmates in college. In their spare time while undergraduates, they developed a software product that regulates traffic on Internet sites..."

**10. (DMD Exercise 1.6)** This problem begins with the text "James McGill, age 68, was recently diagnosed with a particular type of brain tumor..."