In Class Exercises (3/6)

There are 2 production plants, A and B, with capacities 20 and 15 respectively. There are 3 demand centers, 1, 2, 3, with demand of 10 each. The cost of transporting each unit of good from each plant to each demand center is shown below.

	1	2	3
A	3	7	5
B	5	3	3

The following LP minimizes total transportation cost subject to satisfying demand at all three centers and not exceeding the capacity of each plant.

Decision variables: x_{A1} is the amount to be shipped from plant A to region 1, x_{A2} is from plant B to region 2, etc.

$$\begin{aligned}
&\text{minimize:} && 3x_{A1} + 7x_{A2} + 5x_{A3} + 5x_{B1} + 3x_{B2} + 3x_{B3} \\
&\text{subject to:} \\
&\text{(Capacity A)} && x_{A1} + x_{A2} + x_{A3} \leq 20 \\
&\text{(Capacity B)} && x_{B1} + x_{B2} + x_{B3} \leq 15 \\
&\text{(Demand 1)} && x_{A1} + x_{B1} \geq 10 \\
&\text{(Demand 2)} && x_{A2} + x_{B2} \geq 10 \\
&\text{(Demand 3)} && x_{A3} + x_{B3} \geq 10 \\
&\text{(Non-negativity)} && x_{ij} \geq 0 \quad \text{for all } i \in \{A, B\},\ j \in \{1, 2, 3\}
\end{aligned}$$

The optimal solution and shadow prices are as follows:

	1	2	3
A	10	0	5
B	0	10	5

	Capacity A	Capacity B	Demand 1	Demand 2	Demand 3
Shadow Price	0	-2	3	5	5

Exercise 1: Write a sentence to interpret the shadow price of each constraint.

Exercise 2: Write the LP formulation corresponding to the following code.

```
[2]: # Explicitly constructing a simple production planning LP
     import gurobipy as grb
     mod=grb.Model()

     X=mod.addVar(lb=0)
     Y=mod.addVar(lb=0)

     mod.setObjective(30*X+40*Y,sense=grb.GRB.MAXIMIZE)

     mat1=mod.addConstr(2*X+3*Y <=100)
     mat2=mod.addConstr(3*Y<=75)
     labor=mod.addConstr(X+Y<=30)

     # Do not print anything when calling mod.optimize()
     mod.setParam('OutputFlag',False)
     mod.optimize()

     print('Optimal objective: {0:.2f}'.format(mod.ObjVal))
     print('Optimal solution:')
     print('\tX= {0:.2f}'.format(X.x))
     print('\tY= {0:.2f}'.format(Y.x))
     print('Shadow prices:')
     print('\tMaterial 1: {0:.2f}'.format(mat1.PI))
     print('\tMaterial 2: {0:.2f}'.format(mat2.PI))
     print('\tLabor: {0:.2f}'.format(labor.PI))
```

```
Optimal objective: 1150.00
Optimal solution:
        X= 5.00
        Y= 25.00
Shadow prices:
        Material 1: 0.00
        Material 2: 3.33
        Labor: 30.00
```

Exercise 3: Solve the LP in exercise 1 using Gurobi.