Mesoscopic quantum superposition of the generalized cat state: A diffraction limit

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The orthogonality of cat and displaced cat states, underlying Heisenberg limited measurement in quantum metrology, is studied in the limit of a large number of states. The mesoscopic superposition of the generalized cat state is correlated with the corresponding state overlap function, controlled by the sub-Planck structures arising from phase-space interference. The asymptotic expression of this overlap function is evaluated, and the validity of large phase-space support and distinguishability of the constituent states, in which context the asymptotic limit is achieved, are discussed in detail. For a large number of coherent states, uniformly located on a circle, the overlap function significantly matches the diffraction pattern for a circular ring source with uniform angular strength. This is in accordance with the van Cittert–Zernike theorem, where the overlap function, similar to the mutual coherence function, matches a diffraction pattern. The physical situation under consideration is delineated in phase space by utilizing the Husimi Q function.

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I. INTRODUCTION

Cat states and their generalizations are known to achieve Heisenberg limited sensitivity in the estimation of parameters like coordinate or momentum displacement and phase-space rotation [1]. A criterion to distinguish quantum states without classical counterparts from those not possessing the same are studied in [2,3]. For these nonclassical states, subtle interference effects in the phase space [4] lead to sub-Planck structures in their Wigner functions, which in turn allow precision measurement of quantum parameters, bettering the standard quantum limit. Recently, sub-Planck structures in different physical systems have been investigated [5–13]. It has been demonstrated [7,9,10] that the sensitivity of the state used in quantum metrology is directly related to the area of the sub-Planck structures: $\rho = \frac{\hbar^2}{A}$, with A being the action of the effective support of the Wigner function. The interference in phase space is a pure quantum phenomenon, arising from the fact that these states are a superposition of the coherent states (CSs), which themselves are classical. The increase in the number of interfering coherent states in the phase space is akin to the emergence of diffraction in classical optics, when the number of interfering sources becomes large with sufficient phase-space support.

Here, we analyze this diffraction limit of the smallest interference structures and find an exact asymptotic value of the displacement sensitivity. With the assumption of large phase-space support for the estimating state and smallness of the quantum parameters to be estimated, it is found that the asymptotic limit of the sensitivity reaches $|\delta| = \frac{C}{2|\alpha|}$, where C is the first root of J_0, the zeroth-order Bessel function. We explicitly show that this assumption is adequate for realistic values of the physical parameters, i.e., the average photon number and the number of superposed CSs. The numerical analysis depicts how the asymptotic limit of exact overlap function (OF) reaches to the zeroth-order Bessel function for higher-order mesoscopic superpositions. This limiting behavior in the phase-space interference is found to be analogous to the van Cittert–Zernike theorem [14], relating the mutual coherence in classical optics to diffraction. A phase-space distribution (Q function), having only positive regions, reveals the actual physical situation at the point of resemblance between the two theories.

II. RESULTS AND DISCUSSIONS

Cat states and their generalizations play a significant role in quantum optics and quantum computation [15]. A number of experimental schemes exist to produce cat states in laboratory conditions [16]. These "pointer states" [17] often naturally manifest when suitable quantum systems are coupled with a decohering environment. It has been observed that the robustness of these states, made out of classical CSs, is a result of "quantum Darwinism" [18]. We consider a single oscillator with the CS being an eigenstate of a, $a|\alpha\rangle = \alpha|\alpha\rangle$, with annihilation and creation operators a and $a^\dagger$: $[a,a^\dagger] = 1$.

The generalized cat state is composed of CSs, equally phase displaced on a circle:

$$|\mathrm{cat}_{n,\alpha}\rangle = \frac{1}{\sqrt{n}}\sum_{j=1}^{n}\left|e^{\frac{\iota 2\pi j}{n}}\alpha\right\rangle = \frac{1}{\sqrt{n}}\sum_{j=1}^{n} D\left(e^{\frac{\iota 2\pi j}{n}}\alpha\right)|0\rangle, \quad (1)$$

where $|\alpha\rangle = D(\alpha)|0\rangle$, with the displacement operator $D(\alpha) = e^{\alpha a^\dagger - \alpha^\star a}$ and $a|0\rangle = 0$. Here, it is worth mentioning that the CSs are assumed to be distinguishable [19]. The displacements in the coordinate and momenta can be realized through an appropriately displaced cat state [7]: $|\mathrm{cat}^{\delta}_{n,\alpha}\rangle = D(\delta)|\mathrm{cat}_{n,\alpha}\rangle$.

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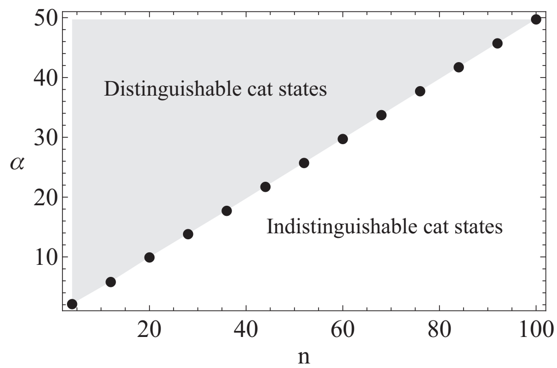

FIG. 1. Parameter domain of the phase-space support (magnitude of α) to maintain the condition of distinguishability. The shaded region designates the domain for distinguishable Schrödinger-cat states. α is the coherent-state parameter.

To check the sensitivity of the estimating state $|\mathrm{cat}_{n,\alpha}\rangle$, we compute the overlap of the same with the displaced state and study the orthogonality conditions,

$$
\begin{aligned}
&\langle \mathrm{cat}_{n,\alpha}|\mathrm{cat}^{\delta}_{n,\alpha}\rangle \\
&= \frac{1}{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\langle 0|D\big(e^{\frac{i2\pi j}{n}}\alpha\big)^{\dagger}D(\delta)D\big(e^{\frac{i2\pi k}{n}}\alpha\big)|0\rangle \\
&= \frac{1}{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\big(e^{i\mathrm{Im}[\delta\alpha^{\star}(e^{-\frac{i2\pi j}{n}}+e^{-\frac{i2\pi k}{n}})+|\alpha|^2 e^{-\frac{i2\pi(k-j)}{n}}]}\big) \\
&\quad\times\big(e^{-\frac{1}{2}|\delta+\alpha(e^{-\frac{i2\pi k}{n}}-e^{-\frac{i2\pi j}{n}})|^2}\big) \qquad (2)
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{1}{n}\sum_{j=1}^{n}\sum_{k=1}^{n}e^{i[2r\cos(\frac{\pi(j-k)}{n})\,\sin(\theta-\frac{\pi(j+k)}{n})+|\alpha|^2\,\sin(\frac{2\pi(j-k)}{n})]} \\
&\quad\times e^{-\frac{1}{2}\{|\delta|^2+2|\alpha|^2[1-\cos(\frac{2\pi(j-k)}{n})]+4r\,\sin(\frac{\pi(j-k)}{n})\sin(\theta-\frac{\pi(j+k)}{n})\}}, \qquad (3)
\end{aligned}
$$

where $r = |\alpha||\delta|$ and $\theta = (\theta_\delta - \theta_\alpha)$, with $\alpha = |\alpha|e^{i\theta_\alpha}$ and $\delta = |\delta|e^{i\theta_\delta}$.

The phase space of the generalized cat state of Eq. (1) is composed of n CSs, equally placed in a circle of radius $|\alpha|$, where large phase-space support means a large magnitude of $|\alpha|$. Now it is worth finding the phase-space support to maintain the distinguishability for a given mesoscopic superposition, i.e., domain of α for a given n? This is delineated in Fig. 1, where we have taken up to a large value of n (e.g., $n = 100$). The upper region of the plot (shaded) depicts the allowed parameter domain of α for mesoscopic superposition of CSs. Table I reveals the values of α (accurate up to the first decimal place), above which the states are distinct. We further observe that the maximum value of the ratio n/α to maintain distinguishability takes the average value $\nu = 2.016$. Thus, one does not need a very large α to generate the said interference structures in phase space. In fact, increasing n is quite difficult in experiments, as it requires a large nonlinearity of the medium. On the contrary, the absolute value of α is directly related to the average photon number of the coherent state, which can be manipulated by controlling the laser beam. Hence, the allowed maximum order of mesoscopic superposition n for a given α, conforming our result, is sufficiently large in reality.

TABLE I. Estimate of the numerically obtained minimum value of phase-space area (proportional to $|\alpha|$) required for a mesoscopic superposition of n CSs.

n	4	12	20	28	36	44	52	60	68	76	84	92	100
α	2.1	5.8	9.9	13.8	17.7	21.7	25.7	29.7	33.7	37.7	41.7	45.7	49.7

It is important to mention that the OF between the initial and displaced cat states can also be represented by the phase-space Wigner distribution:

$$
|\langle \mathrm{cat}_{n,\alpha}|\mathrm{cat}^{\delta}_{n,\alpha}\rangle|^2 = \iint W_{\mathrm{cat}_{n,\alpha}}(x,p)W_{\mathrm{cat}^{\delta}_{n,\alpha}}(x,p)dx\,dp. \qquad (4)
$$

This relation reveals the physical significance of the oscillations of the OF in a particular direction in phase space and connects our result with the mesoscopic superposition structures. The oscillation of the OF is the signature of quantum interference structures of dimension less than Planck's constant, i.e., sub-Planck-scale structures. Each zero of the OF signifies the orthogonality of the original and displaced states, thereby implying the sensitivity limit of Heisenberg limited measurement.

Now, it is intuitive as well as numerically verified by us that the entire contribution of the OF in Eq. (2) or (3) mainly originates from the adjacent components of the original and displaced cat states, i.e., $j \sim k$. Therefore, $|j-k| \ll n$, $\cos[\pi(j-k)/n] \to 1$, and $\sin[\pi(j-k)/n] \to 0$. Then Eq. (3) takes the simpler form

$$
\begin{aligned}
&\langle \mathrm{cat}_{n,\alpha}|\mathrm{cat}^{\delta}_{n,\alpha}\rangle \\
&= \frac{e^{-\frac{1}{2}|\delta|^2}}{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\cos\left[2r\,\sin\left(\theta-\frac{\pi(j+k)}{n}\right)\right]. \qquad (5)
\end{aligned}
$$

The off-diagonal terms in the above expression have a negligible contribution. This assumption bears similar meaning in the classical situation, where an incoherent ring source is assumed, for which the cross correlations between the different points of the source can be neglected. Now, with the assumption of sufficient phase-space support for the estimating state and the smallness of the quantum parameters to be estimated, one can consider only the diagonal terms and obtain

$$
\begin{aligned}
\langle \mathrm{cat}_{n,\alpha}|\mathrm{cat}^{\delta}_{n,\alpha}\rangle &\approx \frac{e^{-\frac{1}{2}|\delta|^2}}{n}\sum_{j=1}^{n}\cos\left[2r\,\sin\left(\theta-\frac{2\pi j}{n}\right)\right] \\
&\approx \frac{1}{n}\sum_{j=1}^{n}\cos\left[2r\,\sin\left(\theta-\frac{2\pi j}{n}\right)\right]. \qquad (6)
\end{aligned}
$$

It needs to be mentioned that the state overlap depends only on $\delta\alpha^{\star}$, which leads to the conclusion that the sensitivity of estimating δ is inversely proportional to $|\alpha|$. It is easily checked that the OF, being of interferometric origin, is only sensitive to the difference in phase: $\langle \mathrm{cat}^{\delta_2}_{n,\alpha}|\mathrm{cat}^{\delta_1}_{n,\alpha}\rangle = e^{i\phi}\langle \mathrm{cat}_{n,\alpha}|\mathrm{cat}^{\delta_1-\delta_2}_{n\alpha}\rangle$. The OF for $n = 2$

$$
|\langle \mathrm{cat}_{2,\alpha}|\mathrm{cat}^{\delta}_{2,\alpha}\rangle|^2 \approx \cos^2(2|\alpha|\delta_\perp), \qquad (7)
$$

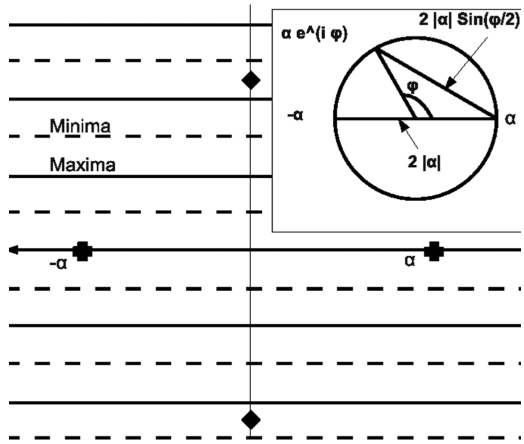

FIG. 2. Analogy with two-source interference: The solid lines show the maximum, and dashed lines show the minimum intensity values. The crosses are the positions of coherent states, and diamonds show the equivalent positions of sources of light which will produce the same pattern at a distance. The inset shows the equivalent position of sources for the state $|\mathrm{cat2}_{\alpha,\phi}\rangle = \frac{|\alpha\rangle + |\alpha e^{\iota\phi}\rangle}{2}$

which matches the known result [7], with $\delta_\perp = |\delta|\sin(\theta_\delta - \theta_\alpha)$ and $\delta_\parallel = |\delta|\cos(\theta_\delta - \theta_\alpha)$. As depicted in Fig. 2, it is interesting to observe that the above expression is analogous to the double-slit interference pattern, where the normalized intensity can be written as $\frac{I}{I_{\max}} = \cos^2(\frac{yb\pi}{s\lambda})$ [20,21]. The path difference between the two waves reaching the observation point is yb/s, where b defines the distance between the two slits, s is the separation between the aperture and the screen, and y corresponds to the vertical coordinate of the detector. The above analogy can be mathematically established by taking λ in the unit of s and redefining the commutation relation, $[a,a^\dagger] = \pi\lambda^{-1}$:

$$|\langle \mathrm{cat}_{2,\alpha}|\mathrm{cat}^\delta_{2,\alpha}\rangle|^2 = \cos^2\left[2\frac{|\alpha|\delta_\perp\pi}{\lambda}\right], \tag{8}$$

where $2|\alpha|$ is the separation of the two coherent-state sources. Use of the phase-shifted cat state, $|\mathrm{cat2}_{\alpha,\phi}\rangle = \frac{|\alpha\rangle + |\alpha e^{\iota\phi}\rangle}{2}$, would yield an interference pattern at an angle $\frac{\phi}{2}$ and *fringe width* $2|\alpha|\sin\frac{\phi}{2}$:

$$|\langle \mathrm{cat2}_{\alpha,\phi}|\mathrm{cat2}^\delta_{\alpha,\phi}\rangle|^2 = \cos^2\left[2|\alpha|\sin\frac{\phi}{2}\left(\delta_\perp\sin\frac{\phi}{2} + \delta_\parallel\cos\frac{\phi}{2}\right)\right]. \tag{9}$$

Introducing a phase between the constituent CSs of a cat state with $n = 2$ gives the state $|\mathrm{cat2}^\phi_\alpha\rangle = \frac{|\alpha\rangle + e^{\iota\phi}|-\alpha\rangle}{2}$. The OF for this state is

$$|\langle \mathrm{cat2}^\phi_\alpha|\mathrm{cat2}^{\phi,\delta}_\alpha\rangle|^2 = \cos^2(2|\alpha|\delta_\perp) - \phi), \tag{10}$$

akin to the phenomenon of "fringe shift" observed in classical optics.

We now derive the state overlap and sensitivity in parameter estimation for a very high order of mesoscopic superpositions. For convenience, we assume n is even:

$$\langle \mathrm{cat}_{n,\alpha}|\mathrm{cat}^\delta_{n,\alpha}\rangle = \frac{2}{n}\sum_{j=1}^{\frac{n}{2}}\cos\left[2r\sin\left(\theta - \frac{2\pi j}{n}\right)\right]. \tag{11}$$

In the asymptotic limit of n, one writes

$$\begin{aligned}
&\lim_{n\to\infty,\ n/\alpha\leqslant\nu}\langle \mathrm{cat}_{n,\alpha}|\mathrm{cat}^\delta_{n,\alpha}\rangle\\
&= \lim_{n\to\infty,\ n/\alpha\leqslant\nu}\frac{2}{n}\sum_{j=1}^{\frac{n}{2}}\cos\left[2r\sin\left(\theta - \frac{2\pi j}{n}\right)\right]\\
&= \lim_{n\to\infty,\ n/\alpha\leqslant\nu}\frac{1}{n}\sum_{j=1}^{n}\cos\left[2r\sin\left(\theta - \frac{2\pi j}{n}\right)\right]\\
&= \int_0^1 \cos[2r\sin(\theta - 2\pi x)]dx\\
&= \frac{1}{2\pi}\int_0^{2\pi}\cos[2r\sin(z)]dz\\
&= J_0(2|\alpha||\delta|).
\end{aligned} \tag{12}$$

It is worth mentioning that, in the above equation, the summation is converted to integration by making (j/n) an integration variable (x) for $n \to \infty$. However, in a realistic example, the domain of the parameter α should always be maintained: $n/\alpha \leqslant \nu$.

In Eq. (12), the result proves our assertion that states can be discriminated for $|\delta| = \frac{C}{2|\alpha|}$ due to orthogonality, where C is a root of the Bessel function (of the first kind) of order zero, i.e., J_0. Here, we evaluate the overlap function for a very high order of mesoscopic superposition. Theoretically, a limit $n \to \infty$ is taken, provided and implied that the states are still distinguishable. It is worth mentioning that, theoretically, the parameter α also does not have any upper limit and, in principle, can go to infinity. Thus, the Bessel function is

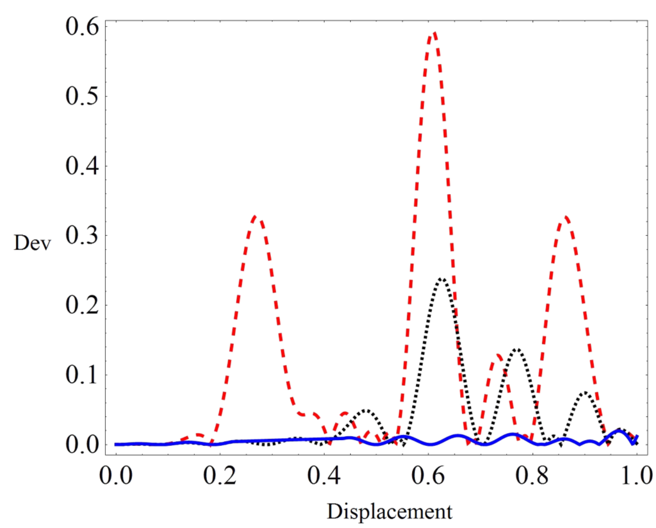

FIG. 3. (Color online) Deviation (Dev) of the amplitude of the overlap function [Eq. (3)] from the zeroth-order Bessel function, with respect to the displacement in phase space for $\alpha = 15$: $n = 8$ (dashed line), $n = 16$ (dotted line), and $n = 30$ (solid line). For a larger value of n, the OF almost coincides with the zeroth-order Bessel function.

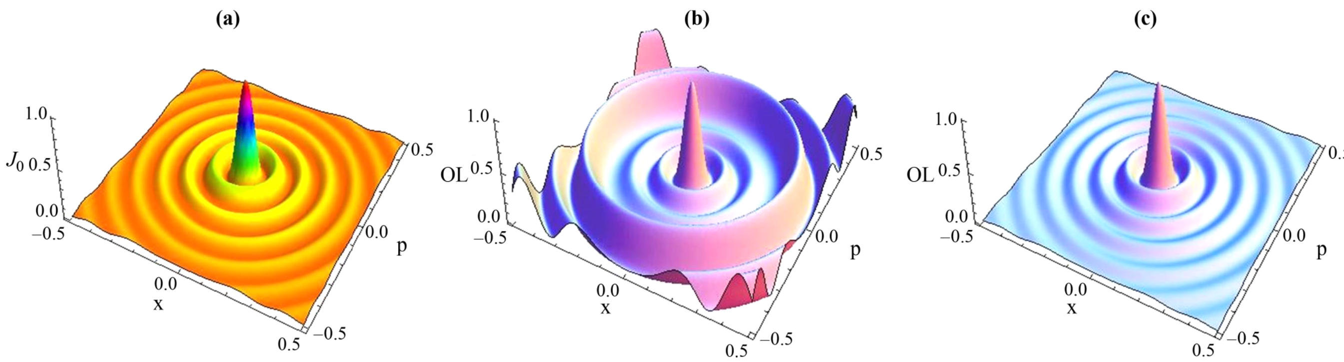

FIG. 4. (Color online) (a) Three-dimensional plot of the Bessel function of zeroth order (J_0) for $\alpha = 15$, (b) 3D plot of overlap (OL) for $\alpha = 15$, $n = 10$, and (c) 3D plot of OL for $\alpha = 15$ for $n = 30$.

obtained without any further restriction. In reality, neither n nor α can go to infinity, and thus, one has to take the practical quantitative estimate of these two physical parameters, as described in Fig. 1 and Table I. The distinguishability condition implies $n/\alpha \leqslant \nu$, where the average value of ν is 2.016.

In Fig. 3, we have plotted the deviation (deviation = OF − Bessel function) with respect to the displacement in phase space for three different values of n with $\alpha = 15$. The best result is obtained for $n = 30$ or for $n/\alpha \approx \nu$, beyond which the constituent CSs become indistinguishable. Thus, for higher-order mesoscopic superposition, our result fits very well with the condition when the phase-space support is sufficient enough. The result is verified by a three-dimensional plotting of the functions in Fig. 4. Figure 4(a) corresponds to the Bessel function for $\alpha = 15$ with respect to the real and imaginary components of the displacement parameter. The same is also performed for the overlap function. The OF for $n = 10$ [Fig. 4(b)] does not match the Bessel function, whereas the oscillations find a remarkable similarity with the Bessel function for $n = 30$ [Fig. 4(c)].

Husimi Q function: So far, we have been discussing the OF, which is a result of superposition of n CSs. However, it becomes a natural question to ask, What is the physical situation of the CSs on a circle for the critical ratio $n/\alpha = \nu$? We have tried to explore the answer by calculating the Husimi Q function. The Q function is a phase-space distribution, which is always positive and does not include any interference structures. Hence, plotting the Q function for a higher-order superposition is less time-consuming. The Q function is calculated and then plotted in Figs. 5(a)–5(c) for $\alpha = 9.9$, $n = 20$; $\alpha = 15.9$, $n = 32$; and $\alpha = 19.9$, $n = 40$, respectively. These cases are evaluated for $n/\alpha = \nu$ and show striking support for our intuition that the CSs are actually just touching each other and start becoming indistinguishable, thereby creating an extended source of ring-shaped light with a radius $\sqrt{2}\alpha$. The above condition has established the required physical situation in classical optics to obtain ring-shaped light source with constant angular source strength.

Thus, the overlap function [Eq. (12)] is the result of superposition of n CSs situated in a ring of radius $\sqrt{2}\alpha$. Hence, the superposition is analogous to the diffraction pattern generated when light passes through the thin ring-shaped opening. The van Cittert–Zernike theorem [14] states that the diffraction problem is identical to the coherence problem, and the two problems result in the same mathematical formalism through the quantity called the complex degree of coherence.

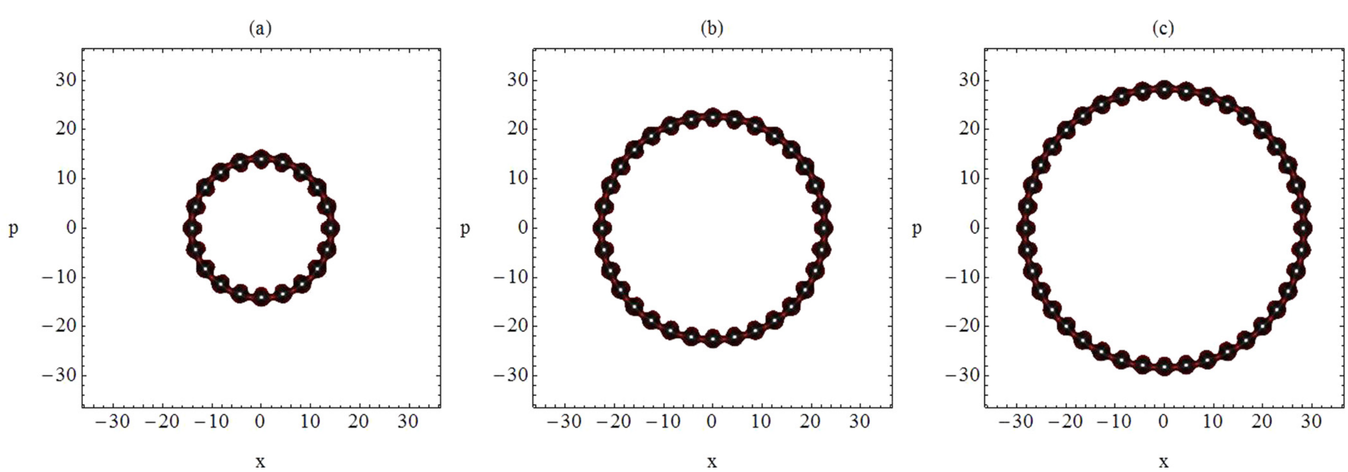

FIG. 5. (Color online) Q function in phase space for different α: (a) $\alpha = 9.9$, $n = 20$; (b) $\alpha = 15.9$, $n = 32$; and (c) $\alpha = 19.9$, $n = 40$. We have plotted, maintaining the critical ratio, $n/\alpha = \nu$, where the CSs are just touching each other.

In this work, n coherent states, symmetrically situated in circle, produce interference ripples in the center. This phase-space interference structure is known to manifest in the overlap function between the original and displaced cat states. We have shown that this interference term is analogous to the diffraction pattern resulting from the equivalent optical sources after proper scaling of the parameters. Thus, the fact that the overlap between the cat states and their shifted forms is of the same form as the diffraction pattern centered at one of the states bears strong resemblance to the van Cittert–Zernike theorem. Here, the normalized mutual coherence function $\gamma_{12}(0)$ for a ring-shaped opening with constant angular source strength can be written explicitly as

$$\begin{aligned}\gamma_{12}(0) &= \frac{\langle E_1(t)E_2(t)^*\rangle_T}{\sqrt{\langle E_1(t)E_1(t)^*\rangle_T\langle E_2(t)E_2(t)^*\rangle_T}}\\ &= J_0\left(\frac{2\pi r_0|\vec{r}_1-\vec{r}_2|}{\lambda R}\right). \end{aligned}\tag{13}$$

Here, $\gamma_{12}(0)$ actually signifies the complex degree of spatial coherence of the two points at the same instance in time, with fields arriving at the observation screen being $E_1(t)$ and $E_2(t)$. r_0 is the radius of the ring, R is the distance of the screen from the opening, and $|\vec{r}_1-\vec{r}_2|$ is the path difference between the points.

The suffix T in the expectation value signifies the time average according to the ergodic hypothesis. The above equation should be compared with the OF for large n [Eq. (12)] for unit distance from the screen to the opening ($R=1$) and for $[a,a^\dagger]=\pi\lambda^{-1}$:

$$\langle \text{cat}_{n,\alpha}|\text{cat}^{\delta}_{n,\alpha}\rangle = J_0\left(\frac{2\pi|\alpha||\delta|}{\lambda}\right).\tag{14}$$

III. CONCLUSIONS

In conclusion, the sensitivity of catlike states to quantum parameter estimation is studied for a large number of constituent CSs. The phase-space support is explored for accessible parameter ranges in a realistic situation. We provide a quantitative estimate of the phase-space support for a given superposition. In the large-n limit, the state OF, determining the orthogonality of cat and displaced cat states, approaches the Bessel function. According to the van Cittert–Zernike theorem, the coherence problem is mathematically identical to the diffraction problem by the complex degree of coherence. The fact that the OF has the same form as the diffraction pattern results in the same expression of normalized mutual coherence function for large n. This is similar to the mutual coherence function of a circular ring, which yields a Bessel function of order zero, matching the theorem of van Cittert and Zernike. In addition, this work opens up several scopes for future studies: (i) Mesoscopic superposition is a purely quantum phenomenon, and the overlap function plays a very crucial role in quantum parameter estimation. The overlap function, without an asymptotic limit, can be investigated for different relative phases (θ_α and θ_δ) of the CS and displacement parameters, which will provide information in various directions in phase space. (ii) Studying mesoscopic superposition in realistic quantum systems, which are not modeled by harmonic oscillator CSs, is quite nontrivial in general. Hence, it has become common practice to study a harmonic oscillator system and use that knowledge to investigate other solvable quantum-mechanical potentials. (iii) Quantum sensitivity has become a very fascinating area of research, where it is known that increasing the value of the CS parameter will make the system more and more sensitive. However, it was completely unknown that the OF would saturate to the Bessel function. This fact can be further utilized to investigate various physical situations of interest.

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[19] Distingushibility is one of the essential criteria for observing the interference structures of nonlocal superposition of localized wave packets. We have considered the superposition of distinguishable CSs. In some other context, if one needs to consider indistinguishable CSs, which themselves overlap, the normalization will be $\sqrt{1/\sum_j^n \sum_k^n \exp[-|\alpha|^2(1-\exp\{-[2i\pi(j-k)]/n\})]}$. For distinguishable states $j=k$, and it becomes $1/\sqrt{n}$, where n is the total number of CSs.

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