MEASUREMENT OF THE HIGHER MOMENTS OF TRANSVERSE MOMENTUM OF CHARGED PARTICLES IN PROTON-PROTON COLLISIONS

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ABSTRACT

We aim to analyse p-p collisions by the studying the transverse momenta of the ejected particles. We reproduce the results obtained by Giuliano Giacalone, Fernando G. Gardim *et al* in their paper on mean transverse momentum fluctuations in heavy-ion collisions [1]. Using the data generated from the Pythia 8 generator, we plot the mean transverse momenta of multiple p-p collisions. We verify the positively skewed deviation of mean transverse momentum $\langle \mathbf{p_T} \rangle$ from Gaussian distribution. First, we divide the data into different multiplicity classes and plot the distribution of $\mathbf{p_T}$ and $\langle \mathbf{p_T} \rangle$. Then we observe how mean and intensive variance of $\mathbf{p_T}$ changes over different multiplicity classes. Finally, we find intensive and standardized skewness of $\mathbf{p_T}$ for different multiplicity classes.

I. INTRODUCTION

For a given collision centrality, the mean transverse momentum $\langle \mathbf{p_T} \rangle$ of emitted particles in ultrarelativistic nucleus-nucleus collisions fluctuates from one event to another. The distribution of $\langle \mathbf{p_T} \rangle$ in event-by-event dynamics reflects the various statistical and dynamical fluctuations.

So, in this project, through our plots, we will show that the probability distribution of $\langle \mathbf{p_T} \rangle$ is **not Gaussian** but has **positive skew**, which arises because of the above-mentioned fluctuations.

We then go on to plot two dimensionless measures of skewness versus different multiplicity classes, namely **standardized skewness** and **intensive skewness**, out of which the first depends on centrality and system size, whereas the second has the property of being independent of the system size. Since these are dimensional quantities, both of these are expected to be less sensitive to analysis details, such as those dependent on the detector.

We shall be using the following definitions as per the **STAR** collaboration.

$$\text{Mean Transverse Momentum} = \langle\langle p_T \rangle\rangle = \langle \frac{\Sigma_{i=1}^{N_{ch}} p_i}{N_{ch}} \rangle \tag{1}$$

where N_{ch} denotes the number of charged particles in an event, p_i is the transverse momentum of the ith particle and angular brackets denote an average over events in a centrality class.

We analyze the variance of dynamical p_T fluctuations which we denote by $\langle \Delta p_i \Delta p_j \rangle$,

$$\langle \Delta p_i \Delta p_j \rangle = \langle \frac{\Sigma_{i,j\neq i}(p_i - \langle\langle p_T \rangle\rangle)(p_j - \langle\langle p_T \rangle\rangle)}{N_{ch}(N_{ch}-1)} \rangle \tag{2}$$

which can also be written as

$$\langle \Delta p_i \Delta p_j \rangle = \langle (\langle p_T \rangle - \langle\langle p_T \rangle\rangle)^2 \rangle \tag{3}$$

and the intensive variance of transverse momentum which is defined as follows

$$\sigma_{p_T} = \frac{\langle \Delta p_i \Delta p_j \rangle^{1/2}}{\langle\langle p_T \rangle\rangle}. \tag{4}$$

The skewness is the third central moment, denoted by $\langle \Delta p_i \Delta p_j \Delta p_k \rangle$ defined as follows

$$\langle \Delta p_i \Delta p_j \Delta p_k \rangle = \langle \frac{\Sigma_{i,j\neq i,k\neq i,j}(p_i - \langle\langle p_T \rangle\rangle)(p_j - \langle\langle p_T \rangle\rangle)(p_k - \langle\langle p_T \rangle\rangle)}{N_{ch}(N_{ch}-1)(N_{ch}-2)} \rangle \tag{5}$$

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which can also be written as

$$\langle \Delta p_i \Delta p_j \Delta p_k \rangle = \langle (\langle p_T \rangle - \langle \langle p_T \rangle \rangle)^3 \rangle \tag{6}$$

Standardized skewness and intensive skewness denoted by γ_{p_T} and Γ_{p_T}, respectively, are defined as follows:

$$\gamma_{p_T} = \frac{\langle \Delta p_i \Delta p_j \Delta p_k \rangle}{\langle \Delta p_i \Delta p_j \rangle^{3/2}} \tag{7}$$

$$\Gamma_{p_T} = \frac{\langle \Delta p_i \Delta p_j \Delta p_k \rangle \langle \langle p_T \rangle \rangle}{\langle \Delta p_i \Delta p_j \rangle^2} \tag{8}$$

The dataset provided is generated with **Pythia 8 Monte Carlo Event Generator**.
Number of events : **2 million**
Collisions System : **p + p** at centre of mass energy **13 TeV**

II. EXPERIMENTAL OBSERVATIONS

A. Transverse Momentum and Mean Transverse Momentum for Each Multiplicity Class

In this section, we have plotted the histograms for the Transverse Momentum **pT** and the Mean Transverse Momentum ⟨**pT**⟩ of proton-proton collisions corresponding to each multiplicity class. The histogram for **pT** is then approximated using an **Exponential** fit, while that of ⟨**pT**⟩ has been approximated using a **Gaussian** fit. Both the quantities **pT** and ⟨**pT**⟩ have statistical fluctuations arising from the finite number of particles in each event. In each of the subsequent subsections corresponding to each of the 6 multiplicity classes, namely **pytree020**, **pytree2040**, **pytree4060**, **pytree6080**, **pytree80100** and **pytree100**, the histograms and the corresponding fits have been plotted. A logarithmic scale has been used on the *y-axis* in order to emphasize the skewness of the data.

1. Multiplicity Class "pytree020"

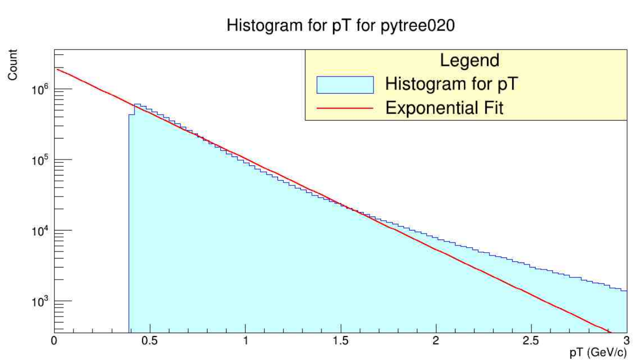

FIG. 1a. (Color Online) Distribution of **pT** for proton-proton collision in the multiplicity class **pytree020**. The solid line is an Exponential fit to the data. Owing to the large size of the data, the data has been arranged in a random order and only the first **40%** of the data has been used for analysis.

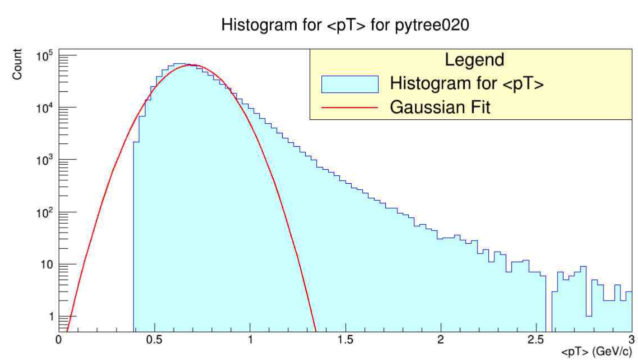

FIG. 1b. (Color Online) Distribution of ⟨**pT**⟩ for proton-proton collision in the multiplicity class **pytree020**. The solid line is a Gaussian fit to the data. Owing to the large size of the data, the data has been arranged in a random order and only the first **40%** of the data has been used for analysis.

2. Multiplicity Class "pytree2040"

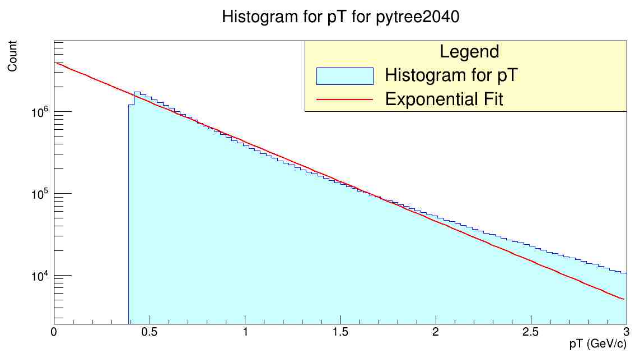

FIG. 2a. (Color Online) Distribution of **pT** for proton-proton collision in the multiplicity class **pytree2040**. The solid line is an Exponential fit to the data.

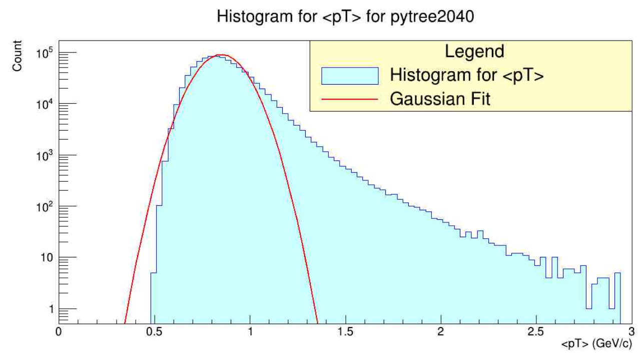

FIG. 2b. (Color Online) Distribution of $\langle \mathbf{pT} \rangle$ for proton-proton collision in the multiplicity class **pytree2040**. The solid line is a Gaussian fit to the data.

3. Multiplicity Class "pytree4060"

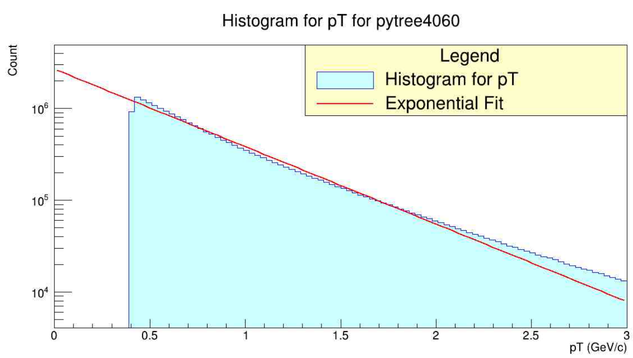

FIG. 3a. (Color Online) Distribution of **pT** for proton-proton collision in the multiplicity class **pytree4060**. The solid line is an Exponential fit to the data.

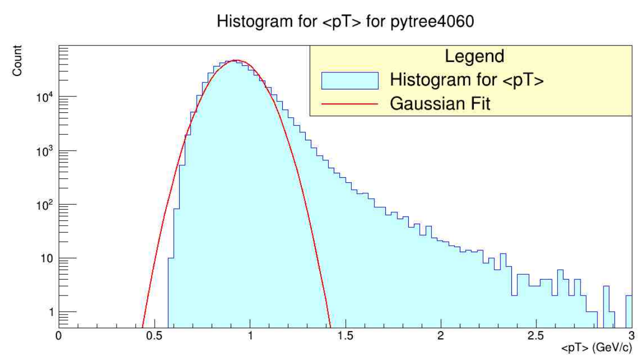

FIG. 3b. (Color Online) Distribution of $\langle \mathbf{pT} \rangle$ for proton-proton collision in the multiplicity class **pytree4060**. The solid line is a Gaussian fit to the data.

4. Multiplicity Class "pytree6080"

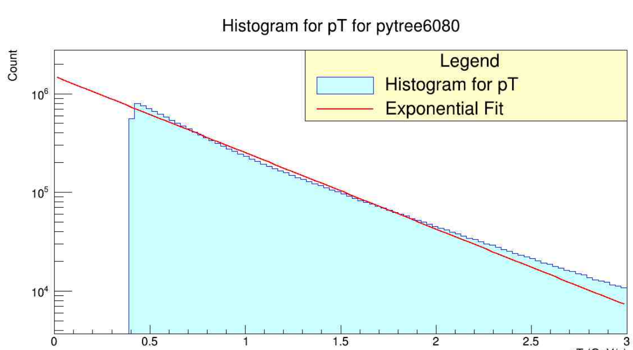

FIG. 4a. (Color Online) Distribution of **pT** for proton-proton collision in the multiplicity class **pytree6080**. The solid line is an Exponential fit to the data.

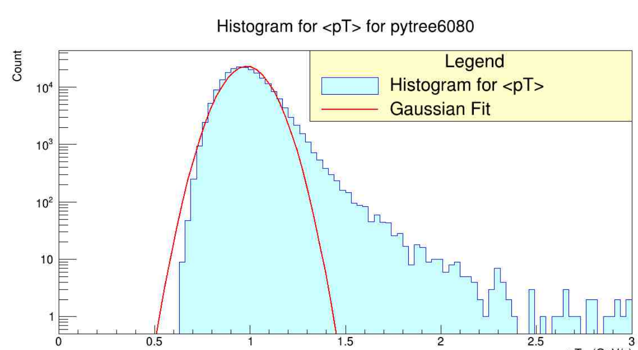

FIG. 4b. (Color Online) Distribution of $\langle \mathbf{pT} \rangle$ for proton-proton collision in the multiplicity class **pytree6080**. The solid line is a Gaussian fit to the data.

5. *Multiplicity Class "pytree80100"*

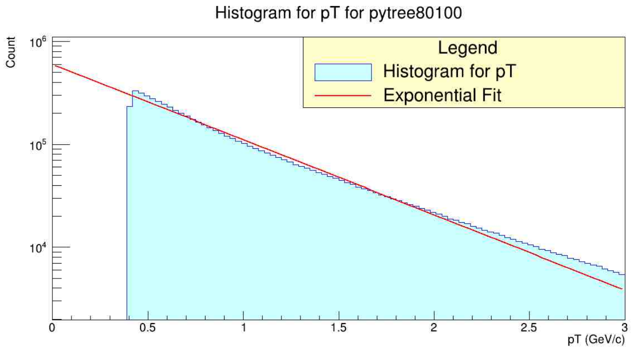

FIG. 5a. (Color Online) Distribution of **pT** for proton-proton collision in the multiplicity class **pytree80100**. The solid line is an Exponential fit to the data.

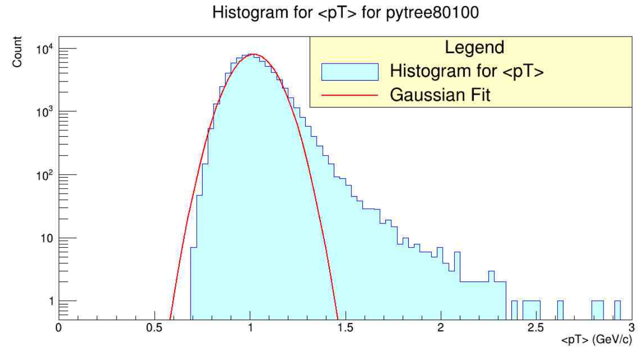

FIG. 5b. (Color Online) Distribution of $\langle \mathbf{pT} \rangle$ for proton-proton collision in the multiplicity class **pytree80100**. The solid line is a Gaussian fit to the data.

6. *Multiplicity Class "pytree100"*

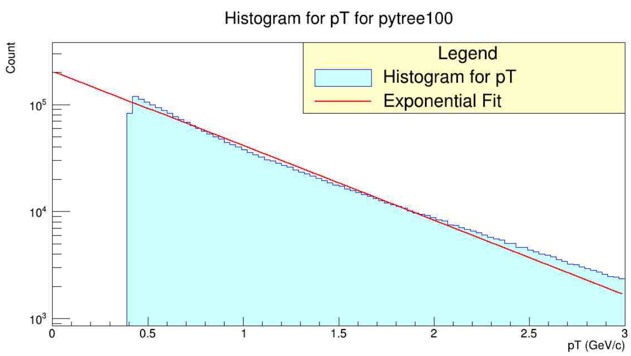

FIG. 6a. (Color Online) Distribution of **pT** for proton-proton collision in the multiplicity class **pytree100**. The solid line is an Exponential fit to the data.

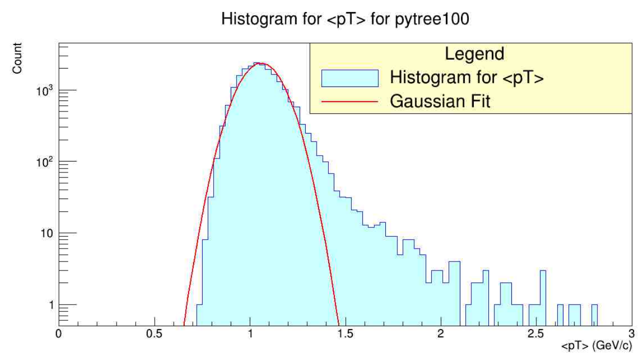

FIG. 6b. (Color Online) Distribution of $\langle \mathbf{pT} \rangle$ for proton-proton collision in the multiplicity class **pytree100**. The solid line is a Gaussian fit to the data.

B. Analysis of Mean, Variance and Skewness Versus Multiplicity Class

From the graphs in FIG. 1a., FIG. 2a., FIG. 3a., FIG. 4a., FIG. 5a. and hyperref[Fig:6a]FIG. 6a., it is clear that the Transverse Momenta of the particles produced in a proton-proton collision follows approximately an **exponential distribution**. Graphs in FIG. 1b., FIG. 2b., FIG. 3b., FIG. 4b., FIG. 5b. and hyperref[Fig:6b]FIG. 6b. reveal that there is some **positive skew** in the distribution of the Mean Transverse Momentum.

In this section, we shall analyse the moments of the distribution of Transverse Momentum. We shall calculate the Mean, Variance and Skewness of the Transverse Momenta for each multiplicity class and study its relation with the multiplicity class. For each of the multiplicity classes, the Mean Transverse Momentum, the Intensive Variance of the Transverse Momentum, the Standardized Skewness of the Transverse Momentum and the Intensive Skewness of the Transverse Momentum, calculated using formulae 1, 4, 7 and 8 respectively have been summarised in the table and the plots below. Note that for *pytree2040*, the number of events is too large for a personal computer at today's scale to handle. Hence, the dataset of this multiplicity class has been randomized and the first **40%** has been used for analysis.

1. *Summary of Data*

The table below summarizes the data.

SUMMARY OF DATA					
Multiplicity Class	**Events**	$\langle \mathbf{p_T} \rangle$ **(GeV/c)**	$\sigma_{\mathbf{p_T}}$	$\gamma_{\mathbf{p_T}}$ **(GeV/c)**	$\mathbf{\Gamma_{p_T}}$
pytree020	952256	0.750912	0.235453	1.22572	5.20581
pytree2040	873322	0.869307	0.174974	1.77742	10.1582
pytree4060	445805	0.940521	0.144591	1.80884	12.51
pytree6080	207990	0.99074	0.130471	3.11451	23.8714
pytree80100	71263	1.03006	0.122603	1.64507	13.4178
pytree100	20981	1.07257	0.132207	4.11805	31.1485

TABLE I. Table Summarizing the Data of Transverse Momenta

2. *Mean Transverse Momentum versus Multiplicity Class*

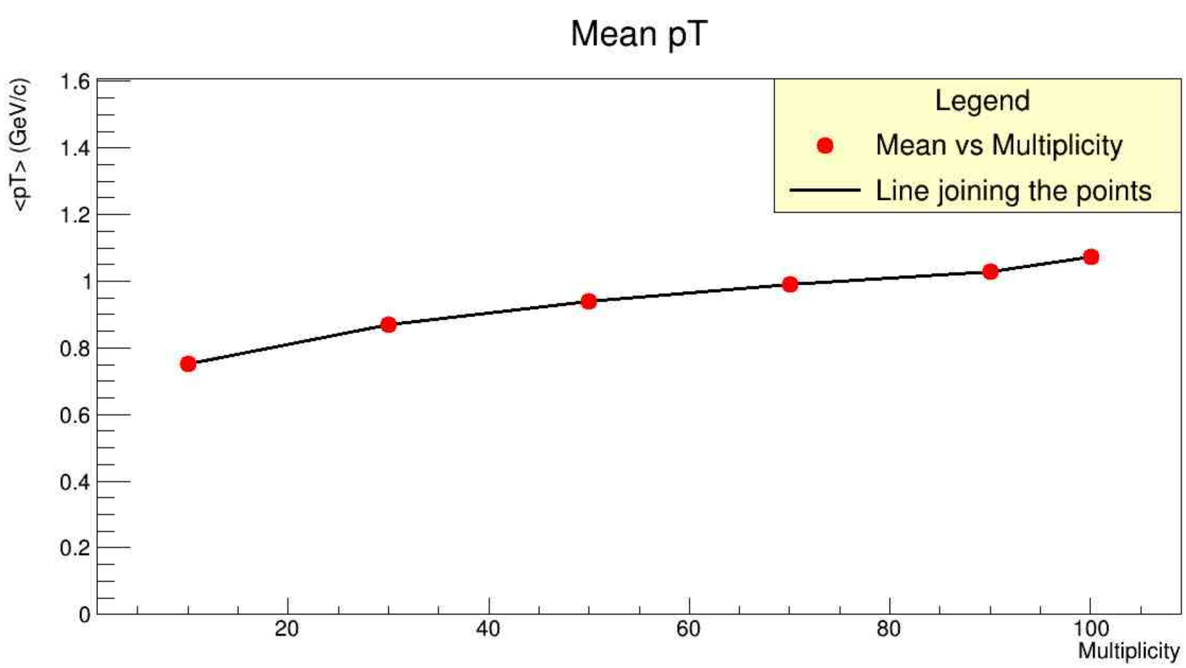

FIG. 6. (Color Online) A plot of **mean** transverse momenta versus multiplicity class. The red dots represent the mean of the transverse momentum. The solid line shows the trend of the mean against the multiplicity class.

3. *Intensive Variance of Transverse Momentum versus Multiplicity Class*

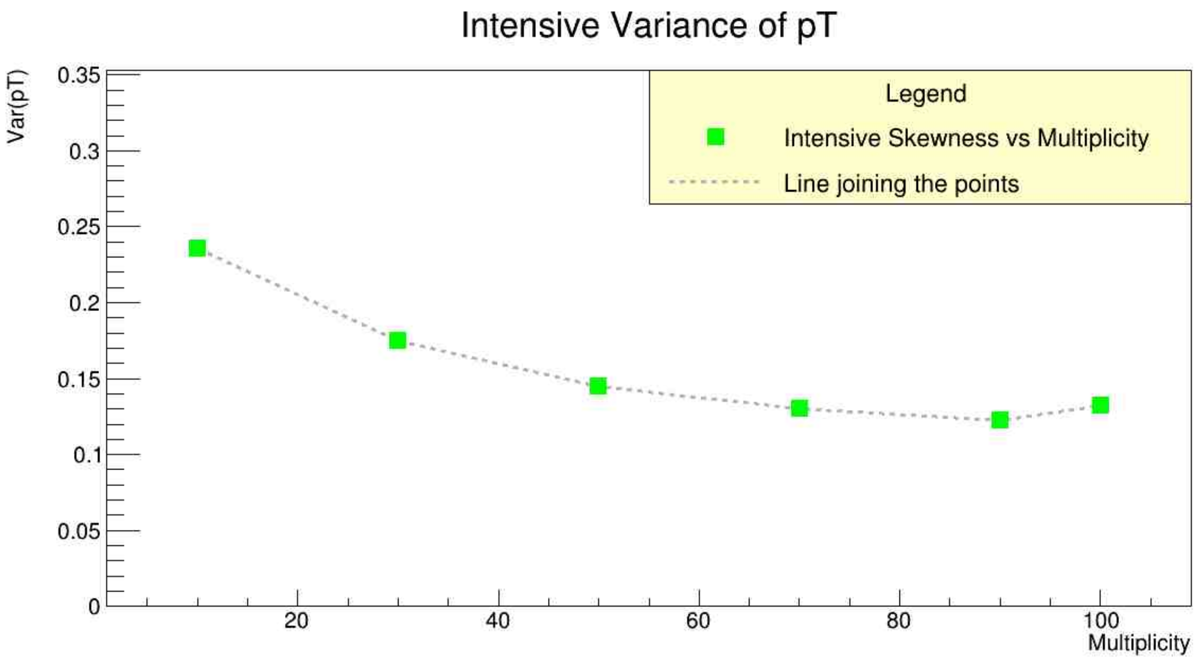

FIG. 7. (Color Online) A plot of the **intensive variance** of transverse momenta versus multiplicity class. The green boxes represent the intensive variance of the transverse momentum. The dashed line shows the trend of the intensive variance against the multiplicity class.

4. *Standardized Skewness of Transverse Momentum versus Multiplicity Class*

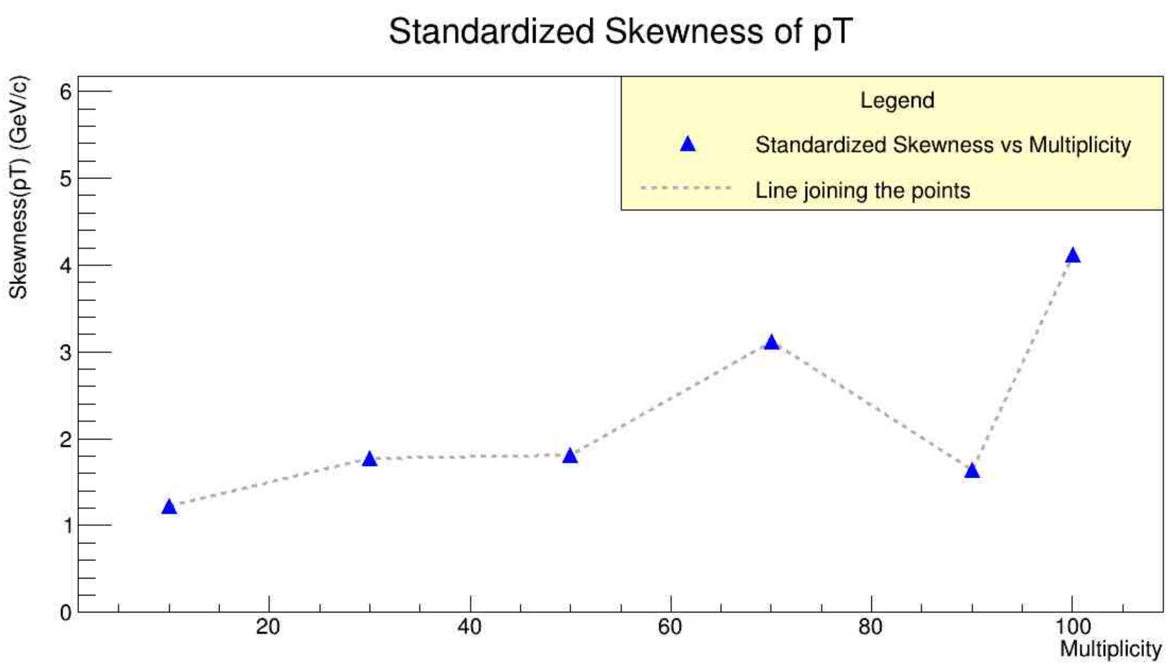

FIG. 8. (Color Online) A plot of the **standardized skewness** of transverse momenta versus multiplicity class. The blue triangles represent the standardized skewness of the transverse momentum. The dashed line shows the trend of the standardized skewness against the multiplicity class.

5. Intensive Skewness of Transverse Momentum versus Multiplicity Class

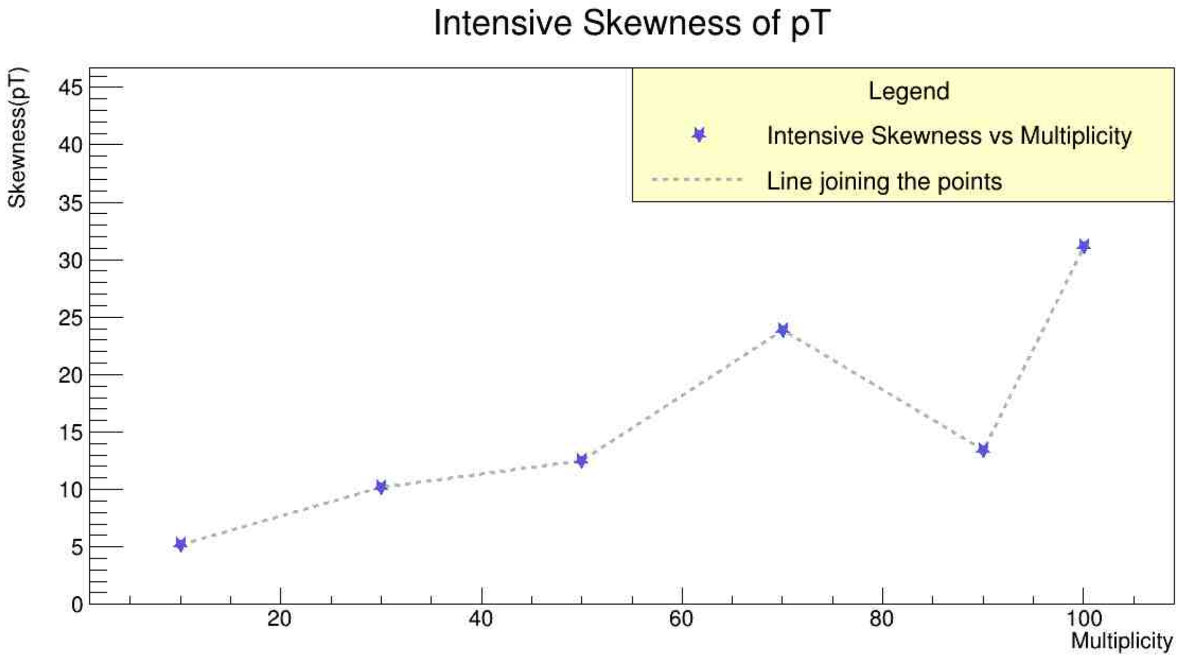

FIG. 9. (Color Online) A plot of the **intensive skewness** of transverse momenta versus multiplicity class. The cyan stars represent the intensive skewness of the transverse momentum. The dashed line shows the trend of the intensive skewness against the multiplicity class.

III. SUMMARY

The study of the mean transverse momentum $\langle \mathbf{p_T} \rangle$ of emitted particles in ultra-relativistic nucleus collisions fluctuates from one event to another. We have shown the various statistical and dynamical fluctuations is been reflected on distribution of the ¡pt¿. It very clear that the function of $\langle \mathbf{p_T} \rangle$ verses collision count tends to have an **Exponential fit** and having a **positive skewness in Gaussian fit**. Hydrodynamics predicts that the event-by-event fluctuations of the mean transverse momentum, $\langle \mathbf{p_T} \rangle$, have positive skew. And fluctuation can be calculated in terms of standard skewness and intensive skewness. At centre of mass energy **13 TeV** for **p+p collision** the standard skewness tends to have values between **1.2 to 4.2 GeV/c** and intensive skewness having values between **5.2 to 31.2** for corresponding multiplicity classes. For the multiplicity class *pytree80100* it shows a dip in value of skewness. This could be for several reasons. Some of then are mentioned below.

(i) The events are simulated using a Monte Carlo Generator, which in some extreme cases produce data sets that are very close to the mean (having very few outliers)

(ii) The number of events in *pytree80100* is much less than the previous multiplicity classes, as is seen from the table

(iii) As seen from FIG. 5b., the histogram of the distribution of $\langle \mathbf{p_T} \rangle$ has a steep decreasing slope to the right side of the peak, which is indicative of higher resemblance to a Gaussian fit. This might occur due to the finite size of the data.

[1] Giuliano Giacalone *et al*, *Skewness of mean transverse momentum fluctuations in heavy-ion collisions* (arXiv:2004.09799 [nucl-th])