Measurement of higher moments of $\cdots$

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Write a proper brief abstarct ...

I. INTRODUCTION

For a given collision centrality, the mean transverse momentum $\langle \mathbf{p_T} \rangle$ of emitted particles in ultrarelativistic nucleus-nucleus collisions fluctuates from one event to another. The distribution of $\langle \mathbf{p_T} \rangle$ in event-by-event dynamics reflects the various statistical and dynamical fluctuations.

So, in this project, through our plots, we will show that the probability distribution of $\langle \mathbf{p_T} \rangle$ is **not Gaussian** but has **positive skew**, which arises because of the above-mentioned fluctuations.

We then go on to plot two dimensionless measures of skewness versus different multiplicity classes, namely **standardized skewness** and **intensive skewness**, out of which the first depends on centrality and system size, whereas the second has the property of being independent of the system size. Since these are dimensional quantities, both of these are expected to be less sensitive to analysis details, such as those dependent on the detector.

We shall be using the following definitions as per the **STAR** collaboration.

$$\text{Mean Transverse Momentum} = \langle\langle p_T \rangle\rangle = \langle \frac{\Sigma_{i=1}^{N_{ch}} p_i}{N_{ch}} \rangle \tag{1}$$

where N_{ch} denotes the number of charged particles in an event, p_i is the transverse momentum of the ith particle and angular brackets denote an average over events in a centrality class.

We analyze the variance of dynamical p_T fluctuations which we denote by $\langle \Delta p_i \Delta p_j \rangle$,

$$\langle \Delta p_i \Delta p_j \rangle = \langle \frac{\Sigma_{i,j\neq i}(p_i - \langle\langle p_T \rangle\rangle)(p_j - \langle\langle p_T \rangle\rangle)}{N_{ch}(N_{ch}-1)} \rangle \tag{2}$$

which can also be written as

$$\langle \Delta p_i \Delta p_j \rangle = \langle (\langle p_T \rangle - \langle\langle p_T \rangle\rangle)^2 \rangle \tag{3}$$

and the intensive variance of transverse momentum which is defined as follows

$$\sigma_{p_T} = \frac{\langle \Delta p_i \Delta p_j \rangle^{1/2}}{\langle\langle p_T \rangle\rangle}. \tag{4}$$

The skewness is the third central moment, denoted by $\langle \Delta p_i \Delta p_j \Delta p_k \rangle$ defined as follows

$$\langle \Delta p_i \Delta p_j \Delta p_k \rangle = \langle \frac{\Sigma_{i,j\neq i,k\neq i,j}(p_i - \langle\langle p_T \rangle\rangle)(p_j - \langle\langle p_T \rangle\rangle)(p_k - \langle\langle p_T \rangle\rangle)}{N_{ch}(N_{ch}-1)(N_{ch}-2)} \rangle \tag{5}$$

which can also be written as

$$\langle \Delta p_i \Delta p_j \rangle = \langle (\langle p_T \rangle - \langle\langle p_T \rangle\rangle)^3 \rangle \tag{6}$$

Standardized skewness and intensive skewness denoted by γ_{p_T} and Γ_{p_T}, respectively, are defined as follows:

$$\gamma_{p_T} = \frac{\langle \Delta p_i \Delta p_j \Delta p_k \rangle}{\langle \Delta p_i \Delta p_j \rangle^{3/2}} \tag{7}$$

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$$\Gamma_{p_T} = \frac{\langle \Delta p_i \Delta p_j \Delta p_k \rangle \langle \langle p_T \rangle \rangle}{\langle \Delta p_i \Delta p_j \rangle^2} \tag{8}$$

The dataset provided is generated with **Pythia 8 Monte Carlo Event Generator**.
Number of events : **2 million**
Collisions System : **p + p** at centre of mass energy **13 TeV**

II. EXPERIMENTAL OBSERVATIONS

A. Transverse Momentum and Mean Transverse Momentum for Each Multiplicity Class

In this section, we have plotted the histograms for the Transverse Momentum **pT** and the Mean Transverse Momentum ⟨**pT**⟩ of proton-proton collisions corresponding to each multiplicity class. The histogram for **pT** is then approximated using an **Exponential** fit, while that of ⟨**pT**⟩ has been approximated using a **Gaussian** fit. Both the quantities **pT** and ⟨**pT**⟩ have statistical fluctuations arising from the finite number of particles in each event. In each of the subsequent subsections corresponding to each of the 5 multiplicity classes, namely **pytree2040**, **pytree4060**, **pytree6080**, **pytree80100** and **pytree100**, the histograms and the corresponding fits have been plotted. A logarithmic scale has been used on the *y-axis* in order to emphasize the skewness of the data.

1. *Multiplicity Class "pytree2040"*

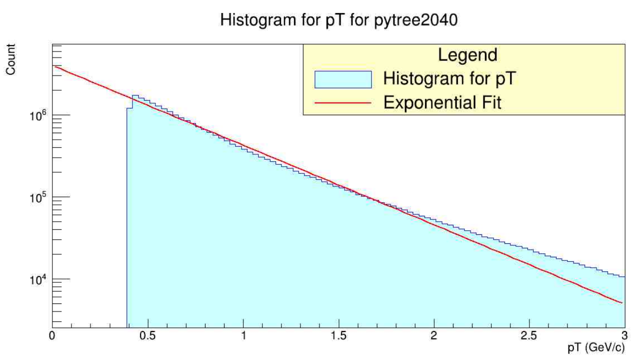

FIG. 1a. (Color Online) Distribution of **pT** for proton-proton collision in the multiplicity class **pytree2040**. The solid line is an Exponential fit to the data.

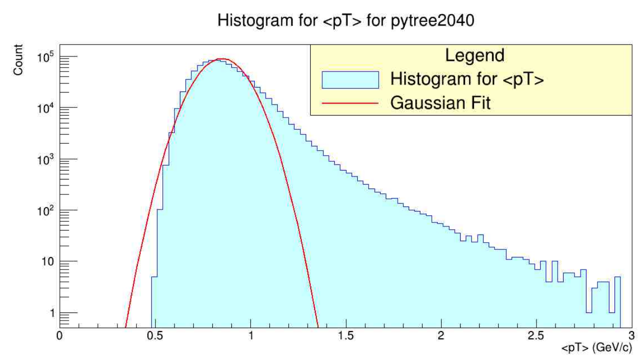

FIG. 1b. (Color Online) Distribution of ⟨**pT**⟩ for proton-proton collision in the multiplicity class **pytree2040**. The solid line is a Gaussian fit to the data.

2. *Multiplicity Class "pytree4060"*

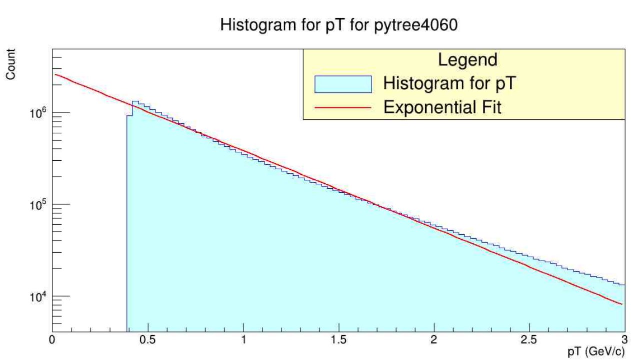

FIG. 2a. (Color Online) Distribution of **pT** for proton-proton collision in the multiplicity class **pytree4060**. The solid line is an Exponential fit to the data.

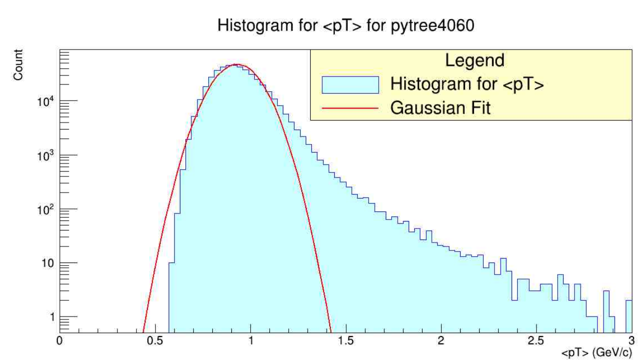

FIG. 2b. (Color Online) Distribution of ⟨**pT**⟩ for proton-proton collision in the multiplicity class **pytree4060**. The solid line is a Gaussian fit to the data.

3. Multiplicity Class "pytree6080"

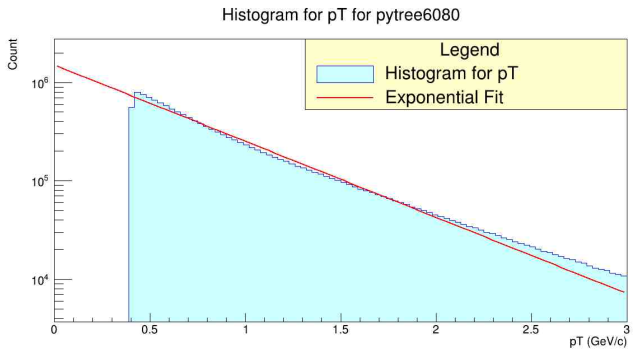

FIG. 3a. (Color Online) Distribution of **pT** for proton-proton collision in the multiplicity class **pytree6080**. The solid line is an Exponential fit to the data.

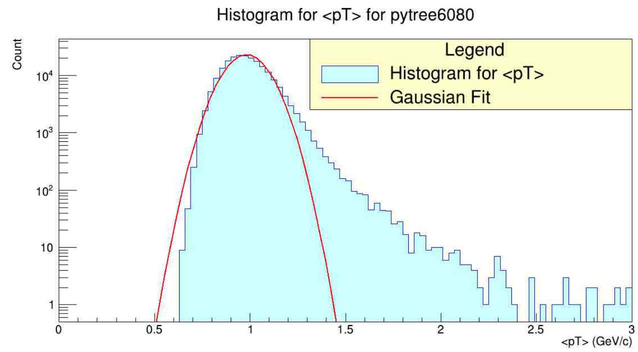

FIG. 3b. (Color Online) Distribution of $\langle \mathbf{pT} \rangle$ for proton-proton collision in the multiplicity class **pytree6080**. The solid line is a Gaussian fit to the data.

4. Multiplicity Class "pytree80100"

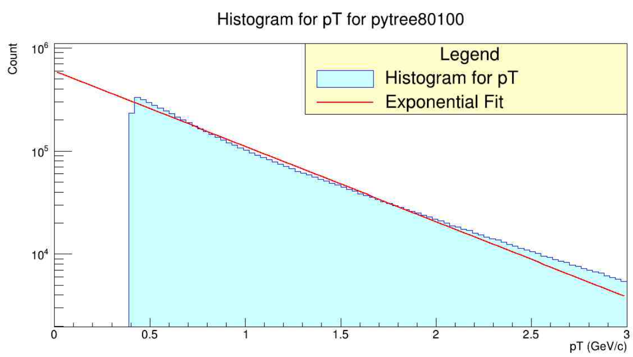

FIG. 4a. (Color Online) Distribution of **pT** for proton-proton collision in the multiplicity class **pytree80100**. The solid line is an Exponential fit to the data.

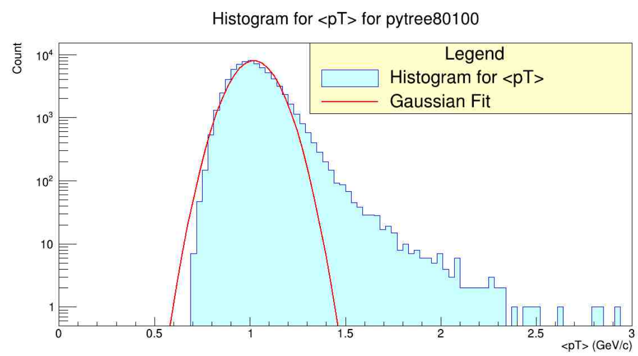

FIG. 4b. (Color Online) Distribution of $\langle \mathbf{pT} \rangle$ for proton-proton collision in the multiplicity class **pytree80100**. The solid line is a Gaussian fit to the data.

5. Multiplicity Class "pytree100"

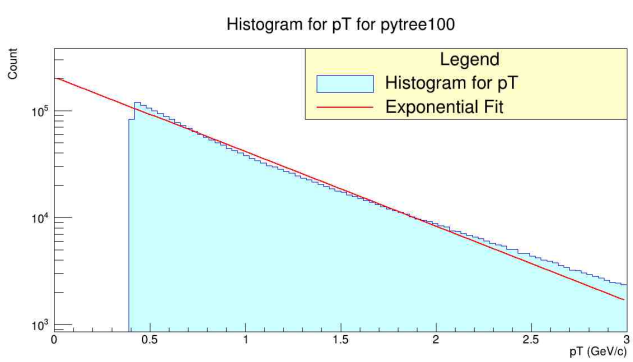

FIG. 5a. (Color Online) Distribution of **pT** for proton-proton collision in the multiplicity class **pytree100**. The solid line is an Exponential fit to the data.

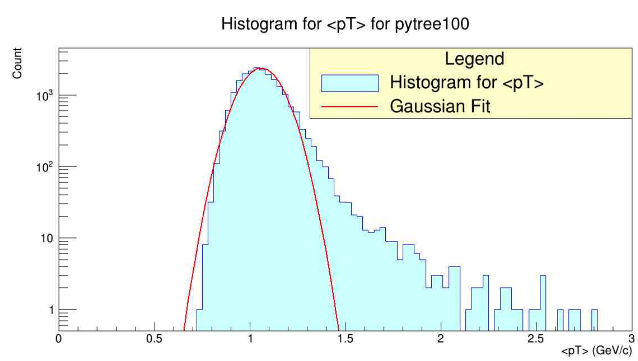

FIG. 5b. (Color Online) Distribution of $\langle \mathbf{pT} \rangle$ for proton-proton collision in the multiplicity class **pytree100**. The solid line is a Gaussian fit to the data.

B. Analysis of Mean, Variance and Skewness Versus Multiplicity Class

From the graphs in FIG. 1a., FIG. 2a., FIG. 3a., FIG. 4a. and FIG. 5a., it is clear that the Transverse Momenta of the particles produced in a proton-proton collision follows approximately an **exponential distribution**. Graphs in FIG. 1b., FIG. 2b., FIG. 3b., FIG. 4b. and FIG. 5b. reveal that there is some **positive skew** in the distribution of the Mean Transverse Momentum.
In this section, we shall analyse the moments of the distribution of Transverse Momentum. We shall calculate the Mean, Variance and Skewness of the Transverse Momenta for each multiplicity class and study its relation with the multiplicity class. For each of the multiplicity classes, the Mean Transverse Momentum, the Intensive Variance of the Transverse Momentum, the Standardized Skewness of the Transverse Momentum and the Intensive Skewness of the Transverse Momentum, calculated using formulae 1, 3, 5 and 6 respectively have been summarised in the table and the plots below.

1. *Summary of Data*

The table below summarizes the data.

SUMMARY OF DATA					
Multiplicity Class	**Events**	$\langle \mathbf{p_T} \rangle$ **(GeV/c)**	$\sigma_{\mathbf{p_T}}$	$\gamma_{\mathbf{p_T}}$ **(GeV/c)**	$\mathbf{\Gamma_{p_T}}$
pytree020	952256	0.750912	0.235453	1.22572	5.20581
pytree2040	873322	0.869307	0.174974	1.77742	10.1582
pytree4060	445805	0.940521	0.144591	1.80884	12.51
pytree6080	207990	0.99074	0.130471	3.11451	23.8714
pytree80100	71263	1.03006	0.122603	1.64507	13.4178
pytree100	20981	1.07257	0.132207	4.11805	31.1485

TABLE I. Table Summarizing the Data of Transverse Momenta

2. Mean Transverse Momentum versus Multiplicity Class

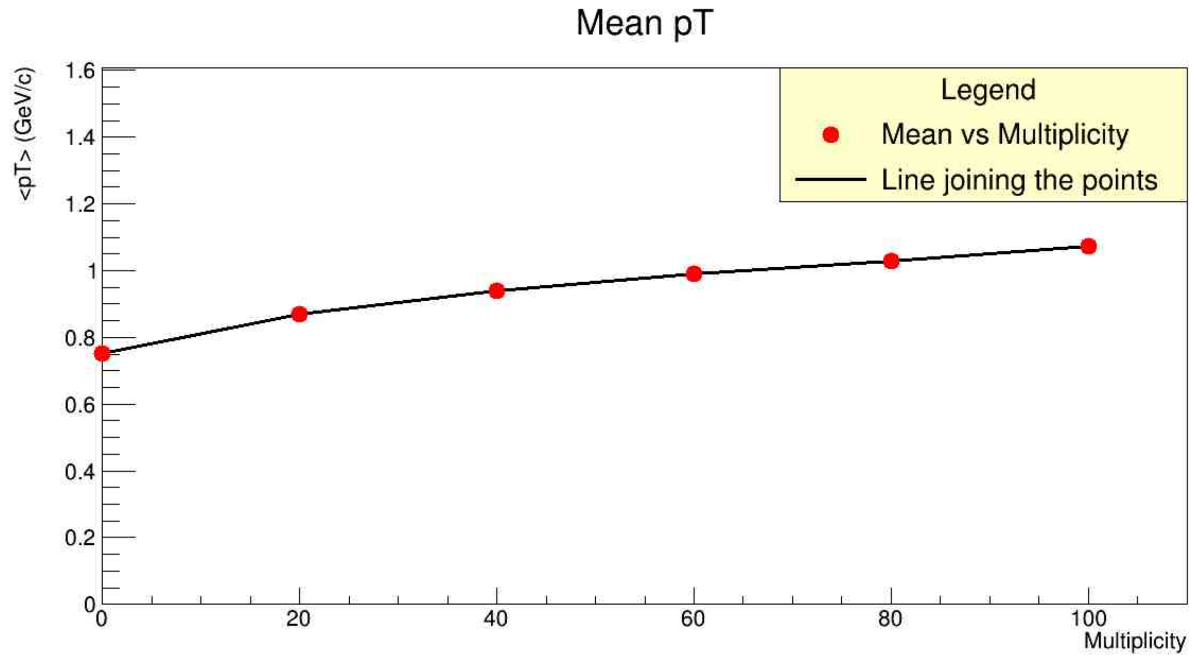

FIG. 6. (Color Online) A plot of **mean** transverse momenta versus multiplicity class. The red dots represent the mean of the transverse momentum. The solid line shows the trend of the mean against the multiplicity class.

3. Intensive Variance of Transverse Momentum versus Multiplicity Class

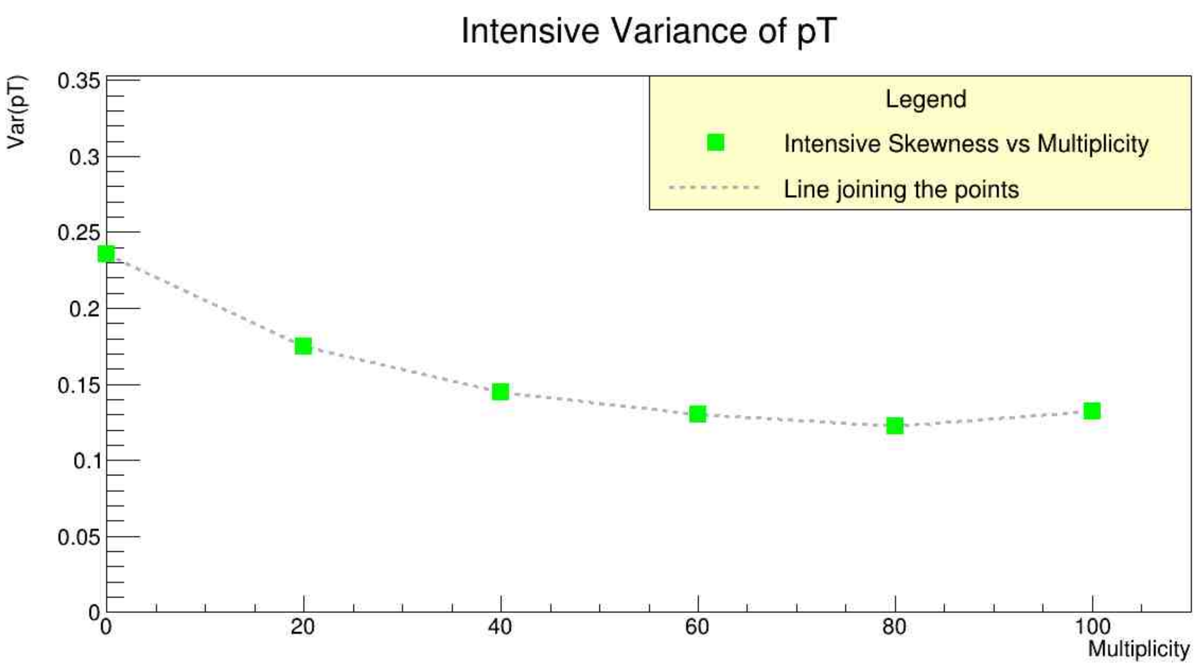

FIG. 7. (Color Online) A plot of the **intensive variance** of transverse momenta versus multiplicity class. The green boxes represent the intensive variance of the transverse momentum. The dashed line shows the trend of the intensive variance against the multiplicity class.

4. Standardized Skewness of Transverse Momentum versus Multiplicity Class

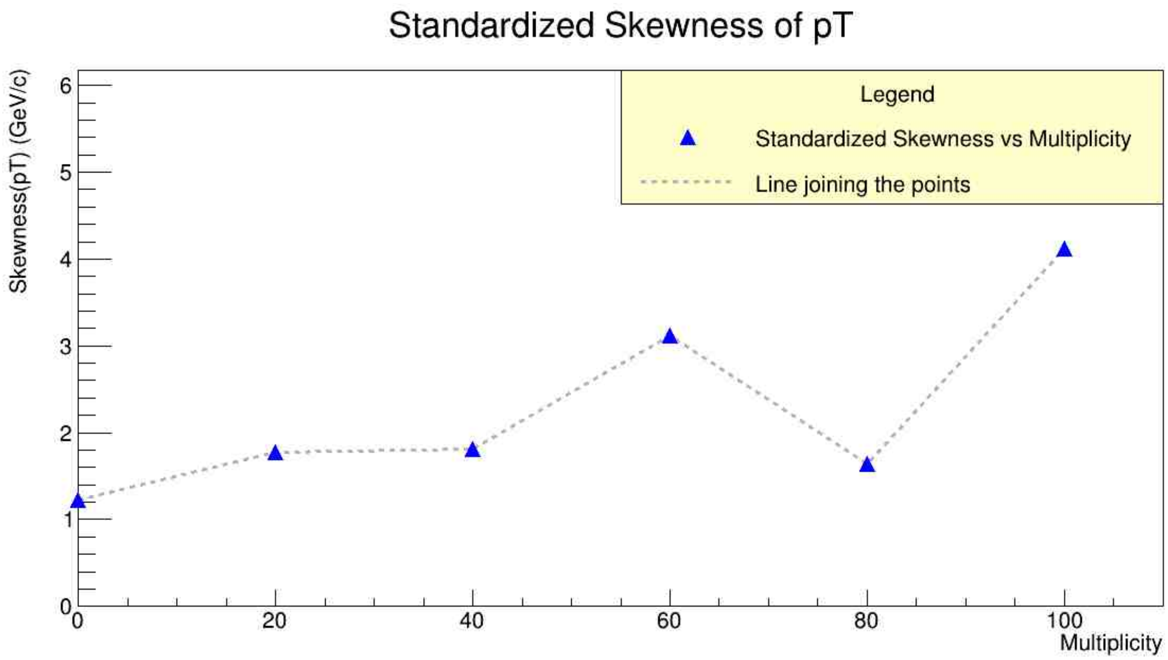

FIG. 8. (Color Online) A plot of the **standardized skewness** of transverse momenta versus multiplicity class. The blue triangles represent the standardized skewness of the transverse momentum. The dashed line shows the trend of the standardized skewness against the multiplicity class.

5. Intensive Skewness of Transverse Momentum versus Multiplicity Class

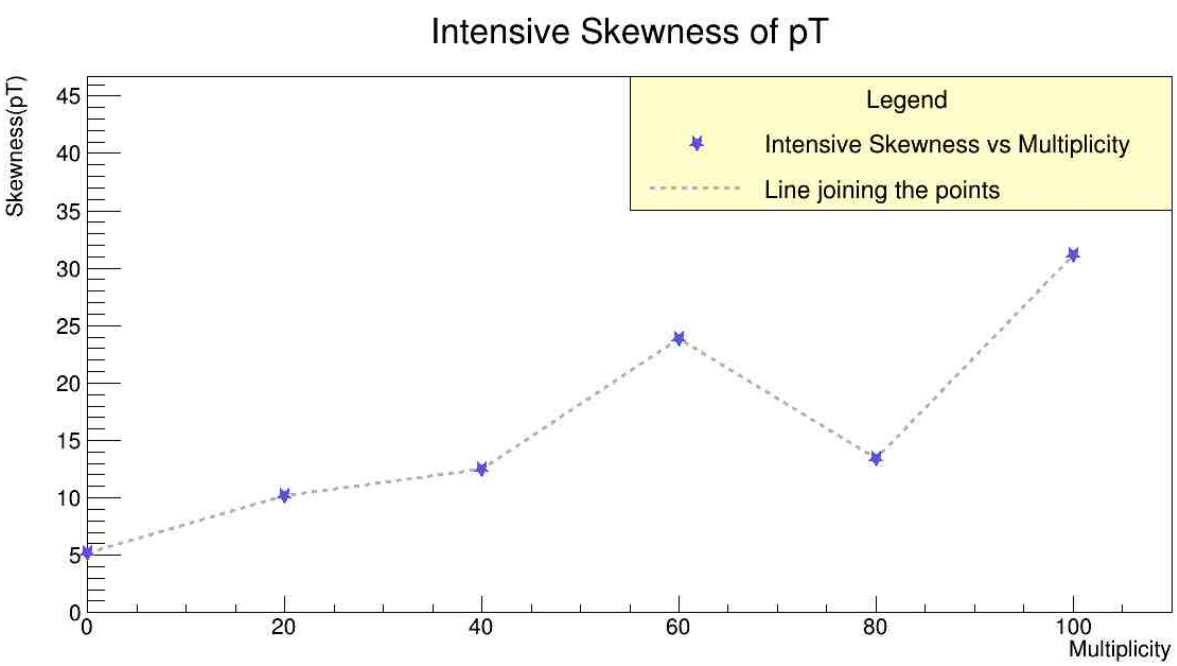

FIG. 9. (Color Online) A plot of the **intensive skewness** of transverse momenta versus multiplicity class. The cyan stars represent the intensive skewness of the transverse momentum. The dashed line shows the trend of the intensive skewness against the multiplicity class.

FIG. 15. (Color online) Put proper captions

III. SUMMARY

The study of $\cdots$

[1] J. Adams *et al.*, (ALICE Collaboration), Nature Physics**13**,535-539 (2017).