Exercise:

Q. $T(n) = 2T(n/2) + n.$

$a = 2 \qquad k = 1$

$b = 2 \qquad p = 0$

$\log_b a = \log_2 (2) = 1$

$\therefore \log_b a = k \;\&\&\; p > -1$

$\therefore \Theta(n^k \log^{p+1} n)$

$= \Theta(n^1 \log^1 n)$

$= \Theta(n \log n).$

Q $T(n) = 2T(n/2) + n\log n$

$a = 2 \quad k = 1$

$b = 2 \quad p = 1$

$\log_b a = \log_2(2) = 1$

$\therefore \log_b a = k \ \&\& \ p > -1$

$\therefore \theta(n^k \log^{p+1} n)$

$= \theta(n^1 \log^2 n)$

$= \theta(n \log^2 n)$

Q. $T(n) = 2T(n/2) + n^2$

$a = 2 \quad k = 2$

$b = 2 \quad p = 0$

$\log_b a = \log_2(2) = 1$

$\therefore \log_b a < k \ \&\& \ p \leq 0$

$\therefore \theta(n^k) = \theta(n^2)$

$\Downarrow$

$O(n^2)$

Q. $T(n) = 8T(n/2) + n^2$

$a = 8 \qquad k = 2$

$b = 2 \qquad p = 0$

$\log_b a = \log_2 8 = \log_2 (2)^3 = 3\log_2 2 = 3$

$\therefore \log_b a > k$

$\therefore \Theta(n^{\log_b a}) = \Theta(n^3)$

Time Complexity $\rightarrow \boxed{O(n^3)}$.