## Quadratic Integer Rings:-

Let D be squarefree integer.

$$\begin{array}{c}
\text{Tr} D \equiv 1 \pmod{4} \text{ tem}, \quad \mathbb{Z} \left[ \frac{1+\sqrt{D}}{2} \right] = \left\{ a + b \left( \frac{1+\sqrt{D}}{2} \right) \mid a, b \in \mathbb{Z} \right\} \\
= \left\{ a + \frac{b}{2} + \frac{b}{2} \sqrt{D} \mid a, b \in \mathbb{Z} \right\} \text{ is a subgring}
\end{array}$$

If 
$$D = 1 \pmod{4}$$
 then  $\omega = \frac{1+\sqrt{D}}{2}$  else  $\omega = \sqrt{D}$ 

$$PD=-1$$
 then we get  $PD=-1$  then we get  $PD=-1$ 

Polynomial Rings:

We have R as a ring (often commutative). and the farm,

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This phynomial is said to be a polynomial rung, if a; ER which is denoted by R[x]

R ig the constant polynomals in R[n].

and a show that RINT is also commitative.

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If R is commetative tean RIn] is also commetative.

Z[x] a polynound ring. Polynound Z/4Z[x] is a consulative ring.

Proposition: Let R be on integral domain and let P(n) and y(N) be hant ero elemets of R[n]. Then

- degree of P(n)q(n) = degree P(n) + degree Q(n)
- (2) the wints of R[n] are just with of R
- niando harpstui no vi [M]A (E)

## Matrine Rings:

Rigaring and a Mn(R) is a matrin Rung of all entries in Mn(R) belongs to Rand 18 of dimension NXN

Mu(R) may or way not be commitative even if R is commetative

If S CR is a subring than Mn (S) < Mn(R) is also a subring

 $M_{\nu}(z) \leq M_{\nu}(Q) \leq M_{\nu}(R) \leq M_{\nu}(C)$ 

Set of Oppertriongular matures is also a substing of Mn (R)

## Group Rings 1-

G = {91,921--.,9,3 be any finite group with operation x R is a countative may with identity 1 \$0

RG is a group ming

Set of ell, aigit azgit --- tang, where aieR

Set of ell,  $a_1g_1 + a_2g_2 + \cdots + a_ng_n$  when  $a_1g_1$  as  $a_1$ :

If  $g_1$  is identity are wary write  $a_1g_1$  as  $a_1$ : e(RG) = RGRG a countative iff G is countative  $ea_1g_1 + ea_1g_2 + \cdots + a_ng_n = g_1 + g_2 + \cdots + g_n = g_n$ Identity of RG is identity of R.  $\Rightarrow (a_1g_1 + a_2g_2 + \cdots + a_ng_n = g_1 + g_2 + \cdots + g_n = g_n + g_n + g_n + g_n + g_n = g_n + g_n + g_n + g_n + g_n = g_n + g_n + g_n + g_n = g_n + g_n + g_n + g_n = g_n + g_n + g_n + g_n = g_n + g_n + g_n + g_n + g_n = g_n + g_$ 

G = DB is a dihedral group of order. 8  $\langle v, s \rangle$   $v^4 = 1$   $S^2 = 1$   $VS = Sr^{-1}$ R = 2

Par Jus à a group surg.

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.> Domain is a ring with no Zero divisors.

Abe a ring and

Abe a domain, i.e., has no zero-divisors. Then if ab = 1

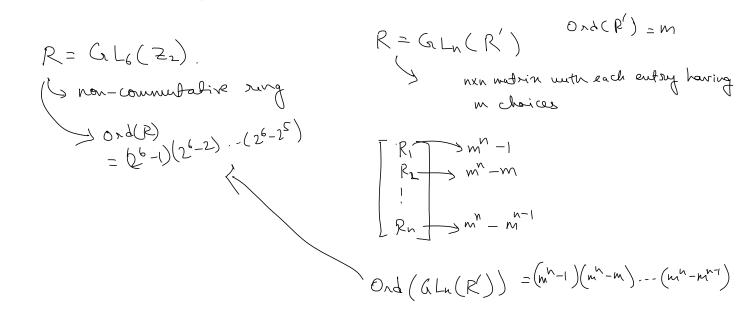
for some a, b \in A. Prore that ba = 1, i.e., a, b are unity in A.

Ami-  $ab=1 \Rightarrow aba=a \Rightarrow aba=0 \Rightarrow a(ba-1)=0$  $\Rightarrow ba=1$ 

0) Let A be a rung and a, b  $\in A$  such that ab=1. Prove that ba and 1-ba are idempotents in A

 $A_{m!} - (ba)^2 = baba = ba$  $(-ba)^2 = (1-ba)(1-ba) = 1-2ba+(ba)^2 = 1-2ba+ba = (-ba)$ 

ab, CEA. A is a ring Such that ab = ca = 1. Prove that C=b and a is a unit of A. +w'. -cb = 1 $\Rightarrow cab = C = b$ . Thus a is a unit



Q) Let R be ring and a, b ∈ R such that ab = ba is a unit of R. Prore that a as b one both with sin R.