



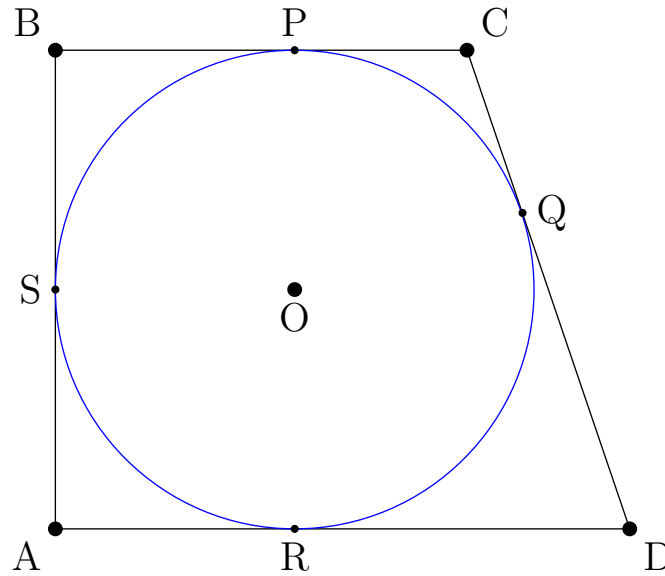
Quiz 3, Date: 16 June, 2024
Instructor: Ankan Kar

Timing: 3:00 PM to 6:00 PM

Score rule as per IMO

All questions carry equal marks, try as much as possible

1. Let $ABCD$ be a trapezium in which $AD \parallel BC$. Also $BA \perp AD$. Now, $ABCD$ has an incircle which touches BC at P and AD at R . Given that length of $PC = 36$ and length of $RD = 49$, find the length PR . (you can refer to the figure below)



2. Let x and y be positive real numbers such that $x = 1 - y$. Prove that

$$x^x y^y + x^y y^x \leq 1.$$

3. Let there exist real numbers a, b, c such that $a^2 + 4b^2 + 16c^2 = 48$ and $ab + 4bc + 2ca = 24$. Then find the value of $a^2 + b^2 + c^2$?

4. Let $x, y, z > 0$ and $xyz = 1$. Then prove that

$$\frac{1+xy}{1+x} + \frac{1+yz}{1+y} + \frac{1+zx}{1+z} \geq 3$$

5. Show that given a subset of $n+1$ elements of $\{1, 2, 3, \dots, 2n\}$, there are two elements in that subset such that one divides the other.

6. We have $x, y, k, n \in \mathbb{N}$. Prove that $k|n$ iff $(x^k - y^k)|(x^n - y^n)$.

End