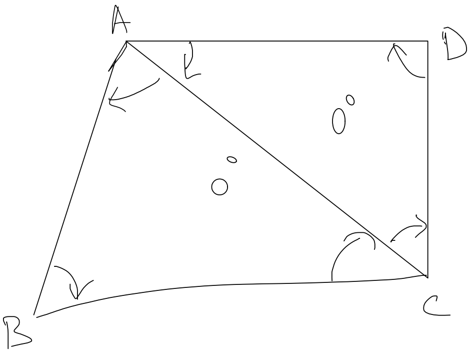
Geometry 8

02 June 2024 18:33

Q> Show that for any distinct points ABCD, we have,

$\measuredangle ABC + \measuredangle BCD + \measuredangle CDA + \measuredangle DAB = 0$

Ans:-



$\Rightarrow 0^\circ$ overall

Q> Find all possible values of,

$x + \frac{4}{x}$ when $x \in \mathbb{R} \setminus \{0\}$

Ans:-

$x \in \mathbb{R}^+ \Rightarrow x + \frac{4}{x} \geq 2\sqrt{4}$

$x \in \mathbb{R}^- \Rightarrow \underbrace{-\left(x + \frac{4}{x}\right)}_{\in \mathbb{R}^+} \geq 2\sqrt{(-x)\left(-\frac{4}{x}\right)} = 2\sqrt{4}$

$\underset{\in \mathbb{R}^+}{-x} + \underset{\in \mathbb{R}^+}{\left(-\frac{4}{x}\right)}$

$\Rightarrow \left(x + \frac{4}{x}\right) \leq -2\sqrt{4}$

Q> For $x < 0$ find the max value of $\frac{3x^2+12}{x}$.

Ans:-

$3x + \frac{12}{x} \Rightarrow \underbrace{-3x - \frac{12}{x}}_{\in \mathbb{R}^+} \geq 2\sqrt{36} = 12$

$\Rightarrow 3x + \frac{12}{x} \leq -12$

Q> $(x-2) + \frac{1}{x^2-16}$, $x > 0, x \neq 4$

$x - 2 > 0 \Rightarrow x > 2$

$x^2 - 16 > 0 \Rightarrow x > 4$

Ans:-

$\underbrace{x - 2 + \frac{1}{x^2-16} \geq 2\sqrt{\frac{x-2}{(x-4)(x+4)}}}_{\text{for } x > 4}$

for $x < 2 \Rightarrow -(x-2) - \frac{1}{x^2-16} \geq 2\sqrt{\frac{x-2}{(x-4)(x+4)}}$

If $x \leq 2 \Rightarrow$ $\quad -(x-2) - \frac{1}{x^2-16} \geq 2\sqrt{\frac{x-2}{(x-4)(x+4)}}$

If $x \in (2,4) \Rightarrow$ AM-GM will not work as one term is positive and one term is negative

Q> $f(x)$ is polynomial of degree 4, s.t. $f(1)=1, f(2)=2, f(3)=3$, $f(4)$, $f(0)=1$. Find $f(5)$.

Ans:- $P(x) = f(x) - x \qquad P(x) = 0$ for $x = 1, 2, 3, 4$.

↳ degree of P is atmost 4.

$\Rightarrow P(x) = a(x-1)(x-2)(x-3)(x-4) \qquad P(0) = 1$

$\Rightarrow 24a = 1$

$\Rightarrow a = \frac{1}{24}$

$\Rightarrow P(x) = \frac{1}{24}(x-1)(x-2)(x-3)(x-4)$

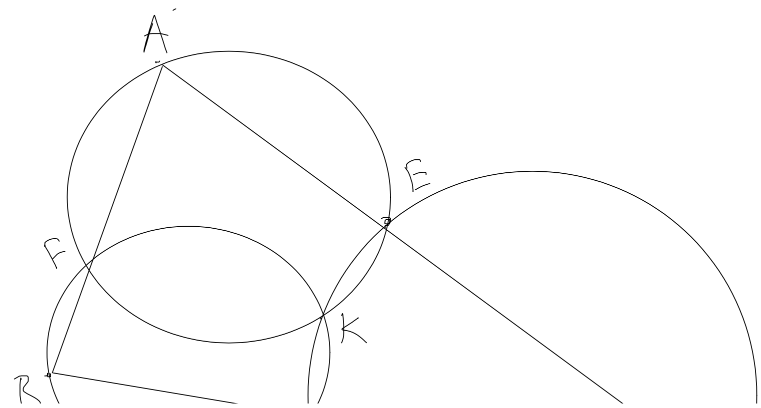
$= f(x) - x$

$\Rightarrow f(5)$ - - - - -

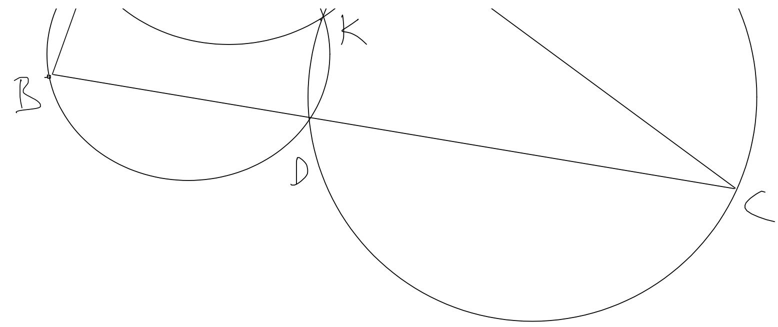
HomeWork

Lemma:- (Miquel Point of a Triangle)

Points D, E, F lie on lines BC, CA, and AB of $\triangle ABC$, respc. Then there exists a point lying on three circles (AEF), (BFD), (CDE).



A, B, X, Y is cyclic
$\Leftrightarrow \measuredangle AXB = \measuredangle AYB$



<u>HomeWork</u>

Q> Points A, B, C lie on a circle with centre O. Show that
$\measuredangle OAC = 90^\circ - \measuredangle CBA$