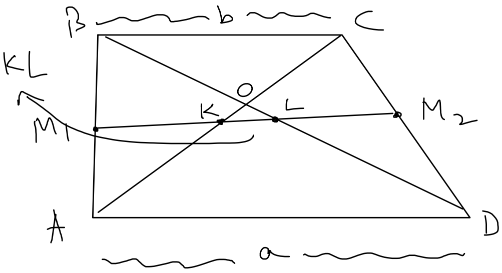
Geometry 11

14 July 2024 10:37

Q> Let the lengths of bases AD and BC of trapezoid ABCD be a and b where $a > b$ then, find the length of the segment that the diagonals intercept on the midline of non-parallel sides

We need to find KL



bases are same

$LM_2 = M_1K = \frac{b}{2}$

$KL + b = M_1M_2$ $\qquad M_1M_2 = \frac{a+b}{2}$ $\qquad \Rightarrow KL = \frac{a+b}{2} - b = \frac{a-b}{2}$

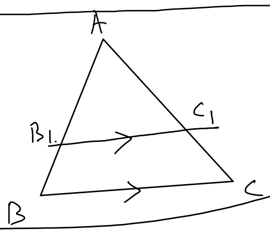
Ans:—

(results are taken from below)

Similar Triangles:—

$\Delta A_1B_1C_1$ and $\Delta A_2B_2C_2$ are similar iff $\angle A = \angle A_1$, $\angle B = \angle B_1$, $\angle C = \angle C_1$

- Then, $A_1B_1 : B_1C_1 : C_1A_1 :: A_2B_2 : B_2C_2 : C_2A_2$
- They are also similar if, $A_1B_1 : B_1C_1 :: A_2B_2 : B_2C_2$ and $\angle A_1B_1C_1 = \angle A_2B_2C_2$

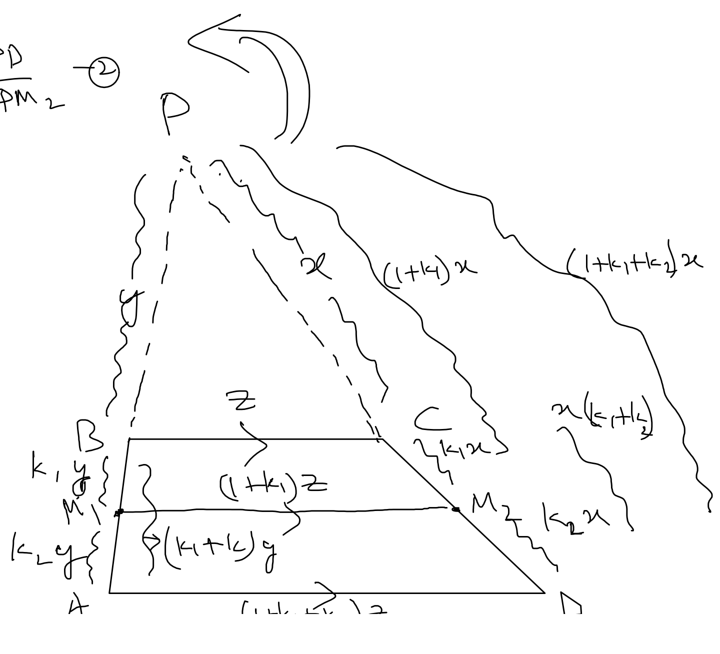


$$\frac{AB_1}{AB} = \frac{B_1C_1}{BC} = \frac{AC_1}{AC}$$

$$\frac{PB}{PM_1} = \frac{PC}{PM_2} \quad -①$$

$$\frac{BM_1}{BA} = \frac{k_1}{k_1+k_2} = \frac{CM_2}{CD}$$

$$\frac{PA}{PM_1} = \frac{PD}{PM_2} \quad -②$$



For the question above:—

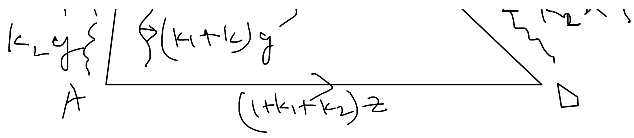
If $k_1 = k_2$,

$M_1M_2 = (1+k_1)z$

$BC_1 = z$

$AD = (1+k_1+k_2)z = (1+2k_1)z$

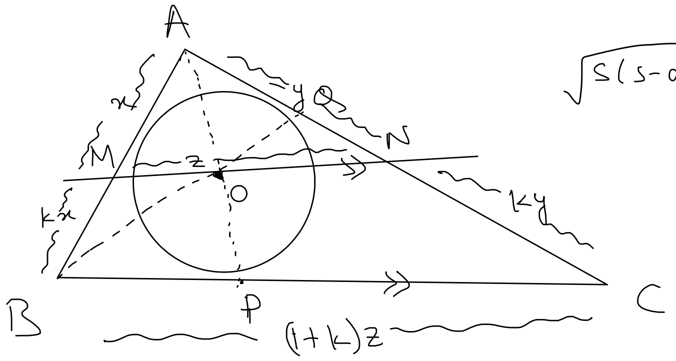
$M_1M_2 = (1+k_1)z$



$$M_1M_2 = (1+k_1)z$$
$$= \frac{2(1+k_1)z}{2}$$
$$= \frac{(2+2k_1)z}{2} = \frac{z+(1+2k_1)z}{2} = \frac{BC+AD}{2} = \frac{a+b}{2}$$

Q) ΔABC has side lengths $AB=12$, $BC=24$, $AC=18$. The line through the incenter of ΔABC parallel to BC intersects AB at M and AC at N. What is the perimeter of ΔAMN.

Ans:-



$$\sqrt{s(s-a)(s-b)(s-c)}$$

$$\frac{AO}{OP}\times\frac{BP}{BC}\times\frac{CQ}{QA}=1 \qquad \frac{CQ}{QA}=2 \qquad \frac{BP}{BC}=\frac{2}{5}$$

$$\Rightarrow \frac{AO}{OP}=\frac{5}{4} \qquad \Rightarrow \frac{OP}{AO}=\frac{4}{5} \Rightarrow \frac{AP}{AO}=\frac{9}{5} \Rightarrow \frac{AO}{AP}=\frac{5}{9}$$

Perimeter of $\Delta AMN = k$ (Perimeter of ΔABC)

s of $\Delta AMN = k$ (s of ΔABC)

$$\left(\sqrt{s(s-a)(s-b)(s-c)}\right)_{\Delta AMN} = \left(\sqrt{ks(ks-ka)(ks-kb)(ks-kc)}\right)_{\Delta ABC}$$

$$\text{area of } \Delta AMN = k^2(\text{area of } \Delta ABC)$$

→ For the conditions to hold MN need not pass through O, it just need to parallel to BC

HomeWork

Q) In the first question of this lecture:- find the length of segment MN

[illegible]

Homework:

Q) In the first question of this lecture:— find the length of segment MN whose endpoints M, N divides AB and CD in the ratio,

$AM : MB = DN : NC = m : n$

Q) ABCD is a parallelogram such that P is on AD and $AP : AD = 1 : p$ and X is the intersection of AC and BP. Prove that

$AX : AC = 1 : (p+1)$