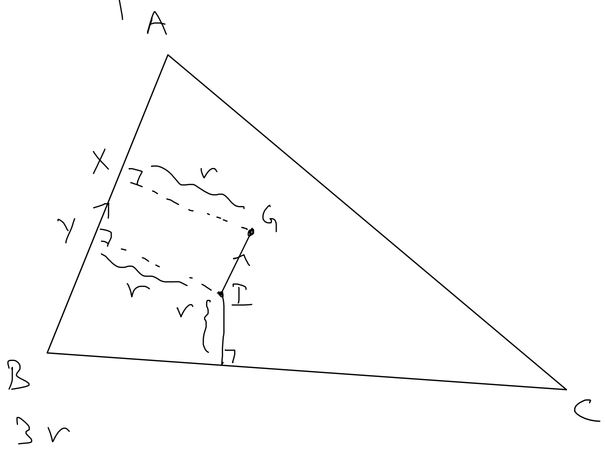
Q) If in ΔABC, $AC = 15$ and $BC = 13$ and $IG \parallel AB$ where I is the incentre and G is the centroid of ΔABC. Find the area of ΔABC.



Ans:- Area of $\Delta ABC = \frac{1}{2} r (AB + BC + CA)$

$= \frac{1}{2} h (AB)$ (h is height of C from AB)

As $IG \parallel AB$, $h = 3(GX)$

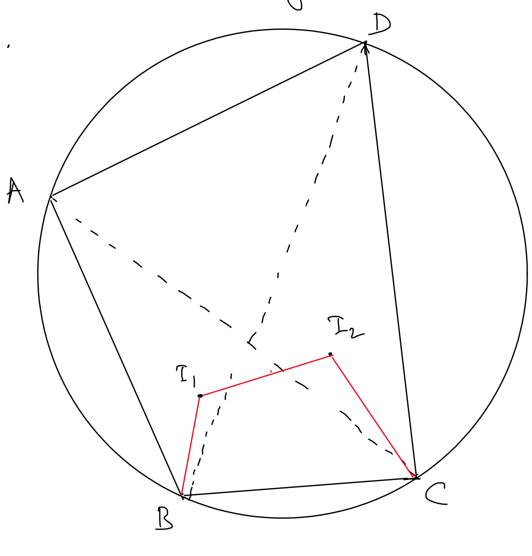
$GX = IY$ as $IG \parallel AB \Rightarrow h = 3(IY) = 3r$

$\frac{1}{\cancel{2}} \cancel{r} (AB + 15 + 13) = \frac{1}{\cancel{2}} 3 \cancel{r} (AB)$

$\Rightarrow 3AB = AB + 15 + 13 \Rightarrow AB = 14$

Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

Q) Let $ABCD$ be a cyclic quadrilateral. Let I_1 and I_2 be the incentres of ΔABC and ΔDBC respectively. Prove that $I_1 I_2 BC$ is also cyclic.



Ans:- We need to show,

$\angle BI_1C = \angle BI_2C$

$\angle BI_1C = 180^\circ - (\angle I_1BC - \angle I_1CB)$

$= 180^\circ - \frac{1}{2}(\angle ABC + \angle ACB)$

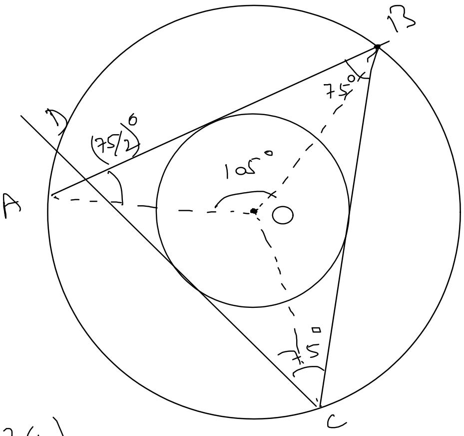
$= 180^\circ - \frac{1}{2}(180^\circ - \angle BAC)$

$= 90^\circ + \frac{1}{2}\angle BAC$

Similarly, $\angle BI_2C = 90^\circ + \frac{1}{2}\angle BDC$

Also, $\angle BAC = \angle BDC$ as BC is common chord. $\Rightarrow \angle BI_1C = \angle BI_2C$

Q> Given a pair of concentric circles chord AB, BC, CD.... of the outer circle are drawn such that they all touch the inner circle. If $\angle ABC = 75^\circ$ how many chords can be drawn before returning to the starting point.



Ans:- $\angle AOB = 105^\circ$

$105k = 360m$

$\hookrightarrow$ we need to find smallest such $k \in \mathbb{N}$

$\Rightarrow 105k \equiv 0 \pmod{360}$

$\Rightarrow 21k = 72m$

$\Rightarrow 7k = 24m \quad \Rightarrow 7k \equiv 0 \pmod{24}$

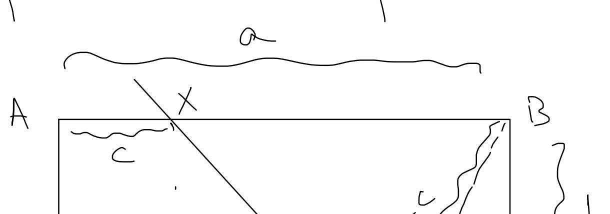
$\Rightarrow$ smallest $\boxed{k = 24}$

Q> The sides x and y of a scalene triangle satisfy $x + \frac{2\Delta}{x} = y + \frac{2\Delta}{y}$ where Δ is the area of the triangle. If $x = 60$ and $y = 63$, what is the length of the largest side of the triangle

Ans:- Apply, $s = x+y+z = 123 + z$ and $x + \frac{2}{x}\sqrt{s(s-x)(s-y)(s-z)} = y + \frac{2}{y}\sqrt{s(s-x)(s-y)(s-z)}$

put $x = 60$, $y = 63$

Q> A rectangular piece of paper has integer side lengths. The paper is folded so that a pair of diagonally opposite vertices coincide and it is found that the crease is of length 65. Find a possible value of the perimeter of the paper.



Ans:- $BX = DY = a - c$

Ans:- $BX = DY = a-c$

$XY = 65$

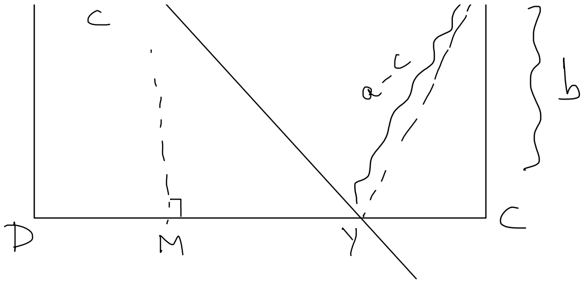
$MY = a-2c$

$(a-2c)^2 + b^2 = 65^2$

$c^2 + b^2 = (a-c)^2$

$a-2c = 16,$

$b = 63$



$65^2 - (a-2c)^2 = (a-c)^2 - c^2$

$\Rightarrow 2a^2 - 6ac + 4c^2 = 65^2$