$$\cos(2\theta) = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta$$

$$\Rightarrow \quad 2\sin^2\theta = 1 - \cos(2\theta)$$

$$\Rightarrow \quad \sin\theta = \pm\sqrt{\frac{1-\cos2\theta}{2}}$$

→ Replacing θ with $\frac{\alpha}{2}$ we get,

$$\boxed{\sin\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1-\cos\alpha}{2}}}$$

$$\boxed{\cos\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1+\cos\alpha}{2}}}$$

$$\tan\left(\frac{\alpha}{2}\right) = \frac{\sin\left(\frac{\alpha}{2}\right)}{\cos\left(\frac{\alpha}{2}\right)}$$

$$= \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}} = \boxed{\frac{\sin\alpha}{1+\cos\alpha} = \frac{1-\cos\alpha}{\sin\alpha}}$$

$$\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta} \quad \Rightarrow \quad (\tan2\theta)\tan^2\theta + 2\tan\theta - \tan2\theta = 0$$

$$\Rightarrow \tan\theta = \frac{-2 \pm \sqrt{4 + 4\tan^2 2\theta}}{2\tan2\theta}$$

$$\Rightarrow \tan\theta = \frac{-1 \pm \sec2\theta}{\tan2\theta} = \frac{-\cos2\theta \pm 1}{\frac{\cos2\theta \sin2\theta}{\cos2\theta}} = \frac{\pm1 - \cos2\theta}{\sin2\theta}$$

$$\tan\theta = \frac{1-\cos2\theta}{\sin2\theta}$$

$$2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$= 2\left(\cos^2\frac{\alpha}{2}\cos^2\frac{\beta}{2} - \sin^2\frac{\alpha}{2}\sin^2\frac{\beta}{2}\right)$$

added and subtracted $\cos\alpha$
↗

$$= \cos\alpha - \cos^2\frac{\alpha}{2} + \sin^2\frac{\alpha}{2} + 2\cos^2\frac{\alpha}{2}\cos^2\frac{\beta}{2} - 2\sin^2\frac{\alpha}{2}\sin^2\frac{\beta}{2}$$

$$= \cos\alpha + \cos^2\frac{\alpha}{2}\left(2\cos^2\frac{\beta}{2} - 1\right) + \sin^2\frac{\alpha}{2}\left(1 - 2\sin^2\frac{\beta}{2}\right)$$

$$= \cos\alpha + \cos^2\frac{\alpha}{2}(\cos\beta) + \sin^2\frac{\alpha}{2}(\cos\beta)$$

$$= \cos\alpha + \cos\beta$$

HomeWork

... $\sin^2\theta + \cos^2\theta$ for $\theta \in [0, 360]$

HomeWork

Q) Find the range of $\sec^2\theta + \cos^2\theta$ for $\theta \in [0, 360]$

Q) Find the range of $\sec\theta + \cos\theta$ for $\theta \in [0, 360]$

Q) Find the range of $\sqrt{1+\tan^2\theta} + \cos\theta$ for $\theta \in [0, 360]$

Q) Find the range of $\sqrt{1+\tan^2\theta} + \sqrt{1-\sin^2\theta}$ for $\theta \in [0, 360]$

Q) Find the range of $\tan\theta \sec\theta$ for $\theta \in [0, 360]$