Geometry 12

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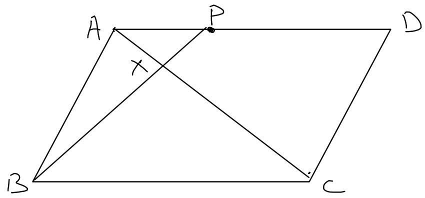
Q) ABCD is a parallelogram such that P is on AD and AP : AD = 1 : p and X is the intersection of AC and BP. Prove that AX : AC = 1 : (p+1)



Ans:— $AP = m \quad AD = pm$

$PD = (p-1)m \qquad BC = pm$

In ΔAPX and ΔBCX, $\dfrac{CX}{AX} = \dfrac{BX}{PX} = \dfrac{BC}{AP}$

$\dfrac{CX}{AX} = \dfrac{p}{1} \qquad CX = CA - AX$

$\Rightarrow \dfrac{CA - AX}{AX} = p \Rightarrow \dfrac{CA}{AX} = p+1 \Rightarrow AX : AC = 1 : (p+1)$

Q) Circle C_1 has its centre O lying on circle C_2. The two circles meet at X and Y. Point Z in the exterior of C_1 lies on circle C_2 and XZ = 13, OZ = 11 and YZ = 7. What is the radius of C_1

Let r be the radius of C_1

$13^2 = 11^2 + 7^2$

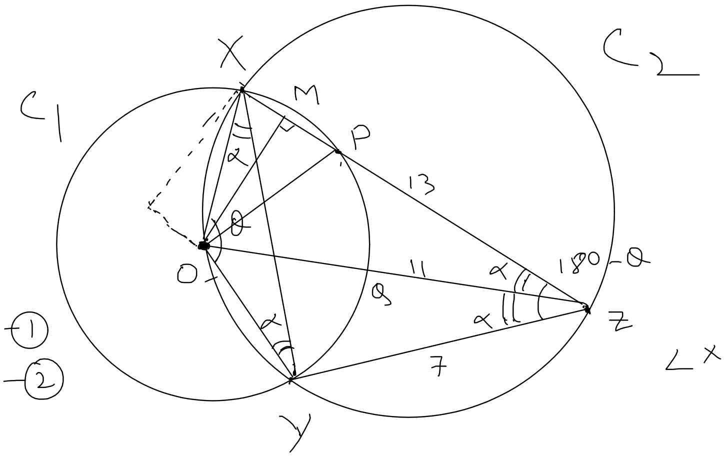
$169 = 121 + 49$

Ans:—

$OX = OY = OP = r = OQ$

$OM \perp XZ$

$\angle OYX = \angle OXY = \angle OZX = \angle OZY$



ΔOXZ

$r^2 = 13^2 + 11^2 - 2 \cdot 13 \cdot 11 \cos\alpha$ — (1)

ΔOZY

$r^2 = 11^2 + 7^2 - 2 \cdot 11 \cdot 7 \cos\alpha$ — (2)

$\angle XZO = \angle OZY$ as $\widehat{XO} = \widehat{OY}$

(1) × 7 − (2) × 13 ⇒

$7r^2 - 13r^2 = 7(13^2 + 11^2) - 13(11^2 + 7^2)$

$\Rightarrow 6r^2 = 13 \times 13^2 - 7 \times 13^2 - 7 \times 11^2 \Rightarrow r^2 = (6 \times 13^2 - 7 \times 11^2)/6$

$\Rightarrow r = \sqrt{(6 \times 13^2 - 7 \times 11^2)/6}$

Q) Circle A has radius 100, Circle B has integer radius $r < 100$ and remains internally tangent to circle A as it rolls once around circumference of circle A. The two circles have the same points of tangency at the beggining and end of circle B's trip. How many possible values can r have?

Ans:— A ↓ $2\pi 100$

B ↓ $2\pi r$

$k(2\pi r) = 2\pi 100$

$k = \frac{100}{r}$

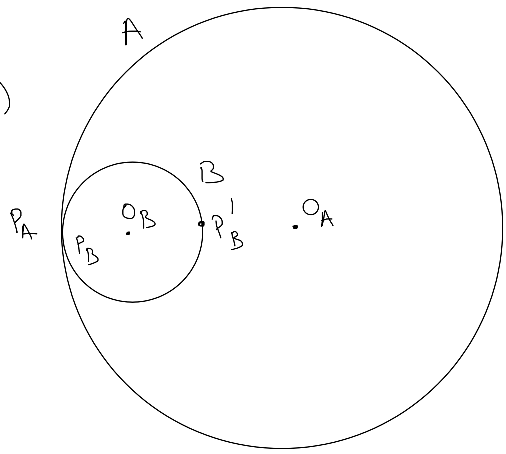
$k \in \mathbb{Z}$

$r = 8$

$100 = 2^2 5^2$

$(2+1)(2+1)$

but 100 is not included



To take a full round and P_B touch with P_A integer multiple of circumference of B should roll

Q) Square AIME has sides of length 10 units. Isosceles triangle GEM has base EM and the area common to ΔGEM and AIME is 80 sq. units. Find the length of altitude to EM in ΔGEM

Ans:— As common area is > 50 G must be outside square

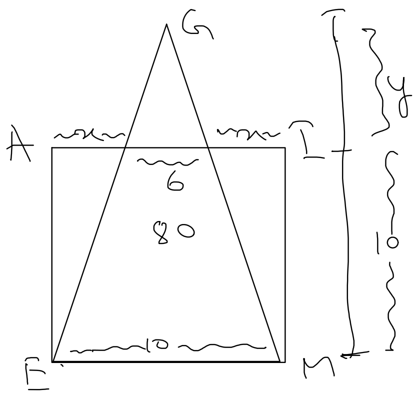
$\cancel{2}\left(\frac{1}{\cancel{2}} x \times 10\right) = 20 \Rightarrow x = 2$

$\frac{6}{10} = \frac{y}{y+10} \Rightarrow 6y + 60 = 10y$

$\Rightarrow 4y = 60$

$\Rightarrow y = 15$

$\Rightarrow$ Altitude $= 25$



HomeWork:—

Q) Point K lies on diagonal BD of parallelogram ABCD. AK intersects lines BC and CD at L and M respectively. Prove that $AK^2 = LK \cdot KM$.