Random Stuff 2

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$\mathbb{Z} \rightarrow$ integers $\mathbb{Q} \rightarrow$ rationals, $\mathbb{R} \rightarrow$ reals, $\mathbb{C} \rightarrow$ complex

$\mathbb{Q}^c \rightarrow$ irrational

$\mathbb{Z} \subset \mathbb{Q}$ $\mathbb{R} \subset \mathbb{C}$

$\circ$) Take two rationals q_1, q_2 where $q_1 < q_2$.

$\underbrace{q_3}_{\text{rational}}$ $\underbrace{q_3'}_{\text{irrational}}$ $\rightarrow$ there will infine irrational and rationals between q_1 and q_2

$(a, \quad) \subset \mathbb{R}$

smallest and lar – element in (a, b) is not defined

$\sup\{(a, b \quad = b$ $\qquad$ $\inf\{(a, b)\} = a$

$\sup\{[a, \quad)\} = b$ $\qquad$ $\inf\{[a, b)\} = a$

$\sup\{(\quad b]\} = b$ $\qquad$ $\inf\{(a, b]\} = a$

$\sup\{ \quad , b]\} = b$ $\qquad$ $\inf\{[a, b]\} = a$

$\sup\{(a, b), (b+1, c)\} = c$

$|(0, 1)| = |\underbrace{\mathbb{R}}^{\text{uncountable}}|$ $\qquad$ $|\overbrace{\mathbb{N}}^{\text{countable}}| < |\mathbb{R}|$

$n_2 \in \mathbb{N}$
$n_1 \in \mathbb{N}$
$(n_2 - n_1) \in \mathbb{N}$

$f : (0, 1) \rightarrow \mathbb{R}$ can be a bijection

$f(x_1) = y_1$ $\quad$ if $y_1 = y_2 \iff x_1 = x_2 \rightarrow$ one-one
$f(x_2) = y_2$

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and $\underset{\text{onto}}{f^{-1}(y_1) = x_1 \text{ exists } \forall y_1 \in \mathbb{R}}$

Suppose $f: \mathbb{N} \rightarrow \mathbb{R}$ is a bijection (one-one and onto)

Intuition

$f(x_1) = f(x_2) \iff x_1 = x_2$

$\forall y \in \mathbb{R}$, $\exists x \in \mathbb{N}$ such that $f(x) = y$

$\Rightarrow \mathbb{R} = \{ f(1), f(2), \ldots \}$

$\rightarrow$ countable but $\mathbb{R}$ is uncountable $\Rightarrow\Leftarrow$

$|P(A)| > |A|$

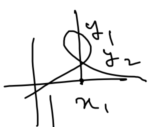
$\rightarrow$ a set that contains all subsets of itself

$A = \{1, 2, 3\}$ $P(A) = \{ \{\phi\}, \{1\}, \{2\}, \{3\}, \{1,2\}, \{2,3\}, \{3,1\}, \{1,2,3\} \}$

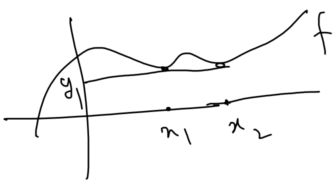
To see a formal proof we need <u>Cantor's Diagonal Argument</u>

$\mathbb{R}^2$

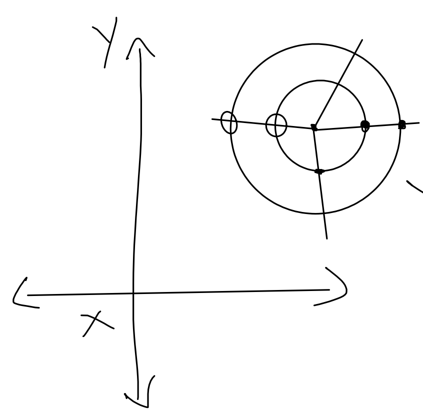
$[0,1] \ncong \mathbb{R}$



f is not a function



f is a function



$\rightarrow$ Both circles are of same cardinality

$f: X \rightarrow Y$
domain (X), codomain (Y)

$f(X)$ is the range

$f(X)$ may not be equal to Y

In function $f(x) = y_1$ and $f(x) = y_2 \Rightarrow y_1 = y_2$

In function $f(x) = y_1$ and $f(x) = y_2 \Rightarrow y_1 = y_2$
(basic difference with map)

$f(x_1) = y_1$, $f(x_2) = y_1 \not\Rightarrow x_1 = x_2$ in function