ZARGON ANKAN

Timing: 4:00 PM to 7:00 PM   Score rule as per IMO

All questions carry equal marks, try as much as possible

1. Prove that for sufficiently large n, the fibonacci number f_n is the integer closest to $\frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n$.

2. Let a, b be real numbers such that $0 \leq a \leq b \leq 1$. Prove that $(ab^2 - ba^2) \leq \frac{1}{4}$.

3. Prove that for $a, b, c > 0$, it is possible to construct a triangle with sides of length a, b, c if and only if $pa^2 + qb^2 > pqc^2$ for any p, q with $p + q = 1$.

4. The numbers in the sequence $101, 104, 109, 116, \ldots$ are of the form $a_n = 100 + n^2$, where $n = 1, 2, 3, \ldots$. For each n, let d_n be the greatest common divisor of a_n and a_{n+1}. Find the maximum value of d_n as n ranges through the positive integers.

5. For natural numbers a, m, n, prove that $\gcd(a^m - 1, a^n - 1) = a^{\gcd(m,n)} - 1$.

6. Find the value of $a \in \mathbb{R}$ such that the sum of squares of the zeros of $x^2 - (a-2)x - (a+1)$ is minimum.

End