Quiz 1, Date: 7 April, 2024 Instructor: Ankan Kar

Timing: 2:00 PM to 4:30 PM Score rule as per IMO

All questions carry equal marks, try as much as possible

- 1. The polynomial $f(x) = x^4 + ax^3 + bx^2 + cx + d$ has real coefficients, and f(2i) = f(2+i) = 0. What is a+b+c+d?
- 2. The equation $z^6 + z^3 + 1 = 0$ has complex roots with argument θ between 90° and 180° in the complex plane. Determine the degree measure of θ .
- 3. Let P(x) be a non-zero polynomial with integer coefficients. If P(n) is divisible by n for each positive integer n, what is the value of P(0)?
- 4. Consider the sequence $1, 3, 3, 3, 5, 5, 5, 5, 5, 7, 7, 7, 7, 7, 7, 7, 7, \dots$ and evaluate its 2016th term.
- 5. Let a and b be natural numbers such that 2a b, a 2b, and a + b are all distinct squares. What is the smallest possible value of b?
- 6. Let p be a prime number greater than 3. Prove that $p^2 1$ is divisible by 24.

End