

# Eigenvalue Dynamics and Minimum Variance Portfolios

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## Data

- Dataset: Fama–French 49 US Industry Portfolios
- Frequencies: Daily and Monthly returns
- Source: [https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data\\_library.html](https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html)

## Tasks

### A. Covariance Matrix Analysis

- On 2022–12–31, estimate sample covariance matrices using daily returns with window sizes  $N = 55, 60, 65, \dots, 500$
- Plot minimum eigenvalue of the covariance matrix versus  $N$
- Plot condition number of the covariance matrix versus  $N$
- For 2005–03–31 to 2025–07–31, using a rolling daily window of  $N = 1000$ :
  - Estimate the sample covariance matrix
  - Plot the maximum eigenvalue through time

### B. Portfolio Backtesting

- Period: 2005–03–31 to 2025–07–31
- Monthly rebalancing using a rolling daily window of  $N = 500$
- Construct long-only minimum variance portfolios using:
  - Sample covariance matrix (Sample.Port)
  - Double-decay EWMA covariance matrix with:
    - \* Correlation half-life: 252
    - \* Volatility half-life: 126
  - Equally weighted portfolio (EQW.Port)
- Apply estimated weights to monthly industry returns
- Plot cumulative returns for all portfolios
- Report for each portfolio:
  - Annualized return
  - Annualized volatility
  - Sharpe ratio (risk-free rate = 0)
- Plot average industry weights through time for Sample.Port
- Repeat the backtest with a maximum weight constraint of 5%