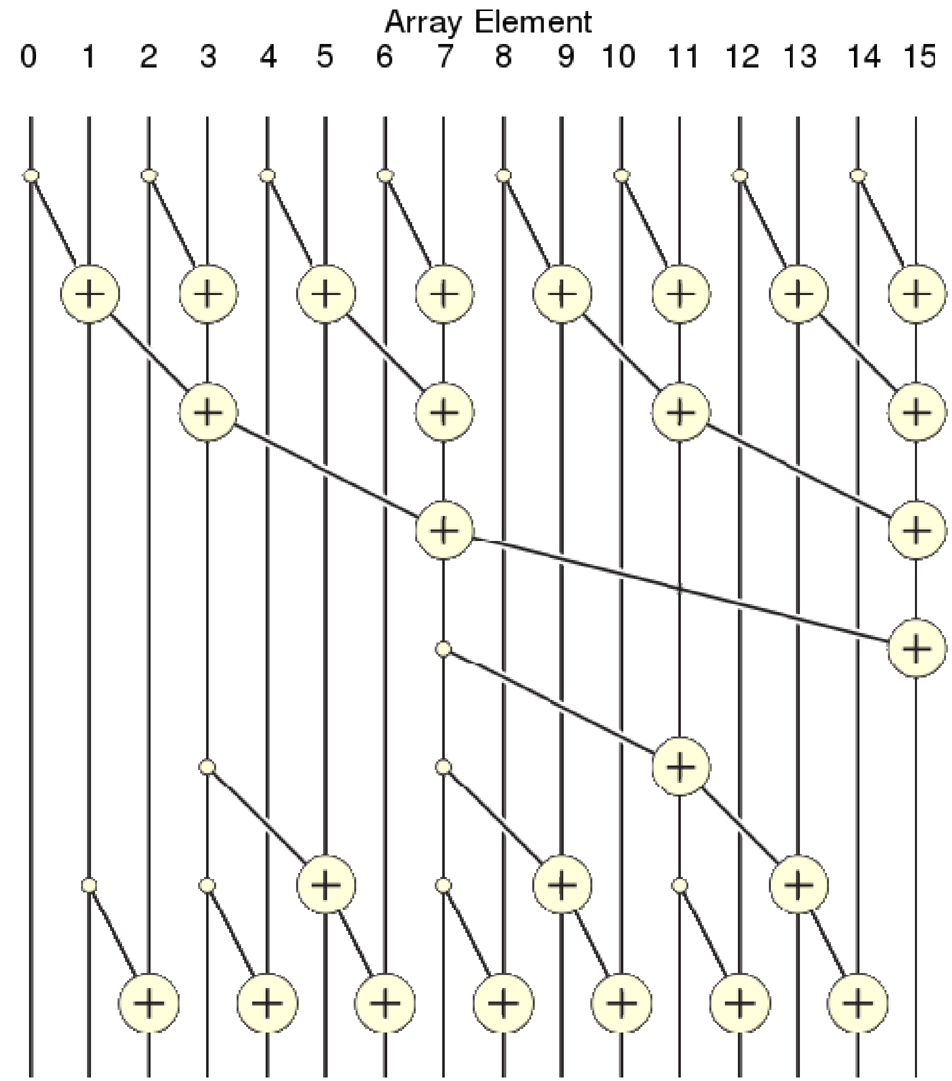
Efficient parallel prefix sum

```
int stride = 1;
while (stride <= blockDim.x) {
   int i = 2*stride*(threadIdx.x+1)-1;
   if (i < 2*blockDim.x)
      sum[i] += sum[i-stride];
   stride *= 2;
   __syncthreads();
}

int stride = blockDim.x/2;
while (stride > 0) {
   int i = 2*stride*(threadIdx.x+1)-1;
   if (i+stride < 2*dimBlock.x)
      sum[i+stride] += sum[i];
   stride /= 2;
   __syncthreads();
}
```

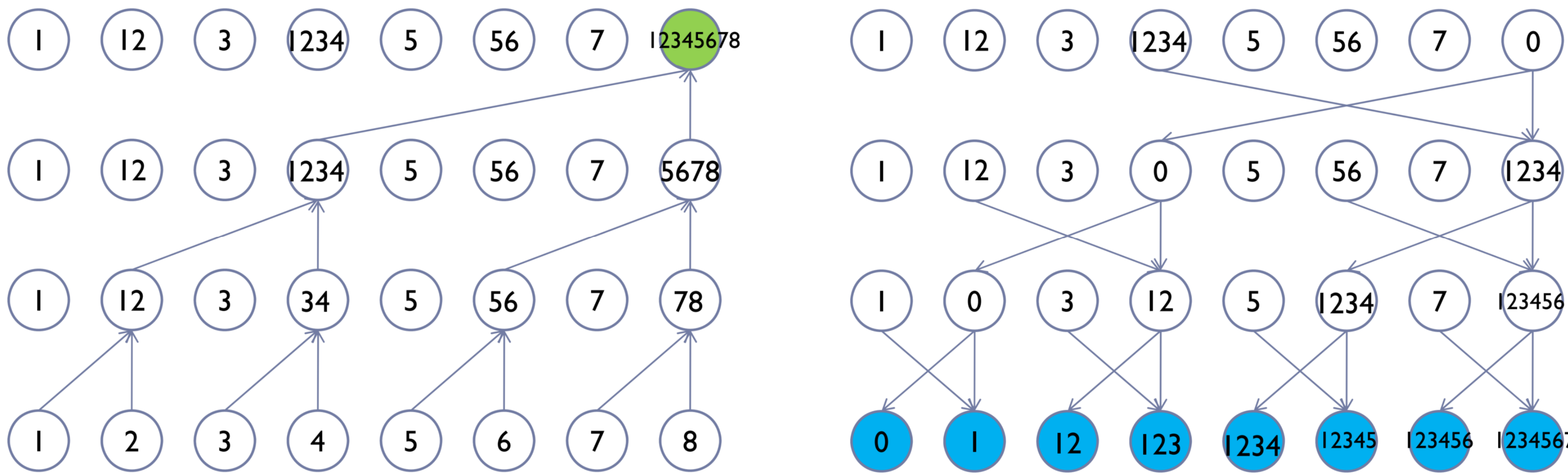


- A thread block computes prefix sum of array `sum` in shared memory.
  - Size of `sum` is 2*(block size).
  - In example, block size = 8.
- In down sweep, threads 0 to (block size) / stride – 1 work in iteration `stride`.
- In up sweep, threads 0 to (block size) / (2*stride) – 1 work in iteration `stride` .

Exclusive scans

- Just like a normal scan, except each input value shouldn't include itself in its output.
  - Ex [1,2,3,4] $\Rightarrow$ [0,1,3,6].
- Up-sweep is the same as in inclusive scan.
- But during down-sweep, first zero out the final output value.
- Then follow a half butterfly pattern downwards.
  - Each right child sums its parents' values.
  - Each left child takes its parent's value.

Exclusive scans



Up-sweep Down-sweep

Up-sweep (reduce):

for $d = 0$ to $\log_2 n - 1$ **do**
    for all $k = 0$ to $n - 1$ by 2^{d+1} in parallel **do**
        $x[k + 2^{d+1} - 1] \leftarrow x[k + 2^d - 1] + x[k + 2^{d+1} - 1]$

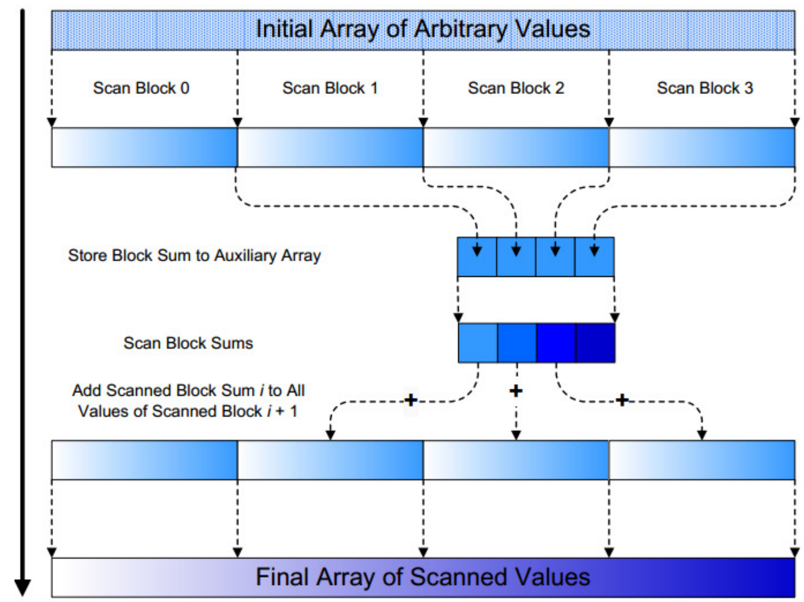
Down-sweep:

$x[n-1] \leftarrow 0$
for $d = \log_2 n - 1$ down to 0 **do**
    for all $k = 0$ to $n - 1$ by 2^{d+1} in parallel **do**
        $t \leftarrow x[k + 2^d - 1]$
        $x[k + 2^d - 1] \leftarrow x[k + 2^{d+1} - 1]$
        $x[k + 2^{d+1} - 1] \leftarrow t + x[k + 2^{d+1} - 1]$

Source: http://courses.me.berkeley.edu/ME290R/S2009/lectures/lec15.PDF

Arbitrary input size

- The inclusive scan algorithm only works for array size ≤ 2*(block size).
- For bigger inputs, break it into segments of size 2*(block size).
- Compute prefix sum on each segment using block algorithm.
- Copy sum of whole segment (stored in `sum[blockDim.x-1]`) to `segment_sum` array.
- Do this for all blocks until they all finish.
  - Ensure blocks finished by ending kernel.
- Compute prefix sum of `segment_sum` array in a second kernel.
- In a third kernel, distribute prefix sums to each segment.
  - Segment increases all values by prefix sum received.



Bank conflicts

- Recall memory address x stored at x % n if shared memory has n banks.
  - Current GPUs have 32 banks.
- Current algorithm has many bank conflicts, causing serialized accesses.

bank 0	0	4	8	12	16
bank 1	1	5	9	13	17
bank 2	2	6	10	14	18
bank 3	3	7	11	15	19

16 banks, stride = 1. 2 way bank conflicts

tid	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
i	1	3	5	7	9	11	13	15	17	19	21	23	25	27	29	31
bank	1	3	5	7	9	11	13	15	1	3	5	7	9	11	13	15

16 banks, stride = 2. 4 way bank conflicts

tid	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
i	3	7	11	15	19	23	27	31	35	39	43	47	51	55	59	63
bank	3	7	11	15	3	7	11	15	3	7	11	15	3	7	11	15

```
...
int i = 2*stride*
   (threadIdx.x+1)-1;
if (i < 2*blockDim.x)
   sum[i] += sum[i-
   stride];
...
```

Removing bank conflicts

- Remove bank conflicts by padding the sum array.
- Store i'th item at address i + floor(i / (# banks)) instead of address i.
  - Do this for reads and writes.
  - Waste some space (~3% with 32 banks), but get faster performance.
- Ex 4 banks.

array	0	1	2	3	4	5	6	7	8	9	10	11		
padded array	0	1	2	3	P	4	5	6	7	P	8	9	10	11

- Padding is a general strategy for removing bank conflicts, though exact scheme depends on problem.

Removing bank conflicts

16 banks, stride = 2. 4 way bank conflicts

tid	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
i	3	7	11	15	19	23	27	31	35	39	43	47	51	55	59	63
bank	3	7	11	15	3	7	11	15	3	7	3	15	3	7	11	15

16 banks, stride = 2, i' = i + floor(i / # banks). No bank conflicts

tid	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
i'	3	7	11	15	20	24	28	32	37	41	45	49	54	58	62	66
bank	3	7	11	15	4	8	12	0	5	9	13	1	6	10	14	2

Segmented scan

Work-efficient segmented scan

Up-sweep:

```
for d=0 to (log₂n - 1) do
   forall k=0 to n-1 by 2^(d+1) do
     if flag[k + 2^(d+1) - 1] == 0:
        data[k + 2^(d+1) - 1] ← data[k + 2^d - 1] + data[k + 2^(d+1) - 1]
     flag[k + 2^(d+1) - 1] ← flag[k + 2^d - 1] || flag[k + 2^(d+1) - 1]
```

Down-sweep:

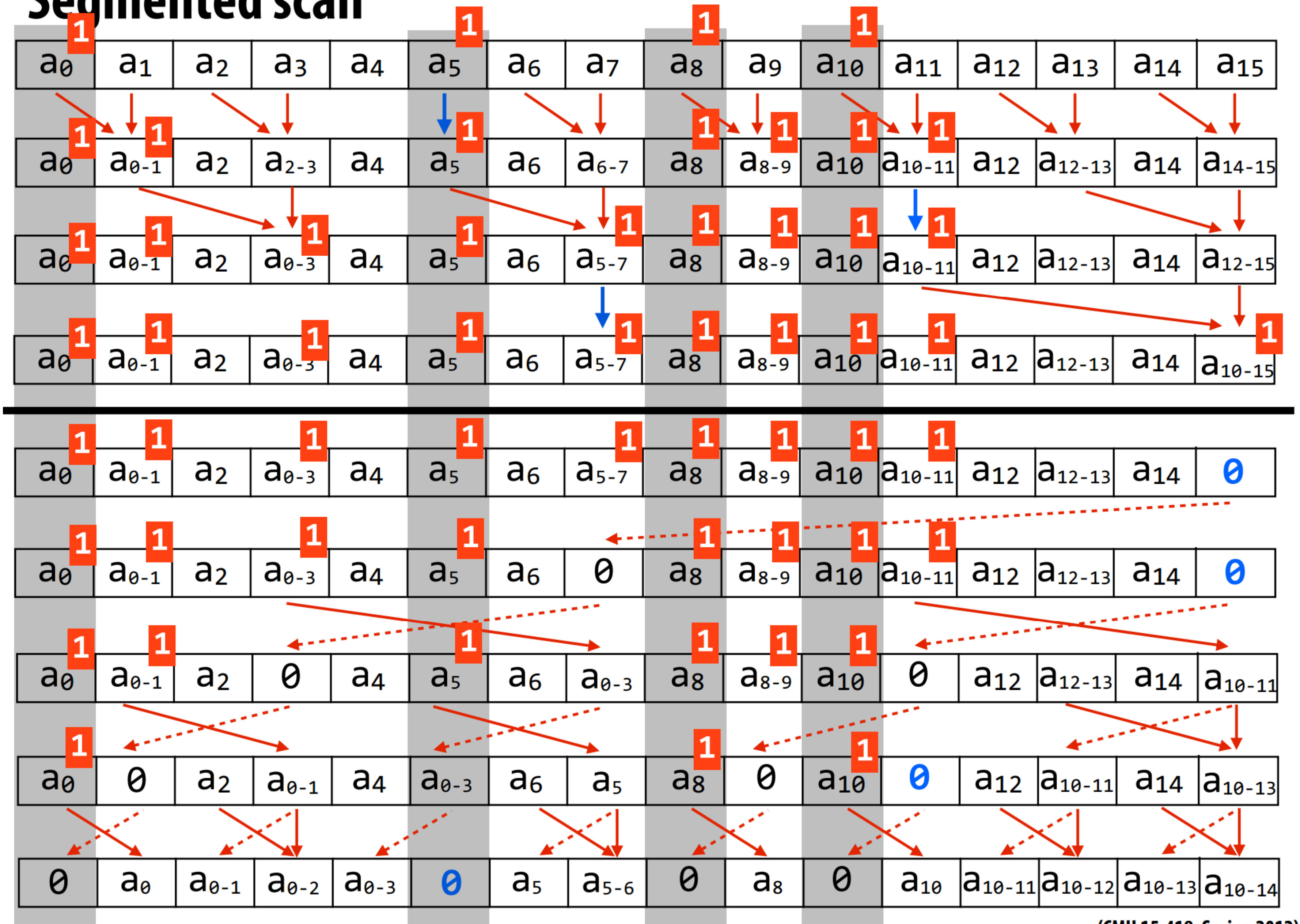
```
data[n-1] ← 0
for d=(log₂n - 1) down to 0 do
   forall k=0 to n-1 by 2^(d+1) do
     tmp ← data[k + 2^d - 1]
     data[k + 2^d - 1] ← data[k + 2^(d+1) - 1]
     if flag_original[k + 2^d] == 1:          // maintain copy of original flags
        data[k + 2^(d+1) - 1] ← 0
     else if flag[k + 2^d - 1] == 1:
        data[k + 2^(d+1) - 1] ← tmp
     else:
        data[k + 2^(d+1) - 1] ← tmp + data[k + 2^(d+1) - 1]
     flag[k + 2^d - 1 ] ← 0
```

(CMU 15-418, Spring 2012)

- Sometimes need to run (exclusive) prefix sum on several segments at once.
- Ex [1 2 3 4] [6 5] [1 3 5] ⇒ [0 1 3 6] [0 6] [0 1 4]
- If do m scans, each of size n individually, then O(n log m) time.
- We do all the scans in O(log (mn)) time.
- Up sweep distributes partial sums.
  - Use flags to delimit segments.
  - No value "crosses" a segment.
- Down sweep collects values from one segment and sums them.

Source: http://www.cs.cmu.edu/afs/cs/academic/class/15418-s12/www/

Segmented scan



(CMU 15-418, Spring 2012)

Application: compaction

- Create array containing elements of input array satisfying a condition.
- Ex Move all odd numbers in *A* to front of *output*.
  - Create filter array that's 1 if element satisfies condition.
  - Prefix sum the filter array.
  - For each element, if it satisfies condition, move it to index given by prefix sum.

A = [1 3 2 4 8 6 5 4 9 7 3]

filter = [1 1 0 0 0 0 1 0 1 1 1]

sums = [1 2 2 2 2 2 3 3 4 5 6]

output = [1 3 5 9 7 3]

Application: string comparison

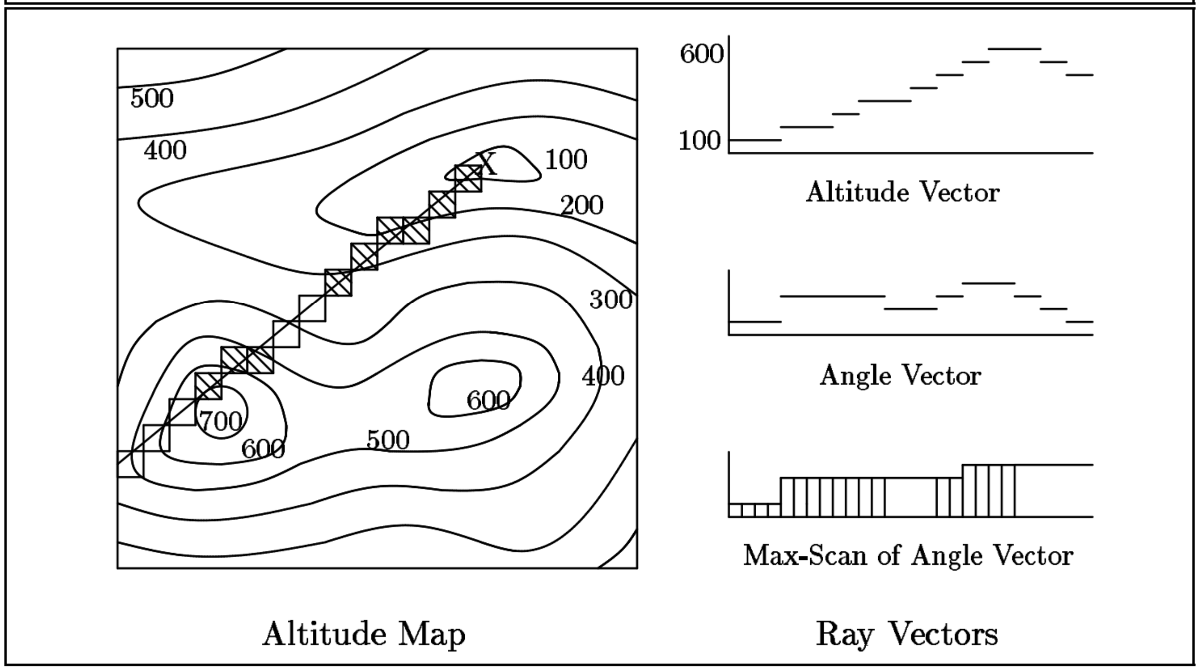
- Compare two strings alphabetically.
- Ex parallax < parallel.
- Let strings be S, T. Let S[i],T[i] denote i'th letter of S,T.

- In parallel, i'th processor compares S[i] to T[i].
  - If S[i]>T[i], set A[i]=1.
  - If S[i]=T[i], set A[i]=0.
  - If S[i]<T[i], set A[i]=-1.
  - If S[i] or T[i] doesn't exist, set A[i]=0.
- Compact A to remove all 0's.
- If output[1]=1, then S>T.
- If output[1]=-1, then T>S.
- If output is empty, then S=T.

- Ex S=parallax, T=parallel, A=[0,0,0,0,0,0,-1,1], output=[-1,1], so T>S.

Application: line of sight

```
procedure line-of-sight(altitude)
  in parallel for each index i
    angle[i] ← arctan(scale × (altitude[i] - altitude[0])/ i)
  max-previous-angle ← max-prescan(angle)
  in parallel for each index i
    if (angle[i] > max-previous-angle[i])
       result[i] ← "visible"
    else
       result[i] ← not "visible"
```



- Given a contour map, an observation point X and a direction, want to know which points are visible.
- First, draw a line from X in the observing direction and record the altitudes along the line in an altitude vector.
- Then for each point calculate its angle, based on its altitude and distance from X.
- Then do a max-scan over the angle vectors.
- A point is visible iff its angle is larger than all the preceding angles.