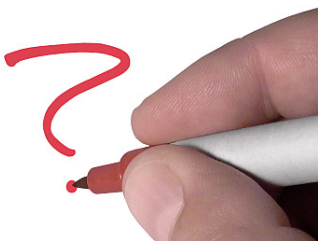
CS243: Introduction to Algorithmic Game Theory

Week 2.1, Dominate Strategy and Truthfulness (Dengji ZHAO)

SIST, ShanghaiTech University, China

Recap: Game Theory

	Participants	Game	Outcome
Game Theory			
Mechanism Design			

Recap: (Simultaneous Move) Game Playing

- A set of n players
- Each player i has a set of strategies S_i
- Let $s = (s_1, \cdots, s_n)$ be the vector of strategies selected by the n players. Also let $s = (s_i, s_{-i})$.
- Let $S = \prod_i S_i$ be the strategy vector space of all players.
- Each $s \in S$ determines the outcome for each player, denote $u_i(s)$ the utility of player i under s.

Recap: (Simultaneous Move) Game Playing

Definition

A strategy vector $s \in S$ is a dominant strategy equilibrium, if for each player i, and each alternate strategy vector $s' \in S$, we have that $u_i(s_i, s'_{-i}) \geq u_i(s'_i, s'_{-i})$

Definition

A strategy vector $s \in S$ is said to be a (pure strategy) Nash equilibrium if for all players i and each alternate strategy $s'_i \in S_i$, we have that $u_i(s_i, s_{-i}) \geq u_i(s'_i, s_{-i})$

Recap: Games

P1 \ P2	Confess	Silent
Confess	4, 4	5, 1
Silent	1, 5	2, 2

Prisoners' Dilemma

Girl \ Boy	B	S
B	6, 5	1, 1
S	2, 2	5, 6

Battle of the Sexes

1 \ 2	H	T
H	-1, 1	1, -1
T	1, -1	-1, 1

Matching Pennies

How to compute strategies?

Learning in Games: Best Response

Best Response

Definition

We say that a change from strategy s_i to s_i' is an improving response for player i if $u_i(s_i', s_{-i}) > u_i(s)$ and best response if s_i' maximizes the players' utility $\max_{s_i' \in S_i} u_i(s_i', s_{-i})$.

Best Response

P1 \ P2	Confess	Silent
Confess	4, 4	5, 1
Silent	1, 5	2, 2

Prisoners' Dilemma

Girl \ Boy	B	S
B	6, 5	1, 1
S	2, 2	5, 6

Battle of the Sexes

1 \ 2	H	T
H	-1, 1	1, -1
T	1, -1	-1, 1

Matching Pennies

Game Design: Mechanism Design

- Auctions (Second Price Auction)

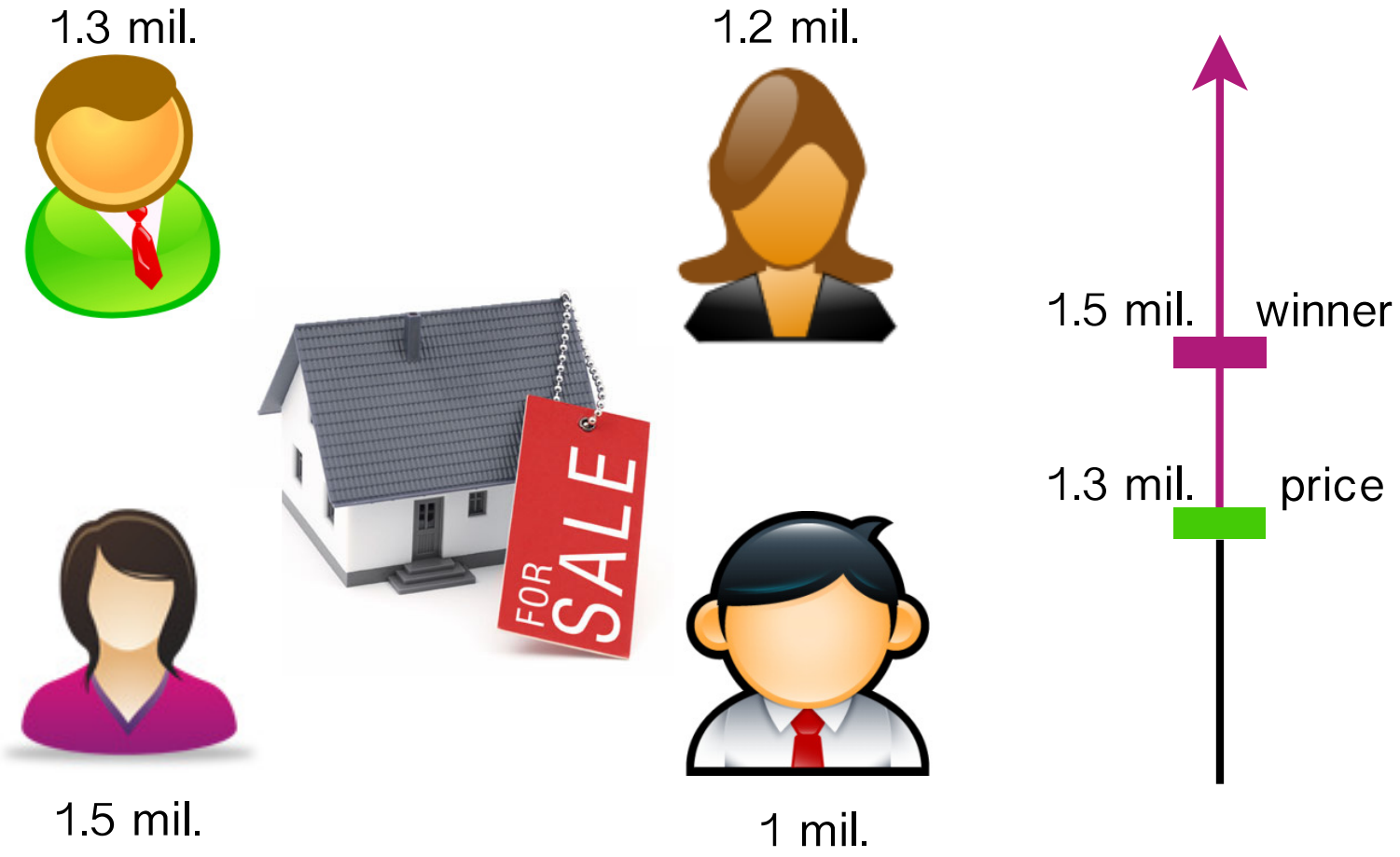
The Setting

- A seller sells an item, e.g. a house.
- A set of n buyers are willing to buy the item, each buyer i has a (private) valuation v_i on the item.

Second Price Auction (Vickrey Auction)

- Each buyer reports her valuation to the seller
- The seller sells the item to the buyer with the highest valuation report
- The seller charges the winner the second highest valuation report

Second Price Auction (Vickrey Auction)



Strategies of the Buyers

- Strategy/Action space:

Strategies of the Buyers

- Strategy/Action space: 
- What is the best strategy for a buyer?

Dominant Strategy in Auction Design: Truthfulness

Definition

An auction is truthful if reporting valuation truthfully is a dominant strategy for all participants/buyers.

Dominant Strategy in Auction Design: Truthfulness

Definition

An auction is truthful if reporting valuation truthfully is a dominant strategy for all participants/buyers.

- Why truthfulness is important?

Dominant Strategy in Auction Design: Truthfulness

Definition

An auction is truthful if reporting valuation truthfully is a dominant strategy for all participants/buyers.

- Why truthfulness is important?
  - Simplify participants' decision making
  - Receive truthful valuation information for other decision making, e.g. maximising social welfare

Dominant Strategy in Auction Design: Truthfulness

Definition

An auction is truthful if reporting valuation truthfully is a dominant strategy for all participants/buyers.

- Why truthfulness is important?
  - Simplify participants' decision making
  - Receive truthful valuation information for other decision making, e.g. maximising social welfare

Question

Is there any weakness of truthfulness?

Challenges

Challenge

Is first price auction truthful?

Challenges

Challenge

Is first price auction truthful?

Question

Is fixed price auction truthful?

- A fixed price is given in advance/public-known.
- All buyers whose reports above the fixed prices will win and pay the fixed price.
- If the number of buyers above the price is more than the number of items to sell, use random tie-breaking.

Advanced Reading

Challenge

How to extend second price auction for single item to multiple items settings? Vickrey-Clarke-Groves (VCG)

- Introduction to Mechanism Design [AGT Chapter 9]