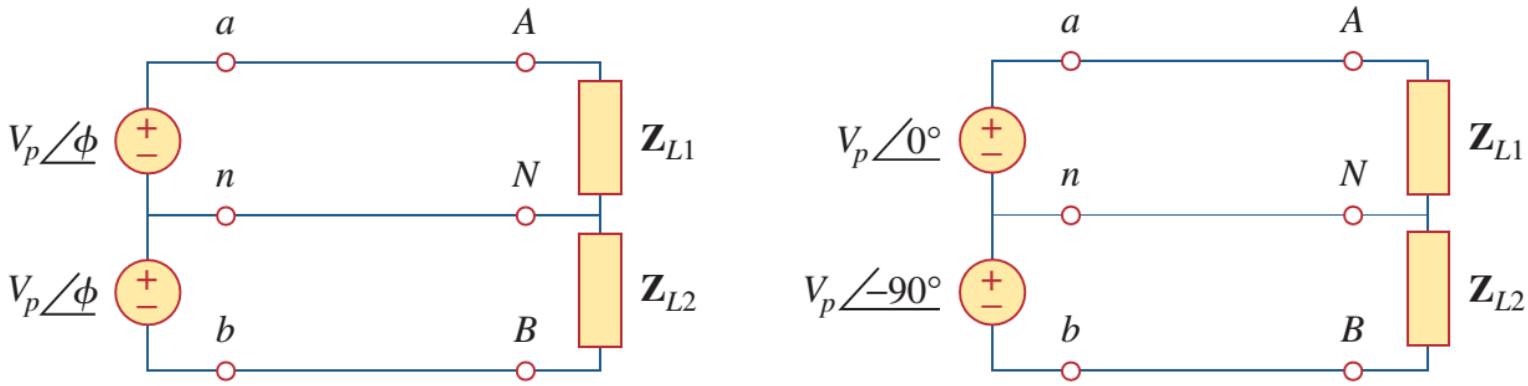
Lecture 10
- Three-Phase Circuits

Single phase vs. Polyphase

- Single-phase power supply
  - For example, two 120V sources with the same phase are connected in series.
  - This allows for appliances to use either 120 or 240V
- Circuits that operate with multiple sources, at the same frequency but *at different phases* are called polyphase.



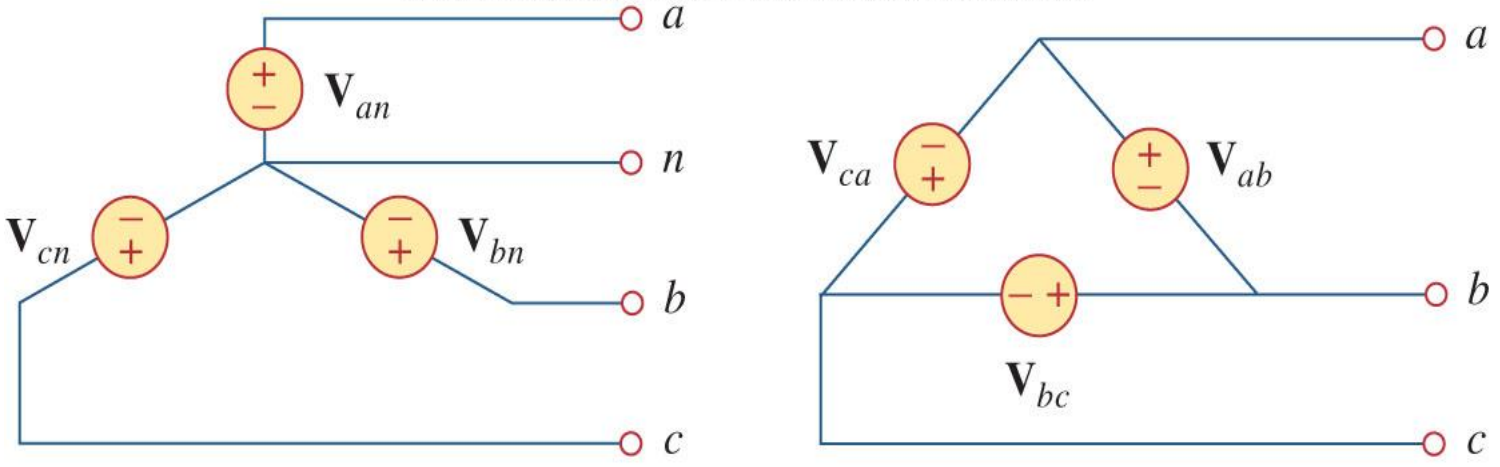
Outline--Three-Phase Circuits

- Balanced Three-Phase System
  - Balanced sources
  - Balanced loads
- Circuit analysis
  - Phase voltage/current
  - Line voltage/current

Balanced Three-Phase Sources

Connecting the Sources

- Three phase voltage sources can be connected by either four or three wire configurations.
  - Four-wire system accomplished using a Y(Wye) connected source.
  - Three-wire configuration accomplished by Delta connected source.



(a) (b)

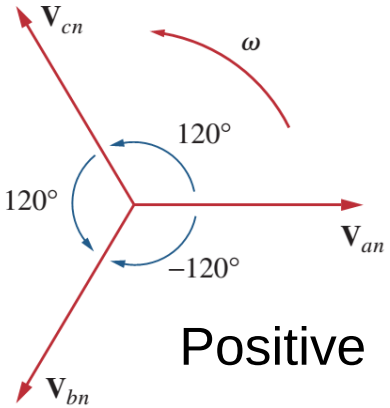
$$V_{an} = V_p \angle 0^\circ$$

$$V_{bn} = V_p \angle -120^\circ$$

$$V_{cn} = V_p \angle -240^\circ = V_p \angle +120^\circ$$

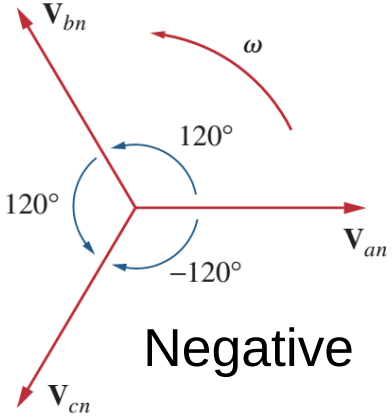
Balanced Three-Phase Sources

- Balanced phase voltage are equal in magnitude and are out of phase with each other by 120deg
- It's easy to know $V_{an}+V_{bn}+V_{cn}=0$
- Two sequences for the phases:



Positive

$$V_{an}=V_p\angle 0^\circ$$
$$V_{bn}=V_p\angle -120^\circ$$
$$V_{cn}=V_p\angle -240^\circ=V_p\angle +120^\circ$$



Negative

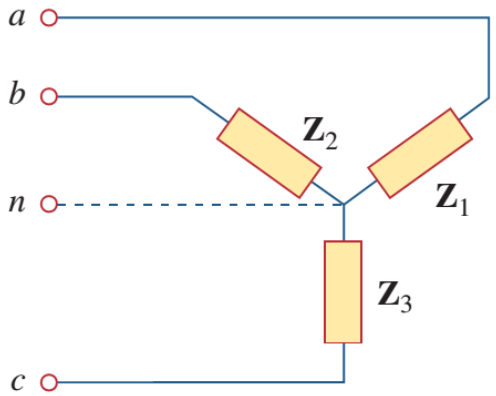
$$V_{an}=V_p\angle 0^\circ$$
$$V_{cn}=V_p\angle -120^\circ$$
$$V_{bn}=V_p\angle -240^\circ=V_p\angle +120^\circ$$

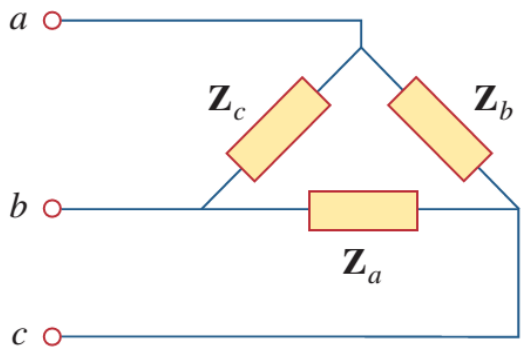
Balanced Loads

- A balanced load means the same impedance for each load.

-- Impedance are equal in magnitude and in phase

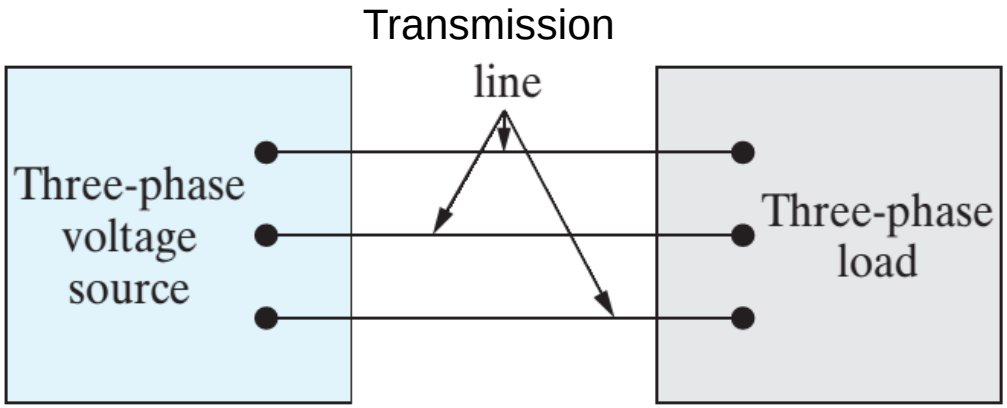
- They may also be connected in either Delta or wye
  - For a balanced wye connected load: $Z_1 = Z_2 = Z_3 = Z_Y$
  - For a balanced delta connected load: $Z_a = Z_b = Z_c = Z_\Delta$





- The load impedance per phase for the above configurations can be interchanged.

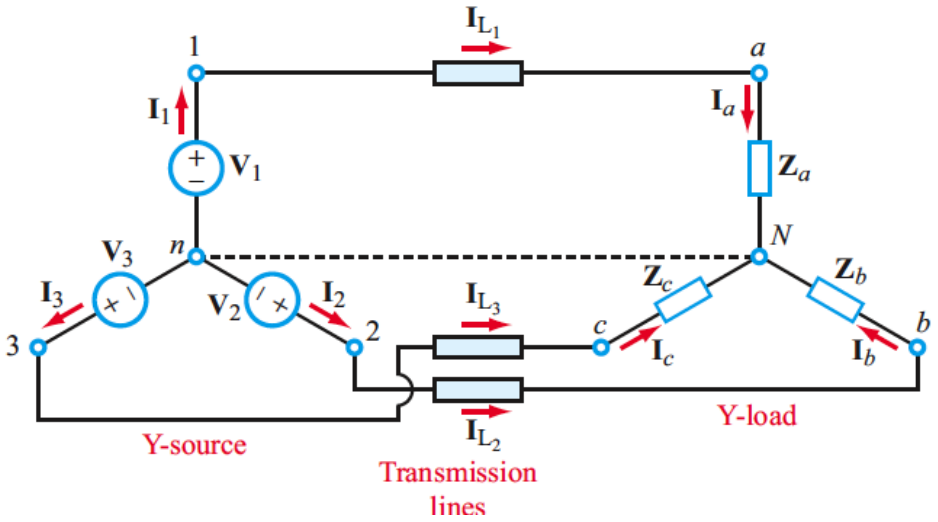
Source-Load configurations



Source	Load
Y	Y
Y	Δ
Δ	Y
Δ	Δ

Source-Load Configurations

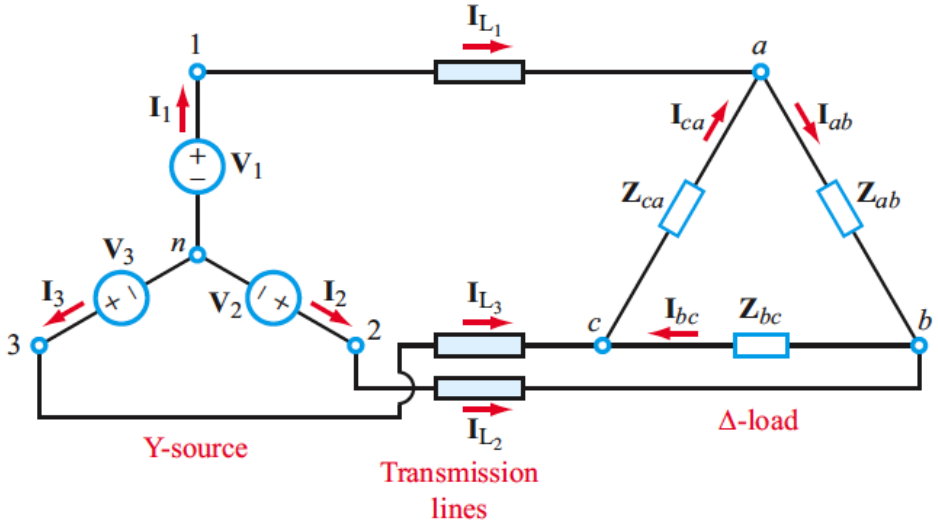
Y-Y



Load Phase Currents
$\mathbf{I}_a, \mathbf{I}_b, \mathbf{I}_c$
(same as line currents $\mathbf{I}_{L_1}$, $\mathbf{I}_{L_2}$, and $\mathbf{I}_{L_3}$)

Load Phase Voltages
$\mathbf{V}_{aN}, \mathbf{V}_{bN}, \mathbf{V}_{cN}$

Y-Delta

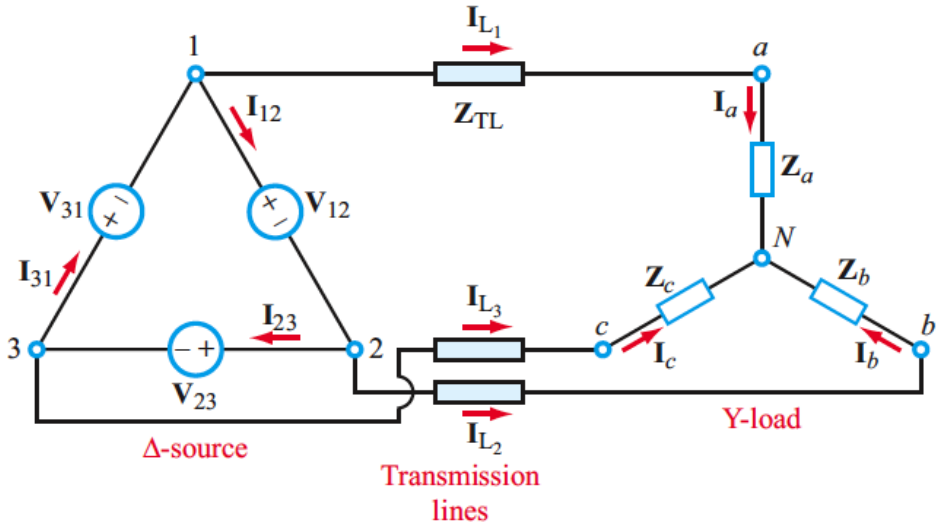


Load Phase Currents
$\mathbf{I}_{ab}, \mathbf{I}_{bc}, \mathbf{I}_{ca}$

Load Phase Voltages
$\mathbf{V}_{ab}, \mathbf{V}_{bc}, \mathbf{V}_{ca}$

Source-Load Configurations (optional)

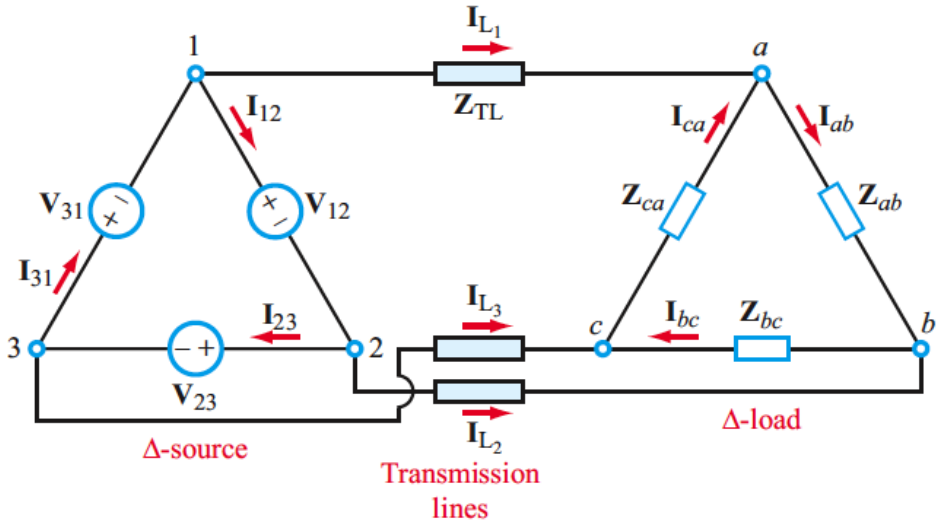
Delta-Y



Load Phase Currents
$\mathbf{I}_a, \mathbf{I}_b, \mathbf{I}_c$
(same as line currents $\mathbf{I}_{L_1}$, $\mathbf{I}_{L_2}$, and $\mathbf{I}_{L_3}$)

Load Phase Voltages
$\mathbf{V}_{aN}, \mathbf{V}_{bN}, \mathbf{V}_{cN}$

Delta-Delta



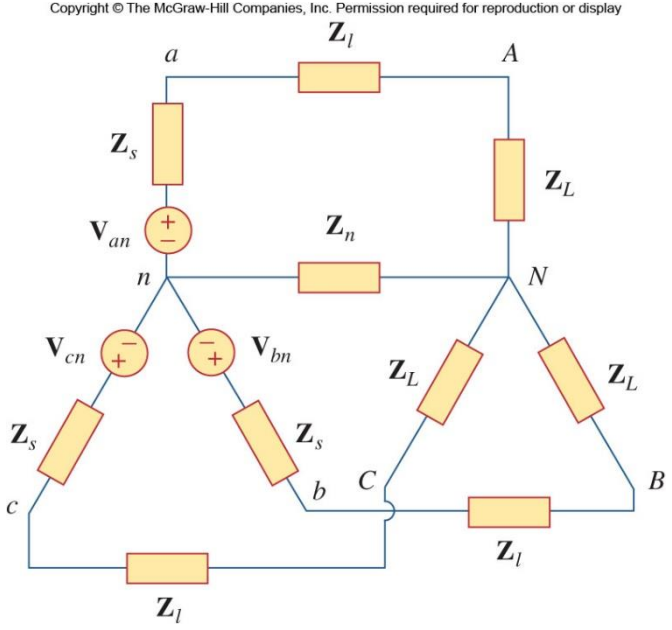
Load Phase Currents
$\mathbf{I}_{ab}, \mathbf{I}_{bc}, \mathbf{I}_{ca}$

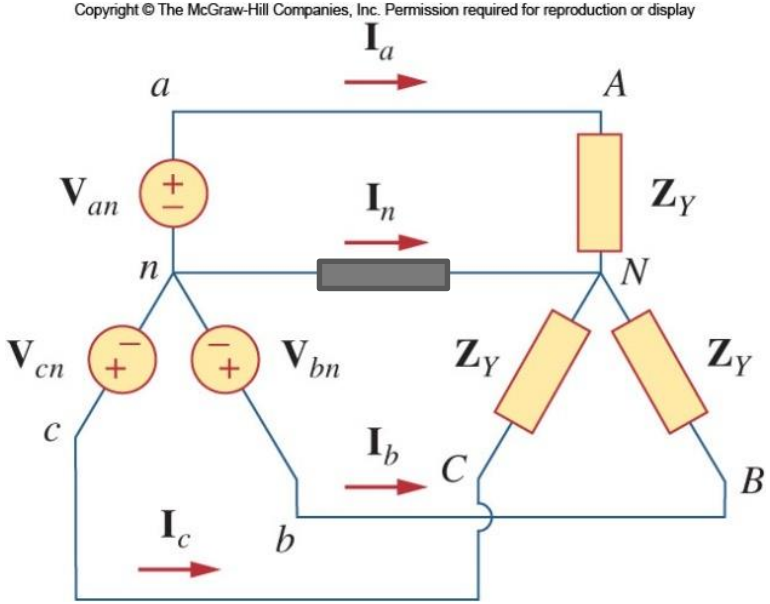
Load Phase Voltages
$\mathbf{V}_{ab}, \mathbf{V}_{bc}, \mathbf{V}_{ca}$
(same as source voltages if $\mathbf{Z}_{TL}$ is negligible)

Balanced Y-Y connection

- The load impedances Z_Y will be assumed to be balanced.
  - This can be the source Z_s, line Z_l and load Z_L together.

$$\mathbf{Z}_Y = \mathbf{Z}_s + \mathbf{Z}_\ell + \mathbf{Z}_L$$



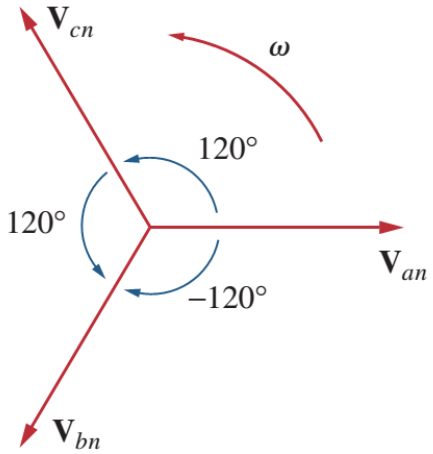


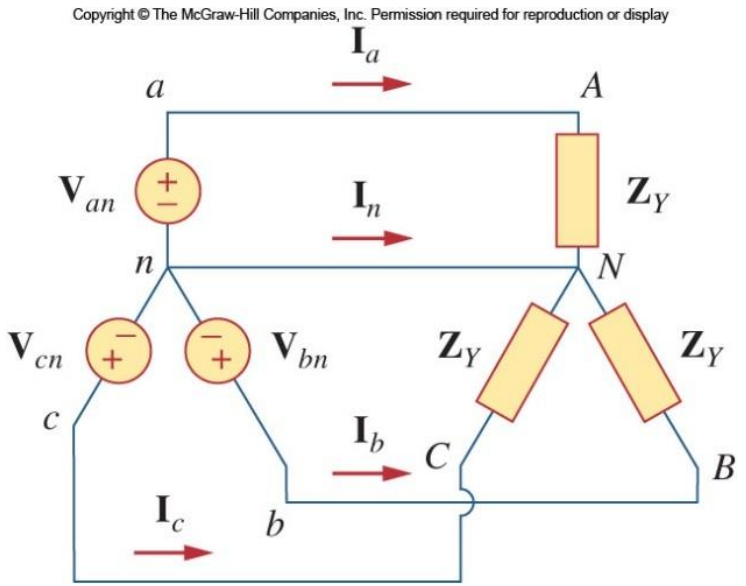
Phase Voltage & Line-to-Line Voltage

- Use the positive sequence:

Phase Voltage $V_{an} = V_p \angle 0^\circ$

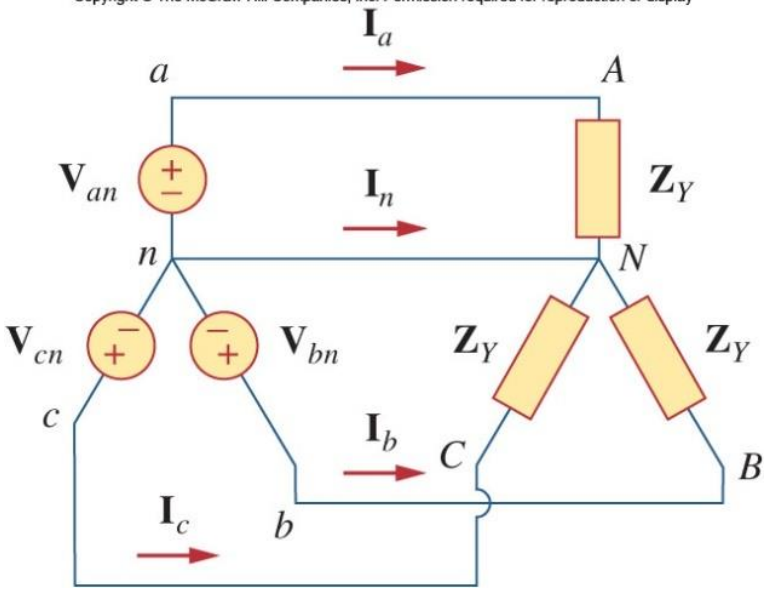
$V_{bn} = V_p \angle -120^\circ \quad V_{cn} = V_p \angle +120^\circ$





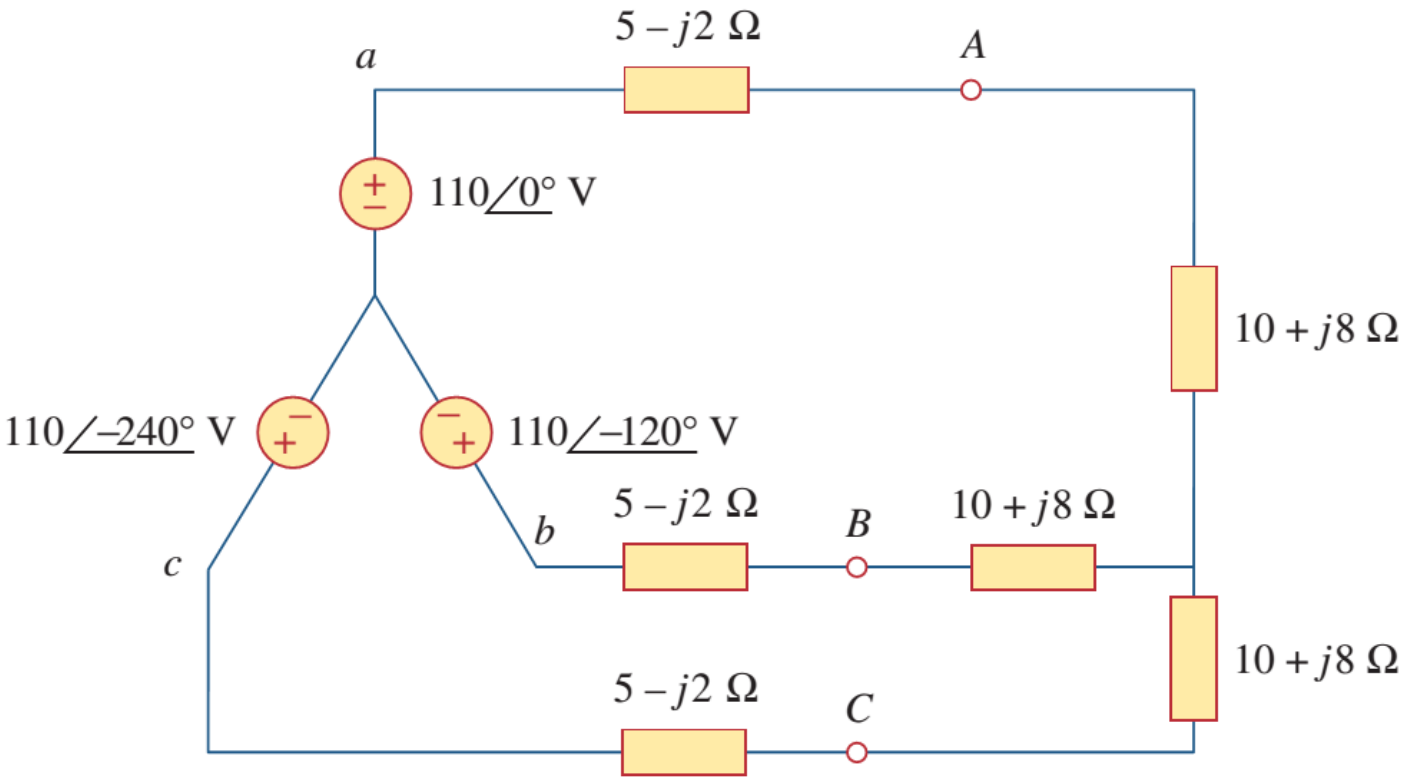
- ***The line to line voltages (or just line voltages in short):***

Line Currents

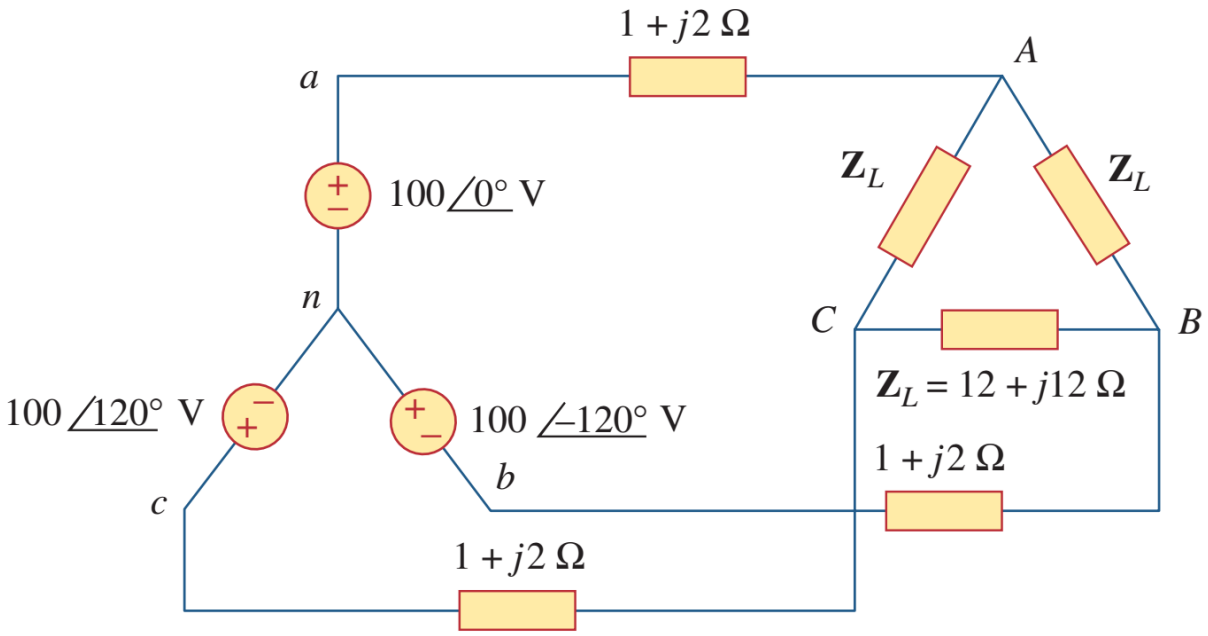


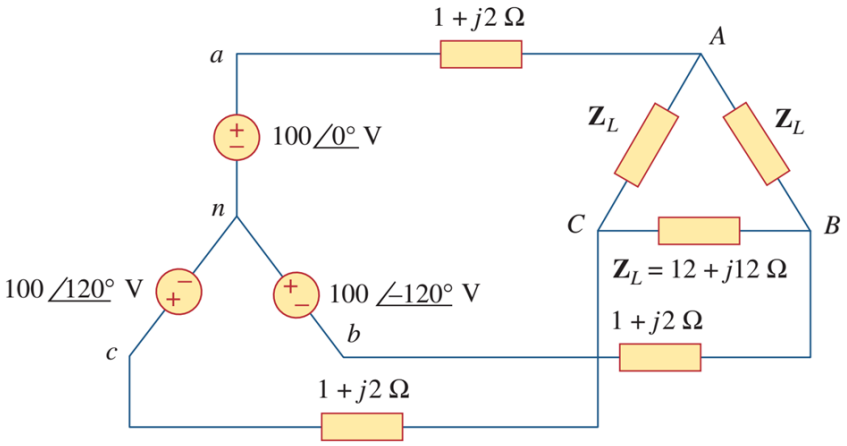
Example

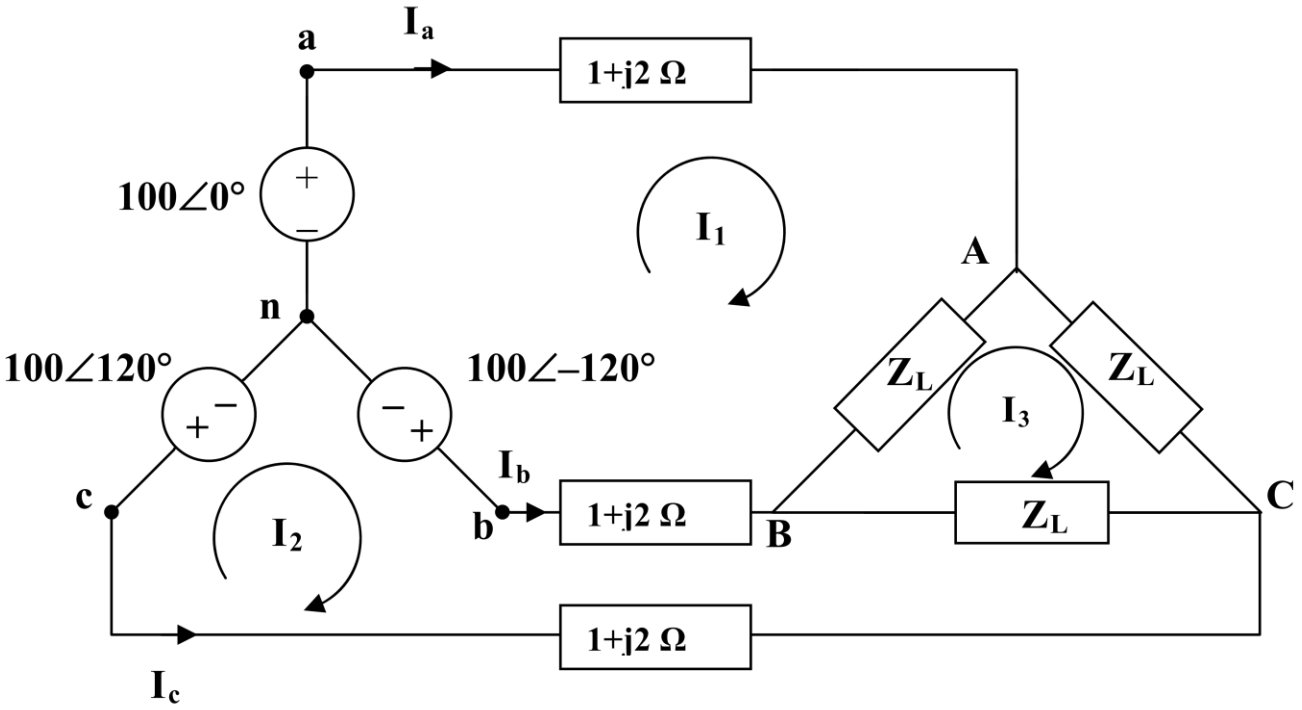
- Calculate the line currents.



Wye-Δ


1 + j2 Ω
a
A
100 ∠0° V
Z_L
Z_L
n
C
B
Z_L = 12 + j12 Ω
100 ∠120° V
100 ∠−120° V
1 + j2 Ω
b
c
1 + j2 Ω


1 + j2 Ω
a
A
100∠0° V
Z_L
Z_L
n
C
B
100∠120° V
100 ∠–120° V
Z_L = 12 + j12 Ω
b
1 + j2 Ω
c
1 + j2 Ω


a
I_a
1+j2 Ω
100∠0°
I_1
A
n
Z_L
Z_L
100∠120°
100∠–120°
I_3
I_b
c
I_2
b
1+j2 Ω
B
Z_L
C
1+j2 Ω
I_c