Student Name: ________________ Student Number: ________________

EE 111 Electric Circuits Midterm-Fall 2021

Nov 11 2021, 8:15 AM – 9:55 AM

6 problems in total (1 A4 crib sheet allowed)

Answer the Questions in English and on Answer Sheets only

Copy and Re-draw the circuits on Answer Sheets for all problems

Two-decimal policy applies for the final answer

1. **(16 points)** For the circuit shown in Fig.1, use mesh-current method to find i_b.

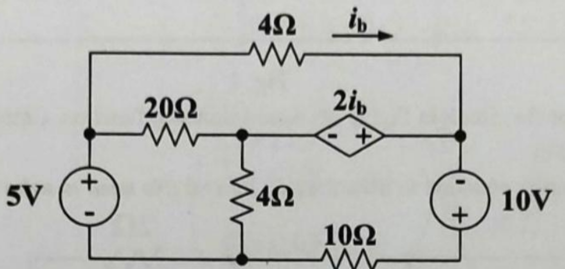

Fig. 1

2. **(16 points)** The **variable dc current source** i_{dc} in Fig.2 can be adjusted. Find the value of i_{dc} so that the power absorbed by the 4A current source is zero. *(Hint: determine if there is any current through the 40-Ω resistor, and then you may use nodal analysis method)*

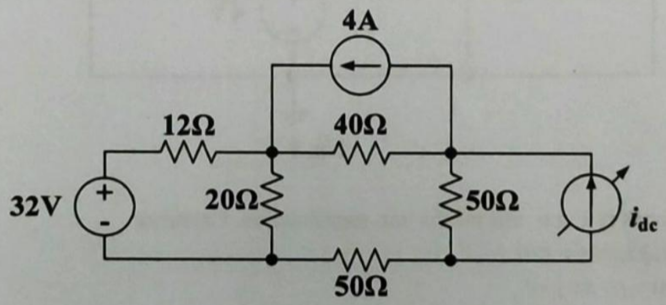

Fig. 2

3. **(16 points)** Find the **Thevenin equivalent OR Norton equivalent** with respect to the terminal a and b for the circuit in Fig.3.

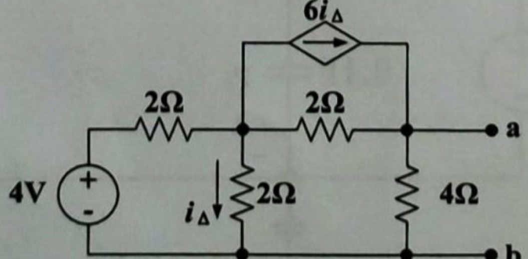

Fig. 3

4. **(16 points)** There is no energy stored in the capacitor when Switch 1 closes at $t = 0$. Twenty milliseconds (20ms) later, Switch 2 is closed, as shown in Fig.4. Find $v_o(t)$ for $t > 0$ *and sketch it in one graph.*

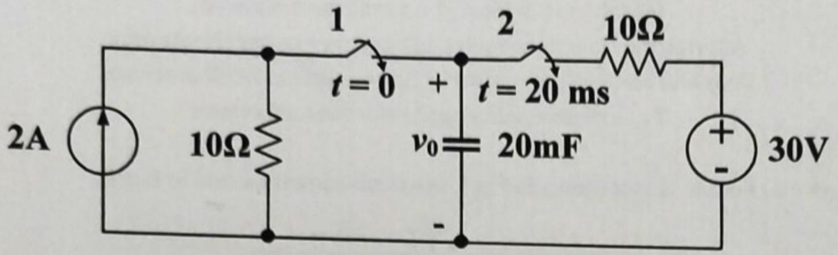

Fig. 4

5. **(16 points)** For the circuit in Fig.5, *u(t)* means unit step function. Calculate

(a) $i_L(0^+)$, $di_L(0^+)/dt$

(b) Find the 2nd-order equation to describe $i_L(t)$ *for t>0* ***(No need to solve it)***

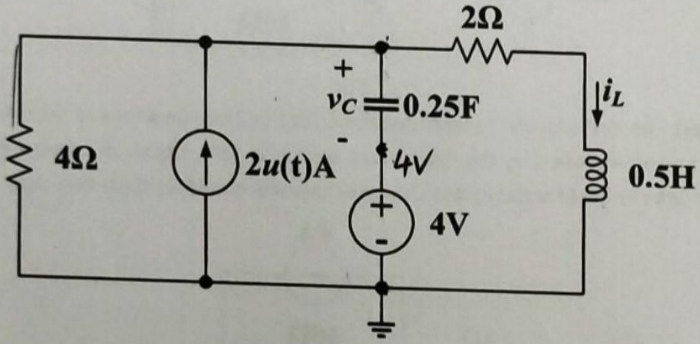

Fig. 5

6. For the circuit in Fig.6, *u(t)* means unit step function. Calculate

(15 points) (a) $i_L(t)$ *for t>0*

(5 points) (b) $v_c(t)$ *for t>0*

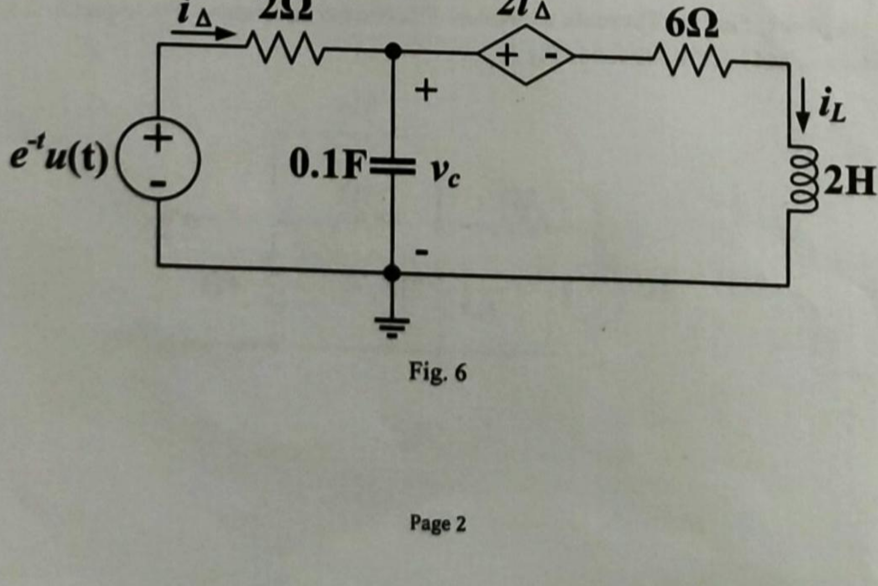

Fig. 6

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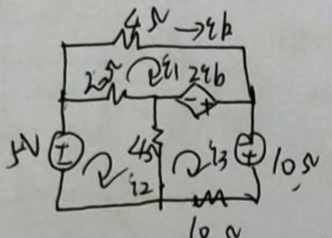
16'

Solution of Problem 1



3'

$i_b = i_1$ 1'

$$\begin{cases} 20(i_b - i_2) + 4i_b + 2i_b = 0 & 3' \\ 20(i_2 - i_b) + 4(i_2 - i_3) - 5 = 0 & 3' \\ 10i_3 + 4(i_3 - i_2) - 2i_b - 10 = 0 & 3' \end{cases}$$

$$\Rightarrow i_b = 0.86\ (A) \quad 3'$$

$$\begin{pmatrix} i_2 = 1.12\ (A) \\ i_3 = 1.16\ (A) \end{pmatrix} \leftarrow 不占分$$

16'

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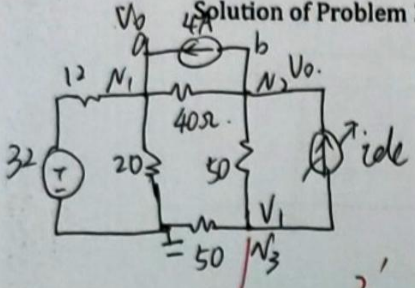
Solution of Problem 2



2'

嫌这个接地

则 $i_{dc} = 6.5A$

Because the power absorbed by 4A is 0W.

the voltage on the terminals a and b is 0V. 2'

Suppose. the voltage on terminal a is V_0, N_3 is V_1

Apply KCL to Nodes 1 and 2 and 3.

$$\begin{cases} \frac{32 - V_0}{12} + 4A = \frac{V_0 - 0}{20\Omega} & 3' \\ -4A + i_{dc} = \frac{V_0 - V_1}{50\Omega} & 3' \\ -i_{dc} + \frac{V_0 - V_1}{50\Omega} = \frac{V_1}{50\Omega} & 3' \end{cases} \Rightarrow \begin{cases} V_0 = 50V \\ V_1 = -200V \\ i_{dc} = 9A \end{cases} \quad 3'$$

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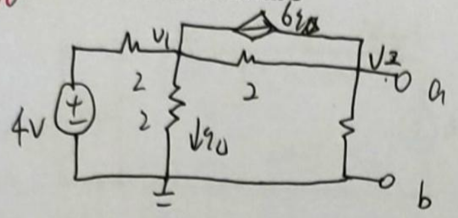
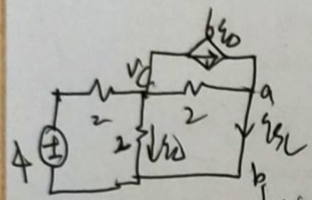
16'

Solution of Problem 3



2'



$$\begin{cases} \frac{v_c}{2} + \frac{v_c}{2} + 6i_0 + \frac{v_c - 4}{2} = 0 \quad 2' \\ i_0 = \frac{v_c}{2} \\ i_{sc} = \frac{v_c}{2} + 6i_0 \quad 2' \end{cases}$$

Apply KCL

$$\begin{cases} \frac{v_1 - 4}{2} + i_0 + \frac{v_1 - v_2}{2} + 6i_0 = 0 \quad 2' \\ \frac{v_1}{2} = i_0 \\ \frac{v_2 - v_1}{2} - 6i_0 + \frac{v_2}{4} = 0 \quad 2' \end{cases}$$

$$\Rightarrow \begin{cases} v_1 = \frac{12}{13}(V) = 0.92(V) \\ v_2 = \frac{56}{13}(V) = 4.31(V) \end{cases}$$

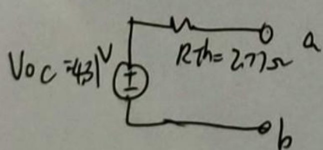
$V_{oc} = V_2 = 4.31(V)$ 2'

$$\Rightarrow \begin{cases} v_c = \frac{4}{9}(V) \\ i_0 = \frac{2}{9}(A) \\ i_{sc} = \frac{14}{9}(A) \approx 1.56(A) \quad 2' \end{cases}$$

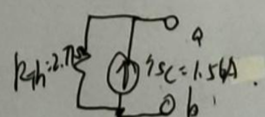
(if you draw Norton equivalent, you need to write it as a decimal.)

$$R_{th} = \frac{v_{oc}}{i_{sc}} = \frac{\frac{56}{13}}{\frac{14}{9}} = \frac{36}{13} \approx 2.77(\Omega) \quad 2'$$

Thevenin ~~equal~~ equivalent



or



2'

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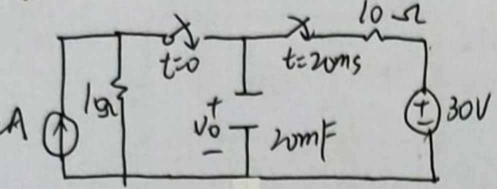
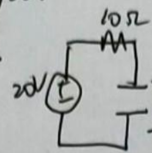
16'

Solution of Problem 4



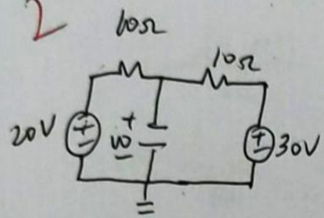
$t=0s$: $V_C(0^+) = V_C(0^-) = 0V$ 1'



$20 = V_o(t) + 10 \cdot C \frac{dV_o(t)}{dt}$

$\frac{dV_o(t)}{dt} + 5V_o(t) = 100$ 2'

2'



$V_o(t) = 20 + e^{-5t}(0-20) = 20 - 20e^{-5t}$ (V) 3'

$t = 20ms$ $V_o(0.02) = 20 - 20e^{-0.1}$ (V)

$t = 20ms$: KCL: $\frac{20-V_o(t)}{10} + \frac{30-V_o(t)}{10} = C\frac{dV_o(t)}{dt}$ 1' 1'

$\frac{dV_o(t)}{dt} + 10V_o(t) = 250$ 2' $V_S = 25$(V) 1'

$V_o(t) = e^{-10(t-0.02)}(V_o(0.02) - V_S) + V_S$

$= e^{-10t+0.2}(20 - 20e^{-0.1} - 25) + 25$

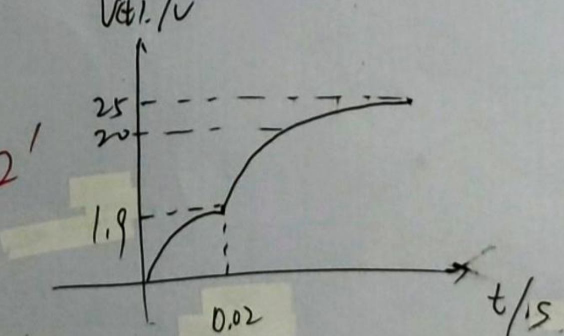
$= 25 - e^{-10t+0.2}(20e^{-0.1}+5)$ (V) 2'

$= 25 - 23.1e^{-10(t-0.02)}$ (V) $t > 20ms$

$$V_o(t) = \begin{cases} 20 - 20e^{-5t} \ (V) & 0 < t < 20ms \\ 25 - 23.1e^{-10(t-0.02)} \ (V) & t > 20ms \end{cases}$$

2'

2'



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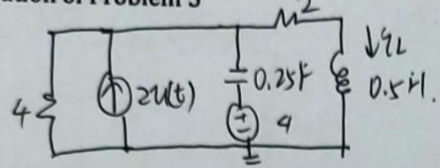
Solution of Problem 5

16'



2'

(a) $t<0$: $i_L(0^+) = i_L(0^-) = 0$ 1' $V_C(0^-) = V_C(0^+) = \cancel{4}\ -4(V)$ 1'

$$L\frac{di_L(0^+)}{dt} + 2i_L(0^+) = 4 + V_C(0^+)$$ $\cancel{3}'$

$$0.5\frac{di_L(0^+)}{dt} = 4 \Rightarrow \frac{di_L(0^+)}{dt} = \cancel{8}\ 0\ (A/s)$$ 单位写不写都行. 2'

for $t>0$

(b)
$$\begin{cases} v_1(t) = V_C(t) + 4 & 2' \\ C\frac{dV_C(t)}{dt} + i_L(t) + \frac{v_1(t)}{4} - 2 = 0 & \cancel{3}' \\ L\frac{di_L(t)}{dt} + 2i_L(t) = v_1(t) & 2' \end{cases}$$

$$\Rightarrow 0.125\frac{d^2 i_L(t)}{dt^2} + 0.625\frac{di_L(t)}{dt} + 1.5 i_L(t) = 2$$ 2'

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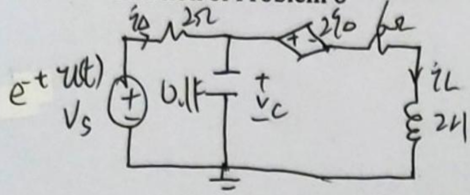
20'

Solution of Problem 6



2'

(1) $\frac{V_s - V_C}{2} = i_\delta$ ① $V_s = e^{-t}u(t)$ ⑥

$i_\delta = i_C + i_L$ ②

$i_C = C\frac{dv_C}{dt} = 0.1\frac{dv_C}{dt}$ ③

$i_L = \frac{V_C - 2i_\delta - v_L}{6}$ ④

$v_L = L\frac{di_L}{dt} = 2\frac{di_L}{dt}$ ⑤

Analysis total 4'

Substitute ① ⑤ ⑥ into ④

$\Rightarrow v_C = 3i_L + \frac{1}{2}V_s + \frac{di_L}{dt}$ ⑦ 1'

$$\frac{dv_C}{dt} = 3\frac{di_L}{dt} - \frac{1}{2}V_s + \frac{d^2 i_L}{dt^2}$$

Substitute ⑦ ① ③ into ② $\frac{V_s}{2} - \frac{1}{2}\left(3i_L + \frac{1}{2}V_s + \frac{di_L}{dt}\right) = 0.1\left(3\frac{di_L}{dt} - \frac{1}{2}V_s + \frac{d^2 i_L}{dt^2}\right) + i_L$

$$\frac{d^2 i_L}{dt^2} + 8\frac{di_L}{dt} + 25 i_L = 3e^{-t}u(t)$$ 1'

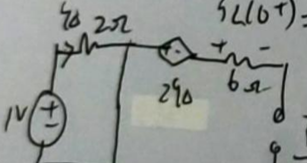
$i_{Lh} = e^{-4t}(A_1\cos 3t + A_2\sin 3t)$ (A). Homogenous Solution. 1'

Particular solution: $i_p = A_3 e^{-t}$ (A) Substitute into equation:

$A_3 e^{-t} - 8A_3 e^{-t} + 25A_3 e^{-t} = 3e^{-t}$ $A_3 = \frac{1}{6}$ 1'

$i_L(t) = \frac{1}{6}e^{-t} + e^{-4t}(A_1\cos 3t + A_2\sin 3t)$ (A)

$i_L(0^+) = i_L(0^-) = 0$ (A) 1' $v_L(0^+)$ is determined by the Following circuit.



$i_\delta = 0.5$ (A) $v_L(0^+) = -2i_\delta = -1$ (V) 1'

$0 = \frac{1}{6} + A_1$

$-1 = 2\left(-\frac{1}{6} + (-4)\cdot(A_1) + 3A_2\right)$ 1' $\Rightarrow A_1 = -\frac{1}{6}$, $A_2 = -\frac{1}{3}$ 1'

$i_L(t) = \frac{1}{6}e^{-t} + e^{-4t}\left(-\frac{1}{6}\cos 3t - \frac{1}{3}\sin 3t\right)$ (A) 2'

(2) $V_C = 3i_L + \frac{1}{2}V_s + \frac{di_L}{dt} = \frac{1}{2}e^{-t} + e^{-4t}\left(-\frac{1}{2}\cos 3t - \sin 3t\right) + \frac{1}{2}e^{-t} - \frac{1}{6}e^{-t} + e^{-4t}\left(\frac{2}{3}\cos 3t + \frac{4}{3}\sin 3t + \frac{1}{2}\sin 3t - \cos 3t\right)$

$= \frac{5}{6}e^{-t} + e^{-4t}\left(-\frac{5}{6}\cos 3t + \frac{5}{6}\sin 3t\right)$ (V) 3'

$= 0.83e^{-t} + e^{-4t}(-0.83\cos 3t + 0.83\sin 3t)$ (V)