CS 101 Fall 2021 - Quiz 8-B

12/07/2021 - 25 Minutes **Name:** **ID number:**

Problem 1 True or False (5×1 pts)

The following questions are True or False questions, you should judge whether each statement is true or false.

Note: You should write down your answers in the box below.

Problem 1.1	Problem 1.2	Problem 1.3	Problem 1.4	Problem 1.5

(1) A DAG has multiple topological sortings if it has multiple sources.

(2) Topological sort can be extended to detect whether a graph has a cycle in $O(|V| + |E|)$ time.

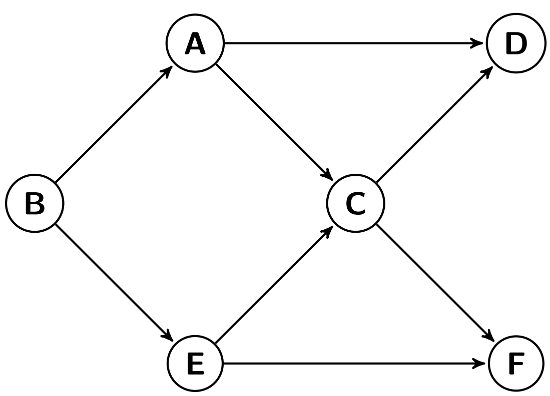
(3) If we add a constraint that each edge can only appear at most once in the shortest path, Dijkstra's algorithm still works for positive-weighted graphs.

(4) Dijkstra's algorithm cannot work for graph with both positive and negative weights but can work for graph whose weights are all negative.

(5) Bellman-Ford algorithm can find the shortest path for all undirected graphs with negative weights.

Problem 2 Topological Sort (2 + 2 pts)

Given the DAG below:



(1) Run topological sort on the given DAG and write down the topological sorting you obtain.

Note: When pushing several vertices into a queue at the same time, push them alphabetically. You are NOT required to show your steps.

(2) How many different topological sortings does the DAG have? Write them down.

Problem 3 Does Shortest Path Change? (3 pts)

Given a shortest path $P = (s, v_1, v_2, \cdots, t)$ from s to t in graph $G = (V, E)$. Now Ge Ziwang adds 1 to the weight of each edge in G i.e. $w(e') = w(e) + 1$. By doing this, Ge Ziwang obtains a new graph $G' = (V, E')$. Is the original shortest path P still guaranteed to be a shortest path from s to t in G'?

If yes, briefly explain why; If not, give a counterexample.

Problem 4 Dijkstra's Algorithm Tiebreak (5 pts)

Consider a directed graph $G = (V, E)$ with positive weights on vertices instead of edges. That is to say, when we visit a node $v \in V$, we need to cost its weight $w(v)$. Now we want to find a shortest path from s to t in such a vertex-weighted graph. How would you apply Dijkstra's algorithm in this setting? Briefly write down your main idea. Assume weights of vertices are all positive.

Hint: Consider how to construct a new graph $G' = (V', E')$ according to the original graph $G = (V, E)$.