CS 101 Fall 2020 - Quiz 4-B

10/15/2020 - 20 Minutes **Name:** **ID number:**

Problem 1(4×2pts): **True or False:** For each statement, write "T" if this statement is correct; write "F" otherwise. Please **write your answers in the box below**.

Statement (1)	Statement (2)	Statement (3)	Statement (4)
T	T	T	F

(1) Merge sort requires $O(n)$ space complexity.

(2) In quicksort (sort in ascending order), if we randomly select the pivot, after the first partition operation, the smallest element of the array can be **anywhere but the last position**.

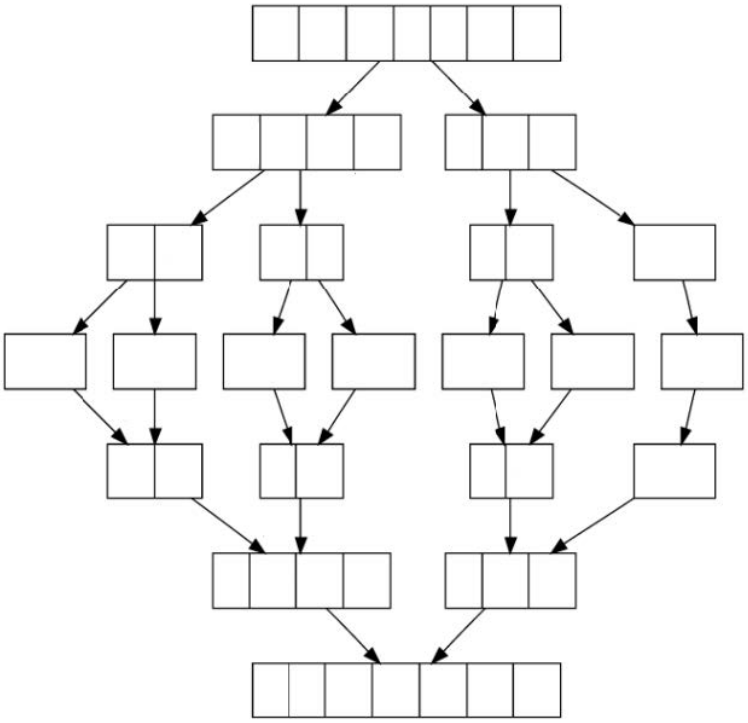
(3) Quicksort algorithm will have $O(n^2)$ time complexity in the worst case.

(4) By applying the partition step of quicksort on an **unsorted** array repeatedly, we can get the $k-th$ biggest number of that array with an **average** time complexity of $O(log(n))$. (k is an arbitrary number)

Problem 2(5pts):

Consider this array: $7, 6, 4, 2, 5, 3, 1$.

(1)(3.5pts) Use **mergesort** to sort this array in ascending order. Show your process in the following figure.



(2)(1.5pts) How many inversions are there in the array? _____

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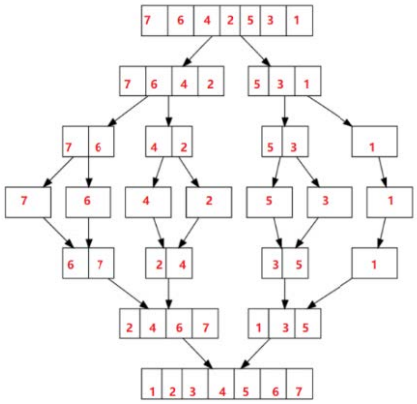
Solution: 1. As follows. 2. 18



Problem 3(3×1pts):

Tom wants to sort his favorite colors in ascending order using quicksort. The original array is:

$$red, cyan, yellow, gray, green, black, blue, white$$

After the first partitioning step, it becomes: ("red" is chosen as pivot)

$$cyan, green, gray, white, black, red, yellow, blue$$

Known that **NO** elements are equal, we can infer that: (Fill the blanks with ">", "<", or "?" if given information is insufficient to judge)

(a) red _____ white (b) yellow _____ gray (c) green _____ black

Solution: 1. > 2. > 3. ?

Problem 4(4pts): Prove that: The time complexity for mergesort is $O(nlog(n))$.

Solution:

(1) $T(n) = 2T(n/2) + O(n)$

(2) $= 2(2T(n/4) + O(n/2)) + O(n)$

(3) $= 4T(n/4) + 2O(n/2) + O(n)$

(4) $= 4T(n/4) + 2O(n)$

(5) $= ...$

(6) $= O(nlog(n))$