Signals and Systems Homework 4

Due Time: 22:00 April 9, 2018

Submitted to blackboard online (photos and electronic documents both allowed) and to the box in front of SIST 1C 403E (the instructor's office).

1. Suppose that we are given the following information about a signal $x[n]$

   1. $x[n]$ is a real and even signal.
   2. $x[n]$ has a period $N = 10$ and Fourier coefficients a_k.
   3. $a_{11} = 5$.
   4. $\frac{1}{10}\sum_{n=0}^{9}|x[n]|^2 = 50$.

   Show that $x[n] = Acos(Bn + C)$, and specify numerical values for the constants A, B and C.

2. Each of the two sequences $x_1[n]$ and $x_2[n]$ has a period $N = 4$, and the corresponding Fourier series coefficients are specified as

$$x_1[n] \longleftrightarrow a_k, \quad x_2[n] \longleftrightarrow b_k$$

where

$$a_0 = a_3 = \frac{1}{2}a_1 = \frac{1}{2}a_2 = 1, \quad b_0 = b_1 = b_2 = b_3 = 1.$$

Using the multiplication property of Fourier series, determine the Fourier series coefficients c_k for the signal $g[n] = x_1[n]x_2[n]$.

3. Let

$$x(t) = \begin{cases} t, & 0 \le t \le 1 \\ 2 - t, & 1 \le t \le 2 \end{cases}$$

be a period signal with fundamental period $T = 2$ and the Fourier coefficients a_k.

(a) Determine the value of a_0.

(b) Determine the Fourier series representation of $dx(t)/dt$.

(c) Use the result of part(b) and the differentiation property of the countinuous-time Fouries series to help determine the Fourier series coefficients of $x(t)$.

4. Let

$$x[n] = \begin{cases} 1, & 0 \leq n \leq 7 \\ 0, & 8 \leq n \leq 9 \end{cases}$$

be a periodic signal with fundamental period $N = 10$ and Fourier sieres coefficients a_k. Also, let

$$g[n] = x[n] - x[n-1]$$

(a) Show that $g[n]$ has a fundamental period of 10.

(b) Determine the Fourier series coefficients of $g[n]$.

(c) Using the Fourier series coefficients of $g[n]$ and the First-Difference property (**page 222, Chapter 3.7.2 of Oppenheim's book), determine a_k for $k \neq 0$.**

5. Consider the following three continuous-time signals with a fundamental period of $T = \frac{1}{2}$:

$$x(t) = \cos(4\pi t)$$

$$y(t) = \sin(4\pi t)$$

$$z(t) = x(t)y(t)$$

(a) Determine the Fourier series coefficients of $x(t)$.

(b) Determine the Fourier series coefficients of $y(t)$.

(c) Use the result of part(a) and (b), along with the multiplication property of the countinuous-time Fourier series , to determine the Fourier series coefficients of $z(t) = x(t)y(t)$.

(d) Determine the Fourier series coefficients of $z(t)$ through direct expansion of $z(t)$ in trigonometric form, and compare your result with that of part(c).

6. Consider the following three dicrete-time signals with a fundamental period of 6:

$$x[n] = 1 + cos(\frac{2\pi}{6}n), \quad y[n] = sin(\frac{2\pi}{6} + \frac{\pi}{4}), \quad z[n] = x[n]y[n]$$

(a) Determine the Fourier series coefficients of $x[n]$.

(b) Determine the Fourier series coefficients of $y[n]$.

(c) Use the result of part(a) and (b), along with the multiplication property of the discrete-time Fourier series , to determine the Fourier series coefficients of $z[n] = x[n]y[n]$.

(d) Determine the Fourier series coefficients of $z[n]$ through direct evaluation, and compare your result with that of part(c).