ShanghaiTech University

CS240 Algorithm Design and Analysis

Spring 2021

Problem Set 3

Due: 23:59, Apr. 7, 2021

1. Submit your solutions to Gradescope (www.gradescope.com).
2. In "Account Settings" of Gradescope, set your FULL NAME to your Chinese name.
3. If you want to submit a handwritten version, scan it clearly.
4. When submitting your homework in Gradescope, match each of your solution to the corresponding problem number.

Note: When proving problem A is NP-complete, please clearly divide your answer into three steps:

(1) Prove that problem A is in NP.

(2) Choose an NP-complete problem B. For any B instance, construct an instance of problem A. Show that the construction runs in polynomial time.

(3) Prove that the yes/no answers to the two instances are the same.

Problem 1:

A **not-all-equal** assignment of a formula in CNF is an assignment for which every clause contains at least 1 true literal and at least 1 false literal. Given a formula ϕ in 4CNF (i.e., every clause contains exactly four literals), the decision problem is whether ϕ has a not-all-equal assignment.

Show that this problem is NP complete.
HINT: Reduction from 3SAT.

Problem 2:

Suppose you are going to schedule courses for the SIST and try to make the number of conflicts no more than K. You are given 3 sets of inputs: $C = \{...\}, S = \{...\}, R = \{\{...\}, \{...\}, ...\}$. C is the set of distinct courses. S is the set of available time slots for all the courses. R is the set of requests from students, consisting of a number of subsets, each of which specifies the courses a student wants to take. A conflict occurs when two courses are scheduled at the same slot even though a student requests both of them. Prove this schedule problem is NP-complete.
Example:

$$K = 1; \quad C = \{a, b, c, d\}, \quad S = \{1, 2, 3\}, \quad R = \{\{a, b, c\}, \{a, c\}, \{b, c, d\}\}$$

An acceptable schedule is:

$$a \to 1; \quad b \to 2; \quad c, d \to 3;$$

Here only one conflict occurs. An unacceptable schedule is:

$$a \to 1; \quad b, c \to 2; \quad d \to 3;$$

Here two ($> K$) conflicts occur.

Problem 3:

SIST allows students to work as TAs but would like to avoid TA cycles. A TA cycle is a list of TAs $(A_1, A_2, \ldots, A_k)$ such that A_1 works as a TA for A_2 in some course, A_2 works as a TA for A_3 in some course, $\cdots$, and finally A_k works as a TA for A_1 in some course. We say a TA cycle is simple if it does not contain the same TA more than once. Given the TA arrangements of SIST, we want to find out whether there is a simple TA cycle containing at least K TAs. Prove this problem is NP-complete.

Problem 4:

Consider the Knapsack problem. We have n items, each with weight a_j and value c_j $(j = 1, ..., n)$. All a_j and c_j are positive integers. The question is to find a subset of the items with total weight at most b such that the corresponding profit is at least k (b and k are also integers). Show that Knapsack is NP-complete by a reduction from Subset Sum.

Problem 5:

Define the balloon-coloring problem: There are n originally white-colored balloons which are different from each other in shape, denoted by $F = \{f_1, f_2, \cdots, f_n\}$ where f_i is one balloon. m children respectively select their favorite balloons out of F according to the balloon shapes. Denote by $S_1, S_2, \cdots, S_m$ their selection results where $S_i \subseteq F$. Note that a balloon can be selected by multiple children, and each child chooses at least two balloons, i.e., $|S_i| \geq 2$. You are required to decide for each balloon whether to color it as red or blue such that each child has at least one red balloon and one blue balloon. Try to prove that to find out whether there is a color scheme satisfying the aforementioned requirement is NP-Complete.

Problem 6:

Consider that a robot travels around the rooms in a building. The rooms are connected by bridges and the lengths of the bridges are all positive integers. There may be multiple bridges between two rooms and there may even be "self-loop" bridges whose two ends are the same room. The robot is allowed to visit the same room for multiple times but not allowed to cross the same bridge for more than once. Given an integer $l \geq 0$, you must determine if there exists a route (i.e., a path) for the robot that starts from room v and then returns to v after travelling a distance of exactly l.

Show that this problem is NP complete.

HINT: Reduction from Subset-Sum.