Lecture 15

-- Laplace Transform in Circuit Analysis

Introduction

- In this lecture, we introduce the concept of modeling circuits in the s domain using the Laplace transform.

- The elegance of using the Laplace transform in circuit analysis lies in (1) transforming the differential equation into an *algebraic* equation; and (2) automatic inclusion of initial conditions in the transformation process, thus providing a complete (transient and steady-state) solution.

Steps in Applying the Laplace transform

- **Transform the circuit from the time domain to the Laplace (*s*) domain, including possible initial conditions.**
- **Solve the circuit using nodal analysis, mesh analysis, source transformation, superposition, or any other analysis technique with which we are familiar.**
- Take the inverse transform of the solution and thus obtain the solution in the time domain.

V-I relations of R,L,C

- R

$$U_R(s) = RI_R(s)$$

- C

$$\boxed{V(s) = \frac{1}{sC} I(s) + \frac{V_0}{s}}$$

$$\boxed{I(s) = sCV(s) - CV_0}$$

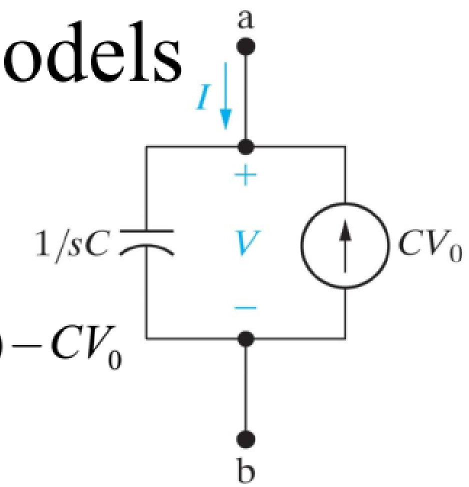
- L

$$\boxed{I(s) = \frac{1}{sL} V(s) + \frac{I_0}{s}}$$

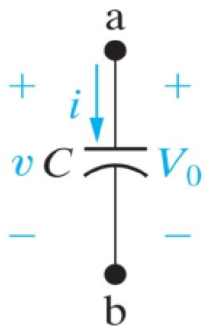
$$\boxed{V(s) = sLI(s) - LI_0}$$

S-domain circuit models for a capacitor

s-Domain Circuit Models

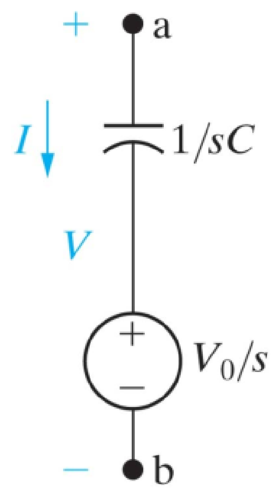


For a capacitor
(with initial conditions)



$$i(t) = C\frac{dv(t)}{dt} \Rightarrow I(s) = sCV(s) - CV_0$$

$$V(s) = \frac{1}{sC}I(s) + \frac{V_0}{s}$$



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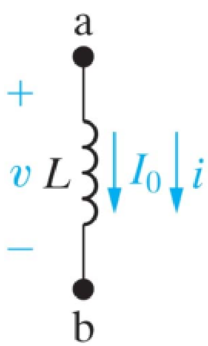
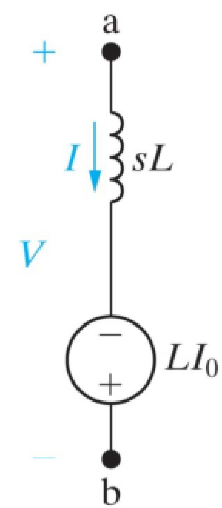
S-domain circuit models for an inductor

s-Domain Circuit Models

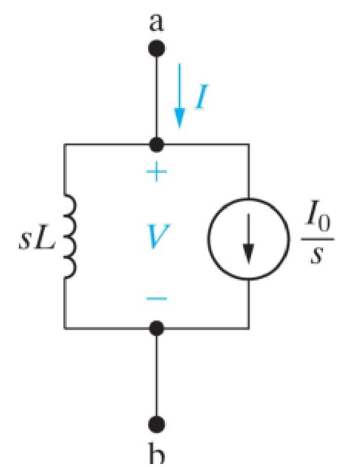
For an inductor
(with initial conditions)



$$v(t) = L\frac{di(t)}{dt} \quad\Rightarrow\quad V(s) = sLI(s) - LI_0$$

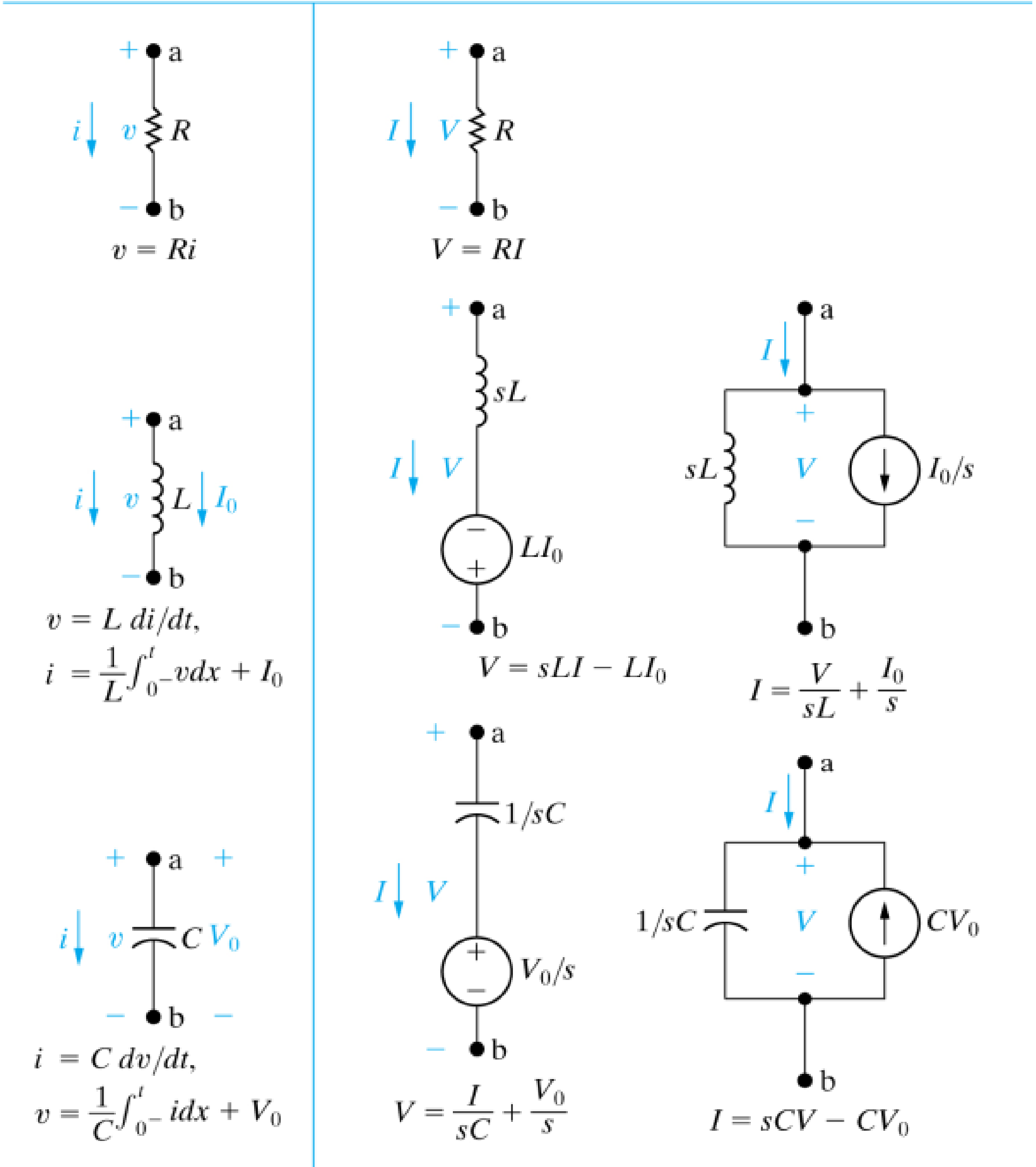


$$I(s) = \frac{1}{sL}V(s) + \frac{I_0}{s}$$



Summary

Time domain	s-domain



Dependent Sources

- The models for dependent sources are easy to develop, drawing from the simple fact that if the Laplace transform of *f(t)* is *F(s)*, then the Laplace transform of *af*(*t*) is *aF*(*s*) — the linearity property.

$$\mathcal{L}[av(t)] = aV(s)$$

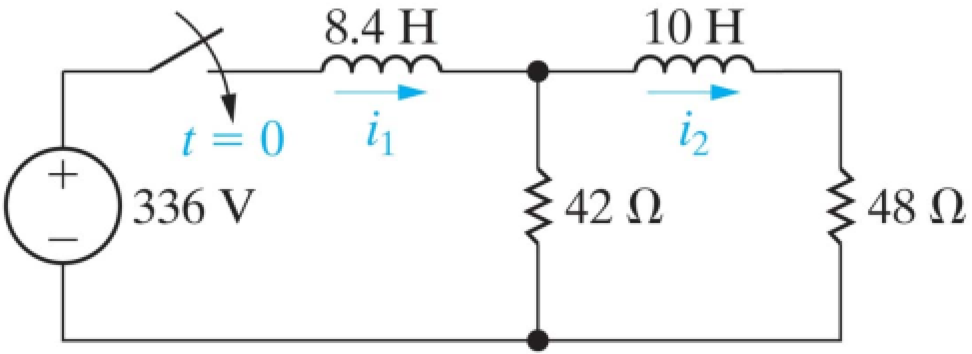
$$\mathcal{L}[ai(t)] = aI(s)$$

Steps in Applying the Laplace transform

- Transform the circuit from the time domain to the Laplace (s) domain, including possible initial conditions.
- Solve the circuit using nodal analysis, mesh analysis, source transformation, superposition, or any other analysis technique with which we are familiar.
- Take the inverse transform of the solution and thus obtain the solution in the time domain.

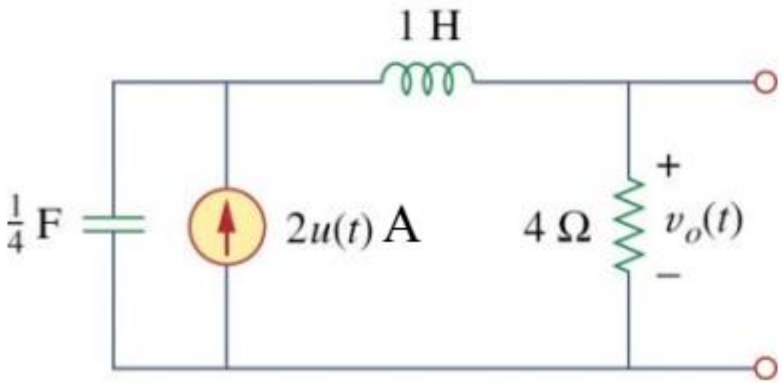
Example 1

Assuming no initial energy storage, find $i_1(t)$ and $i_2(t)$ for $t > 0$.



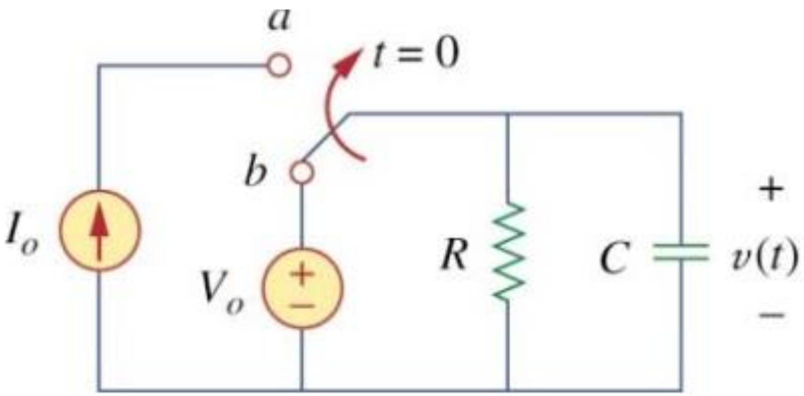
Example 2

Determine $v_0(t)$ *for* $t>0$ *assuming zero initial conditions:*



Example 3

- The switch has been in position *b* for a long time. It is moved to position *a* at $t = 0$. Determine $v(t)$ for $t > 0$.

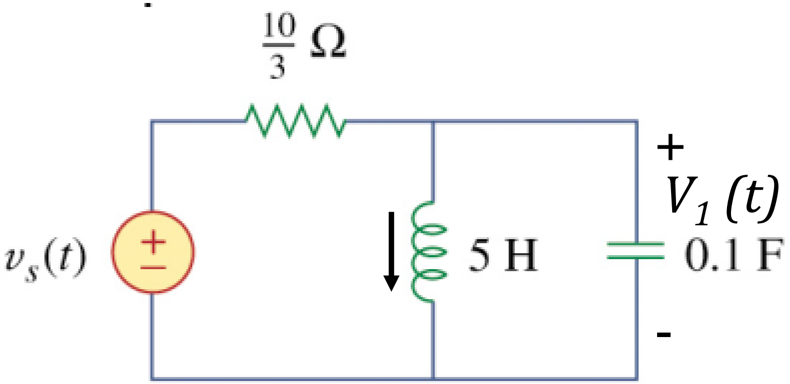


Example 4

- Find (1) the voltage across the capacitor

  (2) current through the inductor

  assuming that $v_s(t) = 10u(t)$ V, and assume that at $t = 0$, *-1 A* flows through the inductor and +5 V is across the capacitor.

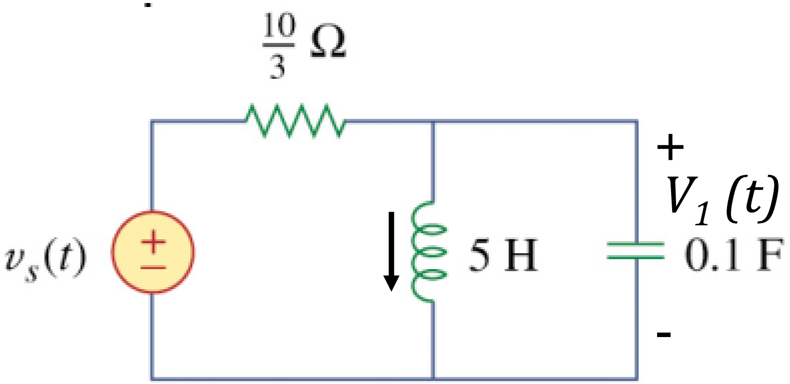


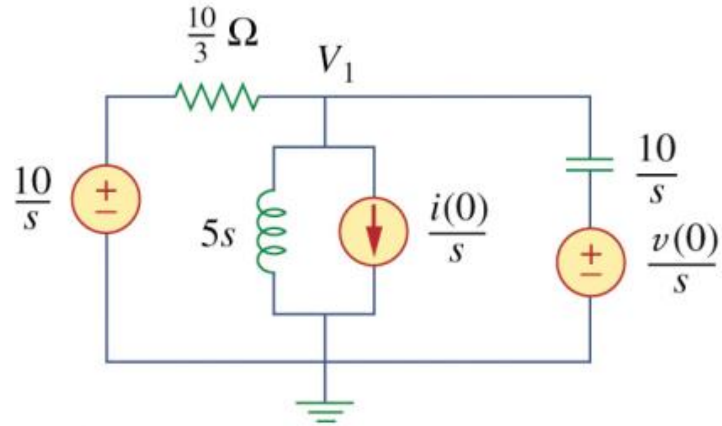
Example 4

- Find (1) the voltage across the capacitor

  (2) current through the inductor

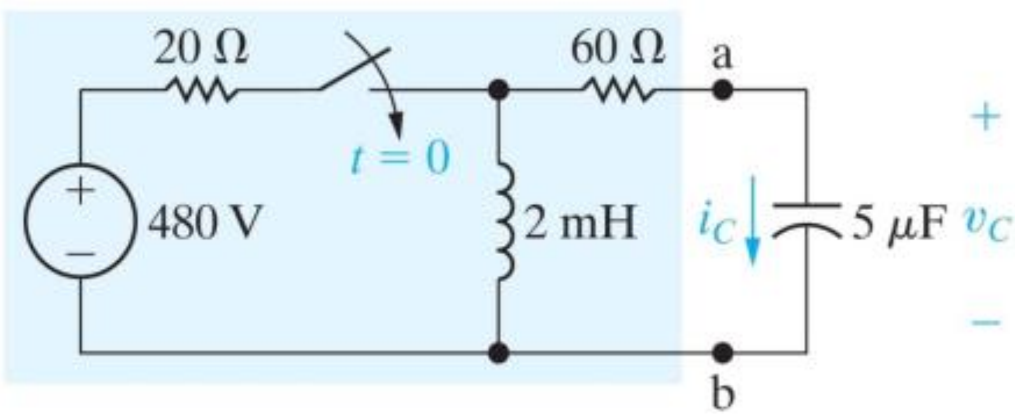
  assuming that $v_s(t) = 10u(t)$ V, and assume that at $t = 0$, *-1 A* flows through the inductor and +5 V is across the capacitor.





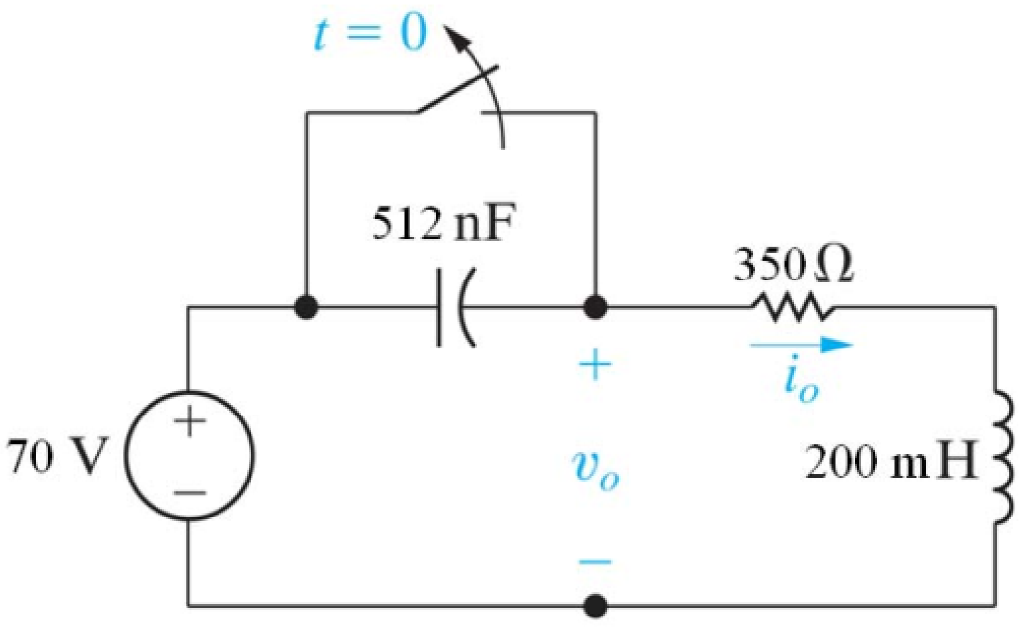
Example 5

- Use Thevenin's equivalent circuit w.r.t terminals *a*-*b* to find current $i_C(t)$.



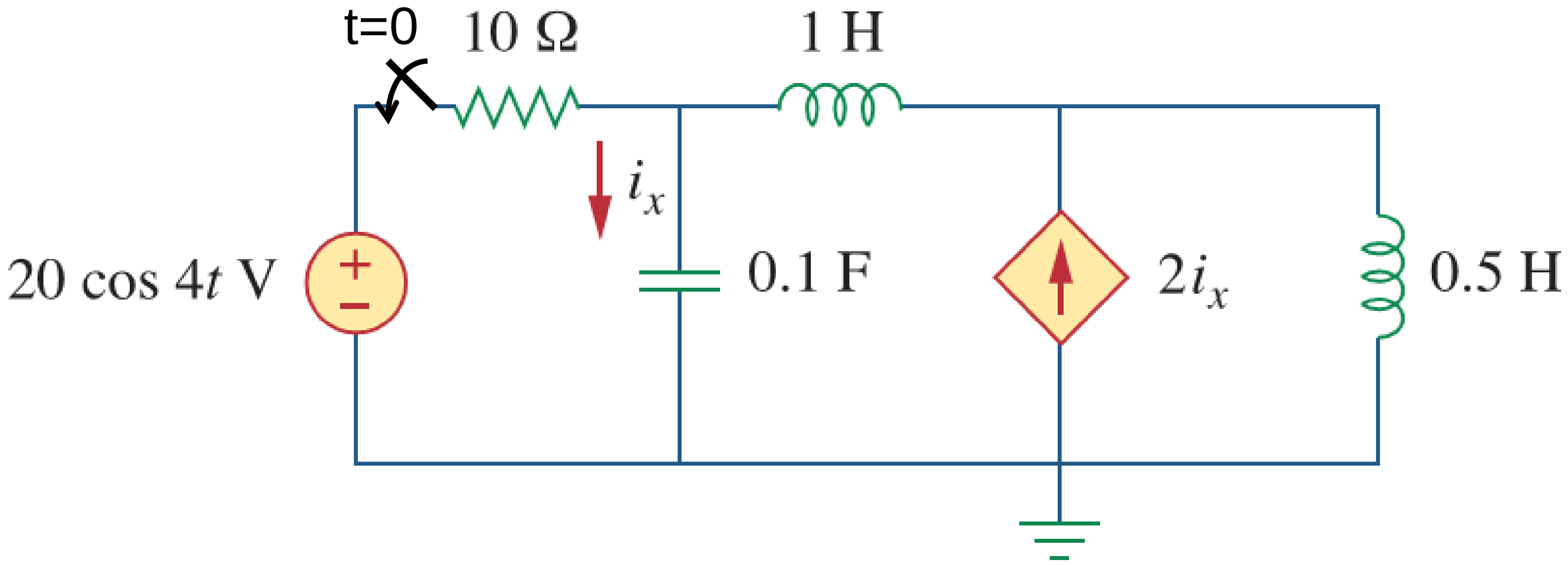
Example 6

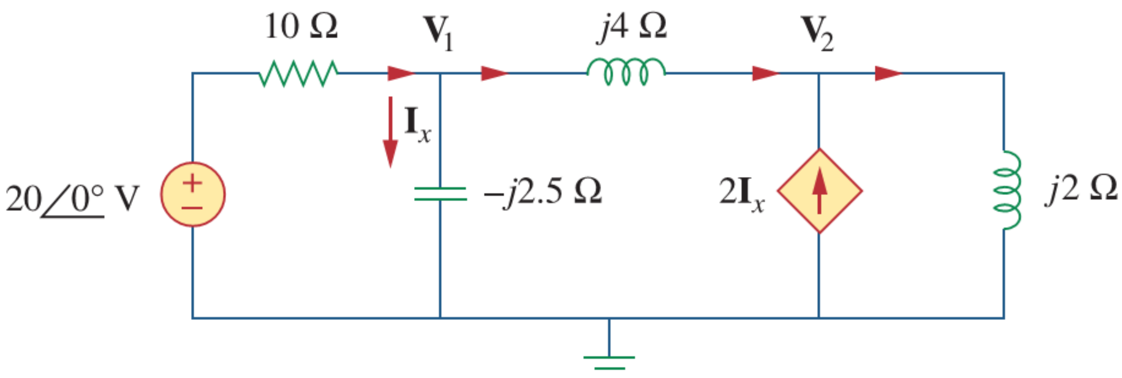
- Find $v_o(t)$



Example 7

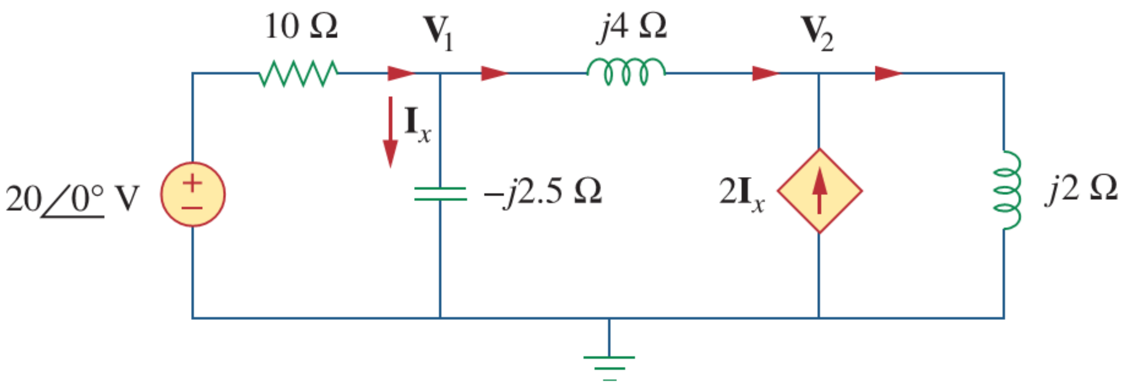
- Example---Find i_x(s.s) assuming no initial energy stored
- **Using phasor method and Laplace transform method**





$$\frac{20 - \mathbf{V}_1}{10} = \frac{\mathbf{V}_1}{-j2.5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4}$$

$$2\mathbf{I}_x + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4} = \frac{\mathbf{V}_2}{j2}$$



$$\frac{20 - \mathbf{V}_1}{10} = \frac{\mathbf{V}_1}{-j2.5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4}$$

$$2\mathbf{I}_x + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4} = \frac{\mathbf{V}_2}{j2}$$

$$i_x = 7.59\cos(4t + 108.4^\circ)\text{ A}$$

Example 8

- There is no initial energy stored in this circuit. Find $i(t)$ if
- $v(t) = e^{-0.6t}\sin 0.8t$ V.