Homework 4

Due date: Nov. 9^{th}, 2021

Turn in your homework in class

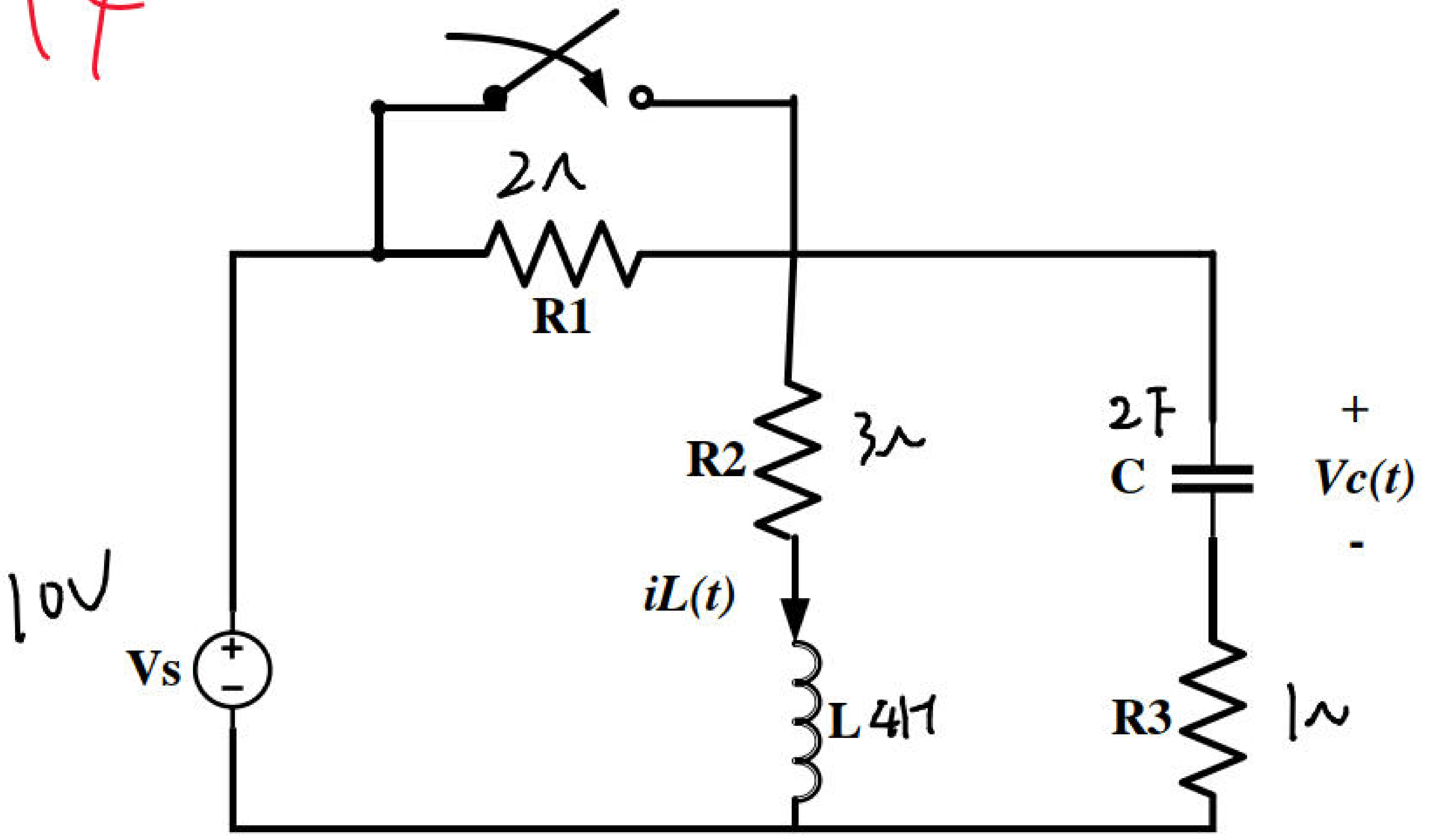
Rules:

- Work on your own. Discussion is permissible, but extremely similar submissions will be judged as plagiarism.
- Please show all intermediate steps: a correct solution without an explanation will get zero credit.
- Please submit on time. No late submission will be accepted.
- Please prepare your submission in English only. No Chinese submission will be accepted.
- If needed, round the number to the nearest hundredths, i.e., rounding it to 2 decimal places.

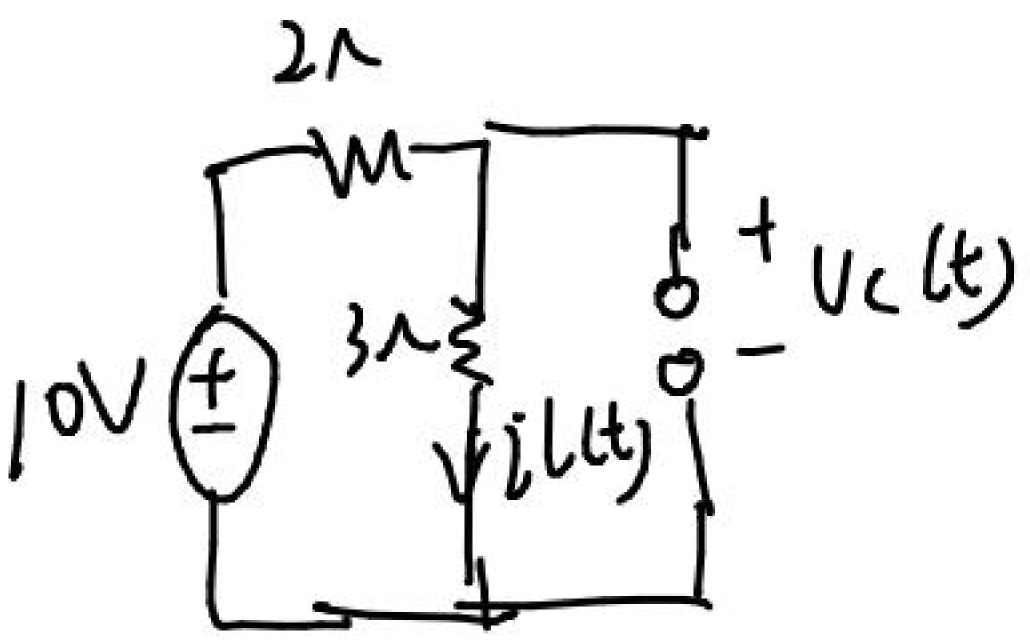
1. For the circuit below, the switch has been open for a long time. At t=0, the switch was closed. Given Vs=10V, R1=2Ω, R2=3Ω, R3=1Ω, L=4H, C=2F

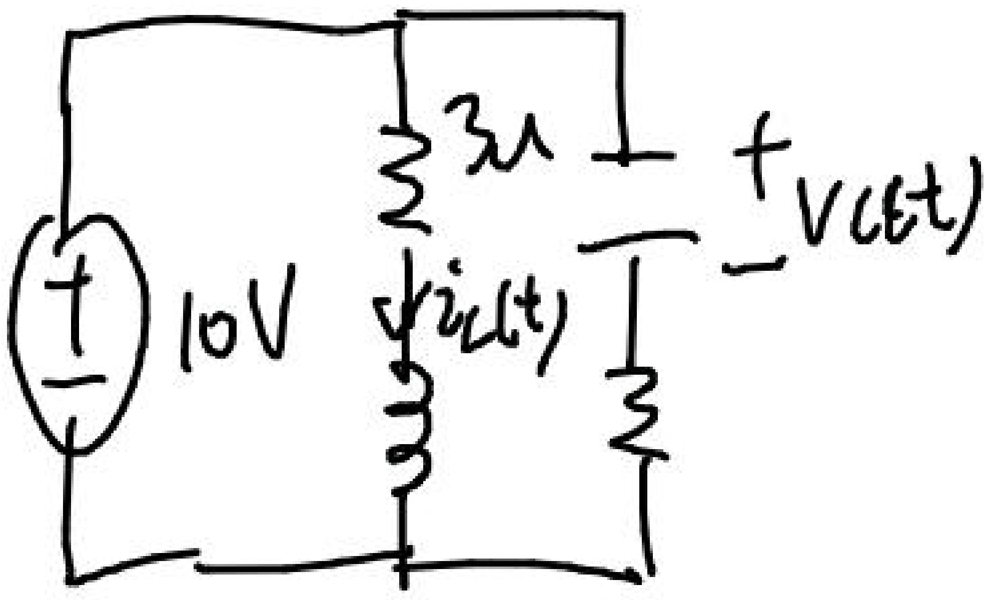
14'

find $Vc(0+)$, $\frac{dVc(0+)}{dt}$, $iL(0+)$, $\frac{diL(0+)}{dt}$.



t<0 Steady State





$V_{R3} = V_s - V_c(0^+)$
$= 4V$ 2'

$i_c = \frac{V_{R3}}{R_3} = 4A$
$= C\frac{dv_c(0^+)}{dt}$

$\frac{dv_c(0^+)}{dt} = 2$ 2'

$i_L(0^-) = 2A$ 2'

$V_c(0^-) = 6V$ 2'

$t > 0$

The feature of capacitors and inductors determine:

$i_L(0^+) = i_L(0^-)$ 1'

$V_c(0^+) = V_c(0^-)$ 1'

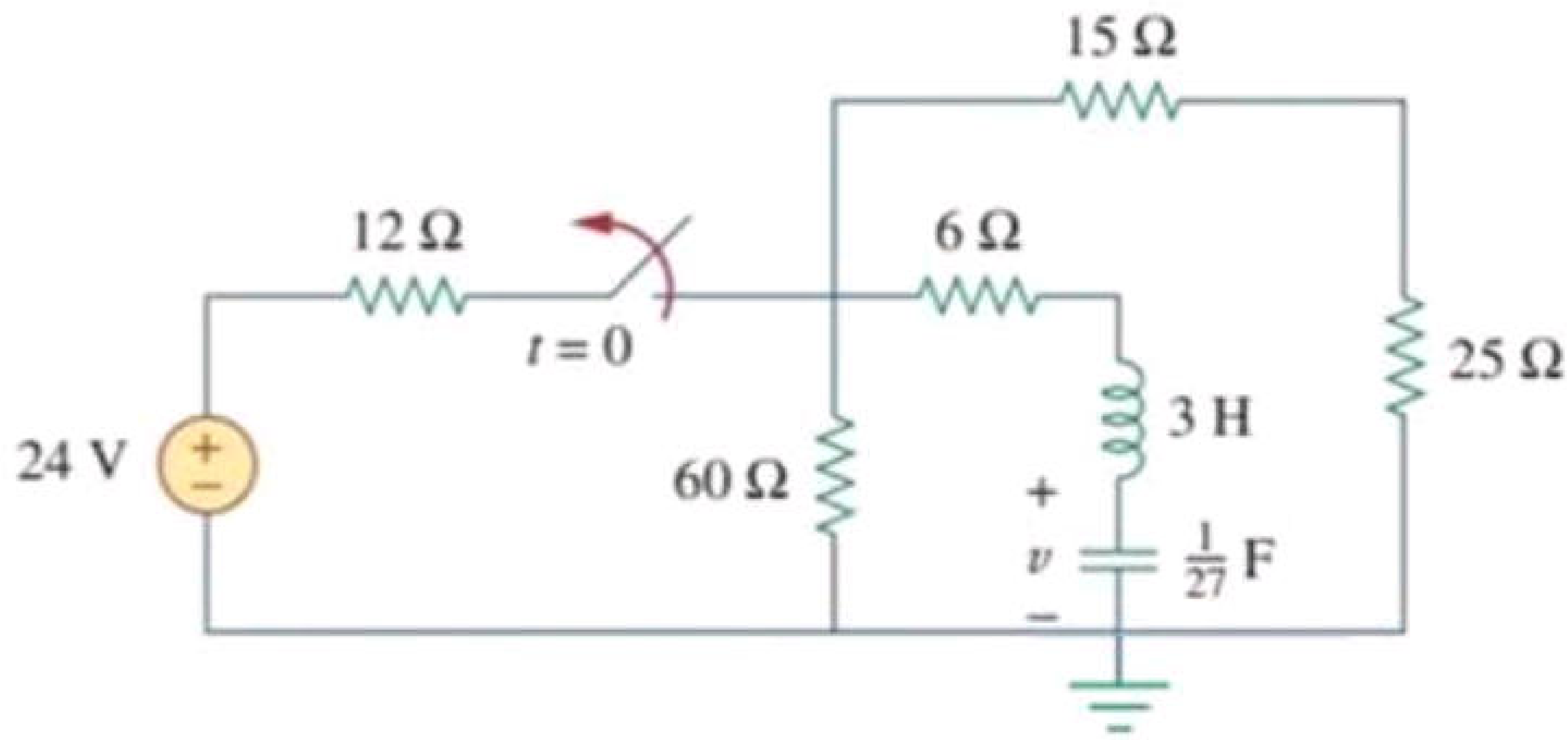
$V_{R2} = i_L(0^+) \cdot R_2$
$= 6V$

$V_L = V_s - V_{R2}$
$= 4V = L\frac{di_L(0^+)}{dt}$ 2'

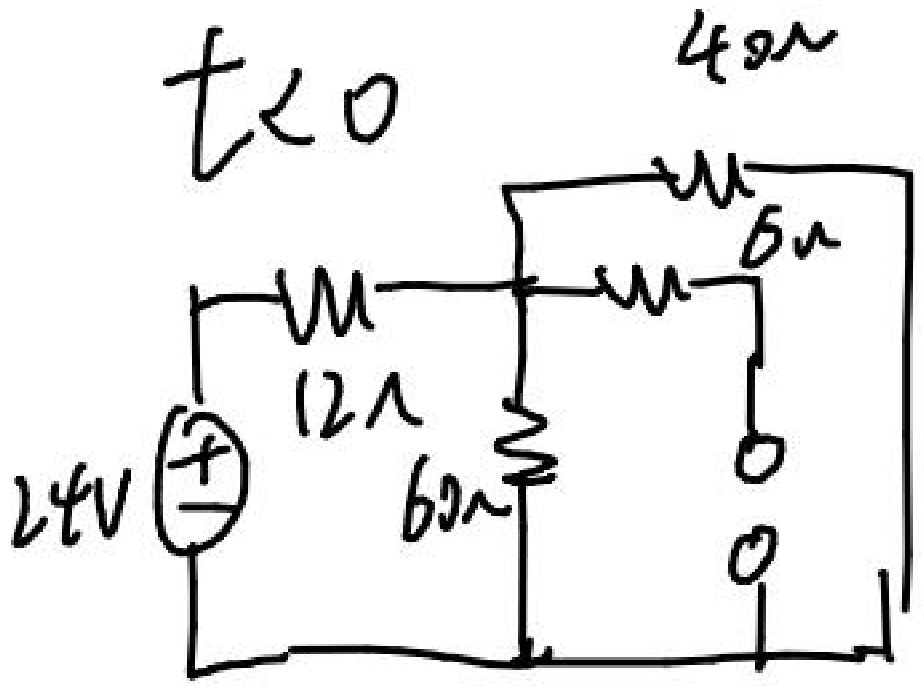
$\frac{di_L(0^+)}{dt} = 1$ 2'

17'

2. For the circuit below. The switch has been closed for a long time. At t=0, the switch was opened. Calculate *v(t)* for t >0.



t<0

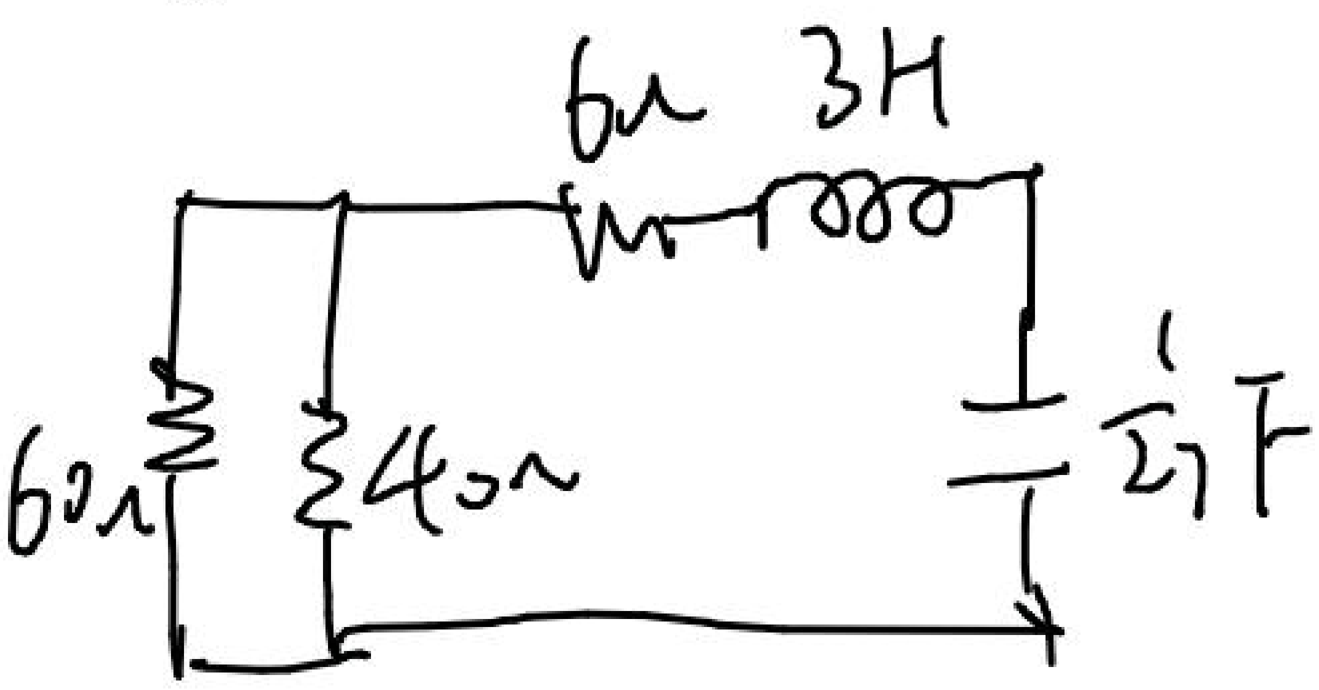


$i_L(0^-) = 0A$ 2'

$$V_C(0^-) = V_S \cdot \frac{40 // 60}{12 + 40 // 60}$$

$= 16V$ 2'

t>0



$R_{all} = 40 // 60 + 6\Omega$ 2'

$= 30\Omega$

$$\frac{d^2V_C(t)}{dt^2} + \frac{R_{all}}{L}\frac{dV_C(t)}{dt} + \frac{1}{LC}V_C(t) = 0$$

$$\frac{d^2V_C(t)}{dt^2} + 10\frac{dV_C(t)}{dt} + 9 = 0$$ 3'

$V_C(t) = A_1 e^{-9t} + A_2 e^{-t}$ 2'

$V_C(0^+) = V_C(0^-) = 16V$

$= A_1 + A_2$ 1'

$$i_C(0^+) = C\frac{dV_C(0^+)}{dt} = i_L(0^+)$$ 1'

$$= \frac{1}{27}(-9A_1 - A_2) = 0A$$

$A_1 = -2 \quad A_2 = 18$ 2'

$v(t) = -2e^{-9t} + 18e^{-t}\ V$

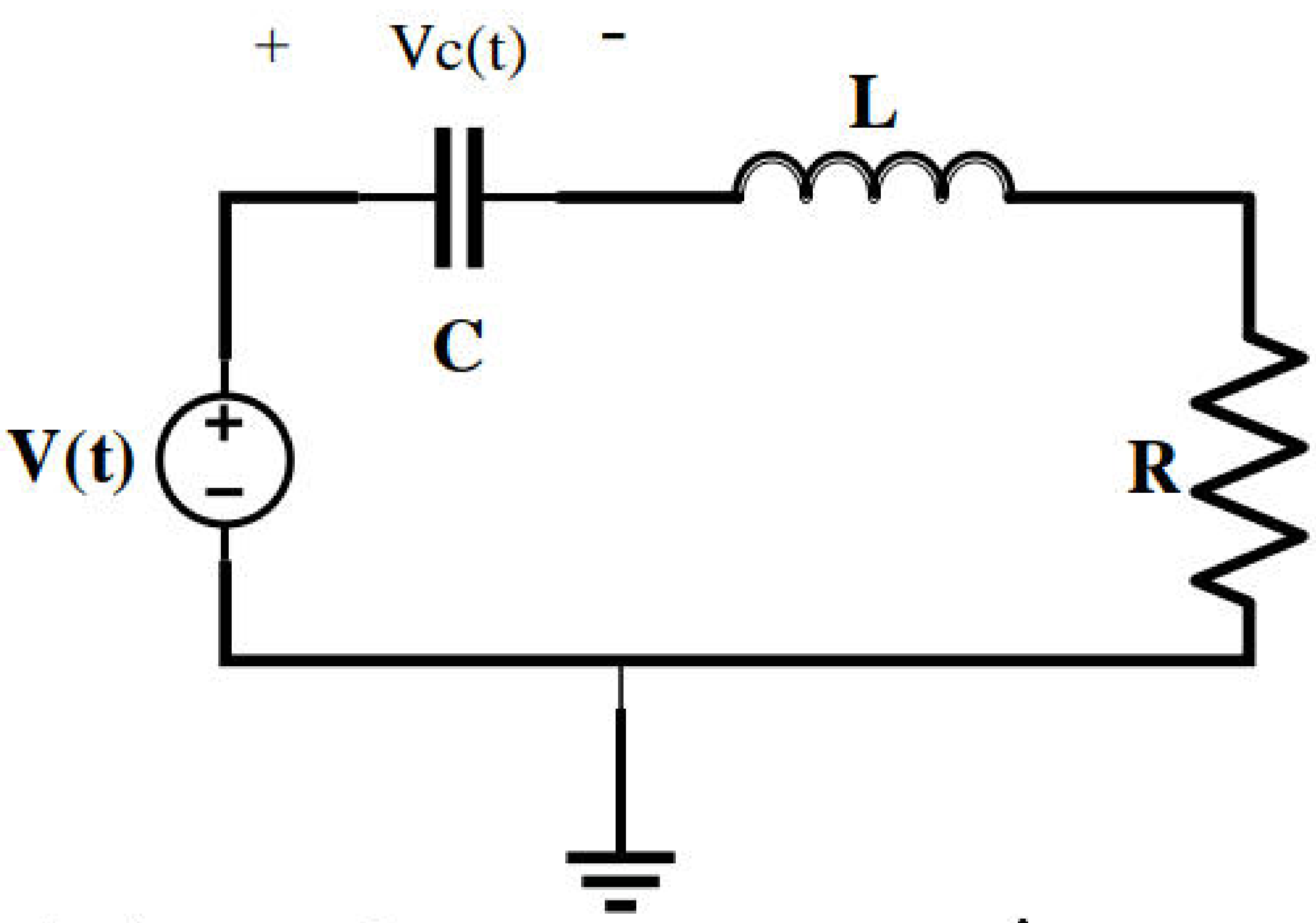
$t>0$ 2'

17

1'

3. For the circuit below: R=100Ω, L=0.25H, C=$\frac{1}{7500}$F. $V(t)=\begin{cases}0, & t<0\\ 10e^{-10t}, & t>0\end{cases}$

Find the expression of Vc(t) for $t > 0$



$$\frac{d^2 v_c(t)}{dt^2} + \frac{R}{L}\frac{dv_c(t)}{dt} + \frac{1}{LC}v_c(t) = \frac{V(t)}{LC}$$

$$\frac{d^2 v_c(t)}{dt^2} + 400\frac{dv_c(t)}{dt} + 30000\,v_c(t) = 3\times10^5 e^{-10t}$$ 3'

y_h: $v_c(t) = A_1 e^{-300t} + A_2 e^{-100t}$ 2'

y_p: V_i not s_1, s_2 $y_p(t) = A_3 e^{-10t}$ 2'

$$100A_3 e^{-10t} - 4000 A_3 e^{-10t} + 30000 A_3 e^{-10t} = 3\times10^5 e^{-10t}$$

$A_3 = \frac{1000}{87}$ 2'

$$V_c(t) = A_1 e^{-300t} + A_2 e^{-100t} + A_3 e^{-10t}$$

$\begin{cases} V_c(0^+) = 0V \ \text{2'} \\ i_C(0^+) = 0A \ \text{2'} \end{cases} \Rightarrow \begin{cases} A_1 + A_2 + A_3 = 0 \\ -300A_1 - 100A_2 - 10A_3 = 0 \end{cases}$

$A_1 = \frac{150}{29}$ $A_2 = -\frac{50}{3}$

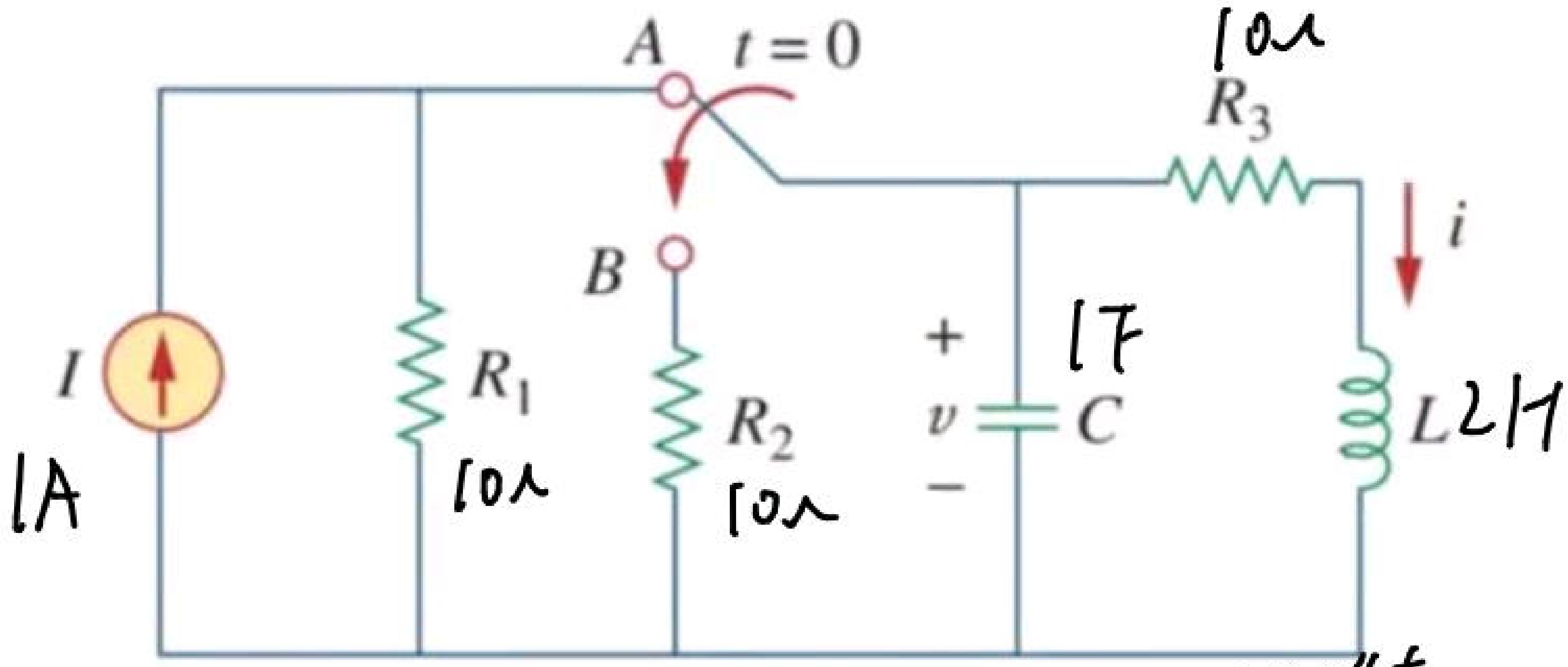
2'

4/7

$V_c(t) = 5.172e^{-300t} - 16.667e^{-100t} + 11.494e^{-10t}$ ✓ 2'

4. In the circuit below, we assume that the switch is at Position A for a long time, but moved to Position B at $t=0$. Given that I=1A, R_1=10Ω, R_2=10Ω, R_3=10Ω, C=1F, L=2H; *calculate $i(t)$ for $t > 0$.*

18'

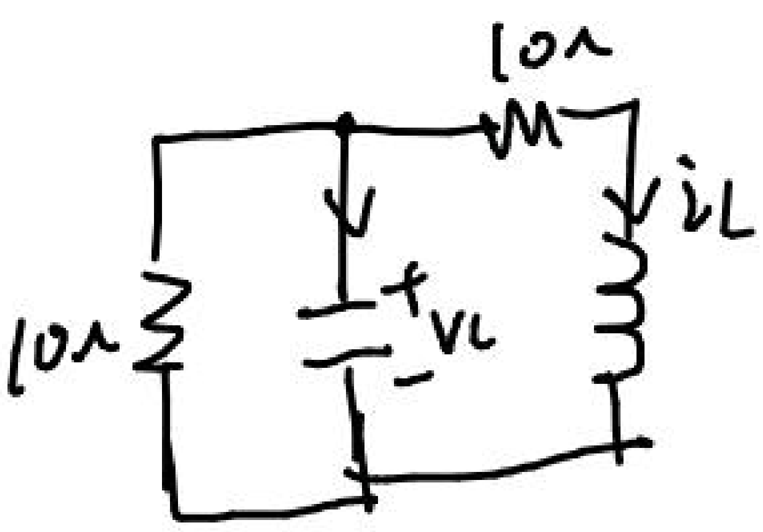


t<0

$V_C(0^-) = 1A \cdot \frac{10}{10+10} \cdot 10\Omega = 5V$ 2'

$i_L(0^-) = 1A \cdot \frac{10}{10+10} A = 0.5A$ 2'

t>0



$$\begin{cases} C\frac{dv_c(t)}{dt} + \frac{v_c(t)}{10\Omega} + \frac{v_c(t) - L\frac{di_L(t)}{dt}}{10\Omega} = 0 \\ v_C(t) = L\frac{di_L(t)}{dt} + 10\Omega \cdot i_L(t) \end{cases}$$ 3' 2'

$$\frac{d^2 i_L(t)}{dt^2} + 5.1\frac{di_L(t)}{dt} + i_L(t) = 0$$ 3'

$i_L(t) = A_1 e^{-0.204t} + A_2 e^{-4.896t}$ A 2'

$i_L(0^+) = 0.5A = A_1 + A_2$ 1'

$V_L(0^+) = V_C(0^+) - i_L(0^+) \cdot R_3 = 0V = (-0.204A_1 - 4.896A_2) \cdot 2$ 1'

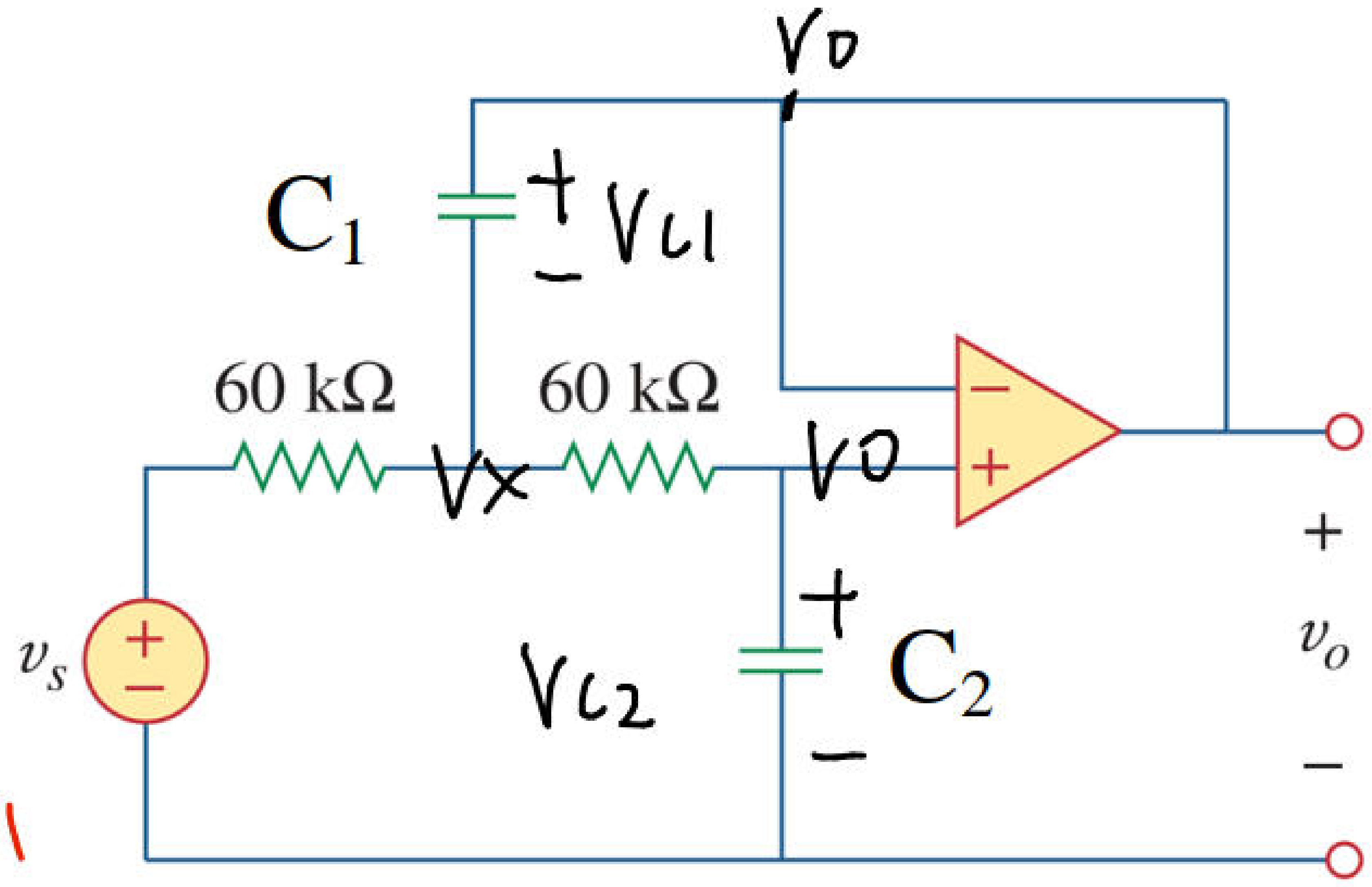
2' $A_1 = 0.522$ $A_2 = -0.022$

$i_L(t) = 0.522e^{-0.204t} - 0.022e^{-4.896t}$ A $(t>0)$

2'

15'

5. For the circuit below. The operational amplifier is working in the linear mode. Given $V_s = u(t)$ V; C_1=C_2=60μF and no initial energy stored in both capacitors. find expression of $v_0(t)$ *for* $0 < t < 0.5$ *sec.*



2' $V_o(t) = V_{c2}(t)$ $\quad V_s(t) = 1V \quad t>0$

$$\frac{V_s - V_x}{60k\Omega} + \frac{V_o - V_x}{60k\Omega} + C\frac{d(V_o - V_x)}{dt} = 0 \quad 2'$$

$$V_o - V_x = V_{c1} \quad 2'$$

$$\frac{V_x - V_o}{60k\Omega} = C\frac{dV_{c2}}{dt} \quad 2'$$

$$V_{C1} = -3.6\frac{dV_{C2}}{dt}$$

$$\frac{d^2V_{c2}(t)}{dt^2} + \frac{5}{9}\frac{dV_{c2}(t)}{dt} + \frac{25}{324}V_{c2}(t) = \frac{25}{324} \quad 3'$$

$$V_{c2}(t) = (A_1 t + A_2)e^{-\frac{5}{18}t} + 1\ V$$

$$V_{C2}(0+) = A_2 + 1 = 0\ V \quad 2'$$

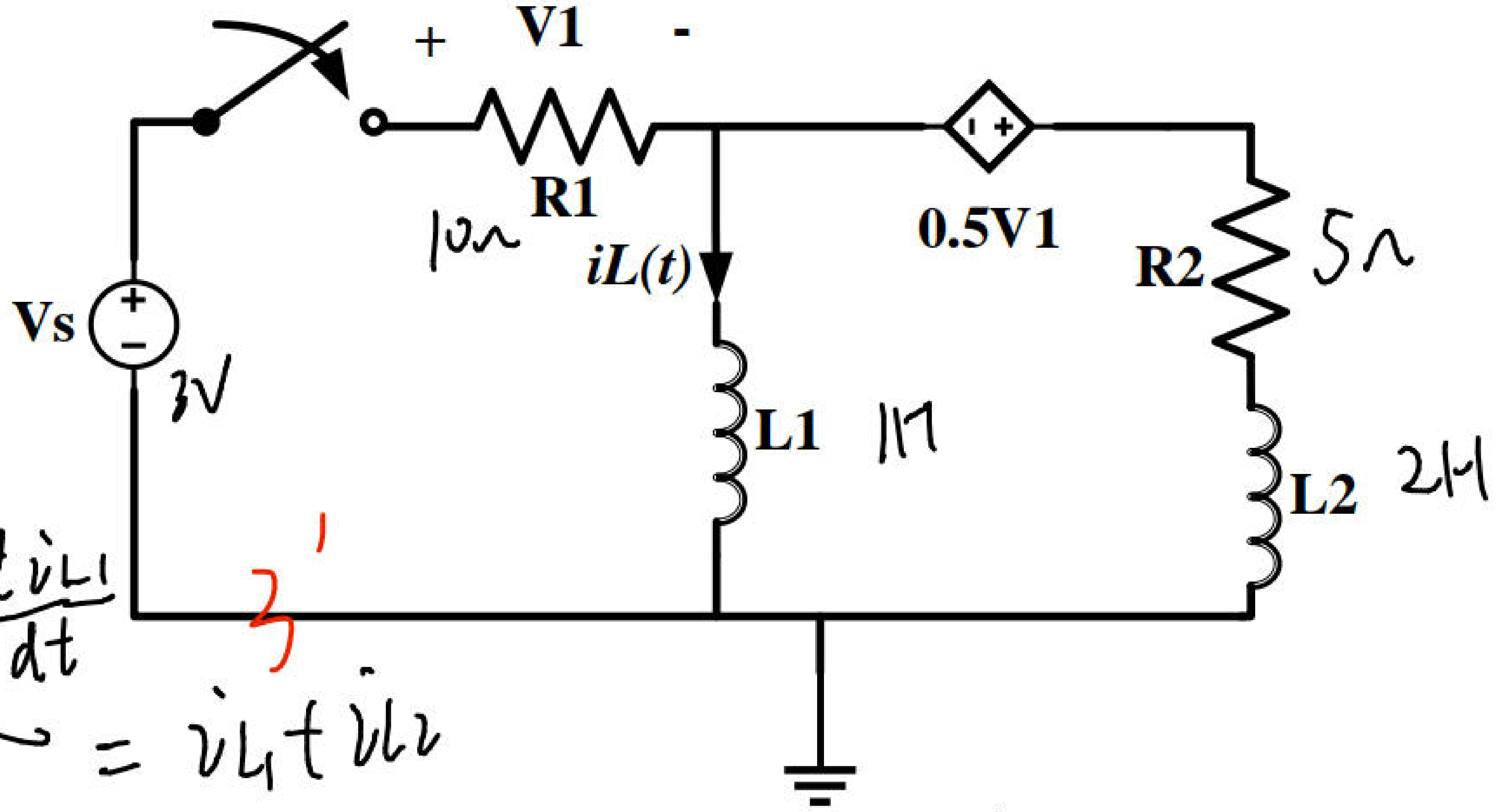
$$i_{C2}(0+) = C_2(A_1 - \tfrac{5}{18}A_2) = 0A \quad 2'$$

$A_2 = -1$ 1' $\quad A_1 = -\frac{5}{18}$

$$V_{c2}(t) = V_o(t) = (-1 - 0.278t)\cdot e^{-0.278t} + 1\ V \quad (0 < t < 0.5\,s) \quad 2'$$

15'

6. For the circuit below. The switch has been open for a long time. At t = 0 the switch is closed. There is no energy stored in inductors L1 and L2. Given R1 = 10Ω, R2 = 5Ω, L1 = 1H, L2 = 2H, Vs = 3V, find $i_L(t)$ *for* $t>0$.



3'

$$\frac{V_s - L_1\frac{di_{L1}}{dt}}{R_1} = i_{L1} + i_{L2}$$

$$V_s + 0.5\left(V_s - L_1\frac{di_{L1}}{dt}\right) = V_s - L_1\frac{di_{L1}}{dt} + i_{L2}\cdot R_2 + L_2\frac{di_{L2}}{dt}$$

$$i_{L2}(t) = \frac{3}{10} - \frac{1}{10}\frac{di_{L1}}{dt} - i_{L1} \quad 2'$$

$$-0.5V_s - 0.5\cdot\frac{di'}{dt} + 5i_{L2} + 2\frac{di_{L2}}{dt} = 0$$

$$-1.5 - 0.5\frac{di'}{dt} + 1.5 - 0.5\frac{di_{L1}}{dt} - 5i_{L1} - 0.2\frac{d^2i_{L1}}{dt^2} - 2\frac{di_{L1}}{dt} = 0$$

$$\frac{d^2i_{L1}}{dt^2} + 15\frac{di_{L1}}{dt} + 25i_{L1} = 0 \quad 2'$$

$$i_{L1}(t) = A_1e^{\frac{-15+5\sqrt{5}}{2}t} + A_2e^{\frac{-15-5\sqrt{5}}{2}t}\ \mathrm{A} \quad 2'$$

$$i_{L1}(0^+) = i_{L1}(0^-) = 0\,\mathrm{A} = A_1 + A_2 \quad 1'$$

$$V_{L1}(0^+) = -1.91A_1 - 13.09A_2 = 3\,\mathrm{V} \quad 1' \Rightarrow A_1 = 0.268,\ A_2 = -0.268 \quad 2'$$

$$i_{L1}(t) = 0.268e^{-1.91t} - 0.268e^{-13.09t}\ \mathrm{A} \quad (t>0)$$

2'