Problem 1

(10 points)

For the continuous-time periodic signal

$$x(t) = 2 + \cos\left(\frac{2\pi}{3}t\right) + 4\sin\left(\frac{5\pi}{3}t\right),$$

determine the fundamental frequency ω_0 and the Fourier series coefficients a_k such that

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}.$$

Problem 2

(20 points)
Suppose we are given the following information about a signal x(t):

1. x(t) is real and odd.
2. x(t) is periodic with period T = 2 and has Fourier coefficients a_k.
3. $a_k = 0$ for $|k| > 1$.
4. $\frac{1}{2}\int_0^2 |x(t)|^2\, dt = 1$.

Specify two different signals that satisfy these conditions.

Problem 3

(20 points)
Consider the following three discrete-time signals with a fundamental period of 6:

$$x[n] = 1 + \cos(\frac{2\pi}{6}n), \qquad y[n] = \sin(\frac{2\pi}{6}n + \frac{\pi}{4}), \qquad z[n] = x[n]y[n].$$

(a) Determine the Fourier series coefficients of $x[n]$.

(b) Determine the Fourier series coefficients of $y[n]$.

(c) Use the results of parts (a) and (b), along with the multiplication property of the discrete-time Fourier series, to determine the Fourier series coefficients of $z[n] = x[n]y[n]$.

(d) Determine the Fourier series coefficients of $z[n]$ through direct evaluation, and compare your result with that of part (c).

Problem 4

(25 points)
Consider a discrete-time LTI system with impulse response

$$h[n] = (\frac{1}{4})^n u[n]$$

Find the Fourier series representation of the output y[n] for each of the following inputs:

(a) $x[n] = \sin(\frac{3\pi}{4}n)$

(b) $x[n] = \cos(\frac{\pi}{4}n) + 2\cos(\frac{\pi}{2}n)$

Problem 5

(25 points)
Consider a continuous-time LTI system with impulse response

$$h(t) = e^{-4|t|}$$

Find the Fourier series representation of the output y(t) for each of the following inputs:

(a) $x(t) = \sum_{n=-\infty}^{+\infty} \delta(t - n)$

(b) x(t) is the periodic wave depicted showed below: