Lecture 5 – Image Segmentation（图像分割）

This lecture will cover:

- **Morphological Image Processing （形态学图像处理）**
  - **Morphological operation**
  - **Morphological algorithms**
- Image Segmentation（图像分割）
  - Point, Line and Edge Detection（点、线和边缘检测）
  - Thresholding（阈值处理）
  - Segmentation using Morphological Watersheds（形态学分水岭分割）

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Mathematical Morphology

- **Language： Set theory （集合论）**

- **Advantages comparing to other spatial or frequency domain methods**
  - Keep more information from image;
  - Parallel processing
  - Insensitive to noise
  - Smooth edge
  - Continuous skeletons

- **Key operations**
  - HMT (Hit or Miss Transformation)(击中与否变换)
  - Dilation （膨胀） and Erosion（腐蚀）

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Steps

To analyze image with mathematical morphological methods,

① Specify the geometrical structural pattern of the object;

② Choose structuring element (SE) based on the pattern

③ Proceed HMT with the selected SE to acquire characteristic image of the object

④ Emphasize the desired information to extract attributes

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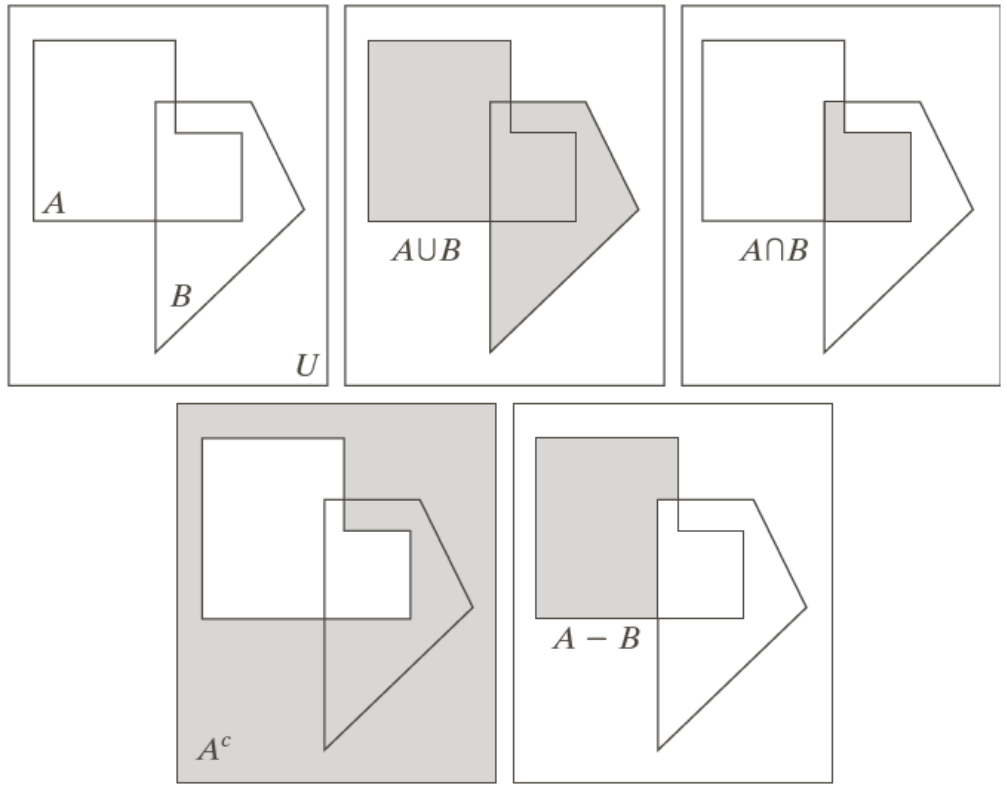
Preliminaries --- Set Operation

An digital image $\boldsymbol{f(x,y)}$ can be considered as a set $\boldsymbol{A}$, if $w = (x, y)$ in 2D integer space $\boldsymbol{Z^2}$, Then

- $\boldsymbol{w \in A}$: w is an element of $\boldsymbol{A}$
- $\boldsymbol{w \notin A}$: w is not an element of $\boldsymbol{A}$
- $\boldsymbol{B = \{w \mid \text{condition}\}}$: all elements which meet the specified condition
  - $\boldsymbol{A \cup B = \{w \mid w \in A \text{ or } w \in B\}}$: union (并集)
  - $\boldsymbol{A \cap B = \{w \mid w \in A \text{ and } w \in B\}}$: intersection (交集)
  - $\boldsymbol{A^c = \{w \mid w \notin A\}}$: complement (补集)
  - $\boldsymbol{A - B = \{w \mid w \in A, w \notin B\}}$: difference (差集)



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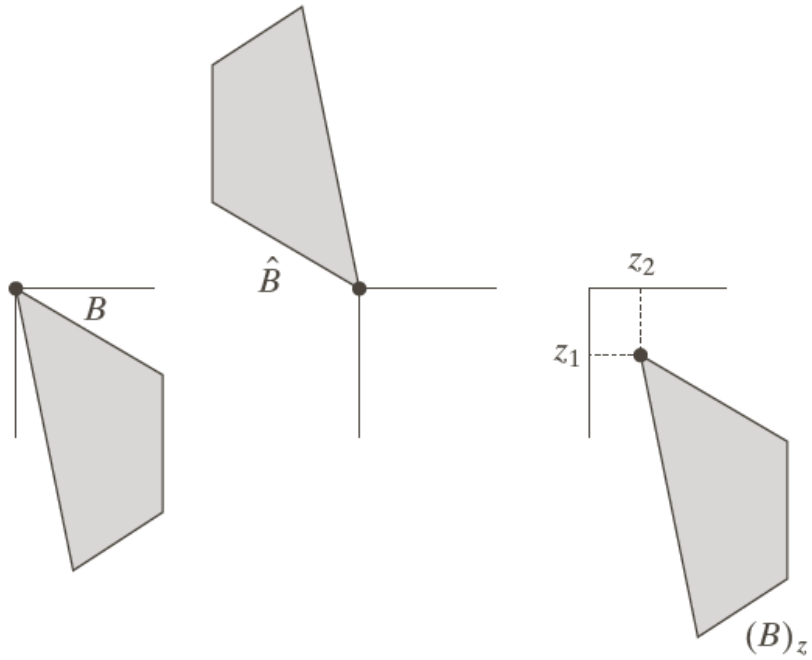
Reflection and Translation

- **Reflection (反射)**

$$\hat{B} = \{w \mid w = -b, b \in B\}$$

- **Translation (平移)**

$$(B)_z = \{w \mid w = b + z, b \in B\},$$
where $z = (z_1, z_2)$



* *Foreground (前景) and Background(背景):*

the sets of pixels in an image defined to be objects and non-objects, respectively

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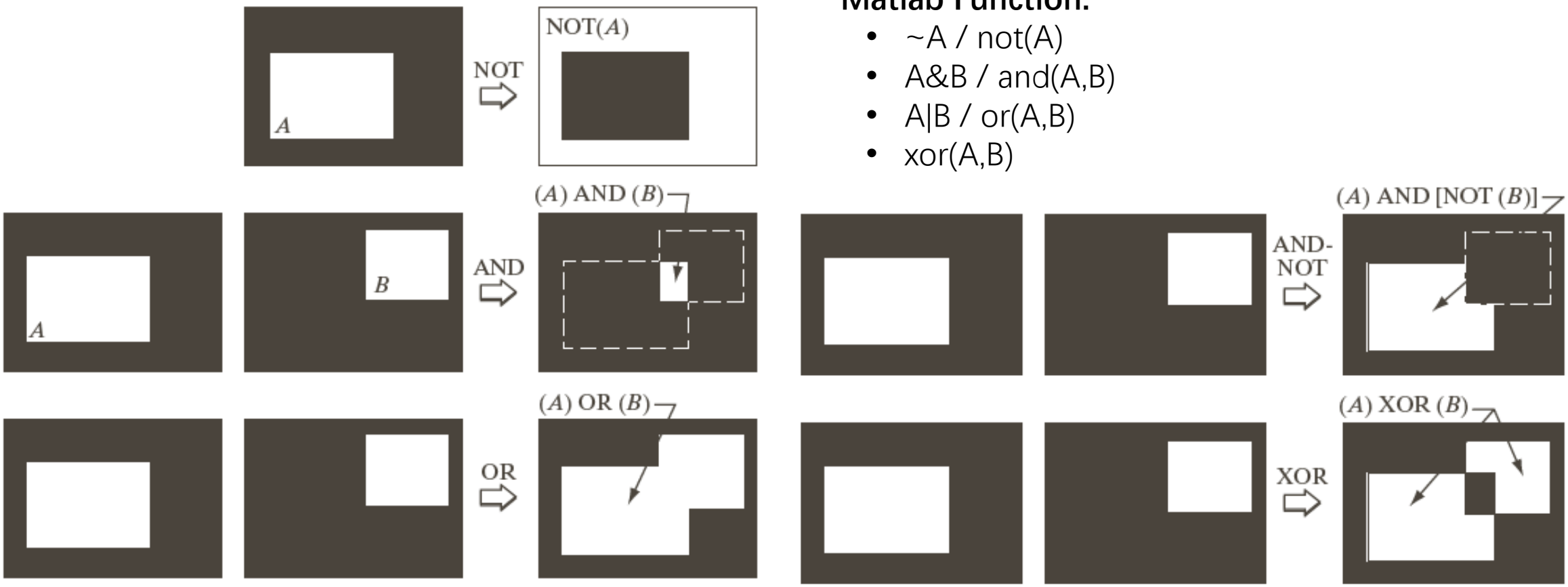
Preliminaries --- Logical Operation



Matlab Function:

- ~A / not(A)
- A&B / and(A,B)
- A|B / or(A,B)
- xor(A,B)

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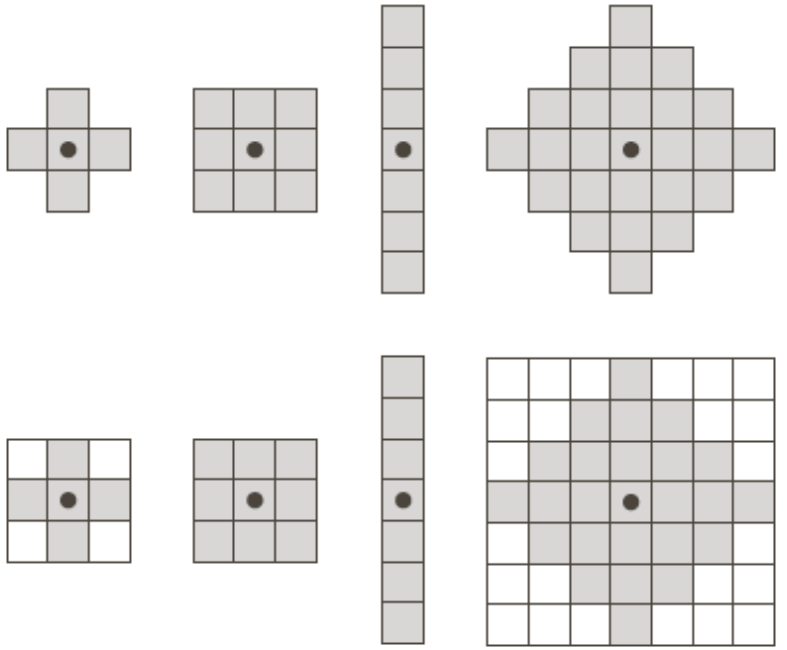
Structuring Element（结构元）

- **Structuring Element (SE) ---** small sets or subimages used to probe an image under study for properties of interest**.**

- **SE Selection**
  - Simpler than the image
  - With boundary
  - Convex

- **Structures**
  - Origin
  - Rectangular



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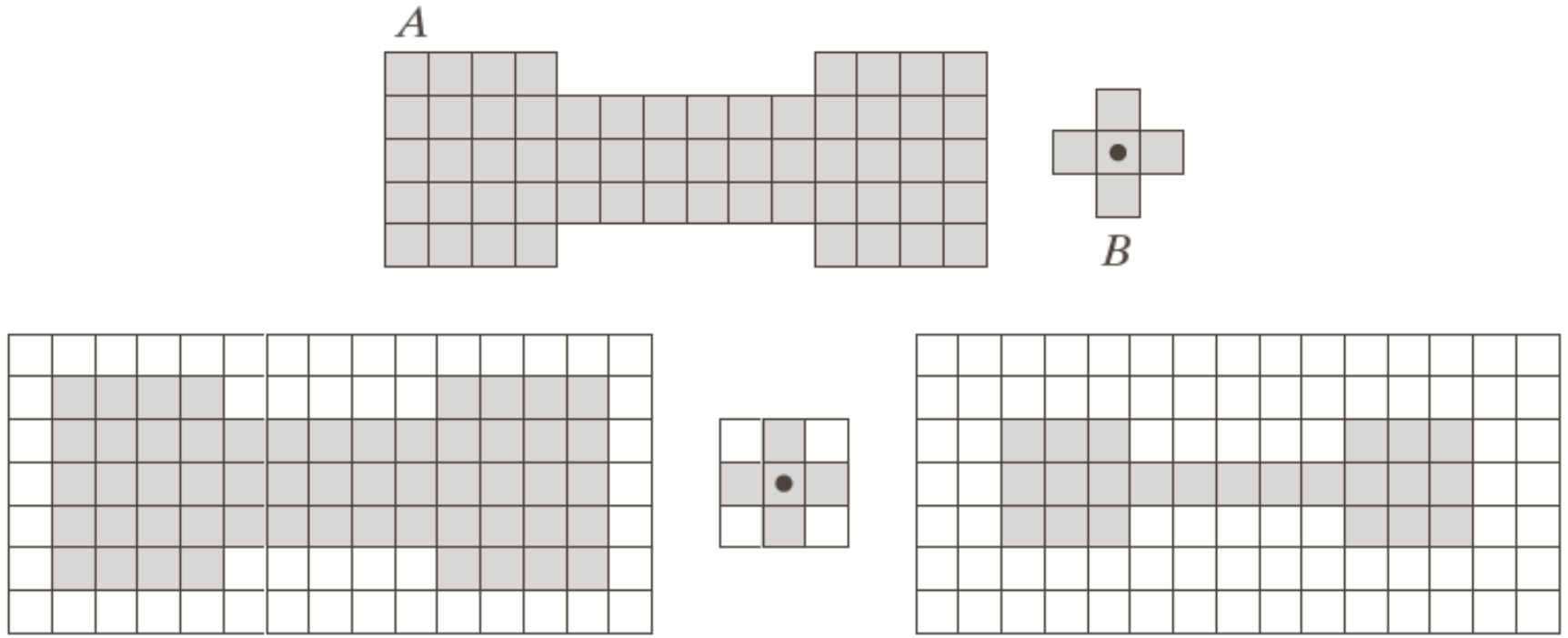
Structuring Element（结构元）

- **Matlab Function:** se = strel(shape, parameters)



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Erosion (腐蚀)

- **Definition:**

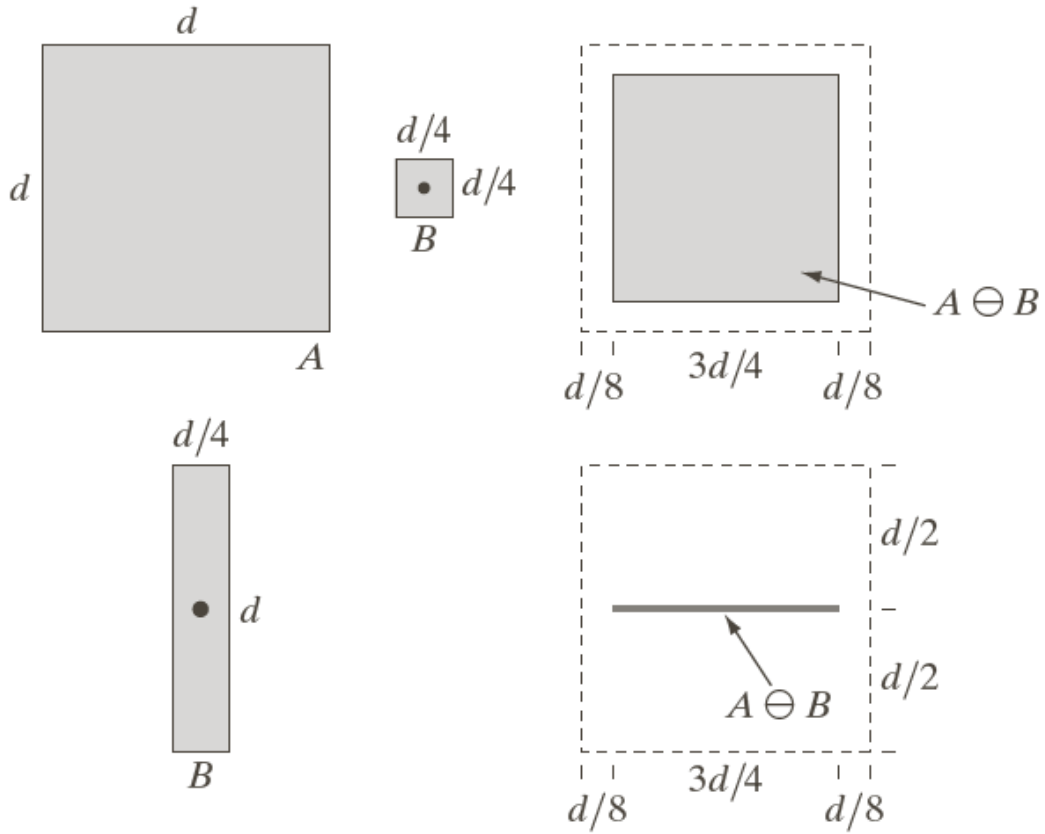
$$A \ominus B = \{z \mid (B)_z \subseteq A\}$$

or

$$A \ominus B = \{z \mid (B)_z \cap A^c = \emptyset\}$$

- **Matlab Function:**

J = imerode(I,SE)



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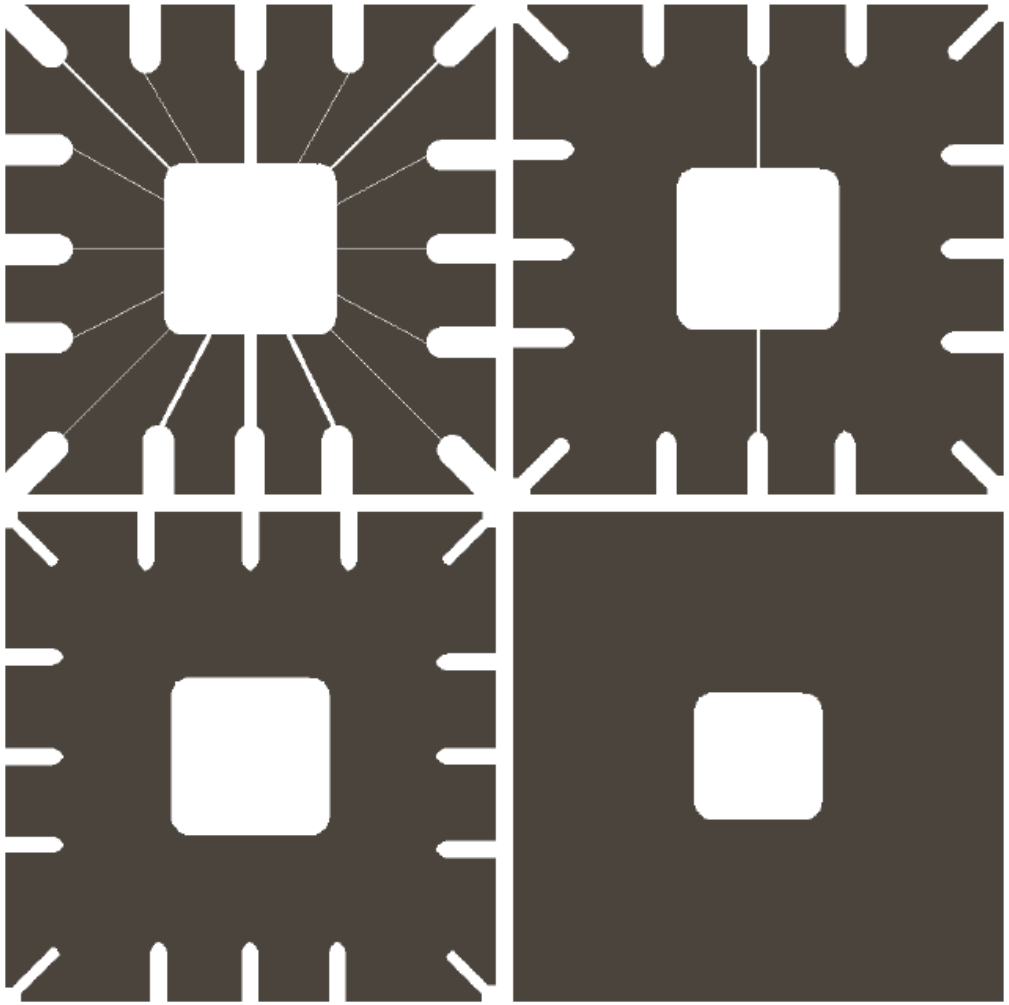
Erosion (腐蚀)



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Dilation (膨胀)

- **Definition:**

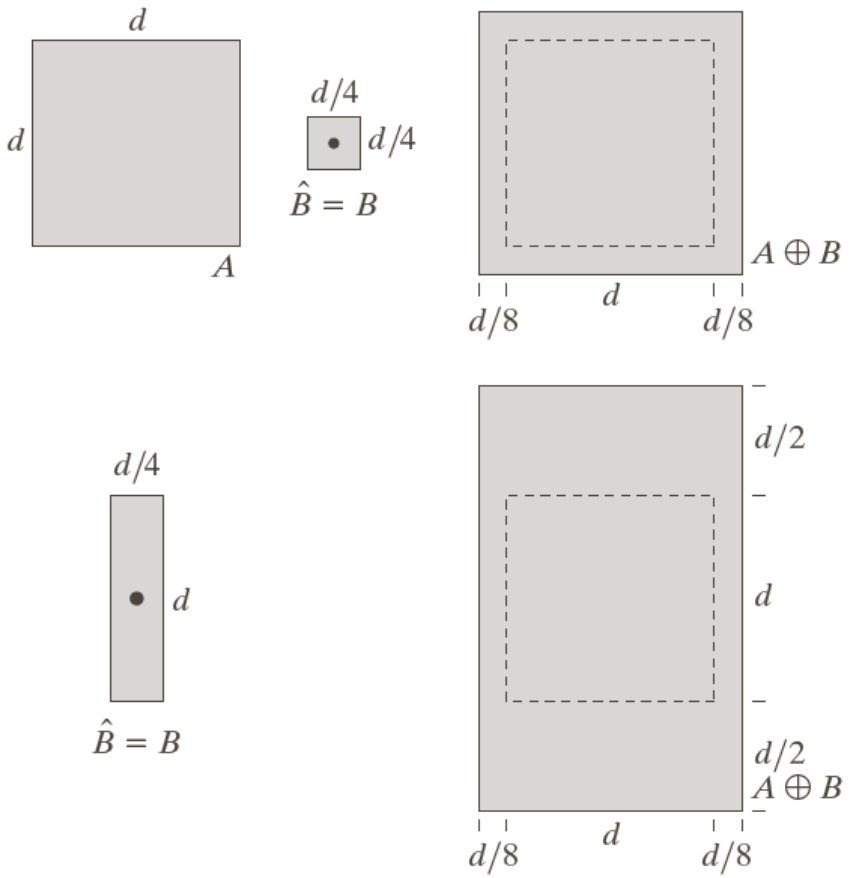
$$A \oplus B = \{z \mid (\hat{B})_z \cap A \neq \emptyset\}$$

or

$$A \oplus B = \{z \mid [(\hat{B})_z \cap A] \subseteq A\}$$

- **Matlab Function:**

J = imdilate(I,SE)



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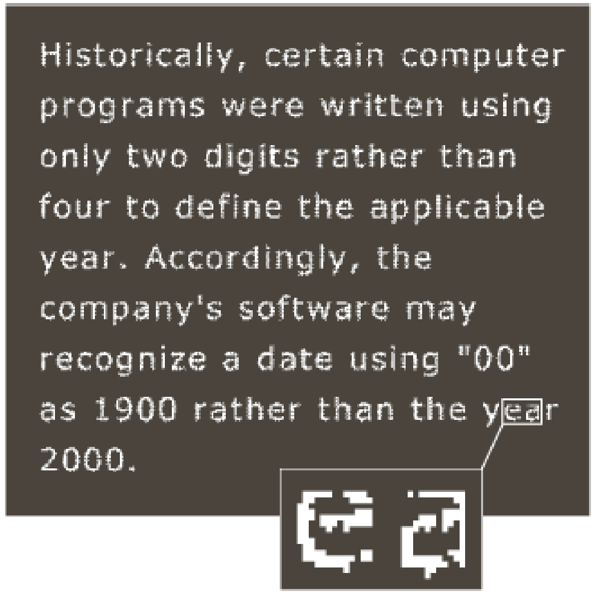
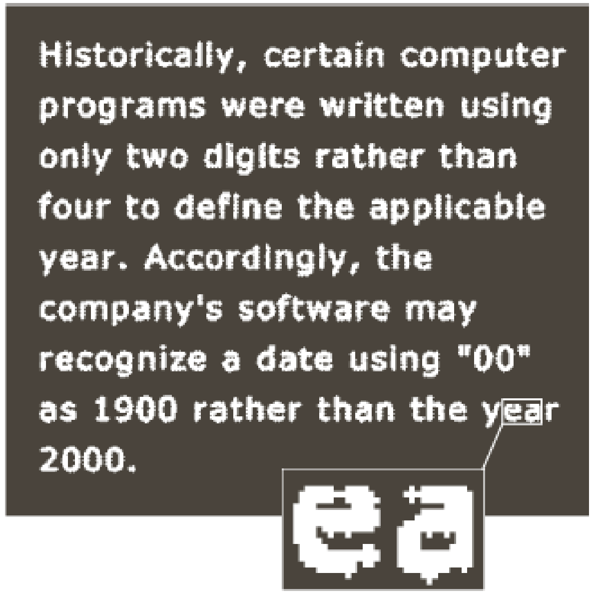
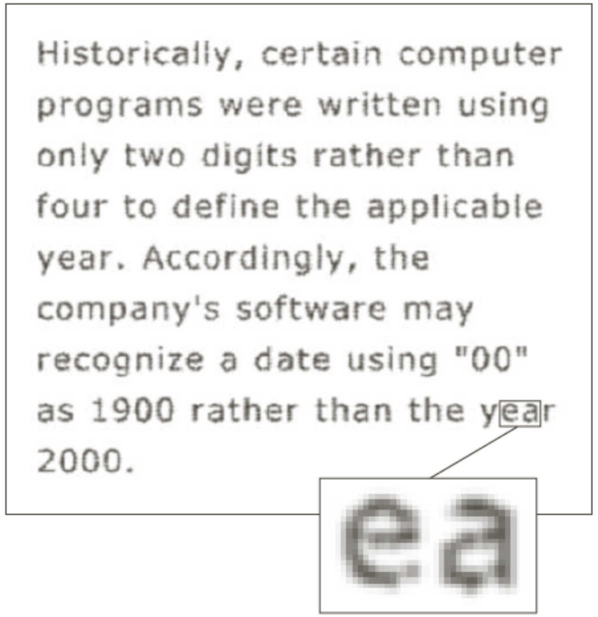
Dilation (膨胀)







0	1	0
1	1	1
0	1	0

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Properties

- **Duality (对偶性)**
  - $(A \ominus B)^c = (A)^c \oplus \hat{B}$
  - $(A \oplus B)^c = (A)^c \ominus \hat{B}$
- **Associativity (结合律)**

$$(A \oplus B) \oplus C = A \oplus (B \oplus C)$$

or

$$(A \oplus B) = A \oplus (B_1 \oplus B_2) = (A \oplus B_1) \oplus B_2$$

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Opening & Closing (开操作和闭操作)

- **Opening（开操作）**
  - Smooth the contour of an object
  - Break narrow isthmuses
  - Eliminate thin protrusions
- **Closing（闭操作）**
  - Smooth the contour of an object
  - Fuse narrow breaks and long thin gulfs
  - Eliminate small holes
  - Fill gaps in the contour

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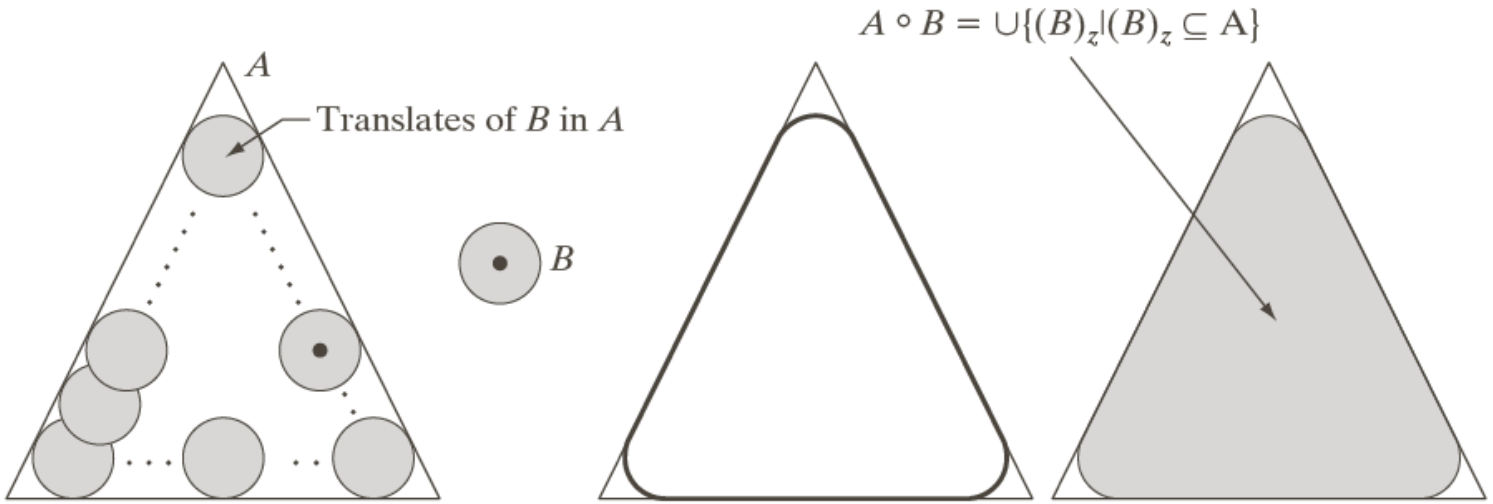
Opening (开操作)

- **Definition:** $A \circ B = (A \ominus B) \oplus B$ **or** $A \circ B = \cup\{(B)_z \mid (B)_z \subseteq A\}$
- **Matlab Function:** J = imopen(I,SE)
- **Properties:**
  - ① $\mathrm{A} \circ \mathrm{B}$ is a subset (subimage) of A
  - ② If C is a subset of D, the $\mathrm{C} \circ \mathrm{B}$ is a subset of $\mathrm{D} \circ \mathrm{B}$
  - ③ $(\mathrm{A} \circ \mathrm{B}) \circ \mathrm{B} = \mathrm{A} \circ \mathrm{B}$



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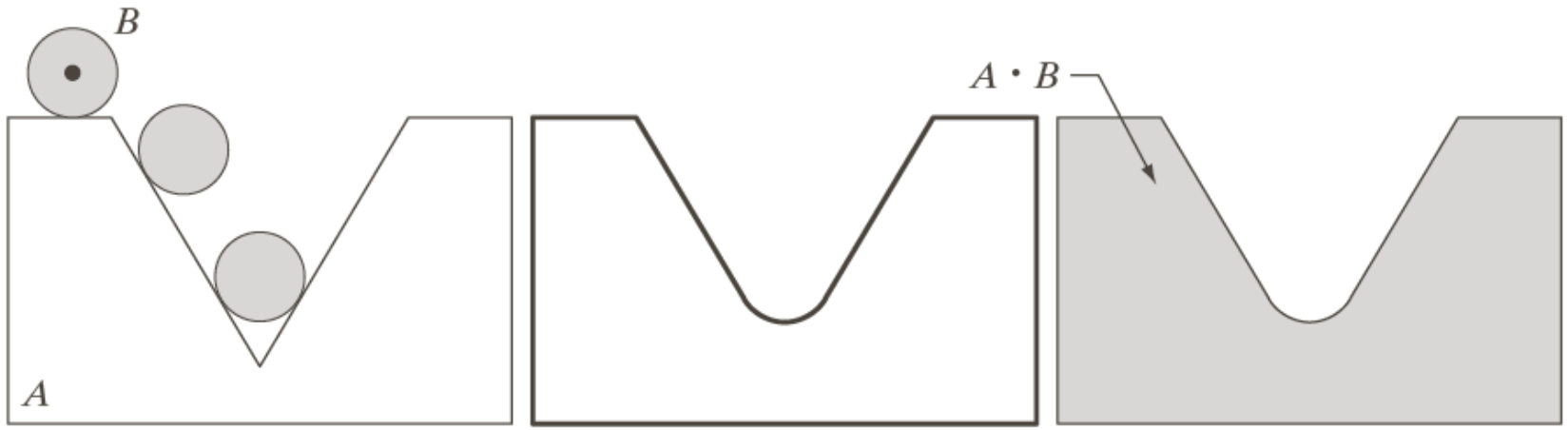
Closing (闭操作)

- **Definition:** $A \bullet B = (A \oplus B) \ominus B$
- **Matlab Function:** J = imclose(I,SE)
- **Properties:**
  - ① A is a subset (subimage) of $A \bullet B$
  - ② If C is a subset of D, the $C \bullet B$ is a subset of $D \bullet B$
  - ③ $(A \bullet B) \bullet B = A \bullet B$



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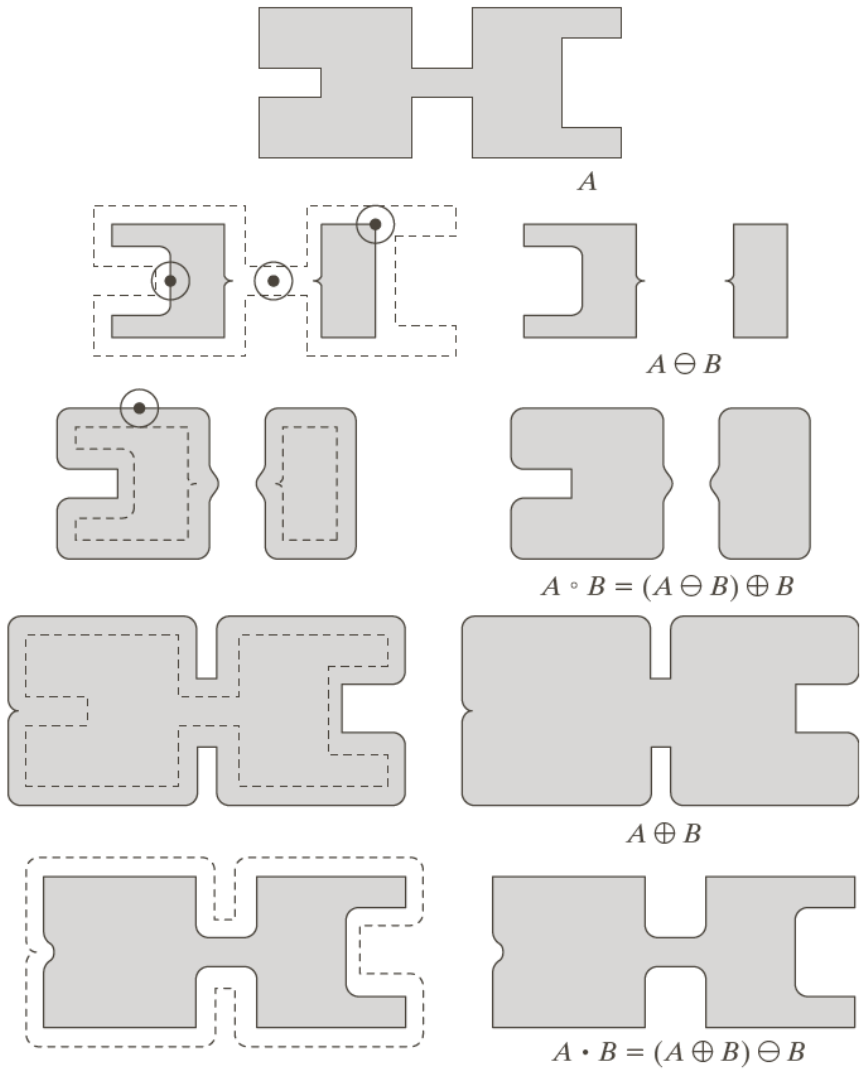
Opening & Closing (开操作和闭操作)



- **Duality (对偶性)**

$$(A \bullet B)^c = (A)^c \circ \hat{B}$$

$$(A \circ B)^c = (A)^c \bullet \hat{B}$$

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Morphological Filtering (形态学滤波)

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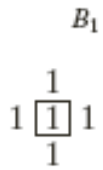
The Hit-or-Miss Transformation (击中或击不中变换)

$$A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2)$$



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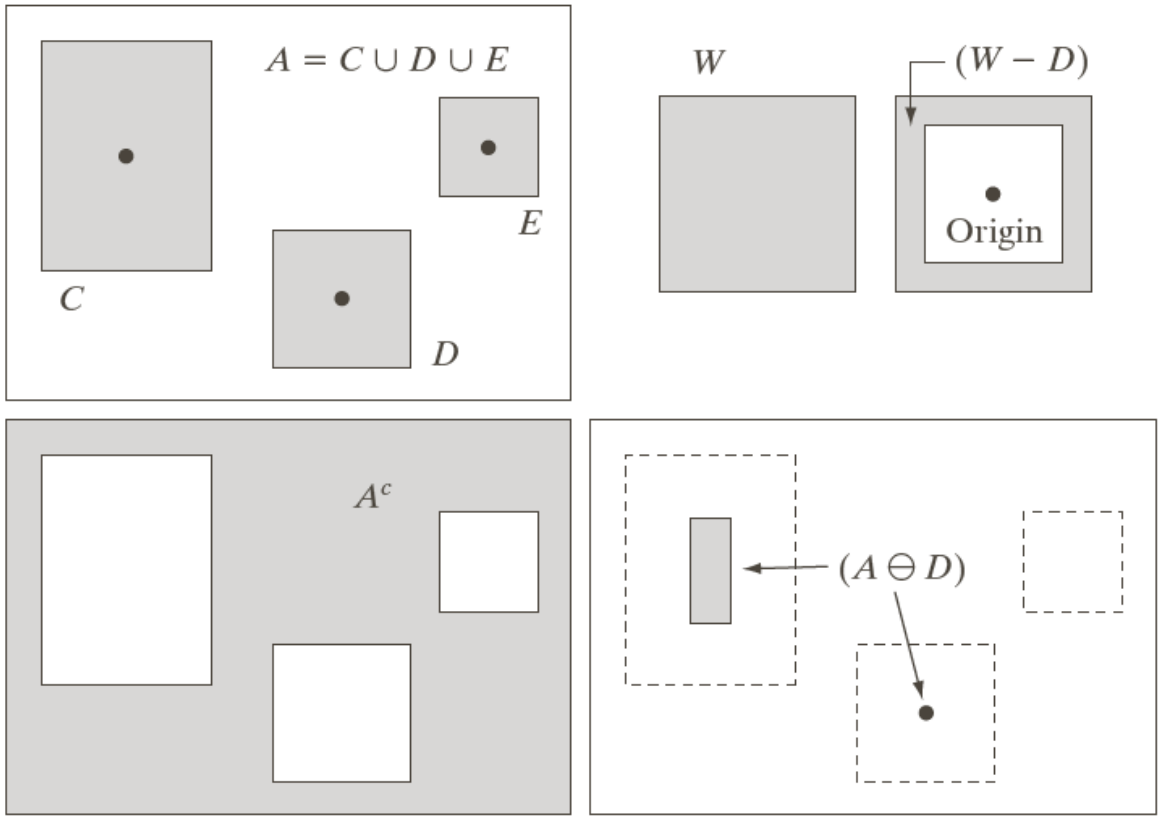
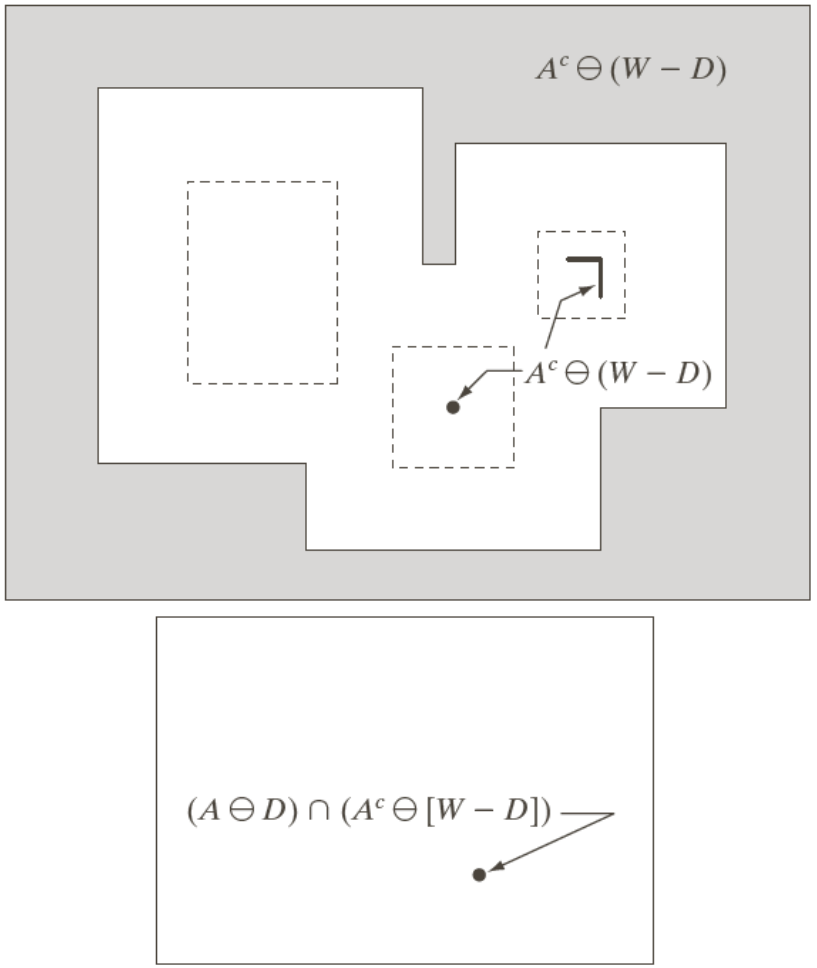
The Hit-or-Miss Transformation (击中或击不中变换)





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The Hit-or-Miss Transformation (击中或击不中变换)

- $A \circledast B = (A \ominus D) \cap [A^c \ominus (W - D)]$

- $A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2)$

  where $B = (B_1, B_2)$,

  B_1 is associated with the object;

  B_2 is associated with the corresponding background.

- $A \circledast B = (A \ominus B_1) - (A \oplus \hat{B}_2)$

  where $(A \oplus \hat{B}_2)^c = (A^c \ominus B_2)$ and $M - N = M \cap N^c$

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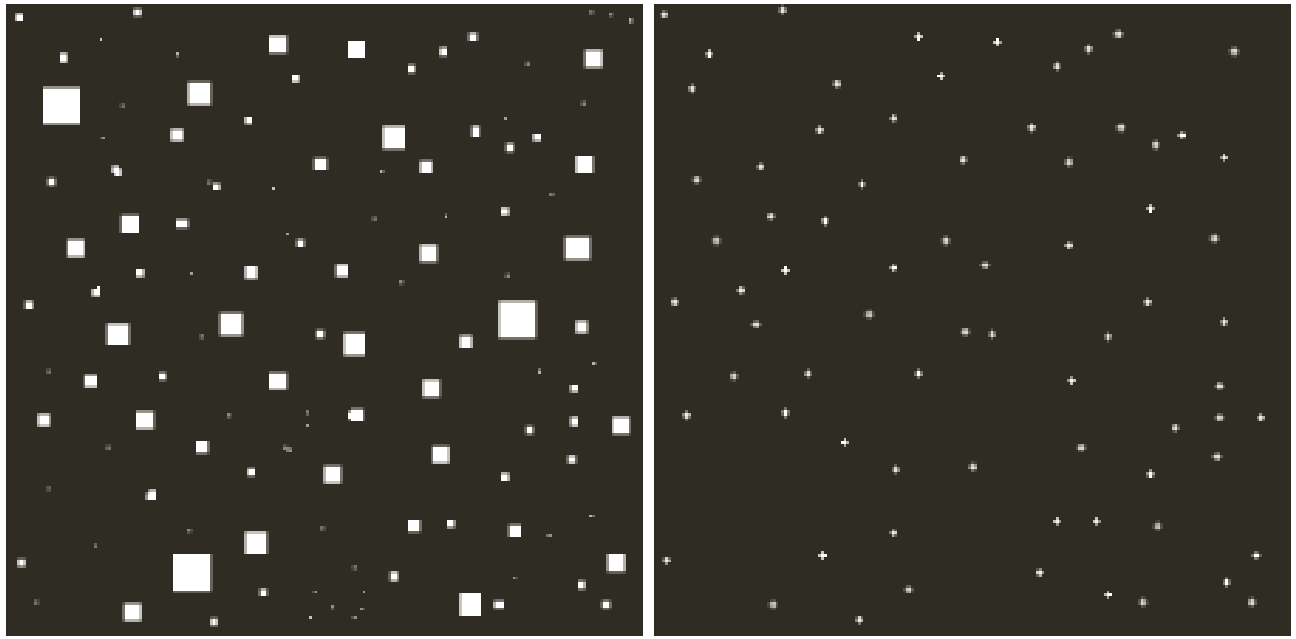
The Hit-or-Miss Transformation (击中或击不中变换)

- **Matlab Function:** J = bwhitmiss(BW,SE1,SE2)

$$B_1 = \begin{matrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{matrix} \qquad B_2 = \begin{matrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{matrix}$$



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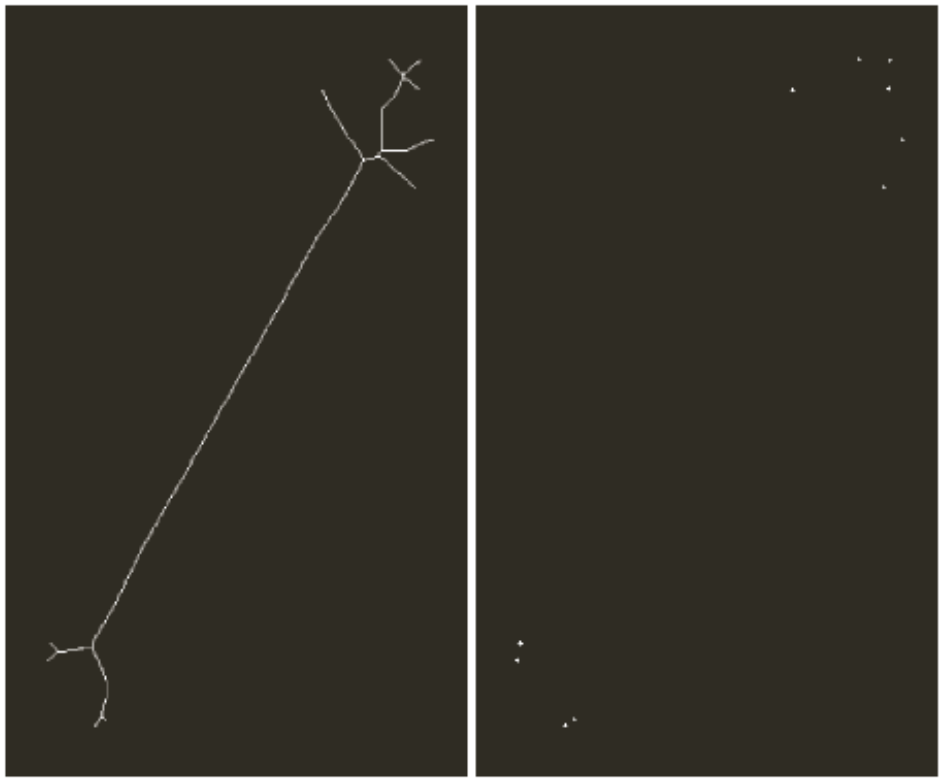
Look-up Table (查找表)

- **Condition: Small SE (2*2, 3*3)**
- **For a 3*3 SE, an index matrix as below can be used**

$$\begin{matrix} 1 & 8 & 64 \\ 2 & 16 & 128 \\ 4 & 32 & 256 \end{matrix}$$



- **Matlab Function:**

lut = makelut(fun,n);

g = applylut(f, lut); or

g = bwlookup(f, lut);

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