CS 101 Fall 2020 - Quiz 12-B

12/24/2020 - 20 Minutes **Name:** **ID number:**

Problem 1(4×2pts): Follow the below steps, show that this problem is in NP:

Given a set of n cities, and distances between each pair of cities, is there a path visit each city exactly once, and the path has distance at most D, for a given D?

Part(A): Construct the verifier.

Part(B): Briefly explain how your verifier works.

Part(C): Show that the verifier works in polynomial time.

Part A: Certificate y is a path through the graph. Check y goes through every city once, and the total length of y is $\leq$ D. If so, output 1, else output 0.

Part B: If an instance has a solution, then there is a path going through each vertex once with total length *le* D.Call the path y and give it to V. Clearly V outputs 1.

Part C: If the graph has n vertices, then all of V's checks can be done in O(n) time.

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Problem 2(7pts): Reduction

For the below problem, choose an NP-complete problem A and for any A instance, construct an instance of the below problem. **You need to ensure the yes/no answers to the two instances are the same, but you do not need to prove it.**

We have 3-coloring problem in class and we've also reduced 3-coloring problem to 4-coloring problem in homework. Now consider 6-coloring problem: Given an undirected graph G, can the nodes be colored in 6 colors so that no adjacent nodes have the same color? Please reduce 4-coloring problem to 6-coloring problem.

(1) For any instance of 4-Coloring problem with graph $G = (V, E)$, we construct an instance of 6-Coloring problem where $G' = (V', E')$. V' is V with two additional vertices $\tilde{v_1}$, $\tilde{v_2}$ and E' is E with edges $(\tilde{v_1}, \tilde{v_2})$ and $(v, \tilde{v_1})$, $(v, \tilde{v_2})$ for every $v \in V$.

(2) Proof (This do not need to be included in answer): We now prove G is a yes-instance of 4-Coloring problem if and only if G' is a yes-instance of the constructed 6-Coloring problem:

- "$\Rightarrow$": if G is a yes-instance of 4-Coloring problem with assignment a, we can just use the assignment for vertices in $V' \bigcap V$ of G' and color the $\tilde{v_1}$ with the 5th color and the $\tilde{v_2}$ with the 6th color. Then the assignment a' is valid for G' which means the constructed 6-Coloring is also a yes-instance.
- "$\Leftarrow$": if G is a yes-instance of the constructed 6-Coloring problem with assignment a, then since the vertices $\tilde{v_1}, \tilde{v_2}$ are directly connected to every other vertices, the set $V' \bigcap V$ can only use 4 different colors. Thus $G = G' - \tilde{v_1}, \tilde{v_2}$ is also a yes-instance.