Lecture 23 Active Shape Model (ASM)

Yuyao Zhang, Xiran Cai PhD

zhangyy8@shanghaitech.edu.cn caixr@shanghaitech.edu.cn

SIST Building 2 302-F/302-C

Course piazza link：
piazza.com/shanghaitech.edu.cn/spring2021/cs270spring2021

上海科技大学
ShanghaiTech University

Outline

- ➢Algorithm 1: Ordinary Procrustes Analysis
- ➢Algorithm 2: Statistical Shape Analysis
- ➢Algorithm 3: Matching model to target


Tim Cootes: Selected Publications by Subject

Statistical Shape Models

Active Shape Models

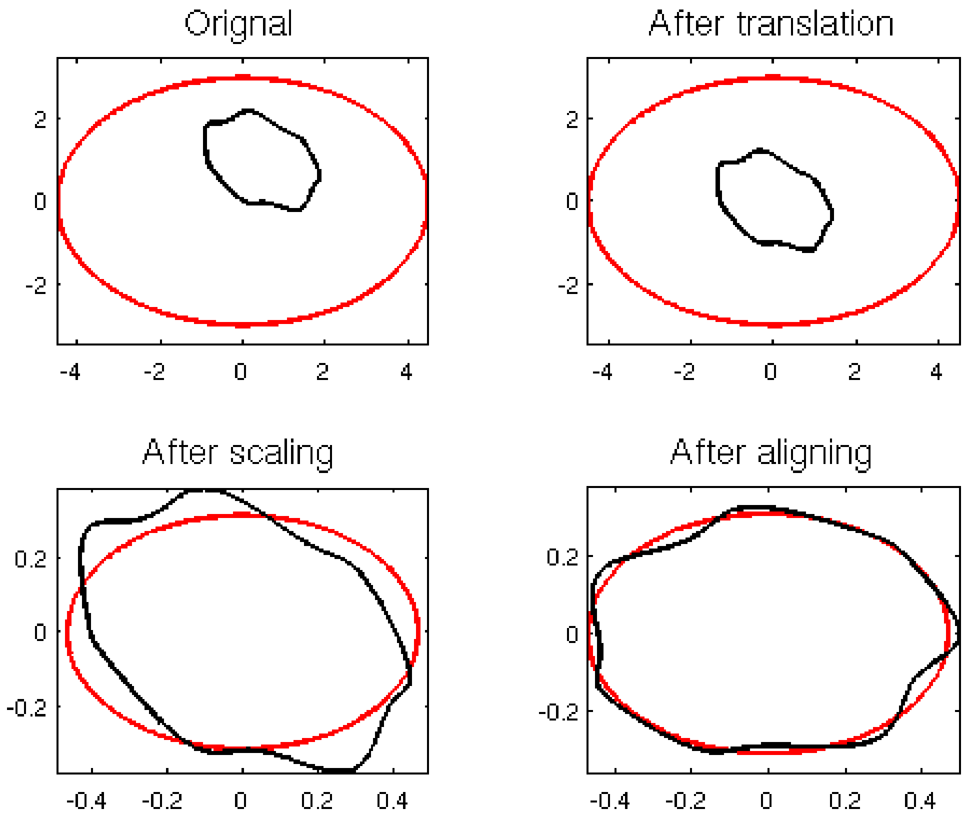
Active Appearance Models

https://personalpages.manchester.ac.uk/staff/timothy.f.cootes/refs_by_subject.html

上海科技大学
ShanghaiTech University

Building models

Algo1: Ordinary Procrustes Analysis



Translation

- Take k points in two dimensions,

$$((x_1, y_1), (x_2, y_2), \cdots, (x_k, y_k))$$

- The mean of these points is $(\bar{x}, \bar{y})$ where

$$\bar{x} = \frac{x_1+x_2+\cdots+x_k}{k}, \ \bar{y} = \frac{y_1+y_2+\cdots+y_k}{k}$$

- Now translate these points so that their mean is translated to the origin

$(x, y) \rightarrow (x - \bar{x}, y - \bar{y})$, giving the point $(x_1 - \bar{x}, y_1 - \bar{y})$, $\cdots \cdots$

Uniform scaling

- The scale component can be removed by scaling the object so that the root mean square distance (*RMSD*) from the points to the translated origin is 1,

$$s = \sqrt{\frac{(\bar{x}_1 - x_1)^2 + (\bar{y}_1 - y_1)^2 + \cdots}{k}}$$

- The scale becomes 1 when the point coordinates are divided by the object's initial scale

$$((\bar{x}_1 - x_1)/s\ ,\ \ (\bar{y}_1 - y_1)/s)$$

- Notice that other methods for defining and removing the scale are sometimes used in the literature.

Rotation

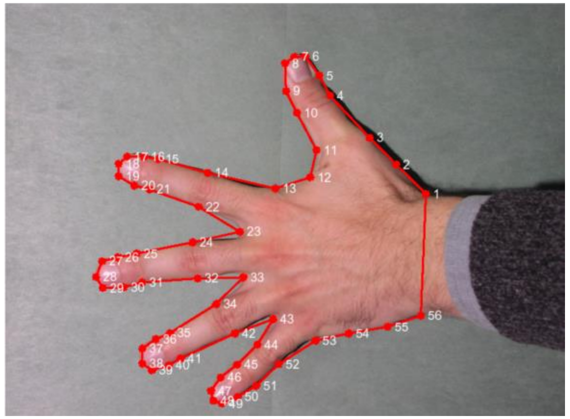
- A rotation by angle θ gives

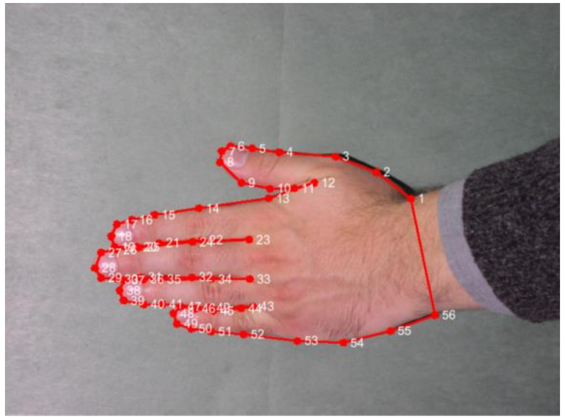
$$(u_1, v_1)=(\cos\theta x_1 - \sin\theta y_1,\ \sin\theta x_1 + \cos\theta y_1)$$

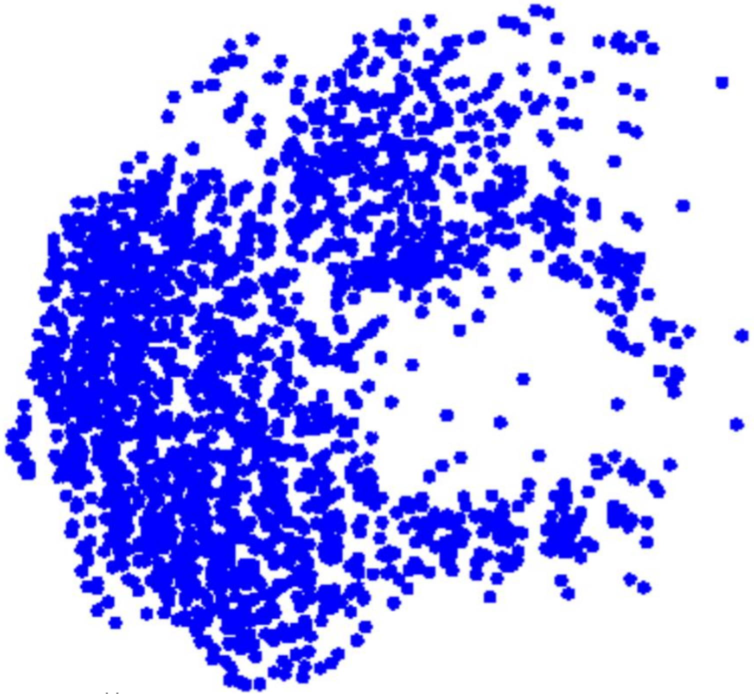
- Where (u, v) are the coordinate of a rotated point. Taking the derivative of $(u_1 - \bar{x}_1)^2 + (v_1 - \bar{y}_1)^2 + \cdots$ with respect to θ and solving for θ when the derivative is zeros gives

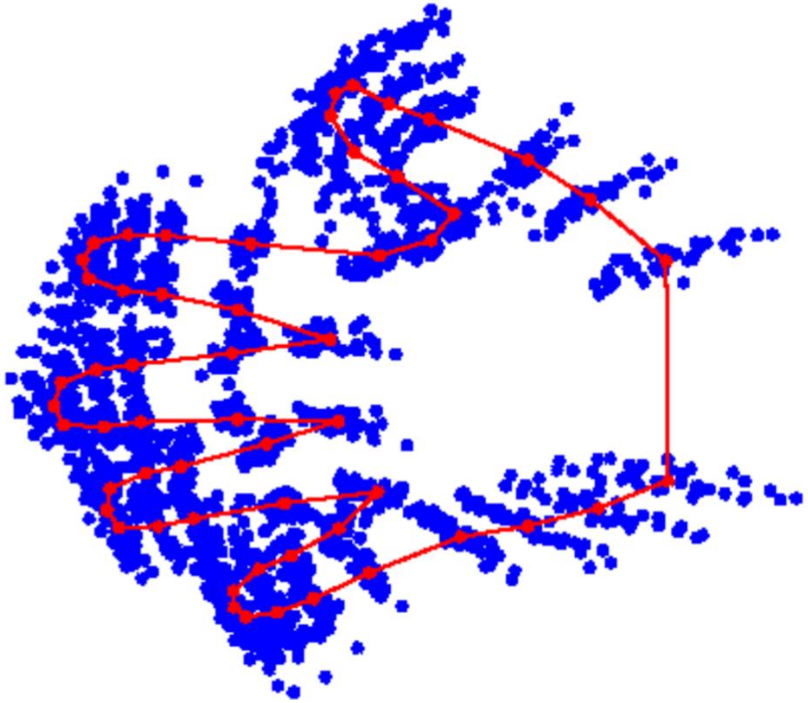
$$\theta = tan^{-1}\left(\frac{\sum_{i=1}^{k}(x_i\bar{y}_i - y_i\bar{x}_i)}{\sum_{i=1}^{k}(x_i\bar{x}_i - y_i\bar{y}_i)}\right)$$

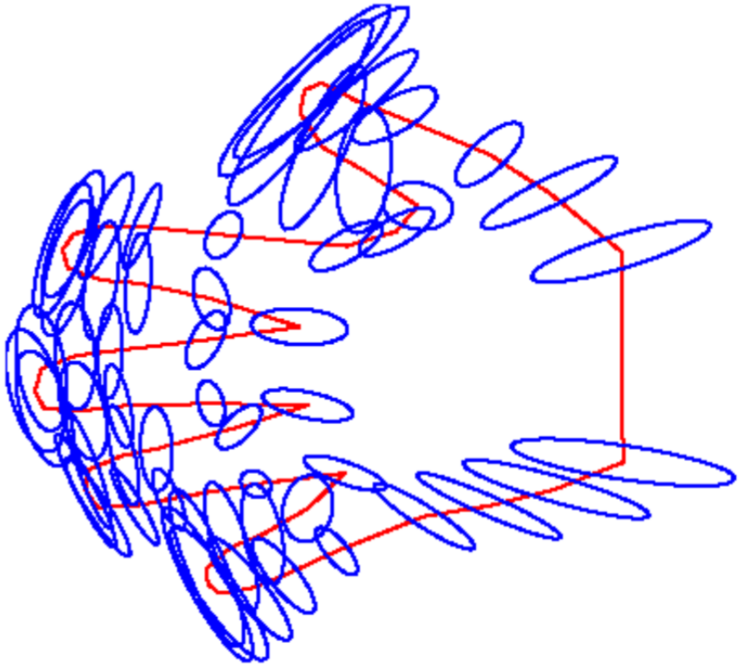
Training points before and after procrustes analysis











Algo2: Statistical Shape Analysis

- Now we have a set of aligned training shapes. Each is described by a 2k-vector of feature point.
- We want to remove the dimensionality of this set to a number n << k.
- The full hand shape model is then built by computing PCA on all model points collectively, which captures correlations of motion between points.

Perform PCA

- Compute mean of data
- Compute covariance of data
- Compute eigenvalues and eigenvector
- Each eigenvalue gives variance of data in the direction of the related eigenvector.
- Choose the largest eigenvalues to account for p% of the total variance.

Shape modes

- For shape synthesis, we will use a linear model.

$n \ll 2k$

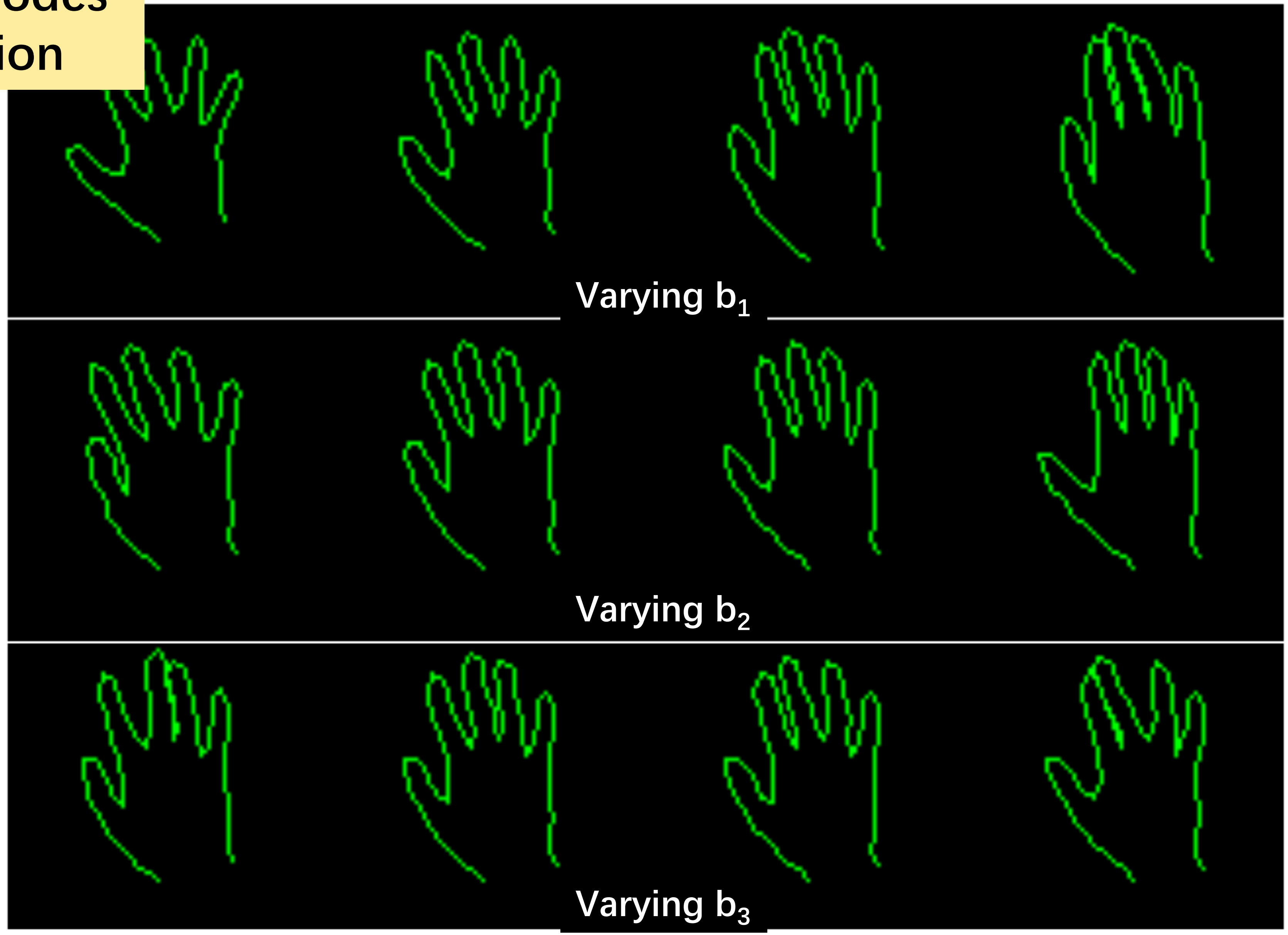
95%

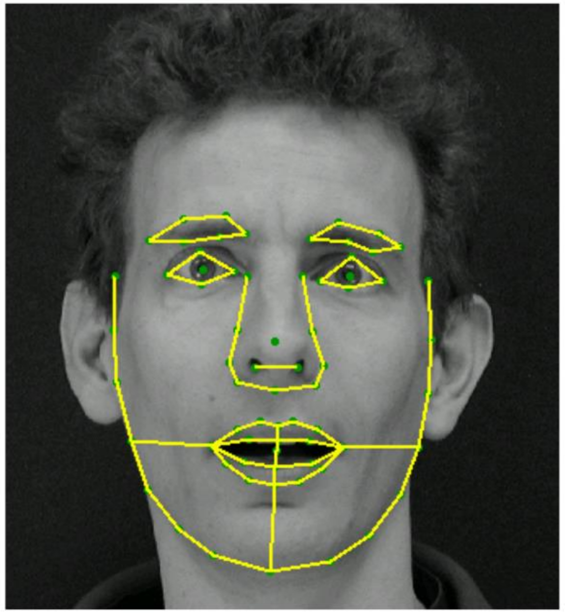
μ $\quad \lambda_k, \ V_k \quad 2k \times 1$

$$\underbrace{X^k}_{2k\times 1} = \mu + \underbrace{b_1}_{1\times 1} \underbrace{V_1}_{2k\times 1} + b_2 V_2 + \cdots b_n V_n$$

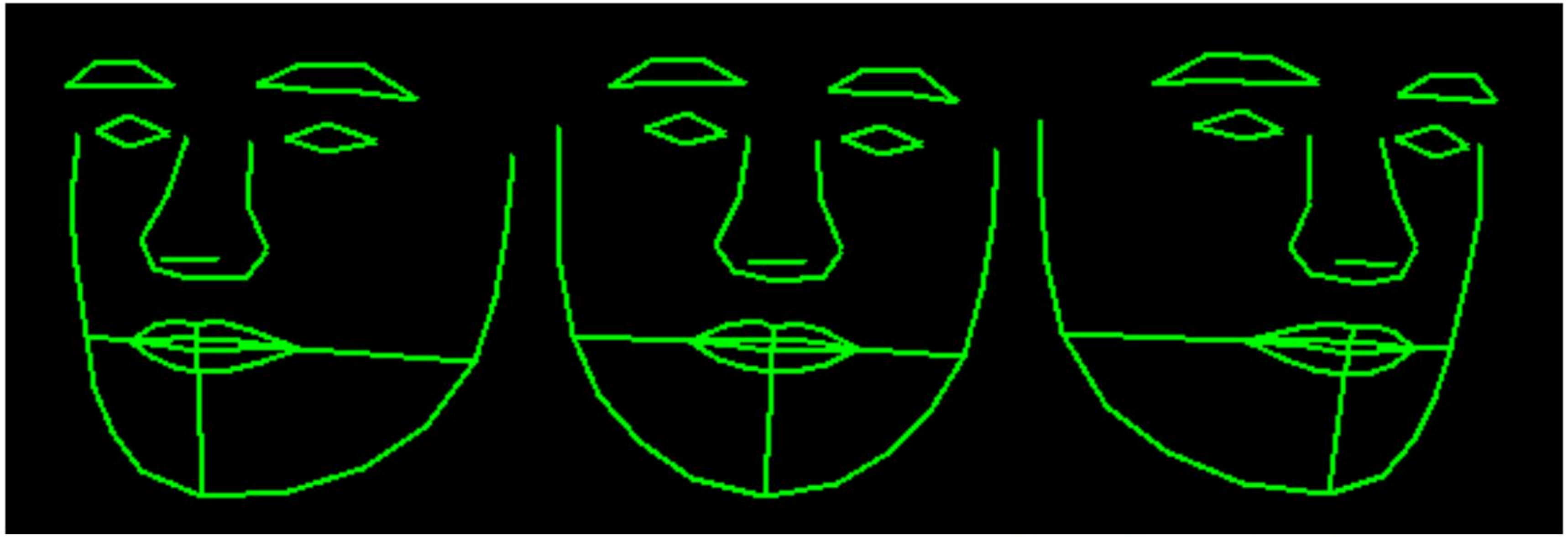
上海科技大学 ShanghaiTech University

First 3 modes of variation





Sample face training image



1st mode of variation

Fitting the curve to new data

- Assume we have a $2k$-vector Y, we want to fit the model: i.e. best translation, rotation, scaling (t, s, θ) and model parameter b.

$$min\|Y - M(\mu + Pb)\|^2$$

$$M(x) = s\begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix} x + t$$

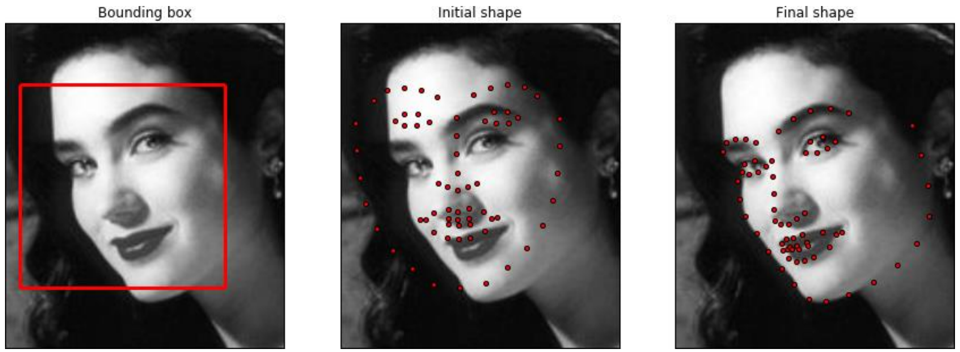
Algo3: Matching model to target

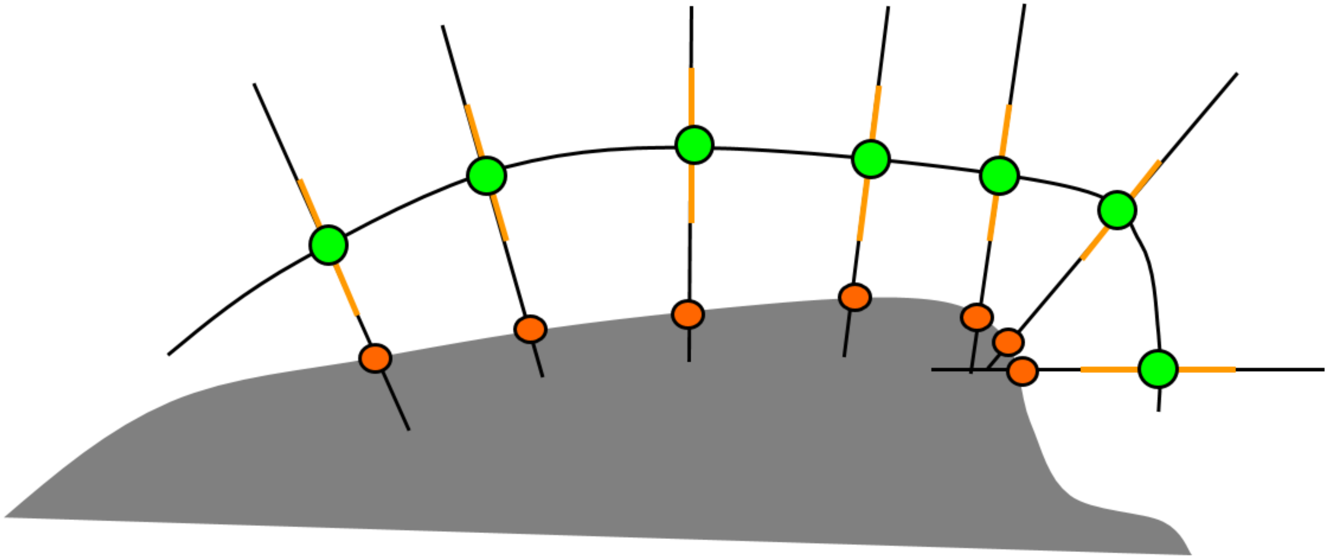
$min\|Y - M(\mu + Pb)\|^2$

- 1. Initialize $b = \mathbf{0}$;
- 2. Generate model points $\mathrm{x} = \mu + Pb$;
- 3. Find (t, s, θ) to best fit Y to X (like in Procrustes analysis)- gives a new Y;
- 4. Project Y into X space $y = M^{-1}Y$;
- 5. Update model parameter $b = P^T(y - \mu)$
- 6. Go to step 2, iterate until convergence.

ASM Search Overview

- How do we know what image point Y should belong to the model?





上海科技大学
ShanghaiTech University

Matching model to target

- Initialize $b = 0, \mathrm{x} = \mu + Pb$;
- Search around each x_i for best nearby image point y_i.
- Algo3: Fit new parameters (t, s, θ, b) to y_i.
- Enforce constraint $|b_i| < 3\sqrt{\lambda_i}$, so that shapes are "reasonable".
- Iterate until convergence.