

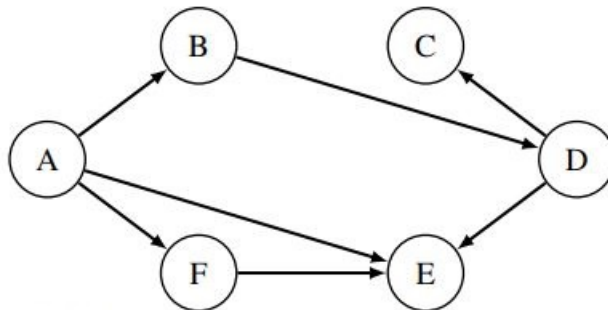
**Problem 1(2+3pts):** Consider a disjoint set with both path compression and union-by-size optimization. When two trees have the same height, the set specified first in the union will be the root of the merged set. The following operations are done:

$union(A, D), find(A), union(C, B), union(E, D), union(F, G), union(C, A), find(B)$

Please write down the result for each *find* operation appeared in the above operations, and draw the disjoint set tree after all operations above is finished.

**Problem 2(3pts):**

Run BFS the following graph starting with vertex A. Please write down the result. Whenever there is a choice of which node to visit next, follow the alphabetical order.



**Problem 3(7pts):** Maze

There is a maze stored in a  $m \times n$  matrix A. If there is an obstacle at point (i, j),  $A[i][j]=1$ ; otherwise  $A[i][j]=0$ .

You're at point (1, 1), and you want to go to (m, n).

	1	2	3	4	5
1	You				
2	1				
3	2				
4	3		7	8	9
5	4	5	6		Exit

Please design an algorithm to find out the shortest path from (1, 1) to (m, n). Briefly explain your algorithm using natural language. For the provided example, the result is 10.