Name: __________ Student ID:__________

Signals and Systems Homework 9

Due Time: 21:59 May 18, 2018
Submitted in-class on Thu (May 17),
or to the box in front of SIST 1C 403E (the instructors office).

1. Let

$$g(t) = x(t) + \alpha x(-t)$$

where

$$x(t) = \beta e^{-t}u(t)$$

and the Laplace transform of $g(t)$ is

$$G(s) = \frac{s}{s^2 - 1}, \qquad -1 < Re\{s\} < 1$$

Determine the values of the constants α and β.

2. Consider two right-sided signals $x(t)$ and $y(t)$ related through the differential equations

$$\frac{dx(t)}{dt} = -2y(t) + \delta(t)$$

and

$$\frac{dy(t)}{dt} = 2x(t)$$

Determine $Y(s)$ and $X(s)$, along with their regions of convergence.

3. Consider an LTI system for which the system function $H(s)$ is rational and has the pole-zero pattern shown in the following figure.

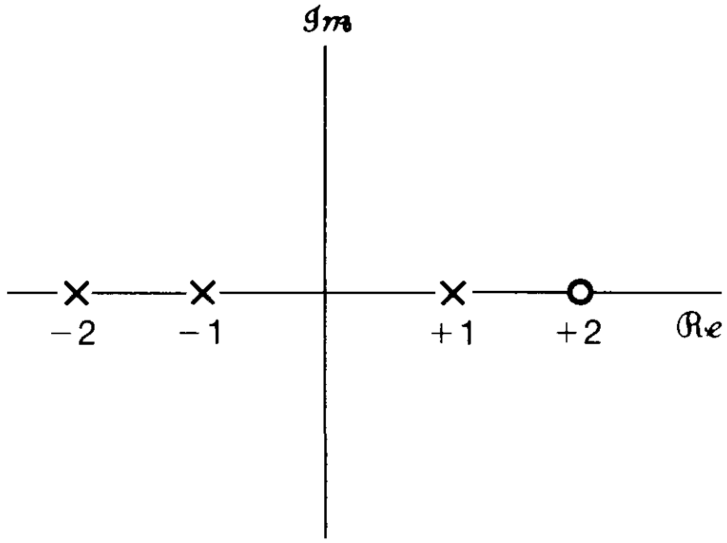

Figure 1:

(a) Indicate all possible ROCs that can be associated with this pole-zero pattern.

(b) For each ROC identified in part (a), specify whether the associated system is stable and/or causal.

4. Consider an LTI system with input $x(t) = e^{-t}u(t)$ and impulse response $h(t) = e^{-2t}u(t)$.

(a) Determine the Laplace transforms of $x(t)$ and $h(t)$.

(b) Using the convolution property, determine the Laplace transform $Y(s)$ of the output $y(t)$.

(c) From the Laplace transform of $y(t)$ as obtained in part (b), determine $y(t)$.

(d) Verify your result in part (c) by explicitly convolving $x(t)$ and $h(t)$.

5. Consider a continuous-time LTI system for which the input $x(t)$ and output $y(t)$ are related by the differential equation

$$\frac{d^2y(t)}{dt^2} - \frac{dy(t)}{dt} - 2y(t) = x(t)$$

Let $X(s)$ and $Y(s)$ denote Laplace transforms of $x(t)$ and $y(t)$, respectively, and let $H(s)$ denote the Laplace transform of $h(t)$, the system impulse response.

(a) Determine $H(s)$ as a ratio of two polynomials in s. Sketch the pole-zero pattern of $H(s)$.

(b) Determine $h(t)$ for each of the following cases:

1. The system is stable.
2. The system is causal.
3. The system is neither stable nor causal.

6. Suppose we are given the following information about a causal and stable LTI system S with impulse response $h(t)$ and a rational system function $H(s)$:

1. $H(1) = 0.2$
2. When the input is $u(t)$, the output is absolutely integrable.
3. When the input is $tu(t)$, the output is not absolutely integrable.
4. The signal $d^2h(t)/dt^2 + 2dh(t)/dt + 2h(t)$ is of finite duration.
5. $H(s)$ has exactly one zero at infinity.

Determine $H(s)$ and its region of convergence.

7. Consider a stable and causal system with a real impulse response $h(t)$ and system function $H(s)$. It is known that $H(s)$ is rational, one of its poles is at $-1 + j$, one of its zeros is at $3 + j$, and it has exactly two zeros at infinity. For each of the following statements, determine whether it is true, whether it is false, or whether there is insufficient information to determine the statement's truth.

(a) $h(t)e^{-3t}$ is absolutely integrable.

(b) The ROC for $H(s)$ is $Re(s) > -1$.

(c) The differential equation relating inputs $x(t)$ and outputs $y(t)$ for S may be written in a form having only real coefficients.

(d) $\lim_{s\to\infty} H(s) = 1$.

(e) $H(s)$ has no fewer than four poles.

(f) $H(jw) = 0$ for at least one finite value of w.

(g) If the input to S is $e^{3t}\sin t$, the output is $e^{3t} cost$.