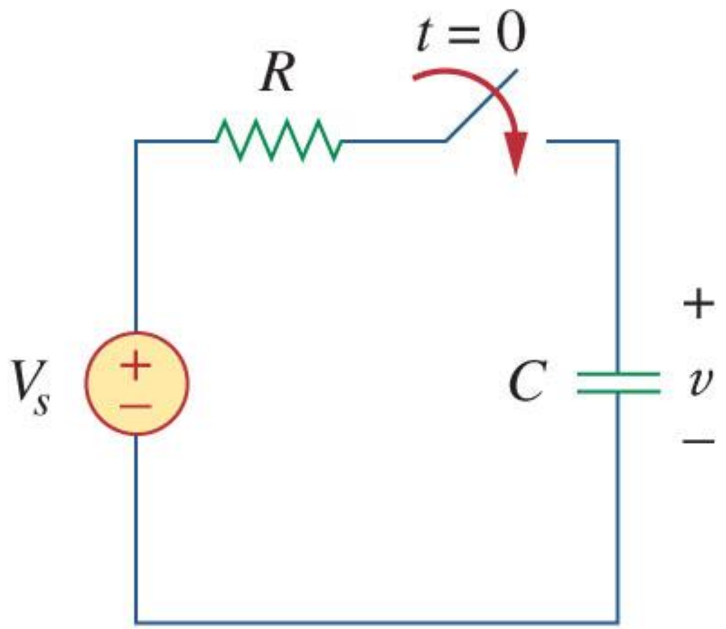
Outline

- Capacitors and inductors
- Natural response of RC/RL circuits
- Step response of RC/RL circuits
- Others

Step Response of RC Circuit

- When a DC source is suddenly applied to a RC circuit, the circuit response is known as the step response.

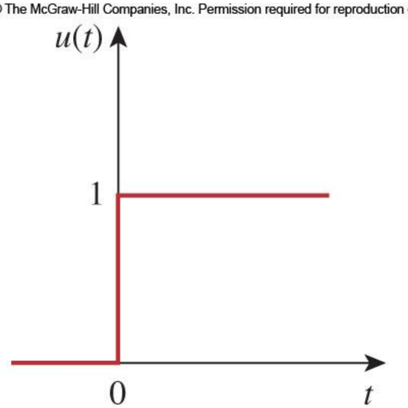


(a)

$$v(0^-) = v(0^+) = v_0$$

The Unit Step *u(t)*

- A step function is one that maintains a constant value before a certain time and then changes to another constant afterwards.



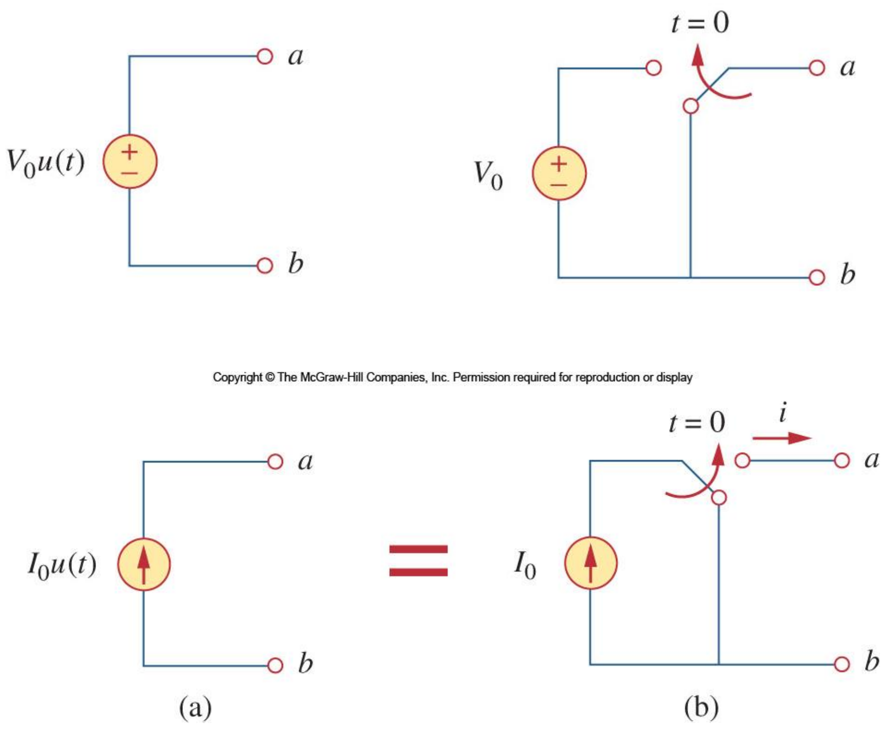
$$u(t) = \begin{cases} 0, & t < 0 \\ 1, & t > 0 \end{cases}$$

switching time may be shifted to $t = t_0$ by

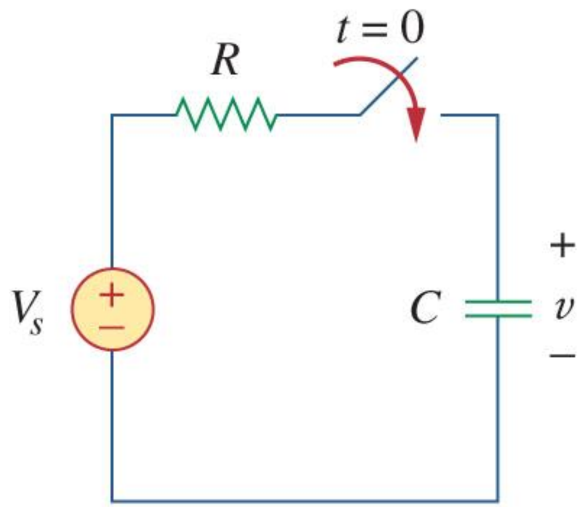
$$u(t - t_0) = \begin{cases} 0, & t < t_0 \\ 1, & t > t_0 \end{cases}$$

Equivalent Circuit of Unit Step

- The unit step function has an equivalent circuit to represent when it is used to switch on a source.

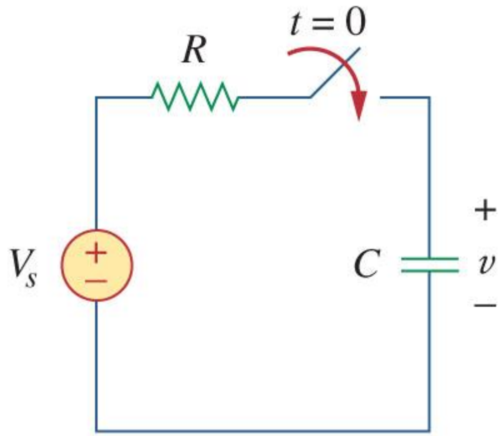


Step Response of the RC Circuit



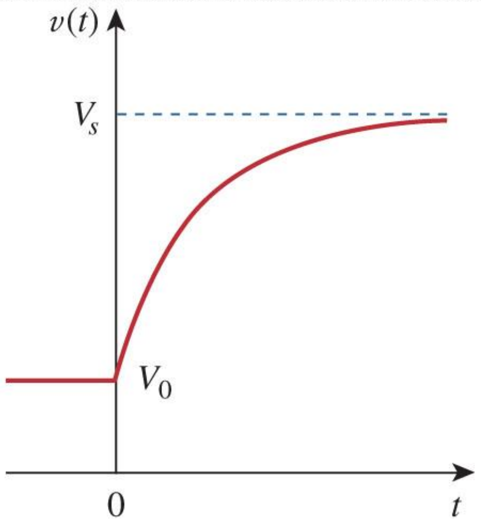
$$v(0^-) = v(0^+) = V_0$$

Step Response of the RC Circuit



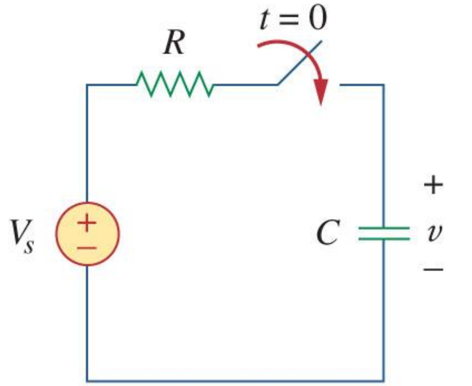
$$v(0^-) = v(0^+) = V_0$$

$$v(t) = \begin{cases} V_0, & t < 0 \\ V_s + (V_0 - V_s)e^{-t/\tau} & t > 0 \end{cases}$$



- This is known as the complete response, or total response.

Complete response



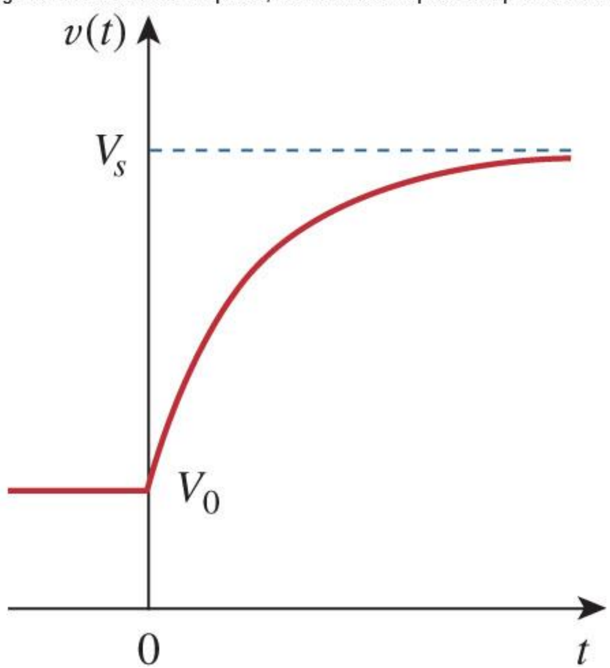
$$v(0^-) = v(0^+) = v_0$$

- The complete response

$$v(t) = \begin{cases} V_0, & t < 0 \\ V_s + (V_0 - V_s)e^{-t/\tau} & t > 0 \end{cases}$$

can be written as:

$$\boxed{v = v_n + v_f}$$



$$\text{Complete response} = \underbrace{\text{natural response}}_{\text{stored energy}} + \underbrace{\text{forced response}}_{\text{independent source}}$$

or

$$v = v_n + v_f$$

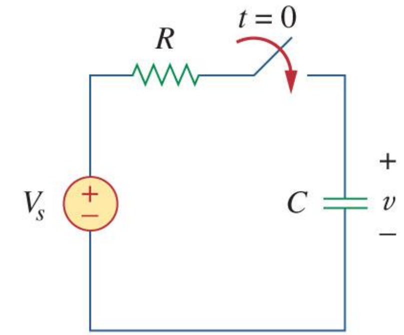
where

$$v_n = V_o e^{-t/\tau}$$

and

$$v_f = V_s(1 - e^{-t/\tau})$$

Another Perspective



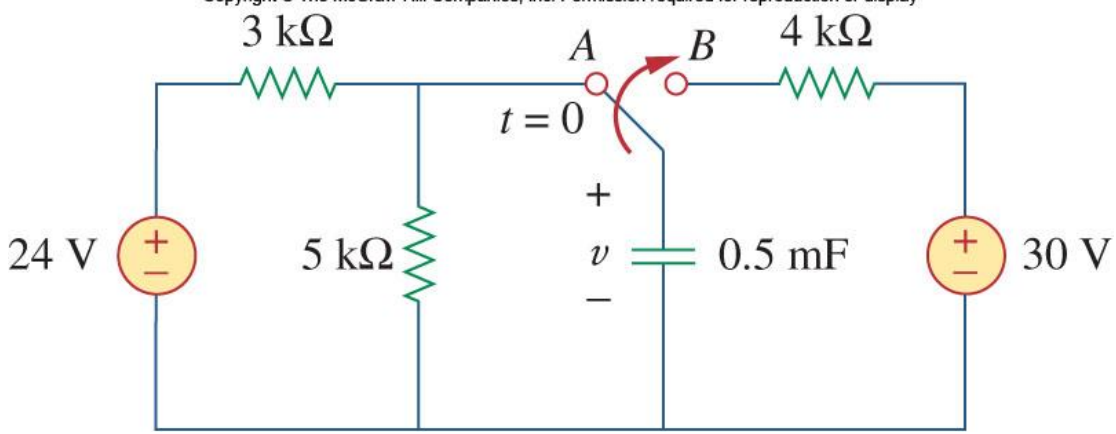
$$v(t)=\begin{cases} V_0, & t<0 \\ V_s+(V_0-V_s)e^{-t/\tau} & t>0 \end{cases}$$

- Another way to look at the response is to break it up into the <u>transient response</u> and the <u>steady state response</u>:

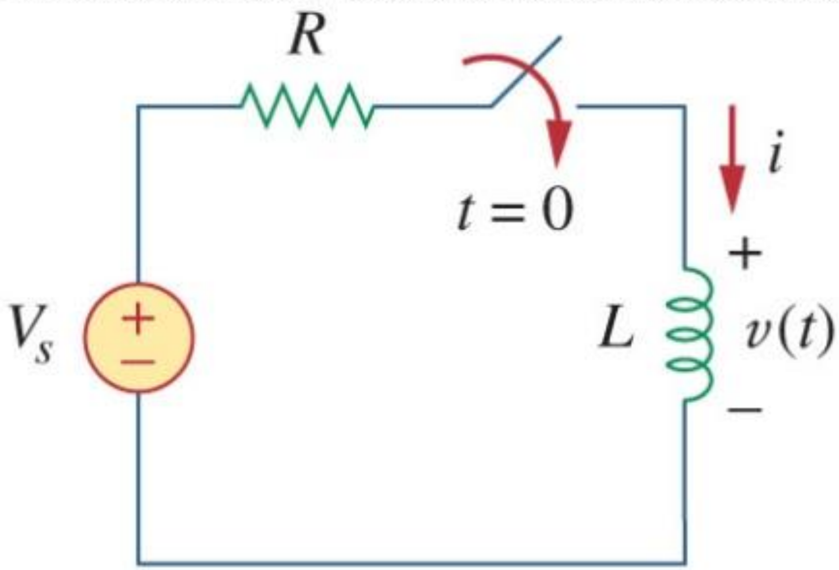
$$v(t) = \underbrace{v(\infty)}_{\text{steady } v_{ss}} + \underbrace{[v(0)-v(\infty)]e^{-t/\tau}}_{\text{transient } v_t}$$

Example

- The switch has been in position A for a long time. At $t = 0$, the switch moves to B. Find $v(t)$.

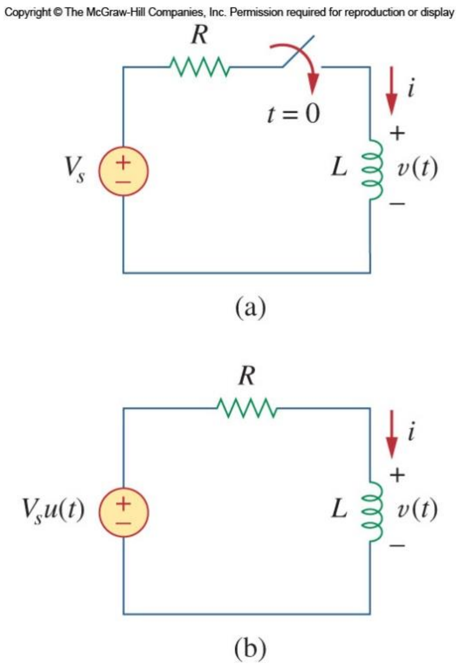


Step Response of the RL Circuit



$$i(0^+) = i(0^-) = I_0$$

Step Response of the RL Circuit



- We will use the transient and steady state response approach.
- We know that the transient response will be an exponential:

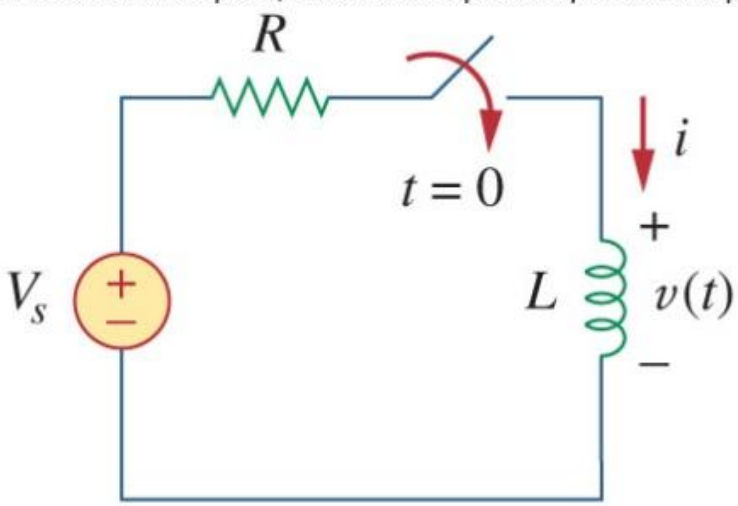
$$i_t = Ae^{-t/\tau}$$

- After a sufficiently long time, the current will reach the steady state:

$$i_{ss} = \frac{V_s}{R}$$

Step Response of RL Circuit

- This yields an overall response of:



$$i = Ae^{-t/\tau} + \frac{V_s}{R}$$

$$i\left(0^+\right) = i\left(0^-\right) = I_0 \qquad A = I_0 - \frac{V_s}{R}$$

$$i\left(t\right) = \frac{V_s}{R} + \left(I_0 - \frac{V_s}{R}\right)e^{-t/\tau}$$

General Procedure of Finding RC/RL Response with D.C. sources

1. **Identify the variable of interest**
   - For RL circuits, it is usually the inductor current $i_L(t)$.
   - For RC circuits, it is usually the capacitor voltage $v_c(t)$.
2. **Determine the initial value of the variable at t_0**
   - Recall that $i_L(t)$ and $v_c(t)$ are continuous variables:

$$i_L(t_0^+) = i_L(t_0^-) \quad \text{and} \quad v_c(t_0^+) = v_c(t_0^-)$$

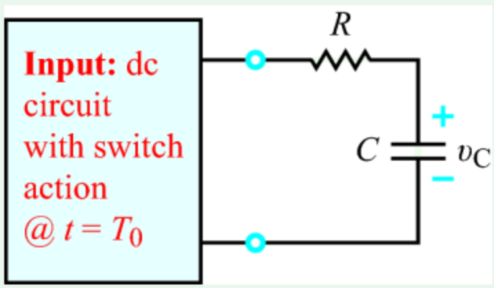
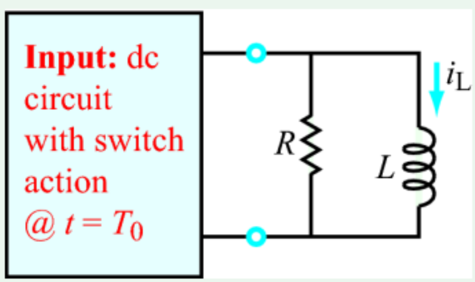
3. **Determine the final value of the variable** (as $t \to \infty$)

If needed, recall an inductor behaves like a short circuit in steady state ($t \to \infty$) & that a capacitor behaves like an open circuit in steady state ($t \to \infty$).

4. **Calculate the time constant for the circuit**
   - $\tau = CR$ for an RC circuit where R is the Thévenin equivalent resistance "seen" by the capacitor.
   - $\tau = L/R$ for an RL circuit, where R is the Thévenin equivalent resistance "seen" by the inductor.

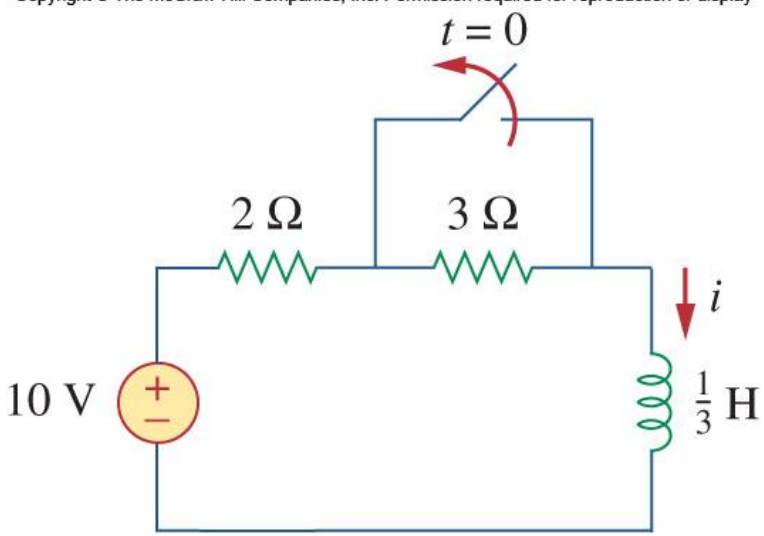
[Source: Berkeley]

Response Form of Basic First-Order Circuits

Circuit	Diagram	Response
RC		$v_C(t) = \left\{ v_C(\infty) + [v_C(T_0) - v_C(\infty)]e^{-(t-T_0)/\tau} \right\}$ $(\tau = RC)$
RL		$i_L(t) = \left\{ i_L(\infty) + [i_L(T_0) - i_L(\infty)]e^{-(t-T_0)/\tau} \right\}$ $(\tau = L/R)$

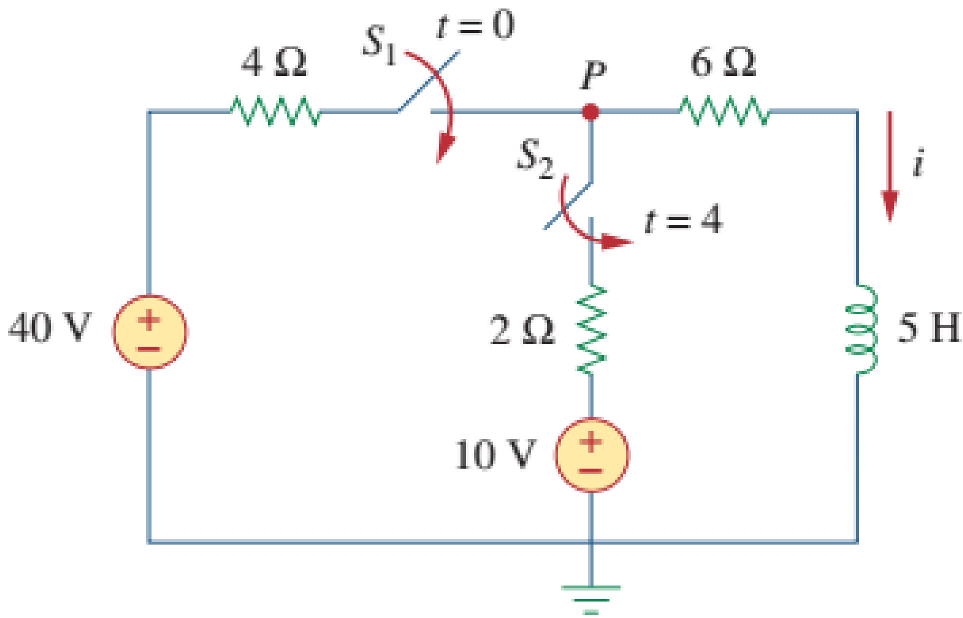
Example

- Find $i(t)$ in the circuit for $t > 0$. Assume that the switch has been closed for a long time.



Sequential switch

At $t = 0$, switch 1 in Fig. 7.53 is closed, and switch 2 is closed 4 s later. Find $i(t)$ for $t > 0$. Calculate i for $t = 2$ s and $t = 5$ s.



We need to consider the three time intervals $t \leq 0$, $0 \leq t \leq 4$, and $t \geq 4$ separately. For $t < 0$, switches S_1 and S_2 are open so that $i = 0$. Since the inductor current cannot change instantly,

$$i(0^-) = i(0) = i(0^+) = 0$$

Practice