Homework 5

Due date: Nov. 25th, 2021

Turn in your homework in class

Rules:

- Work on your own. Discussion is permissible, but extremely similar submissions will be judged as plagiarism.
- Please show all intermediate steps: a correct solution without an explanation will get zero credit.
- Please submit on time. No late submission will be accepted.
- Please prepare your submission in English only. No Chinese submission will be accepted.
- If needed, round the number to the nearest hundredths, i.e., rounding it to 2 decimal places.

1. (a) For the following pairs of sinusoids, determine which one leads and by how much.

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(a) $v(t) = 10\cos(4t - 60°)$ and
$i(t) = 4\sin(4t + 50°)$

(b) $v_1(t) = 4\cos(377t + 10°)$ and
$v_2(t) = -20\cos 377t$

(c) $x(t) = 13\cos 2t + 5\sin 2t$ and
$y(t) = 15\cos(2t - 11.8°)$

(b) Transform the following sinusoids into phasors:

(a) $-20\cos(4t + 135°)$ (b) $8\sin(20t + 30°)$

(c) $20\cos(2t) + 15\sin(2t)$

a)

(a) $i(t) = 40\cos(4t - 40°)$ i(t) leads v(t) $20°$ 2

(b) $V_2(t) = 20\cos(377t + 180°)$

V_2 leads V_1 $170°$ 2

(c) $x(t) = \sqrt{194}\cos(2t - 21.04°)$ y(t) leads x(t) $9.24°$ 2

$= 13.928\cos(2t - 21.04°)$

b)

2 2

(a) $20\angle -45°$ $w = 4$ rad/s (b) $8\angle -60°$ $w = 20$ rad/s

2

(c) $25\cos(2t - 36.87°) = 25\angle -36.87°$ $w = 2$ rad/s

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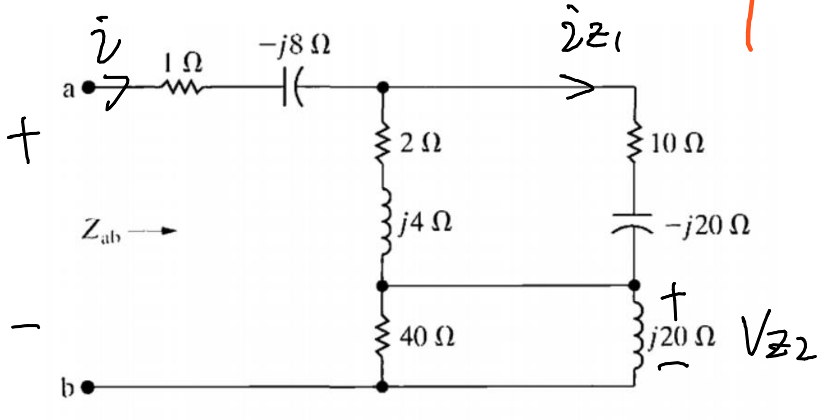
2. For the circuit below:
   1) Calculate the equivalent impedance Z_{ab};
   2) If $V_{ab} = 20\ sin\ (5t + 105°)$,

      i. Calculate current through the 10 Ω resistor, and indicate the reference direction in the circuit diagram;
      ii. Calculate voltage over the j20 Ω inductor and indicate the reference direction in the circuit diagram.



1

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1) $Z_{ab} = 1 - 8j + (2+4j)//(10-20j) + (40)//(20j)$

$= 1 - 8j + 3 + 4j + 8 + 16j = 12 + 12j\ \Omega$ 2

2) $V_{ab} = 20\angle 15°\ V$ 2

The current through the source is

$$i = \frac{V_{ab}}{Z_{ab}} = 1.17\ \angle -30° A$$

$= 1.02 - 0.59j\ A$ 2

2

$i(t) = 1.17\cos(5t - 30°) A$

voltage over $(10-20j)\Omega$ is $V_{z1} = i \cdot (3+4j) = 5.89\angle 23.13° V = (5.41 + 2.31j)V$

$$i_{z1} = \frac{V_{z1}}{10-20j} = (0.02 + 0.26j) A = 0.26\ \angle 86.57° A$$ 2

3) $V_{z2} = i \cdot (8+16j) = 17.59 + 11.62j\ V$

$= 21.08\angle 33.43° V$

$V(t) = 21.08\cos(5t + 33.43°) V$ 2

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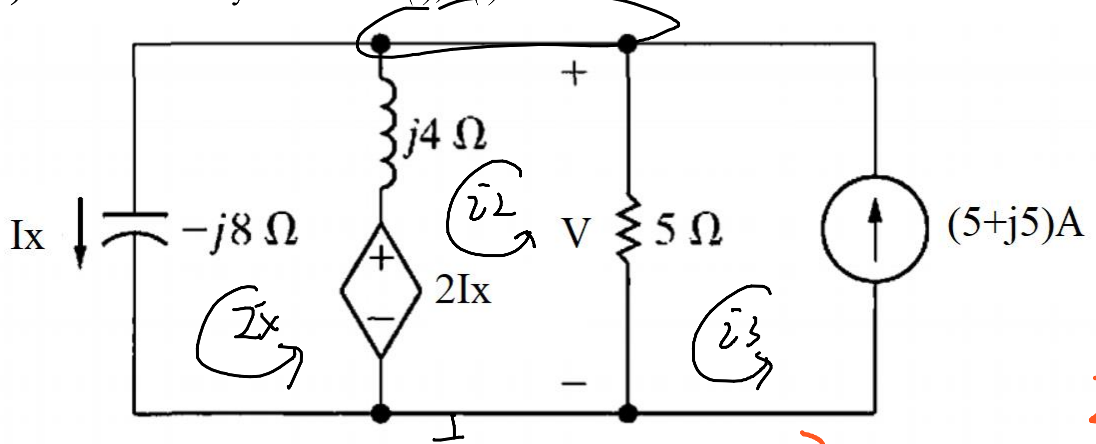
3. For the circuit below, given ω= 2 rad/s
   1) Use nodal analysis to find *Ix(t), V(t)*
   2) Use mesh analysis to find *Ix(t), V(t)*



1) KCL:

$$\begin{cases} 5+5j = I_x + \dfrac{V-2I_x}{4j} + \dfrac{V}{5} \\ I_x = \dfrac{V}{-8j} \end{cases} \Rightarrow \begin{cases} I_x = -4.75+0.23j\,A = 4.76\angle 177.27^\circ A \\ V = 1.81+38.01j\,V = 38.05\angle 87.27^\circ V \end{cases}$$

2 2 2 2

2) KVL:

$$\begin{cases} I_x\cdot(-8j) - 2I_x + 4j\cdot(I_x - i_2) = 0 \\ 4j\cdot(i_2 - I_x) + 2I_x + 5\cdot(i_2 - i_3) = 0 \\ i_3 = 5+5j\ A \end{cases} \Rightarrow \begin{cases} I_x = -4.75+0.23j\,A \\ = 4.76\angle 177.27^\circ A \\ V = 1.81+38.01j\,V \\ = 38.05\angle 87.27^\circ V \end{cases}$$

2 2 2 2 2

$I(t) = 4.76\cos(2t + 177.27^\circ)\,A$ 1

$V(t) = 38.05\cos(2t + 87.27^\circ)\,V$ 1

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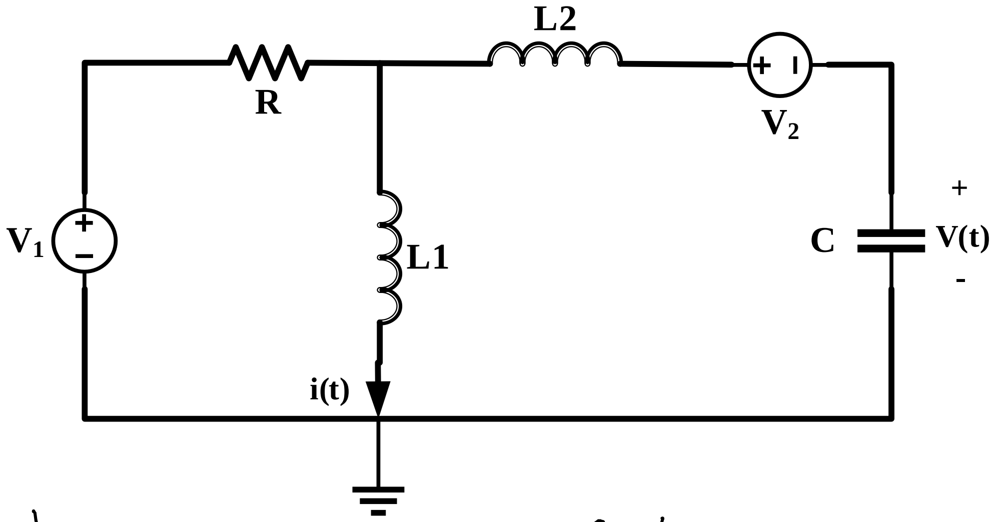
4. For the circuit below:

R=10Ω, L1=10mH, L2=20mH, C=200nF,

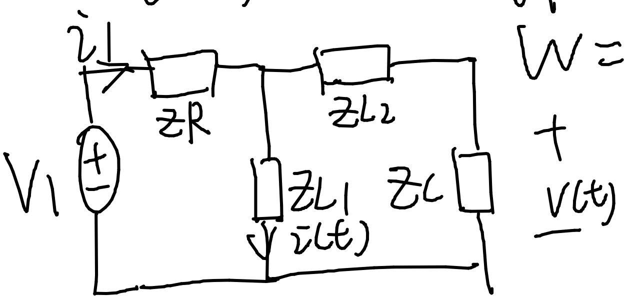
$V_1(t) = 10\sin(5t + 30°)$, $V_2(t) = 4\cos(5t)$

Use superposition theorem to solve the i(t) and V(t)



Only V1 : $V_1 = 10\angle -60° \, V$

$W = 5$



$$Z_{eq} = 10 + (j \cdot 5 \cdot 0.01) \parallel \left(\frac{1}{j \cdot 200 \cdot 10^{-9} \cdot 5} + j \cdot 5 \cdot 0.02 \right)$$

$$= 10 + 0.05j \ \Omega$$ 2

$$i_1 = \frac{V1}{Z_{eq}} = 0.50 - 0.87j \, A = 1.00\angle -60.29° \, A$$

$$V_{ZL1} = 0.05j \cdot i_1(t) = 0.04 + 0.02j \, V = 0.05\angle 29.72° \, V$$

$$i = \frac{V_{ZL1}(t)}{Z_{L1}} = 0.05 - 0.87j \, A = 1.00\angle -60.29° \, A$$ 2

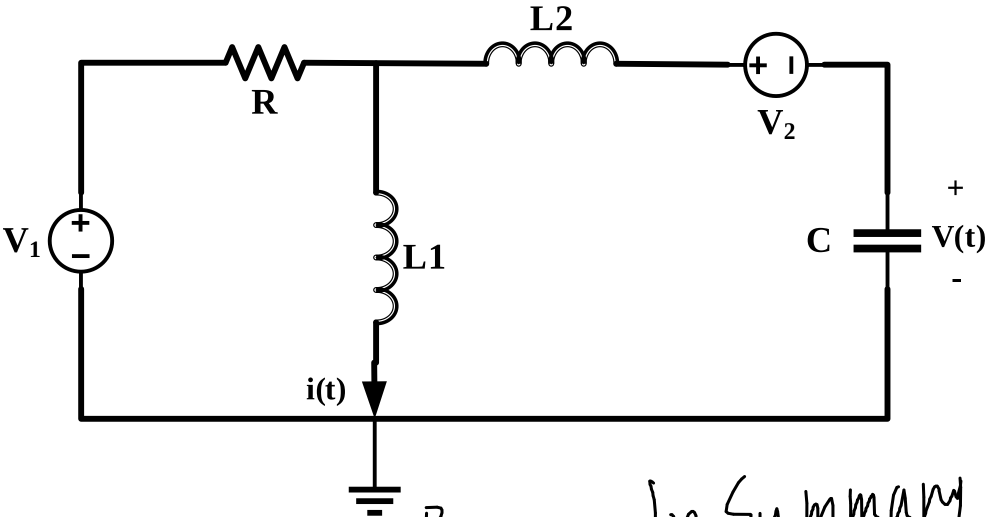
$$V = \frac{V_{ZL1} \cdot Z_C}{Z_C + Z_{L2}} = 0.04 + 0.02j \, V = 0.05\angle 29.72° \, V$$ 2

4. For the circuit below:

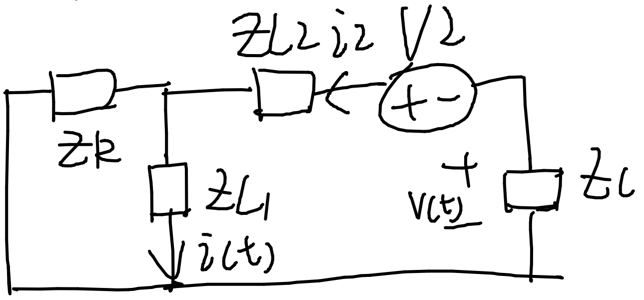
   R=10Ω, L1=10mH, L2=20mH, C=200nF,

   $V_1(t) = 10\sin(5t + 30°)$, $V_2(t) = 4\cos(5t)$

   Use superposition theorem to solve the i(t) and V(t)



Only V_2 $V_2(t) = 4\angle 0° V$



$Zeq = 10 // 0.05j + 0.1j - 10^6 j$

$= 2.5\times10^{-4} - 10^6 j\ \Omega$ 2

$i_2 = \frac{V_2}{zeq} = 10^{-15} + 4\times10^{-6} j\ A$

$V(t) = -i_2 \cdot Z_c = -4 + 10^{-9} j\ V$ 2

$V_{ZL1} = i(t) \cdot (10 // 0.05j)$

$= -2\times10^{-7} + 10^{9} j\ V$

$i(t) = \frac{V_{ZL1}}{Z_{L1}} = 2\times10^{-8} + 4\times10^{-6} j A$ 2

In Summary

$i = i_1 + i_2$

$= 0.496 - 0.869j\ A$

$V = V_1 + V_2$ 1

$= -3.96 + 0.02j\ V$

$i(t) = \cos(5t - 60.286°) A$

$V(t) = 3.96\cos(5t + 179.71°) V$

1

5. Find $i_o(t)$ by using superposition method.

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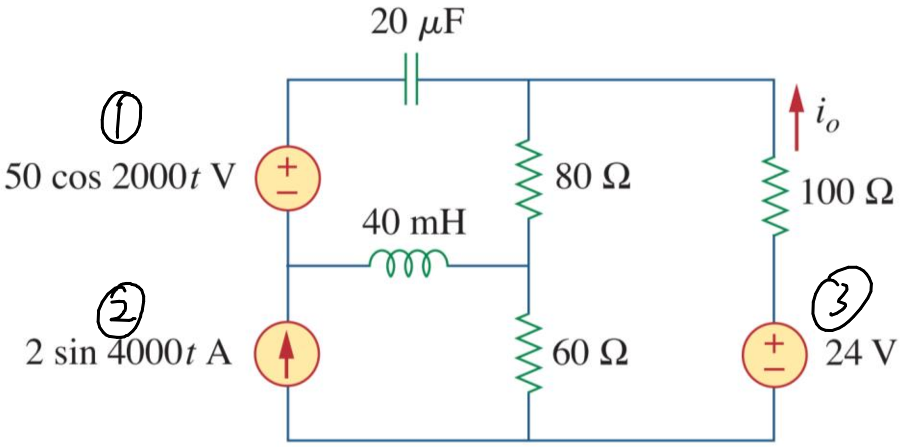

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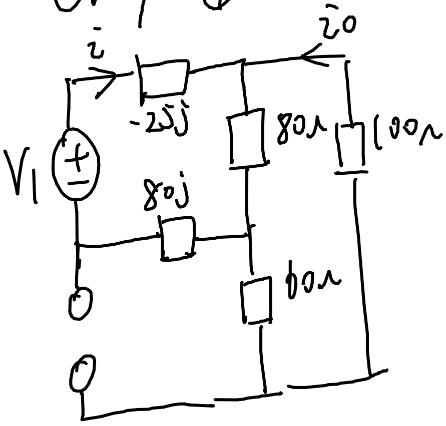


$Z_C = \frac{1}{j \cdot 2000 \cdot 20 \times 10^{-6}} = -25j \ \Omega$ 1

$Z_L = j \cdot 0.04 \cdot 2000 = 80j \ \Omega$ 1

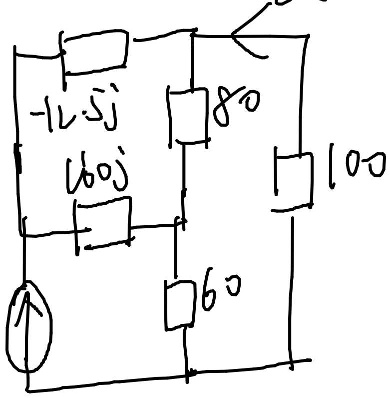
$Z_{eq} = -25j + 80j + 80 // 160 = 55j + \frac{160}{3} \Omega$ 2

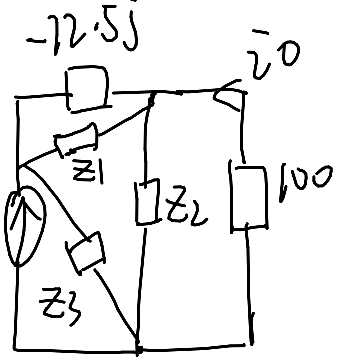
$i = \frac{V_1}{Z_{eq}} = 0.45 - 0.47j \ A = 0.65 \angle -45.88^\circ A$

$i_o = -\frac{1}{3} \cdot i = -0.15 + 0.16j \ A = 0.22 \angle 134.12^\circ A$

2

Only ②:





$Z_C = \frac{1}{j \cdot 20 \times 10^{-6} \cdot 4000} = -12.5j \ \Omega$ 1

$Z_L = j \cdot 0.04 \cdot 4000 = 160j$ 1

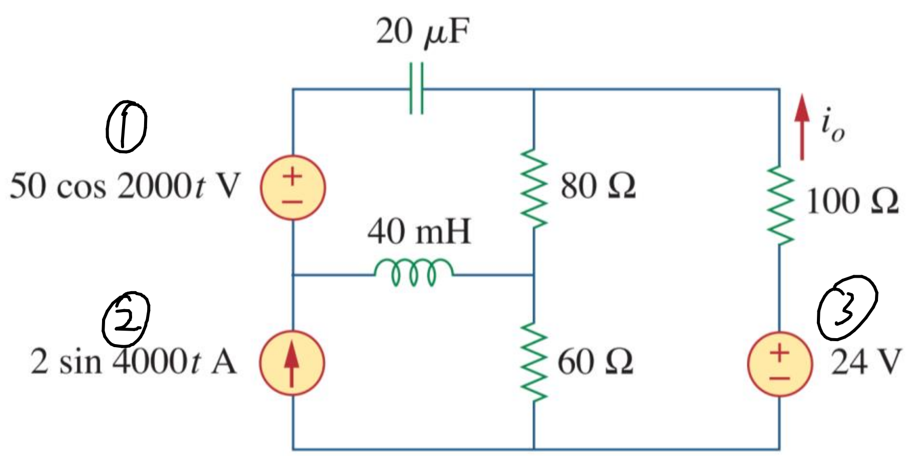
Y → Δ Transform : OR KCL KVL

$Z_1 = \frac{160j \cdot 80 + 80 \cdot 60 + 60 \cdot 160j}{60} = 80 + \frac{1120}{3} j \ \Omega$

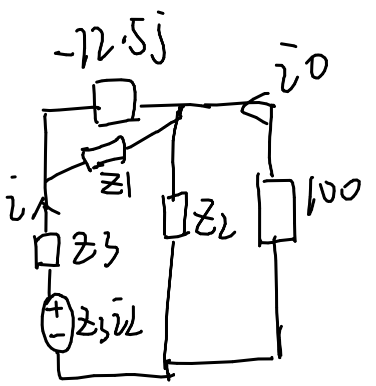
$Z_2 = \frac{4800 + 22400j}{160j} = 140 - 30j \ \Omega$

$Z_3 = \frac{4800 + 22400j}{80} = 60 + 280j$ 2

5. Find $i_o(t)$ by using superposition method.



Y -> D Transform:



$Z_1 = 80 + \frac{1120}{3} j \,\Omega$ $\quad Z_2 = 160 - 30j \,\Omega$ $\quad Z_3 = 60 + 280j$

Source Transformation: $V_2 = Z_3 \cdot i_2 = 560 - 120j$ V

$Z_{eq} = Z_3 + Z_1 // (-12.5j) + Z_2 // 100 = 106.06 + 274.78j \,\Omega$

2

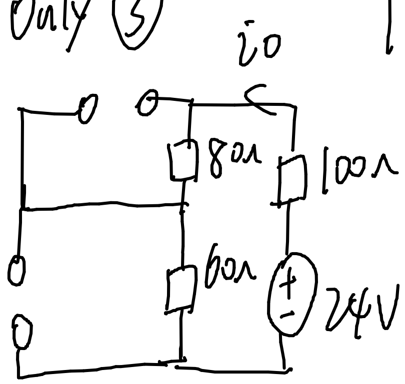
$i = \frac{V_2}{Z_{eq}} = 0.30 - 1.92j$ A

$V_{Z2} = (Z_2 // 100) \cdot i = 8.11 - 114.82j$ V

$i_0 = -\frac{V_{Z2}}{100} = -0.08 + 1.15j$ A $= 1.15 \angle 94.04°$ A

2

Only ③



In Steady state:

$i_0 = 0.1$A

2

Superposition: $i_0(t) = 0.1 + 1.15 \cos(4000t + 94.04°)$
$+ 0.22 \cos(2000t + 134.12°)$ A

2

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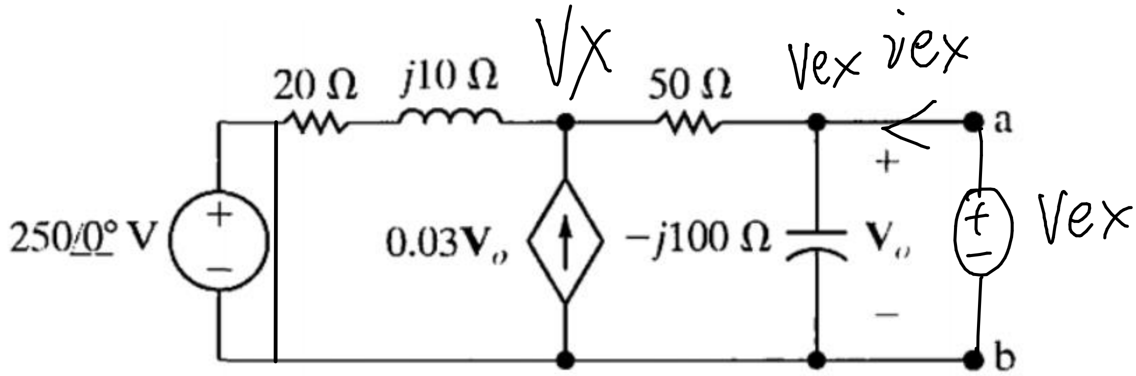
6. For the circuit below. The circuit is working in sinusoidal, single frequency (ω= 2 rad/s), and steady state.
   1) Find the Thevenin AND Norton equivalent circuit at the terminals a and b.
   2) Consider an inductor L=5H is connected to the terminal a and b. Find the current through L $i_L(t)$ and indicate the reference direction in the circuit diagram.



1) KCL: $\dfrac{250 - V_x}{20+10j} + 0.03 \cdot V_o = \dfrac{V_x - V_o}{50} = \dfrac{V_o}{-100j}$ 2

$V_x = (1+0.5j)V_o$

$10-5j - 0.05V_o + 0.03V_o = 0.01j\, V_o$ 2

$V_{Th} = V_o = \dfrac{10-5j}{0.01j+0.02} V = 300-400j\ V = 500\angle -53.13^\circ\ V$

Z_{Th}: Apply External Voltage Vex and KCL

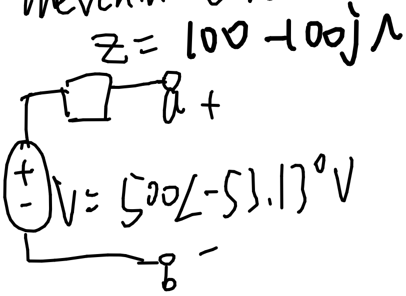
$$\begin{cases} \dfrac{V_{ex}-V_x}{50} + 0.03V_o = \dfrac{V_x}{20+10j} & 2 \\ i_{ex} = \dfrac{V_{ex}}{-100j} + \dfrac{V_{ex}-V_x}{50} & 2 \Rightarrow \\ V_{ex} = V_o & 2 \end{cases}$$

$V_{Th} = 500\angle -53.13^\circ V$ 1

$i_n = 3.536\angle -8.13^\circ A$ 1

$Z_{Th} = 100-100j\ \Omega$ 1

Thevenin Circuit



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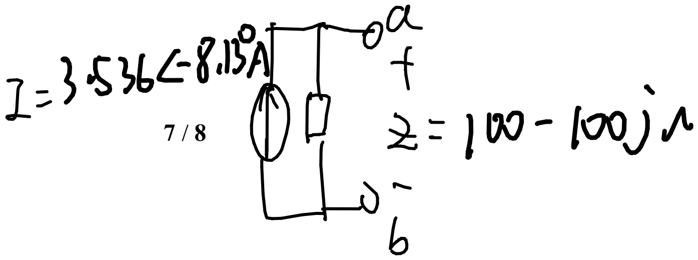
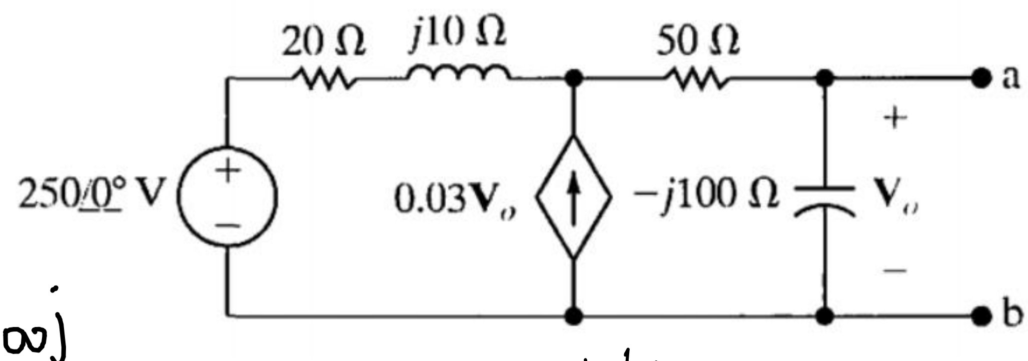
Norton Circuit



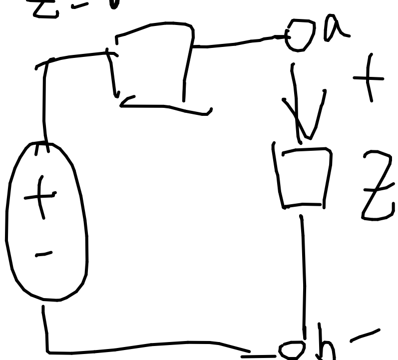
1

6. For the circuit below. The circuit is working in sinusoidal, single frequency (ω= 2 rad/s), and steady state.
   1) Find the Thevenin AND Norton equivalent circuit at the terminals a and b.
   2) Consider an inductor L=5H is connected to the terminal a and b. Find the current through L $i_L(t)$ and indicate the reference direction in the circuit diagram.



2)

$z = 100 - 100j$



$V = 500\angle -53.13^\circ$ V

1

$$i_L = \frac{V_{Th}}{Z_{Th} + Z_L}$$

$$= \frac{500\angle -53.13^\circ}{j \cdot 2 \cdot 5 + 100 - 100j}$$

$$= 3.646 - 0.718j \text{ A} = 3.716\angle -11.14^\circ \text{ A}$$

$$i_L(t) = 3.716 \cos(2t - 11.14^\circ) \text{ A}$$

2

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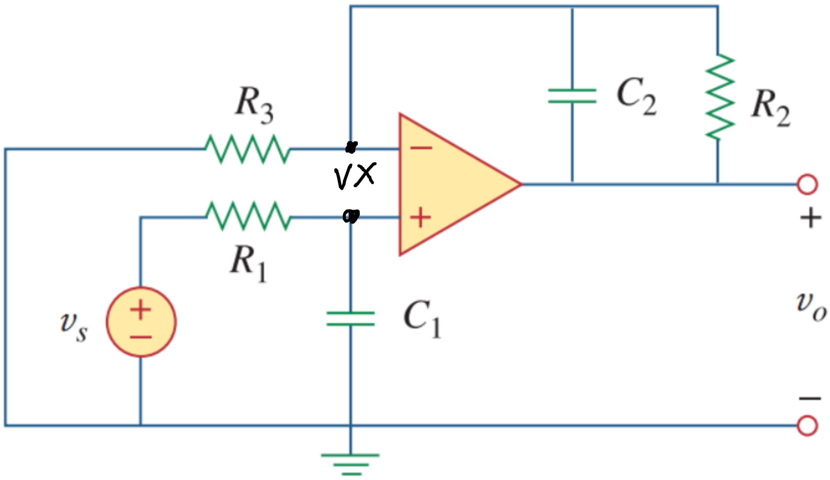
7. For the circuit below. Suppose v_s is a sinusoidal voltage source with the angular frequency ω. Suppose the Op-amp is working in the linear mode. Find the expression for v_o/v_s.



$$\frac{V_s - V_x}{R_1} = V_x \cdot j w C_1$$ 2

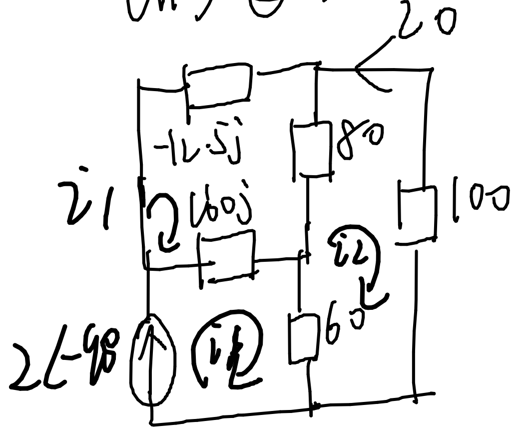
$$\frac{V_o - V_X}{\left(\frac{1}{jwC_2}\right) \| R_2} = \frac{V_X}{R_3}$$ 2

$$V_X = \frac{V_s}{1 + R_1 w C_1 j}$$

$$V_o = 1 + \frac{R_2}{R_3 (R_2 j w C_2 + 1)} V_X$$

$$\frac{V_o}{V_s} = \frac{V_o}{V_X} \cdot \frac{V_X}{V_s} = \frac{1 + \frac{R_2}{R_3(R_2 C_2 w j + 1)}}{1 + R_1 w C_1 j}$$ 2

Only ②:



$$\begin{cases} i_1 \cdot (-12.5j) + (i_1 - i_2)\, 80 + (i_1 + 2j) \cdot 160j = 0 \\ (i_2 - i_1)\, 80 + (i_2 \cdot 100) + (i_2 + 2j)\, 60 = 0 \end{cases}$$

$i_0 = -i_2$

$$\begin{cases} (80 + 147.5j)\, i_1 + (-80)\, i_2 = 320 \\ -80 i_1 + 240\, i_2 = -120j \end{cases}$$

$i_2 =$

$-2 i_1 + 6 i_2 = -3j$

$i_1 = 3 i_2 + \frac{3}{2} j$

$(240 + 442.5j)\, i_2 \quad -221.25 + 120j - 80 i_2$

$= 320$