Problem 1

(30') Compute the Fourier transform of each of the following signals:

(a) $x[n] = \sin\left(\frac{\pi}{2}n\right) + \cos(n)$

(b) $x[n] = (n-1)\left(\frac{1}{3}\right)^{|n|}$

(c) $x[n] = \frac{\sin(\pi(n-2)/2)}{\pi(n-2)}$

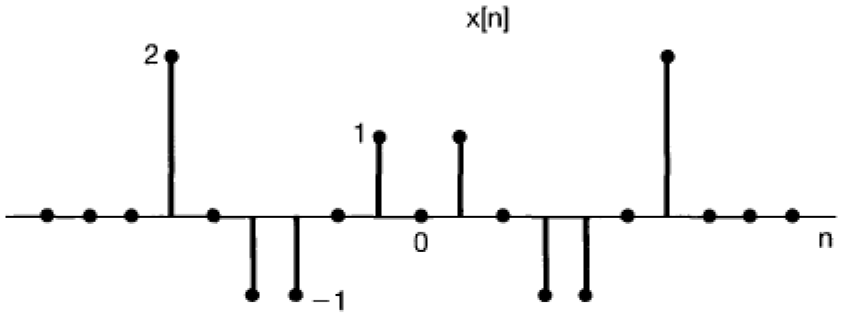
Problem 2

(20') Determine which, if any, of the following signals have Fourier transforms that satisfy each of the following conditions:

1. $\mathrm{Re}\{X(e^{jw})\} = 0.$
2. $\mathrm{Im}\{X(e^{jw})\} = 0.$
3. There exists a real α such that $e^{j\alpha w}X(e^{jw})$ is real.
4. $\int_{-\pi}^{\pi} X(e^{jw})dw = 0.$
5. $X(e^{jw})$ periodic.
6. $X(e^{j0}) = 0.$

Note: You need to justify your answer.

a.



b. $x[n] = \delta[n-1] - \delta[n+1]$

Problem 3

(20') Consider a system consisting of the cascade of two LTI systems with frequency responses

$$H_1(e^{jw}) = \frac{2 - e^{-jw}}{1 + \frac{1}{2}e^{-jw}}$$

and

$$H_2(e^{jw}) = \frac{1}{1 - \frac{1}{2}e^{-jw} + \frac{1}{4}e^{-j2w}}$$

(a) Find the difference equation describing the overall system.
(b) Determine the impulse response of the overall system.

Problem 4

(30') A causal LTI system is described by the difference equation

$$y[n] - ay[n-1] = bx[n] + x[n-1],$$

where a is real and less than 1 in magnitude.

(a) Find a value of b such that the frequency response of the system satisfies

$$\left|H(e^{jw})\right| = 1, \text{ for all w.}$$

This kind of system is called an all-pass system, as it does not attenuate the input e^{jwn} for any value of w. Use the value of b that you have found in the rest of the problem.

(b) Roughly sketch $\angle H\left(e^{jw}\right),\ 0 \le w \le \pi, when\ a = -\frac{1}{2}$

(c) Find and plot the output of this system with $a = -\frac{1}{2}$ when the input is $x[n] = \left(\frac{1}{2}\right)^n u[n]$.