Problem 1

(15 points) Compute the Fourier transform of each of the following signals:

(a)

$$x(t) = [e^{-\alpha t} \cos(\omega_0 t)]u(t), \alpha > 0$$

(b)

$$x(t) = e^{-3|t|} sin(2t)$$

(c)

$$x(t) = \begin{cases} 1 + \cos \pi t, & |t| \leq 1 \\ 0, & |t| > 1 \end{cases}$$

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Problem 2

(15 points) Frequency response of a Linear Time-Invariant system is shown below:

$$H(\omega) = \frac{j\omega + 4}{6 - \omega^2 + 5j\omega}$$

(a) Write out the differential equation that associates system input x(t) with output y(t).

(b) Determine the impulse response $h(t)$ of the system.

(c) Determine the output of the system with input $x(t) = e^{-4t}u(t)$.

Problem 3

(20 points) Ideal low pass filter frequency response is shown. Draw the spectrum of the output signal when input is the following function.

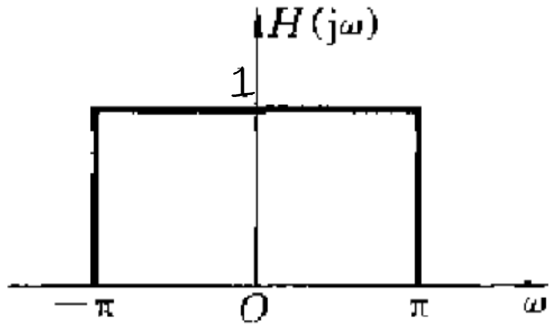

Figure 1: Ideal Low Pass Filter

(a)

$$f(t) = \frac{\sin(\pi t)}{\pi t}$$

(b)

$$f(t) = \begin{cases} 1, |t| \leq 1 \\ 0, |t| > 1 \end{cases}$$

Problem 4

(20 points) The output $y(t)$ of a causal LTI system is related to the input $x(t)$ by the equation

$$\frac{dy(t)}{dt} + 10y(t) = \int_{-\infty}^{+\infty} x(\tau)z(t-\tau)d\tau - x(t)$$

where $z(t) = e^{-t}u(t) + 3\delta(t)$

(a) Find the frequency response $H(j\omega) = Y(j\omega)/X(j\omega)$ of this system.

(b) Determine the impulse response of the system.

Problem 5

(30 points) Let $x(t)$ and $y(t)$ be two real signals. Then the cross-correlation function of $x(t)$ and $y(t)$ is defined as

$$\phi_{xy}(t) = \int_{-\infty}^{+\infty} x(t+\tau)y(\tau)d\tau$$

Similarly, we can define $\phi_{yx}(t), \phi_{xx}(t)$, and $\phi_{yy}(t)$. The last two of these are called the autocorrelation functions of the signals $x(t)$ and $y(t)$, respectively. Let $\Phi_{xy}(j\omega)$, $\Phi_{yx}(j\omega), \Phi_{xx}(j\omega)$, and $\Phi_{yy}(j\omega)$ denote the Fourier transforms of $\phi_{xy}(t), \phi_{yx}(t)$, $\phi_{xx}(t)$, and $\phi_{yy}(t)$, respectively.

(a) Determine the relationship between $\Phi_{xy}(j\omega)$ and $\Phi_{yx}(j\omega)$.
Hint: You may need to prove $\phi_{yx}(t) = \phi_{xy}(-t)$ firstly.

(b) Find an expression for $\Phi_{xy}(j\omega)$ in terms of $X(j\omega)$ and $Y(j\omega)$.

(c) Show that $\Phi_{xx}(j\omega)$ is real and nonnegative for every ω.

(d) Suppose now that $x(t)$ is the input to an LTI system with a real-valued impulse response and with frequency response $H(j\omega)$ and that $y(t)$ is the output. Find expressions for $\Phi_{xy}(j\omega)$ and $\Phi_{yy}(j\omega)$ in terms of $\Phi_{xx}(j\omega)$ and $H(j\omega)$.

(e) Let $x(t)$ be as is illustrated in Figure 2, and let the LTI system impulse response be $h(t) = e^{-at}u(t), a > 0$. Compute $\Phi_{xx}(j\omega), \Phi_{xy}(j\omega)$, and $\Phi_{yy}(j\omega)$ using the results of parts (a)-(d).

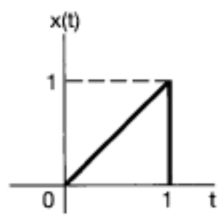

Figure 2: $x(t)$ in 5(e)