Homework 7

Due date:
May 14th, 2018
Turn in your homework in class

Rules:

- Please try to work on your own. Discussion is permissible, but identical submissions are unacceptable!
- Please show all intermediate steps: a correct solution without an explanation will get zero credit.
- Please submit on time. No late submission will be accepted.
- Please prepare your submission in English only. No Chinese submission will be accepted.

1. (9%) Simplify the following expressions and give the answer with rectangular form, polar form and exponential form.

   (a) $\frac{(5+j8)+(-2+j11)*j5}{(6+3j)*(-2-8j)-(3+6j)/(4-j8)}$

   (b) $\frac{(5\angle 60° - 96\angle -105°)*(-20+j8)}{(8-j9)*(10\angle 45°)}$

   (c) $(\frac{-45-j18}{8-j6})^2/\sqrt{(15+j9)/(14+j6)}$

Solutions: **[每个 1.5 分=1.5%*3*2=9%]**

(a) $\frac{209}{435} - j\frac{217}{435}$ or $0.48 - j0.50$ $\quad$ $0.693\angle -46.076°$ or $0.693e^{j(-46.076°)}$ or $0.693e^{j(-0.256\pi)}$

(b) $-10.13 - j14.92$ $\quad$ $18.04\angle -124.17°$ or $18.04e^{j(-124.17°)}$ or $18.04e^{j(-0.69\pi)}$

(c) $-6.99 + j11.53$ $\quad$ $13.49\angle 121.23°$ or $13.49e^{j(121.23°)}$ or $13.49e^{j(0.67\pi)}$

2. (7%) Simplify the following expressions by using phasors:

(a) $i_1(t) = 40\,cos(\omega t - 48°) + 89\,cos(\omega t + 87°)\,A$

(b) $i_2(t) = 88\,sin(\omega t + 65°) - 756\,cos(\omega t + 44°)\,A$

(c) $i_3(t) = 218\,cos(8t) - 950\,sin(8t)\,mA$

(d) $v_1(t) = 64\,sin(8t - 95°) + 24\,sin(8t + 23°)\,V$

(e) $v_2(t) = 50\,sin(100t - 65°) + 45\,cos(100t + 20°) + 30\,sin(100t - 80°)\,mV$

(f) $v_3(t) = 4\,cos(55t + 66°) + 4\,cos(55t - 66°)\,V$

(g) $v_4(t) = 25\,sin(35t) - 50\,cos(35t)\,\mu V$

Solution:

(a) $i_1(t) = 40\cos(\omega t - 48°) + 89\cos(\omega t + 87°)\ A$

$I_1 = 40\angle{-48°} + 89\angle 87° = 66.98\angle 62.06°\ A$ 0.5'

$\Rightarrow i_1(t) = 66.98\cos(\omega t + 62.06°)\ \underline{A}$ 0.5'

(b) $i_2(t) = 88\sin(\omega t + 65°) - 756\cos(\omega t + 44°)\ A$

$I_2 = 88\angle 155° - 756\angle 44° = 791.81\angle{-141.96°}\ \underline{A}$ 0.5'

$\Rightarrow i_2(t) = 791.81\cos(\omega t - 141.96°)\ A$ 0.5'

(c) $i_3(t) = 218\cos 8t - 950\sin 8t\ mA$

$I_3 = 218\angle 0° - 950\angle 90° = 974.69\angle{-77.08°}\ mA$ 0.5'

$\Rightarrow i_3(t) = 974.69\cos(8t - 77.08°)\ \underline{mA}$ 0.5'

(d) $v_1(t) = 64\sin(8t - 95°) + 24\sin(8t + 23°)\ V$

$V_1 = 64\angle{-5°} + 24\angle 113° = 56.83\angle 16.89°\ V$ 0.5'

$\Rightarrow v_1(t) = 56.83\sin(8t + 16.89°)\ \underline{V}$ 0.5'

(e) $v_2(t) = 50\sin(100t - 65°) + 45\cos(100t + 20°) + 30\sin(100t - 80°)\ mV$

$V_2 = 50\angle 25° + 45\angle 20° + 30\angle 10° = 124.36\angle 19.61°\ mV$ 0.5'

$\Rightarrow v_2(t) = 124.36\cos(100t + 19.61°)\ \underline{mV}$ 0.5'

(f) $v_3(t) = 4\cos(55t + 66°) + 4\cos(55t - 66°)\ V$

$V_3 = 4\angle 66° + 4\angle{-66°} = 3.25\ V$ 0.5'

$\Rightarrow v_3(t) = 3.25\ \underline{V}$ 0.5'

(g) $v_4(t) = 25\sin(35t) - 50\cos(35t)\ \mu V$

$V_4 = 25\angle{-90°} - 50\angle 0° = 25\sqrt{5}\angle{-153.43°}\ (= 55.90\angle{-153.43°})\ \mu V$ 0.5'

$\Rightarrow v_4(t) = 25\sqrt{5}\cos(35t - 153.43°)\ \mu V\ (= 55.90\cos(35t - 153.43°)\ \underline{\mu V}$. 0.5'

红色划线处单位少一个就减0.5'，扣完即止.

3. (8%) Find steady state solution of $v(t)$ or $i(t)$ in the following integro differential equations using the phasor approach:
   (a) $v(t) + \int 54v(t)dt = 25\,cos(6t)$.
   (b) $2\frac{dv(t)}{dt} + 8v(t) + 3\int v(t)dt = 50\,sin(8t - 30°)$.
   (c) $8i(t) + \frac{7di(t)}{dt} = 560\,cos(6t + 75°)$.
   (d) $50\int i(t)dt + 2i(t) + \frac{di(t)}{dt} = 6\,cos(3t - 66°)$.

Solution:

(a) $V + \frac{V}{j\omega} = 25\angle 0°$, $\omega = 6$ 1'

$V = 24.66\angle 9.46°$ V 0.5'

$v(t) = 24.66\cos(6t + 9.46°)\ \underline{V}$ 0.5'

(b) $2j\omega V + 8V + 3\frac{V}{j\omega} = 50\angle 60°$, $\omega = 8$ 1'

$V = 2.85\angle -2.89°$ V 0.5'

$v(t) = 2.85\cos(8t - 2.89°)\ \underline{V}$ 0.5'

(c) $8I + 7j\omega I = 560\angle 75°$, $\omega = 6$ 1'

$I = 13.10\angle -4.22°$ A 0.5'

$i(t) = 13.10\cos(6t - 4.22°)\ \underline{A}$ 0.5'

(d) $50\frac{I}{j\omega} + 2I + j\omega I = 6\angle -66°$, $\omega = 3$ 1'

$I = 0.43\angle 15.67°$ A 0.5'

$i(t) = 0.43\cos(3t + 15.67°)\ \underline{A}$ 0.5'

红色划线处单位少一个就减0.5'，扣完即止.

4. (8%) The voltage applied to the circuit shown in Figure 4 at *t=0* is $50\,cos(90t + 36°)\,V$. The circuit resistance is 60Ω and the initial current in the 25mH inductor is zero.
   (a) Find $i(t)$ for $t \geq 0$.
   (b) Write the expressions for the transient and steady-state components of $\mathrm{i(t)}$.
   (c) What are the maximum amplitude, frequency (in radians per second), and phase angle of the steady-state current?
   (d) By how many degrees are the voltage and the steady-state current out of phase?

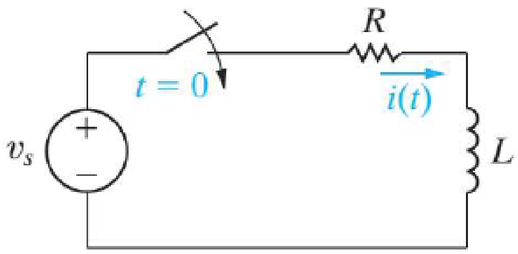


Figure 4

Solution:

(a) The numerical values of the terms are

$\mathrm{V_m} = 50$ **0.5'** R/L=2400 **0.5'** ωL=2.25 **0.5'**

$\sqrt{\mathrm{R}^2 + \omega^2 L^2} = 60.042$ **0.5'**

$\varphi = 36°$ **0.5'** $\theta = \tan^{-1} 2.25/60,$ $\theta = 2.1476°$ **0.5'**

Substitute these values into Equation:

$\mathrm{i} = \left[-0.692\mathrm{e}^{-2400t} + 0.833\cos(90\mathrm{t} - 33.85°)\right] A, \quad t \geq 0$ **1'**

(b) Transient component: $-0.692\mathrm{e}^{-2400t}\,A$ **1'**

Steady-state component: $0.833\cos(90\mathrm{t} - 33.85°)\ \mathrm{A}$ **1'**

(c) maximum amplitude: 0.833A, frequency: 90 rad/s, phase angle: -33.85° **1'**

(d) The current lags the voltage by 33.85° or the voltage leads the current by 33.85° **1'**

5. (10%) Determine the equivalent impedance:
   (a) Z_1 at ω=300rad/s in Figure 5-a.
   (b) Z_2 at 1000Hz with L= 5mH in Figure 5-b.
   (c) Z_3 at 800Hz with L= 2mH in Figure 5-c.
   (d) Z_4 in Figure 5-d.
   (e) Z_5 at $\omega = 10^4$rad/s in Figure 5-e.

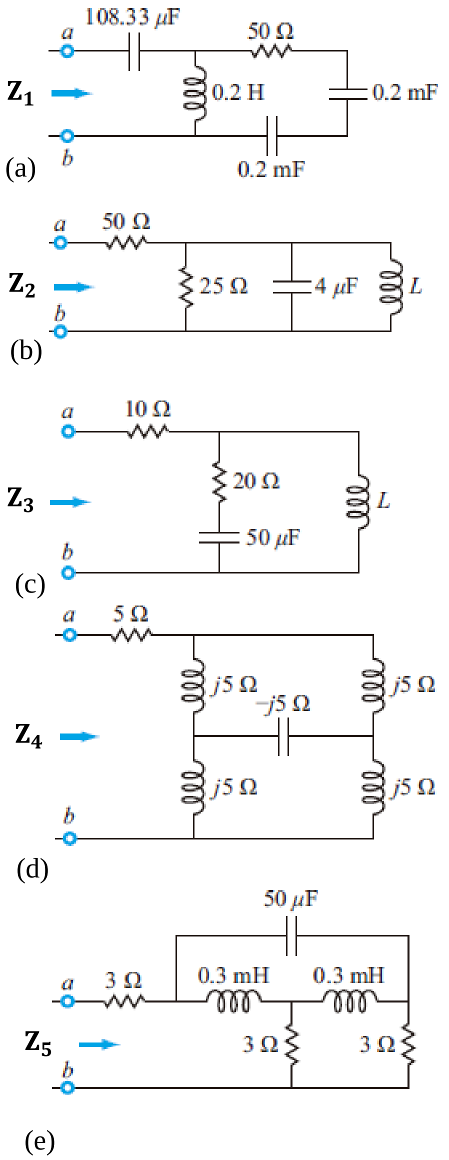
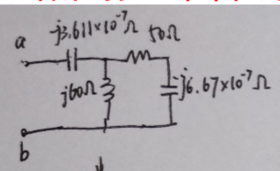


Figure 5

Solutions: **[每个 2’： 5*2’=10%]**

答案每少一个单位则减 1’, 扣完为止

(a)

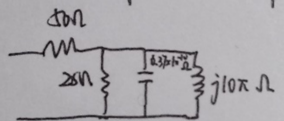


$Z_1 = -j3.611\times10^{-7}\Omega \times \left(j60\Omega \,//\, (50\Omega - j6.67\times10^{-7}\Omega)\right)$

$= 8.88\times10^{-6} - j1.07\times10^{-5}\,\Omega = 1.387\times10^{-5}\angle -50.19^\circ\,\Omega$

$f = 1\text{kHz} \rightarrow \omega = 2\pi f = 2\pi$ krad/s

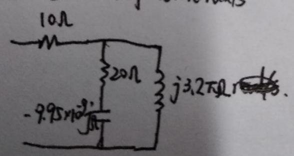
(b)



$Z_2 = 50\Omega + (25\Omega \,//\, {-j6.37\times10^{-10}}\Omega \,//\, j10\pi\Omega)$

$= 50 - j6.37\times10^{-10}\,\Omega = 50\angle -7.30\times10^{-10\circ}\,\Omega$

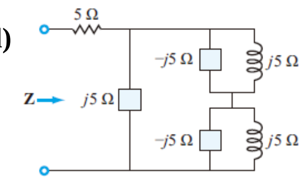
(c) $f = 800\text{Hz} \rightarrow \omega = 2\pi f = 1600\pi$ rad/s



$Z_3 = 10\Omega + \left(j3.2\pi\Omega \,//\, (20\Omega - j9.95\times10^{-9}\Omega)\right)$

$= 14.03 + j8.03\,\Omega = 16.17\angle 29.76^\circ\,\Omega$

(d)



$$\mathbf{Z}_a = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_1\mathbf{Z}_3}{\mathbf{Z}_1}$$

$$= \frac{(j5)(-j5) + (-j5)(j5) + (j5)^2}{j5} = -j5\,\Omega$$

$$\mathbf{Z}_b = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_1\mathbf{Z}_3}{\mathbf{Z}_2} = j5\,\Omega$$

$$\mathbf{Z}_c = \mathbf{Z}_a = -j5\,\Omega.$$

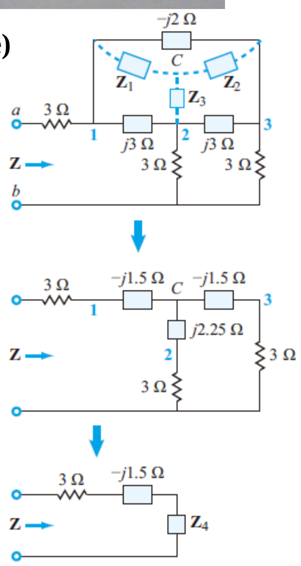
$$\mathbf{Z} = 5 + j5 \,\|\, 2(j5 \,\|\, -j5)$$

$$= 5 + j5 \,\|\, 2\left(\frac{25}{j5 - j5}\right)$$

$$= 5 + j5 \,\|\, \infty$$

$$= (5 + j5) = 5\sqrt{2}\, e^{j45^\circ}$$

(e)



$$\mathbf{Z}_L = j\omega L = j10^4 \times 0.3\times10^{-3} = j3\,\Omega$$

$$\mathbf{Z}_C = \frac{-j}{\omega C} = \frac{-j}{10^4\times50\times10^{-6}} = -j2\,\Omega$$

$$\mathbf{Z}_1 = \frac{j3\times(-j2)}{j3 + j3 - j2} = \frac{6}{j4} = -j1.5\,\Omega$$

$$\mathbf{Z}_2 = \mathbf{Z}_1 = -j1.5\,\Omega$$

$$\mathbf{Z}_3 = \frac{(j3)^2}{j4} = \frac{-9}{j4} = j2.25\,\Omega$$

$$\mathbf{Z}_4 = (3 + j2.25) \,\|\, (3 - j1.5) = (2.077 + j0.115)\,\Omega$$

$$\mathbf{Z} = (3 - j1.5) + (1.99 + j0.62) = (5.077 - j1.385)\,\Omega.$$

6. (9%) Determine $i_x(t)$ by using mesh method in the circuit of Figure 6, given that $v_s(t) = 6cos5 \times 10^5 t$ V.

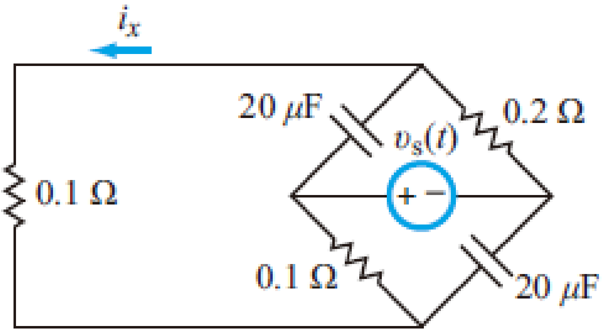
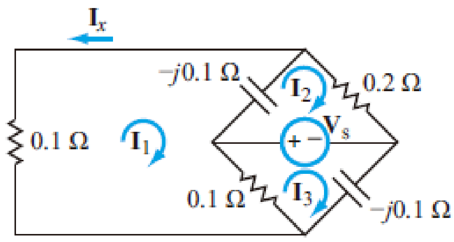


Figure 6

Solution:



At $\omega = 5 \times 10^5$ rad/s,

$$\mathbf{Z}_C = \frac{-j}{\omega C} = \frac{-j}{5 \times 10^5 \times 20 \times 10^{-6}} = -j0.1\ \Omega$$

$$\mathbf{V}_s = 6\angle 0^\circ\ \text{V}$$

Application of the mesh current by inspection method gives:

$$\begin{bmatrix} (0.1+0.1-j0.1) & j0.1 & -0.1 \\ j0.1 & (0.2-j0.1) & 0 \\ -0.1 & 0 & (0.1-j0.1) \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \mathbf{I}_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 6 \\ -6 \end{bmatrix}$$

Solution of matrix equation gives 方程组每个式子 1’，共 3’

$$\mathbf{I}_1 = (6.79 - j23.77)\ \text{A}$$

$$\mathbf{I}_2 = (15.85 - j4.53)\ \text{A}$$

$$\mathbf{I}_3 = -(14.72 + j38.49)\ \text{A}$$ 解出三个答案，每个 1’，共 3’

$$\mathbf{I}_x = -\mathbf{I}_1 = (6.79 - j23.77)\ \text{A} = 24.72e^{-j74.06^\circ}\ \text{A}$$ 1’

$$i_x(t) = 24.72\cos(5 \times 10^5 t - 74.06^\circ)\ \text{A}.$$ 2’

如果最后的单位(A)少了，扣 1’，扣完即止

7. (9%) Find the value of ω at which $v_s(t)$ and $i_s(t)$ in the circuit of Figure 7 are in-phase (in-phase means that there is no imaginary part in the total Z_{eq}).

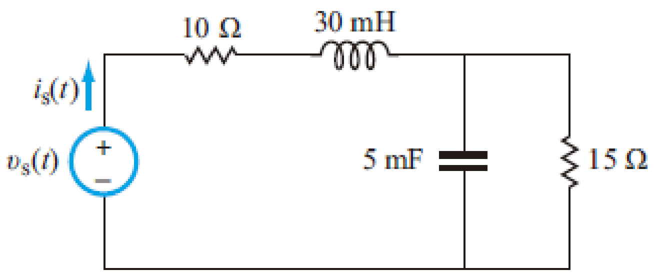
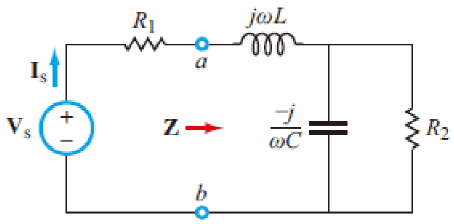


Figure 7

Solution: Transforming the circuit to the phasor domain leads to the circuit in Fig. P7.75(a).



2'

Fig. P7.75(a)

For $\mathbf{I}_s$ to be in-phase with $\mathbf{V}_s$, it is necessary that the impedance of the circuit to the right of terminals (a, b) be purely real.

$$
\begin{aligned}
\mathbf{Z} &= j\omega L + R_2 \parallel \left(\frac{-j}{\omega C}\right) \\
&= j\omega L - \frac{jR_2/\omega C}{R_2 - \frac{j}{\omega C}} \\
&= \left(j\omega L\left(R_2 - \frac{j}{\omega C}\right) - j\frac{R_2}{\omega C}\right) \times \left[\frac{1}{R_2 - \frac{j}{\omega C}}\right] \\
&= \left[\frac{L}{C} + j\left(\omega L R_2 - \frac{R_2}{\omega C}\right)\right]\left[\frac{\omega C}{R_2\omega C - j}\right] \\
&= \left[\frac{L}{C} + j\left(\frac{\omega^2 LCR_2 - R_2}{\omega C}\right)\right]\left[\frac{\omega C(R_2\omega C + j)}{R_2^2\omega^2C^2 + 1}\right] \\
&= \frac{\omega L + j(\omega^2 LCR_2 - R_2)}{\omega C} \times \frac{\omega C(R_2\omega C + j)}{R_2^2\omega^2C^2 + 1} \\
&= [\omega^2 LCR_2 - (\omega^2 LCR_2 - R_2)] + \frac{j[\omega L + R_2\omega C(\omega^2 LCR_2 - R_2)]}{R_2^2\omega^2C^2 + 1}.
\end{aligned}
$$
3'

Equating the imaginary component to zero gives

$$\omega L + R_2\omega C(\omega^2 LCR_2 - R_2) = 0$$
1'

$$L + R_2^2 LC^2\omega^2 - R_2^2 C = 0$$

$$\omega = \sqrt{\frac{R_2^2 C - L}{R_2^2 C^2 L}}$$
2'

$$= \left[\frac{(15)^2 \times 5 \times 10^{-3} - 30 \times 10^{-3}}{(15)^2 \times (5 \times 10^{-3})^2 \times 30 \times 10^{-3}}\right]^{1/2} = 80.55 \text{ rad/s.}$$
1'

少单位则减 1'，扣完为止

8. (9%) The input signal in the op-amp circuit of Figure 8 is given by

$$v_{in}(t) = 0.5cos2000t \text{ V}.$$

Obtain an expression for $v_{out}(t)$ and then evaluate it for $R_1 = 2k\Omega, R_2 = 10k\Omega$, and $C = 0.1\ \mu$F.

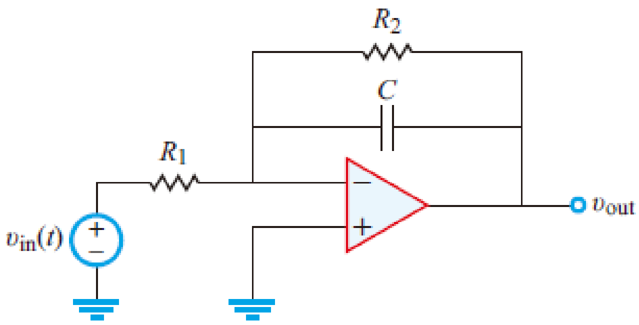
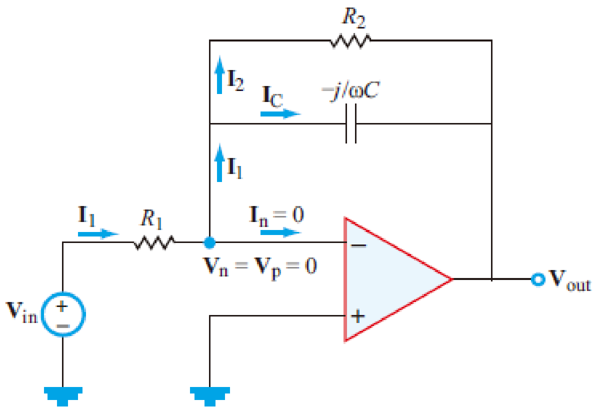


Figure 8

Solution:



2'

$\mathbf{V}_{in} = 0.5\angle 0^\circ$ V.

Since $\mathbf{V}_n = 0$,

$$\mathbf{I}_1 = \frac{\mathbf{V}_{in}}{R_1}.$$

Also,

$$\begin{aligned} \mathbf{I}_1 &= \mathbf{I}_2 + \mathbf{I}_C \\ &= \frac{\mathbf{V}_n - \mathbf{V}_{out}}{R_2} + \frac{\mathbf{V}_n - \mathbf{V}_{out}}{-j/\omega C} \\ &= -\mathbf{V}_{out}\left(\frac{1}{R_2} + j\omega C\right). \end{aligned}$$ 2'

Hence,

$$\begin{aligned} \mathbf{V}_{out} &= -\left(\frac{R_2}{R_1}\right)\left(\frac{1}{1 + j\omega R_2 C}\right)\mathbf{V}_{in} \\ &= -\left(\frac{R_2}{R_1}\right)\frac{1 - j\omega R_2 C}{1 + \omega^2 R_2^2 C^2}\mathbf{V}_{in}. \end{aligned}$$ 3'

For $\mathbf{V}_{in} = 0.5$ V, $R_1 = 2$ kΩ, $R_2 = 10$ kΩ, $\omega = 2000$ rad/s, and $C = 0.1\ \mu$F,

$$\begin{aligned} \mathbf{V}_{out} &= -0.5(1 - j2) \\ &= 0.5\sqrt{5}\, e^{j180^\circ} \cdot e^{-j63.4^\circ} \\ &= 1.12 e^{j116.6^\circ} \text{ V}. \end{aligned}$$ 1'

$$v_{out}(t) = 1.12\cos(2000t + 116.6^\circ) \quad \text{(V)}.$$ 1'

少单位则减 1'，扣完为止

9. (9%) The circuit in Figure 9 is in the phasor domain. Determine and plot its Thevenin equivalent circuit at terminals *(a,b)*.

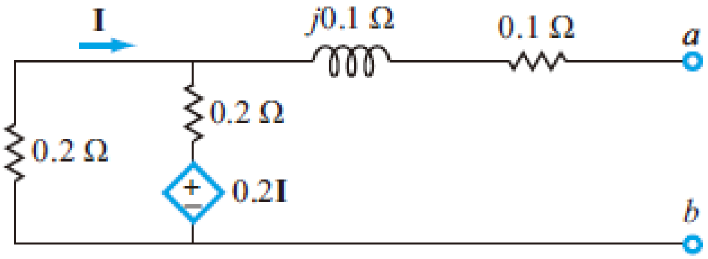
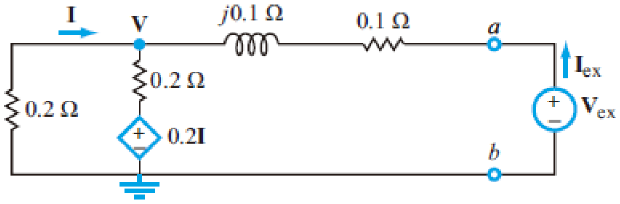


Figure 9

Solution: Since the circuit has no independent sources,

$$\mathbf{V}_{\mathrm{Th}} = 0.$$ 1'



1'

To determine $\mathbf{Z}_{\mathrm{Th}}$, we add an external source $\mathbf{V}_{\mathrm{ex}}$ and compute $\mathbf{I}_{\mathrm{ex}}$.

At node $\mathbf{V}$:

$$\frac{\mathbf{V}}{0.2} + \frac{\mathbf{V} - 0.2\mathbf{I}}{0.2} + \frac{\mathbf{V} - \mathbf{V}_{\mathrm{ex}}}{0.1 + j0.1} = 0.$$ 2'

Also,

$$\mathbf{I} = -\frac{\mathbf{V}}{0.2}.$$

Solution leads to

$$\mathbf{V} = \left[\frac{25 - j15}{85}\right]\mathbf{V}_{\mathrm{ex}} = [0.294 - j0.176]\mathbf{V}_{\mathrm{ex}}.$$ 1'

$$\mathbf{I}_{\mathrm{ex}} = \frac{\mathbf{V}_{\mathrm{ex}} - \mathbf{V}}{0.1(1+j)}$$

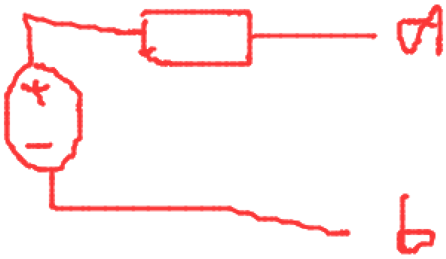
$$= \frac{10}{1+j}[1 - 0.294 + j0.176]\mathbf{V}_{\mathrm{ex}} = (4.41 - j2.65)\mathbf{V}_{\mathrm{ex}}.$$ 1'

$$\mathbf{Z}_{\mathrm{Th}} = \frac{\mathbf{V}_{\mathrm{ex}}}{\mathbf{I}_{\mathrm{ex}}} = \frac{1}{4.41 - j2.65} = (0.17 + j0.10)\ \Omega.$$ 1'

共 3'

$Z_{TH} = (0.17 + 0.10)\Omega$

$V_{TH} = 0$ V



2'

10. (12%) Find v_0 in the circuit of Figure 10 using superposition.

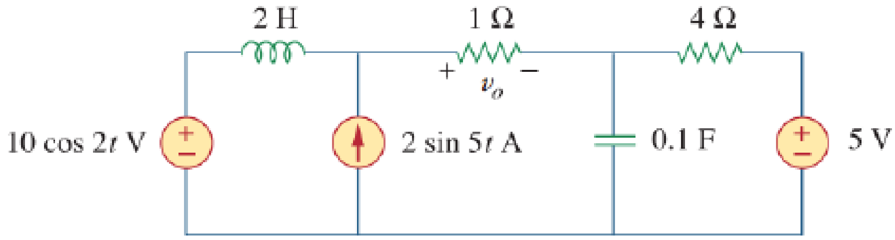
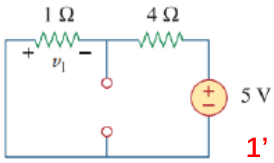


Figure 10

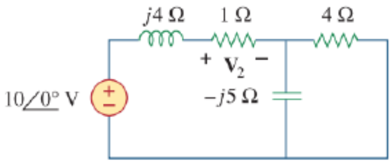
Solution:



1'

(a)

$$-v_1 = \frac{1}{1+4}(5) = 1 \text{ V}$$ 2'

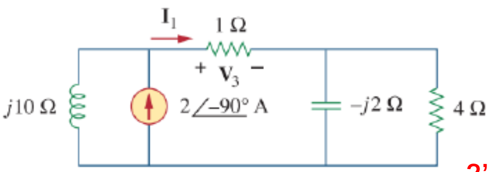


2'

(b)

$$\mathbf{V}_2 = \frac{1}{1+j4+\mathbf{Z}}(10\angle 0^\circ) = \frac{10}{3.439+j2.049} = 2.498\angle -30.79^\circ$$

$$v_2 = 2.498\cos(2t - 30.79^\circ)$$ 2'



2'

(c)

$$\mathbf{I}_1 = \frac{j10}{j10 + 1 + \mathbf{Z}_1}(2\angle -90^\circ) \text{ A}$$

$$\mathbf{V}_3 = \mathbf{I}_1 \times 1 = \frac{j10}{1.8 + j8.4}(-j2) = 2.328\angle -80^\circ \text{ V}$$

$$v_3 = 2.33\cos(5t - 80^\circ) = 2.33\sin(5t + 10^\circ) \text{ V}$$ 2'

$$v = v_1 + v_2 + v_3 = -1 + 2.498\cos(2t - 30.79^\circ) + 2.33\sin(5t + 10^\circ) \text{ V}$$ 1'

少单位则减 1'，扣完为止

11. (10%) The input circuit shown in Figure 11 contains two sources, given by

$$i_s(t) = 2\cos 10^3 t\ A$$
$$v_s(t) = 8\sin 10^3 t\ V$$

This input circuit is to be connected to a load circuit that provides optimum performance when the impedance Z of the input circuit is purely real. The circuit includes a "matching" element whose *type* and *magnitude* should be chosen to realize that condition. What should those attributes (type and magnitude) be?

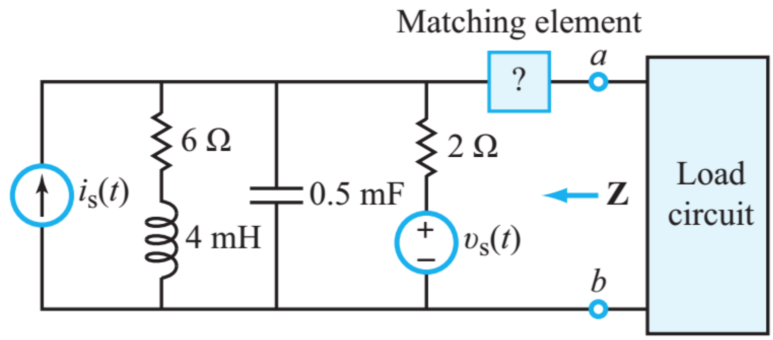
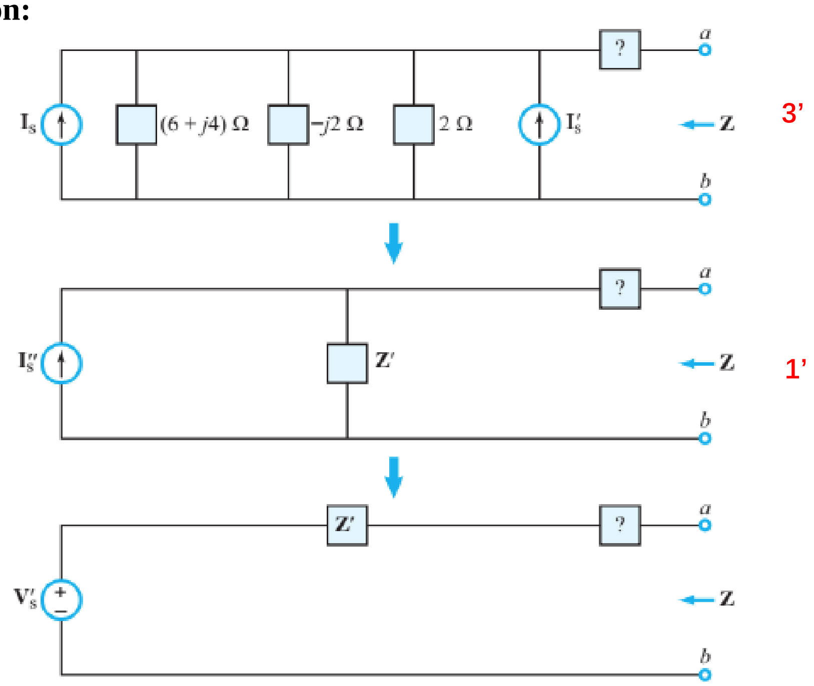


Figure 11

Solution:



The matching element $\mathbf{Z}_x$ has to cancel the imaginary part of $\mathbf{Z}'$. Hence

$$\mathbf{Z}_x = +j0.76\ \Omega. \quad 1'$$

$$\mathbf{I}'_s = \frac{\mathbf{V}_s}{2} = \frac{-j8}{2} = -j4\ \text{A} \quad 1'$$

$$\mathbf{I}''_s = \mathbf{I}_s + \mathbf{I}'_s = (2 - j4)\ \text{A} \quad 1'$$

$$\mathbf{Z}' = (6 + j4) \parallel (-j2) \parallel 2 = (1.1 - j0.76)\ \Omega \quad 1'$$

$$\mathbf{V}_{\text{Th}} = \mathbf{V}'_s = \mathbf{I}''_s\mathbf{Z}' = (2 - j4)(1.1 - j0.76) = -(0.84 + j5.92)\ \text{V}.$$

So it has to be an inductor L such that 1'

$$\omega L = 0.76,$$

or

$$L = \frac{0.76}{10^3} = 0.76\ \text{mH}. \quad 1'$$

少单位则减 1'，扣完为止