1. Compute the Fourier transform of each of the following signals.

(a) $x[n] = u[n-2] - u[n-6]$

(b) $x[n] = \left(\frac{1}{2}\right)^{-n} u[-n-1]$

(c) $x[n] = \sin\left(\frac{5\pi}{3}n\right) + \cos(\frac{7\pi}{3}n)$

2. Consider a casual LTI system described by the difference equation

$$y[n] + \frac{1}{2}y[n-1] = x[n]$$

(a) Determine the frequency response $H(e^{jw})$ of this system.

(b) What is the response of the system to the following inputs?

(i) $x[n] = \left(\frac{1}{2}\right)^{n} u[n]$

(ii) $x[n] = \delta[n] + \frac{1}{2}\delta[n-1]$

(c) Find the response to the inputs with the following Fourier transforms:

(i) $X(e^{j\omega}) = \frac{1-\frac{1}{4}e^{-j\omega}}{1+\frac{1}{2}e^{-j\omega}}$

(ii) $X(e^{j\omega}) = 1 + 2e^{-3j\omega}$

3. Let $X(e^{j\omega})$denote the Fourier transform of the signal x[n] depicted in the following figure. Perform the following calculations without explicitly evaluating $X(e^{j\omega})$:

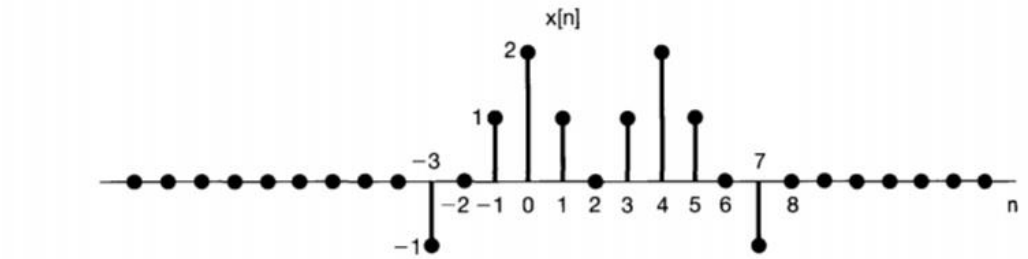

Fig P5.23

(a) Evaluate $X(e^{j0})$

(b) Find $\measuredangle X(e^{j\omega})$

(c) Evaluate $\int_{-\pi}^{\pi} X(e^{j\omega})\, d\omega$

(d) Find $X(e^{j\pi})$

(e) Determine and sketch the signal whose Fourier transform is $Re\{X(e^{j\omega})\}$

4.(a) Suppose we want to design a discrete-time LTI system which has the property that if the input is

$$x[n] = \left(\frac{1}{2}\right)^{n} u[n] - \frac{1}{4}\left(\frac{1}{2}\right)^{n-1} u[n-1]$$

then the output is

$$y[n] = \left(\frac{1}{3}\right)^n u[n]$$

(i) Find the impulse response and frequency response of a discrete-time LTI system that has the foregoing property.

(ii) Find a difference equation relating $x[n]$ and $y[n]$ that characterizes the system.

(b) Suppose that a system has the response $\left(\frac{1}{4}\right)^n u[n]$ to the input $(n+2)\left(\frac{1}{2}\right)^n u[n]$. If the output of this system is $\delta[n] - \left(-\frac{1}{2}\right)^n u[n]$, what is the input?