Lecture 3
Circuit Theorems

Outline

- Linearity property

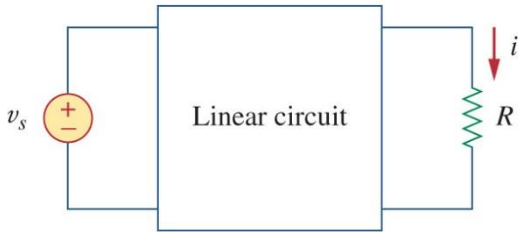
- Superposition

- Thevenin’s theorem
- Source transformation
- Norton’s theorem

- Power transfer

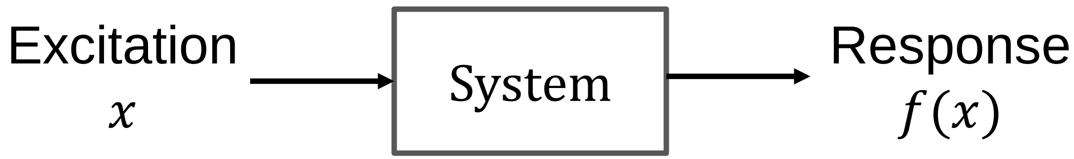
Linear Circuit

- A linear circuit consists of only linear elements (resistors, capacitors and inductors), linear dependent sources, and independent sources.

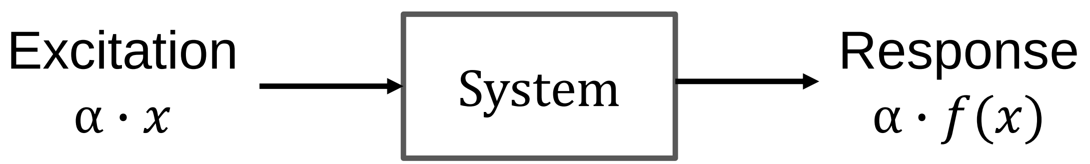


- In a circuit,
  - Excitation: Sources
  - Response: Voltage or current in the branches

Linearity Property



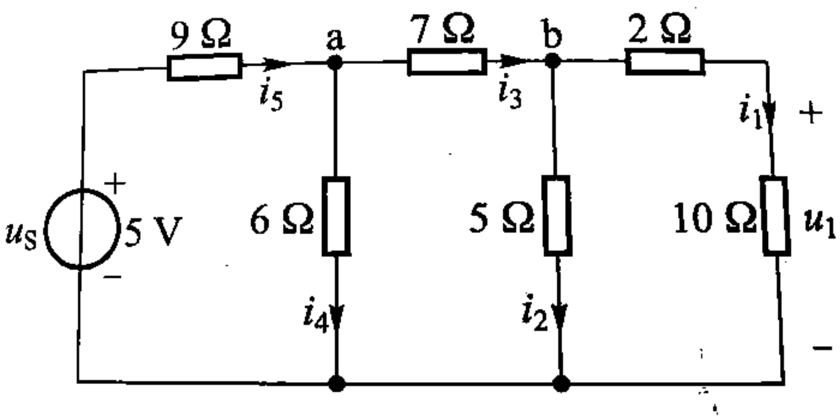
- Linearity is a combination of
  - homogeneity (scaling) property



α is constant

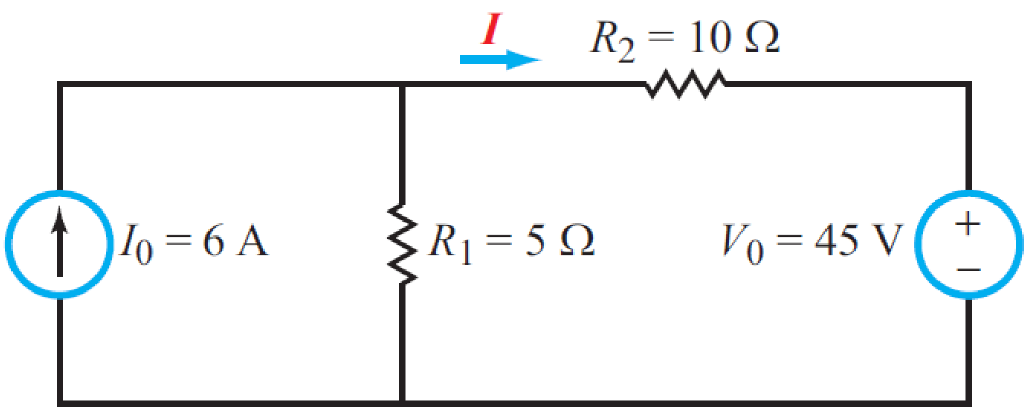
  - additivity property

Example of homogeneity (scaling) property

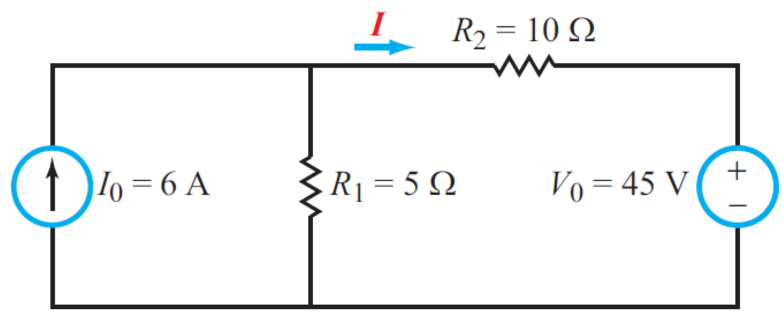


Superposition

- The superposition principle states that the voltage across (or current through) an element in *a linear circuit* is the algebraic sum of the voltages across (or currents through) that element due to each **independent source** acting alone.



Applying Superposition



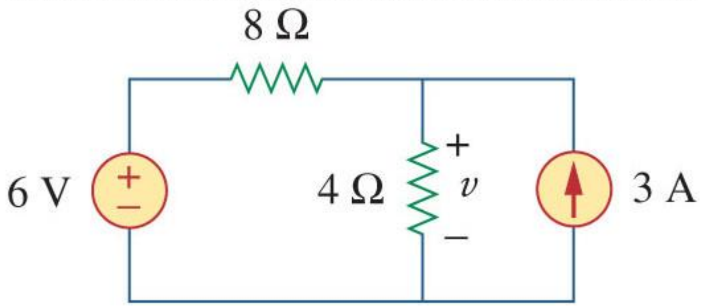
- The steps are:
  1. Turn off all other **independent** sources except for the source of interest. Find the output (voltage or current) due to that active source.
     - Turn off means to replace **independent** voltage source by short circuit (0 V), **independent** current source by open circuit (0 A).
  2. Repeat step 1 for each **independent** source.
  3. Find the total contribution by adding algebraically all the contributions due to the **independent** sources.

Note that

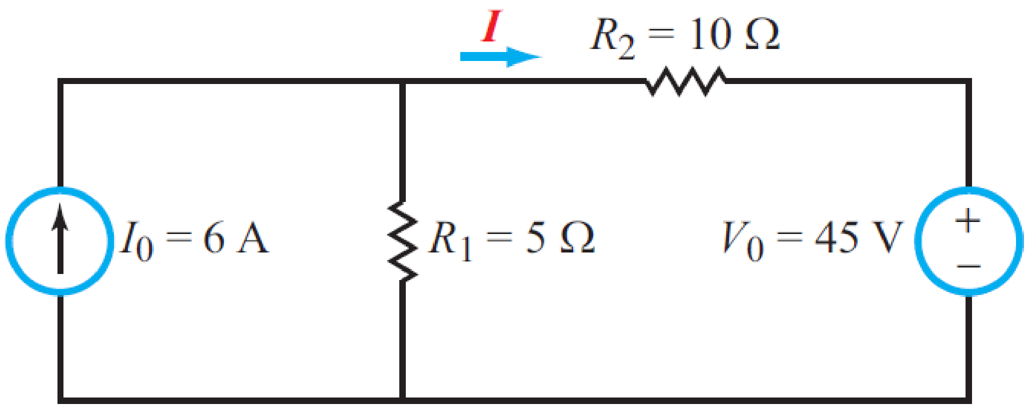
1) Using superposition means applying one independent source at a time.
2) Dependent sources are left alone.

Open Circuit and Short Circuit

- Turn off an independent voltage source means
  - v=0
  - Replace by wire
  - Short circuit
- Turn off an independent current source means
  - i=0
  - Cut off the branch
  - Open circuit

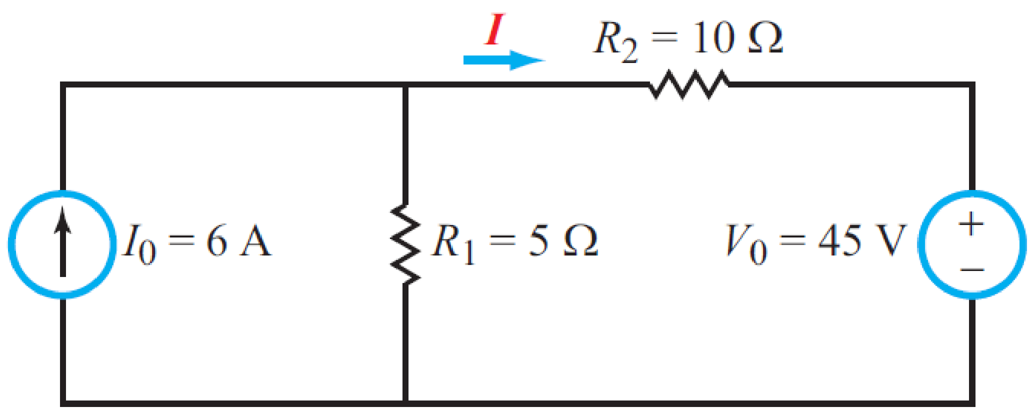


Example: Superposition



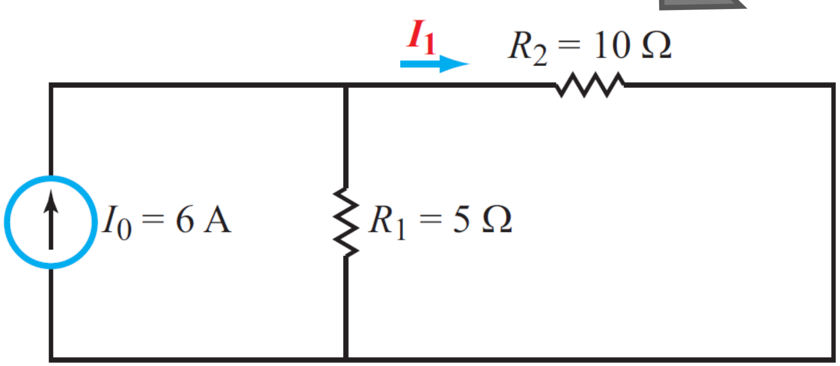
[Source: Berkeley]

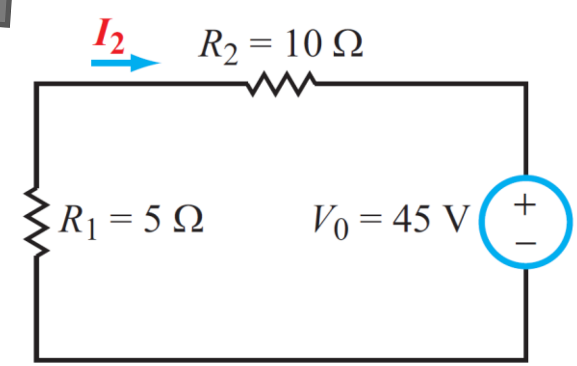
Example: Superposition



Contribution from I_0 alone

Contribution from V_0 alone





$I_1 = 2$ A

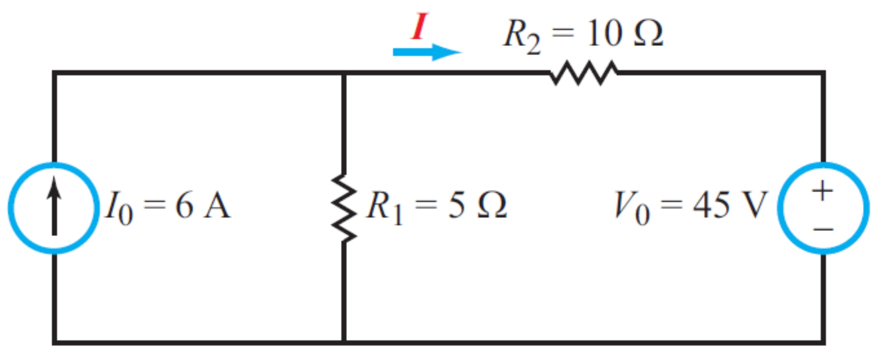
$I_2 = -3$ A

$$\boxed{I = I_1 + I_2 = 2 - 3 = -1 \text{ A}}$$

[Source: Berkeley]

Why Superposition?

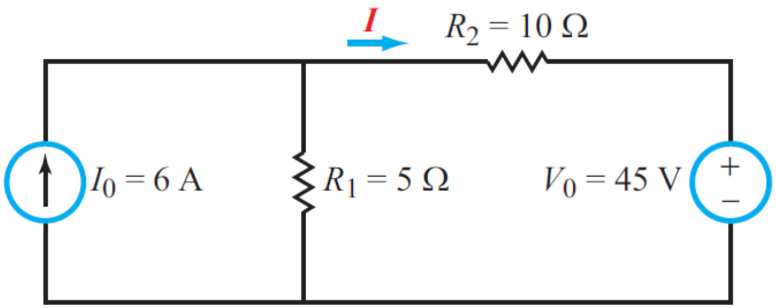
- Because it entails solving a circuit multiple times, this source-superposition method may not be attractive.
- But it is useful to evaluate the sensitivity of a response to specific sources in the circuit.



$$I = aI_0 + bV_0$$

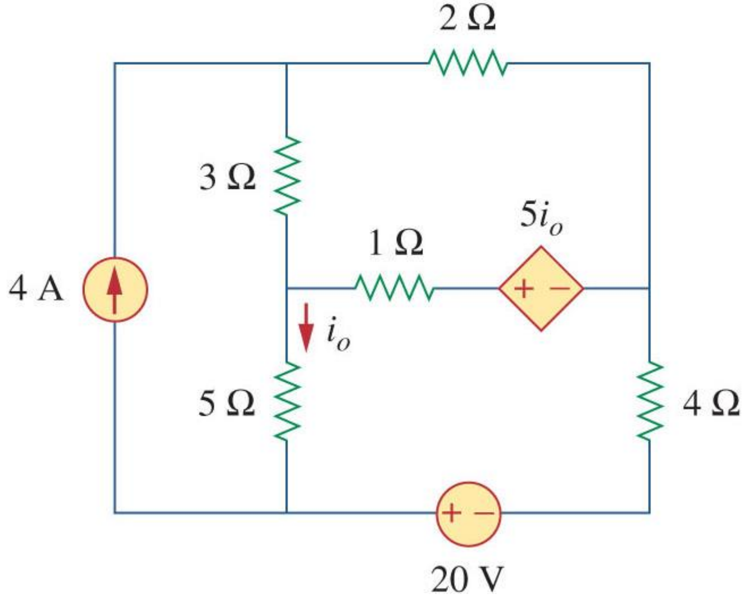
How about Power absorbed by R_2

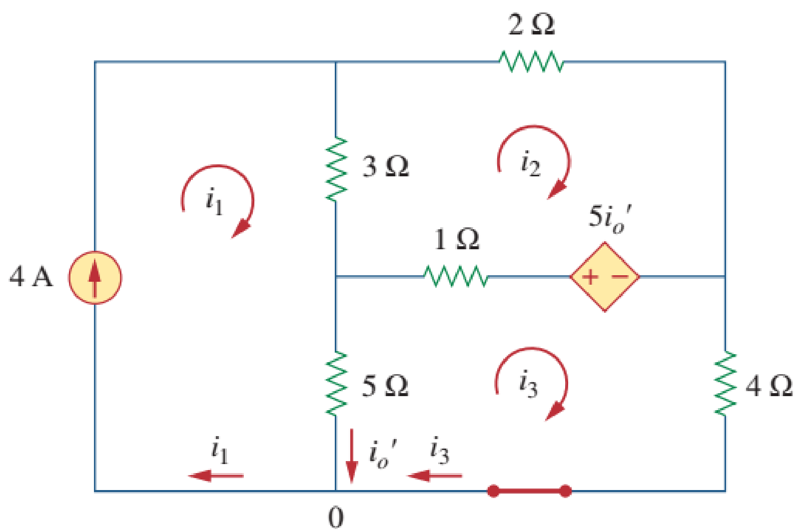
- Power due to $I_0, P_1 =?$
- Power due to $V_0, P_2 =?$
- Power due to both V_0 and $I_0, P =?$

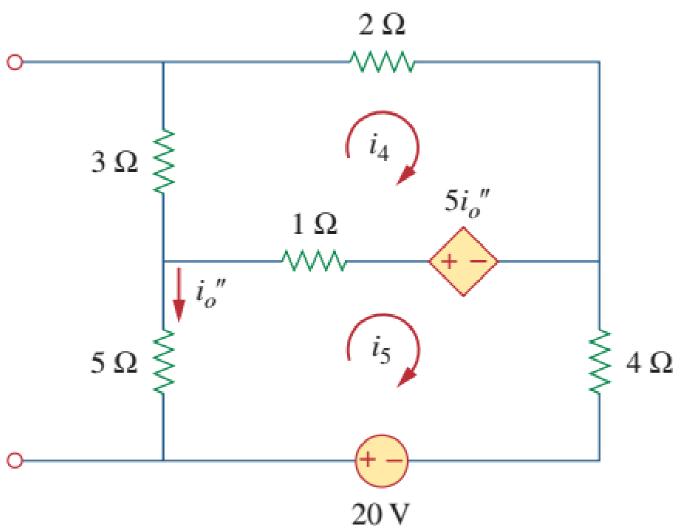


Practice 1

- Find i_0 in the circuit shown below.

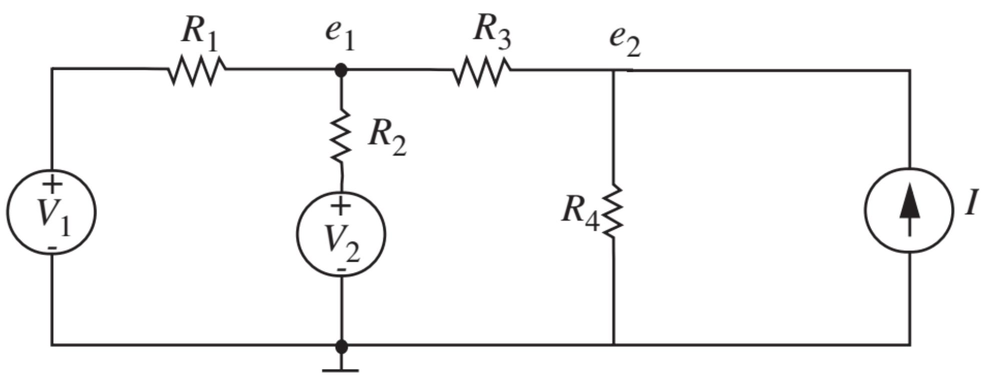



2 Ω
3 Ω
i_2
i_1
$5i_o'$
1 Ω
4 A
5 Ω
i_3
4 Ω
i_1
i_o'
i_3
0


2 Ω
i_4
3 Ω
$5i_o''$
1 Ω
i_o''
5 Ω
i_5
4 Ω
20 V

Practice 2

- Express node voltage e_1 as a function of two voltage sources V_1, V_2 and one current source I.



[Source: MIT]

Outline

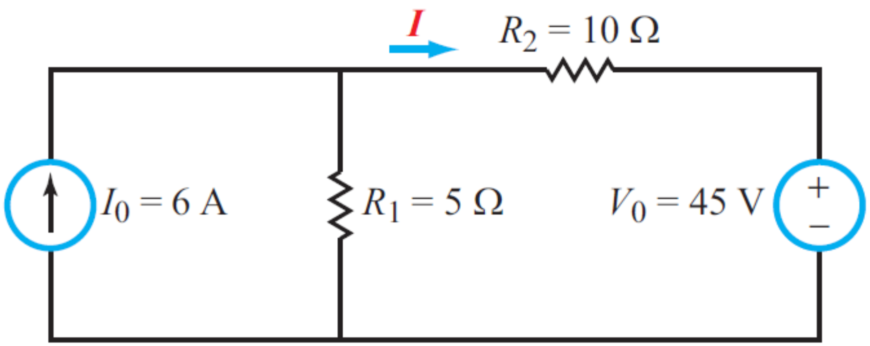
- Linearity property

- Superposition

- Thevenin’s theorem
- Source transformation
- Norton’s theorem

- Power transfer

Example



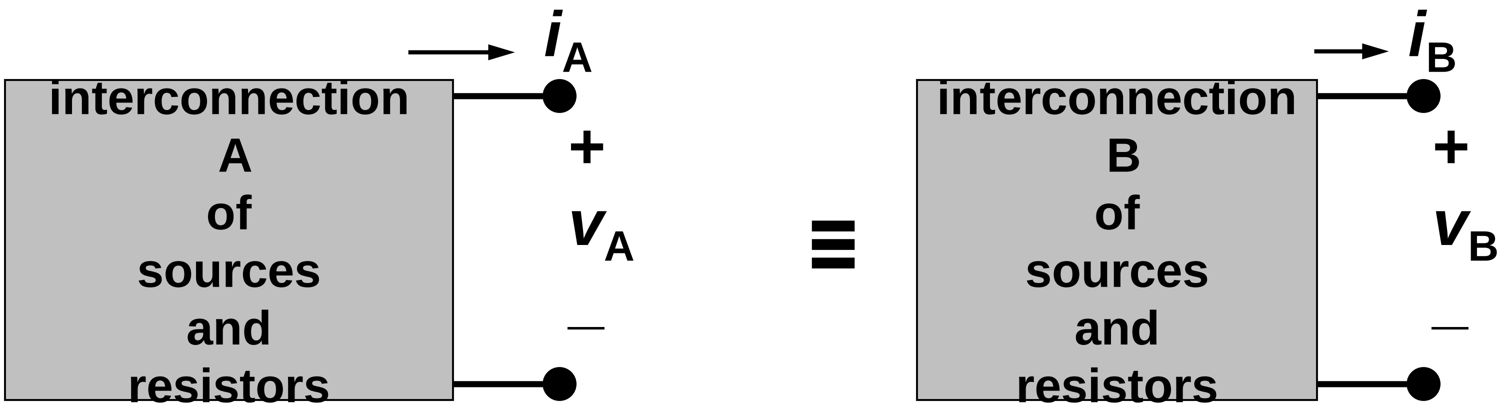
- Q1: If $R_2 = 1\Omega, I = ?$
- Q2: What if $R_2 = 5\Omega, I = ?$
- ??

Thevenin's Theorem – Motivation

- In many circuits, only one element (called *the load*) is variable while others are fixed.
  - An example is the household outlet: different appliances may be plugged into the outlet, each presenting a different resistance.
  - Ordinarily one has to re-analyze the circuit for the load change.
  - This problem can be avoided by circuit theorem (e.g. Thevenin's theorem), which provides a technique to replace the fixed part of the circuit with an **equivalent** circuit.

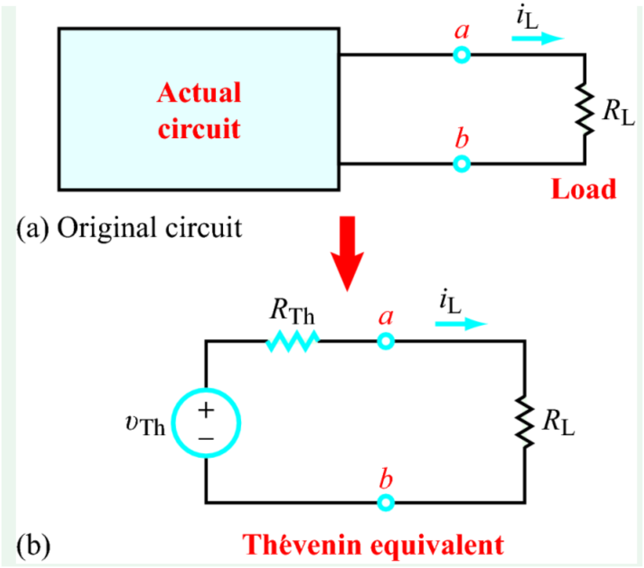
Equivalent Circuit Concept

- A network of voltage sources, current sources, and resistors can be replaced by an **equivalent circuit** which has identical **terminal properties** (I-V characteristics) without affecting the operation of the rest of the circuit.

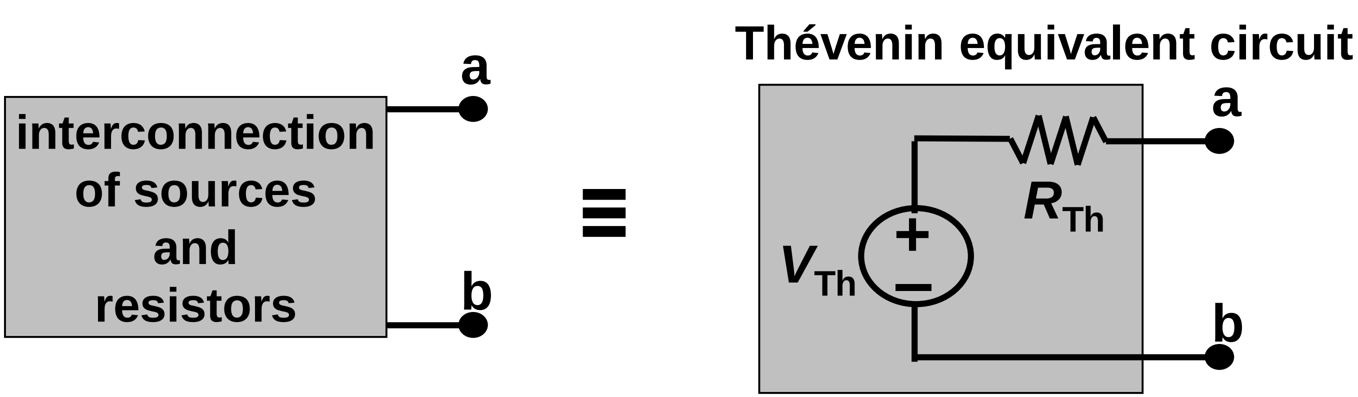


$$i_A(v_A) = i_B(v_B)$$

Thevenin's Theorem (1880s, Leon Thevenin, French)



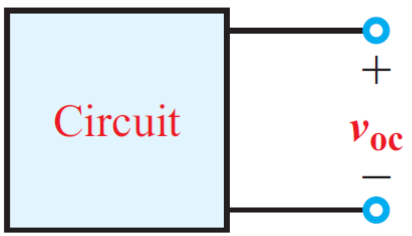
- Thevenin's theorem states that a two terminal circuit (including resistors, linear dependent sources, and independent sources.) may be replaced with a voltage source in series with a resistor:



How Do We Find Thévenin Equivalent Circuits?

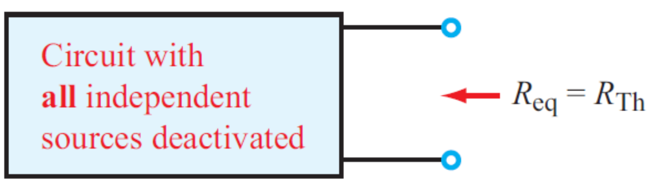
Method 1: Equivalent Resistance

1. Analyze circuit to find v_{oc}
2. Deactivate all **independent** sources by replacing voltage sources with short circuits and current sources with open circuits.
3. Simplify circuit to find equivalent resistance.



(a) $v_{Th} = v_{oc}$

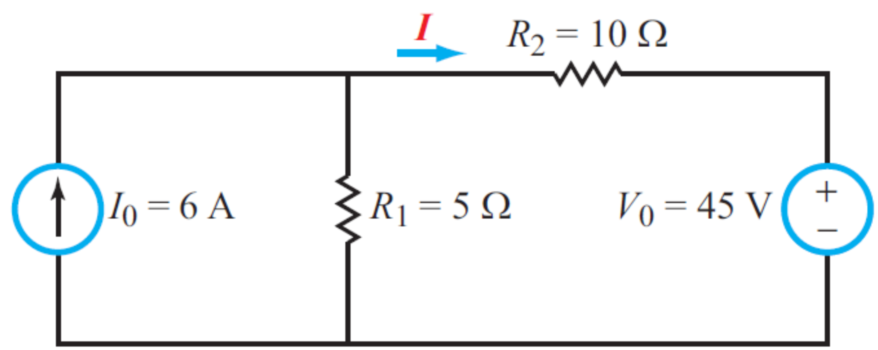
Equivalent-Resistance Method



Note: This method does not apply to circuits that contain dependent sources.

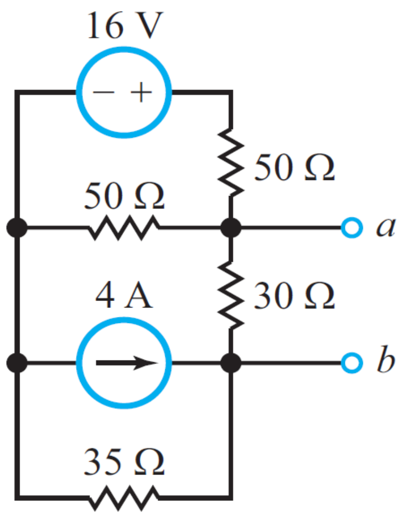
Example

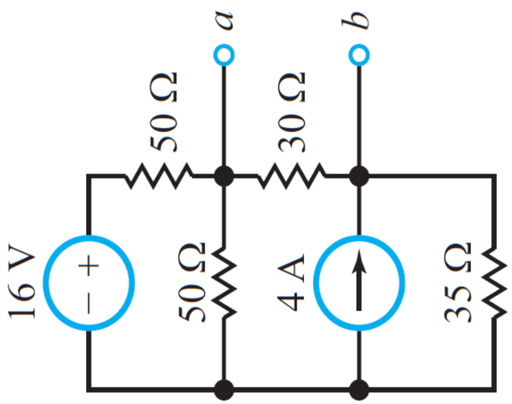
- Use Thévenin Equivalent Circuits
- Q1: If $R_2 = 1\Omega, I = ?$
- Q2: What if $R_2 = 5\Omega, I = ?$



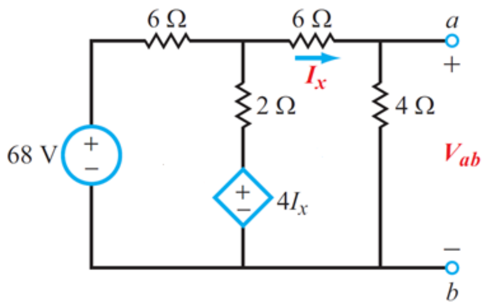
Practice
Thévenin Equivalent Circuit

(Circuit has no dependent sources)





How Do We Find Thévenin Equivalent Circuits ?



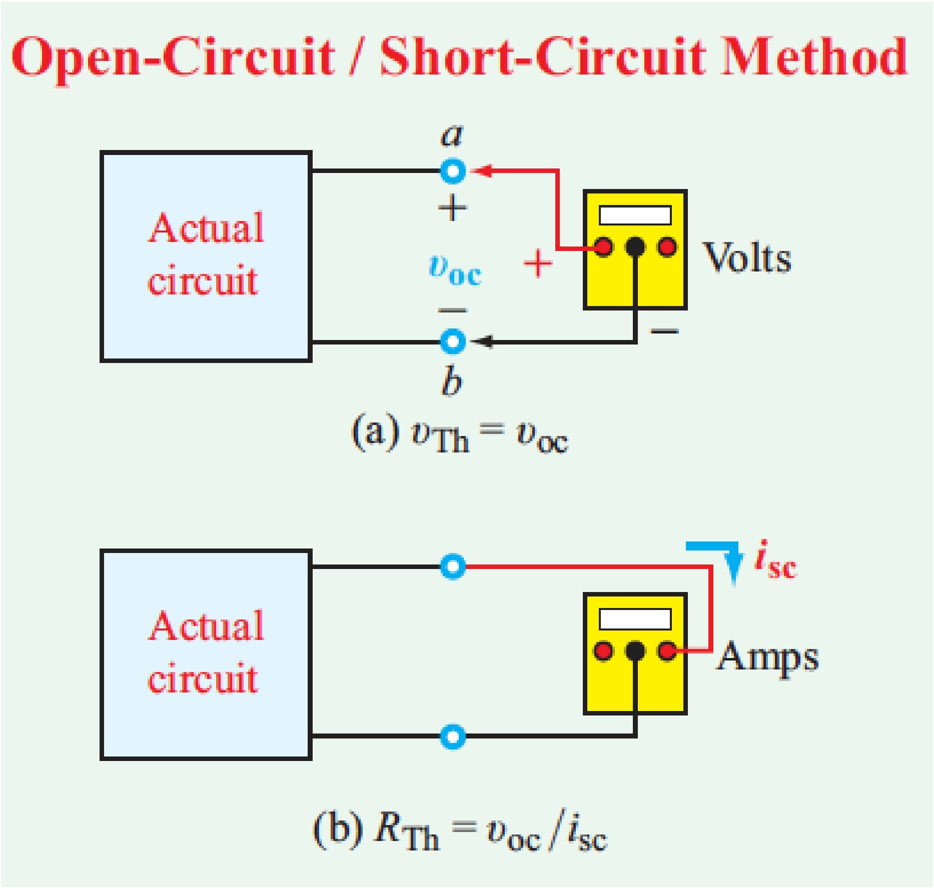
Method 2: Open/short circuit

1. Analyze circuit to find v_{oc}
2. Analyze circuit to find i_{sc}

$$v_{\mathrm{Th}} = v_{\mathrm{oc}}$$

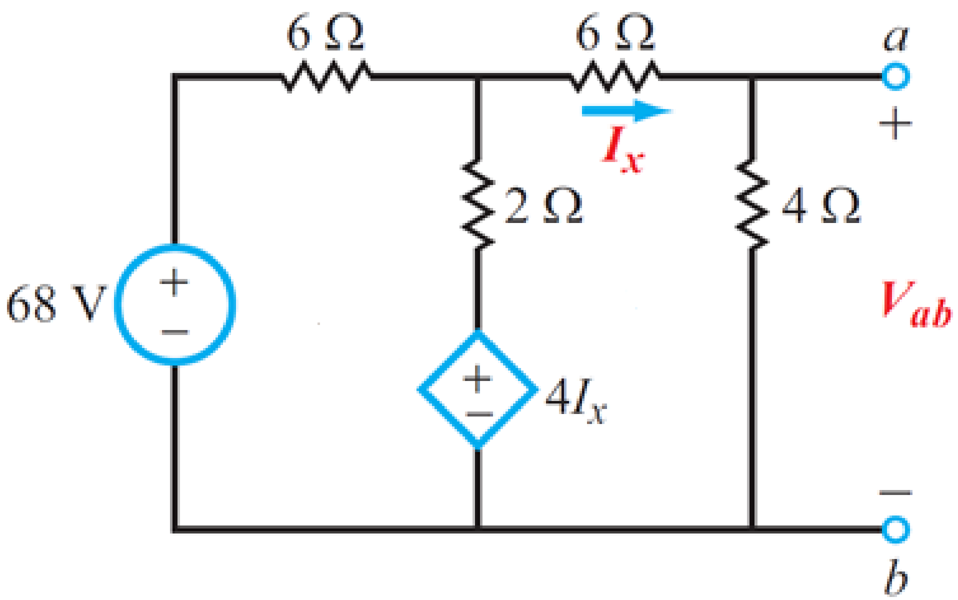
$$R_{\mathrm{Th}} = \frac{v_{\mathrm{Th}}}{i_{\mathrm{sc}}}$$

Note: This method is applicable to any linear circuit, whether or not it contains dependent sources.

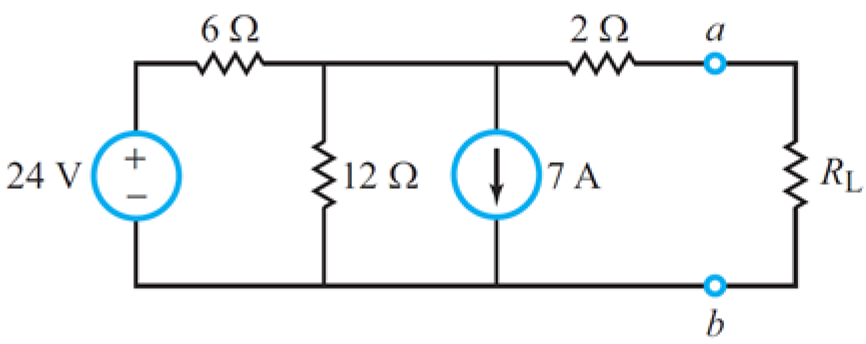


[Source: Berkeley]

Example



Practice

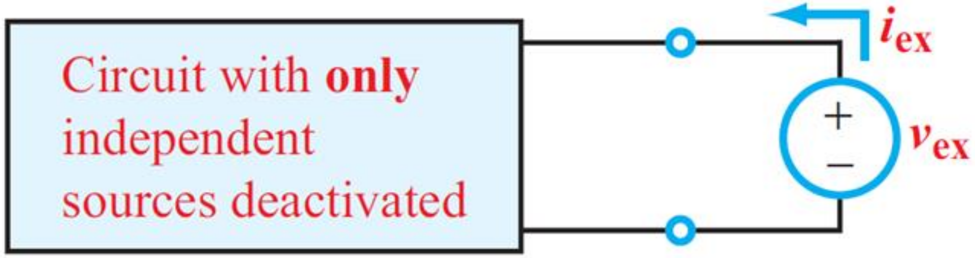


How Do We Find Thévenin Equivalent Circuits?

Method 3:

Step 1. **Still** $v_{\mathrm{Th}} = v_{\mathrm{OC}}$

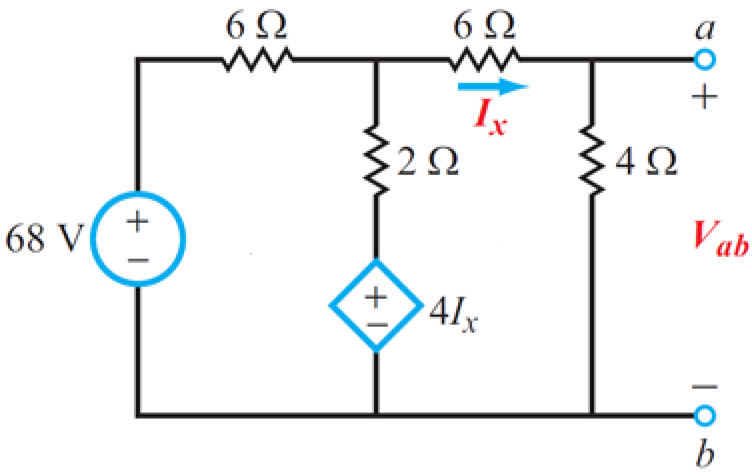
Step 2. **External-Source Method**

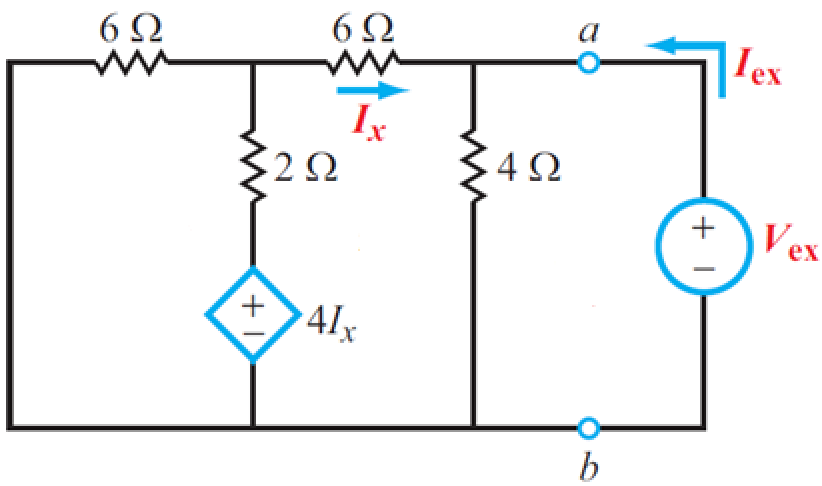


If a circuit contains both dependent and independent sources, R_{Th} can be determined by (a) deactivating independent sources (only), (b) adding an external source v_{ex}, and then (c) solving the circuit to determine i_{ex}. The solution is $R_{\mathrm{Th}} = v_{\mathrm{ex}}/i_{\mathrm{ex}}$.

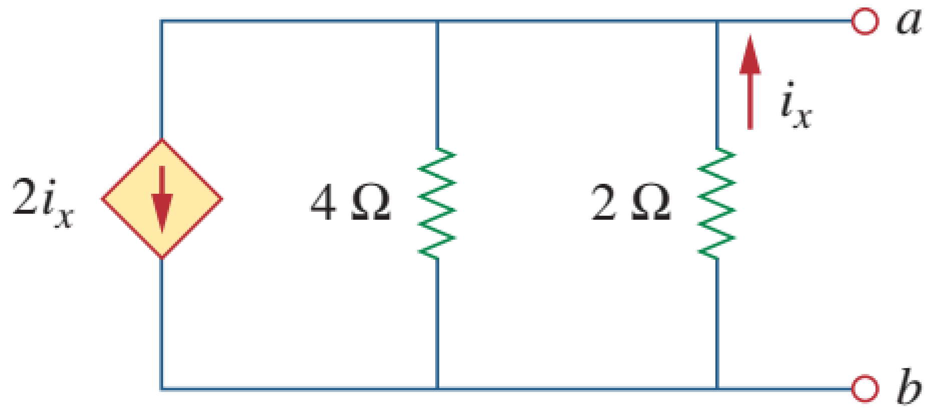
[Source: Berkeley]

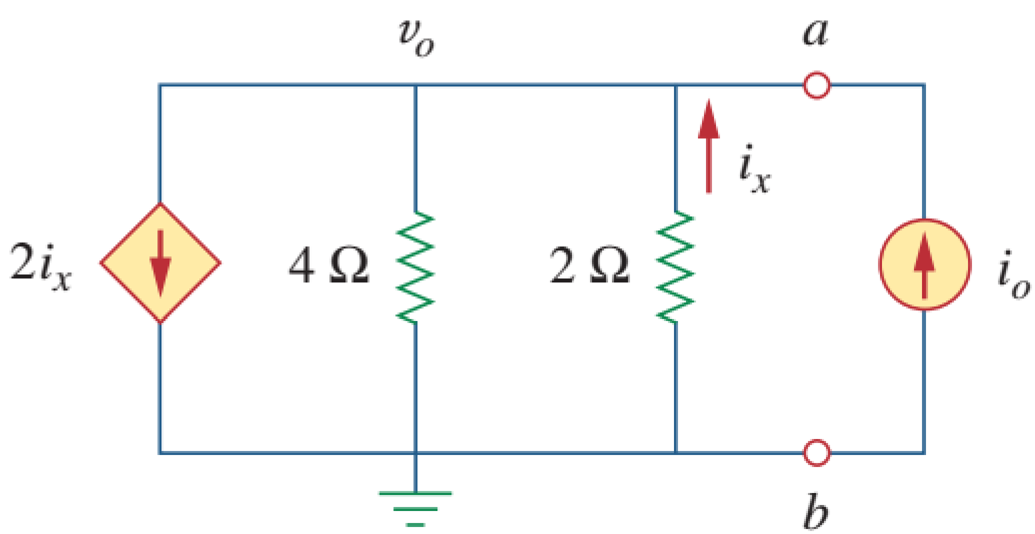
Example





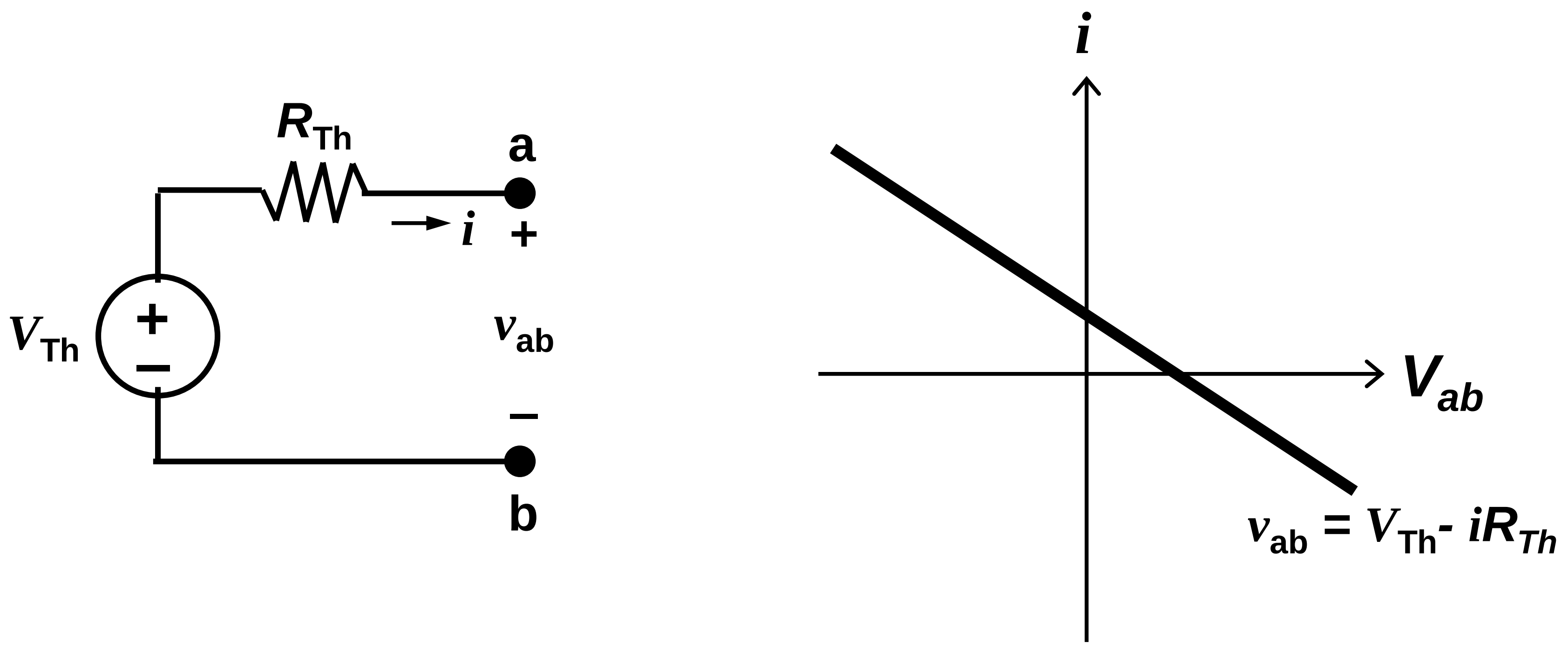
Example





(b)

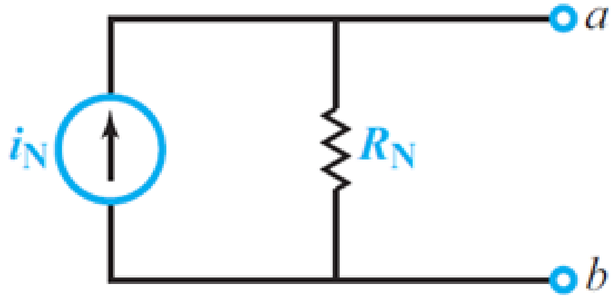
I-V Characteristic of Thévenin Equivalent



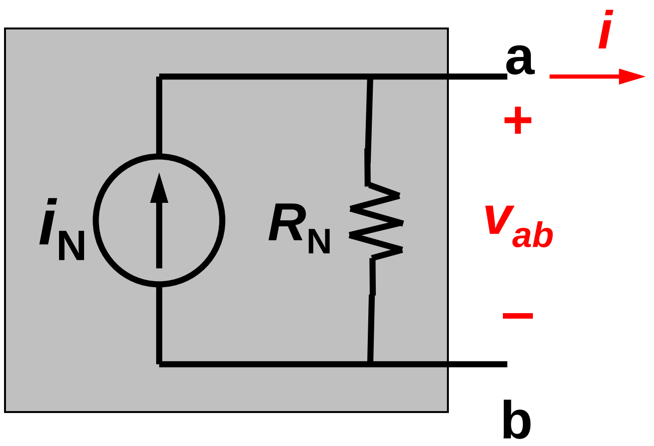
[Source: Berkeley]

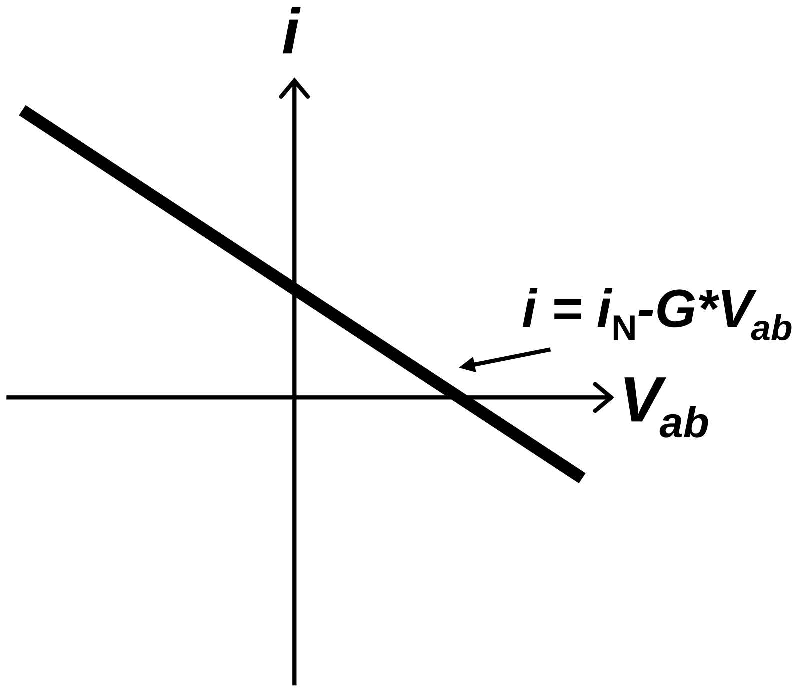
Norton's Theorem

Norton equivalent circuit

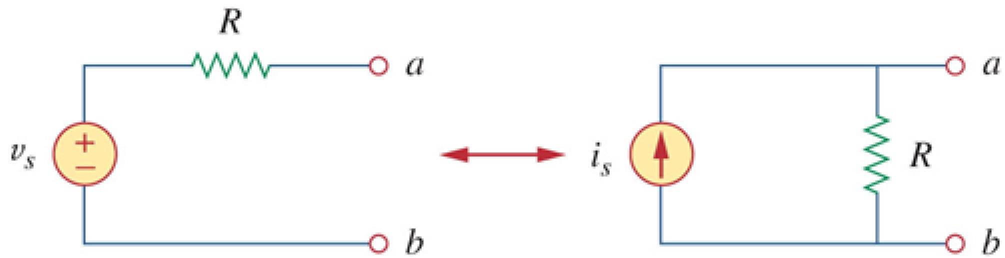


I-V Characteristic of Norton Equivalent





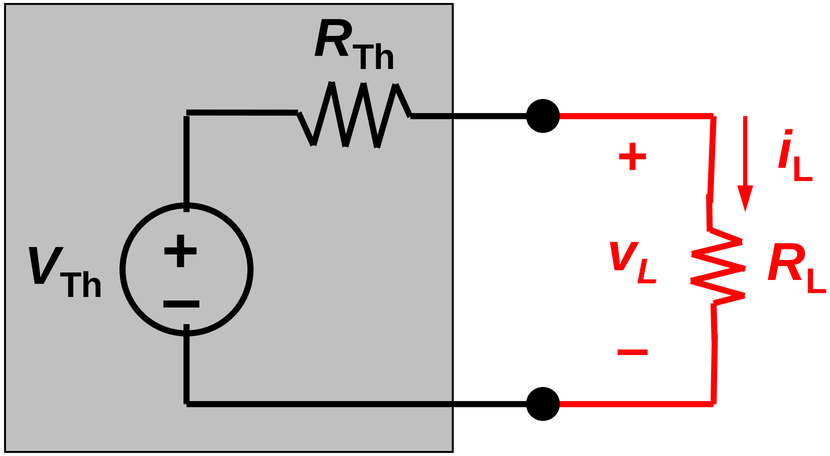
Source Transformation

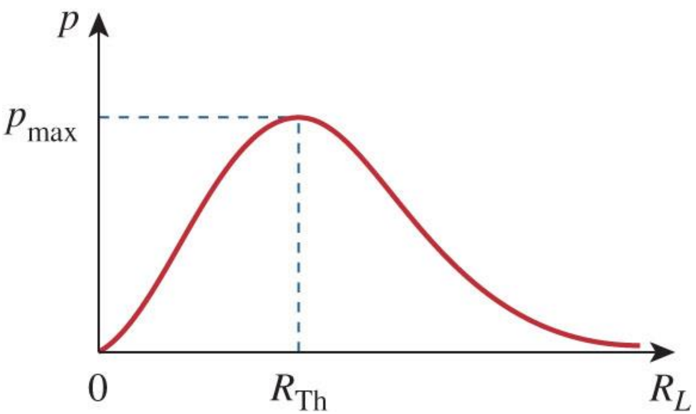


- A source transformation is the process of replacing “a voltage source v_s in series with a resistor *R*” **by** “a current source i_s in parallel with a resistor *R*”, or vice versa. $\boldsymbol{V_s/i_s = R}$

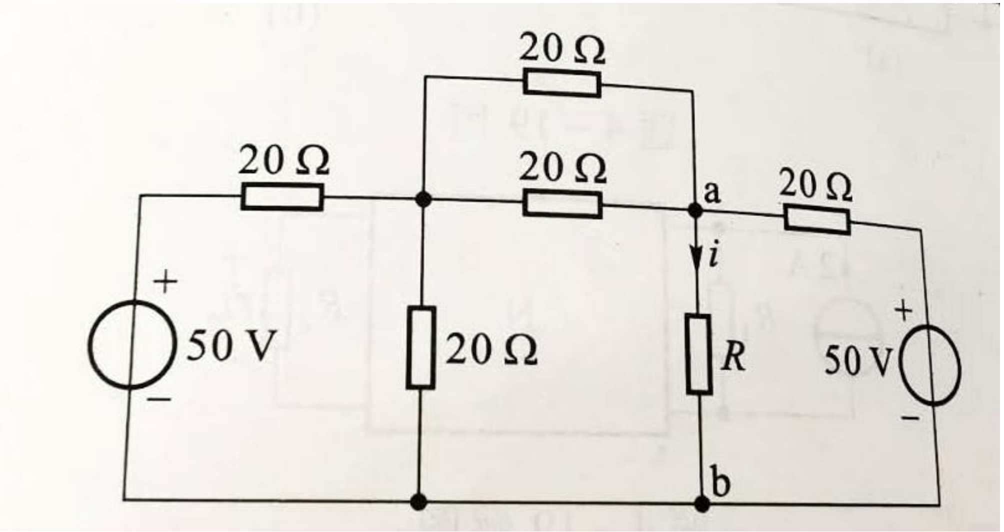
- These transformations work because the two sources have equivalent behavior at their terminals:
  - If the sources are turned off, resistance at the terminals are both R
  - If the terminals are short circuited, the currents is the same.

Max Power Transfer



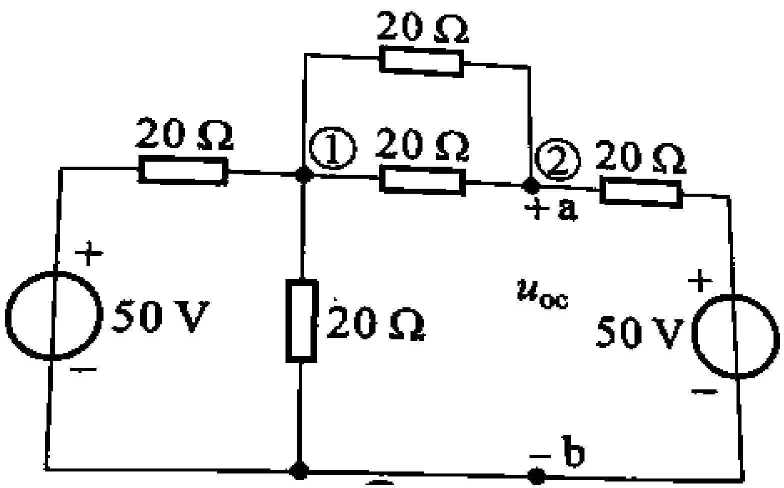


Example



(1) Calculate the value of R, at which maximum power transferred to R holds.

(2) Calculate the percentage/ratio: P_R/P_{total}


20 Ω
20 Ω
①
20 Ω
②
20 Ω
+ a
+
50 V
20 Ω
u_{oc}
+
50 V
−
−
− b