CS 243: Homework #4

Due on 11:59 a.m. Nov 29, 2017

Dengji ZHAO

ShanghaiTech

Contents

Problem 1: Cost Sharing

(a) 1 credit

Consider the 3-player game (N, c) given by the following cost function:

$$c\{1\} = 3; \quad c\{2\} = 4; \quad c\{3\} = 7 \quad c\{1,2\} = 5; \quad c\{1,3\} = 8; \quad c\{2,3\} = 10 \quad c\{1,2,3\} = 11$$

Is (2,3,6) in the core of the game? Give the proof.

Solution
Yes.
Proof:
First, 2+3+6 = 11 = c(N)
Next, $2 \leq c\{1\} = 3 \quad 3 \leq c\{2\} = 4 \quad 6 \leq c\{3\} = 7 \quad 2+3 \leq c\{1,2\} = 5 \quad 2+6 \leq c\{1,3\} = 8$ $3+6 \leq c\{2,3\} = 10$

(b) 2 credit

If $c\{1,2,3\} = 12$ (other costs remain unchanged), give a vector that is in the core of the game or prove that the core is empty.

Solution
The core is empty.
Proof: Assume that the vector is (x,y,z). If the vector is in the core of the game, we must have

$$x + y \leq 5$$

$$x + z \leq 8$$

$$y + z \leq 10$$

By adding the above three inequalities and dividing both sides by 2, we obtain that $x + y + z \leq 11.5 < c\{1,2,3\}$. Therefore the vector cannot be budget balanced.

Problem 2: GSP

(a) 2.5 credits

Three advertisers $\{1,2,3\}$ bid for two ad slots $\{A, B\}$. The average revenue (valuation) per click are \$6, \$4, \$3 for the three bidders, respectively, and the click through rate of the ad slot are 500 and 300 clicks per hour respectively (independent of bidders).

Draw the bipartite graph with nodes indicating advertisers and ad slots, and edges indicating values per hour if the advertiser is allocated to the slot linked by the edge. Under the General Second Price auction, assume all advertisers bid truthfully, what are the allocation and the prices charged for each advertiser?

Solution

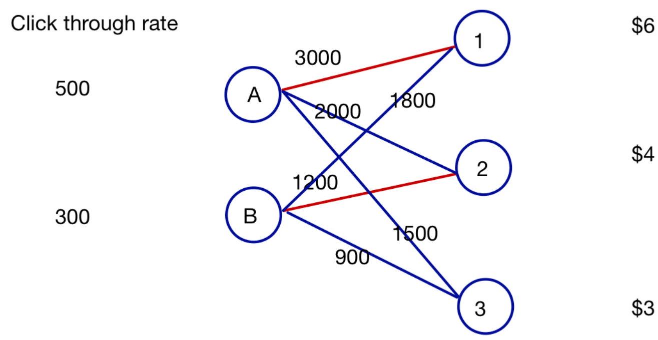
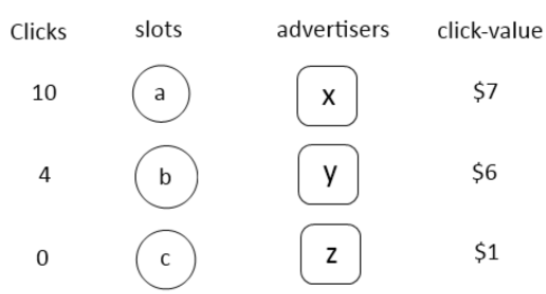


Figure 1: bipartite graph

Advertiser 1 will be allocated to slot A and Advertiser 2 will be allocated to slot B. Advertiser 1 will pay $4*500=$2000, and Advertiser 2 will pay $3*300=$900

(b) 2 credits

The GSP auction is showed as follows with 3 advertisers x, y, z and 3 slots a, b, c.



Is $b_x = 5, b_y = 4, b_z = 2$ a Nash equilibrium under GSP? Is it an unique Nash equilibrium?

Solution

Yes, it is an equilbrium. It is not unique. For example, 3,5,1 for each advertiser.

Problem 3: Public Goods

A mayor wants to decide whether or not to build a bridge according to the valuations of the citizens. The bridge has a cost c to build and each citizen enjoy a private value $v_i \geq 0$ if the bridge is built. The bridge is built if and only if $\sum_i v_i > c$, but all citizens have to share the cost.

(a) 2 credits

Suppose the mayor applies the VCG mechanism for this problem: ask each citizen to report her valuation and then choose the decision and their payments according to their reports. If the bridge is built, what is the price p_i each citizen i has to pay?

Solution

$$p_i = \begin{cases} 0 & \sum_{j \neq i} v_j > c \\ 0 - (\sum_{j \neq i} v_j - c) = c - \sum_{j \neq i} v_j & \sum_{j \neq i} v_j \leq c \end{cases}$$

(b) 2 credits

Show that the total payment $P = \sum_i p_i \leq c$ in this mechanism.

$P = \sum_i p_i$
$\leq \sum_i (c - \sum_{j \neq i} v_j) = n \cdot c - (n-1) \sum_i v_i$
$\leq n \cdot c - (n-1) \cdot c = c.$