Implicit Geometry Modeling

Implicit surfaces & SDF & HW 5

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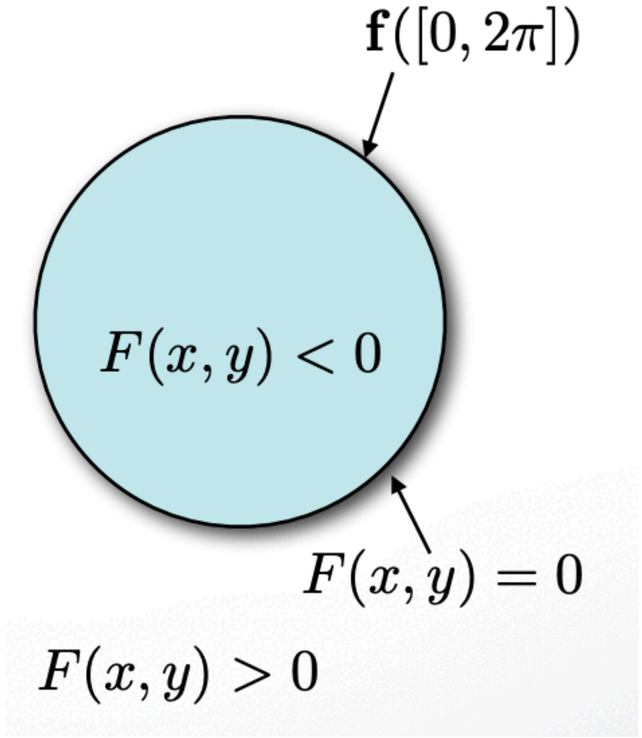
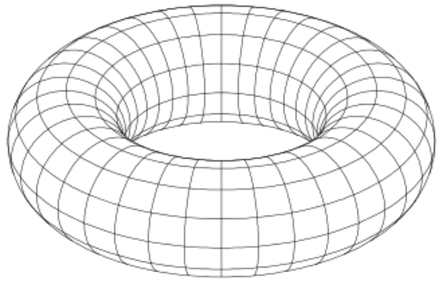
Surface Representation

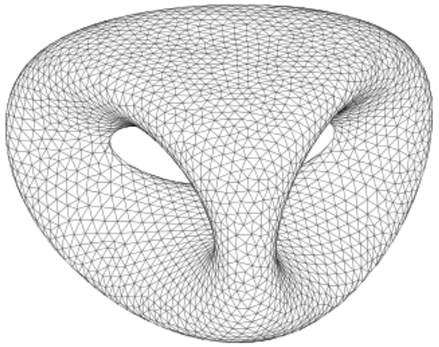
- Explicit: $f(x) = (rcos(x), rsin(x))$
  - Find the coordinates directly by the given parameters.
  - Triangle mesh, points, splines, …
- Implicit: $F(x, y) = \sqrt{x^2 + y^2} - r$
  - Geometry is implied in the given equation.
  - SDF, scalar field, …

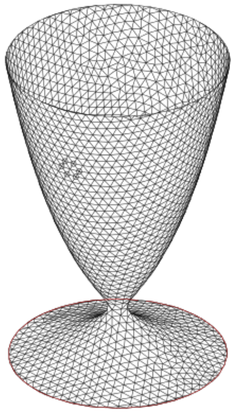


Implicit Surface

- An implicit surface is a surface in Euclidean space defined by an equation.
- In general, $F(x, y, z) : \mathbb{R}^3 \rightarrow \mathbb{R}$. $F(x, y, z) = v$ represents a surface.
- Normal vector: (F_x, F_y, F_z) => gradient
- Normal curvature: $\kappa_n = \dfrac{v^T H v}{\|grad\ F\|}$, H is the Hessian matrix of F.

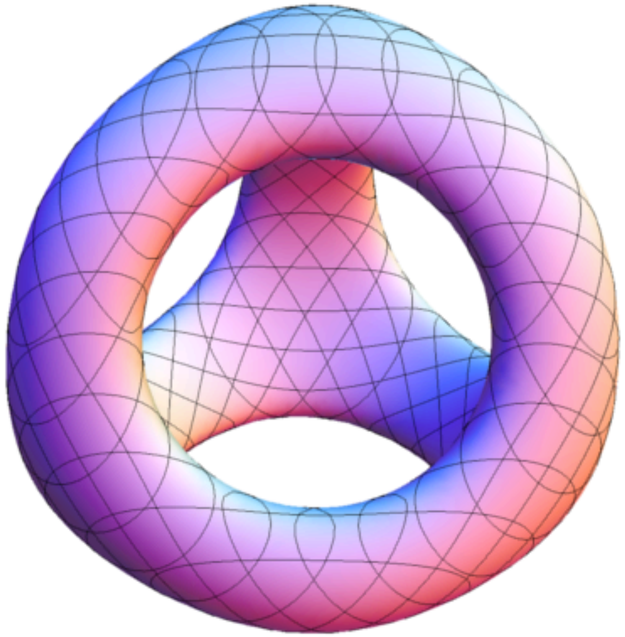




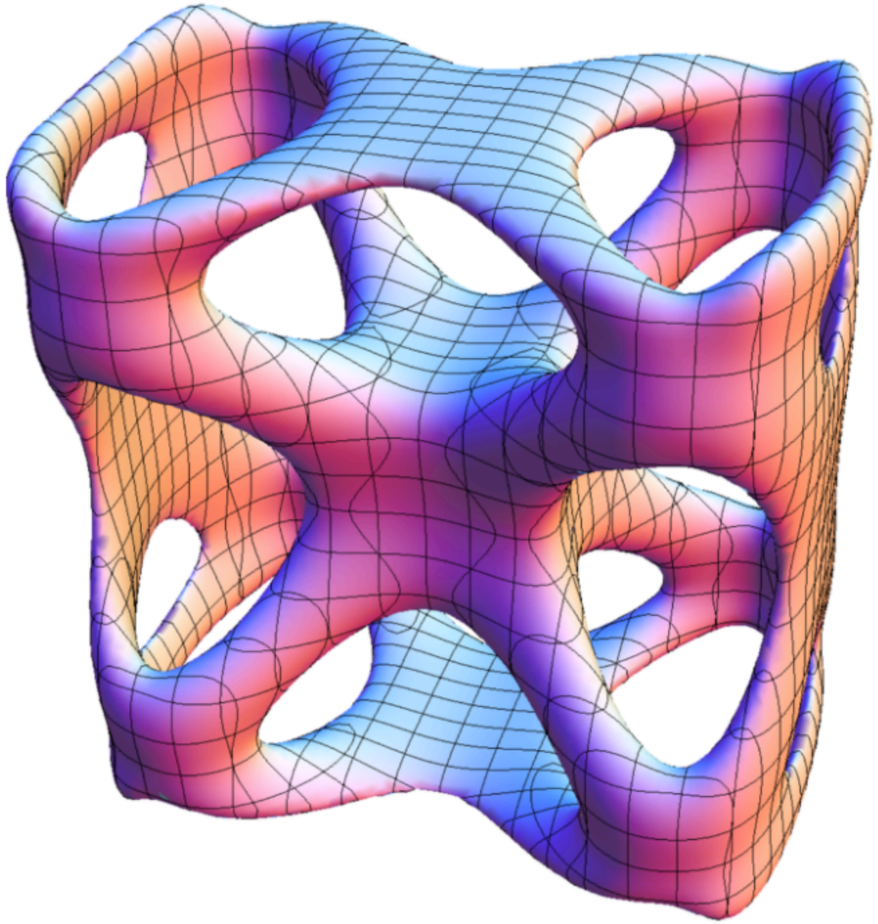


Implicit Surface Examples

$$\{(x,y,z) \in \mathbb{R}^3 ; \quad (x-2)^2(x+2)^2 + (y-2)^2(y+2)^2 + (z-2)^2(z+2)^2 + 3(x^2y^2 + x^2z^2 + y^2z^2) + 6 * x * y * z - 10(x^2 + y^2 + z^2) + 22 = 0\}$$

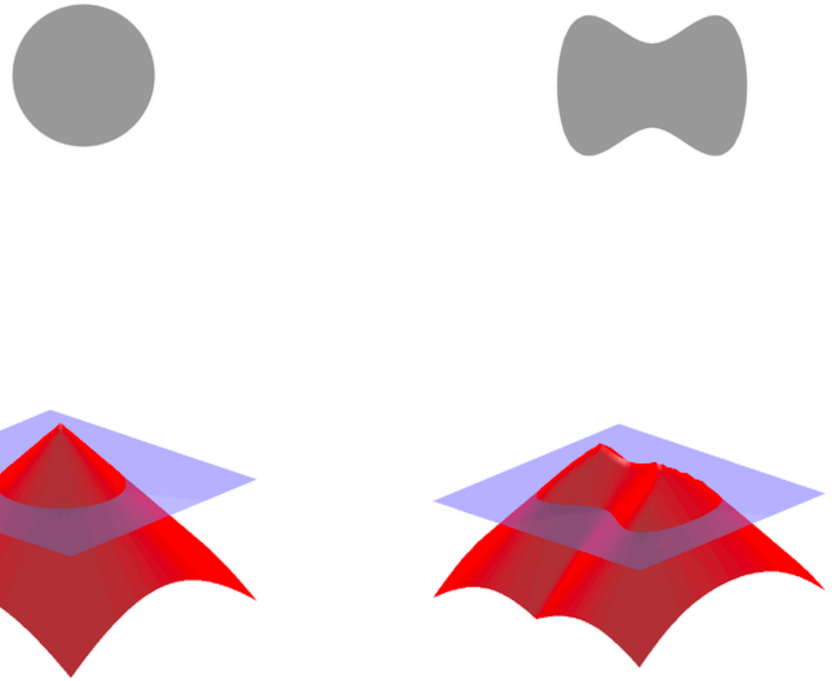


$$\{(x,y,z) \in \mathbb{R}^3 ; \quad (3(x-1)x^2(x+1) + 2y^2)^2 + (z^2 - 0.85)^2 * (3(y-1)y^2(y+1) + 2z^2)^2 + (x^2 - 0.85)^2 * (3(z-1)z^2(z+1) + 2x^2)^2 + (y^2 - 0.85)^2 * -0.12 = 0\}$$



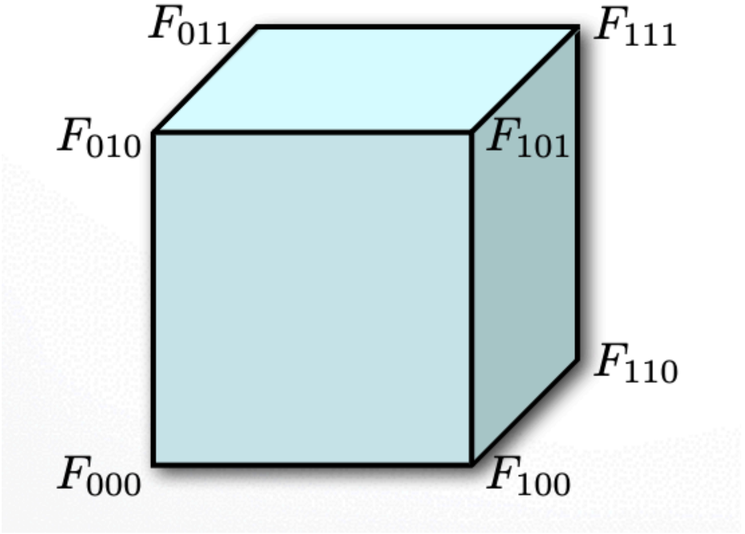
Signed Distance Function (SDF)

- Suppose Ω is a closed subset.
- $f(x) = d(x, \partial\Omega)$ if $x \in \Omega$
- $f(x) = -d(x, \partial\Omega)$ if $x \in \Omega^C$
- Examples: see figures on the right
  - Above: $\Omega \subset \mathbb{R}^2$
  - Below: corresponding $f(x)$
    - $f(x) = 0$: blue plane
  - $d(\cdot)$: Euclidean distance



SDF Discretization

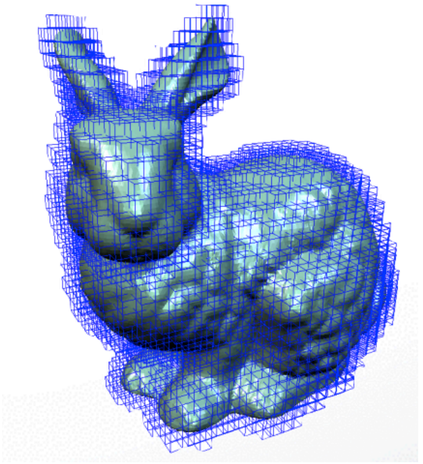
- Regular Cartesian 3D grid
- Compute SDF value at each vertex of cells
- For each point inside cells, use tri-linear interpolation to compute its value.



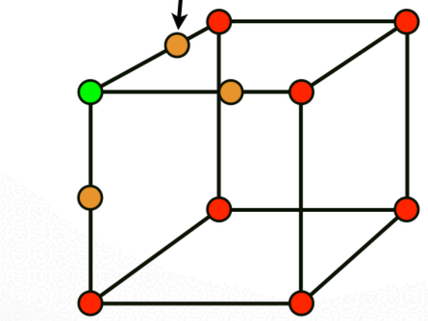
$$
\begin{array}{llll}
F_{000} & (1-u) & (1-v) & (1-w) \; + \\
F_{100} & u & (1-v) & (1-w) \; + \\
F_{010} & (1-u) & v & (1-w) \; + \\
F_{001} & (1-u) & (1-v) & w \; + \\
\vdots & & & \\
F_{111} & u & v & w
\end{array}
$$

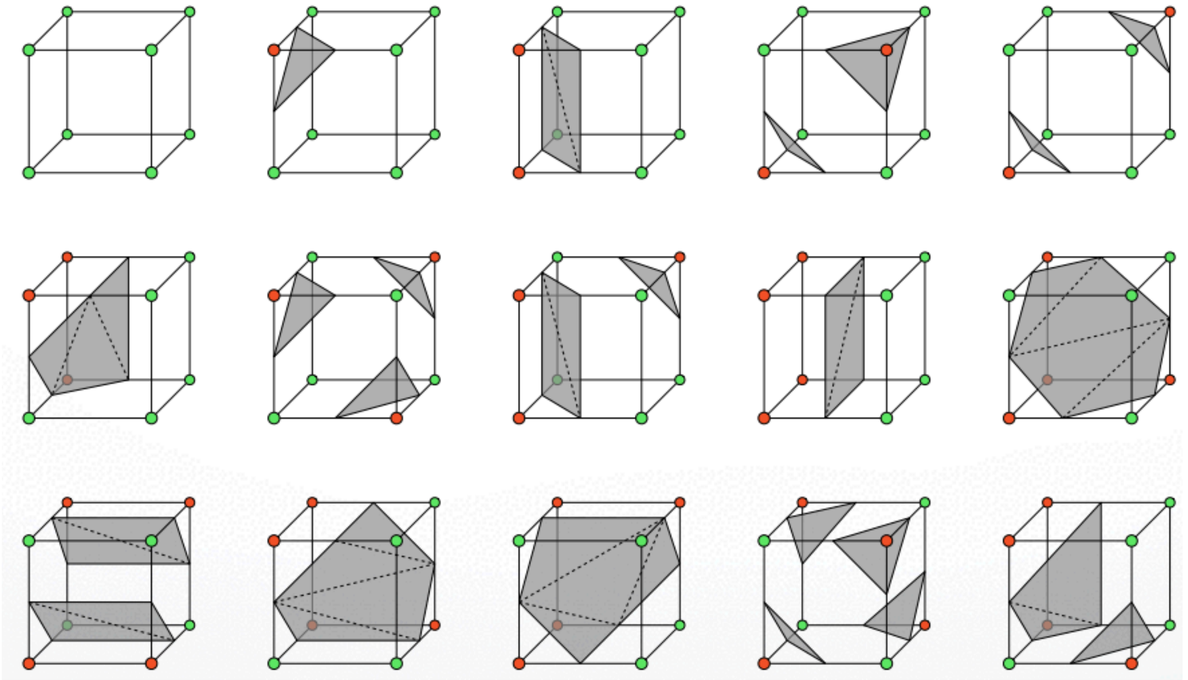
SDF to Mesh

- Extract points on the iso-surface (level set) of F(x), then perform triangulation.
- Marching Cubes
  - Classify each cell vertex (inside or outside).
  - Classify all cells (inside/outside/ intersecting).
  - For intersecting cells, find intersection points on the cell by linear interpolation along edges. Then look up table for patch configuration.



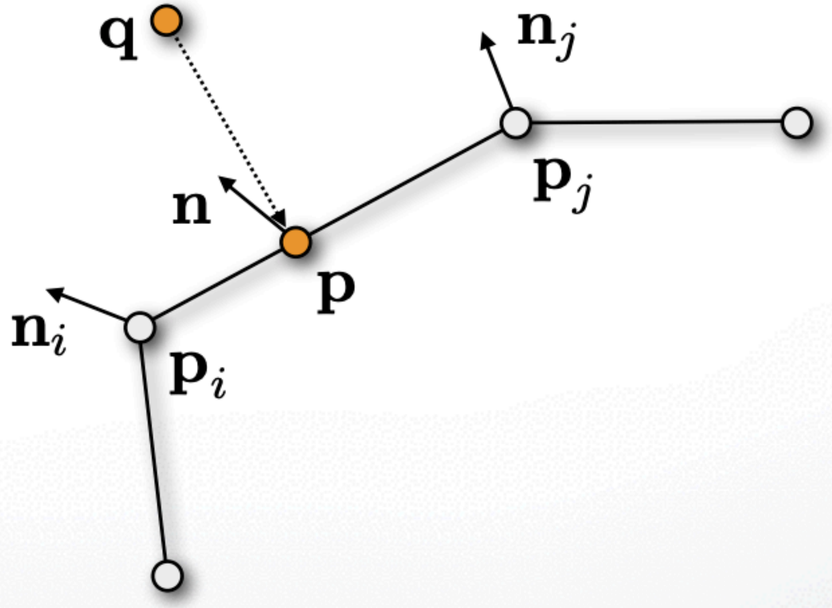
$$\frac{\mathbf{x}_{i,j,k} \cdot |F_{i,j,k+1}| \; + \; \mathbf{x}_{i,j,k+1} \cdot |F_{i,j,k}|}{|F_{i,j,k}| + |F_{i,j,k+1}|}$$





Mesh to SDF

- Find closest mesh triangle and the closest point
  - Write ΔABC as $T(s,t) = (1-s-t)A + sB + tC.$
  - Distance: $Q(s,t) = \|T(s,t) - q\|$ (quadratic)
  - Find the minimal $Q(s*, t*)$, take $T(s*, t*)$ as the closest point on the triangle.
- Inside/ outside
  - $(q-p)^T n < 0$ => inside



Applications of SDF

- KinectFusion, DynamicFusion, Fusion4D, …


$t = 2s$


$t = 10s$


$t = 32s$

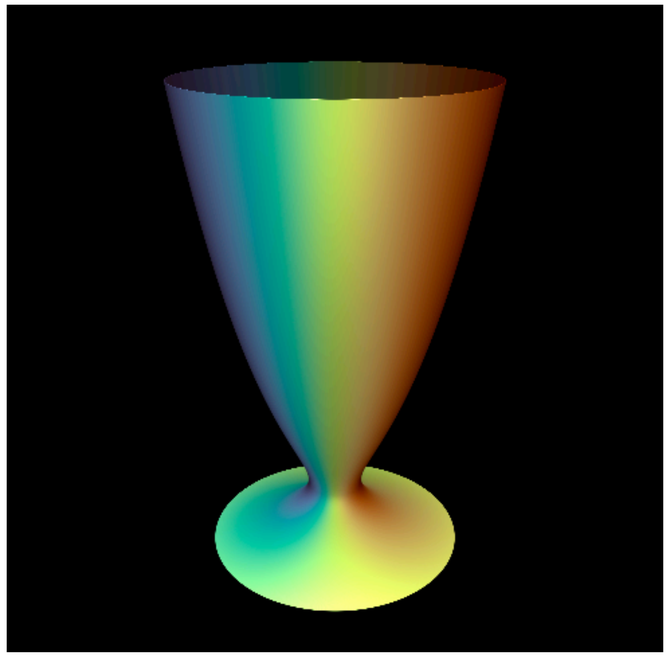
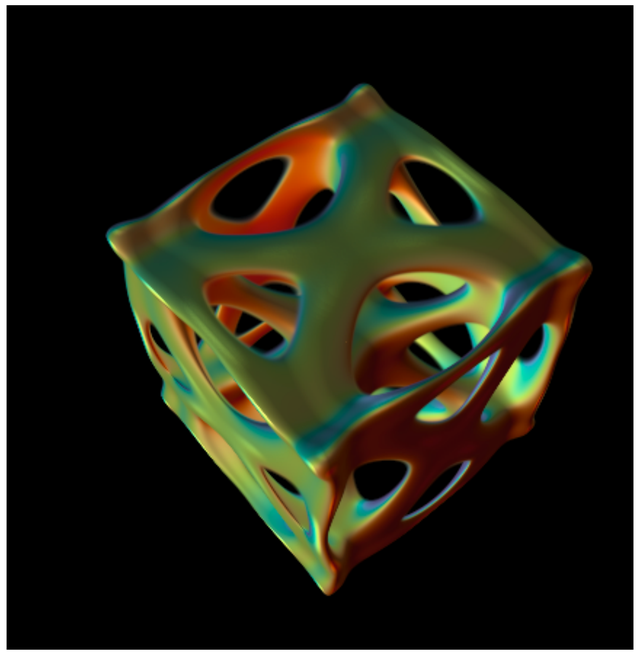
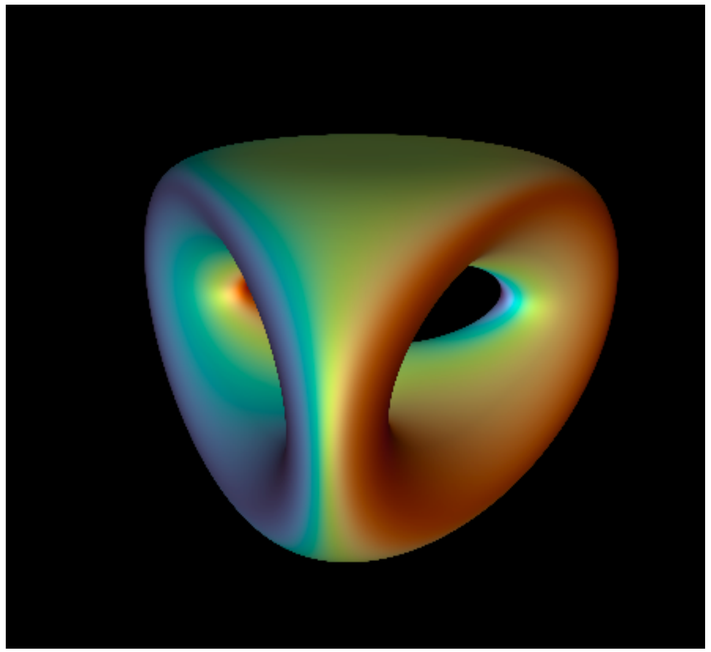

$t = 39s$


$t = 55s$


$t = 60s$

HW5

- Visualize iso-surfaces via Volume Rendering
- Input implicit functions, try to visualize normals, curvatures, Phong shading…
- Design your own transfer function

References

- CSCI 599: Explicit & Implicit Surfaces. Prof. Hao Li.
- SDF: https://en.wikipedia.org/wiki/Signed_distance_function
- Implicit Surface: https://en.wikipedia.org/wiki/Implicit_surface
- Interesting implicit surfaces in $\mathbb{R}^3$: https://math.stackexchange.com/questions/46212/interesting-implicit-surfaces-in-mathbbr3
- Distance between Point and Triangle in 3D: https://www.geometrictools.com/Documentation/DistancePoint3Triangle3.pdf
- DynamicFusion: Reconstruction and Tracking of Non-rigid Scenes in Real-Time
- Fusion4D: Real-time Performance Capture of Challenging Scenes

Thanks! Q&A