

Signals and Systems Homework 10

Due Time: 21:59 May 25, 2018

**Submitted in-class on Thu (May 24),
or to the box in front of SIST 1C 403E (the instructor's office).**

1. Using partial-fraction expansion and the fact that

$$a^n u[n] \longleftrightarrow \frac{1}{1 - az^{-1}}, |z| > |a|$$

find the inverse z-transform of

$$X(z) = \frac{1 - \frac{1}{3}z^{-1}}{(1 - z^{-1})(1 + 2z^{-1})}, |z| > 2$$

2. Consider a left-sided sequence $x[n]$ has z-transform

$$X(z) = \frac{1}{(1 - \frac{1}{2}z^{-1})(1 - z^{-1})}$$

- (a) Write $X(z)$ as a ratio of polynomials in z instead of z^{-1} .
- (b) Using a partial-fraction expansion, express $X(z)$ as a sum of terms, where each term represents a pole from your answer in part (a).
- (c) Determine $x[n]$.

3. Consider an even sequence $x[n]$ ($x[n] = x[-n]$) with rational z-transform $X(z)$.

- (a) From the definition of the z-transform, show that $X(z) = X(\frac{1}{z})$.
- (b) From your results in part (a), show that if a pole(zero) of $X(z)$ occurs at $z = z_0$, then a pole(zero) must also occur at $z = \frac{1}{z_0}$.
- (c) Verify the results in part (b) for each of the following sequences:
 - (i) $\sigma[n+1] + \sigma[n-1]$
 - (ii) $\sigma[n+1] - \frac{5}{2}\sigma[n] + \sigma[n-1]$

4. The input $x(t)$ and output $y(t)$ of a causal LTI system are related through the block-diagram representation shown in the following figure.

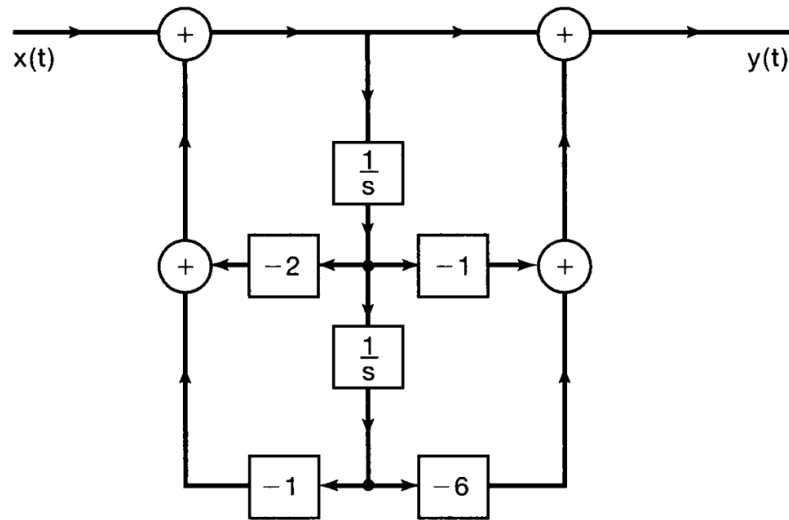


Figure 1:

- Determine a differential equation relating $y(t)$ and $x(t)$.
- Is this system stable?

5. Consider a fourth-order causal LTI system S whose system function is specified as

$$H(s) = \frac{1}{(s^2 - s + 1)(s^2 + 2s + 1)}$$

- (a) Draw a block diagram representation for S as a *cascade* interconnection of two second-order system, each of which is represented in direct form. There should be no multiplications by nonreal coefficients in the resulting block diagram.
- (b) Draw a block diagram representation for S as a *parallel* interconnection of two second-order system, each of which is represented in direct form. There should be no multiplications by nonreal coefficients in the resulting block diagram.