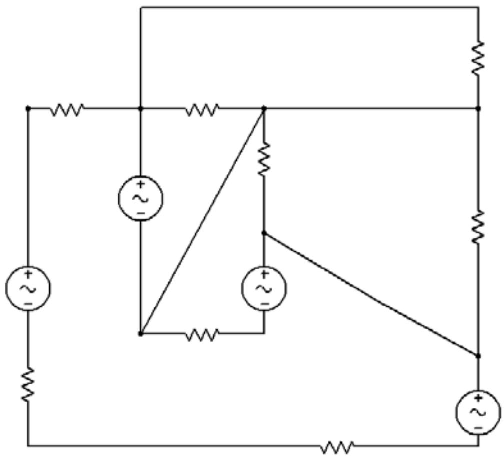
# Homework 2

Due date:
Mar.19th, 2018
Turn in your homework in class

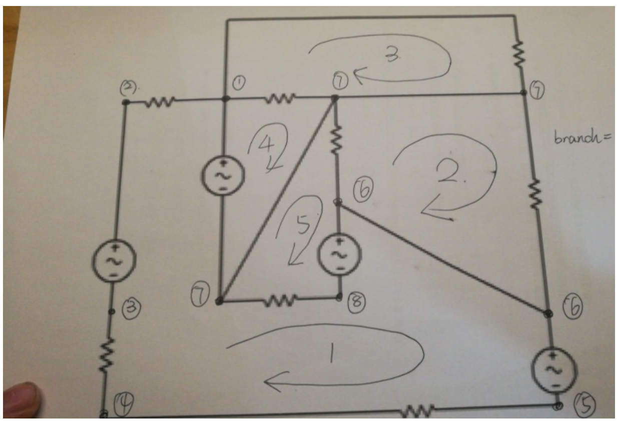
Rules:

- Work on your own. Discussion is permissible, but extremely similar submissions will be judged as plagiarism.
- Please show all intermediate steps: a correct solution without an explanation will get zero credit.
- Please submit on time. No late submission will be accepted.
- Please prepare your submission in English only. No Chinese submission will be accepted.

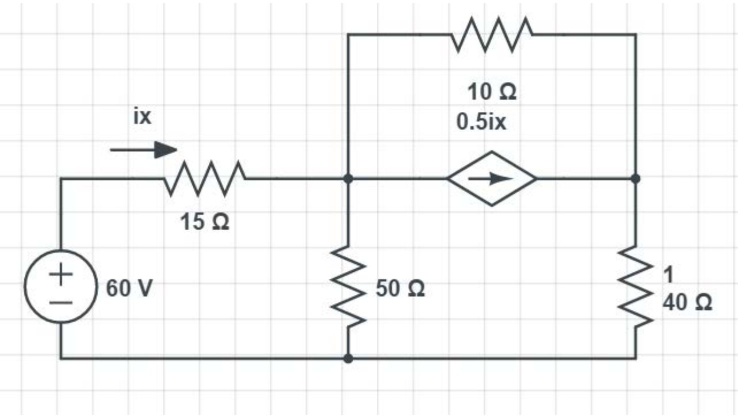
1.Determine the number of independent loops, branches and nodes.



Solution:
5 independent loops, 8 nodes, 12 branches.

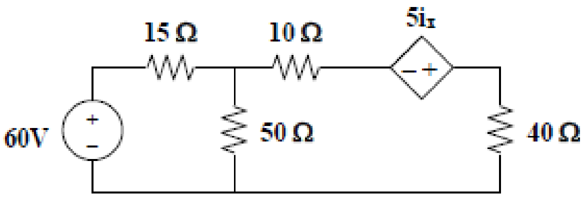


2. Using source transformation, find the value of $i_x$ in the circuit.

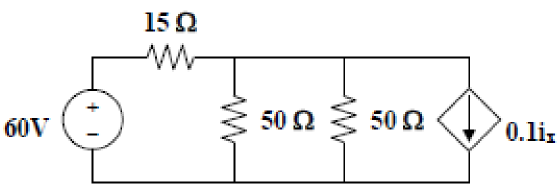



Solution:

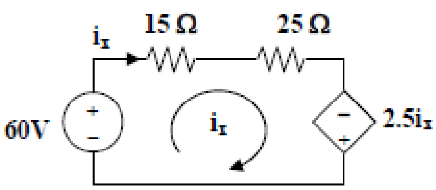
As shown in Fig. (a), we transform the dependent current source to a voltage source,




(a)




(b)

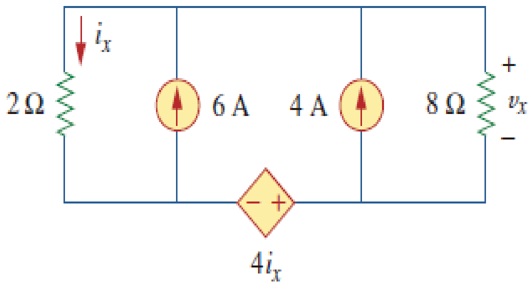



(c)

In Fig. (b), $50||50 = 25$ ohms. Applying KVL in Fig. (c),

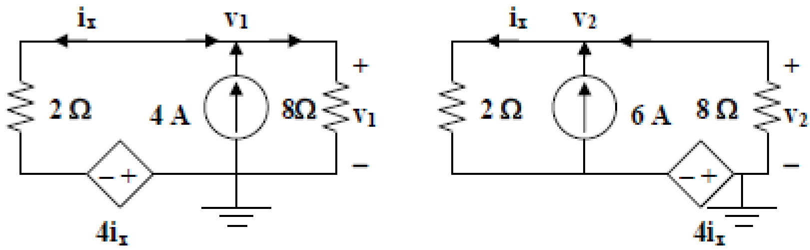
$-60 + 40i_x - 2.5i_x = 0$, or $i_x = \mathbf{1.6\ A}$

3. Using superposition, find the value of $v_x$.




Solution:

Let $v_x = v_1 + v_2$, where $v_1$ and $v_2$ are due to the 4-A and 6-A sources respectively.




(a) (b)

To find $v_1$, consider the circuit in Fig. (a).

$$v_1/8 - 4 + (v_1 - (-4i_x))/2 = 0 \text{ or } (0.125+0.5)v_1 = 4 - 2i_x \text{ or } v_1 = 6.4 - 3.2i_x$$

But, $i_x = (v_1 - (-4i_x))/2$ or $i_x = -0.5v_1$. Thus,

$$v_1 = 6.4 + 3.2(0.5v_1), \text{ which leads to } v_1 = -6.4/0.6 = -10.667$$

To find $v_2$, consider the circuit shown in Fig. (b).

$$v_2/8 - 6 + (v_2 - (-4i_x))/2 = 0 \text{ or } v_2 + 3.2i_x = 9.6$$

But $i_x = -0.5v_2$. Therefore,

$$v_2 + 3.2(-0.5v_2) = 9.6 \text{ which leads to } v_2 = -16$$

Hence, $v_x = -10.667 - 16 = \mathbf{-26.67V}$.

Checking,

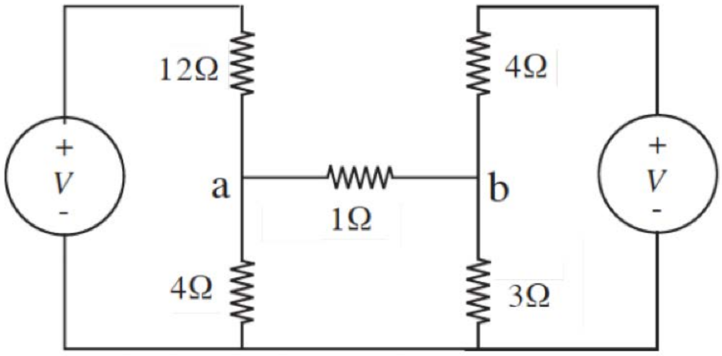
$$i_x = -0.5v_x = 13.333\text{A}$$

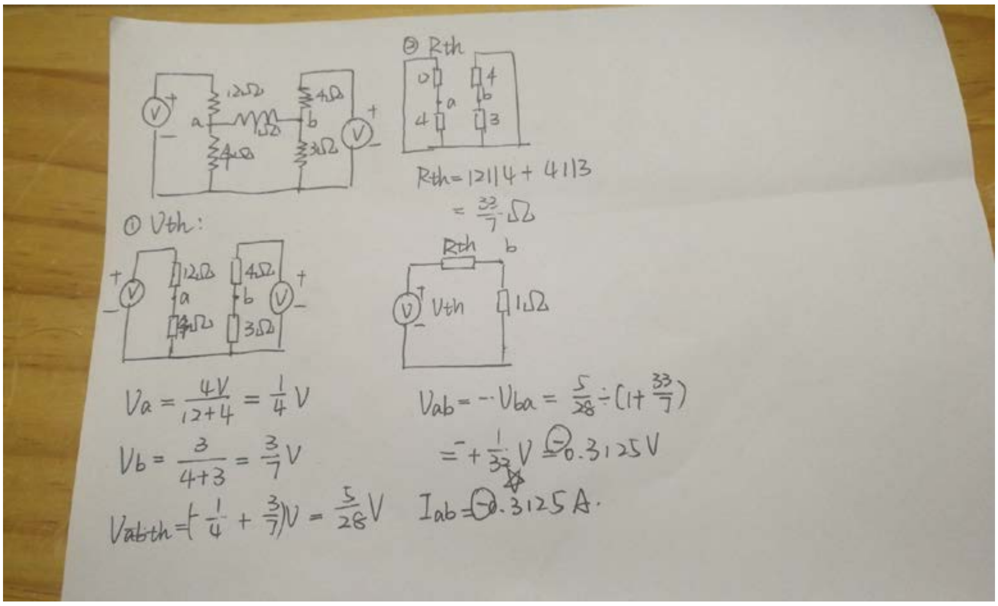
Now all we need to do now is sum the currents flowing out of the top node.

$$13.333 - 6 - 4 + (-26.67)/8 = 3.333 - 3.333 = 0$$

4. Use Thévenin method to find the voltage and current between two nodes a and b, Vab and Iab.

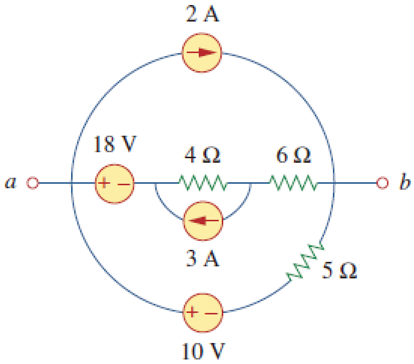
(Hint: Find the Thevenin equivalent circuit between node a and b, excluding the 1 ohm resistance. Afterwards, find Vab.)



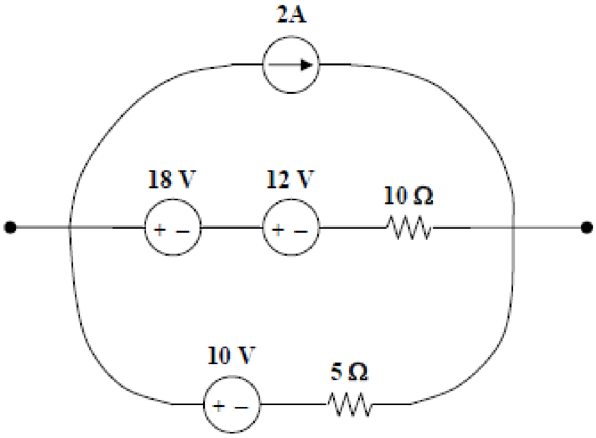
5. Try to find the Thevenin's and Norton's equivalent circuits of these following figures.

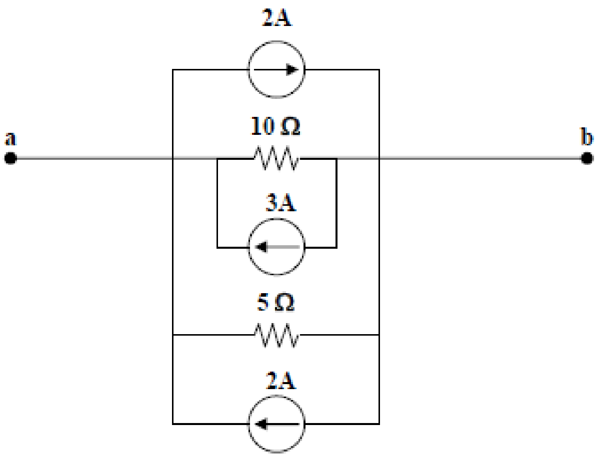
(1) Find the Norton and Thevenin equivalent with respect to the terminals a-b in the circuit **only with independent source**.(Hint: you can use source transformation to simplify the procedure.)

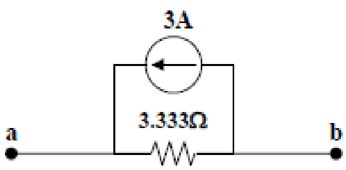


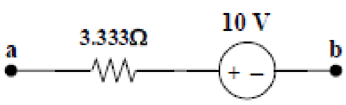

Solution:

The circuit can be reduced by source transformations.



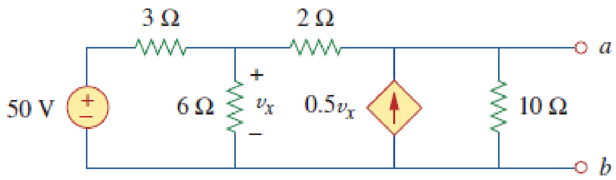



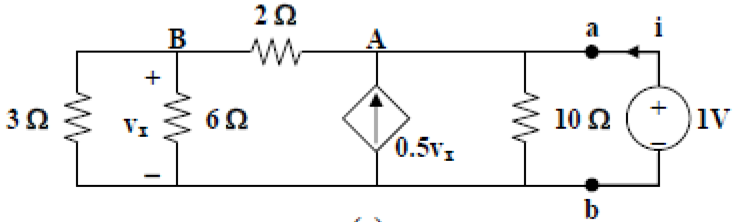

**Norton Equivalent Circuit** **Thevenin Equivalent Circuit**

(2) Find the Norton and Thevenin equivalent in the circuit with respect to the terminals a-b **with dependent and independent source.**




Solution:

To find $R_{Th}$, remove the 50V source and insert a 1-V source at a – b, as shown in Fig. (a).




**(a)**

We apply nodal analysis. At node A,

$$i + 0.5v_x = (1/10) + (1 - v_x)/2, \text{ or } i + v_x = 0.6 \quad (1)$$

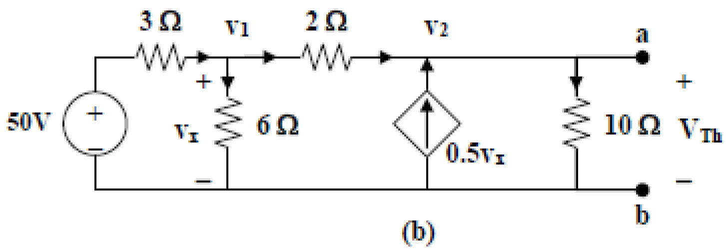
At node B,

$$(1 - v_o)/2 = (v_x/3) + (v_x/6), \text{ and } v_x = 0.5 \quad (2)$$

From (1) and (2), $i = 0.1$ and

$$R_{Th} = 1/i = \mathbf{10\ ohms}$$

To get $V_{Th}$, consider the circuit in Fig. (b).




**(b)**

At node 1, $(50 - v_1)/3 = (v_1/6) + (v_1 - v_2)/2, \text{ or } 100 = 6v_1 - 3v_2$ (3)

At node 2, $0.5v_x + (v_1 - v_2)/2 = v_2/10, \ v_x = v_1, \text{ and } v_1 = 0.6v_2$ (4)

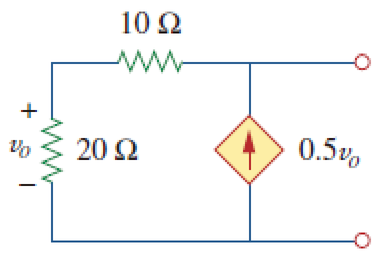
From (3) and (4),

$$v_2 = V_{Th} = \mathbf{166.67\ V}$$

$$I_N = V_{Th}/R_{Th} = \mathbf{16.667\ A}$$

$$R_N = R_{Th} = \mathbf{10\ ohms}$$

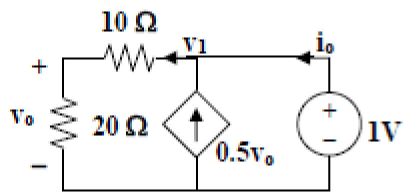
(3) Find the Norton equivalent circuit in the figure **only with dependent source**.




Solution:

Because there are no independent sources, $I_N = I_{sc} = 0$ **A**

$R_N$ can be found using the circuit below.



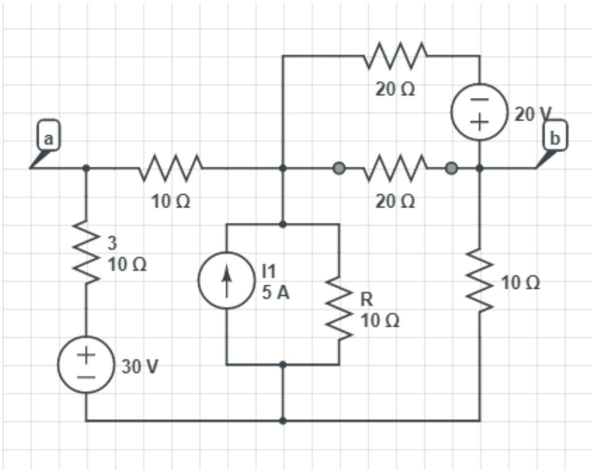

Applying KCL at node 1, $v_1 = 1$, and $v_o = (20/30)v_1 = 2/3$

$i_o = (v_1/30) - 0.5v_o = (1/30) - 0.5\text{x}2/3 = 0.03333 - 0.33333 = -0.3$ A.

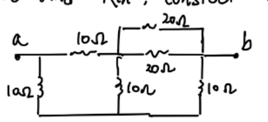
Hence,

$R_N = 1/(-0.3) = $ **–3.333 ohms**

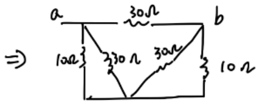
6. In practice, Thevenin or Norton equivalent are used to make complicated circuit clearer. For the circuit in the figure, find the Thevenin equivalent between terminals a and b.




To find $R_{th}$, consider the circuit in Figure (a)




(a)

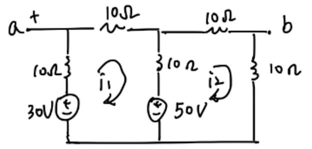



(b)

$20 \parallel 20 = 10\Omega$, Transform the wye sub-network to a delta as shown in Figure (b).

$10 \parallel 30 = 7.5\Omega$, $R_{th} = R_{ab} = 30 \parallel (7.5+7.5) = 10\Omega$

To find $V_{th}$, we transform the 20-V (to a current source in a parallel with the 20Ω resistor and then back into a voltage source in series with the parallel combination of the two 20Ω resistors) and the 5-A sources. We obtain the circuit shown in Figure (c).




For loop 1, $-30 + 50 + 30i_1 - 10i_2 = 0$ or $-2 = 3i_1 - i_2$ ①

For loop 2, $-50 - 10 + 30i_2 - 10i_1 = 0$ or $6 = -i_1 + 3i_2$ ②

Solving (1) and (2) $i_1 = 0$, $i_2 = 2A$

Applying KVL to the output loop, $-V_{ab} - 10i_1 + 30 - 10i_2 = 0$. $V_{ab} = 0V$

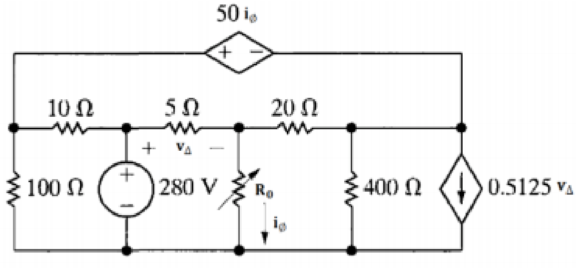
$V_{th} = V_{ab} = 10\,V$

7. The variable resistor in the circuit in the figure below is adjusted for maximum power transfer to $R_0$.

(1) Find the numerical value of $R_0$.

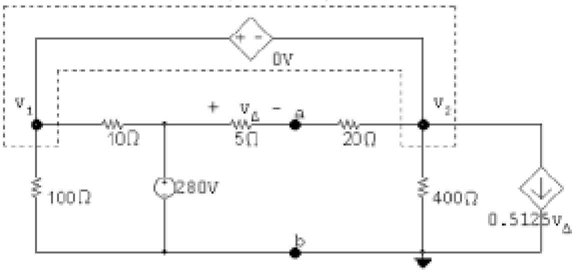
(2) Find the maximum power delivered to $R_0$.

(3) How much power does the 280V source deliver to the circuit when $R_0$ is adjusted to the value found in (1)?




[a] First find the Thévenin equivalent with respect to $R_o$.

Open circuit voltage: $i_\phi = 0$; $50i_\phi = 0$



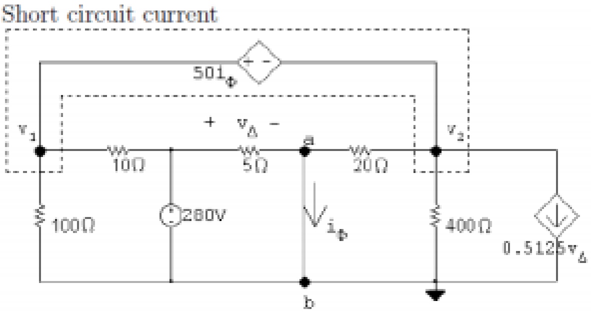

$$\frac{v_1}{100} + \frac{v_1 - 280}{10} + \frac{v_1 - 280}{25} + \frac{v_1}{400} + 0.5125v_\Delta = 0$$

$$v_\Delta = \frac{(280 - v_1)}{25}5 = 56 - 0.2v_1$$

$$v_1 = 210 \text{ V}; \qquad v_\Delta = 14 \text{ V}$$

$$V_{\text{Th}} = 280 - v_\Delta = 280 - 14 = 266 \text{ V}$$

Short circuit current




$$\frac{v_1}{100} + \frac{v_1 - 280}{10} + \frac{v_2}{20} + \frac{v_2}{400} + 0.5125(280) = 0$$

$$v_\Delta = 280 \text{ V}$$

$$v_2 + 50i_\phi = v_1$$

$$i_\phi = \frac{280}{5} + \frac{v_2}{20} = 56 + 0.05v_2$$

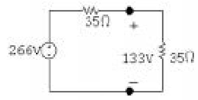
$$v_2 = -968 \text{ V}; \qquad v_1 = -588 \text{ V}$$

$$i_\phi = i_{\text{sc}} = 56 + 0.05(-968) = 7.6 \text{ A}$$

$$R_{\text{Th}} = V_{\text{Th}}/i_{\text{sc}} = 266/7.6 = 35\,\Omega$$

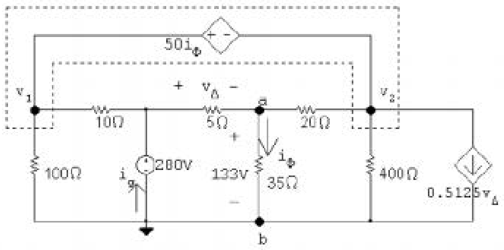
$$\therefore \quad R_o = 35\,\Omega$$

[b]




$$p_{\max} = (133)^2/35 = 505.4 \text{ W}$$

[c]




$$\frac{v_1}{100} + \frac{v_1 - 280}{10} + \frac{v_2 - 133}{20} + \frac{v_2}{400} + 0.5125(280 - 133) = 0$$

$$v_2 + 50i_\phi = v_1; \qquad i_\phi = 133/35 = 3.8 \text{ A}$$

Therefore, $v_1 = -189$ V and $v_2 = -379$ V; thus,

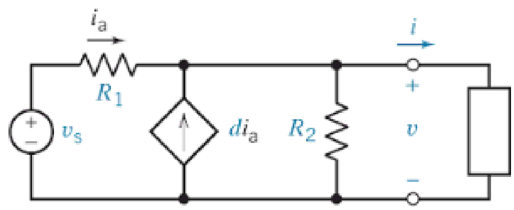
$$i_g = \frac{280 - 133}{5} + \frac{280 + 189}{10} = 76.30 \text{ A}$$

$$p_{280\text{V}} \text{ (dev)} = (280)(76.3) = 21{,}364 \text{ W}$$

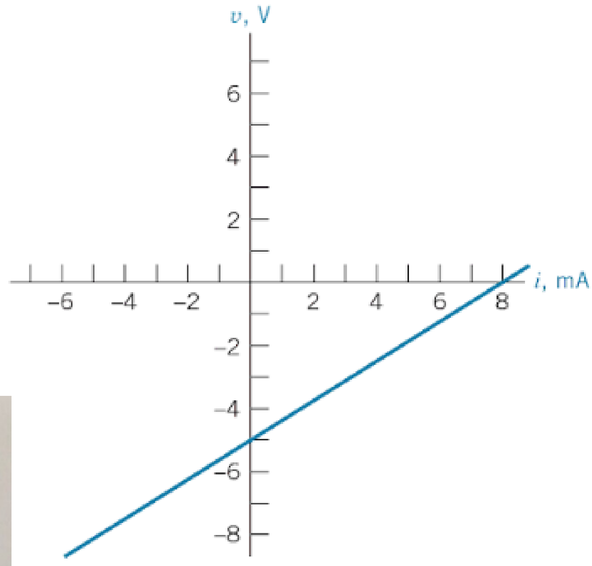
8. Xiaoming Wang, an SPST student, did an experiment on an electric circuit as shown in figure (a). He wanted to find the relationship between output voltage and output current. He found that the relationship is linear as shown in figure (b). To further find out the relationship among parameters inside the circuit, he would like to ask his SIST friends for help. So as an SIST student, please use all circuit theorems to help him.

The circuit shown in Figure has four unspecified circuit parameters : $v_s$, $R_1$, $R_2$ and $d$, ($R_1 = R_2 = R$) where $d$ is the gain of the dependent current source. ***Solve R and Vs as the function of d*** so that the output voltage and current has the the V-I characteristics shown in Figure (b).

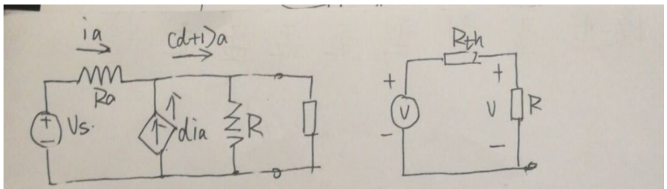
(Hint: You can solve the problem by understanding the meaning of slope and v-intercept in the figure. Use what you have learnt before. You will know how the linearity the circuit works.)



(a)



(b)



① $V_{th} = (d+1) i_a R$

$i_a = \frac{V_s - V_{th}}{R_a}$

$i_a R_a = V_s - (d+1) R i_a$

$i_a = \frac{V_s}{dR+R+R_a}$

$\therefore V_{th} = \frac{(d+1)RV_s}{(d+1)R+R_a}$

② $I_{sc} = (d+1) i_a$ . ($i_a = \frac{V_s}{R_a}$)

$= (d+1)\frac{V_s}{R_a}$

$\Rightarrow R_{th} = \frac{RR_a}{(d+1)R+R_a}$

③ $V_{th} = -5V = \frac{(d+1)RV_s}{(d+1)R+R_a}$

$R_{th} = -\frac{5}{8}k\Omega = \frac{RR_a}{(d+1)R+R_a}$

$R_a = R = R$.

$\frac{(d+1)V_s}{d+2} = -5$ $\quad -\frac{5}{8}\times 1000 = \frac{R}{d+2}$

$V_s = \frac{-5(d+2)}{d+1}(V)$ $\quad R = -\frac{5}{8}(d+2)(k\Omega)$

V = -Rth*i+Vth