Lecture 16
--Review

Outline

- Circuit Basics
- Temporal Analysis
- AC circuits
- Laplace Transform

Outline 2

- Circuit Basics

---- PSC, KCL, KVL

---- Circuit theorems

Important circuit analysis skills for circuits in time domain(DC and temporal analysis), circuits in phasor domain, circuits in s-domain.

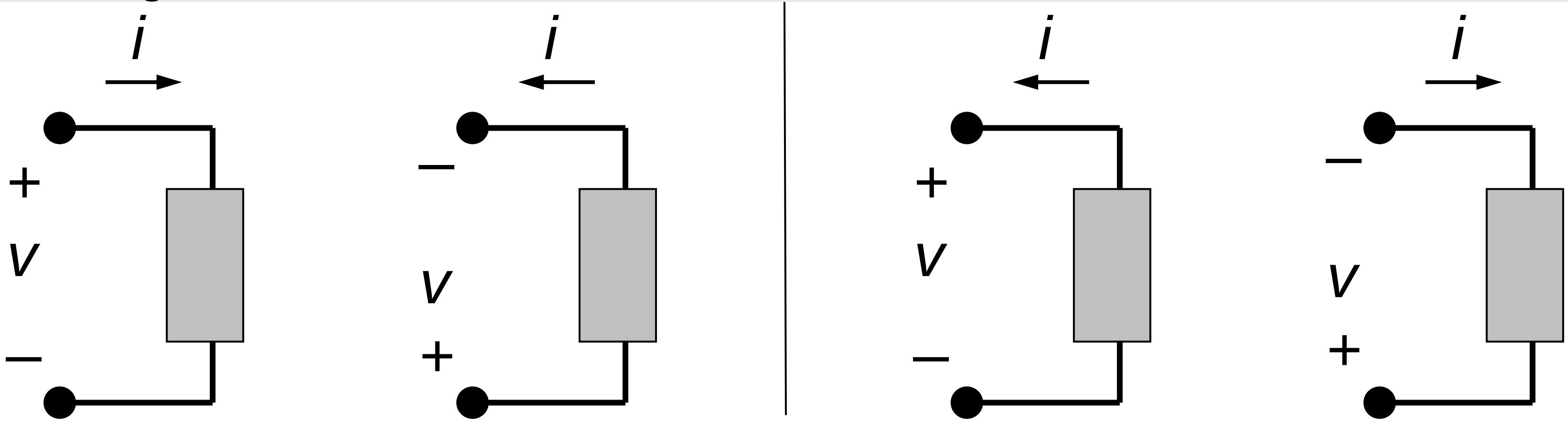
- Temporal Analysis

--1st-order, 2nd-order circuits

- AC circuits
- Laplace Transform

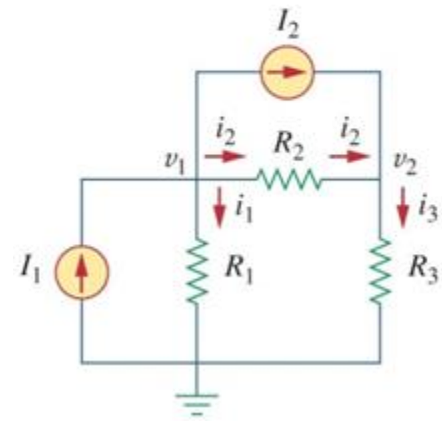
Passive Sign Convention

Whenever the reference direction for the current in an element is in the direction of the reference voltage drop across the element, use positive sign in any expression that relates the voltage to the current. Otherwise, use a negative sign.

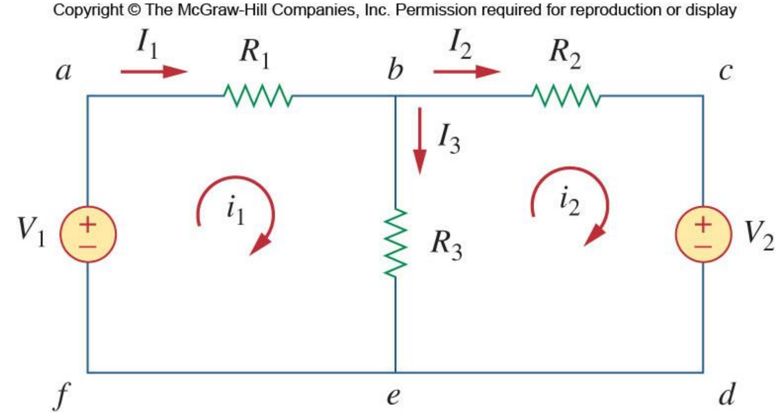


- If $p > 0$, power is absorbed by the element.
  - electrical energy into heat (resistors in toasters), light (light bulbs), or acoustic energy (speakers); by storing energy (charging a battery).
- If $p < 0$, power is extracted from the element.

- Node Analysis
  - Node voltage is the unknown
  - Solve by KCL
  - Special case: Floating voltage source



- Mesh Analysis
  - Loop current is the unknown
  - Solve by KVL
  - Special case: Current source

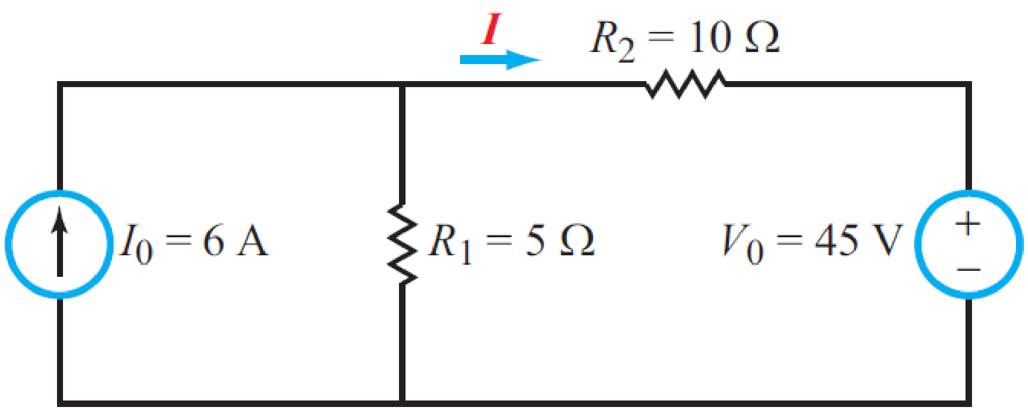


Circuit theorem

- Linearity property
- Superposition
- Thevenin’s theorem
- Source transformation
- Norton’s theorem

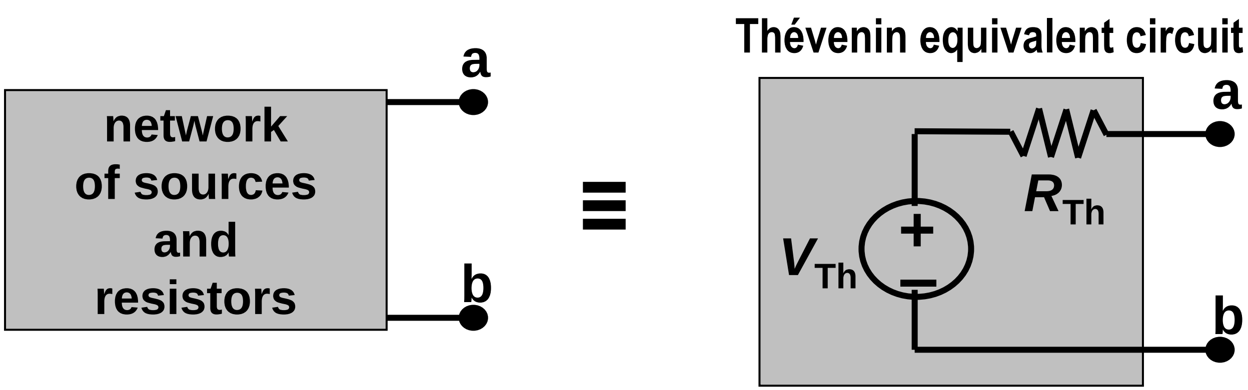
Superposition

- The superposition principle states that the voltage across (or current through) an element in *a linear circuit* is the algebraic sum of the voltages across (or currents through) that element due to each independent source acting alone.



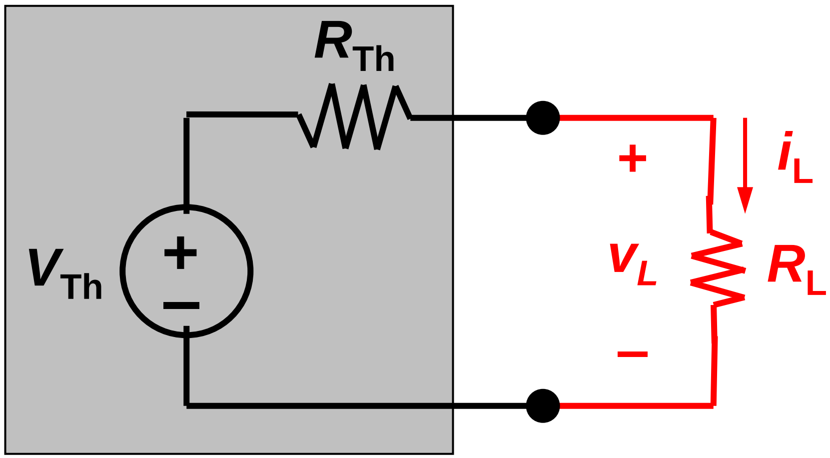
Thevenin's Theorem

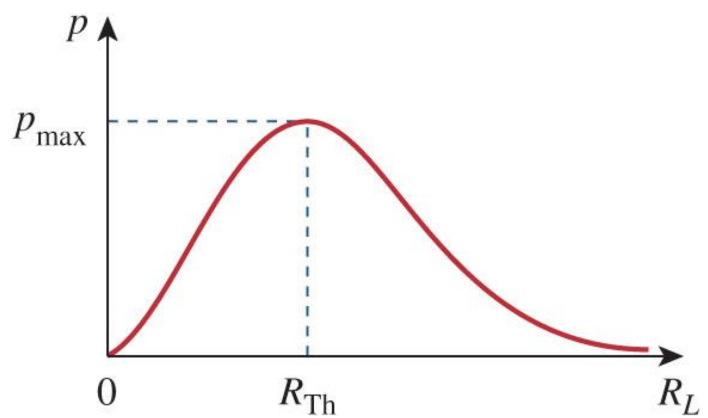
- In many circuits, one element will be variable (called *the load*), while others are fixed.
  - Ordinarily one has to re-analyze the circuit for load change.
  - This problem can be avoided by circuit theorem (e.g. Thevenin's theorem), which provides a technique to replace the fixed part of the circuit with an **equivalent** circuit.



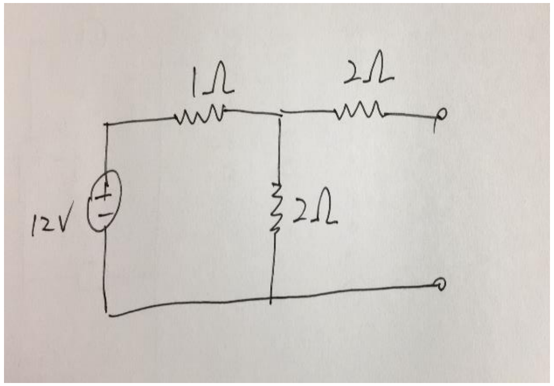
3 methods

Max Power Transfer

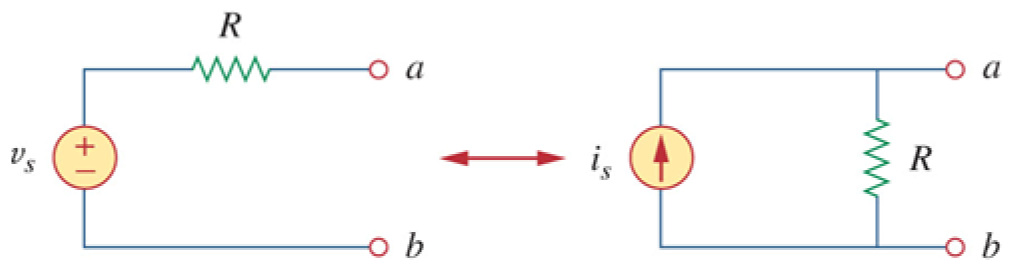




Percentage?



Source Transformation

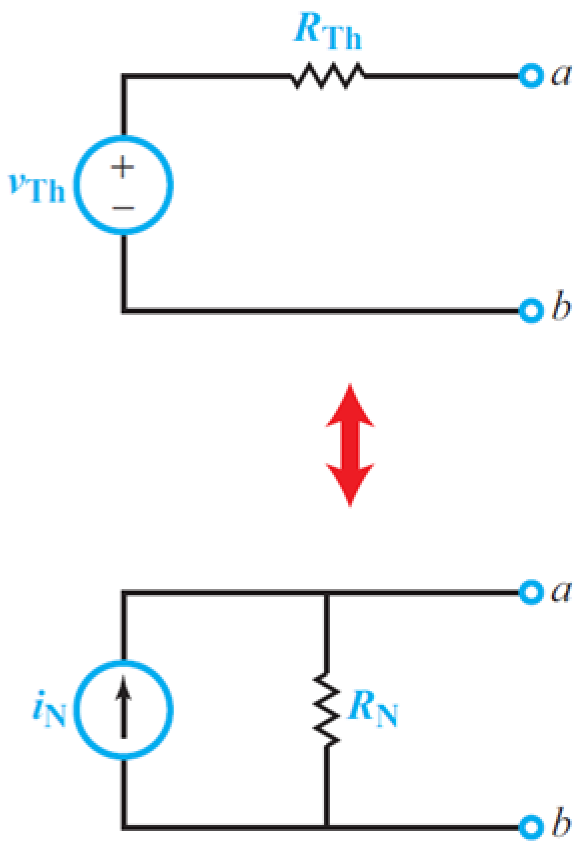


- A source transformation is the process of replacing a voltage source v_s in series with a resistor R by a current source i_s in parallel with a resistor R, or vice versa.

Norton’s Theorem

Thévenin equivalent circuit

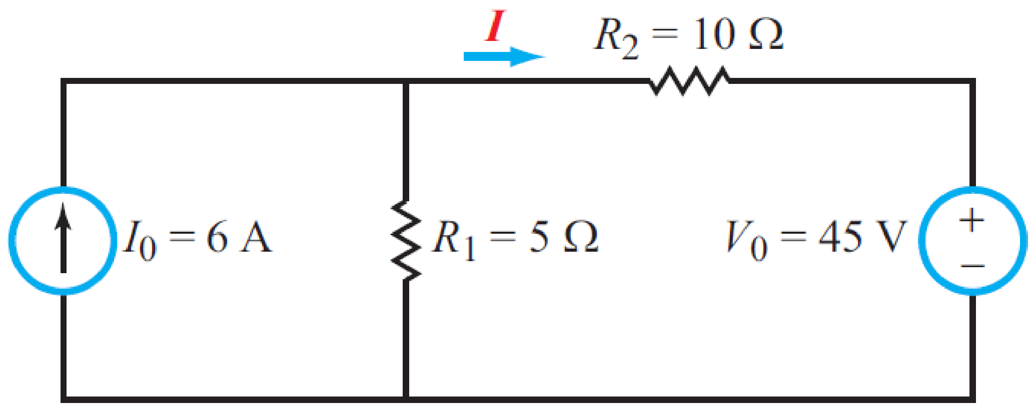
Norton equivalent circuit



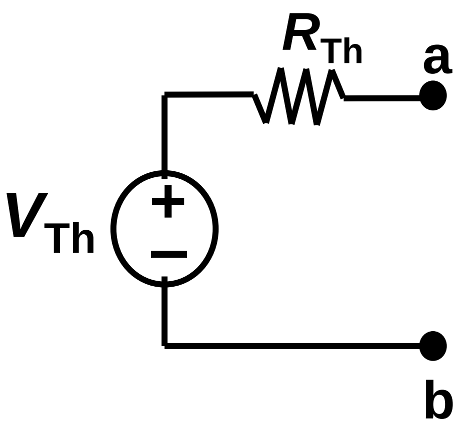
[Source: Berkeley]

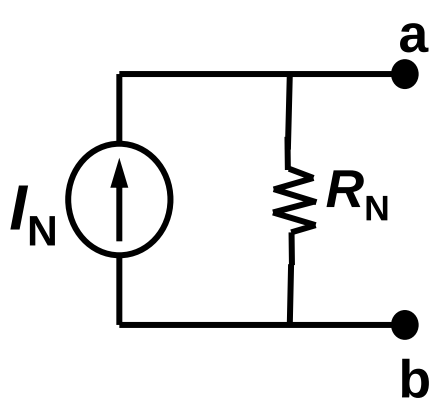
Summary

- Superposition
  - Voltage off → SC
  - Current off → OC



- Thevenin and Norton Equivalent Circuits
  - Solve for OC voltage
  - Solve for SC current



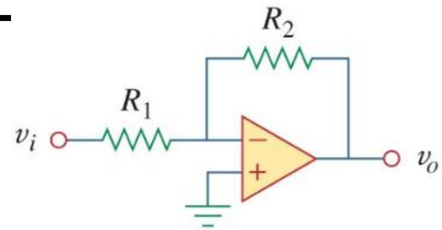


$$I_N = \frac{V_{Th}}{R_{Th}}$$

$$R_N = R_{Th}$$

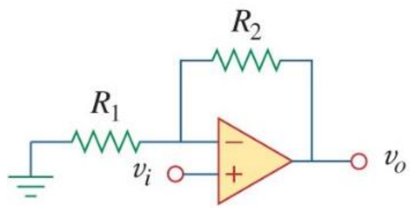
OA

Op amp circuit	Name/output-input relationship



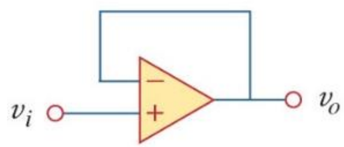
Inverting amplifier

$$v_o = -\frac{R_2}{R_1} v_i$$



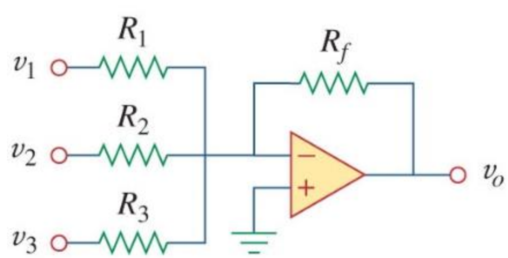
Noninverting amplifier

$$v_o = \left(1 + \frac{R_2}{R_1}\right) v_i$$



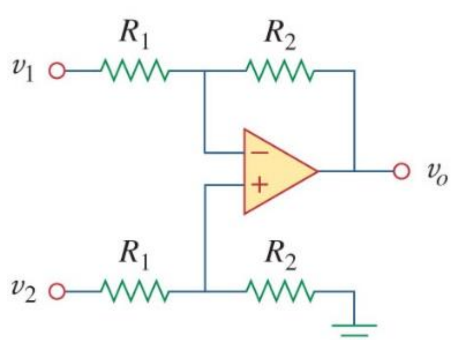
Voltage follower

$$v_o = v_i$$



Summer

$$v_o = -\left(\frac{R_f}{R_1} v_1 + \frac{R_f}{R_2} v_2 + \frac{R_f}{R_3} v_3\right)$$



Difference amplifier

$$v_o = \frac{R_2}{R_1}(v_2 - v_1)$$

Part 2 Temporal Analysis

Summary of Capacitors and Inductors

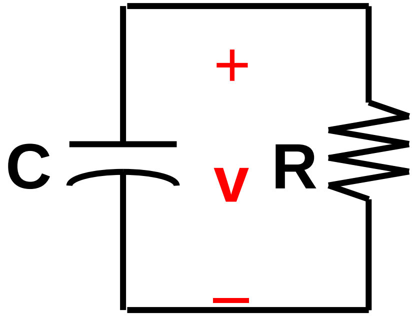
Table 5-4: Basic properties of R, L, and C.

Property	R	L	C
i–υ relation	$i = \frac{\upsilon}{R}$	$i = \frac{1}{L}\int_{t_0}^{t} \upsilon\, dt' + i(t_0)$	$i = C\,\frac{d\upsilon}{dt}$
υ-i relation	$\upsilon = iR$	$\upsilon = L\,\frac{di}{dt}$	$\upsilon = \frac{1}{C}\int_{t_0}^{t} i\, dt' + \upsilon(t_0)$
p (power transfer in)	$p = i^2 R$	$p = Li\,\frac{di}{dt}$	$p = C\upsilon\,\frac{d\upsilon}{dt}$
w (stored energy)	0	$w = \frac{1}{2}Li^2$	$w = \frac{1}{2}C\upsilon^2$
Series combination	$R_{eq} = R_1 + R_2$	$L_{eq} = L_1 + L_2$	$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$
Parallel combination	$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$	$\frac{1}{L_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$	$C_{eq} = C_1 + C_2$
dc behavior	no change	short circuit	open circuit
Can υ change instantaneously?	yes	yes	no
Can i change instantaneously?	yes	no	yes

[Source: Berkeley]

Natural Response Summary

RC Circuit



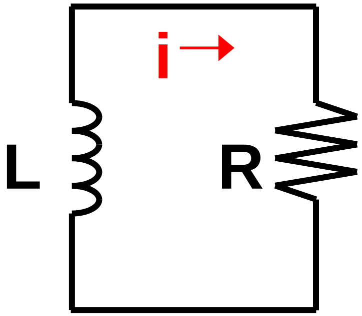
- Capacitor voltage cannot change instantaneously

$$v(0^-) = v(0^+)$$

$$v(t) = v(0)e^{-t/\tau}$$

- time constant $\tau = RC$

RL Circuit



- Inductor current cannot change instantaneously

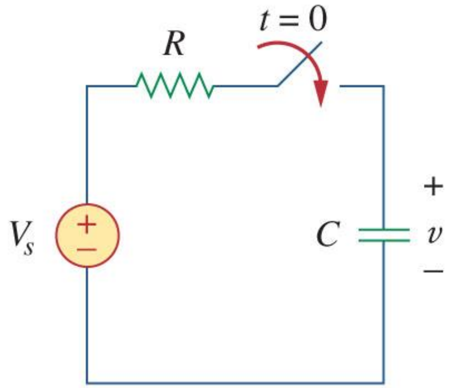
$$i(0^-) = i(0^+)$$

$$i(t) = i(0)e^{-t/\tau}$$

- time constant $\tau = \frac{L}{R}$

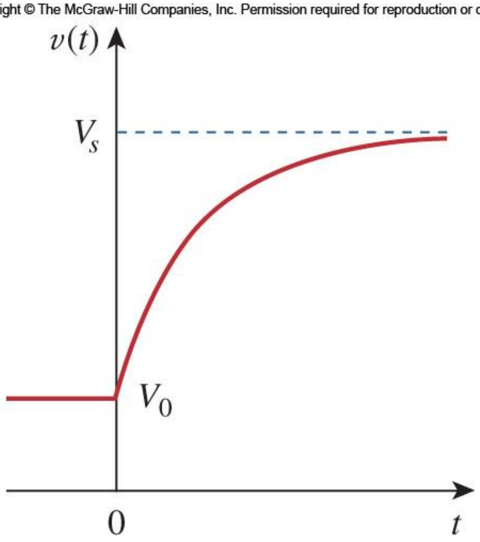
[Source: Berkeley]

Step Response of the RC Circuit



$$v(0^-) = v(0^+) = v_0$$

$$v(t) = \begin{cases} V_0, & t < 0 \\ V_s + (V_0 - V_s)e^{-t/\tau} & t > 0 \end{cases}$$



- This is known as the complete response, or total response.

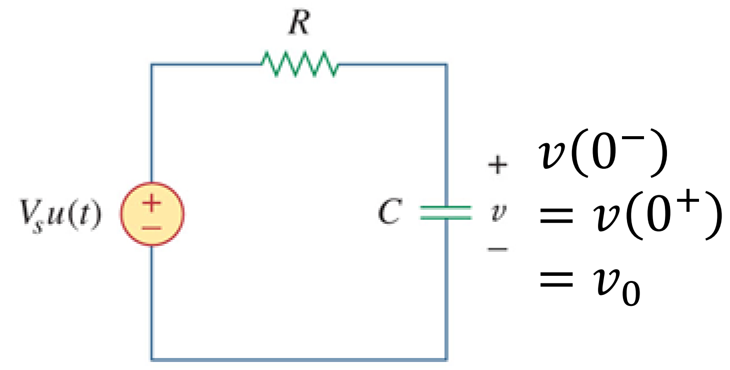
Forced Response

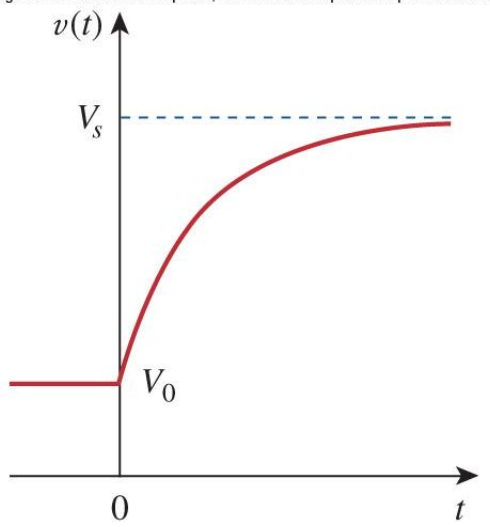
- The complete response

$$v(t)=\begin{cases} V_0, & t<0 \\ V_s+(V_0-V_s)e^{-t/\tau} & t>0 \end{cases}$$

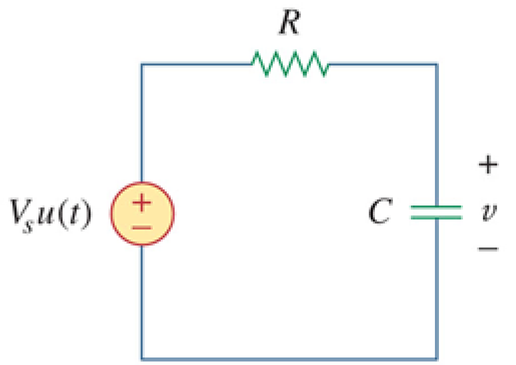
can be written as:

$$\boxed{v=v_n+v_f}$$





Another Perspective



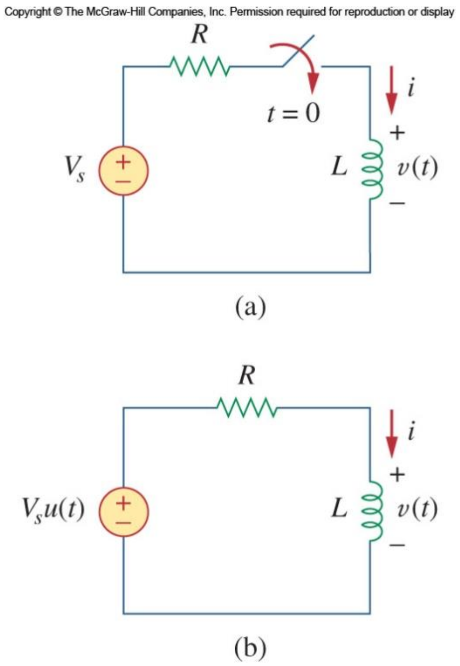
$$v(t)=\begin{cases} V_0, & t<0 \\ V_s+(V_0-V_s)e^{-t/\tau} & t>0 \end{cases}$$

- Another way to look at the response is to break it up into the <u>transient response</u> and the <u>steady state response</u>:

$$v(t)=\underbrace{v(\infty)}_{\text{steady } v_{ss}}+\underbrace{[v(0)-v(\infty)]e^{-t/\tau}}_{\text{transient } v_t}$$

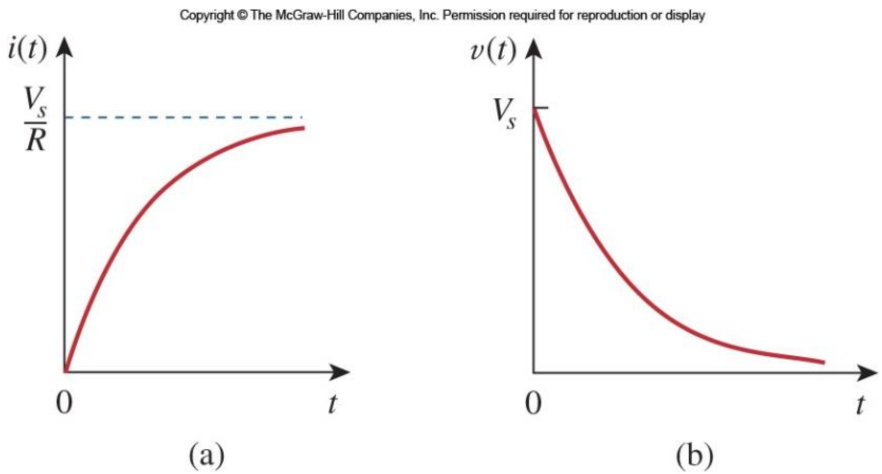
Can be extended as a "three-elements" method

Step Response of the RL Circuit



- We will use the transient and steady state response approach.

$$i(t) = \frac{V_s}{R} + \left(I_0 - \frac{V_s}{R}\right) e^{-t/\tau}$$



General First-Order Circuits

General Second-Order Circuits

- The principles of the approach to solve the series and parallel forms of RLC circuits can **be applied** to general second-order circuits, by taking the following five steps:

  1. First determine the initial conditions, $x(0)$ and $dx(0)/dt$.

  2. Applying KVL and KCL, to find the general second-order differential equation to describe $x(t)$. 3.**Depending on the roots of C.E. , the form of the general solution (3 cases) of homogeneous equation can be determined.**

  4. We obtain the **particular solution** by **observation/calculation**, **specially** for a DC/step response

$$x_{p.s.}(t)=x(\infty)$$

  5. The total response = general solution + particular solution.

$$X(t)=x_{p.s.}(t)+ x_{g.s.}(t)$$

  6. Using the initial conditions to determine the constants of $X(t)$.

General solution for second-order circuits for $t \geq 0$.

$x(t)$ = unknown variable (voltage or current)

Differential equation: $x'' + ax' + bx = c$

Initial conditions: $x(0)$ and $x'(0)$

Final condition: $x(\infty) = \dfrac{c}{b}$

$$\boxed{\alpha = \frac{a}{2} \qquad \omega_0 = \sqrt{b}}$$

Overdamped Response $\alpha > \omega_0$

$$x(t) = [A_1 e^{s_1 t} + A_2 e^{s_2 t} + x(\infty)]$$

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} \qquad s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$

[redacted]

Critically Damped $\alpha = \omega_0$

$$x(t) = [(B_1 + B_2 t)e^{-\alpha t} + x(\infty)]$$

[redacted]

Underdamped $\alpha < \omega_0$

$x(t) = [D_1 \cos \omega_d t + D_2 \sin \omega_d t$ [redacted] $]e^{-\alpha t} + x(\infty)$

[redacted]

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$$

$x(t)$ = unknown variable (voltage or current)

Differential equation:	$x'' + ax' + bx = c$
Initial conditions:	$x(0)$ and $x'(0)$
Final condition:	$x(\infty) = \frac{c}{b}$

[Important]

1.This table works well when c is a constant, as $x(\infty)$ is actually a particular solution (特解) of the equation.

2. When c is a function of time (t), such as $c = 5t$; $c = t^2+3$; $c = e^{-t}$; you should also be able to solve the equation (Requirement of the course).

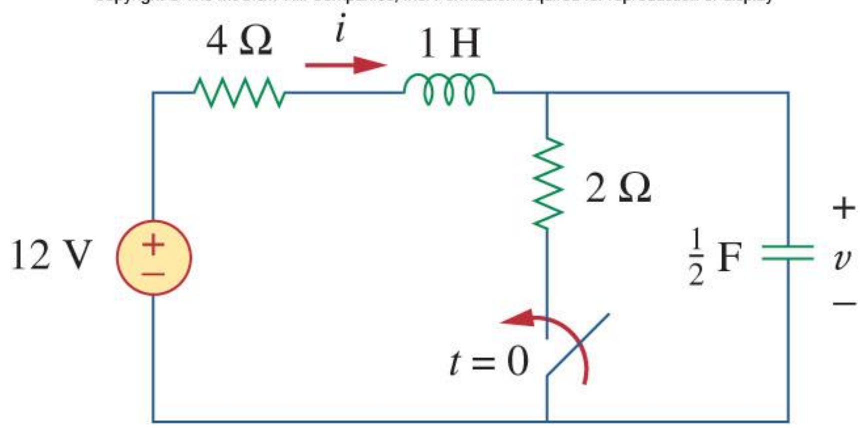
General RLC Circuits

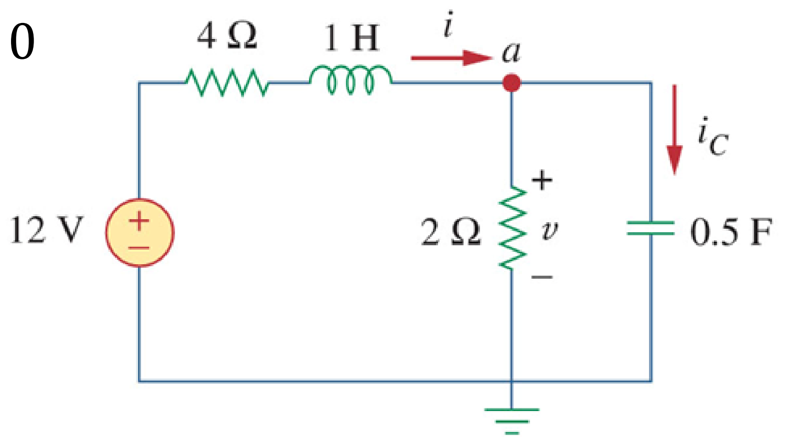
- Find the complete response v for $t > 0$ in the circuit.

1. Initial conditions

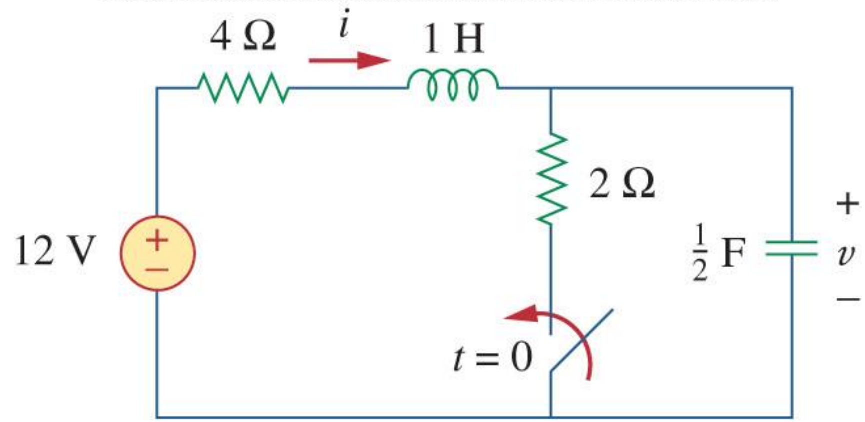
$$v(0^+) = v(0^-) = 12V,\ i(0^+) = i(0^-) = 0$$

$$\frac{dv(0^+)}{dt} = \frac{i_C(0^+)}{C} = \frac{-12/2}{0.5} = -12\ V/s$$





General RLC Circuits



- Find the complete response v for $t > 0$ in the circuit.

1. Initial conditions

$$v(0^+) = v(0^-) = 12V,\ i(0^+) = i(0^-) = 0$$

$$\frac{dv(0^+)}{dt} = \frac{i_C(0^+)}{C} = \frac{-12/2}{0.5} = -12\ V/s$$

2. KCL at node *a*: $i = \frac{v}{2} + 0.5\frac{dv}{dt}$

KVL on left mesh: $4i + 1\frac{di}{dt} + v = 12$

$\Rightarrow$ $\frac{d^2v}{dt^2} + 5\frac{dv}{dt} + 6v = 24$ $\Rightarrow$ General Solution $v_t(t) = A_1e^{-2t} + A_2e^{-3t}$

3. Particular Solution : Steady-state response $v_{ss}(t) = 4V$

4. Put together : $v(t) = 4 + \mathrm{A}_1e^{-2t} - \mathrm{A}_2e^{-3t}$

5. Using initial conditions to determine A_1 , A_2

Self-test-General RLC Circuit

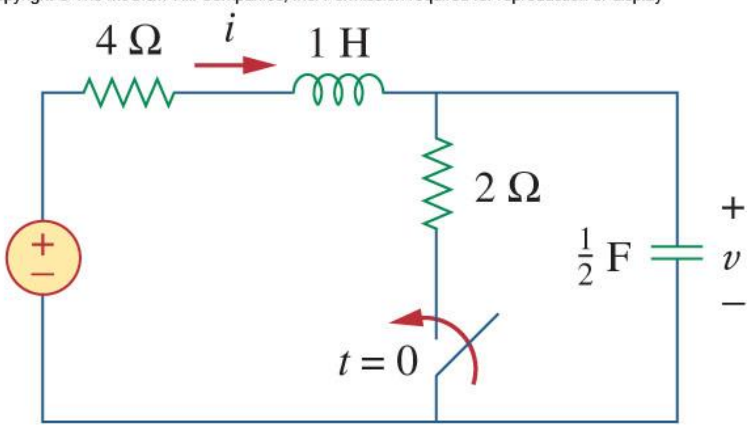
- Find the complete response v for $0 < t < 1$ in the circuit.

Using time-domain method and Laplace transform method

V=*12t*

Or

V=e^{-2t}



Outline 3

- Circuit Basics

---- PSC, KCL, KVL

---- Circuit theorems

Important circuit analysis skills for circuits in time domain(DC and temporal analysis), circuits in phasor domain, circuits in s-domain.

- Temporal Analysis

--1st-order, 2nd-order circuits

- AC circuits

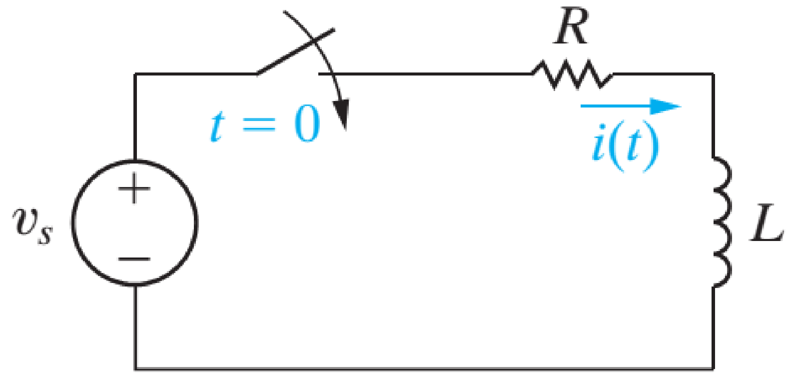
--Phasor, Sinusoidal S.S. Analysis, AC power, 3-Phase Circuits,

--Mutual inductance, Frequency Response(transfer function, Bode Plots, Resonance, Filters)

- Laplace Transform

AC Steady-State Analysis by Phasor Method

$$v_s = V_m \cos(\omega t + \phi)$$



?

$$i = \boxed{\frac{-V_m}{\sqrt{R^2 + \omega^2 L^2}} \cos(\phi - \theta) e^{-(R/L)t}} + \boxed{\frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \cos(\omega t + \phi - \theta)}$$

Transient response

Steady-state response

Sinusoid-Phasor Transformation

$$v(t) = V_m \cos(\omega t + \phi) \quad \Leftrightarrow \quad \mathbf{V} = V_m \underline{/\phi}$$

(Time-domain representation) (Phasor-domain representation)

- Applying a derivative to a phasor yields:

$$\frac{dv}{dt} \Leftrightarrow j\omega V$$

(Time domain) (Phasor domain)

- Applying an integral to a phasor yields:

$$\int v dt \Leftrightarrow \frac{V}{j\omega}$$

(Time domain) (Phasor domain)

Review: Impedance and Admittance

Resistor	$\mathbf{Z} = R$	$\mathbf{Y} = 1/R$
Inductor	$\mathbf{Z} = j\omega L$	$\mathbf{Y} = 1/j\omega L$
Capacitor	$\mathbf{Z} = 1/j\omega C$	$\mathbf{Y} = j\omega C$

Impedance is voltage/current

$$\mathbf{Z} = R + jX$$

R = resistance = Re(Z)

X = reactance = Im(Z)

Admittance is current/voltage

$$\mathbf{Y} = \frac{1}{\mathbf{Z}} = G + jB$$

G = conductance = Re(Y)

B = susceptance = Im(Y)

AC Phasor Analysis General Procedure

Step 1: Adopt cosine reference

$$v_s(t) = 12\sin(\omega t - 45^\circ)$$
$$= 12\cos(\omega t - 45^\circ - 90^\circ) = 12\cos(\omega t - 135^\circ)\text{ V.}$$

$$\mathbf{V}_s = 12e^{-j135^\circ}\text{ V.}$$

Step 2: Transform circuit to phasor domain

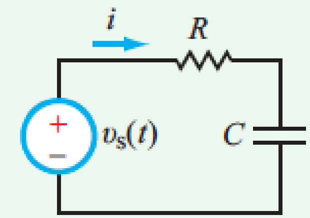
Step 3: $\boxed{\text{Cast KCL and/or KVL}}$ equations in phasor domain

$$\mathbf{Z}_R\mathbf{I} + \mathbf{Z}_C\mathbf{I} = \mathbf{V}_s,$$

which is equivalent to

$$\left(R + \frac{1}{j\omega C}\right)\mathbf{I} = 12e^{-j135^\circ}.$$

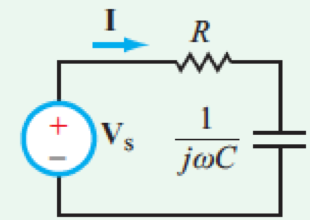
Step 1
Adopt Cosine Reference
(Time Domain)



$v_s(t) = 12\sin(\omega t - 45^\circ)$ (V)

Step 2
Transfer to Phasor Domain

$i \rightarrow \mathbf{I}$
$v \rightarrow \mathbf{V}$
$R \rightarrow \mathbf{Z}_R = R$
$L \rightarrow \mathbf{Z}_L = j\omega L$
$C \rightarrow \mathbf{Z}_C = 1/j\omega C$



$\mathbf{V}_s = 12e^{-j135^\circ}$ (V)

Step 3
Cast Equations in
Phasor Form

$$\mathbf{I}\left(R + \frac{1}{j\omega C}\right) = \mathbf{V}_s$$

AC Phasor Analysis General Procedure

Step 4: Solve for unknown variable

$$\mathbf{I} = \frac{12e^{-j135°}}{R + \frac{1}{j\omega C}} = \frac{j12\omega C e^{-j135°}}{1 + j\omega RC}.$$

Using the specified values, namely $R = \sqrt{3}$ kΩ, $C = 1$ μF, and $\omega = 10^3$ rad/s,

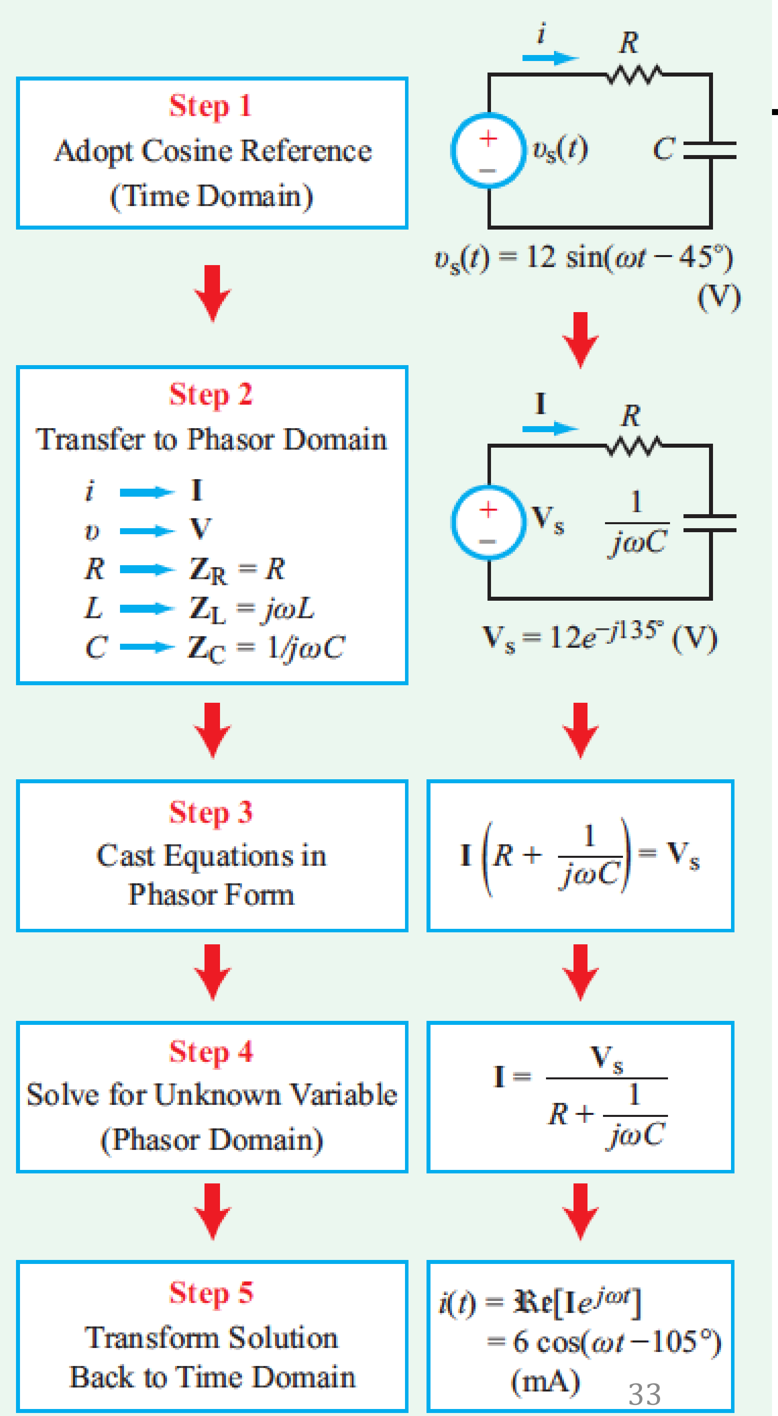
$$\mathbf{I} = \frac{j12 \times 10^3 \times 10^{-6} e^{-j135°}}{1 + j10^3 \times \sqrt{3} \times 10^3 \times 10^{-6}} = \frac{j12e^{-j135°}}{1 + j\sqrt{3}} \text{ mA.}$$

$$\mathbf{I} = \frac{12e^{-j135°} \cdot e^{j90°}}{2e^{j60°}} = 6e^{j(-135°+90°-60°)} = 6e^{-j105°} \text{ mA.}$$

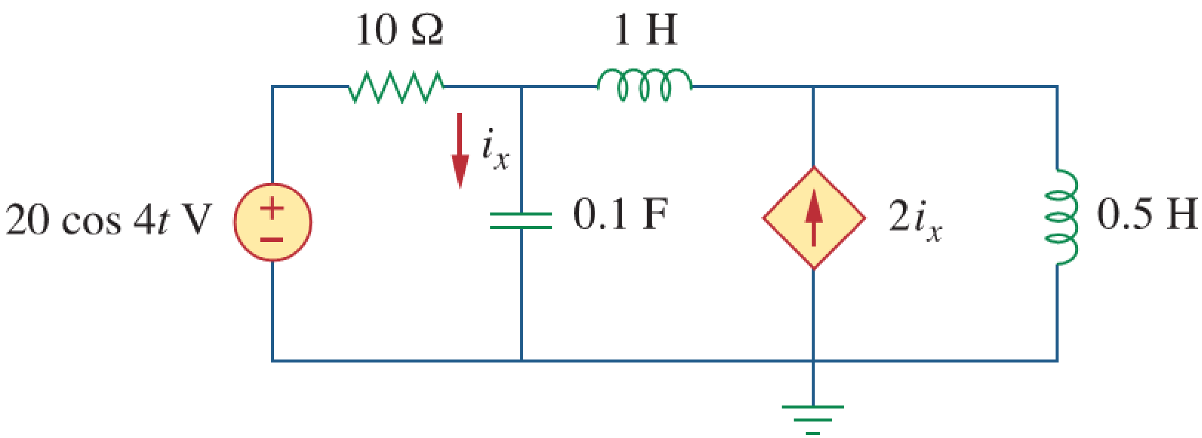
Step 5: Transform solution back to time domain

To return to the time domain, we apply the fundamental relation between a sinusoidal function and its phasor counterpart, namely

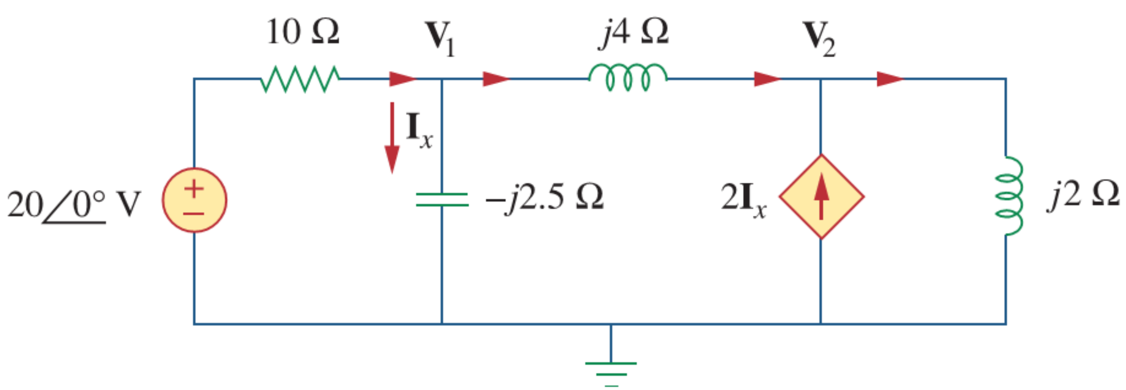
$$i(t) = \mathfrak{Re}[\mathbf{I}e^{j\omega t}] = \mathfrak{Re}[6e^{-j105°} e^{j\omega t}] = 6\cos(\omega t - 105°) \text{ mA.}$$



Example-Nodal Analysis



Find i_x



$$\frac{20 - \mathbf{V}_1}{10} = \frac{\mathbf{V}_1}{-j2.5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4}$$

$$2\mathbf{I}_x + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4} = \frac{\mathbf{V}_2}{j2}$$

$$i_x = 7.59\cos(4t + 108.4^\circ)\ \mathrm{A}$$

Power in AC Circuits

Complex Power

$$v(t) = V_m \cos(\omega t + \theta_v) \Longrightarrow \mathbf{V} = V_m \angle \theta_v$$

$$i(t) = I_m \cos(\omega t + \theta_i) \Longrightarrow \mathbf{I} = I_m \angle \theta_i$$

We observe that:

$$\frac{1}{2}\mathbf{V}\mathbf{I}^* = \frac{1}{2} V_m I_m \angle(\theta_v - \theta_i)$$

$$= \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) + j\frac{1}{2} V_m I_m \sin(\theta_v - \theta_i)$$

- Define a *single* power metric

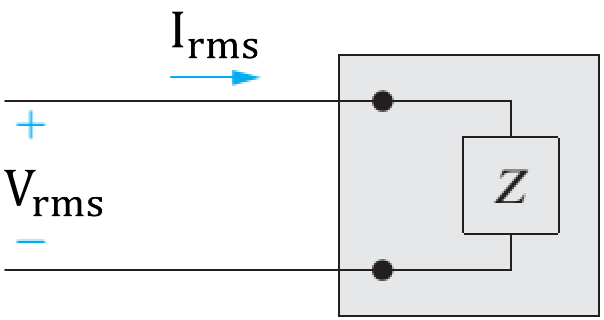
$$\boxed{\mathbf{S} = \frac{1}{2}\mathbf{V}\mathbf{I}^* = \mathbf{V}_{\text{rms}}\mathbf{I}_{\text{rms}}^* = V_{rms} I_{rms} \angle(\theta_v - \theta_i) = P + jQ}$$

Unit: volt-amp (VA)

Complex power is important in power analysis because it contains *all* the information pertaining to the power absorbed by a given load.

Another Way to Calculate Complex Power

$$\boxed{\mathbf{S} = I_{\text{rms}}^2 \mathbf{Z} = \frac{V_{\text{rms}}^2}{\mathbf{Z}^*} = \mathbf{V}_{\text{rms}} \mathbf{I}_{\text{rms}}^*}$$



$$\mathbf{V}_{\text{rms}} = \mathbf{I}_{\text{rms}} Z$$

$$\begin{aligned}\mathbf{S} &= \mathbf{V}_{\text{rms}} \mathbf{I}_{\text{rms}}^* \\ &= \mathbf{V}_{\text{rms}} \left(\frac{\mathbf{V}_{\text{rms}}}{Z}\right)^* \\ &= \frac{|\mathbf{V}_{\text{rms}}|^2}{Z^*}\end{aligned}$$

$$\begin{aligned}\mathbf{S} &= \mathbf{V}_{\text{rms}} \mathbf{I}_{\text{rms}}^* \\ &= \mathbf{I}_{\text{rms}} Z \mathbf{I}_{\text{rms}}^* \\ &= |\mathbf{I}_{\text{rms}}|^2 Z \\ &= |\mathbf{I}_{\text{rms}}|^2 (R + jX) \\ &= |\mathbf{I}_{\text{rms}}|^2 R + j|\mathbf{I}_{\text{rms}}|^2 X \\ &= I_{rms}^2 R + j I_{rms}^2 X\end{aligned}$$

$$\boxed{\begin{aligned} P &= \text{Re}(\mathbf{S}) = I_{\text{rms}}^2 R \\ Q &= \text{Im}(\mathbf{S}) = I_{\text{rms}}^2 X \end{aligned}}$$

$$\mathbf{S} = \frac{1}{2}\mathbf{V}\mathbf{I}^* = \mathbf{V}_{\mathrm{rms}}\mathbf{I}^*_{\mathrm{rms}} = V_{rms}I_{rms}\angle(\theta_v - \theta_i) = P + jQ$$

$$P = \mathrm{Re}(\mathbf{S}) = I^2_{\mathrm{rms}}R$$

$$Q = \mathrm{Im}(\mathbf{S}) = I^2_{\mathrm{rms}}X$$

- Average (or real) power

$$P = \mathrm{Re}[\frac{1}{2}\mathbf{V}\mathbf{I}^*]$$

Unit: Watts

- Reactive power

$$Q = \mathrm{Im}[\frac{1}{2}\mathbf{V}\mathbf{I}^*]$$

Unit: Volt Amperes Reactive (VARs)

- Apparent power

$$s = |\mathbf{S}| = \frac{1}{2}V_mI_m = V_{rms}I_{rms}$$

Unit: volt-amp (VA)

$$\text{Complex Power} = \mathbf{S} = P + jQ = \mathbf{V}_{\text{rms}}(\mathbf{I}_{\text{rms}})^*$$

$$= |\mathbf{V}_{\text{rms}}|\,|\mathbf{I}_{\text{rms}}|\underline{/\theta_v - \theta_i}$$

$$\text{Apparent Power} = S = |\mathbf{S}| = |\mathbf{V}_{\text{rms}}|\,|\mathbf{I}_{\text{rms}}| = \sqrt{P^2 + Q^2}$$

$$\text{Real Power} = P = \text{Re}(\mathbf{S}) = S\cos(\theta_v - \theta_i)$$

$$\text{Reactive Power} = Q = \text{Im}(\mathbf{S}) = S\sin(\theta_v - \theta_i)$$

$$\text{Power Factor} = \frac{P}{S} = \cos(\theta_v - \theta_i)$$

Example

2-8 题图 2-8 所示电路中，已知 $R_1=5\Omega$，$R_2=3\Omega$，$L=10\text{mH}$，$C=100\mu\text{F}$，$u_S(t)=10\sqrt{2}\cos 1000t\,\text{V}$，$i_S(t)=2\sqrt{2}\cos(1000t+30^\circ)\,\text{A}$。求电压源，电流源各自发出的有功功率和无功功率。

解 题图 2-8 所示电路的相量模型如题图 2-8(a)所示，设电压源电流和电流源端电压的参考方向如题图 2-8(a)所示。流出电压源的电流

$$\dot{I}=\frac{10\underline{/0^\circ}}{5-\text{j}10}-2\underline{/30^\circ}=1.347\underline{/-171.5^\circ}(\text{A})$$

$\dot{U}\cdot\dot{I}$

电压源发出的有功功率和无功功率为

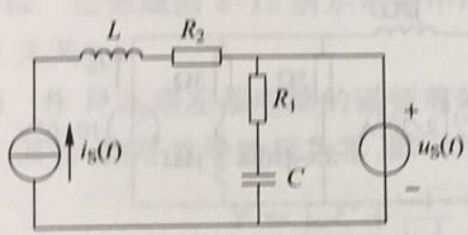
$$P_{u发}=10\times1.347\times\cos(0-(-171.5^\circ))=-13.3(\text{W})$$

$$Q_{u发}=10\times1.347\times\sin(0-(-171.5^\circ))=1.99(\text{var})$$

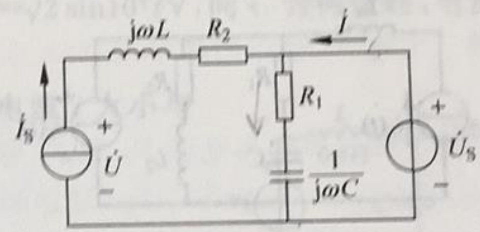
$P=-V\cdot I$
$=13.3$

电流源的端电压

$$\dot{U}=(3+\text{j}10)\dot{I}_S+\dot{U}_S=20.97\underline{/75.7^\circ}(\text{V})$$



题图 2-8



题图 2-8(a)

电流源发出的有功功率和无功功率为

$$P_{i发}=20.97\times2\times\cos(75.7^\circ-30^\circ)=29.3(\text{W})$$

$$Q_{i发}=20.97\times2\times\sin(75.7^\circ-30^\circ)=30.0(\text{var})$$

3-Phase Circuits

Outline--Three-Phase Circuits

- Balanced Three-Phase System
  - Balanced sources
  - Balanced loads
- Circuit analysis
  - Phase voltage/current
  - Line voltage/current
  - **Power calculation**
- Unbalanced Three-Phase Loads

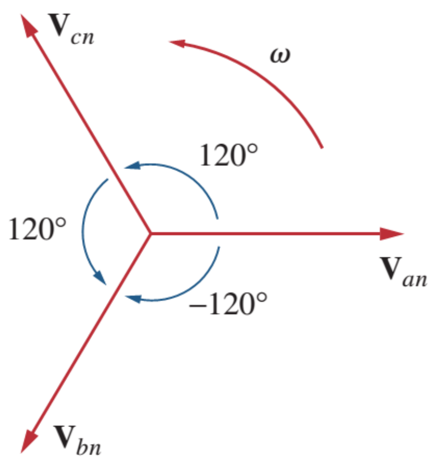
Phase Voltage & Line-to-Line Voltage

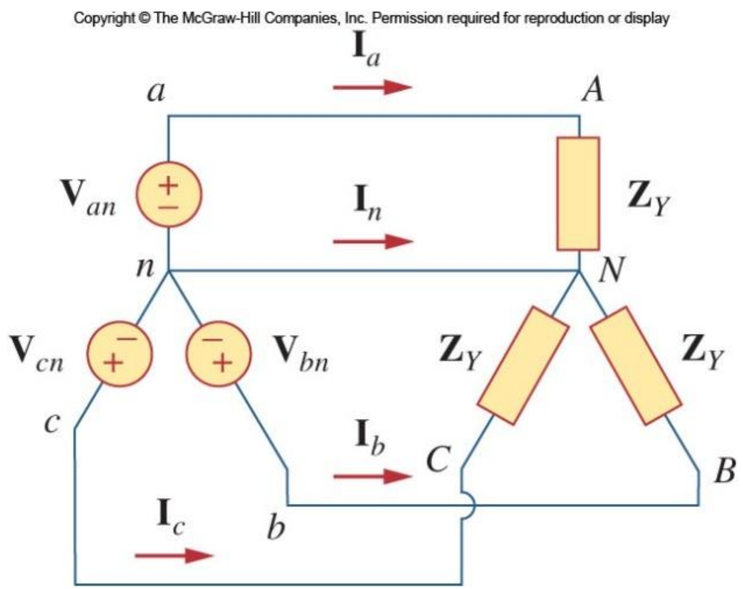
- Use the positive sequence:

Phase Voltage $\quad V_{an} = V_p \angle 0^\circ$

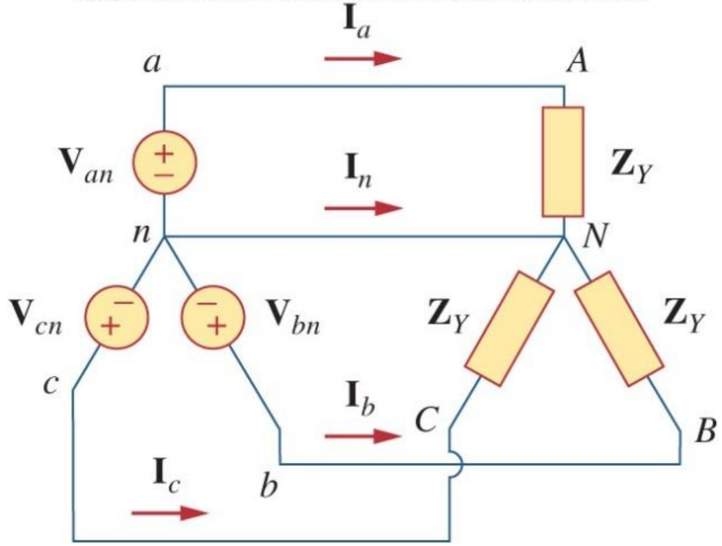
$$V_{bn} = V_p \angle -120^\circ \quad V_{cn} = V_p \angle +120^\circ$$

- The line to line (or line in short) voltages:



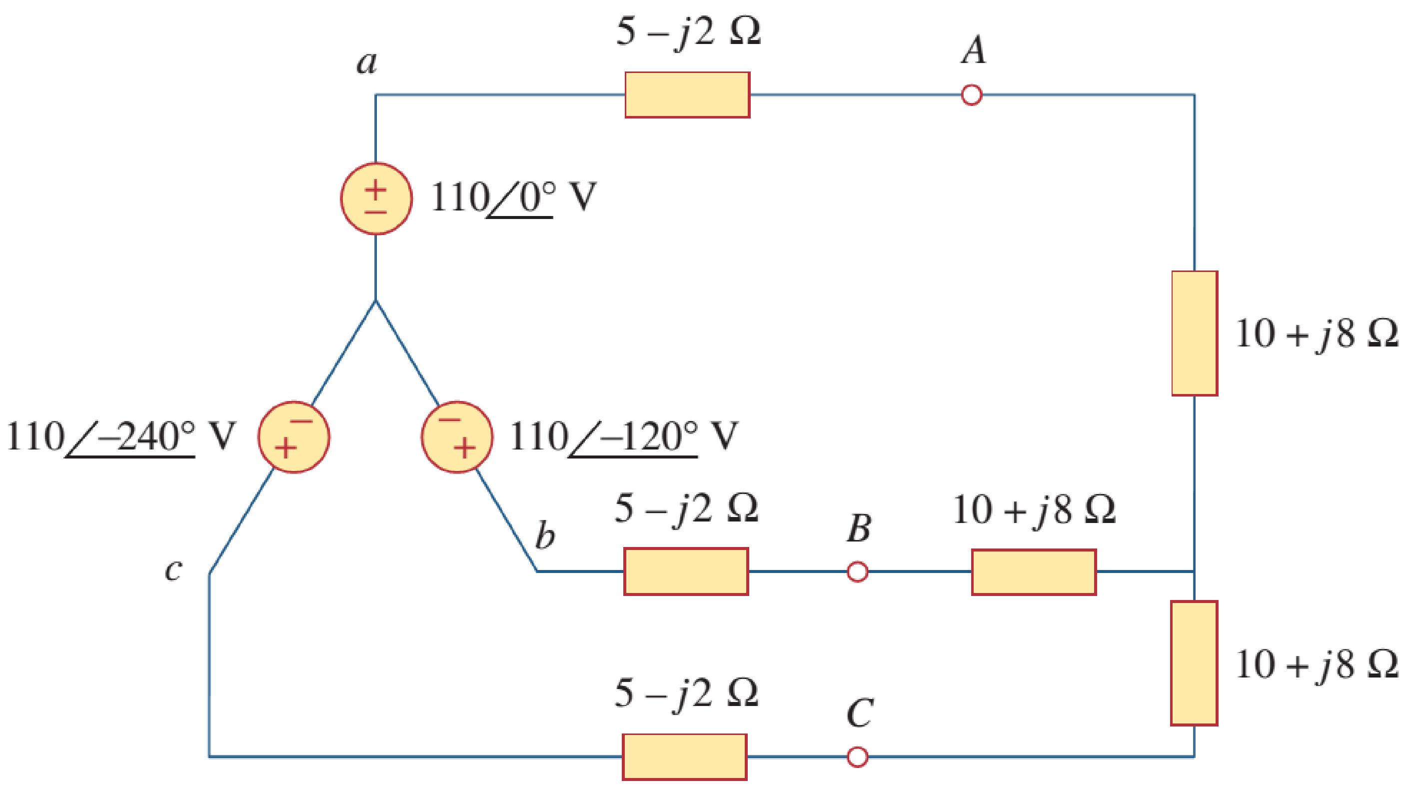


Line Currents

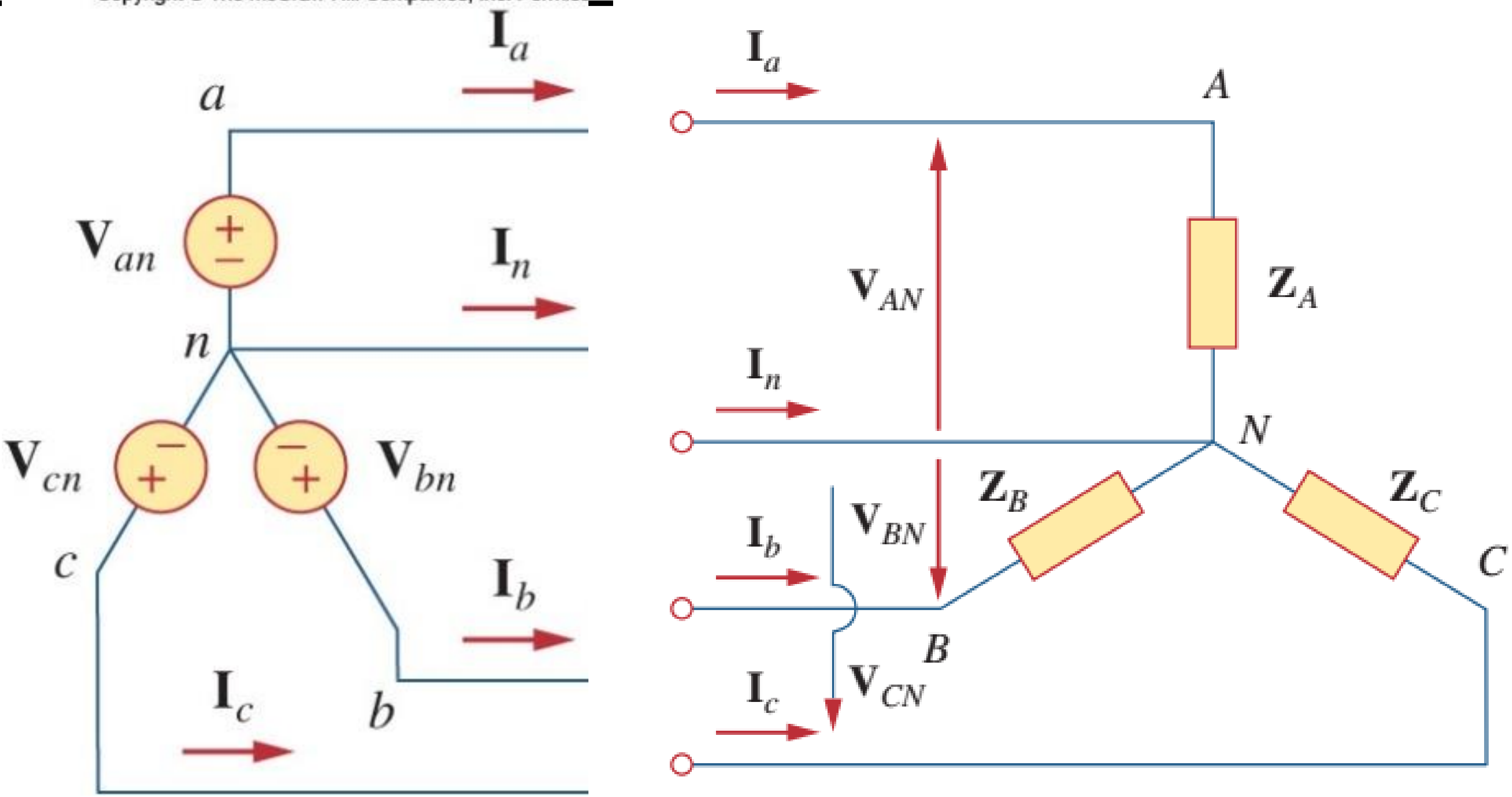


Example

- Calculate the line currents.



E



The unbalanced Y-load of Fig. 12.23 has balanced voltages of 100 V and the *acb* sequence. Calculate the line currents and the neutral current. Take $\mathbf{Z}_A = 15\ \Omega$, $\mathbf{Z}_B = 10 + j5\ \Omega$, $\mathbf{Z}_C = 6 - j8\ \Omega$.

The unbalanced Y-load of Fig. 12.23 has balanced voltages of 100 V and the *acb* sequence. Calculate the line currents and the neutral current. Take $\mathbf{Z}_A = 15\ \Omega$, $\mathbf{Z}_B = 10 + j5\ \Omega$, $\mathbf{Z}_C = 6 - j8\ \Omega$.

Solution:

Using Eq. (12.59), the line currents are

$$\mathbf{I}_a = \frac{100\underline{/0^\circ}}{15} = 6.67\underline{/0^\circ}\ \text{A}$$

$$\mathbf{I}_b = \frac{100\underline{/120^\circ}}{10 + j5} = \frac{100\underline{/120^\circ}}{11.18\underline{/26.56^\circ}} = 8.94\underline{/93.44^\circ}\ \text{A}$$

$$\mathbf{I}_c = \frac{100\underline{/-120^\circ}}{6 - j8} = \frac{100\underline{/-120^\circ}}{10\underline{/-53.13^\circ}} = 10\underline{/-66.87^\circ}\ \text{A}$$

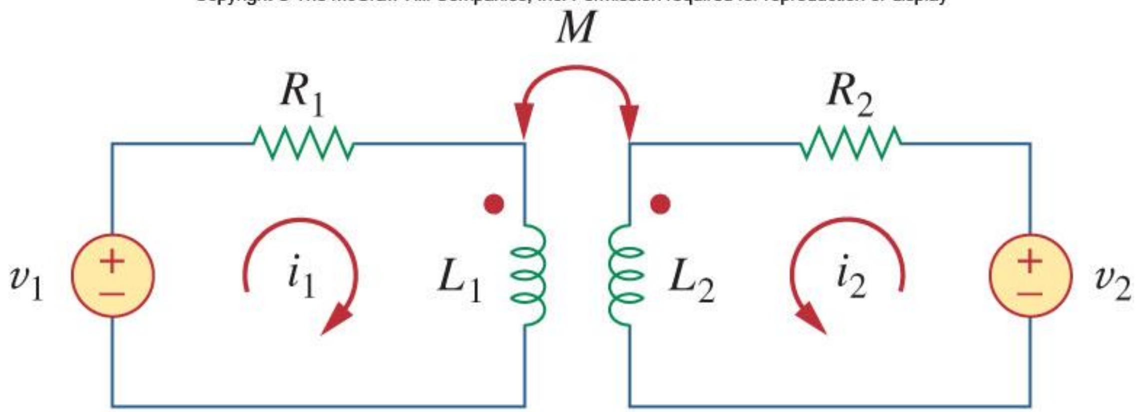
Using Eq. (12.60), the current in the neutral line is

$$\begin{aligned}\mathbf{I}_n = -(\mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c) &= -(6.67 - 0.54 + j8.92 + 3.93 - j9.2) \\ &= -10.06 + j0.28 = 10.06\underline{/178.4^\circ}\ \text{A}\end{aligned}$$

Mutual Inductance

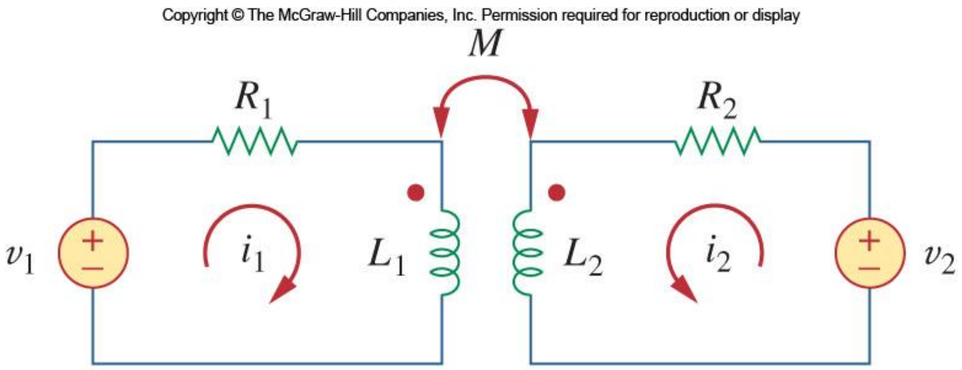
Magnetically Coupled Circuits

- L_1, L_2: self-inductances
- M: mutual inductance
- Dots: indicating polarity of mutually induced voltages.

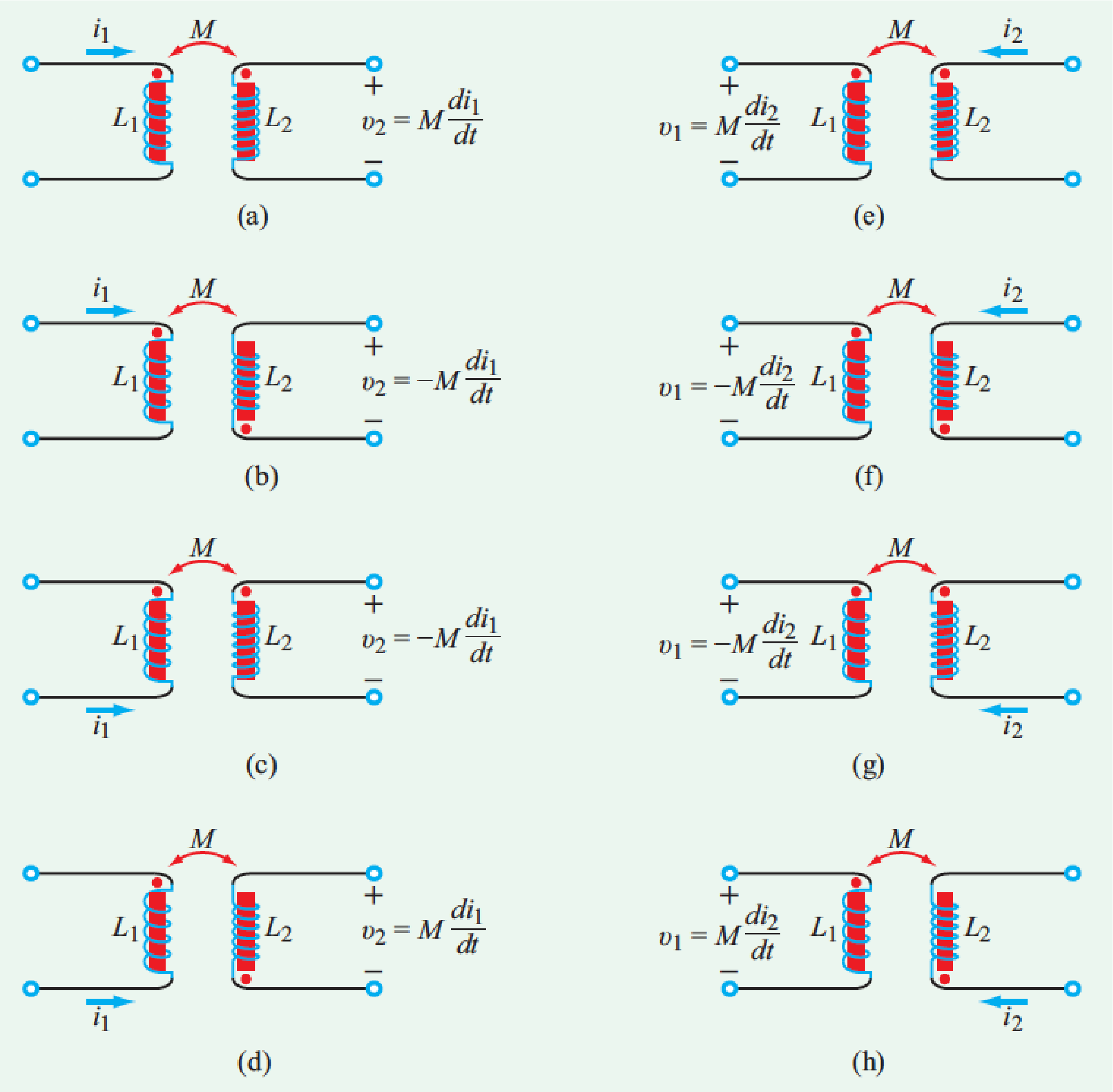


Analysis

- Relate v_1, v_2 with i_1 and i_2.
  - In time domain
  - In phasor domain



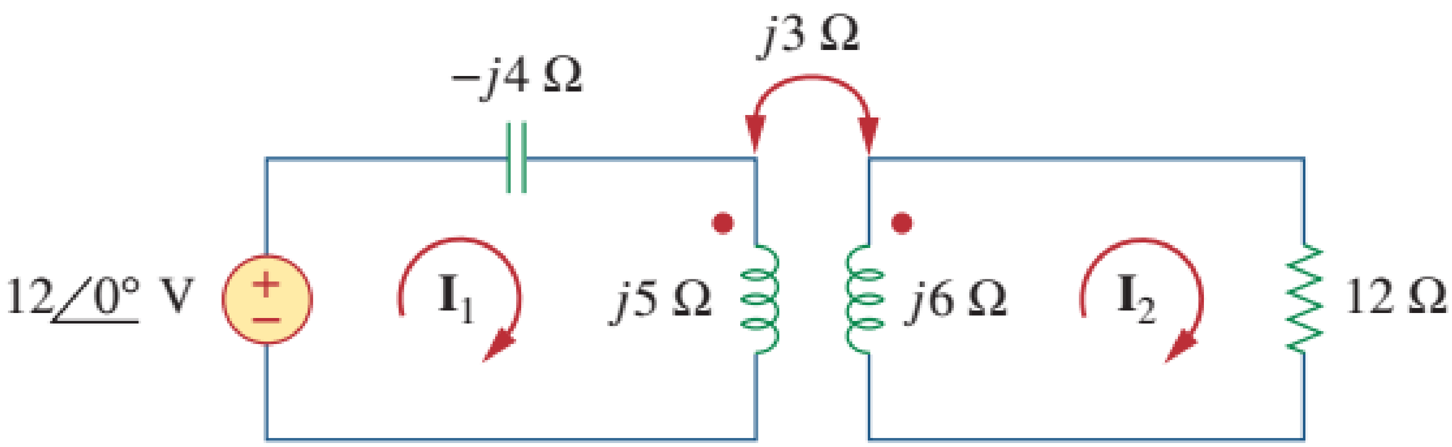
Dot Convention: Defines Directions of Windings



[Source: Berkeley]

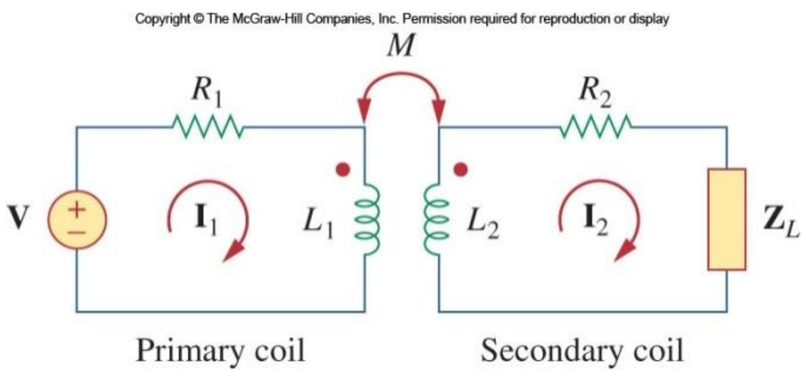
Exercise

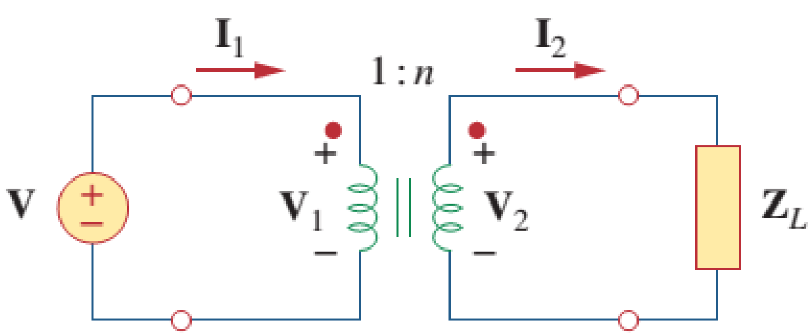
- Calculate the phasor currents $\mathbf{I}_1$, and $\mathbf{I}_2$



Ideal Transformers

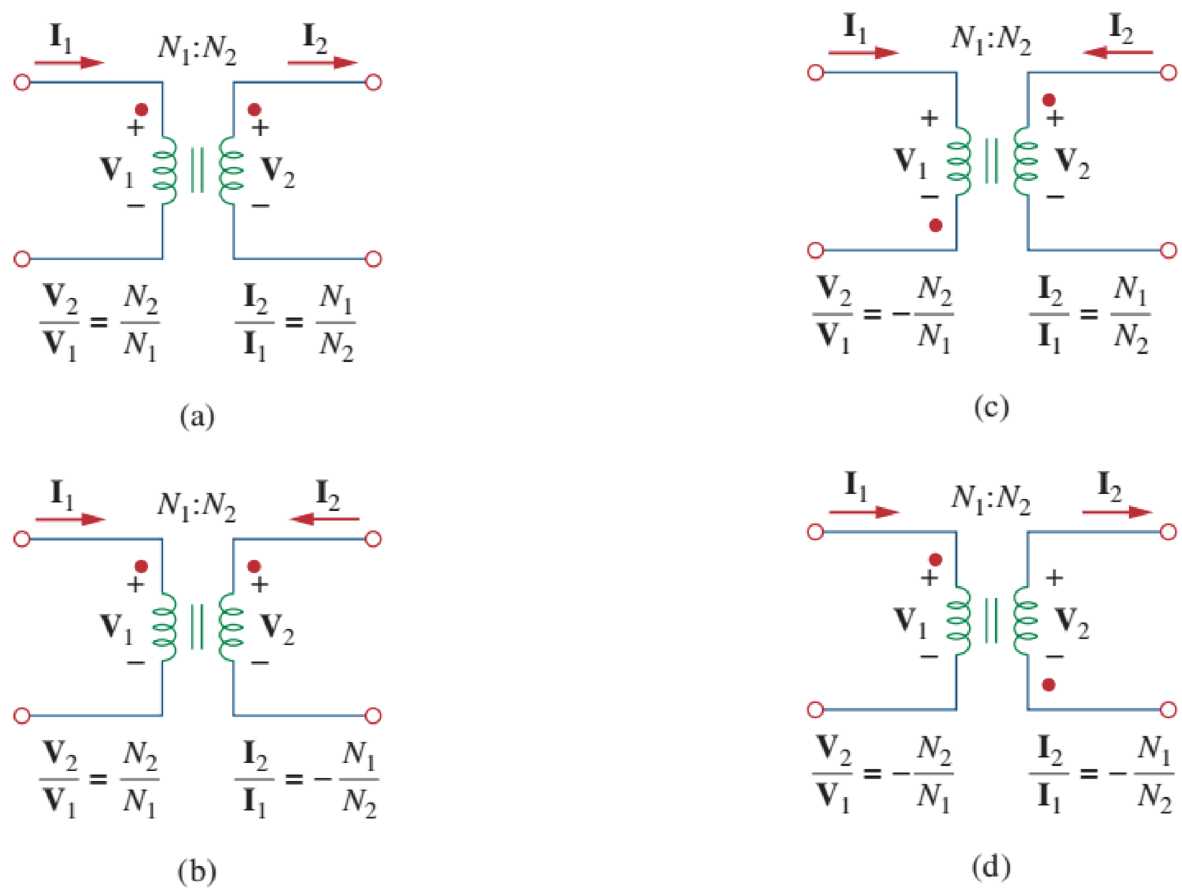
- The ideal transformer has:
  - Coils with very large reactance
    $(L_1, L_2, M \to \infty)$
  - Coupling coefficient k=1.
  - Primary and secondary coils are lossless, $R_1 = R_2 = 0$.





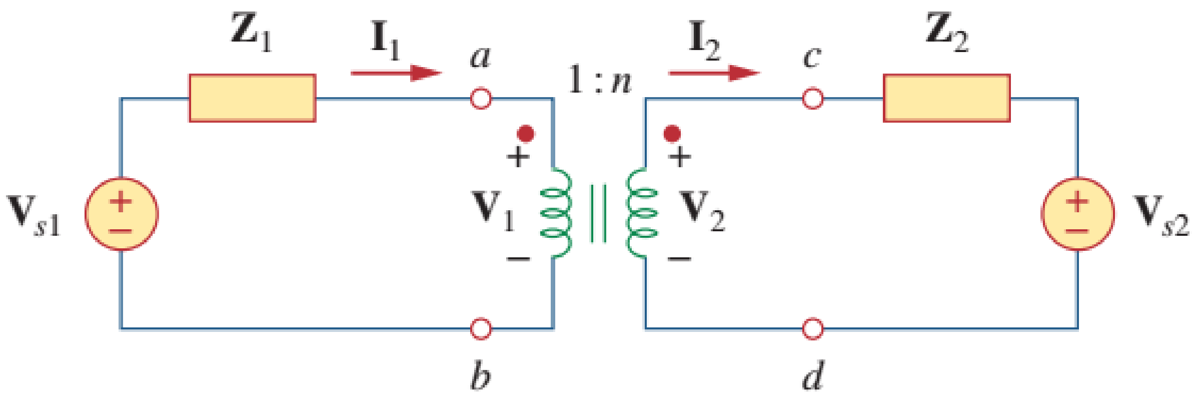
$$\frac{\mathbf{V_2}}{\mathbf{V_1}} = \frac{N_2}{N_1} = n$$

1. If $\mathbf{V}_1$ and $\mathbf{V}_2$ are *both* positive or both negative at the dotted terminals, use $+n$ in Eq. (13.52). Otherwise, use $-n$.
2. If $\mathbf{I}_1$ and $\mathbf{I}_2$ *both* enter into or both leave the dotted terminals, use $-n$ in Eq. (13.55). Otherwise, use $+n$.



Ideal Transformers

$$\frac{\mathbf{V_2}}{\mathbf{V_1}} = \frac{N_2}{N_1} = n$$

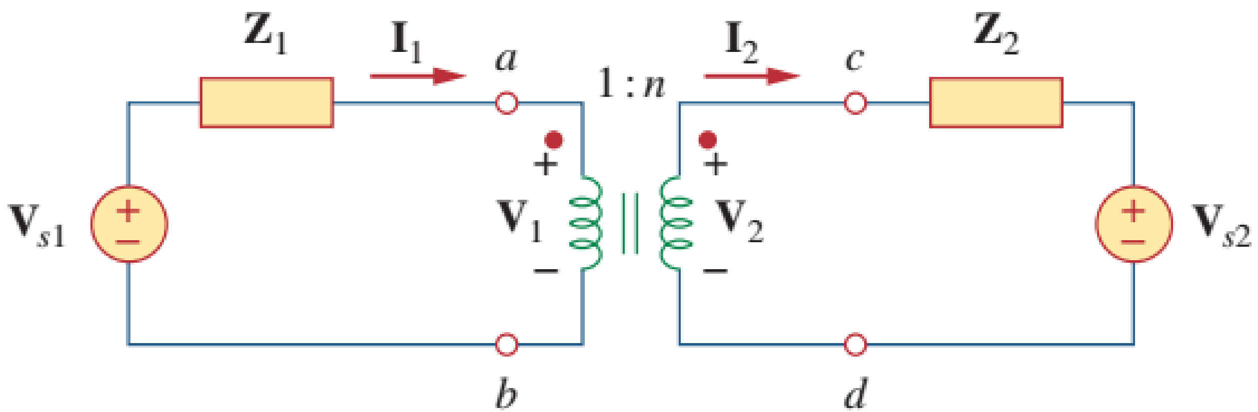


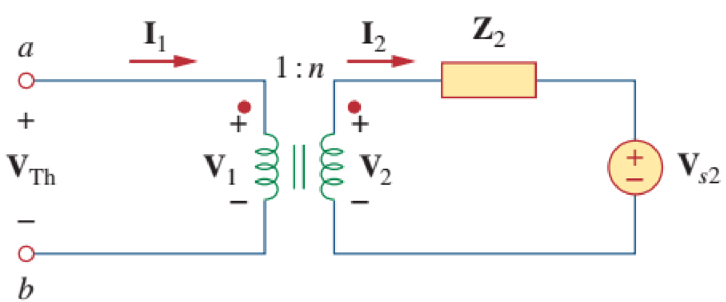
- The current is related as:

Ideal Transformers

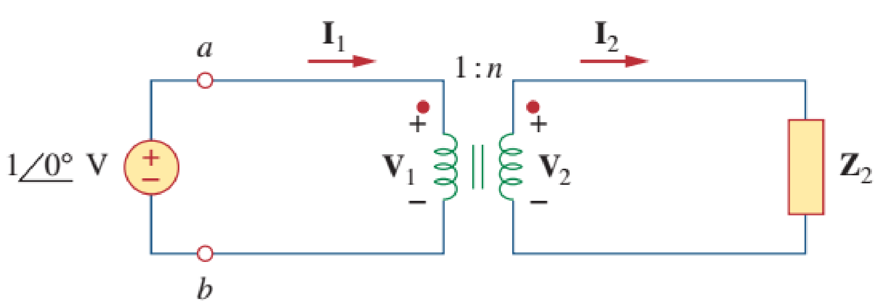
$$\frac{\mathbf{V_2}}{\mathbf{V_1}} = \frac{N_2}{N_1} = n$$

- Reflected impedance and source

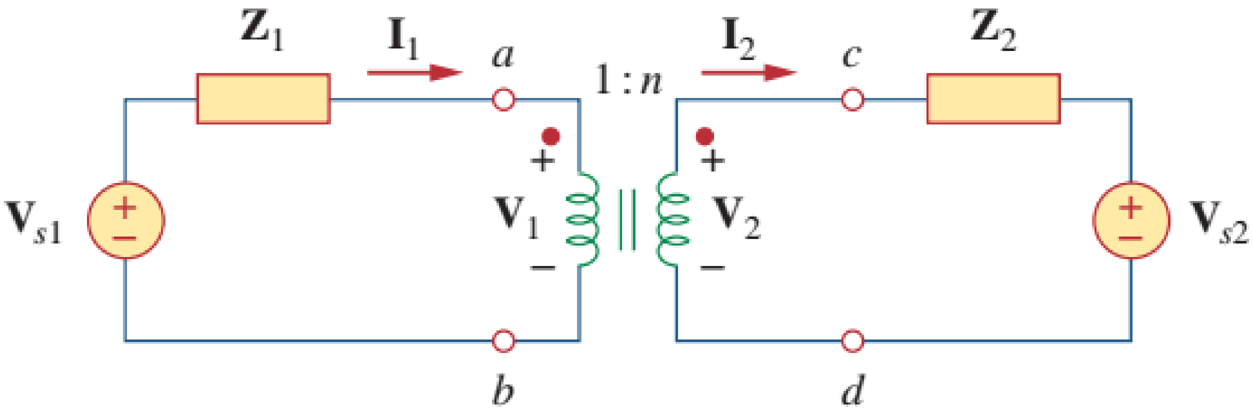


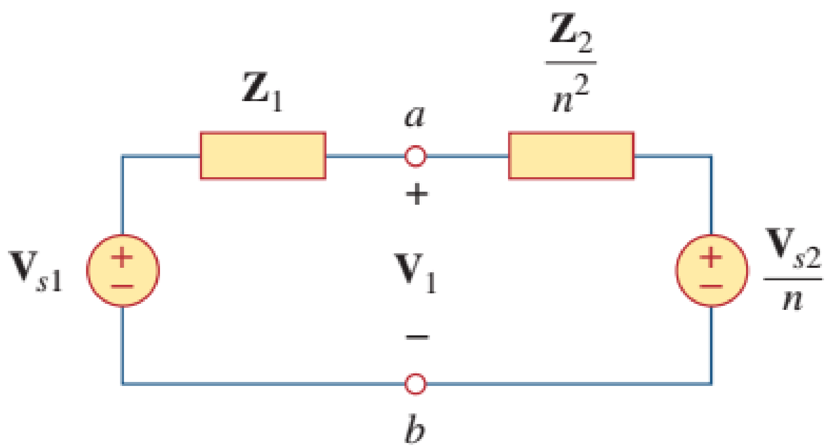


(a)



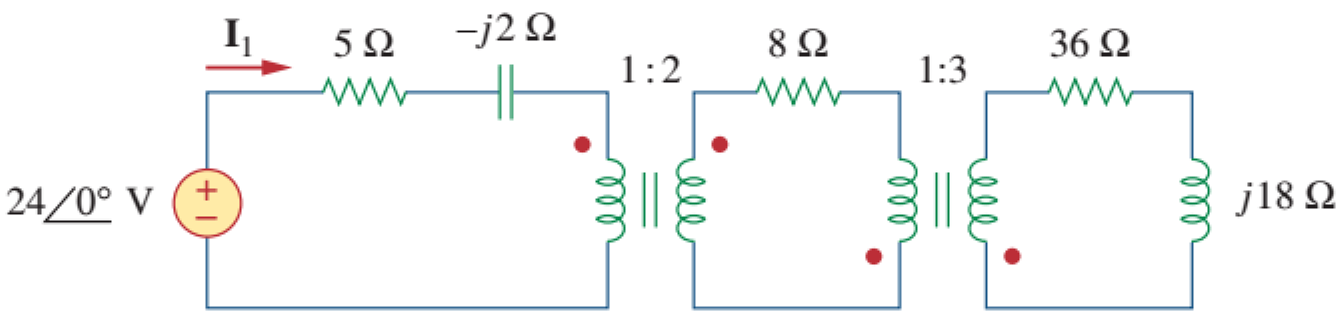
(b)


Z1
I1
a
1 : n
I2
c
Z2
Vs1
V1
V2
Vs2
b
d


Z2/n2
Z1
a
Vs1
V1
Vs2/n
b

Practice

- Find reflected impedance and $\mathbf{I}_1$

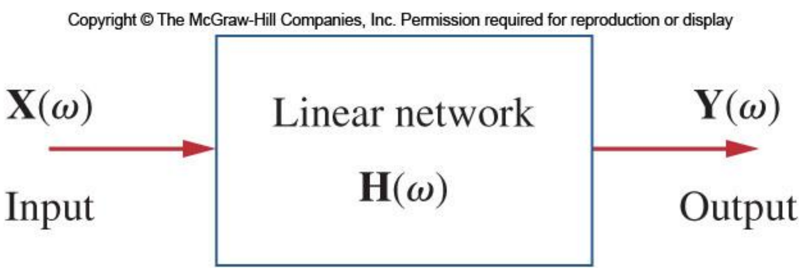


Frequency response

- Transfer function
- Bode plots (or diagram)
- Resonance
- Filter

Transfer Function

- The transfer function *H(ω)* is the frequency-dependent ratio of a forced function *Y(ω)* to the forcing function *X(ω)*.



$$H(\omega)=\frac{Y(\omega)}{X(\omega)}$$

$$H(\omega)=\text{Voltage gain}=\frac{V_o(\omega)}{V_i(\omega)}$$

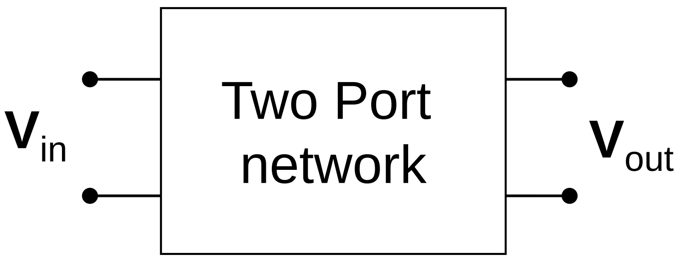
$$H(\omega)=\text{Current gain}=\frac{I_o(\omega)}{I_i(\omega)}$$

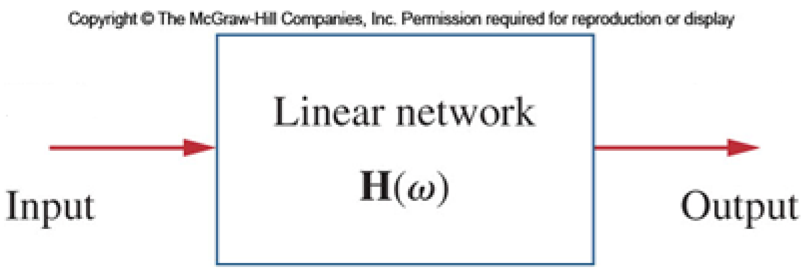
$$H(\omega)=\text{Transfer impedance}=\frac{V_o(\omega)}{I_i(\omega)}$$

$$H(\omega)=\text{Transfer admittance}=\frac{I_o(\omega)}{V_i(\omega)}$$

Transfer Function – Voltage Gain

- Complex quantity
- Both magnitude and phase are function of frequency

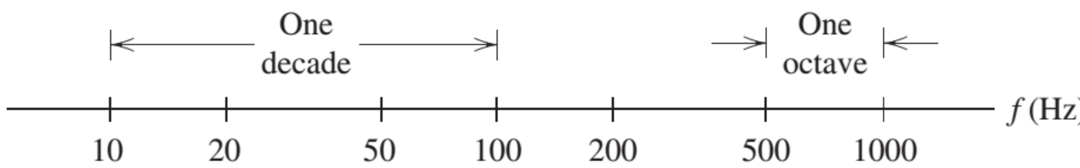




$$\mathbf{H}(f) = \frac{\mathbf{V_{out}}}{\mathbf{V_{in}}} = \frac{V_{out}}{V_{in}} \angle \left(\theta_{out} - \theta_{in}\right)$$

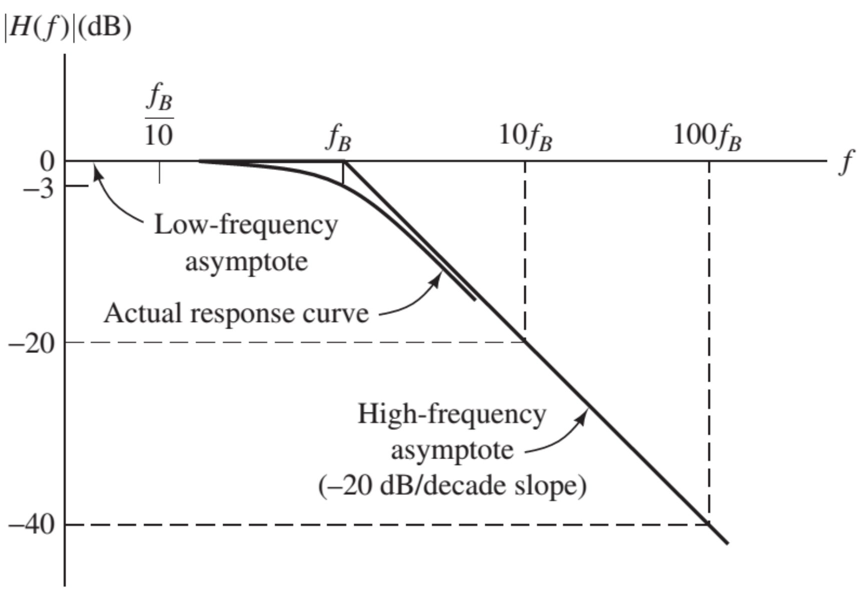
$$\mathbf{H(f)} = H(f)\angle\theta$$

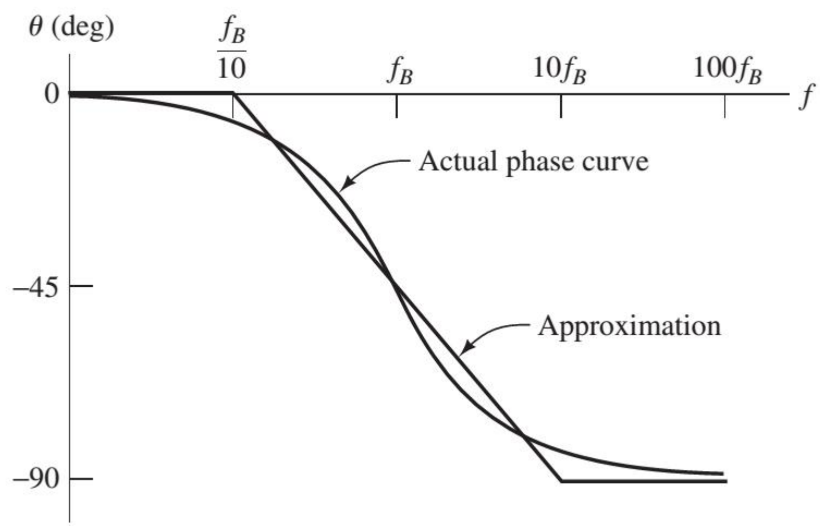
Bode Plots



Plotting the frequency response, magnitude or phase, on plots with

- Frequency X in log scale
- Y scale in dB (for magnitude) or degree (for phase)





Bode Plots

- Bode plot is particularly useful for displaying transfer function-- a general form is displayed as:

$$\mathbf{H}(\omega) = \frac{K(j\omega)^{\pm 1}(1 + j\omega/z_1)[1 + j2\zeta_1\omega/\omega_k + (j\omega/\omega_k)^2]\cdots}{(1 + j\omega/p_1)[1 + j2\zeta_2\omega/\omega_n + (j\omega/\omega_n)^2]\cdots}$$

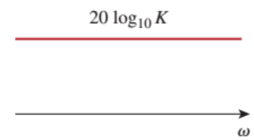
In constructing a Bode plot, we plot each factor separately and then add them graphically. The factors can be considered one at a time and then combined additively because of the logarithms involved. It is this mathematical convenience of the logarithm that makes Bode plots a powerful engineering tool.

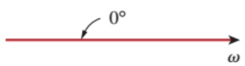
TABLE 14.3

Summary of Bode straight-line magnitude and phase plots.

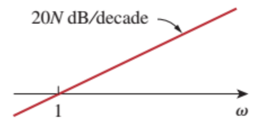
Factor | **Magnitude** | **Phase**

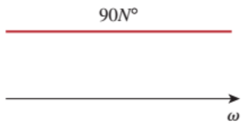
Factor: K



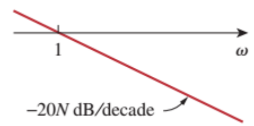


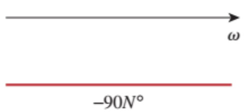
Factor: $(j\omega)^N$



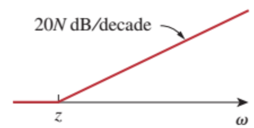


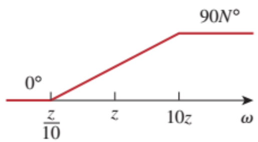
Factor: $\dfrac{1}{(j\omega)^N}$



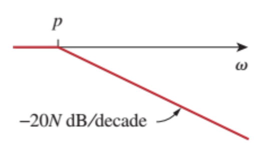


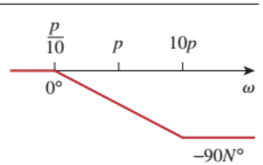
Factor: $\left(1 + \dfrac{j\omega}{z}\right)^N$



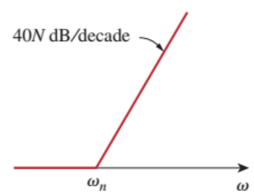


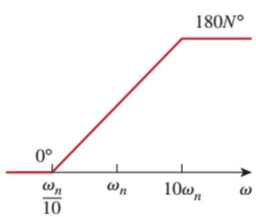
Factor: $\dfrac{1}{(1 + j\omega/p)^N}$



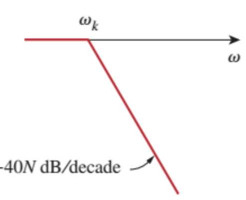


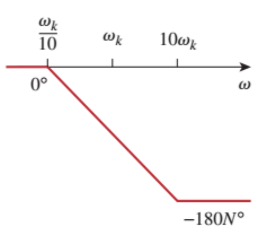
Factor: $\left[1 + \dfrac{2j\omega\zeta}{\omega_n} + \left(\dfrac{j\omega}{\omega_n}\right)^2\right]^N$





Factor: $\dfrac{1}{[1 + 2j\omega\zeta/\omega_k + (j\omega/\omega_k)^2]^N}$

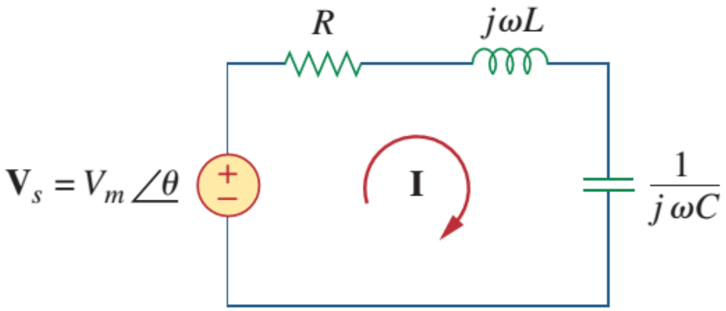




Series Resonance

- A series resonant circuit consists of an inductor and capacitor in series.

$$H(\omega) = \frac{V}{I} = \mathbf{Z} = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

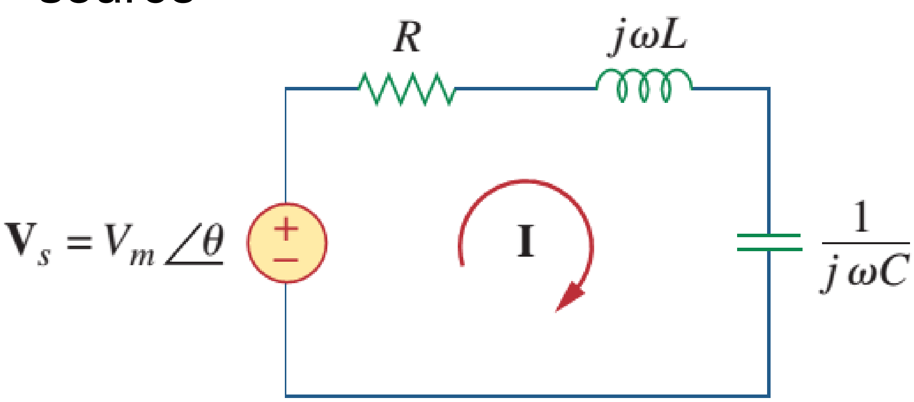


  - Resonance occurs when the imaginary part of *Z* is zero.
  - The value of ω that satisfies this is called the resonant frequency.

$$\boxed{\omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/s}}$$

Series Resonance

- At resonance:
  - The impedance is purely resistive
  - The voltage V_s and the current I are in phase
  - The magnitude of the transfer function is minimum
  - The inductor and capacitor voltages can be much more than the source

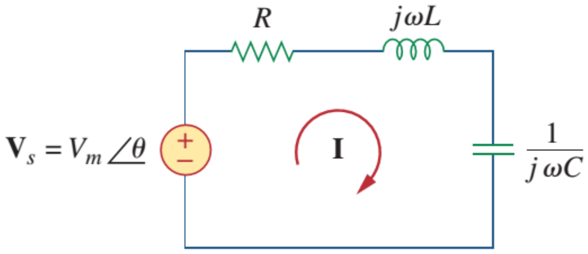


$$|\mathbf{V}_L| = \frac{V_m}{R}\omega_0 L$$

$$|\mathbf{V}_C| = \frac{V_m}{R}\frac{1}{\omega_0 C}$$

$$H(\omega) = \frac{V}{I} = \mathbf{Z} = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

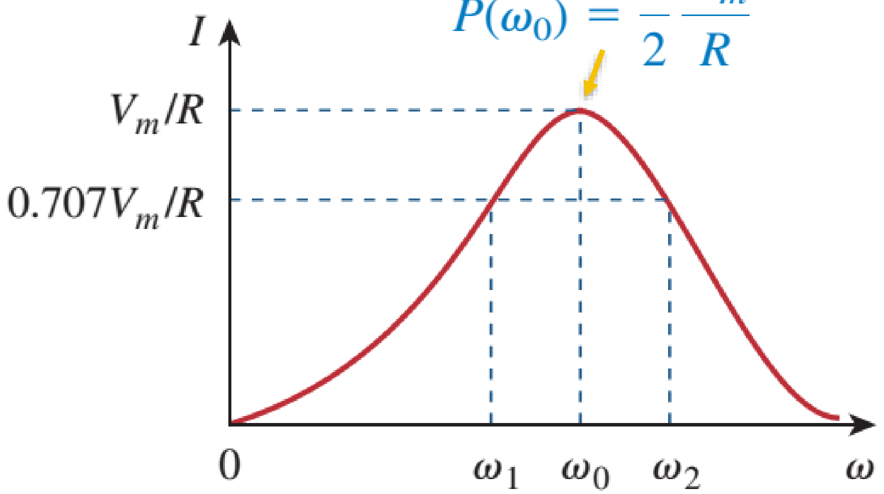
Half-Power Frequencies



- the current magnitude:

$$I = |\mathbf{I}| = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$

$$P(\omega_0) = \frac{1}{2}\frac{V_m^2}{R}$$



$$\omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/s}$$

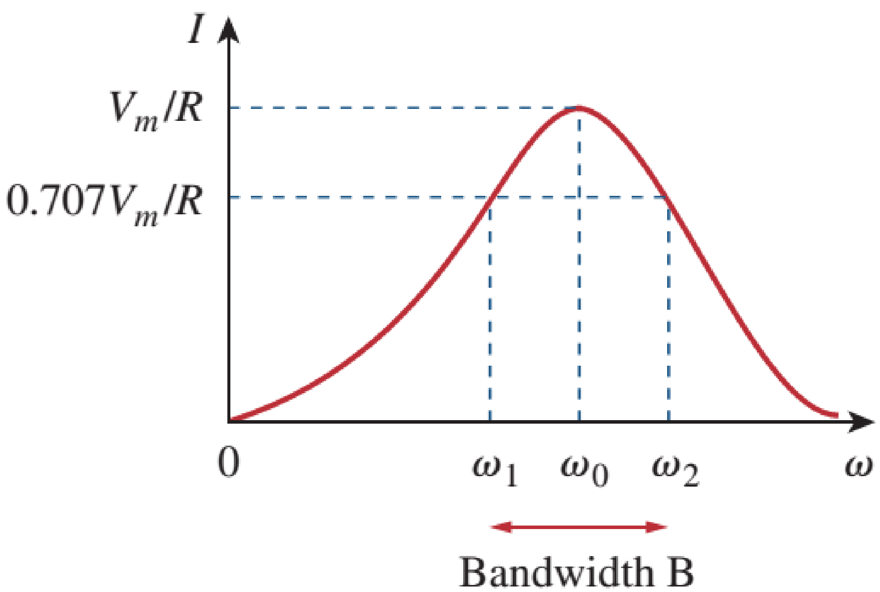
$$P(\omega_1) = P(\omega_2) = \frac{1}{2}P(\omega_0)$$

$$\omega_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\omega_0 = \sqrt{\omega_1 \omega_2}$$

Bandwidth



$$\omega_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

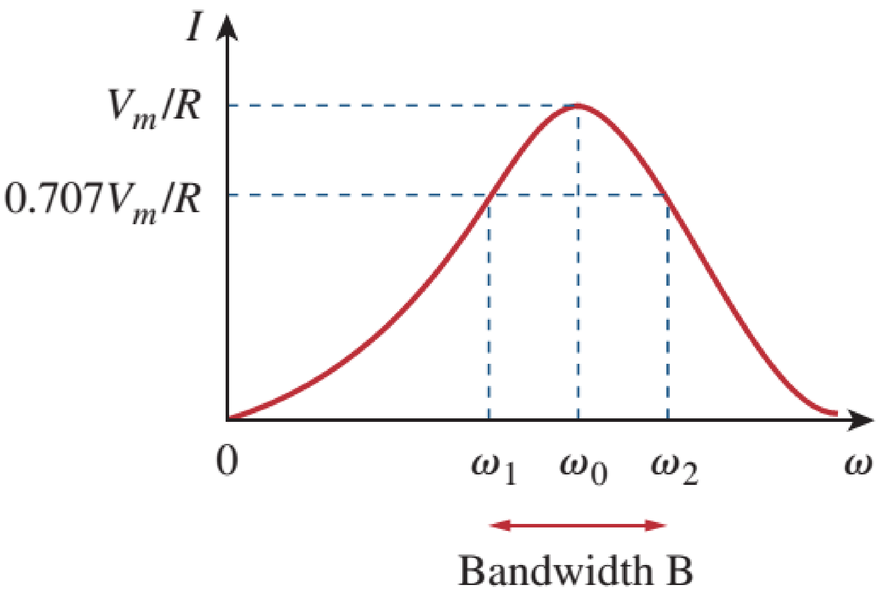
$$\omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

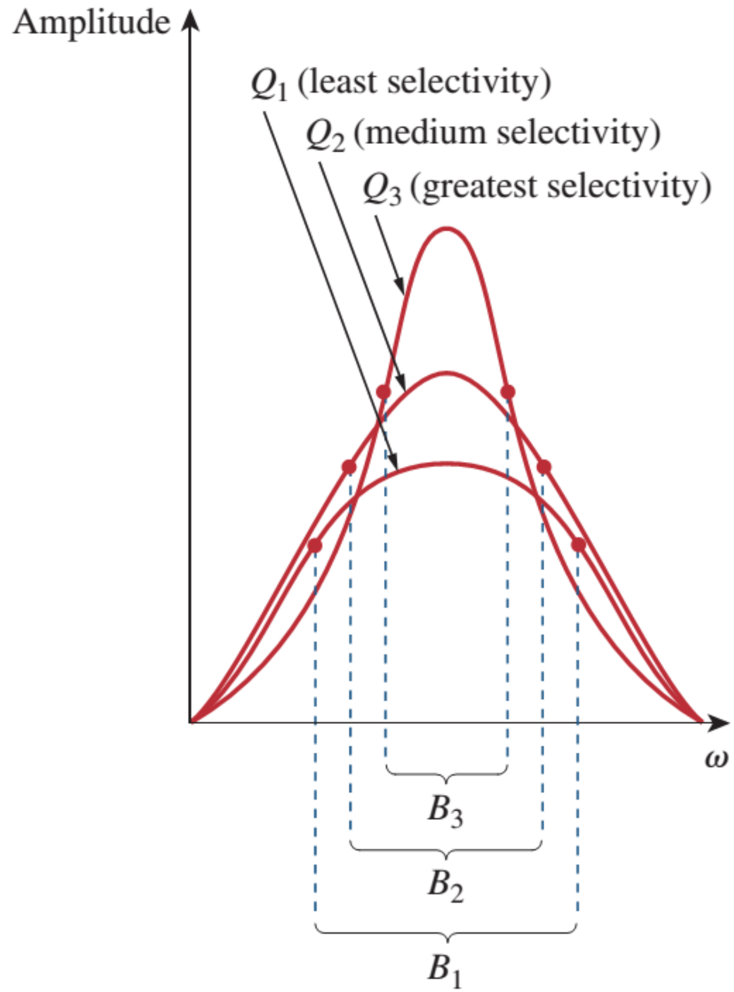
$$B = \omega_2 - \omega_1 = \frac{R}{L}$$

- Bandwidth: the difference between the two half-power frequencies

Quality Factor Q

- Quality factor Q: measure the "sharpness" of the resonance.



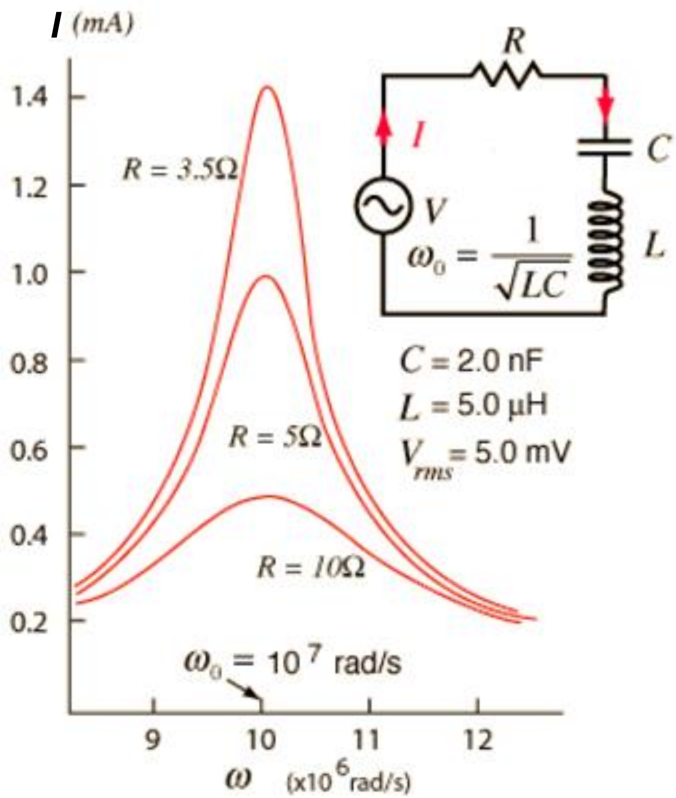


The smaller the B, the higher the Q.

$$\boxed{Q = \frac{\omega_0}{B}} \qquad \omega_0 = \frac{1}{\sqrt{LC}} \qquad B = \omega_2 - \omega_1 = \frac{R}{L}$$

$$\boxed{Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR}}$$

Quality Factor *Q*



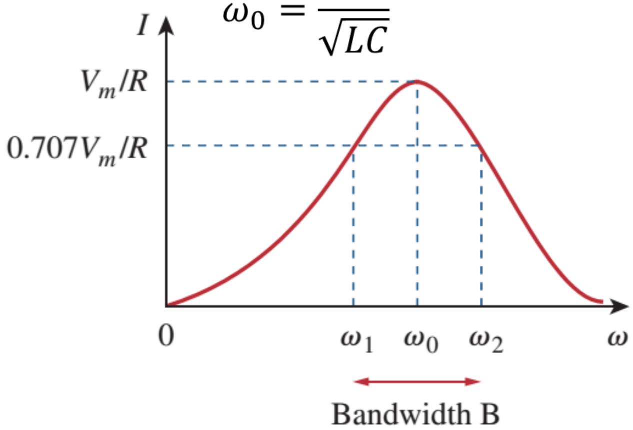
[Source: Georgia State U]

$$Q = \frac{\omega_0}{B}$$

$$\boxed{Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 C R}}$$

Approximation of Half-Power Frequencies

$$\omega_0 = \frac{1}{\sqrt{LC}}$$



$$\omega_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\frac{R}{L} = B \qquad B = \frac{\omega_0}{Q}$$

$$\omega_1 = -\frac{\omega_0}{2Q} + \omega_0\sqrt{1 + \frac{1}{4Q^2}}$$

$$\omega_2 = \frac{\omega_0}{2Q} + \omega_0\sqrt{1 + \frac{1}{4Q^2}}$$

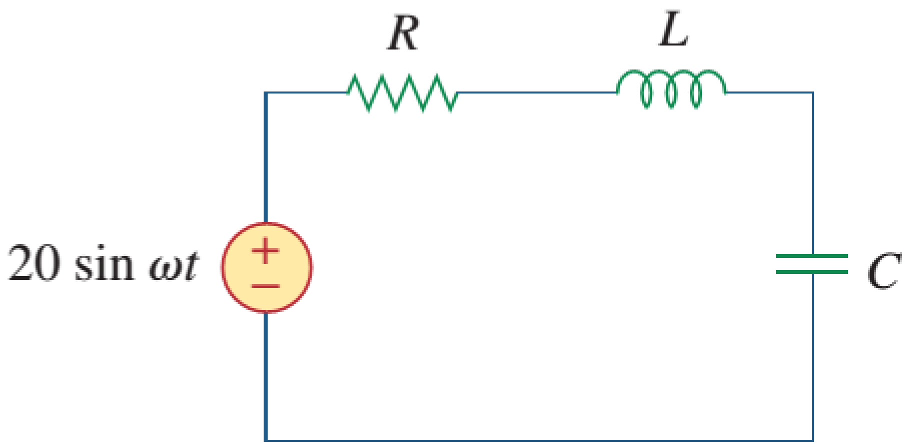
- For high-Q ($Q \geq 10$) circuits, half-power frequencies can be approximated as

$$\omega_1 \simeq \omega_0 - \frac{B}{2}, \qquad \omega_2 \simeq \omega_0 + \frac{B}{2}$$

Example

In the circuit, $R = 2\Omega$, $L = 1\text{mH}$ and $C = 0.4\mu\text{F}$

- Find resonant frequency ω_0.
- Calculate *Q* and bandwidth *B.*
- Find half-power frequencies.
- Determine the amplitude of the current at $\omega_{0,}$ ω_1 and ω_2.



$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-3} \times 0.4 \times 10^{-6}}} = 50 \text{ krad/s}$$

$$B = \omega_2 - \omega_1 = 2 \text{ krad/s}$$

$$Q = \frac{\omega_0}{B} = \frac{50}{2} = 25$$

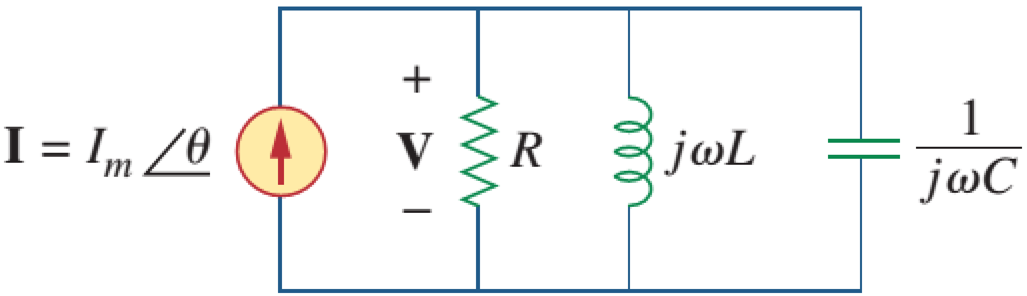
At $\omega = \omega_0$,

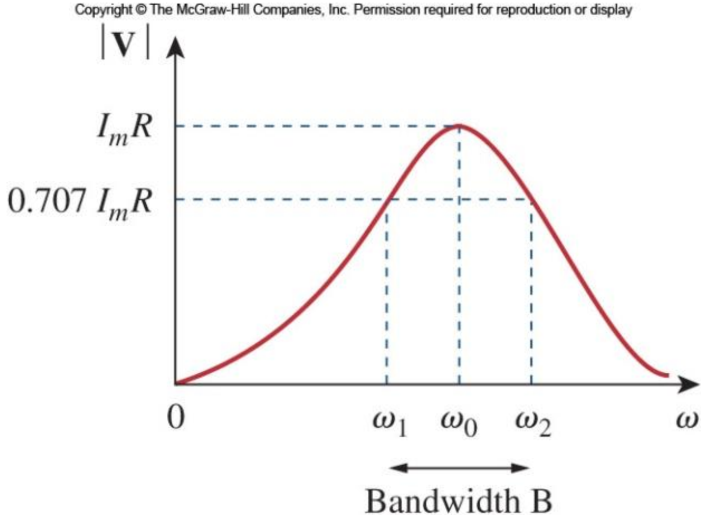
$$I = \frac{V_m}{R} = \frac{20}{2} = 10 \text{ A}$$

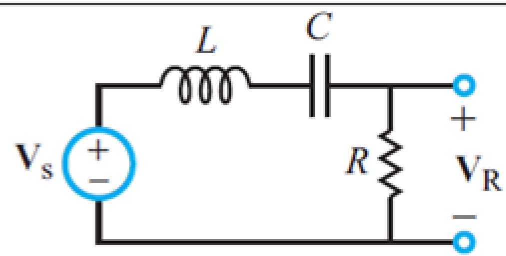
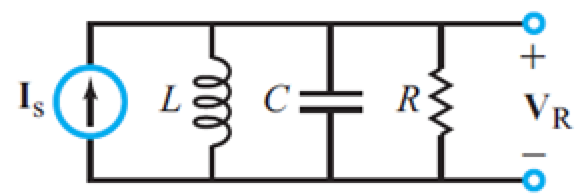
At $\omega = \omega_1, \omega_2$,

$$I = \frac{V_m}{\sqrt{2}R} = \frac{10}{\sqrt{2}} = 7.071 \text{ A}$$

Parallel resonance





	Series	Parallel
RLC Circuit		
Transfer Function	$\mathbf{H} = \dfrac{\mathbf{V}_R}{\mathbf{V}_s}$	$\mathbf{H} = \dfrac{\mathbf{V}_R}{\mathbf{I}_s}$
Resonant Frequency, ω_0	$\dfrac{1}{\sqrt{LC}}$	$\dfrac{1}{\sqrt{LC}}$
Bandwidth, B	$\dfrac{R}{L}$	$\dfrac{1}{RC}$
Quality Factor, Q	$\dfrac{\omega_0}{B} = \dfrac{\omega_0 L}{R}$	$\dfrac{\omega_0}{B} = \dfrac{R}{\omega_0 L}$
Lower Half-Power Frequency, ω_1	$\left[-\dfrac{1}{2Q} + \sqrt{1 + \dfrac{1}{4Q^2}}\right]\omega_0$	$\left[-\dfrac{1}{2Q} + \sqrt{1 + \dfrac{1}{4Q^2}}\right]\omega_0$
Upper Half-Power Frequency, ω_2	$\left[\dfrac{1}{2Q} + \sqrt{1 + \dfrac{1}{4Q^2}}\right]\omega_0$	$\left[\dfrac{1}{2Q} + \sqrt{1 + \dfrac{1}{4Q^2}}\right]\omega_0$

Notes: (1) The expression for Q of the series RLC circuit is the inverse of that for Q of the parallel circuit. (2) For $Q \geq 10$, $\omega_1 \simeq \omega_0 - \frac{B}{2}$, and $\omega_2 \simeq \omega_0 + \frac{B}{2}$.

[Source: Berkeley]

Filter

- Passive Filter
- Active Filter

Please refer to : Filter lecture notes

Laplace Transform