T1.(a) The Laplace transform of $x(t)$ is

$$X(s) = \int_0^{\infty} (e^{-2t} + e^{-3t})e^{-3t} dt$$

$$= \left[\frac{-e^{-(s+2)t}}{s+2}\right]\Big|_0^{\infty} + \left[\frac{-e^{-(s+3)t}}{s+3}\right]\Big|_0^{\infty}$$

$$= \frac{1}{s+2} + \frac{1}{s+3} = \frac{2s+5}{s^2+5s+6}, ROC: Re\{s\} > -2$$

Step 3 of 24

Draw the pole-zero plot for the transfer function.

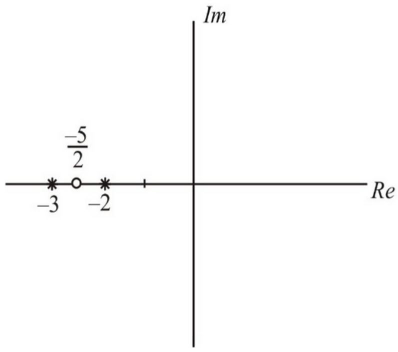


Figure 1

(b)

$$\because [sin\omega_0 t]u(t) \overset{L}{\leftrightarrow} \frac{\omega_0}{s^2+\omega_0^2}, ROC: Re\{s\} > 0$$

$$\because e^{s_0 t}x(t) \overset{L}{\leftrightarrow} X(s-s_0)$$

$$\therefore X(s) = \frac{1}{s+4} + \frac{5}{(s+5)^2+25} = \frac{s^2+15s+70}{s^3+14s^2+90s+200} \quad ROC: Re\{s\} > -4$$

Step 7 of 24

Draw the pole-zero plot for the transfer function.

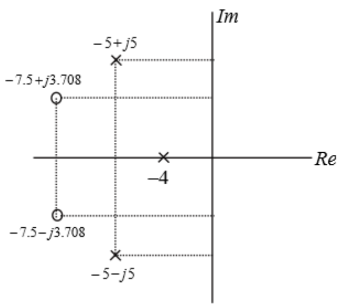


Figure 2

(c)

$$X(s) = 1 + \frac{1}{s}, ROC: Re\{s\} > 0$$

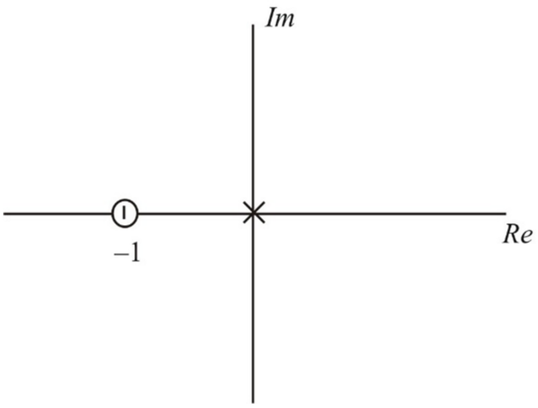


Figure 9

(d)

$$te^{-2|t|} = te^{-2t}u(t) + te^{2t}u(-t) \overset{L}{\leftrightarrow} \frac{1}{(s+2)^2} - \frac{1}{(s-2)^2} = \frac{-8s}{(s^2-4)^2}, ROC: -2 < Re\{s\} < 2$$

Step 12 of 24

Draw the pole-zero plot for the transfer function.

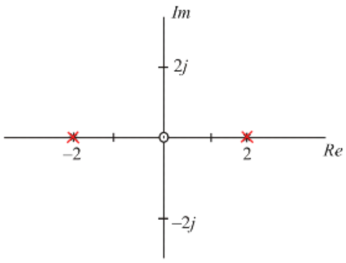
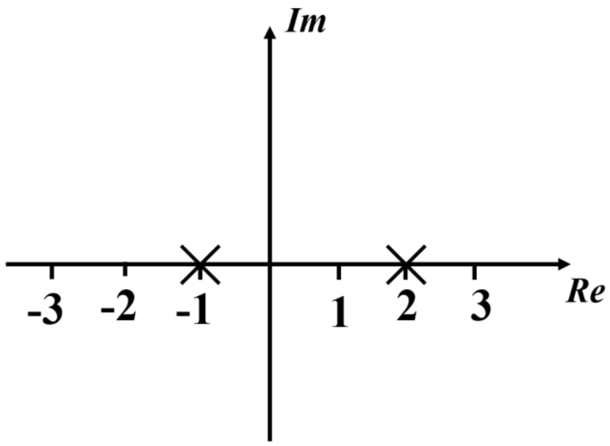


Figure 4

T2.(a)

$$s^2Y(s) - sY(s) - 2Y(s) = X(s)$$
$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s^2 - s - 2}$$



(b)

(i) Stable→ $ROC: -1 < Re\{s\} < 2,\ \ h(t) = -\frac{1}{3}e^{2t}u(-t) - \frac{1}{3}e^{-t}u(t)$

(i) Causal→ $ROC: Re\{s\} > 2,\ \ h(t) = \frac{1}{3}e^{2t}u(t) - \frac{1}{3}e^{-t}u(t)$

(i) Neither stable nor causal→ $ROC: Re\{s\} < -1,\ \ h(t) = -\frac{1}{3}e^{2t}u(-t) + \frac{1}{3}e^{-t}u(-t)$

T3. $From\ 2, \because u(t) \overset{L}{\leftrightarrow} \frac{1}{s}$

$\therefore H(s) = 0 \ when \ s = 0$

$also, from \ 3, tu(t) \overset{L}{\leftrightarrow} \frac{1}{s^2}$

$\therefore H(s) \ has \ only \ 1 \ zero \ at \ s = 0$

$from \ 4, p(t) = \frac{d^2h(t)}{dt^2} + 2\frac{dh(t)}{dt} + 2h(t) \ is \ finite \ duration,$

$\therefore P(s) \ has \ no \ poles, in \ other \ words, there's \ no \ s \ in \ the \ denomiator \ of \ P(s)$

$use \ Laplace \ transformation, P(s) = s^2H(s) + 2sH(s) + 2H(s)$

$H(s) = \frac{P(s)}{s^2 + 2s + 2}$

$\because P(s) \ has \ no \ poles, and \ from \ 5, the \ order \ of \ s \ in \ numerator \ is \ one \ less \ than \ the \ s \ in \ denomiator$

$\therefore H(s) = \frac{As}{s^2 + 2s + 2}$

$where \ A \ is \ a \ constant,$

$from \ 1, H(1) = 0.2$

$\therefore A = 1$

$\therefore H(s) = \frac{s}{s^2 + 2s + 2}$

$\therefore poles \ are \ -1 \pm j$

$\because h(t) \ is \ stable \ and \ causal$

$\therefore ROC: Re\{s\} > -1$

T4. (a)

$$H(s) = (2s^2 + 4s - 6)H_1(s)$$
$$\therefore Y(s) = (2s^2 + 4s - 6)Y_1(s)$$
$$\therefore y(t) = 2\frac{d^2y_1(t)}{dt^2} + 4\frac{dy_1(t)}{dt} - 6y_1(t)$$

(b)

$$\because Y_1(s) = \frac{F(s)}{s}, f(t) = dy_1(t)/dt$$

(c)

$$\because F(s) = \frac{E(s)}{s}, e(t) = \frac{df(t)}{dt} = \frac{d^2y_1(t)}{dt^2}$$

(d)

$$y(t) = 2e(t) + 4f(t) - 6y_1(t)$$

T5.

$$\frac{d^2y(t)}{dt^2}+5\frac{dy}{dt}+6y(t)=x(t)$$

Use Laplace transformation, we get $s^2Y(s)-sy(0^-)-y'(0^-)+5sY(s)-5y(0^-)+6Y(s)=X(s)$

(a)

$$X(s)=\frac{1}{s+4}$$

$$Y_{zs}(s)=\frac{X(s)}{s^2+5s+6}=\frac{1}{(s+2)(s+3)(s+4)}=\frac{1/2}{s+2}-\frac{1}{s+3}+\frac{1/2}{s+4}$$

$$y_{zs}(t)=\frac{1}{2}e^{-2t}u(t)-e^{-3t}u(t)+\frac{1}{2}e^{-4t}u(t)$$

(b)

$$Y_{zi}(s)=\frac{(s+5)y(0^-)+y'(0^-)}{s^2+5s+6}=\frac{1}{s+2}$$

$$y_{zi}(t)=e^{-2t}u(t)$$

(c)

$$y(t)=y_{zi}(t)+y_{zs(t)}=\frac{3}{2}e^{-2t}u(t)-e^{-3t}u(t)+\frac{1}{2}e^{-4t}u(t)$$