Week 13 Quiz

March 18th 2020

1 Lecture 25

Given $X, Y \in \mathbb{S}^n_{++}$, $\forall z \in \mathbb{R}^n, z^T X z > 0, z^T Y z > 0$.
For $\theta_1, \theta_2 \geq 0$, then $\forall z$,

$$\begin{aligned} z^T(\theta_1 X + \theta_2 Y)z &= \theta_1 z^T X z + \theta_2 z^T Y z \\ &\geq 0 + 0 \\ &= 0 \end{aligned}$$

If $\theta_1 = \theta_2 = 0$, $z^T(\theta_1 X + \theta_2 Y)z = 0 \notin \mathbb{S}^n_{++}$. So $\mathbb{S}^n_{++}$ is not a convex cone.

2 Lecture 26

$\forall Y_1, Y_2 \in C$, we can get $\forall \theta \in (0,1), \theta Y_1 + (1-\theta)Y_2 \in C$.
$\forall x_1, x_2 \in f^{-1}(C), f(x_1) = X_1, f(x_2) = X_2$, and $\forall \theta \in (0,1)$

$$\begin{aligned} f(\theta x_1 + (1-\theta)x_2) &= A(\theta x_1 + (1-\theta)x_2) + b \\ &= \theta A x_1 + \theta b + (1-\theta)Ax_2 + (1-\theta)b \\ &= \theta f(x_1) + (1-\theta) f(x_2) \\ &= \theta X_1 + (1-\theta)X_2 \\ &\in C \end{aligned}$$

So $f^{-1}(C)$ is convex.