T1.(a) $x[n]=u[n-2]-n[n-6]=\delta[n-2]+\delta[n-3]+\delta[n-4]+\delta[n-5]$

$\therefore X\left(e^{j\omega}\right)=e^{-2j\omega}+e^{-3j\omega}+e^{-4j\omega}+e^{-5j\omega}$

(b) $X\left(e^{j\omega}\right)=\sum_{n=-\infty}^{-1}\left(\frac{1}{2}\right)^{-n}e^{-j\omega n}=\sum_{n=1}^{\infty}\left(\frac{1}{2}e^{j\omega}\right)^{n}=\frac{e^{j\omega}}{2(1-\frac{1}{2}e^{j\omega})}$

(c) $x[n]=-\sin\left(\frac{\pi}{3}n\right)+\cos\left(\frac{\pi}{3}n\right)=-\frac{1}{2j}\left[e^{\frac{j\omega n}{3}}-e^{\frac{-j\omega n}{3}}\right]+\frac{1}{2}\left[e^{\frac{j\omega n}{3}}+e^{\frac{-j\omega n}{3}}\right]$,

therefore, $X\left(e^{j\omega}\right)=-\frac{\pi}{j}\left[\delta\left(\omega-\frac{\pi}{3}\right)-\delta\left(\omega+\frac{\pi}{3}\right)\right]+\pi[\delta\left(\omega-\frac{\pi}{3}\right)+\delta(\omega+\frac{\pi}{3})]$

T2.(a) $Y\left(e^{j\omega}\right)=X\left(e^{j\omega}\right)H\left(e^{j\omega}\right)$

$$\therefore H\left(e^{j\omega}\right)=\frac{Y\left(e^{j\omega}\right)}{X(e^{j\omega})}=\frac{1}{1+\frac{1}{2}e^{-j\omega}}$$

(b)(i) In this case, $X\left(e^{j\omega}\right)=\frac{1}{1-\frac{1}{2}e^{-j\omega}}$

$$\therefore Y\left(e^{j\omega}\right)=\frac{1}{1-\frac{1}{2}e^{-j\omega}}\cdot\frac{1}{1+\frac{1}{2}e^{-j\omega}}=\frac{1/2}{1-\frac{1}{2}e^{-j\omega}}+\frac{1/2}{1+\frac{1}{2}e^{-j\omega}}$$

$$y[n]=\frac{1}{2}\left(\frac{1}{2}\right)^{n}u[n]+\frac{1}{2}\left(-\frac{1}{2}\right)^{n}u[n]$$

(b)(ii) In this case, $X\left(e^{j\omega}\right)=1+\frac{1}{2}e^{-j\omega}$

$\therefore Y\left(e^{j\omega}\right)=1$

$y[n]=\delta[n]$

(c)(i) $Y\left(e^{j\omega}\right)=\left[\frac{1-\frac{1}{4}e^{-j\omega}}{1+\frac{1}{2}e^{-j\omega}}\right]\left[\frac{1}{1+\frac{1}{2}e^{-j\omega}}\right]=\frac{1}{\left(1+\frac{1}{2}e^{-j\omega}\right)^{2}}-\frac{\frac{1}{4}e^{-j\omega}}{\left(1+\frac{1}{2}e^{-j\omega}\right)^{2}}$

$$\therefore y[n]=(n+1)\left(-\frac{1}{2}\right)^{n}u[n]-\frac{1}{4}(n)\left(-\frac{1}{2}\right)^{n-1}u[n-1]$$

(ii) $Y\left(e^{j\omega}\right)=\left[1+2e^{-3j\omega}\right]\left[\frac{1}{1+\frac{1}{2}e^{-j\omega}}\right]=\frac{1+2e^{-3j\omega}}{1+\frac{1}{2}e^{-j\omega}}$

$$\therefore y[n]=\left(-\frac{1}{2}\right)^{n}u[n]+2\left(-\frac{1}{2}\right)^{n-3}u[n-3]$$

T3.(a) $X\left(e^{j0}\right)=\sum_{n=-\infty}^{\infty}x[n]=6$

(b) Note that $y[n]=x[n+2]$ is even, thus $Y(e^{j\omega})$ is real and even, $\therefore \measuredangle Y\left(e^{j\omega}\right)=0$,

$\because Y\left(e^{j\omega}\right)=e^{2j\omega}X\left(e^{j\omega}\right)$, $\therefore \measuredangle X\left(e^{j\omega}\right)=-2\omega$.

(c) $\because 2\pi x[0]=\int_{-\pi}^{\pi}X\left(e^{j\omega}\right)d\omega$, $\therefore \int_{-\pi}^{\pi}X\left(e^{j\omega}\right)d\omega=4\pi$

(d) $X(e^{j\pi})=\sum_{n=-\infty}^{\infty}x[n](-1)^{n}=2$

(e) $\because Ev\{x[n]\}\leftarrow FT\rightarrow Re\{X\left(e^{j\omega}\right)\}$,

$\therefore\ desired\ signal\ is\ Ev\{x[n]\} = \frac{(x[n]+x[-n])}{2}, which\ is:$

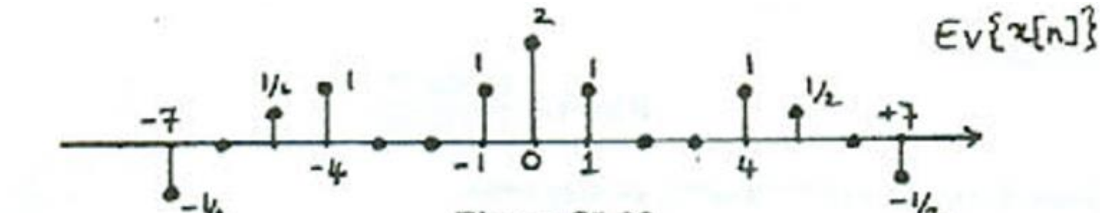

Figure S5.23

T4.(a)(i) $\because X(e^{j\omega}) = \frac{1}{1-\frac{1}{2}e^{-j\omega}} - \frac{\frac{1}{4}e^{-j\omega}}{1-\frac{1}{2}e^{-j\omega}}$, $Y(e^{j\omega}) = \frac{1}{1-\frac{1}{3}e^{-j\omega}}$

$$\therefore H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{1-\frac{1}{2}e^{-j\omega}}{\left(1-\frac{1}{3}e^{-j\omega}\right)\left(1-\frac{1}{4}e^{-j\omega}\right)}$$

$$\therefore h[n] = 3\left(\frac{1}{4}\right)^n u[n] - 2\left(\frac{1}{3}\right)^n u[n]$$

(ii) $\because \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{1-\frac{1}{2}e^{-j\omega}}{\left(1-\frac{1}{3}e^{-j\omega}\right)\left(1-\frac{1}{4}e^{-j\omega}\right)}$

$$\therefore \left[1-\frac{7}{12}e^{-j\omega}+\frac{1}{12}\left(e^{-j\omega}\right)^2\right]Y(e^{j\omega}) = \left[1-\frac{1}{2}e^{-j\omega}\right]X(e^{j\omega})$$

$$\therefore y[n]-\frac{7}{12}y[n-1]+\frac{1}{12}y[n-2] = x[n]-\frac{1}{2}x[n-1]$$

(b) $X(e^{j\omega}) = \frac{1}{\left(1-\frac{1}{2}e^{-j\omega}\right)^2} + \frac{1}{1-\frac{1}{2}e^{-j\omega}} = \frac{2\left(1-\frac{1}{4}e^{-j\omega}\right)}{\left(1-\frac{1}{2}e^{-j\omega}\right)^2}$, $Y(e^{j\omega}) = \frac{1}{1-\frac{1}{4}e^{-j\omega}}$

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{\left(1-\frac{1}{2}e^{-j\omega}\right)^2}{2\left(1-\frac{1}{4}e^{-j\omega}\right)^2}$$

when $Y(e^{j\omega}) = \frac{\frac{1}{2}e^{-j\omega}}{1+\frac{1}{2}e^{-j\omega}}, X(e^{j\omega}) = \frac{e^{-j\omega}\left(1-\frac{1}{4}e^{-j\omega}\right)^2}{\left(1-\frac{1}{2}e^{-j\omega}\right)^2\left(1+\frac{1}{2}e^{-j\omega}\right)} = \frac{9/16}{\left(1+\frac{1}{2}e^{-j\omega}\right)} + \frac{5/16}{\left(1-\frac{1}{2}e^{-j\omega}\right)} + \frac{1/8}{\left(1-\frac{1}{2}e^{-j\omega}\right)^2}$

$$\therefore x[n] = \frac{9}{16}\left(-\frac{1}{2}\right)^{n-1}u[n-1] + \frac{5}{16}\left(\frac{1}{2}\right)^{n-1}u[n-1] + \frac{1}{8}n\left(\frac{1}{2}\right)^{n-1}u[n-1]$$