ShanghaiTech University

CS101 Algorithms and Data Structures

Fall 2021

Homework 12

Due date: 23:59, January 3, 2022

1. Please write your solutions in English.
2. Submit your solutions to gradescope.com.
3. Set your FULL NAME to your Chinese name and your STUDENT ID correctly in Account Settings.
4. If you want to submit a handwritten version, scan it clearly. Camscanner is recommended.
5. When submitting, match your solutions to the according problem numbers correctly.
6. No late submission will be accepted.
7. Violations to any of the above may result in zero grade.
8. In this homework, all the proofs need three steps. The demands and an example are given on the next page. If you do not organize your answer in the standard format, you will not get any point.

Demand of the NP-complete Proof

When proving problem A is NP-complete, please clearly divide your answer into three steps:

1. Prove that problem A is in NP.
2. Choose an NP-complete problem B and for any B instance, construct an instance of problem A.
3. Prove that the yes/no answers to the two instances are the same.

Proof Example

Suppose you are going to schedule courses for the SIST and try to make the number of conflicts no more than K. You are given 3 sets of inputs: $C = \{\cdots\}, S = \{\cdots\}, R = \{\{\cdots\}, \{\cdots\}, \cdots\}$. C is the set of distinct courses. S is the set of available time slots for all the courses. R is the set of requests from students, consisting of a number of subsets, each of which specifies the course a student wants to take. A conflict occurs when two courses are scheduled at the same slot even though a student requests both of them. Prove this schedule problem is NP-complete.

1. Firstly, for any given schedule as a certificate, we can traverse every student's requests and check whether the courses in his/her requests conflicts and count the number of conflicts, and at last check if the total number is fewer than K, which can be done in polynomial time. Thus the given problem is in NP.
2. We choose 3-coloring problem which is a NP-complete problem. For any instance of 3-coloring problem with graph G, we can construct an instance of the given problem: let every node v becomes a course, thus construct C; let every edge (u, v) becomes a student whose requests is $\{u, v\}$, thus construct R; let each color we use becomes a slot, thus construct S; at last let K equals to 0.
3. We now prove G is a yes-instance of 3-coloring problem if and only if (C, S, R, K) is a yes-instance of the given problem:
   - "$\Rightarrow$": if G is a yes-instance of 3-coloring problem, then schedule the courses according to their color. Since for each edge (u, v), u and v will be painted with different color, then for each student, his/her requests will not be scheduled to the same slot, which means the given problem is also a yes-instance.
   - "$\Leftarrow$": if (C, S, R, K) is a yes-instance of the given problem, then painting the nodes in G according to their slots. Since $K = 0$, then for every student, there is no conflict between their requests, which suggests that for every edge (u, v), u and v will not be painted with the same color. It is also a yes-instance of 3-coloring problem.

Therefore, the given problem is NP-complete.

1: (3'+3'+4') Multiple Choice

The following questions are multiple choice questions, each question may have one or more correct answers. Select all correct answers.

Note: You should write those answers ***in the box*** *below.*

Question 1	Question 2	Question 3

Question 1. *A problem in NP is NP-complete if:*

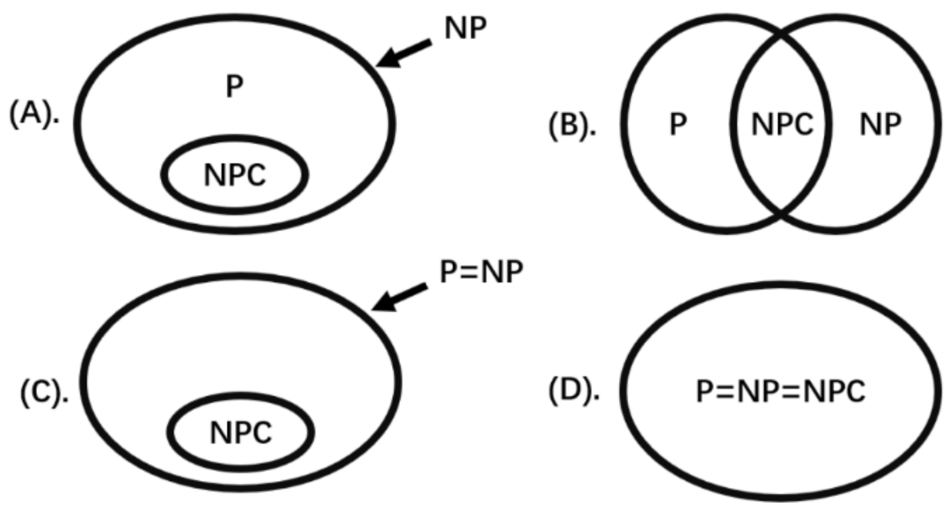
(A) It can be reduced to the 3-SAT problem in polynomial time.

(B) The 3-SAT problem can be reduced to it in polynomial time.

(C) It can be reduced to any other problem in NP in polynomial time.

(D) Some problem in NP can be reduced to it in polynomial time.

Question 2. *Suppose we have found an algorithm which correctly solves the vertex cover problem in polynomial time. Then under this circumstance, which one of the following Venn diagrams correctly represents the relationship among the complexity classes P, NP and NP Complete (NPC)?*



Question 3. *Assume problem A reduces to problem B in polynomial time. Which of the following choice(s) is/are correct?*

(A) If A is in ***P****. Then B is in* ***P****.*

(B) If B is in ***P****. Then A is in* ***P****.*

(C) If A is ***NP****-hard. Then B is* ***NP****-hard.*

(D) If B is ***NP****-hard. Then A is* ***NP****-hard.*

2: (10') 4-COLOR

Given an undirected graph and 4 different colors, can we color the nodes so that no adjacent nodes have the same color? Show that the 4-COLOR problem is NP-complete. **(Hint: Reduce from 3-COLOR)**

3: (10') Reduction from Independent Set

Given a set E and m subsets of $E, S_1, S_2, ..., S_m$, is there a way to select k of the m subsets such that the selected subsets are pairwise disjoint?
Show that this problem is NP complete.
HINT: Reduction from Independent Set.

4: (10') Exact 4-SAT problem

In the Exact 4-SAT problem, the input is a set of clauses, each of which is a disjunction of exactly four literals, and such that each variable occurs at most once in each clause. The goal is to find a satisfying argument, if one exists. Prove that Exact 4-SAT is NP-complete by a reduction from 3-SAT.