EE150 Signals and Systems
– Part 6: Laplace Transform (LT)

$\downarrow$ Week 9, Tue, 20180424

CTFT

(Continuous-Time) Fourier transform is extremely useful for studying signal and LTI systems. Dirichlet (sufficient) conditions for CTFT exists:

1. $x(t)$ is absolutely integrable
2. finite number of ... extrema ... finite interval ...
3. finite discontinuity ... finite interval ...

However, not all signals have CTFT!

Try to find a transform which is more general than CTFT, and can be applied to larger class of signals.

Laplace Transform

Eigen-function e^{st}: $H(s) \equiv \int_{-\infty}^{\infty} h(\tau) e^{-s\tau} d\tau$

$$e^{st} \longrightarrow \boxed{\text{LTI}} \longrightarrow H(s)e^{st}$$

Laplace transform (LT) of $x(t)$: complex $s = \sigma + j\omega$

$$X(s) := \int_{-\infty}^{\infty} x(t) e^{-st} dt \tag{1}$$

CTFT eigen-function: $e^{j\omega t}$ ($s = j\omega$: pure imaginary)

$$\Longrightarrow \quad \begin{aligned} X(s) &= FT\{x(t)e^{-\sigma t}\}, \\ X(s)\big|_{s=j\omega} &= FT\{x(t)\}. \end{aligned}$$

LT

Note: Definition in (1) is called Bilateral LT

Unilateral LT:

$$X(s) := \int_{0^-}^{\infty} x(\tau) e^{-st} dt$$

practical since usually we deal with right-sided signals

Right-sided signal: $x(t) = 0$, $\forall t < t_0$ for some t_0

Example

$x_1(t) = e^{-at}u(t)$, $a \in \mathbb{R}$

$$\begin{aligned} X_1(s) &= \int_{-\infty}^{\infty} e^{-at}u(t)e^{-st}dt \\ &= \int_{0}^{\infty} e^{-(a+\sigma)t}u(t)e^{-j\omega t}dt \\ &= \frac{1}{a+\sigma+j\omega}, \quad a+\sigma>0 \\ &= \frac{1}{a+s}, \quad Re(s) > -a \end{aligned}$$

Integral converges only when $Re(s) > -a$

Example

$x_2(t) = -e^{-at}u(-t)$, $a \in \mathbb{R}$

$$\begin{aligned} X_2(s) &= -\int_{-\infty}^{0} e^{-at}e^{-st}dt \\ &= -\int_{0}^{\infty} e^{(s+a)t}dt \\ &= \frac{1}{s+a}, \quad Re(s) < -a \end{aligned}$$

Same LT, different convergence region!

If $a \in \mathbb{C}$, then convergence region $Re(s) < Re(-a)$

Region of Convergence

Region of (conditional) Convergence (ROC): region of s for which

$$\int_{-\infty}^{\infty} x(t)e^{-st}dt \quad \text{converges}$$

Region of (absolute) Convergence (ROC): region of s for which

$$\int_{-\infty}^{\infty} |x(t)e^{-st}|dt \quad \text{converges}$$

Note for some $x(t)$, these two regions might be different.

If $x(t)e^{-\sigma t}$ satisfies the first condition in Dirichlet conditions, these two regions are identical. Usually we assume this holds.

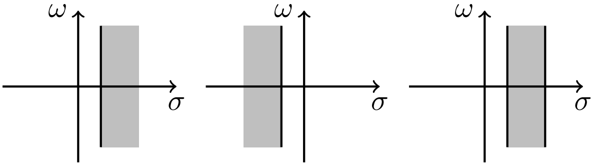
Properties of ROC

Property 1

ROC consists of strips in s-plane.

$s = \sigma + j\omega$:

$$\int_{-\infty}^{\infty} |x(t)e^{-st}|dt = \int_{-\infty}^{\infty} |x(t)|e^{-\sigma t}dt$$



The boundary $Re(s) = \sigma_0$ might be or not be in ROC

Properties of ROC

Polynomial $P(s)$: $P(s) = a_0 + a_1 s + \cdots a_n s^n$

Rational $X(s)$: ratio $P(s)/Q(s)$ of two polynomials $P(s)$ and $Q(s)$

Zero (for rational X): s such that $X(s) = 0$

Pole (for rational X): s such that $X(s) = \infty$

Property 2

ROC of rational X does not contain any pole.

Properties of ROC

Property 3

If $x(t)$ is of finite duration and absolutely integrable, then ROC is the entire s-plane.

$$\int_a^b |x(t)e^{-st}|dt = \int_a^b |x(t)|e^{-\sigma t}dt \leq M_{a,b,\sigma} \int_a^b |x(t)|dt,$$

$$\text{where} \quad M_{a,b,\sigma} = \max_{t\in[a,b]} e^{-\sigma t} < +\infty$$

$\uparrow$ Week 9, Tue, 20180424

$\downarrow$ Week 9, Thu, 20180426

Properties of ROC

Right-sided signal: $x(t) = 0$, $\forall t < t_0$ for some t_0

Property 4

If $x(t)$ is right-sided, and if a line $Re(s) = \sigma_0$ is in ROC, then ROC contains all s such that $Re(s) \geq \sigma_0$.

$$\begin{aligned}
\int_{t_0}^{\infty} |x(t)e^{-st}|dt &= \int_{t_0}^{\infty} |x(t)e^{-\sigma_0 t}|e^{-(\sigma-\sigma_0)t}dt \\
&\leq e^{-(\sigma-\sigma_0)t_0} \int_{t_0}^{\infty} |x(t)e^{-\sigma_0 t}|dt \\
&< +\infty
\end{aligned}$$

Properties of ROC

Left-sided signal: $x(t) = 0, \forall t > t_0$ for some t_0

Similarly

Property 5

If $x(t)$ is left-sided, and if a line $Re(s) = \sigma_0$ is in ROC, then ROC contains all s such that $Re(s) \leq \sigma_0$.

Properties of ROC

Two-sided signal: of infinite extent for both $t > 0$ and $t < 0$

Property 6

If $x(t)$ is two-sided, ROC is a strip (can be empty).

$$
\begin{gathered}
x(t) = x_R(t) + x_L(t), \\
\int_{-\infty}^{\infty} |x(t)| dt = \int_{-\infty}^{\infty} |x_R(t)| dt + \int_{-\infty}^{\infty} |x_L(t)| dt, \\
ROC = ROC_R \bigcap ROC_L
\end{gathered}
$$

Properties of ROC

A signal must fall into one of the following (see Properties 3-6): of finite duration, right-sided, left-sided, two-sided.

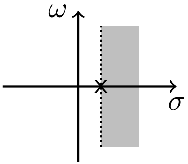
Hence ROC must be a single strip: the whole plane, a right-plane, a left-plane, a bounded strip.

Properties of ROC

Rational $X(s)$, from Property 2, ROC does not contain any pole.

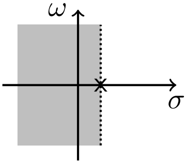
- right to the rightmost pole
- left to the leftmost pole
- a strip between two consecutive poles

1. If $x(t)$ right-sided and $X(s)$ rational, then ROC:
   the region to the right of the rightmost pole.



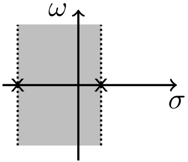
Properties of ROC

2. If $x(t)$ left-sided and $X(s)$ rational, then ROC: the region to the left of the leftmost pole.

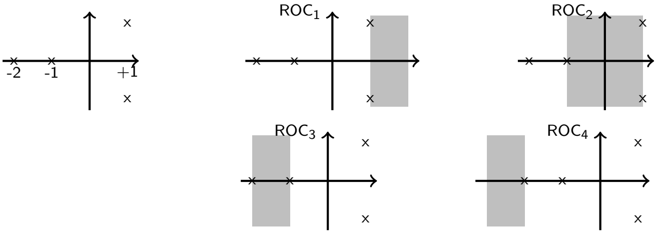


Properties of ROC

3. If $x(t)$ two-sided and $X(s)$ rational, then ROC: a strip between two consecutive poles



Convergence Example: 4-pole rational $X(s)$ shown below, possible ROCs are:



ROC

$$X(s) = \frac{1}{s-2} + \frac{1}{s+3}$$

Given

$$ROC : -3 < Re(s) < 2$$

- Observe ROC is $\{-3 < Re(s)\} \cap \{Re(s) < 2\}$
- Therefore $x(t) = e^{-3t}u(t) - e^{2t}u(-t)$
- Q: Try inverting other two possibilities for ROC

Inverse LT

$$
\begin{aligned}
X(s) = X(\sigma + j\omega) &= \int_{-\infty}^{\infty} x(t) e^{-\sigma t} e^{-j\omega t} dt \\
&= F\{x(t)e^{-\sigma t}\}
\end{aligned}
$$

$$
\begin{aligned}
\Rightarrow x(t) &= F^{-1}\{X(\sigma + j\omega)\} \cdot e^{\sigma t} \\
&= e^{\sigma t} \cdot \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\sigma + j\omega) e^{j\omega t} d\omega \\
&= \frac{1}{2\pi} \int_{\omega=-\infty}^{\omega=\infty} X(\sigma + j\omega) e^{(\sigma + j\omega)t} \frac{d(\sigma + j\omega)}{j}
\end{aligned}
$$

$$
\Rightarrow x(t) = \frac{1}{2\pi j} \int_{\sigma - j\infty}^{\sigma + j\infty} X(s) e^{st} ds
$$

Inverse LT

Again, this formal approach is more complex

Try to use partial-fraction expansion together with table of common functions for finding L^{-1}

Properties of LT

Section	Property	Signal	Laplace Transform	ROC
		$x(t)$	$X(s)$	R
		$x_1(t)$	$X_1(s)$	R_1
		$x_2(t)$	$X_2(s)$	R_2
9.5.1	Linearity	$ax_1(t) + bx_2(t)$	$aX_1(s) + bX_2(s)$	At least $R_1 \cap R_2$
9.5.2	Time shifting	$x(t - t_0)$	$e^{-st_0}X(s)$	R
9.5.3	Shifting in the s-Domain	$e^{s_0 t}x(t)$	$X(s - s_0)$	Shifted version of R (i.e., s is in the ROC if $s - s_0$ is in R)
9.5.4	Time scaling	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{s}{a}\right)$	Scaled ROC (i.e., s is in the ROC if s/a is in R)
9.5.5	Conjugation	$x^*(t)$	$X^*(s^*)$	R
9.5.6	Convolution	$x_1(t) * x_2(t)$	$X_1(s)X_2(s)$	At least $R_1 \cap R_2$
9.5.7	Differentiation in the Time Domain	$\frac{d}{dt}x(t)$	$sX(s)$	At least R
9.5.8	Differentiation in the s-Domain	$-tx(t)$	$\frac{d}{ds}X(s)$	R
9.5.9	Integration in the Time Domain	$\int_{-\infty}^{t} x(\tau)d(\tau)$	$\frac{1}{s}X(s)$	At least $R \cap \{\mathcal{Re}\{s\} > 0\}$

Initial- and Final-Value Theorems

9.5.10 If $x(t) = 0$ for $t < 0$ and $x(t)$ contains no impulses or higher-order singularities at $t = 0$, then

$$x(0^+) = \lim_{s\to\infty} sX(s)$$

$$\lim_{t\to\infty} x(t) = \lim_{s\to 0} sX(s)$$

$\uparrow$ Week 9, Thu, 20180426

$\downarrow$ Week 10, Thu, 20180503

Properties of Unilateral LT

Similar to CTFT, but ROC needs to be considered.

1. Initial- and Final-Value Theorems: under proper conditions

$$x(0^+) = \lim_{s \to \infty} sX(s)$$
$$\lim_{t \to \infty} x(t) = \lim_{s \to 0} sX(s)$$

   These two theorems are useful to check whether your Unilateral LT or Inverse Unilateral LT is correct.

Proof: Initial Value Theorem

Let $\epsilon > 0$ be any fixed number;
Let $\delta(\epsilon) > 0$ be such that $|x(t) - x(0^+)| < \epsilon$, when $0 < t < \delta(\epsilon)$

$$\begin{aligned} sX(s) &= \int_0^\infty sx(t)e^{-st}dt \\ &= \int_{-\infty}^\infty x(\frac{\tau}{s})e^{-\tau}d\tau \\ &= \int_0^{s\delta} x(\frac{\tau}{s})e^{-\tau}d\tau + \int_{s\delta}^\infty x(\frac{\tau}{s})e^{-\tau}d\tau \end{aligned}$$

Show that as $s \to \infty$

- first integral is between $[x(0^+) - \epsilon, x(0^+) + \epsilon]$
- second integral $\to 0$

since $\epsilon > 0$ is arbitrary this completes the proof

Properties of LT

E.g.

$$x(t) = e^{-2t}u(t) + e^{-t}\cos(3t)u(t)$$

$$L(x(t)) = X(s) = \frac{2s^2 + 5s + 12}{(s^2 + 2s + 10)(s + 2)}, \quad Rs\{s\} > -1$$

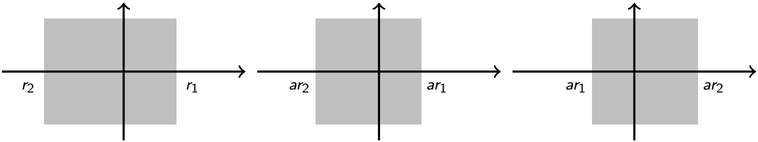
$$x(0^+) = 2; \quad \lim_{s\to\infty} sX(s) = 2$$

$$\lim_{t\to\infty} x(t) = 0; \quad \lim_{s\to 0} sX(s) = 0$$

Properties of LT

ROC may be changed for some properties.
e.g. time scaling $x_1(t) = x(at) \leftrightarrow \frac{1}{|a|}X(\frac{s}{a})$, ROC: $R_1 = aR$

ROC of $X(s)$ ROC of $X_1(s)$: $a = 0.8$, $a = -0.8$



Properties of LT

Multiplication to 'convolution':

$$x(t)y(t) \to \frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} X(r)Y(s-r)dr$$

$$\begin{aligned}
\int_{-\infty}^{\infty} x(t)y(t)e^{-st}dt &= \int_{-\infty}^{\infty}\left(\frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} X(r)e^{rt}dr\right) y(t)e^{-st}dt \\
&= \frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} X(r)\left(\int_{-\infty}^{\infty} y(t)e^{-(s-r)t}dt\right) dr \\
&= \frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} X(r)Y(s-r)dr
\end{aligned}$$

Rational LT

$$X(s) = \frac{P(s)}{Q(s)}, \quad \text{simplest fraction,}$$

$$Q(s) = \prod_{i=1}^{I} (s - s_i)^{p_i}, \quad s_i\text{'s are distinct}$$

Then

$$X(s) = R(s) + \sum_{i=1}^{I} \sum_{k=1}^{p_i} \frac{C_{i,k}}{(s - s_i)^k},$$

where $R(s)$ is a polynomial of s, $deg(R) = deg(P) - deg(Q)$

Rational LT

Proof: It suffices to consider $X(s) = 1/Q(s)$ (check).
Now by induction: it suffices to consider the expansions of

- $\frac{1}{(s-a)^k} \cdot \frac{1}{s-a}$: solved
- $\frac{1}{(s-a)^k} \cdot \frac{1}{s-b}$:

$$
\begin{aligned}
\frac{1}{(s-a)^1} \cdot \frac{1}{s-b} &= \frac{c_1}{s-a} + \frac{c_0}{s-b}, \\
\frac{1}{(s-a)^2} \cdot \frac{1}{s-b} &= \frac{c_1}{(s-a)^2} + \frac{1}{s-a}\frac{c_0}{s-b} \\
&= \frac{c_1}{(s-a)^2} + \frac{c_1 c_0}{s-a} + \frac{c_0^2}{s-b}, \\
\frac{1}{(s-a)^3} \cdot \frac{1}{s-b} &= \ldots
\end{aligned}
$$

Rational LT

How to find the expansion?
(1) By method of undetermined coefficients:

$$X(s) = \frac{4s}{(s+2)^2(s-4)} = \frac{A}{s+2} + \frac{B}{(s+2)^2} + \frac{C}{s-4}$$

$$\begin{aligned}
s = 0, &\implies 0 = \frac{A}{2} + \frac{B}{4} - \frac{C}{4} \\
s = -1, &\implies \frac{4}{5} = A + B - \frac{C}{5} \\
s = 1, &\implies -\frac{4}{27} = \frac{A}{3} + \frac{B}{9} - \frac{C}{3} \\
&\implies A = -\frac{4}{9},\ B = \frac{4}{3},\ C = \frac{4}{9},
\end{aligned}$$

Rational LT

How to find the expansion?
(2) By limiting arguments:

$$\frac{4s}{(s+2)^2(s-4)} = \frac{A}{s+2} + \frac{B}{(s+2)^2} + \frac{C}{s-4}$$

$$C = \lim_{s \to 4} (s-4)X(s) = \lim_{s \to 4} \frac{4s}{(s+2)^2} = \frac{4}{9},$$

$$B = \lim_{s \to -2} (s+2)^2 X(s) = \lim_{s \to -2} \frac{4s}{s-4} = \frac{4}{3},$$

$$A = \lim_{s \to -2} (s+2) \left(X(s) - \frac{B}{(s+2)^2} \right) = \lim_{s \to -2} \frac{8}{3(s-4)} = -\frac{4}{9}.$$

$\uparrow$ Week 10, Thu, 20180503

$\downarrow$ Week 11, Thu, 20180510

Some LT Pairs

TABLE 9.2 LAPLACE TRANSFORMS OF ELEMENTARY FUNCTIONS

Transform pair	Signal	Transform	ROC
1	$\delta(t)$	1	All s
2	$u(t)$	$\frac{1}{s}$	$\mathcal{R}e\{s\} > 0$
3	$-u(-t)$	$\frac{1}{s}$	$\mathcal{R}e\{s\} < 0$
4	$\frac{t^{n-1}}{(n-1)!}u(t)$	$\frac{1}{s^n}$	$\mathcal{R}e\{s\} > 0$
5	$-\frac{t^{n-1}}{(n-1)!}u(-t)$	$\frac{1}{s^n}$	$\mathcal{R}e\{s\} < 0$
6	$e^{-\alpha t}u(t)$	$\frac{1}{s+\alpha}$	$\mathcal{R}e\{s\} > -\alpha$
7	$-e^{-\alpha t}u(-t)$	$\frac{1}{s+\alpha}$	$\mathcal{R}e\{s\} < -\alpha$
8	$\frac{t^{n-1}}{(n-1)!}e^{-\alpha t}u(t)$	$\frac{1}{(s+\alpha)^n}$	$\mathcal{R}e\{s\} > -\alpha$
9	$-\frac{t^{n-1}}{(n-1)!}e^{-\alpha t}u(-t)$	$\frac{1}{(s+\alpha)^n}$	$\mathcal{R}e\{s\} < -\alpha$
10	$\delta(t-T)$	e^{-sT}	All s
11	$[\cos\omega_0 t]u(t)$	$\frac{s}{s^2+\omega_0^2}$	$\mathcal{R}e\{s\} > 0$
12	$[\sin\omega_0 t]u(t)$	$\frac{\omega_0}{s^2+\omega_0^2}$	$\mathcal{R}e\{s\} > 0$
13	$[e^{-\alpha t}\cos\omega_0 t]u(t)$	$\frac{s+\alpha}{(s+\alpha)^2+\omega_0^2}$	$\mathcal{R}e\{s\} > -\alpha$
14	$[e^{-\alpha t}\sin\omega_0 t]u(t)$	$\frac{\omega_0}{(s+\alpha)^2+\omega_0^2}$	$\mathcal{R}e\{s\} > -\alpha$
15	$u_n(t) = \frac{d^n\delta(t)}{dt^n}$	s^n	All s
16	$u_{-n}(t) = \underbrace{u(t) * \cdots * u(t)}_{n \text{ times}}$	$\frac{1}{s^n}$	$\mathcal{R}e\{s\} > 0$

Some LT Properties

Causality:

LTI system: Causal $h(t) = 0,\ t < 0$

ROC is a right-half plane

Note: the converse statement is not true

e.g. $e^{-(t+1)}u(t+1) \leftrightarrow \frac{e^s}{s+1}$, $Re(s) > -1$, non-causal

LTI + Causal + Rational $H(s)$: ROC to the right of the rightmost pole

Some LT Properties

Stability:

LTI system is stable iff ROC of $H(s)$ includes $j\omega$-axis ($Re(s) = 0$)

Proof: stable, BIBO

$$y(t) = \int h(v)x(t-v)dv \quad \text{bounded for all bounded } x$$

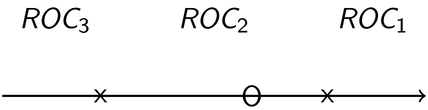
$$\implies \int |h(v)|dv \quad \text{exists}$$

the other direction is trivial

Some LT Properties

e.g.

$$H(s) = \frac{s-1}{(s+1)(s-2)}$$



ROC_1: causal, not stable

ROC_2: not causal, stable

ROC_3: not causal, not stable

Some LT Properties

LTI + Causal + Rational $H(s)$:
stable iff all poles lie in the left-half of the s-plane

(all poles have negative real parts)

Example 9.27

LTI + Stable + Causal system with impulse response $h(t)$ and system function $H(s)$. Suppose $H(s)$ is rational, contain a pole at $s = -2$, and does not have a zero at the origin. The location of all other poles and zeros is unknown.Determine whether each of the following statements is true, false, or insufficient to determine.

(a) $FT\{h(t)e^{3t}\}$ converges

(b) $\int_{-\infty}^{\infty} h(t)dt = 0$

Example

(c) $t \cdot h(t)$ is the impulse response of a causal and stable system.

(d) $dh(t)/dt$ contains at least one pole in its LT.

(e) $H(s) = H(-s)$

(f) $\lim_{s\to\infty} H(s) = 2$

Answer:

(a) False, $FT\{h(t)e^{3t}\} = H(s)|_{s=-3}$. But $s = -3$ is not in the ROC as ...

Example

(b) False. The integration$= H(0) = 0$. But $H(s)$ does not have a zero at origin.

(c) True. $LT\{t \cdot h(t)\}$ has a ROC the same as that of $H(s)$. As $H(s)$'s ROC includes $j\omega$-axis (why?), the corresponding system is also stable. Since $h(t) = 0$ for $t < 0$ (why?), $th(t) = 0$ for $t < 0$. So the system is also causal.

(d) True. The LT of $dh(t)/dt$ is $sH(s)$. So the original pole of $H(s)$ at $s = -2$ will not be cancelled by the multiplication of s. $\rightarrow H(s)$ also has a pole at $s = -2$.

Example

(e) False. It implies $s = 2$ is also a pole. Then, $j\omega$-axis is not in the ROC (why?) and $H(s)$ cannot be a stable system.

(f) It cannot be ascertained. We need to know the order of the numerator and denominator of $H(s)$

$\uparrow$ Week 11, Thu, 20180510

$\downarrow$ Week 12, Tue, 20180515

LTI System Characterized by LCC Differential Eqn

LTI system characterized by Linear Constant-Coefficient (LCC) Differential Equation:

$$\sum_{i=0}^{N} a_i \frac{d^i}{dt^i} y(t) = \sum_{k=0}^{M} b_k \frac{d^k}{dt^k} x(t)$$

$$\xleftrightarrow{\quad LT \quad} \quad \sum_{i=0}^{N} a_i s^i Y(s) = \sum_{k=0}^{M} b_k s^k X(s)$$

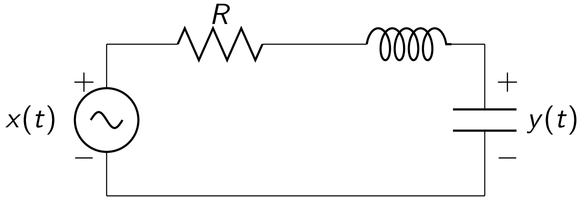
$$\therefore H(s) = \frac{\sum b_k s^k}{\sum a_i s^i}$$

$\rightarrow$ $H(s)$ is rational for a system by LCC Differential Equation

LTI System Characterized by LCC Differential Eqn

Ex. 9.24

Voltage drops at input and output are $x(t)$ and $y(t)$ respectively



Kirchhoff's voltage law + Faraday's law of induction

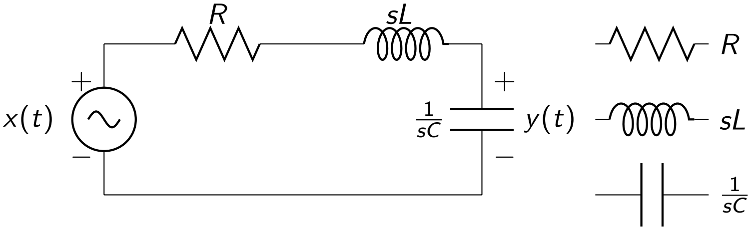
$$x(t) = RC \cdot \frac{dy(t)}{dt} + LC \cdot \frac{d^2y(t)}{dt^2} + y(t)$$

$$X(s) = RCsY(s) + LCs^2Y(s) + Y(s)$$

$$\rightarrow H(s) = \frac{1/(LC)}{s^2 + (R/L)s + 1/(LC)}$$

LTI System Characterized by LCC Differential Eqn

Other approach: using the impedance of R, L and C



$$V_L = L\frac{d}{dt}i_L, \qquad i_c = \frac{d\theta}{dt} = c\frac{d}{dt}V_c$$

$$\therefore\ Y(s) = \frac{1/(sC)}{R + sL + 1/(sC)}X(s)$$

LTI System Characterized by LCC Differential Eqn

Note:
Only LCC Differential Equation is not complete to specify an LTI system.
Need extra information like causality, stability to find the ROC and consequently the impulse response.

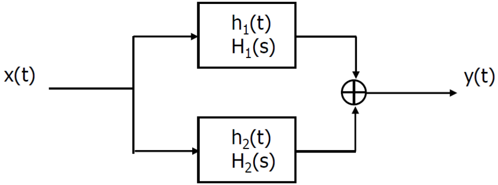
System function for interconnections of LTI Systems

- Parallel Interconnection:
  Consider the parallel connection of two systems

$$h(t) = h_1(t) + h_2(t)$$

Then from the linearity of LT,

$$H(s) = H_1(s) + H_2(s)$$



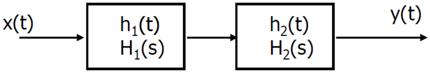
System function for interconnections of LTI Systems

- Series Interconnection:
  Similarly, the impulse response of the series connection is

$$h(t) = h_1(t) * h_2(t)$$

The resultant system function is then

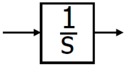
$$H(s) = H_1(s)H_2(s)$$



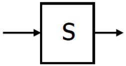
System function for interconnections of LTI Systems

Block Diagram Representation for Causal LTI System Described by Differential Equations and Rational System Function

- Integration:



- Differentiation:



E.g. Consider a causal second-order system with system function:

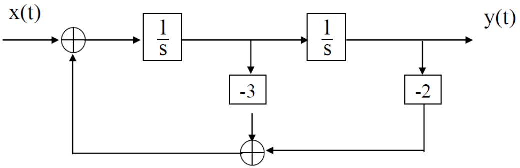
$$H(s) = \frac{1}{(s+1)(s+2)} = \frac{1}{s^2 + 3s + 2}$$

System function for interconnections of LTI Systems

Block Diagram Representation for Causal LTI System (cont.)

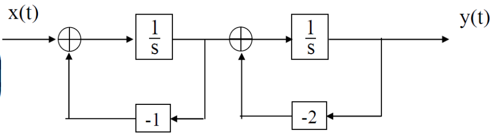
$$\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) = x(t)$$

- Direct Form:



- Series Form:

$$H(s) = \left(\frac{1}{s+1}\right)\left(\frac{1}{s+2}\right)$$



- Parallel Form:

$$H(s) = \frac{1}{s+1} - \frac{1}{s+2}$$