**CS 101 Fall 2021 - Quiz 8-B**

**12/07/2021 - 25 Minutes** **Name:** **ID number:**

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**Problem 1 True or False (5×1 pts)**

The following questions are True or False questions, you should judge whether each statement is true or false.

*Note: You should write down your answers in the box below.*

| Problem 2.1 | Problem 2.2 | Problem 2.3 | Problem 2.4 | Problem 2.5 |
|---|---|---|---|---|
| T | T | T | F | F |

(1) A DAG has multiple topological sortings if it has multiple sources.

(2) Topological sort can be extended to detect whether a graph has a cycle in $O(|V| + |E|)$ time.

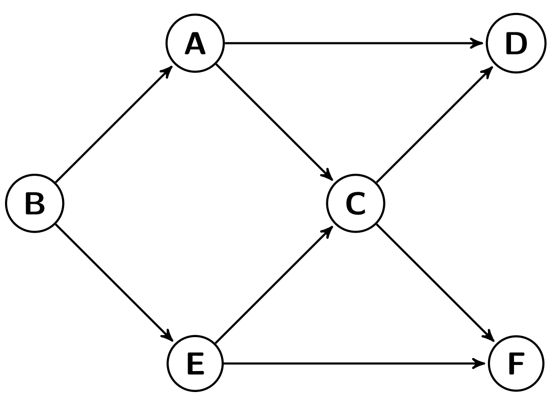
(3) If we add a constraint that each edge can only appear at most once in the shortest path, Dijkstra's algorithm still works for positive-weighted graphs.

(4) Dijkstra's algorithm cannot work for graph with both positive and negative weights but can work for graph whose weights are all negative.

(5) Bellman-Ford algorithm can find the shortest path for all undirected graphs with negative weights.

**Problem 2 Topological Sort (2 + 2 pts)**

Given the DAG below:




(1) Run topological sort on the given DAG and write down the topological sorting you obtain.

*Note: When pushing several vertices into a queue at the same time, push them alphabetically. You are NOT required to show your steps.*

BAECDF

(2) How many different topological sortings does the DAG have? Write them down.

4 : BAECDF, BAECFD, BEACDF, BEACFD

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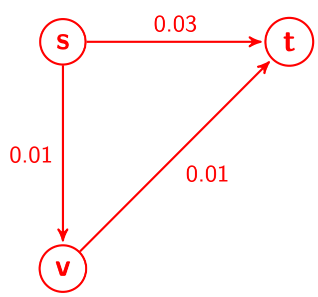
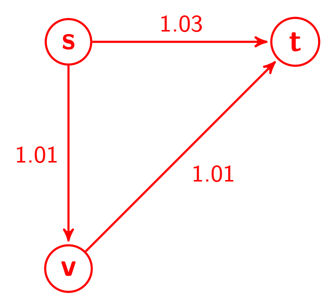
**Problem 3 Does Shortest Path Change? (3 pts)**

Given a shortest path $P = (s, v_1, v_2, \cdots, t)$ from $s$ to $t$ in graph $G = (V, E)$. Now Ge Ziwang adds 1 to the weight of each edge in $G$ i.e. $w(e') = w(e) + 1$. By doing this, Ge Ziwang obtains a new graph $G' = (V, E')$. Is the original shortest path $P$ still guaranteed to be a shortest path from $s$ to $t$ in $G'$?

**If yes, briefly explain why; If not, give a counterexample.**




$P = (s, v, t)$ in the left graph $G$, but the shortest path in the right graph $G'$ will change to $(s, t)$.

**Problem 4 Dijkstra's Algorithm Tiebreak (5 pts)**

Consider a directed graph $G = (V, E)$ with positive weights on vertices instead of edges. That is to say, when we visit a node $v \in V$, we need to cost its weight $w(v)$. Now we want to find a shortest path from $s$ to $t$ in such a vertex-weighted graph. How would you apply Dijkstra's algorithm in this setting? Briefly write down your main idea. Assume weights of vertices are all positive.

*Hint: Consider how to construct a new graph $G' = (V', E')$ according to the original graph $G = (V, E)$.*

Solution1: Construct a new graph $G' = (V', E')$ according to the original graph $G = (V, E)$. Start with $V' = \emptyset$ and $E' = \emptyset$. For each vertex $v \in V$ of weight $w(v)$, add two new vertices $v'$ and $v''$ to $V'$ and add a new edge $(v', v'')$ to $E'$ of weight $w(v)$. For each edge $e = (u, v) \in E$, add a new edge $(u'', v')$ of weight 0 to $E'$. Run Dijkstra's algorithm on the new graph $G'$ from $s'$ to $t''$ to obtain the shortest path $P'$, which can be converted to the shortest path $P$ in $G$ by combining $v', v''$ in $P'$ to $v$ in $P$.

Solution2: Construct a new graph $G' = (V', E')$ according to the original graph $G = (V, E)$. Start with $V' = V$ and $E' = \emptyset$. For each edge $e = (u, v) \in E$, add a new edge $(u, v)$ of weight $(w(u) + w(v))/2$ to $E'$. Run Dijkstra's algorithm on the new graph $G'$ from $s$ to $t$ to obtain the shortest path.