EE 111 Homework 6

Due date: May. 13th, 2019

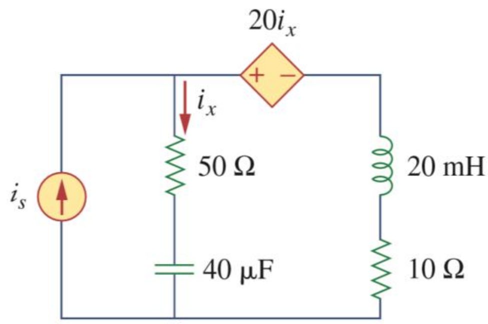
Turn in your homework in class

Rules:

- Work on your own. Discussion is permissible, but similar submissions will be judged as plagiarism.
- Please show all intermediate steps: a correct solution without an explanation will get zero credit.
- Please submit on time. No late submission will be accepted.
- Please prepare your submission in English only. No Chinese submission will be accepted.

1. Instantaneous and Average Power (10')

For the circuit, $i_s = 3cos2 \times 10^3 t$A. Find the average power absorbed by the 50Ω resistor



Solution:

$$20mH \rightarrow j\omega L = j \times 2 \times 10^3 \times 20 \times 10^{-3} = j40$$

$$40\mu F \rightarrow \frac{1}{j\omega C} = \frac{1}{j \times 2 \times 10^3 \times 40 \times 10^{-6}} = -j12.5$$

Nodal analysis:

$$-3 + \frac{V_0 - 20I_X}{10 + j40} + \frac{V_0 - 0}{50 - j12.5} = 0$$

$$I_X = \frac{V_0 - 0}{50 - j12.5}$$

$$(0.0203\text{-}j0.0105)V_0 = 3$$

$$V_0 = 3 / (\ 0.0228\angle\text{-}27.4190^\circ) = 131.33\angle 27.4190^\circ V$$

As for the power, all we need is the magnitude of the rms value of I_X .

$$|I_X| = \frac{131.33}{51.54} = 2.548\text{A}$$

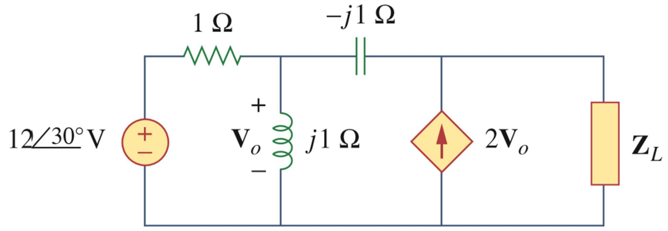
$$|I_X|_{\text{rms}} = \frac{2.548}{\sqrt{2}} = 1.8017\text{A}$$

Then the average power absorbed by the 50Ω resistor can be calculated.

$$P_{\text{avg}} = 1.8017^2 \times 50 = 162.31\text{W}$$

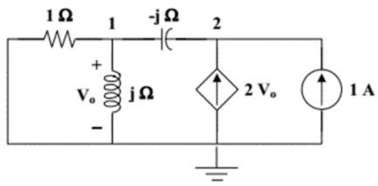
2. Maximum Average Power Transfer (12')

In the circuit, find the value of Z_L that will absorb the maximum power and the value of the maximum power. (note: the voltage is Vm)



Solution:

Calculate the Z_{th}, insert a 1A current source at the load terminals, and use nodal analysis



$$\frac{V_0}{1}+\frac{V_0}{j}=\frac{V_2-V_0}{-j}\rightarrow V_0=jV_2 \qquad (1)$$

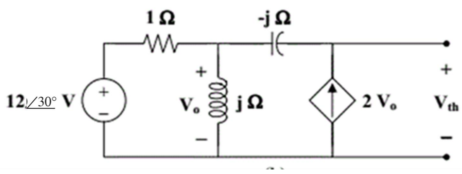
$$1+2V_0=\frac{V_2-V_0}{-j}\rightarrow 1=jV_2-(2+j)V_0 \qquad (2)$$

Substituting (1) into (2), $V_2=\dfrac{1}{1\text{-}j}$

$$Z_{\text{th}}=\frac{V_2}{1}=\frac{1+j}{2}=0.5+j0.5$$

$$Z_{\text{L}}=Z_{\text{th}}^{*}=0.5\text{-}j0.5\Omega$$

Calculate the V_{th},



$$12\angle 30^{\circ}=6\sqrt{3}+\text{j}6 \qquad 2V_0+\frac{6\sqrt{3}+\text{j}6-V_0}{1}=\frac{V_0}{j}\rightarrow V_0=-\frac{6\sqrt{3}+\text{j}6}{1+j}$$

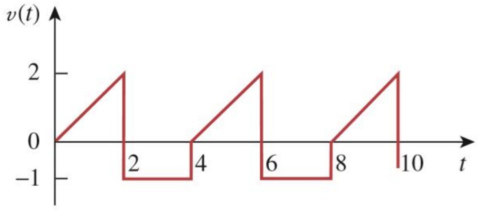
$$\text{-}V_0\text{-}(\text{-j}\times 2V_0)+V_{\text{th}}=0\rightarrow V_{\text{th}}=(1\text{-j}2)V_0=-6\frac{2+\sqrt{3}+(1-2\sqrt{3})j}{1+j}$$

$$P_{\max}=\frac{|V_{\text{th}}|^2}{8R_L}=\frac{\left(6\times\frac{\sqrt{(2+\sqrt{3})^2+(1-2\sqrt{3})^2}}{\sqrt{2}}\right)^2}{8\times 0.5}=90W$$

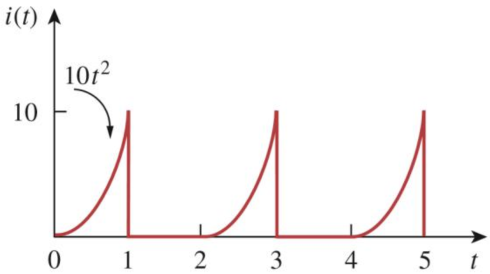
3.**Effective or RMS Value (10')**

Compute the RMS value of the waveform depicted in the figures.

(1)



(2)



Solution:

(1)

$$v(t) = \begin{cases} t, 0 < t < 2 \\ -1, 2 < t < 4 \end{cases}$$

$$V_{\text{rms}}^2 = \frac{1}{4}[\int_0^2 t^2 dt + \int_2^4 (-1)^2 dt] = \frac{1}{4}[\frac{8}{3} + 2] = 1.1667$$

$$V_{\text{rms}} = 1.08V$$

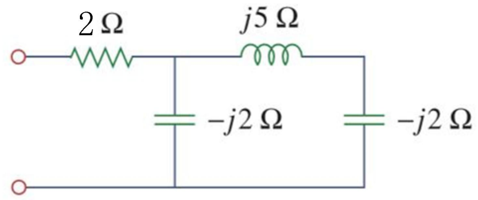
(2)

$$I_{\text{rms}}^2 = \frac{1}{2}[\int_0^1 (10t^2)^2 dt + \int_1^2 0^2 dt] = 50\int_0^1 t^4 dt = 10$$

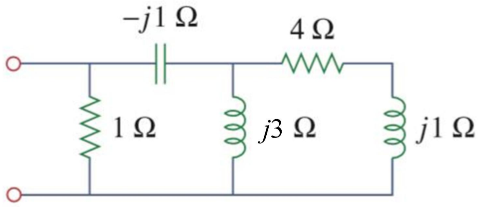
$$I_{\text{rms}} = 3.162A$$

4.Apparent Power and Power Factor (12')

Obtain the power factor for each of the circuits. Specify each power factor as leading or lagging.



(a)



(b)

Solution:

(a)

$$\text{-j2} \,\|\, (j5 - j2) = -j2 \,\|\, -j3 = -j6$$

$$Z_T = 2 - j6 = 6.32\angle\text{-}71.57^{\circ}\Omega$$

pf= cos(-71.57°)=0.32

(leading)

(b)

$$\text{j3} \,\|\, (4{+}j) = \frac{j12-3}{4+j4} = 1.13 + \text{j}1.88$$

$$Z = 1 \,\|\, (1.13 + j1.88 - j1) = \frac{1.13 + j0.88}{2.13 + j0.88} = 0.6215\angle 15.46^{\circ}\Omega$$

pf= cos(15.46°)=0.96

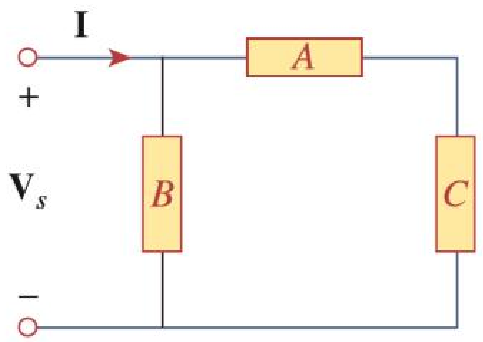
(lagging)

5.Complex Power （14'）

In the circuit, device A receives 2 kW at 0.8 pf lagging, device B receives 3 kVAat 0.8 pf leading, while device C is inductive and consumes 1 kW and receives 7500 VAR.

(a) Determine the power factor of the entire system.

(b) Find **I** given that $V_s = 120\angle 45^\circ$ V rms.



Solution:

$$S_A = 2000 + j\frac{2000}{0.8} \times 0.6 = 2000 + j1500, \qquad (2')$$

$$S_B = 3000 \times 0.8 - j3000 \times \sqrt{1-0.8^2} = 2400 - j1800, \qquad (2')$$

$$S_C = 1000 + j7500, \qquad (2')$$

$$S = S_A + S_B + S_C = 5400 + j7200 \qquad (1')$$

(a)

$$\text{pf} = \frac{5400}{\sqrt{5400^2 + 7200^2}} = 0.6, \quad (2'), lagging \quad (1')$$

(b)

$$S = V_{rms} I_{rms}^* \to I_{rms}^* = \frac{5400 + j7200}{120\angle 45^o} = 75\angle 8.1^o A \qquad (2')$$

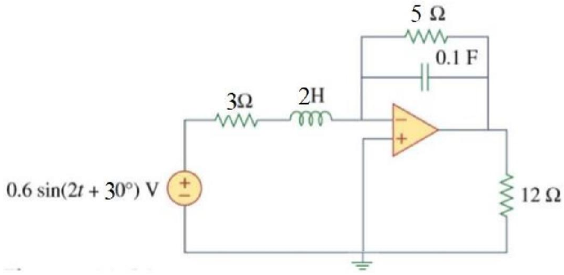
$$I_{rms} = 75\angle -8.1^o A \qquad (2')$$

6.Power Factor （14’）

For the op amp circuit, calculate:

(a) the complex power delivered by the voltage source

(b) the average power dissipated in the 12Ω resistor

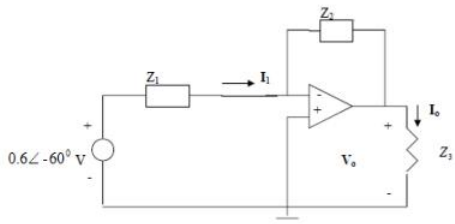


Solution:

$$jwL = j2\times 4 = j4\Omega, \frac{1}{jwC} = \frac{1}{j2\times 0.1} = -j5\Omega$$

$$5//(-j5) = \frac{-j5\times 5}{5-j5} = \frac{5}{2} - j\frac{5}{2}\Omega$$

The frequency-domain version of the circuit is shown below.



$$Z_1 = 3 + j4\Omega, \qquad (2')$$

$$Z_2 = \frac{5}{2} - j\frac{5}{2}\Omega,\ Z_3 = 12\Omega \qquad (2')$$

(a)

$$I_1 = \frac{0.6\angle -60^0 - 0}{3+j4} = 0.12\angle -113.1^o A, \qquad (2')$$

$$S = \frac{1}{2}V_s I_1^* = \frac{1}{2}\times 0.6\angle -60^0 \times 0.12\angle +113.1^o = 0.036\angle \cancel{-173.1^o}\ 53.1^\circ\ mVA \qquad (2')$$

(b)

$$V_o = -\frac{Z_2}{Z_1}V_s, \qquad (2')$$

$$I_o = \frac{V_o}{Z_3} = -\frac{\frac{5}{2} - j\frac{5}{2}}{12(3+j4)}(0.6\angle -60) = 0.035\angle -158.1^o A \qquad (2')$$

$$P = \frac{1}{2}|I_o|^2 R = 0.5\times 0.035^2 \times 12 = 7.35\, mW \qquad (2')$$

7.Balanced Three-Phase Voltages （8'）

A balanced Y-Y four-wire system has phase voltages

$$V_{an} = 120\angle 0^{o} V, V_{bn} = 120\angle -120^{0} V, V_{cn} = 120\angle 120^{0}\,\mathrm{V}$$

The load impedance per phase is $19 + j13\ \Omega$, and the line impedance per phase is $1 + j2\ \Omega$.Solve for the line currents and neutral current.

Solution:

$$I_a = \frac{V_{an}}{Z_L + Z_Y} = \frac{120\angle 0^o}{20 + j15} = 4.8\angle -36.87^o A \qquad (3')$$

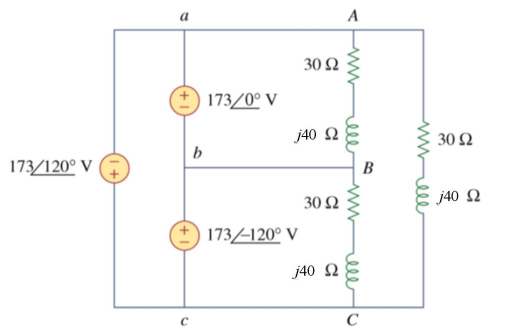
$$I_b = I_a\angle -120^o = 4.8\angle -156.87^o A \qquad (2')$$

$$I_c = I_a\angle 120^o = 4.8\angle 83.87^o A \qquad (2')$$

As a balanced system, $I_n = 0A$ (1')

8.Balanced Delta-Delta Connection (10')

For the $\Delta - \Delta$ circuit, calculate the phase and line currents.



Solution:

$$Z_\Delta = 30 + j40 = 50\angle 53.1^o \Omega \qquad (2')$$

The phase currents are

$$I_{AB} = \frac{V_{ab}}{Z_\Delta} = \frac{173\angle 0^o}{50\angle 53.1^o} = 3.46\angle -53.1^o A \qquad (2')$$

$$I_{BC} = I_{AB}\angle -120^o = 3.46\angle -173.1^o A \qquad (1')$$

$$I_{CA} = I_{AB}\angle 120^o = 3.46\angle 66.9^o A \qquad (1')$$

The line currents are

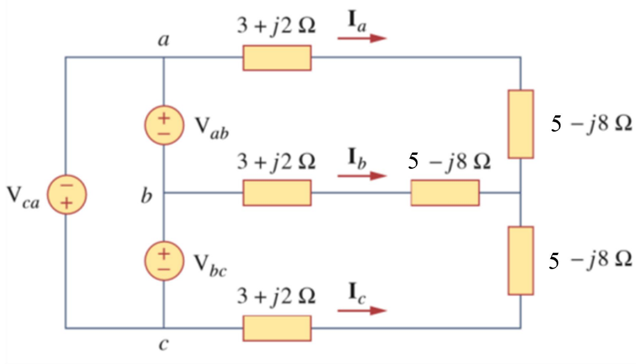
$$I_{\mathrm{a}} = I_{AB} - I_{CA} = I_{AB}\sqrt{3}\angle -30^o = 5.99\angle -83.1^o A \qquad (2')$$

$$I_b = I_a\angle -120^o = 5.99\angle -203.1^o A \qquad (1')$$

$$I_b = I_a\angle 120^o = 5.99\angle 36.9^o A \qquad (1')$$

9.Balanced Delta-Wye Connection (10')

In the circuit, if $V_{ab} = 440\angle 0^o V, V_{bc} = 440\angle -120^o V, V_{ca} = 440\angle 120^o V$ find the line currents



Solution:

Convert the delta-connected source to an equivalent wye-connected source and consider the single-phase equivalent.

$$I_a = \frac{440\angle(0^0 - 30^0)}{\sqrt{3}Z_Y}, \qquad (2')$$

where $Z_Y = 3 + j2 + 5 - j8 = 8 - j6 = 10\angle -36.9^o \Omega$, (2')

$$I_a = \frac{440\angle(0^0 - 30^0)}{\sqrt{3}\times 10\angle\text{-}36.9^o} = 25.404\angle 6.9^o A, \qquad (2')$$

$$I_b = I_a\angle -120^o = 25.404\angle\text{-}113.1^o A, \qquad (2')$$

$$I_a = I_a\angle 120^o = 25.404\angle 126.9^o A \qquad (2')$$