上海科技大学

2017-2018 学年第 1 学期本科生第 2 次期中考试卷

姓名：______________

学院：______________

学号：______________

开课单位：信息学院

授课教师：周平强

考试科目：《电路基础》

课程代码：EE111

考试时间：2016 年 11 月 28 日 10 点 15 分 – 12 点 15 分。

考试成绩录入表：

题目	1	2	3	4	5	总分
计分						
复核						

评卷人签名： **复核人签名：**

日期： **日期：**

编写说明：

1. 要求评卷人和复核人不能是同一人。
2. 试卷内页和答题纸编排格式由各学院和出题教师根据实际需要自定，每页须按顺序标注页码（除封面外），要求排版清晰、美观，便于在页面左侧装订。为方便印刷归档，建议使用 A4 双面印刷（学校有印刷一体机提供）。
3. 主考教师编写试卷时尽可能保证试题科学、准确、合理，如考试过程中发现试题有误，主考教师需负责现场解释，此类情况学校将作为教学评估记录的一部分。

Problem 1 (15 pts) — First-Order RL Circuit Analysis

The current and voltage at the terminals of the inductor in Fig. 1 are

$$i(t) = (4 + 4e^{-40t})\text{A}, t \geq 0;\ v(t) = -80e^{-40t}\text{V}, t \geq 0.$$

a) Find the numerical values of V_s, R, L, and I_0 (the initial current of L at $t = 0$);

b) How many milliseconds after the switch has been closed does the energy stored in the inductor reach 9 Joule?

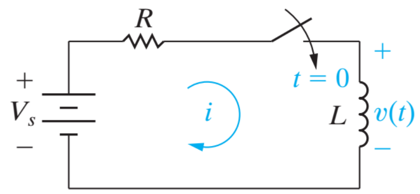


Fig. 1 for Problem 1.

Your answer:

$$i = \frac{V_s}{R} + \left(I_o - \frac{V_s}{R}\right)e^{-(R/L)t} \qquad v = (V_s - I_oR)e^{-(R/L)t}$$

$$\therefore\ \frac{V_s}{R} = 4; \qquad I_o - \frac{V_s}{R} = 4$$

$$V_s - I_oR = -80; \qquad \frac{R}{L} = 40$$

$$\therefore\ I_o = 4 + \frac{V_s}{R} = 8\,\text{A}$$

Now since $V_s = 4R$ we have

$$4R - 8R = -80; \qquad R = 20\,\Omega$$

$$V_s = 80\,\text{V}; \qquad L = \frac{R}{40} = 0.5\,\text{H}$$

[b] $i = 4 + 4e^{-40t}; \qquad i^2 = 16 + 32e^{-40t} + 16e^{-80t}$

$$w = \frac{1}{2}Li^2 = \frac{1}{2}(0.5)[16 + 32e^{-40t} + 16e^{-80t}] = 4 + 8e^{-40t} + 4e^{-80t}$$

$$\therefore\ 4 + 8e^{-40t} + 4e^{-80t} = 9 \quad \text{or} \quad e^{-80t} + 2e^{-40t} - 1.25 = 0$$

Let $x = e^{-40t}$:

$x^2 + 2x - 1.25 = 0$; Solving, $x = 0.5$; $x = -2.5$

But $x \geq 0$ for all t. Thus,

$$e^{-40t} = 0.5; \qquad e^{40t} = 2; \qquad t = 25\ln 2 = 17.33\,\text{ms}$$

Problem 2 (20 pts) — First-order RL Circuit Analysis

Before it closes at time $t = 0$, the switch in the circuit shown in Fig. 2 has been open for a long time. Find the current through the inductor $i_o(t)$ for $t \geq 0$.

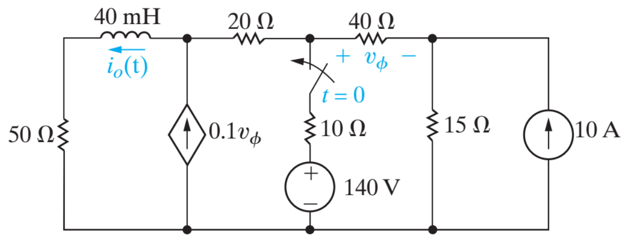
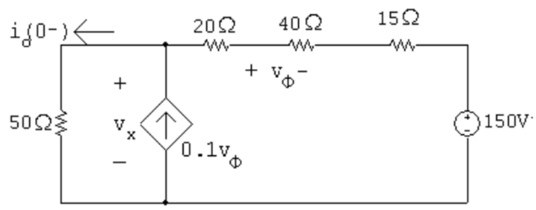


Fig. 2 for Problem 2.

For $t < 0$



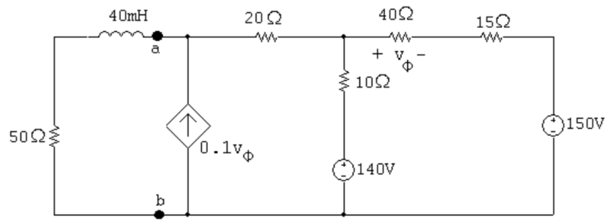
$$\frac{v_x}{50} - 0.1v_\phi + \frac{v_x - 150}{75} = 0$$

$$v_\phi = \frac{40}{75}(v_x - 150)$$

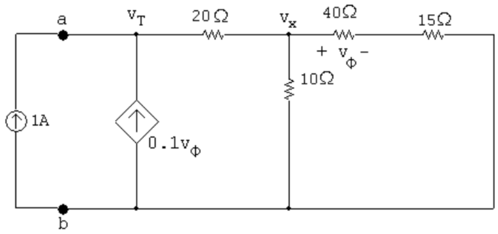
Solving,

$$v_x = 300\,\mathrm{V}; \qquad i_o(0^-) = \frac{v_x}{50} = 6\,\mathrm{A}$$

$t > 0$



Find Thévenin equivalent with respect to a, b. Use a test source to find the Thévenin equivalent resistance:



$$-1 - 0.1v_\phi + \frac{v_\mathrm{T} - v_x}{20} = 0$$

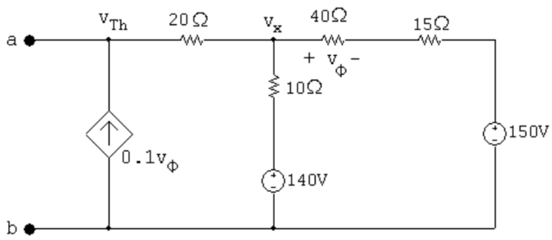
$$\frac{v_x - v_\mathrm{T}}{20} + \frac{v_x}{10} + \frac{v_x}{55} = 0$$

$$v_\phi = \frac{40}{55}v_x$$

Solving,

$$v_\mathrm{T} = 74\,\mathrm{V} \qquad \text{so} \qquad R_\mathrm{Th} = \frac{v_\mathrm{T}}{1\,\mathrm{A}} = 74\,\Omega$$

Find the open circuit voltage with respect to a, b:



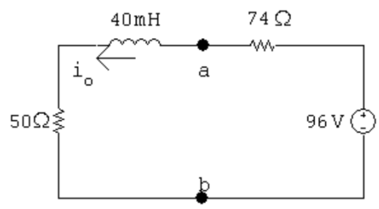
$$-0.1v_\phi + \frac{v_\mathrm{Th} - v_x}{20} = 0$$

$$\frac{v_x - v_\mathrm{Th}}{20} + \frac{v_x - 140}{10} + \frac{v_x - 150}{55} = 0$$

$$v_\phi = \frac{40}{55}(v_x - 150)$$

Solving,

$$v_\mathrm{Th} = 96\,\mathrm{V}$$



$$i_o(\infty) = 96/124 = 0.774\,\mathrm{A}$$

$$\tau = \frac{40 \times 10^{-3}}{124} = 0.3226\,\mathrm{ms}; \qquad 1/\tau = 3100$$

$$i_o = 0.774 + (6 - 0.774)e^{-3100t} = 0.774 + 5.226e^{-3100t}\,\mathrm{A}, \qquad t \geq 0$$

Problem 3 (20 pts) — Second-order RLC circuit

The initial value of the voltage v in the circuit shown in Fig. 4 is zero, and the initial value of the capacitor current, $i_c(0^+)$, is 45mA. The expression for the capacitor current is known to be $i_c(t) = A_1 e^{-200t} + A_2 e^{-800t}, t \geq 0^+$. $R = 250\Omega$. Find

a) The values of L, C, A_1 and A_2.

b) The express for $v(t), t \geq 0$.

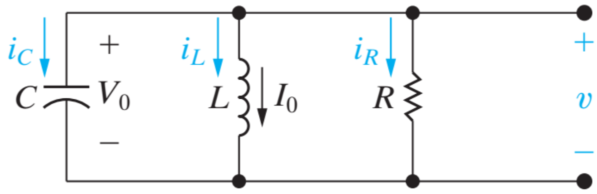


Fig. 5 for Problem 5.

[a] $2\alpha = 1000; \qquad \alpha = 500\,\text{rad/s}$

$$2\sqrt{\alpha^2 - \omega_o^2} = 600; \qquad \omega_o = 400\,\text{rad/s}$$

$$C = \frac{1}{2\alpha R} = \frac{1}{2(500)(250)} = 4\,\mu F$$

$$L = \frac{1}{\omega_o^2 C} = \frac{1}{(400)^2(4\times 10^{-6})} = 1.5625\,\text{H}$$

$$i_{\text{C}}(0^+) = A_1 + A_2 = 45\,\text{mA}$$

$$\frac{di_{\text{C}}}{dt} + \frac{di_{\text{L}}}{dt} + \frac{di_{\text{R}}}{dt} = 0$$

$$\frac{di_{\text{C}}(0)}{dt} = -\frac{di_{\text{L}}(0)}{dt} - \frac{di_{\text{R}}(0)}{dt}$$

$$\frac{di_{\text{L}}(0)}{dt} = \frac{0}{1.5625} = 0\,\text{A/s}$$

$$\frac{di_{\text{R}}(0)}{dt} = \frac{1}{R}\frac{dv(0)}{dt} = \frac{1}{R}\frac{i_{\text{C}}(0)}{C} = \frac{45\times 10^{-3}}{(250)(4\times 10^{-6})} = 45\,\text{A/s}$$

$$\therefore \quad \frac{di_{\text{C}}(0)}{dt} = 0 - 45 = -45\,\text{A/s}$$

$$\therefore \quad 200A_1 + 800A_2 = 45; \qquad A_1 + A_2 = 0.045$$

Solving, $A_1 = -15\,\text{mA}; \qquad A_2 = 60\,\text{mA}$

$$\therefore \quad i_{\text{C}} = -15e^{-200t} + 60e^{-800t}\,\text{mA}, \qquad t \geq 0^+$$

[b] By hypothesis

$$v = A_3 e^{-200t} + A_4 e^{-800t}, \qquad t \geq 0$$

$$v(0) = A_3 + A_4 = 0$$

$$\frac{dv(0)}{dt} = \frac{45\times 10^{-3}}{4\times 10^{-6}} = 11{,}250\,\text{V/s}$$

$$-200A_3 - 800A_4 = 11{,}250; \qquad \therefore \quad A_3 = 18.75\,\text{V}; \qquad A_4 = -18.75\,\text{V}$$

$$v = 18.75e^{-200t} - 18.75e^{-800t}\,\text{V}, \qquad t \geq 0$$

Problem 4 (15 pts) — Second-order RLC circuit

Determine the Thevenin equivalent circuit of the circuit shown in Fig. 5 at terminals (a, b), given that $v_s(\mathrm{t}) = 12\cos 2500t\,\mathrm{V}$, $i_s(\mathrm{t}) = 0.5\cos(2500t - 30°)\mathrm{A}$.

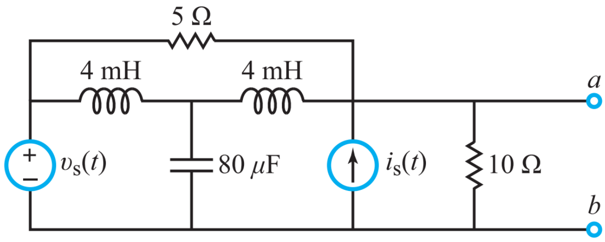
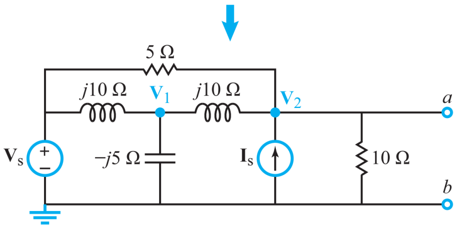


Fig. 5 for Problem 5.

Solution:



(a)

At $\omega = 2500$ rad/s,

$$\mathbf{Z}_\mathrm{L} = j\omega L = j2500 \times 4 \times 10^{-3} = j10\ \Omega$$

$$\mathbf{Z}_\mathrm{C} = \frac{-j}{\omega C} = \frac{-j}{2500 \times 80 \times 10^{-6}} = -j5\ \Omega.$$

Also,

$$\mathbf{V}_\mathrm{s} = 12\angle 0°\ \mathrm{V},$$

$$\mathbf{I}_\mathrm{s} = 0.5\angle -30°\ \mathrm{A}.$$

To obtain the Thévenin equivalent circuit, we can either apply impedance and source transformations to simplify the circuit or apply one of the analysis techniques. We opt to apply nodal analysis to determine $\mathbf{V}_\mathrm{oc}$ at terminals (a, b).

At node $\mathbf{V}_1$:

$$\frac{\mathbf{V}_1 - \mathbf{V}_\mathrm{s}}{j10} + \frac{\mathbf{V}_1}{-j5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j10} = 0$$

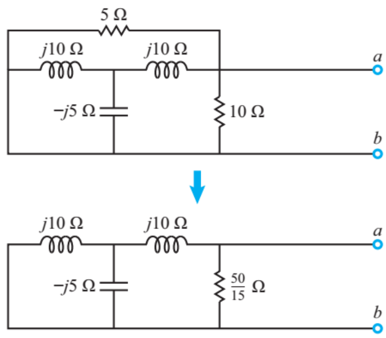
At node $\mathbf{V}_2$:

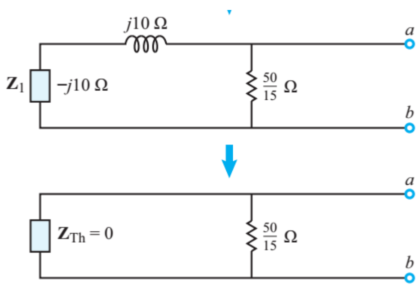
$$\frac{\mathbf{V}_2 - \mathbf{V}_1}{j10} + \frac{\mathbf{V}_2 - \mathbf{V}_\mathrm{s}}{5} + \frac{\mathbf{V}_2}{10} - \mathbf{I}_\mathrm{s} = 0$$

Upon inserting the values for $\mathbf{V}_\mathrm{s}$ and $\mathbf{I}_\mathrm{s}$ and then solving for $\mathbf{V}_1$ and $\mathbf{V}_2$, we determine that

$$\mathbf{V}_\mathrm{Th} = \mathbf{V}_\mathrm{oc} = \mathbf{V}_2 = -12\ \mathrm{V}.$$

To determine $\mathbf{Z}_\mathrm{Th}$, we eliminate the sources and then simplify the circuit.

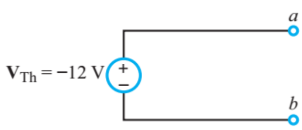




$$\mathbf{Z}_1 = \frac{(j10)(-j5)}{j10 - j5} = -j10\ \Omega$$

$$\mathbf{Z}_\mathrm{Th} = (j10 - j10) \parallel \frac{50}{15} = 0.$$

Hence, the Thévenin equivalent circuit is:



Problem 5 (30 pts) — Second-order RLC circuit

In the circuit shown in Fig. 6, the switch was closed at $t = 0$ and re-opened at $t = 0.5$s. Determine the response $i_L(\mathrm{t})$ for $t \geq 0$. Before $t = 0$, there is no energy stored in the inductor and capacitor.

Assume that $V_s = 18$V, $R_s = 1\Omega$, $R_1 = 5\Omega$, $R_2 = 2\Omega$, $L = 2$H, and $C_1 = \frac{1}{17}$F.

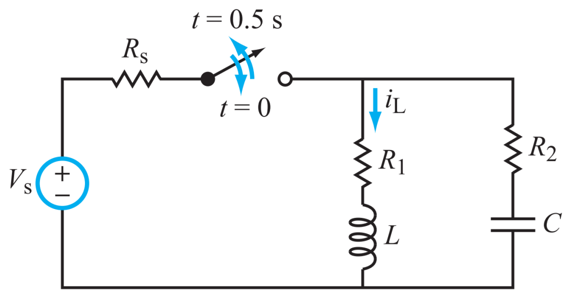


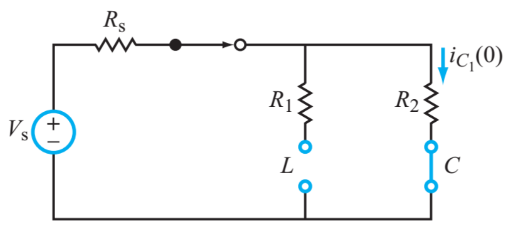
Fig. 6 for Problem 6.

Time Segment 1: $0 \leq t \leq 0.5$ s

Prior to $t = 0$, the circuit contained no sources. Hence,

$$i_{L_1}(0) = i_{L_1}(0^-) = 0, \qquad \text{[open-circuit equivalent]}$$

$$\upsilon_{C_1}(0) = \upsilon_{C_1}(0^-) = 0. \qquad \text{[short-circuit equivalent]}$$



(a) At $t = 0$

At $t = 0$ (Fig. (a)):

$$i_{C_1}(0) = \frac{V_s}{R_s + R_2} = \frac{18}{1+2} = 6 \text{ A}, \tag{1}$$

$$\upsilon'_{C_1}(0) = \frac{i_{C_1}(0)}{C} = \frac{6}{\frac{1}{17}} = 102 \text{ V/s}. \tag{2}$$

At $0 \leq t \leq 0.5$ s (Fig. (b)):

$$-V_s + R_s i_a + i_b R_2 + \upsilon_{C_1} = 0, \qquad \text{[outer loop]} \tag{3}$$

$$i_b = C\frac{d\upsilon_{C_1}}{dt} = C\upsilon'_{C_1}. \tag{4}$$

Using Eq. (4) in Eq. (3) and solving for i_a gives

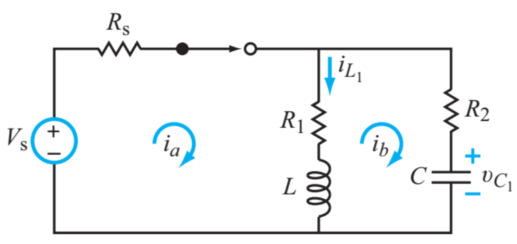
$$i_a = \frac{V_s - \upsilon_{C_1} - R_2 C \upsilon'_{C_1}}{R_s}. \tag{5}$$

The left loop equation is:

$$-V_s + R_s i_a + R_1(i_a - i_b) + L(i'_a - i'_b) = 0. \tag{6}$$

The derivative of Eq. (5) gives

$$i'_a = \frac{-\upsilon'_{C_1} - R_2 C \upsilon''_{C_1}}{R_s}. \tag{7}$$



(b) At $0 \leq t \leq 0.5$ s

Using Eqs. (4), (5), and (7) in (6) gives:

$$-V_s + (V_s - v_{C_1} - R_2 C v'_{C_1}) + \frac{R_1}{R_s}(V_s - v_{C_1} - R_2 C v'_{C_1}) - R_1 C v'_{C_1} + \frac{L}{R_s}[-v'_{C_1} - R_2 C v''_{C_1}] - LC v''_{C_1} = 0. \qquad (8)$$

Collecting like terms leads to:

$$v''_{C_1}\left[LC\left(1+\frac{R_2}{R_s}\right)\right] + v'_{C_1}\left[R_2 C + \frac{R_1 R_2 C}{R_s} + R_1 C + \frac{L}{R_s}\right] + v_{C_1}\left[1+\frac{R_1}{R_s}\right] = \frac{R_1 V_s}{R_s}. \qquad (9)$$

or equivalently

$$v''_{C_1} + a v'_{C_1} + b v'_{C_1} = c, \qquad (10)$$

where

$$a = \frac{R_s(R_1+R_2)C + R_1 R_2 C + L}{(R_s+R_2)LC} = \frac{1(5+2)(\frac{1}{17}) + 5\times 2\times(\frac{1}{17}) + 2}{(1+2)\times 2\times\frac{1}{17}} = 8.5,$$

$$b = \frac{R_s + R_1}{(R_s+R_2)LC} = \frac{1+5}{(1+2)\times 2\times\frac{1}{17}} = 17,$$

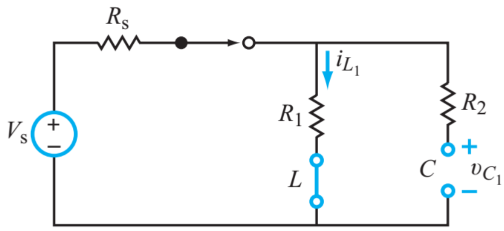
$$c = \frac{R_1 V_s}{(R_s+R_2)LC} = \frac{5\times 18}{(1+2)\times 2\times\frac{1}{17}} = 255.$$

$$\alpha = \frac{a}{2} = 4.25 \text{ Np/s},$$

$$\omega_0 = \sqrt{b} = \sqrt{17} = 4.12 \text{ rad/s},$$

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -4.25 + \sqrt{4.25^2 - 17} = -3.22 \text{ Np/s},$$

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -4.25 - \sqrt{4.25^2 - 17} = -5.28 \text{ Np/s}.$$



(c) At $t = \infty$ (had the switch remained closed)

Had the switch remained closed, at $t = \infty$, the circuit becomes as shown in Fig. (c), in which case

$$v_{C_1}(\infty) = i_{L_1} R_1 = \frac{V_s R_1}{R_s + R_1} = \frac{18\times 5}{1+5} = 15 \text{ V}.$$

From Table 6-2,

$$A_1 = \frac{v'_{C_1}(0) - s_2[v_{C_1}(0) - v_{C_1}(\infty)]}{s_1 - s_2} = \frac{102 + 5.28[0-15]}{-3.22+5.28} = 11.05 \text{ V},$$

$$A_2 = -\left[\frac{v'_{C_1}(0) - s_1[v_{C_1}(0) - v_{C_1}(\infty)]}{s_1 - s_2}\right] = \frac{102 + 3.22[0-15]}{-3.22+5.28} = -26.05 \text{ V}.$$

Hence, $v_C(t)$ is given by

$$\begin{aligned} v_{C_1}(t) &= v_{C_1}(\infty) + A_1 e^{s_1 t} + A_2 e^{s_2 t} \\ &= 15 + 11.05 e^{-3.22t} - 26.05 e^{-5.28t} \qquad \text{(V)}, \qquad \text{for } 0 \le t \le 0.5 \text{ s}. \qquad (11) \end{aligned}$$

From Fig. (b), the current $i_{L_1}(t)$ is given by

$$i_{L_1}(t) = i_a - i_b. \qquad (12)$$

Using Eqs. (4) and (5) in Eq. (12) gives:

$$\begin{aligned} i_{L_1}(t) &= \frac{V_s}{R_s} - \frac{v_{C_1}}{R_s} - \frac{R_2 C}{R_s} v'_{C_1} - C v'_{C_1} \\ &= \frac{V_s}{R_s} - \frac{v_{C_1}}{R_s} - C\left(1 + \frac{R_2}{R_s}\right) v'_{C_1}. \qquad (13) \end{aligned}$$

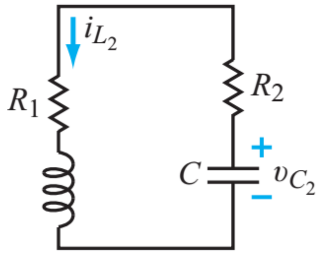
From Eq. (11),

$$\begin{aligned} v'_{C_1}(t) &= -3.22 \times 11.05 e^{-3.22t} + 5.28 \times 26.05 e^{-5.28t} \\ &= -35.59 e^{-3.22t} + 137.59 e^{-5.28t} \qquad \text{(V/s)}. \qquad (14) \end{aligned}$$

Using Eqs. (11) and (14) in Eq. (13), and then simplifying terms, leads to

$$i_{L_1}(t) = [3 - 4.77 e^{-3.22t} + 1.77 e^{-5.28t}] \qquad \text{(A)}, \qquad \text{for } 0 \le t \le 0.5 \text{ s}. \qquad (15)$$

Time Segment 2: $t > 0.5$ s



(d) After $t = 0.5$ s

After re-opening the switch, the circuit becomes a series RLC circuit as shown in Fig. (d). Since the circuit no longer contains sources,

$$i_{L_2}(\infty) = 0,$$
$$\upsilon_{C_2}(\infty) = 0.$$

From Table 6-1, the damping factors are:

$$\alpha = \frac{R}{2L} = \frac{R_1 + R_2}{2L} = \frac{5+2}{2\times 2} = \frac{7}{4} = 1.75 \text{ Np/s},$$
$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{2\times\frac{1}{17}}} = 2.92 \text{ rad/s}.$$

Since $\alpha < \omega_0$, the response will be underdamped:

$$i_{L_2}(t) = [D_1\cos\omega_d(t-0.5) + D_2\sin\omega_d(t-0.5)]e^{-\alpha(t-0.5)}, \quad (16)$$

where

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = 2.33 \text{ rad/s},$$

and the expression in Eq. (16) was shifted in time by 0.5 s. At $t = 0.5$ s, we require that:

$$i_{L_1}(0.5) = i_{L_2}(0.5), \quad (17a)$$
$$\upsilon_{C_1}(0.5) = \upsilon_{C_2}(0.5). \quad (17b)$$

Equating the expressions given by Eqs. (15) and (16) at $t = 0.5$ s gives:

$$3 - 4.78e^{-3.22\times 0.5} + 1.78e^{-5.28\times 0.5} = D_1,$$

which gives

$$D_1 = 2.17 \text{ V}. \quad (18)$$

From the circuit in Fig. (d),

$$\begin{aligned}\upsilon_{C_2}(t) &= (R_1+R_2)i_{L_2} + Li'_{L_2}\\ &= 7[D_1\cos\omega_d(t-0.5) + D_2\sin\omega_d(t-0.5)]e^{-\alpha(t-0.5)}\end{aligned}$$

$$\begin{aligned}&+ 2[-\omega_d D_1\sin\omega_d(t-0.5) + \omega_d D_2\cos\omega_d(t-0.5)\\ &- \alpha D_1\cos\omega_d(t-0.5) - \alpha D_2\sin\omega_d(t-0.5)]e^{-\alpha(t-0.5)}. \quad (19)\end{aligned}$$

At $t = 0.5$ s, Eqs. (11) and (19) give:

$$\upsilon_{C_1}(0.5) = 15 + 11.07e^{-3.22\times 0.5} - 26.07e^{-5.28\times 0.5} = 15.35 \text{ V}, \quad (20a)$$
$$\begin{aligned}\upsilon_{C_2}(0.5) &= 7D_1 + 2\omega_d D_2 - 2\alpha D_1\\ &= 7\times 2.17 + 2\times 2.33D_2 - 2\times 1.75\times 2.17 = 7.6 + 4.66D_2. \quad (20b)\end{aligned}$$

Equating Eq. (20a) to Eq. (20b) leads to

$$D_2 = 1.66 \text{ V}.$$

Hence,

$$i_{L_2}(t) = [2.17\cos 2.33(t-0.5) + 1.66\sin 2.33(t-0.5)]e^{-1.75(t-0.5)} \quad \text{(A)},$$
$$\text{for } t \geq 0.5 \text{ s}. \quad (21)$$

The expressions given by Eqs. (15) and (21) constitute the complete solution.