Signals and Systems Homework 8

Due Time: 21:59 May 11, 2018

Submitted in-class on Thu (May 10),

or to the box in front of SIST 1C 403E (the instructors office).

1. Consider the signal

$$x(t) = e^{-5t}u(t-1)$$

and denote its Laplace transform by $X(s)$.

(a) Using $X(s) = \int_{-\infty}^{+\infty} x(t)e^{-st}dt$, evaluate $X(s)$ and specify its region of convergence.

(b) Determine the values of the finite numbers A and t_0 such that the Laplace transform $G(s)$ of

$$g(t) = Ae^{-5t}u(-t-t_0)$$

has the same algebraic form as $X(s)$. What is the region of convergence corresponding to $G(s)$?

2. For the Laplace transform of

$$x(t) = \begin{cases} e^t \sin 2t, & t \le 0 \\ 0, & t > 0 \end{cases}$$

indicate the location of its poles and its region of convergence.

3. How many signals have a Laplace transform that may be expressed as

$$\frac{(s-1)}{(s+2)(s+3)(s^2+s+1)}$$

in its region of convergence? Please write down their region of convergence.

4. Given that

$$e^{-at}u(t) \overset{\mathscr{L}}{\longleftrightarrow} \frac{1}{s+a}, \quad \mathscr{R}\{s\} > \mathscr{R}\{-a\}$$

determine the inverse Laplace transform of

$$X(s) = \frac{2(s+2)}{s^2+7s+12} \quad \mathscr{R}\{s\} > -3$$

5. A causal LTI system S with impulse response $h(t)$ has its input $x(t)$ and output $y(t)$ related through a linear constant-coefficient differential equation of the form

$$\frac{d^3y(t)}{dt^3} + (1+\alpha)\frac{d^2y(t)}{dt^2} + \alpha(\alpha+1)\frac{dy(t)}{dt} + \alpha^2 y(t) = x(t).$$

   (a) If

   $$g(t) = \frac{dh(t)}{dt} + h(t).$$

   how many poles does $G(s)$ have?

   (b) For what real values of the parameter α is S guaranteed to be stable?

6. Consider a signal $y(t)$ which is related to two signals $x_1(t)$ and $x_2(t)$ by

$$y(t) = x_1(t-2) * x_2(-t+3)$$

where

$$x_1(t) = e^{-2t}u(t) \quad \text{and} \quad x_2(t) = e^{-3t}u(t)$$

Given that

$$e^{-at}u(t) \overset{\mathscr{L}}{\longleftrightarrow} \frac{1}{s+a}, \quad \mathscr{R}\{s\} > -a,$$

use properties of the Laplace transform to determine the Laplace transform $Y(s)$ of $y(t)$.

7. The system function of a causal LTI system is

$$H(s) = \frac{s+1}{s^2+2s+2}.$$

Determine and sketch the response $y(t)$ when the input is

$$x(t) = e^{-|t|}, \quad -\infty < t < \infty.$$