CS290: Introduction to Algorithmic Game Theory: Sponsored Search Auctions

Week 8.1 Sponsored Search Auction (Dengji ZHAO)

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Recap: Myerson's Optimal Auction

- Given the bids $\mathbf{b}$ and the distribution of agents' valuations $\mathbf{F}$, compute virtual bids $b'_i = \phi_i(b_i) = b_i - \frac{1-F_i(b_i)}{f_i(b_i)}$.
- Run VCG on the virtual bids $\mathbf{b}'$ to get allocation $\mathbf{x}'$ and payment $\mathbf{p}'$.
- Output $\mathbf{x} = \mathbf{x}'$ and $\mathbf{p}$ with $p_i = \phi_i^{-1}(p'_i)$.

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Profit maximisation

Myerson's Optimal Auction maximises the seller's profit.

- Quiz 12: digital goods?

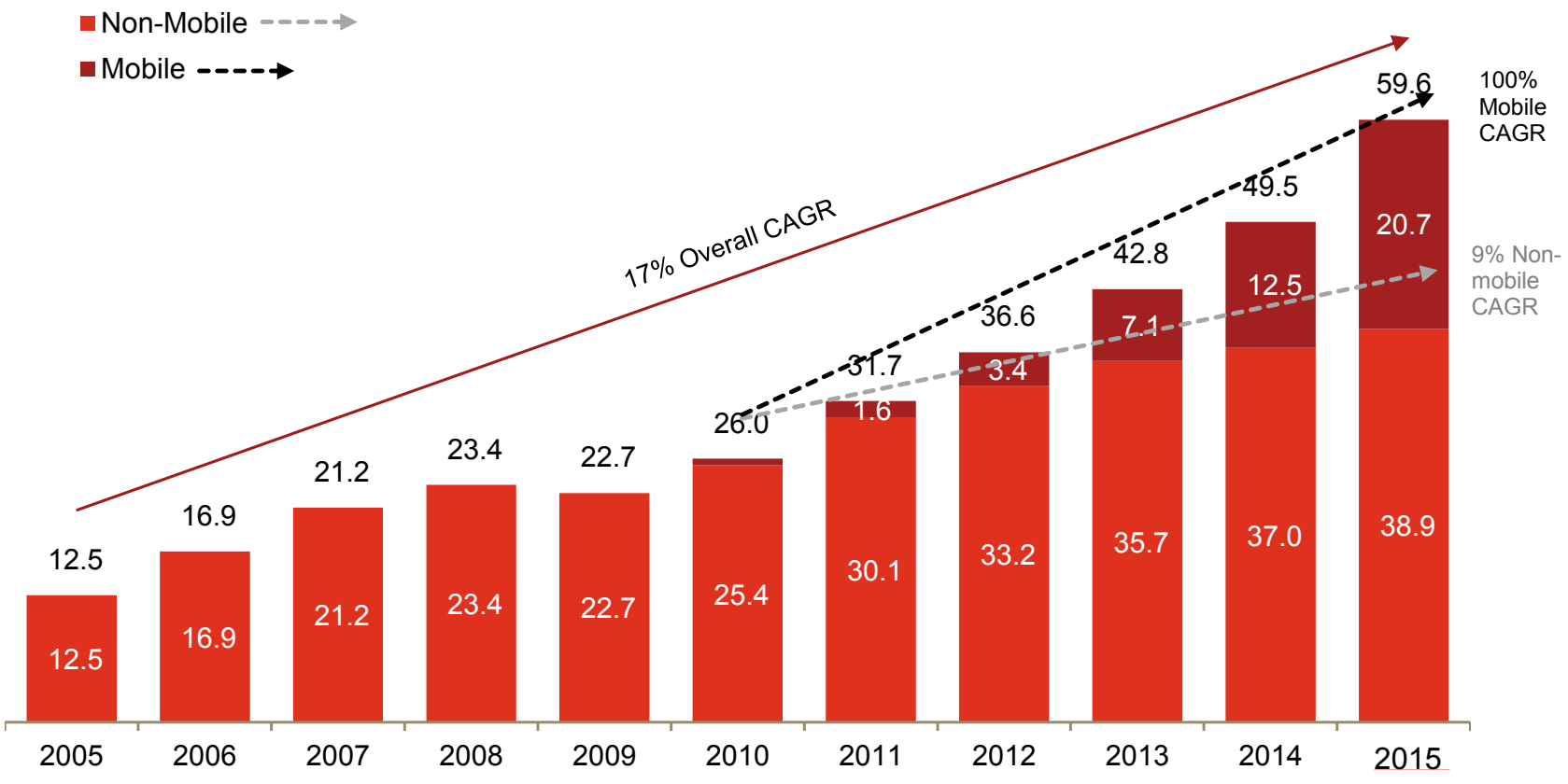
Sponsored Search Auctions

Sponsored Search Auction

- Used to sell ads slots by search engines such as Google, Baidu.
- Profit maximisation for the search engines?

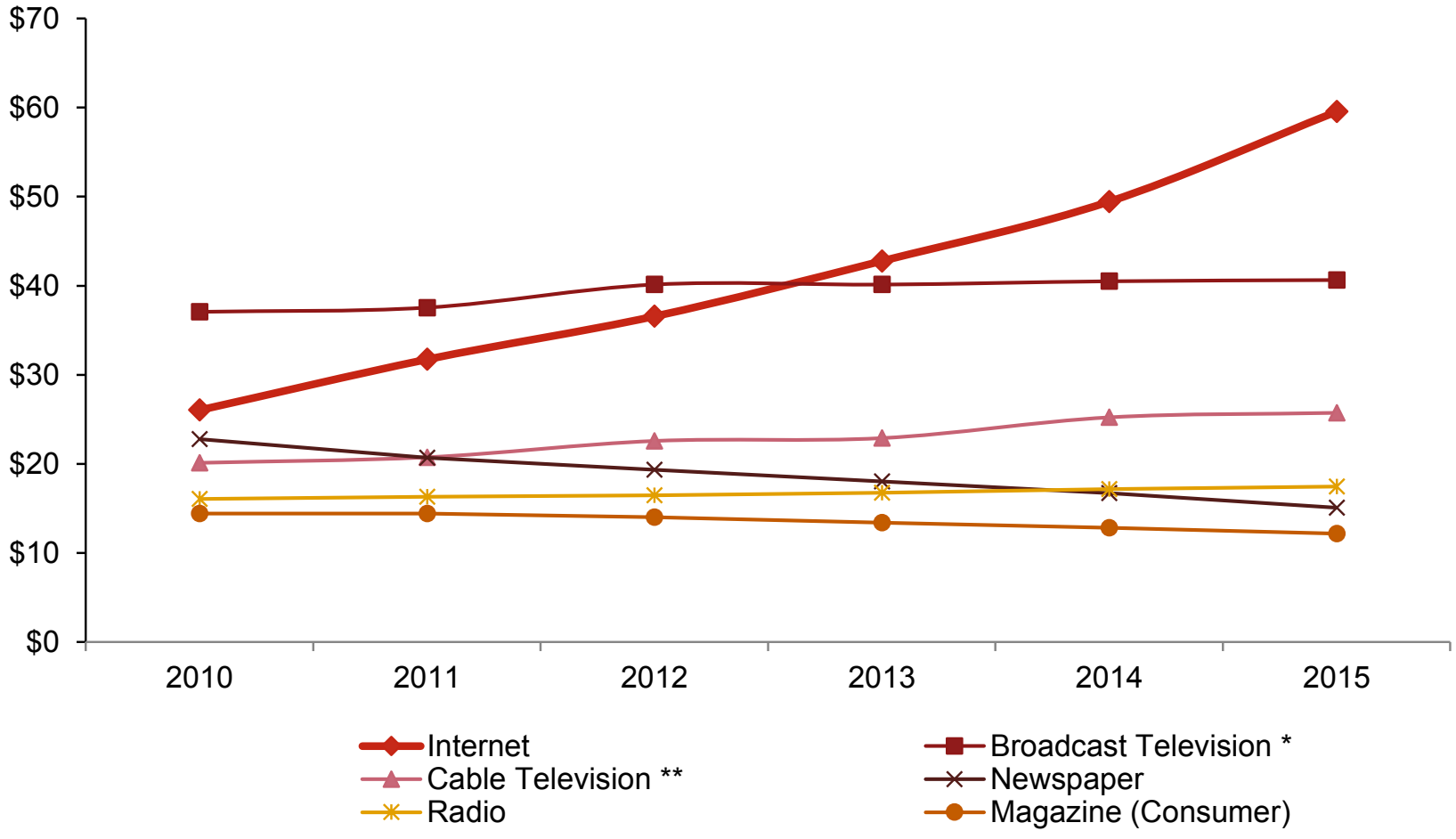
The Sponsored Search Market

Annual Revenue 2005-2015 ($ billions)



Source: IAB/PwC Internet Ad Revenue Report, FY 2015

The Sponsored Search Market

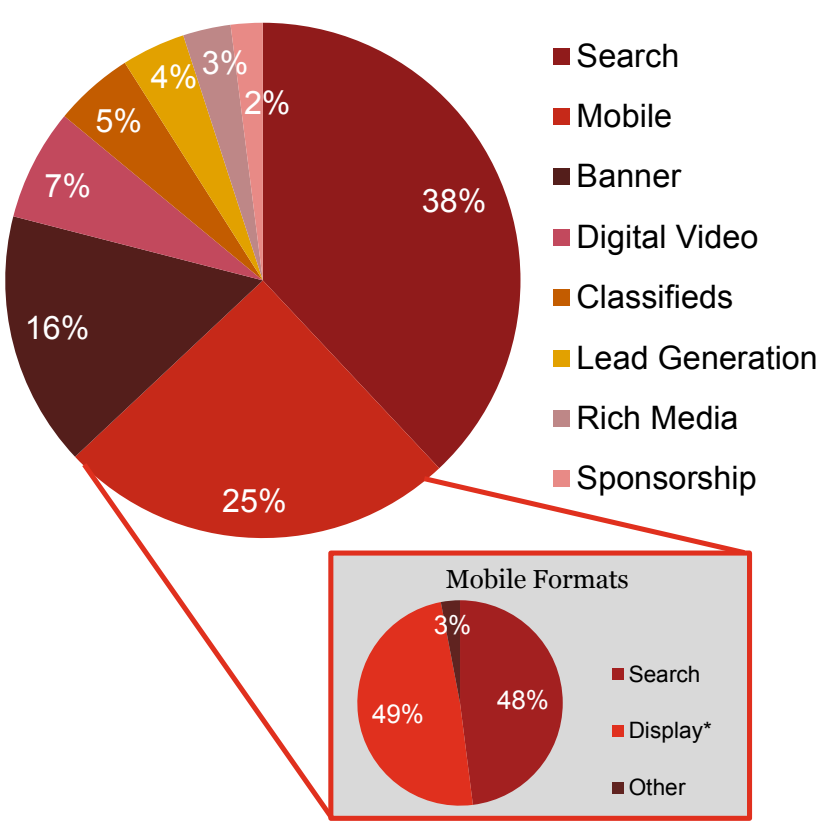


Sources: IAB/PwC Internet Ad Revenue Report, FY 2015; PwC

The Sponsored Search Market

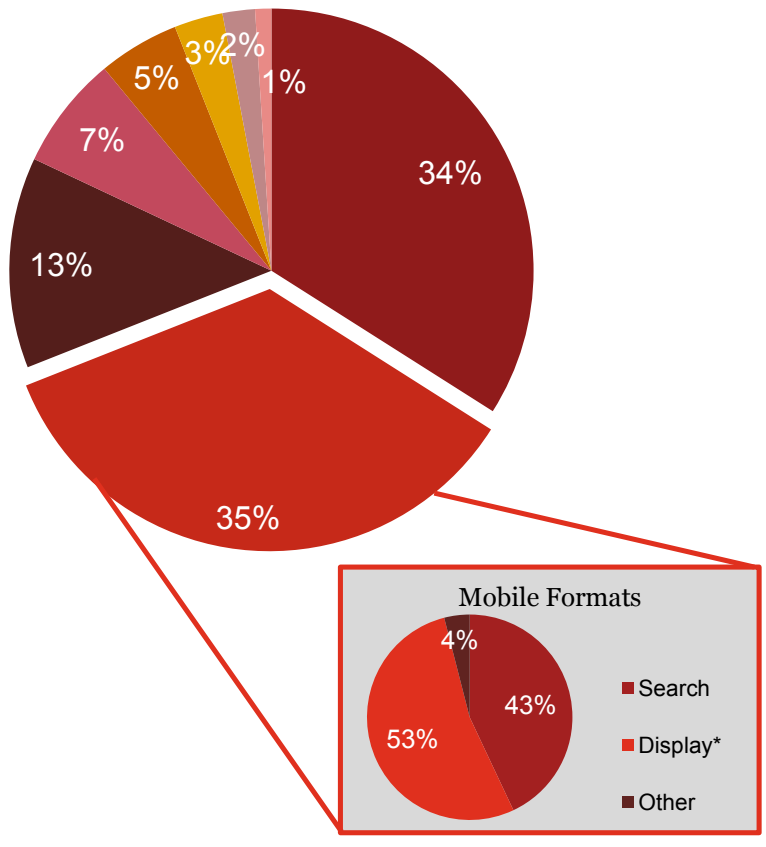
Ad formats – full year 2014

Total - $49.5 billion**



Ad formats – full year 2015

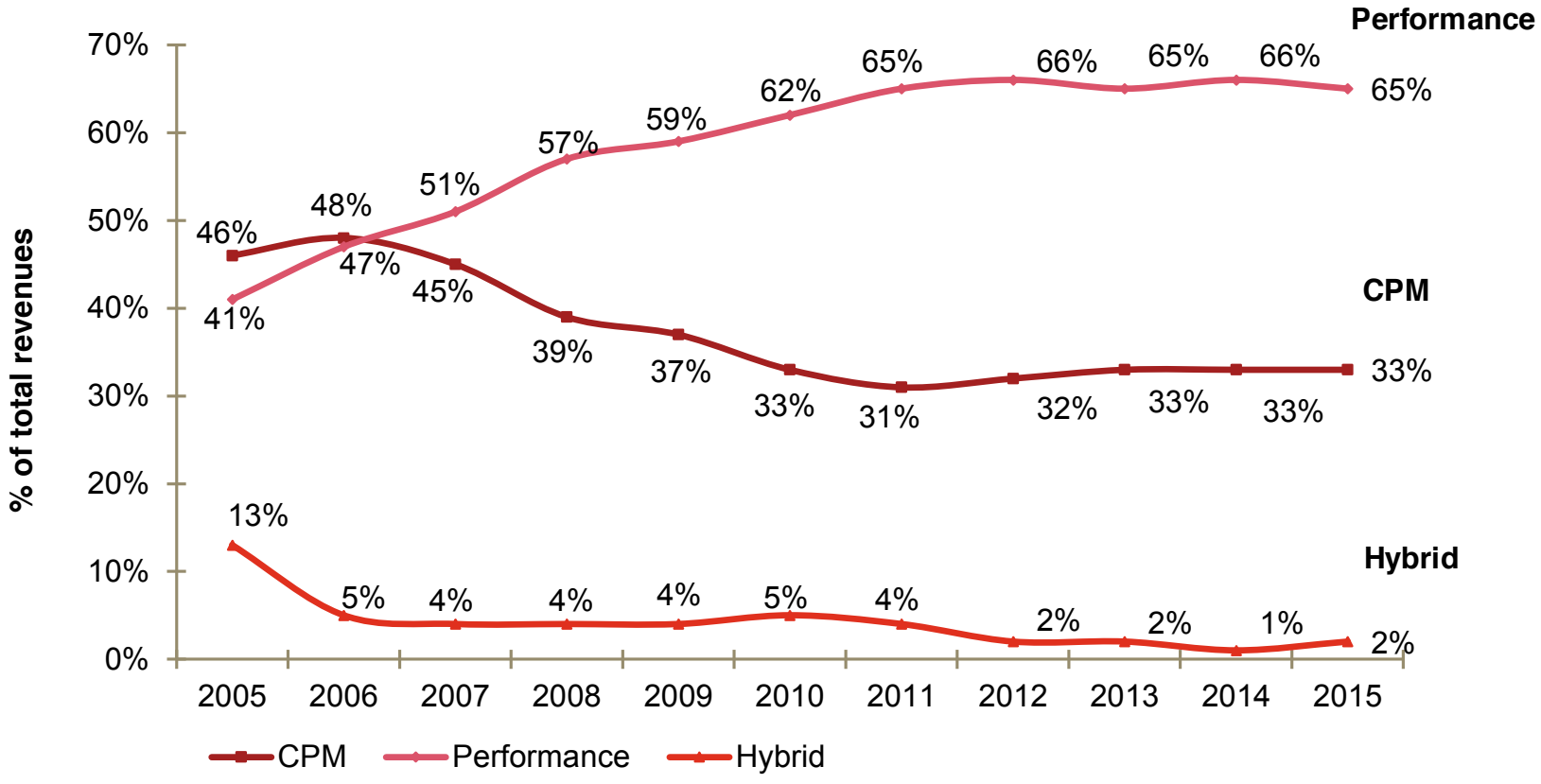
Total - $59.6 billion**



Source: IAB/PwC Internet Ad Revenue Report, FY 2015

The Sponsored Search Market

Internet ad revenues by pricing model*



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The Basic Model

- A set of advertisers/bidders (n), each specify a list of pairs of keywords and bids as well as a total budget (daily/weekly/monthly).
- A search engine with $m < n$ number of ad slots. The search engine estimate a click through rate α_{ij}, the probability that a user will click on the ith slot when it is occupied by bidder j. Assume that $\alpha_{ij} \geq \alpha_{i+1j}$ for $i = 1, ..., m - 1$.
- The search engine also assigns a weight w_j to each advertiser j. The weight can be thought of as a relevance or quality metric.

Generalized Second Price (GSP) Auctions

For each search of a keyword, GSP does the following to allocate ads:

- Rank advertisers by their score $b_j w_j$.
- The highest score gets the first slot, the second highest score gets the second slot and so on.
- A bider pays per click the lowest bid necessary to retain his position.

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Two different variants:

1. Rank by bid (used by Overture): assume that $w_j = 1$
2. Rank by revenue (used by Google): assume that $w_j = \alpha_{1j}$

Efficiency in A Static Setting

- How to maximize social welfare?

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$$
\begin{aligned}
\max \quad & \sum_{i=1}^{k}\sum_{j=1}^{n} \alpha_{ij} v_j x_{ij} \\
\text{s.t.} \quad & \sum_{j=1}^{n} x_{ij} \leq 1 \qquad \forall i = 1, \ldots, k \\
& \sum_{i=1}^{k} x_{ij} \leq 1 \qquad \forall j = 1, \ldots, n \\
& x_{ij} \geq 0 \qquad \forall i = 1, \ldots, k, \ \forall j = 1, \ldots, n
\end{aligned}
$$

where $x_{ij} = 1$ if bidder j is assigned to slot i and zero otherwise.

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- What will be the payment under VCG?

The VCG Payments

- Consider three bidders $1, 2, 3$ with $v_1 > v_2 > v_3$ for one keyword and two slots.
- Suppose that $\alpha_{ij} = \mu_i$ with $\mu_1 > \mu_2$ (CTR are bidder independent).

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- Quiz 13: what are the VCG payments for bidders $1, 2$?

The VCG Payments

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- Quiz 14: What are the GSP payments for bidders $1, 2$?

Revenue Maximization

- How to maximize search engine's revenue?

The Dynamic Setting

- What will happen if the game is repeated?

Advanced Reading

- AGT Chapter 28. Sponsored Search Auctions