

Edge detection



[Winter in Kraków photographed by Marcin Ryczek](#)

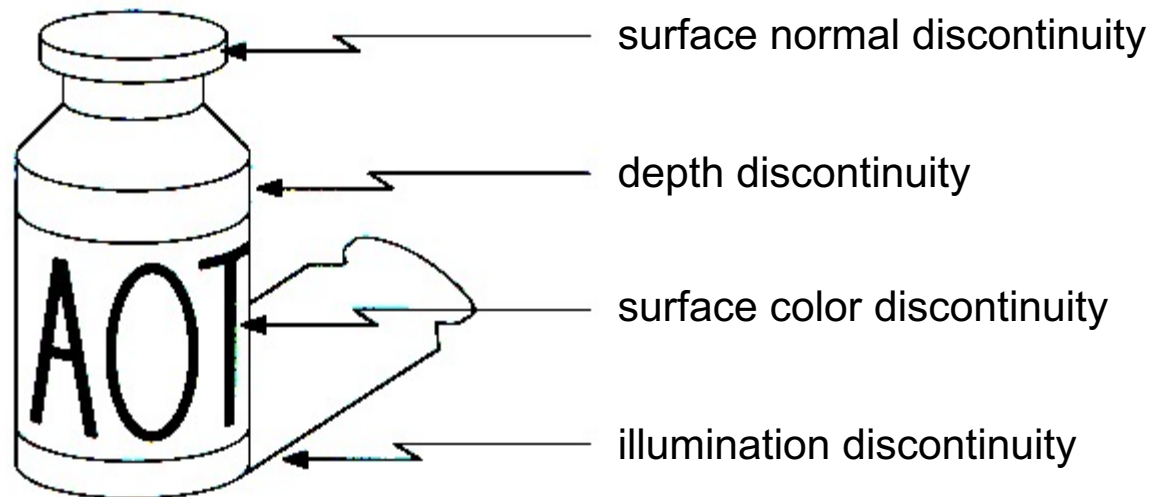
Edge detection

- **Goal:** Identify sudden changes (discontinuities) in an image.
 - Intuitively, most semantic and shape information from the image can be encoded in the edges
 - More compact than pixels
- **Ideal:** artist's line drawing (but artist is also using object-level knowledge)



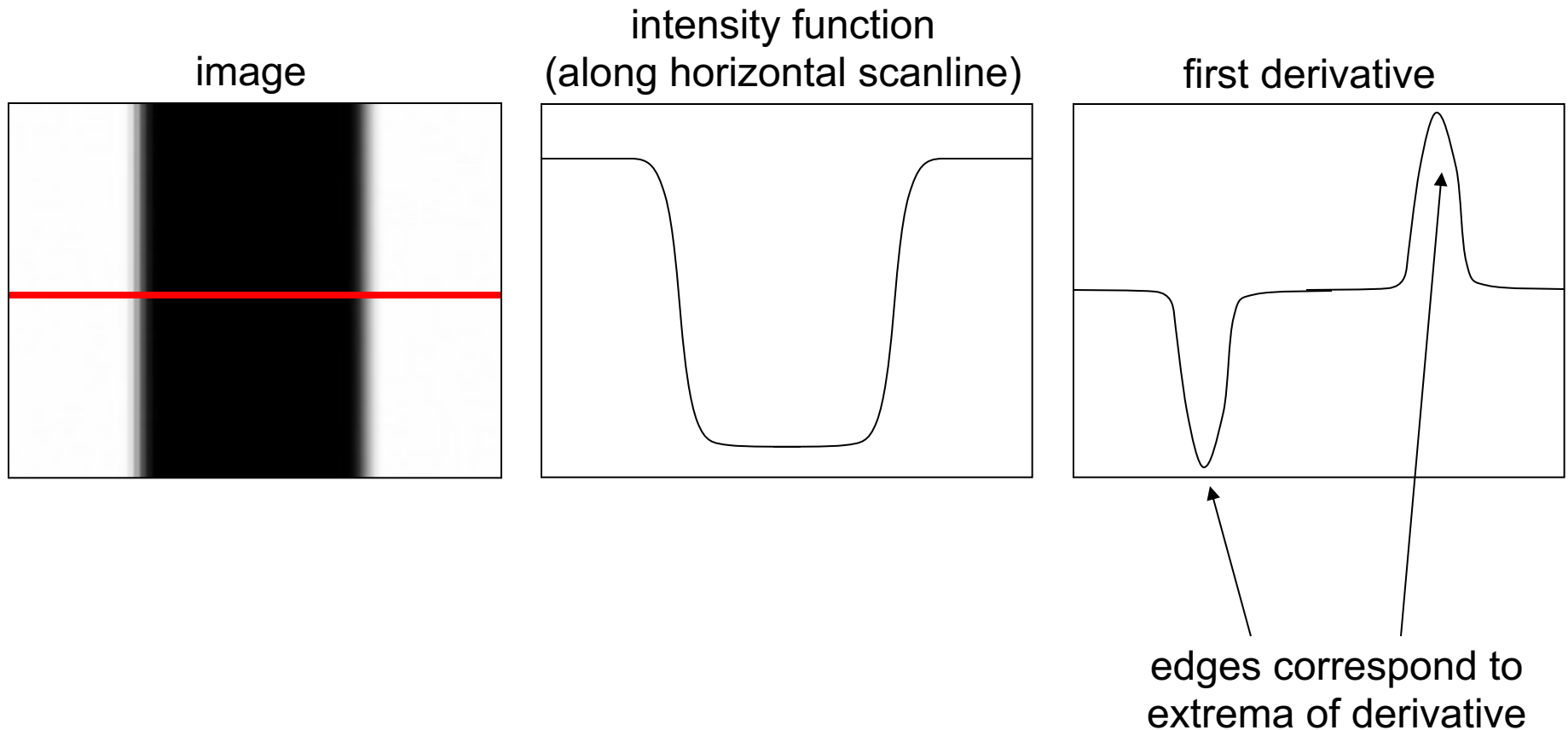
Origin of edges

Edges are caused by a variety of factors:



Edge detection

- An edge is a place of rapid change in the image intensity function



Derivatives with convolution

For 2D function $f(x,y)$, the partial derivative is:

$$\frac{\partial f(x, y)}{\partial x} = \lim_{\varepsilon \rightarrow 0} \frac{f(x + \varepsilon, y) - f(x, y)}{\varepsilon}$$

For discrete data, we can approximate using finite differences:

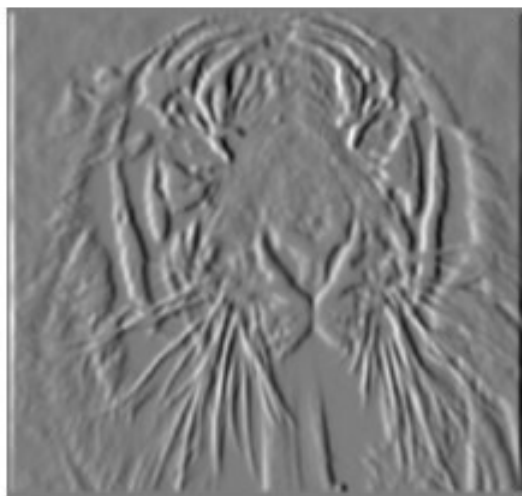
$$\frac{\partial f(x, y)}{\partial x} \approx \frac{f(x + 1, y) - f(x, y)}{1}$$

To implement the above as convolution, what would be the associated filter?

Partial derivatives of an image

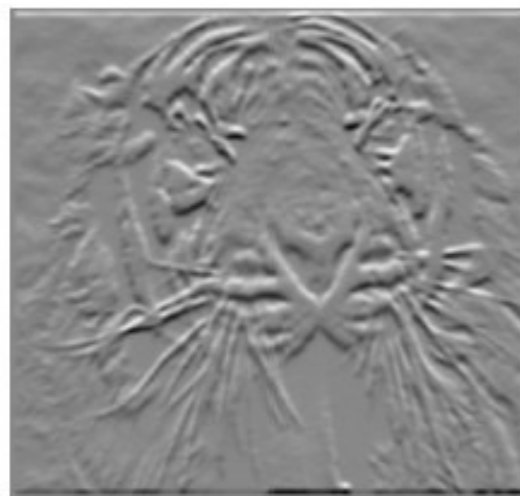


$$\frac{\partial f(x, y)}{\partial x}$$



-1	1
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$$\frac{\partial f(x, y)}{\partial y}$$



-1	1
1	-1

or

Which shows changes with respect to x?

Finite difference filters

Other approximations of derivative filters exist:

Prewitt: $M_x =$

-1	0	1
-1	0	1
-1	0	1

 ; $M_y =$

1	1	1
0	0	0
-1	-1	-1

Sobel: $M_x =$

-1	0	1
-2	0	2
-1	0	1

 ; $M_y =$

1	2	1
0	0	0
-1	-2	-1

Roberts: $M_x =$

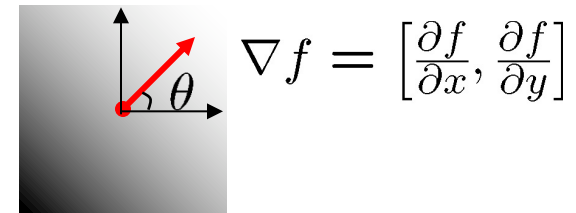
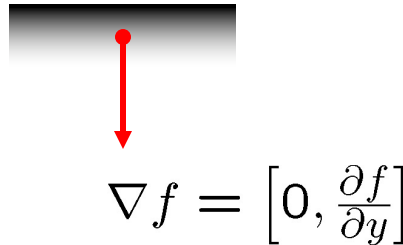
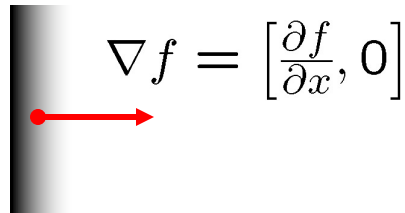
0	1
-1	0

 ; $M_y =$

1	0
0	-1

Image gradient

The gradient of an image: $\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right]$



The gradient points in the direction of most rapid increase in intensity

- How does this direction relate to the direction of the edge?

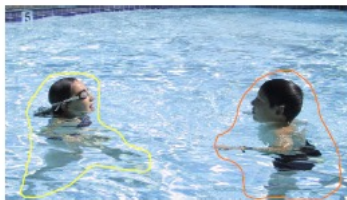
The gradient direction is given by $\theta = \tan^{-1} \left(\frac{\partial f}{\partial y} / \frac{\partial f}{\partial x} \right)$

The edge strength is given by the gradient magnitude

$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

Application: Gradient-domain image editing

- Goal: solve for pixel values in the target region to match gradients of the source region while keeping background pixels the same



sources/destinations



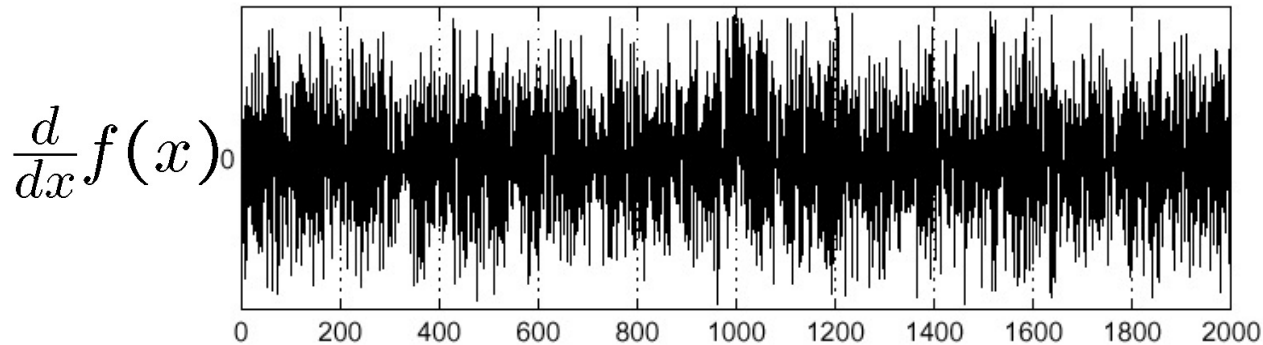
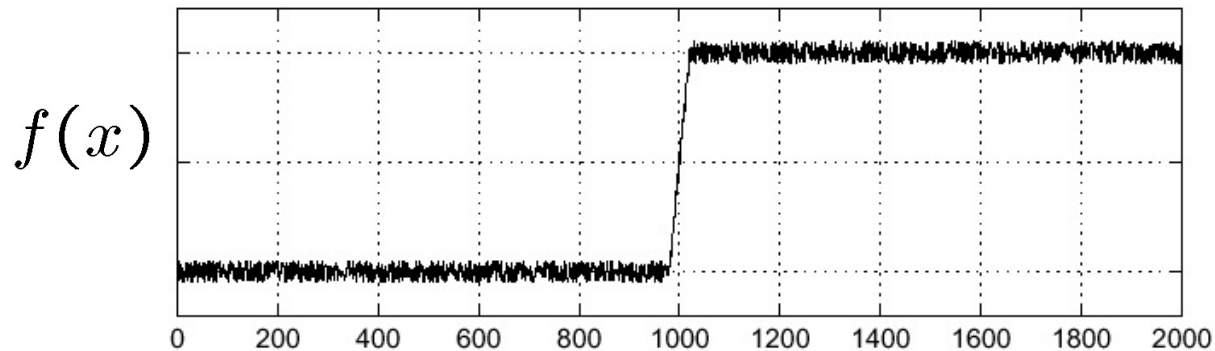
cloning



seamless cloning

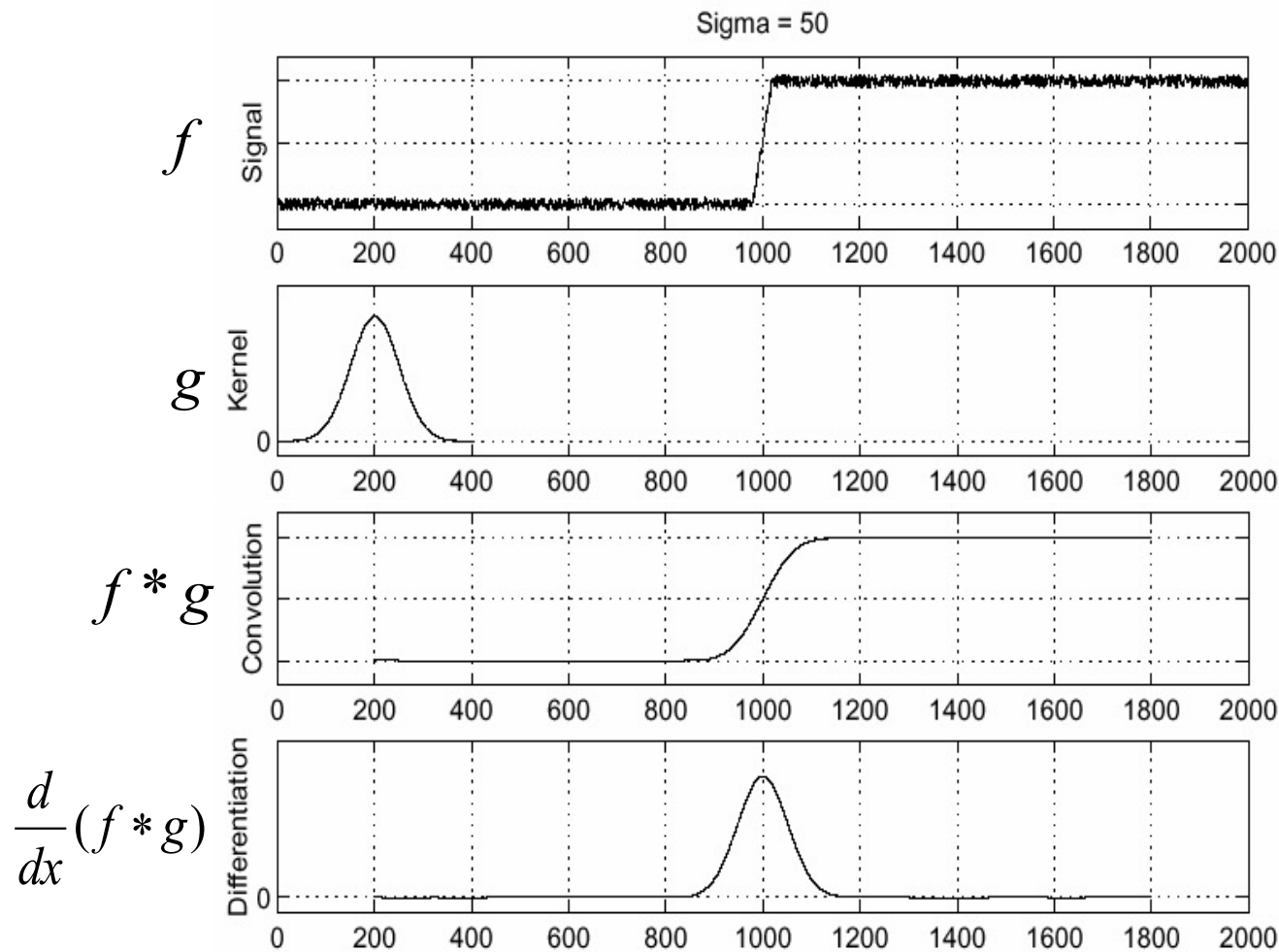
Effects of noise

Consider a single row or column of the image



Where is the edge?

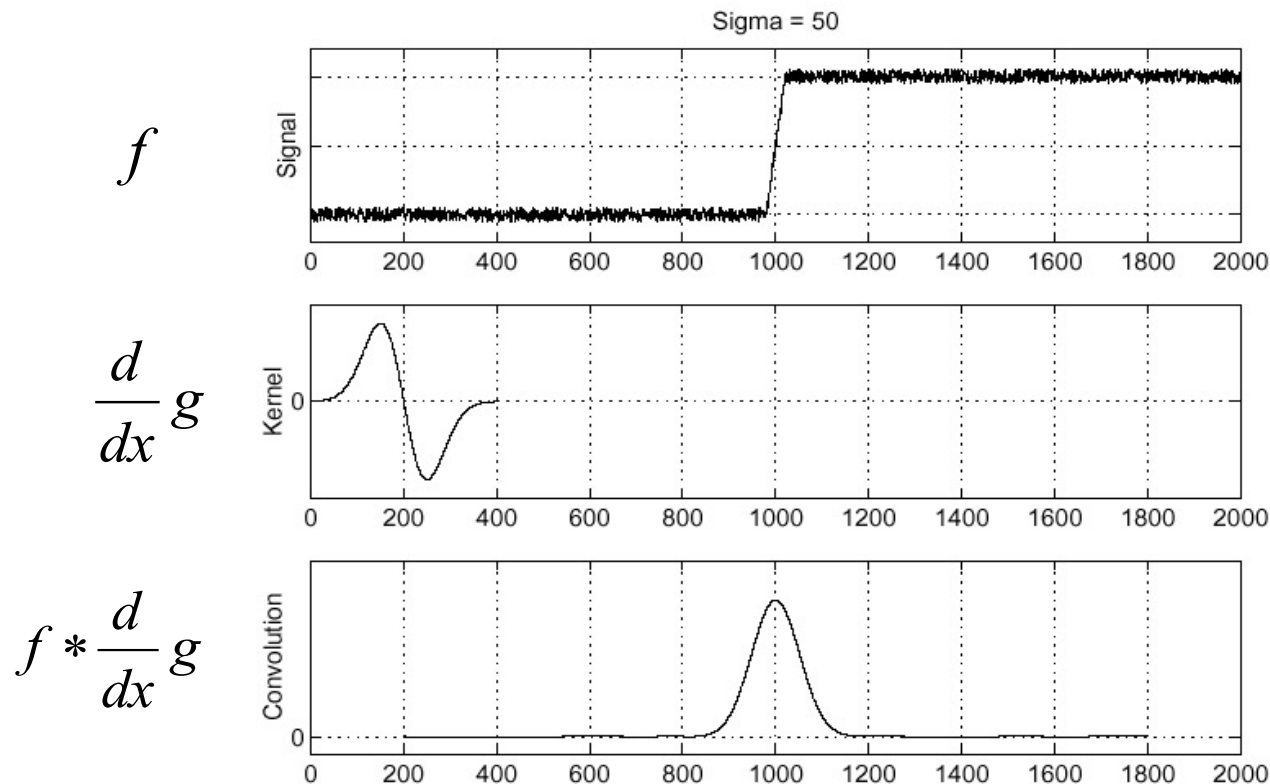
Solution: smooth first



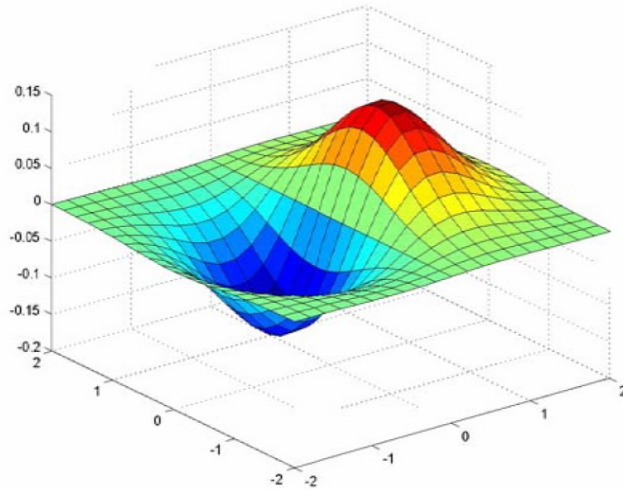
- To find edges, look for peaks in $\frac{d}{dx}(f * g)$

Derivative theorem of convolution

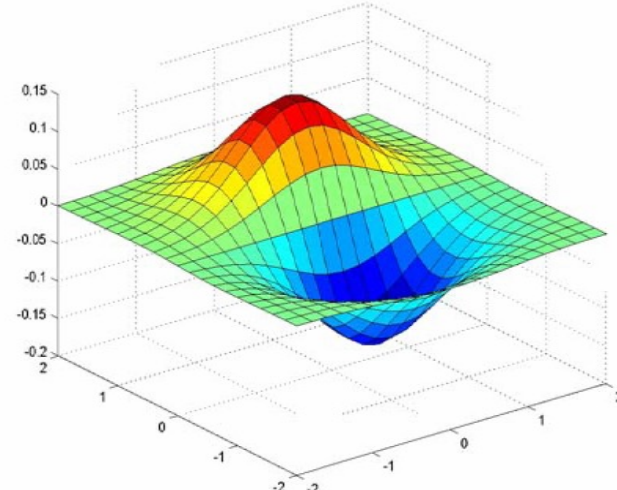
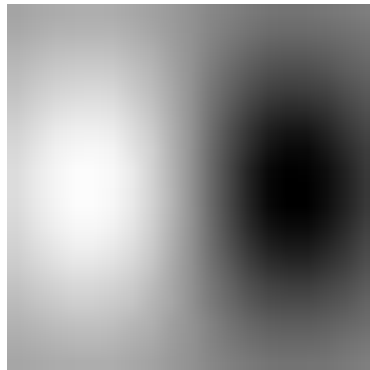
- Differentiation is convolution, and convolution is associative: $\frac{d}{dx}(f * g) = f * \frac{d}{dx}g$
- This saves us one operation:



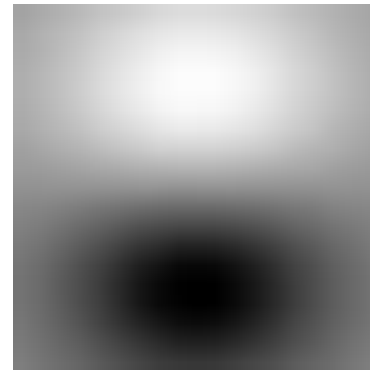
Derivative of Gaussian filters



x-direction

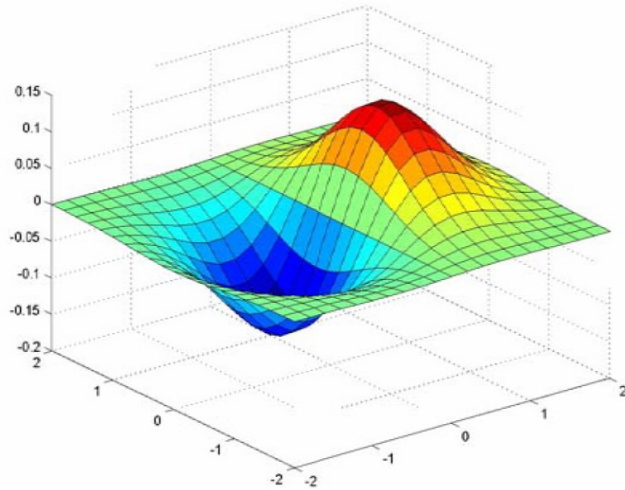


y-direction

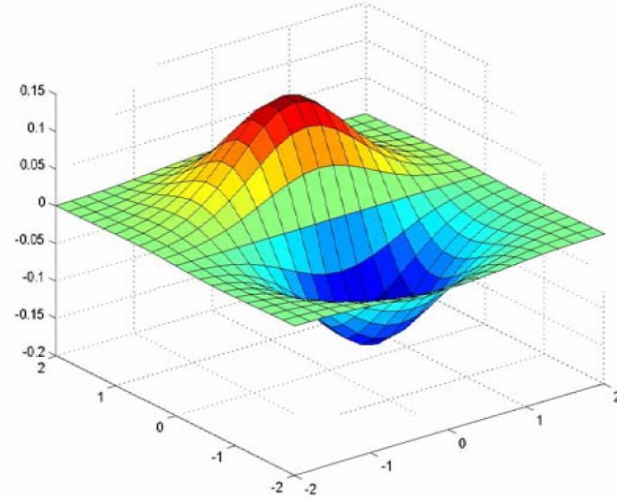
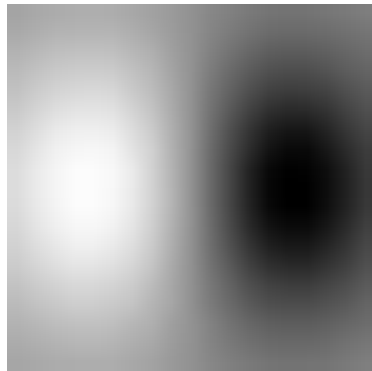


Which one finds horizontal/vertical edges?

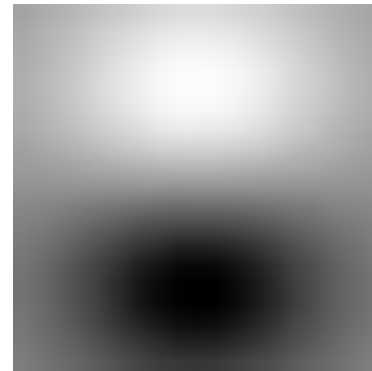
Derivative of Gaussian filters



x-direction



y-direction



Are these filters separable?

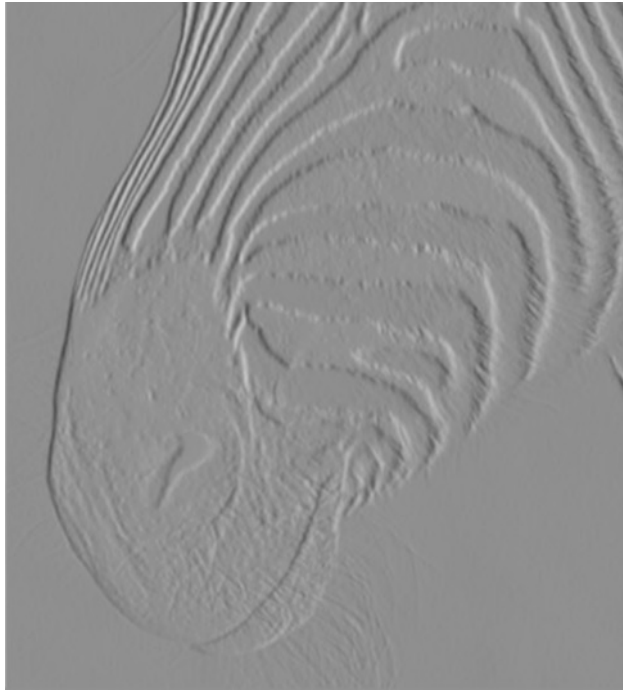
Recall: Separability of the Gaussian filter

$$\begin{aligned} G_{\sigma}(x, y) &= \frac{1}{2\pi\sigma^2} \exp^{-\frac{x^2 + y^2}{2\sigma^2}} \\ &= \left(\frac{1}{\sqrt{2\pi}\sigma} \exp^{-\frac{x^2}{2\sigma^2}} \right) \left(\frac{1}{\sqrt{2\pi}\sigma} \exp^{-\frac{y^2}{2\sigma^2}} \right) \end{aligned}$$

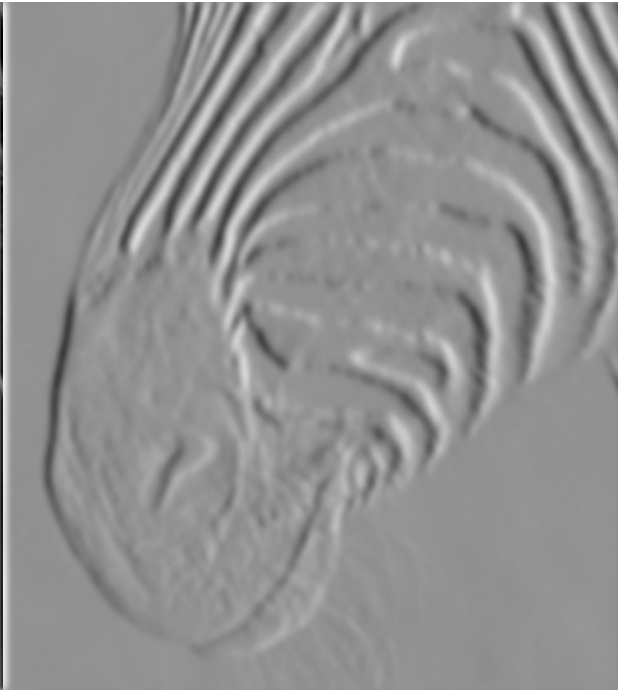
The 2D Gaussian can be expressed as the product of two functions, one a function of x and the other a function of y

In this case, the two functions are the (identical) 1D Gaussian

Scale of Gaussian derivative filter



1 pixel



3 pixels



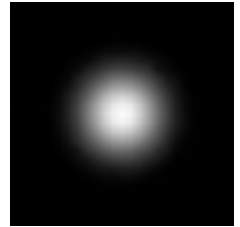
7 pixels

Smoothed derivative removes noise, but blurs edge. Also finds edges at different “scales”

Review: Smoothing vs. derivative filters

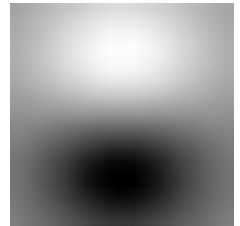
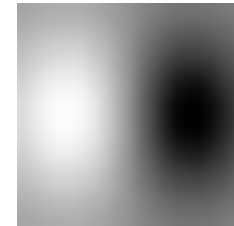
Smoothing filters

- Gaussian: remove “high-frequency” components; “low-pass” filter
- Can the values of a smoothing filter be negative?
- What should the values sum to?
 - **One:** constant regions are not affected by the filter



Derivative filters

- Derivatives of Gaussian
- Can the values of a derivative filter be negative?
- What should the values sum to?
 - **Zero:** no response in constant regions



Build a canny edge detector



Build a canny edge detector

Step1: Filter out any noise using a Gaussian kernel if necessary

Step2: Find the intensity gradient of the image.

a. Apply a pair of convolution masks(in x and y directions):

$$G_x = \begin{bmatrix} -1 & 0 & +1 \\ -2 & 0 & +2 \\ -1 & 0 & +1 \end{bmatrix}$$
$$G_y = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ +1 & +2 & +1 \end{bmatrix}$$

Build a canny edge detector

b. Compute the gradient magnitude and direction with:

$$G = \sqrt{G_x^2 + G_y^2}$$
$$\theta = \arctan\left(\frac{G_y}{G_x}\right)$$

OpenCV rounds the direction to one of four possible angles(namely 0, 45, 90, or 135)

Build a canny edge detector



norm of the gradient

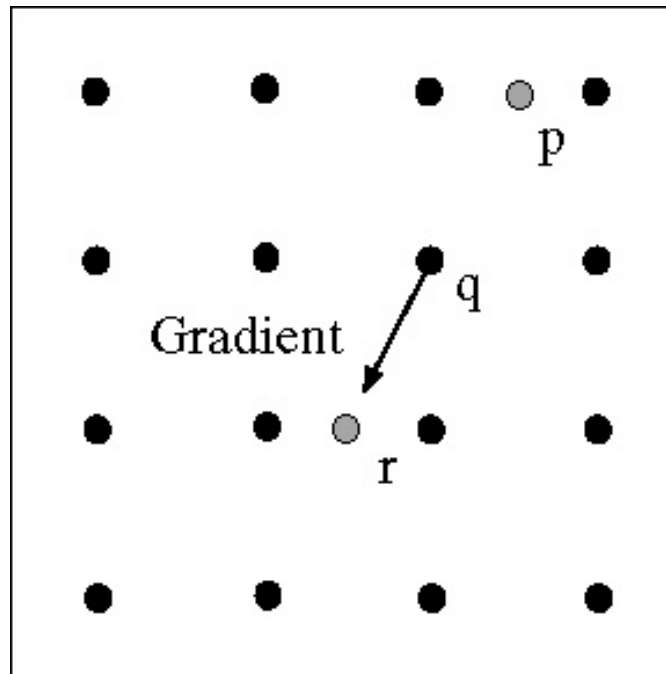
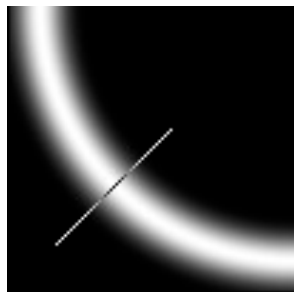
Build a canny edge detector



How to turn these thick regions of the gradient into curves?

Threshold the norm of the gradient

Non-maximum suppression



Check if pixel is local maximum along gradient direction, select single max across that direction

Non-maximum suppression

Case 1. The gradient direction is 0:

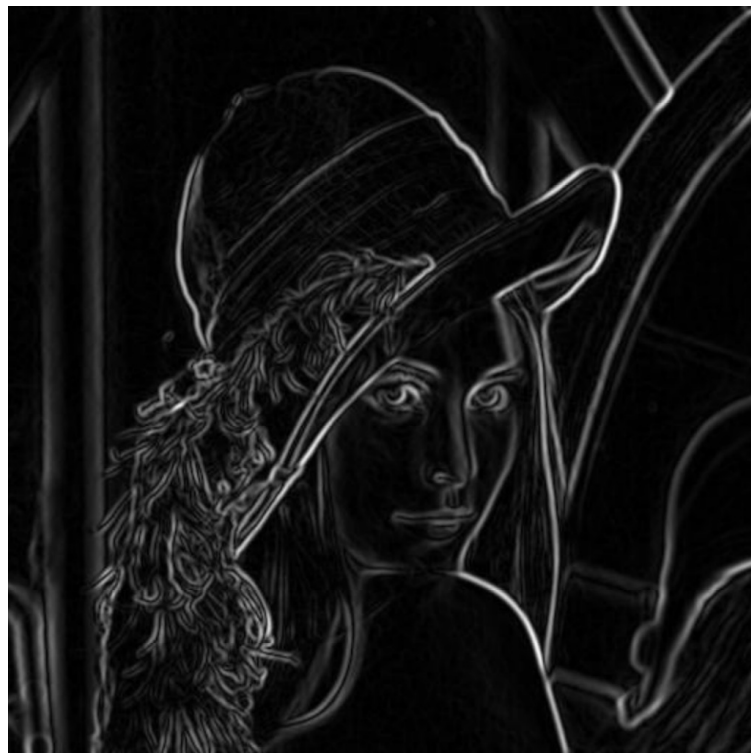
Find a maximum from $\text{Image}(x-1, y)$, $\text{Image}(x, y)$ and $\text{Image}(x+1, y)$, set the maximum pixel as edge pixel

Case 2. The gradient direction is 45:

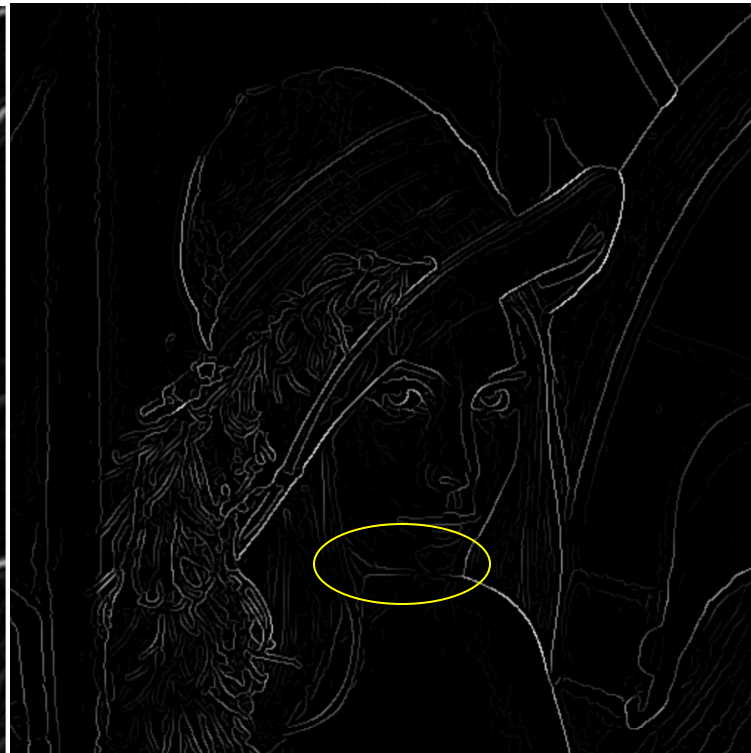
Find a maximum from $\text{Image}(x-1, y-1)$, $\text{Image}(x, y)$ and $\text{Image}(x+1, y+1)$, set the maximum pixel as edge pixel

Non-maximum suppression

before



after



Another problem: pixels along this edge don't survive after this procedure

Hysteresis

Canny does use two thresholds(upper and lower)

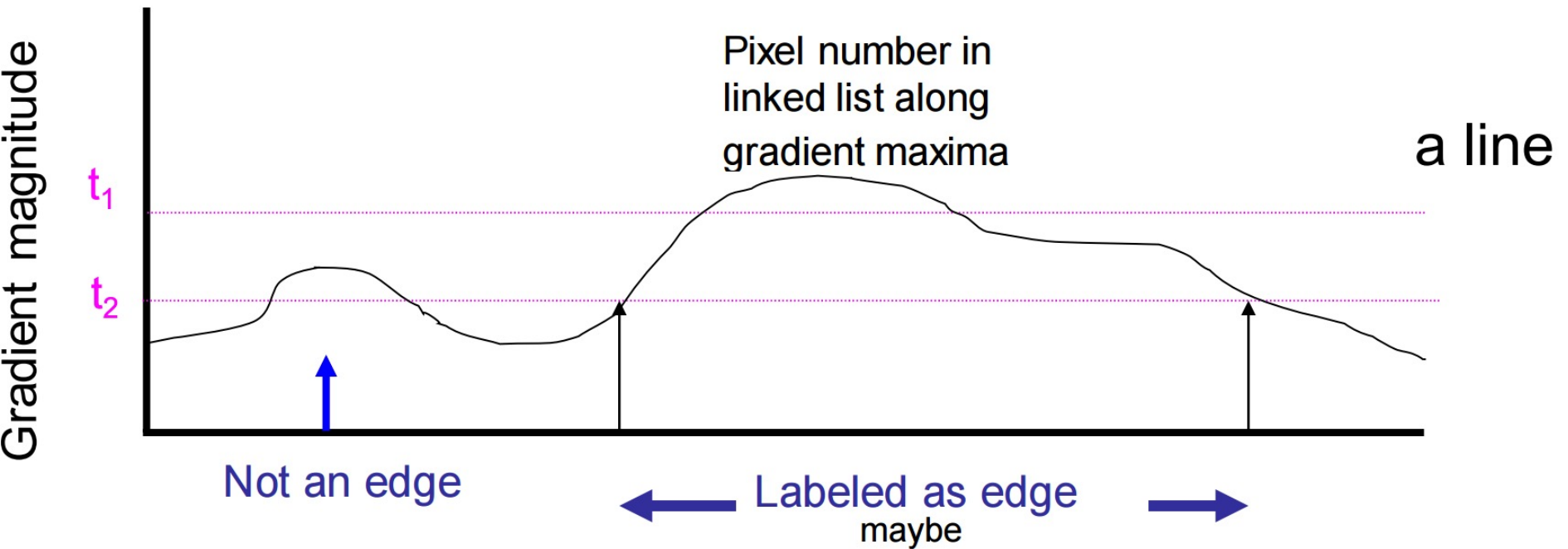
- a. If a pixel gradient is higher than the upper threshold, the pixel is accepted as an edge
- b. If a pixel gradient is below the lower threshold, then it is rejected

Hysteresis

c. If the pixel gradient is between the two thresholds, then it will be accepted only if it is connected to a pixel that is above the upper threshold

Canny recommend a upper: lower ratio between 2:1 and 3:1

Hysteresis



Hysteresis thresholding



original image



**high threshold
(strong edges)**



**low threshold
(weak edges)**



hysteresis threshold

Recap: Canny edge detector

Smoothing with Gaussian (denosing)

Compute x and y gradient images

1. Find magnitude and orientation of gradient
2. **Non-maximum suppression:**
 - Thin wide “ridges” down to single pixel width
3. **Linking and thresholding (hysteresis):**
 - Define two thresholds: low and high
 - Use the high threshold to start edge curves and the low threshold to continue them

MATLAB: `edge(image, 'canny');`

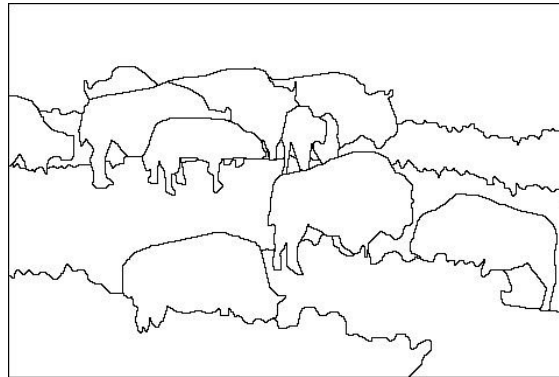
J. Canny, [***A Computational Approach To Edge Detection***](#), IEEE Trans. Pattern Analysis and Machine Intelligence, 8:679-714, 1986.

Image gradients vs. meaningful contours

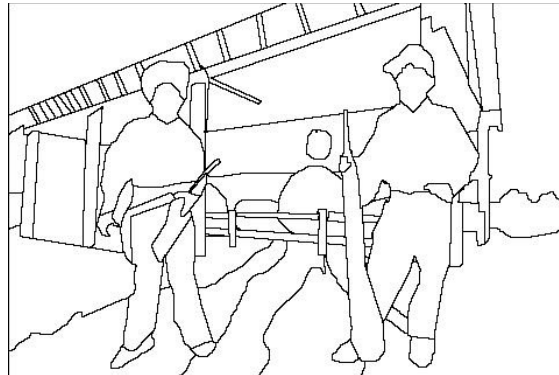
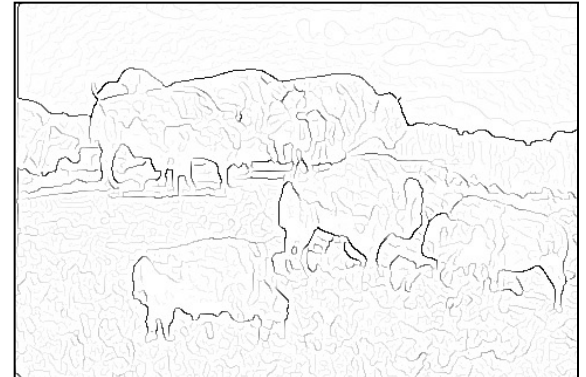
image



human segmentation



gradient magnitude

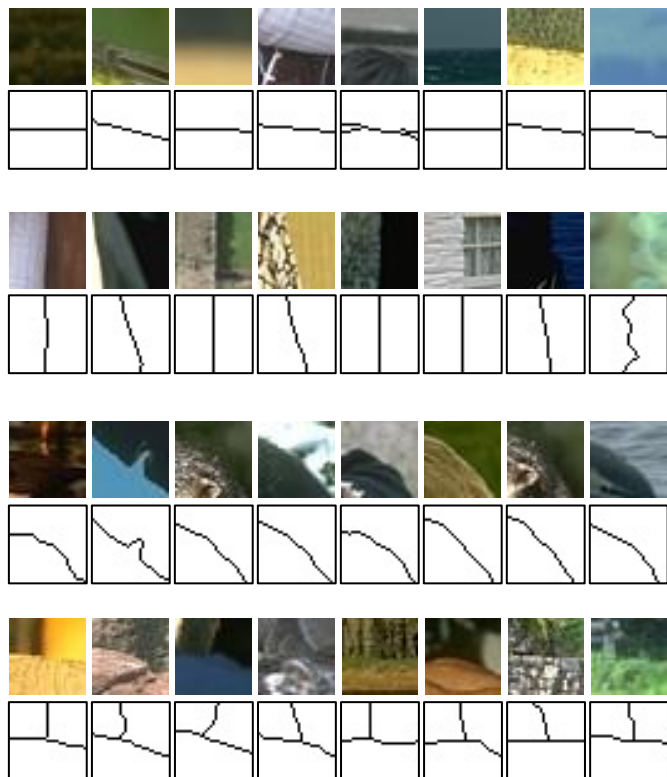


Berkeley segmentation database:

<http://www.eecs.berkeley.edu/Research/Projects/CS/vision/grouping/segbench/>

Data-driven edge detection

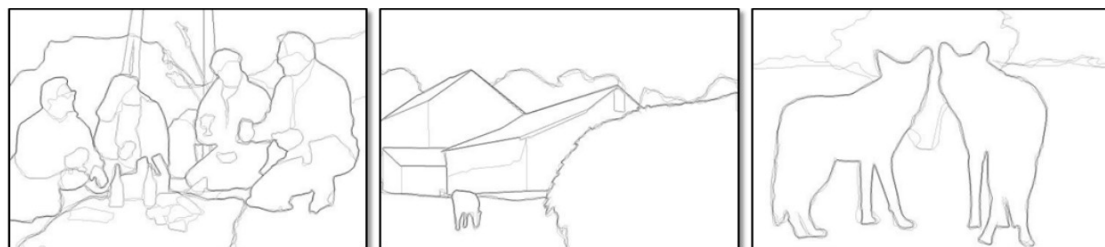
Training data



Input images



Ground truth



Output

