ShanghaiTech University

CS101 Algorithms and Data Structures

Fall 2020

Homework 5

Due date: 23:59, October 19, 2020

1. Please write your solutions in English.
2. Submit your solutions to gradescope.com.
3. Set your FULL NAME to your Chinese name and your STUDENT ID correctly in Account Settings.
4. If you want to submit a handwritten version, scan it clearly. Camscanner is recommended.
5. When submitting, match your solutions to the according problem numbers correctly.
6. No late submission will be accepted.
7. Violations to any of the above may result in zero grade.
8. Problem 0 gives you a template on how to organize your answer, so please read it carefully.

Problem 0: Notes and Example

Notes

1. Some problems in this homework requires you to design Divide and Conquer algorithm. When grading these problems, we will put more emphasis on how you reduce a problem to a smaller size problem with Divide and Conquer strategy.
2. Your answer for these problems should include:
   (a) Algorithm Design
   (b) Time Complexity Analysis
3. In Algorithm Design, you should describe your algorithm for each step clearly.
4. Unless required, writing pseudocode is optional. If you write pseudocode, please give some additional descriptions if the pseudocode is not obvious.
5. You are recommended to do this homework with LaTeX.

Example: Binary Search
Given a sorted array a of n elements, write a function to search a given element x in a.

Algorithm Design: We basically ignore half of the elements just after one comparison.

1. Compare x with the middle element.
2. If x matches with middle element, we return the mid index.
3. Else If x is greater than the mid element, then x can only lie in right half subarray after the mid element. So we recur for right half.
4. Else (x is smaller) recur for the left half.

Pseudocode(Optional):
$left$ and $right$ are indexes of the leftmost and rightmost elements in given array a respectively.

```
1:  function BINARYSEARCH(a, value, left, right)
2:      if right < left then
3:          return not found
4:      end if
5:      mid ← ⌊(right − left)/2⌋ + left
6:      if a[mid] = value then
7:          return mid
8:      end if
9:      if value < a[mid] then
10:         return binarySearch(a, value, left, mid-1)
11:     else
12:         return binarySearch(a, value, mid+1, right)
13:     end if
14: end function
```

Time Complexity Analysis: During each recursion, the calculation of mid and comparison can be done in constant time, which is $O(1)$. We ignore half of the elements after each comparison, thus we need $O(\log n)$ recursions.

$$T(n) = T(n/2) + O(1)$$

Therefore, $T(n) = \log n$

1: (2'+1'+1') Single Choice

The following questions are single choice questions, each question has **only one** correct answer. Select the correct answer.

Note: You should write those answers in the box below.

Question 1	Question 2	Question 3

Question 1. *What is the common data structure used in implementation of* ***Breadth First Search****?* _____ *What is the common data structure used in implementation of* ***Depth First Search****?* _____

(A) Stack

(B) Queue

(C) Hash Table

(D) Tree

Question 2. *Given the following pseudo-code, which kind of traversal is it doing?*

```
1: function ORDER(node)
2:     if node has left child then
3:         order(node.left)
4:     end if
5:     if node has right child then
6:         order(node.right)
7:     end if
8:     visit(node)
9: end function
```

(A) Preorder traversal

(B) Postorder traversal

(C) Inorder traversal

(D) None of above

Question 3. *Which traversal strategy should we use if we want to print the hierachical structure ?*

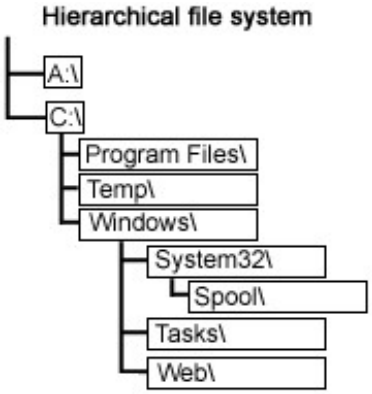


Figure 1: an example of hierachical structure

(A) Preorder traversal

(B) Postorder traversal

(C) Inorder traversal

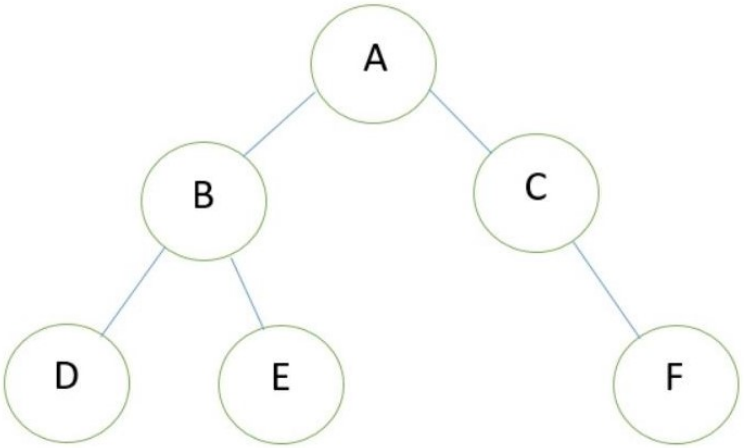
(D) None of above

2: (5') Running BFS & DFS

Please run BFS and DFS(preorder) on the following tree.

Note:

1. *You should select proper data structure for BFS and DFS first (1')*
2. *Please show what is currently in your data structure at each step (2')*
3. *When you want to put more than one child of some node onto your data structure, please push/enqueue them alphabetically.*
4. *Write down the order of sequence after you finish running BFS and DFS. (2')*



(a) BFS

(b) DFS

3: (5')Skyline problem

We can abstract a street into a straight line and assume that the bottoms of all the buildings on one side of the street lie on the x-axis, and the two-dimensional skyline of the buildings can be drawn as follows:

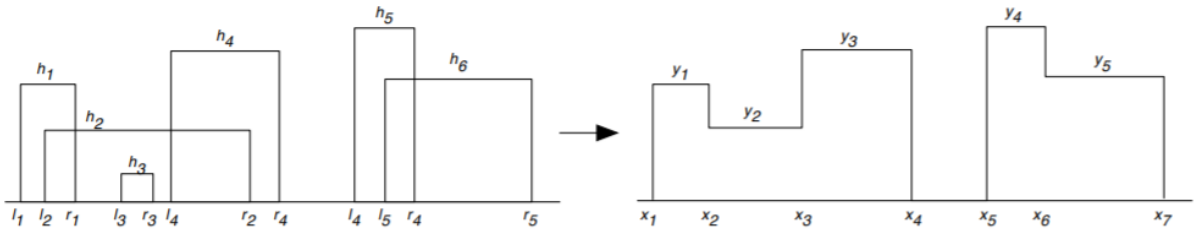


Figure 2: Skyline problem

Building B_i is represented by a triple (l_i, h_i, r_i), where l_i and r_i denote the left and right x coordinates of the building, respectively, and h_i denotes the building's height. A skyline is composed of x coordinates and the heights in the following format:

$$(x_1, y_1, x_2, y_2, x_3...)$$

meaning that at x_1 we draw a building at height y_1 until x_2 at which point we draw a line up or down to height y_2 and then continue horizontally until x_3 and so on.

(a) Firstly, let's see how to merge. There are two skylines, one is composed of n buildings, and the other is composed of m buildings. For the $m+n$ buildings, you need to show how to merge the two skylines into one new skyline in $O(m+n)$ steps.

(b) Now, let's design a complete algorithm. There are n buildings on one side of the street, you need to design an algorithm to output their skyline. The time complexity limit is $O(n \log n)$.

4: (5') Finding Majority

Suppose you're a TA for the CS101 course, and you need to help the TA group solve the following problem. They have a collection of n students' code solutions for the programming task, suspecting them of academic plagiarism. It's difficult to judge whether two code solutions are equivalent, but the TA group has a high-tech "plagiarism detection machine" that takes two code solutions and determines whether they are equivalent after performing some operations.

Their question is the following: among the collection of n code solutions, is there a set of more than $n/2$ of them that are all equivalent to one another? Assume that the only feasible operations you can do with these code solutions are to pick two of them and plug them into the plagiarism detection machine. Show that, with the help of the plagiarism detection machine, how to decide the answer to their question with only $O(n \log n)$ invocations of the machine.

5: (5') Polynomial Multiplication

Design an efficient algorithm for the following problem:
Input: n numbers $\{a_1, \cdots, a_n\}$
Goal: Compute the polynomial with $\{a_1, \cdots, a_n\}$ as its roots. In other words, compute coefficients $\{b_0, \cdots, b_n\}$ so that $(x - a_1) \cdot (x - a_2) \cdots (x - a_n) = b_0 + b_1 x + \cdots + b_n x^n$.
(Hint: Try divide and conquer & use $O(nlogn)$ time **polynomial multiplication algorithm** as a blackbox)

Describe your algorithm(3'), write the recurrence for the running time(1') and solve the recurrence to compute the time complexity(Do not only write the answer)(2').