HW6

Problem 1 Shown in **Figure 1**(a) is the frequency response $H(j\omega)$ of a continuous-time filter referred to as a lowpass differentiator. For each of the input signals $x(t)$ below, determine the filtered output signal $y(t)$.

(a) $x(t) = cos(2\pi t + \theta)$

(b) $x(t) = cos(4\pi t + \theta)$

(c) $x(t)$ is a half-wave rectified sine wave of period, as sketched in **Figure 1**(b).

$$x(t) = \begin{cases} sin(2\pi t), & m \leq t \leq (m + \frac{1}{2}) \\ 0, & (m + \frac{1}{2}) \leq t \leq m \text{ for any integer } m \end{cases}$$

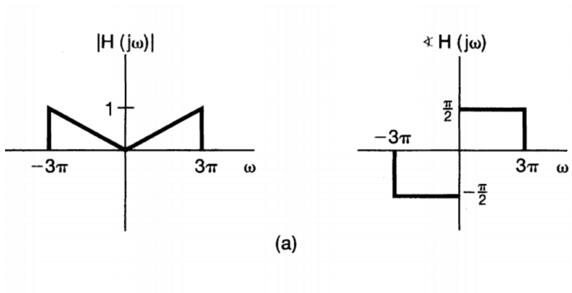

Figure 1(a)

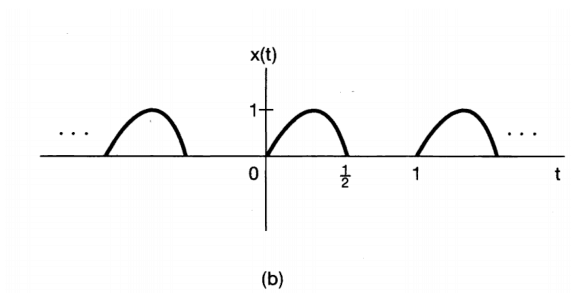

Figure 1(b)

Problem 2 The output $y(t)$ of a causual LTI system is related to the input $x(t)$ by the differential equation

$$\frac{dy(t)}{dt} + 2y(t) = x(t)$$

(a) Determine the frequency response

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)}$$

of the system, and sketch its Bode plot.

(b) Specify, as a function of frequency, the group delay associated with this system.

(c) If the input has its Fourier transform as follows, determine $Y(j\omega)$(the Fourier transform of the output) and the output $y(t)$

(i) $X(j\omega) = \frac{1+j\omega}{2+j\omega}$

(ii) $X(j\omega) = \frac{2+j\omega}{1+j\omega}$

(iii) $X(j\omega) = \frac{1}{(2+j\omega)(1+j\omega)}$

Problem 3 Consider the discrete-time sequency x[n] =cos[nπ/5]. Find two different continous-time signals that would produce this squence when sampled at a frequency of fs = 500 Hz.

Problem 4 Shown in **Figure 4** is a system in which the sampling signal is an impulse train with alternating sign. The Fourier transform of the input signal is as indicated in the figure.

(a) For $\Delta < \pi/(2\omega_M)$, sketch the Fourier transform of $x_p(t)$ and $y(t)$.

(b) For $\Delta < \pi/(2\omega_M)$, determine a system that will recover $x(t)$ from $x_p(t)$.

(c) For $\Delta < \pi/(2\omega_M)$, determine a system that will recover $x(t)$ from $y(t)$.

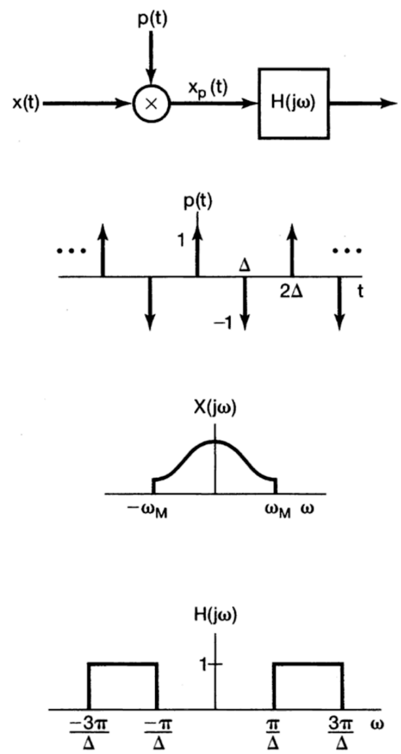

Figure 4

Problem 5 Consider the system shown in **Figure 5**.

Assume that the input is bandlimited, $X_a(\omega) = 0$ for $|\omega| > 2\pi \cdot 1000$.

(a) What constraints must be placed on M, T_1 and T_2 in order for $y_a(t)$ to be equal to $x_a(t)$?

(b) If $f_1 = f_2 = 20\text{kHz}$ and $M = 4$, find an expression for $y_a(t)$ in terms of $x_a(t)$.

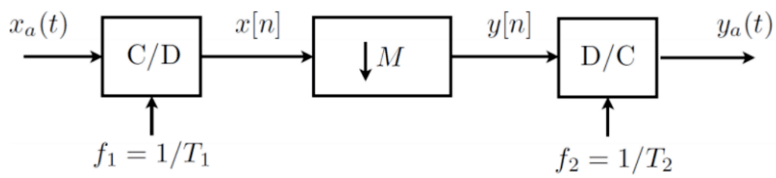

Figure 5