CS 101 Fall 2020 - Quiz 2-B

10/10/2020 - 20 Minutes **Name:** **ID number:**

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(1) (8 Points) Here is a sorting algorithm in the following.

```
Procedure Sort(A):
  for i = 1 to A.length - 1:
      for j = A.length downto i + 1:
          if A[j] < A[j - 1]
              key = A[j]
              A[j] = A[j - 1]
              A[j - 1] = key
      // Mark
```

- (3 Points) Which sorting algorithm does it describe?
- (5 Points) Given a list as `[31, 4, 59, 26, 41, 58]`, we use the above procedure to sort it. Write down what will the list be like each time when the procedure meets the `Mark`.

- Bubble Sort.
- `[4, 31, 26, 59, 41, 58]`
  `[4, 26, 31, 41, 59, 58]`
  `[4, 26, 31, 41, 58, 59]`
  `[4, 26, 31, 41, 58, 59]`
  `[4, 26, 31, 41, 58, 59]`

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(2) (7 Points) Suppose that we have a hash table with n slots, with collisions resolved by chaining, and suppose that n keys are inserted into the table. Each key is equally likely to be hashed to each slot. Let M be the maximum number of keys in any slot after all keys have been inserted.

- (3 Points) Calculate the probability Q_k that exactly k keys hash to a particular slot.
- (4 Points) Let P_k be the probability that $M = k$, that is, the probability that the slot containing the most keys contains k keys. Show that $P_k \le nQ_k$.

- Suppose we select a specific set of k keys. The probability that these k keys are inserted into the particular slot and that all other keys are inserted elsewhere is

$$\left(\frac{1}{n}\right)^k \left(1 - \frac{1}{n}\right)^{n-k}$$

Since there are $\binom{n}{k}$ ways to choose k keys, we get

$$Q_k = \left(\frac{1}{n}\right)^k \left(1 - \frac{1}{n}\right)^{n-k} \binom{n}{k}$$

- Let X_i be the number of keys that hash to slot i, and A_i be the event that $X_i = k$, i.e., that exactly k keys hash to slot i. From the above we have $\mathsf{Pr}\{\mathsf{A_i}\} = Q_k$. Then,

$$\begin{aligned}
P_k &= \mathsf{Pr}\{M = k\} \\
&= \mathsf{Pr}\left\{\max_i X_i = k\right\} \\
&\le \mathsf{Pr}\left\{\exists i, X_i = k\right\} \\
&= \mathsf{Pr}\left\{A_1 \cup A_2 \cup \cdots \cup A_n\right\} \\
&\le \mathsf{Pr}\left\{A_1\right\} + \mathsf{Pr}\left\{A_2\right\} + \cdots + \mathsf{Pr}\left\{A_n\right\} \\
&= nQ_k
\end{aligned}$$