

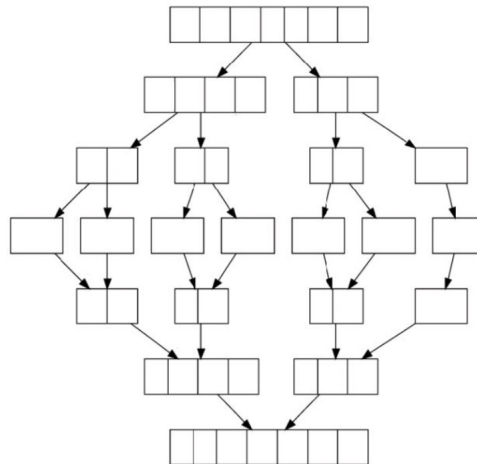
(8 Points) Problem 1: True or False: For each statement, write “T” if this statement is correct; write “F” otherwise. Please write your answers in the box below.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)

- (1) Merge sort requires $O(n)$ space complexity.
- (2) In quick sort (with n distinct elements and sort in ascending order, $n \geq 2$), if we randomly select the pivot, after the first partition operation, the smallest element of the array can be anywhere.
- (3) Quick sort algorithm will have $O(n^2)$ time complexity in the worst case.
- (4) Insertion sort never compares the same two elements twice.
- (5) Quick sort runs in best case $\Theta(\log n)$ time for certain inputs.
- (6) Randomly choosing pivots can be used to reduce the probability of quick sort taking the worst case running time.
- (7) Merge sort has a worst case runtime that is asymptotically better than quick sort's worst case runtime.
- (8) Insertion sort (ascending) has the best time complexity on an already sorted list among all sorting methods.

(6 Points) Problem 2: Consider this array: 7, 6, 4, 2, 5, 3, 1.

- (1) (4 pts) Use **mergesort** to sort this array in ascending order. Show your process in the following figure.
- (2) (2 pts) How many inversions are there in the array?



(3 Points) Problem 3:

Tom wants to sort his favorite colors in ascending order using quicksort. The original array is:

red, cyan, yellow, gray, green, black, blue, white

After the first partitioning step, it becomes: (“red” is chosen as pivot)

white, cyan, yellow, gray, red, black, green, blue

Known that NO elements are equal, we can infer that: (Fill the blanks with “>”, “<”, or “?” if given information is insufficient to judge)

- (a) red____blue (b) yellow____gray (c) green____cyan

(8 Points) Problem 4: The following is a pseudo code

```
// ASSUMPTION: ALL ELEMENTS IN ARR[] ARE DISTINCT
int function(int arr[], int left, int right, int k)
{
    // If k is smaller than number of elements in array
    if (k > 0 && k <= right - left + 1) {
        // partition function moves all elements smaller than pivot to left of it
        // and greater elements to right in O(right - left + 1) time
        // and return the final position of the pivot
        int pos = partition(arr, left, right);
        // If position is same as k
        if (pos - left == k - 1)
            return arr[pos];
        // If position is bigger, recur for left subarray
        if (pos - left > k - 1)
            return function(arr, left, pos - 1, k);
        // Else recur for right subarray
        return function(arr, pos + 1, right, k - pos + left - 1);
    }
    // If k is bigger than number of elements in array, there is no correct result
    return INT_MAX;
}
```

- (3 Points) What is the output when $arr = [4, 3, 2, 5, 1]$, $left = 0$, $right = 4$, $k = 4$?
- (3 Points) What does the function intend to do?
- (2 Points) Analyze the **worst** time complexity of the function above. Please explain your answer.