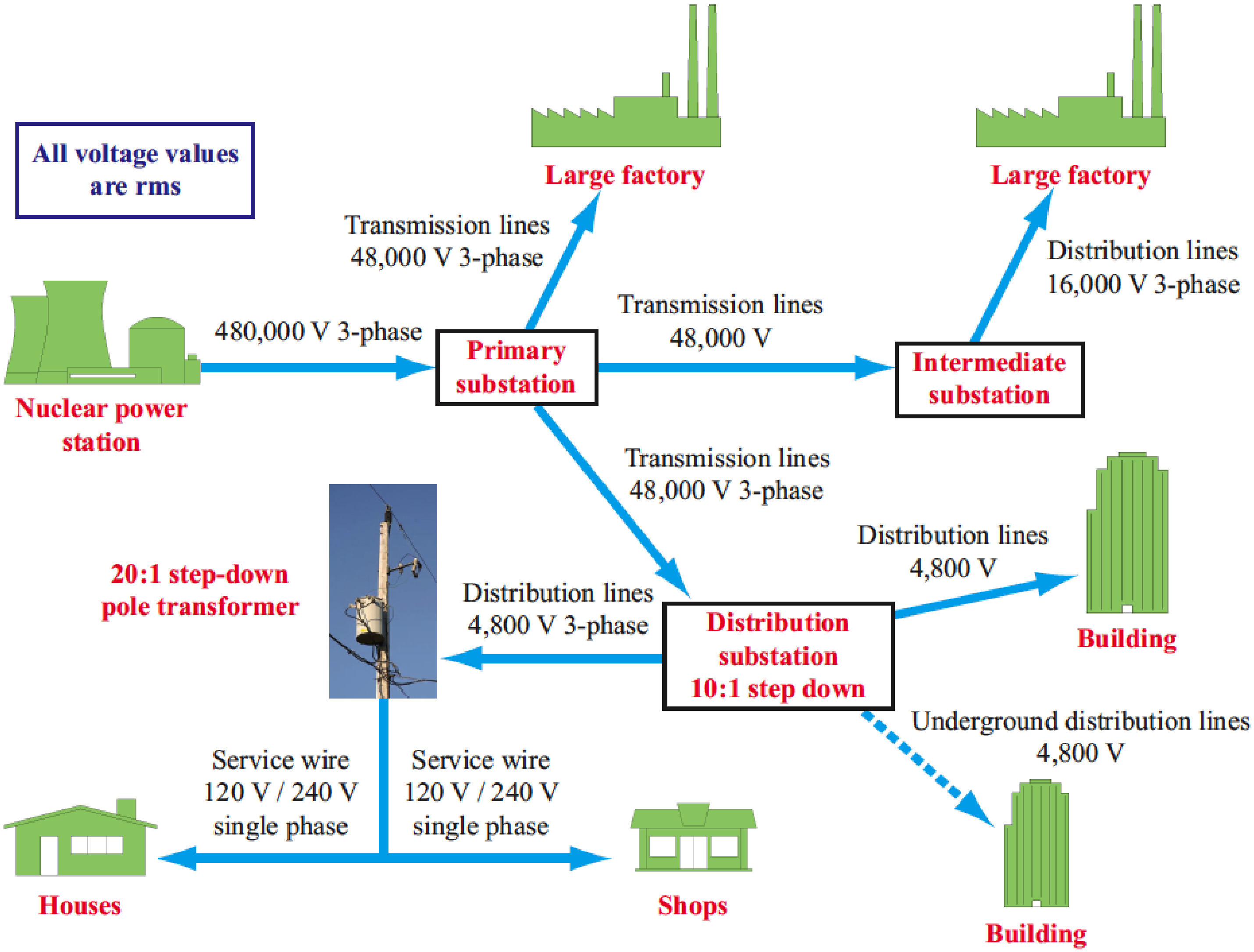
Lecture 10

- Three-Phase Circuits/Transformers

Outline--Three-Phase Circuits

- Balanced Three-Phase System
  - Balanced sources
  - Balanced loads
- Circuit analysis
  - Phase voltage/current
  - Line voltage/current

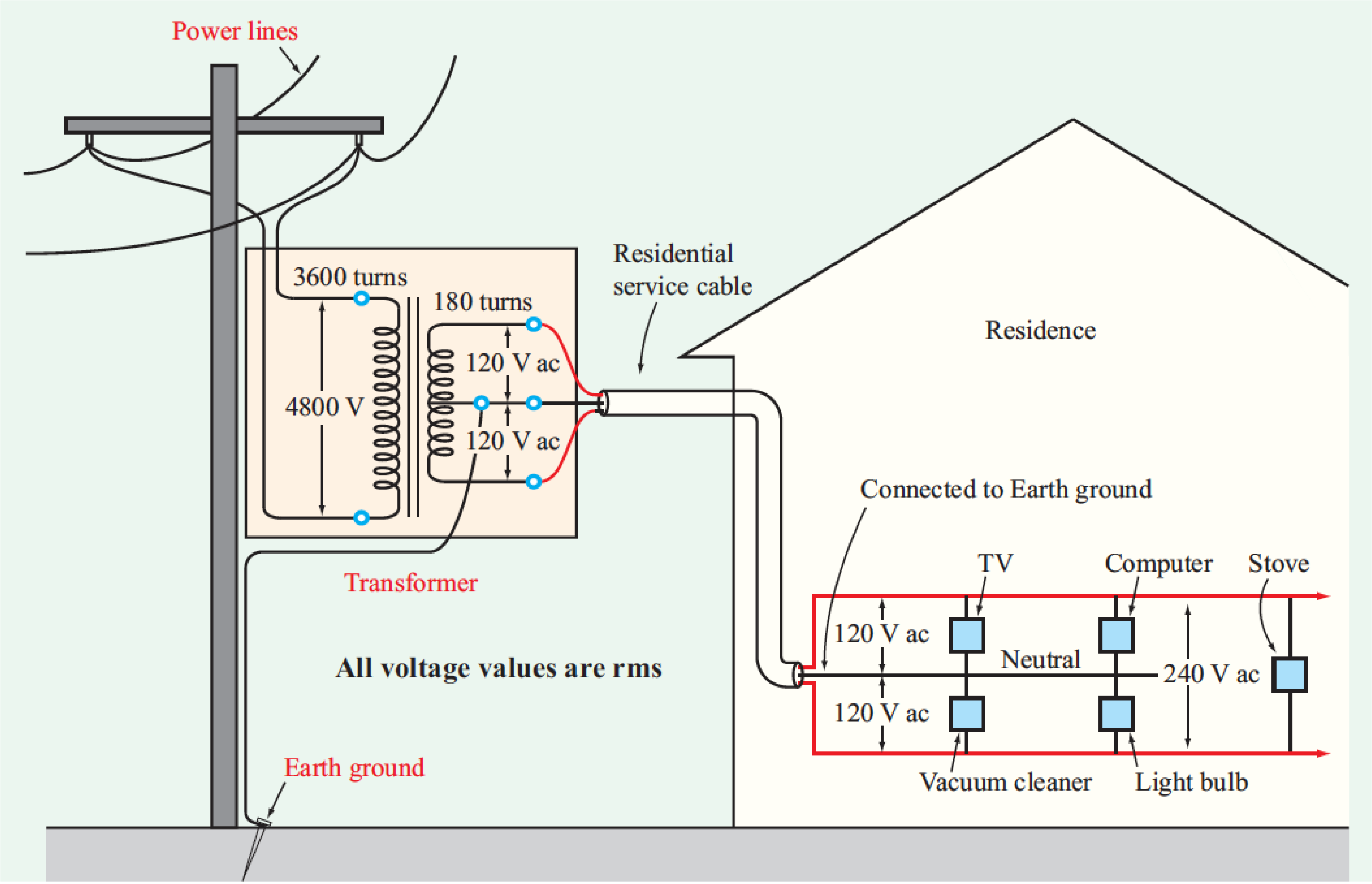
Three-Phase System (in USA)



[Source: Berkeley]

Figure 10-1: Typical electrical power grid.

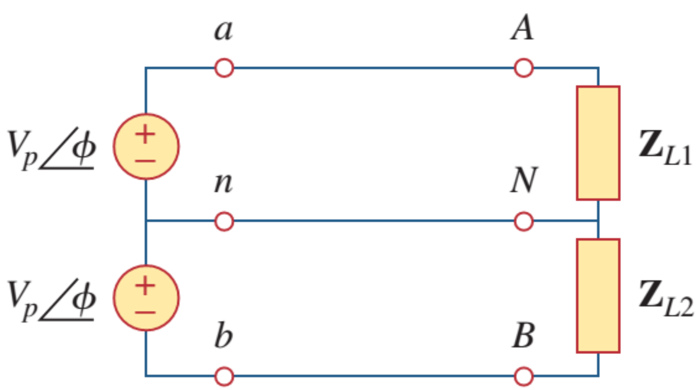
A 4800-V rms single-phase connected to residential user through a 20 : 1 step-down transformer

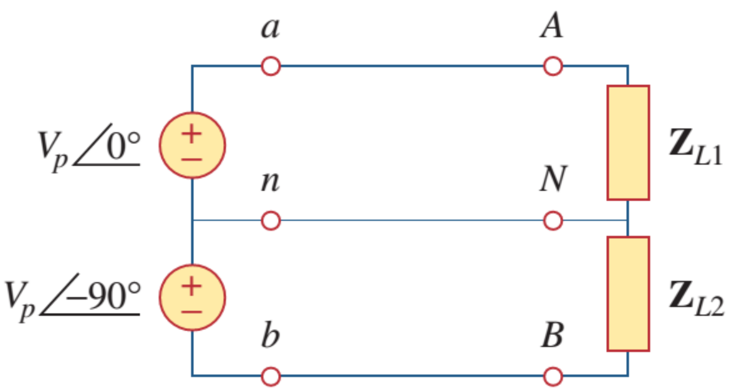


[Source: Berkeley]

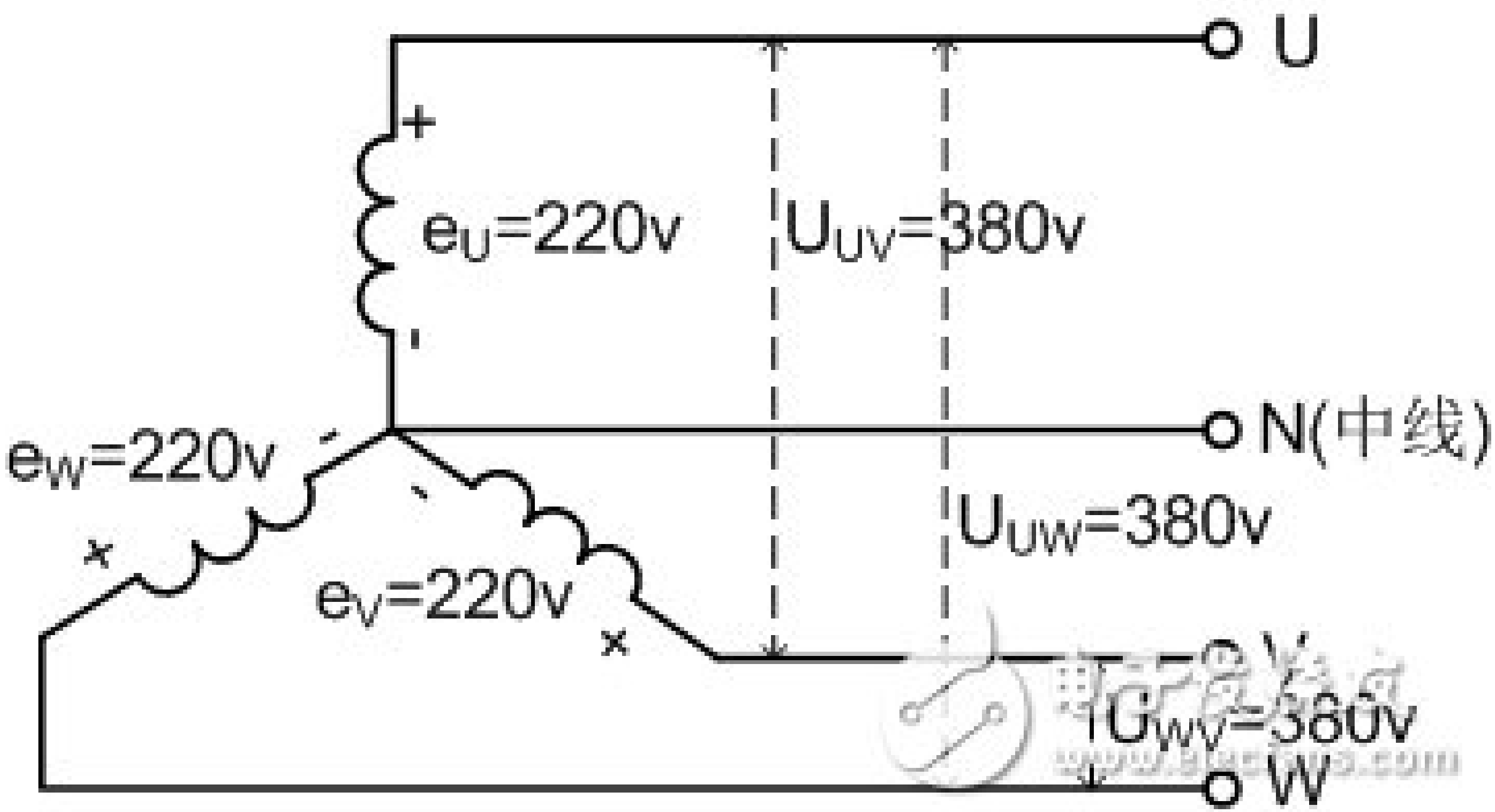
Single Phase vs. Polyphase

- Households have single-phase power supply
  - This typically in a three wire form, where two 120V sources with the same phase are connected in series.
  - This allows for appliances to use either 120 or 240V
- Circuits that operate at the same frequency but with multiple sources at different phases are called <u>polyphase</u>.



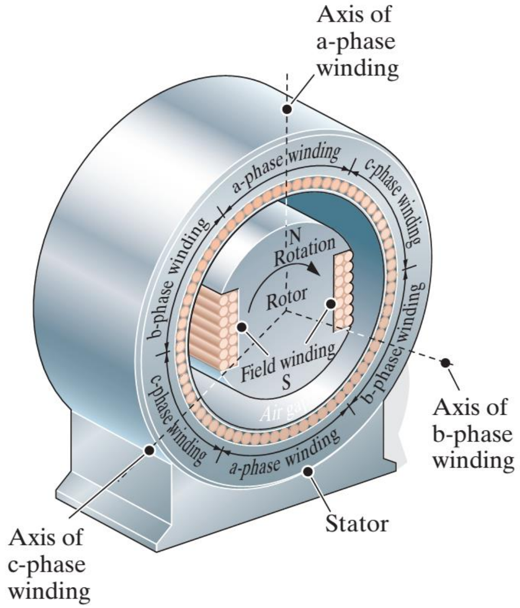


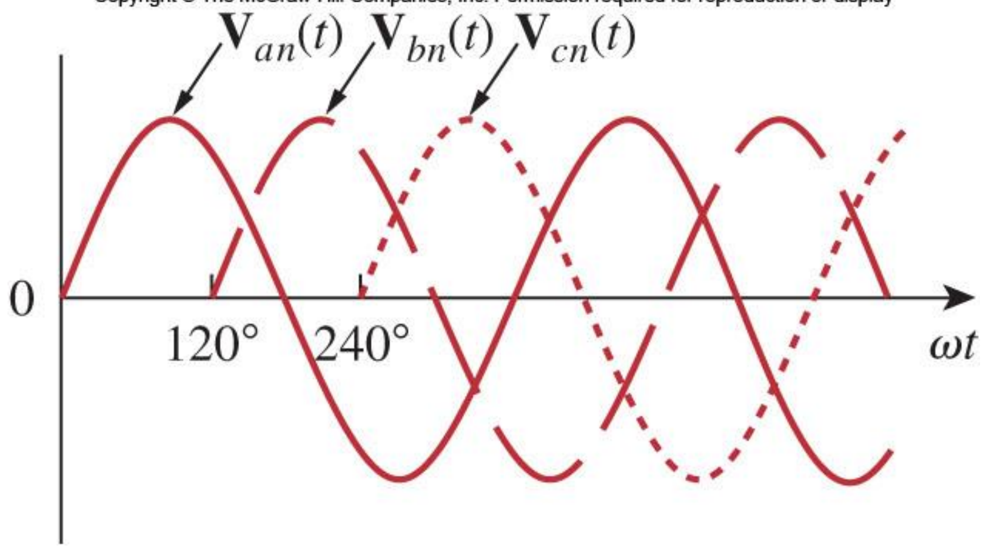
Three-phase four-wire system in China

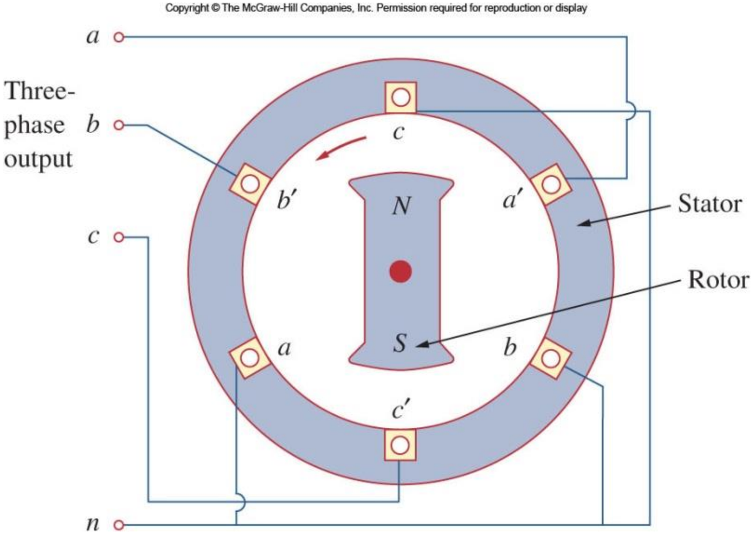


Three-Phase Sources

- Three phase voltages are typically produced by a three-phase AC generator.
- The output voltages look like below.

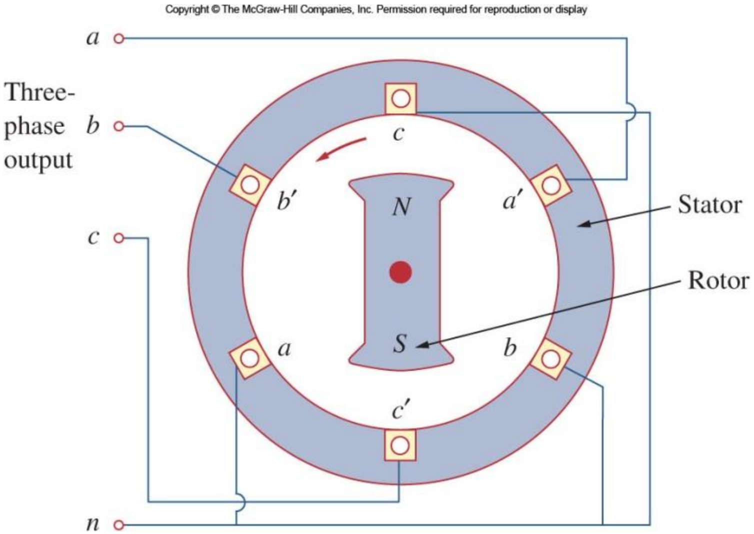




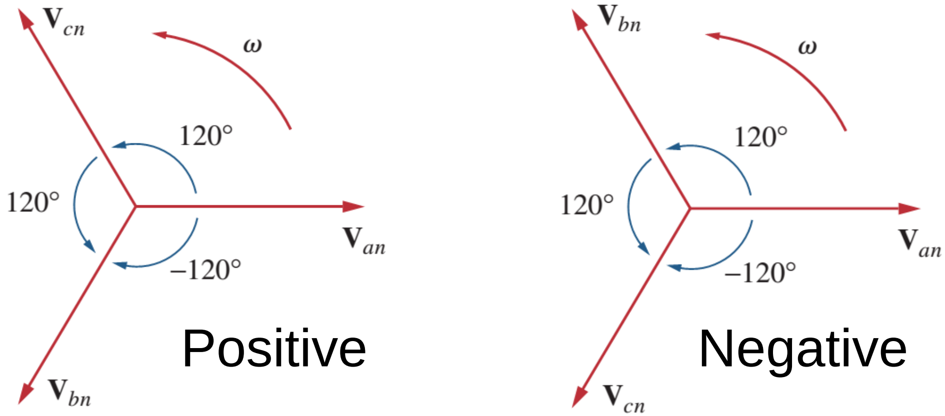


Balanced Three-Phase Sources

- Balanced phase voltage are equal in magnitude and are out of phase with each other by 120deg
- It's easy to know $V_{an}+V_{bn}+V_{cn}=0$
- Two sequences for the phases:



$$|V_{an}|=|V_{bn}|=|V_{cn}|$$



Positive

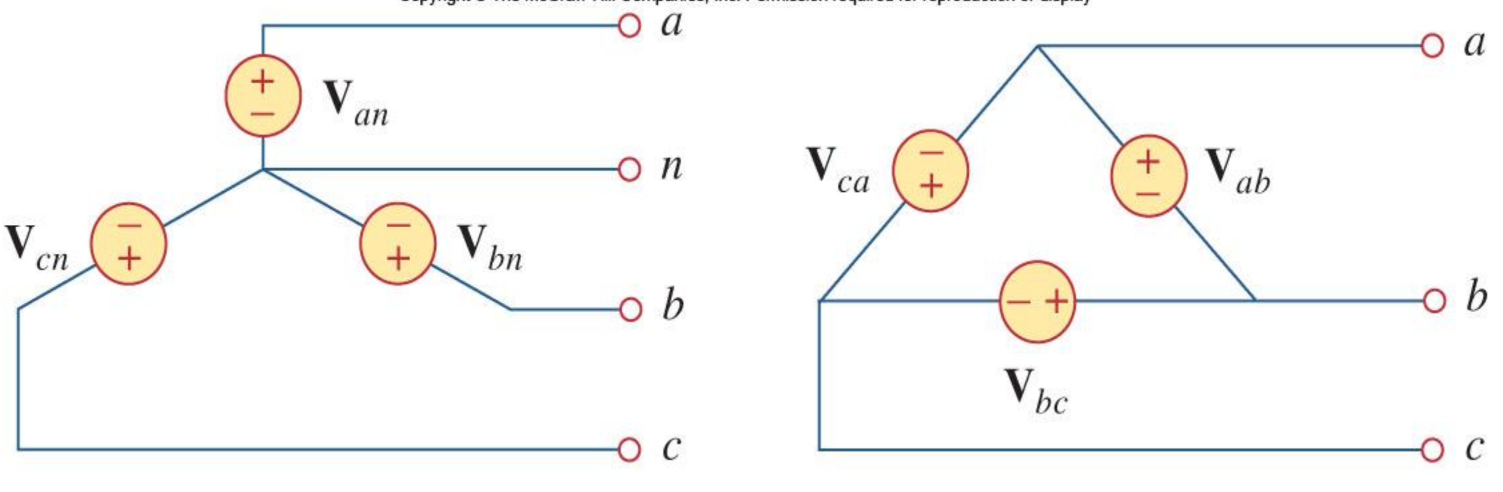
$$V_{an}=V_p\angle 0^\circ$$
$$V_{bn}=V_p\angle -120^\circ$$
$$V_{cn}=V_p\angle -240^\circ=V_p\angle +120^\circ$$

Negative

$$V_{an}=V_p\angle 0^\circ$$
$$V_{cn}=V_p\angle -120^\circ$$
$$V_{bn}=V_p\angle -240^\circ=V_p\angle +120^\circ$$

Connecting the Sources

- Three phase voltage sources can be connected by either three or four wire configurations.
  - Three-wire configuration accomplished by Delta connected source.
  - Four-wire system accomplished using a Y(Wye) connected source.



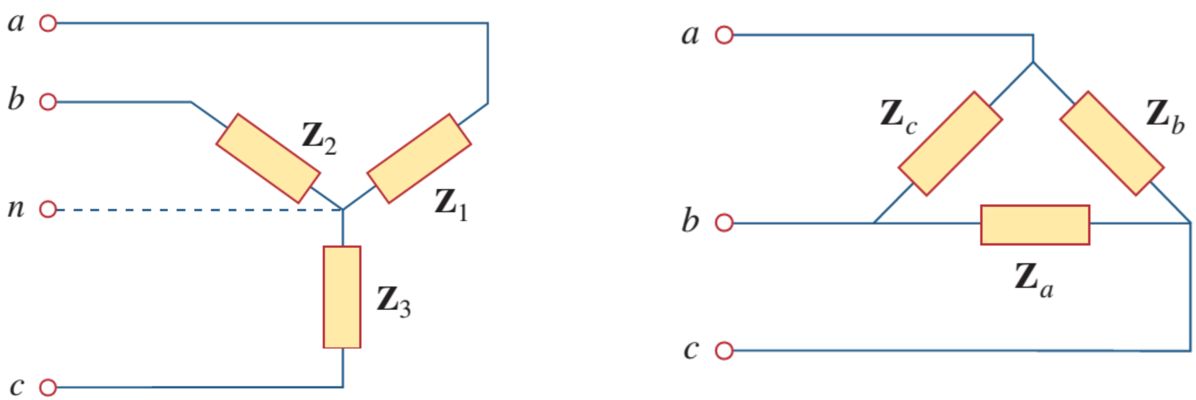
(a) (b)

Balanced Loads

- A <u>balanced</u> load is one that has the same impedance presented to all three voltage sources.

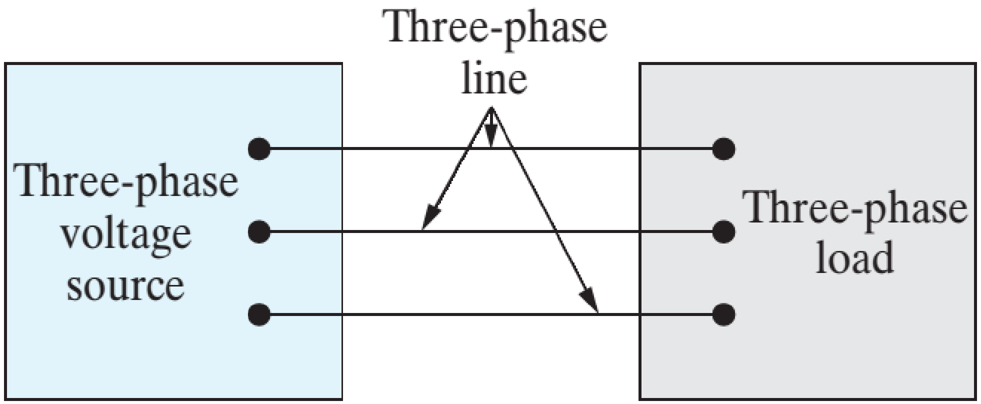
-- *Phase impedance are equal in magnitude and in phase*

- They may also be connected in either Delta or wye
  - For a balanced wye connected load: $Z_1 = Z_2 = Z_3 = Z_Y$
  - For a balanced delta connected load: $Z_a = Z_b = Z_c = Z_\Delta$



- The load impedance per phase for the two load configurations can be interchanged.

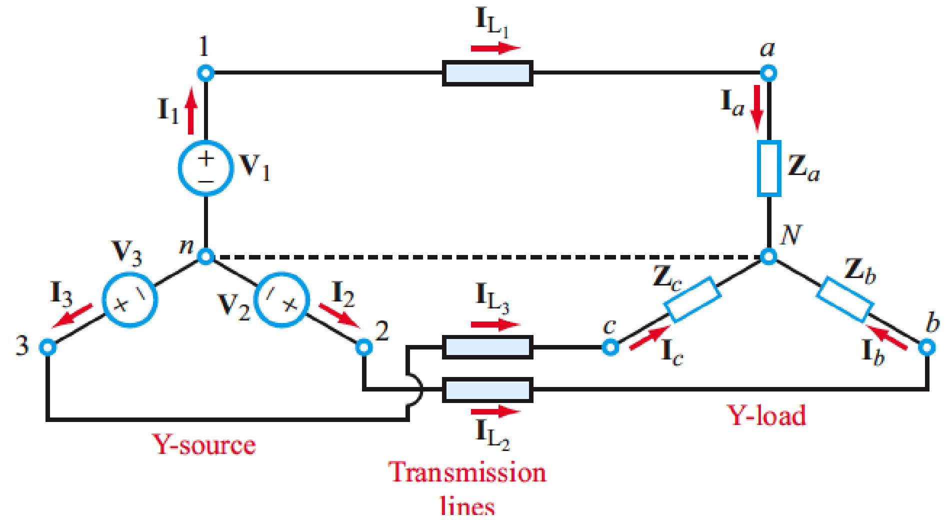
Source-Load configurations



Source	Load
Y	Y
Y	Δ
Δ	Y
Δ	Δ

Source-Load Configurations

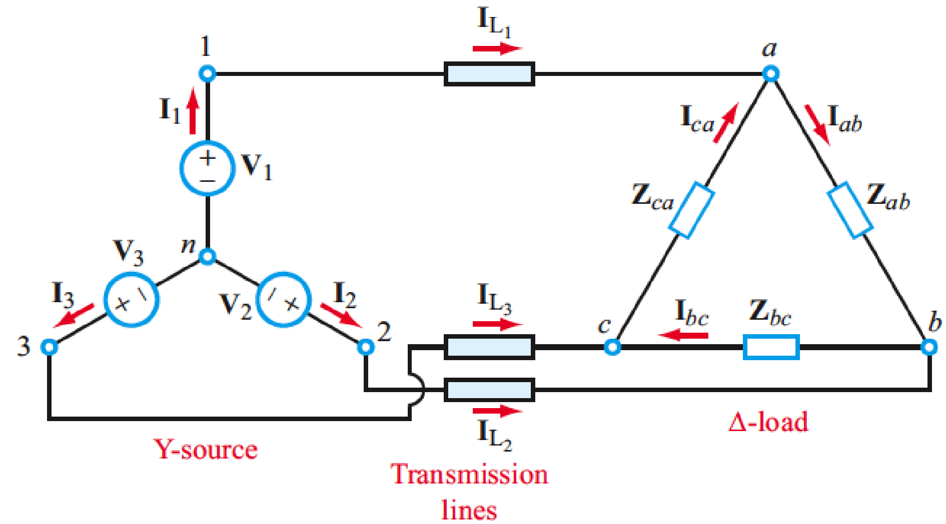
Y-Y



Load Phase Currents
$\mathbf{I}_a, \mathbf{I}_b, \mathbf{I}_c$
(same as line currents
$\mathbf{I}_{L_1}, \mathbf{I}_{L_2}$, and $\mathbf{I}_{L_3}$)

Load Phase Voltages
$\mathbf{V}_{aN}, \mathbf{V}_{bN}, \mathbf{V}_{cN}$

Y-Delta

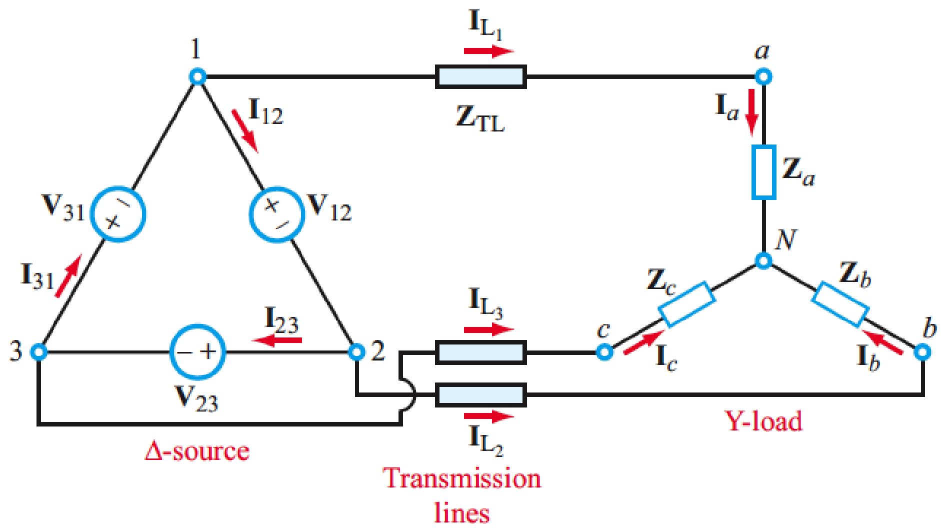


Load Phase Currents
$\mathbf{I}_{ab}, \mathbf{I}_{bc}, \mathbf{I}_{ca}$

Load Phase Voltages
$\mathbf{V}_{ab}, \mathbf{V}_{bc}, \mathbf{V}_{ca}$

Source-Load Configurations

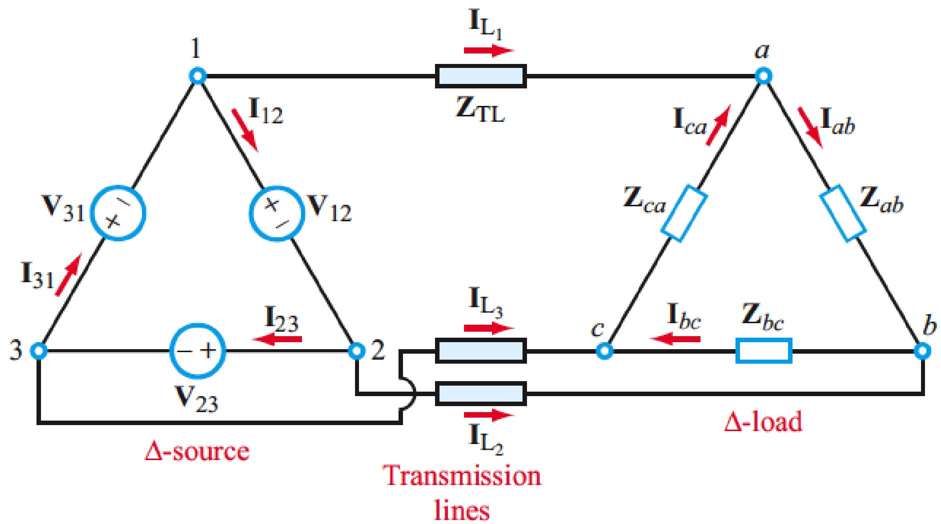
Delta-Y



Load Phase Currents
$\mathbf{I}_a, \mathbf{I}_b, \mathbf{I}_c$
(same as line currents
$\mathbf{I}_{L_1}, \mathbf{I}_{L_2}$, and $\mathbf{I}_{L_3}$)

Load Phase Voltages
$\mathbf{V}_{aN}, \mathbf{V}_{bN}, \mathbf{V}_{cN}$

Delta-Delta



Load Phase Currents
$\mathbf{I}_{ab}, \mathbf{I}_{bc}, \mathbf{I}_{ca}$

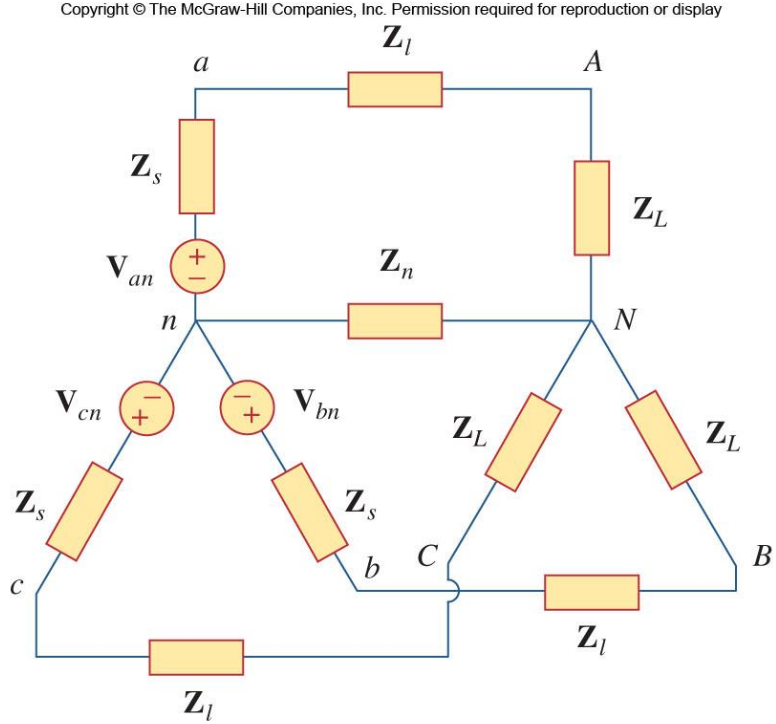
Load Phase Voltages
$\mathbf{V}_{ab}, \mathbf{V}_{bc}, \mathbf{V}_{ca}$
(same as source voltages
if $\mathbf{Z}_{TL}$ is negligible)

Balanced Y-Y connection

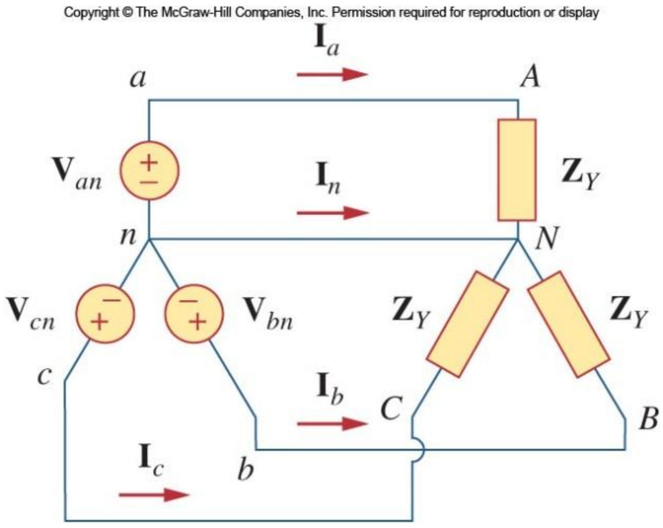
- Any three-phase system can be reduced to an equivalent Y-Y system.
- The load impedances Z_Y will be assumed to be balanced.
  - This can be the source Z_s, line Z_l and load Z_L together.

$$\mathbf{Z}_Y = \mathbf{Z}_s + \mathbf{Z}_\ell + \mathbf{Z}_L$$

  - How about Z_n ?



Line-to-Line Voltage

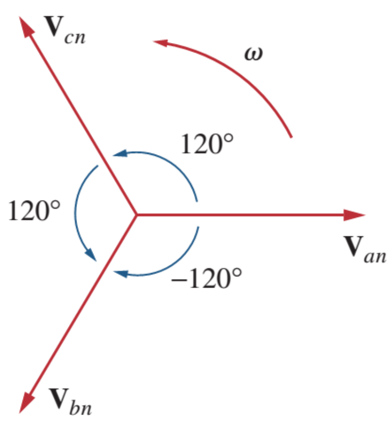


- Use the positive sequence:

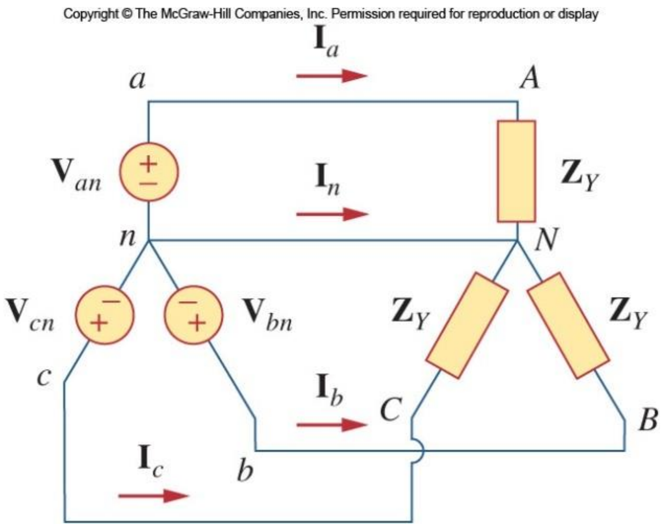
$$V_{an} = V_p \angle 0^\circ$$

$$V_{bn} = V_p \angle -120^\circ \quad V_{cn} = V_p \angle +120^\circ$$

The line to line (or line in short) voltages:

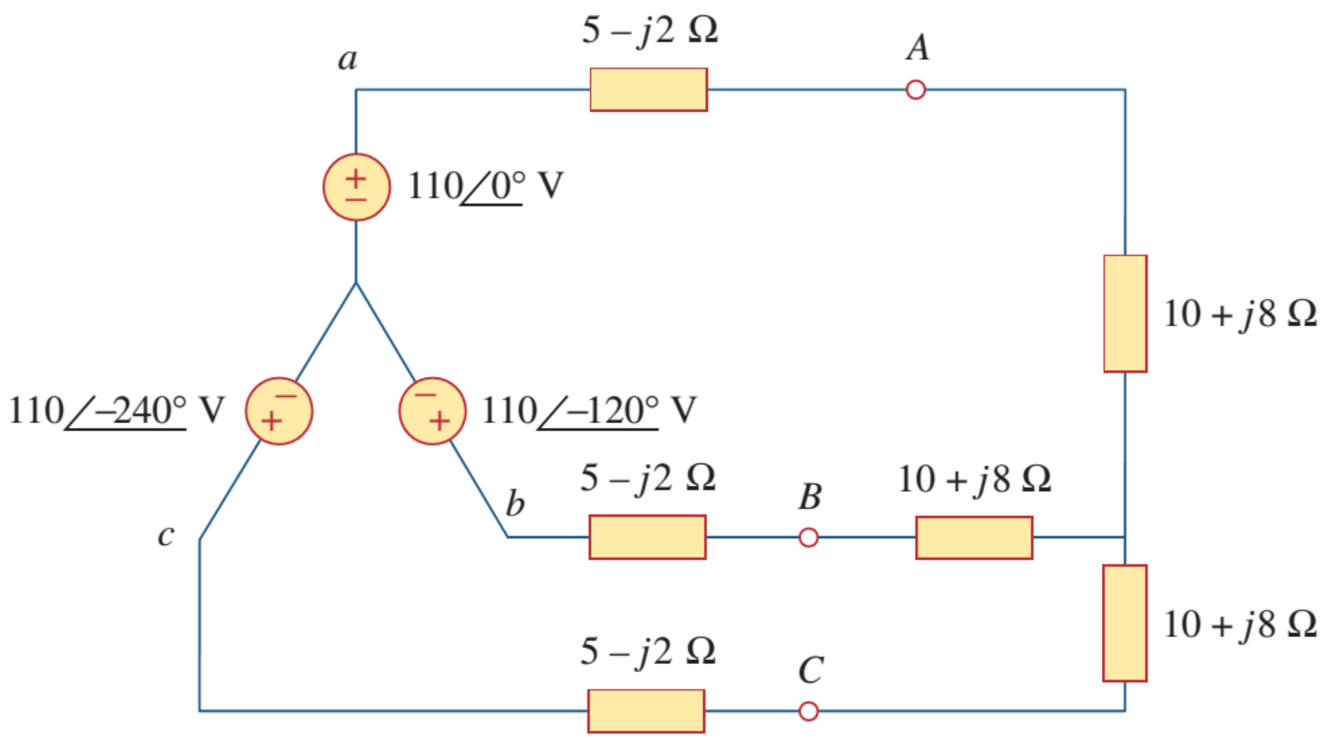


Line Currents

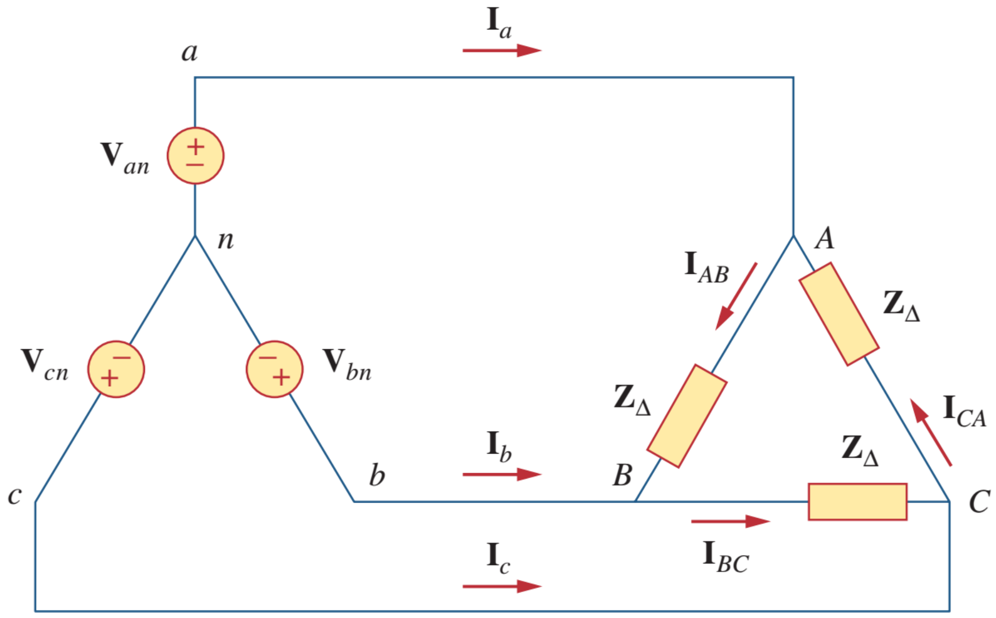


Example

- Calculate the line currents.



Wye-Δ



$$\mathbf{V}_{an} = V_p\angle 0^\circ$$

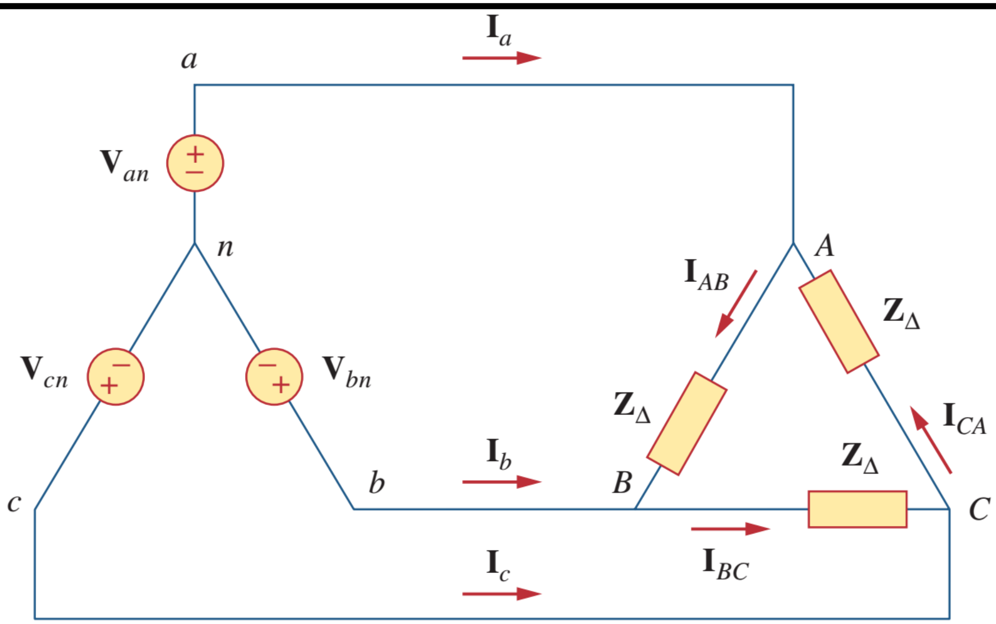
$$\mathbf{V}_{bn} = V_p\angle -120^\circ, \qquad \mathbf{V}_{cn} = V_p\angle +120^\circ$$

$$\mathbf{V}_{ab} = \sqrt{3}V_p\angle 30^\circ = \mathbf{V}_{AB}, \qquad \mathbf{V}_{bc} = \sqrt{3}V_p\angle -90^\circ = \mathbf{V}_{BC}$$

$$\mathbf{V}_{ca} = \sqrt{3}V_p\angle -150^\circ = \mathbf{V}_{CA}$$

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_\Delta}, \qquad \mathbf{I}_{BC} = \frac{\mathbf{V}_{BC}}{\mathbf{Z}_\Delta}, \qquad \mathbf{I}_{CA} = \frac{\mathbf{V}_{CA}}{\mathbf{Z}_\Delta}$$

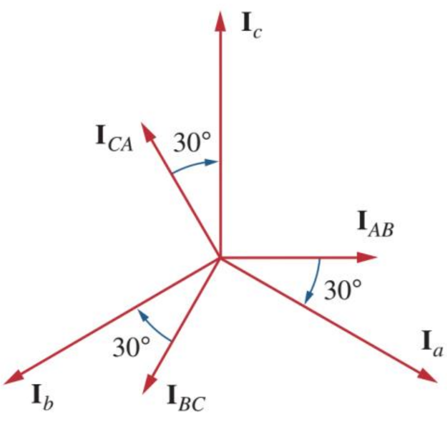
Wye-Δ



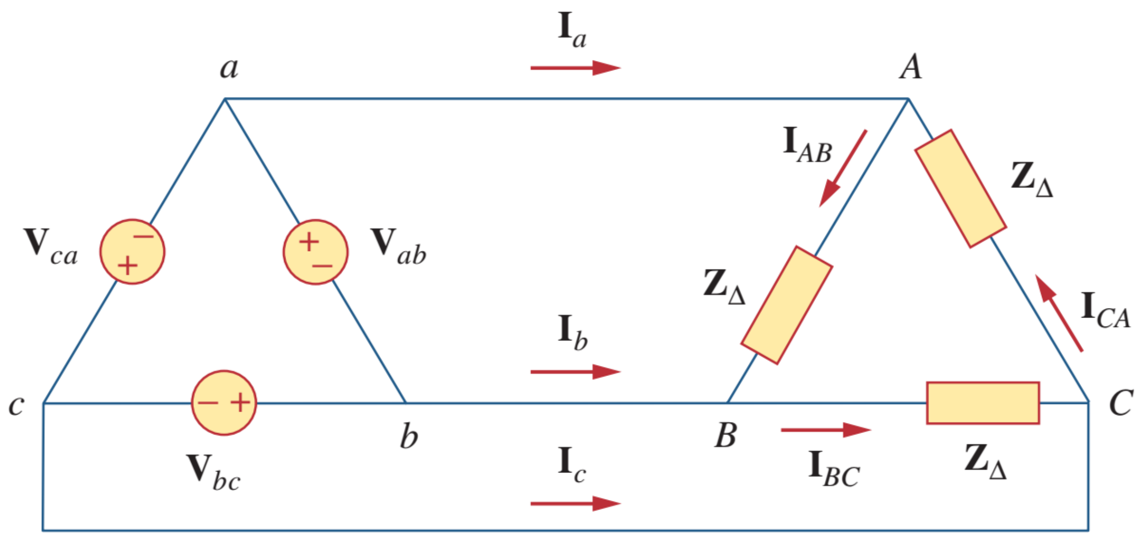
$$\mathbf{I}_a = \mathbf{I}_{AB} - \mathbf{I}_{CA}, \qquad \mathbf{I}_b = \mathbf{I}_{BC} - \mathbf{I}_{AB}, \qquad \mathbf{I}_c = \mathbf{I}_{CA} - \mathbf{I}_{BC}$$

Since $\mathbf{I}_{CA} = \mathbf{I}_{AB} \angle -240^\circ$,

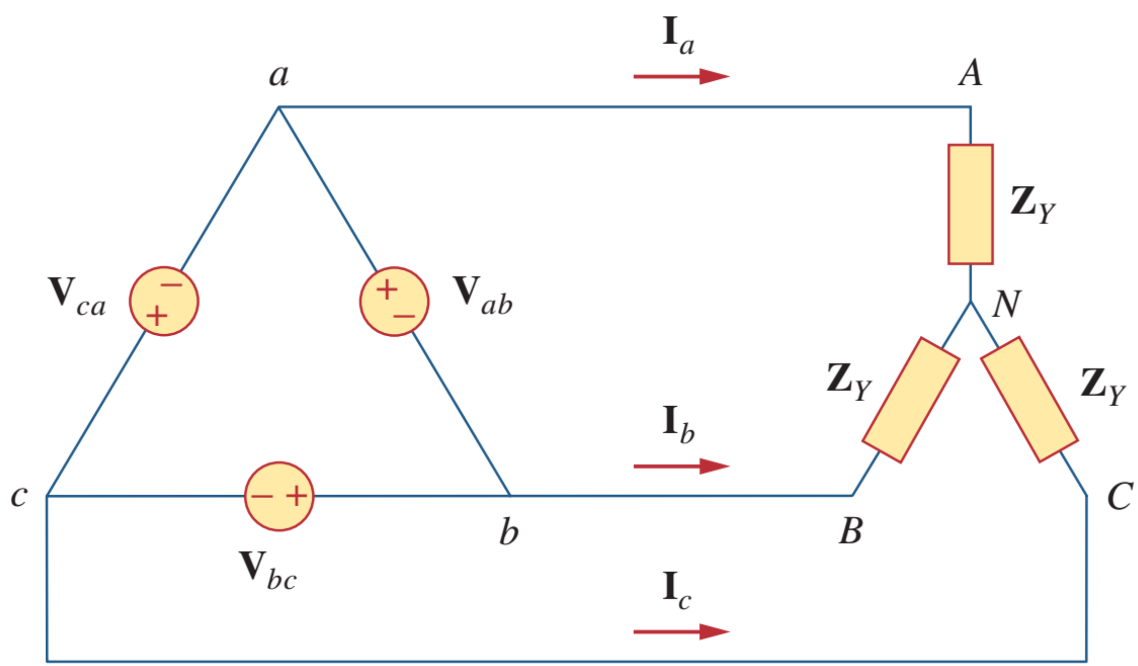
$$\begin{aligned}\mathbf{I}_a &= \mathbf{I}_{AB} - \mathbf{I}_{CA} = \mathbf{I}_{AB}(1 - 1\angle -240^\circ) \\ &= \mathbf{I}_{AB}(1 + 0.5 - j0.866) = \mathbf{I}_{AB}\sqrt{3}\angle -30^\circ\end{aligned}$$



- Δ-Δ


I_a
a
A
I_AB
Z_Δ
V_ca
V_ab
Z_Δ
I_CA
I_b
c
C
b
B
V_bc
I_c
I_BC
Z_Δ

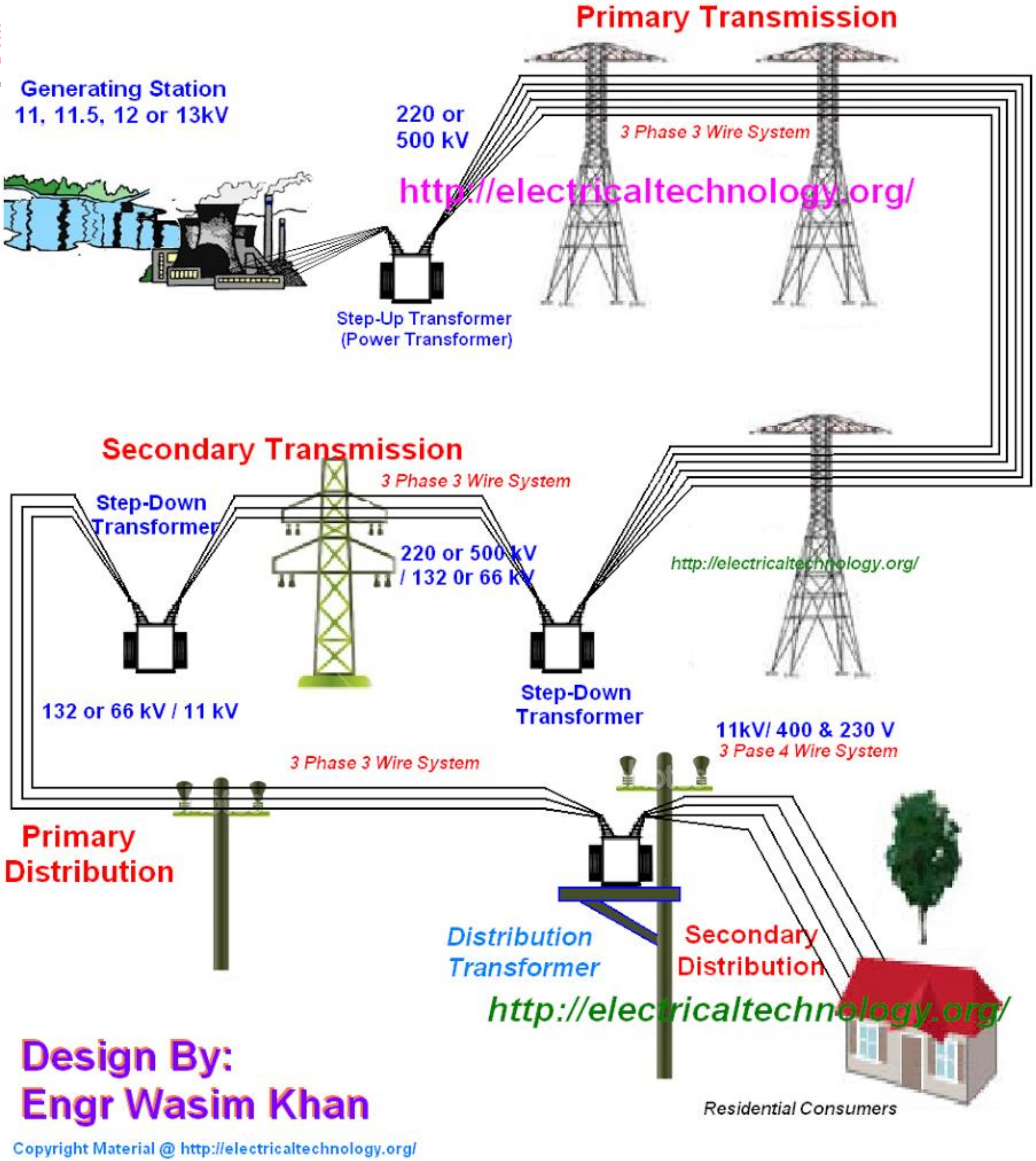
Δ-Wye


I_a
a
A
Z_Y
V_ca
V_ab
N
Z_Y
Z_Y
I_b
c
C
b
B
V_bc
I_c

Connection	Phase voltages/currents	Line voltages/currents
Y-Y	$\mathbf{V}_{an} = V_p\angle 0^\circ$ $\mathbf{V}_{bn} = V_p\angle -120^\circ$ $\mathbf{V}_{cn} = V_p\angle +120^\circ$ Same as line currents	$\mathbf{V}_{ab} = \sqrt{3}V_p\angle 30^\circ$ $\mathbf{V}_{bc} = \mathbf{V}_{ab}\angle -120^\circ$ $\mathbf{V}_{ca} = \mathbf{V}_{ab}\angle +120^\circ$ $\mathbf{I}_a = \mathbf{V}_{an}/\mathbf{Z}_Y$ $\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ$ $\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ$
Y-Δ	$\mathbf{V}_{an} = V_p\angle 0^\circ$ $\mathbf{V}_{bn} = V_p\angle -120^\circ$ $\mathbf{V}_{cn} = V_p\angle +120^\circ$ $\mathbf{I}_{AB} = \mathbf{V}_{AB}/\mathbf{Z}_\Delta$ $\mathbf{I}_{BC} = \mathbf{V}_{BC}/\mathbf{Z}_\Delta$ $\mathbf{I}_{CA} = \mathbf{V}_{CA}/\mathbf{Z}_\Delta$	$\mathbf{V}_{ab} = \mathbf{V}_{AB} = \sqrt{3}V_p\angle 30^\circ$ $\mathbf{V}_{bc} = \mathbf{V}_{BC} = \mathbf{V}_{ab}\angle -120^\circ$ $\mathbf{V}_{ca} = \mathbf{V}_{CA} = \mathbf{V}_{ab}\angle +120^\circ$ $\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle -30^\circ$ $\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ$ $\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ$
Δ-Δ	$\mathbf{V}_{ab} = V_p\angle 0^\circ$ $\mathbf{V}_{bc} = V_p\angle -120^\circ$ $\mathbf{V}_{ca} = V_p\angle +120^\circ$ $\mathbf{I}_{AB} = \mathbf{V}_{ab}/\mathbf{Z}_\Delta$ $\mathbf{I}_{BC} = \mathbf{V}_{bc}/\mathbf{Z}_\Delta$ $\mathbf{I}_{CA} = \mathbf{V}_{ca}/\mathbf{Z}_\Delta$	Same as phase voltages $\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle -30^\circ$ $\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ$ $\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ$
Δ-Y	$\mathbf{V}_{ab} = V_p\angle 0^\circ$ $\mathbf{V}_{bc} = V_p\angle -120^\circ$ $\mathbf{V}_{ca} = V_p\angle +120^\circ$ Same as line currents	Same as phase voltages $\mathbf{I}_a = \dfrac{V_p\angle -30^\circ}{\sqrt{3}\mathbf{Z}_Y}$ $\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ$ $\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ$

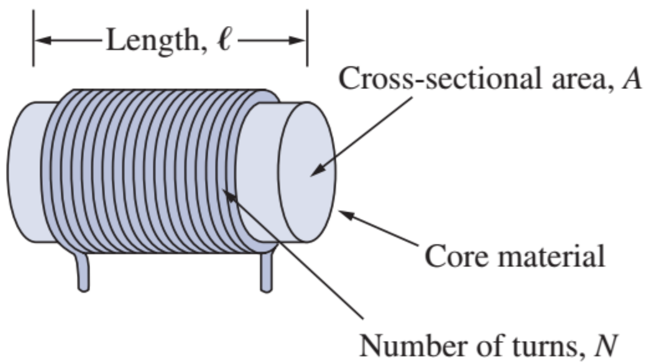
Outline-Transformers

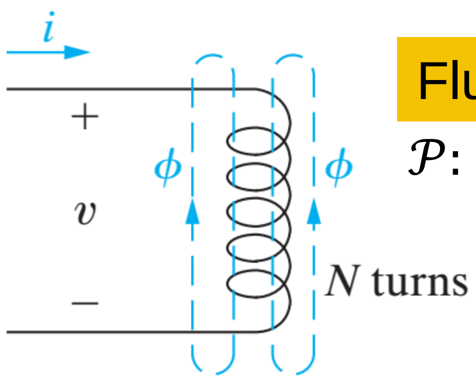
- Mutual inductance (review)
- Transformers


Primary Transmission
Generating Station
11, 11.5, 12 or 13kV
220 or
500 kV
3 Phase 3 Wire System
http://electricaltechnology.org/
Step-Up Transformer
(Power Transformer)
Secondary Transmission
3 Phase 3 Wire System
Step-Down
Transformer
220 or 500 kV
/ 132 0r 66 kV
http://electricaltechnology.org/
132 or 66 kV / 11 kV
Step-Down
Transformer
11kV/ 400 & 230 V
3 Pase 4 Wire System
3 Phase 3 Wire System
Primary
Distribution
Distribution
Transformer
Secondary
Distribution
http://electricaltechnology.org/
Design By:
Engr Wasim Khan
Residential Consumers
Copyright Material @ http://electricaltechnology.org/

Review: Self Inductance

- Self inductance: reaction of the inductor to the change in current through itself.





Flux $\phi = \mathcal{P}Ni$

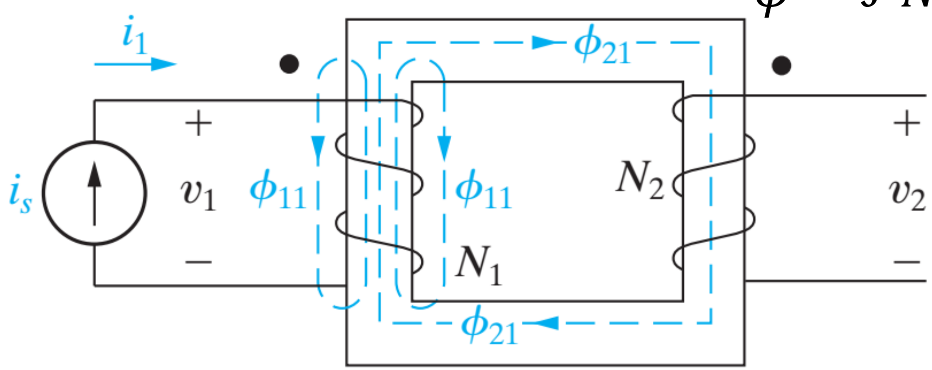
$\mathcal{P}$: permeance

$$v = N\frac{d\phi}{dt} = L\frac{di}{dt}, L = N\frac{d\phi}{di}$$

Mutual Inductance

- Mutual inductance: reaction of the inductor to change in current through another inductor.

$$\phi = \mathcal{P}Ni$$



$$\phi_1 = \phi_{11} + \phi_{21}$$

$$\phi_{11} = \mathcal{P}_{11}N_1 i_1, \quad \phi_{21} = \mathcal{P}_{21}N_1 i_1,$$

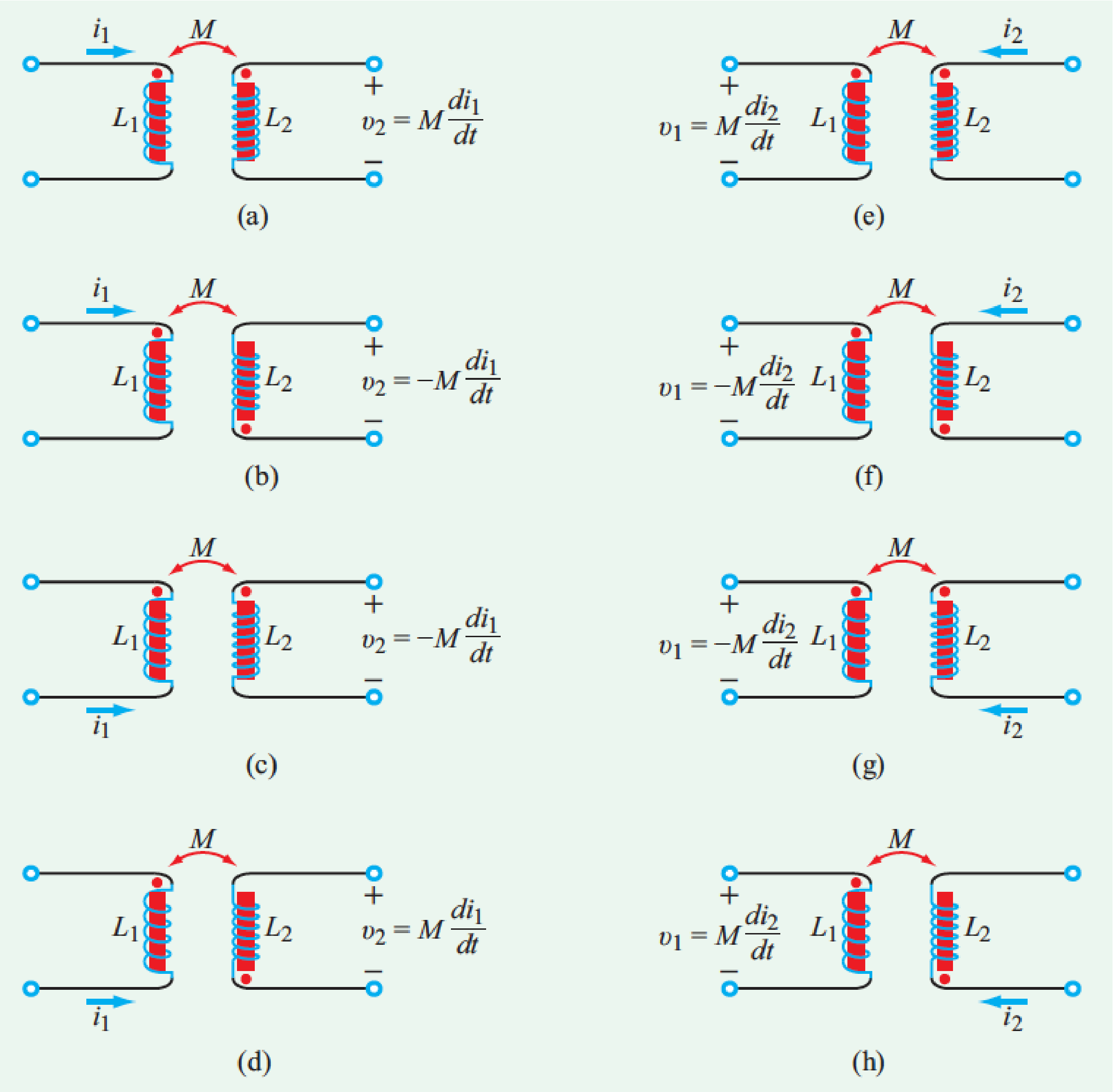
$$\phi_1 = \mathcal{P}_1 N_1 i_1$$

$$v_1 = \frac{d(N_1\phi_1)}{dt} = N_1\frac{d}{dt}(\phi_{11} + \phi_{21})$$
$$= N_1^2(\mathcal{P}_{11} + \mathcal{P}_{21})\frac{di_1}{dt} = N_1^2\mathcal{P}_1\frac{di_1}{dt}$$

$$v_2 = \frac{d(N_2\phi_{21})}{dt} = N_2\frac{d}{dt}(\mathcal{P}_{21}N_1 i_1)$$
$$= N_2 N_1 \mathcal{P}_{21}\frac{di_1}{dt}$$
$$= M_{21}\frac{di_1}{dt}$$

$$\phi = \mathcal{P}Ni$$

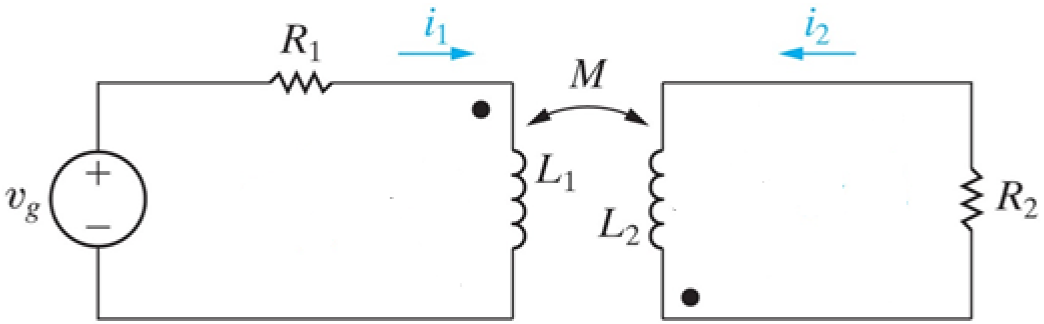
Dot Convention: Defines Directions of Windings



[Source: Berkeley]

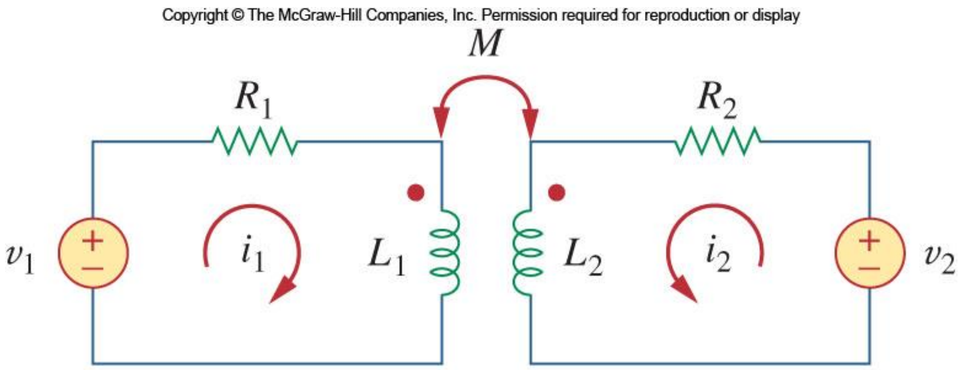
Mutual Inductance: General Case

- Two circuits lined by a magnetic field
  - L_1, L_2: self-inductances
  - M: mutual inductance
  - Dots: indicating polarity of mutually induced voltages.



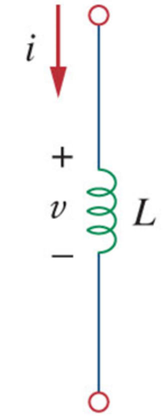
Exercise

- Relate v_1, v_2 with $i_1$and i_2.
  - In time domain
  - In phasor domain



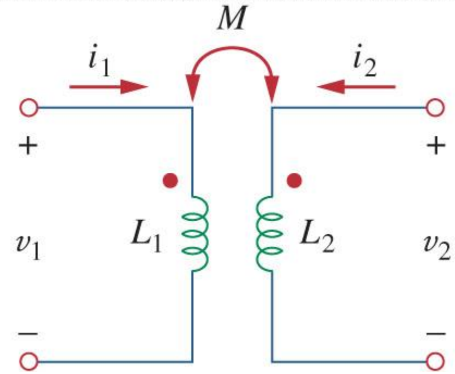
Energy in a Coupled Circuit

- The energy stored in an inductor is



- For coupled inductors, the total energy stored is

$$w = \frac{1}{2}L_1 i_1^{\,2} + \frac{1}{2}L_2 i_2^{\,2} \pm M i_1 i_2$$



  - The positive sign is selected when the currents both enter or leave the dotted terminals.

Coupling Coefficient *k*

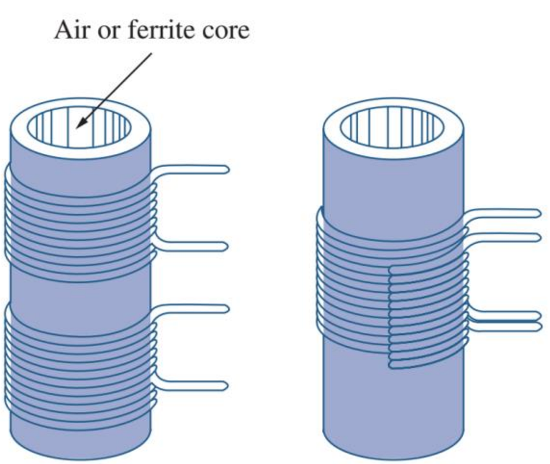
- The system cannot have negative energy

$$\frac{1}{2}L_1 i_1^{\,2} + \frac{1}{2}L_2 i_2^{\,2} - M i_1 i_2 \geq 0 \quad \Rightarrow \quad \boxed{M \leq \sqrt{L_1 L_2}}$$

- Define a parameter describes how closely M approaches upper limit.

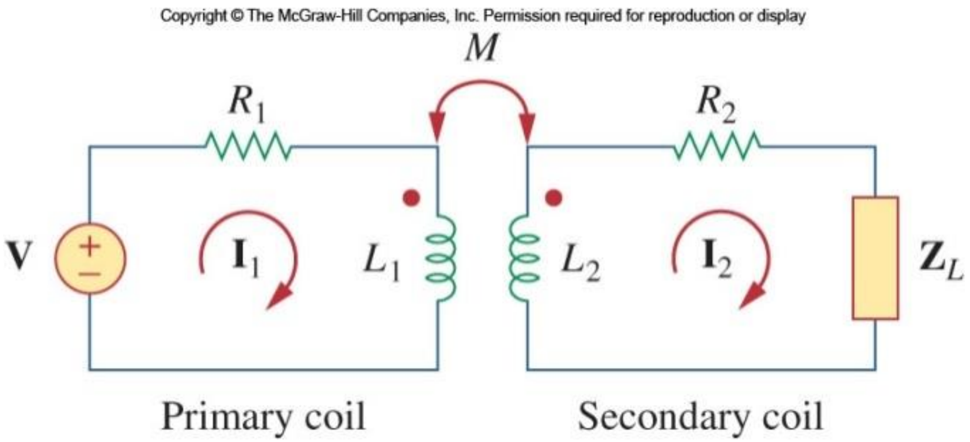
$$\boxed{k = \frac{M}{\sqrt{L_1 L_2}}}$$

  - Coupling coefficient, $0 \leq k \leq 1$.
  - determined by the physical configuration of the coils.



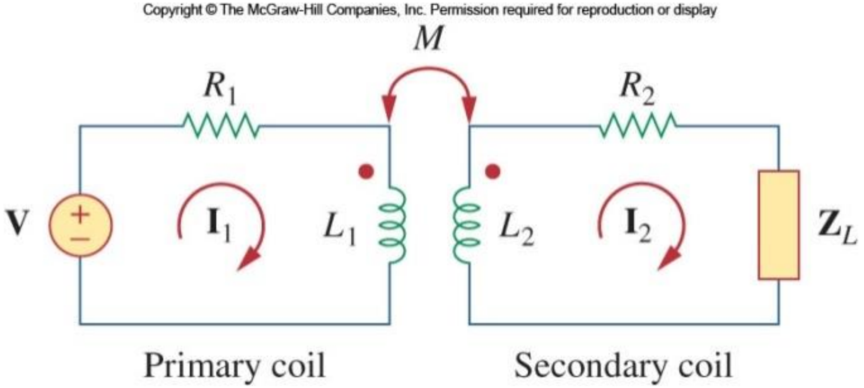
Linear Transformers

- A transformer is a magnetic device that takes advantage of mutual inductance.
  - Called linear if the coils are wound on a magnetically linear material, i.e. permeability μ is constant.



Transformer Impedance

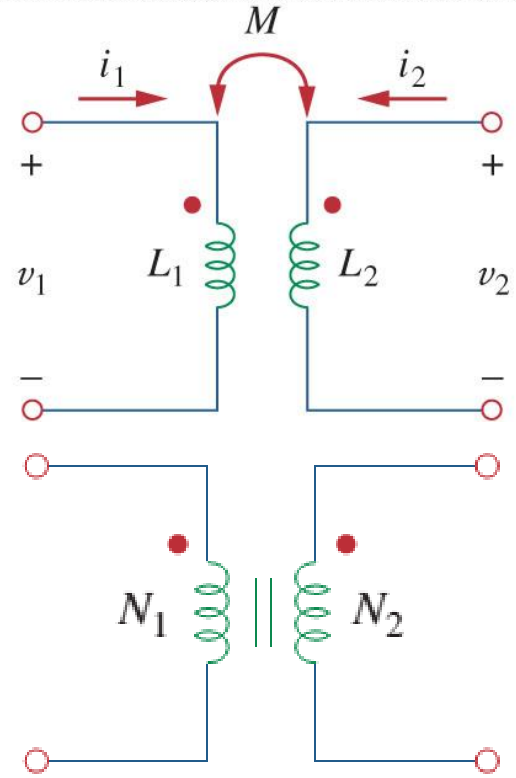
- An important parameter to know for a transformer is how the input impedance Z_{in} is seen from the source.
  - Z_{in} is important because it governs the behavior of the primary circuit.



$$\mathbf{Z}_{\text{in}} = \frac{\mathbf{V}}{\mathbf{I}_1} = R_1 + j\omega L_1 + \frac{\omega^2 M^2}{R_2 + j\omega L_2 + \mathbf{Z}_L}$$

Reflected impedance from secondary to primary

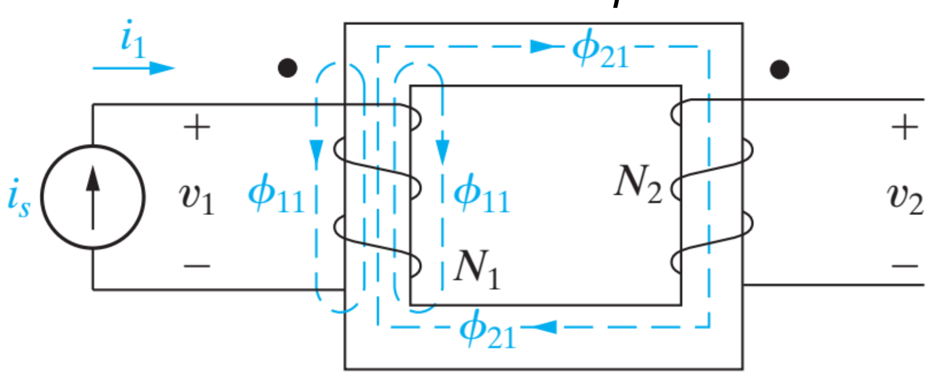
Ideal Transformers



- The ideal transformer has:
  - Coils with very large reactance

    $(L_1, L_2, M \to \infty)$
  - Coupling coefficient k=1.
  - Primary and secondary coils are lossless, $R_1 = R_2 = 0$.

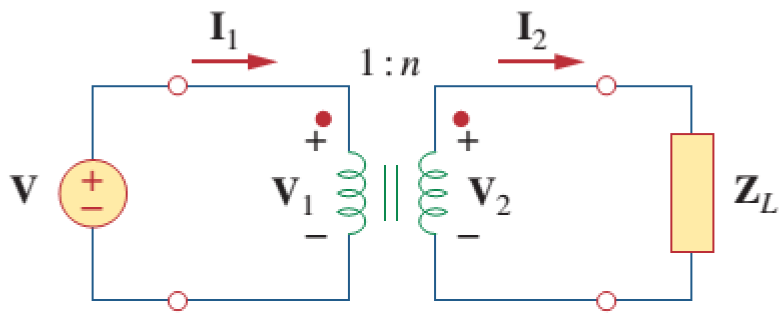
$$\phi = \mathcal{P}Ni$$



$$\frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{N_2}{N_1} = n$$

Ideal Transformers II

$$\frac{\mathbf{V_2}}{\mathbf{V_1}} = \frac{N_2}{N_1} = n$$



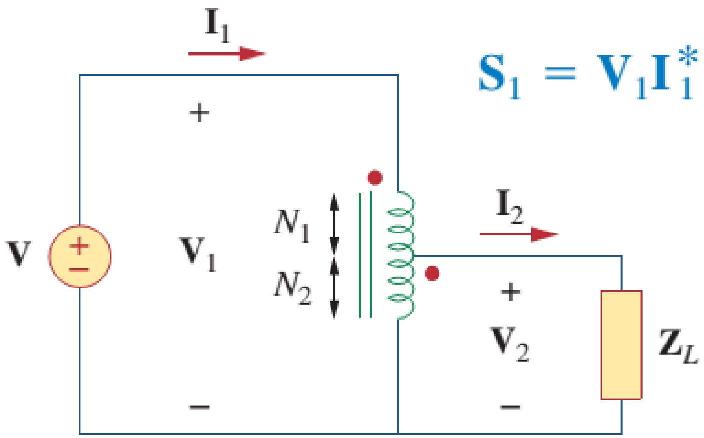
- The current is related as:

- Reflected impedance

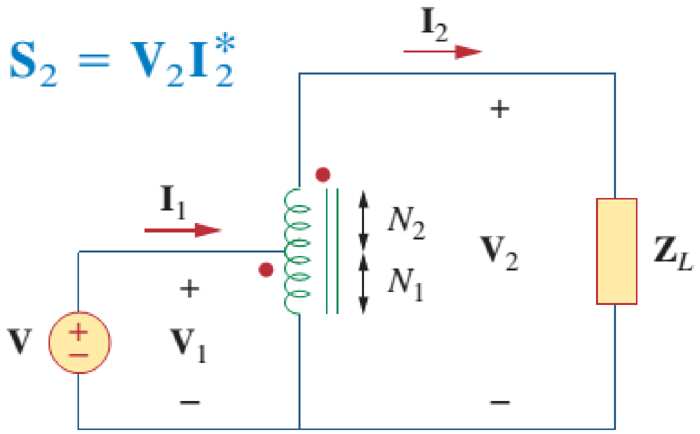
$$\mathbf{Z}_{\text{in}} = \frac{\mathbf{V}_1}{\mathbf{I}_1} =$$

Ideal Autotransformer

- Autotransformer uses one winding for primary & secondary
  - It does not offer isolation!



$$\mathbf{S}_1 = \mathbf{V}_1\mathbf{I}_1^* = \mathbf{S}_2 = \mathbf{V}_2\mathbf{I}_2^*$$



$$\boxed{\frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{N_1 + N_2}{N_2} = 1 + \frac{N_1}{N_2}}$$

$$\boxed{\frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{N_1}{N_1 + N_2}}$$

$$\frac{\mathbf{I}_1}{\mathbf{I}_2} = \frac{N_2}{N_1 + N_2}$$

$$\frac{\mathbf{I}_1}{\mathbf{I}_2} = \frac{N_1 + N_2}{N_1} = 1 + \frac{N_2}{N_1}$$