Homework 3

Due date:
Mar.26th, 2018
Turn in your homework in class

Rules:

- Work on your own. Discussion is permissible, but extremely similar submissions will be judged as plagiarism.
- Please show all intermediate steps: a correct solution without an explanation will get zero credit.
- Please submit on time. No late submission will be accepted.
- Please prepare your submission in English only. No Chinese submission will be accepted.

1. Find the equivalent resistance in the circuit $R_{ab}$ in Fig. 1 by using Y-Δ transformation.

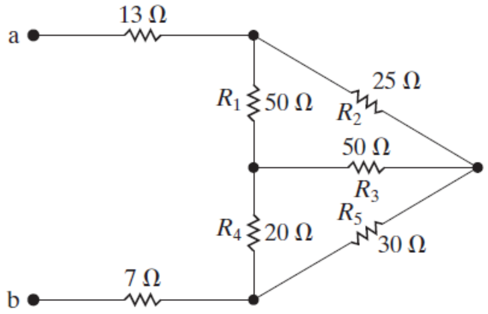



**Figure 1**

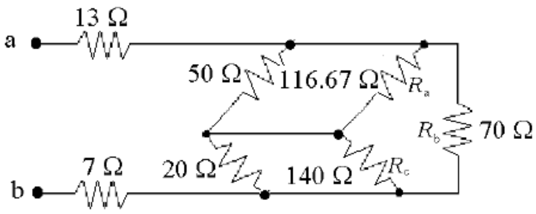
Use the figure below to transform the Y to an equivalent Δ:

$$R_a = \frac{(25)(30) + (25)(50) + (30)(50)}{30} = \frac{3500}{30} = 116.67\,\Omega$$

$$R_b = \frac{(25)(30) + (25)(50) + (30)(50)}{50} = \frac{3500}{50} = 70\,\Omega$$

$$R_c = \frac{(25)(30) + (25)(50) + (30)(50)}{25} = \frac{3500}{25} = 140\,\Omega$$

Then we can obtain such a figure:




$$70\|[(50\|116.67) + (20\|140)] = 30\,\Omega$$

Right side of the circuit: 70||[(50||116.67)+(20||140)]=30Ω

$R_{ab}$=13+7+30=50Ω

2. For the circuit in Fig. 2, determine the value of R such that the maximum power delivered to the load is 3 mW.

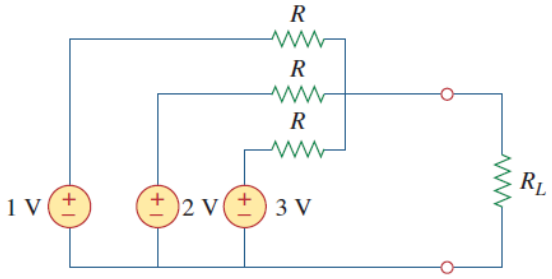
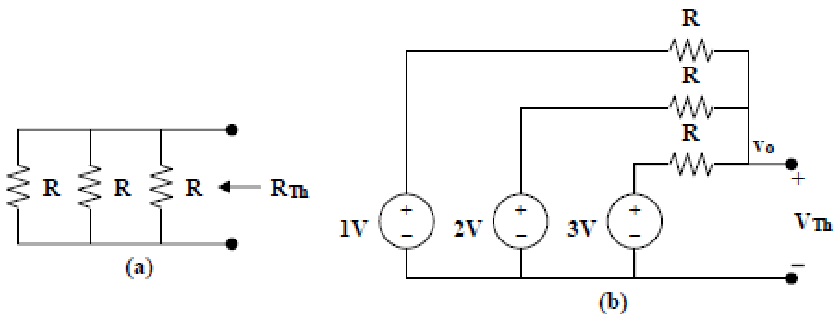



**Figure 2**




$$R_{Th} = \frac{R}{3}$$

$$((1 - v_o)/R) + ((2 - v_o)/R) + ((3 - v_o)/R) = 0$$

$$v_o = 2 = V_{Th}$$

Maximum power:

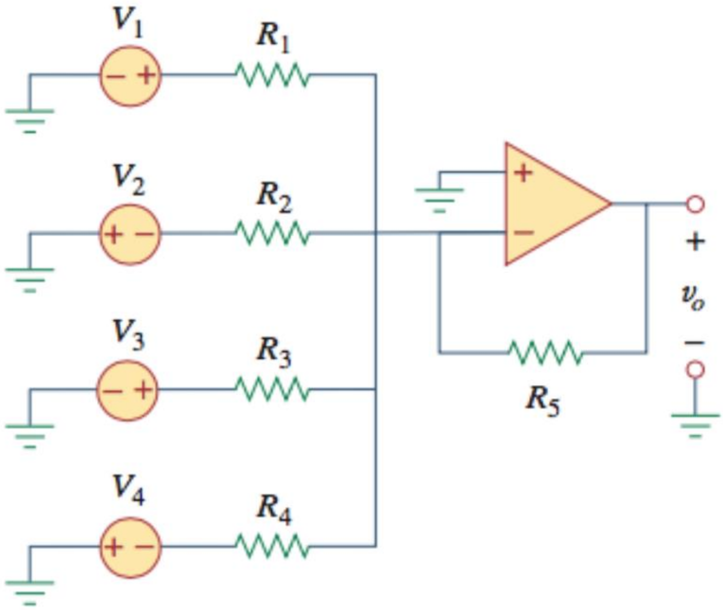
$$R_L = R_{Th} = R/3$$

$$P_{max} = [(V_{Th})^2/(4R_{Th})] = \mathbf{3\ mW}$$

$$R_{Th} = [(V_{Th})^2/(4P_{max})] = 4/(4xP_{max}) = 1/P_{max} = R/3$$

$$R = 3/(3x10^{-3}) = \mathbf{1\ k\Omega}$$

3. Calculate $v_0$ in this circuit.

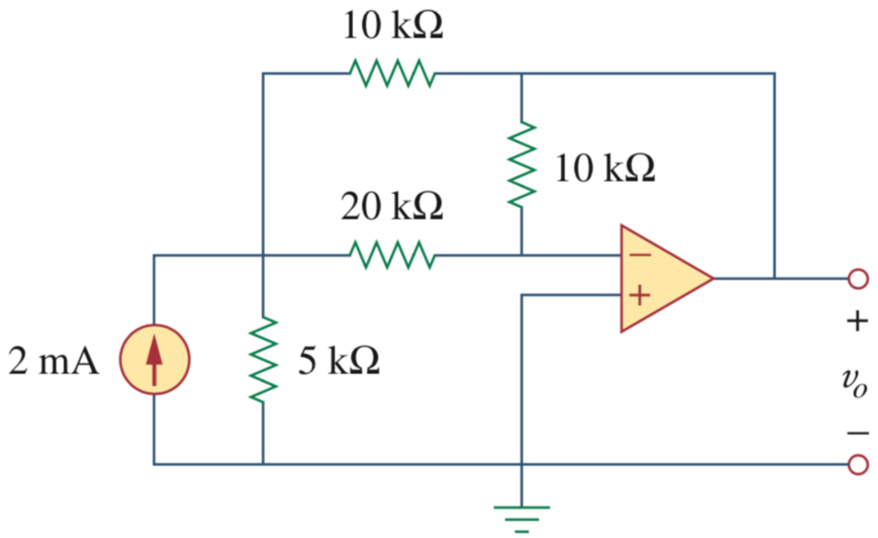



Solution：

$$\frac{V_O}{R_5} = \frac{0 - V_1}{R_1} + \frac{0 + V_2}{R_2} + \frac{0 - V_3}{R_3} + \frac{0 + V_4}{R_4}$$

$$\Rightarrow \ V_O = \left(-\frac{V_1}{R_1} + \frac{V_2}{R_2} - \frac{V_3}{R_3} + \frac{V_4}{R_4}\right) R_5$$

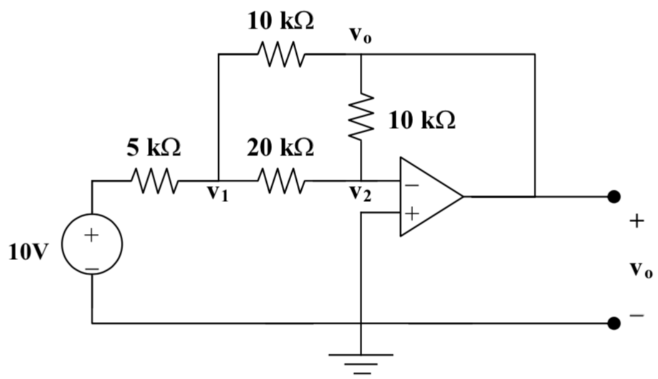
4. Determine the output voltage $v_0$ in the circuit below.




**Chapter 5, Solution 14.**

Transform the current source as shown below. At node 1,

$$\frac{10 - v_1}{5} = \frac{v_1 - v_2}{20} + \frac{v_1 - v_o}{10}$$

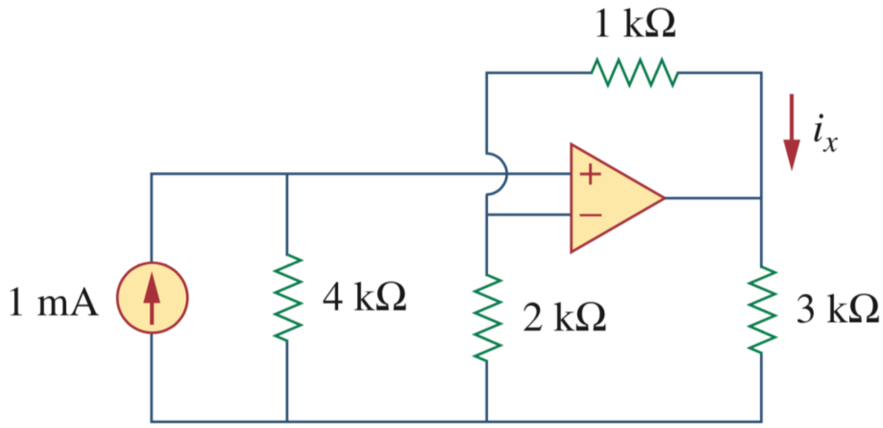



But $v_2 = 0$. Hence $40 - 4v_1 = v_1 + 2v_1 - 2v_o \longrightarrow$ $40 = 7v_1 - 2v_o$ (1)

At node 2, $\frac{v_1 - v_2}{20} = \frac{v_2 - v_o}{10}$, $v_2 = 0$ or $v_1 = -2v_o$ (2)

From (1) and (2), $40 = -14v_o - 2v_o \longrightarrow$ $v_o = \mathbf{-2.5V}$

5. Refer to the op amp circuit in Fig below. Calculate $i_x$ and the power absorbed by the 3-kΩ resistor.




This is a noninverting amplifier.

$$v_o = \left(1+\frac{1}{2}\right)v_i = \frac{3}{2}v_i$$

Since the current entering the op amp is 0, the source resistor has a 0 V potential drop. Hence $v_i = 4V$.

$$v_o = \frac{3}{2}(4) = 6V$$

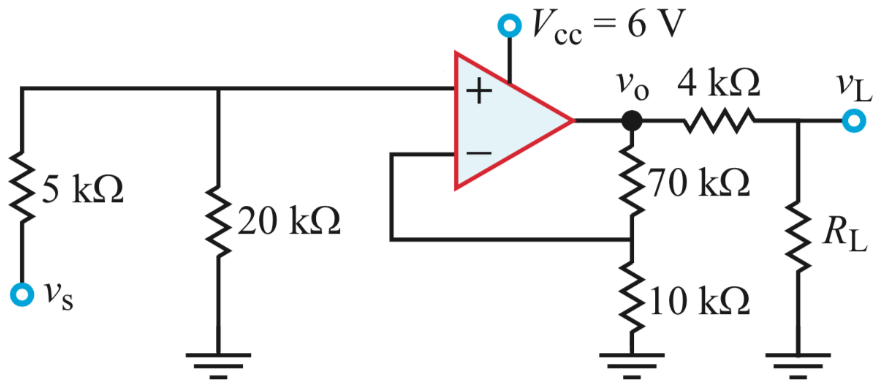
Power dissipated by the 3kΩ resistor is

$$\frac{v_o^2}{R} = \frac{36}{3k} = \mathbf{12mW}$$

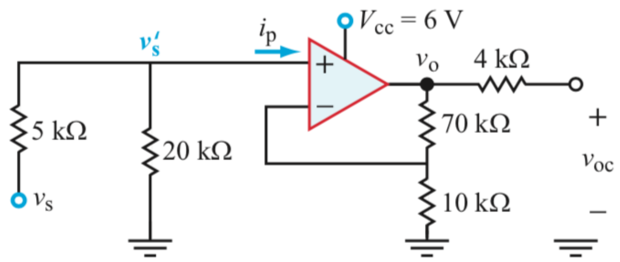
$$i_x = \frac{v_a - v_o}{R} = \frac{4-6}{1k} = \mathbf{-2mA}.$$

**12mW, –2mA**

6. For the circuit of the Fig below, what should the resistance of $R_L$ be so as to have the maximum transfer of power into it?




**Solution:** We seek to find the Thévenin circuit as seen by $R_L$. With $R_L$ removed, we need to find the open-circuit voltage $\upsilon_{oc}$:




Since there is no voltage drop across the 4-kΩ resistor,

$$\upsilon_{oc} = \upsilon_o.$$

At the input side, in view of $i_p = 0$, voltage division gives:

$$\upsilon_s' = \frac{\upsilon_s \times 20\text{k}}{(5+20)\text{k}} = 0.8\upsilon_s.$$

As a noninverting amplifier,

$$\upsilon_o = \frac{(70+10)\text{k}}{10\text{k}}\,\upsilon_s' = 8\upsilon_s' = 8 \times 0.8\upsilon_s = 6.4\upsilon_s.$$

Hence,

$$\upsilon_{Th} = \upsilon_{oc} = \upsilon_o = 6.4\upsilon_s.$$

Next, we seek to find $R_{Th}$ by calculating the short-circuit current at the output. Replacing $R_L$ with a short circuit, the current through it is simply

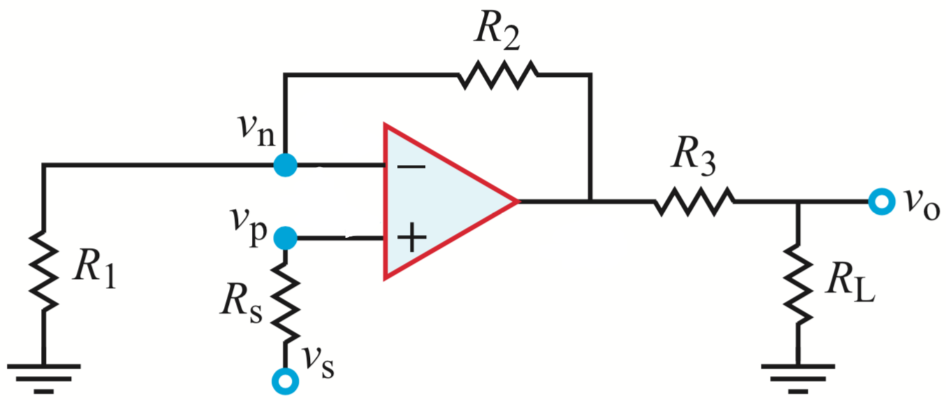
$$i_{sc} = \frac{\upsilon_o}{4\text{ k}\Omega} = \frac{\upsilon_{Th}}{4\text{ k}\Omega}.$$

Hence,

$$R_{Th} = \frac{\upsilon_{oc}}{i_{sc}} = 4\text{ k}\Omega.$$

To maximize power transfer into $R_L$, its value should be 4 kΩ.

7. Obtain an expression for the voltage gain $G=v_0/v_s$ foe the circuit in Fig below.




**Solution:** Since $i_n = 0$ (ideal op-amp constraint),

$$v_n = \frac{v_o' R_1}{R_1 + R_2}.$$

Also, $v_p = v_s$ (because $i_p = 0$), and $v_p = v_n$.

Hence,

$$v_o' = \left(\frac{R_1 + R_2}{R_1}\right) v_s.$$

At the output side, voltage division gives:

$$v_o = \frac{v_o' R_L}{R_3 + R_L}.$$

Hence,

$$v_o = \left(\frac{R_1 + R_2}{R_1}\right) v_s \times \frac{R_L}{R_3 + R_L} = \left[\frac{R_L}{R_1}\left(\frac{R_1 + R_2}{R_3 + R_L}\right)\right] v_s.$$

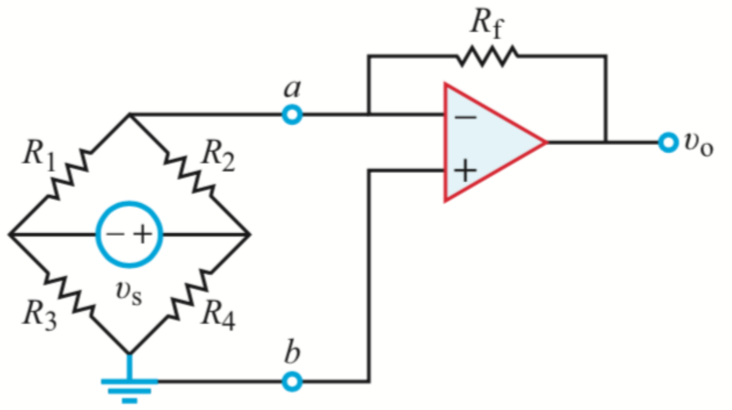
$$G = \frac{v_o}{v_s} = \frac{R_L(R_1 + R_2)}{R_1(R_3 + R_L)}.$$

8. In the circuit of Fig below, a bridge circuit is connected at the input side of an inverting op-amp circuit.

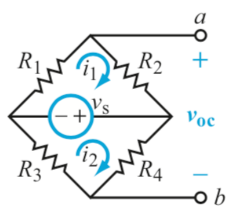
(a) Obtain the Thevenin equivalent at terminals (a , b) for the bridge circuit.

(b) Use the result in (a) to obtain an expression for $G = \upsilon_O/\upsilon_S$.

(c) Evaluate G for $R_1 = R_4 = 100\Omega$, $R_2 = R_3 = 101\Omega$, and $R_f = 100\text{k}\Omega$.




**Solution: (a)** The Thévenin equivalent circuit at $(a, b)$:




$$\upsilon_s + i_1(R_1 + R_2) = 0$$

or

$$i_1 = \frac{-\upsilon_s}{R_1 + R_2}.$$

Also,

$$-\upsilon_s + i_2(R_3 + R_4) = 0$$

and

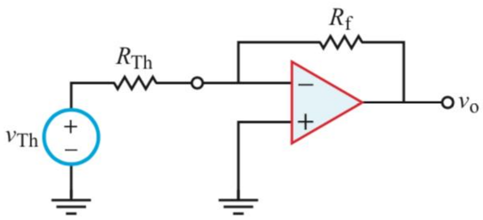
$$i_2 = \frac{\upsilon_s}{R_3 + R_4}.$$

$$\begin{aligned}\upsilon_{Th} = \upsilon_{oc} &= i_1 R_2 + i_2 R_4 \\ &= \frac{-\upsilon_s R_2}{R_1 + R_2} + \frac{\upsilon_s R_4}{R_3 + R_4} = \frac{[R_4(R_1 + R_2) - R_2(R_3 + R_4)]\upsilon_s}{(R_1 + R_2)(R_3 + R_4)}.\end{aligned} \tag{1}$$

Suppressing $\upsilon_s$ (by replacing it with a short circuit) leads to

$$\begin{aligned}R_{Th} &= (R_1 \parallel R_2) + (R_3 \parallel R_4) \\ &= \frac{R_1 R_2}{R_1 + R_2} + \frac{R_3 R_4}{R_3 + R_4} = \frac{R_1 R_2(R_3 + R_4) + R_3 R_4(R_1 + R_2)}{(R_1 + R_2)(R_3 + R_4)}.\end{aligned}$$

**(b)** For the new circuit:




$$v_o = -\frac{R_f}{R_{Th}} v_{Th} \qquad \text{(inverting amplifier)} \tag{3}$$

Inserting Eqs. (1) and (2) into (3) leads to

$$G = \frac{v_o}{v_s} = \frac{-R_f[R_4(R_1+R_2) - R_2(R_3+R_4)]}{R_1R_2(R_3+R_4) + R_3R_4(R_1+R_2)}$$

**(c)** For $R_1 = R_4 = 100\ \Omega$, $R_2 = R_3 = 101\ \Omega$, and $R_f = 10^5\ \Omega$,

$$G = \frac{-10^5[100(100+101) - 101(100+101)]}{100\times 101(100+101) + 100\times 101(100+101)}$$
$$= 4.9505 \simeq 5.$$