Problem 1 (7 points)

For each of the 7 devices in the circuit of Fig. 1, determine whether the device is a supplier or a recipient of power and how much power it is supplying or receiving.

Note: Use the passive sign convention.

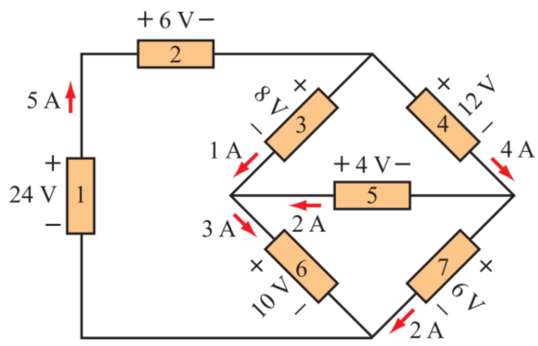


Fig. 1 for Problem 1.

Solution:

	Voltage (V)	Current (A)	Power (W)	Power Supplier (S) or Recipient (R)?
Device 1	24	-5	-120	S
Device 2	6	5	-30	R
Device 3	8	1	8	R
Device 4	12	4	48	R
Device 5	4	-2	-8	S
Device 6	10	3	30	R
Device 7	6	2	12	R

Problem 2 (12 points)

Find I in the circuit of Fig. 2.

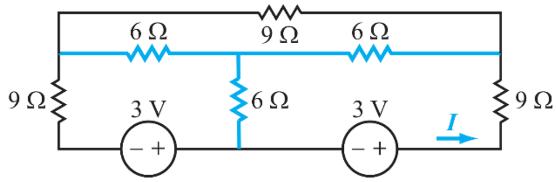
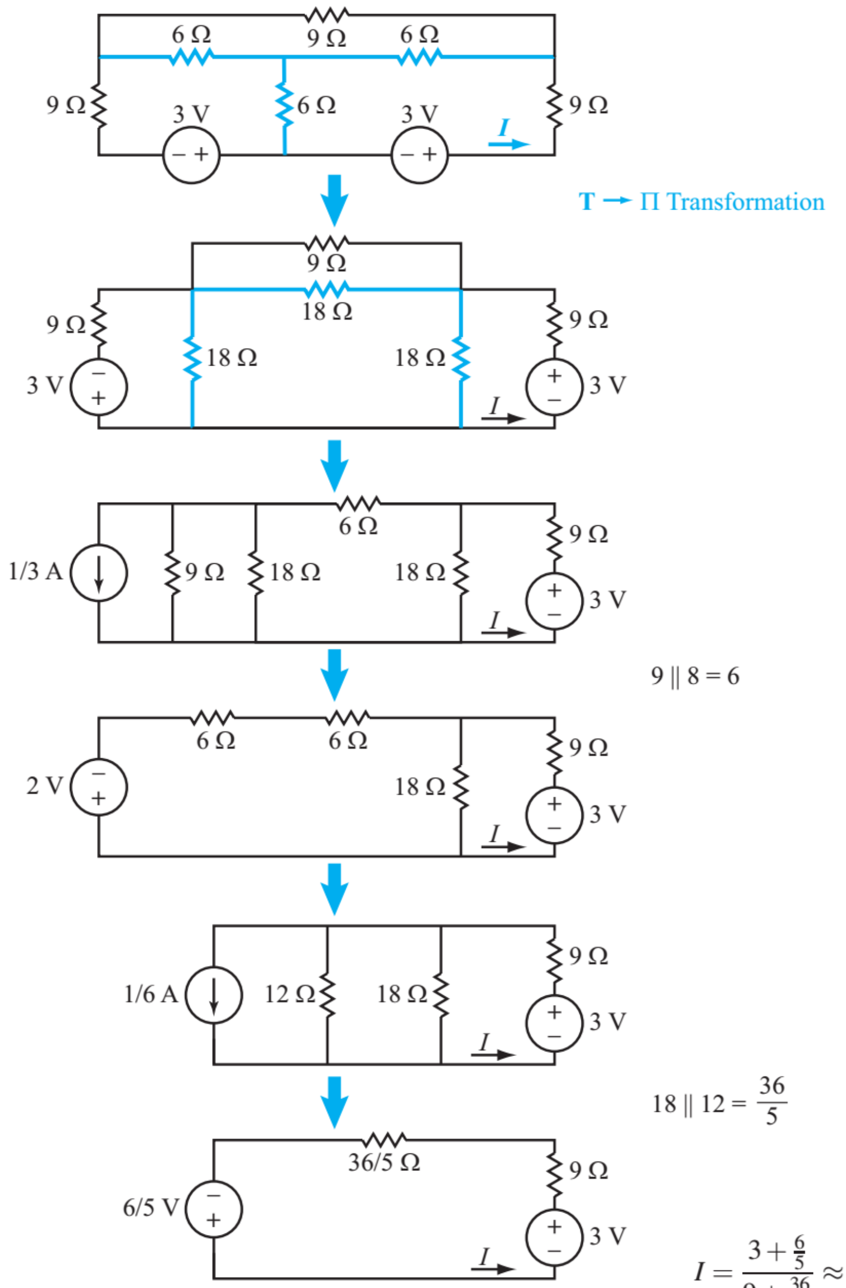


Fig. 2 for Problem 2.

Solution:



$$9 \parallel 8 = 6$$

$$18 \parallel 12 = \frac{36}{5}$$

$$I = \frac{3+\frac{6}{5}}{9+\frac{36}{5}} \approx 0.26 \text{ A.}$$

Problem 3 (12 points)

Find current I_x in the circuit of Fig.3.

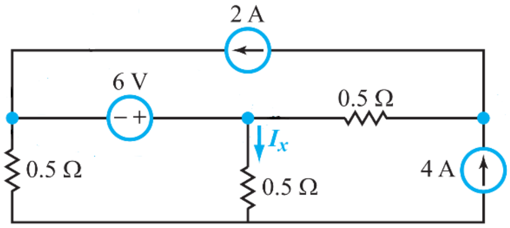
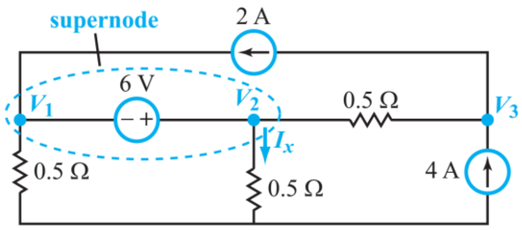


Fig. 3 for Problem 3.



Solution: The presence of a voltage source between designated nodes 1 and 2 makes the combination of nodes 1 and 2 a supernode. Hence,

$$\frac{V_1}{0.5} - 2 + \frac{V_2}{0.5} + \frac{V_2 - V_3}{0.5} = 0. \tag{1}$$

For node 3,

$$\frac{V_3 - V_2}{0.5} - 4 + 2 = 0, \tag{2}$$

and the auxiliary equation is

$$V_2 - V_1 = 6. \tag{3}$$

Combining the three equations leads to:

$$V_1 = -2 \text{ V}, \qquad V_2 = 4 \text{ V}, \qquad V_3 = 5 \text{ V}.$$

Hence,

$$I_x = \frac{V_2}{0.5} = \frac{4}{0.5} = 8 \text{ A}.$$

Problem 4 (12 points)

In the circuit of Fig. 4(a), when $U = 1\text{V}$，$I = -0.2\text{A}$, when $U = 7\text{V}$, $I = 1\text{A}$.
Find the current I_1 in Fig. 4(b).

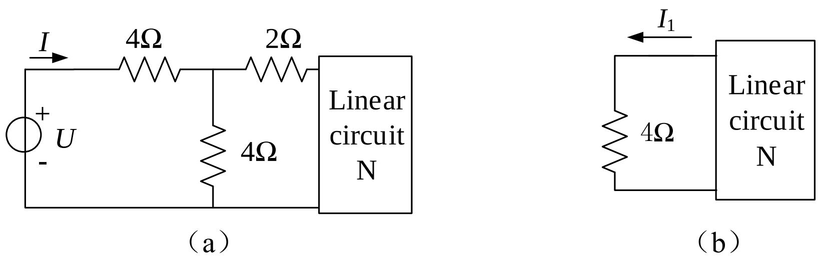


Fig. 4 for Problem 4. The linear circuits in these two subfigures are the same.

Solution:

I is a linear function of U, easy to find that $\text{I} = 0.2\text{U} - 0.4$.

When U=0, I=-0.4A.

So I_1 = 0.8A.

Problem 5 (15 points)

For the circuit in Fig. 5,

a) Find the Thevenin equivalent circuit at terminals (a, b) as seen by the load resistor R_L.

b) Choose R_L so that the current flowing through it is 0.16mA. 0.2K

c) Choose R_L so that the power delivered to it is maximum. How much power will that be?

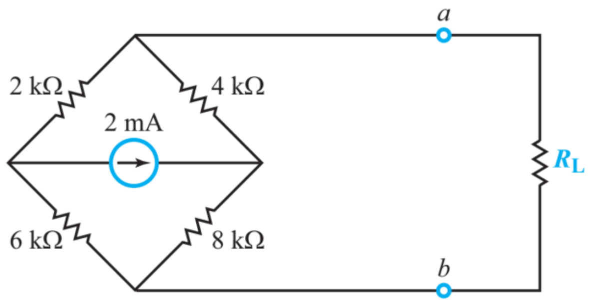
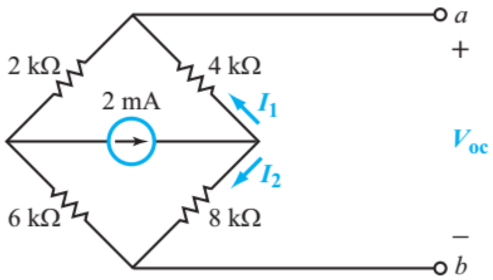


Fig. 5 for Problem 5.

Solution: We need to find the Thévenin equivalent circuit at terminals (a, b), as if R_L were not present.



The current source will divide among I_1 and I_2 such that

$$(4+2)I_1 = (8+6)I_2$$

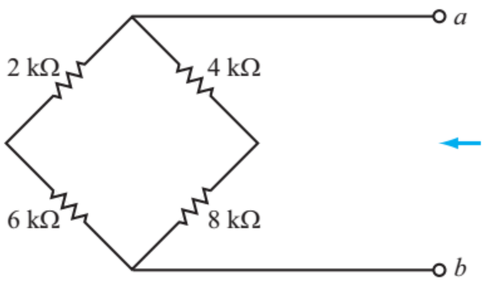
Also, $I_1 + I_2 = 2$ mA.

The solution yields:

$$I_1 = 1.4 \text{ mA}, \qquad I_2 = 0.6 \text{ mA}.$$

$$V_{oc} = (-4I_1 + 8I_2) \times 10^3$$
$$= -4 \times 1.4 + 8 \times 0.6 = -0.8 \text{ V}.$$

To find R_{Th}, we suppress the current source and simplify the circuit:



$R_{Th} = 8\text{ k}\Omega \parallel 12\text{ k}\Omega = 4.8\text{ k}\Omega$

b) $R_L = 200\Omega$

c) $R_L = 4.8k\Omega$, max power = $\frac{(0.4V)^2}{4.8k\Omega} = 33\mu\text{W}$.

Problem 6 (12 points)

In the circuit shown in Fig. 6, find the gain $G = \frac{v_o}{v_s}$.

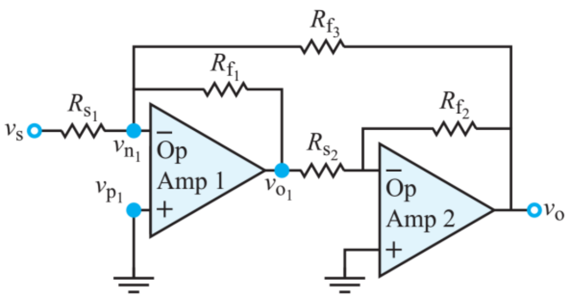


Fig. 6 for Problem 6.

Solution: For the second op amp,

$$v_o = \left(-\frac{R_{f_2}}{R_{s_2}}\right) v_{o_1} \tag{1}$$

For the first op amp,

$$\frac{v_{n_1} - v_s}{R_{s_1}} + \frac{v_{n_1} - v_{o_1}}{R_{f_1}} + \frac{v_{n_1} - v_o}{R_{f_3}} = 0$$

Also,

$$v_{n_1} = v_{p_1} = 0.$$

Hence,

$$-\frac{v_s}{R_{s_1}} - \frac{v_{o_1}}{R_{f_1}} - \frac{v_o}{R_{f_3}} = 0 \tag{2}$$

Simultaneous solution of (1) and (2) leads to

$$v_o = \frac{v_s}{R_{s_1}} \left[\frac{R_{f_1} R_{f_2} R_{f_3}}{R_{f_3} R_{s_2} - R_{f_1} R_{f_2}}\right].$$

Problem 7 (10 points)

The ideal operational amplifier circuit shown in Fig. 7 is driven by a input ramp signal

$$v_I(t) = \begin{cases} 0\ V, & t < 0 \\ 1000t\ V, & t \geq 0 \end{cases}$$

Assume that the capacitor voltage is zero for $t < 0$. What are the value of output voltage $v_I(t)$ at $t = 1ms$?

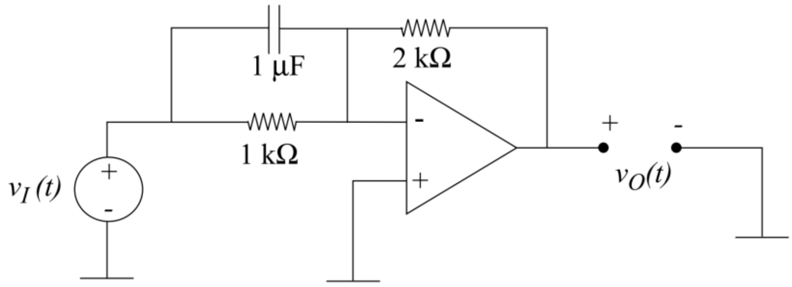


Fig. 7 for Problem 7.

Solution:

KCL at node v^-:

$$\frac{v_I(t) - 0}{1000} + C\frac{dv_I(t)}{dt} + \frac{v_0(t) - 0}{2000} = 0, \text{ since } v^- = v^+ = 0$$

$$\text{Therefore, } v_I(t) = 1000t, \frac{dv_I(t)}{dt} = 1000, \text{ so}$$

$$v_0(t) = -2000 \cdot t - 2\ [volts]$$

$$v_0(t = 1ms) = -4Volts$$

Problem 8 (20 points)

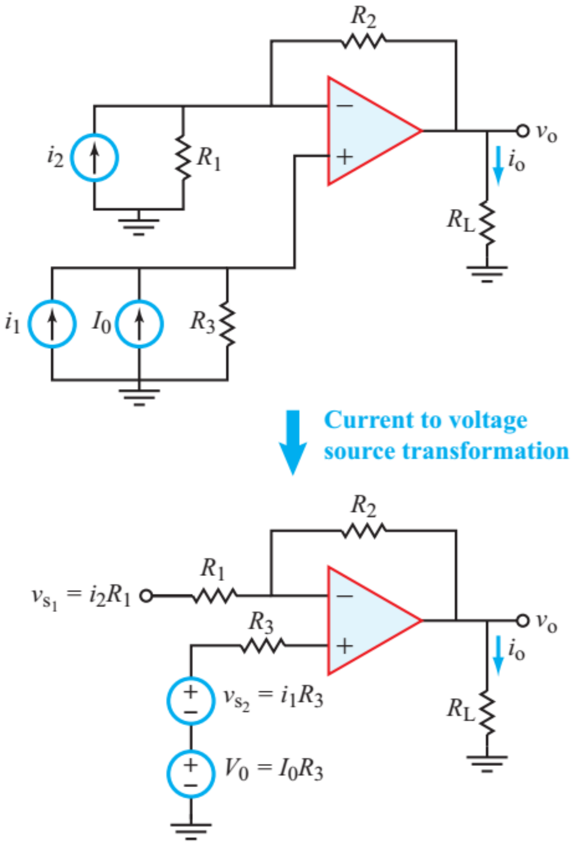
Design an op-amp circuit that can perform the operation

$$i_o = (30i_1 - 8i_2 + 0.6)\ \text{A}$$

Where i_1 and i_2 are two input current sources.

Solution: Below is just one possible solution.

<u>Remember to include the internal resistance of current sources when you calculate the gain!!!</u>



$$\begin{aligned} v_o &= \left(-\frac{R_2}{R_1}\right) v_{s_1} + \left(\frac{R_2+R_1}{R_1}\right)[v_{s_2}+V_0] \\ &= \left(-\frac{R_2}{R_1}\right) i_2R_1 + \left(\frac{R_2+R_1}{R_1}\right)[i_1R_3+I_0R_3] \end{aligned}$$

Choose $R_1 = 2\ \text{k}\Omega$, $R_2 = 8\ \text{k}\Omega$. Hence,

$$v_o = -8\times 10^3 i_2 + 5[i_1R_3 + I_0R_3]$$

Choose $R_3 = 6\ \text{k}\Omega$, os that the coefficient of i_1 is 30×10^3.
Choose $I_0 = 0.02\ \text{A} = 20\ \text{mA}$, so that $5I_0R_3 = 0.6\times 10^3$.
Hence,

$$v_o = -8\times 10^3 i_2 + 30\times 10^3 i_1 + 0.6\times 10^3 \qquad \text{(V)}$$

$$i_o = \frac{v_o}{R_L}$$

Choose $R_L = 1\ \text{k}\Omega$,

$$i_o = (30i_1 - 8i_2 + 0.6)\ \text{A}.$$