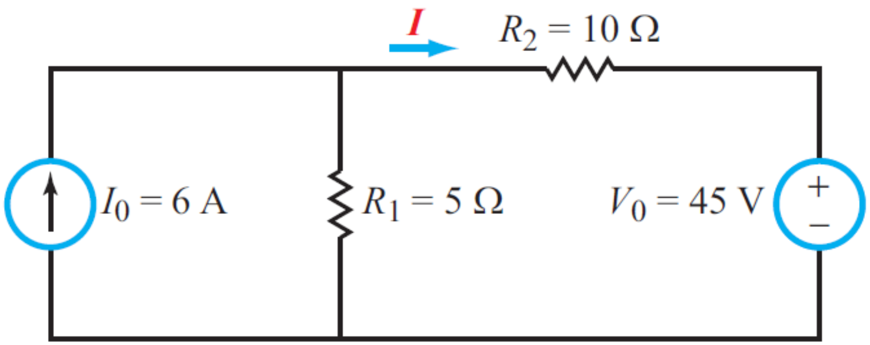
Lecture 3
Circuit Theorems

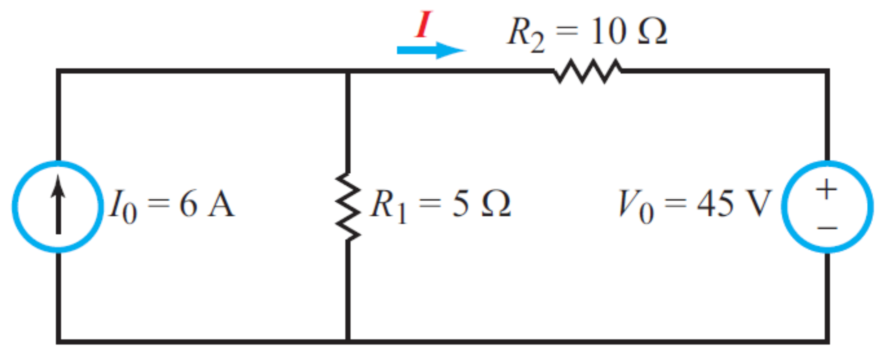
Exercise

- Q1: I =?



[Source: Berkeley]

Extension 1



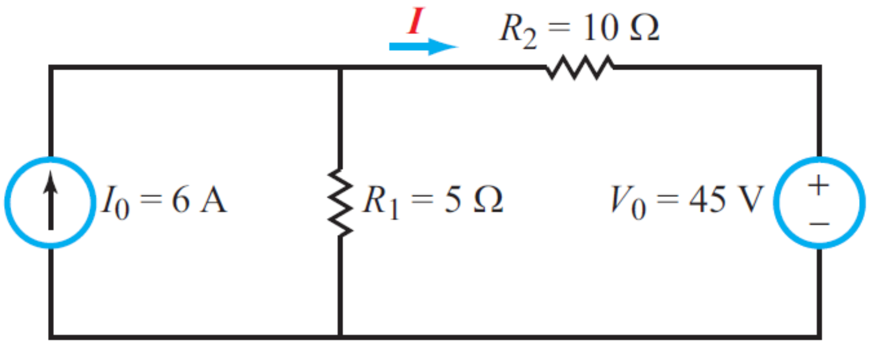
- Q2:

  If $I_0 = 12\text{A}$, $V_0 = 45V, I = ?$
- Q3:

  If $I_0 = 6\text{A}$, $V_0 = 90V, I = ?$

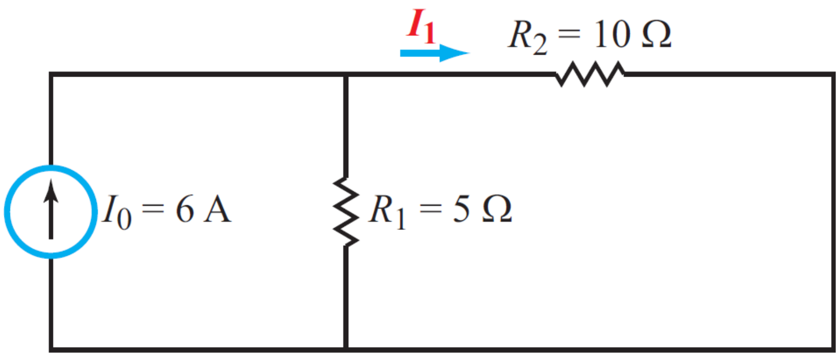
- Q4: $I = aI_0 + bV_0 + c$?

Extension 2

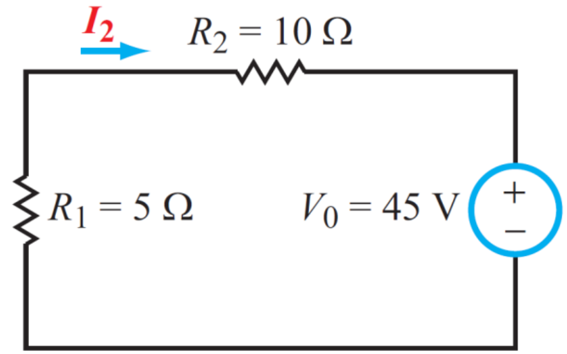


- Q5: If $I_0 = 6A, V_0 = 0,\ \ I_1 = ?$
- Q6: If $I_0 = 0, V_0 = 45V,\ I_2 = ?$
- Q7: $I = I_1 + I_2$?

Contribution from I_0 alone

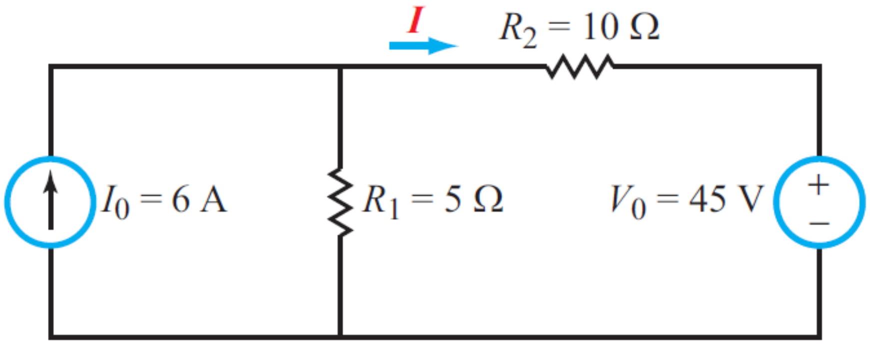


Contribution from V_0 alone



Extension 3

- Q7: If $R_2 = 1\Omega, I =?$
- Q8: What if $R_2 = 5\Omega$?



Outline

- Linearity property
- Superposition
- Thevenin's theorem
- Source transformation
- Norton's theorem

Linearity Property

Excitation x → System → Response $f(x)$

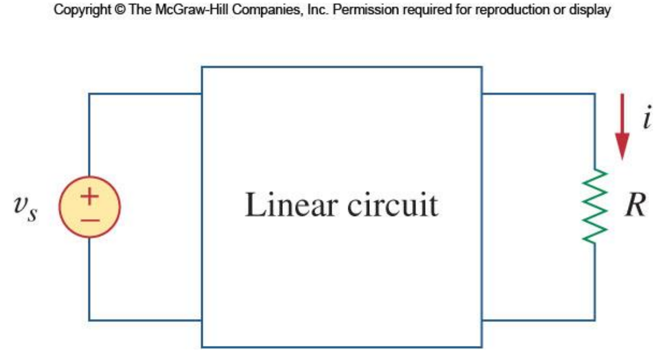
- Linearity is a combination of
  - homogeneity (scaling) property

    Excitation $\alpha \cdot x$ → System → Response $\alpha \cdot f(x)$ α is constant

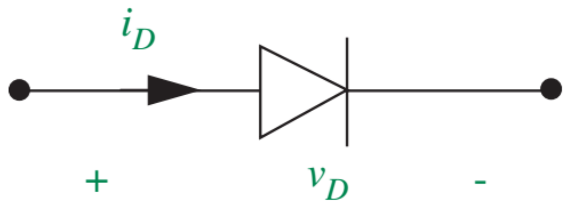
  - additivity property

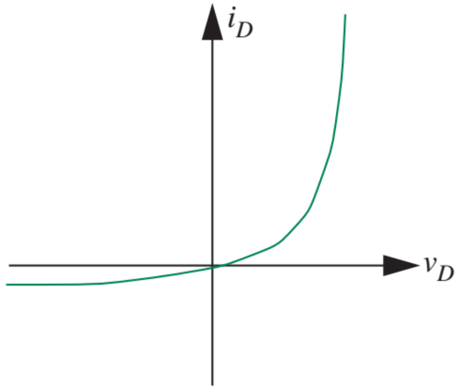
    Excitation $x_1 + x_2$ → System → Response $f(x_1) + f(x_2)$

Linear Circuit



- In a circuit,
  - Excitation: Sources
  - Response: Voltage or current
- A linear circuit consists of only linear elements (resistors, capacitors and inductors), linear dependent sources, and independent sources.
  - Linear means I-V characteristic of elements/sources are straight lines when plotted.



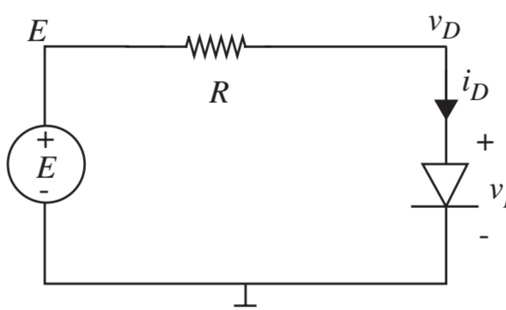


- Is the power relation linear?

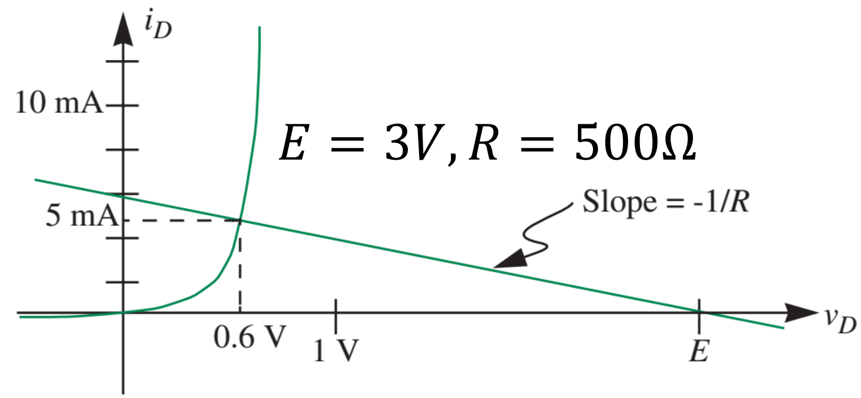
Nonlinear Circuit Analysis

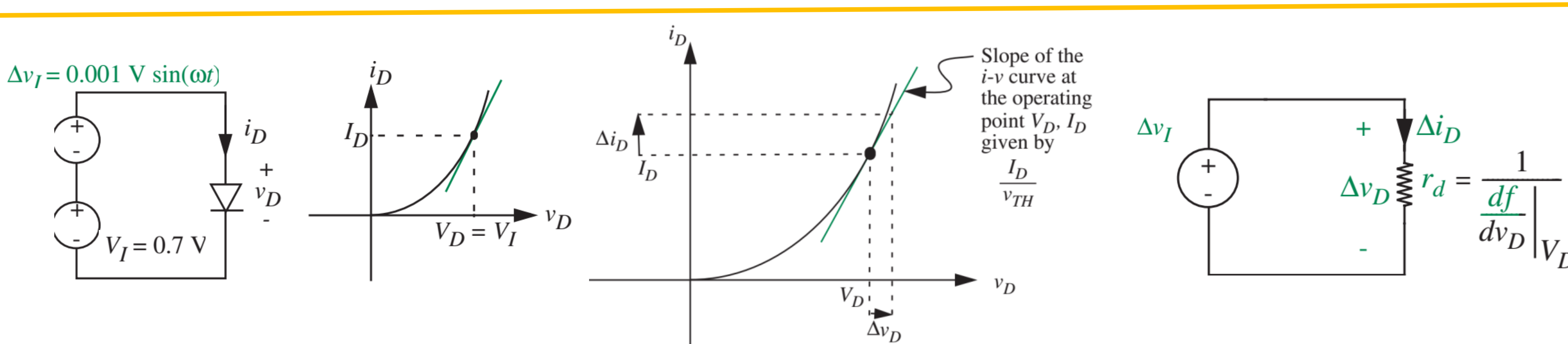
- NOT covered by this course
- Analytical solution $\dfrac{v_D - E}{R} + i_D = 0$
- Graphical solution
- Incremental analysis

Refer to the reading notes from MIT on the course page.



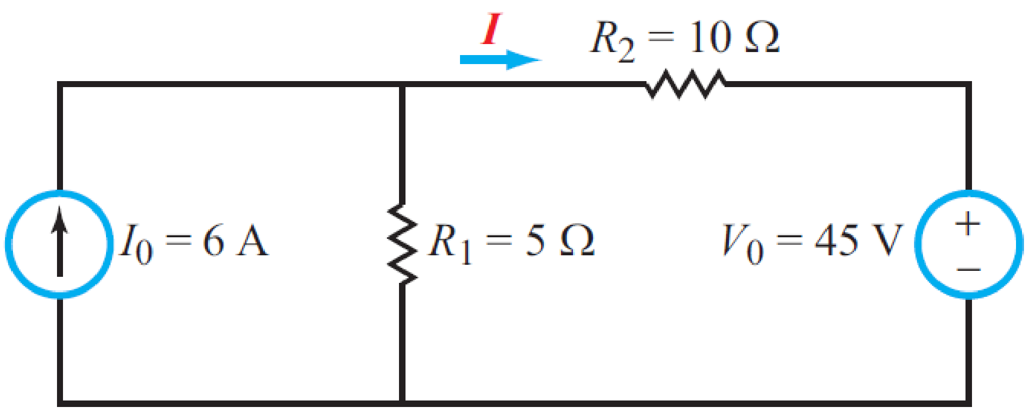
$$i_D = I_s(e^{v_D/V_{TH}} - 1)$$



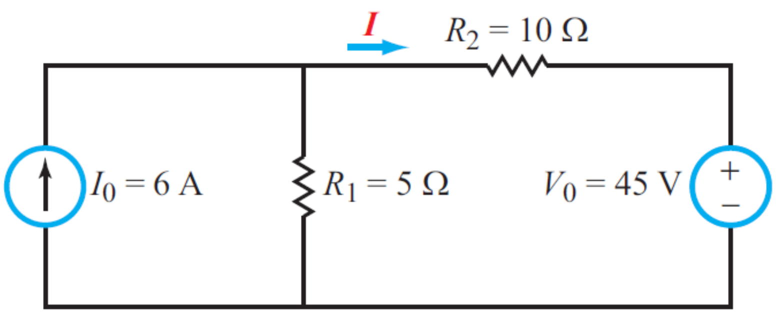


Superposition

- The superposition principle states that the voltage across (or current through) an element in *a linear circuit* is the algebraic sum of the voltages across (or currents through) that element due to each independent source acting alone.



Applying Superposition



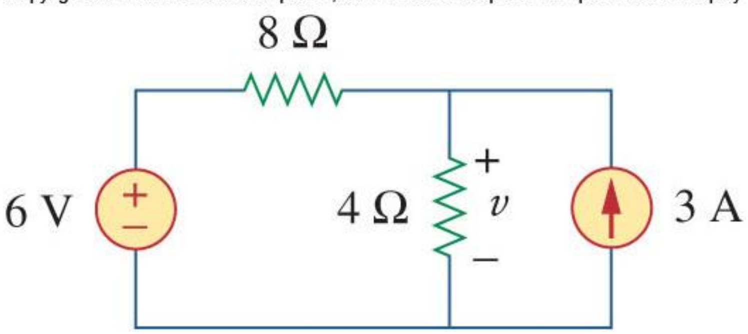
- The steps are:
  1. Turn off all independent sources except one source. Find the output (voltage or current) due to that active source.
     - Replace independent voltage source by short circuit (0 V), independent current source by open circuit (0 A).
  2. Repeat step 1 for each of the other independent sources.
  3. Find the total contribution by adding algebraically all the contributions due to the independent sources.

Note that

1) Using superposition means applying one independent source at a time.
2) Dependent sources are left alone.

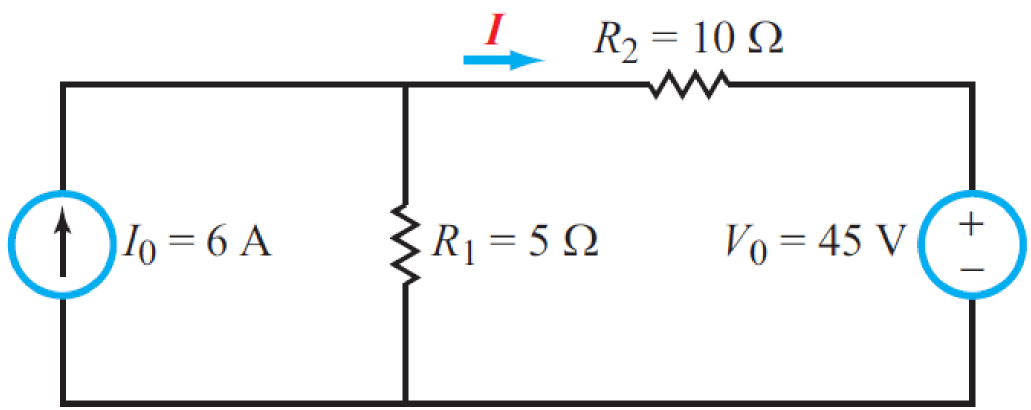
Open Circuit and Short Circuit

- Open circuit
  - i=0, i.e., cut off the branch
- Short circuit
  - v=0, i.e., replace the element by wire



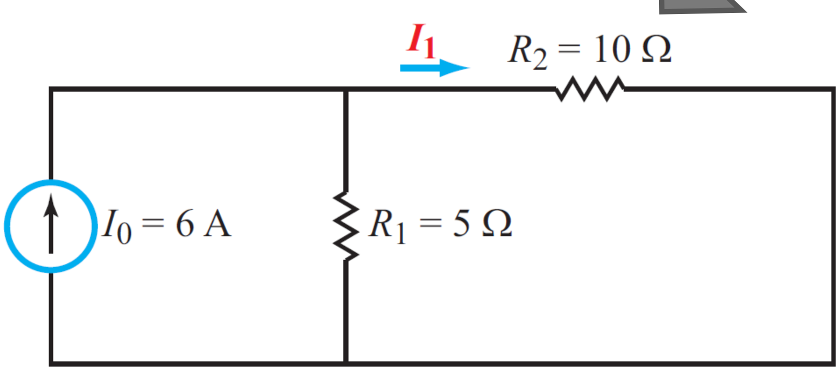
- Turn off an independent voltage source means
  - V=0
  - Replace by wire
  - Short circuit
- Turn off an independent current source means
  - i=0
  - Cut off the branch
  - open circuit

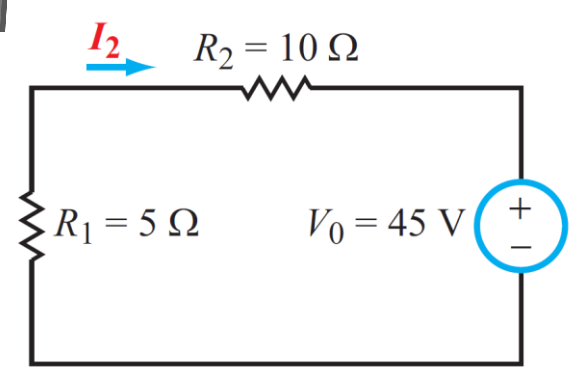
Example: Superposition



Contribution from I_0 alone

Contribution from V_0 alone





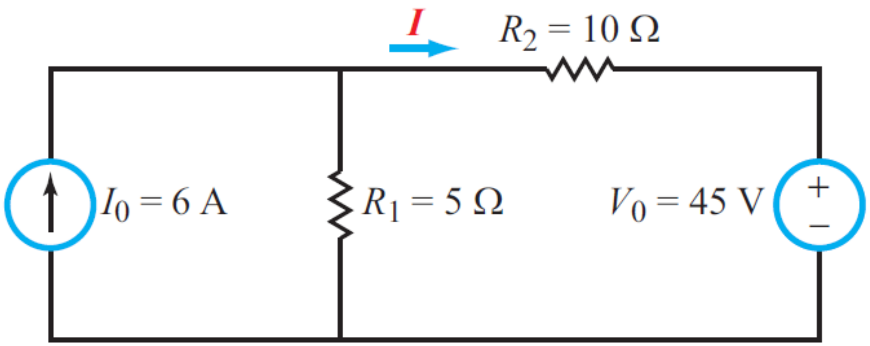
$I_1 = 2$ A

$$\boxed{I = I_1 + I_2 = 2 - 3 = -1\text{ A}}$$

$I_2 = -3$ A

[Source: Berkeley]

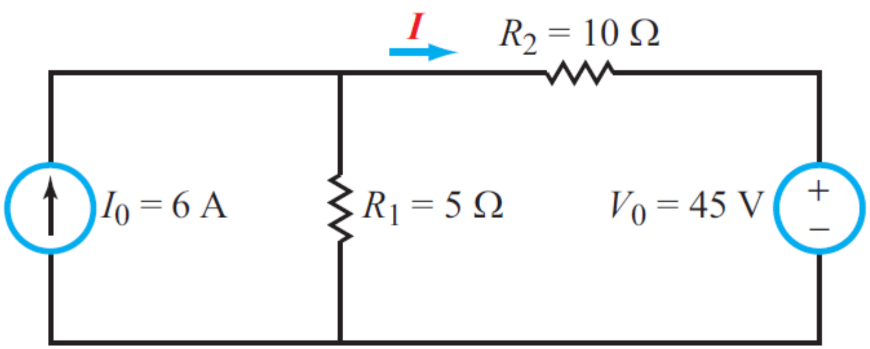
How about Power by R_2



- Power due to $I_0, P_1 =?$
- Power due to $V_0, P_2 =?$
- Power due to both V_0 and $I_0, P =?$

Why Superposition?

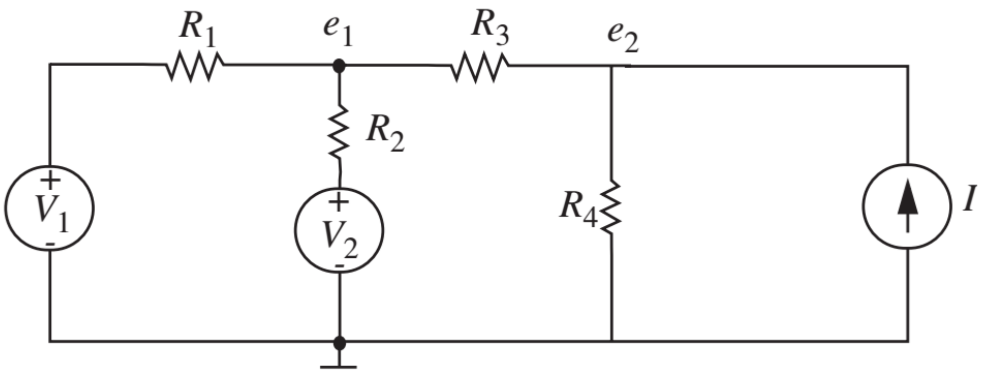
- Because it entails solving a circuit multiple times, this source-superposition method may not be attractive.
- But it is useful to evaluate the sensitivity of a response to specific sources in the circuit.



$$I = aI_0 + bV_0$$

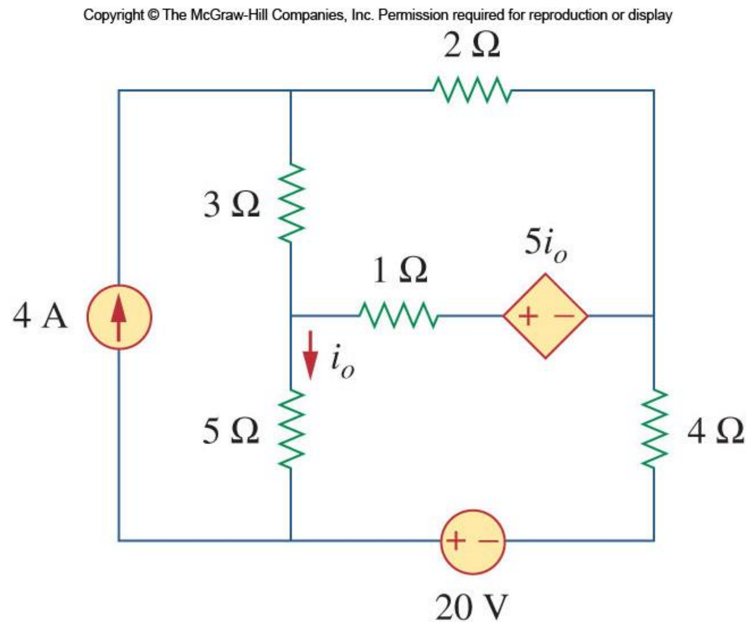
Practice 1

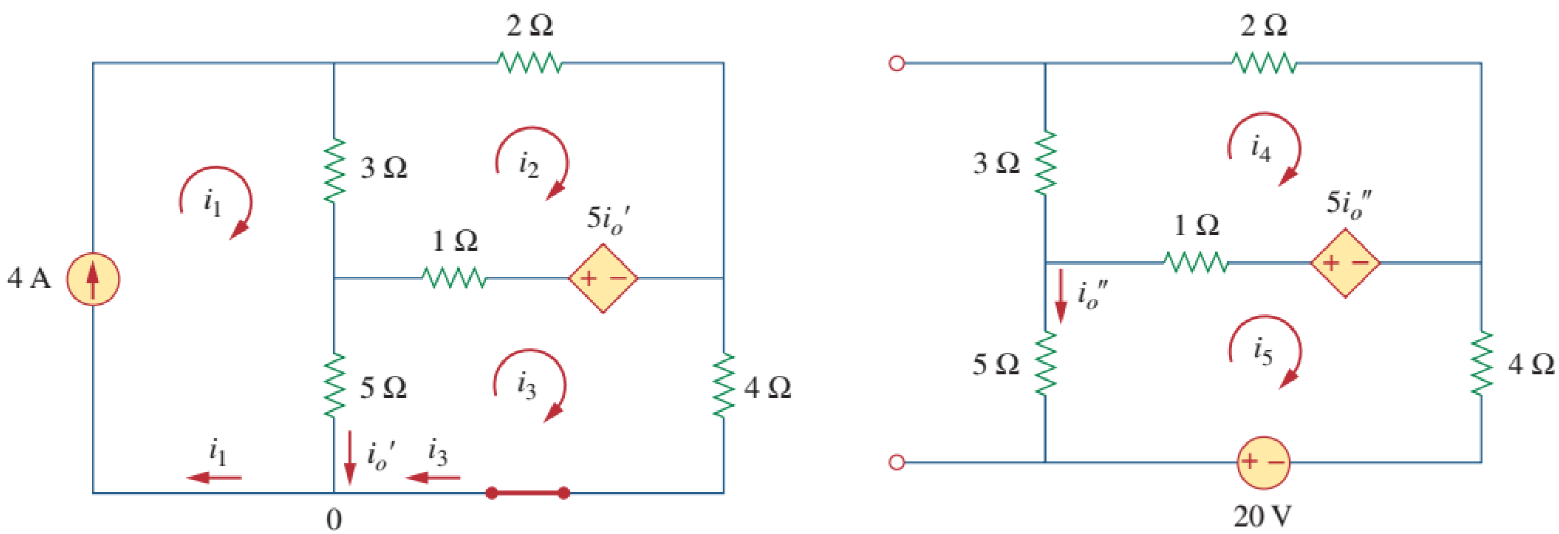
- Express node voltage e_1 as a function of two voltage sources V_1, V_2 and one current source *I*.



Practice 2

- Find i_0 in the circuit shown below.

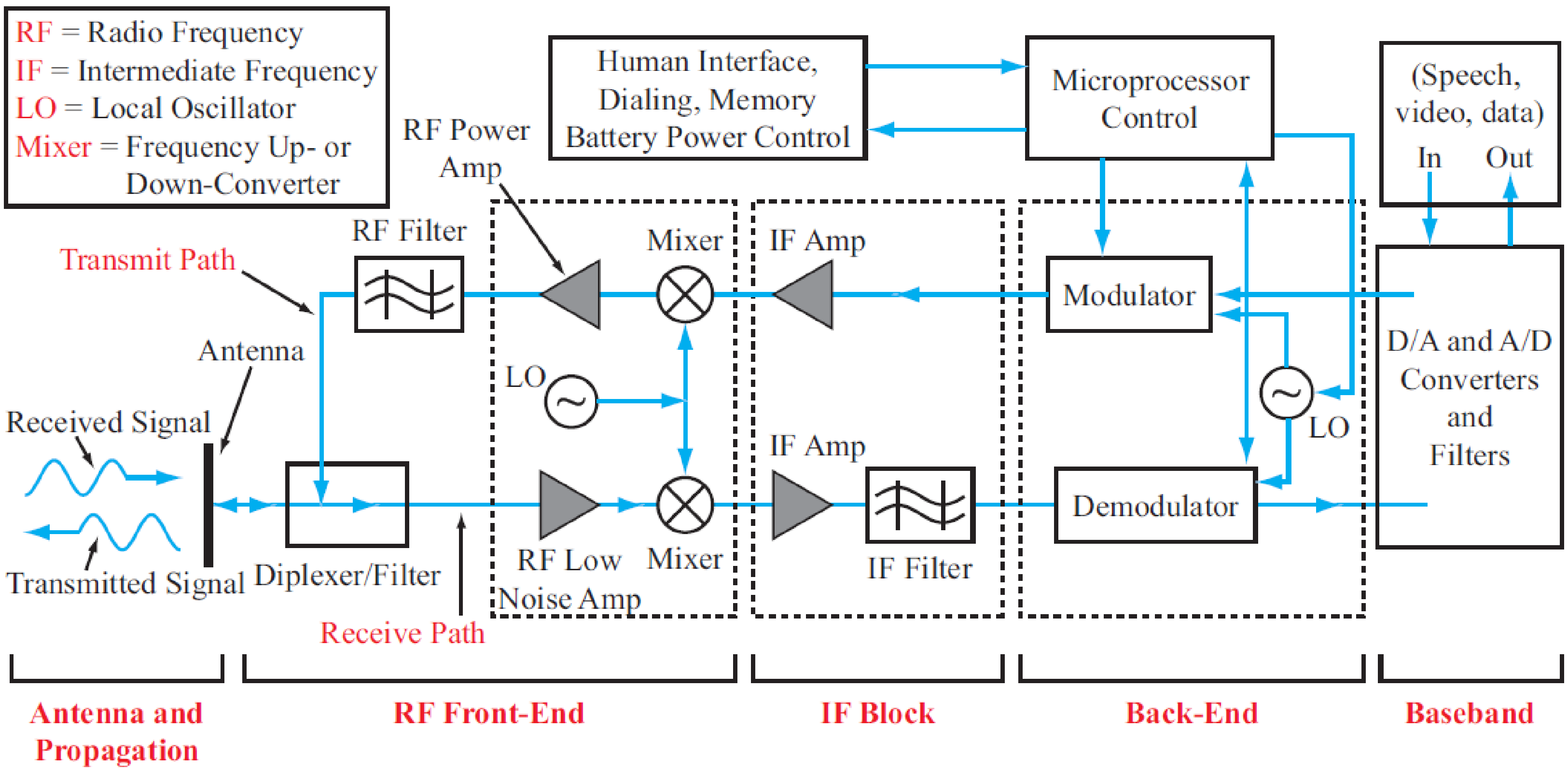



2 Ω
3 Ω
i_1
i_2
1 Ω
$5i_o'$
4 A
5 Ω
i_3
4 Ω
i_1
i_o'
i_3
0
2 Ω
3 Ω
i_4
1 Ω
$5i_o''$
i_o''
5 Ω
i_5
4 Ω
20 V

Outline

- Linearity property
- Superposition
- Thevenin's theorem
- Source transformation
- Norton's theorem

Thevenin's Theorem – Motivation 1

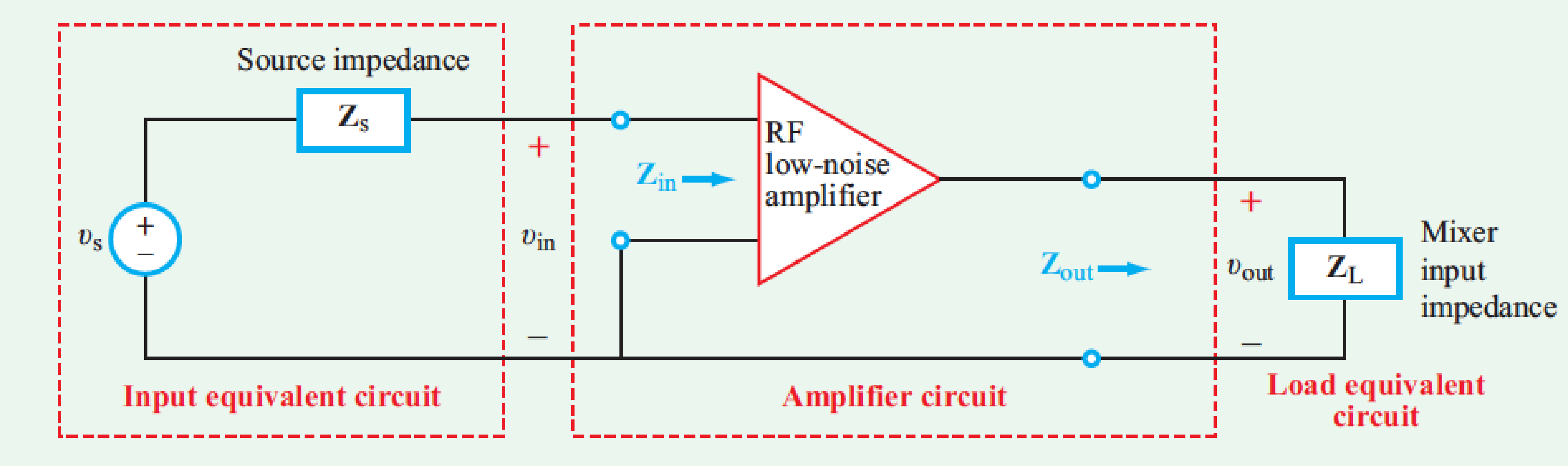


Circuit systems can be complex. How does an engineer handle such a complex architecture?

[Source: Berkeley]

Equivalent Circuit Representation

- Fortunately, many circuits are linear.
- Simple equivalent circuits may be used to represent complex circuits.

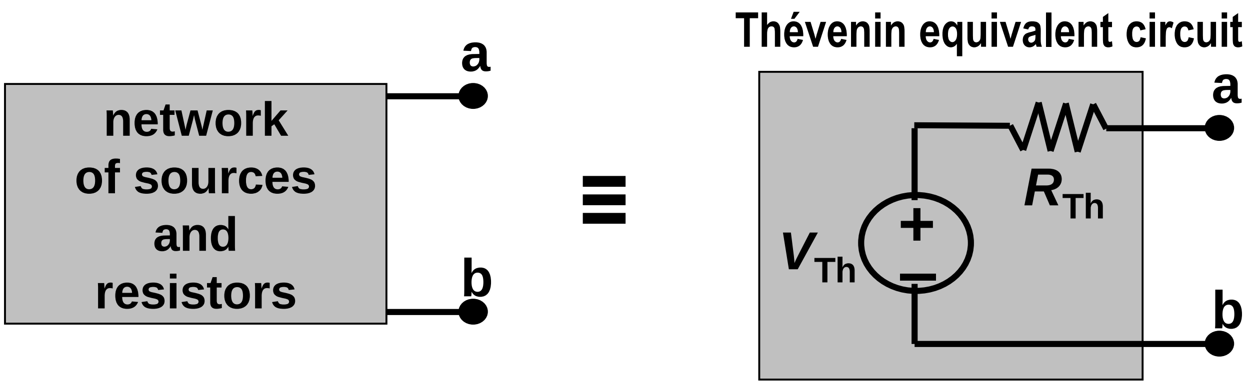


Isolating the amplifier, while keeping it in the context of its input and output neighbors, facilitates both the analysis and design processes.

[Source: Berkeley]

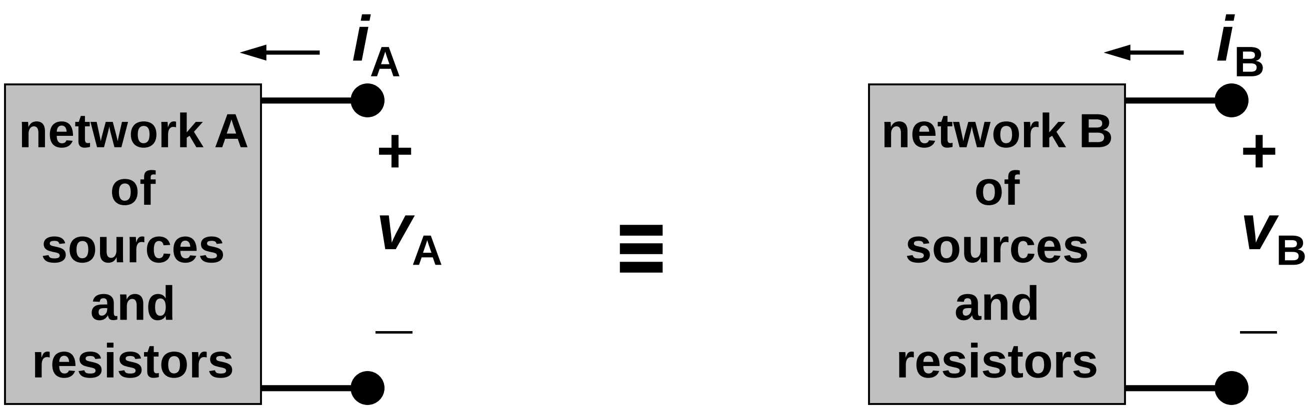
Thevenin's Theorem – Motivation 2

- In many circuits, one element will be variable (called *the load*), while others are fixed.
  - An example is the household outlet: many different appliances may be plugged into the outlet, each presenting a different resistance.
  - Ordinarily one has to re-analyze the circuit for load change.
  - This problem can be avoided by circuit theorem (e.g. Thevenin's theorem), which provides a technique to replace the fixed part of the circuit with an **equivalent** circuit.



Equivalent Circuit Concept

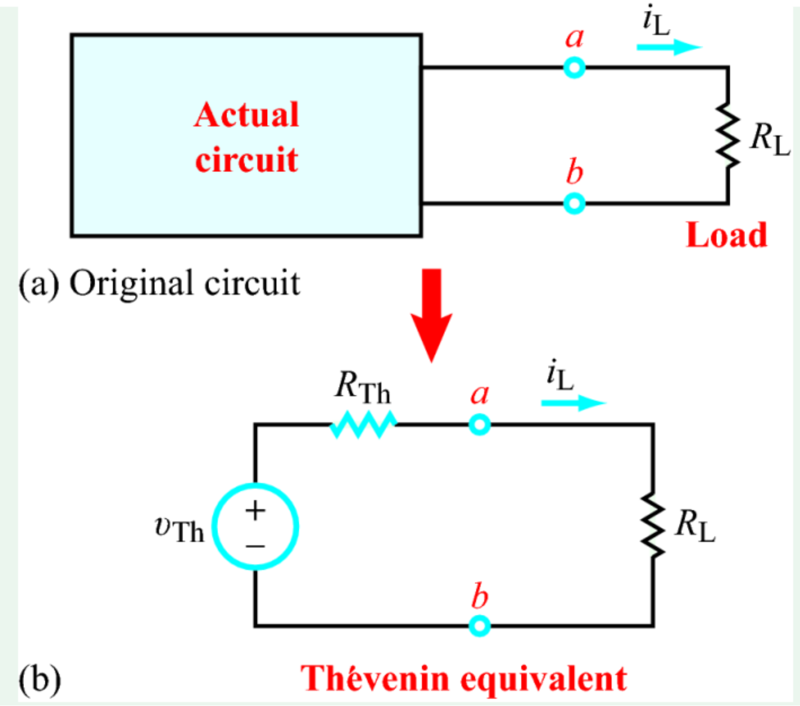
- A network of voltage sources, current sources, and resistors can be replaced by an **equivalent circuit** which has identical terminal properties (*I*-*V* characteristics) without affecting the operation of the rest of the circuit.



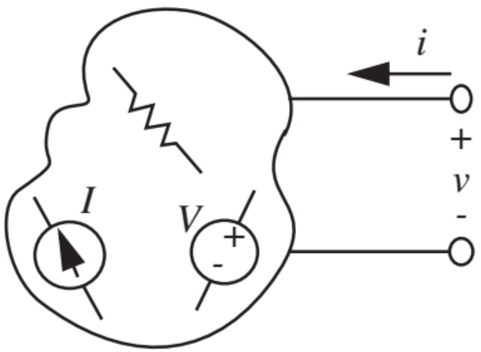
$$i_A(v_A) = i_B(v_B)$$

Thevenin's Theorem (1880s, Leon Thevenin, French)

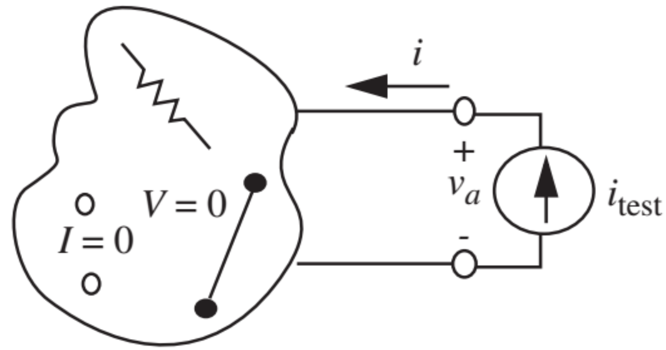
- Thevenin's theorem states that a linear two terminal circuit may be replaced with a voltage source in series with a resistor:



Derivation of Thevenin Network by Superposition

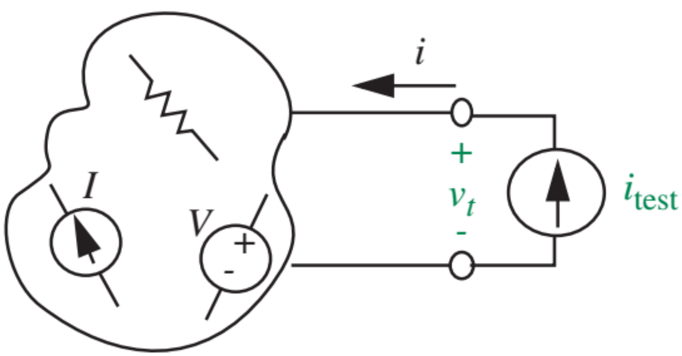


Apply external excitation i_{test}

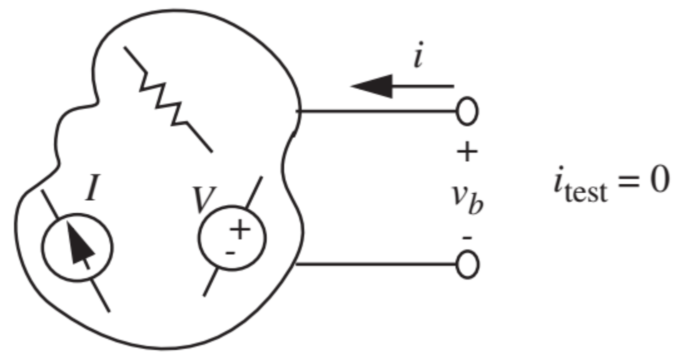


Zero internal ind. sources

$$v_a = i_{test} R_t$$



$$v_t = v_a + v_b = v_{oc} + i_{\text{test}} R_t.$$



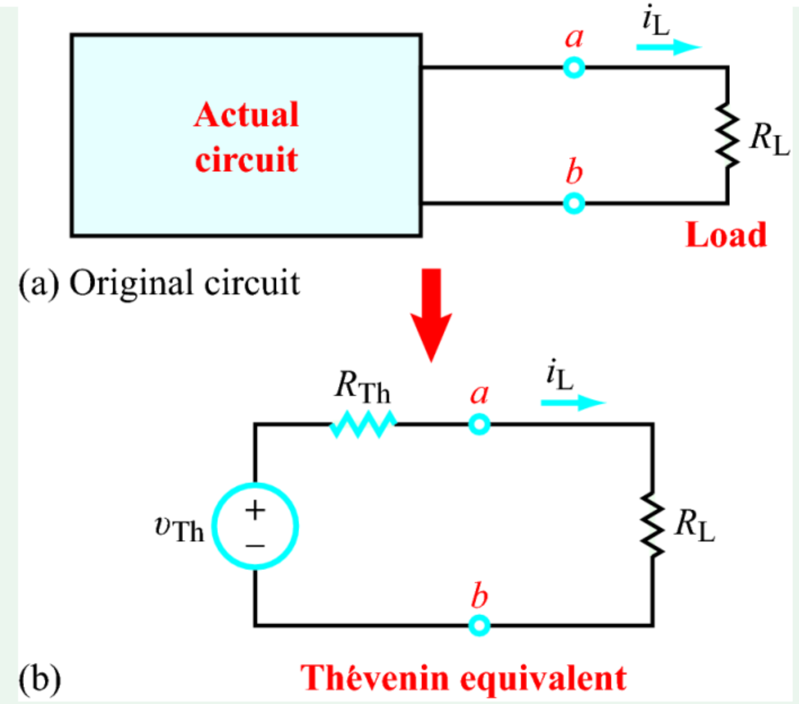
Zero external test source

$$v_b = v_{oc}$$

Thevenin's Theorem (1880s, Leon Thevenin, French)

- Thevenin's theorem states that a linear two terminal circuit may be replaced with a voltage source in series with a resistor:

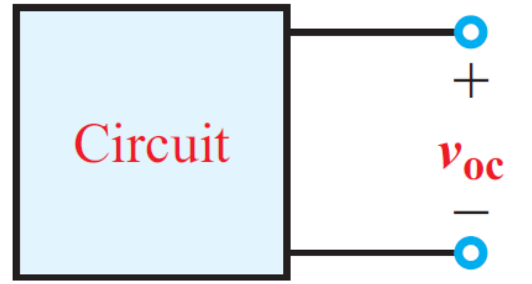
The voltage source's value is equal to the <u>open circuit voltage</u> at the terminals.
The resistance is equal to the resistance measured at the terminals <u>when the independent sources are turned off</u>.



How Do We Find Thévenin Equivalent Circuits?

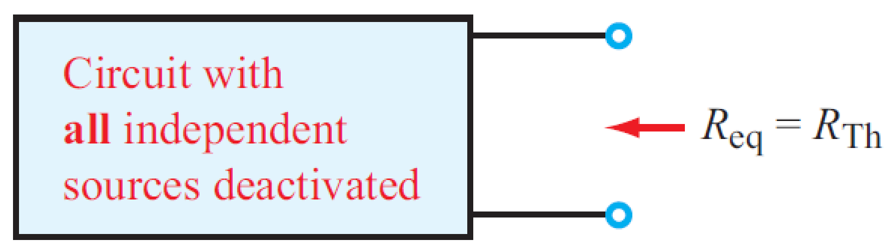
Method 1: Equivalent Resistance

1. Analyze circuit to find v_{oc}
2. Deactivate all independent sources by replacing voltage sources with short circuits and current sources with open circuits.
3. Simplify circuit to find equivalent resistance.



(a) $v_{Th} = v_{oc}$

Equivalent-Resistance Method

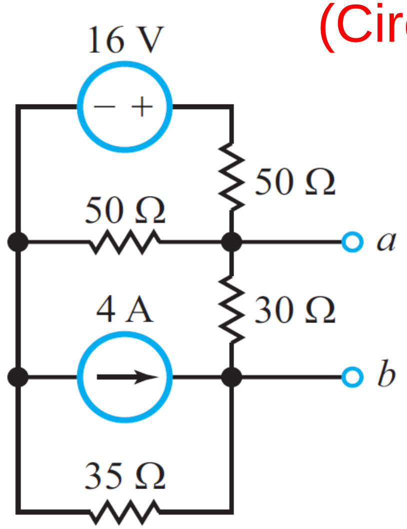


Note: This method does not apply to circuits that contain dependent sources.

Practice

Find R_{Th}

(Circuit has no dependent sources)



How Do We Find Thévenin Equivalent Circuits ?

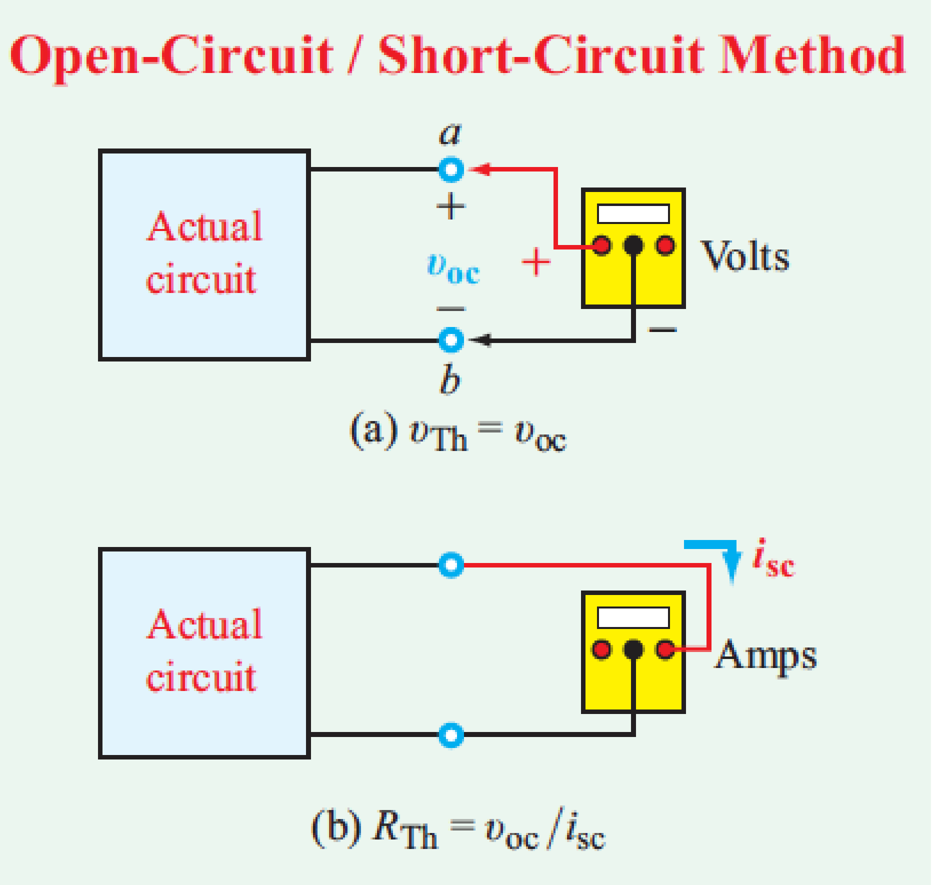
Method 2: Open/short circuit

1. Analyze circuit to find v_{oc}
2. Analyze circuit to find i_{sc}

$$v_{\mathrm{Th}} = v_{\mathrm{oc}}$$

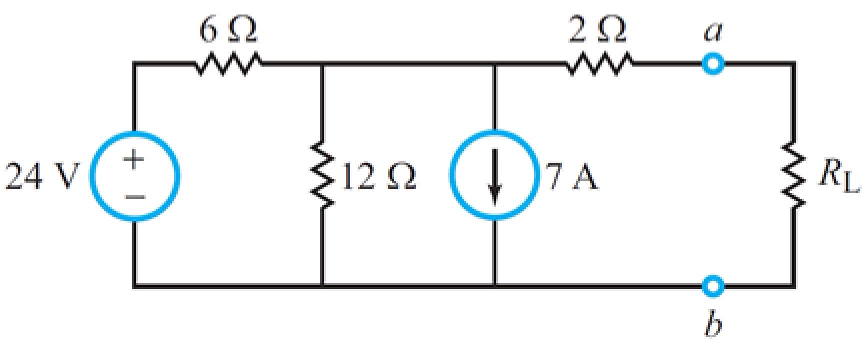
$$R_{\mathrm{Th}} = \frac{v_{\mathrm{Th}}}{i_{\mathrm{sc}}}$$

Note: This method is applicable to any linear circuit, whether or not it contains dependent sources.



[Source: Berkeley]

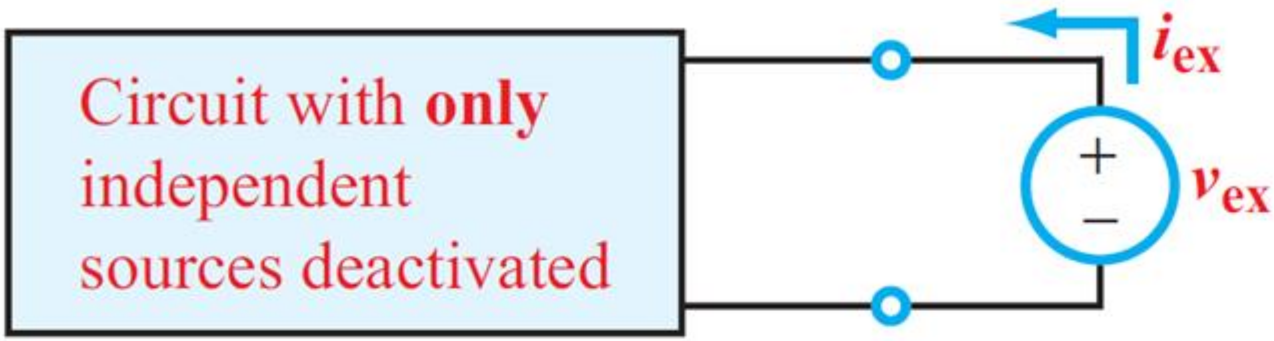
Example



How Do We Find Thévenin Equivalent Circuits?

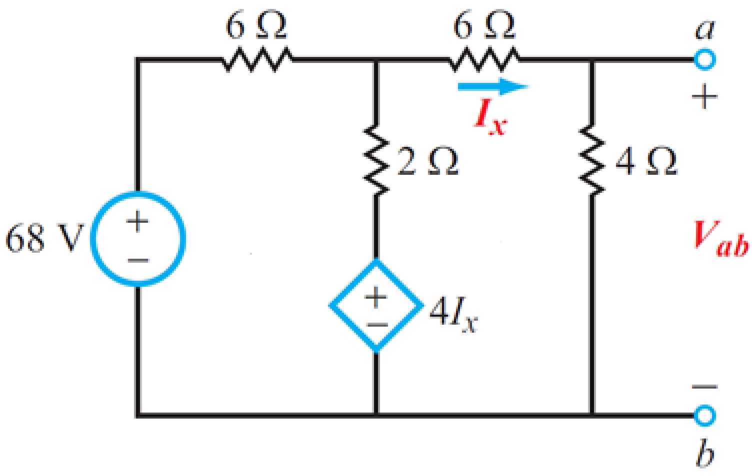
Method 3:

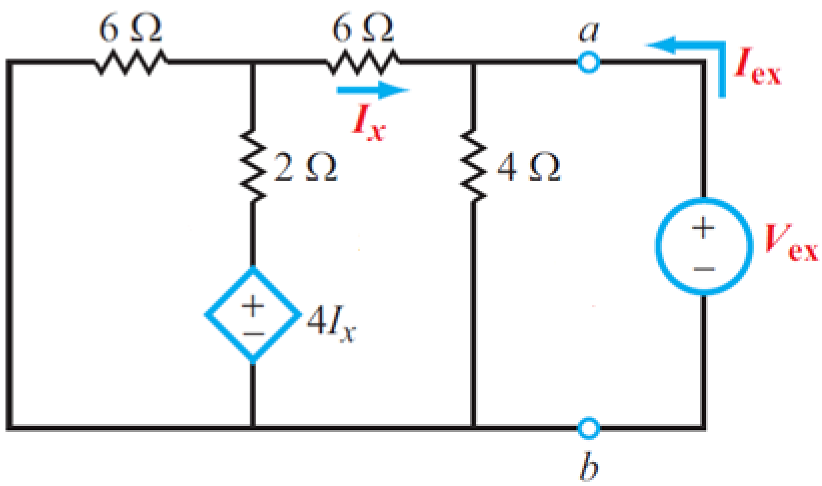
External-Source Method



If a circuit contains both dependent and independent sources, R_{Th} can be determined by (a) deactivating independent sources (only), (b) adding an external source v_{ex}, and then (c) solving the circuit to determine i_{ex}. The solution is $R_{Th} = v_{ex}/i_{ex}$.

Example

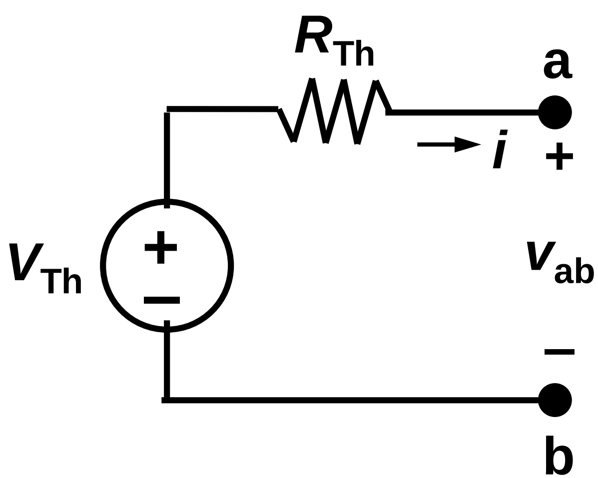


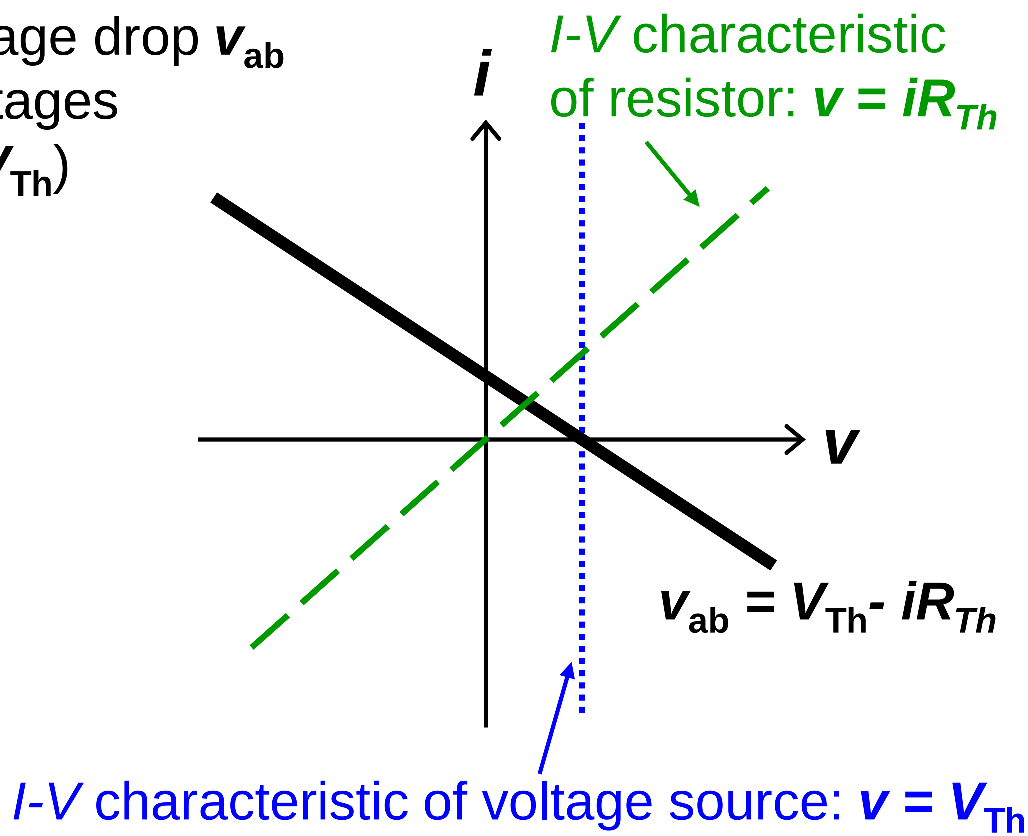


I-V Characteristic of Thévenin Equivalent

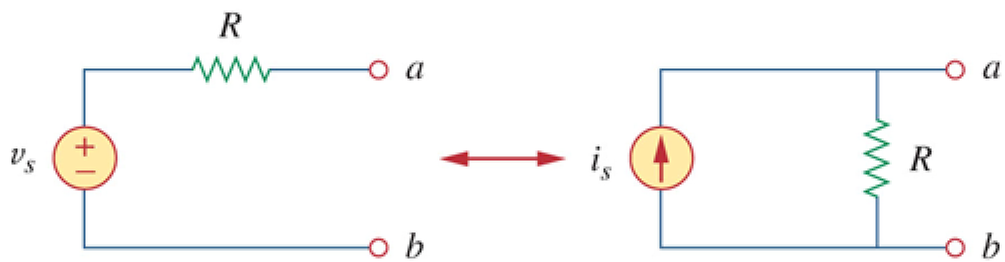
- The *I-V* characteristic for the series combination of elements is obtained by adding their voltage drops.

For a given current $\boldsymbol{i}$, the voltage drop $\boldsymbol{v_{ab}}$ is equal to the sum of the voltages dropped across the source ($\boldsymbol{V_{Th}}$) and across the resistor ($\boldsymbol{iR_{Th}}$)



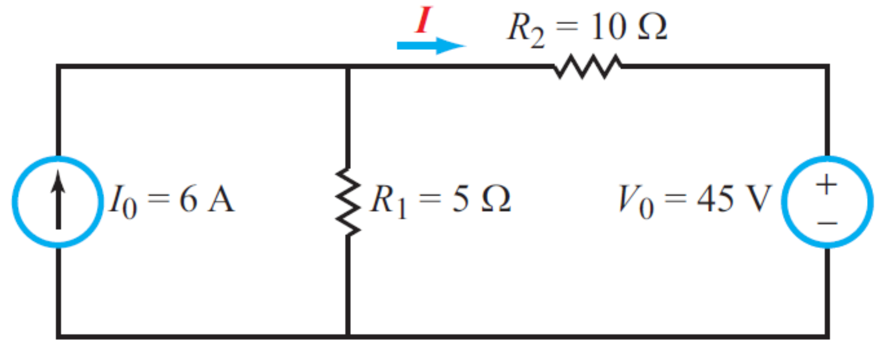


Source Transformation



- A source transformation is the process of replacing a voltage source v_s in series with a resistor R by a current source i_s in parallel with a resistor R, or vice versa.
- These transformations work because the two sources have equivalent behavior at their terminals:
  - If the sources are turned off, resistance at the terminals are both R
  - If the terminals are short circuited, the currents need to be the same.

Revisit Extension 3

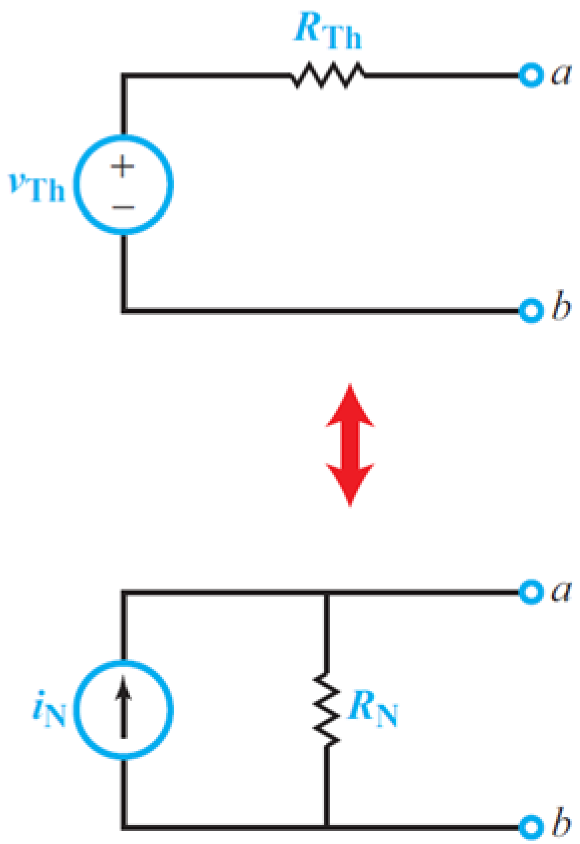


- Q7: If $R_2 = 1\Omega, I =$?
- Q8: What if $R_2 = 5\Omega$?

Norton's Theorem

Thévenin equivalent circuit

Norton equivalent circuit

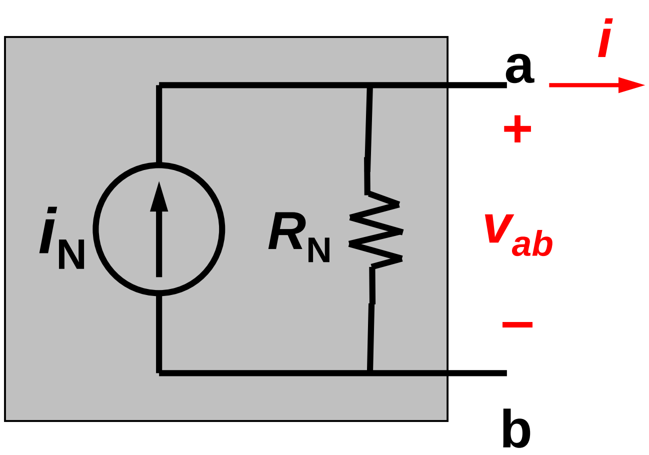


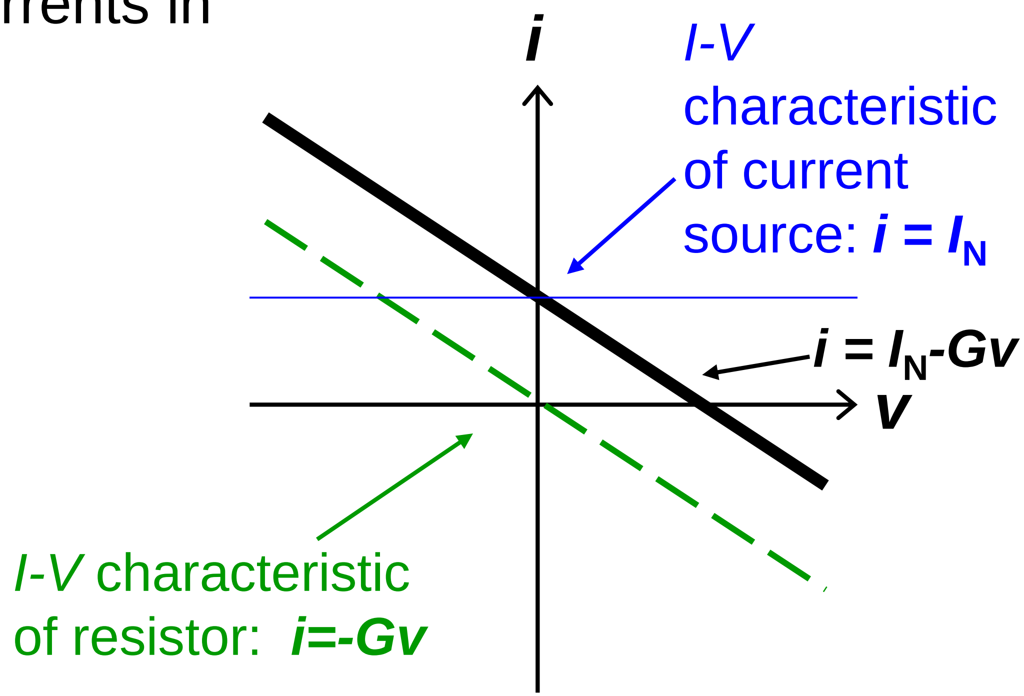
[Source: Berkeley]

I-V Characteristic of Norton Equivalent

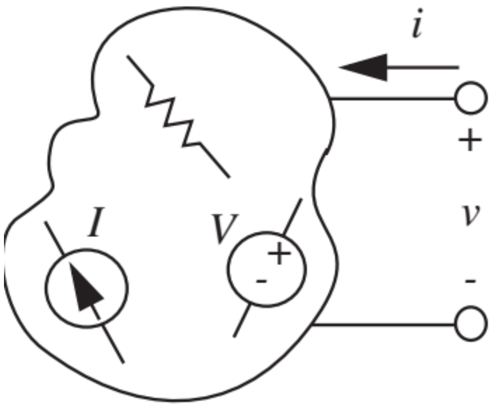
- The *I-V* characteristic for the parallel combination of elements is obtained by adding their currents:

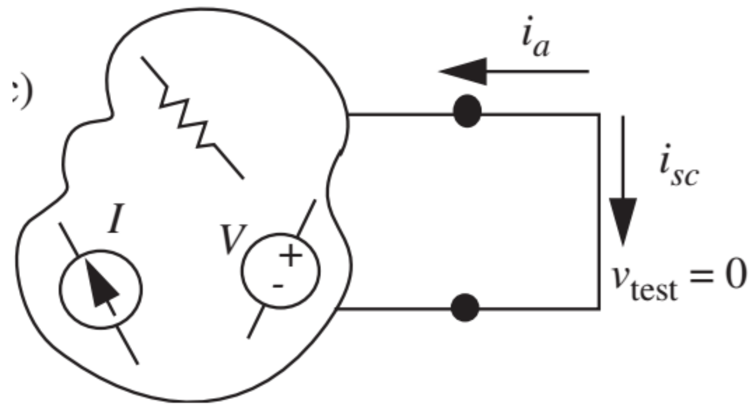
For a given voltage v_{ab}, the current i is equal to the sum of the currents in each of the two branches:

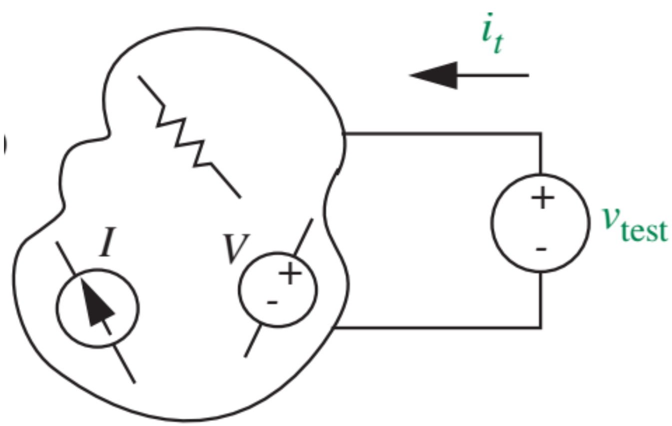


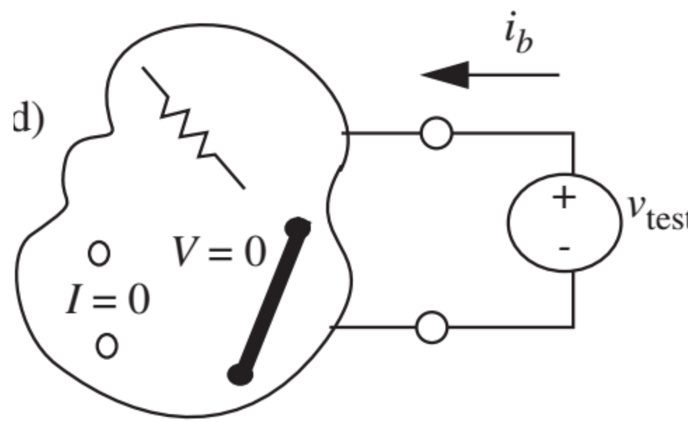


Derivation of Norton Network



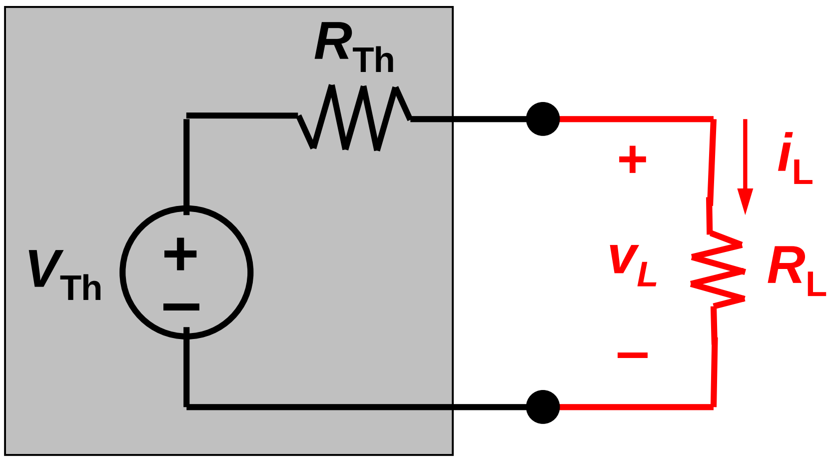


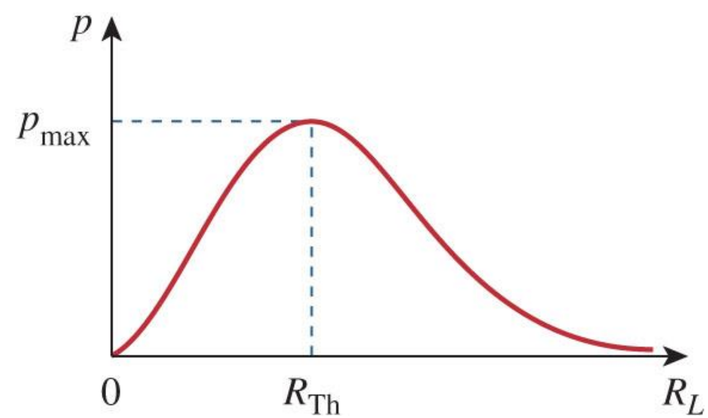




$$i_t = i_a + i_b = -i_{sc} + \frac{v_{test}}{R_t}.$$

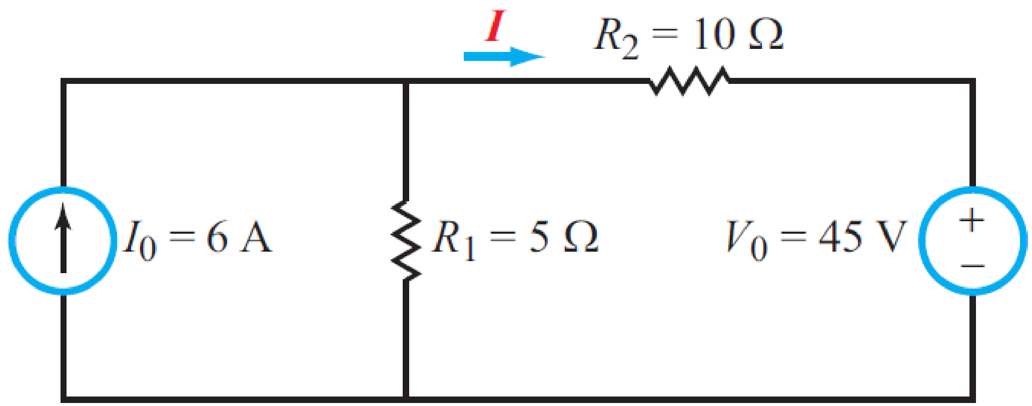
Max Power Transfer



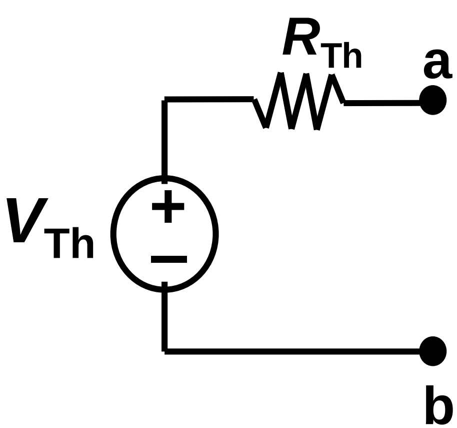


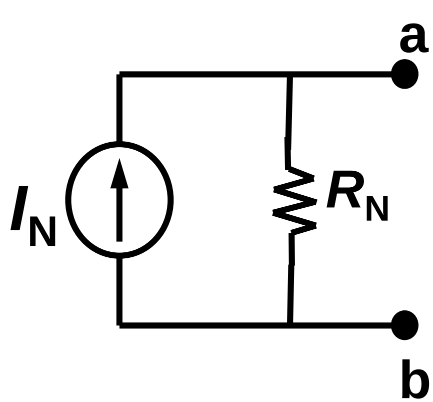
Summary

- Superposition
  - Voltage off → SC
  - Current off → OC



- Thevenin and Norton Equivalent Circuits
  - Solve for OC voltage
  - Solve for SC current





$$I_N = \frac{V_{Th}}{R_{Th}}$$

$$R_{\mathrm{N}} = R_{\mathrm{Th}}$$