Problem 1

(20 points) Determine the values of P_∞ and E_∞ for each of the following signals:

(a) $x_2(t) = e^{j(2t+\frac{\pi}{4})}$

(b) $x_1[n] = \left(\frac{1}{2}\right)^n u[n]$

每算错一个–5分

Solution

(a) (10 Points) $x_2(t) = e^{j\left(2t+\frac{\pi}{4}\right)}$, $|x_2(t)| = 1$. Therefore, $E_\infty = \int_{-\infty}^{\infty} |x_2(t)|^2 \, dt = \int_{-\infty}^{+\infty} dt = \infty$,

$P_\infty = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x_2(t)|^2 \, dt = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} dt = \lim_{T\to\infty} 1 = 1$

(b) (10 Points) $x_1[n] = \left(\frac{1}{2}\right)^n u[n], |x_1(n)|^2 = \left(\frac{1}{4}\right)^n u[n]$.

Therefore, $E_\infty = \sum_{n=-\infty}^{+\infty} |x_1[n]|^2 = \sum_{n=0}^{+\infty} \left(\frac{1}{4}\right)^n = \frac{4}{3}$

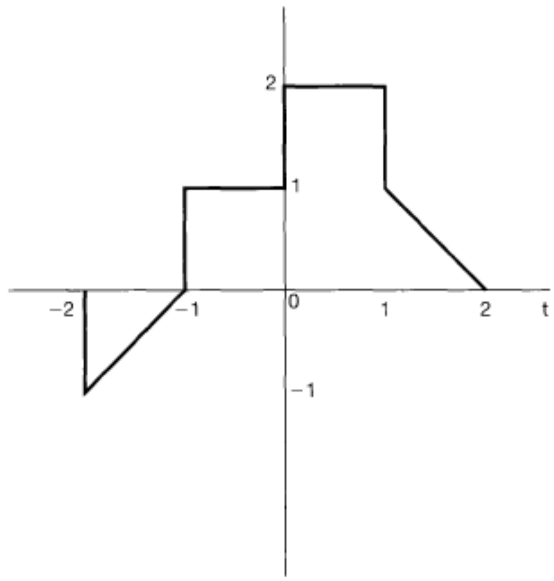
$P_\infty = 0, \quad$ because $E_\infty < \infty$

Problem 2

(20 points) A continuous-time signal $x(t)$ is shown in the following figure. Sketch and label carefully each of the following signals:

(a) $x(2t+1)$

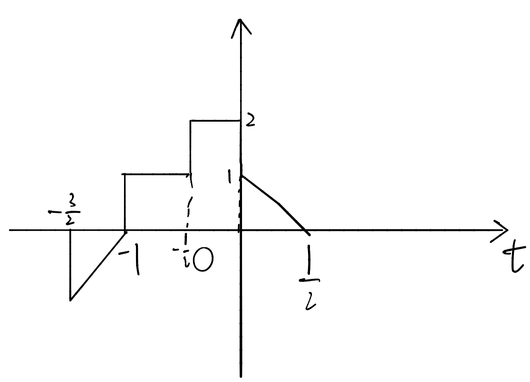
(b) $x(t)\left[\delta\left(t+\frac{3}{2}\right)-\delta\left(t-\frac{3}{2}\right)\right]$



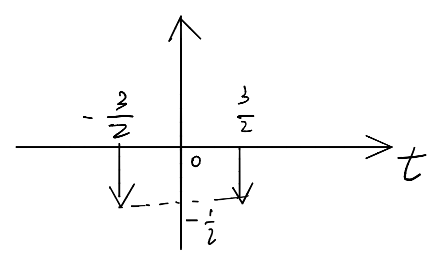
画错−10分

每画错一边−5分

(a)



(b)



Problem 3

(15 points) Determine whether or not each of the following continuous-time signals is periodic. If the signal is periodic, determine its fundamental period.

(a) $x(t) = 3\cos\left(4t + \frac{\pi}{3}\right)$

(b) $x(t) = \mathrm{E}_v\left\{|sin(4\pi t)|\, u(t)\right\}$

(c) $x(t) = e^{j(\pi t - 1)}$

判断周期性2分，化简公式2分，计算周期1分

Solution

(a) (5 points) Periodic, period $= \frac{\pi}{2}$

(b) (5 points) Periodic, period $= \frac{1}{4}$

(c) (5 points) Periodic, period $= 2\pi/\pi = 2$.

Problem 4

(25 points) In this chapter, we introduced a number of general properties of systems. In particular, a system may or may not be

(1) Memoryless

(2) Invertible

(3) Causal

(4) Stable

(5) Time invariant

(6) Linear

Determine which of the properties hold for each of the following continuous-time systems. Justify your answers. In each example, $y(t)$ denotes the system output and $x(t)$ is the system input.

(a)

$$y(t) = [\cos(3t)]x(t)$$

判断性质每个一分

证明线性和时不变性每个1.5分

(b)

$$y(t) = \begin{cases} 0, & t < 0 \\ x(t) + x(t-2), & t \geq 0 \end{cases}$$

(c)

$$y(t) = \begin{cases} 0, & x(t) < 0 \\ x(t) + x(t-2), & x(t) \geq 0 \end{cases}$$

Solution

(a) (10 points) The system is memoryless, causal, stable, linear.
Time-invariant:

$$y(t-t_0) = [\cos(3t-3t_0)]x(t-t_0) \neq [\cos(3t)]x(t-t_0)$$

Linear:
Suppose $x_3(t) = ax_1(t) + bx_2(t)$,

$$y_3(t) = [\cos(3t)]x_3(t) = a\cos(3t)x_1(t) + b\cos(3t)x_2(t) = ay_1(t) + by_2(t)$$

(b) (7.5 points) The system is causal, stable, linear.
Time-invariant:

$$y(t-t_0) = \begin{cases} 0, & t < t_0 \\ x(t-t_0) + x(t-t_0-2), & t \geq t_0 \end{cases} \neq \begin{cases} 0, & t < 0 \\ x(t-t_0) + x(t-t_0-2), & t \geq 0 \end{cases}$$

Linear: If $x_3(t) = ax_1(t) + bx_2(t)$,

$$y_3(t) = \begin{cases} 0, & t < 0 \\ x_3(t) + x_3(t-2), & t \geq 0 \end{cases} = \begin{cases} 0, & t < 0 \\ ax_1(t) + bx_2(t) + ax_1(t-2) + bx_2(t-2), & t \geq 0 \end{cases} = ay_1(t) + by_2(t)$$

Problem 4 continued on next page. . .

(c) (7.5 points) The system is causal, stable, time-invariant.

Time-invariant:

$$y(t-t_0) = \begin{cases} 0, & x(t-t_0) < 0 \\ x(t-t_0) + x(t-t_0-2), & x(t-t_0) \geq 0 \end{cases} = \begin{cases} 0, & x(t-t_0) < 0 \\ x(t-t_0) + x(t-t_0-2), & x(t-t_0) \geq 0 \end{cases}$$

(system with input x$(t-t_0)$)

Linear: If $x_3(t) = ax_1(t) + bx_2(t)$,

$$y_3(t) = \begin{cases} 0, & x_3(t) < 0 \\ x_3(t) + x_3(t-2), & x_3(t) \geq 0 \end{cases} = \begin{cases} 0, & ax_1(t) + bx_2(t) < 0 \\ ax_1(t) + bx_2(t) + ax_1(t-2) + bx_2(t-2), & ax_1(t) + bx_2(t) \geq 0 \end{cases}$$

$$\neq ay_1(t) + by_2(t) = \begin{cases} 0, & x_1(t) < 0 \\ ax_1(t) + ax_1(t-2), & x_1(t) \geq 0 \end{cases} + \begin{cases} 0, & x_2(t) < 0 \\ bx_2(t) + bx_2(t-2), & x_2(t) \geq 0 \end{cases}$$

Problem 5

(20 points) Let $x[n]$ be a discrete-time signal, and let

$$y_1[n] = x[2n] \quad \text{and} \quad y_2[n] = \begin{cases} x[n/2], & \text{n even} \\ 0, & \text{n odd} \end{cases}$$

The signals $y_1[n]$ and $y_2[n]$ respectively represent in some sense the speeded up and slowed down versions of $x[n]$. However, it should be noted that the discrete-time notions of speeded up and slowed down have subtle differences with respect to their continuous-time counterparts. Consider the following statements:

(1) If $x[n]$ is periodic, then $y_1[n]$ is periodic.

(2) If $y_1[n]$ is periodic, then $x[n]$ is periodic.

(3) If $x[n]$ is periodic, then $y_2[n]$ is periodic.

(4) If $y_2[n]$ is periodic, then $x[n]$ is periodic.

判断2分。解释3分，其中一三四问写出周期相等得1分，写出周期比1/2k为得2分，未写出k倍扣一分，第二问举出反例得3分。

For each of these statements, determine whether it is true, and if so, determine the relationship between the fundamental periods of the two signals considered in the statement. If the statement is not true, produce a counterexample to it.

Solution

(1) (5 points) True. Suppose $x[n] = x[n + k_1 N_1]$, $y_1[n + k_2 N_2] = x\left[2(n + k_2 N_2)\right] = x[2n + k_1 N_1]$. Thus, $2k_2 N_2 = k_1 N_1$ and $\frac{N_2}{N_1} = \frac{k_1}{2k_2}$, with $k_1 \leq 2k_2$ and $k_1, k_2 \in \mathcal{N}^+$, Since k_1 and k_2 takes any poisitive integer at the same time, $\frac{N_2}{N_1} = \frac{k_1}{2k_2} = \frac{1}{2k}$ or 1, $k \in \mathcal{N}^+$

(2) (5 points) False. $y_1[n]$ is periodic does not imply $x[n]$ is periodic. e.g. let $x[n] = \begin{cases} 1, & \text{n even} \\ (1/2)^n, & \text{n odd} \end{cases}$ Then $y_1[n] = x[2n]$ is periodic but $x[n]$ is clearly not periodic.

(3) (5 points) True. $x[n] = x[n + k_1 N_1]$, when n is an even number, $y_2[n] = y_2\left[n + k_2 N_2^{'}\right] = x[\frac{n+k_2 N_2^{'}}{2}] = x[\frac{n}{2}] = x[\frac{n}{2} + k_1 N_1]$. Thus, $\frac{N_2^{'}}{N_1} = \frac{2k_1}{k_2}$, $k_2 \leq 2k_1$. Similar with (a), $\frac{N_2^{'}}{N_1} = 2k$ or 1. When n is odd, period of $y_2[n]$ is 2. Therefore, $\frac{N_2}{N_1} = 2k$ or 1, $k, k_1, k_2 \in \mathcal{N}^+$

(4) (5 points) True. $y_2[n + k_2 N_2] = y_2[n]; x\left[n + k_1 N_1\right] = x[n]$ and $\frac{N_1}{N_2} = \frac{k_2}{2k_1} = \frac{1}{2k}$ or 1, with $k_2 \leq 2k_1$ and $k, k_1, k_2 \in \mathcal{N}^+$.