Problem 1

(20 points) A causal LTI filter has the frequency response $H(j\omega)$ shown in Figure 1. For each of the input signals given below, determine the filtered output signal $y(t)$.

(a) $x(t) = e^{jt}$

(b) $x(t) = (sin\omega_0 t)u(t)$

(c) $X(j\omega) = \frac{1}{(j\omega)(6+j\omega)}$

(d) $X(j\omega) = \frac{1}{2+j\omega}$

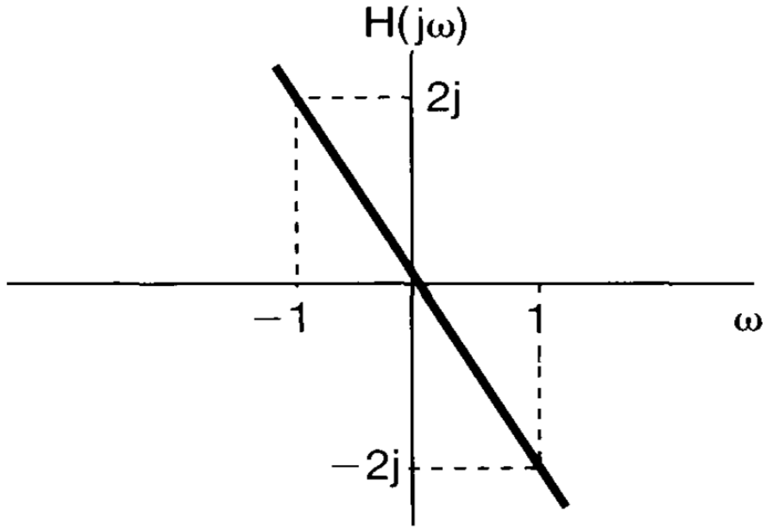

Figure 1: Problem 1

Problem 2

(20 points) The output $y(t)$ of a causal LTI system is related to the input $x(t)$ by the differential equation

$$\frac{dy(t)}{dt} + 2y(t) = x(t) \tag{1}$$

(a) Determine the frequency response

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} \tag{2}$$

of the system, and sketch its Bode plot.

(b) Specify, as a function of frequency, the group delay associated with this system.

(c) If $x(t) = e^{-t}u(t)$, determine $Y(j\omega)$, the Fourier transform of the output.

Problem 3

(20 points) The frequency which, under the sampling theorem, must be exceeded by the sampling frequency is called the *Nyquist rate*. Determine the Nyquist rate corresponding to each of the following signals:

(a) $x(t) = 1 + cos(2000\pi t) + sin(4000\pi t)$

(b) $x(t) = \frac{sin(4000\pi t)}{\pi t}$

(c) $x(t) = \left(\frac{sin(4000\pi t)}{\pi t}\right)^2$

Problem 4

(10 points) Consider the discrete-time sequence $x[n] = cos[n\pi/4]$, find two different continuous-time signals that would produce this sequence when sampled at a frequency of $f_s = 10$ kHz.

Problem 5

(30 points) Suppose that we would like to slow a segment of speech to one-half its normal speed. The speech signal $s_a(t)$ is assumed to have no energy outside of 5 kHz, and is sampled at a rate of 10 kHz, yielding the sequence

$$s[n] = s_a(nT_s) \tag{3}$$

The following system is proposed to create the slowed-down speech signal. Assume that $S_a(\omega)$ is as shown in the following figure:

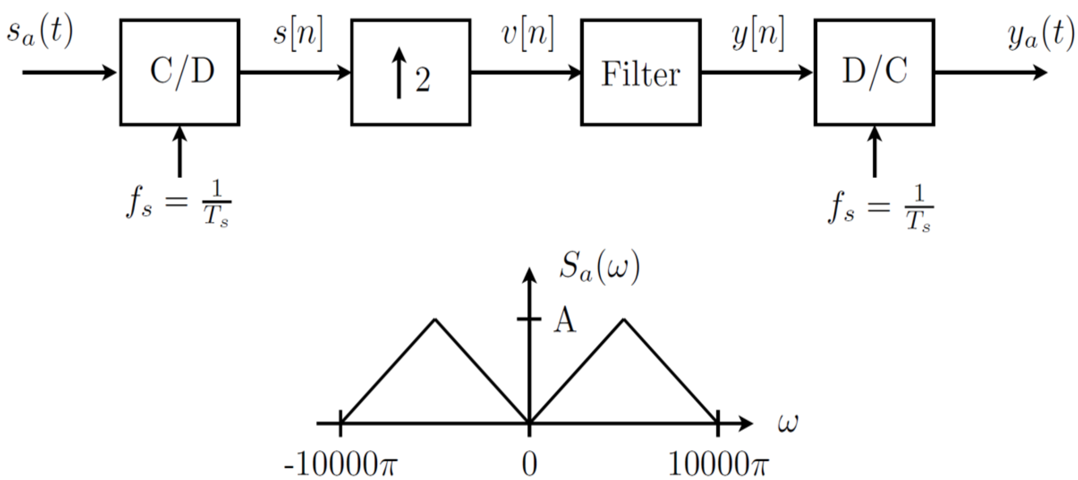

Figure 2: Problem 5

(a) Find the spectrum of $v[n]$.

(b) Suppose that the discrete-time filter is described by the difference equation:

$$y[n] = v[n] + \frac{1}{2}(v[n-1] + v[n+1]) \tag{4}$$

Find the frequency response of the filter and describe its effect on $v[n]$.

(c) What is $Y_a(\omega)$ in terms of $X_a(\omega)$? Does $y_a(t)$ correspond to slowed-down speech?