

# Profit Maximization in Mechanism Design

Optimal Auction Design. Myerson 1981

ch 13.2

## Vickrey auction with reservation price $r$

单物品：VCG = second price auction

- Two bidders
- $b_1, b_2 \in [0, 1]$  Random/uniform
- $\min(b_1, b_2)$
- $Pr(\min(b_1, b_2) > x) = (1 - x)^2$
- $F(x) = 1 - (1 - x)^2$
- $f(x) = F'(x) = 2 - 2x$
- $E(\min(b_1, b_2)) = \int_0^1 x f(x) dx = \int_0^1 x(2 - 2x) = x^2 - 2/3 x^3 \Big|_0^1 = 1/3$

### Reserve Price

- $VCG_r : \max(b_1, b_2) \geq r \rightarrow \text{sell}$
- $\text{payment} = \max(r, \min(b_1, b_2))$
- $P(\text{pay } r) = r * (1 - r) * 2$
- $E(\text{payment}) = r * r * (1 - r) * 2 + x^2 - 2/3 x^3 \Big|_r^1 = r^2 - 4/3 r^3 + 1/3$

明显好于VCG 且1/2最优

### Virtual Valuation

$$\phi_i(v_i) = v_i - \frac{1-F(v_i)}{f(v_i)}$$

1. Collect all bids  $b_i$
2. Compute  $\phi_i(b_i)$
3. Apply VCG on  $\phi_i(b_i)$ , get allocation  $x_i$ , payment  $P_i$
4. Final allocation is  $x_i$ , final payment is  $\phi_i^{-1}(P_i)$

$$v_i \in [0, 1)$$

$$\phi_i(b_i) = b_i - (1 - b_i) = 2b_i - 1 \in [-1, 1]$$

$$2b_i - 1 = 0$$

$$b_i = 1/2$$

卖软件：保留价vcg-> fixed price auction

$$\frac{1-F(v_i)}{f(v_i)} \text{ 单调非减能保证truthful}$$