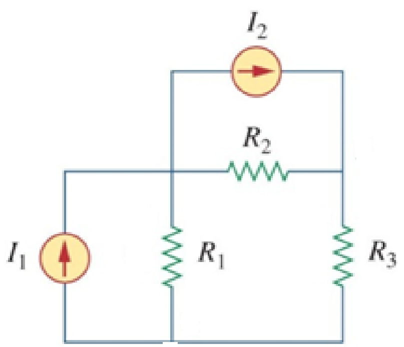
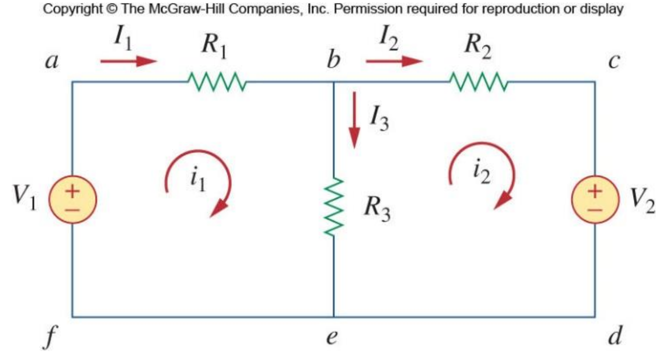
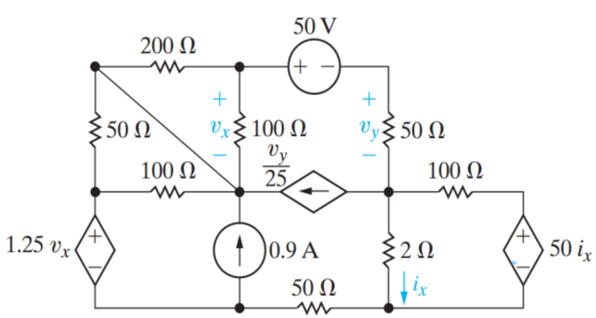
Lecture 5
- RC/RL First-Order Circuits

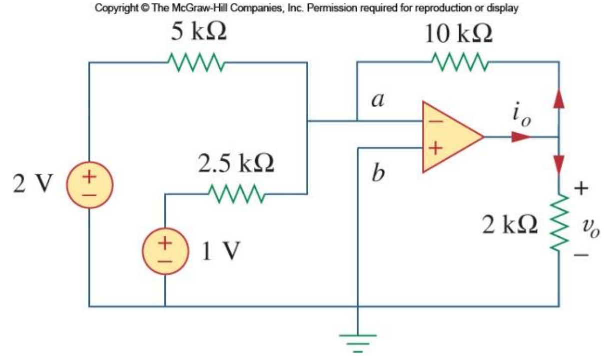
Beginning of Temporal Behavior Analysis of Circuits

- Till now we discussed <u>static</u> analysis of a circuit
  - Responses at a given time depend only on inputs at that time.
  - Circuit responds to input changes infinitely fast.







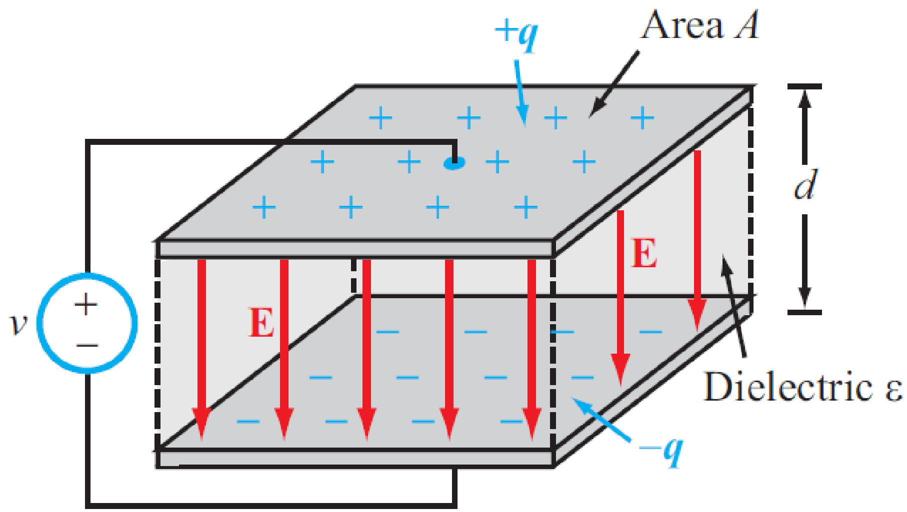


Outline

- Capacitors and inductors
- Natural response of RC/RL circuits
- Step response of RC/RL circuits
- Others

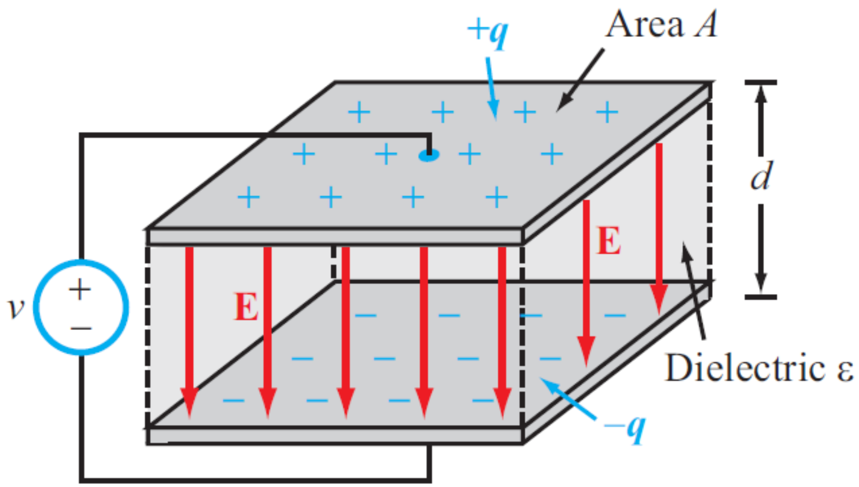
Capacitors

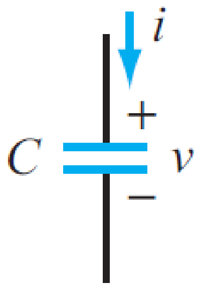
Storage element that stores energy in electric field



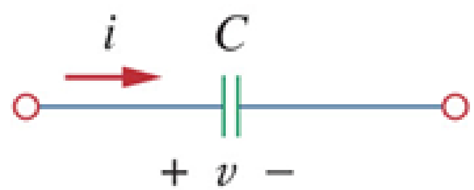
Parallel plate capacitor

V-I Relationship of Capacitors





Stored Energy

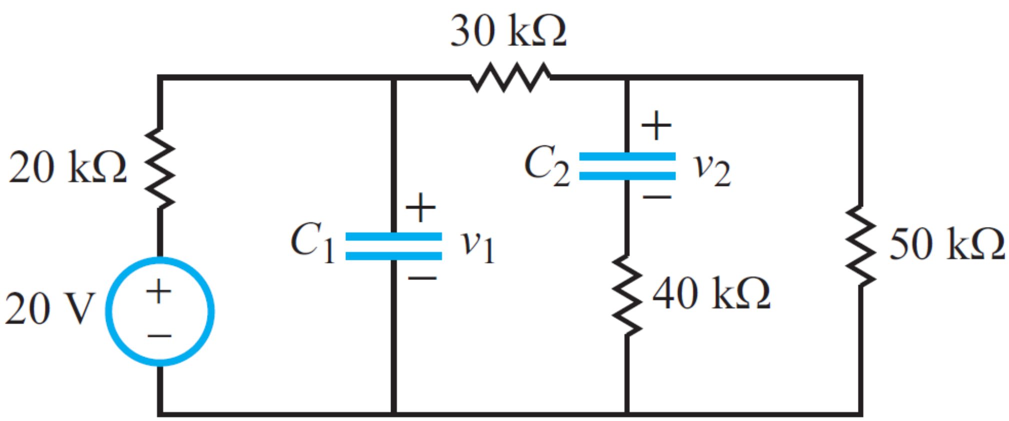


$$i = C\frac{dv}{dt}$$

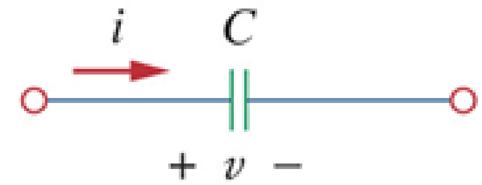
- The instantaneous power delivered to the capacitor is

- The energy stored in a capacitor is:

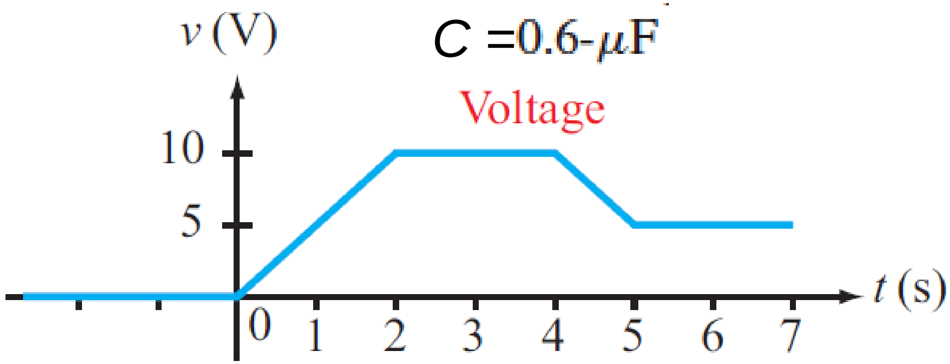
Example-1

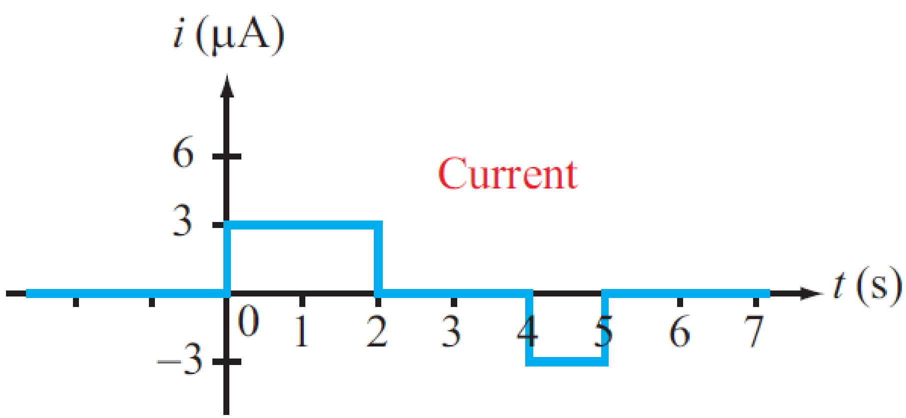


Example-2 Capacitor Response

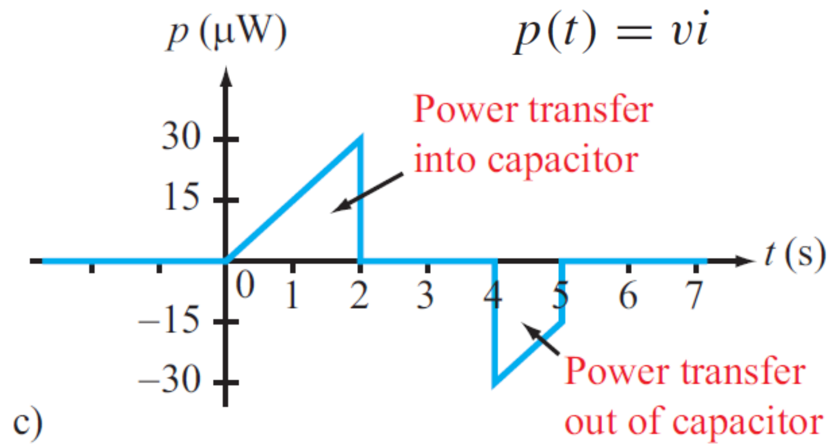


$C = 0.6\text{-}\mu\text{F}$

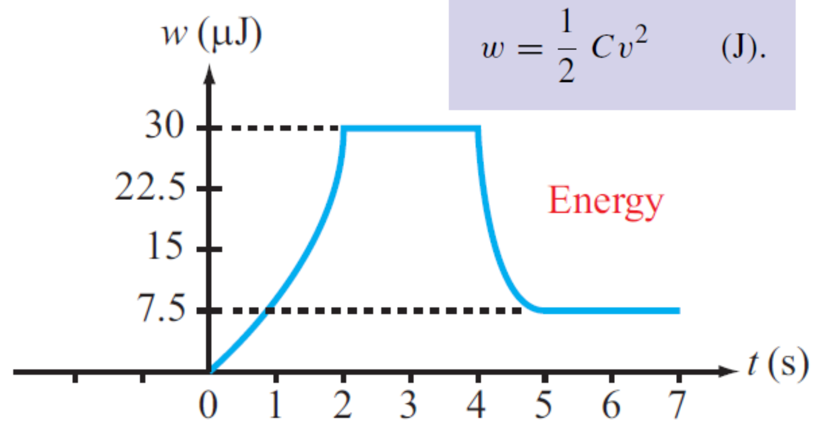




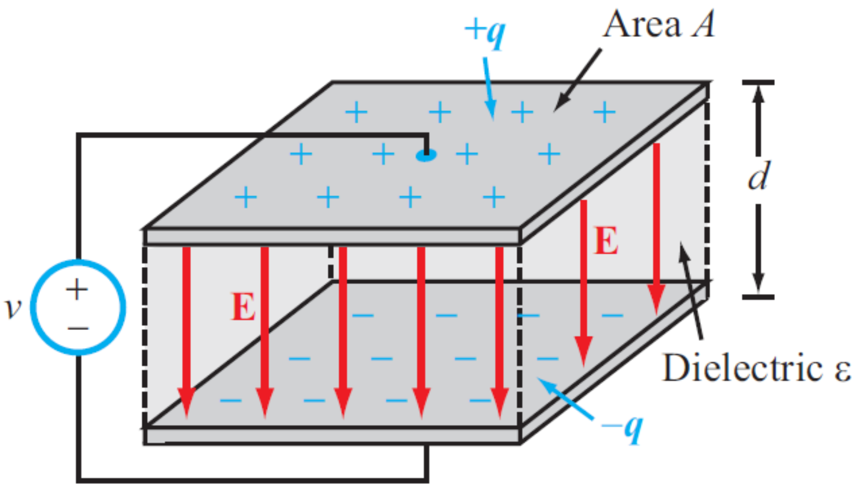
$p(t) = vi$

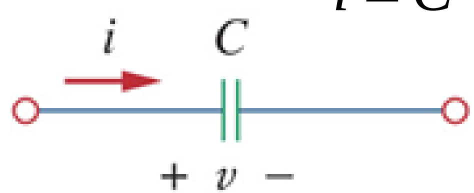


$$w = \frac{1}{2} C v^2 \qquad \text{(J).}$$

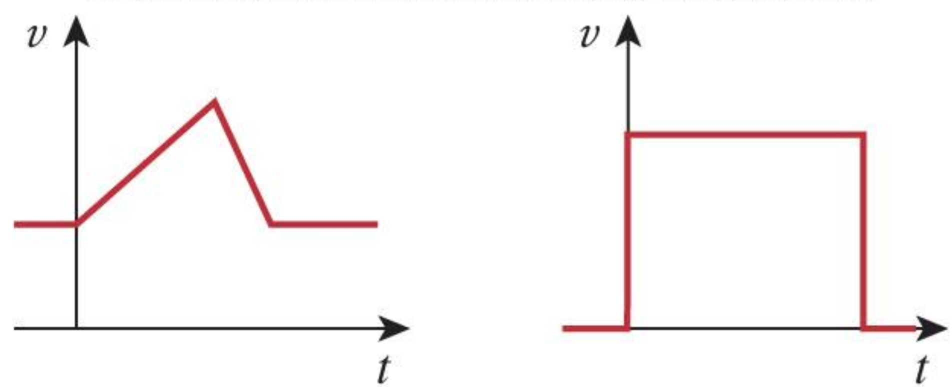


Important Property of Capacitors





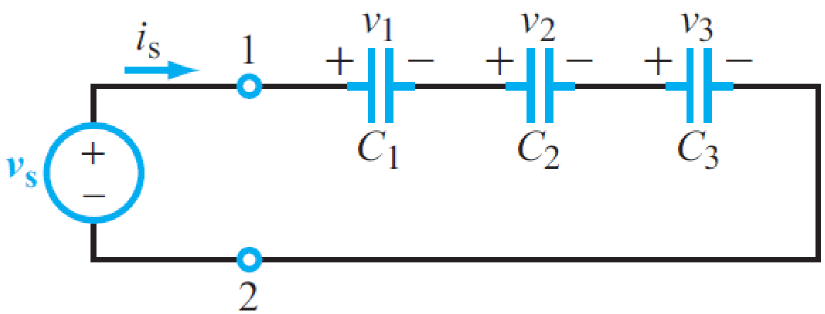
$$i = C\frac{dv}{dt}$$

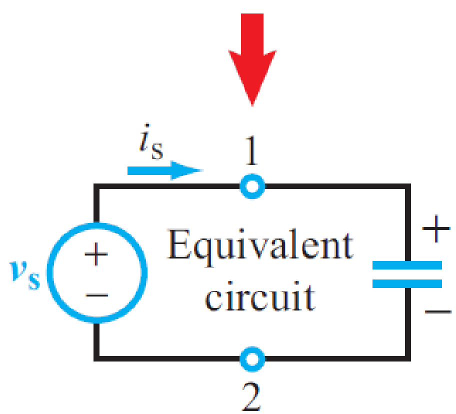


(a) (b)

Capacitors in Series

Combining In-Series Capacitors

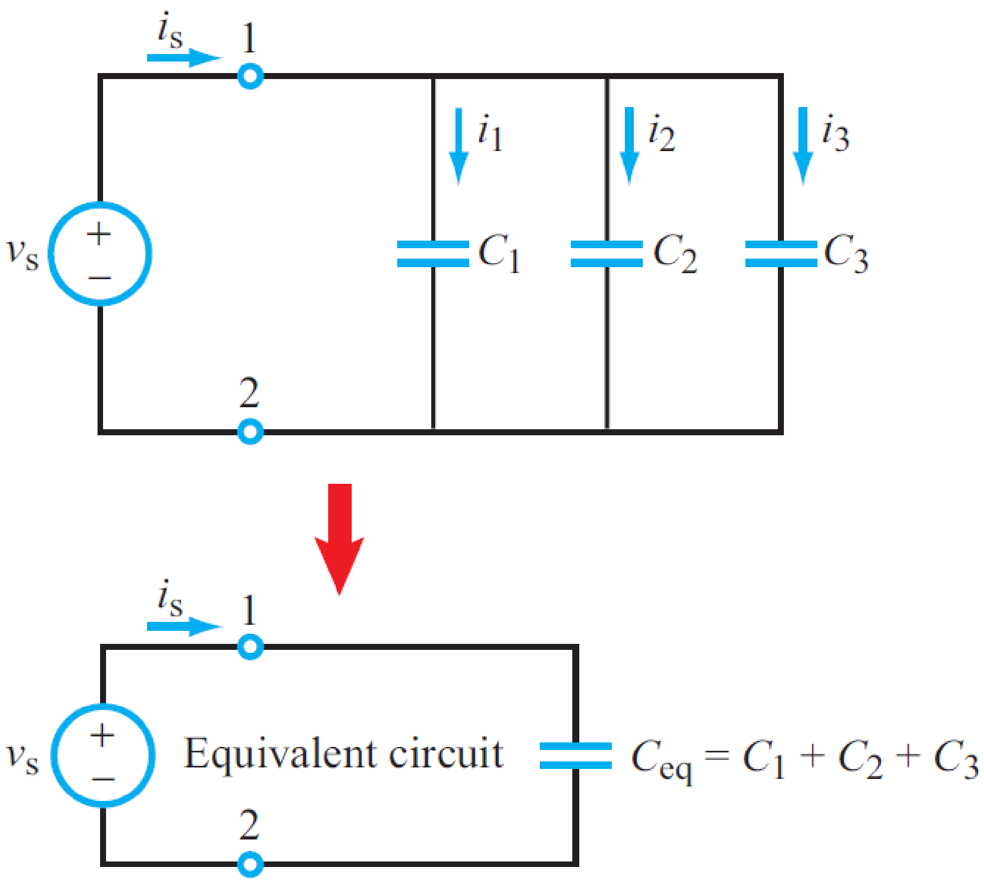




$$C_{eq} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)^{-1}$$

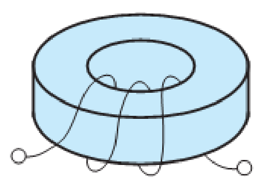
[Source: Berkeley]

Capacitors in Parallel

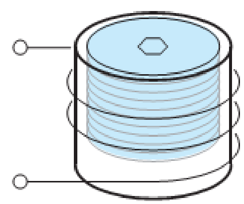


Inductors

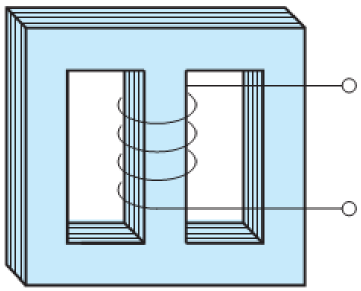
- A storage element that stores energy in magnetic field.
  - They have applications in power supplies, transformers, radios, TVs, radars, and electric motors.
- Any conductor has inductance, but the effect is typically enhanced by coiling the wire up.



(a) Toroidal inductor

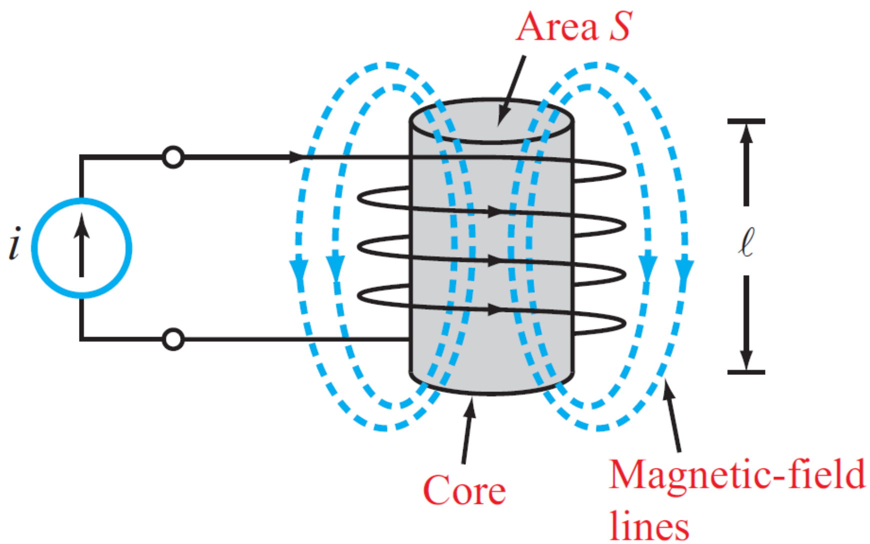


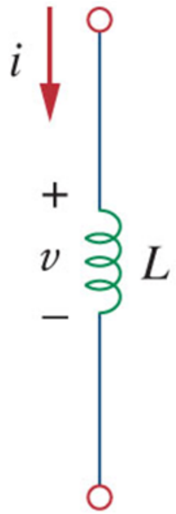
(b) Coil with an iron-oxide slug that can be screwed in or out to adjust the inductance



(c) Inductor with a laminated iron core

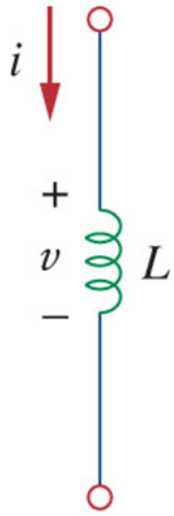
V-I Relationship of Inductors





$$v = L\frac{di}{dt}$$

Energy Stored in an Inductor

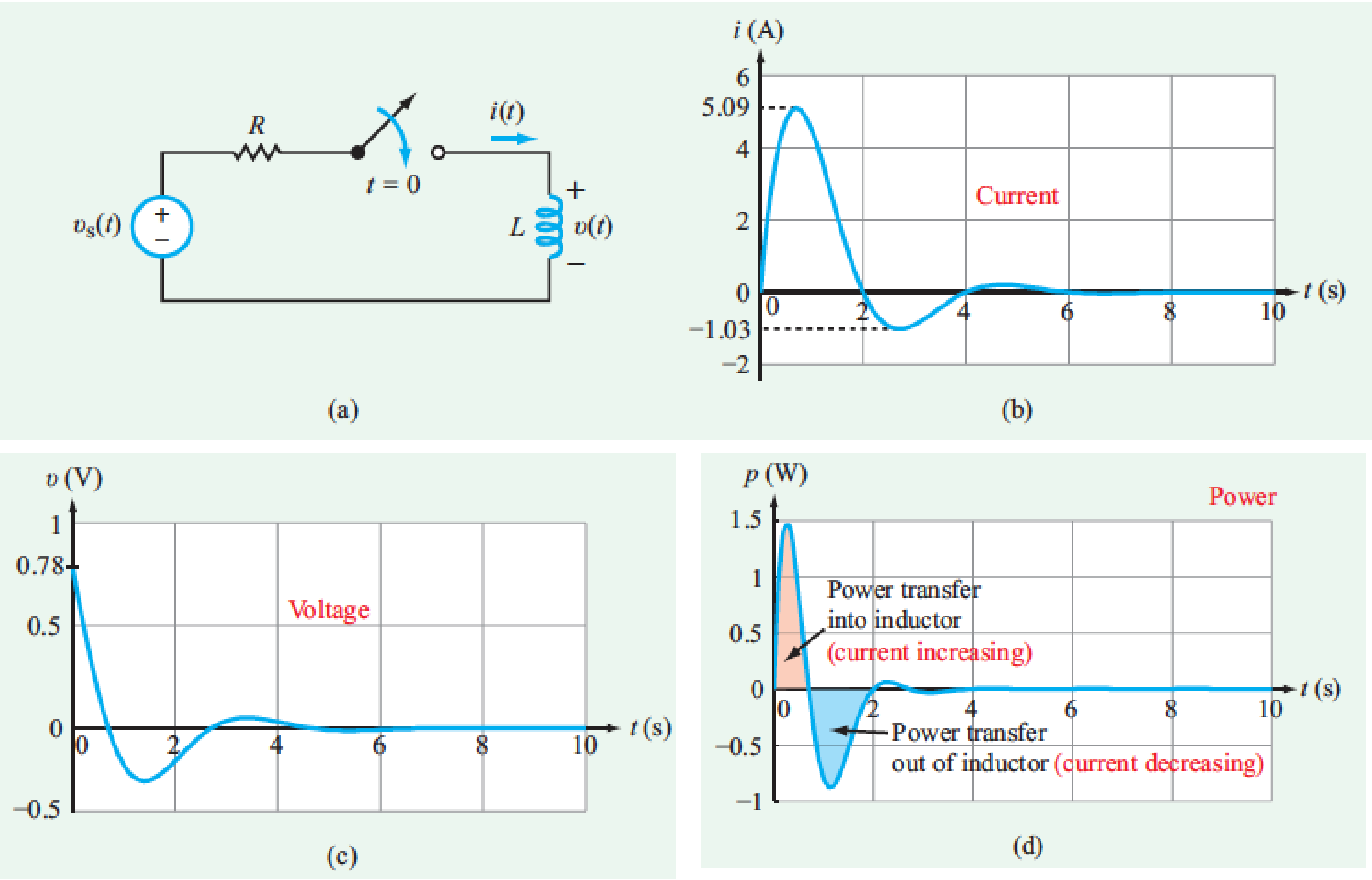


$$v = L\frac{di}{dt}$$

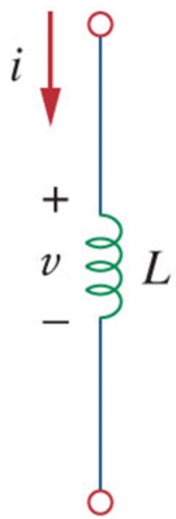
- The power delivered to the inductor is:

- The energy stored is:

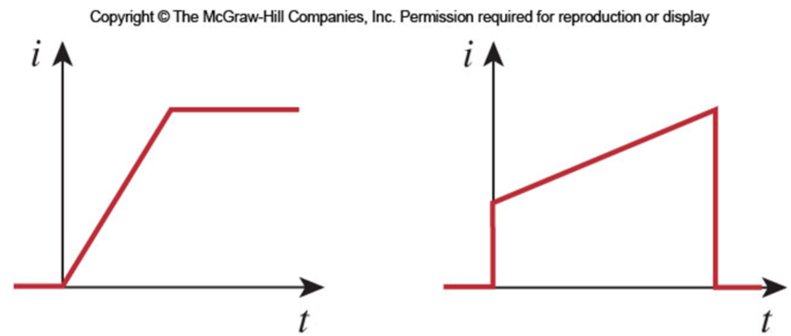
Inductor Response



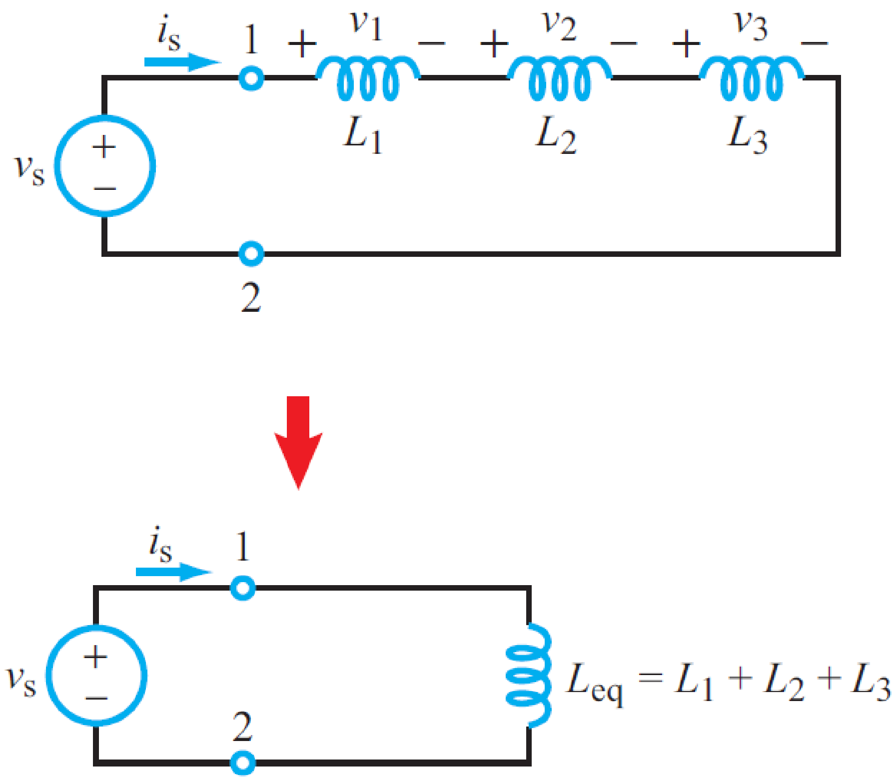
Important Property of Inductors



$$v = L\frac{di}{dt}$$

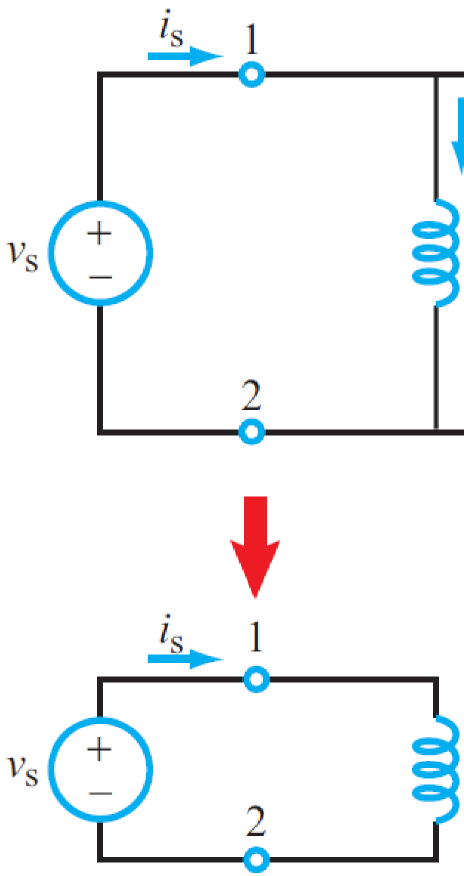


Inductors in Series



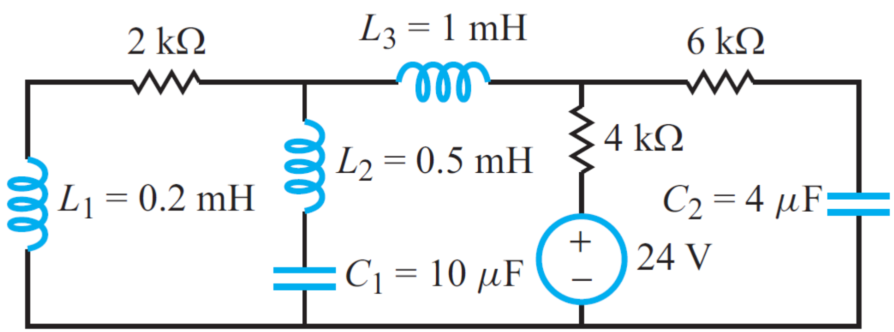
Inductors in Parallel

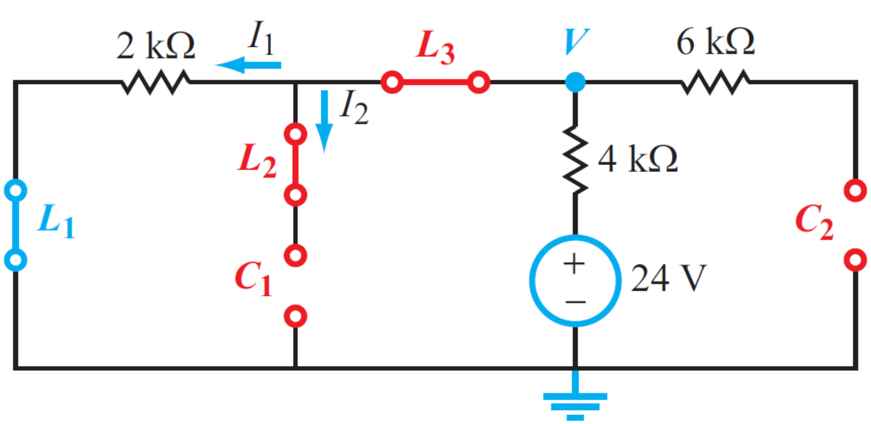
Combining In-Parallel Inductors



$$L_{eq} = \left(\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}\right)^{-1}$$

Example


2 kΩ
$L_3 = 1$ mH
6 kΩ
4 kΩ
$L_2 = 0.5$ mH
$L_1 = 0.2$ mH
$C_2 = 4\ \mu$F
$C_1 = 10\ \mu$F
24 V
+
–


2 kΩ
I_1
L_3
V
6 kΩ
I_2
L_2
4 kΩ
L_1
C_2
C_1
24 V
+
–

Summary of Resistors, Capacitors and Inductors

Table 5-4: Basic properties of R, L, and C.

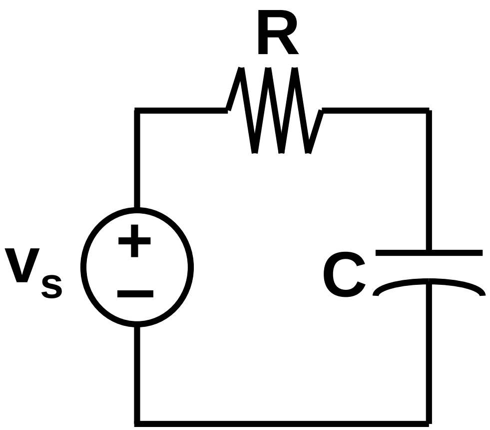
Property	R	L	C
i–υ relation	$i = \dfrac{\upsilon}{R}$	$i = \dfrac{1}{L}\displaystyle\int_{t_0}^{t} \upsilon\, dt' + i(t_0)$	$i = C\,\dfrac{d\upsilon}{dt}$
υ-i relation	$\upsilon = iR$	$\upsilon = L\,\dfrac{di}{dt}$	$\upsilon = \dfrac{1}{C}\displaystyle\int_{t_0}^{t} i\, dt' + \upsilon(t_0)$
p (power transfer in)	$p = i^2 R$	$p = Li\,\dfrac{di}{dt}$	$p = C\upsilon\,\dfrac{d\upsilon}{dt}$
w (stored energy)	0	$w = \dfrac{1}{2}Li^2$	$w = \dfrac{1}{2}C\upsilon^2$
Series combination	$R_{eq} = R_1 + R_2$	$L_{eq} = L_1 + L_2$	$\dfrac{1}{C_{eq}} = \dfrac{1}{C_1} + \dfrac{1}{C_2}$
Parallel combination	$\dfrac{1}{R_{eq}} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$	$\dfrac{1}{L_{eq}} = \dfrac{1}{L_1} + \dfrac{1}{L_2}$	$C_{eq} = C_1 + C_2$
dc behavior	no change	short circuit	open circuit
Can υ change instantaneously?	yes	yes	no
Can i change instantaneously?	yes	no	yes

Outline

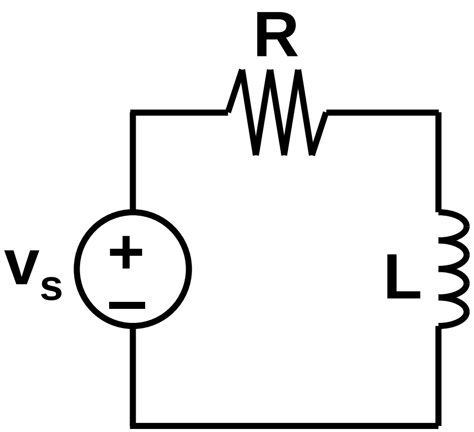
- Capacitors and inductors
- Natural response of RC/RL circuits
- Step response of RC/RL circuits
- Others

RC and RL Circuits

- A circuit that contains only sources, resistors and a capacitor is called an ***RC circuit***.



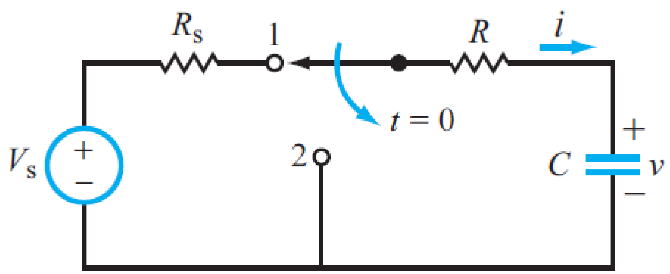
- A circuit that contains only sources, resistors and an inductor is called an ***RL circuit***.



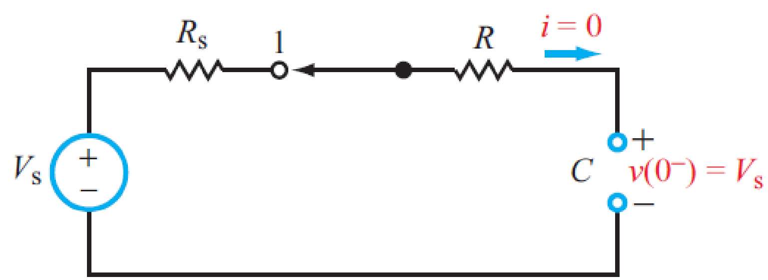
[Source: Berkeley]

Natural Response of a Charged Capacitor

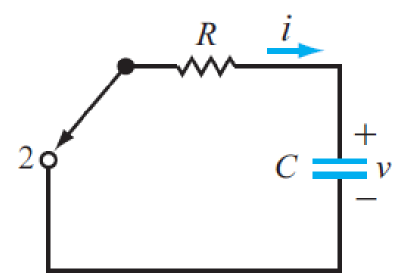
Behavior (*i.e.*, current and voltage) when stored energy in the inductor or capacitor is released to the resistive part of the network (containing no independent sources).



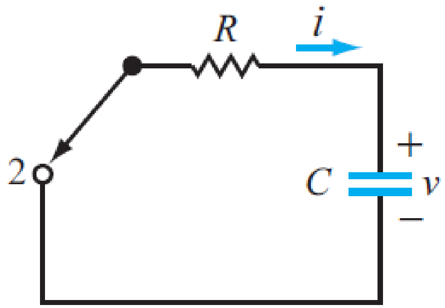
(a) $t = 0^-$ is the instant just before the switch is moved from terminal 1 to terminal 2;



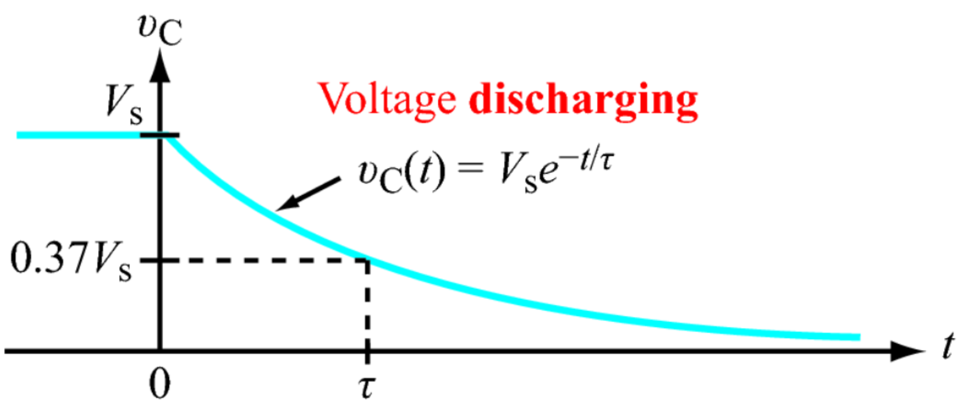
(b) $t = 0$ is the instant just after it was moved, $t = 0$ is synonymous with $t = 0^+$.

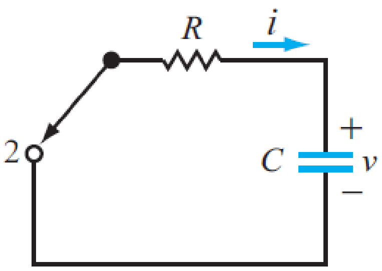


Natural Response of a Charged Capacitor

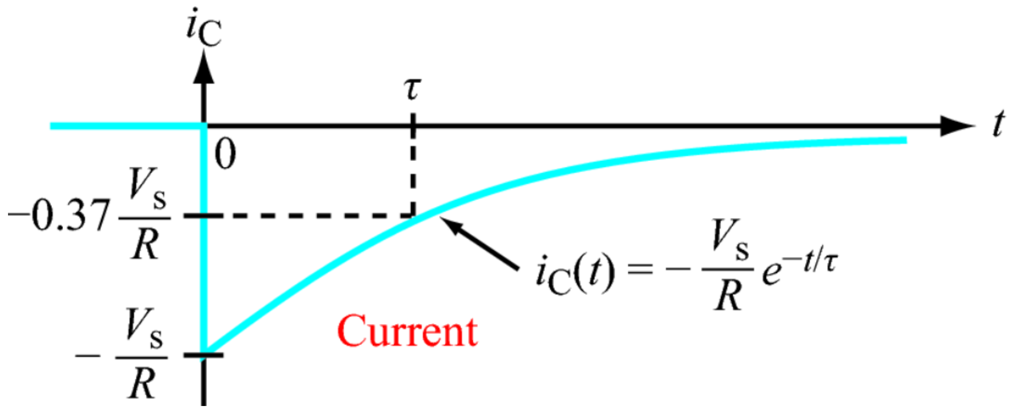


Natural Response of RC





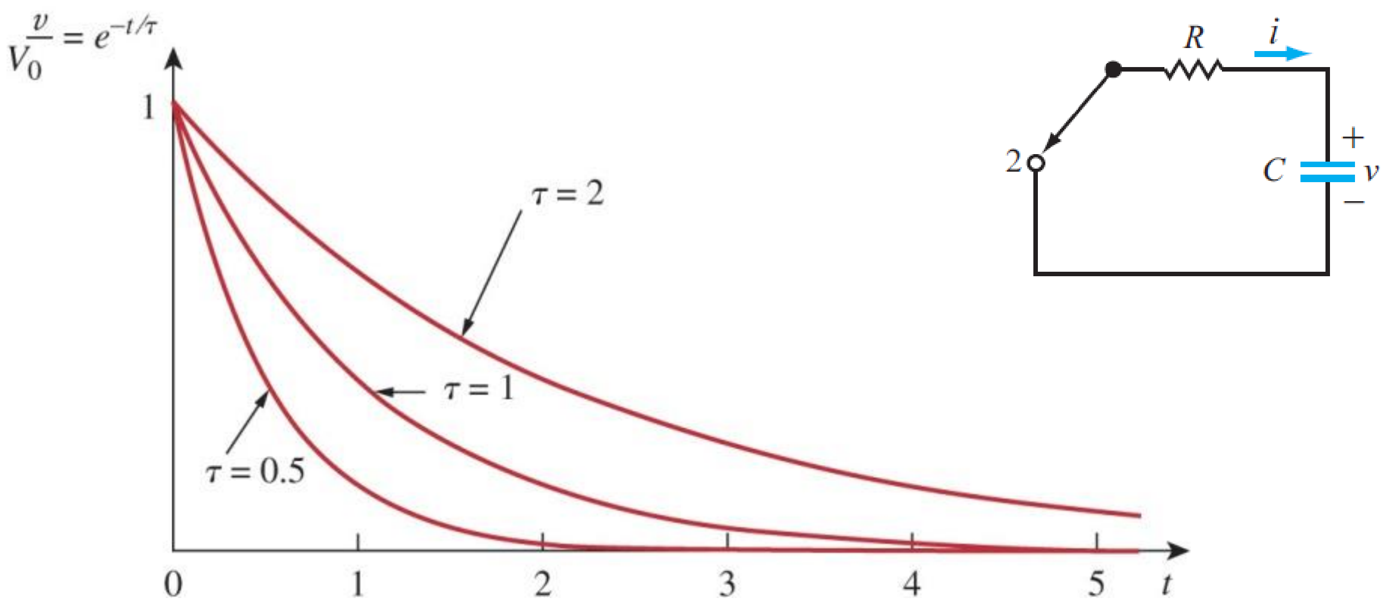
Time constant: $\tau = RC$



[Source: Berkeley]

Time Constant τ ($= RC$)

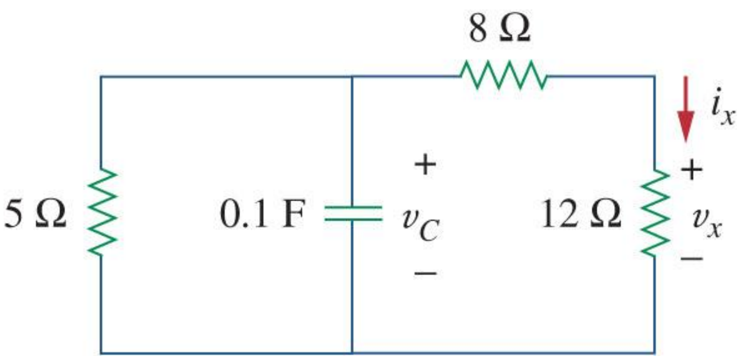
- A circuit with a small time constant has a fast response and vice versa.



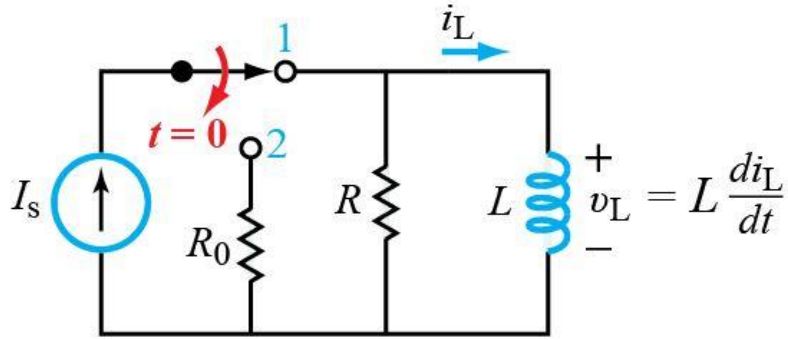
[Source: Berkeley]

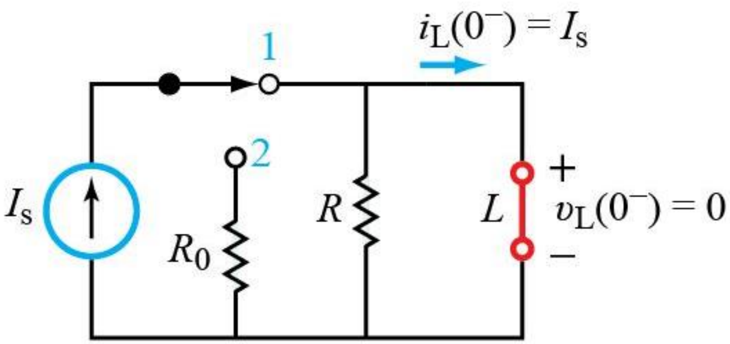
Example

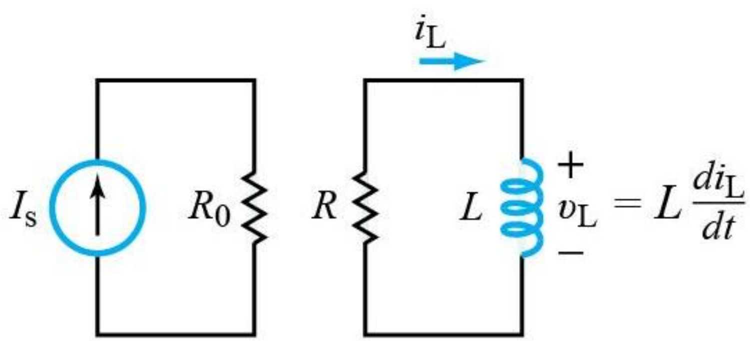
- In the circuit below, let $v_C(t=0)=15\text{V}$. Find v_C, v_x, and i_x for $t>0$.



Natural Response of the RL Circuit

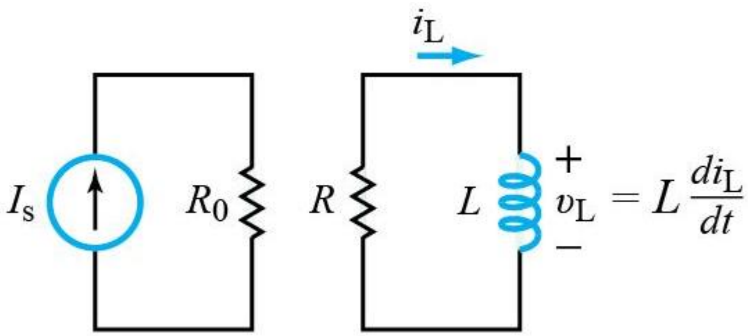




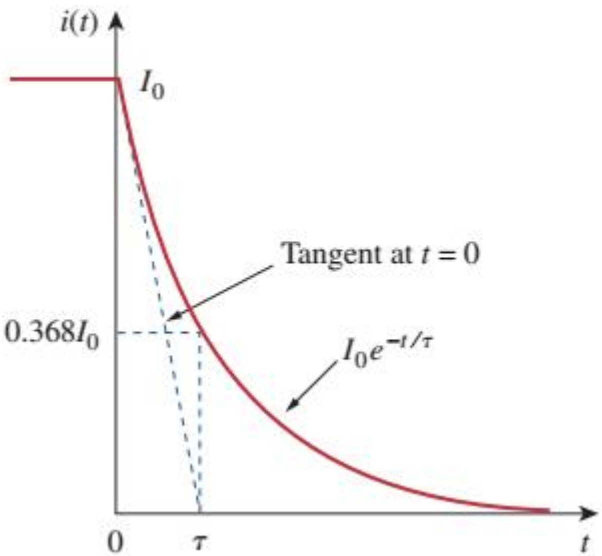


[Source: Berkeley]

Natural Response of the RL Circuit



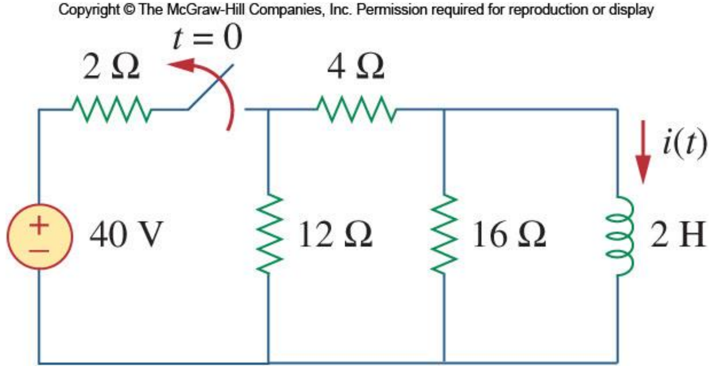
Natural Response of the RL Circuit



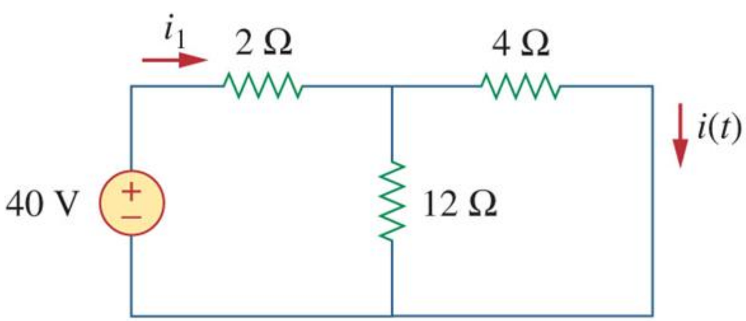
[Source: Berkeley]

Example

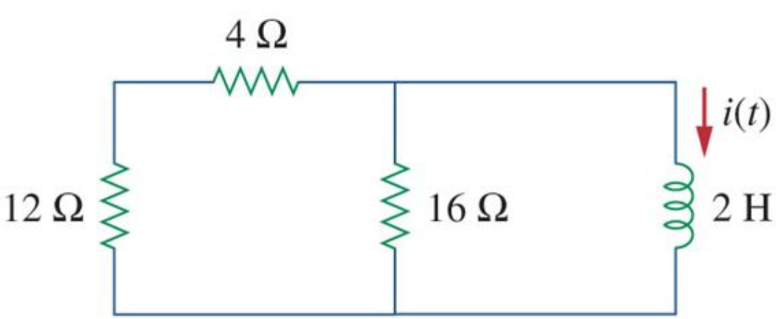
- The switch in the circuit below has been closed for a long time. At $t = 0$, the switch is opened. Calculate $i(t)$ for $t > 0$.



When $t < 0$

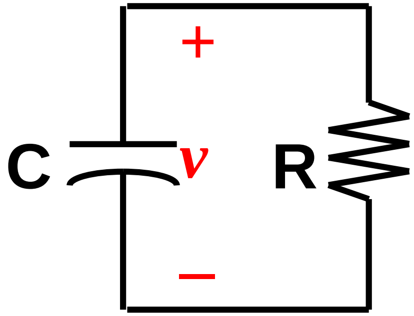


When $t > 0$



Natural Response Summary

RC Circuit



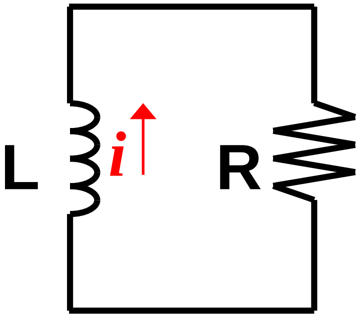
- Capacitor voltage cannot change instantaneously

$$v(0^-) = v(0^+)$$

$$v(t) = v(0)e^{-t/\tau}$$

- time constant $\tau = RC$

RL Circuit



- Inductor current cannot change instantaneously

$$i(0^-) = i(0^+)$$

$$i(t) = i(0)e^{-t/\tau}$$

- time constant $\tau = \dfrac{L}{R}$

[Source: Berkeley]