Lecture 2-2 Basic relationship between pixels

Yuyao Zhang, Xiran Cai PhD

zhangyy8@shanghaitech.edu.cn caixr@shanghaitech.edu.cn

SIST Building 2 302-F/302-C

Course piazza link: piazza.com/shanghaitech.edu.cn/spring2021/cs270spring2021

Outline

- **Neighbors of Pixel**
- **Relationship between Pixels**
  - Adjacency
  - Connectivity
  - Regions
  - Boundaries
- **Distance measures**
  - Euclidean distance
  - City-block distance
  - Chessboard distance

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Neighbors of Pixel

If a pixel p at coordinate (x, y)

- **$N_4(p)$**
- **$N_D(p)$**
- **$N_8(p)$**

Neighbors of Pixel

If a pixel p at coordinate (x, y)

- **$N_4(p)$**

  (x+1, y), (x-1, y), (x, y+1), (x, y-1)

- **$N_D(p)$**

- **$N_8(p)$**

	q_1	
q_2	p	q_3
	q_4	

Neighbors of Pixel

If a pixel p at coordinate (x, y)

- **$N_4(p)$**

  (x+1, y), (x-1, y), (x, y+1), (x, y-1)

- **$N_D(p)$**

  (x+1, y+1), (x+1, y-1), (x-1, y+1), (x-1, y-1)

- **$N_8(p)$**

r_1		r_2
	p	
r_3		r_4

Neighbors of Pixel

If a pixel p at coordinate (x, y)

- **$N_4(p)$**

  (x+1, y), (x-1, y), (x, y+1), (x, y-1)

- **$N_D(p)$**

  (x+1, y+1), (x+1, y-1), (x-1, y+1), (x-1, y-1)

- **$N_8(p) : N_4(p) \cup N_D(p)$**

r_1	q_1	r_2
q_2	p	q_3
r_3	q_4	r_4

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Adjacency

To define adjacency of pixels, we need identify

- **Type of Neighbor**

  $N_4(p)$, $N_D(p)$, $N_8(p)$

- **The set of intensity values V**
  - Binary image: V = {1}
  - Gray-scale image: V = [L_{min}, L_{max}]

q_1	p	q_2

0	0	0
0	1	1
0	0	0

Adjacency in a binary image

	q_1	
q_2	p	q_3

0	39	0
11	13	16
0	0	0

Adjacency in a gray-scale image

Adjacency

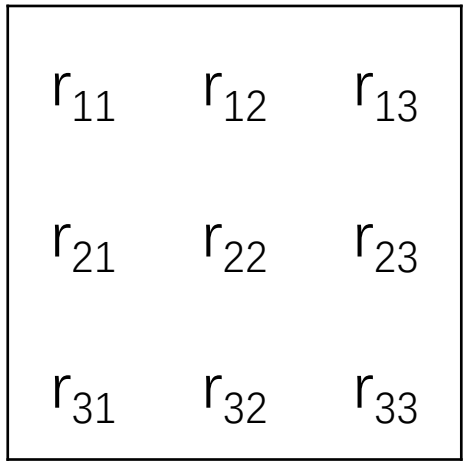
Types of Adjacency:

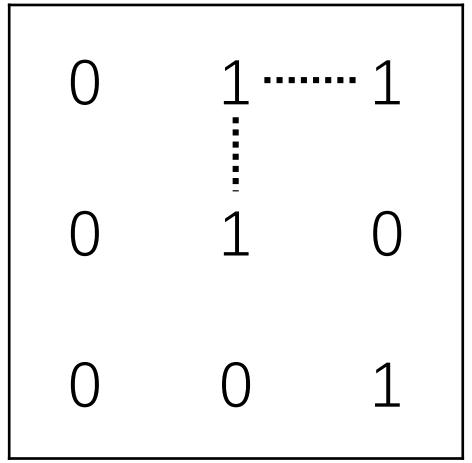
- **4-adjacency**
- **8-adjacency**
- **M-adjacency (mixed adjacency)**

Adjacency

Types of Adjacency:

- **4-adjacency**
  - p, q $\in$ V
  - q $\in N_4(p)$
- 8-adjacency
- M-adjacency (mixed adjacency)

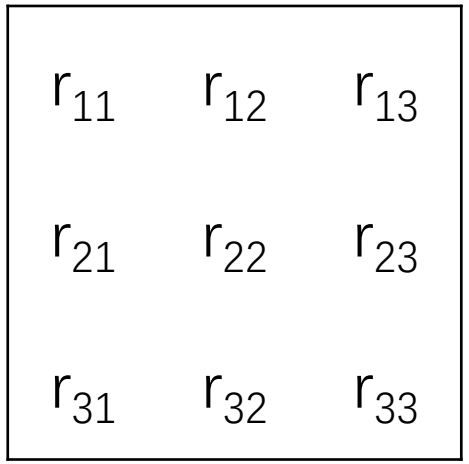


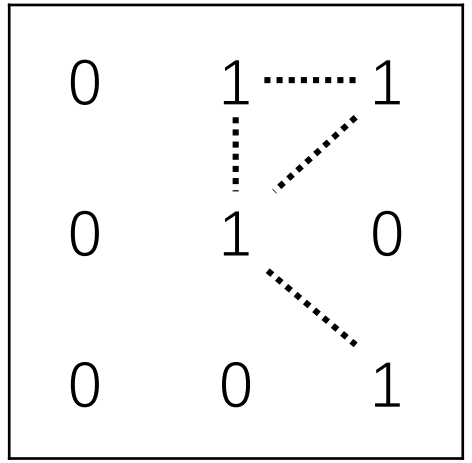


Adjacency

Types of Adjacency:

- 4-adjacency
- **8-adjacency**
  - p, q ∈ V
  - q ∈ $N_8(p)$
- M-adjacency (mixed adjacency)

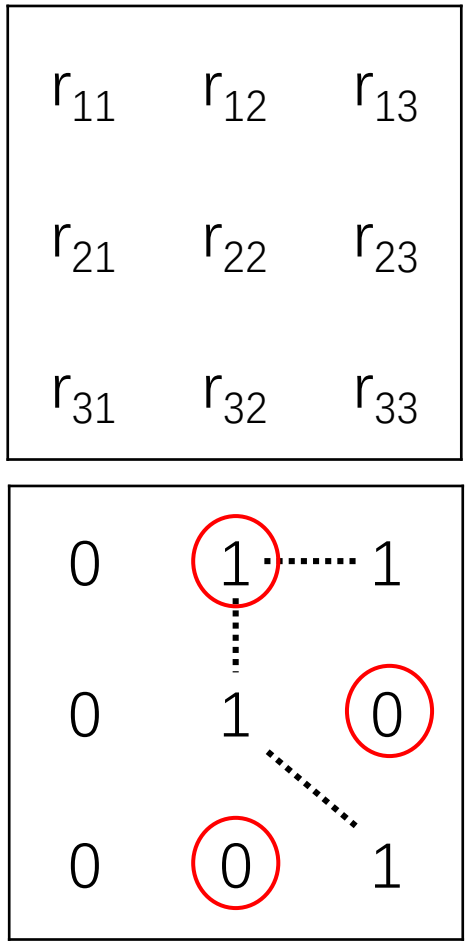




Adjacency

Types of Adjacency:

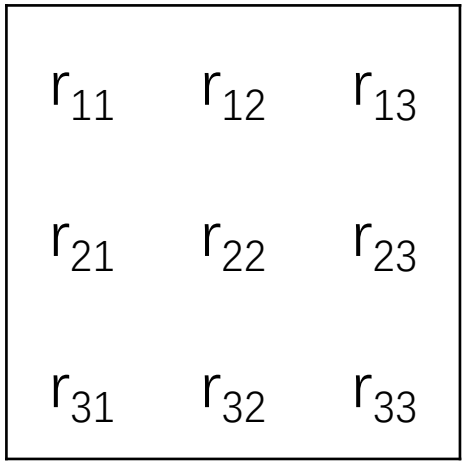
- 4-adjacency
- 8-adjacency
- **M-adjacency (mixed adjacency)**
  - p, q $\in V$
  - q $\in N_4(p)$ or
    q $\in N_D(p)$ and $N_4(p) \cap N_4(q) \notin V$

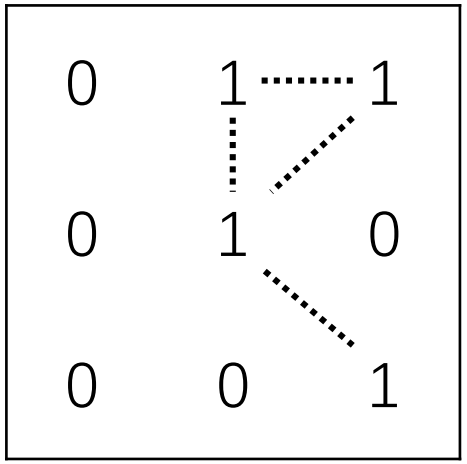


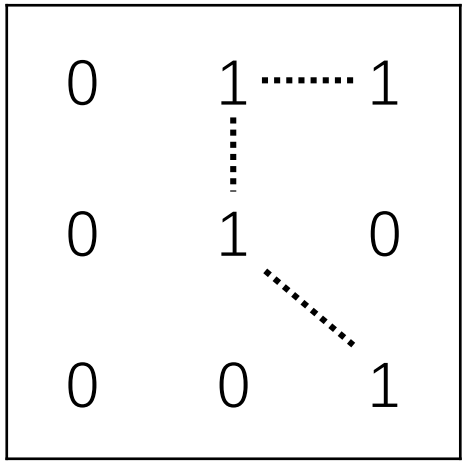
Connectivity

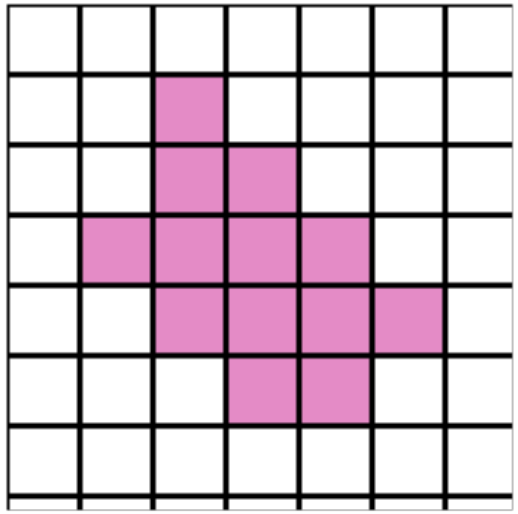
Important concept used in establishing boundaries of objects and components of regions in an image

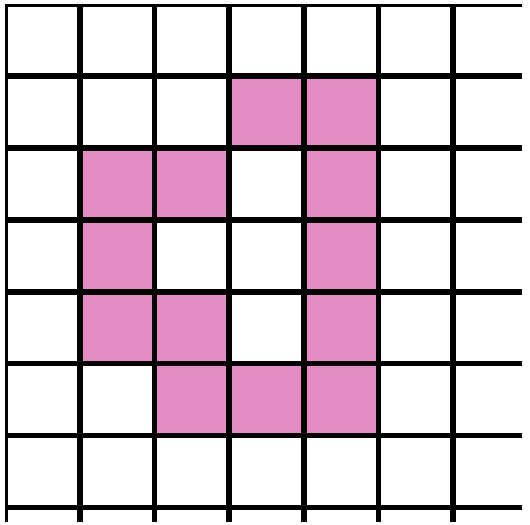
- *Path*
- *Connected*
- *Connected component*
- *Connected set*

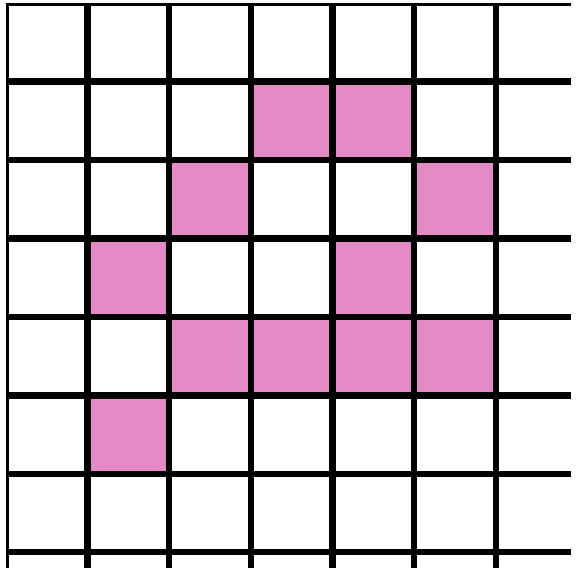


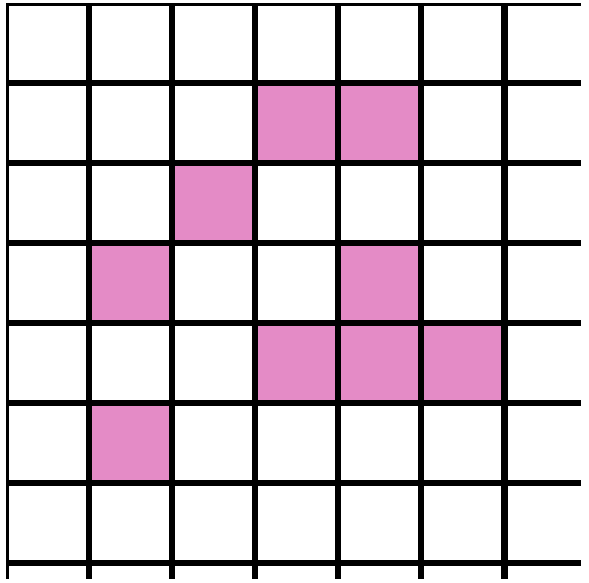








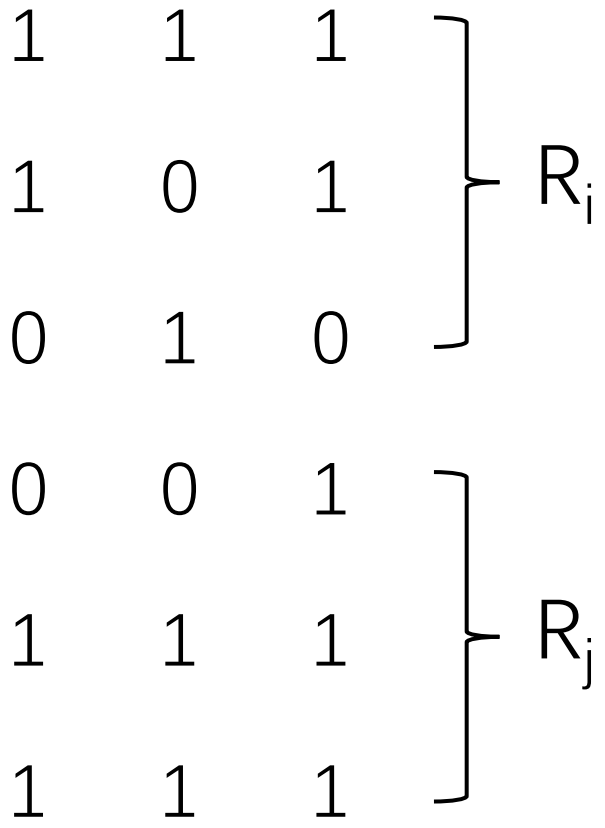




Region

R: a subset of an image which is also a connected set

- *Adjacent region*
- *Disjoint region*



Pixels

- **Neighbors of Pixel**
- **Relationship between Pixels**
  - Adjacency
  - Connectivity
  - Regions
  - Boundaries
- **Distance measures**
  - Euclidean distance
  - City-block distance
  - Chessboard distance

Distance Measures

For pixels p, q and z, with coordinates (x, y), (s, t) and (v, w), $\boldsymbol{D}$ is a **distance function or metric** if

- $\boldsymbol{D}(p, q) \geq 0$ ($\boldsymbol{D}(p, q) = 0$ only if $p = q$)
- $\boldsymbol{D}(p, q) = \boldsymbol{D}(q, p)$
- $\boldsymbol{D}(p, z) \leq \boldsymbol{D}(p, q) + \boldsymbol{D}(q, z)$

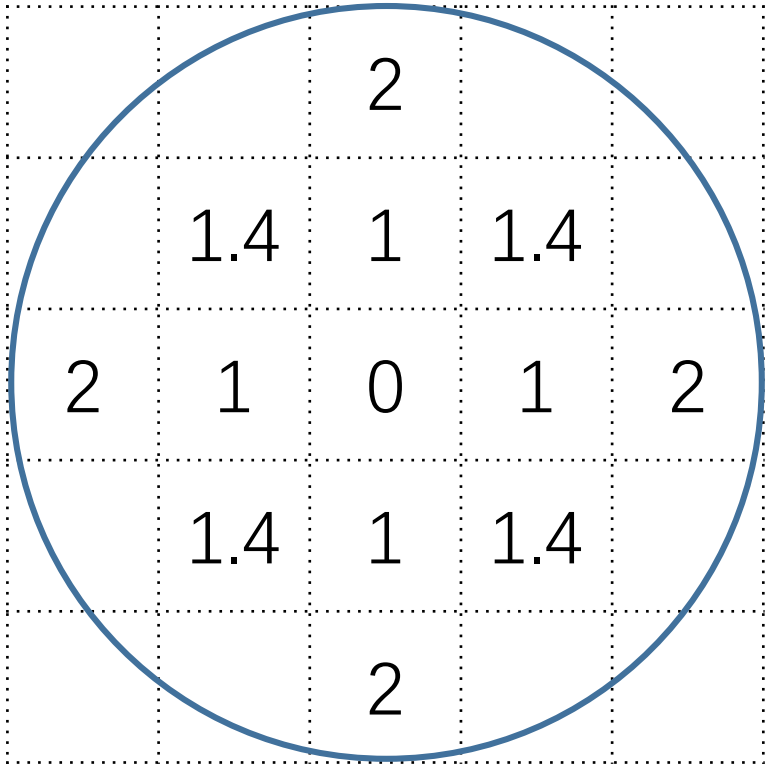
Distance Measures

- **Euclidean distance:**

$$D_e(p, q) = [(x-s)^2+(y-t)^2]^{1/2}$$

- City-block distance:

- Chessboard distance



Distance Measures

- Euclidean distance：

$$D_e(p, q) = [(x-s)^2+(y-t)^2]^{1/2}$$

- **City-block distance:**

$$D_4(p, q) = |x-s|+|y-t|$$

- Chessboard distance

		2		
	2	1	2	
2	1	0	1	2
	2	1	2	
		2		

Distance Measures

- Euclidean distance:

$$D_e(p, q) = [(x-s)^2+(y-t)^2]^{1/2}$$

- City-block distance:

$$D_4(p, q) = |x-s|+|y-t|$$

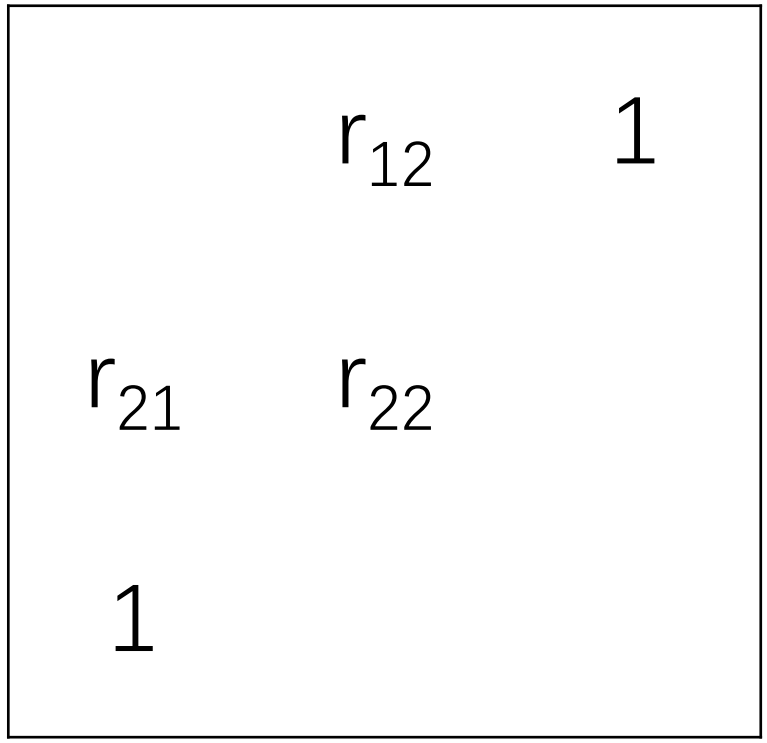
- **Chessboard distance**

$$D_8(p, q) = \max(|x-s|, |y-t|)$$

2	2	2	2	2
2	1	1	1	2
2	1	0	1	2
2	1	1	1	2
2	2	2	2	2

Distance Measures

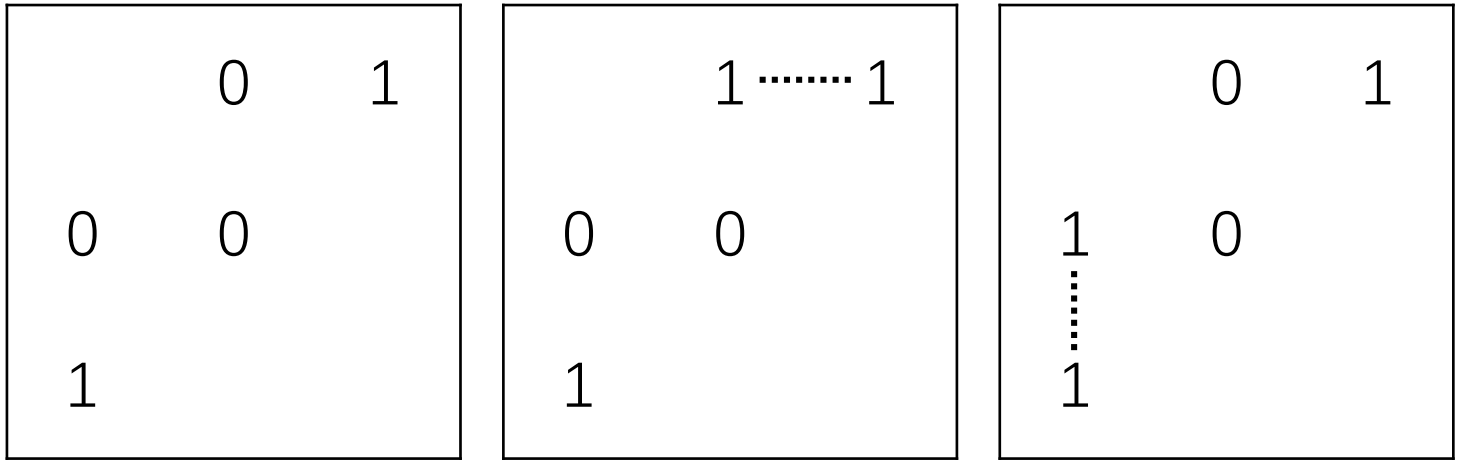
D_m distance is defined as the shortest m-path between the point



D_m = ?

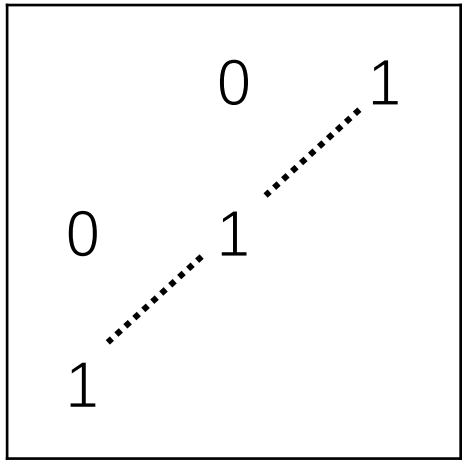
Distance Measures

No m-path between the point

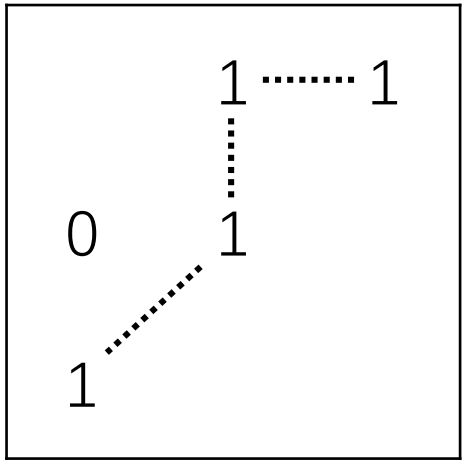


Distance Measures

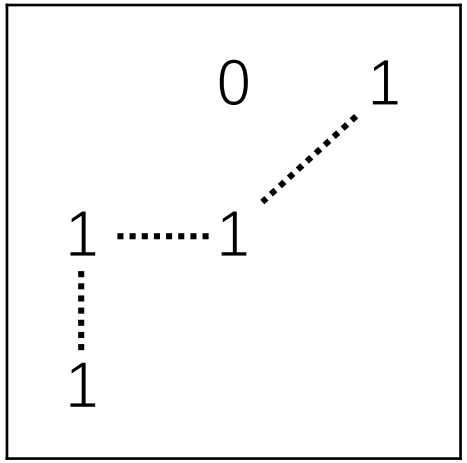
D_m distance is different by the values of r_{12}, r_{21} and r_{22}



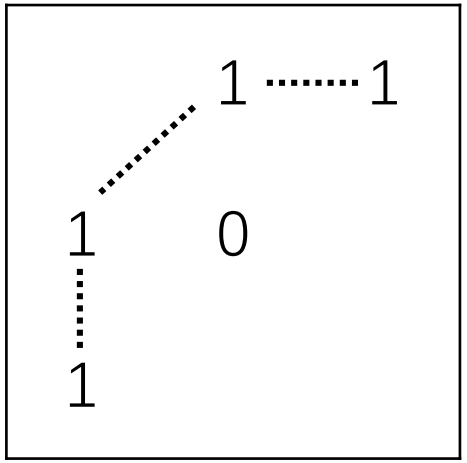
$D_m = 2$



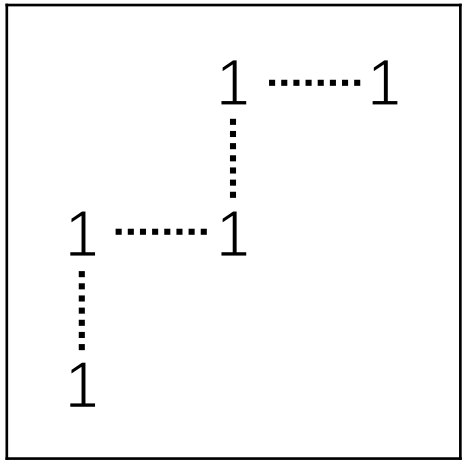
$D_m = 3$



$D_m = 3$



$D_m = 3$



$D_m = 4$

Take home message

- **1. Relationship between pixels.**
- **2. Core problem: types of adjacency.**
- **3. Connection and distance measurements.**