Homework 8

Due date: 18:30 of Dec.30th, 2021

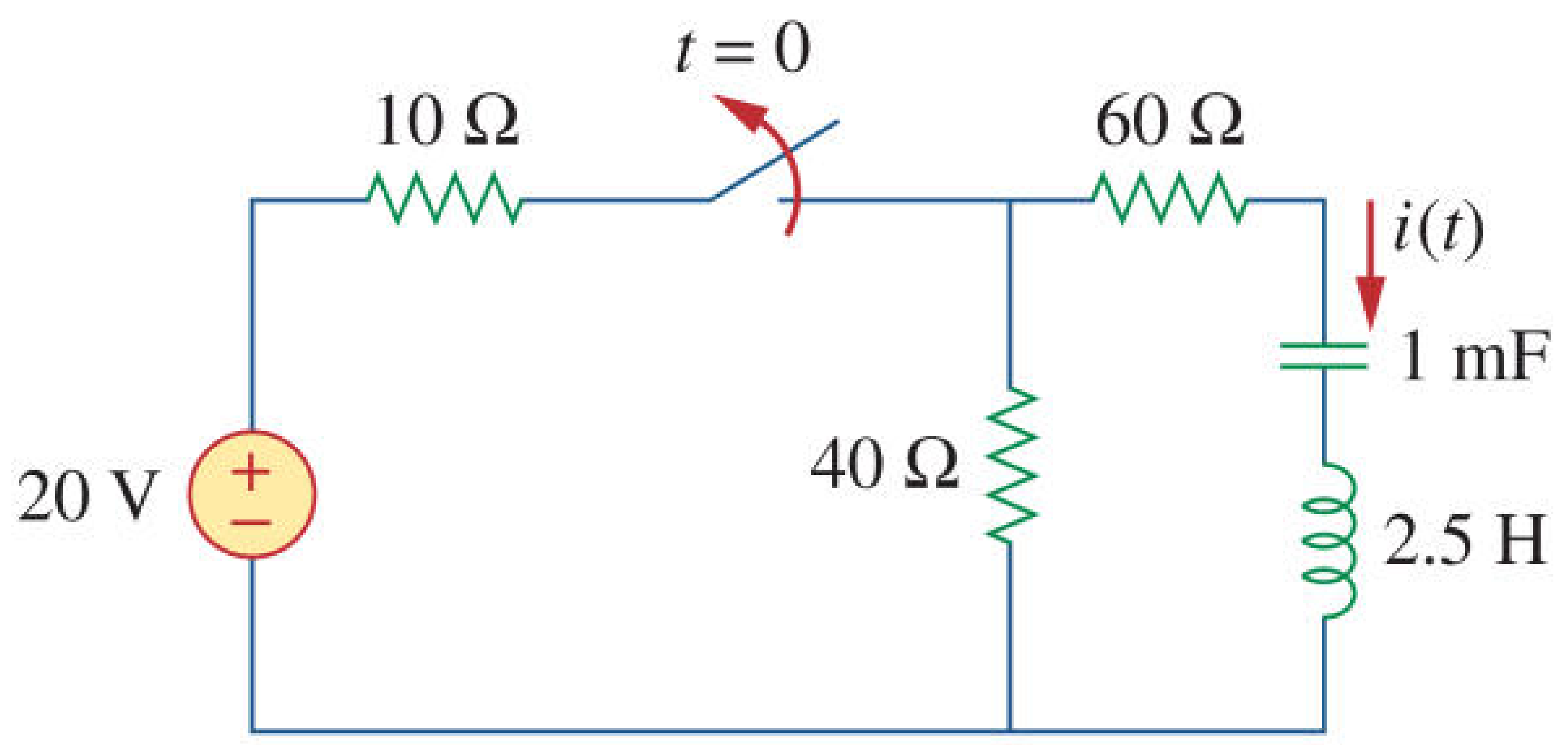
Turn in your homework to Tutorial Course Classroom 1B- 110

Rules:

- Work on your own. Discussion is permissible, but extremely similar submissions will be judged as plagiarism.
- Please show all intermediate steps: a correct solution without an explanation will get zero credit.
- Please submit on time. No late submission will be accepted.
- Please prepare your submission in English only. No Chinese submission will be accepted.
- If needed, round the number to the nearest hundredths, i.e., rounding it to 2 decimal places.

16'

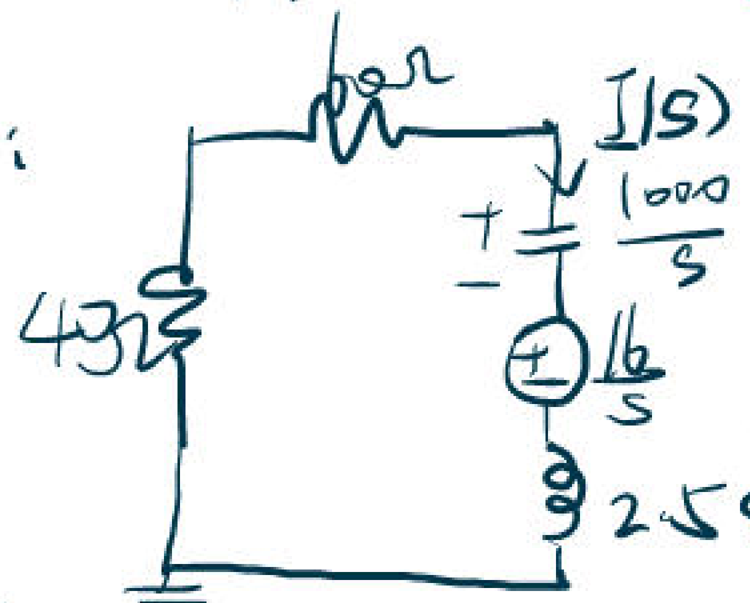
1. Use Laplace domain method to find *i(t)* for *t>0* in the circuit below.



$i_L(0^-) = 0\,A$ 2'

$V_c(0^-) = 20 \times \frac{40}{40+10} = 16\,V$ 2'

s-domain:



2' 1'

Apply KVL: $100\,I(s) + \frac{1000}{s} I(s) + \frac{16}{s} + 2.5s\,I(s) = 0$ 3'

$$I(s) = \frac{-\frac{16}{s}}{100 + \frac{1000}{s} + 2.5s}$$ 2'

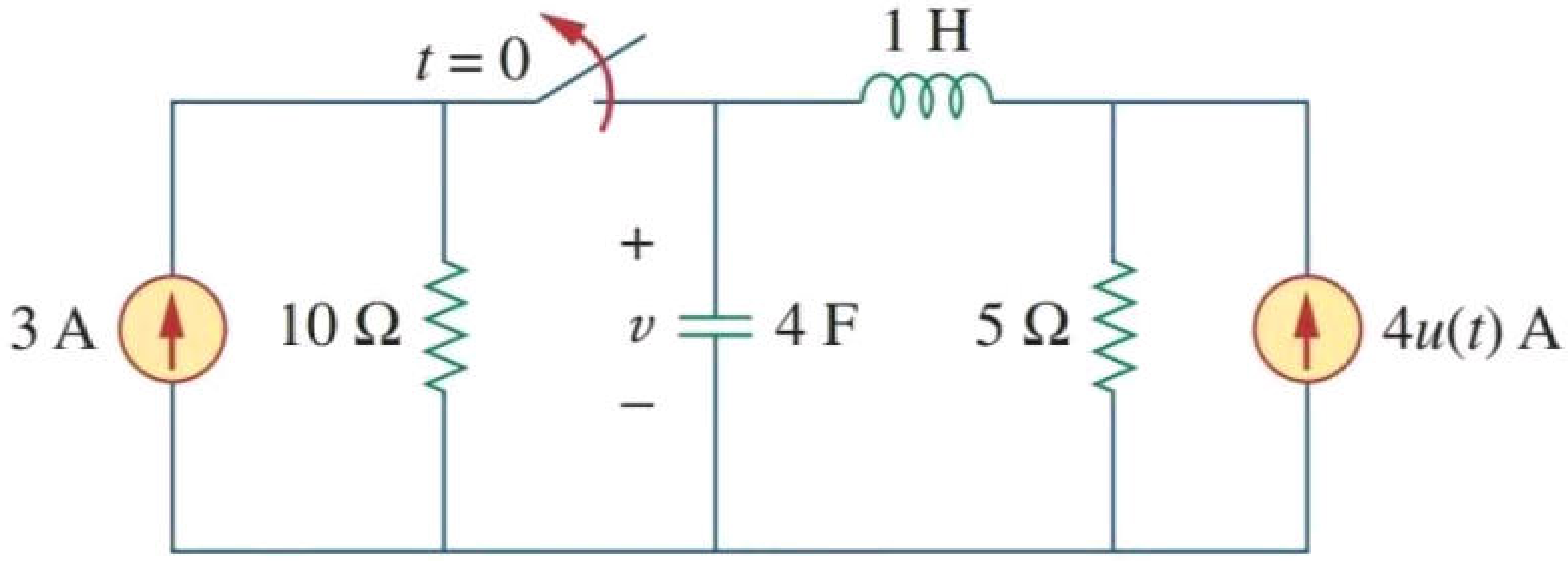
$$= \frac{-6.4}{(s+20)^2}$$ 2'

$$i(t) = \left[-6.4\,t\,e^{-20t} \ (A)\right] u(t), \ t > 0$$

2'

22'

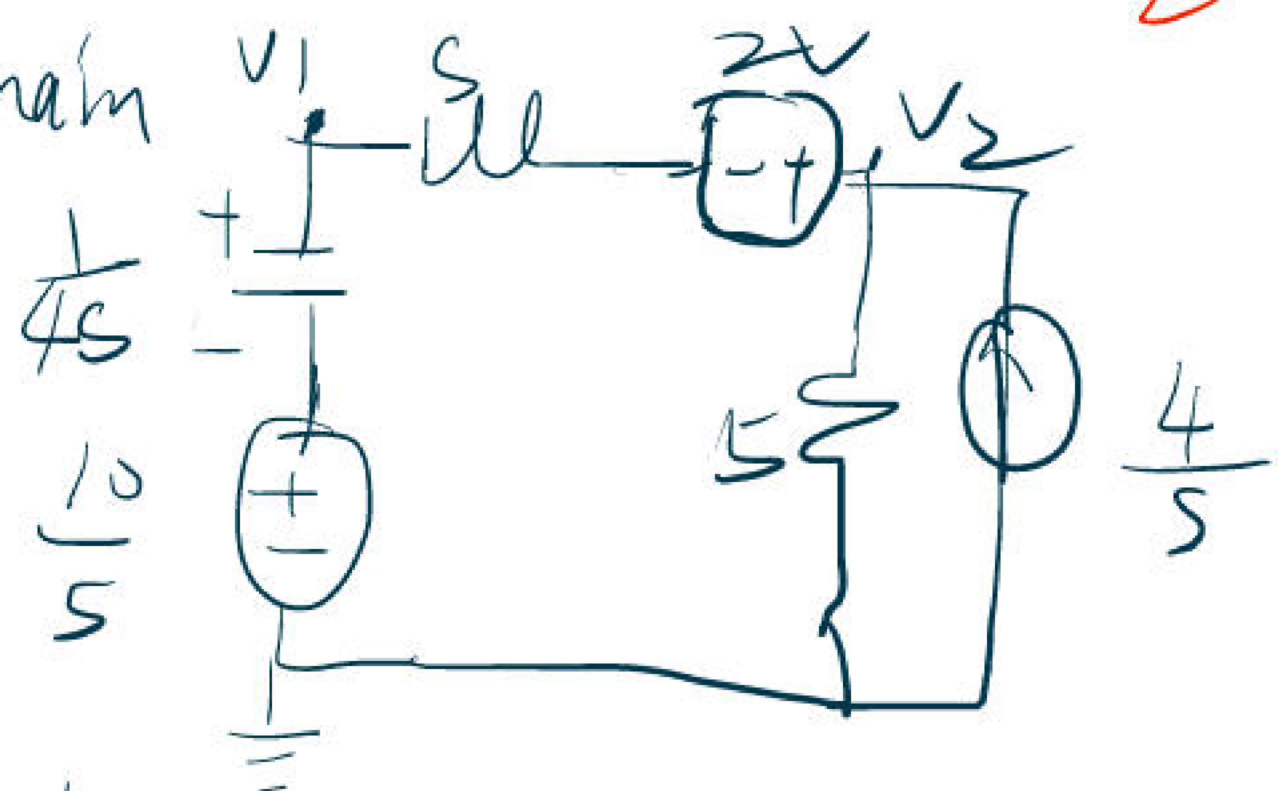
2. The switch has been closed for a long time and use Laplace domain method to find *v(t)* for *t* > 0 in the circuit in the figure below.



$i_L(0^-) = 3 \times \frac{10}{10+5} = 2A$ 2'

$V_c(0^-) = i_L(0^-) \times 5 = 10V$ 2'

S-domain



2'

Apply KCL: 2'

$$\frac{V_1+2-V_2}{s} + \frac{4}{s} = \frac{V_2}{5}$$

2'

$$\frac{\frac{10}{s}-V_1}{\frac{1}{4s}} = \frac{V_1+2-V_2}{s}$$

$$\Rightarrow V_1 = V_c(s) = \frac{40s+198+\frac{20}{s}}{4s^2+20s+1}$$

$$\Rightarrow V_1 = \frac{10s^2+49.5s+5}{s(s^2+5s+\frac{1}{4})} = \frac{10s^2+49.5s+5}{s\left(s-\frac{-5+2\sqrt{6}}{2}\right)\left(s-\frac{-5-2\sqrt{6}}{2}\right)}$$ 2'

$$= \frac{A}{s} + \frac{B}{s-\frac{-5+2\sqrt{6}}{2}} + \frac{C}{s-\frac{-5-2\sqrt{6}}{2}}$$

$$A = sV_C(s)\Big|_{s=0} = 20 \qquad 2'$$

$$B = \left(s - \frac{-5+2\sqrt{6}}{2}\right) V_c(s)\Big|_{s=\frac{-5+2\sqrt{6}}{2}} = -10.205 \qquad 2'$$

$$C = \left(s - \frac{-5-2\sqrt{6}}{2}\right) V_c(s)\Big|_{s=\frac{-5-2\sqrt{6}}{2}} = 0.205 \qquad 2'$$

$$\Rightarrow V_C(s) = \frac{20}{s} + \frac{-10.205}{s+0.0505} + \frac{0.205}{s+4.949} \qquad 2'$$

$$\Rightarrow V_c(t) = [20 - 10.205e^{-0.0505t} + 0.205e^{-4.949t}]u(t) \ (V)$$

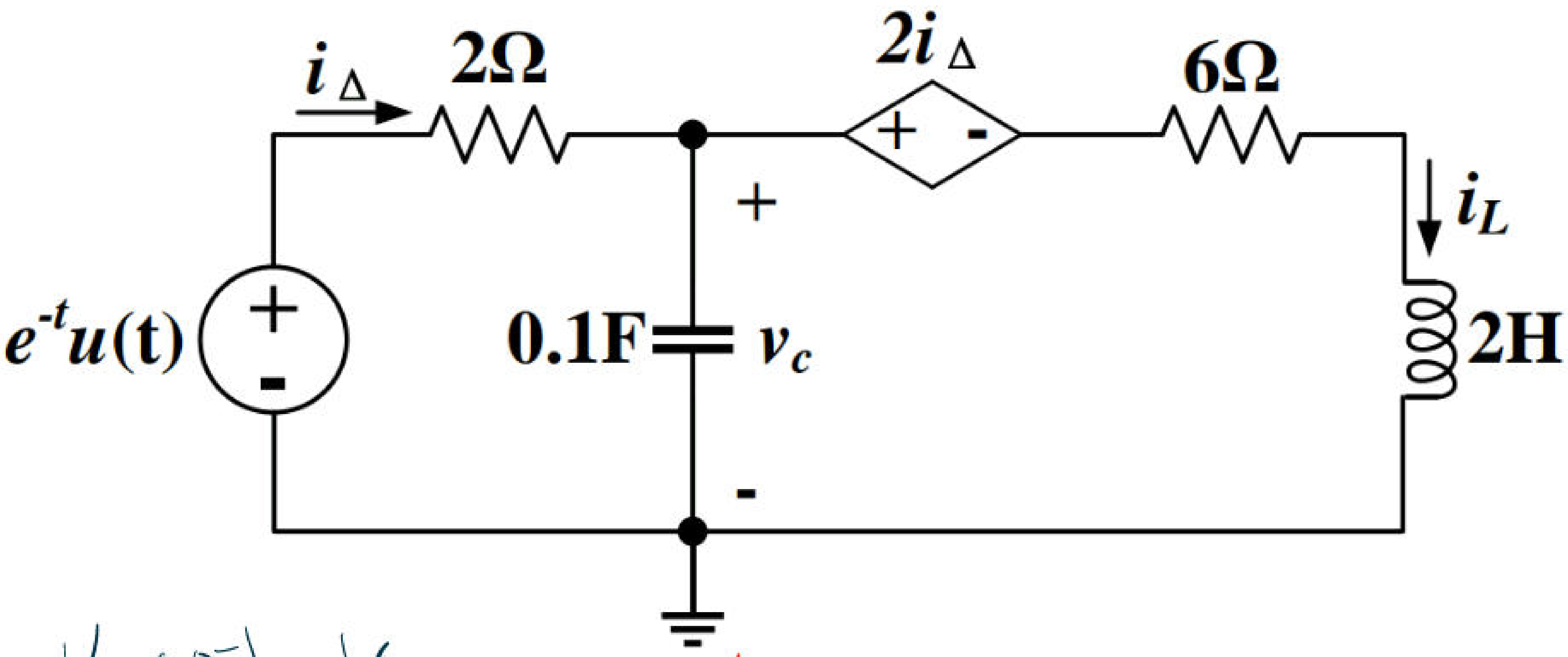
$2'$ $\quad t>0$

36'

3. For the circuit below, $u(t)$ means unit step function. Use Laplace domain method to calculate

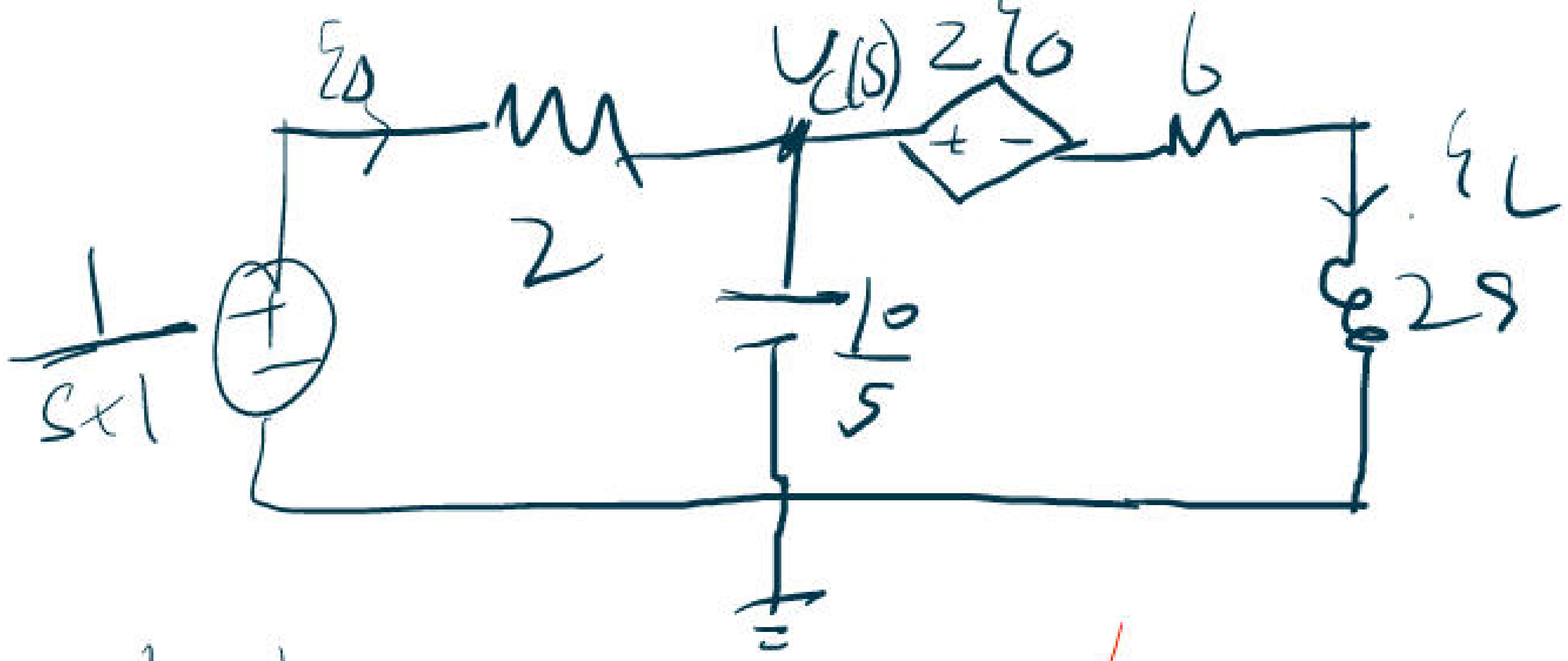
(a) $i_L(t)$ for t >0

(b) $v_c(t)$ for t >0



$V_C(0^-) = V_0 = 0$ 2'

$i_L(0^-) = I_0 = 0$ 2'



2'

Apply KCL: 3'

(a)

$$\frac{V_C(s) - \frac{1}{s+1}}{2} = -I_\Delta(s) \quad ①$$

$$-I_\Delta(s) + \frac{V_C(s)}{\frac{10}{s}} + \frac{V_C(s) - 2I_\Delta(s)}{6+2s} = 0 \quad ②$$ 3'

$$\frac{V_C(s) - 2I_\Delta(s)}{6+2s} = I_L(s) \quad ③$$ 2'

$$\Rightarrow I_L(s) = \frac{5-s}{2(s+1)(s^2+8s+25)}$$ 2'

$$= \frac{1}{2}\left[\frac{K_1}{s+1} + \frac{K_2}{s-(-4+3j)} + \frac{K_2^*}{s-(-4-3j)}\right]$$ 2'

Then $\alpha = -4 \quad \omega = 3$ 2'

$$k_1 = \frac{5-s}{2(s^2+8s+25)}\Bigg|_{s=-1} = \frac{1}{6} \qquad 2'$$

$$k_2 = \frac{5-s}{2(s+1)(s-(-4-3j))}\Bigg|_{s=-4+3j} = 0.186\angle 116.565^\circ \qquad 2'$$

$$\Rightarrow I_L(t) = \left[0.167e^{-t} + 0.373e^{-4t}\cos(3t+116.565^\circ)\right]u(t)\ (A)$$

$2'$ $\qquad t>0$

(b) Also from ①②③

$$\Rightarrow V_C(s) = \left[\frac{(3+s)(5-s)}{(s+1)(s^2+8s+25)} + \frac{1}{s+1}\right]\frac{1}{2} \qquad 2'$$

$$= \frac{5s+20}{(s+1)(s^2+8s+25)} \qquad 2'$$

$$2' \qquad = \frac{k_1}{s+1} + \frac{k_2}{s-(-4+3j)} + \frac{k_2^*}{s-(-4-3j)}$$

$$k_1 = (s+1)V_C(s)\Big|_{s=-1} = \frac{5}{6} \qquad \alpha=-4,\ \omega=3$$

$2'$

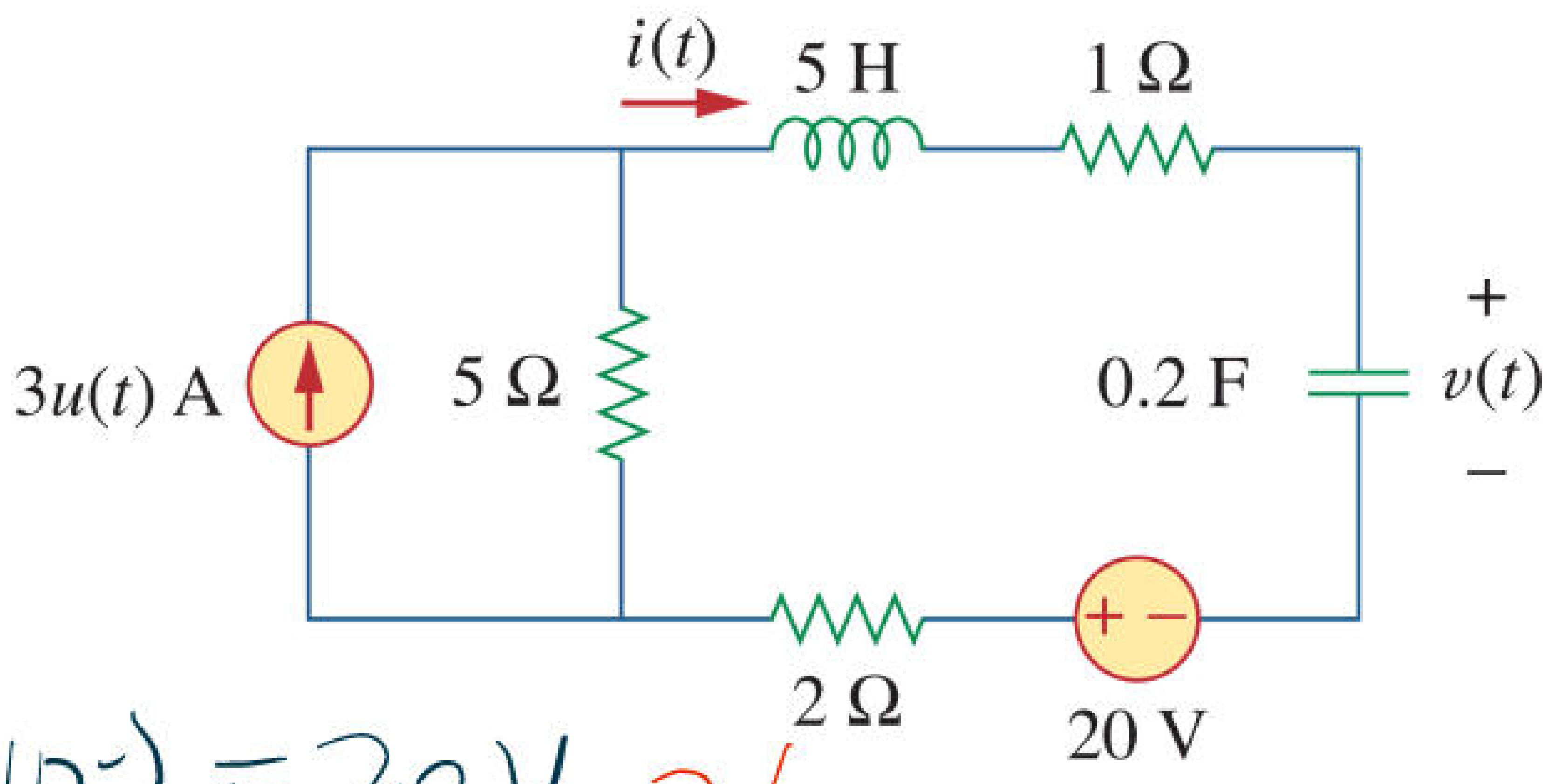
$$k_2 = \left[s-(-4+3j)\right]V_C(s)\Big|_{s=-4+3j} = \frac{5j}{-6-6j} = 0.589\angle -135^\circ$$

$$\Rightarrow V_C(t) = \left[0.833e^{-t} + 1.178e^{-4t}\cos(3t-135^\circ)\right]u(t)\ (V)$$

$2'$ $\qquad t>0$

26'

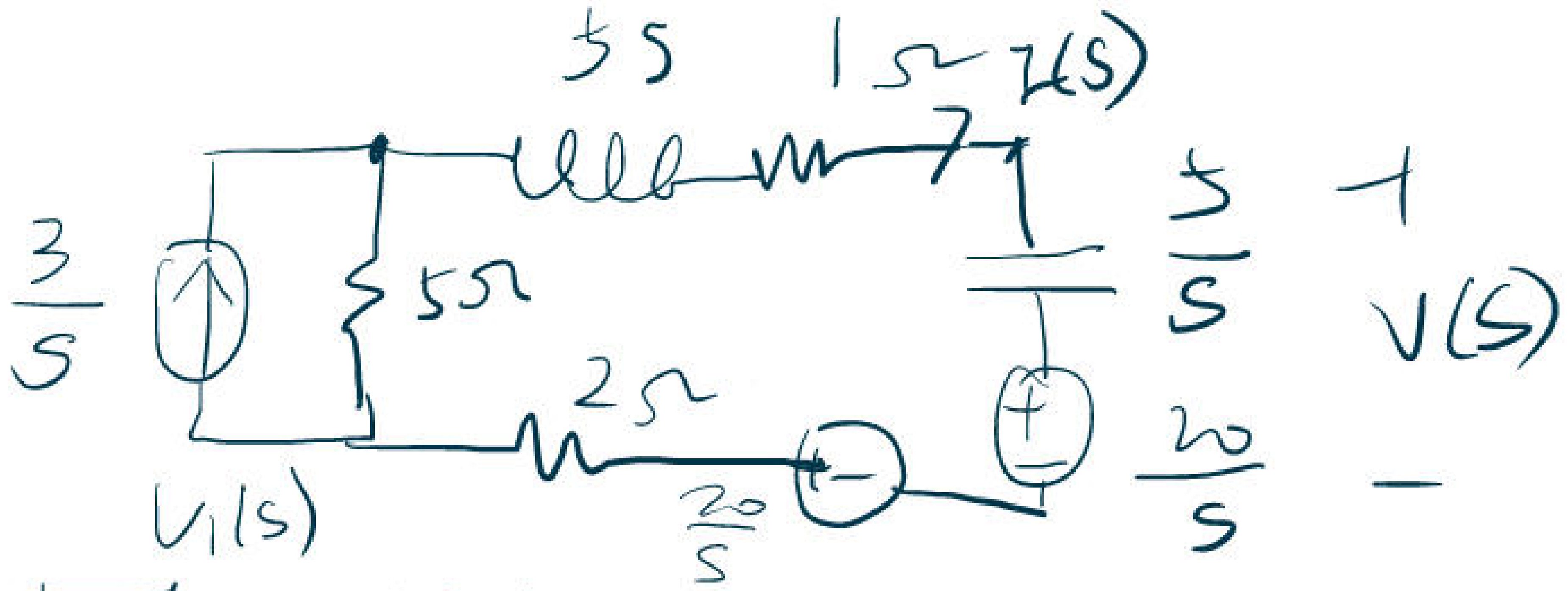
4. Use Laplace domain method to obtain *v(t) and i(t)* for t>0 in the circuit below.

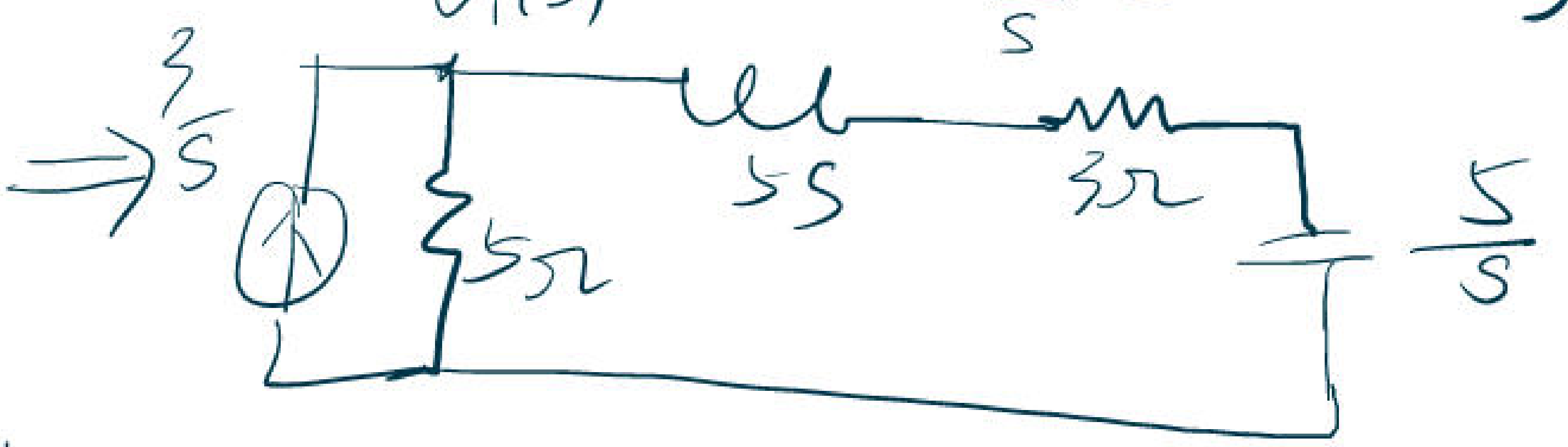


$V_C(0^-) = 20V$ 2'

$i_L(0^-) = 0A$ 2'

s-domain





2'

Apply KCL: $\frac{V_1(s)}{5} - \frac{3}{s} + \frac{V_1(s)}{5s+3+\frac{5}{s}} = 0 \Rightarrow V_1(s) = \frac{15s^2+9s+15}{s(s^2+1.6s+1)}$ 2' 2'

$I(s) = \frac{V_1(s)}{5s+3+\frac{5}{s}} = \frac{3}{s^2+1.6s+1} = \frac{K_1}{s-(-0.8+0.6j)} + \frac{K_1^*}{s-(-0.8-0.6j)}$ 2'

$K_1 = [s-(-0.8+0.6j)]\, I(s) \Big|_{s=-0.8+0.6j} = 2.5\angle -90^\circ$ $\begin{cases} \alpha = 0.8 \\ \omega = 0.6 \end{cases}$ 1' 1'

$\Rightarrow i(t) = \{5e^{-0.8t} \cos(0.6t - 90^\circ)\}u(t)\ (A)$ $t>0$ 2' 2'

$$V(s) = V_1(s)\frac{\frac{5}{s}}{5s+3+\frac{5}{s}} + \frac{20}{s} = \frac{100s^2+160s+175}{s(5s^2+8s+5)} = \frac{20s^2+32s+35}{s(s^2+1.6s+1)} = \frac{K_1}{s} + \frac{K_2}{s-(-0.8+0.6j)} + \frac{K_2^*}{s-(-0.8-0.6j)}$$

2'

$$K_1 = sV(s)\Big|_{s=0} = 35$$

2'

$$K_2 = \left(s-(-0.8+0.6j)\right)V(s)\Big|_{s=-0.8+0.6j} = 12.5\angle 126.87^\circ$$

2'

Since $\alpha = -0.8$ $\omega = 0.6$

$$\Rightarrow v(t) = \left[35 + 25e^{-0.8t}\cos(0.6t + 126.87^\circ)\right]u(t)\ (V) \quad t>0$$

2'