

Notes for Final Exam P3

Solution:

1. For the instance T of Average-sum, giving a subset R of T as a certificate, check whether the sum of R is the average of T , i.e. $h(T)$.
2. We reduce Subset-sum to Average-sum. Let (S, t) be an instance of Subset-sum. Let $S' = \{x \in S \mid x \leq t\}$, and suppose $|S'| = n$. we add $y = (n+1)t - \sum_{i=1}^n x_i$, i.e. $y = (n+1)t - \sum_{x_i \in S'} x_i$ to S' , then we obtain the instance T , i.e. $T = S' \cup \{y\}$, we can find $h(T) = t$. We claim that the Subset-sum instance (S, t) has a solution if and only if the Average-Sum instance T has a solution.
3. assume there's a subset R of S which sums to t . Then R is also a subset of T (the number larger than t will not be in R), so there's a subset of T which sums to $h(T)$. Next, assume there's a subset R of T which sums to t . Now, if $y > t$, then $y \notin R$, and so we have a subset R of S summing to t . Otherwise, $y \leq t$, which implies $\sum_{i=1}^n x_i \geq nt$. But since every $x_i \leq t$ and there are n values, this implies every $x_i = t$. Thus, any $x_i \in S$ a solution to Subset-sum.

criteria for Q3 (3+3+4)

1. NP proof

the full points at least include the certificate, i.e. one subset R of set T , and compare the sum of the subset R with the average of the set T .

Some wrong answers:

- None
- Iterate all the subsets and check the sum or check the problem itself, i.e. check whether there is a subset with sum to $h(T)$.
- just say compute $h(T)$ takes polynomial time.

2. construction

you should reduce the subset_sum to average_sum, i.e. given any instance (S, t) of a subset_sum, try to construct an instance (T) of average-sum. Hence, any construction that gives the target of subset_sum, such that $t = h(T)$ is wrong. In principle, it is the opposite direction of the reduction construction. These answers will lose all the remaining points.

The correct construction is that given (S, t) , you should try to construct another set T s.t. it satisfies some conditions.

Hence, all the construction which constructs the new set will get one point. (Such as add some elements, remove some elements, change some elements)

If you successfully consider the goal is to make the average of the new set be t , you will get 2 points here. Additionally, if you have considered removing some elements which are larger than t , you will get 3 points.

3. proof of iff

If you forget to check the additional element or cannot describe it clearly when proving Average_sum yes \rightarrow subset sum yes, you will lose 1 or 2 points.

exam

☒ Resolved ☐ Unresolved



葛煦 5 months ago

Should y be $(n+1)t - \sum_{x_i \in S'} x_i$ instead? $(n+1)t - \sum_{i=1}^n x_i$ might be negative.

helpful! | 0



杨飞明 5 months ago

Since $1 \leq i \leq n$ for x_i and $|S'| = n$, here in fact it indicates what you means. Still, thanks for your suggestion. I will change the description to make it more clear.

good comment | 0

