Electric Circuits

Discussion 1

TA: Kefei Wu, Ke Zhang

Spring 2018

Outline

- Introduction
- Homework 1
- Review part
  - Basic Concepts : Charge, Energy and Power
  - Methods to analysis electric circuit: KCL & KVL, Nodal and Mesh
  - Circuit Theorems: Main ideas & their transformations
- Extension part
  - Y – Δ Transformations

Introduction

- Professor: Chaofeng Ye
- TAs: Kefei Wu, Ke Zhang
- Lectures:
  - Monday, 10:15AM – 11:55AM
  - Wednesday, 10:15AM – 11:55AM (EVEN week only!)
- Discussion: 19:30-21:00, Friday
- Office Hour: Just After the Discussion time.
- QQ(recommended) and Email Questioning is always available.

  wukf@shanghaitech.edu.cn 512150874

  zhangke1@shanghaitech.edu.cn 1296620420

Workload/Grading Policy

- 10 homework assignments: 20%
- 8 lab assignments: 25%
  - No late HW or Lab reports accepted!
- 2 midterms (15% x 2) + 1 final exam (20%)
  - Midterm 1: week 6 (tentative)
  - Midterm 2: week 11 (tentative)
  - Notify the instructor immediately if you miss an exam due to an unforeseeable event, and submit a note from your physician in case of illness.
  - NO make-up exams!
- Quizzes (5%)
  - Quizzes are held in <u>classes</u> and will not be announced. There won't be any makeup quizzes.

请务必遵守学术道德规范!

- 单次作业或者实验抄袭
  - 抄袭与被抄袭者该次作业/实验均计零分，课程总成绩打九折。
- 累计两次作业或者实验抄袭
  - 抄袭与被抄袭者相应作业/实验计零，课程总成绩均打七折。
- 累计三次作业或实验抄袭者，或者考试作弊者
  - 课程总成绩计零，同时上报信息学院学术委员会公开处理。

Homework

- 1. Submission
  - Monday in class (Before 10:15)
  - No LATE submission will be accepted.
  - Appealing against the homework score.

- 2. Integrity
  - Cheating in exams
  - Copying in homework

- 3. Collection
  - Your homework should be saved for total submission after the Final Exams (which values for 5% of regular grade).

Homework

- 4.Format
  - A4 (Printed version better)
  - Bind all your paper!
  - Transform the fraction(or π and other constants) to the float.
  - Units!
  - Process!
  - Keep your homework clean.

$$\begin{cases} i_1 = \dfrac{1}{360150}\,(A) \\ i_x = -1.027766209\times10^{-3}\,(A) \end{cases}$$

牛逼啊兄弟.. 小数点后面位数够了.

摸头.jpg

$\therefore i_6 = i_1 - i_x = 1.03054283\times10^{-3}\,(A)$

$V_{23} = 30 i_1 = \dfrac{1}{12005}\,(V) = 8.33\times10^{-5}\,(V)$

$P = \dfrac{3}{500} = 0.006\,(W)$

-3

Discussion Class

- 1. Time Adjustments.
  - Monday 12^{th} -13^{th} class in 1D-106 SIST
  - Tuesday 7^{th} -8^{th} class in 1D-106 SIST
  - Friday 12^{th} -13^{th} class in 1D-106 SIST
- 2.Quiz
- 3.Experience
  - What do you need for the following study in EE111?
- 4.Content

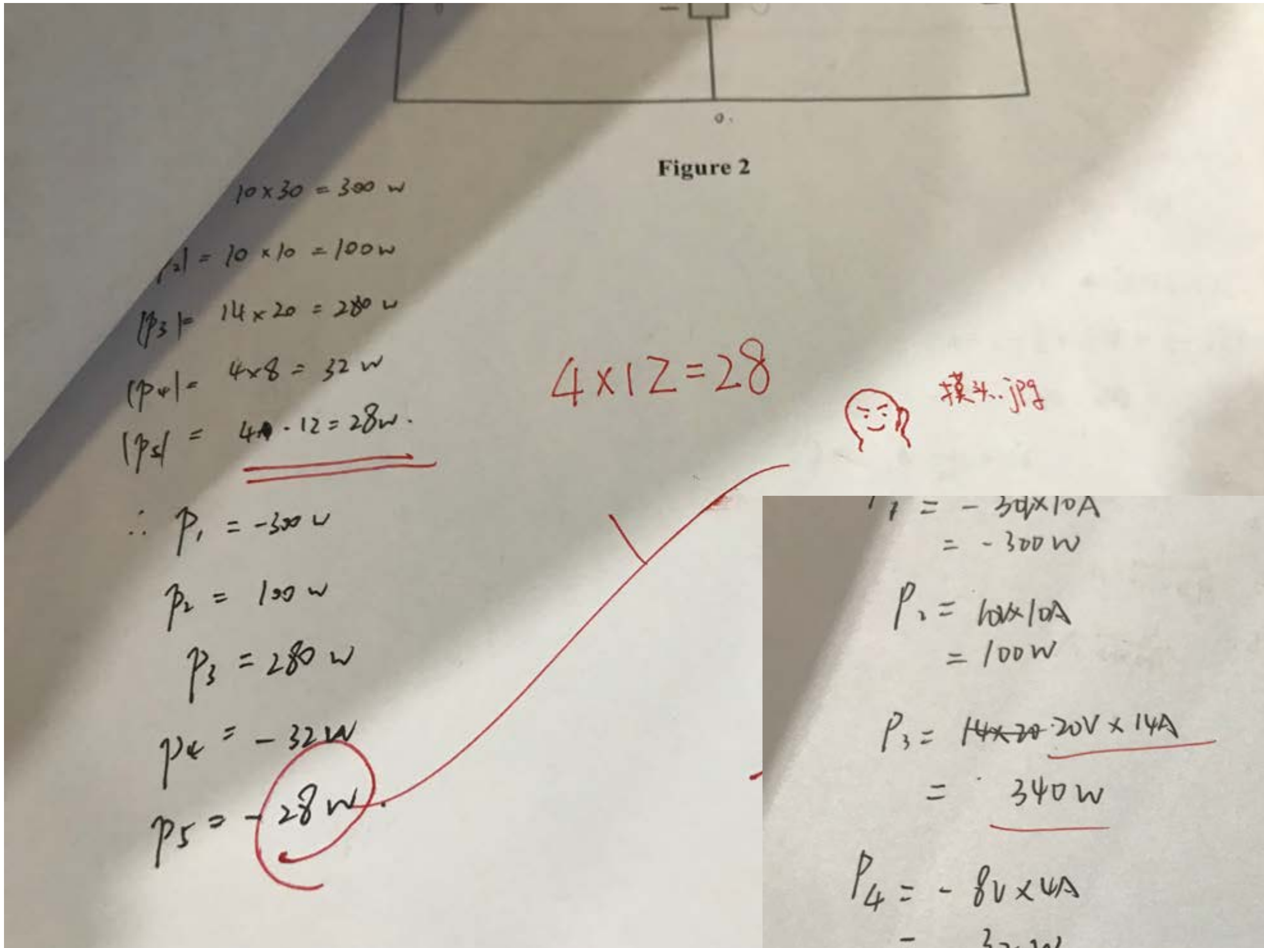
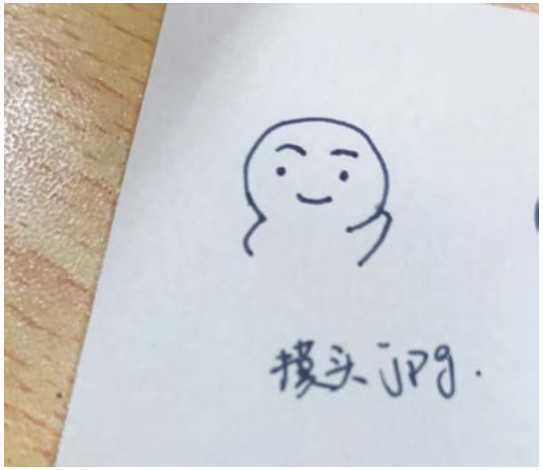
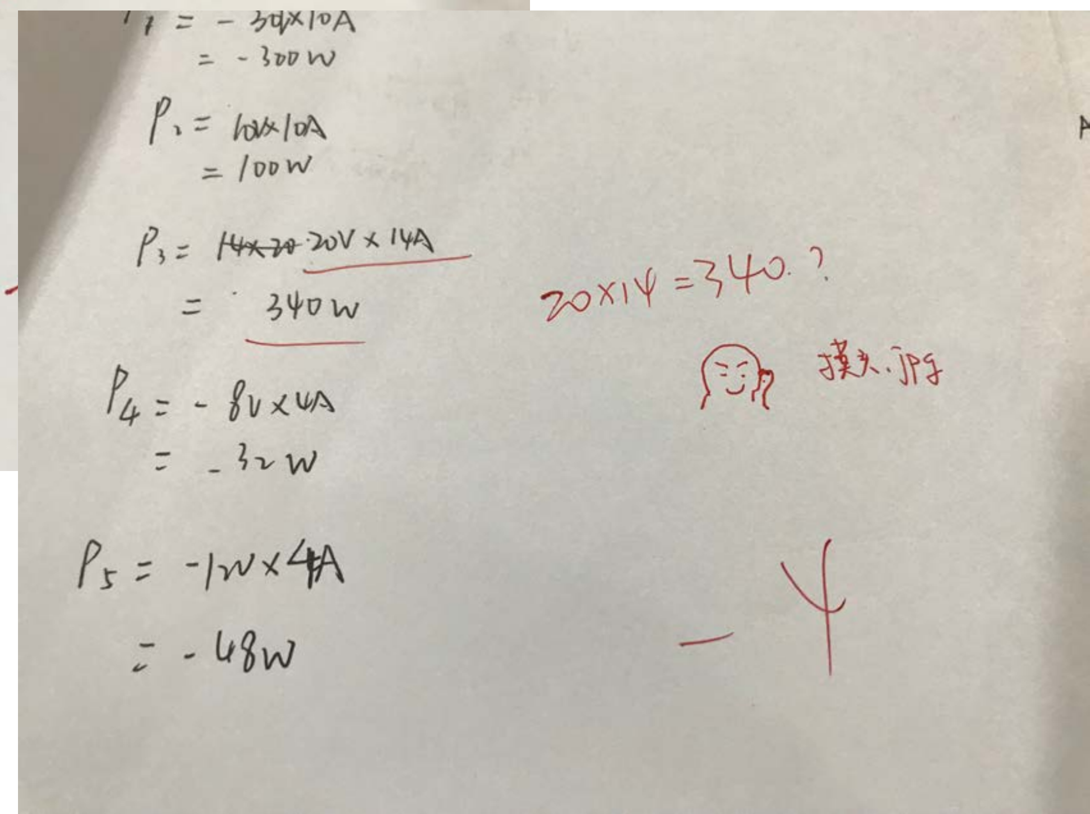


Figure 2
10×30 = 300 W
|P2| = 10×10 = 100W
|P3| = 14×20 = 280 W
|P4| = 4×8 = 32 W
|P5| = 4A · 12 = 28W.
4×12=28
摸头.jpg
∴ P1 = -300 W
P2 = 100 W
P3 = 280 W
P4 = -32W
P5 = -28W


摸头.jpg.


= -300W
P2 = 10V×10A
= 100W
P3 = 20V × 14A
= 340W
20×14 = 340. ?
摸头.jpg
P4 = -8V×4A
= -32W
P5 = -12V×4A
= -48W

Homework 1

(a) $q(t) = 1.7t(1 - e^{-1.2t})\ nC$

(b) $q(t) = 0.2t\sin(120\pi t) + \cos(2e^{-\sin t})\ mC$

(c) $i(t) = 4e^{-t} - 3e^{-2t}\ mA, q(0) = 0.2C$

(d) $i(t) = 12e^{-3t}\cos(40\pi t)\ nA, q(0) = 2.28\ pC$

Homework 1

2. Find the current i_1 and i_2 shown in Figure 1.

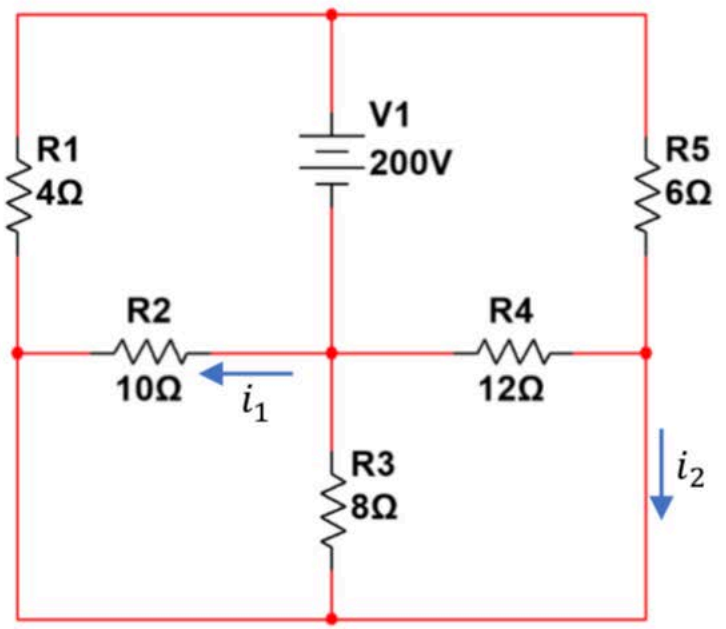


Figure 1

Homework 1

3. Find the power absorbed by each of the elements from p_1 to p_5 with the following circuit shown in Figure 2.

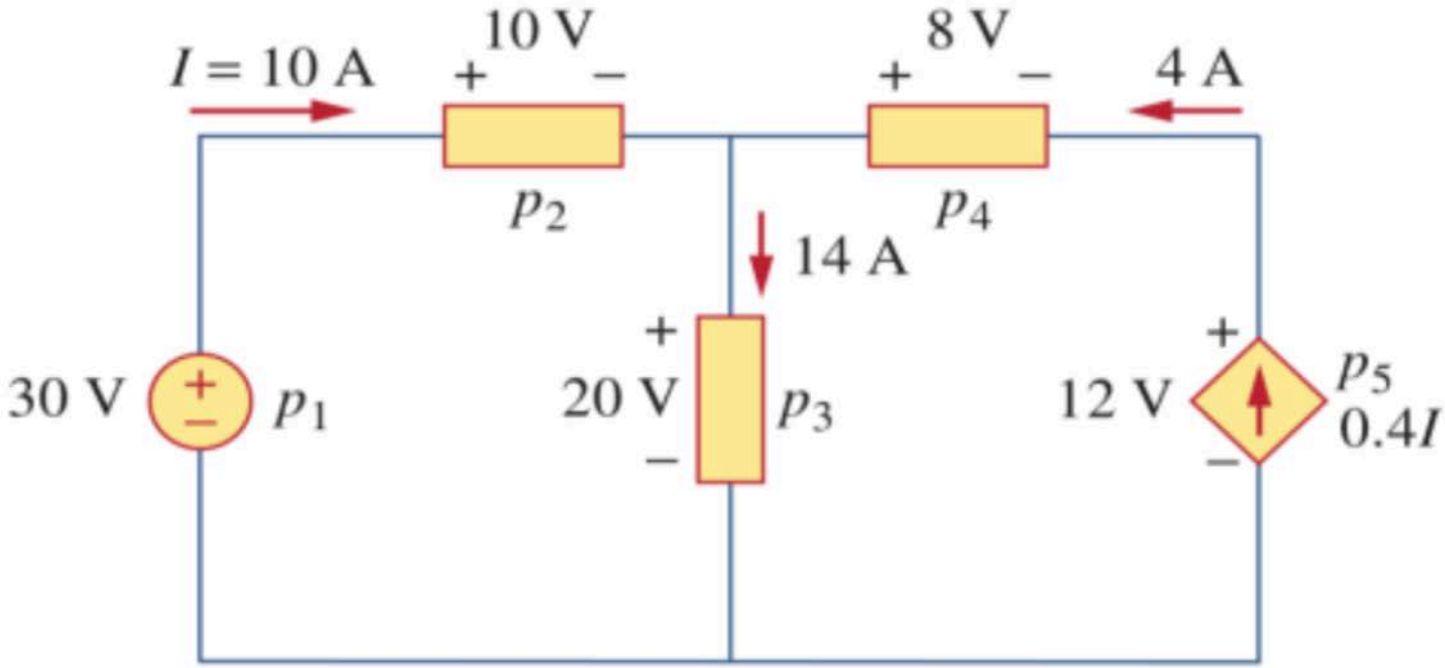


Figure 2

Homework 1

4. Find i_1, i_6, v *and power on the voltage source of* $8V$ with the circuit shown in the Figure 3.

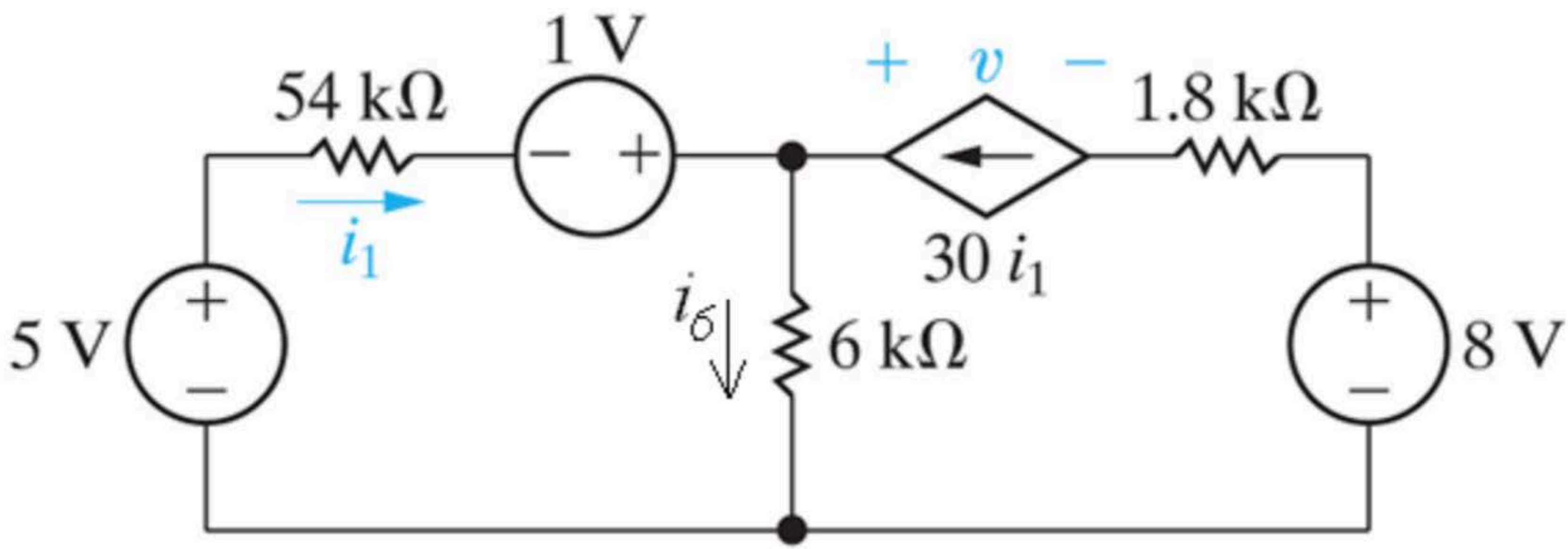


Figure 3

Homework 1

5. Use nodal analysis to find V_0 in the circuit of Figure 4.

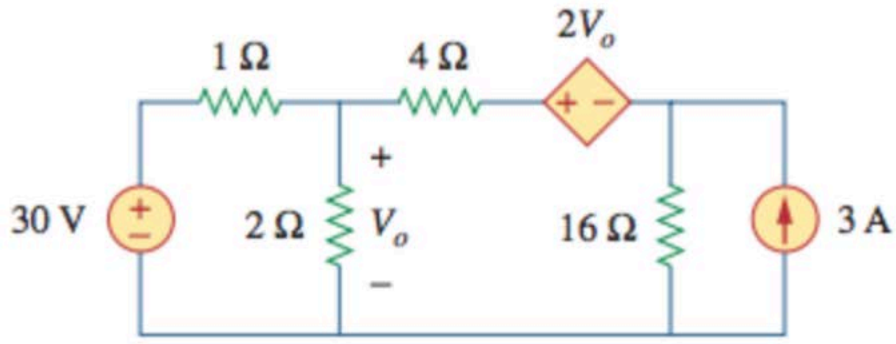


Figure 4

Homework 1

6. Apply mesh analysis to find I_X in the circuit of Figure 5.

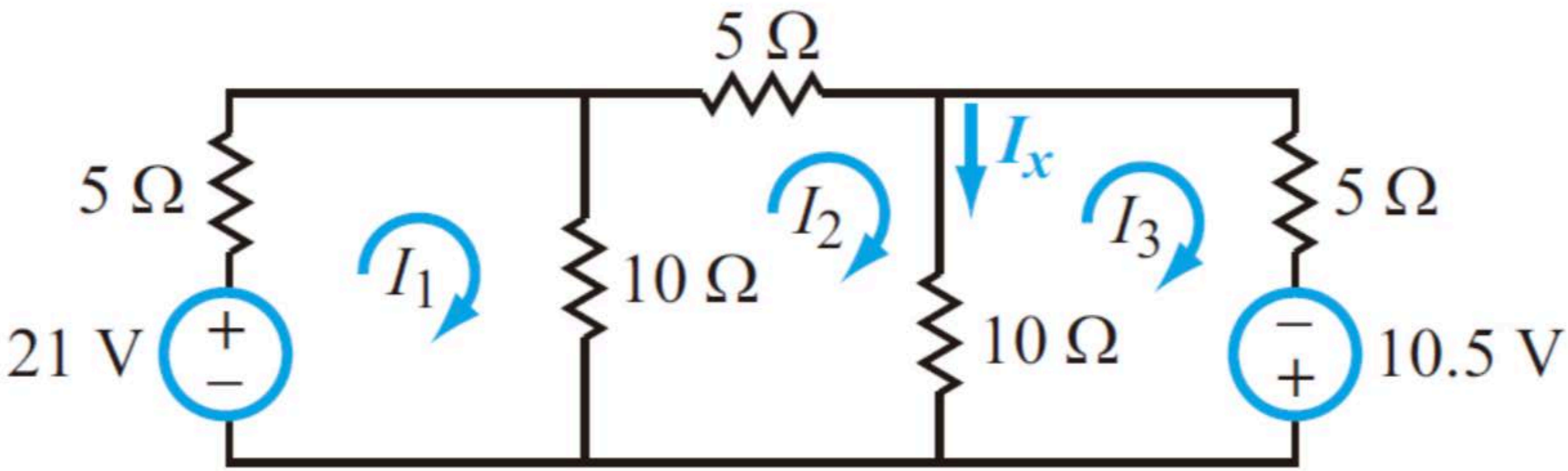


Figure 5

Homework 1

7. Determine **A** if $V_{out}/V_S = 9$ in the circuit of Figure 6.

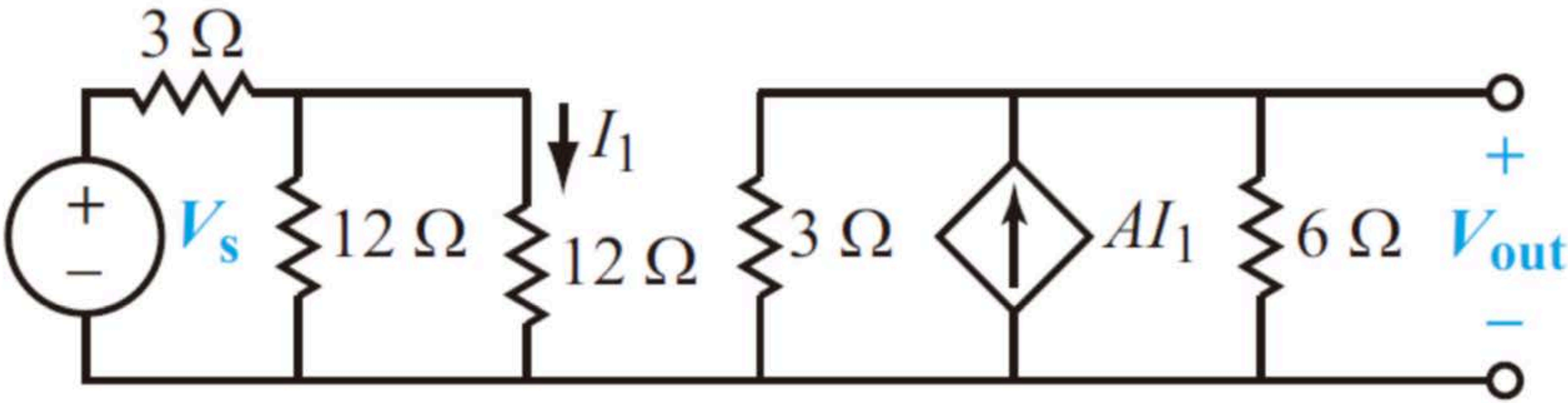


Figure 6

Basic concepts

Relationship between

- Charge, Current and Time
- Energy, Charge and Voltage
- Voltage and Current
- Power and Energy

Basic concepts

$$I = \frac{NetCharge\ \ crossing\ \ surface\ \ in\ \ time\ \ \Delta t}{\Delta t} = \frac{dq}{dt}$$

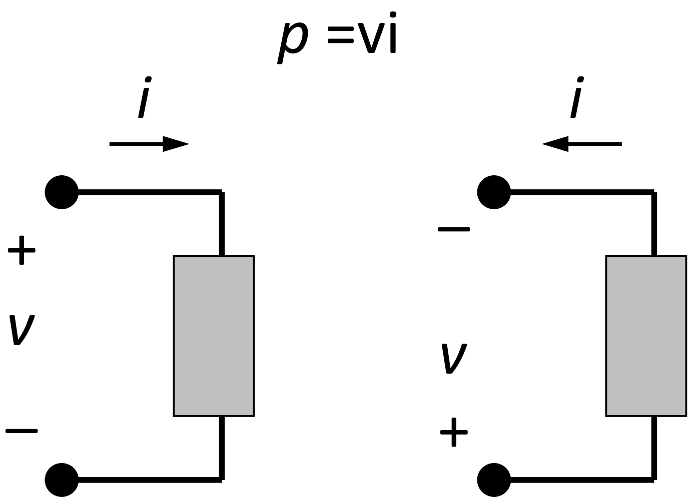
$$v = \frac{dE}{dq} \qquad\qquad E = \Delta q \cdot V_{AB}, \qquad V_{AB} = V_A - V_B$$

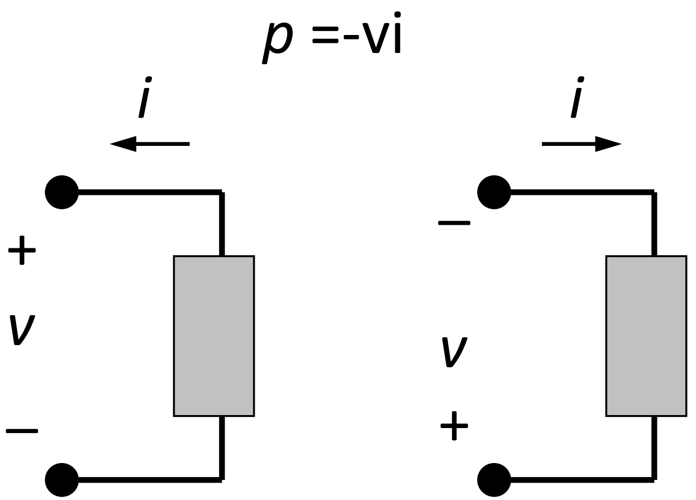
$$p = \frac{dE}{dt} = \frac{dE}{dq} \cdot \frac{dq}{dt} = v \cdot i$$

Basic concepts

- Voltage is a relative quantity between node to node
- Voltage is the reason to form Current

Sign Convention





Basic concepts

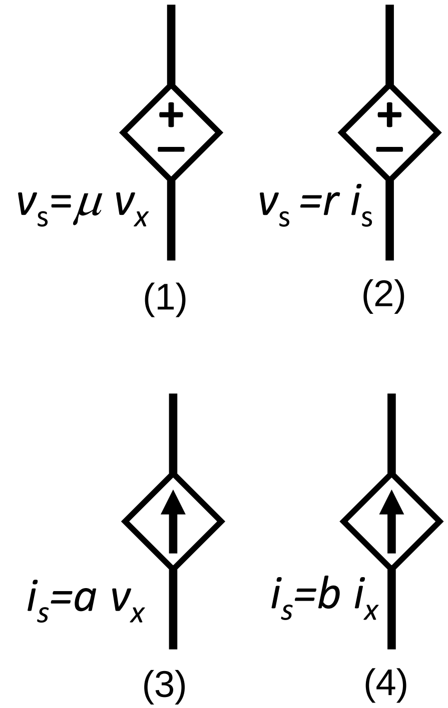
dependent source

(1) Voltage-Controlled Voltage Source(VCVS)

(2) Current-Controlled Voltage Source(CCVS)

(1) Voltage-Controlled Current Source(VCCS)

(1) Current-Controlled Current Source(CCVS)



Circuit analysis

Branch: represents a single element or a two-terminal element

Node: the point of connection between two or more branches

- two nodes actually becomes one node if there is no branch between them!

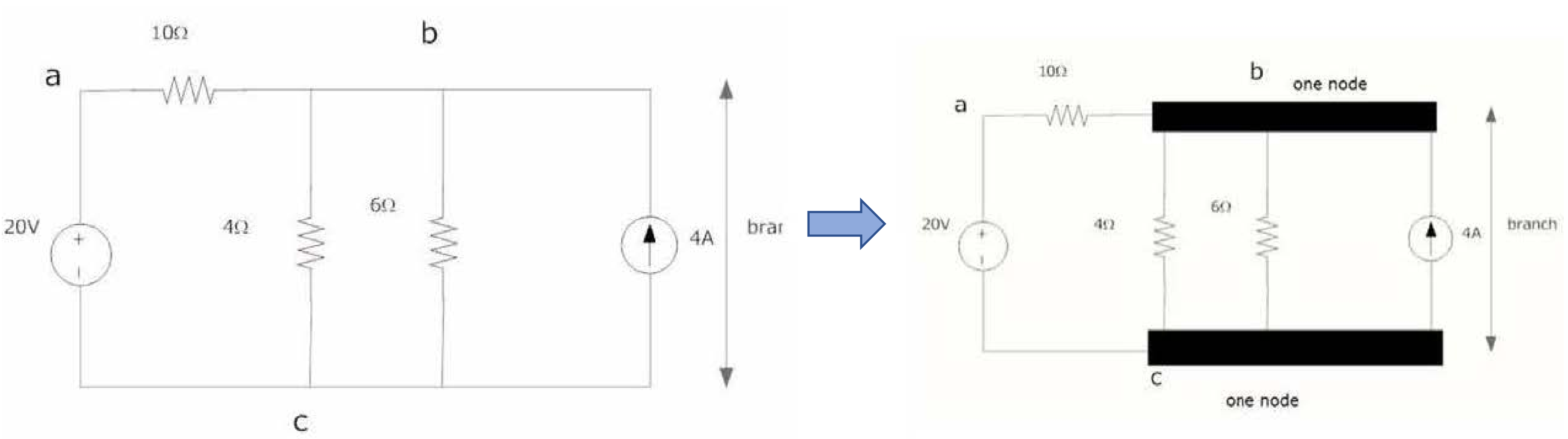
Loop: any closed path in circuit, starting from a node and returning to it without any repeated node.

- Independent loop(mesh): contains at least one branch which is not a part of any other independent loop

Circuit analysis

Relation

- A network with *b* branches, *n* nodes and *l* independent loops will satisfy the fundamental theorem of network topology **$b = l + n - 1$**



- Branch:5, Node:3, Independent Loop:3

Nodal Analysis – Three Steps

- Given a circuit with *n* nodes：
  1. Select a node as the reference (i.e., **ground**) node. Define the node voltages (except reference node and the ones set by the voltage sources).

  2. Apply **KCL** at nodes with unknown voltage, expressing current in terms of the node voltages

     Special cases: floating voltage sources?

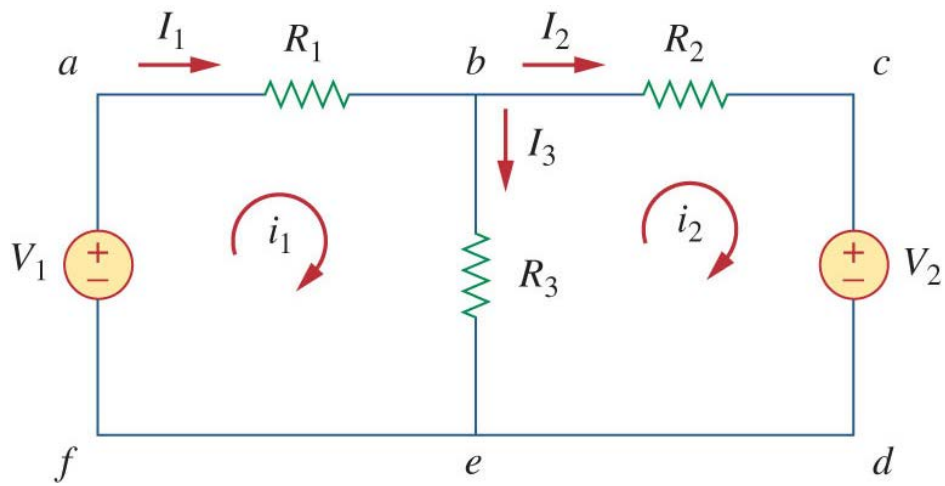
  3. Solve the resulting simultaneous equations to obtain the unknown node voltages.

Note: **series** connected elements of current source could be ignored

Mesh Analysis Steps – Three Steps

- Mesh analysis follows these steps:
  1. Assign mesh currents $i_1, i_2, \ldots i_n$ to the n meshes
  2. Apply **KVL** to each of the n mesh currents.
  3. Solve the resulting n simultaneous equations to get the mesh currents.

  Note: **parallel** connected elements of voltage source could be ignored



Which method could I choose?

Nodal analysis

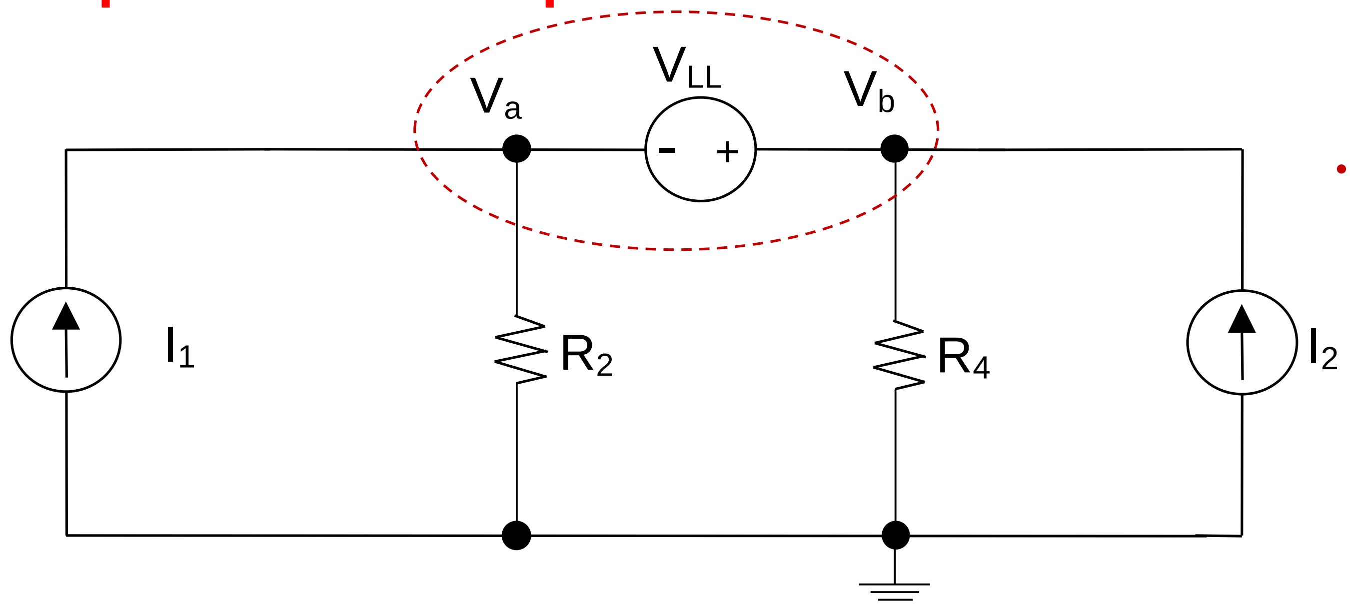
- If the network contains
  - ✓ Many parallel connected elements
  - ✓ Current sources
  - ✓ Supernodes
  - ✓ Circuits with fewer nodes than meshes
- If node voltages are what are being solved for
- Non-planar circuits can only be solved using nodal analysis

Which method could I choose?

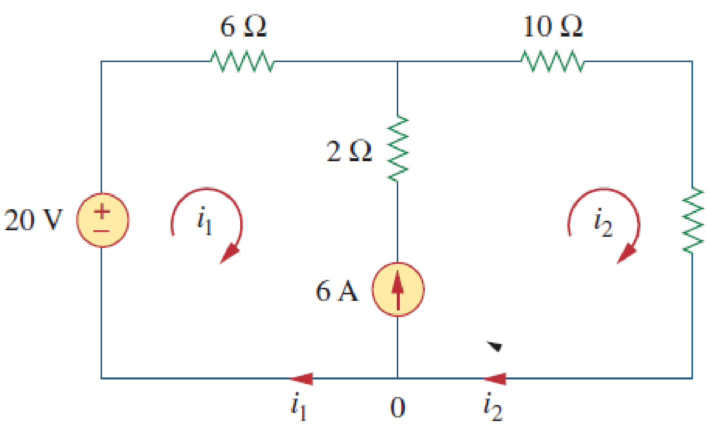
Mesh analysis

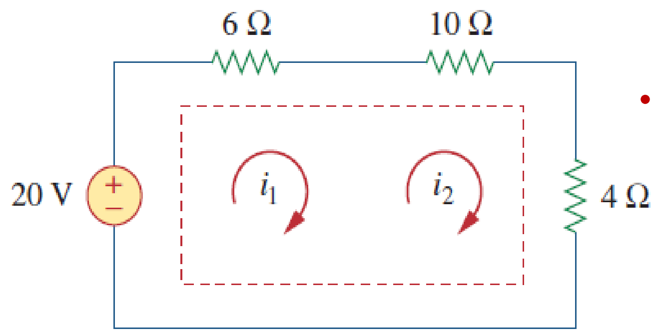
- If the network contains
  - ✓ Many series connected elements
  - ✓ Voltage sources
  - ✓ Supermesh
  - ✓ Circuits with fewer meshes than nodes
- If branch or mesh current is what are being solved for
- Non-planar circuits cannot only be solved

Supernode and Supermesh



- Contains only voltage source

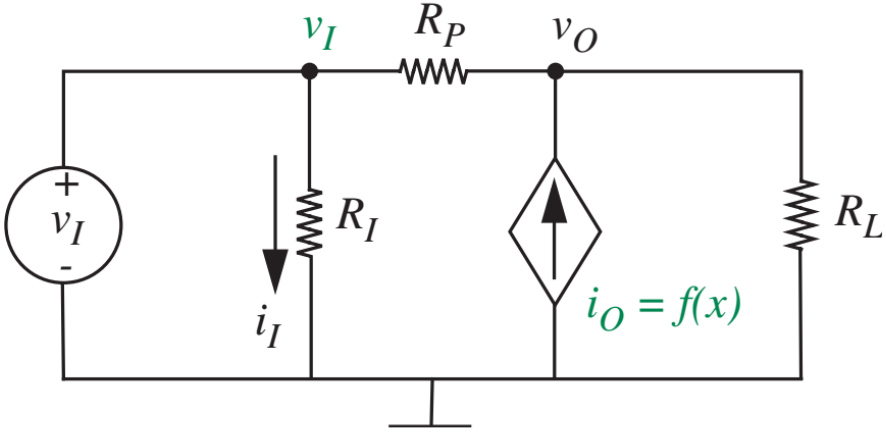




- Contains only current source

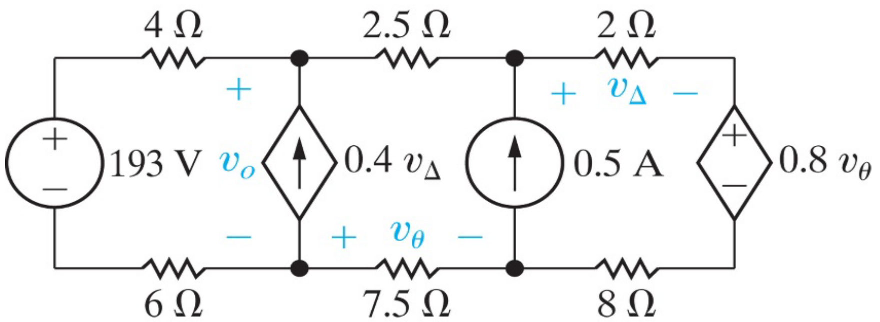
Nodal analysis with dependent source

- regard the dependent source as an independent source but add constraint equation. This is an equation that relates the dependent source's variable to the voltage or current that the source depends on in the circuit.

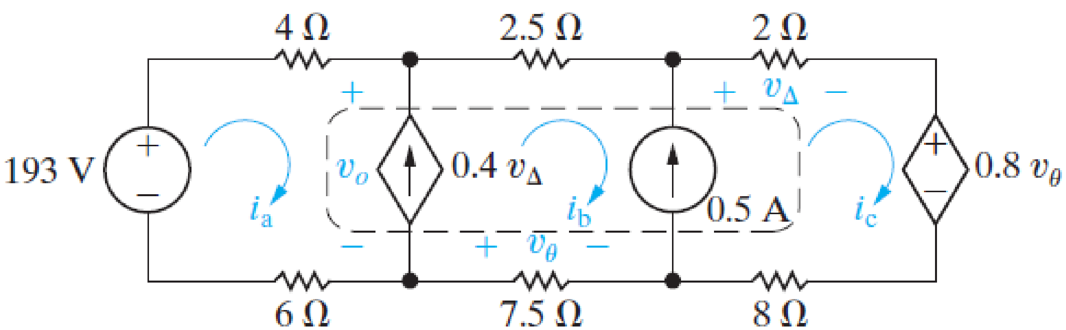


Exercise

- Which method is better?



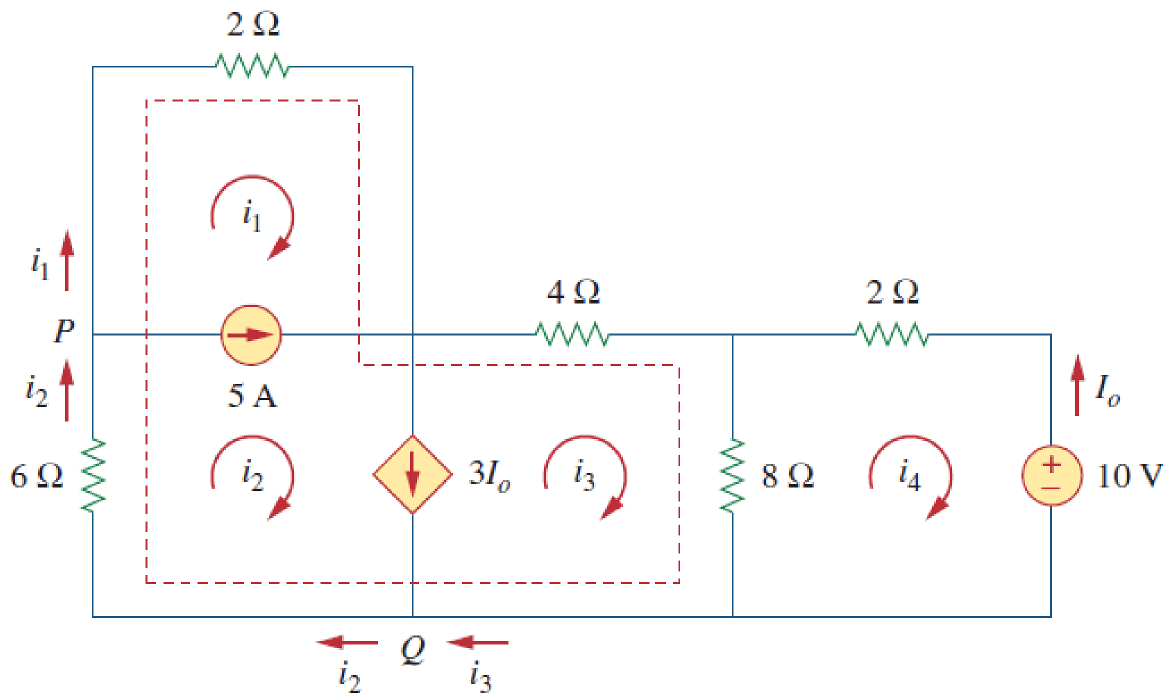
- 3 nodal voltage equations
- 2 constraint equations



- 1 supermesh
- 4 constraint equations

Exercise

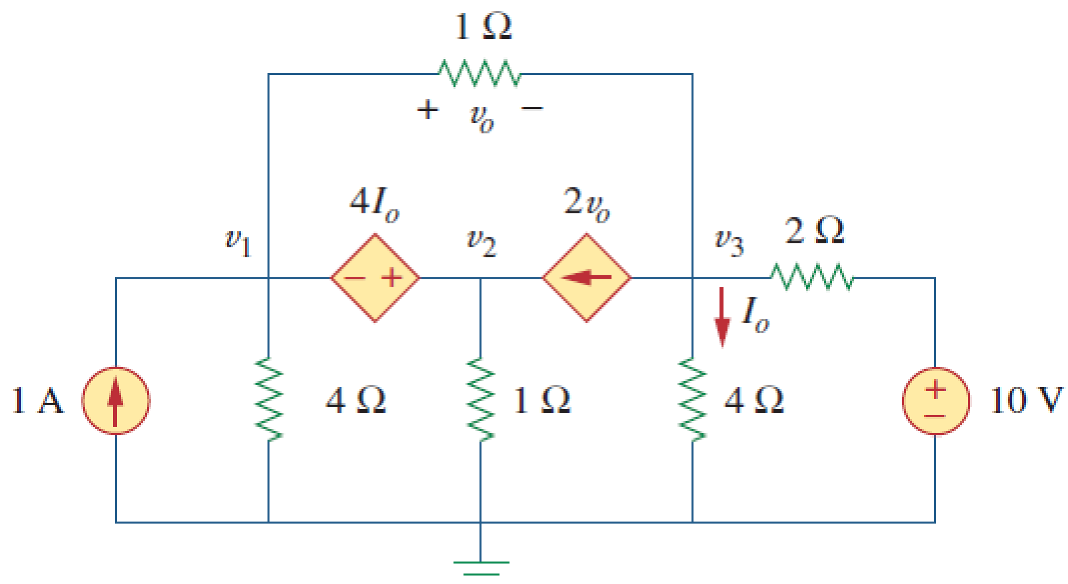
- Find i_1 to i_4 using mesh analysis.



- $i_1 = -7.5A, i_2 = -2.5A, i_3 = 3.93A, i_4 = 2.143A$

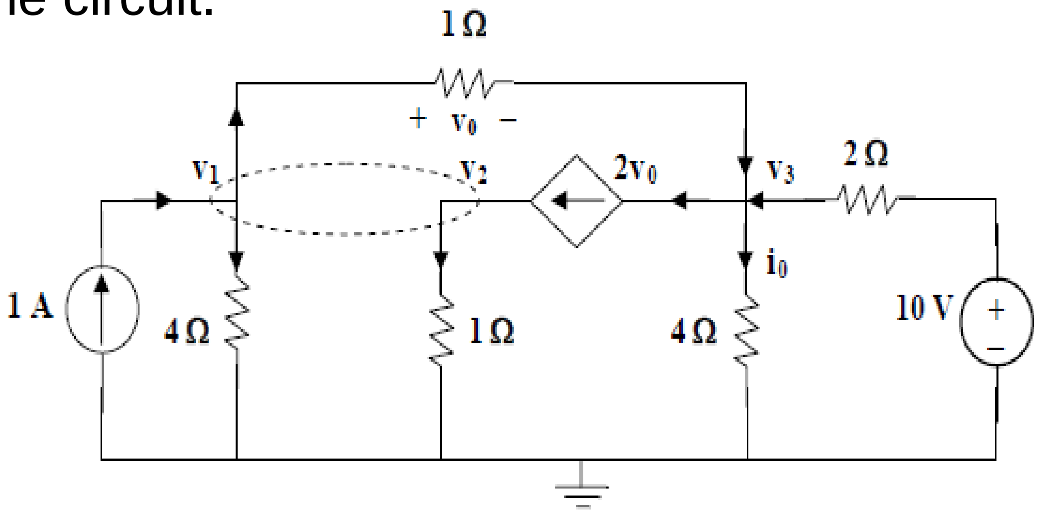
Exercise

- Find the node voltages for the circuit.



Exercise

- Find the node voltages for the circuit.

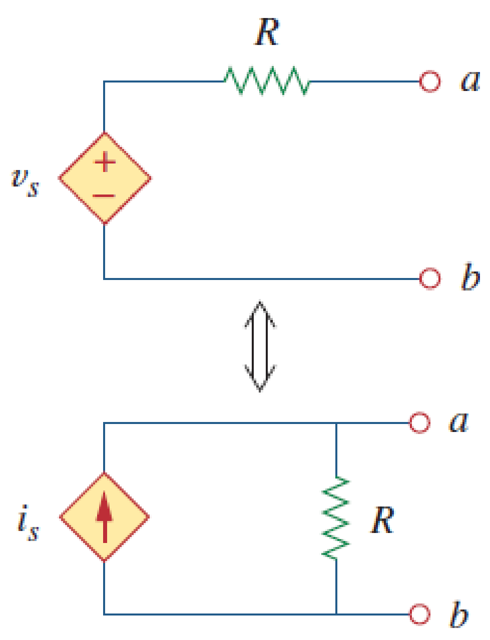


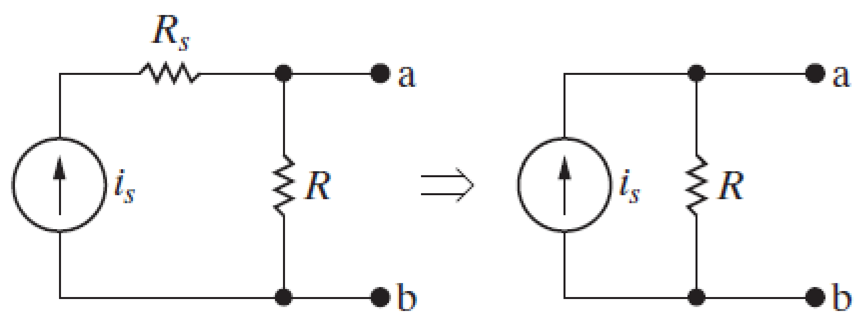
$$
\begin{cases}
1 + 2v_0 = \dfrac{v_1}{4} + \dfrac{v_2}{1} + \dfrac{v_1 - v_3}{1} \\
v_0 = v_1 - v_3 \\
2v_0 + \dfrac{v_3}{4} = v_1 - v_3 + \dfrac{10 - v_3}{2} \\
v_2 = v_1 + 4i_0 \\
i_0 = \dfrac{v_3}{4}
\end{cases}
$$

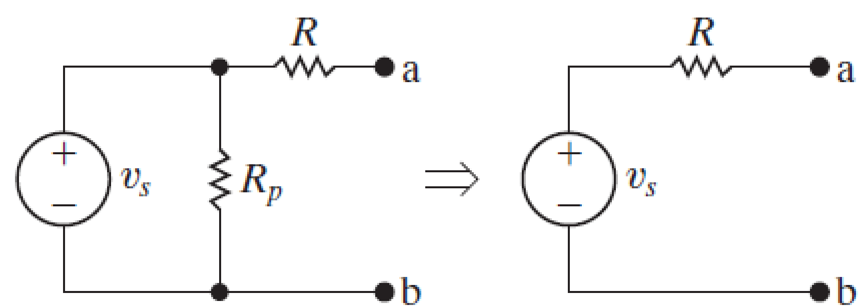
- $v_1 = 4.97V, v_2 = 4.85V, v_3 = -0.12V$

Source Transformation

- Very useful to simplify circuits
- Suitable for both independent and dependent sources.



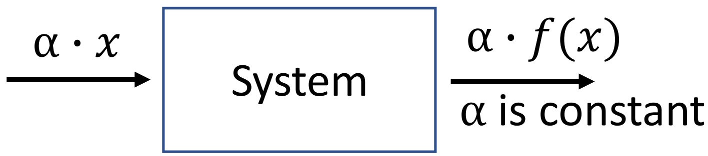


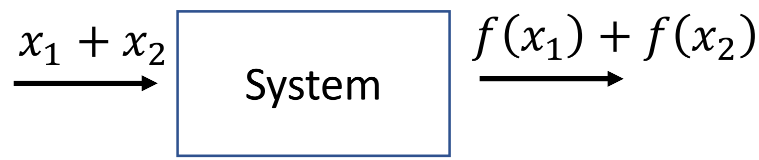


Circuit Theorems

Linear property

- homogeneity (scaling) property + additivity property



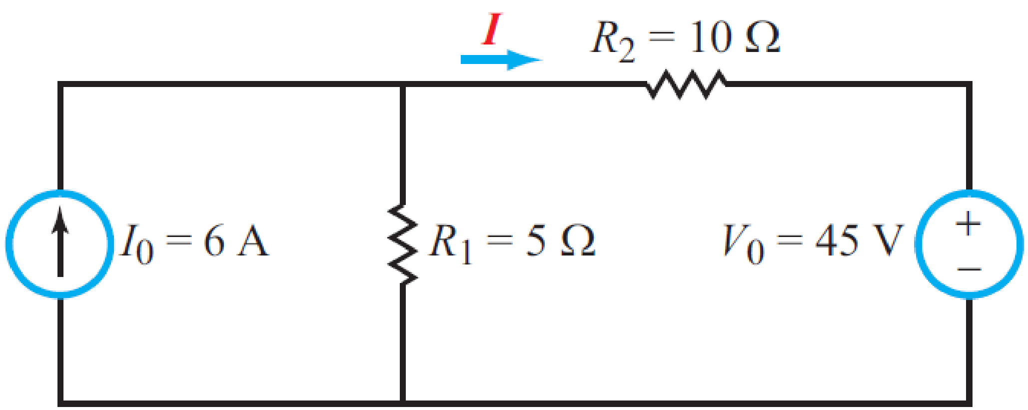


- suitable for circuits that consist of only linear elements (resistors, capacitors and inductors), linear dependent sources, and independent sources.

Circuit Theorems

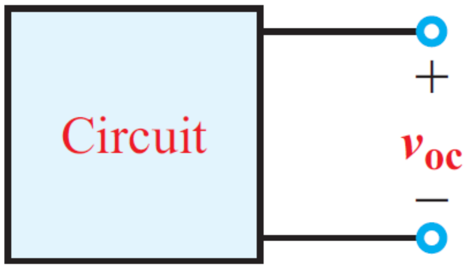
Superposition

- the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltages across (or currents through) that element due to each independent source acting alone.



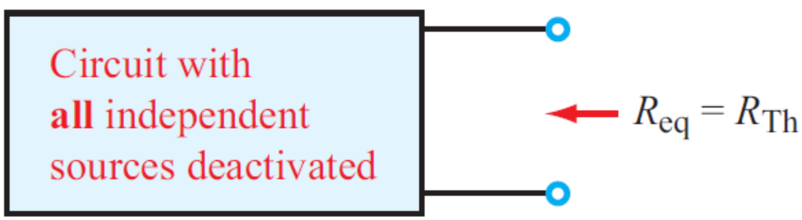
Circuit Theorems

Thevenin's Theorem

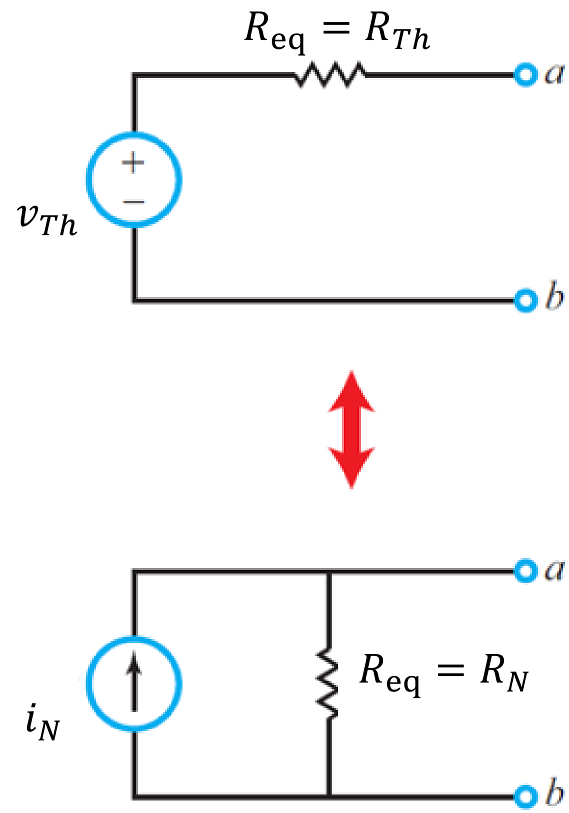


(a) $v_{\mathrm{Th}} = v_{\mathrm{oc}}$

Equivalent-Resistance Method



Norton's Theorem



Circuit Theorems

How to solve v_{Th}、 i_N、 R_{Th}

- Open/short circuit

✓ Analyze circuit to find v_{oc}、 i_{sc} (apply the previous : basic KCL KVL relation/ linear property/ superposition/ nodal or mesh)

$$v_{\mathrm{Th}} = v_{\mathrm{oc}} \qquad i_N = i_{sc} \qquad R_{Th} = \frac{v_{Th}}{i_N}$$

Circuit Theorems

How to solve v_{Th}、 i_N、 R_{Th}

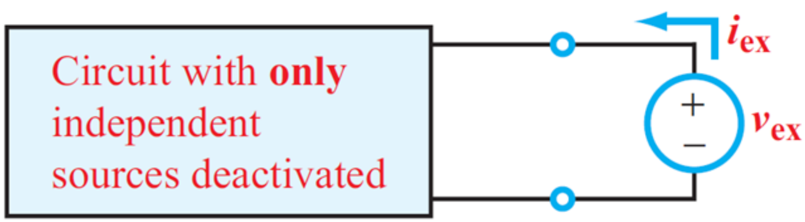
- Directly solve R_{eq}

✓ circuit has no dependent sources: Deactivate all independent sources

i.e., voltage source is shorted/ current source is opened(zero setting).

✓ circuit has dependent sources: external-source method

first deactivate independent sources, then add external source

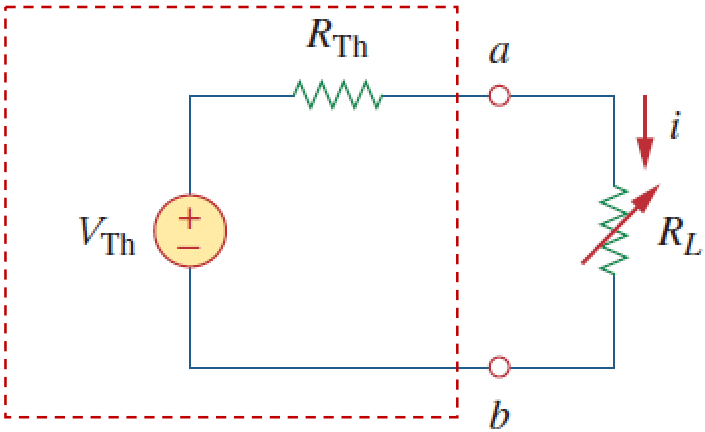


$$R_{Th} = \frac{v_{ex}}{i_{ex}}$$

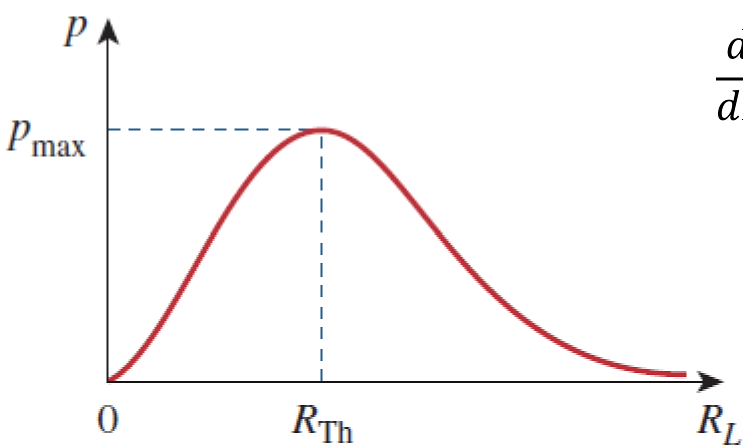
- There may exist negative resistance, this is possible for a dependent source.

Maximum Power Transfer

In many applications, we expect to maximize the power delivered to a load. Resistive network could be described as a Thevenin equivalent.



$$p = i^2 R_L = \left(\frac{V_{Th}}{R_{Th} + RL}\right)^2 R_L$$



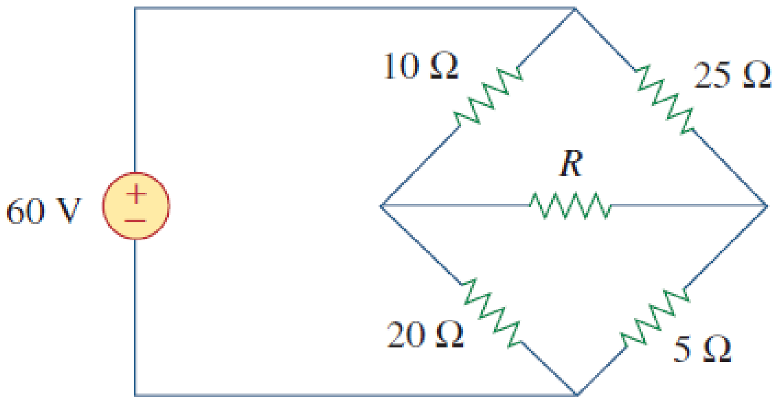
$$\frac{dp}{dR_L} = V_{Th}^2 \left[\frac{(R_{Th} + RL)^2 - RL \cdot 2((RT_h + RL)}{((RT_h + RL)^4}\right] = 0$$

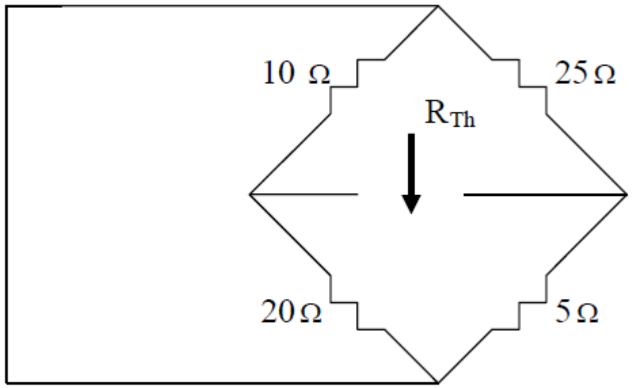
$$\longrightarrow R_L = R_{Th}$$

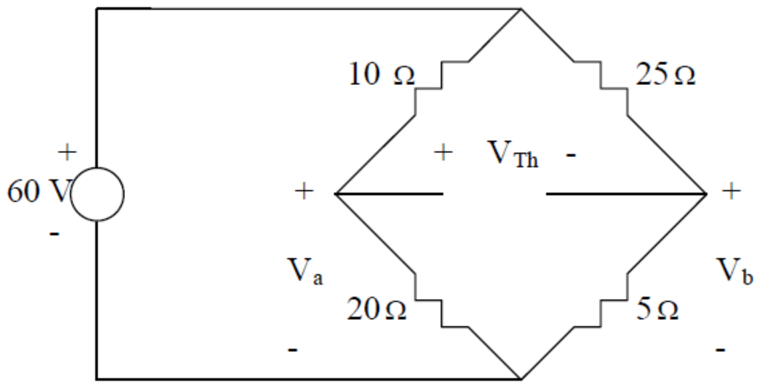
$$\longrightarrow p_{max} = \frac{V_{Th}^2}{4R_{Th}}$$

Exercise

- Determine the maximum power that can be delivered to the variable resistor R



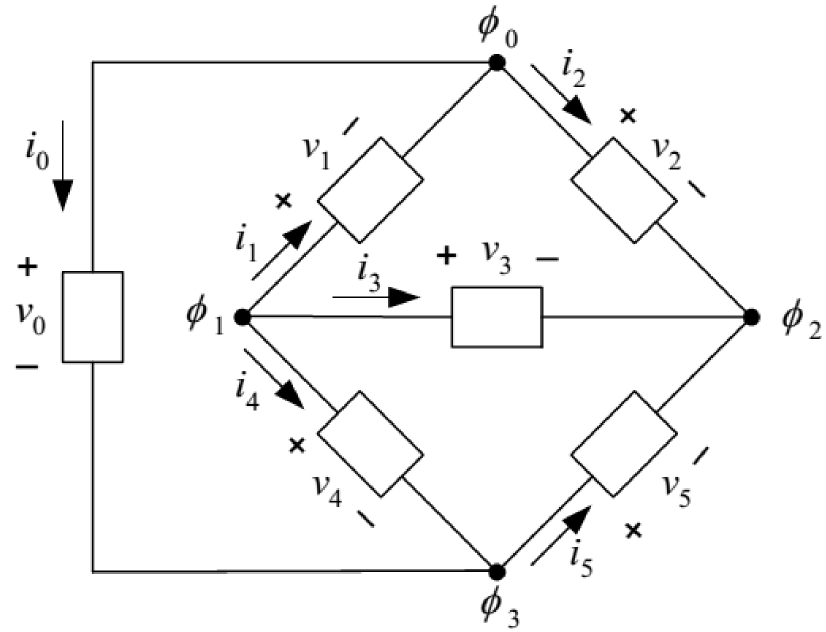




- $R_{Th} = 10.833\Omega, V_{Th} = 30V, p_{max} = 20.77W$

Tellegen's Theorem

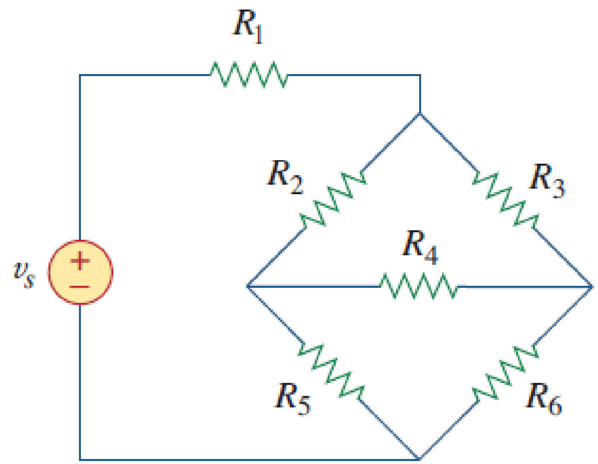
$$\sum_{n=0}^{5} i_n v_n = 0.$$

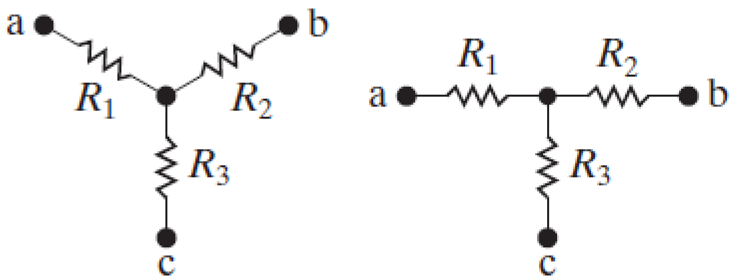


- In an electrical network, the summation of instantaneous power in all the branches is equal to zero.
- Extremely general; it is valid for any lumped network that contains any elements, *linear or nonlinear*, *passive or active*, *time-varying or time-invariant*.
- Can be used to check your computation result if you are not so sure.

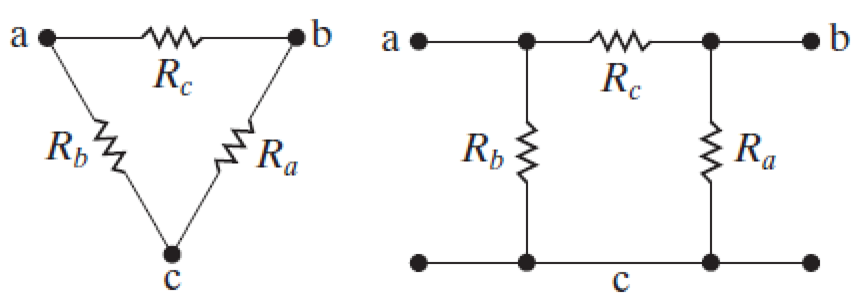
Extension: Y – Δ Transformations

- When the resistors are neither in parallel nor in series?
- Consider the bridge circuit on the left side, circuits of such type can be simplified by using three-terminal equivalent networks.





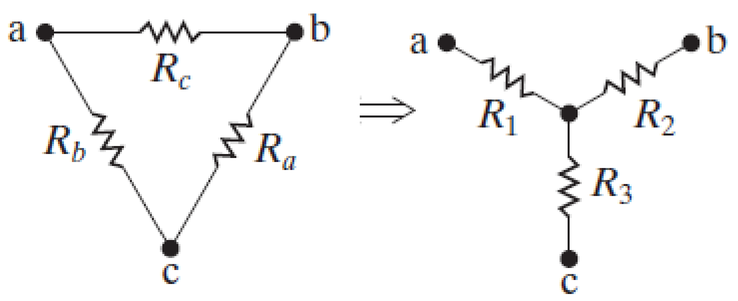
Y(T)



Δ(π)

Extension: Δ to Y Transformations

- $R_{ab} = R_c \,//\, (R_a + R_b) = R_1 + R_2$



$$R_{\mathrm{ab}} = \frac{R_c(R_a + R_b)}{R_a + R_b + R_c} = R_1 + R_2,$$

$$R_{\mathrm{bc}} = \frac{R_a(R_b + R_c)}{R_a + R_b + R_c} = R_2 + R_3,$$

$$R_{\mathrm{ca}} = \frac{R_b(R_c + R_a)}{R_a + R_b + R_c} = R_1 + R_3.$$

$$\Longrightarrow$$

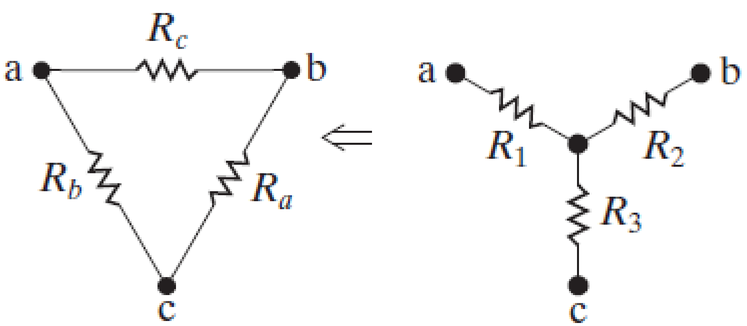
$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c},$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c},$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}.$$

Extension: Y to Δ Transformations

- Dividing the left-side equation by the relative former equation



$$R_1R_2 + R_2R_3 + R_3R_1 = \frac{R_aR_bR_c(R_a + Rb + R_c)}{(R_a + Rb + R_c)^2}$$

$$= \frac{R_aR_bR_c}{R_a + Rb + R_c}$$

$$\Longrightarrow$$

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1},$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2},$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}.$$

Exercise

- Determine the equivalent resistance