# The Boosting Approach to Machine Learning

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# Boosting

- General method for improving the accuracy of any given learning algorithm.

- Works by creating a series of challenge datasets s.t. even modest performance on these can be used to produce an overall high-accuracy predictor.

  - Works amazingly well in practice --- Adaboost and its variations one of the top 10 algorithms.

  - Backed up by solid foundations.

# Readings:

- The Boosting Approach to Machine Learning: An Overview. Rob Schapire, 2001
- Theory and Applications of Boosting. NIPS tutorial.

  http://www.cs.princeton.edu/~schapire/talks/nips-tutorial.pdf

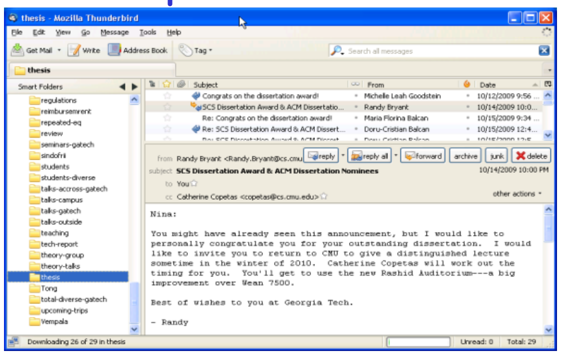
# Plan for today:

- Motivation.
- A bit of history.
- Adaboost: algo, guarantees, discussion.
- Focus on supervised classification.

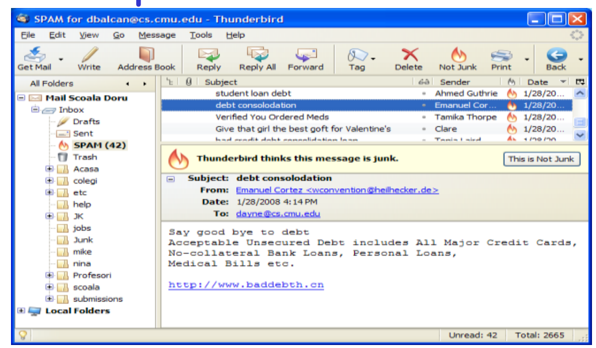
# An Example: Spam Detection

- E.g., classify which emails are spam and which are important.

Not spam




spam

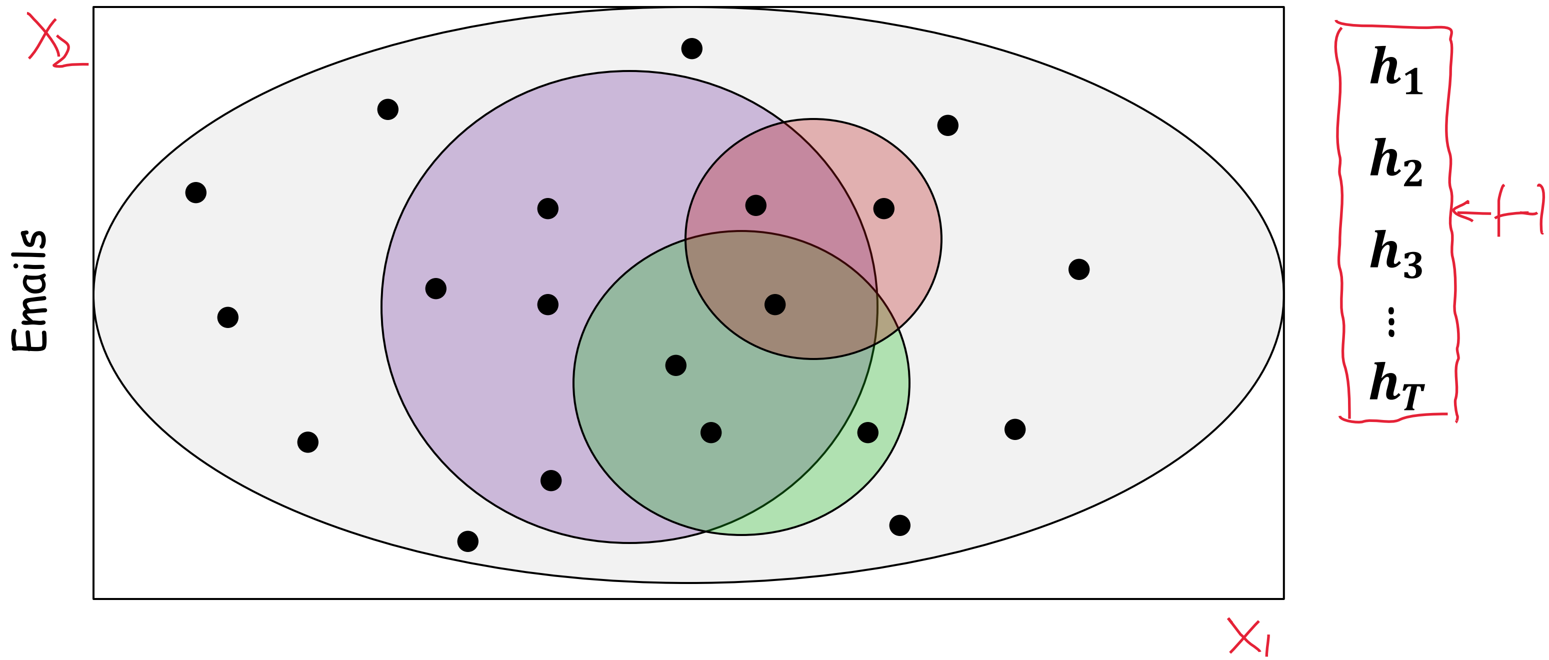



Key observation/motivation:

- Easy to find rules of thumb that are often correct.
  - E.g., "If buy now in the message, then predict spam."
  - E.g., "If say good-bye to debt in the message, then predict spam."
- Harder to find single rule that is very highly accurate.

# An Example: Spam Detection

- Boosting: meta-procedure that takes in an algo for finding rules of thumb (weak learner). Produces a highly accurate rule, by calling the weak learner repeatedly on cleverly chosen datasets.




- apply weak learner to a subset of emails, obtain rule of thumb
- apply to 2nd subset of emails, obtain 2nd rule of thumb
- apply to 3rd subset of emails, obtain 3rd rule of thumb
- repeat T times; combine weak rules into a single highly accurate rule.

# Boosting: Important Aspects

**How to choose examples on each round?**

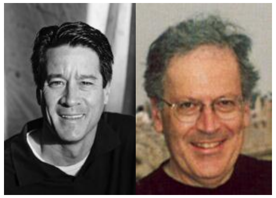
- Typically, concentrate on “hardest” examples (those most often misclassified by previous rules of thumb)

**How to combine rules of thumb into single prediction rule?**

- take (weighted) majority vote of rules of thumb

Historically….

# Weak Learning vs Strong/PAC Learning



- [Kearns & Valiant '88]: defined weak learning: being able to predict better than random guessing (error $\leq \frac{1}{2} - \gamma$) , consistently.

- Posed an open pb: "Does there exist a boosting algo that turns a weak learner into a strong PAC learner (that can produce arbitrarily accurate hypotheses)?"

- Informally, given "weak" learning algo that can consistently find classifiers of error $\leq \frac{1}{2} - \gamma$, a boosting algo would provably construct a single classifier with error $\leq \epsilon$.

# Weak Learning vs Strong/PAC Learning

## Strong (PAC) Learning

- $\exists$ algo *A*
- $\forall\, c \in \cancel{H}\ C$
- $\forall D$
- $\forall\, \epsilon > 0$
- $\forall\, \delta > 0$
- *A* produces h s.t.: $\text{poly}(\frac{1}{\epsilon}, \frac{1}{\delta})$

$\forall h \in H$ $\Pr[err(h) \geq \epsilon] \leq \delta$

$\Pr[err(h) < \epsilon] > 1 - \delta$

## Weak Learning

- $\exists$ algo *A*
- $\exists \gamma > 0$
- $\forall\, c \in \cancel{H}\ C$
- $\forall D$
- $\forall\, \epsilon > \frac{1}{2} - \gamma$
- $\forall\, \delta > 0$
- *A* produces h s.t.

$\forall h \in H,$ $\Pr[err(h) \geq \epsilon] \leq \delta$

- [Kearns & Valiant '88]: defined weak learning & posed an open pb of finding a boosting algo.

# Surprisingly....

## Weak Learning =Strong (PAC) Learning

Original Construction [Schapire '89]:

- poly-time boosting algo, exploits that we can learn a little on every distribution.
  - A modest booster obtained via calling the weak learning algorithm on 3 distributions.

    $H = MV(h_1, h_2, h_3)$

    Error $= \beta < \frac{1}{2} - \gamma \rightarrow$ error $3\beta^2 - 2\beta^3$

    $err_S(H) \leq 3\beta^2 - 2\beta^3 \leq \beta < \frac{1}{2} - \gamma$

  - Then amplifies the modest boost of accuracy by running this somehow recursively.
- Cool conceptually and technically, not very practical.

# An explosion of subsequent work

## Background (cont.)

- [Freund & Schapire '95]:
  - introduced "AdaBoost" algorithm
  - strong practical advantages over previous boosting algorithms
- experiments and applications using AdaBoost:

[Drucker & Cortes '96]
[Jackson & Craven '96]
[Freund & Schapire '96]
[Quinlan '96]
[Breiman '96]
[Maclin & Opitz '97]
[Bauer & Kohavi '97]
[Schwenk & Bengio '98]
[Schapire, Singer & Singhal '98]
[Abney, Schapire & Singer '99]
[Haruno, Shirai & Ooyama '99]
[Cohen & Singer' 99]
[Dietterich '00]
[Schapire & Singer '00]
[Collins '00]
[Escudero, Màrquez & Rigau '00]
[Iyer, Lewis, Schapire, Singer & Singhal '00]
[Onoda, Rätsch & Müller '00]
[Tieu & Viola '00]
[Walker, Rambow & Rogati '01]
[Rochery, Schapire, Rahim & Gupta '01]
[Merler, Furlanello, Larcher & Sboner '01]
⋮

- continuing development of theory and algorithms:

[Breiman '98, '99]
[Schapire, Freund, Bartlett & Lee '98]
[Grove & Schuurmans '98]
[Mason, Bartlett & Baxter '98]
[Schapire & Singer '99]
[Cohen & Singer '99]
[Freund & Mason '99]
[Domingo & Watanabe '99]
[Mason, Baxter, Bartlett & Frean '99, '00]
[Duffy & Helmbold '99, '02]
[Freund & Mason '99]
[Ridgeway, Madigan & Richardson '99]
[Kivinen & Warmuth '99]
[Friedman, Hastie & Tibshirani '00]
[Rätsch, Onoda & Müller '00]
[Rätsch, Warmuth, Mika, Onoda, Lemm & Müller '00]
[Allwein, Schapire & Singer '00]
[Friedman '01]
[Koltchinskii, Panchenko & Lozano '01]
[Collins, Schapire & Singer '02]
[Demiriz, Bennett & Shawe-Taylor '02]
[Lebanon & Lafferty '02]
⋮

# Adaboost (Adaptive Boosting)

"A Decision-Theoretic Generalization of On-Line Learning and an Application to Boosting"

[Freund-Schapire, JCSS'97]

Godel Prize winner 2003

# Informal Description Adaboost

- Boosting: turns a weak algo into a strong (PAC) learner.

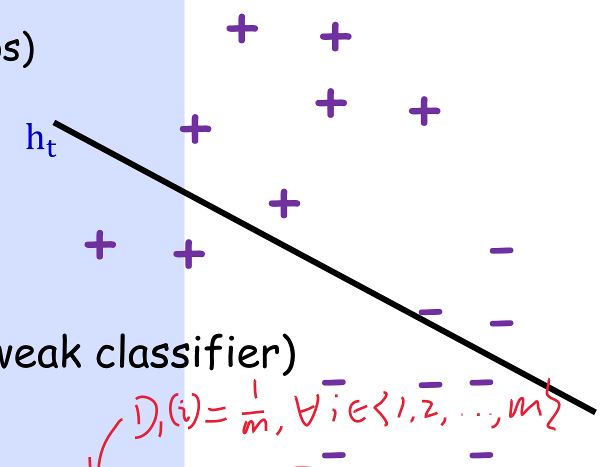
**Input**: $S=\{(x_1,y_1), \ldots,(x_m,y_m)\}$; $x_i \in X, y_i \in Y = \{-1,1\}$

weak learning algo $A$ (e.g., Naïve Bayes, decision stumps)

- For $t=1,2, \ldots ,T$
  - Construct $D_t$ on $\{x_1, \ldots, x_m\}$
  - Run $A$ on $D_t$ producing $h_t: X \to \{-1,1\}$ (weak classifier)

$h_t$ ; $\epsilon_t = P_{x_i \sim D_t}(h_t(x_i) \neq y_i)$ error of $h_t$ over $D_t$

- Output $H_{final}(x) = \mathrm{sign}(\sum_{t=1} \alpha_t h_t(x))$




$D_1(i) = \frac{1}{m}, \forall i \in \{1,2,\ldots,m\}$

$t=1$: $\epsilon_t = \sum_{i=1}^{m} \frac{1}{m} \mathbb{1}_{(h_t(x_i) \neq y_i)}$

$t \geq 2$: $\epsilon_t = \sum_{i=1}^{m} D_t(i) \mathbb{1}_{(h_t(x_i) \neq y_i)}$

Roughly speaking $D_{t+1}$ increases weight on $x_i$ if $h_t$ incorrect on $x_i$ ; decreases it on $x_i$ if $h_t$ correct.

$\epsilon_t = P_{r_D}(h_t(x) \neq y)$

# Adaboost (Adaptive Boosting)

- Weak learning algorithm $A$.
- For $t=1,2, \ldots ,T$
  - **Construct $\mathbf{D_t}$ on $\{\mathbf{x_1}, \ldots, \boldsymbol{x}_\mathbf{m}\}$**
  - Run $A$ on $D_t$ producing $h_t : X \to \{-1, +1\}$

Constructing $D_t$

- $D_1$ uniform on $\{x_1, \ldots, x_m\}$ [i.e., $D_1(i) = \frac{1}{m}$]
- Given $D_t$ and $h_t$ set

$$D_{t+1}(i) = \frac{D_t(i)}{Z_t} \, e^{\{-\alpha_t\}} \quad \text{if } y_i = h_t(x_i)$$

$$D_{t+1}(i) = \frac{D_t(i)}{Z_t} \, e^{\{\alpha_t\}} \quad \text{if } y_i \neq h_t(x_i)$$

$$D_{t+1}(i) = \frac{D_t(i)}{Z_t} \, e^{\{-\alpha_t y_i \, h_t(x_i)\}}$$

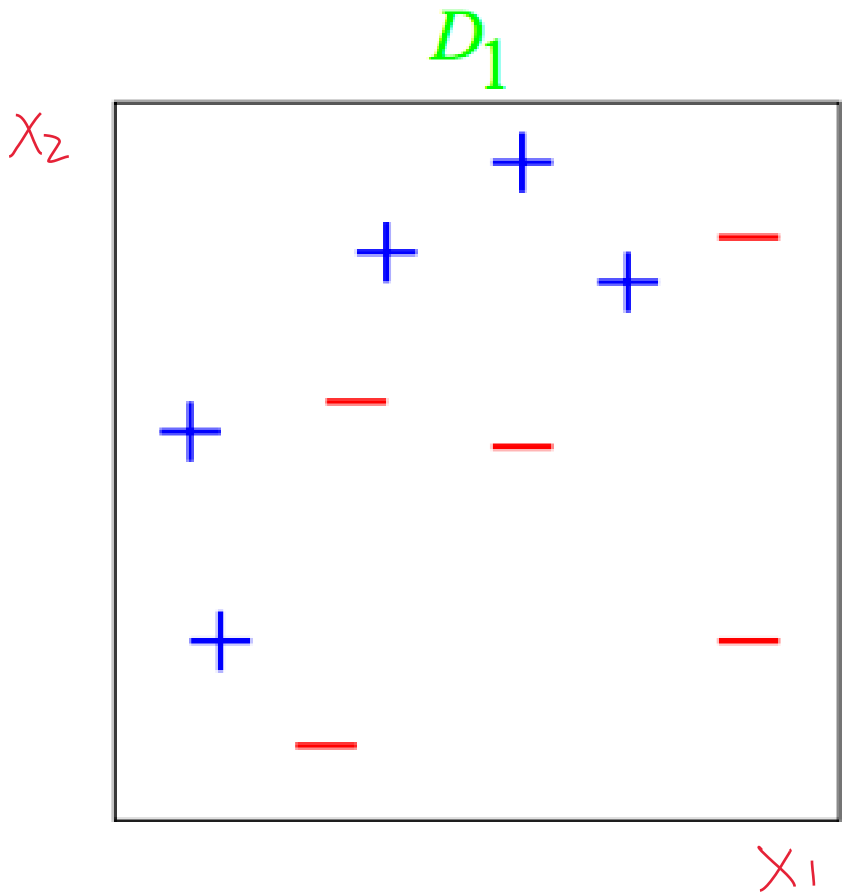
$$\alpha_t = \frac{1}{2} \ln\left(\frac{1-\epsilon_t}{\epsilon_t}\right) > 0$$

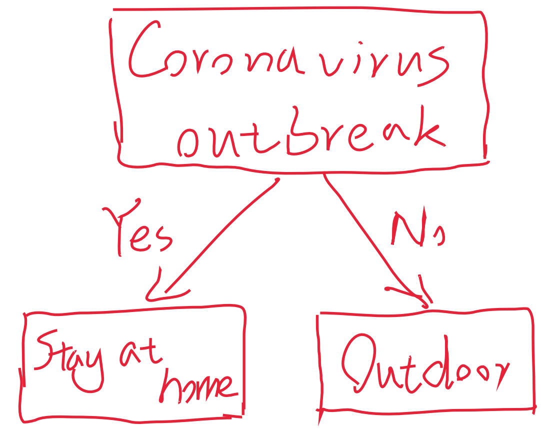
$D_{t+1}$ puts half of weight on examples $x_i$ where $h_t$ is incorrect & half on examples where $h_t$ is correct

Final hyp: $H_{\text{final}}(x) = \text{sign}(\sum_t \alpha_t h_t(x))$

# Adaboost: A toy example

Weak classifiers: vertical or horizontal half-planes (a.k.a. decision stumps)



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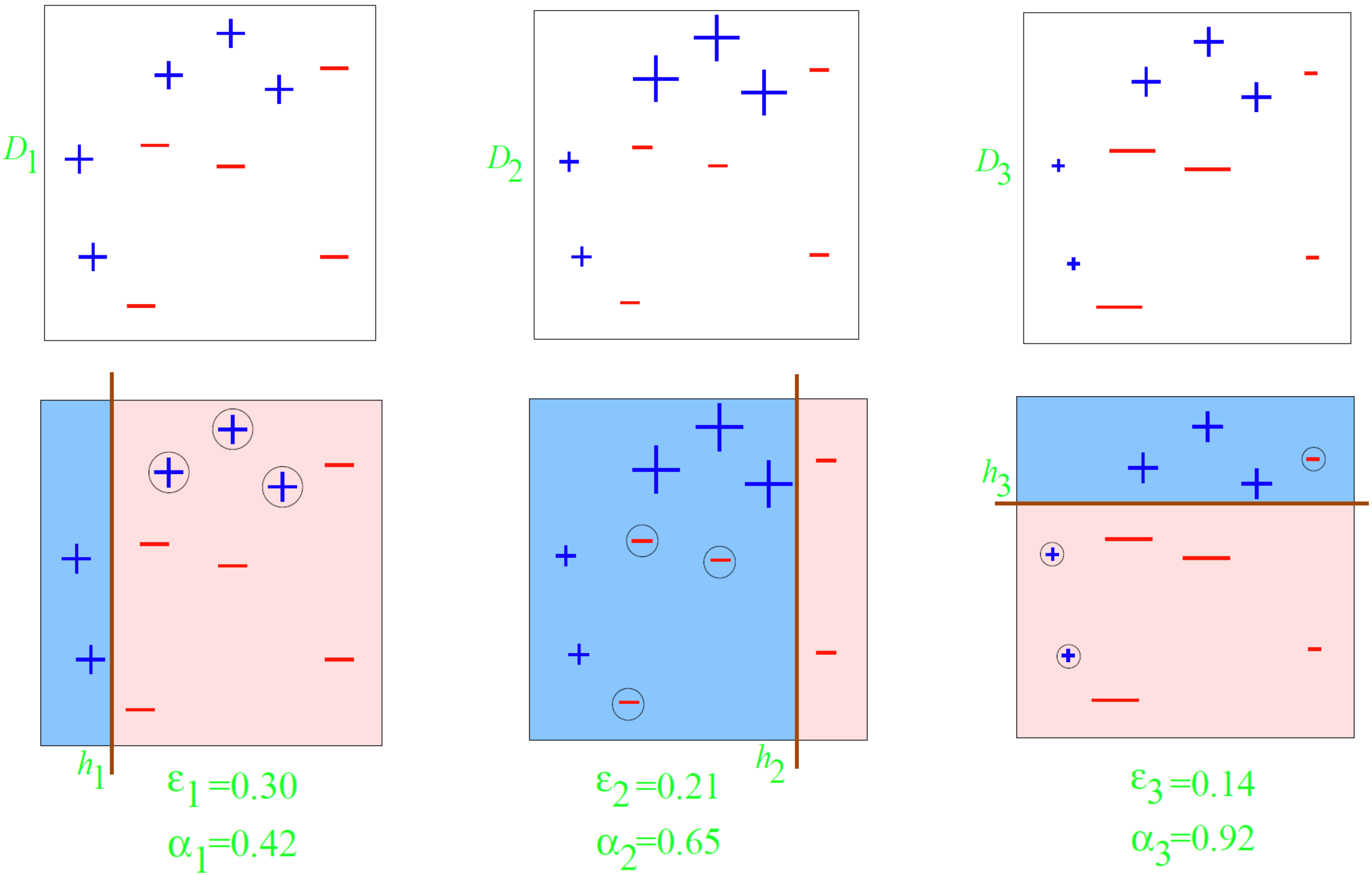

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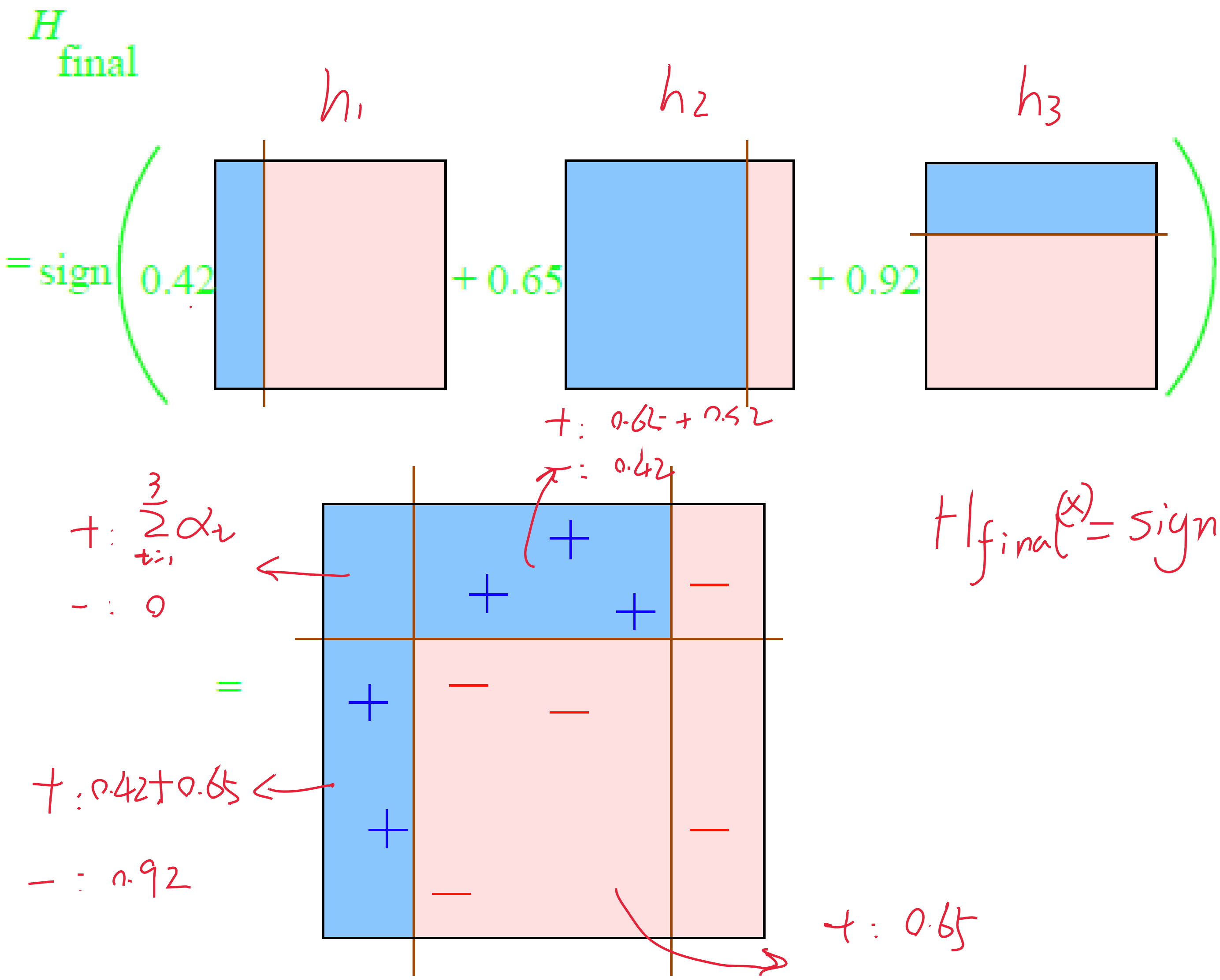


$+: 0.65 + 0.92$

$-: 0.42$

$+: \sum_{t=1}^{3} \alpha_t$

$-: 0$

$+: 0.42 + 0.65$

$-: 0.92$

$+: 0.65$

$-: 0.42 + 0.92$

$$H_{final}(x) = sign\left(\sum_{t=1}^{3} \alpha_t h_t(x)\right)$$

# Adaboost (Adaptive Boosting)

- Weak learning algorithm *A*.
- For $t=1,2,\ldots,T$
  - **Construct $D_t$ on $\{x_1, \ldots, x_m\}$**
  - Run *A* on $D_t$ producing $h_t$

<u>Constructing $D_t$</u>

- $D_1$ uniform on $\{x_1, \ldots, x_m\}$ [i.e., $D_1(i) = \frac{1}{m}$]
- Given $D_t$ and $h_t$ set

$$D_{t+1}(i) = \frac{D_t(i)}{Z_t} e^{\{-\alpha_t\}} \text{ if } y_i = h_t(x_i)$$

$$D_{t+1}(i) = \frac{D_t(i)}{Z_t} e^{\{\alpha_t\}} \text{ if } y_i \neq h_t(x_i)$$

$$D_{t+1}(i) = \frac{D_t(i)}{Z_t} e^{\{-\alpha_t y_i h_t(x_i)\}}$$

$$\alpha_t = \frac{1}{2} \ln\left(\frac{1-\epsilon_t}{\epsilon_t}\right) > 0$$

$D_{t+1}$ puts half of weight on examples $x_i$ where $h_t$ is incorrect & half on examples where $h_t$ is correct

<u>Final hyp</u>: $H_{final}(x) = \text{sign}(\sum_t \alpha_t h_t(x))$

# Nice Features of Adaboost

pros

- Very general: a meta-procedure, it can use any weak learning algorithm!!! (e.g., Naïve Bayes, decision stumps)
- Very fast (single pass through data each round) & simple to code, no parameters to tune. (except # iterations (T))
- Shift in mindset: goal is now just to find classifiers a bit better than random guessing.
- Grounded in rich theory.

Cons:
1. perform not well: (1) insufficient data (2) over-complicated (base learner) (3) too weak
2. sensitive to noise/outliers

- Relevant for big data age: quickly focuses on "core difficulties", well-suited to distributed settings, where data must be communicated efficiently [Balcan-Blum-Fine-Mansour COLT'12].

# Analyzing Training Error

**Theorem** $\epsilon_t = 1/2 - \gamma_t$ (error of $h_t$ over $D_t$)

$$err_S(H_{final}) \leq \exp\left[-2\sum_t \gamma_t^2\right]$$

So, if $\forall t, \gamma_t \geq \gamma > 0$, then $err_S(H_{final}) \leq \exp[-2\,\gamma^2 T]$

The training error drops exponentially in T!!!

To get $err_S(H_{final}) \leq \epsilon$, need only $T = O\left(\frac{1}{\gamma^2}\log\left(\frac{1}{\epsilon}\right)\right)$ rounds

## Adaboost is adaptive

- Does not need to know $\gamma$ or T a priori
- Can exploit $\gamma_t \gg \gamma$

# Understanding the Updates & Normalization

**Claim**: $D_{t+1}$ puts half of the weight on $x_i$ where $h_t$ was incorrect and half of the weight on $x_i$ where $h_t$ was correct.

$\alpha_t = \frac{1}{2}\ln\left(\frac{1-\epsilon_t}{\epsilon_t}\right)$

Recall $D_{t+1}(i) = \frac{D_t(i)}{Z_t}\, e^{\{-\alpha_t y_i h_t(x_i)\}}$

Probabilities are equal!

$\epsilon_{t+1} = \sum_{i: y \neq h(x_i)} D_{t+1}(i)$ $\qquad D_{t+1}(i)$ $\qquad \sum_i D_t(i) = 1$

$$\Pr_{D_{t+1}}[y_i \neq h_t(x_i)] = \sum_{i:y_i \neq h_t(x_i)} \frac{D_t(i)}{Z_t} e^{\alpha_t} = \epsilon_t \frac{1}{Z_t} e^{\alpha_t} = \frac{\epsilon_t}{Z_t}\sqrt{\frac{1-\epsilon_t}{\epsilon_t}} = \frac{\sqrt{\epsilon_t(1-\epsilon_t)}}{Z_t}$$

$$\Pr_{D_{t+1}}[y_i = h_t(x_i)] = \sum_{i:y_i = h_t(x_i)} \frac{D_t(i)}{Z_t} e^{-\alpha_t} = \frac{1-\epsilon_t}{Z_t} e^{-\alpha_t} = \frac{1-\epsilon_t}{Z_t}\sqrt{\frac{\epsilon_t}{1-\epsilon_t}} = \frac{\sqrt{(1-\epsilon_t)\epsilon_t}}{Z_t}$$

$$Z_t = \sum_{\substack{\cancel{i:y_i = h_t(x_i)} \\ i=1}}^{m} D_t(i) e^{-\alpha_t y_i h_t(x_i)} = \sum_{i:y_i = h_t(x_i)} D_t(i) e^{-\alpha_t} + \sum_{i:y_i \neq h_t(x_i)} D_t(i) e^{\alpha_t}$$

$$= (1-\epsilon_t)e^{-\alpha_t} + \epsilon_t e^{\alpha_t} = 2\sqrt{\epsilon_t(1-\epsilon_t)}$$

# Analyzing Training Error: Proof Intuition

**Theorem** $\epsilon_t = 1/2 - \gamma_t$ (error of $h_t$ over $D_t$)

$$err_S(H_{final}) \le \exp\left[-2\sum_t \gamma_t^2\right]$$

- On round $t$, we increase weight of $x_i$ for which $h_t$ is wrong.
- If $H_{final}$ incorrectly classifies $x_i$,
  - Then $x_i$ incorrectly classified by (wtd) majority of $h_t$'s.
  - Which implies final prob. weight of $x_i$ is large.

    Can show probability $\ge \frac{1}{m}\left(\frac{1}{\prod_t Z_t}\right)$

- Since sum of prob. $= 1$, can't have too many of high weight.

  Can show # incorrectly classified $\le m \left(\prod_t Z_t\right)$.

  And $\left(\prod_t Z_t\right) \to 0$ .

# Analyzing Training Error: Proof Math

Step 1: unwrapping recurrence: $D_{T+1}(i) = \frac{1}{m}\left(\frac{\exp(-y_i f(x_i))}{\prod_t Z_t}\right)$

where $f(x_i) = \sum_t \alpha_t h_t(x_i)$. [Unthresholded weighted vote of $h_i$ on $x_i$ ]

Step 2: $\mathrm{err}_S(H_{final}) \leq \prod_t Z_t$.

Step 3: $\prod_t Z_t = \prod_t 2\sqrt{\epsilon_t(1-\epsilon_t)} = \prod_t \sqrt{1-4\gamma_t^2} \leq e^{-2\sum_t \gamma_t^2}$

# Analyzing Training Error: Proof Math

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where $f(x_i) = \sum_t \alpha_t h_t(x_i)$. $\in \{-1, 1\}$

$\notin \{-1, 1\}$

Recall $D_1(i) = \frac{1}{m}$ and $D_{t+1}(i) = D_t(i)\,\frac{\exp(-y_i\alpha_t h_t(x_i))}{Z_t}$

$$D_{T+1}(i) = \frac{\exp(-y_i\alpha_T h_T(x_i))}{Z_T} \times \underline{D_T(i)}$$

$$= \frac{\exp(-y_i\alpha_T h_T(x_i))}{Z_T} \times \underline{\frac{\exp(-y_i\alpha_{T-1} h_{T-1}(x_i))}{Z_{T-1}} \times D_{T-1}(i)}$$

…….

$$= \frac{\exp(-y_i\alpha_T h_T(x_i))}{Z_T} \times \cdots \times \frac{\exp(-y_i\alpha_1 h_1(x_i))}{Z_1}\,\frac{1}{m}$$

$D_{T+1} \prod_t Z_t =$

$\frac{1}{m} \exp(-y_i f(x_i))$

$$= \frac{1}{m}\,\frac{\exp(-y_i(\alpha_1 h_1(x_i) + \cdots + \alpha_T h_T(x_T)))}{Z_1 \cdots Z_T} = \frac{1}{m}\,\frac{\exp(-y_i f(x_i))}{\prod_t Z_t}$$

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Step 1: unwrapping recurrence: $D_{T+1}(i) = \frac{1}{m}\left(\frac{\exp(-y_i f(x_i))}{\prod_t Z_t}\right)$

where $f(x_i) = \sum_t \alpha_t h_t(x_i)$. $H_{final}(x) = sign(f(x))$

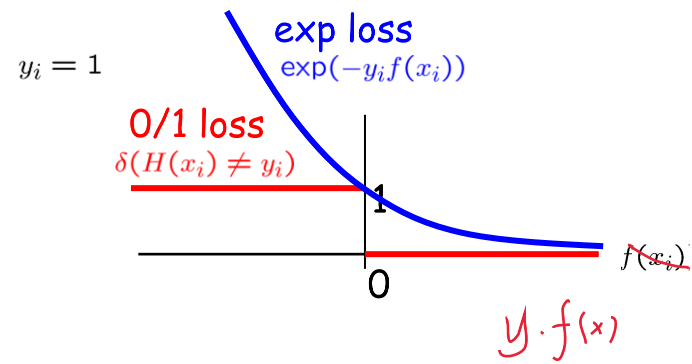
Step 2: $\mathrm{err}_S(H_{final}) \leq \prod_t Z_t$.

$$\mathrm{err}_S(H_{final}) = \frac{1}{m}\sum_i 1_{y_i \neq H_{final}(x_i)}$$

$$= \frac{1}{m}\sum_i 1_{y_i f(x_i) \leq 0}$$

$$\leq \frac{1}{m}\sum_i \exp(-y_i f(x_i))$$

$$= \underbrace{\left(\sum_i D_{T+1}(i)\right)}_{=1} \prod_t Z_t = \prod_t Z_t.$$



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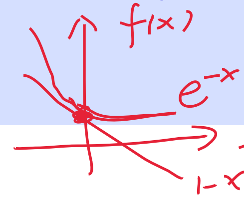
$1 - x \le e^{-x}$




$\sqrt{1-x} \le e^{-\frac{x}{2}}$

Step 2: $\mathrm{err}_S(H_{final}) \le \prod_t Z_t$.

$\varepsilon_t = \frac{1}{2} - \gamma_t$

Step 3: $\prod_t Z_t = \prod_t 2\sqrt{\epsilon_t(1-\epsilon_t)} = \prod_t \sqrt{1 - 4\gamma_t^2} \le e^{-2\sum_t \gamma_t^2}$

Note: recall $Z_t = (1-\epsilon_t)e^{-\alpha_t} + \epsilon_t e^{\alpha_t} = 2\sqrt{\epsilon_t(1-\epsilon_t)}$

$\alpha_t$ minimizer of $\alpha \to (1-\epsilon_t)e^{-\alpha} + \epsilon_t e^{\alpha}$ **Quiz**

$\alpha_t = \operatorname{argmin}_{\alpha} (1-\varepsilon_t)e^{-\alpha} + \varepsilon_t e^{\alpha}$ $\quad \alpha_t = \frac{1}{2}\ln\left(\frac{1-\varepsilon_t}{\varepsilon_t}\right)$

# Analyzing Training Error: Proof Intuition

**Theorem** $\epsilon_t = 1/2 - \gamma_t$ (error of $h_t$ over $D_t$)

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- Since sum of prob. $= 1$, can't have too many of high weight.

  Can show # incorrectly classified $\leq m \left(\prod_t Z_t\right)$.

  And $\left(\prod_t Z_t\right) \to 0$.

# Generalization Guarantees

**Theorem** $err_S(H_{final}) \le \exp\left[-2\sum_t \gamma_t^2\right]$ where $\epsilon_t = 1/2 - \gamma_t$

How about generalization guarantees?

Original analysis [Freund&Schapire'97]

$O(dT \log(dT))$

- H space of weak hypotheses; d=VCdim(H) $\longleftrightarrow$ VC dim(G) = D

$H_{final}$ is a weighted vote, so the hypothesis class is:

G={all fns of the form $\text{sign}(\sum_{t=1}^{T} \alpha_t h_t(x))$ }

$O\left(\sqrt{\frac{D + \ln\frac{1}{\delta}}{m}}\right)$

**Theorem** [Freund&Schapire'97]

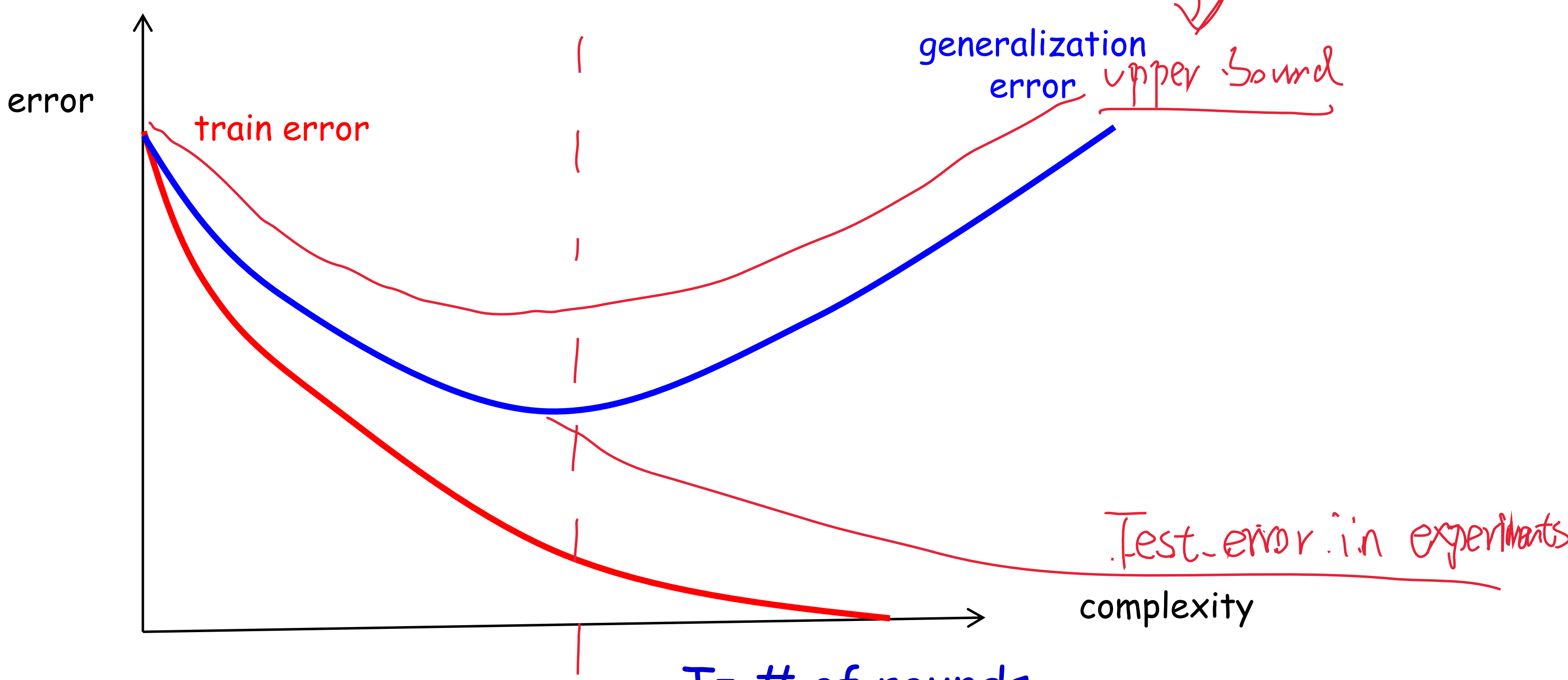
$\forall\, g \in G, err(g) \le err_S(g) + \tilde{O}\left(\sqrt{\frac{Td}{m}}\right)$ T= # of rounds

**Key reason**: $\text{VCdim}(G) = \tilde{O}(dT)$ plus typical VC bounds.

# Generalization Guarantees

**Theorem** [Freund&Schapire'97]

$$\forall\, g \in co(H), err(g) \leq err_S(g) + \tilde{O}\left(\sqrt{\frac{Td}{m}}\right)$$ where d=VCdim(H)




T= # of rounds

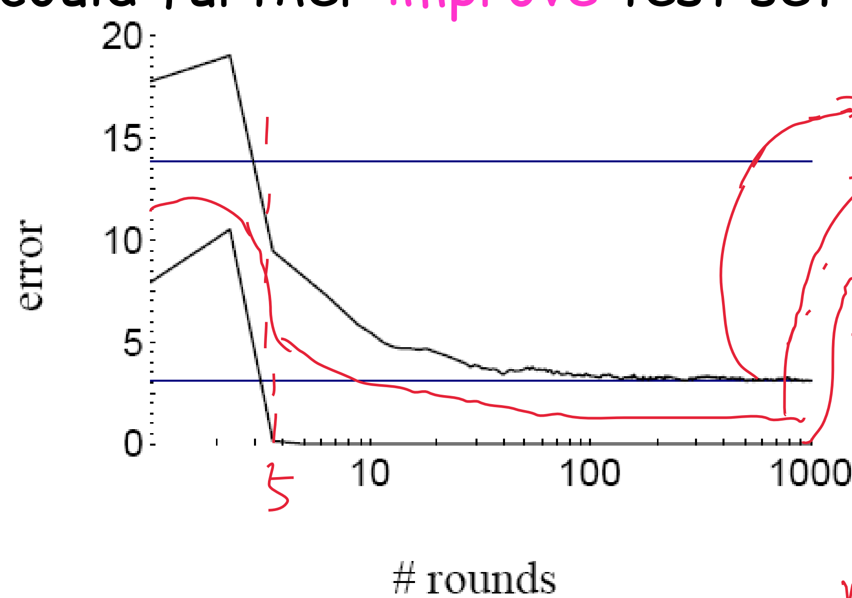
# Generalization Guarantees

generalization error

unbiased estimate of

- Experiments with boosting showed that the test error of the generated classifier usually does not increase as its size becomes very large.

- Experiments showed that continuing to add new weak learners after correct classification of the training set had been achieved could further improve test set performance!!!




$Pr_D(yf(x) \le 0)$ (Test error)

$Pr_S(yf(x) \le \theta)$, $(\theta > 0)$

$Pr_S(yf(x) \le 0)$ (Training error)

$y \ne H_{final}(x) \in \{-1, 1\}$

margin $\equiv \underline{\underline{y f(x)}}$
$\in [-1, 1]$

# Generalization Guarantees

- Experi[illegible] [illegible]th boosting showed that the test error of the gene[illegible] [illegible]assifier usually does not [illegible] as its size bec[illegible] [illegible]y large.

- Experime[illegible] showed that continuing to add [illegible] [illegible]eak learners after correct classification of the tr[illegible]ning set had been achieved could further improve test set performance!!!

- These results seem to contradict FS'87 bound and Occam's razor (in [illegible] [illegible]chieve good test error the [illegible] [illegible]fier should be as simple as [illegible]

# How can we explain the experiments?

R. Schapire, Y. Freund, P. Bartlett, W. S. Lee. present in "*Boosting the margin: A new explanation for the effectiveness of voting methods*" a nice theoretical explanation.

**Key Idea:**

Training error does not tell the whole story.

We need also to consider the classification confidence!!

# History of Boosting

$\forall c \in C$

**L.G. Valiant (1984)**

$\boxed{P_r(err_D(h) \le \epsilon) > 1-\delta}$

*Genesis of* Strong *PAC Learning Model*

Valiant 1988 : Weak $P_r(err_D(h) \le \frac{1}{2}-\gamma) > 1-\delta$

C

**R.E Schapire (1990)**

**Y. Freund (1995)**

Weak Learn = Strong Learn

*Concept of Boosting appears*

**Schapire & Freund (1995,1997)**

1. Algorithm

2 Traing / Generalization error bound

*Adaboost is born*

**L. Breiman(1996) Bagging**

**Freund & Schapire (1996)**

**R. Quinlan (1996)**

**Breiman(1996, 1997)**

*Experiments with Adaboost*

New generalization bound

**Schapire, Freund, P. Bartlett, Lee (1997)**

**Schapire, Y. Singer (1998)**

**Schapire, Singer, Freund, Iyer (1998)**

**Friedman, Hastie, Tibshirani (1998)**

*Attempts to explain why Adaboost works Improvements*