Quiz 6

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1. Initialize $\boldsymbol{\theta}$
2. Repeat
3. E-step: Use $\boldsymbol{X}$ and current $\boldsymbol{\theta}$ to calculate $P(\boldsymbol{Z}|\boldsymbol{X}, \boldsymbol{\theta})$
4. M-step: Replace current $\boldsymbol{\theta}$ by

$$\boldsymbol{\theta} \leftarrow arg\max_{\boldsymbol{\theta'}} Q(\boldsymbol{\theta'}|\boldsymbol{\theta}) + logP(\boldsymbol{\theta'})$$

where $Q(\boldsymbol{\theta'}|\boldsymbol{\theta}) = E_{P(\boldsymbol{Z}|\boldsymbol{X},\boldsymbol{\theta})}[logP(\boldsymbol{X}, \boldsymbol{Z}|\boldsymbol{\theta'})]$
5. Until convergence

Only M-step is changed, because in MAP, we have

$$\begin{aligned}
&E_{P(\boldsymbol{Z}|\boldsymbol{X},\boldsymbol{\theta})}[log(P(\boldsymbol{X}, \boldsymbol{Z}|\boldsymbol{\theta'})P(\boldsymbol{\theta'}))] \\
&= E_{P(\boldsymbol{Z}|\boldsymbol{X},\boldsymbol{\theta})}[logP(\boldsymbol{X}, \boldsymbol{Z}|\boldsymbol{\theta'})] + E_{P(\boldsymbol{Z}|\boldsymbol{X},\boldsymbol{\theta})}[logP(\boldsymbol{\theta'})] \\
&= E_{P(\boldsymbol{Z}|\boldsymbol{X},\boldsymbol{\theta})}[logP(\boldsymbol{X}, \boldsymbol{Z}|\boldsymbol{\theta'})] + logP(\boldsymbol{\theta'}) \\
&= Q(\boldsymbol{\theta'}|\boldsymbol{\theta}) + logP(\boldsymbol{\theta'})
\end{aligned}$$