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# Machine Learning, 2021 Fall

## Quiz 2 solution

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### 1 [Lecture 9-10]

Bias, measures the distance of the average response of the model from the true value, high bias implies this underfitting.

Variance: measures the generalizability of model beyond the training data, high variance implies overfitting.

Noise: the inaccuracy of data.

Expected error: the expected error between model and true mapping function.

### 2 [Lecture 11-12]

True, we can always rescale the parameters to make the margin = 1.

False, you need to pick penalty term C.

### 3 [Lecture 13]

We will use the dataset below to learn a decision tree which predicts if people pass machine learning (Yes or No), based on their previous GPA (High, Medium, or Low) and whether or not they studied.

| GPA | Studied | Passed |
|-----|---------|--------|
| L   | F       | F      |
| L   | T       | T      |
| M   | F       | F      |
| M   | T       | T      |
| H   | F       | T      |
| H   | T       | T      |

For this problem, you can write your answers using  $\log_2$ , but it may be helpful to note that  $\log_2 3 \approx 1.6$

1. What is the entropy  $H(\text{Passed})$  ?

ANSWER:

$$\begin{aligned} H(\text{Passed}) &= -\left(\frac{2}{6} \log_2 \frac{2}{6} + \frac{4}{6} \log_2 \frac{4}{6}\right) \\ H(\text{Passed}) &= -\left(\frac{1}{3} \log_2 \frac{1}{3} + \frac{2}{3} \log_2 \frac{2}{3}\right) \\ H(\text{Passed}) &= \log_2 3 - \frac{2}{3} \approx 0.92 \end{aligned}$$

2. What is the entropy  $H(\text{Passed} \mid \text{GPA})$  ?

ANSWER:

$$\begin{aligned} H(\text{Passed} \mid \text{GPA}) &= -\frac{1}{3} \left( \frac{1}{2} \log_2 \frac{1}{2} + \frac{1}{2} \log_2 \frac{1}{2} \right) - \frac{1}{3} \left( \frac{1}{2} \log_2 \frac{1}{2} + \frac{1}{2} \log_2 \frac{1}{2} \right) - \frac{1}{3} (1 \log_2 1) \\ H(\text{Passed} \mid \text{GPA}) &= \frac{1}{3} (1) + \frac{1}{3} (1) + \frac{1}{3} (0) \\ H(\text{Passed} \mid \text{GPA}) &= \frac{2}{3} \approx 0.66 \end{aligned}$$

3. What is the entropy  $H(\text{Passed} \mid \text{Studied})$  ?

ANSWER:

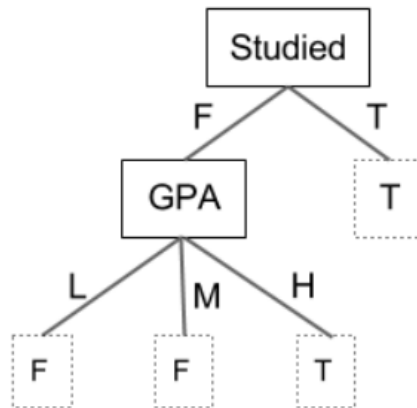
$$H(\text{Passed} \mid \text{Studied}) = -\frac{1}{2} \left( \frac{1}{3} \log_2 \frac{1}{3} + \frac{2}{3} \log_2 \frac{2}{3} \right) - \frac{1}{2} (1 \log_2 1)$$

$$H(\text{Passed} \mid \text{Studied}) = \frac{1}{2} \left( \log_2 3 - \frac{2}{3} \right)$$

$$H(\text{Passed} \mid \text{Studied}) = \frac{1}{2} \log_2 3 - \frac{1}{3} \approx 0.46$$

4. Draw the full decision tree that would be learned for this dataset. You do not need to show any calculations.

ANSWER: We want to split first on the variable which maximizes the information gain  $H(\text{Passed}) - H(\text{Passed} \mid A)$ . This is equivalent to minimizing  $H(\text{Passed} \mid A)$ , so we should split on "Studied" first.



#### 4 [Lecture 14]

effective number is  $N/k$ , if the neighborhoods were nonoverlapping, there would be  $N/k$  classes and we would fit one parameter (a mean) in each class.

(If some students do not answer effective number, but analysis that "as  $k$  decreasing, model overfitting, larger variance and lower bias", they can receive at most 0.2 points).

#### 5 [Lecture 15]

(a) Cut(A, B): Minimize the weights of connections between groups

Assoc(A, B): Maximize the weights of connections within groups

Ncut(A, B): Consider the connectivity between groups relative to the density of each group

Nassoc(A, B): Consider the connectivity within groups relative to the density of each group

(b) Because they consider the intra-cluster density

#### 6 [Lecture 16]

The left singular vectors measures document-concept similarity, the right singular vectors measures term-concept similarity.