# EE 111 Homework 3

Due date: Apr. $1^{st}$, 2019

Turn in your homework in class

Rule:

- Work on your own. Discussion is permissible, but similar submissions will be judged as plagiarism.
- Please show all intermediate steps: a correct solution without an explanation will get zero credit.
- Please submit on time. No late submission will be accepted.
- Please prepare your submission in English only. No Chinese submisson will be accepted.

1. (8pt) In the circuit shown in Fig.1 below

$$v(t) = 72e^{-100t}V, t > 0$$

$$i(t) = 9e^{-100t}mA, t > 0$$

   (a) Find the values of $R$ and $C$.

   (b) Calculate the time constant $\tau$.

   (c) Determine the time required for the voltage to decay half its initial value at $t = 0$.

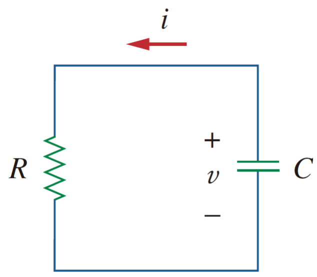



Fig. 1

**Solution:**

(a)

$$R = \frac{v(t)}{i(t)} = \frac{72e^{-100t}}{9e^{-100t}} = 8k\Omega \quad (1')$$

$$\tau = RC = \frac{1}{100}s$$

$$C = \frac{1}{100R} = \frac{1}{100 \times 8 \times 10^3} = 1.25\mu F \quad (2')$$

(b)

$$\tau = \frac{1}{100} = 10ms \quad (1')$$

(c)

$$v(0) = 72V \quad (1')$$

$$v(t) = \frac{1}{2}v(0) \quad (1')$$

$$72e^{-100t} = 36$$

$$t = \frac{ln2}{100} = 6.93ms \quad (2')$$

2. (10pt) Assuming that the switch in Fig.2 has been in position $A$ for a long time and is moved to position $B$ at $t = 0$, Then at $t = 1$ second, the switch moves from $B$ to $C$. Find $v_C(t)$ for $t \geq 0$.

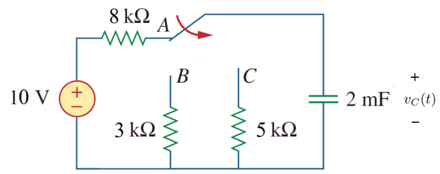



Fig. 2

**Solution:**

$$v_C(0) = 10V \tag{1'}$$

For $0 < t < 1$,

$$\tau_1 = R_B C = 3000 \times 2^{-3} = 6s \tag{1'}$$

$$v_C(t) = v_C(0)e^{-\frac{t}{\tau_1}} = 10e^{-\frac{t}{6}}V \tag{2'}$$

When $t = 1s$,

$$v_C(1) = 10e^{-\frac{1}{6}}V \tag{1'}$$

For $t > 1$,

$$\tau_2 = R_C C = 5000 \times 2^{-3} = 10s \tag{1'}$$

$$v_C(t) = v_C(1)e^{-\frac{t-1}{\tau_2}} = 10e^{-\frac{1}{6}}e^{-\frac{t-1}{10}} = 9.355e^{-\frac{t}{10}}V \tag{2'}$$

$$\therefore v_C(t) = \begin{cases} 10e^{-\frac{t}{6}}V, & 0 \leq t < 1 \\ 9.355e^{-\frac{t}{10}}V, & t \geq 1 \end{cases} \tag{2'}$$

3. (8pt) For the circuit in Fig.3, find $i_o$ for $t > 0$.

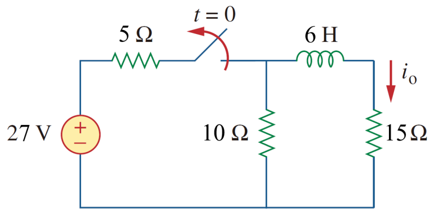



Fig. 3

**Solution:**

$$i_o(0) = \frac{27}{10||15+5} \cdot \frac{10}{15+10} = \frac{54}{55}A \qquad (2')$$

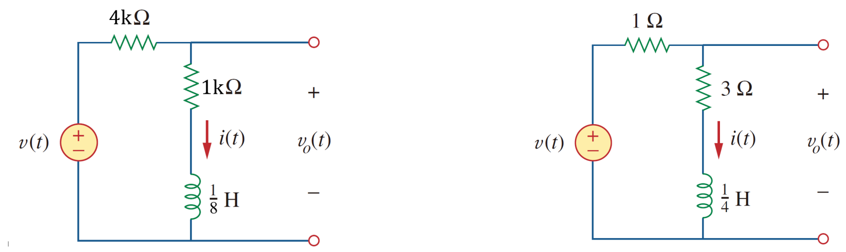
$$\tau = \frac{L}{R} = \frac{6}{10+15} = \frac{6}{25}s \qquad (2')$$

$$i_o(t) = i_o(0)e^{-\frac{t}{\tau}} = \frac{54}{55}e^{-\frac{25t}{6}}A \qquad (4')$$

4. (10pt) Consider the circuit of Fig.4.

   (a) Find $v_o(t)$ in Fig.4(a) if $i(0) = 6A$ and $v(t) = 0V$.

   (b) Find $v_o(t)$ in Fig.4(b) if $i(0) = 6A$ and $v(t) = 24u(t)V$.




(a) (b)

Fig. 4

**Solution:**

(a) For $t > 0$,

$$i(t) = i(0)e^{-\frac{R}{L}t = 6e^{-\frac{5000}{1/8}t} = 6e^{-40000t}} A \quad (2')$$

$$v_o(t) = -4000i(t) = -24000e^{-40000t}V$$

$$\therefore v_o(t) = -24000e^{-40000t}u(t)V \quad (2')$$

(b) Apply KVL to the mesh,

$$-24 + 1 \cdot i(t) + 3 \cdot i(t) + \frac{1}{4}\frac{di(t)}{dt} = 0 \quad (2')$$

Simplify it,

$$\frac{di(t)}{dt} + 16i(t) = 96$$

$$i(t) = e^{-\int 16dt}(\int 96e^{\int 16dt}dt + C) = 6 + Ce^{-16t}A \quad (1')$$

$$i(0) = 6 + C = 6 \rightarrow C = 0 \rightarrow i(t) = 6A \quad (1')$$

$$v_o(t) = 24 - 1 \cdot i(t) = 24 - 6 = 18V \quad (2')$$

5. (10pt) For the circuit in Fig.5,

$$v = 40e^{-25t}V$$

and

$$i = 10e^{-25t}A, \quad t > 0$$

(a) Find $L$ and $R$.

(b) Determine the time constant.

(c) Calculate the initial energy in the inductor.

(d) What fraction of the initial energy is dissipated in 20 ms?

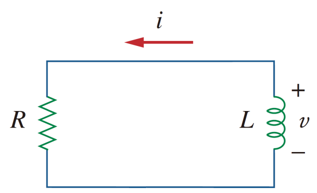



Fig. 5

**Solution:**

(a)

$$R = \frac{v}{i} = \frac{40e^{-25t}}{10e^{-25t}} = 4\Omega \qquad (1')$$

$$\tau = \frac{L}{R} = \frac{1}{25}s$$

$$L = R\tau = 4/25 = 0.16H \qquad (2')$$

(b)

$$\tau = \frac{1}{25} = 0.04s \qquad (2')$$

(c)

$$w(0) = \frac{1}{2}Li^2(0) = \frac{1}{2}\frac{4}{25} \times 10^2 = 8J \qquad (2')$$

(d) The value of the energy remaining at $20ms$ is given by:

$$w(20) = \frac{1}{2}Li^2(20) = \frac{1}{2}\frac{4}{25} \times (10e^{-25\times 0.02})^2 = 8e^{-1} \qquad (2')$$

So the fraction of the energy dissipated in the first $20ms$ is given by:

$$\frac{8 - 8e^{-1}}{8} \cdot 100\% = (1 - e^{-1}) \cdot 100\% = 63.21\% \qquad (1')$$

10pt

6. The switch in Fig.6 has been in position $a$ for a long time. At $t = 0$, it moves to position b. Calculate $i(t)$ for $t > 0$.

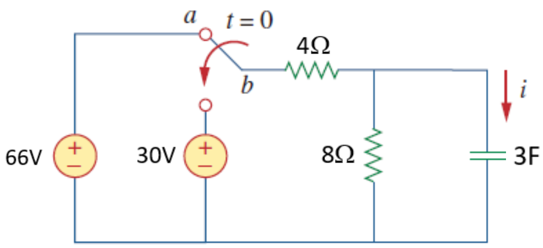
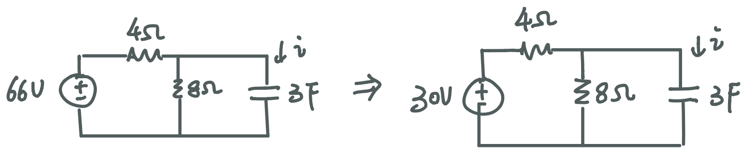



Fig. 6




$R_{eq} = 4 \parallel 8 = \frac{8}{3}\,\Omega$

$\tau = R \cdot C = 3 \times \frac{8}{3} = 8\ s$ 2'

$V(0) = \frac{8}{4+8} \times 66 = 44\ V$ 1'

$V(\infty) = \frac{8}{4+8} \times 30 = 20\,V$ 1'

$V(t) = V(\infty) + [V(0) - V(\infty)] \cdot e^{-t/\tau}$

$= 20 + (44 - 20) \cdot e^{-t/8}$

$= 20 + 24 \cdot e^{-t/8}\ V$ 3'

$i(t) = C\,\frac{dv(t)}{dt}$

$= 3 \times 24 \times e^{-\frac{t}{8}} \cdot (-\frac{1}{8})$

$= -9\,e^{-0.125t}\,u(t)\ A$ 3'

10pt

7. The switch in Fig. 7 has been in position a for a long time, at $t = 0$, it moves to position b. Find $v(t)$ for $t < 0$ and $t > 0$ in the circuit.

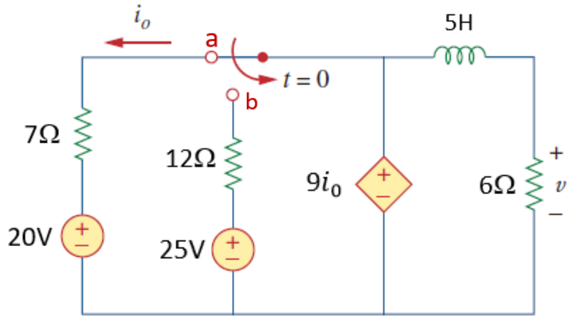
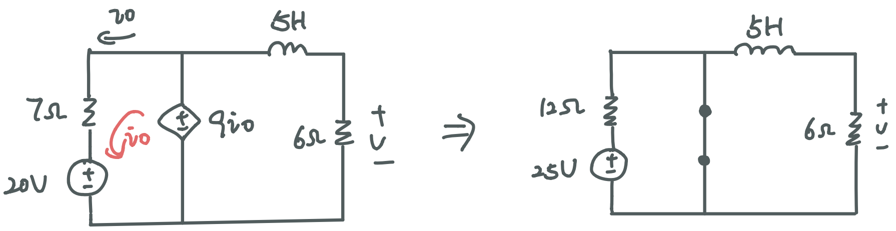



Fig. 7




(4) ① $7i_o + 20 - 9i_o = 0 \Rightarrow i_o = 10\,A$ 2'

5H inductance short cut

for $t<0$, $V = 9i_o = 90\,V$ 2'

$i = \frac{V}{6} = 15\,A$

(6) ② for $t>0$

$i(0) = 15$, $i(\infty) = 0$

$\tau = \frac{L}{R_{th}} = \frac{5}{6}\,s$ } 2'

$\therefore i(t) = i(\infty) + [i(0) - i(\infty)] \cdot e^{-t/\tau}$

$= 15 \cdot e^{-1.2t}\,A$ 2'

$V(t) = 6 \cdot i(t) = 90\,e^{-1.2t}\,u(t)\,V$ 2'

$$\therefore\ V(t) = \begin{cases} 90\,V & \text{for } t<0 \\ 90 \cdot e^{-1.2t}\,V & \text{for } t>0 \end{cases}$$

10pt

8. If the input pulse in Fig.8 is applied to the circuit in Fig. 8 (b), determine the response $i(t)$ while $i(0) = 0.75A$.

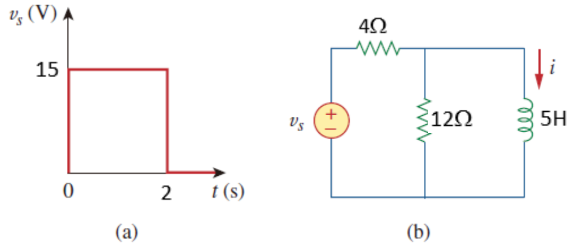



Fig. 8

① for $0<t<2$, $i(0)=0$, $i(\infty)=\frac{15}{4}=3.75$

$R_{th} = 4 \| 12 = 3\ \Omega$

$\tau = \frac{L}{R_{th}} = \frac{5}{3}\ s$ 2'

$i(t) = i(\infty) + [i(0) - i(\infty)] \cdot e^{-t/\tau}$ 3'

$= 3.75 - 3 \cdot e^{-0.6t}$ A

② for $t>2$, $i(\infty)=0$

now $V_s = 0$ 2'

$i(t) = i(\infty) + [i(2) - i(\infty)] \cdot e^{-\frac{t-2}{\tau}}$ $2 \times \frac{3}{5}$ $\frac{6}{5}$

$= i(2) \cdot e^{-\frac{t-2}{\tau}}$ 3'

$= (3.75 - 3 \cdot e^{-1.2}) \cdot e^{-0.6t+1.2}$ A

$= 3.75\, e^{-0.6t+1.2} - 3e^{-0.6t}$ A

$$\therefore \quad i(t) = \begin{cases} 3.75 - 3 \cdot e^{-0.6t}\ \text{A} & 0<t<2 \\ 3.75 e^{-0.6t+1.2} - 3e^{-0.6t}\ \text{A} & t>2 \end{cases}$$

10pt

9. If $v(0) = 6\,V$, find $v_0(t)$ for $t > 0$ in the op amp circuit in Fig.9. Let $R$ = 3kΩ and $C = \frac{1}{3}$ μF.

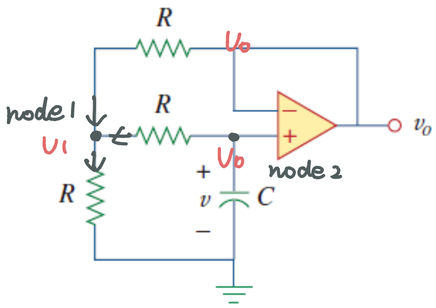



Fig. 9

for node 1:

$$\frac{v_o - v_1}{R} + \frac{v_o - v_1}{R} = \frac{v_1}{R}$$

$$\Rightarrow \quad v_1 = \frac{2}{3} v_o \qquad 2'$$

for node 2:

$$C \frac{dv_o}{dt} + \frac{v_o - v_1}{R} = 0 \qquad 2'$$

过程 2'

$$-RC \frac{dv_o}{dt} = v_o - v_1 = \frac{1}{3} v_o$$

$$\frac{dv_o}{dt} = -\frac{v_o}{3RC}$$

$$\frac{dv_o}{v_o} = -\frac{1}{3RC} dt$$

$$\ln v_o = -\frac{1}{3RC} t + C$$ 常数 C

当 $t=0$ 时, $v_o(0) = e^{-\frac{t}{3RC}} \cdot e^C = e^C$

$$v_o(t) = e^{-\frac{t}{3RC}} \cdot v_o(0)$$ 常数 $e^C$

$\because \quad v_o(0) = 6\,V$

$$\tau = 3RC = 3 \times 3000 \times \frac{1}{3} \times 10^{-6} = 3 \times 10^{-3} s \qquad 2'$$

$$\therefore \quad v_o(t) = 6\, e^{-1000t/3}\, u(t)\ V \qquad 2'$$

14 pt

10. At the time the double-pole switch in the circuit shown in Fig. 10 is closed, the initial voltages on the capacitors are 12V and 4V, as shown. Find the numerical expressions for $v_0(t)$, $v_2(t)$, and $v_f(t)$ that are applicable, as long as the ideal op amp operates in its linear range.

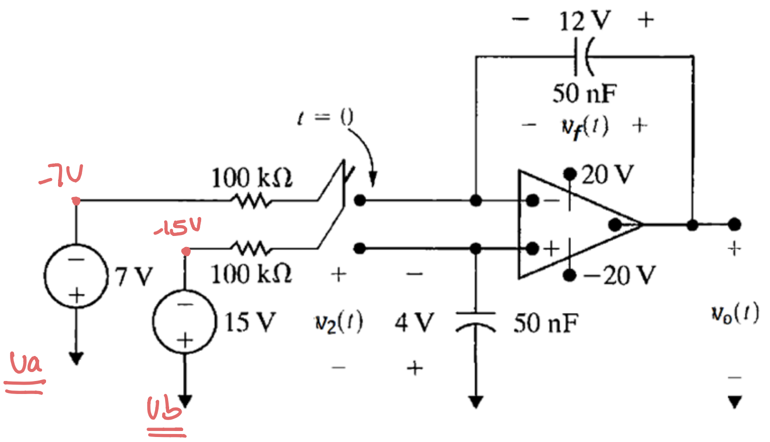
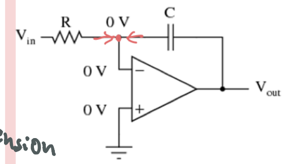



Fig. 10

equation for an integrating amplifier:

$$V_o = \frac{1}{RC}\int_0^t (U_b - U_a)\,dx + V_o(0)$$

extension




$$v_o = -\frac{1}{RC}\int_0^t v_i(\tau)\,d\tau$$

$i = C\frac{dv}{dt}$

$\frac{V_{in}}{R} + C\frac{dV_o}{dt} = 0$

$V_o = -\int \frac{1}{R\cdot C} \cdot V_{in}$

(4) for $V_o(t)$  $RC = 10 \times 10^3 \times 0.05 \times 10^{-6} = 5\text{ ms}$

$\therefore \frac{1}{RC} = 200$  2'

$\because V_b - V_a = -15 - (-7) = -8\text{ V}$

$V_o(0) = -4 - (-12) = 8\text{ V}$

$\Rightarrow V_o(t) = \frac{1}{RC}\int_0^t (V_b - V_a)\,dx + V_o(0)$

$= 200\int_0^t -8\,dx + 8$

$= (-1600t + 8)\text{ V} \quad 0 \le t \le t_{sat}$  2' #

(4) for $V_2(t)$  $V_2(0^+) = -4V,\ V_2(\infty) = -15V,\ \tau = 5\text{ ms}$  2'

$\therefore V_2(t) = V_2(\infty) + [V_2(0^+) - V_2(\infty)]e^{-t/RC}$

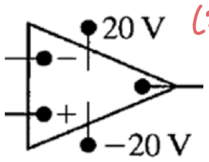
$= -15 + (-4 + 15)e^{-200t}$

$= -15 + 11e^{-200t}\text{ V} \quad 0 \le t \le t_{sat}$  2' #

(4) for $V_f(t)$  $\because V_f + V_2 = V_o$  2'

$\Rightarrow V_f(t) = V_o(t) - V_2(t)$

$= 23 - 1600t - 11e^{-200t}\text{ V} \quad 0 \le t \le t_{sat}$  2' #




(2) for $-1600t_{sat} + 8 = -20 \Rightarrow t_{sat} = 17.5\text{ ms}$ # 2'

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