CS 101 Fall 2020 - Quiz 12-A

12/22/2020 - 20 Minutes **Name:** **ID number:**

Problem 1(2+3+2pts): Follow the below steps, show that this problem is in NP:

Given a graph with n nodes and a number k, are there k nodes that form a clique? (vertices in a clique are all connected to each other)

Part(A): Construct the verifier.

Part(B): Briefly explain how your verifier works.

Part(C): Show that the verifier works in polynomial time.

Part A: Certificate y is a set of k nodes in x. Check each pair of the k nodes is connected by an edge. If so, output 1. Otherwise, output 0.

Part B: If an instance has a solution, then there are k nodes that are mutually connected. Call this set y and give it to V. Clearly V outputs 1.

Part C: Checking k nodes are mutually connected takes $O(k^2)$ time.

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Problem 2(8pts): Reduction

For the below problem, choose an NP-complete problem A and for any A instance, construct an instance of the below problem. **You need to ensure the yes/no answers to the two instances are the same, but you do not need to prove it.**

We have learned 3-coloring problem in class. Now consider this problem: Given an undirected graph G, can the nodes be colored in 5 colors so that no adjacent nodes have the same color? Please reduce 3-coloring problem to 5-coloring problem.

(1) For any instance of 3-Coloring problem with graph $G = (V, E)$, we construct an instance of 5-Coloring problem where $G' = (V', E')$. V' is V with two additional vertices $\tilde{v_1}$, $\tilde{v_2}$ and E' is E with edges $(\tilde{v_1}, \tilde{v_2})$ and $(v, \tilde{v_1})$, $(v, \tilde{v_2})$ for every $v \in V$.

(2) Proof (This do not need to be included in answer): We now prove G is a yes-instance of 3-Coloring problem if and only if G' is a yes-instance of the constructed 5-Coloring problem:

- "$\Rightarrow$": if G is a yes-instance of 3-Coloring problem with assignment a, we can just use the assignment for vertices in $V' \bigcap V$ of G' and color the $\tilde{v_1}$ with the 4th color and the $\tilde{v_2}$ with the 5th color. Then the assignment a' is valid for G' which means the constructed 5-Coloring is also a yes-instance.
- "$\Leftarrow$": if G is a yes-instance of the constructed 5-Coloring problem with assignment a, then since the vertices $\tilde{v_1}, \tilde{v_2}$ are directly connected to every other vertices, the set $V' \bigcap V$ can only use 3 different colors. Thus $G = G' - \tilde{v_1}, \tilde{v_2}$ is also a yes-instance.