AC Steady-State Analysis

Discussion7

Quick Review

- Sinusoids with Phasors
- Impedance and combinations
- KCL/KVL in frequency domain
- Miscellaneous

Sum of Sinusoidal Functions

If

$$v = v_1 + v_2 + \cdots + v_n$$

where $v_i$sare sinusoidal voltages of the same frequency,

then

$$\mathbf{V} = \mathbf{V}_1 + \mathbf{V}_2 + \cdots + \mathbf{V}_n$$

$$v = v_1 + v_2 + \cdots + v_n$$

$$\Updownarrow$$

$$V_m \cos(\omega t + \theta) = V_{m1} \cos(\omega t + \theta_1) + V_{m2} \cos(\omega t + \theta_2) + \cdots + V_{mn} \cos(\omega t + \theta_n)$$

$$\Updownarrow$$

$$\mathrm{Re}\left(V_m e^{j\theta} e^{j\omega t}\right) = \mathrm{Re}\left(V_{m1} e^{j\theta_1} e^{j\omega t}\right) + \cdots + \mathrm{Re}\left(V_{mn} e^{j\theta_n} e^{j\omega t}\right)$$

$$\Updownarrow \ \mathbf{V}_k = V_{mk} e^{j\theta_k}$$

$$\mathrm{Re}\left(\mathbf{V} e^{j\omega t}\right) = \mathrm{Re}\left((\mathbf{V}_1 + \cdots + \mathbf{V}_n) e^{j\omega t}\right)$$

$$\Updownarrow$$

$$\mathbf{V} = \mathbf{V}_1 + \mathbf{V}_2 + \cdots + \mathbf{V}_n$$

Example

If $y_1 = 20\cos(\omega t - 30°)$ and $y_2 = 40\cos(\omega t + 60°)$, express $y = y_1 + y_2$ as a single sinusoidal function.

1. Use trigonometric identities
2. Use the phasor concept

$$
\begin{aligned}
y &= (20\cos 30 + 40\cos 60)\cos\omega t \\
&\quad + (20\sin 30 - 40\sin 60)\sin\omega t \\
&= 37.32\cos\omega t - 24.64\sin\omega t.
\end{aligned}
$$

$$y = 44.72\cos(\omega t + 33.43^\circ)$$

$$
\begin{aligned}
\mathbf{Y} &= \mathbf{Y}_1 + \mathbf{Y}_2 \\
&= 20\underline{/-30^\circ} + 40\underline{/60^\circ} \\
&= (17.32 - j10) + (20 + j34.64) \\
&= 37.32 + j24.64 \\
&= 44.72\underline{/33.43^\circ}.
\end{aligned}
$$

Kirchhoff's Laws in the Frequency Domain

- Let $v_1, v_2, \cdots v_n$ be the voltages around a closed loop. Then according to KCL

$$v_1 + v_2 + \cdots + v_n = 0$$

$$\mathbf{V}_1 + \mathbf{V}_2 + \cdots + \mathbf{V}_n = 0$$

Similarly, KCL holds for phasors:

$$i_1 + i_2 + \cdots + i_n = 0$$

$$\boxed{\mathbf{I}_1 + \mathbf{I}_2 + \cdots + \mathbf{I}_n = 0,}$$

AC Phasor Analysis General Procedure

Step 1: Adopt cosine reference

$$v_s(t) = 12\sin(\omega t - 45^\circ)$$
$$= 12\cos(\omega t - 45^\circ - 90^\circ) = 12\cos(\omega t - 135^\circ)\ \text{V}.$$
$$\mathbf{V}_s = 12e^{-j135^\circ}\ \text{V}.$$

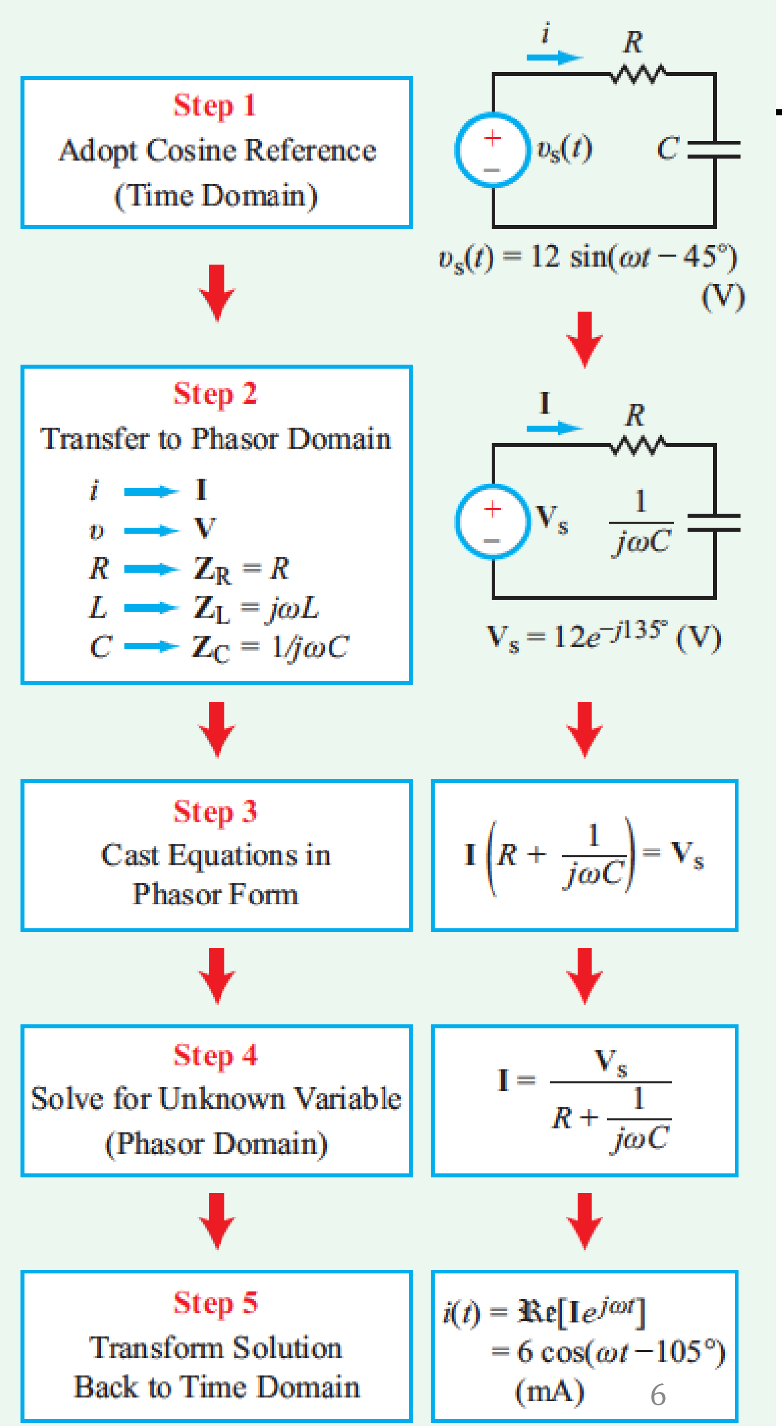
Step 2: Transform circuit to phasor domain

Step 3: Cast KCL and/or KVL equations in phasor domain

$$\mathbf{Z}_R\mathbf{I} + \mathbf{Z}_C\mathbf{I} = \mathbf{V}_s,$$

which is equivalent to

$$\left(R + \frac{1}{j\omega C}\right)\mathbf{I} = 12e^{-j135^\circ}.$$



AC Phasor Analysis General Procedure

Step 4: Solve for unknown variable

$$\mathbf{I} = \frac{12e^{-j135^\circ}}{R + \frac{1}{j\omega C}} = \frac{j12\omega C e^{-j135^\circ}}{1 + j\omega RC}.$$

Using the specified values, namely $R = \sqrt{3}$ kΩ, $C = 1\ \mu$F, and $\omega = 10^3$ rad/s,

$$\mathbf{I} = \frac{j12 \times 10^3 \times 10^{-6} e^{-j135^\circ}}{1 + j10^3 \times \sqrt{3} \times 10^3 \times 10^{-6}} = \frac{j12e^{-j135^\circ}}{1 + j\sqrt{3}} \text{ mA.}$$

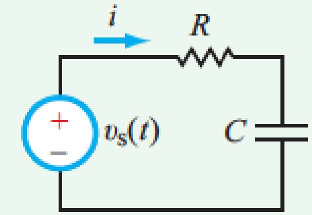
$$\mathbf{I} = \frac{12e^{-j135^\circ} \cdot e^{j90^\circ}}{2e^{j60^\circ}} = 6e^{j(-135^\circ + 90^\circ - 60^\circ)} = 6e^{-j105^\circ} \text{ mA.}$$

Step 5: Transform solution back to time domain

To return to the time domain, we apply the fundamental relation between a sinusoidal function and its phasor counterpart, namely

$$i(t) = \Re\mathfrak{e}[\mathbf{I}e^{j\omega t}] = \Re\mathfrak{e}[6e^{-j105^\circ} e^{j\omega t}] = 6\cos(\omega t - 105^\circ) \text{ mA.}$$

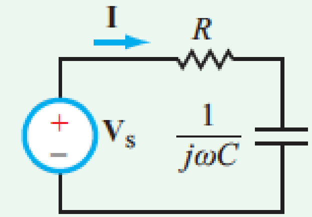
Step 1
Adopt Cosine Reference (Time Domain)



$v_s(t) = 12\sin(\omega t - 45^\circ)$ (V)

Step 2
Transfer to Phasor Domain

- $i \rightarrow \mathbf{I}$
- $v \rightarrow \mathbf{V}$
- $R \rightarrow \mathbf{Z}_R = R$
- $L \rightarrow \mathbf{Z}_L = j\omega L$
- $C \rightarrow \mathbf{Z}_C = 1/j\omega C$



$\mathbf{V}_s = 12e^{-j135^\circ}$ (V)

Step 3
Cast Equations in Phasor Form

$$\mathbf{I}\left(R + \frac{1}{j\omega C}\right) = \mathbf{V}_s$$

Step 4
Solve for Unknown Variable (Phasor Domain)

$$\mathbf{I} = \frac{\mathbf{V}_s}{R + \frac{1}{j\omega C}}$$

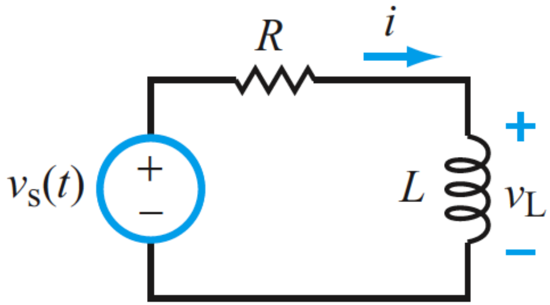
Step 5
Transform Solution Back to Time Domain

$$i(t) = \Re\mathfrak{e}[\mathbf{I}e^{j\omega t}] = 6\cos(\omega t - 105^\circ) \text{ (mA)}$$

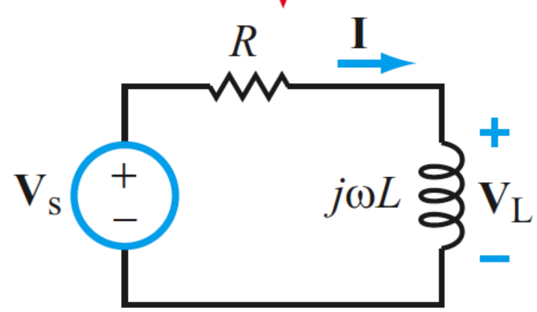
Example: RL Circuit

$$v_s(t) = 15\sin(4\times 10^4 t - 30^\circ)\ \text{V}.$$

Also, $R = 3\ \Omega$ and $L = 0.1$ mH. Obtain an expression for the voltage across the inductor.



(a) Time domain



(b) Phasor domain

Solution:

Step 1: Convert $v_s(t)$ to the cosine reference

$$v_s(t) = 15\sin(4\times 10^4 t - 30^\circ) = 15\cos(4\times 10^4 t - 120^\circ)\ \text{V},$$

$$\mathbf{V}_s = 15e^{-j120^\circ}\ \text{V}.$$

Step 2: Transform circuit to the phasor domain

Step 3: Cast KVL in phasor domain

$$R\mathbf{I} + j\omega L\mathbf{I} = \mathbf{V}_s.$$

Step 4: Solve for unknown variable

$$\mathbf{I} = \frac{\mathbf{V}_s}{R + j\omega L} = \frac{15e^{-j120^\circ}}{3 + j4 \times 10^4 \times 10^{-4}}$$
$$= \frac{15e^{-j120^\circ}}{3 + j4} = \frac{15e^{-j120^\circ}}{5e^{j53.1^\circ}} = 3e^{-j173.1^\circ} \text{ A.}$$

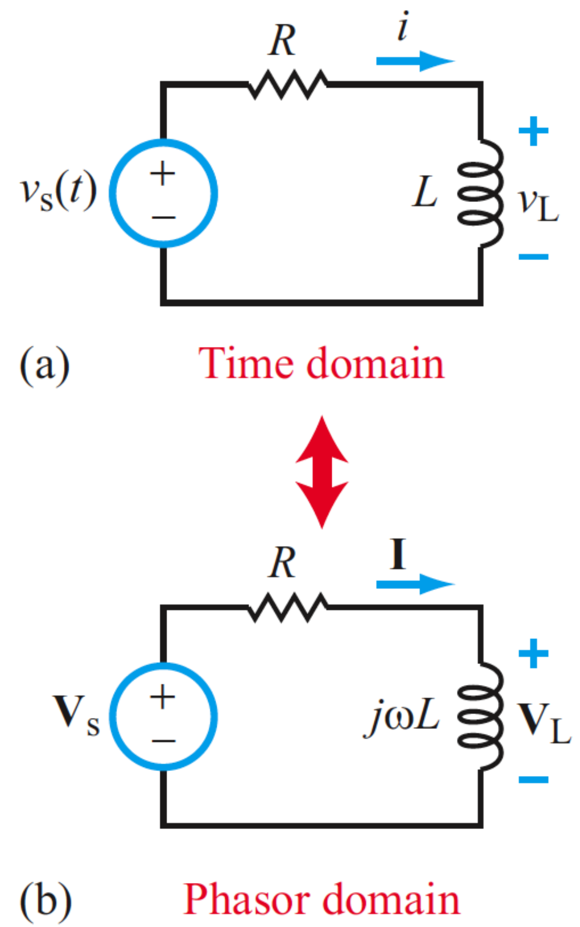
The phasor voltage across the inductor is related to $\mathbf{I}$ by

$$\begin{aligned} \mathbf{V}_L &= j\omega L\mathbf{I} \\ &= j4 \times 10^4 \times 10^{-4} \times 3e^{-j173.1^\circ} \\ &= j12e^{-j173.1^\circ} \\ &= 12e^{-j173.1^\circ} \cdot e^{j90^\circ} = 12e^{-j83.1^\circ} \text{ V,} \end{aligned}$$

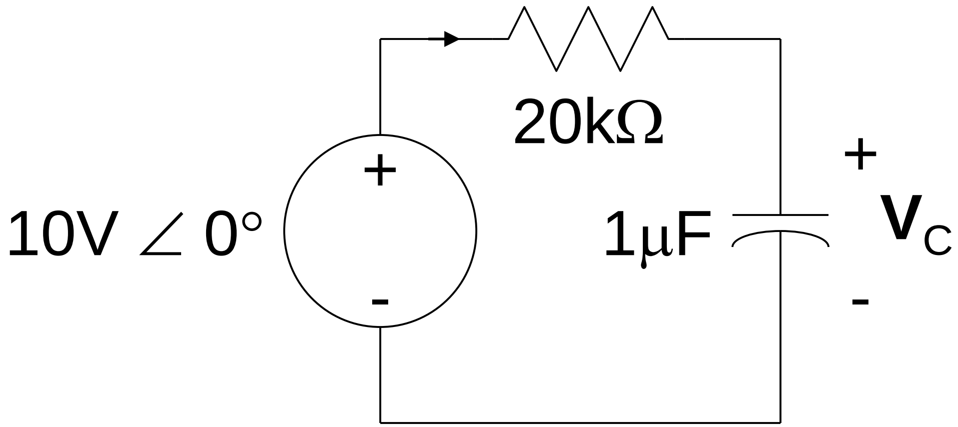
where we replaced j with e^{j90°.

Step 5: Transform solution to the time domain

$$\begin{aligned} \upsilon_L(t) &= \mathfrak{Re}[\mathbf{V}_L e^{j\omega t}] \\ &= \mathfrak{Re}[12e^{-j83.1^\circ} e^{j4\times 10^4 t}] \\ &= 12\cos(4 \times 10^4 t - 83.1^\circ) \text{ V.} \end{aligned}$$



Exercise1



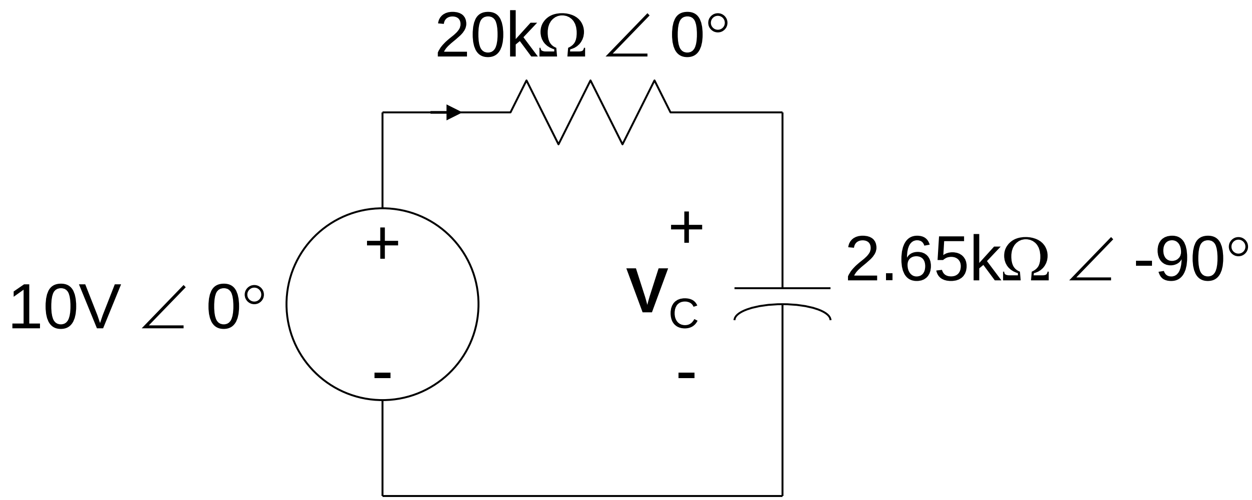
f=60 Hz, $\mathbf{V}_C$=?

1. f=60 Hz, $\mathbf{V}_C$=?
2. ω = 10， find $\mathbf{V}_C$

First compute impedances for resistor and capacitor:

$\mathbf{Z}_R$ = R= 20kΩ = 20kΩ ∠ 0°

$\mathbf{Z}_C$ = 1/*j* (2πf x 1μF) = 2.65kΩ ∠ -90°

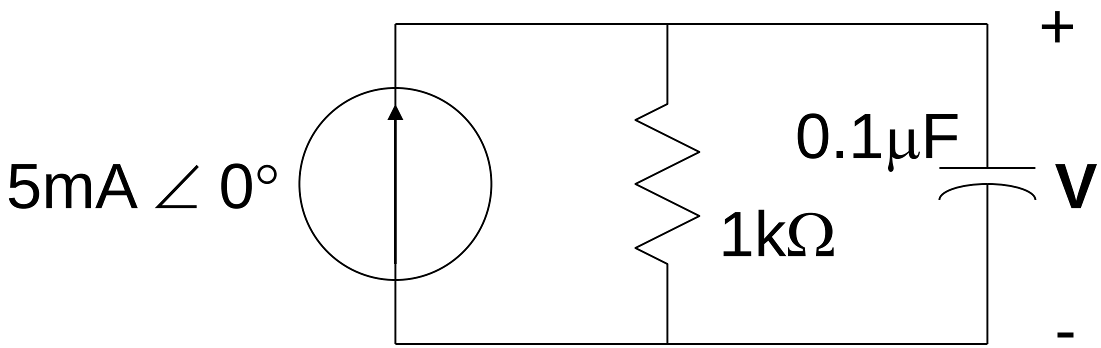


Now use the voltage divider to find $\mathbf{V}_C$:

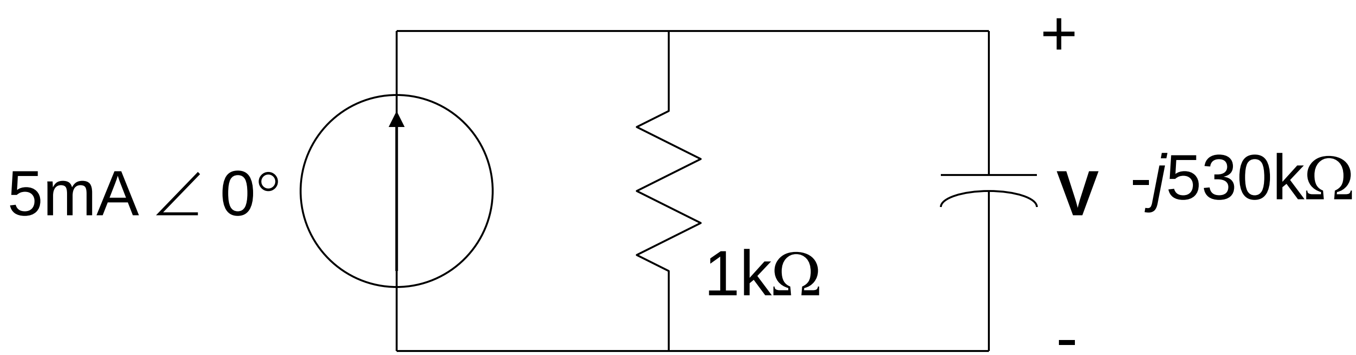
$$\mathbf{V}_C = 10\text{V} \angle 0° \left(\frac{2.65\text{k}\Omega \angle -90°}{2.65\text{k}\Omega \angle -90° + 20\text{k}\Omega \angle 0°} \right)$$

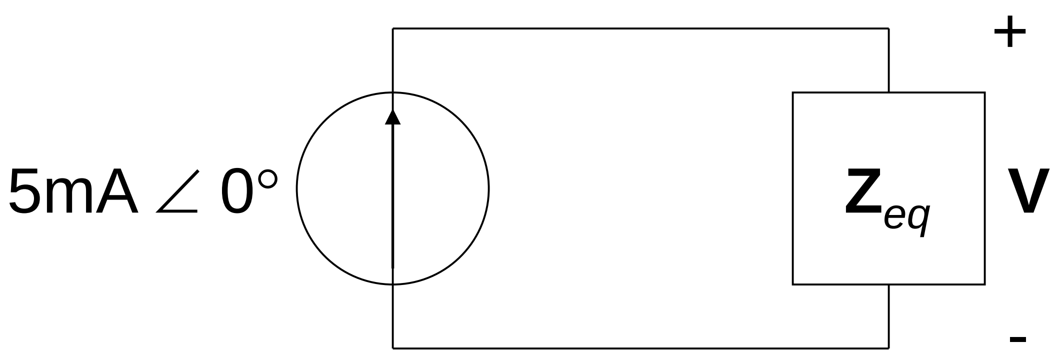
$$\mathbf{V}_C = 1.31\text{V} \angle -82.4°$$

Exercise2



Find $v(t)$ for $\omega=2\pi \times 3000$





$$\mathbf{Z}_{eq} = \frac{1000 \times (-j530)}{1000 - j530} = \frac{10^3 \angle 0° \times 530 \angle -90°}{1132 \angle -27.9°}$$

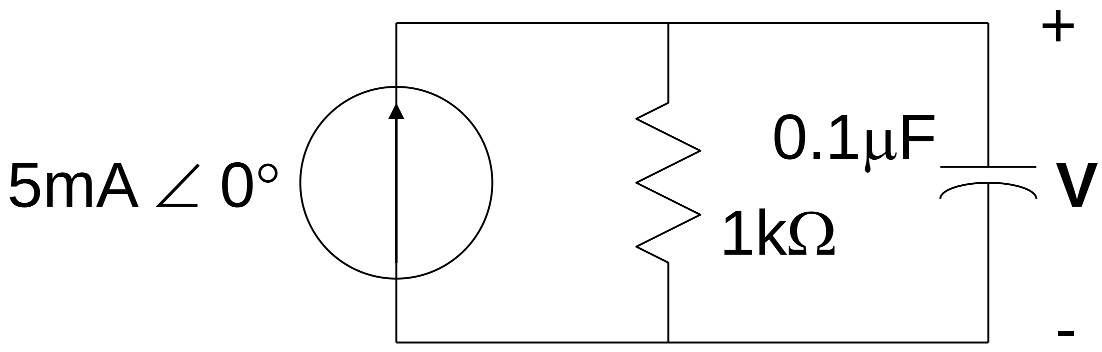
$$\mathbf{Z}_{eq} = 468.2\Omega \angle -62.1°$$

$$\mathbf{V} = \mathbf{I}\mathbf{Z}_{eq} = 5\text{mA} \angle 0° \times 468.2\Omega \angle -62.1°$$

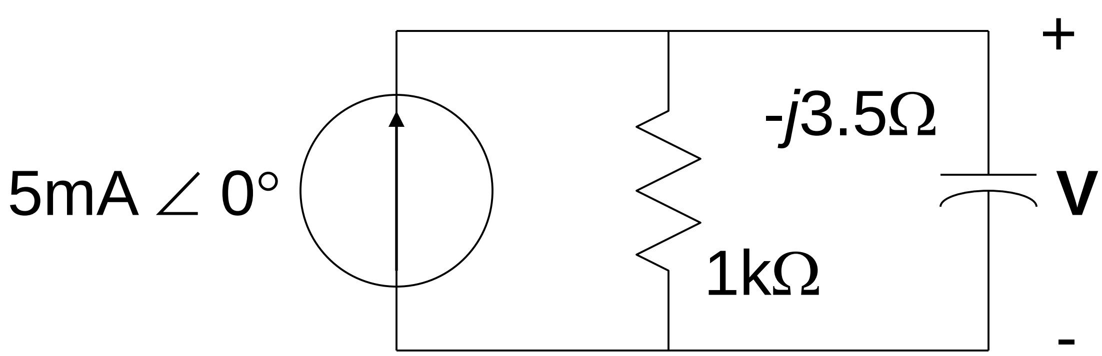
$$\mathbf{V} = 2.34\text{V} \angle -62.1°$$

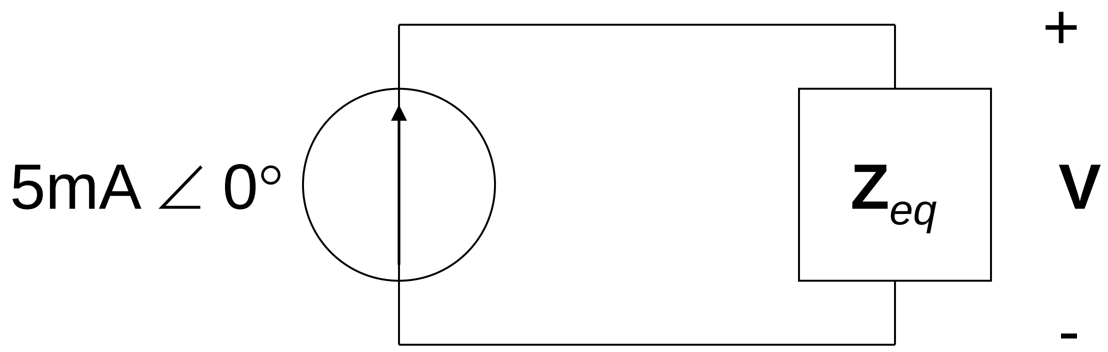
$$v(t) = 2.34\cos(2\pi 3000\text{t} - 62.1°)\text{V}$$

Change the Frequency



Find $v(t)$ for $\omega=2\pi\ 455000$





$$\mathbf{Z}_{eq} = \frac{1000 \times (-j3.5)}{1000 - j3.5} = \frac{10^3 \angle 0° \times 3.5 \angle -90°}{1000 \angle -0.2°}$$

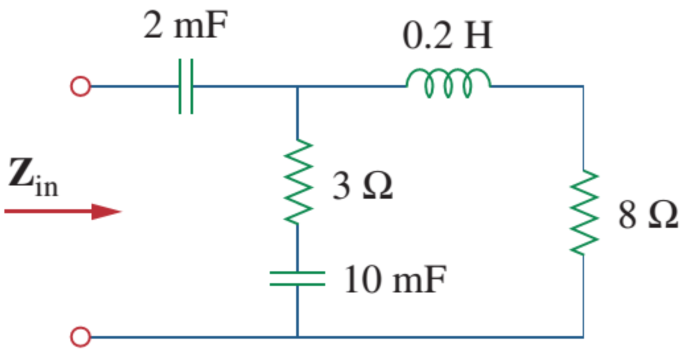
$$\mathbf{Z}_{eq} = 3.5 \angle -89.8° \Omega$$

$$\mathbf{V} = \mathbf{I}\mathbf{Z}_{eq} = 5\angle 0° \text{mA} \times 3.5 \angle -89.8° \Omega \quad \mathbf{V} = 17.5 \angle -89.8° \text{mV}$$

$$v(t) = 17.5 \cos(2\pi 455000\text{t} - 89.8°) \text{mV}$$

Exercise3

- Find the input impedance of the circuit below. $\omega = 50$rad/s.



$$\mathbf{Z}_1 = \frac{1}{j\omega C} = \frac{1}{j50 \times 2 \times 10^{-3}} = -j10\ \Omega$$

$$\mathbf{Z}_2 = 3 + \frac{1}{j\omega C} = 3 + \frac{1}{j50 \times 10 \times 10^{-3}} = (3 - j2)\ \Omega$$

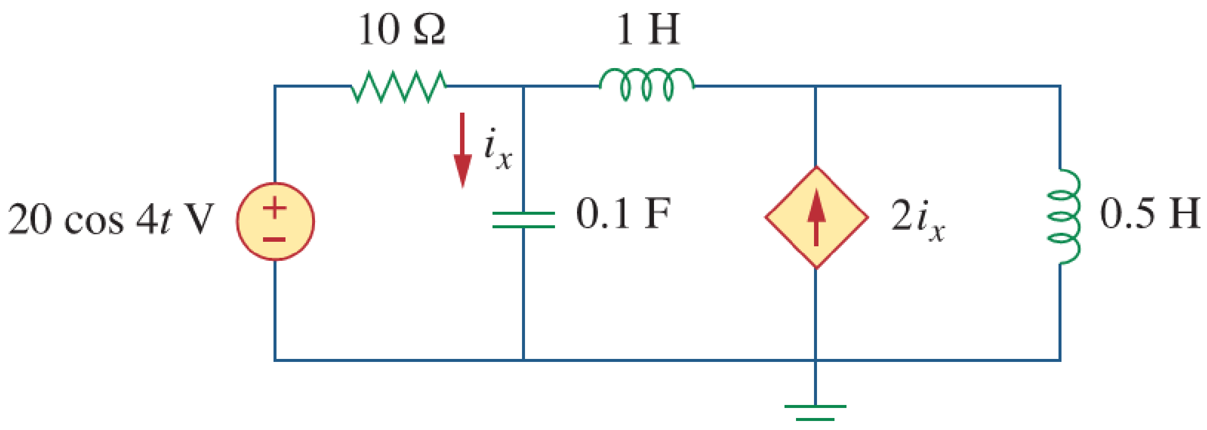
$$\mathbf{Z}_3 = 8 + j\omega L = 8 + j50 \times 0.2 = (8 + j10)\ \Omega$$

$$\mathbf{Z}_{\text{in}} = \mathbf{Z}_1 + \mathbf{Z}_2 \parallel \mathbf{Z}_3 = -j10 + \frac{(3 - j2)(8 + j10)}{11 + j8}$$

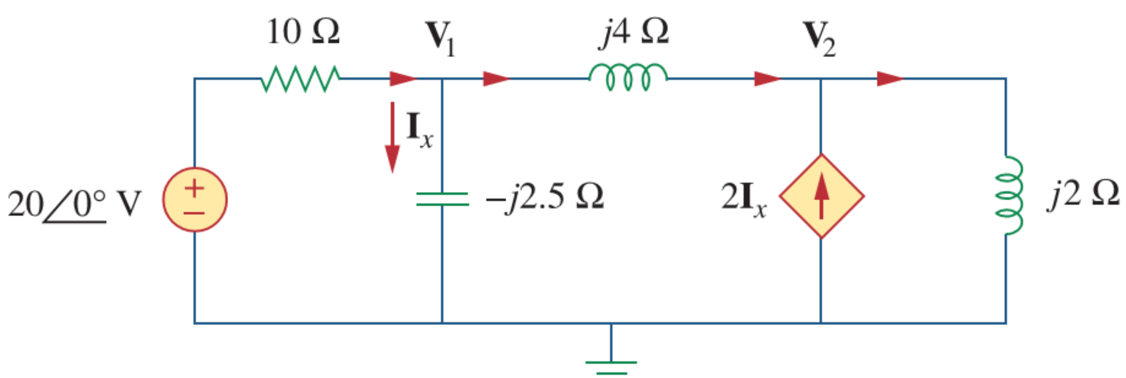
$$= -j10 + \frac{(44 + j14)(11 - j8)}{11^2 + 8^2} = -j10 + 3.22 - j1.07\ \Omega$$

Nodal Analysis

- Note that AC sources appear as DC sources with their values expressed as their amplitude.



Find i_x

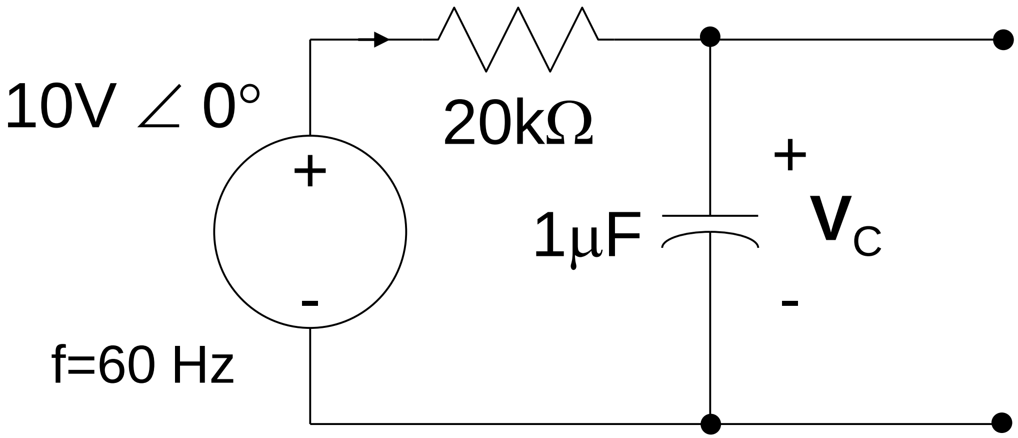


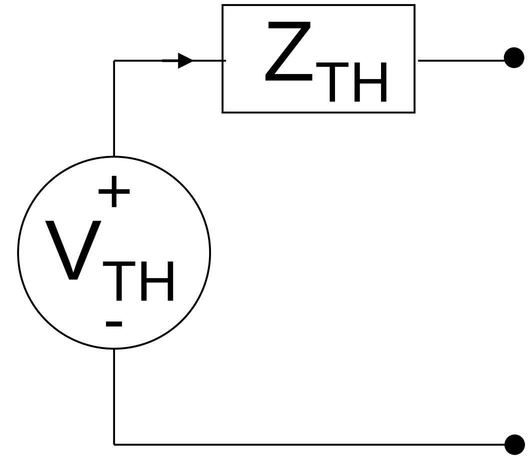
$$\frac{20 - \mathbf{V}_1}{10} = \frac{\mathbf{V}_1}{-j2.5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4}$$

$$2\mathbf{I}_x + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4} = \frac{\mathbf{V}_2}{j2}$$

$$i_x = 7.59\cos(4t + 108.4^\circ)\text{ A}$$

Thevenin Equivalent





$\mathbf{Z}_R$ = R= 20kΩ = 20kΩ ∠ 0°

$\mathbf{Z}_C$ = 1/*j* (2πf x 1μF) = 2.65kΩ ∠ -90°

$$\mathbf{V}_{TH} = \mathbf{V}_{OC} = 10\text{V} \angle 0° \left(\frac{2.65\text{k}\Omega \angle -90°}{2.65\text{k}\Omega \angle -90° + 20\text{k}\Omega \angle 0°} \right) = 1.31 \angle -82.4$$

$$\mathbf{Z}_{TH} = \mathbf{Z}_R \,||\, \mathbf{Z}_C = {}^{\circ}\left(\frac{20\text{k}\Omega \angle 0° \cdot 2.65\text{k}\Omega \angle -90°}{2.65\text{k}\Omega \angle -90° + 20\text{k}\Omega \angle 0°} \right) = 2.62 \angle -82.4$$

Something need to notice

- A sinusiod function can be equivalently transformed to a phasor $v(t) = V_m \cos(\omega t + \phi) \Leftrightarrow \mathbf{V} = V_m \angle \phi$

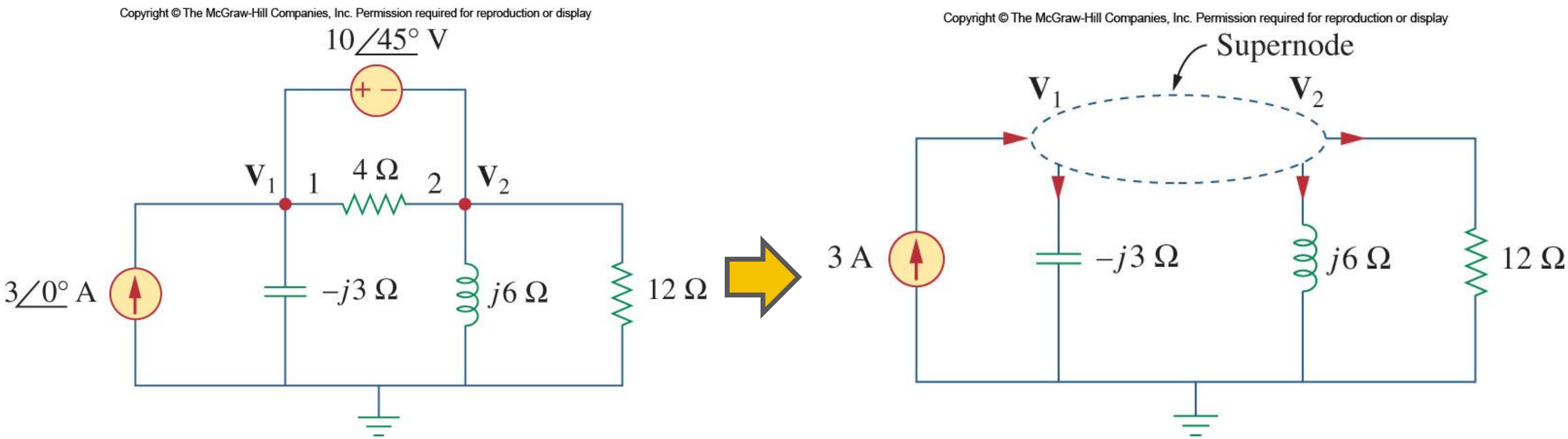
- Get familiar with the relations and caculation for complex numbers and phasors

- When we describe the impedance, we need to specify at what frequency we are describing the impedance.

Outline

- **Most analytical techniques we used before can be applied to frequency-domain circuits !!**
- Nodal/Mesh analysis
- Superposition
- Source transformation/Thevenin/Norton
- Applications

Nodal Analysis - Supernode



Example 7-12: Nodal Analysis

Apply the nodal-analysis method to determine $i_L(t)$ in the circuit of Fig. 7-25(a). The sources are given by:

$$v_{s_1}(t) = 12\cos 10^3 t \text{ V},$$

$$v_{s_2}(t) = 6\sin 10^3 t \text{ V}.$$

Solution: We first will demonstrate how to solve this problem using the standard nodal-analysis method, and then we will solve it again by applying the by-inspection method.

Nodal-Analysis Method

Our first step is to transform the given circuit to the phasor domain. Accordingly,

$$\mathbf{Z}_C = \frac{1}{j\omega C} = \frac{-j}{10^3 \times 0.25 \times 10^{-3}} = -j4\ \Omega,$$

$$\mathbf{Z}_L = j\omega L = j10^3 \times 10^{-3} = j1\ \Omega,$$

$$\mathbf{V}_{s_1} = 12 \text{ V},$$

and

$$\mathbf{V}_{s_2} = -j6 \text{ V},$$

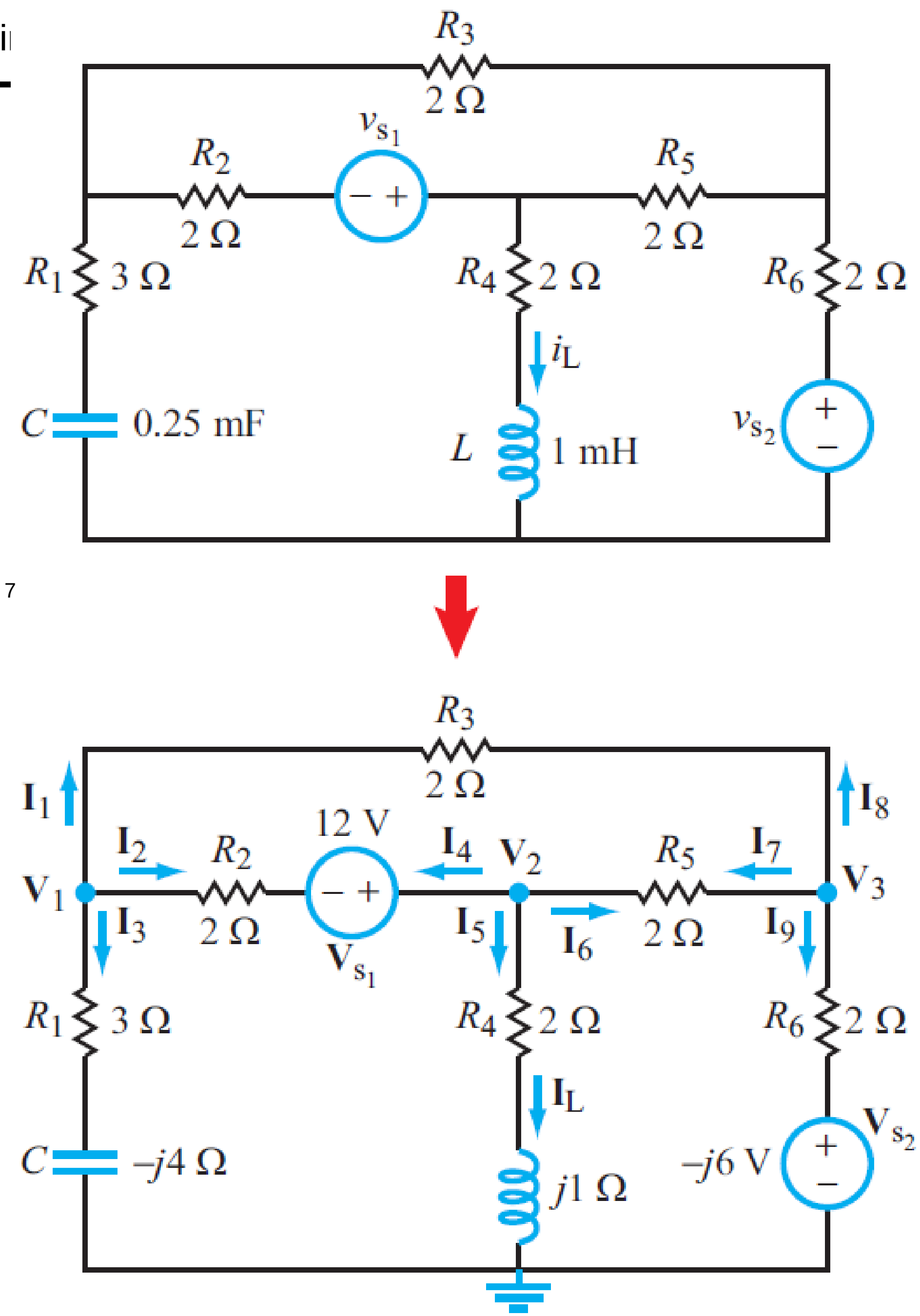
$$\mathbf{I}_1 = \frac{\mathbf{V}_1 - \mathbf{V}_3}{R_3} = \frac{\mathbf{V}_1 - \mathbf{V}_3}{2},$$

$$\mathbf{I}_2 = \frac{\mathbf{V}_1 - \mathbf{V}_2 + \mathbf{V}_{s_1}}{R_2} = \frac{\mathbf{V}_1 - \mathbf{V}_2 + 12}{2},$$

$$\mathbf{I}_3 = \frac{\mathbf{V}_1}{R_1 + \mathbf{Z}_C} = \frac{\mathbf{V}_1}{3 - j4}.$$

Inserting the expressions for $\mathbf{I}_1$ to $\mathbf{I}_3$ in Eq. (7.109) and then rearranging the terms leads to

$$\left(\frac{1}{2} + \frac{1}{2} + \frac{1}{3 - j4}\right)\mathbf{V}_1 - \frac{1}{2}\mathbf{V}_2 - \frac{1}{2}\mathbf{V}_3 = -6.$$



Lecture 7

Node 1

Cont.

$$(2.24 + j0.32)\mathbf{V}_1 - \mathbf{V}_2 - \mathbf{V}_3 = -12 \qquad \text{(node 1)}.$$

Similarly, at node 2,

$$\frac{\mathbf{V}_2 - \mathbf{V}_1 - 12}{2} + \frac{\mathbf{V}_2}{2 + j1} + \frac{\mathbf{V}_2 - \mathbf{V}_3}{2} = 0,$$

which can be simplified to

$$-\mathbf{V}_1 + (2.8 - j0.4)\mathbf{V}_2 - \mathbf{V}_3 = 12 \qquad \text{(node 2)},$$

and at node 3,

$$\frac{\mathbf{V}_3 - \mathbf{V}_2}{2} + \frac{\mathbf{V}_3 - \mathbf{V}_1}{2} + \frac{\mathbf{V}_3 + j6}{2} = 0,$$

or

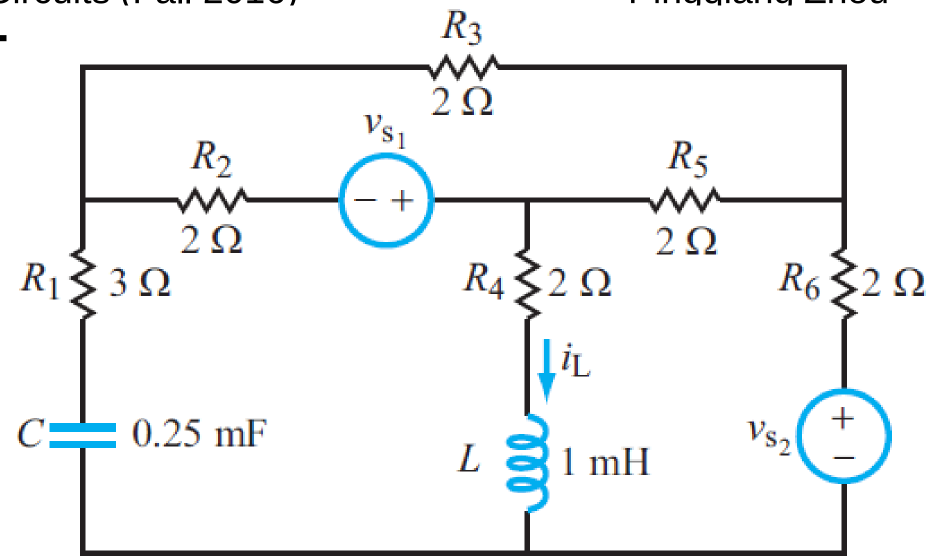
$$-\mathbf{V}_1 - \mathbf{V}_2 + 3\mathbf{V}_3 = -j6 \qquad \text{(node 3)}.$$

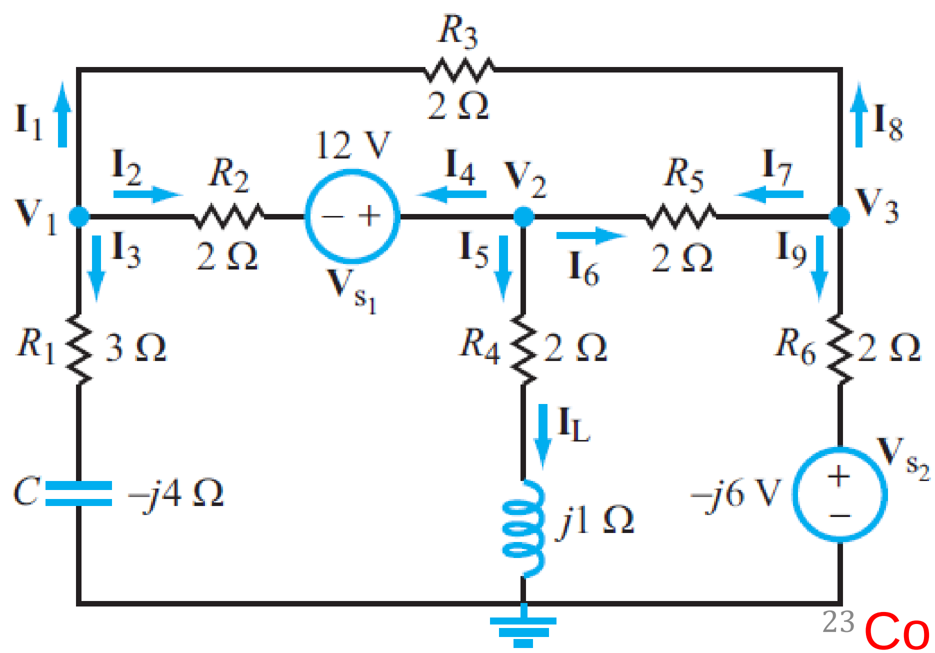
Equations (7.112) to (7.114) now are ready to be cast in matrix form:

$$\begin{bmatrix} (2.24 + j0.32) & -1 & -1 \\ -1 & (2.8 - j0.4) & -1 \\ -1 & -1 & 3 \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \\ \mathbf{V}_3 \end{bmatrix} = \begin{bmatrix} -12 \\ 12 \\ -j6 \end{bmatrix}.$$

Matrix inversion, either manually or by MATLAB® provides the solution:

$$\mathbf{V}_1 = -(4.72 + j0.88) \text{ V},$$
$$\mathbf{V}_2 = (2.46 - j0.89) \text{ V},$$
$$\mathbf{V}_3 = -(0.76 + j2.59) \text{ V}.$$





Cont

Example 7-12: Nodal Analysis (cont.)

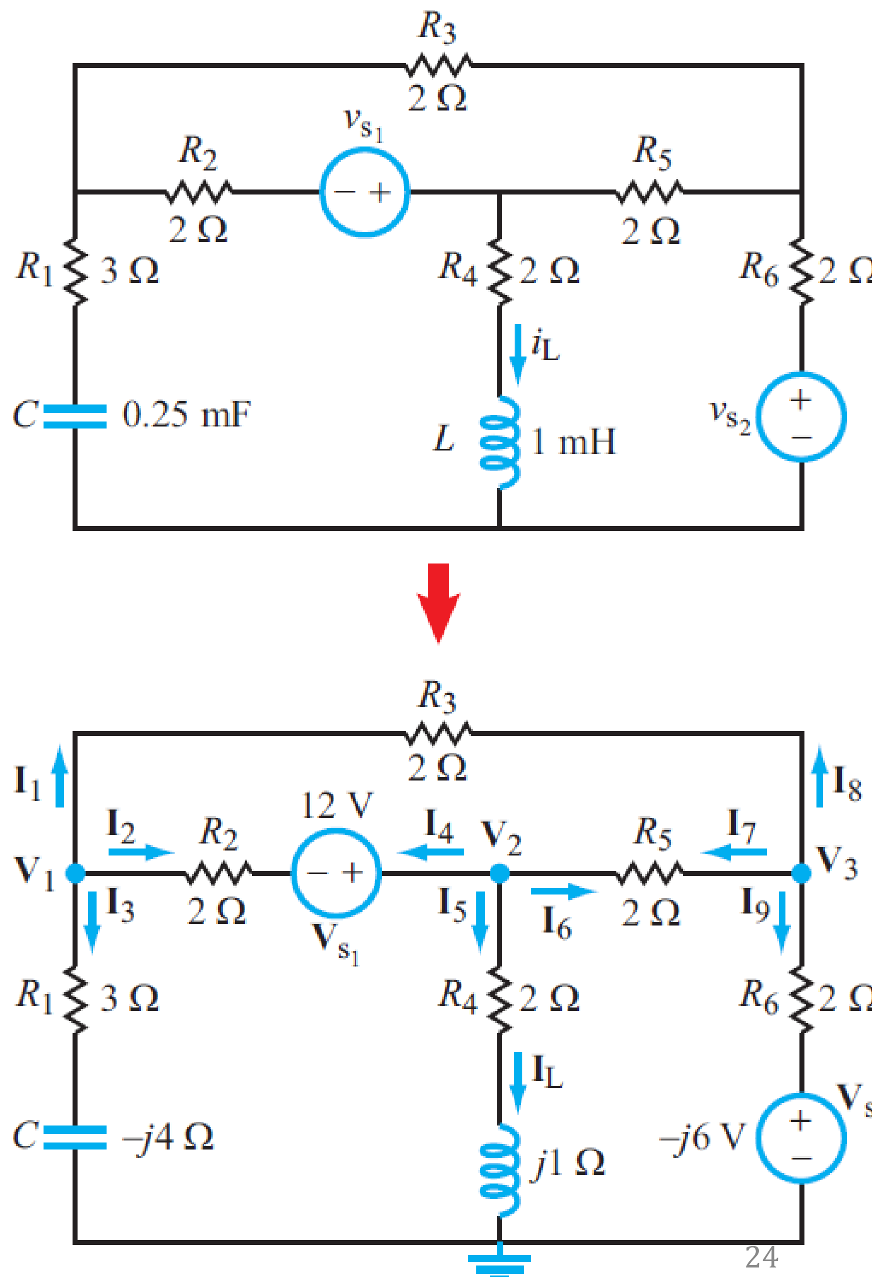
$$\mathbf{V}_3 = -(0.76 + j2.59)\ \text{V}.$$

Hence,

$$\mathbf{I}_\text{L} = \frac{\mathbf{V}_2}{2 + j1} = \frac{2.46 - j0.89}{2 + j1}$$
$$= 0.81 - j0.85 = 1.17e^{-j46.5^\circ}$$

and its corresponding time-domain counterpart is

$$\boxed{\begin{aligned} i_\text{L}(t) &= \mathfrak{Re}[\mathbf{I}_\text{L} e^{j1000t}] \\ &= \mathfrak{Re}[1.17e^{-j46.4^\circ} e^{j1000t}] \\ &= 1.17\cos(1000t - 46.5^\circ)\ \text{A}. \end{aligned}}$$



Mesh Analysis

- Just as in KCL, the KVL analysis also applies to phasor and frequency domain circuits. In KVL, supermesh analysis is also valid.

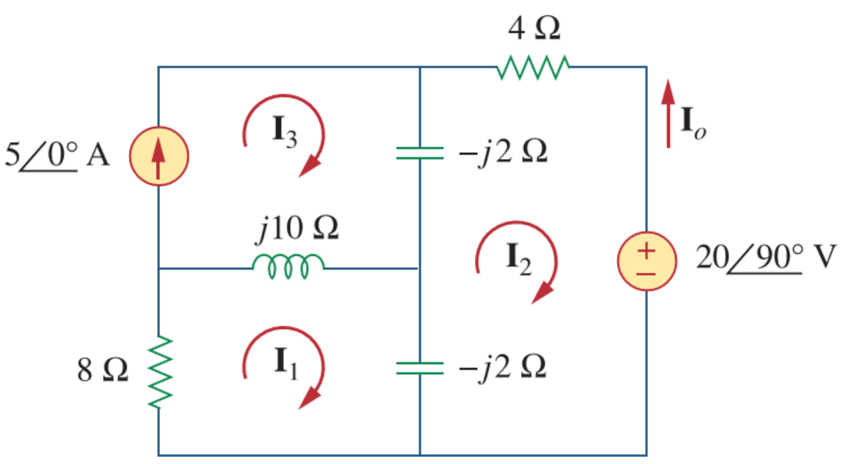


Figure 10.7
For Example 10.3.

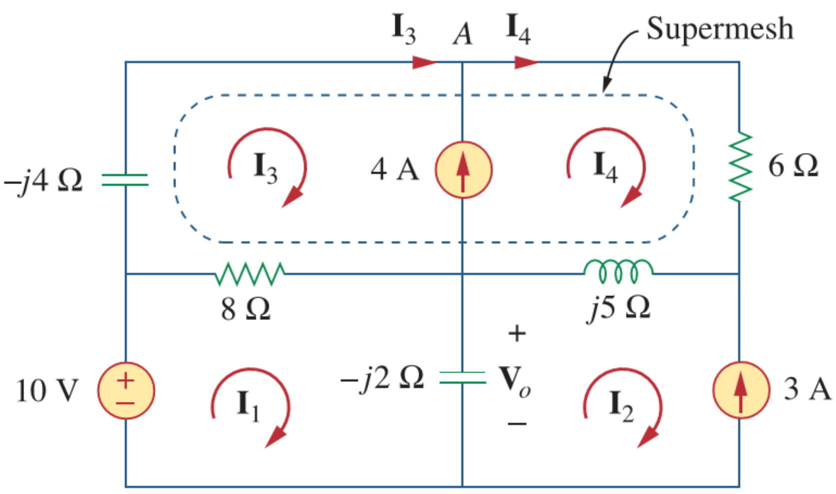


Figure 10.10
Analysis of the circuit in Fig. 10.9.

Example 7-14: Mesh Analysis by Inspection

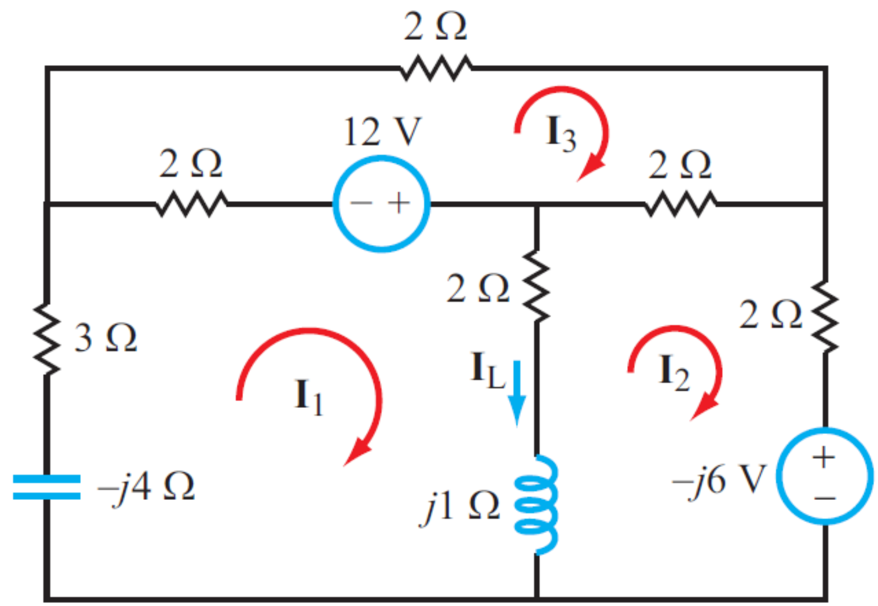
$$\begin{bmatrix} \mathbf{Z}_{11} & \mathbf{Z}_{12} & \mathbf{Z}_{13} \\ \mathbf{Z}_{21} & \mathbf{Z}_{22} & \mathbf{Z}_{23} \\ \mathbf{Z}_{31} & \mathbf{Z}_{32} & \mathbf{Z}_{33} \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \mathbf{I}_3 \end{bmatrix} = \begin{bmatrix} \mathbf{V}_{t_1} \\ \mathbf{V}_{t_2} \\ \mathbf{V}_{t_3} \end{bmatrix}$$

$\mathbf{Z}_{kk}$ = sum of all impedances in loop k

$\mathbf{Z}_{k\ell}$ = $\mathbf{Z}_{\ell k}$ = ***negative*** of impedance(s) shared by loop k and ℓ, with $k \neq \ell$

$\mathbf{I}_k$ = phasor current of loop k

$\mathbf{V}_{t_k}$ = total of phasor voltage sources contained in loop k, with the polarity defined as positive if $\mathbf{I}_k$ flows from (−) to (+) through the source.



In view of these definitions, the matrix equation for the circuit in Fig. 7-28 is given by

$$\begin{bmatrix} (7-j3) & -(2+j1) & -2 \\ -(2+j1) & (6+j1) & -2 \\ -2 & -2 & 6 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \mathbf{I}_3 \end{bmatrix} = \begin{bmatrix} 12 \\ j6 \\ -12 \end{bmatrix}.$$

Matrix inversion leads to

$$\mathbf{I}_1 = (0.43 + j0.86) \text{ A},$$

$$\mathbf{I}_2 = (-0.38 + j1.71) \text{ A},$$

and

$$\mathbf{I}_3 = (-1.98 + j0.86) \text{ A}.$$

$$\mathbf{I}_L = \mathbf{I}_1 - \mathbf{I}_2 = (0.43 + j0.86) - (-0.38 + j1.71)$$
$$= 0.81 - j0.85 = 1.17e^{-j46.4^\circ} \text{ A},$$

and its time-domain counterpart is

$$i_L(t) = \mathfrak{Re}[\mathbf{I}_L e^{j\omega t}] = \mathfrak{Re}[1.17e^{-j46.4^\circ} e^{j1000t}]$$
$$= 1.17 \cos(1000t - 46.4^\circ) \text{ A}.$$

Outline

- ~~Nodal/Mesh analysis~~
- Superposition
- Source transformation/Thevenin/Norton
- Applications

Superposition

- Since AC circuits are linear, it is also possible to apply the principle of superposition.

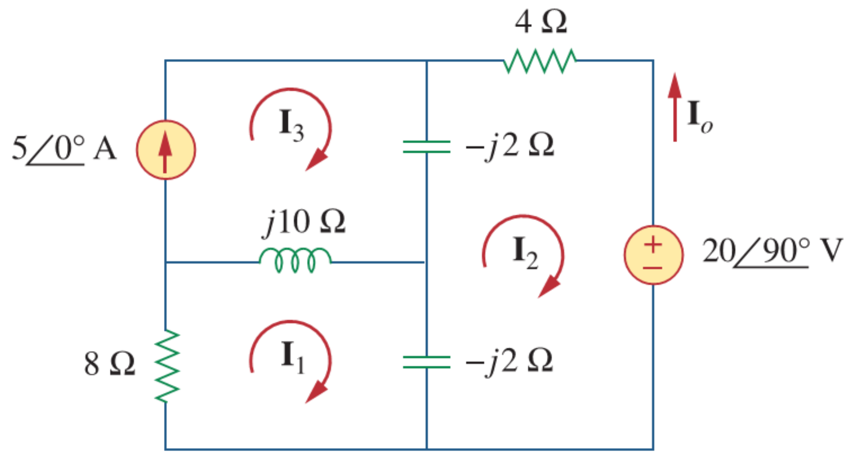
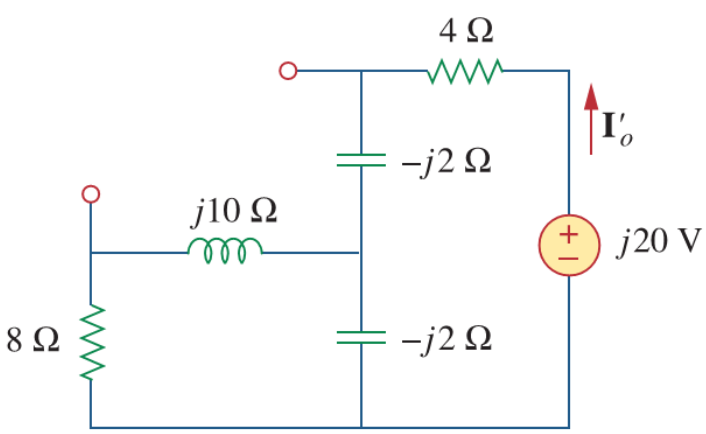
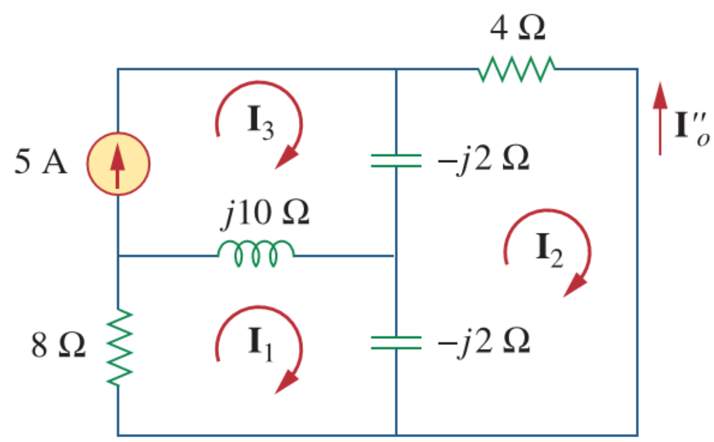
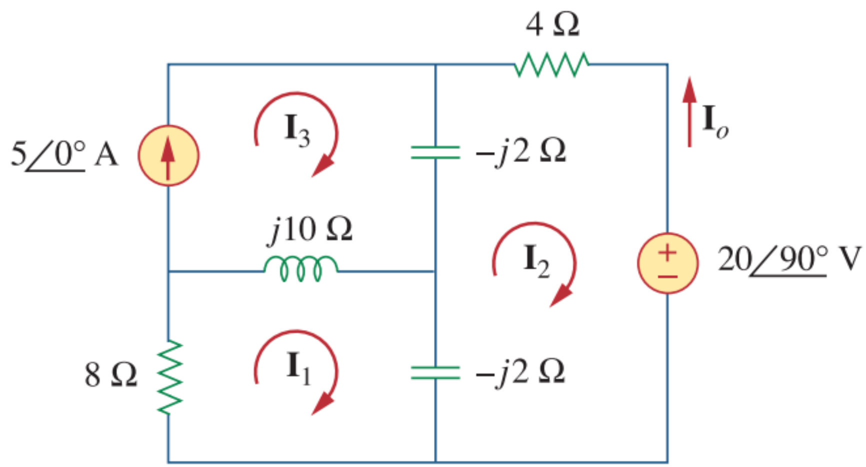


Figure 10.7
For Example 10.3.







where $\mathbf{I}'_o$ and $\mathbf{I}''_o$ are due to the voltage and current sources, respectively. To find $\mathbf{I}'_o$, consider the circuit in Fig. 10.12(a). If we let $\mathbf{Z}$ be the parallel combination of $-j2$ and $8 + j10$, then

$$\mathbf{Z} = \frac{-j2(8 + j10)}{-2j + 8 + j10} = 0.25 - j2.25$$

and current $\mathbf{I}'_o$ is

$$\mathbf{I}'_o = \frac{j20}{4 - j2 + \mathbf{Z}} = \frac{j20}{4.25 - j4.25}$$

or

$$\mathbf{I}'_o = -2.353 + j2.353 \qquad \textbf{(10.5.2)}$$

To get $\mathbf{I}''_o$, consider the circuit in Fig. 10.12(b). For mesh 1,

$$(8 + j8)\mathbf{I}_1 - j10\mathbf{I}_3 + j2\mathbf{I}_2 = 0 \qquad \textbf{(10.5.3)}$$

For mesh 2,

$$(4 - j4)\mathbf{I}_2 + j2\mathbf{I}_1 + j2\mathbf{I}_3 = 0 \qquad \textbf{(10.5.4)}$$

For mesh 3,

$$\mathbf{I}_3 = 5 \qquad \textbf{(10.5.5)}$$

From Eqs. (10.5.4) and (10.5.5),

$$(4 - j4)\mathbf{I}_2 + j2\mathbf{I}_1 + j10 = 0$$

Expressing $\mathbf{I}_1$ in terms of $\mathbf{I}_2$ gives

$$\mathbf{I}_1 = (2 + j2)\mathbf{I}_2 - 5 \qquad \textbf{(10.5.6)}$$

Substituting Eqs. (10.5.5) and (10.5.6) into Eq. (10.5.3), we get

$$(8 + j8)[(2 + j2)\mathbf{I}_2 - 5] - j50 + j2\mathbf{I}_2 = 0$$

or

$$\mathbf{I}_2 = \frac{90 - j40}{34} = 2.647 - j1.176$$

Current $\mathbf{I}''_o$ is obtained as

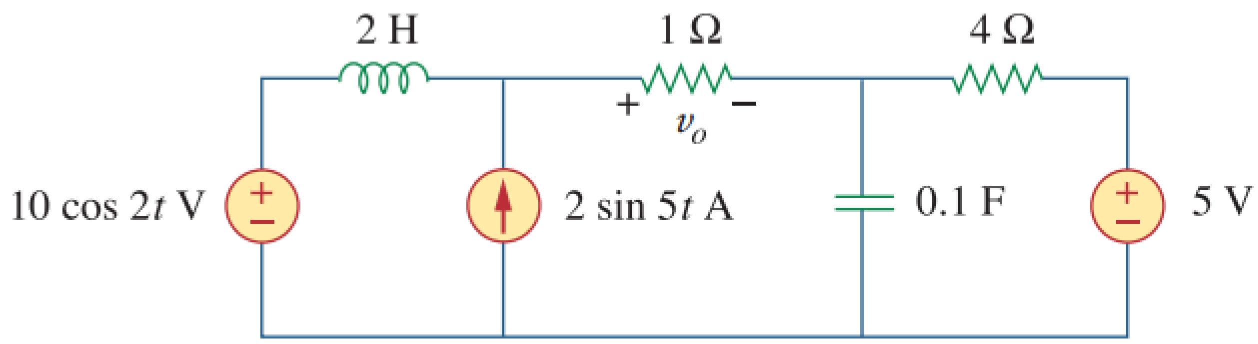
$$\mathbf{I}''_o = -\mathbf{I}_2 = -2.647 + j1.176 \qquad \textbf{(10.5.7)}$$

From Eqs. (10.5.2) and (10.5.7), we write

$$\mathbf{I}_o = \mathbf{I}'_o + \mathbf{I}''_o = -5 + j3.529 = 6.12\angle 144.78^\circ \text{ A}$$

Superposition II

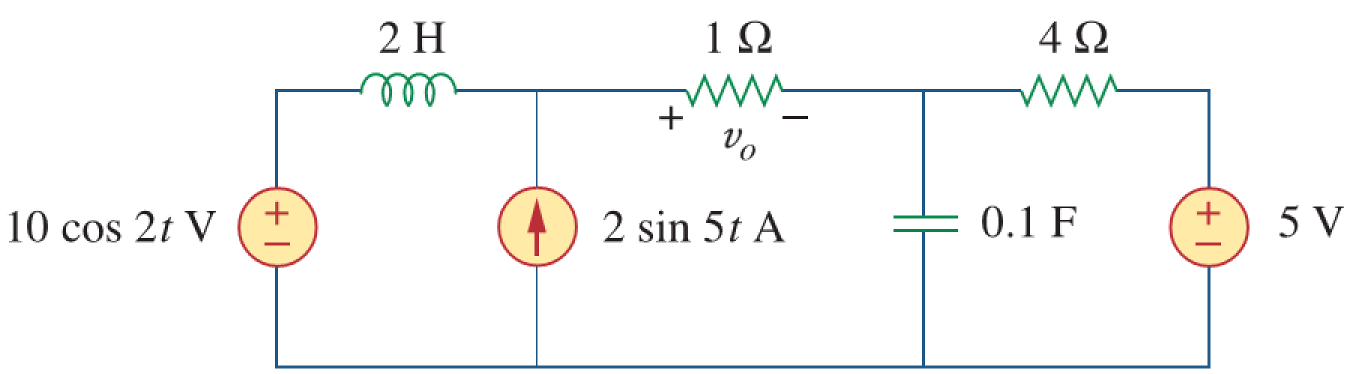
Find v_o of the circuit of Fig. 10.13 using the superposition theorem.

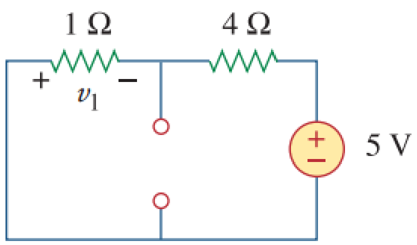


- What happens if the sources are at different frequencies?

Superposition II

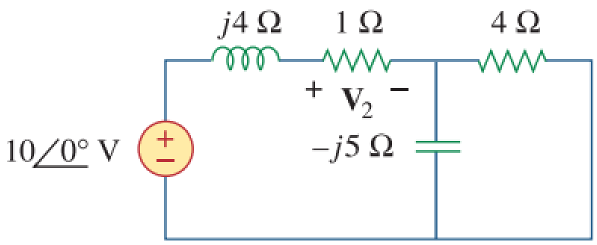
- Particularly important when the circuit has sources operating at different frequencies.
  - The complication is that each source must have its own frequency domain equivalent circuit, because each element has a different impedance at different frequencies.
  - Also, the resulting voltages and current must be converted back to time domain before being added. This is because there is an exponential factor $e^{j\omega t}$ implicit in sinusoidal analysis.





(a)

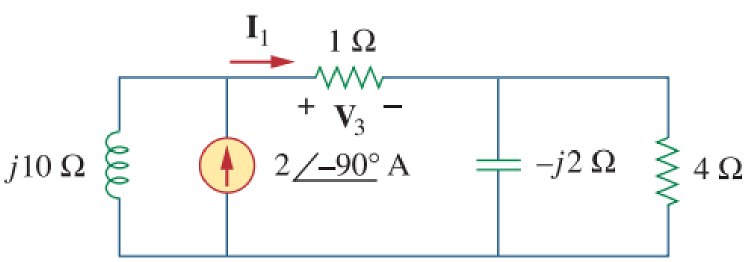
$$-v_1 = \frac{1}{1+4}(5) = 1\ \text{V}$$



(b)

$$\mathbf{V}_2 = \frac{1}{1 + j4 + \mathbf{Z}}(10\underline{/0^\circ}) = \frac{10}{3.439 + j2.049} = 2.498\underline{/-30.79^\circ}$$

$$v_2 = 2.498\cos(2t - 30.79^\circ)$$



(c)

$$\mathbf{I}_1 = \frac{j10}{j10 + 1 + \mathbf{Z}_1}(2\underline{/-90^\circ})\ \text{A}$$

$$\mathbf{V}_3 = \mathbf{I}_1 \times 1 = \frac{j10}{1.8 + j8.4}(-j2) = 2.328\underline{/-80^\circ}\ \text{V}$$

$$v_3 = 2.33\cos(5t - 80^\circ) = 2.33\sin(5t + 10^\circ)\ \text{V}$$

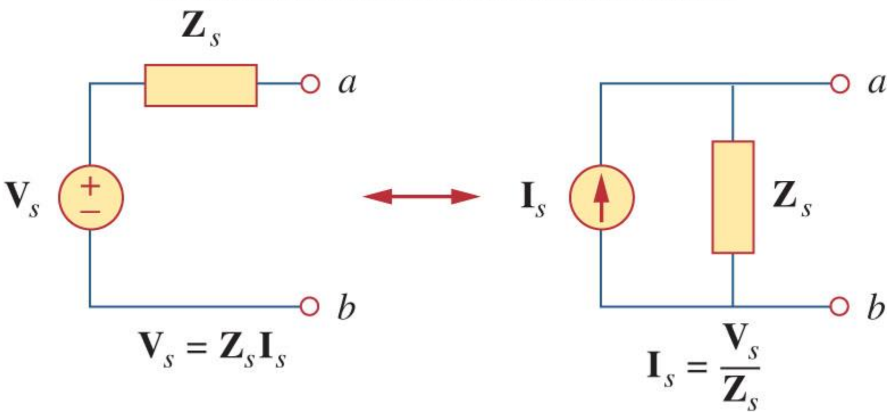
Outline

- ~~Nodal/Mesh analysis~~
- ~~Superposition~~
- Source transformation/Thevenin/Norton
- Applications

Source Transformation

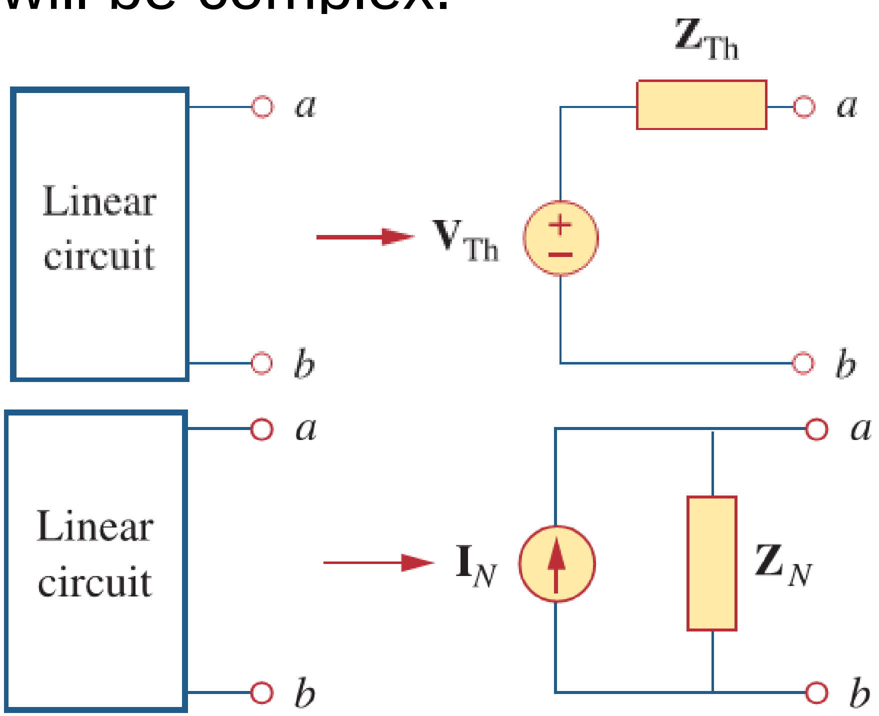
- Source transformation in frequency domain involves transforming a voltage source in series with an impedance to a current source in parallel with an impedance.
- Or vice versa:

$$V_s = Z_s I_s \quad \Leftrightarrow \quad I_s = \frac{V_s}{Z_s}$$



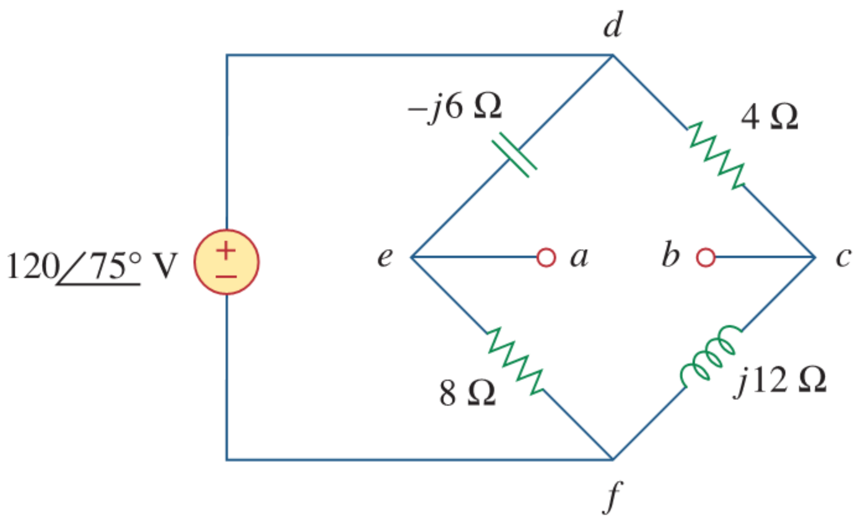
Thevenin and Norton Equivalency

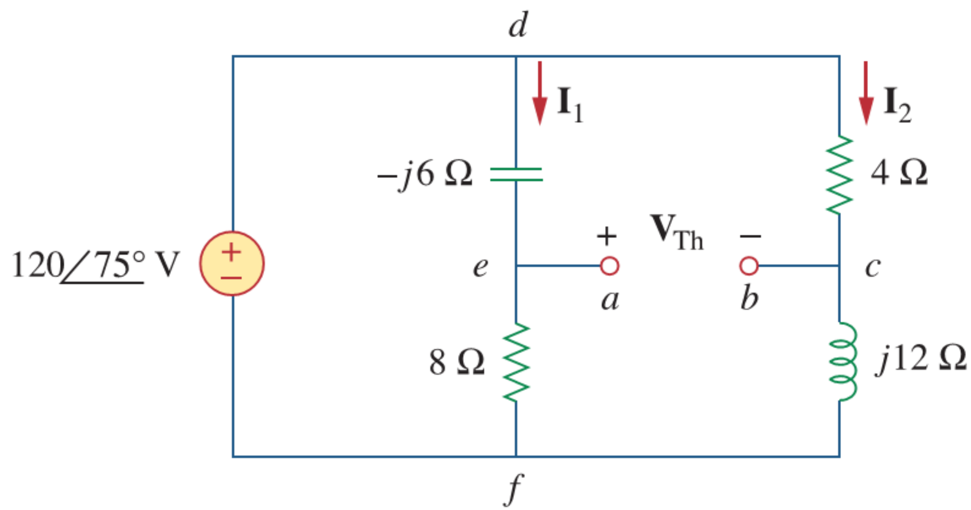
- Both Thevenin and Norton's theorems are applied to AC circuits the same way as DC.
- The only difference is the fact that the calculated values will be complex.

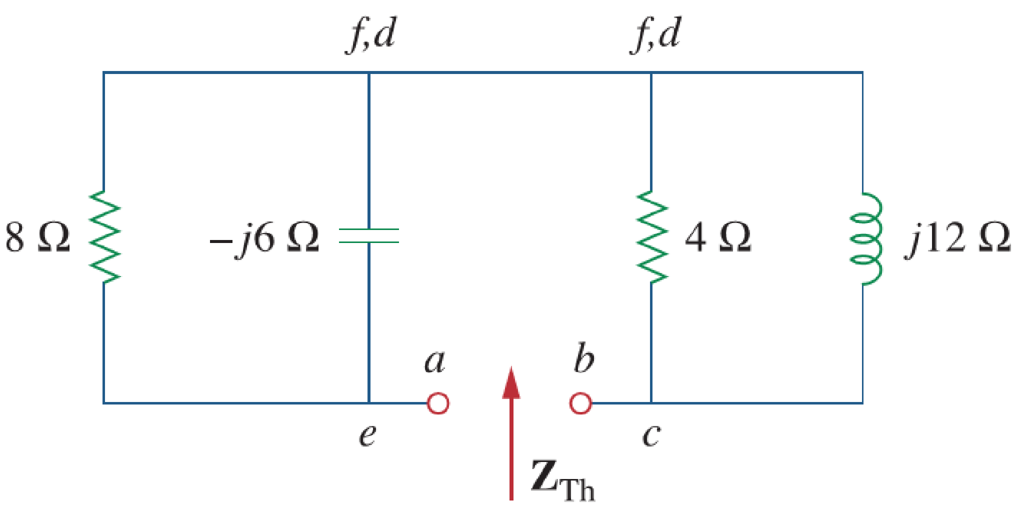


$$\mathbf{V}_{\mathrm{Th}} = \mathbf{Z}_N \mathbf{I}_N, \qquad \mathbf{Z}_{\mathrm{Th}} = \mathbf{Z}_N$$

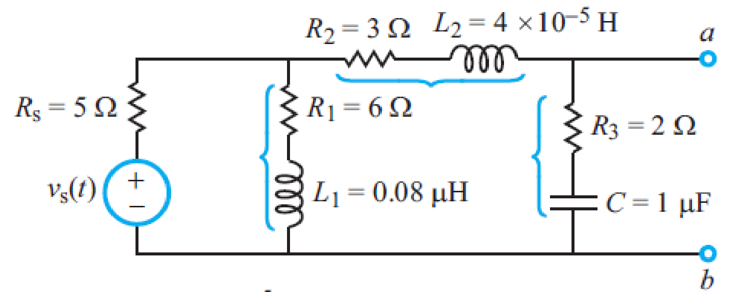
Example


d
−j6 Ω
4 Ω
120⁄75° V
e
a
b
c
8 Ω
j12 Ω
f

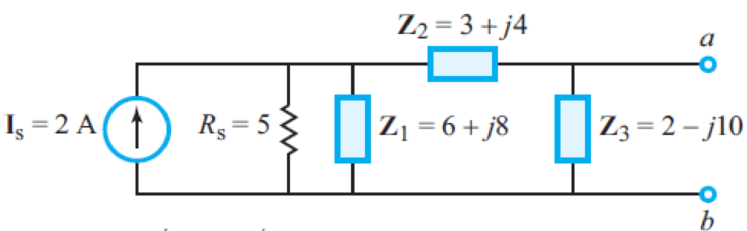

d
I1
I2
−j6 Ω
4 Ω
+
VTh
−
120⁄75° V
e
a
b
c
8 Ω
j12 Ω
f


f,d
f,d
8 Ω
−j6 Ω
4 Ω
j12 Ω
a
b
e
c
ZTh

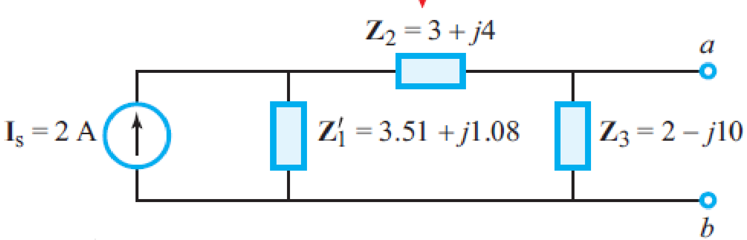
Example 7-9: Thévenin Circuit



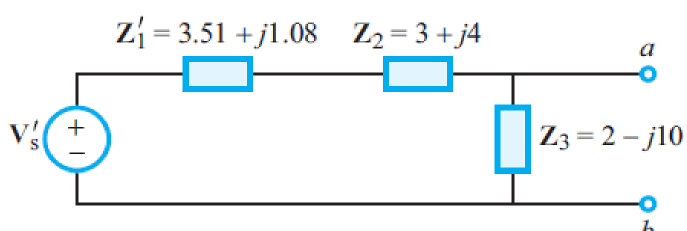
(a) $v_s(t) = 10 \cos 10^5 t$ (V)



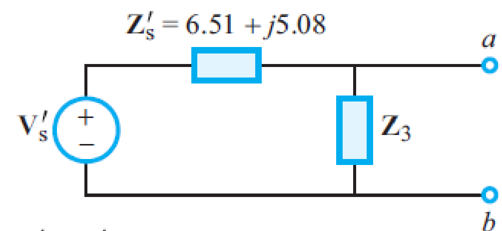
(b) $\mathbf{I}_s = \mathbf{V}_s / R_s = 10/5 = 2$ A



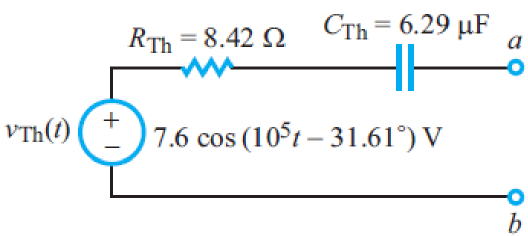
(c) $\mathbf{Z}_1' = R_s \parallel \mathbf{Z}_1$



(d) $\mathbf{V}_s' = \mathbf{I}_s \mathbf{Z}_s' = (7.02 + j2.16)$ V



(e) $\mathbf{Z}_s' = \mathbf{Z}_1' + \mathbf{Z}_2$



(f) Thévenin equivalent

$$\mathbf{Z}_{Th} = \mathbf{Z}_s' \parallel \mathbf{Z}_3$$

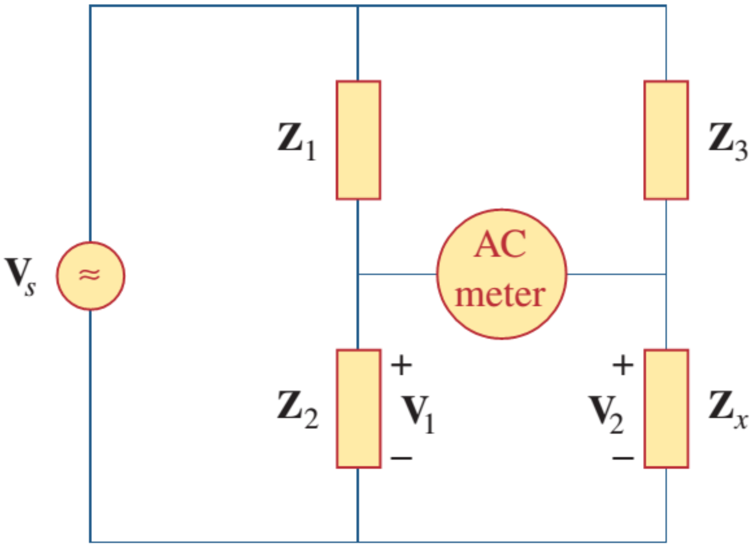
$$= \frac{(6.51 + j5.08)(2 - j10)}{(6.51 + j5.08) + (2 - j10)} = (8.42 - j1.59)\ \Omega$$

$$\boxed{R_{Th} = 8.42\ \Omega, \qquad C_{Th} = \frac{1}{1.59\omega} = 6.29\ \mu\text{F}}$$

Outline

- ~~Nodal/Mesh analysis~~
- ~~Superposition~~
- ~~Source transformation/Thevenin/Norton~~
- Applications

Application: AC Bridges



$$\mathbf{V}_1 = \frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}\mathbf{V}_s = \mathbf{V}_2 = \frac{\mathbf{Z}_x}{\mathbf{Z}_3 + \mathbf{Z}_x}\mathbf{V}_s$$

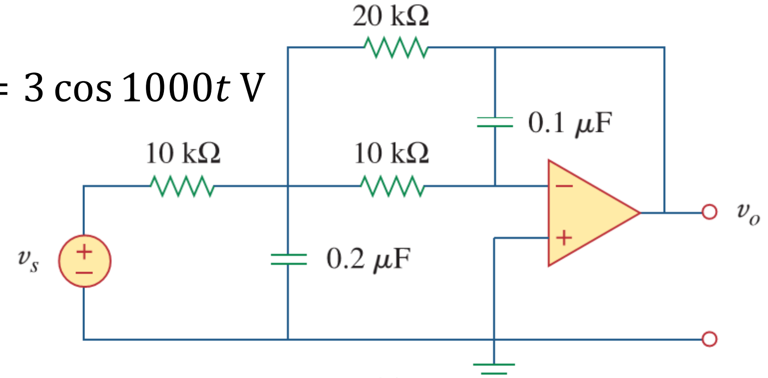
$$\boxed{\mathbf{Z}_x = \frac{\mathbf{Z}_3}{\mathbf{Z}_1}\mathbf{Z}_2}$$

AC Op Amp Circuits

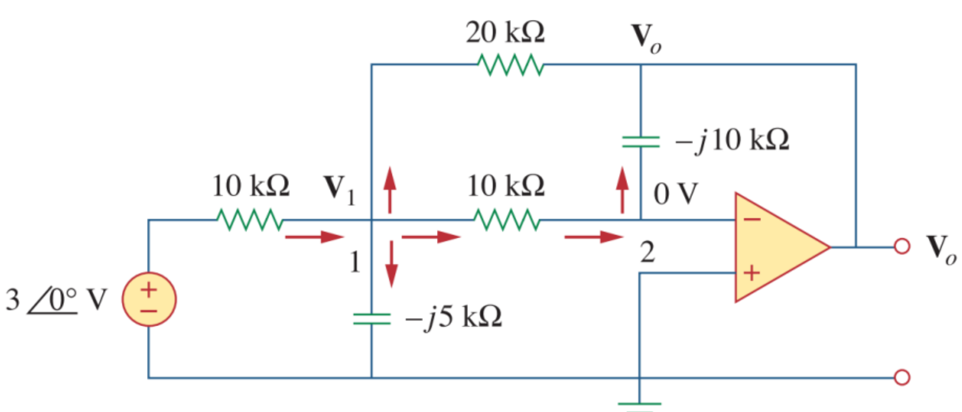
Question 1: Are op amps used in ac circuits?

Answer 1: Yes.

$$v_s = 3\cos 1000t \text{ V}$$



(a)



(b)

Solution:

We first transform the circuit to the frequency domain, as shown in Fig. 10.31(b), where $\mathbf{V}_s = 3\underline{/0^\circ}$, $\omega = 1000$ rad/s. Applying KCL at node 1, we obtain

$$\frac{3\underline{/0^\circ} - \mathbf{V}_1}{10} = \frac{\mathbf{V}_1}{-j5} + \frac{\mathbf{V}_1 - 0}{10} + \frac{\mathbf{V}_1 - \mathbf{V}_o}{20}$$

or

$$6 = (5 + j4)\mathbf{V}_1 - \mathbf{V}_o \tag{10.11.1}$$

At node 2, KCL gives

$$\frac{\mathbf{V}_1 - 0}{10} = \frac{0 - \mathbf{V}_o}{-j10}$$

which leads to

$$\mathbf{V}_1 = -j\mathbf{V}_o \tag{10.11.2}$$

Substituting Eq. (10.11.2) into Eq. (10.11.1) yields

$$6 = -j(5 + j4)\mathbf{V}_o - \mathbf{V}_o = (3 - j5)\mathbf{V}_o$$

$$\mathbf{V}_o = \frac{6}{3 - j5} = 1.029\underline{/59.04^\circ}$$

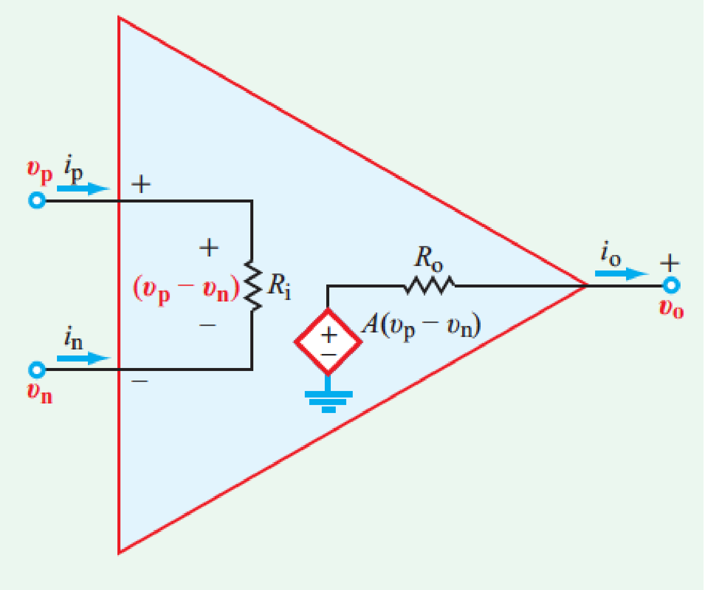
Hence,

$$v_o(t) = 1.029\cos(1000t + 59.04^\circ)\ \text{V}$$

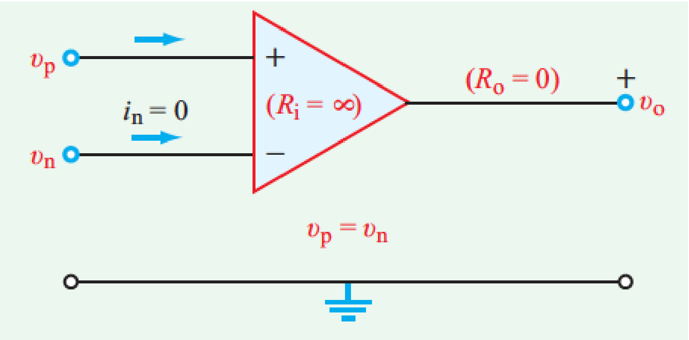
Question 2: Is the ideal op-amp model applicable to ac circuits?

Answer 2: The ideal op-amp model is based on the assumption that the open-loop gain A is very large ($> 10^4$), which is true at dc and low frequencies, but not necessarily so at high frequencies. The range of frequencies over which A is large depends on the specific op-amp design. As we shall see later on

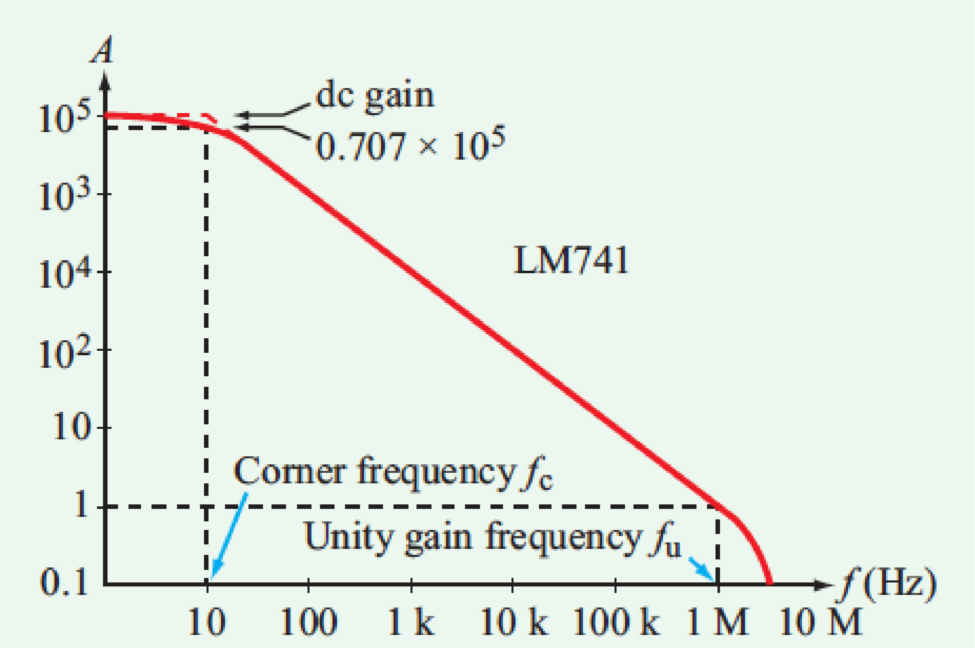
AC Op-Amp



(a) Op-amp equivalent circuit



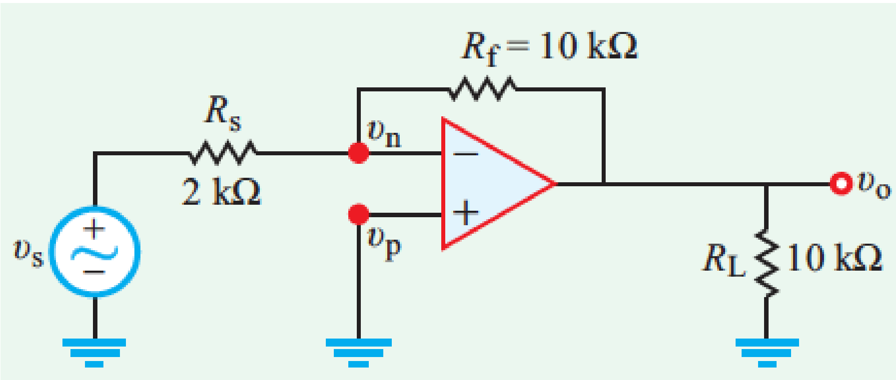
(b) Ideal op-amp model



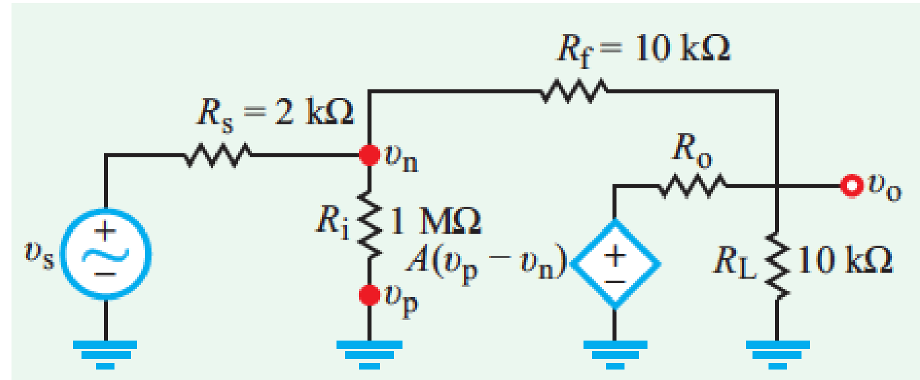
Open-loop gain A versus frequency for the LM741 op amp.

[See more at http://www.matni.com/Arabic/Elec-Info/LM741%20DETAILS/741.html](http://www.matni.com/Arabic/Elec-Info/LM741%20DETAILS/741.html)

Example



(a) Inverting amplifier circuit



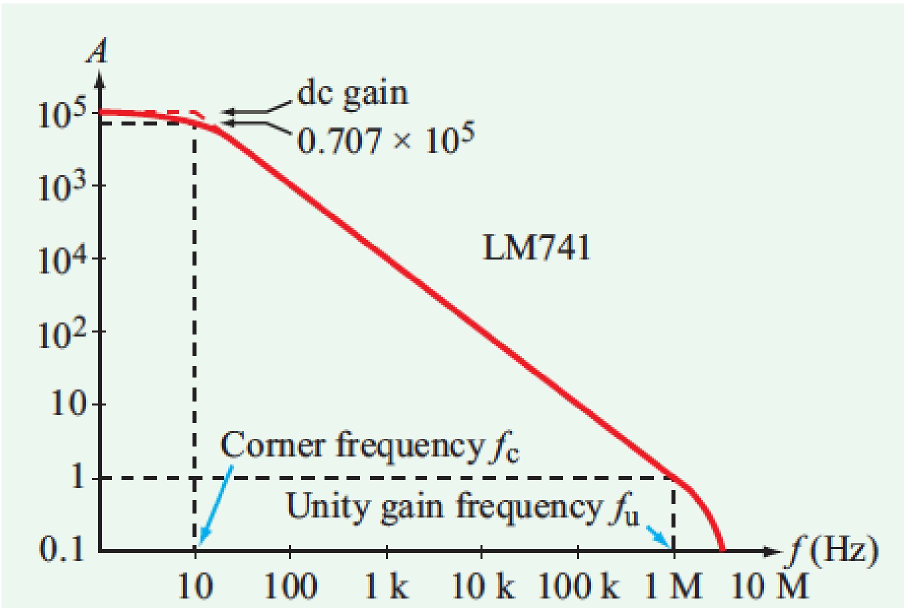
(b) Equivalent circuit model

$$\frac{v_n - v_s}{R_s} + \frac{v_n}{R_i} + \frac{v_n - v_o}{R_f} = 0,$$

$$\frac{v_o - v_n}{R_f} + \frac{v_o - A(v_p - v_n)}{R_o} + \frac{v_o}{R_L} = 0.$$

$$G = \frac{v_o}{v_s} = \frac{R_f}{R_s}$$

$$\left[\frac{R_s R_i (R_o - A R_f)}{(R_L R_o + R_f R_L + R_f R_o)(R_i R_f + R_s R_f + R_s R_i) - R_s R_i (R_o - A R_f)}\right]$$



Open-loop gain A versus frequency for the LM741 op amp.

f (Hz)	A	G	Error
0 (dc)	10^5	-4.997	0.06%
100	10^4	-4.970	0.6%
1 k	10^3	-4.714	5.7%
10 k	10^2	-3.111	37.8%
100 k	10	-0.707	85.9%
1 M	1	-0.081	98.4%

The error is defined as $G_{\text{ideal}} = -5$

$$\%\ \text{error} = \left(\frac{G_{\text{ideal}} - G}{G_{\text{ideal}}} \right) \times 100.$$

Audio: dc to 1 kHz: Minor distortion

Video: Up to 1 MHz: Serious distortion

Conclusion: The LM741 model is not suitable for video signals; it is necessary to use an Op-amp model with a higher corner frequency.