Name: _____ **Student ID:**_____

Signals and Systems Homework 3

Due Time: 21:59 March 30, 2018
Submitted in-class on Thu (Thu),
or to the box in front of SIST 1C 403E (the instructors office).

The process of solving a problem is a must. You can't score by giving only the result.

1. (15') Consider a continuous-time ideal lowpass filter S whose frequency response is

$$H(j\omega) = \begin{cases} 1, & |\omega| \leq 100 \\ 0, & |\omega| > 100 \end{cases}$$

When the input to this filter is a signal x(t) with fundamental period $T = \pi/6$ and Fourier series coefficients a_k, it is found that

$$x(t) \xrightarrow{S} y(t) = x(t)$$

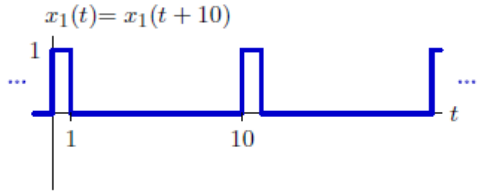
For what values of k is it guaranteed that $a_k = 0$?

2. (20') Consider a causal continuous-time LTI system whose frequency response is $H(j\omega) = \frac{1}{j\omega+4}$. Find the Fourier series representation of the output $y(t)$ for each of the following inputs:

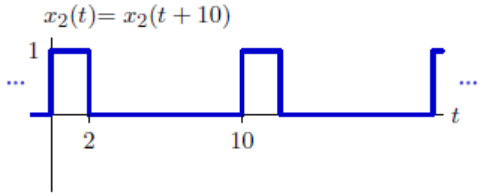
   (a) $x(t) = \cos 2\pi t$

   (b) $x(t) = \sin 4\pi t + \cos(6\pi t + \pi/4)$

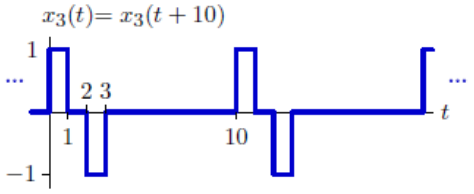
3. (a) (6') Determine the Fourier series coefficients a_k for $x_1(t)$ shown below.



(b) (6') Determine the Fourier series coefficients b_k for $x_2(t)$ shown below.



(c) (6') Determine the Fourier series coefficients c_k for $x_3(t)$ shown below.



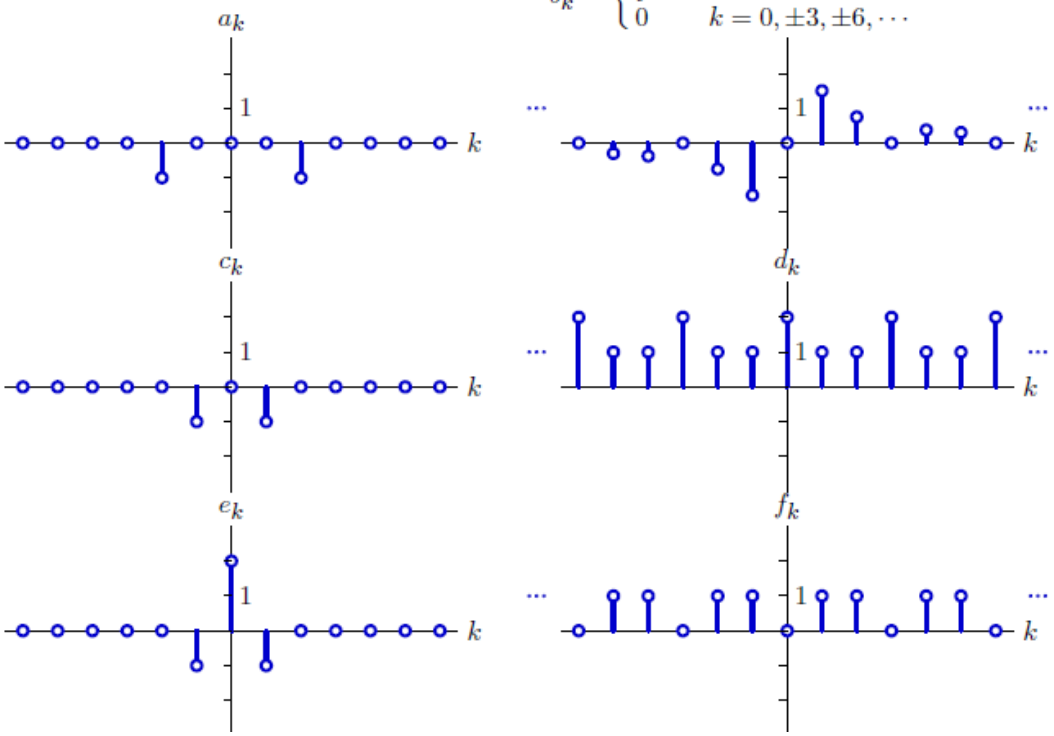
4. (15') Determine the CT signals with the following Fourier series coefficients. Assume that the signals are periodic in $T = 4$. Give an expression that is valid for $0 \leq t < 4$ (other values can be found by periodic extension).

$$a_k = \begin{cases} jk & |k| < 3 \\ 0 & \text{otherwise} \end{cases}$$

5. (20') Matching problem. You must explain why.

Consider the following Fourier series coefficients.

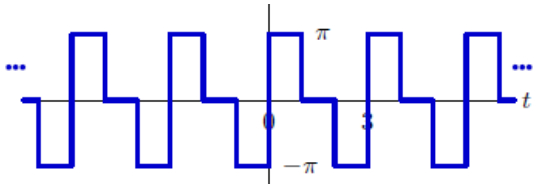
$$b_k = \begin{cases} \frac{3}{j2k} & k = \pm 1, \pm 2, \pm 4, \pm 5, \pm 7, \ldots \\ 0 & k = 0, \pm 3, \pm 6, \cdots \end{cases}$$



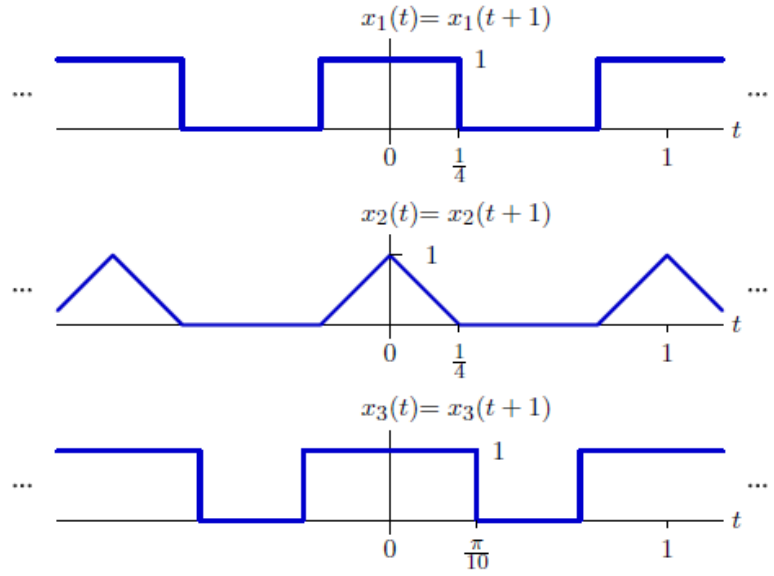
(a) Which coefficients (if any) corresponds to the following periodic signals?

$$x_1(t) = 2 - 2\cos\left(\frac{2\pi}{3}t\right)$$

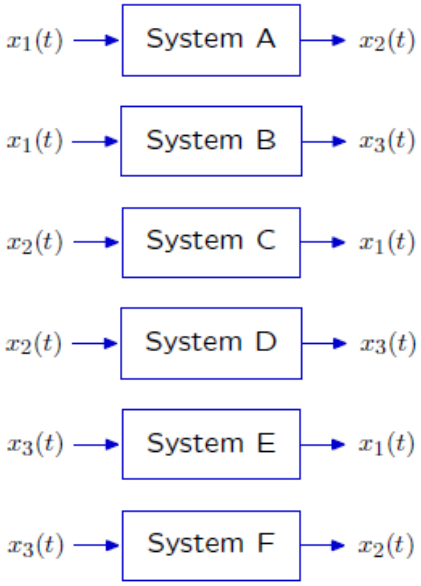
(b) Which (if any) set corresponds to the following periodic signal with period $T = 3$?



6. (12') Input/Output pairs The following signals are periodic with period $T = 1$.



Determine if the following systems could or could not be LTI.



Give a list of the systems that could *NOT* be LTI and *EXPLAIN WHY*, if your list is empty, write 'None'.

Hint: We can use the 'filter' idea as follows. First calculate the Fourier series coefficients. Then ask if each Fourier series coefficient in the output is a scaled version of the corresponding coefficient in the input.

$$x_2(t) \leftrightarrow b_k = \frac{4\sin^2(\pi k/4)}{\pi^2 k^2} = \begin{cases} 1/4, & k = 0 \\ \frac{2}{\pi^2 k^2}, & |k| = 1, 3, 5, 7, 9, 11, 13... \\ \frac{4}{\pi^2 k^2}, & |k| = 2, 6, 10, 14, ... \\ 0, & |k| = 4, 8, 12, 16 \end{cases}$$