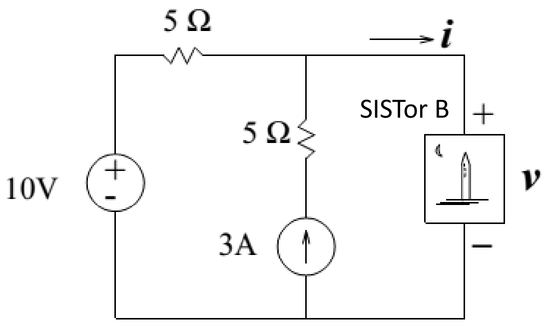
Problem 1 (8 points)

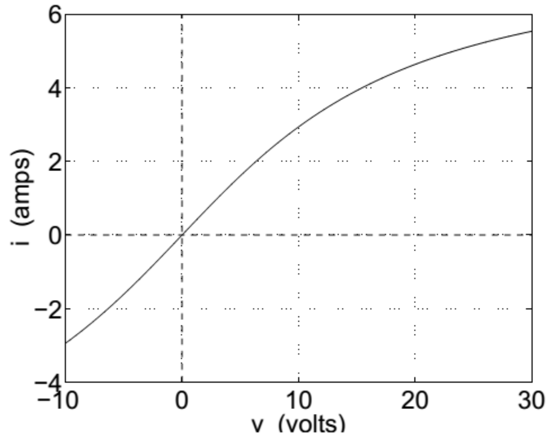
You must show your detailed work to get full credit.

The resistive network shown in Fig.1(a) is connected to an element SISTor B. The SISTor is a nonlinear device with *i* - *v* characteristic shown in Fig.1(b). Determine

1) the current i drawn by the SISTor and
2) the voltage v across the SISTor.



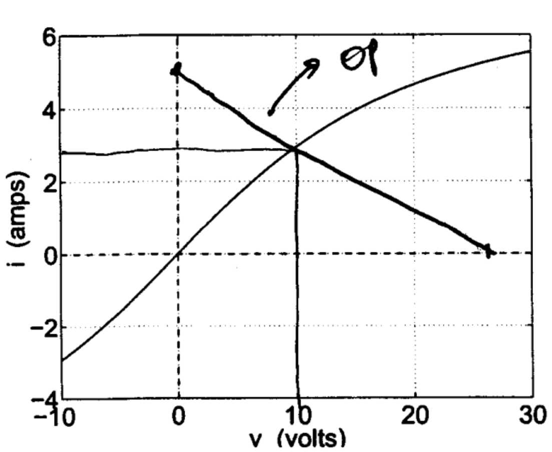
(a)

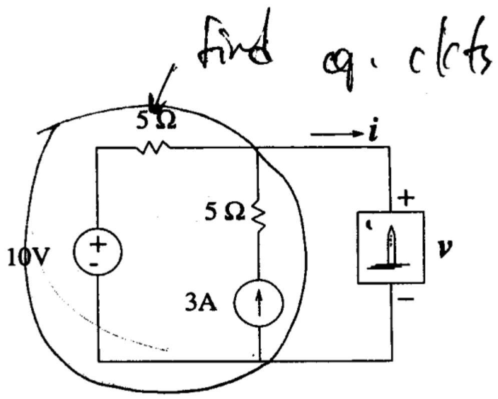


(b) i - v curve for SISTor B.

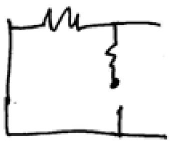
Fig. 1 for Problem 1.

Your answer:

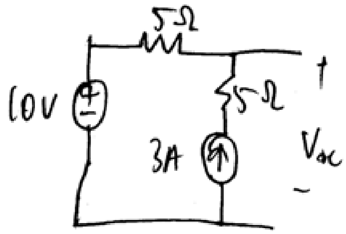




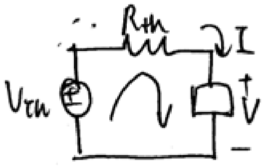
find $R_{th} \Rightarrow$ short all indep sources



$R_{eq} = R_{th} = 5\,\Omega$



$3A = \frac{V_{oc}-10}{5} \Rightarrow V_{oc} = 25V$



$-V_{th} + IR_{th} + V = 0$

$V_{th} = IR_{th} + V$

① $I = 0 \Rightarrow V = V_{th} = 25V$

② $V = 0 \Rightarrow I = 5A$

$i =$ 3A	$v =$ 10V

Problem 2 (8 points) — Diodes

You must show your detailed work to get full credit.

In the circuit shown in Fig.2, assume all the diodes are ideal with threshold voltage equals to 700mV. Find V_a and then explain why.

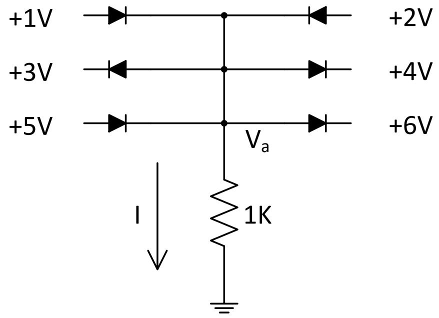


Fig. 2 for Problem 2.

Your answer:

if D_6 is on, $V_a = 6.7V$ .so D_6 is off.

if D_1 is on, $V_a = 0.7$, ~~so~~ then D_5 is on,

the $V_a = 4.3V$, so ~~D_5~~ D_1 will be off, so D_1 is off

D_3 is the same as D_1 , so D_3 is off

if D_4 is on, $V_a = 4.7V$, but V_T is $0.7V$,

so $V_{a\,max}$ is $4.3V < 4.7V$, so D_4 is off

if D_2 is on. $V_a = 2.7V$, $2.7V < 4.3V$, so D_2 is on.

in a short, $\boxed{D_2 . D_5 \text{ is on, others are off}}$.

so $V_a = 4.3\ V$ ($2.7V \sim 4.3V$ are all right)

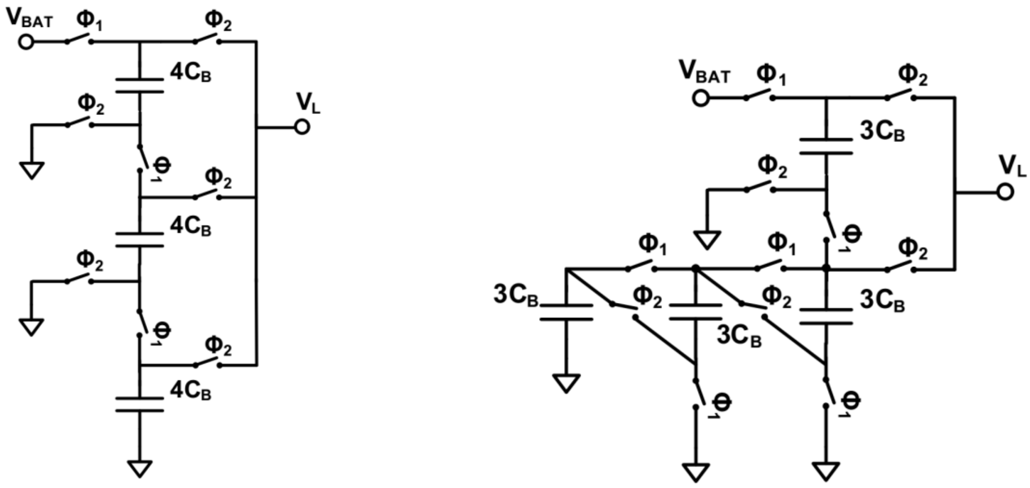
Problem 3 (9 points)

You must show your detailed work to get full credit.

Fig. 3 shows two Switched Capacitor (SC) Converter circuits which convert input DC voltage V_{BAT} to output voltage V_L

a) The conversion ratio is defined as n = $\frac{V_{BAT}}{V_L}$.
b) All the switches in these circuits are controlled by a periodic square wave with 50% duty cycle: During high voltage phase, the ϕ_1 switches are turned on, meanwhile the ϕ_2 switches are turned off; during low voltage phase, the ϕ_2 switches are turned on, but the ϕ_1 switches are turned off.
c) Assuming that the capacitors can be fully charged in a half cycle.

Find the conversion ratios n_1 and n_2 for the two SC converters shown in Fig. 3.



(a) $\boldsymbol{n_1} = \frac{V_{BAT}}{V_L} = \mathbf{3/1}$ (b) $\boldsymbol{n_2} = \frac{V_{BAT}}{V_L} = 4/3$

Fig. 3 for Problem 3.

Your answer:

$$\boldsymbol{n_1} = \frac{V_{BAT}}{V_L} = \mathbf{3/1}$$

$$\boldsymbol{n_2} = \frac{V_{BAT}}{V_L} = 4/3$$

Problem 4 (15 points) — First-Order Circuit Analysis

You must show your detailed work to get full credit.

The circuit shown in Fig. 4 contains two switches, both of which had been open for a long time before $t = 0$. Switch 1 closes at $t = 0$, and Switch 2 closes at $t = 5\text{s}$.

Determine $v_C(t)$ for $t \geq 0$, given that $V_0 = 24$ V, $R_1 = R_2 = 16\text{k}\Omega$, and $C = 250\mu\text{F}$. Assume that $v_C(0) = 0$.

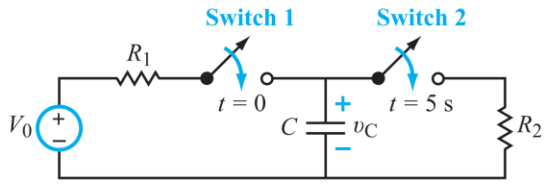
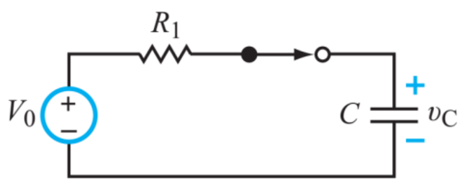


Fig. 4 for Problem 4.

Time Segment 1: $0 \leq t \leq 5$ s

$$\tau_1 = R_1 C = 16 \times 10^3 \times 250 \times 10^{-6} = 4 \text{ s}.$$

$$\begin{aligned} v_{C_1}(t) &= v_{C_1}(\infty) + (v_{C_1}(t) - v_{C_1}(\infty))e^{-t/\tau_1} \\ &= V_0 + (0 - V_0)e^{-0.25t} \\ &= 24(1 - e^{-0.25t}), \qquad \text{for } 0 \leq t \leq 5 \text{ s}. \end{aligned}$$



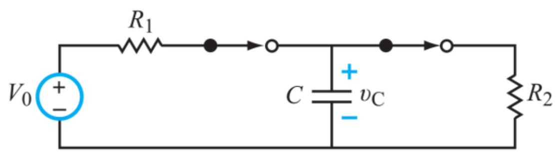
Time Segment 2: $t \geq 5$ s

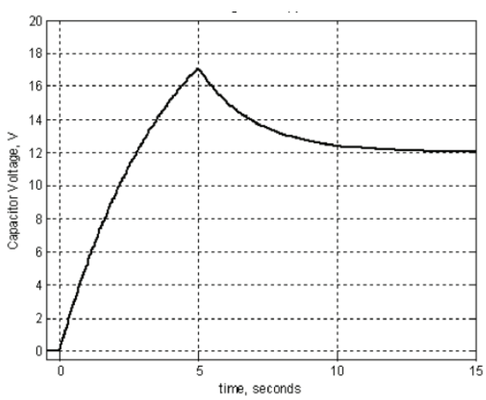
Through source transformation, it is easy to see that R_1 and R_2 should be combined in parallel. Hence:

$$\tau_2 = \left(\frac{R_1 R_2}{R_1 + R_2}\right) C = 8 \times 10^3 \times 250 \times 10^{-6} = 2 \text{ s}.$$

$$v_{C_2}(t) = v_{C_2}(\infty) + [v_{C_2}(5 \text{ s}) - v_{C_2}(\infty)]e^{-(t-5)/\tau_2}$$

$$\begin{aligned} v_{C_2}(\infty) &= \frac{V_0 R_2}{R_1 + R_2} = \frac{24 \times 16}{16 + 16} = 12 \text{ V}. \\ v_{C_2}(5 \text{ s}) &= v_{C_1}(5 \text{ s}) = 24(1 - e^{-0.25 \times 5}) = 17.12 \text{ V} \\ v_{C_2}(t) &= 12 + [17.12 - 12]e^{-0.5(t-5)} \\ &= 12 + 5.12e^{-0.5(t-5)}, \qquad \text{for } t \geq 5 \text{ s}. \end{aligned}$$





Problem 5 (18 points) – General Second-Order Circuit Analysis

You must show your detailed work to get full credit.

Determine $i_L(t)$ in the op-amp circuit of Fig. 4 for $t \geq 0$, where $V_s = 1\text{mV}$, $R_1 = 10\text{k}\Omega$, $R_2 = 1\text{M}\Omega$, $R_3 = 100\Omega$, $\text{L} = 5\text{H}$ and $C = 1\mu\text{F}$. Assume the op-amp is ideal.

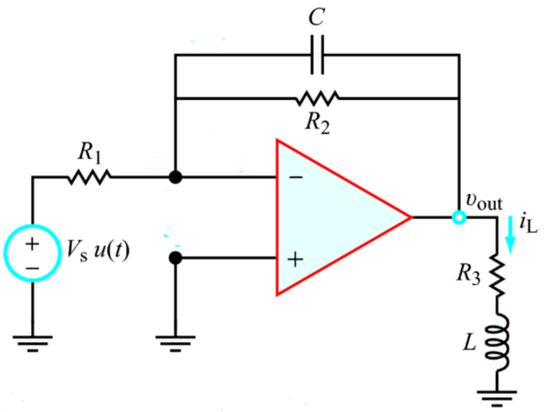
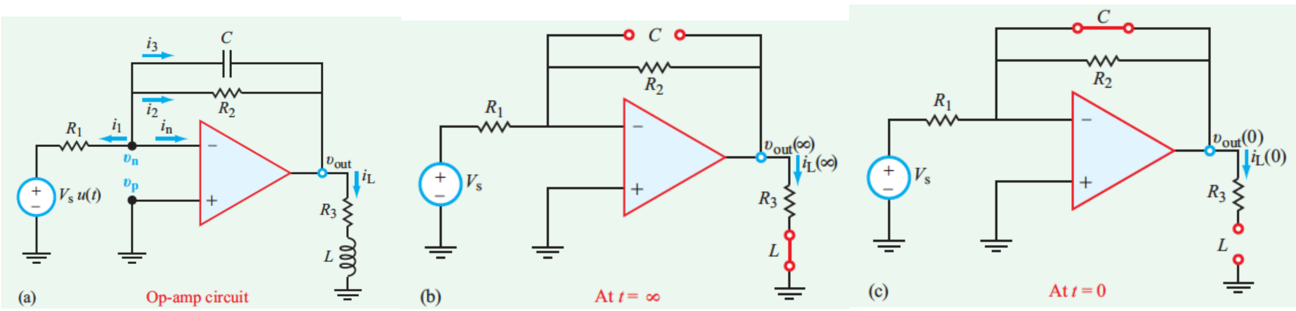


Fig. 5 for Problem 5.

Solution:



Solution: KCL at node v_n gives

$$i_1 + i_n + i_2 + i_3 = 0,$$

or equivalently,

$$\frac{v_n - V_s}{R_1} + i_n + \frac{v_n - v_{out}}{R_2} + C\frac{d}{dt}(v_n - v_{out}) = 0.$$

Since $v_n = v_p = 0$, $i_n = 0$, and

$$v_{out} = R_3 i_L + L\frac{di_L}{dt},$$

$$\frac{R_3}{R_2} i_L + \left(\frac{L}{R_2} + R_3 C\right)\frac{di_L}{dt} + LC\frac{d^2 i_L}{dt^2} = -\frac{V_s}{R_1}.$$

Rearranging, we have

$$i_L'' + a i_L' + b i_L = c,$$

where

$$a = \frac{L + R_2 R_3 C}{R_2 LC} = 21,$$

$$b = \frac{R_3}{R_2 LC} = 20,$$

and

$$c = \frac{-V_s}{R_1 LC} = -0.02.$$

The damping behavior of i_L is determined by how the magnitude of α compares with that of ω_0:

$$\alpha = \frac{a}{2} = 10.5 \text{ Np/s},$$
$$\omega_0 = \sqrt{b} = \sqrt{20} = 4.47 \text{ rad/s}.$$

Since $\alpha > \omega_0$, i_L will exhibit an overdamped response given by

$$i_L(t) = [A_1 e^{s_1 t} + A_2 e^{s_2 t} + i_L(\infty)]\, u(t),$$

with

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -1.0,$$
$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -20.$$

$$i_L(0) = i_L(0^-) = 0,$$

which means that the inductor behaves like an open circuit at $t = 0$, as depicted in Fig. 6-18(c). Also, since the voltage υ_C across the capacitor was zero before $t = 0$, it has to remain at zero at $t = 0$, which is why it has been replaced with a short circuit in Fig. 6-18(c). Consequently, $\upsilon_{\text{out}}(0) = 0$, $\upsilon_L(0) = 0$, and

$$i'_L(0) = \frac{1}{L}\,\upsilon_L(0) = 0.$$

From Table 6-2, with $x = i_L$,

$$A_1 = \frac{i'_L(0) - s_2[i_L(0) - i_L(\infty)]}{s_1 - s_2}$$
$$= \frac{0 + 20(0+1)}{-1+20} \times 10^{-3} = 1.05 \text{ mA} \qquad (6.106)$$

and

$$A_2 = -\left[\frac{i'_L(0) - s_1[i_L(0) - i_L(\infty)]}{s_1 - s_2}\right]$$
$$= -\left[\frac{0 + 1(0+1)}{-1+20}\right] \times 10^{-3} = -0.053 \text{ mA}. \qquad (6.107)$$

The final expression for $i_L(t)$ is then given by

$$i_L(t) = [1.05e^{-t} - 0.053e^{-20t} - 1] \text{ mA}, \qquad \text{for } t \geq 0.$$

At $t = \infty$, the circuit assumes the equivalent configuration shown in Fig. 6-18(b), which is an inverting amplifier with an output voltage

$$\upsilon_{\text{out}}(\infty) = -\frac{R_2}{R_1}\, V_s.$$

Hence,

$$i_L(\infty) = \frac{\upsilon_{\text{out}}(\infty)}{R_3} = -\frac{R_2 V_s}{R_1 R_3} = -1 \text{ mA}.$$

The expression for $i_L(t)$ becomes

$$i_L(t) = [A_1 e^{-t} + A_2 e^{-20t} - 10^{-3}]. \qquad (6.105)$$

Problem 6 (12 points) – AC Circuit Analysis

You must show your detailed work to get full credit.

The impedance Z_L in the circuit shown in Fig. 7 is adjusted for maximum average power transfer to Z_L. The internal impedance of the sinusoidal voltage source is $4 + j7\ \Omega$.

a) Determine Z_L.

b) What is the maximum average power delivered to Z_L?

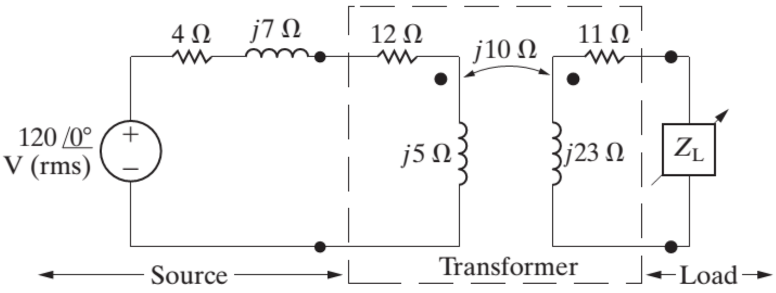
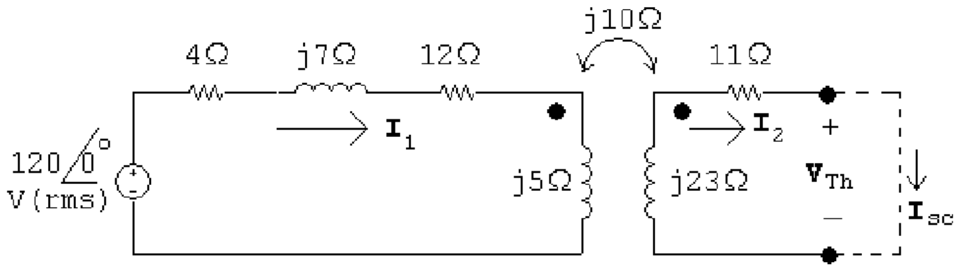


Fig. 6 for Problem 6.

Solution:

First, find the Thevenin Equivalent circuit.



Open circuit:

$$\mathbf{V}_{\mathrm{Th}} = \frac{120}{16 + j12}(j10) = 36 + j48\,\mathrm{V}$$

Short circuit:

$$(16 + j12)\mathbf{I}_1 - j10\mathbf{I}_{\mathrm{sc}} = 120$$

$$-j10\mathbf{I}_1 + (11 + j23)\mathbf{I}_{\mathrm{sc}} = 0$$

Solving,

$$\mathbf{I}_{\mathrm{sc}} = 2.4\,\mathrm{A}$$

$$Z_{\mathrm{Th}} = \frac{36 + j48}{2.4} = 15 + j20\,\Omega$$

$$\therefore \quad Z_{\mathrm{L}} = Z_{\mathrm{Th}}^{*} = 15 - j20\,\Omega$$

$$\mathbf{I}_{\mathrm{L}} = \frac{\mathbf{V}_{\mathrm{Th}}}{Z_{\mathrm{Th}} + Z_L} = \frac{36 + j48}{30} = 1.2 + j1.6\,\mathrm{A(rms)}$$

$$P_{\mathrm{L}} = |\mathbf{I}_{\mathrm{L}}|^2(15) = 60\,\mathrm{W}$$

Problem 7 (12 points) – Transfer Response

You must show your detailed work to get full credit.

For the op-amp circuit shown in Fig. 7,

(a) Obtain an expression for transfer response $\mathrm{H}(\omega) = V_o/V_s$ in standard form. Given that $R_1 = R_2 = 100\,\Omega, C_1 = 10\ \mu\text{F}$ and $C_2 = 0.4\ \mu\text{F}$.

(b) What type of filter is it? What is its maximum gain?

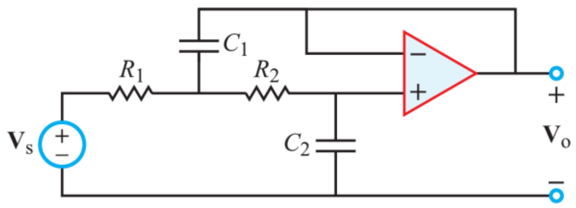
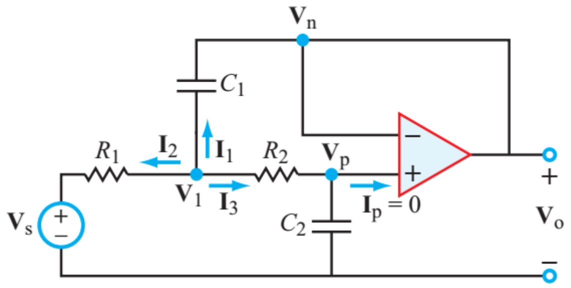


Fig. 7 for Problem 7.

Your answer:



(a) At node $\mathbf{V}_1$:

$$\mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3 = 0,$$

or equivalently

$$\frac{\mathbf{V}_1 - \mathbf{V}_o}{1/j\omega C_1} + \frac{\mathbf{V}_1 - \mathbf{V}_s}{R_1} + \frac{\mathbf{V}_1}{R_2 + 1/j\omega C_2} = 0.$$

Also,

$$\mathbf{V}_p = \mathbf{V}_n = \mathbf{V}_o,$$

and by voltage division

$$\mathbf{V}_p = \frac{\mathbf{V}_1/j\omega C_2}{R_2 + 1/j\omega C_2}.$$

Simultaneous solution leads to:

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{1}{1 + j\omega(R_1 + R_2)C_2 + (j\omega\sqrt{R_1R_2C_1C_2})^2}$$
$$= \frac{1}{1 + j2\xi\,\omega/\omega_c + (j\omega/\omega_c)^2},$$

with

$$\omega_c = \frac{1}{\sqrt{R_1R_2C_1C_2}} = \frac{1}{\sqrt{100 \times 100 \times 10^{-5} \times 0.4 \times 10^{-6}}} = 5000 \text{ rad/s},$$
$$\xi = \frac{(R_1 + R_2)C_2\omega_c}{2} = 100 \times 0.4 \times 10^{-6} \times 5000 = 0.2.$$

(b) Low pass filter, with max gain equals one.

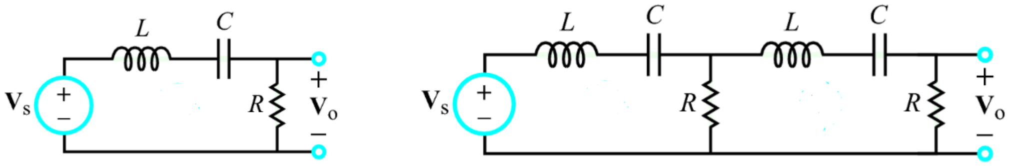
Problem 8 (18 points) – Filters

You must show your detailed work to get full credit.

Given that $R = 2\Omega$, $\mathrm{L} = 10\mathrm{mH}$, and $C = 1\ \mu\mathrm{F}$,

(a) Determine the center frequency ω_a, bandwidth B_a and quality factor Q_a of the single-stage series RLC filter shown in Fig. 8(a).

(b) Determine the center frequency ω_b, bandwidth B_b and quality factor Q_b of the two-stage series RLC filter shown in Fig. 8(b).



(a) (b)

Fig. 8 for Problem 8.

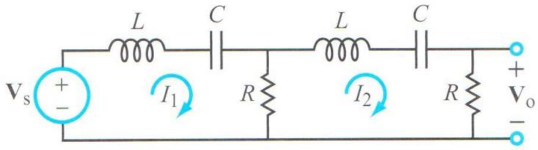
Your answer:

(a) $\boldsymbol{\omega_a = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-2}\times 10^{-6}}} = 10^4}$ **rad/s**

$$\boldsymbol{Q_a = \frac{\omega_a L}{R} = \frac{10^4\times 10^{-2}}{2} = 50}$$

$$\boldsymbol{B_a = \frac{\omega_a}{Q_a} = 200}$$

(b)



The loop equations for mesh currents $\mathbf{I}_1$ and $\mathbf{I}_2$ are

$$-\mathbf{V}_s + \mathbf{I}_1\left(j\omega L + \frac{1}{j\omega C} + R\right) - R\mathbf{I}_2 = 0$$

and

$$-R\mathbf{I}_1 + \mathbf{I}_2\left(2R + j\omega L + \frac{1}{j\omega C}\right) = 0.$$

Simultaneous solution of the two equations leads to

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_s}$$

$$= \frac{\omega^2 R^2 C^2}{\omega^2 R^2 C^2 - (1-\omega^2 LC)^2 - j3\omega RC(1-\omega^2 LC)}$$

$$= \frac{\omega^2 R^2 C^2[\omega^2 R^2 C^2 - (1-\omega^2 LC)^2 + j3\omega RC(1-\omega^2 LC)]}{[\omega^2 R^2 C^2 - (1-\omega^2 LC)^2]^2 + 9\omega^2 R^2 C^2 (1-\omega^2 LC)^2}.$$

Resonance occurs when the imaginary part of $\mathbf{H}(\omega)$ is zero, which is satisfied either when $\omega = 0$ (which is a trivial resonance) or when $\omega = 1/\sqrt{LC}$. Hence, the two-stage circuit has the same resonance frequency as a single-stage circuit.

Using the specified values of R, L, and C, we can calculate the magnitude $M(\omega) = |\mathbf{H}(\omega)|$ and plot it as a function of ω. The result is displayed in Fig. 9-18(b). From the spectral plot, we have

$$\omega_{c_1} = 9963 \text{ rad/s},$$

$$\omega_{c_2} = 10037 \text{ rad/s},$$

$$B_2 = \omega_{c_2} - \omega_{c_1} = 10037 - 9963 = 74 \text{ rad/s},$$

and

$$Q_2 = \frac{\omega_0}{B_2} = \frac{10^4}{74} = 135,$$

where B_2 is the bandwidth of the two-stage BP-filter response. The two-stage combination increases the quality factor from 50 to 135.