Name: _____ **Student ID:**_____

Signals and Systems Homework 2

Due Time: 23:59 March 23, 2018
Submitted in-class on Thu (Mar 22),
or to the box in front of SIST 1C 403E (the instructors office).

1. Let
$$x[n] = \delta[n] + 2\delta[n-1] - \delta[n-3] \quad \text{and} \quad h[n] = 2\delta[n+1] + 2\delta[n-1]$$
Compute and plot each of the following convolutions:
   (a) $y_1[n] = x[n] * h[n]$
   (b) $y_2[n] = x[n+2] * h[n]$
   (c) $y_3[n] = x[n] * h[n+2]$

2. For each of the following pairs of waveforms, use the convolution integral to find the response $y(t)$ of the LTI system with impulse response $h(t)$ to the input $x(t)$. Sketch your results.
   (a) $x(t)$ and $h(t)$ are as in Figure 1(a).
   (b) $x(t)$ and $h(t)$ are as in Figure 1(b).
   (c) $x(t)$ and $h(t)$ are as in Figure 1(c).

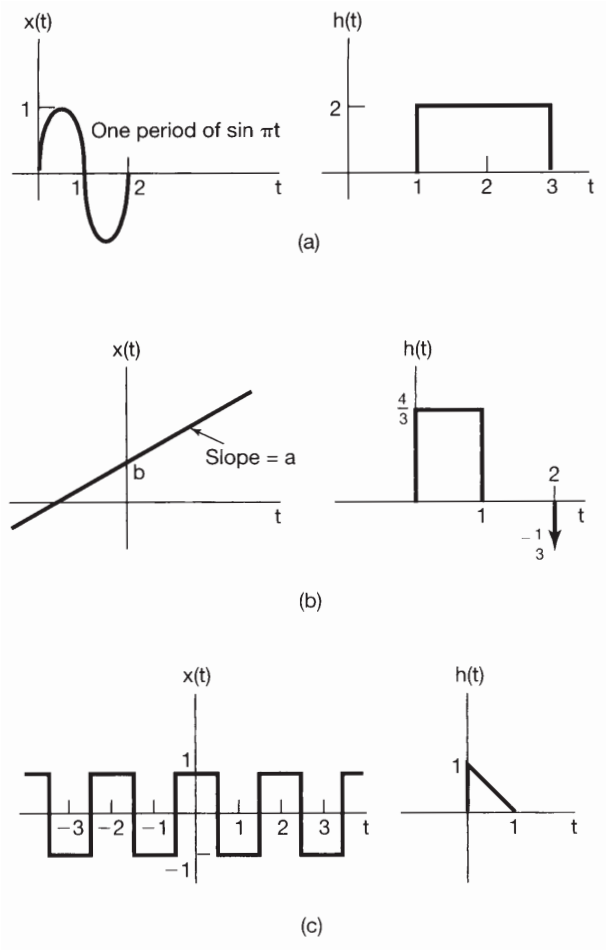

Figure 1:

3. Determine and sketch the convolution of the following two signals:
$$x(t) = \begin{cases} t+1, & 0 \le t \le 1 \\ 2-t, & 1 < t \le 2 \\ 0, & \text{elsewhere} \end{cases}$$
$$h(t) = \delta(t+2) + 2\delta(t+1)$$

4. Suppose that

$$x(t) = \begin{cases} 1, & 0 \le t \le 1 \\ 0, & \text{elsewhere} \end{cases}$$

and $h(t) = x(t/\alpha)$, where $0 < \alpha \le 1$.

(a) Determine and sketch $y(t) = x(t) * h(t)$.

(b) If $dy(t)/dt$ contains only three discontinuities, what is the value of α?

5. Let

$$x(t) = u(t-3) - u(t-5) \quad \text{and} \quad h(t) = e^{-3t}u(t)$$

(a) Compute $y(t) = x(t) * h(t)$.

(b) Compute $g(t) = (dx(t)/dt) * h(t)$.

(c) How is $g(t)$ related to $y(t)$?

6. Let $h(t)$ be the triangular pulse shown in Figure 2(a), and let $x(t)$ be the impulse train depicted in Figure 2(b). That is

$$x(t) = \sum_{k=-\infty}^{+\infty} \delta(t - kT).$$

Determine and sketch $y(t) = x(t) * h(t)$ for the following values of T:
(a). $T = 4$ (b). $T = 2$ (c). $T = 3/2$ (d). $T = 1$

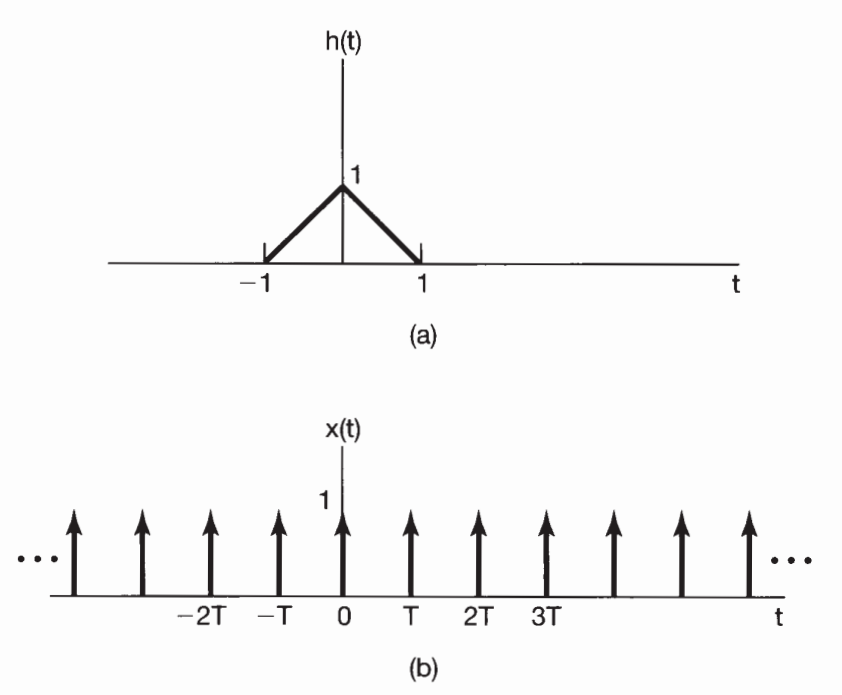

Figure 2:

7. Let the signal

$$y[n] = x[n] * h[n],$$

where

$$x[n] = 3^n u[-n-1] + \left(\frac{1}{3}\right)^n u[n]$$

and

$$h[n] = \left(\frac{1}{4}\right)^n u[n+3]$$

(a) Determine $y[n]$ without utilizing the distributive property of convolution.

(b) Determine $y[n]$ utilizing the distributive property of convolution.

8. An analog system has the input-output relation

$$y(t) = \int_0^t e^{-(t-\tau)} x(\tau) d\tau \quad t \geq 0$$

and zero otherwise. The input is $x(t)$ and $y(t)$ is the output.

(a) Is this a linear time-invariant system? If so, can you determine without any computation the impulse response of the system? Explain.

(b) Is this system causal? Explain.

(c) Find the unit-step response $s(t)$ and from it find the impulse response $h(t)$. Is this a stable system? Explain.

(d) Find the response due to a pulse $x(t) = u(t) - u(t-1)$.