Homework 3

Due time: 18:30 on Oct. 28th, 2021

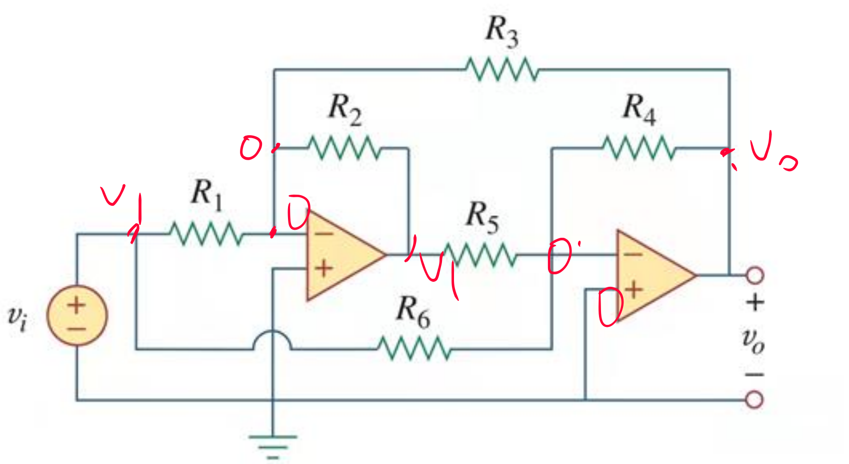
Turn in your homework in class or to tutorial classroom (1B110)

Rules:

- Work on your own. Discussion is permissible, but extremely similar submissions will be judged as plagiarism.
- Please show all intermediate steps: a correct solution without an explanation will get zero credit.
- Please submit on time. No late submission will be accepted.
- Please prepare your submission in English only. No Chinese submission will be accepted.
- If needed, round the number to the nearest hundredths, i.e., rounding it to 2 decimal places.

12'

1. For the circuit below, assume the operational amplifiers are both working in their linear mode, determine the gain v_o/v_i of the circuit using resistance $\boldsymbol{R_1}$ ***to*** $\boldsymbol{R_6}$.



$$\begin{cases} \dfrac{-v_i}{R_1} = \dfrac{v_1'}{R_2} + \dfrac{v_o}{R_3} & 3' \\[2ex] \dfrac{v_i}{R_6} + \dfrac{v_1'}{R_5} + \dfrac{v_o}{R_4} = 0 & 3' \end{cases}$$

$$\Rightarrow v_1' = -\left(\frac{R_2}{R_1}v_i + \frac{R_2}{R_3}v_o\right) \quad 2'$$

$$\frac{v_i}{R_6} + \frac{v_o}{R_4} - \frac{R_2}{R_1R_5}v_i - \frac{R_2}{R_3R_5}v_o = 0.$$

$$\left(\frac{1}{R_6} - \frac{R_2}{R_1R_5}\right)v_i = \left(\frac{R_2}{R_3R_5} - \frac{1}{R_4}\right)v_o \quad 2'$$

$$\frac{v_o}{v_i} = \frac{\frac{1}{R_6} - \frac{R_2}{R_1R_5}}{\frac{R_2}{R_3R_5} - \frac{1}{R_4}} = \frac{R_1R_3R_4R_5 - R_2R_3R_4R_6}{R_1R_2R_4R_6 - R_1R_3R_5R_6}$$

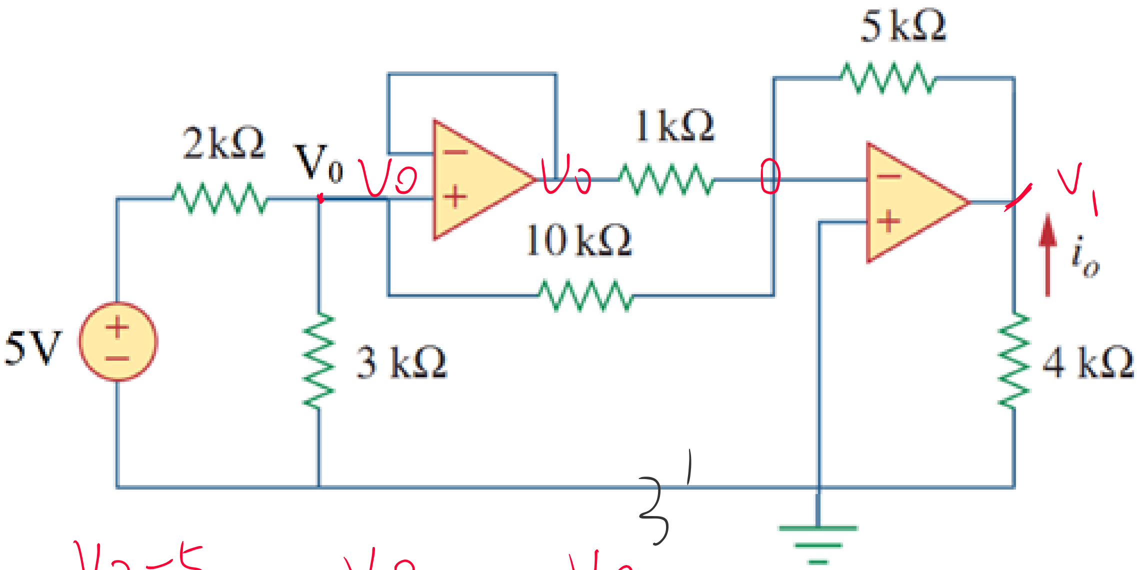
2'

14'

2. For the circuit below, assume the operational amplifiers are both working in their linear mode,

（1） Calculate $\mathbf{V_0}$ in the op amp circuit.

（2） Calculate i_0 in the op amp circuit.



3'

(1) $\frac{V_o - 5}{2000} + \frac{V_o}{3000} + \frac{V_o}{10000} = 0 \Rightarrow V_o = \frac{75}{28}(V)$

$= 2.68 (V)$

2'

(2) $\frac{-V_o}{1000} - \frac{V_o}{10000} - \frac{V_1}{5000} = 0$ 3' $\Rightarrow V_1 = -\frac{825}{56}(V)$

2'

$i_0 = \frac{-V_1}{4000} = \frac{825}{224000} = 3.68 \times 10^{-3} (A)$

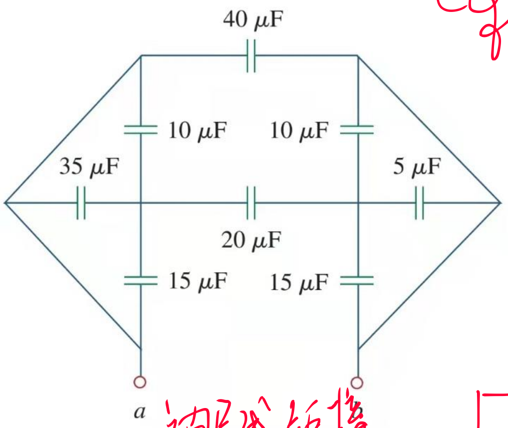
2'

2'

3. Find equivalent capacitance C_{ab} and inductance L_{ab} for the following two networks:

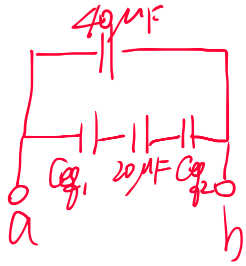
(1) $C_{eq_1} = 35\mu F // 15\mu F // 10\mu F$

$= 35+15+10 = 60\mu F$ 3'



$C_{eq_2} = 15\mu F // 10\mu F // 5\mu F$

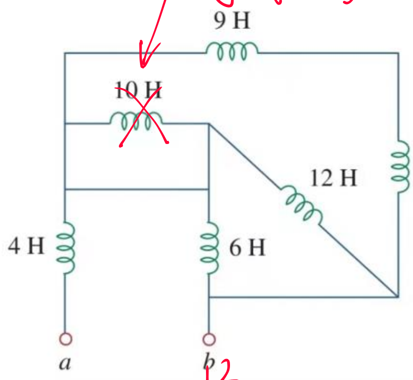
$= 15+10+5 = 30\mu F$ 3'



$C_{ab} = \dfrac{1}{\frac{1}{C_{eq_1}} + \frac{1}{20} + \frac{1}{C_{eq_2}}} + 40 = 50\mu F$

2'

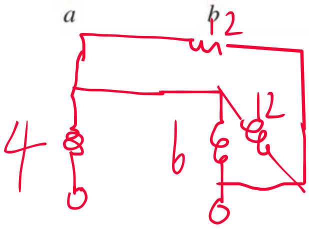
被导线短接



(2)

$L_{eq} = 6//12//(9+3)+4 = 7(H)$

3'



14'

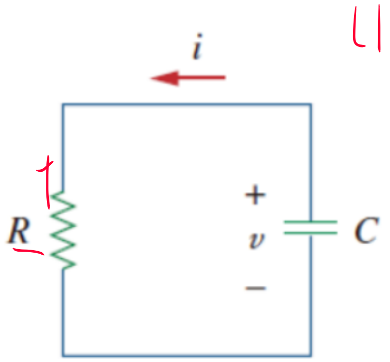
4. For the circuit below

$v(t) = 5e^{-50t}V, \quad t > 0$

$i(t) = 150e^{-50t}mA, \quad t > 0$

(1) Find R and C in the circuit.

(2) Calculate the energy dissipated on R during the time slot of $0 < t < 0.1$s



(1) natural responce: 2' 2'

$v(t) = V_0 e^{-\frac{t}{\tau}} \Rightarrow \tau = \frac{1}{50}(s) \quad V_0 = 5(V)$

$i(t) = \frac{V(t)}{R} = \frac{V_0 e^{-\frac{t}{\tau}}}{R} \Rightarrow R = \frac{100}{3}(\Omega)$ 2'

$= 33.3(\Omega)$

$-\frac{1}{\tau} = -\frac{1}{RC} \Rightarrow C = 6\times10^{-4}(F)$ 2'

2'

(2) $W = Pt = v(t)\, i(t)\, t$ 2'

$= \int_0^{0.1} 5e^{-50t} \cdot 0.15 e^{-50t}\, dt$

$= \int_0^{0.1} 0.75\, e^{-100t}\, dt$

$= -7.5\times10^{-3} e^{-100t} \Big|_0^{0.1}$

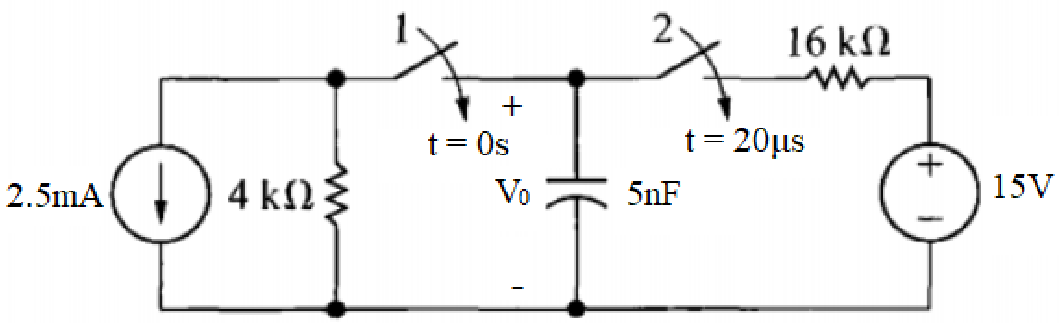
$= -7.5\left(e^{-10} - 1\right)\times10^{-3}$

$= 7.5\times10^{-3}\,(J)$

2'

20

5. For the circuit below:
There is no energy stored in the capacitor in the circuit before $t = 0$s.
When $t = 0$s, Switch 1 is closed. When $t = 20\mu$s, Switch 2 is closed.
Find $V_0(t)$ for $t \geq 0$.



$\tau = RC = 4\times10^3 \times 5\times10^{-9} = 2\times10^{-5}\,(s)$ 2'

$0<t<20\mu s$: $V_0(0) = V_0(0^-) = V_0(0^+) = 0\,(V)$ 2'

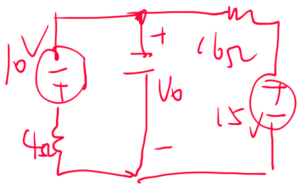
$V_0(\infty) = -2.5\times10^{-3}\times4\times10^3 = -10\,(V)$ 2'

$V_0(t) = V_0(\infty) + [V_0(0) - V_0(\infty)]e^{-\frac{t}{\tau}}$ 2'

$= -10 + 10e^{-\frac{t}{2\times10^{-5}}}\,(V)$ 2'

for $t \geq 20\mu s$:

let $t = 20\mu s \Rightarrow V_0'(0) = -10 + 10e^{-1}\,(V) = -6.32\,(V)$ 2'



KVL:

$V_0'(\infty) = \frac{25}{(16+4)\times10^3}\times4\times10^3 - 10 = -5\,(V)$ 2'

$\tau' = R'C = (4\parallel16)\times10^3\times5\times10^{-9} = 1.6\times10^{-5}\,(s)$ 2'

$V_0'(t) = -5 + [-10+10e^{-1}+5]e^{-\frac{t-2\times10^{-5}}{\tau'}}$

$= -5 + (-5+10e^{-1})e^{-\frac{t-2\times10^{-5}}{1.6\times10^{-5}}}\,(V)$

$= -5 - 1.32e^{-\frac{t-2\times10^{-5}}{1.6\times10^{-5}}}\,(V)$ 2'

注意写单位

$$\Rightarrow V_0(t) = \begin{cases} -10+10e^{-\frac{t}{2\times10^{-5}}} & \checkmark\ 0<t<20\mu s \\ -5-1.32e^{-\frac{t-2\times10^{-5}}{1.6\times10^{-5}}} & \checkmark\ t \geq 20\mu s \end{cases}$$

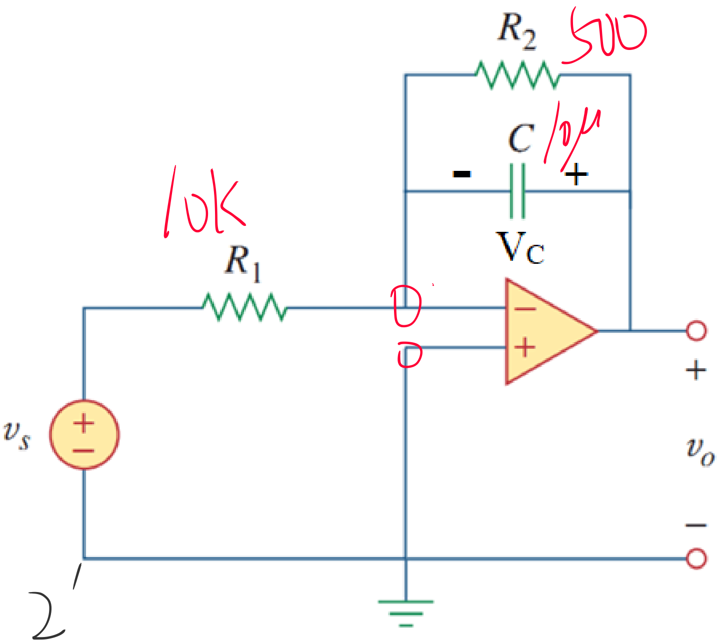
2'

13'

6. For the circuit below. assume the operational amplifier is always working in its linear mode, $V_C(0-)= 5V$, $R_1= 10k\Omega$, $R_2= 500\Omega$, $C=10\mu F$

$$V_S(t) = \begin{cases} 0, & t \le 0 \\ e^{-200t}, & t > 0 \end{cases}$$

Find output voltage of the Op Amp $V_0(t)$ for $t>0$.



$V_0 = V_C$ 2'

$\frac{-V_s}{R_1} = \frac{V_0}{R_2} + C\frac{dV_c}{dt}$ 3'

$-10^{-4} e^{-200t} = 2\times10^{-3} V_0 + 10^{-5}\frac{dV_0}{dt}$

$-10 e^{-200t} = 200 V_0 + \frac{dV_0}{dt}$ 2'

$V_0 = e^{-200t}\left(\int_0^t -10 e^{-200t}\cdot e^{200t}\,dt + C\right) = (-10t+C)e^{-200t}$

Since $V_0(0) = V_C(0) = V_C(0^-) = 5V$ 2'

$\Rightarrow C = 5$ 2'

$\Rightarrow V_0(t) = (-10t+5)e^{-200t}$ (V) $(t>0)$

2'

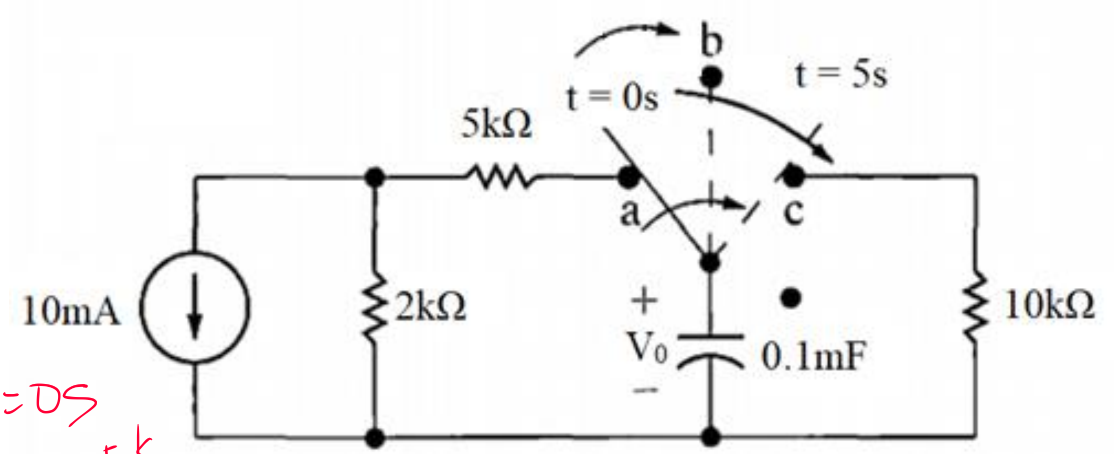
16'

7. For the circuit below:

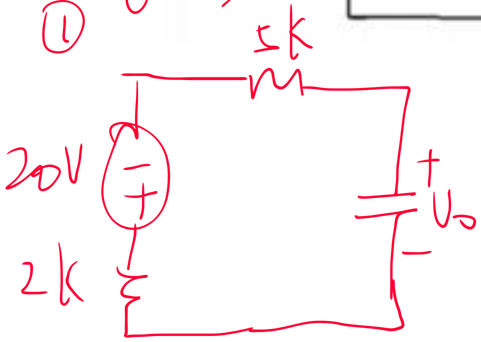
The switch in the circuit has been in *position a* for a long time.

At t = 0s , it moves instantaneously to *position b*, where it remains for 5 s before moving instantaneously to *position c*.

Find the expressions for $V_0(t)$ for $t \geq 0$.



① $t=0s$



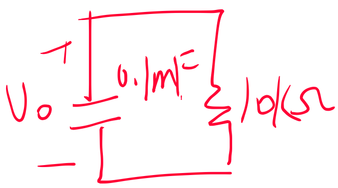
$V_0(0) = -2\times10^3 \times 10\times10^{-3} = -20(V)$ 2'

2'

② $0<t<5s$: the circuit no close, thus $V_0(t)$ remains -20V

③ $5s \leq t$ $V_0(t) = V_0(5^-) = V_0(5^+) = -20\,(V)$ 2' Since no source $V_0(\infty) = 0(V)$ 2'

$i_0(t) = C\dfrac{dv_0(t)}{dt}$ 2' $\tau = RC = 1\,(s)$ 2'



$\Rightarrow V_0(t) = -20e^{-(t-5)}\,(V)$ $t \geq 5s$ 2'

$$\Rightarrow V_0(t) = \begin{cases} -20\text{ V} & 0 \leq t < 5s \\ -20e^{-(t-5)}\text{ V} & t \geq 5s \end{cases}$$

2'