

Remember that your work is graded on the quality of your writing and explanation as well as the validity.

**Problem 1 Notes of discussion (1 pts):**

I promise that I will complete this QUIZ independently, and will not use any electronic products or paper-based materials during the QUIZ, nor will I communicate with other students during this QUIZ.

True or False: I have read the notes and understood them.

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**Problem 2 True or False(5×1pts):**

The following questions are True or False questions, you should judge whether each statement is true or false and write the answer(T/F) in the box below.

*Note: You should write those answers in the box below.*

Question 2.1	Question 2.2	Question 2.3	Question 2.4	Question 2.5
T	T	F	T	T

**2.1** The time complexity of graph traversal on a connected graph is  $\Theta(E)$  where E is the number of edges.

**2.2** BFS can be used to give a path to any node in the graph which has the smallest number of edges.

**2.3** Adjacency matrix is better than adjacency list when inserting a new vertex.

**2.4** We can use DFS to determine whether a graph is bipartite.

**2.5** A directed graph with n vertices has at least n edges to ensure the whole graph is strongly connected.

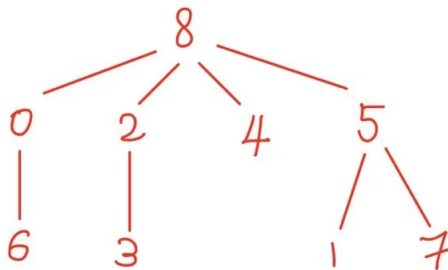
**Problem 3 Disjoint Set(3 pts):**

Consider a disjoint set with both path compression and union-by-size optimization. When two trees have the same height, the set specified first in the union will be the root of the merged set. The following operations are done:

union(2,3);union(1,2);union(5,7);union(8,4);union(7,2);  
find(1);union(0,6);union(4,6);union(6,3);find(2);

Please write down the result for each *find* operation appeared in the above operations, and draw the disjoint set tree after all operations above is finished.

5, 8



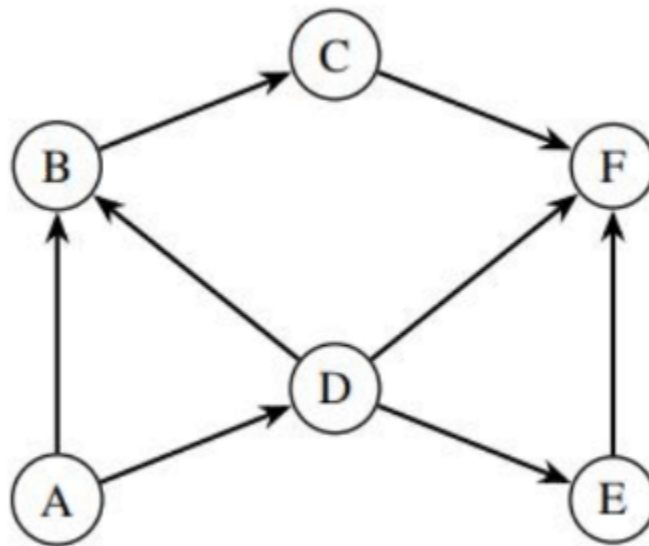
**Problem 3 Graph Algorithms(3+3 pts):**

(1) Design an algorithm to solve the problem below efficiently and give a worst case runtime bound in  $\Theta$  notation in terms of  $V$  and  $E$ .

Problem: determine whether there is a path between from node  $s$  to node  $t$  in a directed graph.

Run DFS or BFS starting from  $s$ . After that, check whether  $t$  is visited. Time:  $\Theta(|V| + |E|)$

(2) Run DFS on the following graph starting with vertex A. Please write down the result. Whenever there is a choice of which node to visit next, follow the alphabetical order.



A B C F D E

**Problem 4 Maze(5 pts):**

There is a maze stored in a  $m \times n$  matrix  $A$ . If there is an obstacle at point  $(i, j)$ ,  $A[i][j] = 1$ ; otherwise  $A[i][j] = 0$ . You're at point  $(1, 1)$ , and you want to go to  $(m, n)$ .

	1	2	3	4	5
1	You				
2	1				
3	2				
4	3		7	8	9
5	4	5	6		Exit

Please design an algorithm to find out the shortest path from  $(1, 1)$  to  $(m, n)$ . Briefly explain your algorithm using natural language. For the provided example, the result is 10.

The input matrix can be seen as a undirected graph, each point  $(i, j)$  as a vertex, and there are edges between each point with value 0 and each of its neighbor with value 0. Use BFS to search this graph starting at point  $(1, 1)$ . For each node appeared in the fringe, record its parent. When we reach point  $(m, n)$ , stop, then we can calculate the shortest path by the recorded parent.