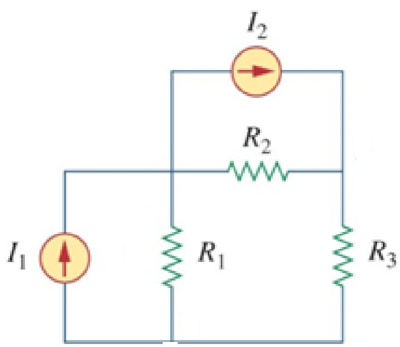
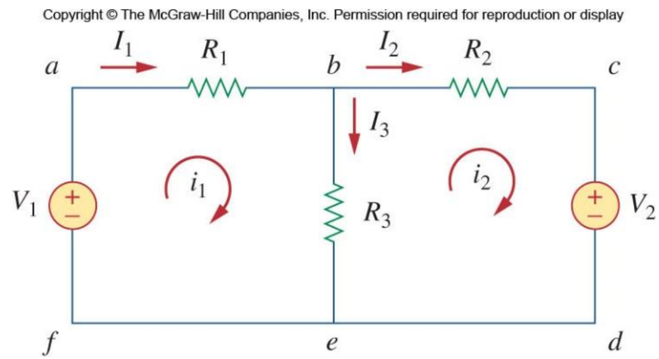
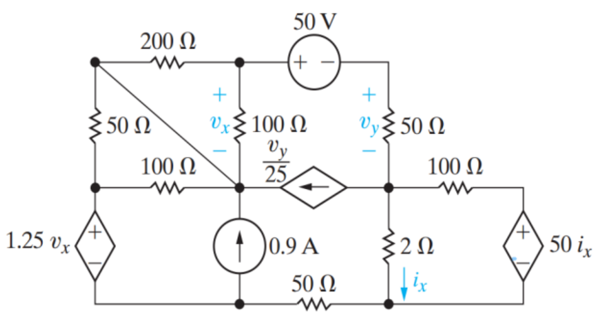
Lecture 5
- RC/RL First-Order Circuits

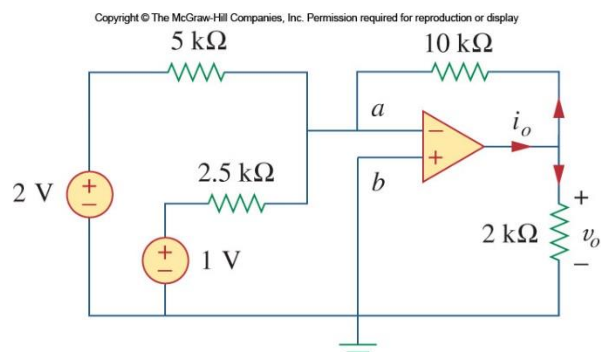
Temporal Behavior of Circuit Responses

- Till now we discussed <u>static</u> analysis of a circuit
  - Responses at a given time depend only on inputs at that time.
  - Circuit responds to input changes infinitely fast.



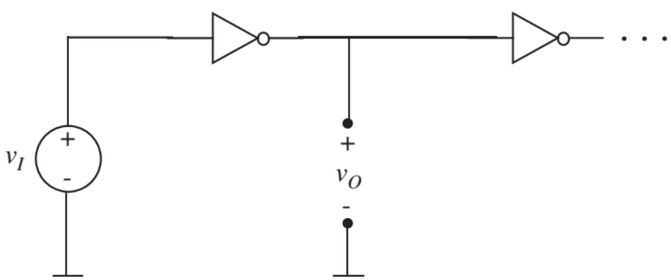


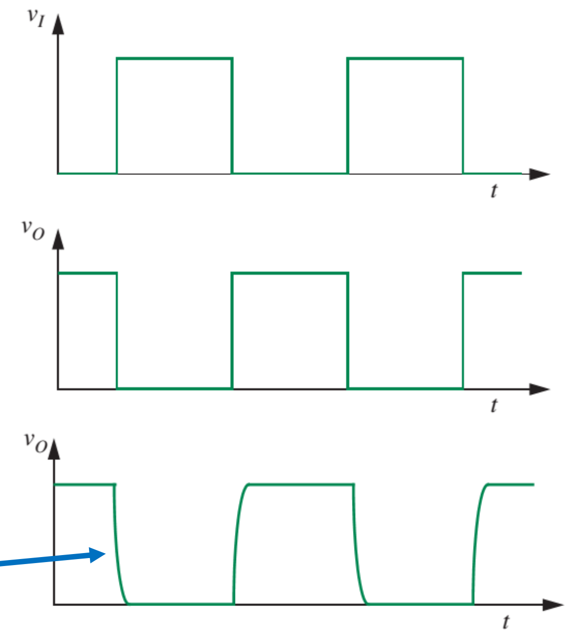




Temporal Behavior of Circuit Responses

- From now on we start to discuss dynamic circuit
  - **Time-varying** sources and responses





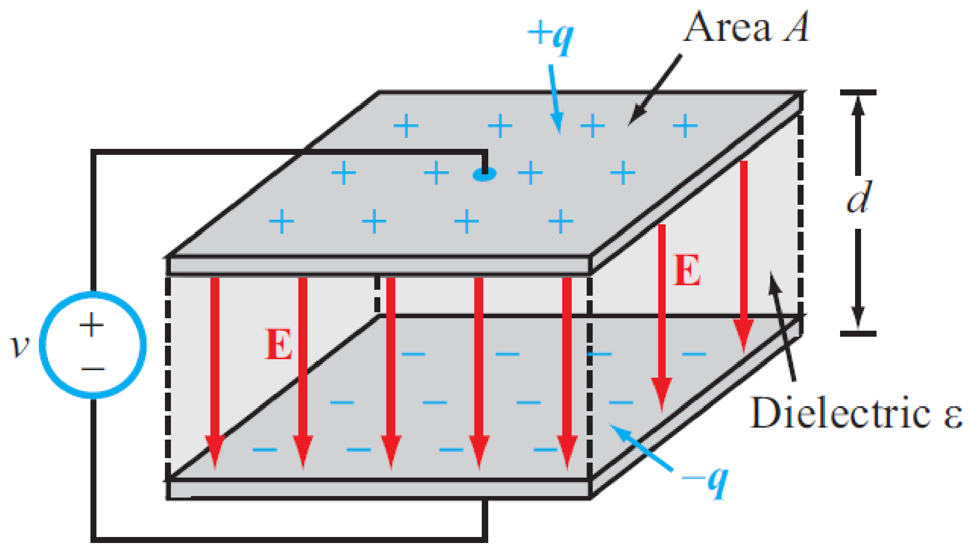
We have to introduce capacitors and inductors to explain such temporal behavior.

Outline

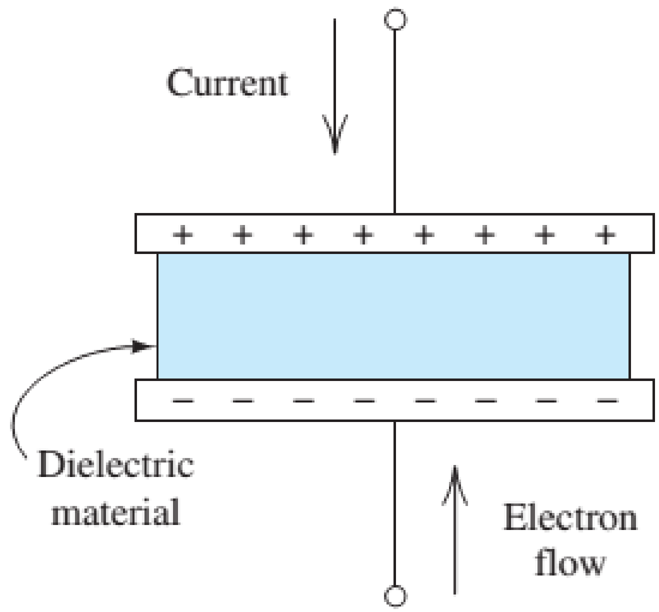
- Capacitors and inductors
- Natural response of RC/RL circuits
- Step response of RC/RL circuits

Capacitors

Storage element that stores energy in electric field



Parallel plate capacitor

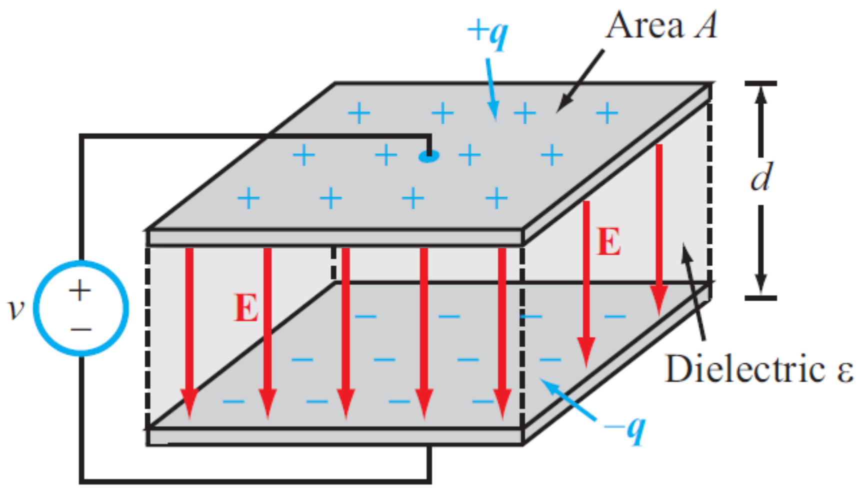


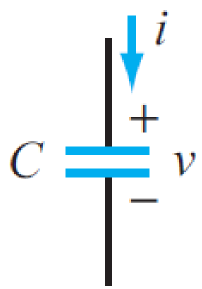
(a) As current flows through a capacitor, charges of opposite signs collect on the respective plates

Does DC voltage generate current <u>flow through</u> a capacitor?

Does AC voltage generate current <u>flow through</u> a capacitor?

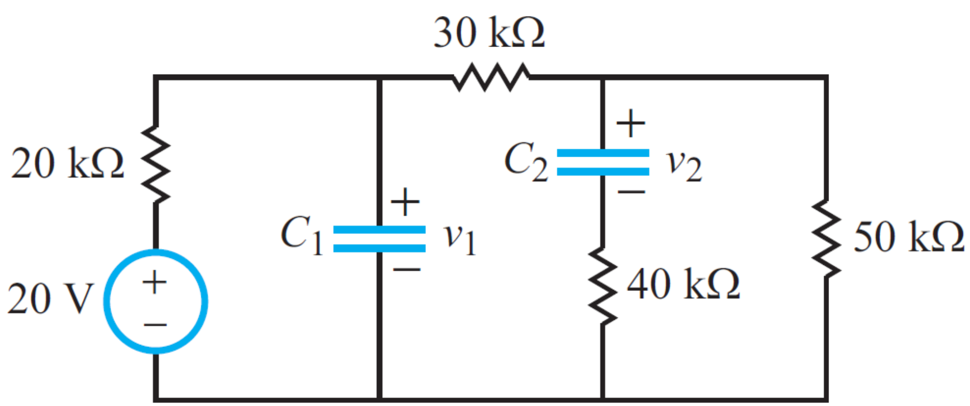
V-I Relationship of Capacitors



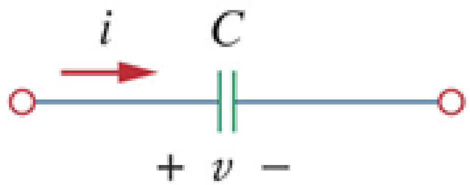


[Source: Berkeley]

Example



Stored Energy

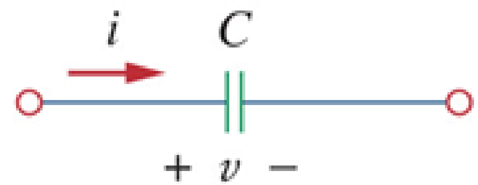


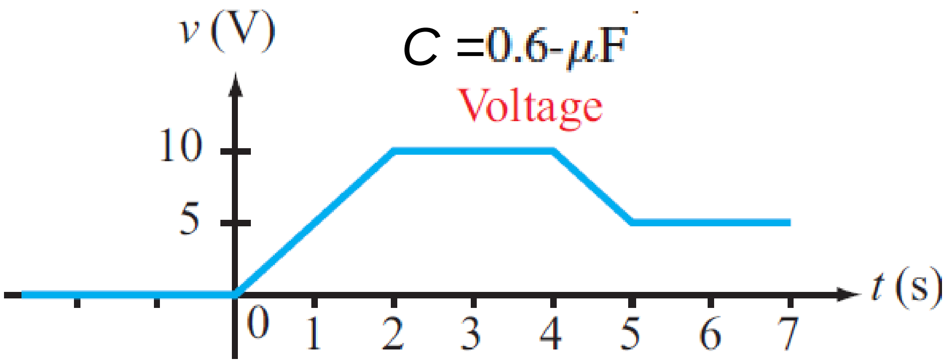
$$i = C\frac{dv}{dt}$$

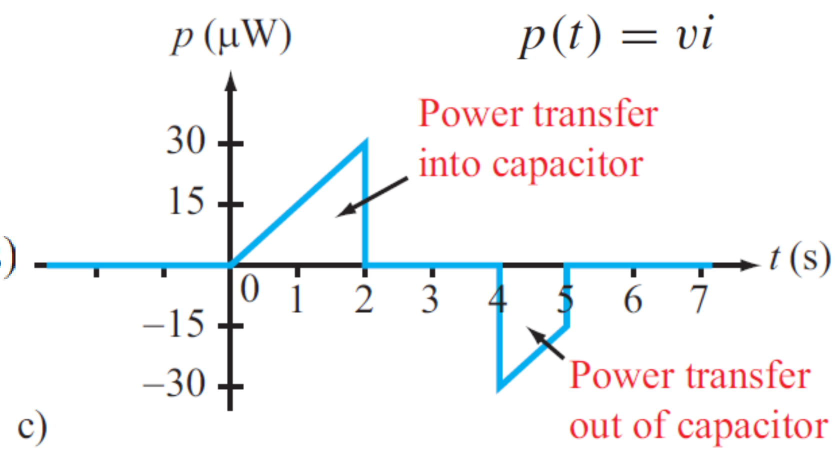
- The instantaneous power delivered to the capacitor is

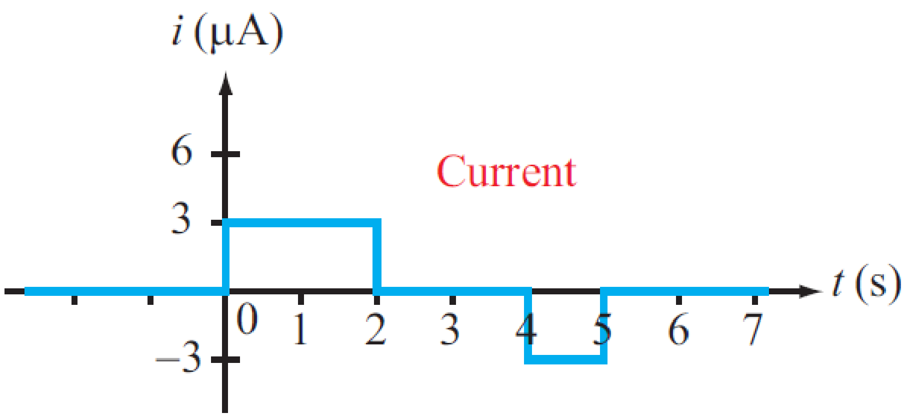
- The energy stored in a capacitor is:

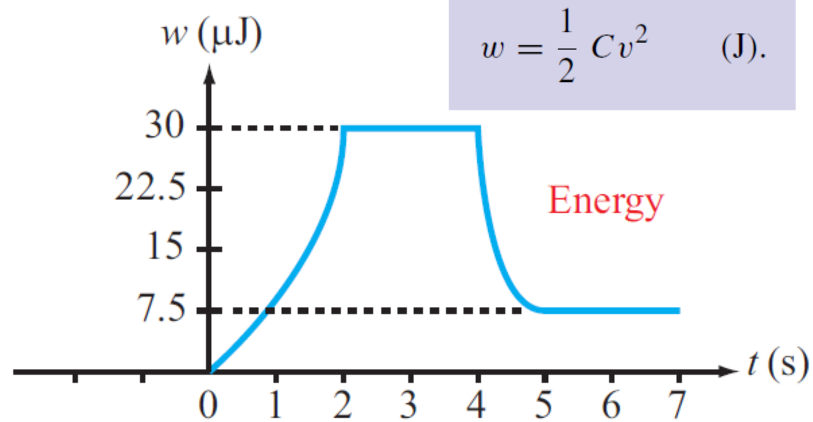
Capacitor Response





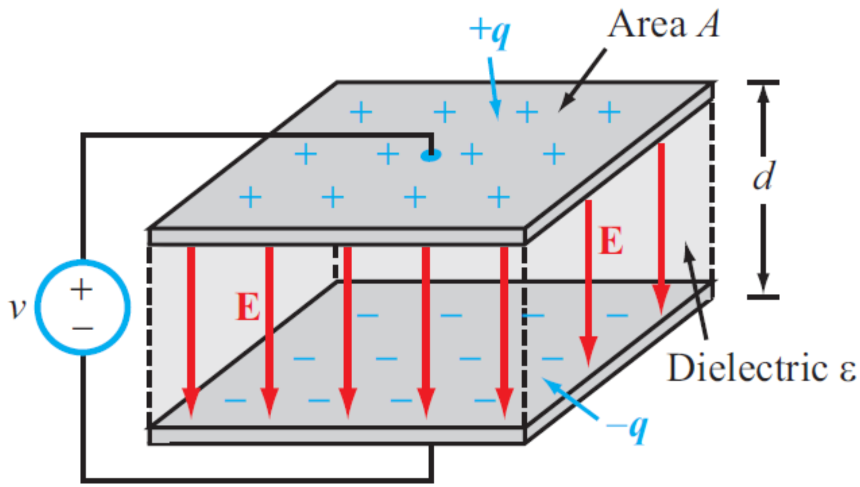


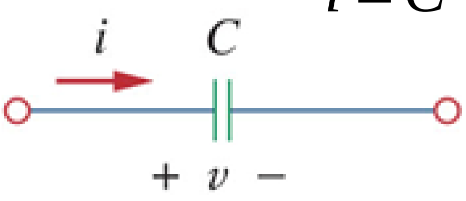




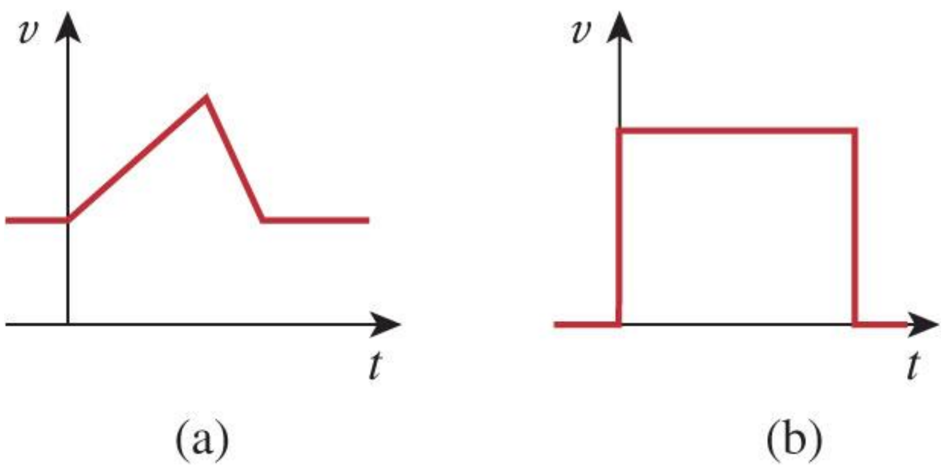
$$w = \frac{1}{2} C v^2 \qquad \text{(J).}$$

Important Property of Capacitors





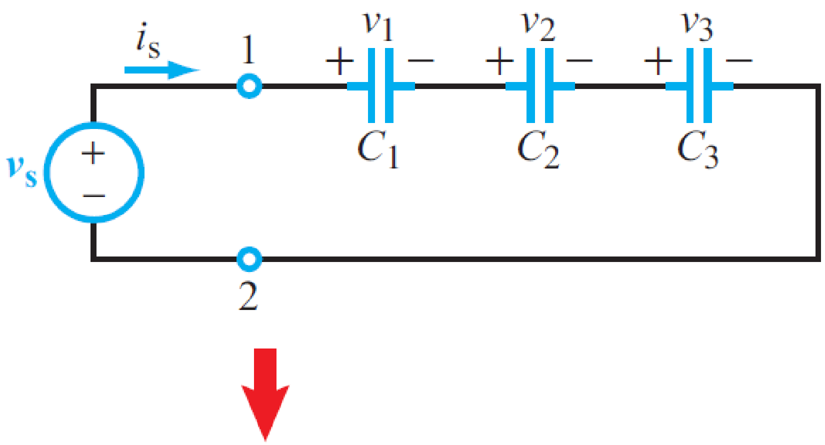
$$i = C\frac{dv}{dt}$$

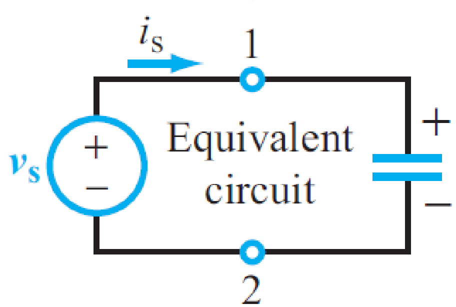


[Source: Berkeley]

Capacitors in Series

Combining In-Series Capacitors

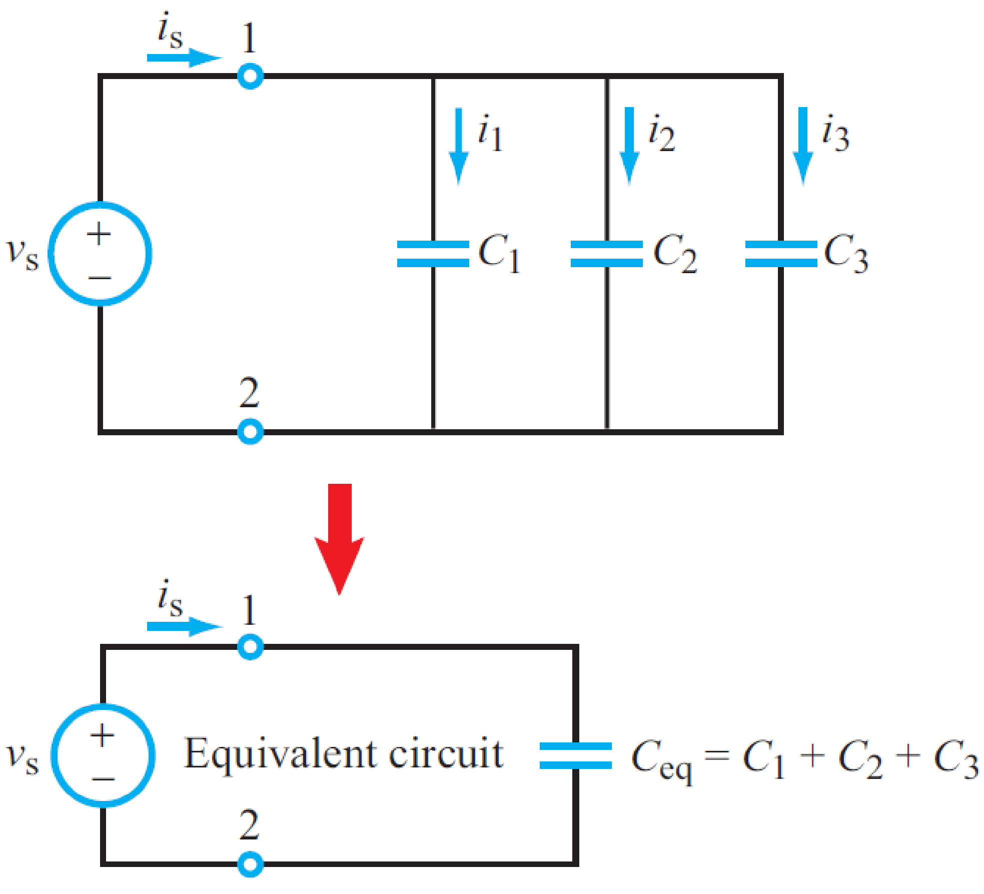




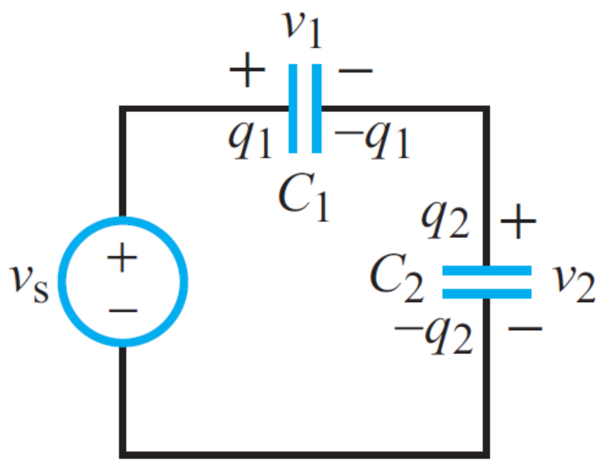
$$C_{eq} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)^{-1}$$

[Source: Berkeley]

Capacitors in Parallel



Voltage Division

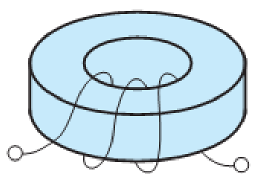


$$v_1 = \left(\frac{C_2}{C_1 + C_2}\right) v_s$$

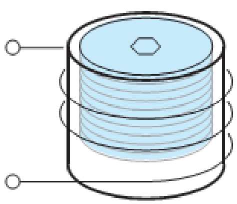
$$v_2 = \left(\frac{C_1}{C_1 + C_2}\right) v_s$$

Inductors

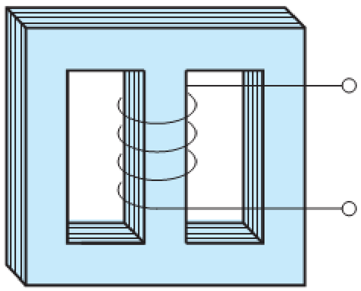
- A storage element that stores energy in magnetic field.
  - They have applications in power supplies, transformers, radios, TVs, radars, and electric motors.
- Any conductor has inductance, but the effect is typically enhanced by coiling the wire up.



(a) Toroidal inductor



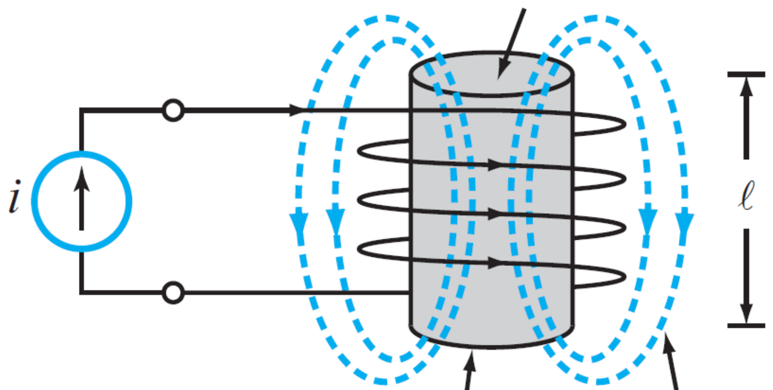
(b) Coil with an iron-oxide slug that can be screwed in or out to adjust the inductance

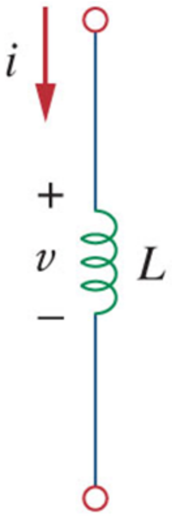


(c) Inductor with a laminated iron core

$$L = \frac{N^2 \mu S}{l}$$

V-I Relationship of Inductors



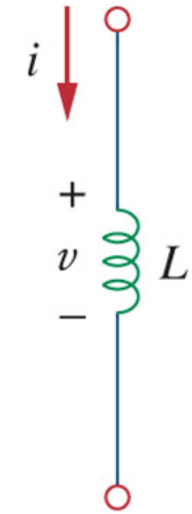


$$v = L\frac{di}{dt}$$

[Source: Berkeley]

Energy Stored in an Inductor

- The power delivered to the inductor is:

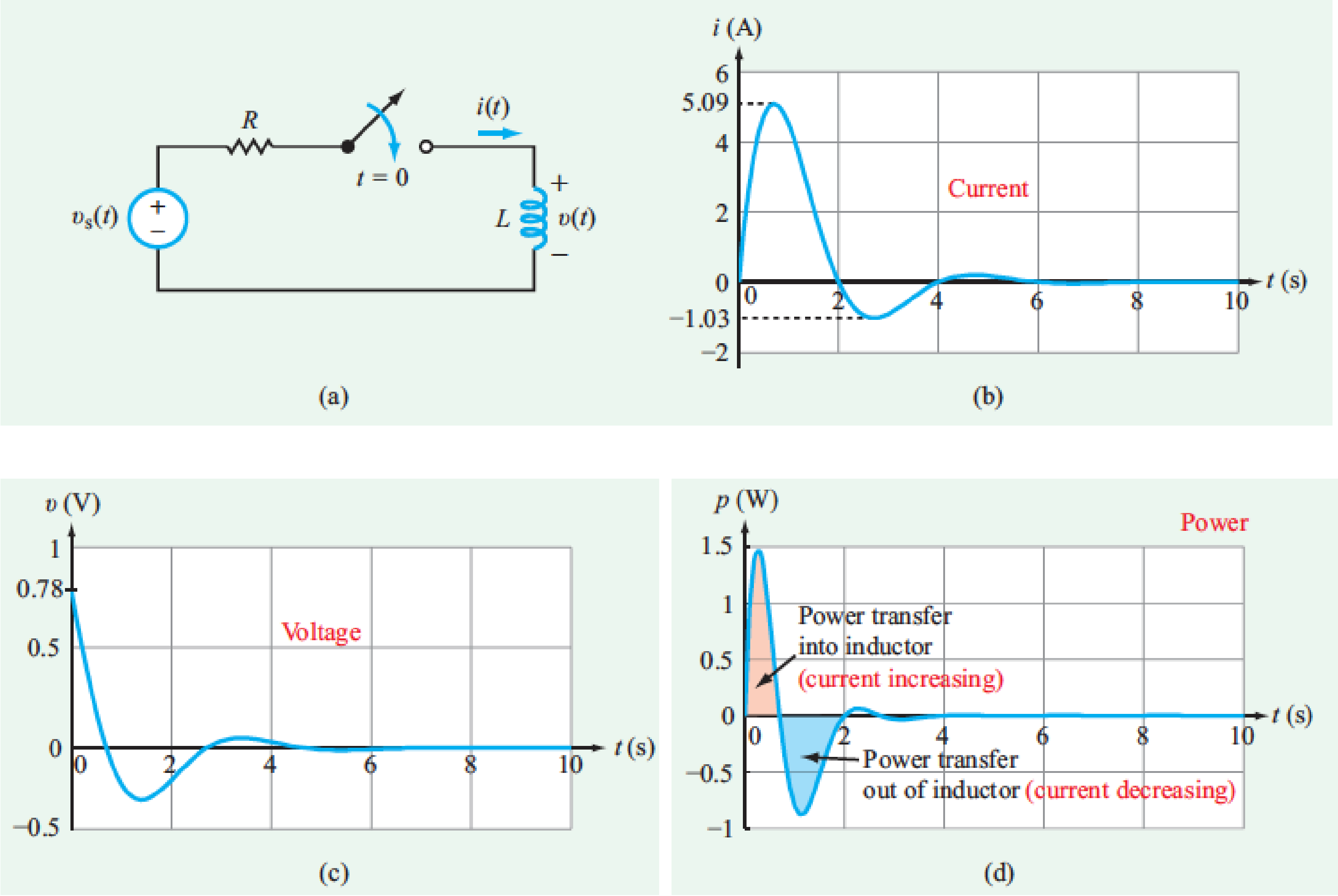


$$v = L\frac{di}{dt}$$

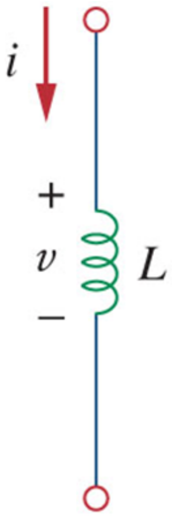
- The energy stored is:

Inductor Response

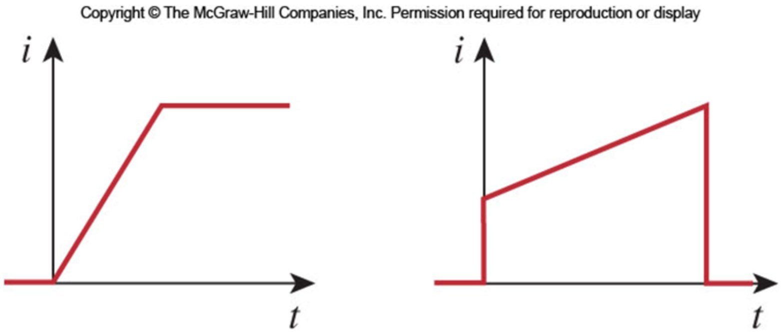
$$i(t) = 10e^{-0.8t}\sin(\pi t/2)\ \text{A}, \qquad (\text{for } t \geq 0)$$



Important Property of Inductors

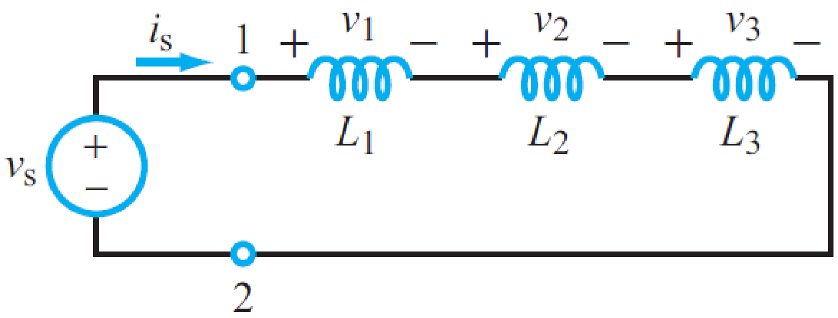


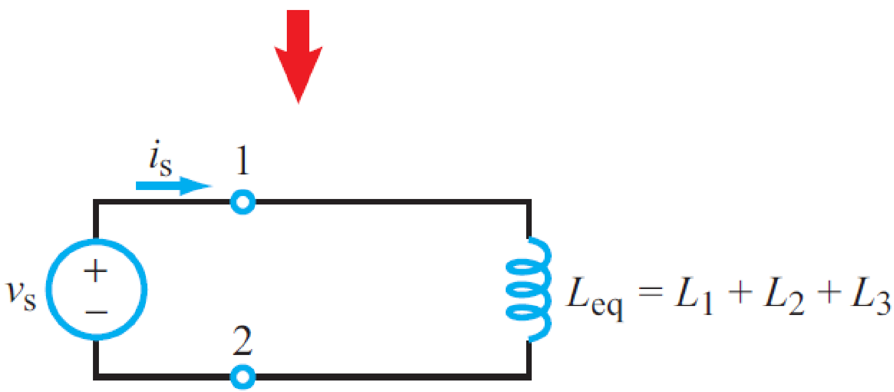
$$v = L\frac{di}{dt}$$



[Source: Berkeley]

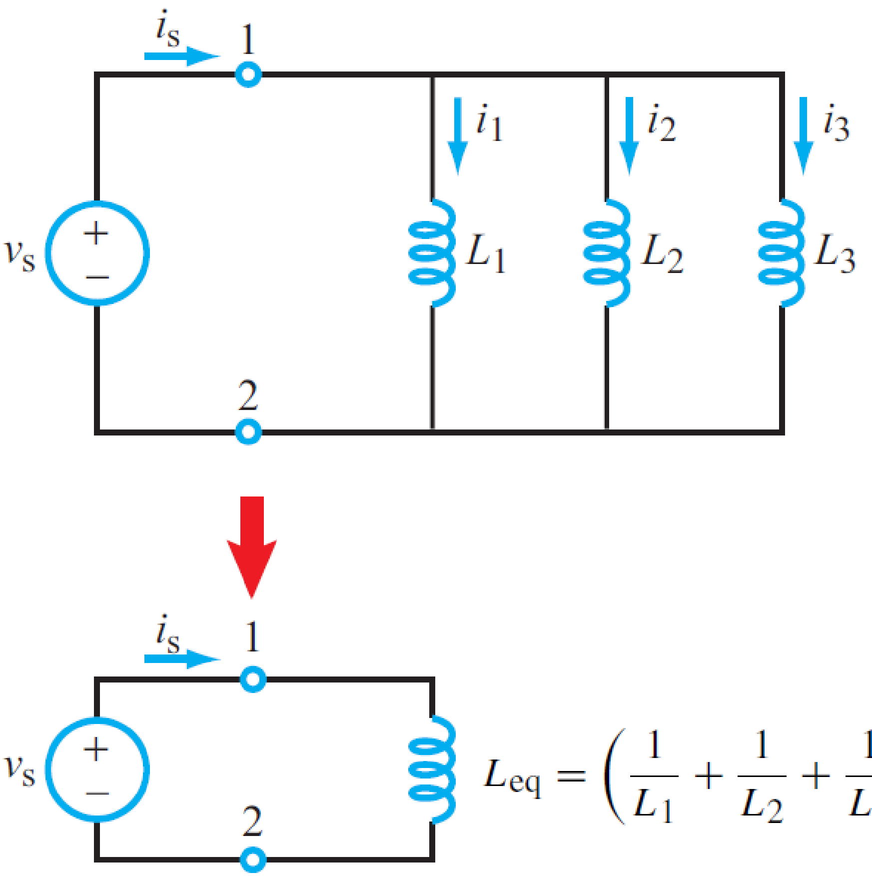
Inductors in Series





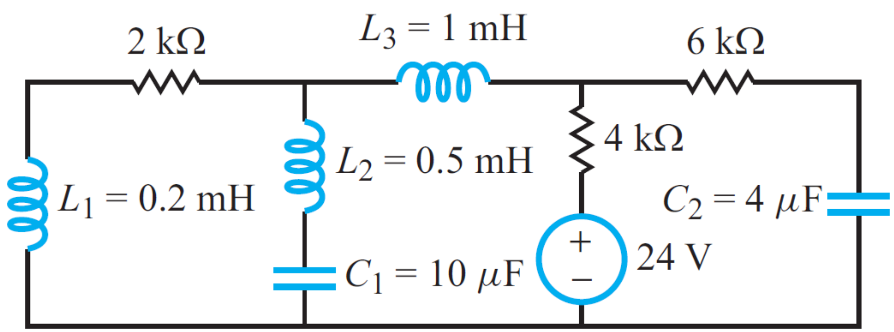
Inductors in Parallel

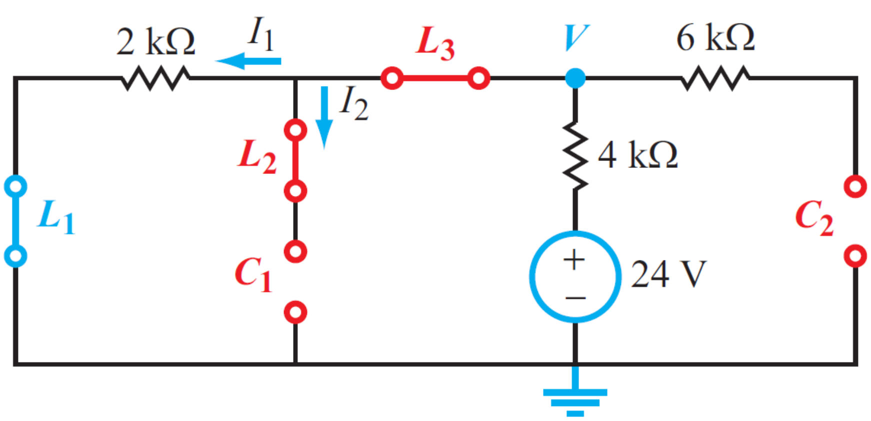
Combining In-Parallel Inductors



$$L_{eq} = \left(\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}\right)^{-1}$$

Example


2 kΩ
L3 = 1 mH
6 kΩ
4 kΩ
L2 = 0.5 mH
L1 = 0.2 mH
C2 = 4 μF
+
–
24 V
C1 = 10 μF


2 kΩ
I1
L3
V
6 kΩ
I2
L2
4 kΩ
L1
C2
C1
+
–
24 V

Summary of Capacitors and Inductors

Table 5-4: Basic properties of R, L, and C.

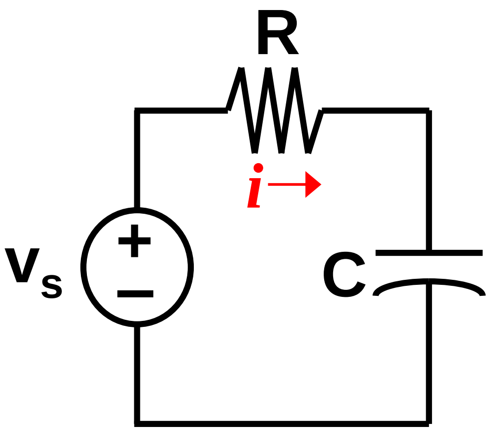
Property	R	L	C
i–υ relation	$i = \frac{\upsilon}{R}$	$i = \frac{1}{L}\int_{t_0}^{t} \upsilon\, dt' + i(t_0)$	$i = C\,\frac{d\upsilon}{dt}$
υ-i relation	$\upsilon = iR$	$\upsilon = L\,\frac{di}{dt}$	$\upsilon = \frac{1}{C}\int_{t_0}^{t} i\, dt' + \upsilon(t_0)$
p (power transfer in)	$p = i^2R$	$p = Li\,\frac{di}{dt}$	$p = C\upsilon\,\frac{d\upsilon}{dt}$
w (stored energy)	0	$w = \frac{1}{2}Li^2$	$w = \frac{1}{2}C\upsilon^2$
Series combination	$R_{eq} = R_1 + R_2$	$L_{eq} = L_1 + L_2$	$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$
Parallel combination	$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$	$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}$	$C_{eq} = C_1 + C_2$
dc behavior	no change	short circuit	open circuit
Can υ change instantaneously?	yes	yes	no
Can i change instantaneously?	yes	no	yes

Outline

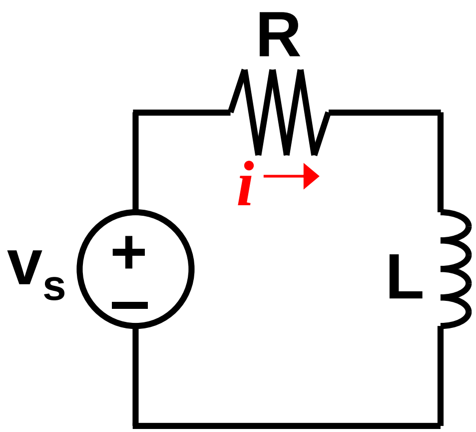
- Capacitors and inductors
- Natural response of RC/RL circuits
- Step response of RC/RL circuits

RC and RL Circuits

- A circuit that contains only sources, resistors and a capacitor is called an ***RC circuit***.

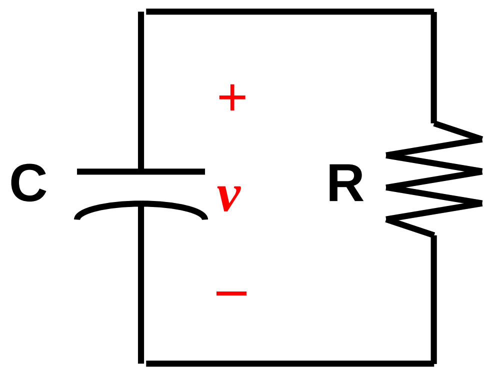


- A circuit that contains only sources, resistors and an inductor is called an ***RL circuit***.



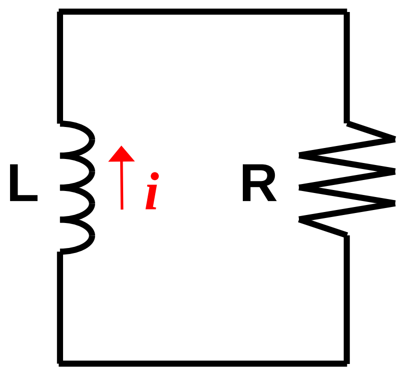
RC and RL Circuits

RC Circuit



- Capacitor voltage cannot change instantaneously
- In steady state, a capacitor behaves like an open circuit

RL Circuit

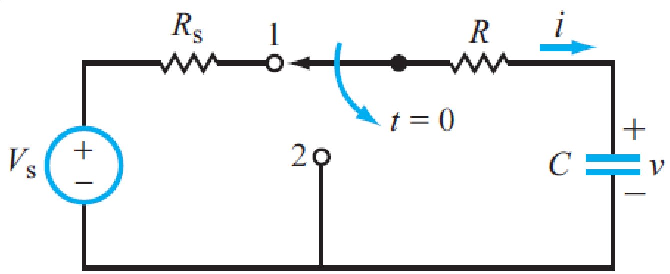


- Inductor current cannot change instantaneously
- In steady state, an inductor behaves like a short circuit.

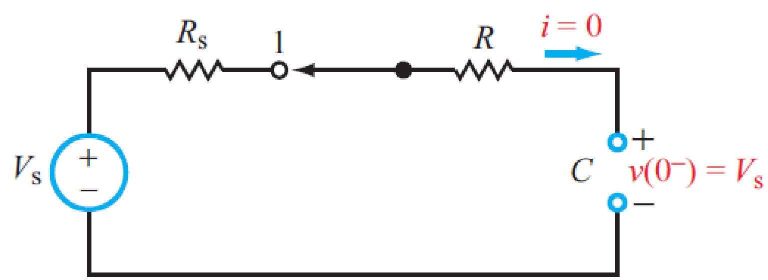
[Source: Berkeley]

Natural Response of a Charged Capacitor

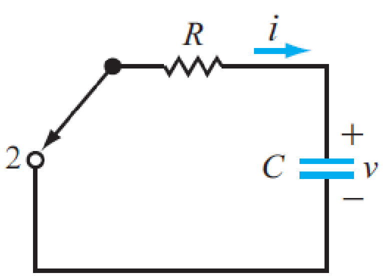
Behavior (*i.e.*, current and voltage) when stored energy in the inductor or capacitor is released to the resistive part of the network (containing no independent sources).



(a) $t = 0^-$ is the instant just before the switch is moved from terminal 1 to terminal 2;

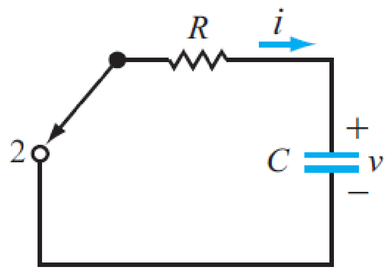


(b) $t = 0$ is the instant just after it was moved, $t = 0$ is synonymous with $t = 0^+$.

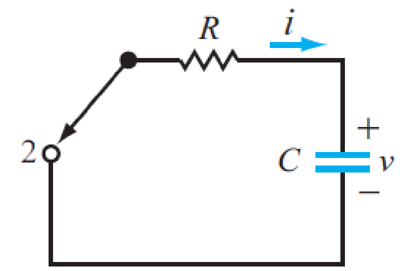


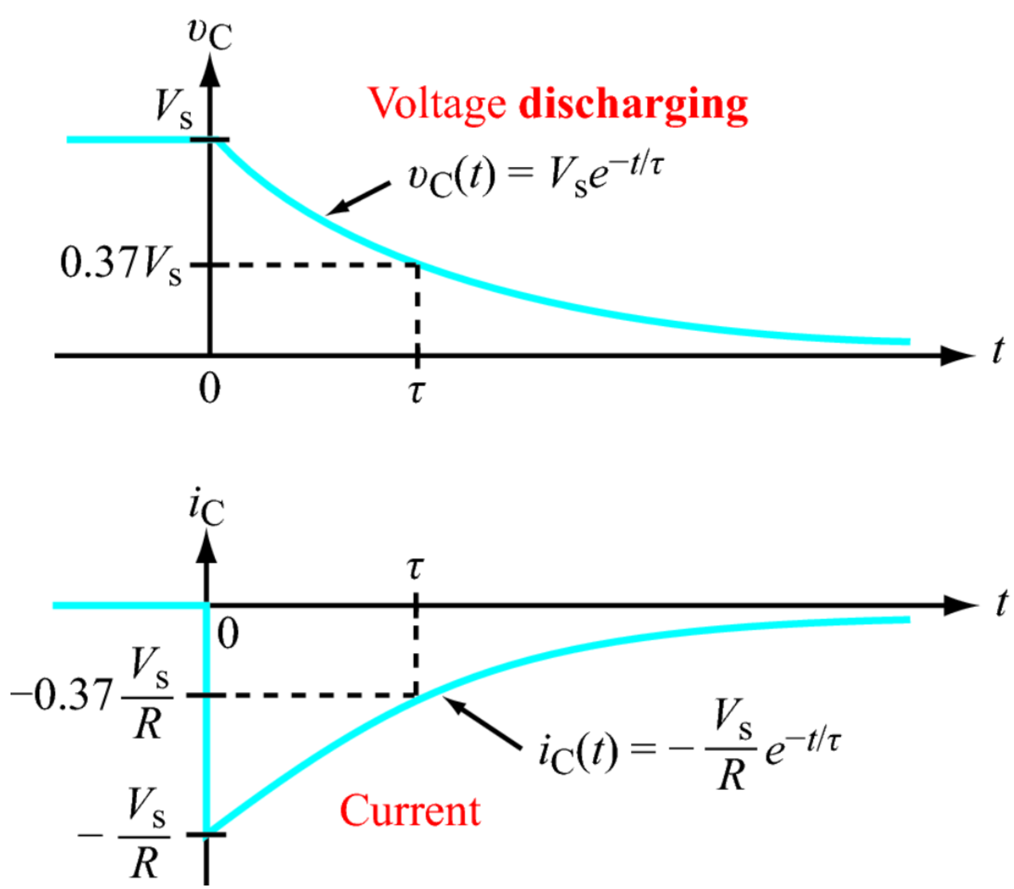
[Source: Berkeley]

Natural Response of a Charged Capacitor



Natural Response of RC



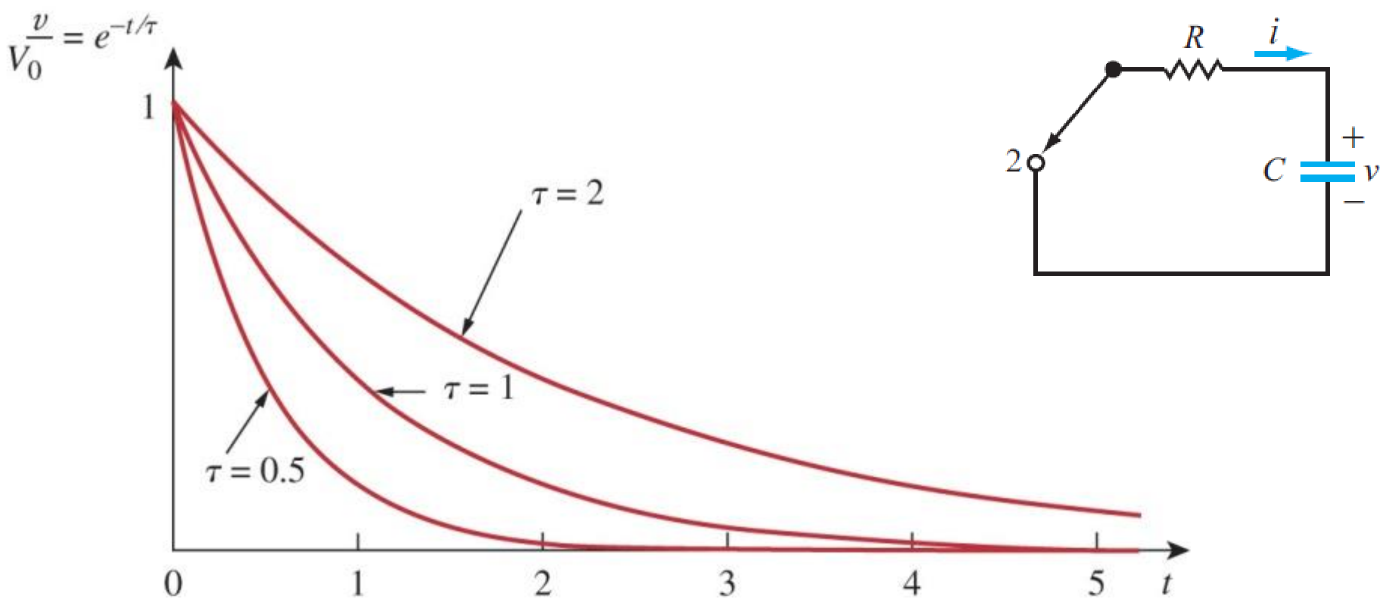


Time constant: $\tau = RC$

[Source: Berkeley]

Time Constant τ (= RC)

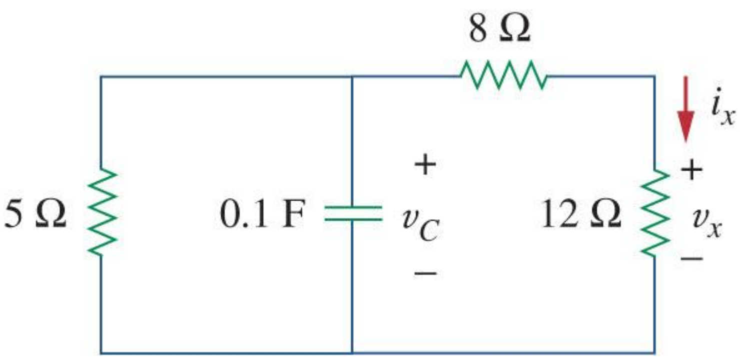
- A circuit with a small time constant has a fast response and vice versa.



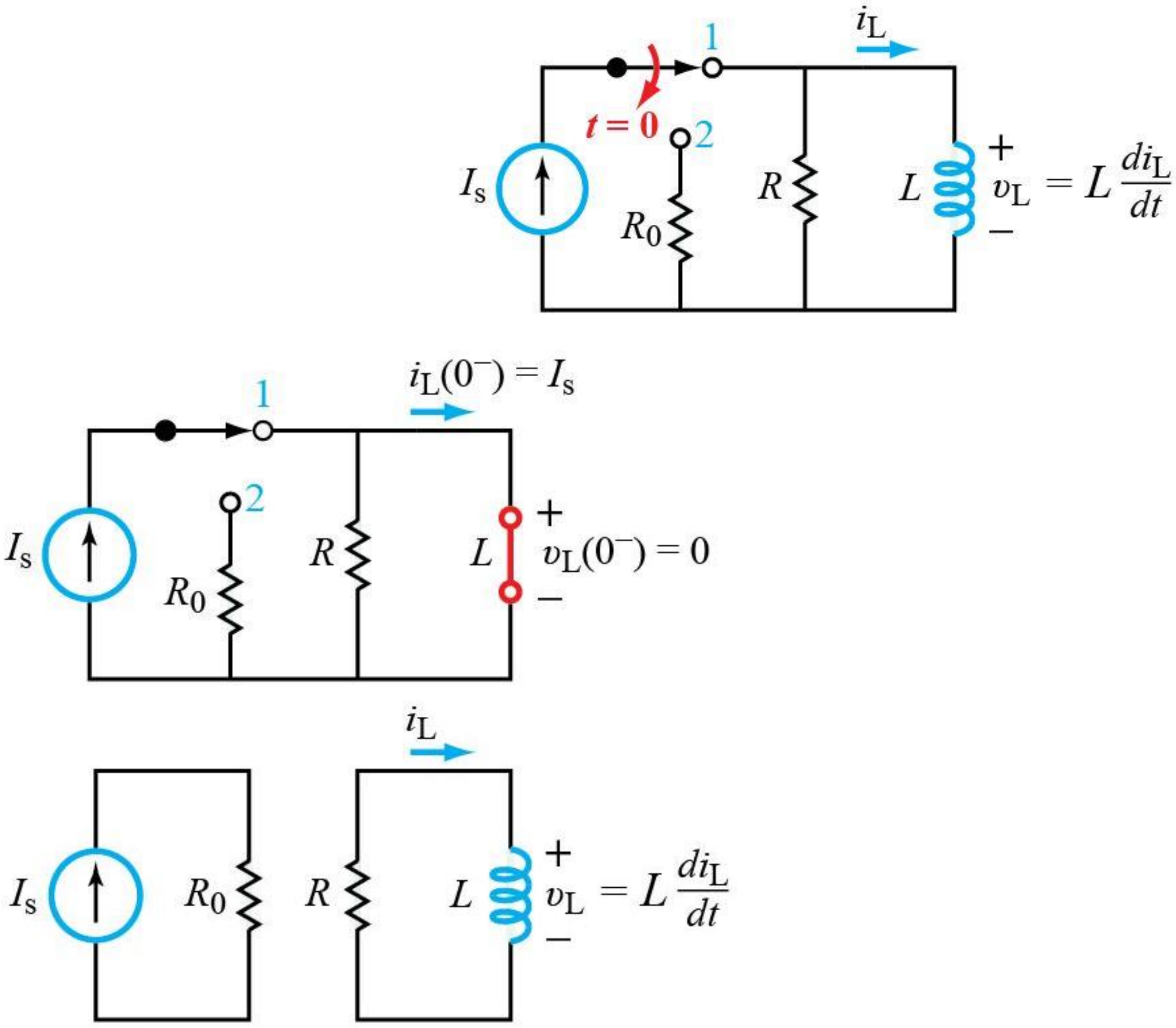
[Source: Berkeley]

Example

- In the circuit below, let $v_C(0) = 15\text{V}$. Find v_C, v_x, and i_x for $t > 0$.

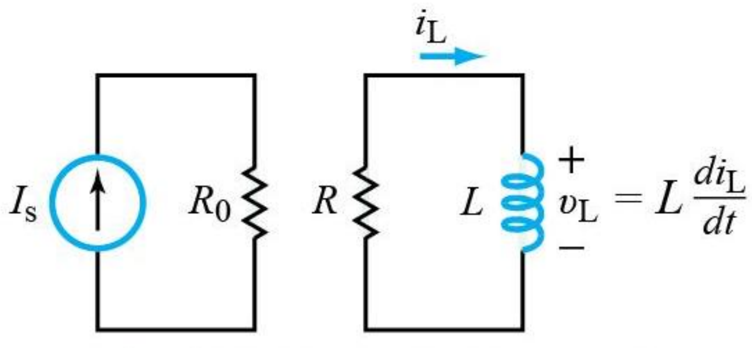


Natural Response of the RL Circuit



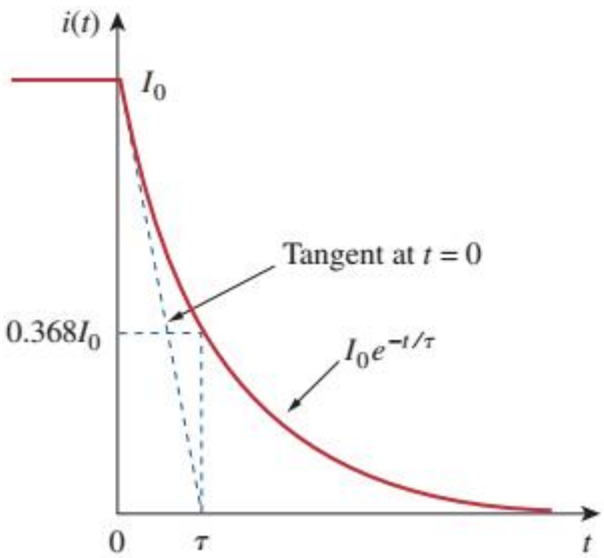
[Source: Berkeley]

Natural Response of the RL Circuit



[Source: Berkeley]

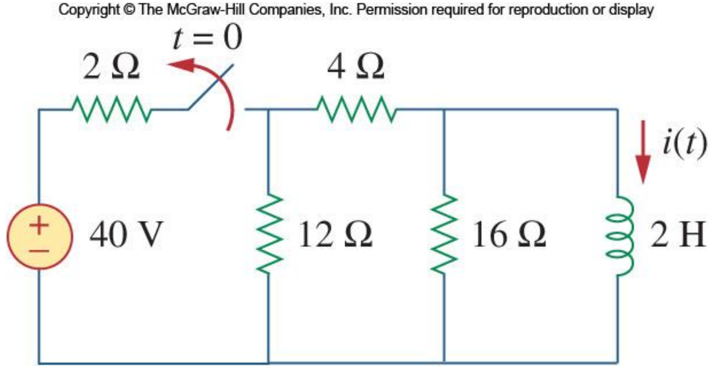
Natural Response of the RL Circuit



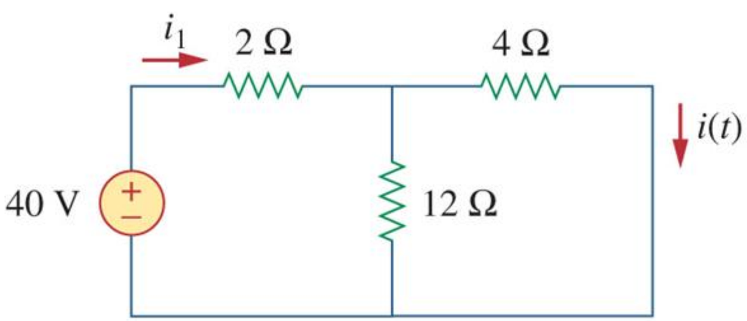
[Source: Berkeley]

Example

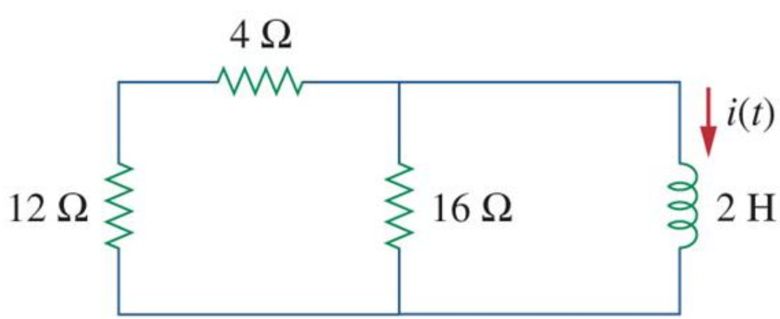
- The switch in the circuit below has been closed for a long time. At $t = 0$, the switch is opened. Calculate $i(t)$ for $t > 0$.



When $t < 0$

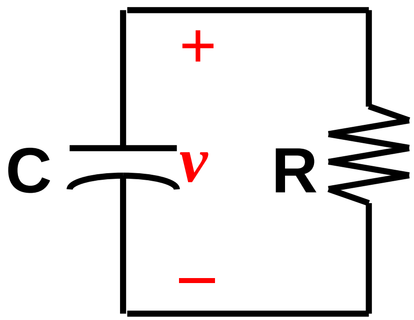


When $t > 0$



Natural Response Summary

RC Circuit



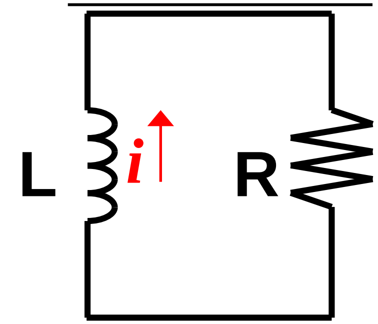
- Capacitor voltage cannot change instantaneously

$$v(0^-) = v(0^+)$$

$$v(t) = v(0)e^{-t/\tau}$$

- time constant $\tau = RC$

RL Circuit



- Inductor current cannot change instantaneously

$$i(0^-) = i(0^+)$$

$$i(t) = i(0)e^{-t/\tau}$$

- time constant $\tau = \dfrac{L}{R}$

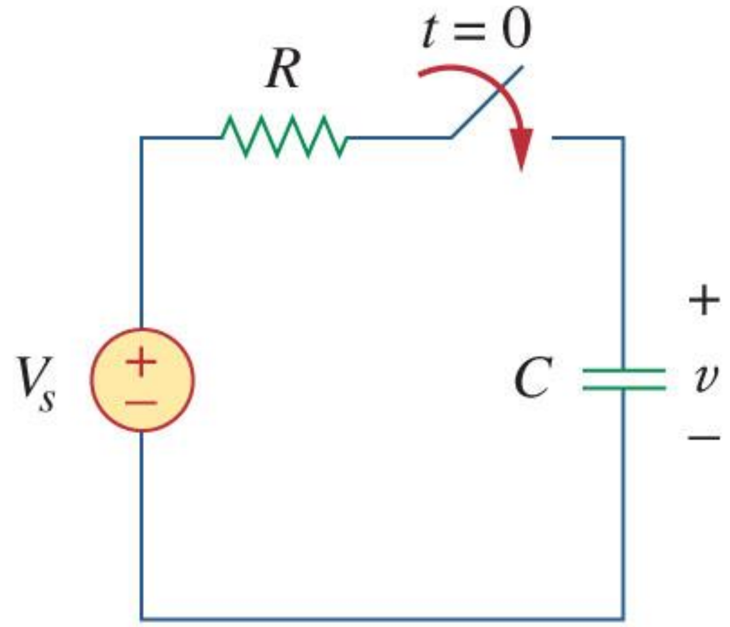
[Source: Berkeley]

Outline

- Capacitors and inductors
- Natural response of RC/RL circuits
- Step response of RC/RL circuits

Step Response of RC Circuit

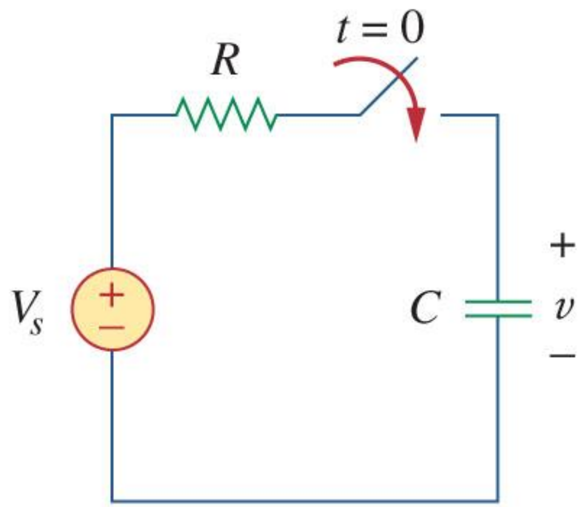
- When a DC source is suddenly applied to a RC circuit, the circuit response is known as the step response.



(a)

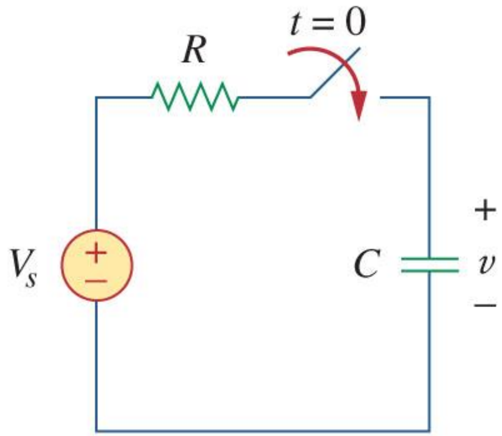
$$v(0^-) = v(0^+) = v_0$$

Step Response of the RC Circuit



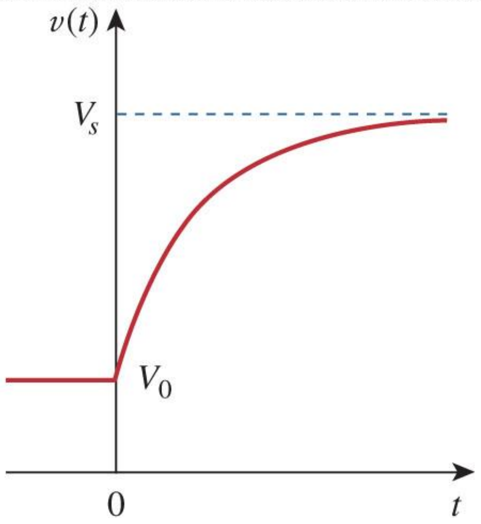
$$v(0^-) = v(0^+) = v_0$$

Step Response of the RC Circuit



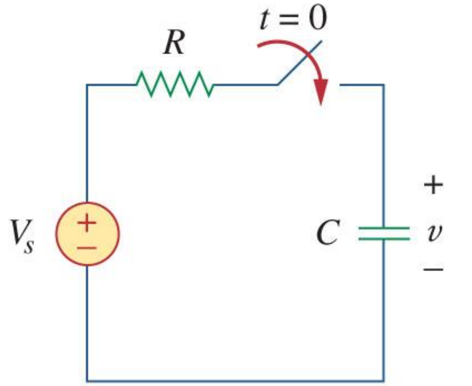
$$v(0^-) = v(0^+) = v_0$$

$$v(t) = \begin{cases} V_0, & t < 0 \\ V_s + (V_0 - V_s)e^{-t/\tau} & t > 0 \end{cases}$$



- This is known as the complete response, or total response.

Forced Response



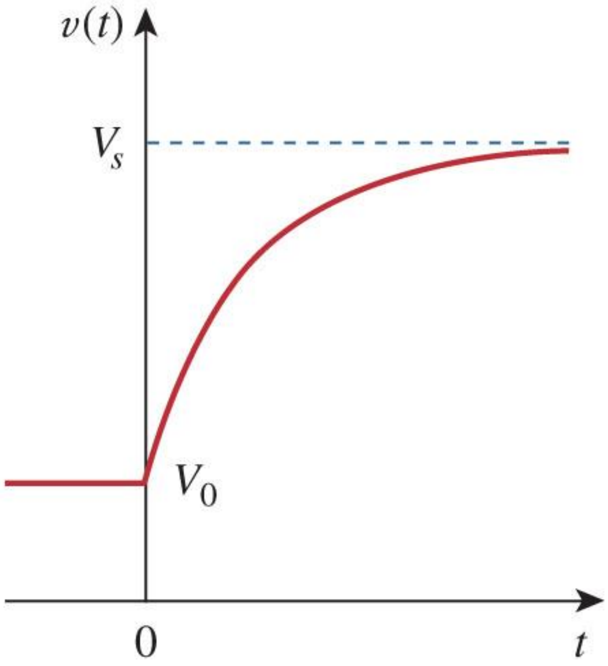
$$v(0^-) = v(0^+) = v_0$$

- The complete response

$$v(t) = \begin{cases} V_0, & t < 0 \\ V_s + (V_0 - V_s)e^{-t/\tau} & t > 0 \end{cases}$$

can be written as:

$$\boxed{v = v_n + v_f}$$



$$\text{Complete response} = \underset{\text{stored energy}}{\text{natural response}} + \underset{\text{independent source}}{\text{forced response}}$$

or

$$v = v_n + v_f$$

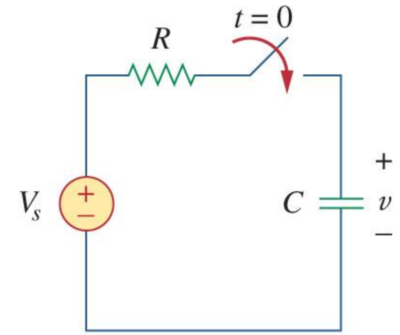
where

$$v_n = V_o e^{-t/\tau}$$

and

$$v_f = V_s(1 - e^{-t/\tau})$$

Another Perspective



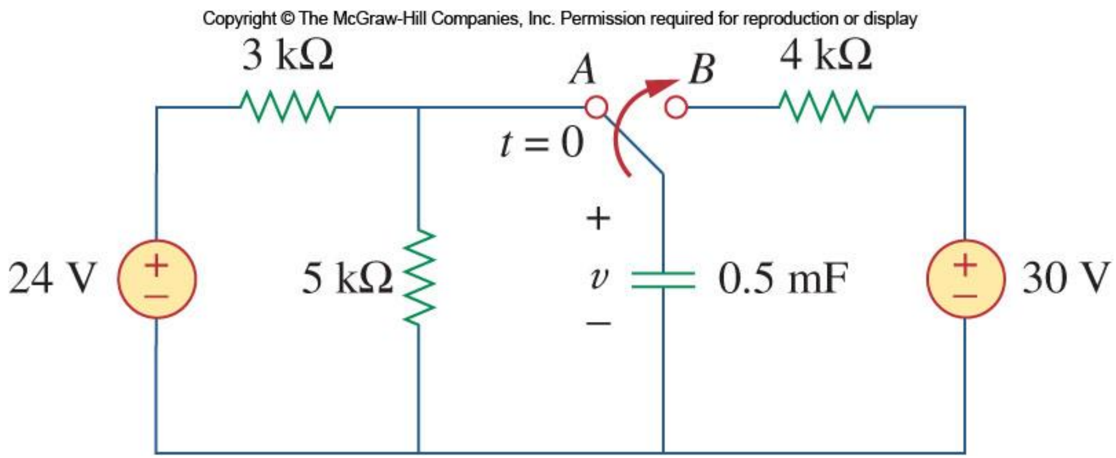
$$v(t)=\begin{cases} V_0, & t<0 \\ V_s+(V_0-V_s)e^{-t/\tau} & t>0 \end{cases}$$

- Another way to look at the response is to break it up into the transient response and the steady state response:

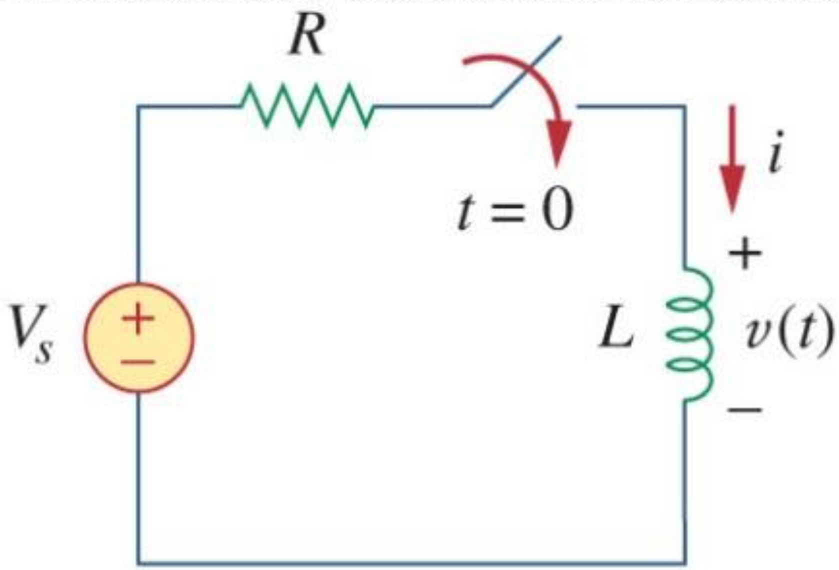
$$v(t) = \underbrace{v(\infty)}_{\text{steady } v_{ss}} + \underbrace{[v(0)-v(\infty)]e^{-t/\tau}}_{\text{transient } v_t}$$

Example

- The switch has been in position A for a long time. At $t = 0$, the switch moves to B. Find $v(t)$.

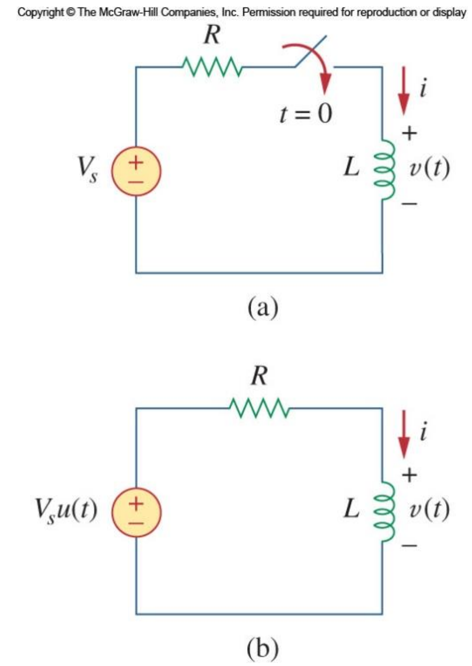


Step Response of the RL Circuit



$$i(0^+) = i(0^-) = I_0$$

Step Response of the RL Circuit



(a)
(b)

- We will use the transient and steady state response approach.
- We know that the transient response will be an exponential:

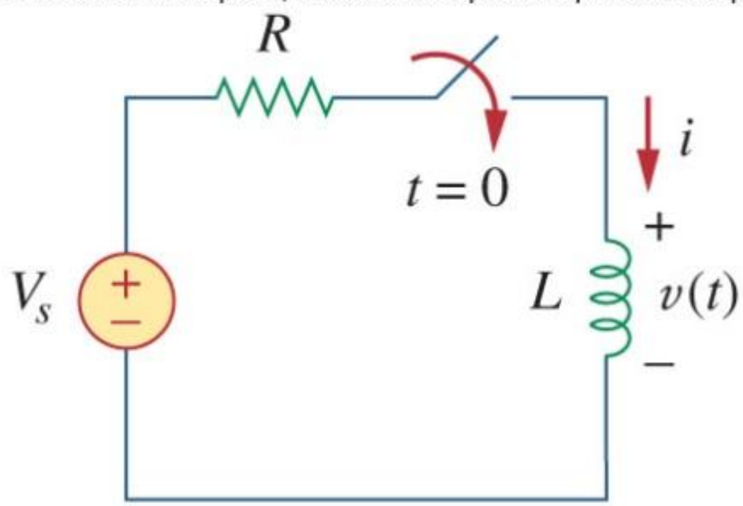
$$i_t = Ae^{-t/\tau}$$

- After a sufficiently long time, the current will reach the steady state:

$$i_{ss} = \frac{V_s}{R}$$

Step Response of RL Circuit

- This yields an overall response of:



$$i = Ae^{-t/\tau} + \frac{V_s}{R}$$

$$i\left(0^{+}\right) = i\left(0^{-}\right) = I_0 \qquad A = I_0 - \frac{V_s}{R}$$

$$i(t) = \frac{V_s}{R} + \left(I_0 - \frac{V_s}{R}\right)e^{-t/\tau}$$

Response of a Circuit

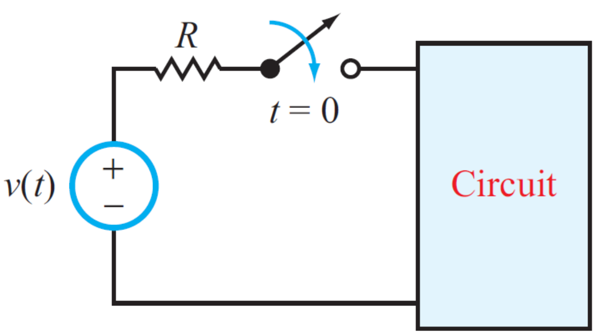
- ***Circuit (dynamic) response***
  - the reaction of a certain voltage or current in the circuit to change, such as the adding of a new source, the elimination of a source, in the circuit configuration.
- ***Transient response***
  - Behavior when voltage or current source are **suddenly** applied to or removed from the circuit due to switching.
  - Temporary behavior

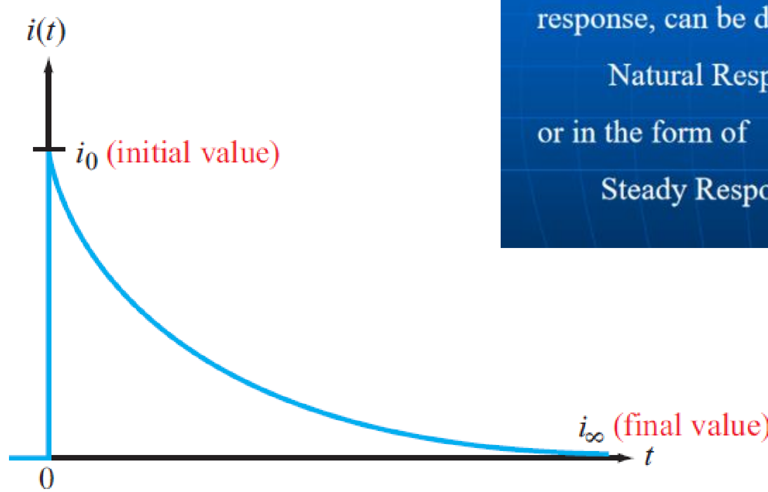
The solution of a linear circuit, called dynamic response, can be decomposed into

Natural Response + Forced Response

or in the form of

Steady Response + Transient Response

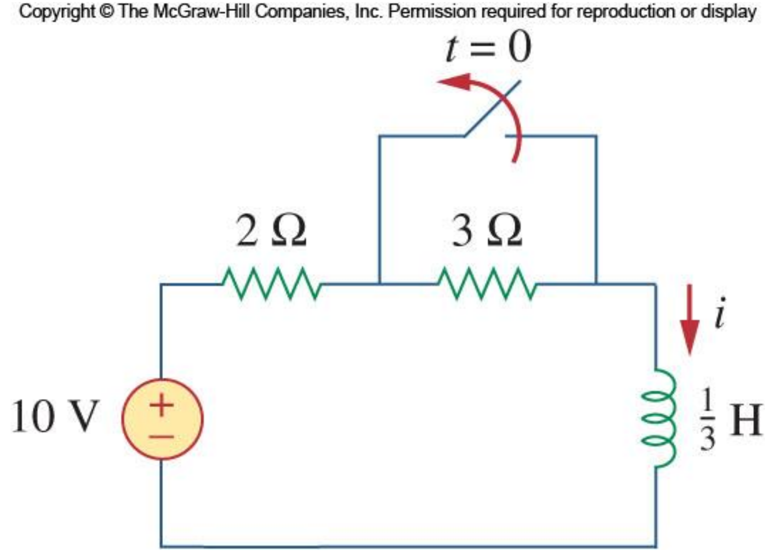




[Source: Berkeley]

Example

- Find $i(t)$ in the circuit for $t > 0$. Assume that the switch has been closed for a long time.



General Procedure for Finding RC/RL Response

1. **Identify the variable of interest**
   - For RL circuits, it is usually the inductor current $\boldsymbol{i_L(t)}$.
   - For RC circuits, it is usually the capacitor voltage $\boldsymbol{v_c(t)}$.
2. **Determine the initial value (at $t = t_0^-$ and t_0^+) of the variable**
   - Recall that $i_L(t)$ and $v_c(t)$ are continuous variables:

     $$i_L(t_0^+) = i_L(t_0^-) \quad \text{and} \quad v_c(t_0^+) = v_c(t_0^-)$$

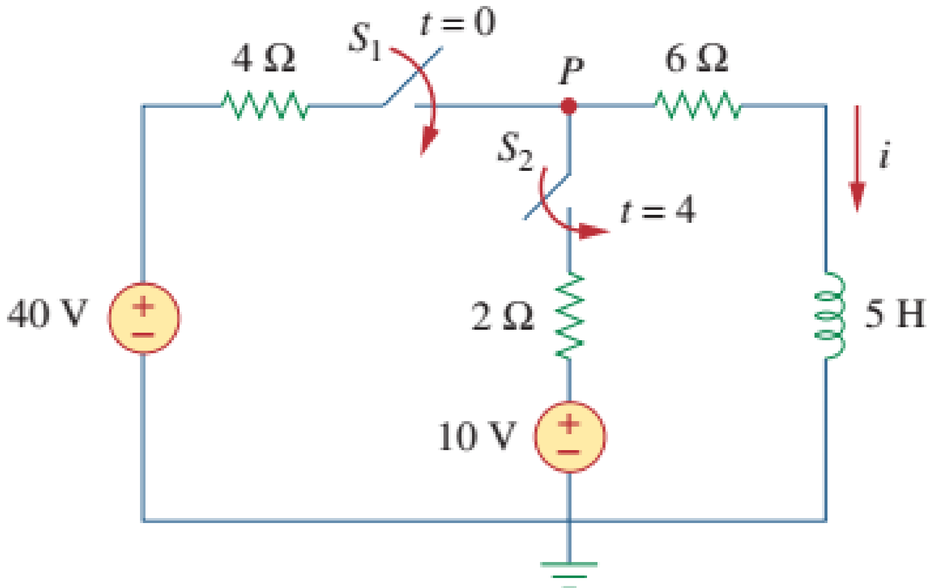
   - Assuming that the circuit reached steady state before t_0: use the fact that an inductor behaves like a short circuit in steady state or that a capacitor behaves like an open circuit in steady state.

Procedure (cont'd)

3. **Calculate the final value of the variable** (as $t \rightarrow \infty$)
   - Again, make use of the fact that an inductor behaves like a short circuit in steady state ($t \rightarrow \infty$) or that a capacitor behaves like an open circuit in steady state ($t \rightarrow \infty$).
4. **Calculate the time constant for the circuit**
   - $\tau = CR$ for an RC circuit where R is the Thévenin equivalent resistance "seen" by the capacitor.
   - $\tau = L/R$ for an RL circuit, where R is the Thévenin equivalent resistance "seen" by the inductor.

Sequential switch

At $t = 0$, switch 1 in Fig. 7.53 is closed, and switch 2 is closed 4 s later. Find $i(t)$ for $t > 0$. Calculate i for $t = 2$ s and $t = 5$ s.

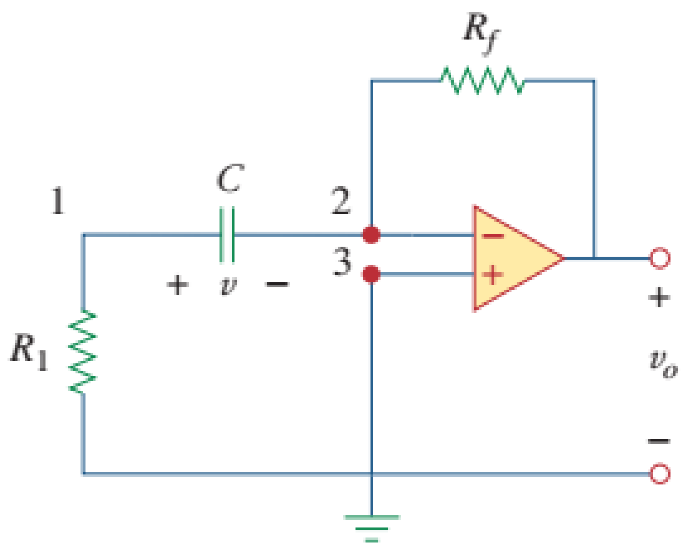


We need to consider the three time intervals $t \le 0$, $0 \le t \le 4$, and $t \ge 4$ separately. For $t < 0$, switches S_1 and S_2 are open so that $i = 0$. Since the inductor current cannot change instantly,

$$i(0^-) = i(0) = i(0^+) = 0$$

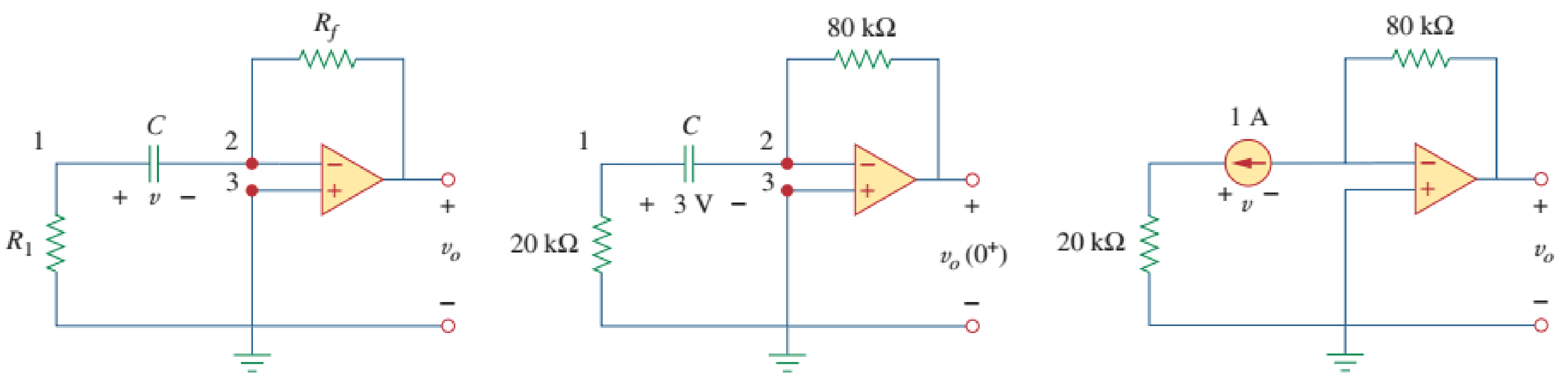
First order op-amp circuit

For the op amp circuit in Fig. 7.55(a), find v_o for $t > 0$, given that $v(0) = 3$ V. Let $R_f = 80$ kΩ, $R_1 = 20$ kΩ, and $C = 5$ μF.



(a)

For the op amp circuit in Fig. 7.55(a), find v_o for $t > 0$, given that $v(0) = 3$ V. Let $R_f = 80\ \text{k}\Omega$, $R_1 = 20\ \text{k}\Omega$, and $C = 5\ \mu\text{F}$.



(a) (b) (c)

Calculate $v(t)$ for 0<t<2s given $v(0)=2V$, R=5ohm, C=0.4F

$V_s = e^{-t}$?

$V_s = e^{-t/2}$?