

# Solution of Quiz4

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## Problem 1 : Regularization

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Batch Normalization: Show that the BN operator is differentiable. You can use the conclusion that elementary functions are differentiable.

solution:

BN formula:

$$\mu_B \leftarrow \frac{1}{m} \sum_{i=1}^m x_i$$

$$\sigma_B^2 \leftarrow \frac{1}{m} \sum_{i=1}^m (x_i - \mu_B)^2$$

$$\hat{x}_i \leftarrow \frac{x_i - \mu_B}{\sqrt{\sigma_B^2 + \epsilon}}$$

$$y_i \leftarrow \gamma \hat{x}_i + \beta \equiv \text{BN}_{\gamma, \beta}(x_i)$$

$$\frac{\partial y_i}{\partial x_i} = \gamma \cdot \frac{\partial \hat{x}_i}{\partial x_i} = \gamma \cdot \left( \frac{1}{\sqrt{\sigma_B^2 + \epsilon}} \right)$$

According to the chain rule, if a sub function (elementary function) is differentiable and the outer function is differentiable, then its composite function is also differentiable.

1. List the BN formula (including the rescaling part) or clearly describe the process of BN (including the rescaling part). (4)
2. Give the derivative function step by step or give it in chain rule form. (4)
3. Clearly prove and say the a sub function (elementary function) is differentiable and the outer function is differentiable to get the final conclusion. (2)

### Problem 2. ResNet (10 points)

Consider a ResNet block, where  $F$  is a concatenation of two  $3 \times 3$  conv + ReLU layers:

$$y = F(x) + x$$

Assume  $x$ , intermediate layer and  $y$  are  $16 \times 16 \times 64$  feature maps. Modify  $F$  based on the bottleneck structure (Lecture 06, P46) such that the number of parameters of the block is reduced by 50-60%.

本题步骤:

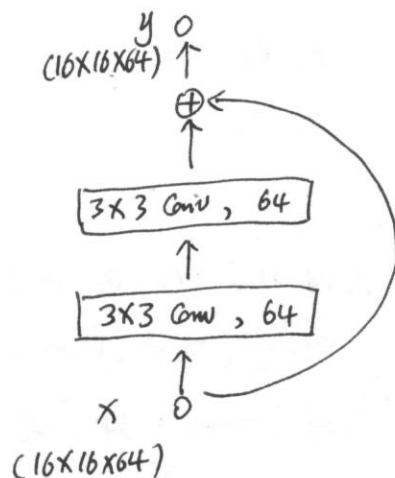
画出B的网络结构图  
写出A,B所用参数数量  
求解二次不等式

给分:

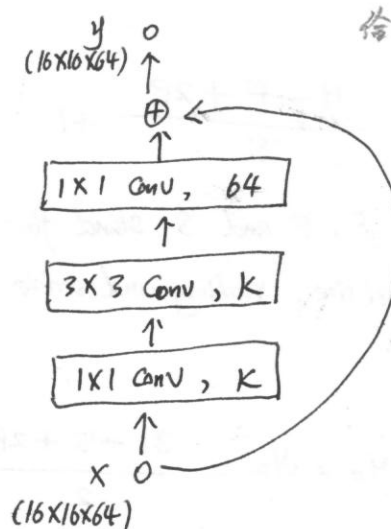
- ① 画出或写出B的网络结构 2分
- ② 写出A,B所用参数数量各2分
- ③ 写对不等式 2分
- ④ 求出结果且指定  $k$  范围为  $[51, 57]$  的整数 2分

Solution:

A. 2  $3 \times 3$  Conv + ReLU layers:



B. The bottleneck structure:



$A_{\text{parameter\_number}} =$

$$3 \times 3 \times 64 \times 64 + 3 \times 3 \times 64 \times 64$$

$$= 36864 \times 2 = 73728$$

OR with bias

$$(36864 + 64) \times 2 = 73856$$

To reduce the number of parameters by 50~60%,

$$40\% \leq \frac{9K^2 + 128K}{73728} \leq 50\%$$

By solving above inequation,

$$K \in [50.57, 57.28] \text{ (no bias)}$$

$$\text{OR } K \in [50.43, 57.17] \text{ (w. bias)}$$

$$\therefore \{51, 52, 53, 54, 55, 56, 57\}$$

$$B_{\text{parameter\_number}} = 1 \times 1 \times K \times 64$$

$$+ 3 \times 3 \times K \times K$$

$$+ 1 \times 1 \times K \times 64$$

$$= 9K^2 + 128K$$

$$\text{OR } 9K^2 + 128K + 3 \times 64$$

$$40\% \leq \frac{9K^2 + 128K + 3 \times 64}{73856} \leq 50\%$$