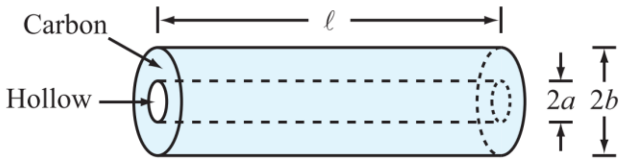
Problem 1 – Short Answers (20 points)

1) A resistor of length ℓ consists of a hollow cylinder of radius *a* surrounded by a layer of carbon that extends from *r* = *a* to *r* = *b*. Develop an expression for the resistance *R*.



Carbon resistor of Problem 1.1

Solution:

(a) $R = \frac{\ell}{\sigma A}$.

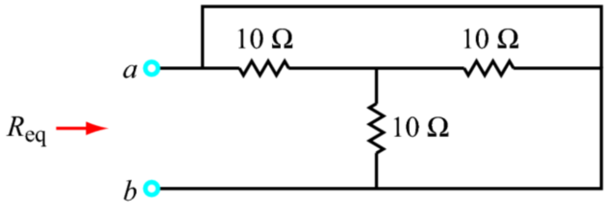
The area through which current can flow is the cross section consisting of carbon. Hence,

$$A = \pi b^2 - \pi a^2.$$

Thus,

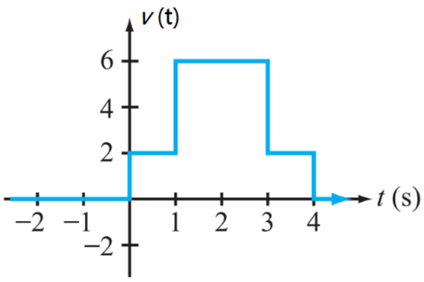
$$R = \frac{\ell}{\sigma\pi(b^2 - a^2)}.$$

2) Determine the equivalent resistance between terminals (a,b):



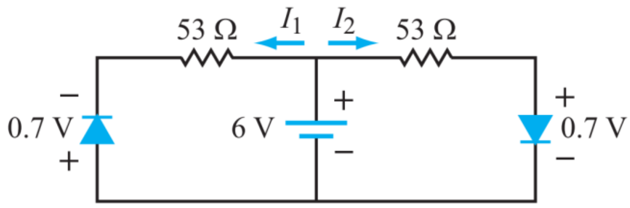
0.

3) Provide expressions in terms of step functions for the waveform



$$\upsilon_5(t) = 2u(t) + 4u(t-1) - 4u(t-3) - 2u(t-4)$$

4) Determine I_1 and I_2 in the circuit of Fig. P2.67. Assume V_F = 0.7 V for both diodes.



Solution: The diode in the left-hand loop is reverse biased, so

$$I_1 = 0.$$

In the right-hand loop, the diode is forward biased. Hence,

$$I_2 = \frac{6 - 0.7}{53} = 0.1 \text{ A}.$$

5) The excitation function for all four of the circuits shown in Figure 3. is

$$v_s(t) = 0 \qquad t < 0$$
$$v_s(t) = 10V \quad t \geq 0$$

For each of the circuits, select the time function on the right that corresponds in magnitude and shape to the output, $v_o(t)$. Assume that all capacitors and inductors have zero initial states, (the appropriate state variable is zero for t less than zero). If no matching response exists, say so and explain briefly. All responses are made up of "**straight lines**" and "**exponentials**." You may choose a time function more than once.

Solution:

(A) $\rightarrow \quad v_0(t) = 10V(1 - e^{-t/\tau})\,;\tau = R \cdot C$

(B) $\rightarrow \quad v_0(t) = 10V\left(\frac{R}{R+R}\right)(1 - e^{-}t/\tau)\,;\tau = R \cdot C$

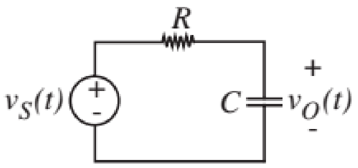
(C) $\rightarrow \quad v_0(t)$: finally $= 10V$; initially $= 0$

$v_0(t) = 10(1 - e^{-t/\tau})\,;\tau = L/R$

(D) $\rightarrow \quad \frac{V_S}{R} + C\,\frac{dV_0}{dt} = 0 \quad \Rightarrow \quad V_0 = \frac{-10}{RC} \cdot t$, within the linear region of the op. amp.

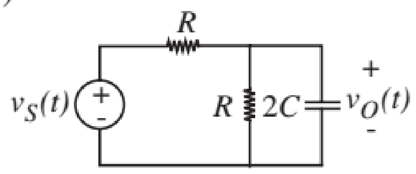
3, 7, 3, 4.

(a)



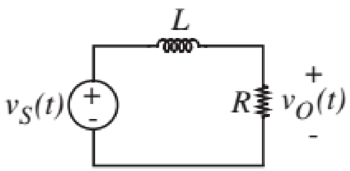
Response number ______

(b)



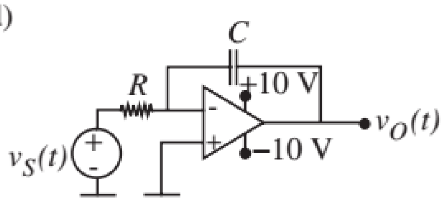
Response number ______

(c)



Response number ______

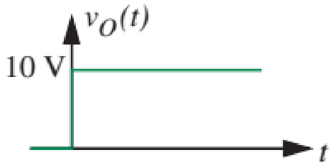
(d)



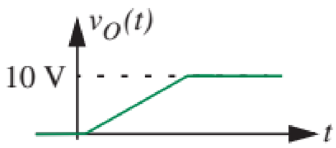
Op-amp saturation at ±10 V

Response number ______

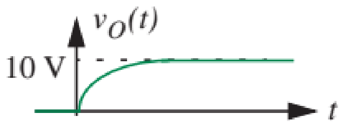
(1)



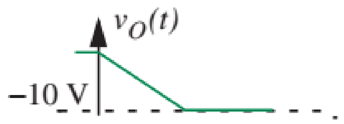
(2)



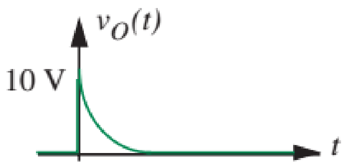
(3)



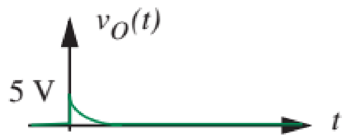
(4)



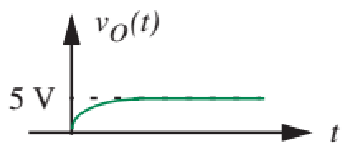
(5)



(6)



(7)



(8)

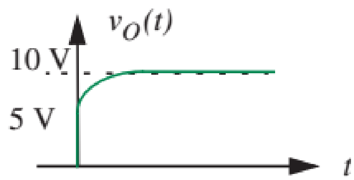
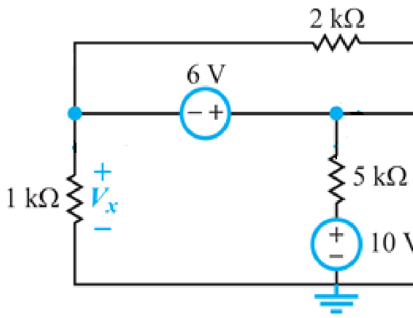


Figure 3.

Problem 2 (12 points) – Determine V_x in the circuit



Solution: The combination of nodes V_1 and V_2 constitutes a supernode. Hence, for the supernode

$$\frac{V_1}{10^3} + \frac{V_1 - V_3}{2 \times 10^3} + \frac{V_2 - 10}{5 \times 10^3} + \frac{V_2 - V_3}{6 \times 10^3} = 0 \tag{1}$$

For node V_3,

$$\frac{V_3 - V_1}{2 \times 10^3} + \frac{V_3 - V_2}{6 \times 10^3} + \frac{V_3}{4 \times 10^3} = 0. \tag{2}$$

The auxiliary equation is

$$V_2 - V_1 = 6 \text{ V}. \tag{3}$$

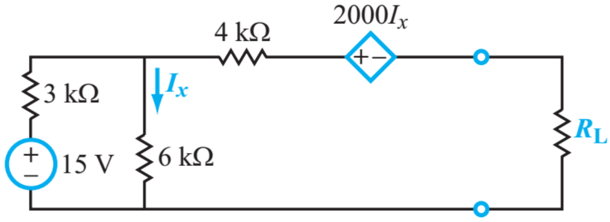
The three equations can be solved to yield

$$V_1 = 0.38 \text{ V}, \qquad V_2 = 6.38 \text{ V}, \qquad V_3 = 1.37 \text{ V}.$$

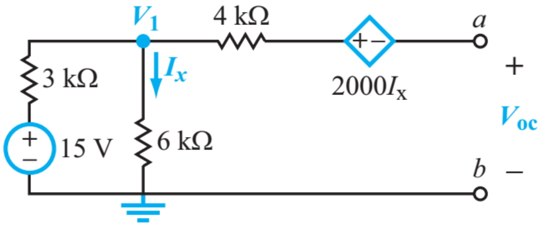
Hence,

$$V_x = V_1 = 0.38 \text{ V}.$$

Problem 3 (18 points) Determine the maximum power that can be extracted by the load resistor from the circuit shown below.



Solution: To find the Thévenin equivalent circuit, we start by determining $V_{Th} = V_{oc}$.



Voltage division:

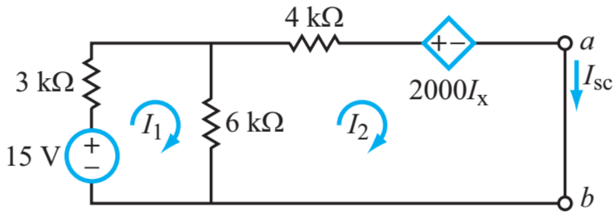
$$V_1 = \frac{15}{(3+6)\text{k}} \times 6\text{k} = 10 \text{ V}$$
$$I_x = \frac{V_1}{6\text{k}} = \frac{10}{6} \text{ mA.}$$

The dependent voltage source is:

$$2000 I_x = 2 \times \frac{10}{6} \times 10^3 \times 10^{-3} = \frac{20}{6} \text{ V.}$$

With (a, b) an open circuit, no current flows through the 4-kΩ resistor. Hence, there is no voltage drop across it.

$$V_{Th} = V_{oc} = V_1 - 2000 I_x = 10 - \frac{20}{6} = \frac{40}{6} = 6.67 \text{ V.}$$



Next, we find I_{sc}:

$$-15 + 3\text{k}I_1 + 6\text{k}(I_1 - I_2) = 0$$
$$6\text{k}(I_2 - I_1) + 4\text{k}I_2 + 2000 I_x = 0$$

Also,

$$I_x = I_1 - I_2$$

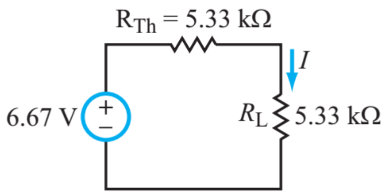
Solution yields:

$$I_1 = 2.5 \text{ mA}, \qquad I_2 = 1.25 \text{ mA}.$$

$$I_{sc} = I_2 = 1.25 \text{ mA}.$$

$$R_{Th} = \frac{V_{oc}}{I_{sc}} = \frac{6.67}{1.25 \times 10^{-3}} = 5.33 \text{ k}\Omega.$$

Hence, $R_L = 5.33$ kΩ extracts maximum power.



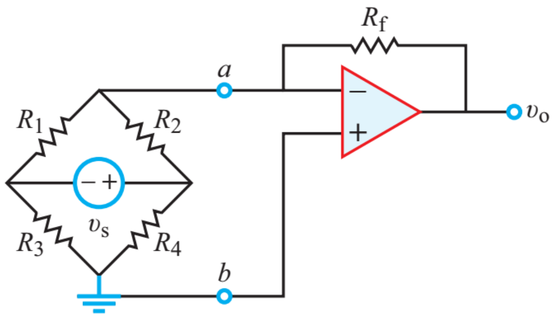
$$I = \frac{6.67}{2 \times 5.33} = 0.625 \text{ mA}$$

$$P_{max} = I^2 R_L = (0.625 \times 10^{-3})^2 \times 5.33 \times 10^3 = 2.09 \qquad \text{(mW)}.$$

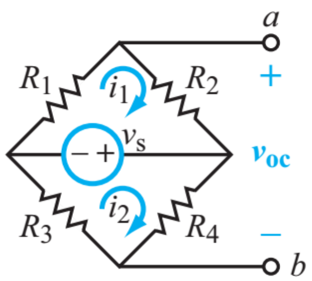
Problem 4 (18 points) – In the circuit below, a bridge circuit is connected at the input side of an inverting op-amp circuit.

a) Obtain the Thevenin equivalent at terminals (a,b) for the bridge circuit.

b) Use the result in (a) to obtain an expression for G = υ_o/υ_s.



Solution: (a) The Thévenin equivalent circuit at (a,b):



$$\upsilon_s + i_1(R_1 + R_2) = 0$$

or

$$i_1 = \frac{-\upsilon_s}{R_1 + R_2}.$$

Also,

$$-\upsilon_s + i_2(R_3 + R_4) = 0$$

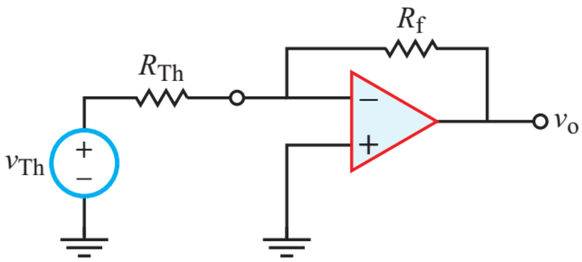
and

$$i_2 = \frac{\upsilon_s}{R_3 + R_4}.$$

$$\begin{aligned}\upsilon_{Th} = \upsilon_{oc} &= i_1 R_2 + i_2 R_4 \\ &= \frac{-\upsilon_s R_2}{R_1 + R_2} + \frac{\upsilon_s R_4}{R_3 + R_4} = \frac{[R_4(R_1 + R_2) - R_2(R_3 + R_4)]\upsilon_s}{(R_1 + R_2)(R_3 + R_4)}.\end{aligned} \tag{1}$$

Suppressing υ_s (by replacing it with a short circuit) leads to

$$\begin{aligned}R_{Th} &= (R_1 \parallel R_2) + (R_3 \parallel R_4) \\ &= \frac{R_1 R_2}{R_1 + R_2} + \frac{R_3 R_4}{R_3 + R_4} = \frac{R_1 R_2 (R_3 + R_4) + R_3 R_4 (R_1 + R_2)}{(R_1 + R_2)(R_3 + R_4)}.\end{aligned}$$

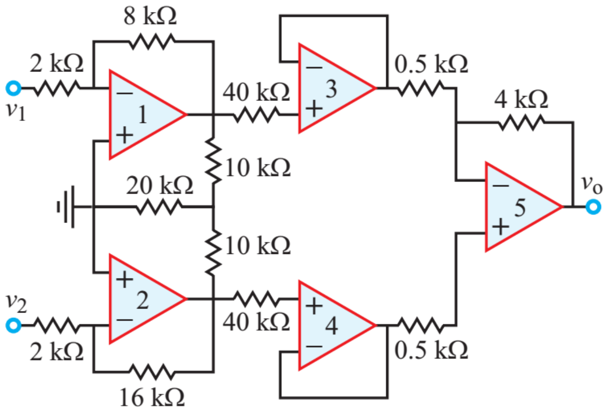


$$\upsilon_o = -\frac{R_f}{R_{Th}}\upsilon_{Th} \qquad \text{(inverting amplifier)} \tag{3}$$

Inserting Eqs. (1) and (2) into (3) leads to

$$G = \frac{\upsilon_o}{\upsilon_s} = \frac{-R_f[R_4(R_1+R_2)-R_2(R_3+R_4)]}{R_1R_2(R_3+R_4)+R_3R_4(R_1+R_2)}$$

Problem 5 (12 points) Relate v_o in the circuit to v_1 and v_2.



Solution:

Op amp 1: $$v_{o_1} = \left(-\frac{8}{2}\right) v_1 = -4v_1$$

Op amp 2: $$v_{o_2} = \left(-\frac{16}{2}\right) v_2 = -8v_2$$

Op amp 3: $v_{o_3} = v_{o_1}$ (voltage follower)

Op amp 4: $v_{o_4} = v_{o_2}$ (voltage follower)

Op amp 4: $$v_o = -\left(\frac{4}{0.5}\right) v_{o_3} + \left(\frac{4+0.5}{0.5}\right) v_{o_4} \quad \text{(difference amplifier)}$$

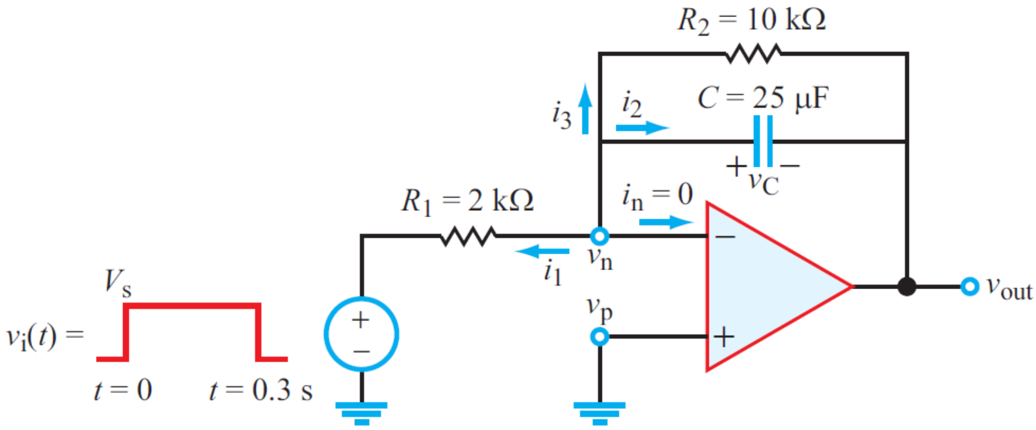
$$= 32v_1 - 72v_2.$$

Problem 6 (20 points) – First-order circuit analysis

You must show your work to get full credit.

The op-amp circuit shown in Fig. 7 is subjected to an input pulse of amplitude $V_s = 2.4V$ and duration $T_0 = 0.3s$. Determine and plot the output voltage $V_{out}(t)$ for $t \geq 0$, assuming that the capacitor was uncharged before $t = 0$.

Hint: $e^{-1.2} = 0.3$.



Your answer:

$$C\frac{dv_{out}}{dt} + \frac{v_{out}}{R_2} = -\frac{v_i(t)}{R_1}, \quad \tau = R_2C = 0.25s.$$

For $0 \leq t \leq 0.3s$, $v_{out} = -12(1 - e^{-4t})V$, $v_{out}(0.3) = -8.4V$.

For $t > 0.3s$, $v_{out} = -8.4\left(1 - e^{-4(t-0.3)}\right)V$.