# CS243: Homework #2

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# Problem 1: Game Playing

## (a) 1 credit

Alice and Bob are choosing a movie to watch together at home. They can only choose either a tragedy, a comedy or a documentary. Alice is preparing/choosing a movie and Bob going out to buy some popcorn or coke. Since the popcorn shop and coke shop are in different directions, Bob can only choose to buy one of them. Bob prefers coke to popcorn and he hates tragedy very much. Alice prefers popcorn to coke and she loves tragedy. Although she doesn't like comedy when having some coke, she enjoys watching comedy while having some popcorn very much. The payoff of the game is modelled as follows:

| Alice \ Bob | Popcorn | Coke |
|---|---|---|
| Comedy | 4,4 | 0,2 |
| Documentary | 2,0 | 2,2 |
| Tragedy | 3,0 | 1,0 |

Is there any pure strategy Nash equilibria? Is there any dominant strategy? If yes, list all of them.

## (b) 1 credit

One hour later, Alice's mum came home and brought ten chicken wings. Alice and Bob will share the ten chicken wings and they both want to have as many as possible. Therefore, they played a game to decide: Each of them tells Alice's mum privately how many chicken wings they want. Suppose Alice wants $a$ wings and Bob wants $b$ wings. If $a + b \leq 10$, they receive all what they want. If $a + b > 10$, then the person who wants fewer wings gets what he/she wants while the other receives the rest of the wings. Is (5,5) a nash equilibrium? Is it unique? Prove your answer.

## (c) 1 credit

The neighbourhood committee wants to show a great film in an opening area but needs a volunteer to arrange it with no payment. Suppose the volunteer have a utility of 3 while the other people who watch the film will have a utility of 5. Suppose the neighbourhood has a very large number of population. Is there any Nash equilibrium (including mixed strategies)? Explain your answer.
Assumption: we assume that all people apply the same strategy.

# Problem 2: Auctions

## (a)1 credit

Tom wants to hire a temporary worker to do some work. Each worker has a private cost for doing the work. Help Tom to design a Vickrey-like auction in which workers report their costs and the auction chooses a worker and a payment. Truthful reporting should be a dominant strategy in the auction and the auction should also minimizes the cost for the hiring.

## (b)1 credit

Tom is selling $k$ identical copies of a digital good (e.g. a software). Suppose that there are $n > k$ bidders and each bidder wants at most one copy. Can you help Tom to design a second-price-like auction? Prove that your auction design is truthful.

### (c)1 credit

Consider an auction for selling one item: the item is allocated to the highest bidders and all bidders have to pay what they have bid. Is this auction truthful, efficient and individually rational? Give the proof or counter examples.

## Problem 3: VCG

An auctioneer is selling five items $M = a, b, c, d, e$. There are seven buyers $\{1, 2, 3, 4, 5, 6, 7\}$ with the following seven bids.
$B_1 = (a, b, 9)$,
$B_2 = (b, e, 12)$,
$B_3 = (c, d, e, 10)$,
$B_4 = (c, 6)$,
$B_5 = (a, c, 13)$,
$B_6 = (b, c, e, 16)$,
$B_7 = (b, d, e, 18)$.

### (a)1 credit

What is the allocation of applying VCG?

### (b)1 credit

What are their payments under VCG (each buyer pays the harm he caused to the others due to his participation)?

## Problem 4: Social choice

Consider there are 5 candidates $a, b, c, d, e$ for the president of a student union in ShanghaiTech with the following five preferences from five voters:

$$a \succ_1 b \succ_1 c \succ_1 d \succ_1 e$$

$$c \succ_2 a \succ_2 d \succ_2 e \succ_2 b$$

$$b \succ_3 c \succ_3 d \succ_3 a \succ_3 e$$

$$c \succ_4 b \succ_4 d \succ_4 a \succ_4 e$$

$$d \succ_5 b \succ_5 c \succ_5 a \succ_5 e$$

### (a)0.5 credit

Consider a social choice function that assigns 1 score to each candidate ranked at top-3 in each preference and 0 score to the rest, and choose the winner with the highest total score (with random tie-breaking). Who will be the winner?

### (b)1.5 credit

Is the above social choice function truthful? Give the proof or counter examples.