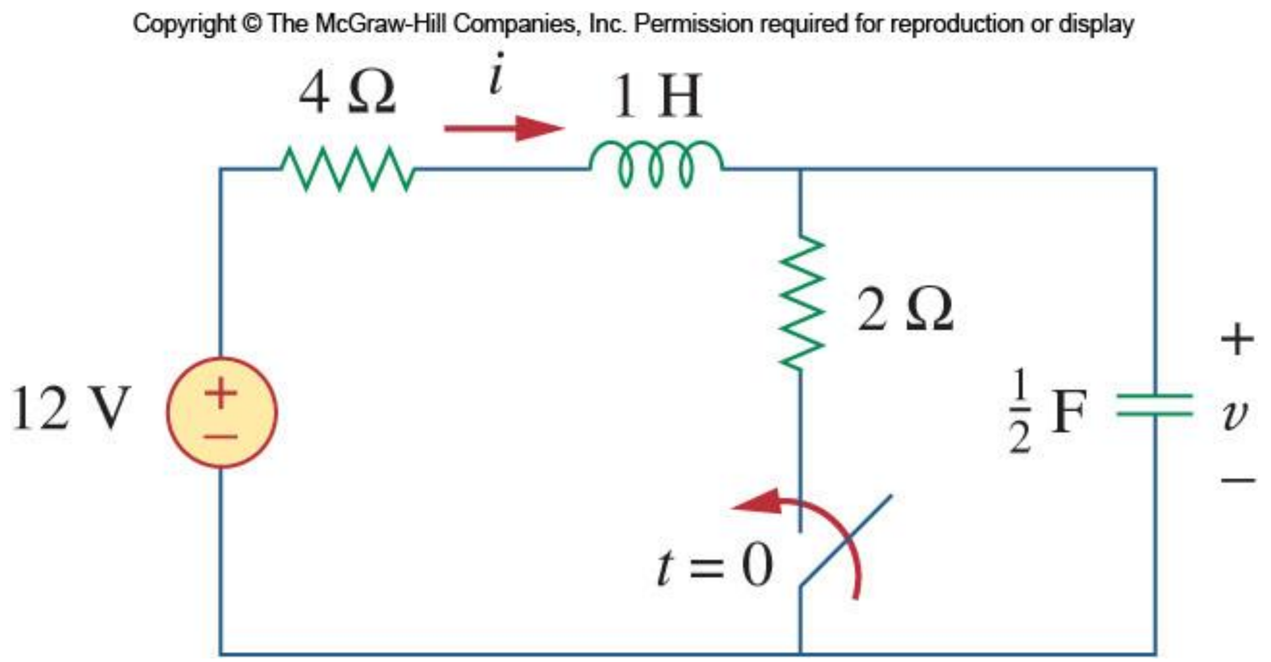
General Second-Order Circuits

- An example



General Second-Order Circuits

- The principles of solving the series/parallel forms of RLC circuits can **be applied** to **general second-order circuits**, by taking the following six steps:

  1. First determine the initial conditions, $x(0)$ *and* $dx(0)/dt$.

  2. Applying KVL and KCL, to find the general second-order differential equation to describe $x(t)$. 3.**Depending on the roots of C.E. , the form of the general solution $x_{g.s.}(t)$ (3 cases) of homogeneous equation can be determined.**

  4. We obtain the **particular solution** by observation/calculation, **specially** for a DC/step response

$$x_{p.s.}(t)=x(\infty)$$

  5. The total response = general solution + particular solution.

$$X(t)=x_{p.s.}(t)+ x_{g.s.}(t)$$

  6. Using the initial conditions to determine the constants of $X(t)$.

General solution for second-order circuits for $t \geq 0$.

$x(t)$ = unknown variable (voltage or current)

Differential equation: $x'' + ax' + bx = c$

Initial conditions: $x(0)$ and $x'(0)$

Final condition: $x(\infty) = \dfrac{c}{b}$

$$\boxed{\alpha = \frac{a}{2} \qquad \omega_0 = \sqrt{b}}$$

Overdamped Response $\alpha > \omega_0$

$$x(t) = [A_1 e^{s_1 t} + A_2 e^{s_2 t} + x(\infty)]$$

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} \qquad s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$

[redacted]

Critically Damped $\alpha = \omega_0$

$$x(t) = [(B_1 + B_2 t)e^{-\alpha t} + x(\infty)]$$

[redacted]

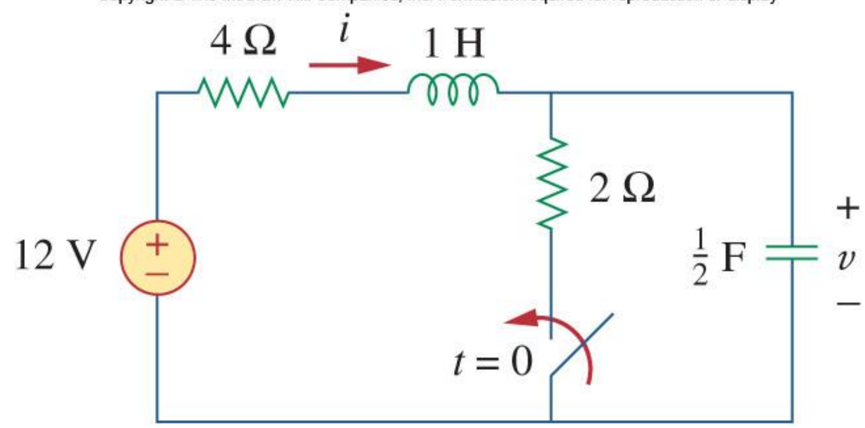
Underdamped $\alpha < \omega_0$

$x(t) = [D_1 \cos \omega_d t + D_2 \sin \omega_d t$ [redacted] $]e^{-\alpha t} + x(\infty)$

[redacted]

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$$

General RLC Circuits

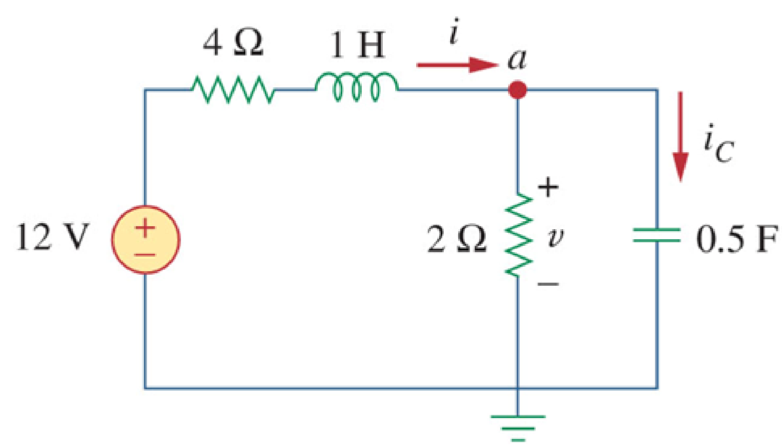


- Find the complete response $v(t)$ for $t > 0$ in the circuit.

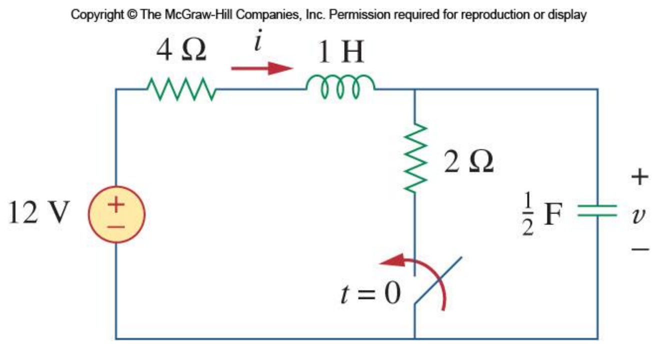
1. Initial conditions

$$v(0^+) = v(0^-) = 12V,\ i(0^+) = i(0^-) = 0$$

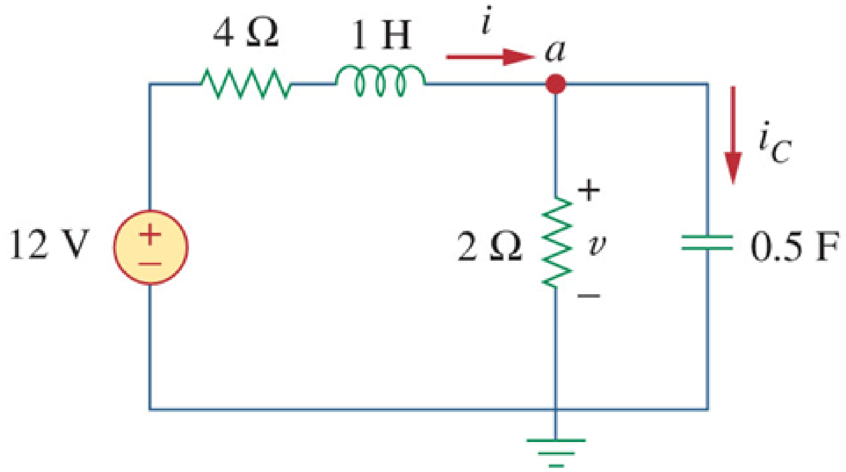
$$\frac{dv(0^+)}{dt} = \frac{i_C(0^+)}{C} = \frac{-12/2}{0.5} = -12\ V/s$$



General RLC Circuits



- Find the complete response $v(t)$ for $t > 0$ in the circuit.



2. KCL at node *a*: $i = \frac{v}{2} + 0.5\frac{dv}{dt}$

   KVL on left mesh: $4i + 1\frac{di}{dt} + v = 12$

$$\Rightarrow \quad \frac{d^2v}{dt^2} + 5\frac{dv}{dt} + 6v = 24$$

$$\frac{d^2v}{dt^2} + 5\frac{dv}{dt} + 6v = 24$$

3. General Solution:

⇨ General Solution $v_t(t) = A_1e^{-2t} + A_2e^{-3t}$

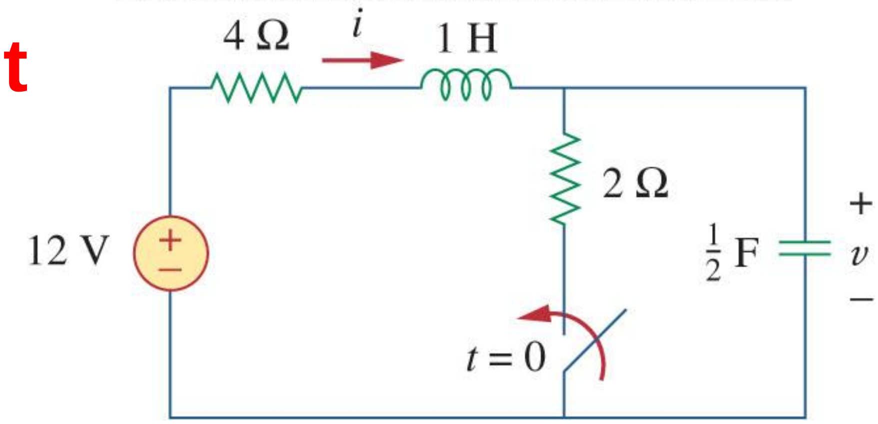
4. Particular Solution : Steady-state response $v_{ss}(t) = 4V$

5. Put together : $v(t) = 4 + \mathrm{A}_1e^{-2t} - \mathrm{A}_2e^{-3t}$

6. Using initial conditions to determine A_1 , A_2

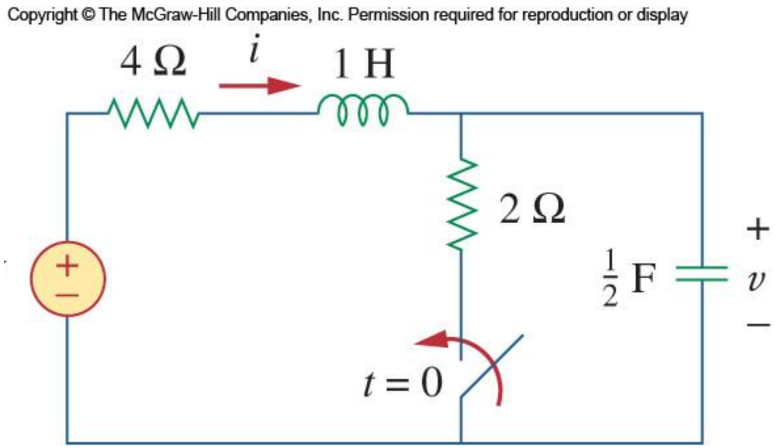
Self-test-General RLC Circuit

- Find the complete response $i(t)$ for $t > 0$ in the circuit.



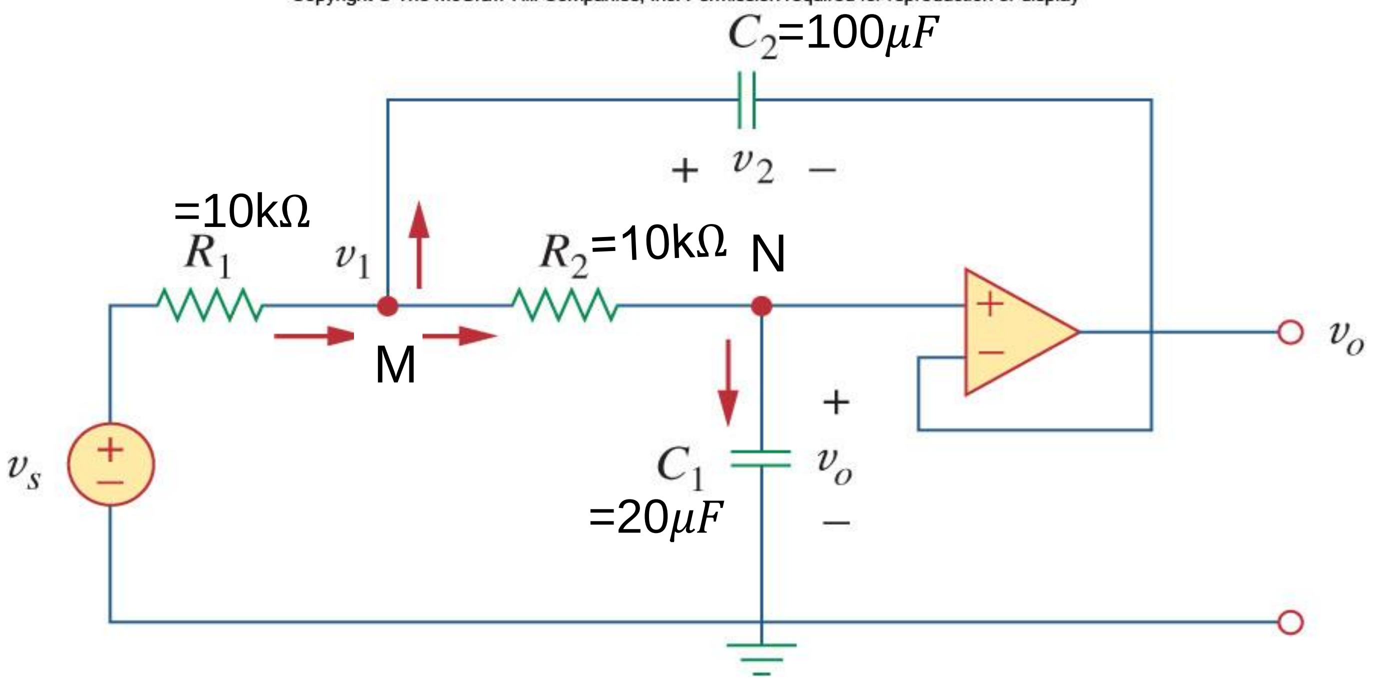
Find *v(t) for t>0*

$e^{-t}u(t)$



Example of 2nd-order op-amp circuits

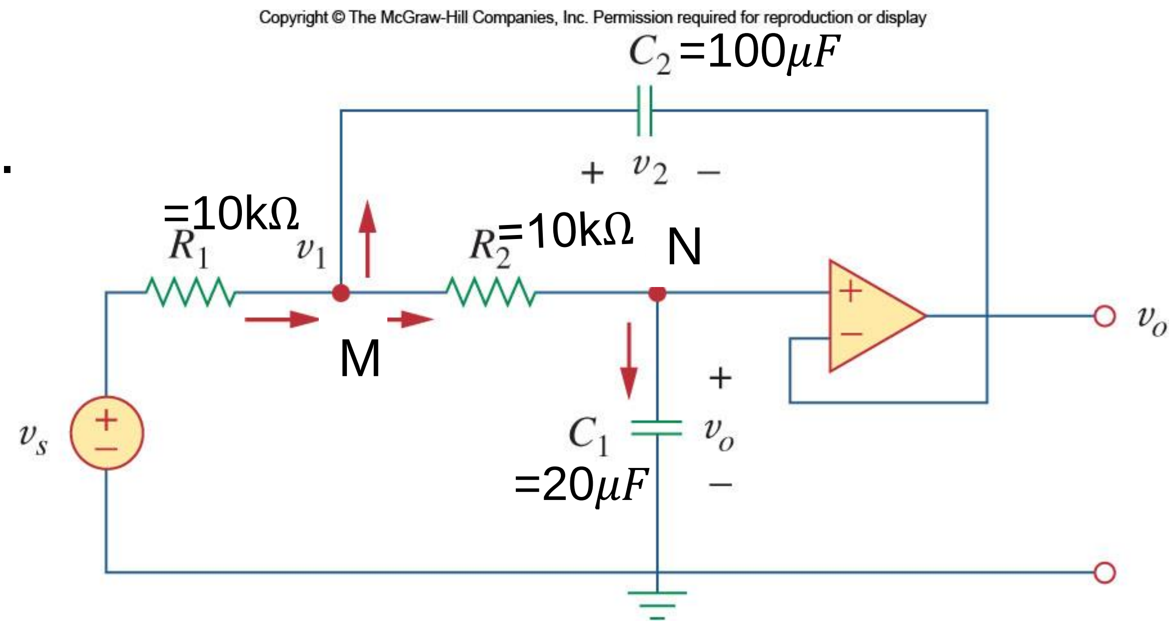
- Find v_o for $t > 0$
  when $v_s = 10u(t)mV$.



Initial conditions: $v_o(0^+) = 0$, $C_1 \frac{dv_o(0^+)}{dt} = \frac{v_1(0^+) - v_o(0^+)}{R_2} = \frac{v_2(0^+)}{R_2} = 0$

Example of 2nd-order op-amp circuits

- Find v_o for $t > 0$ when $v_s = 10u(t)mV$.



KCL at node M:

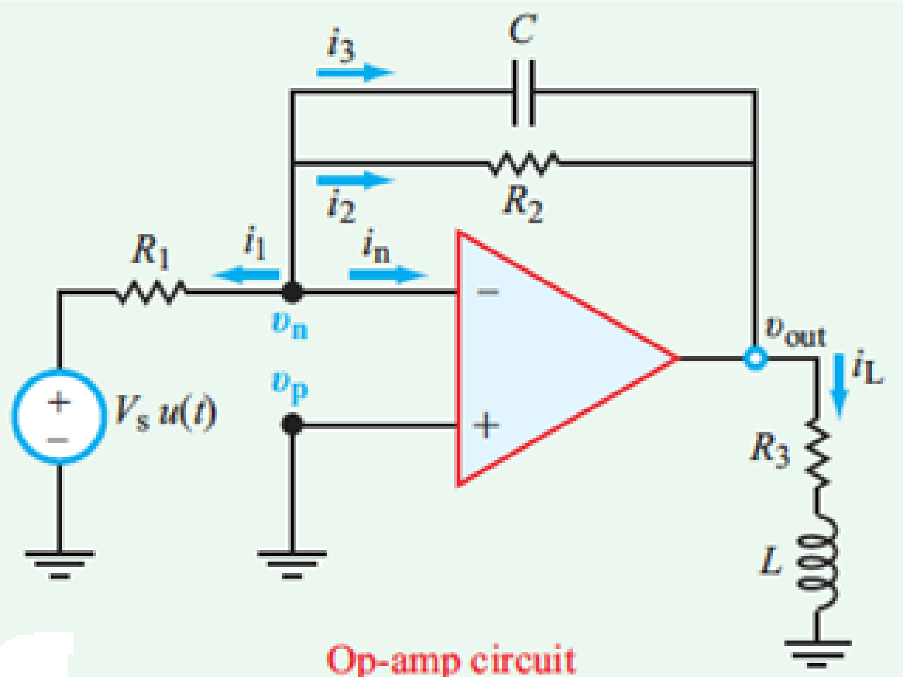
$$\frac{v_s - v_1}{R_1} = C_2\frac{dv_2}{dt} + \frac{v_1 - v_o}{R_2}$$

KCL at node N:

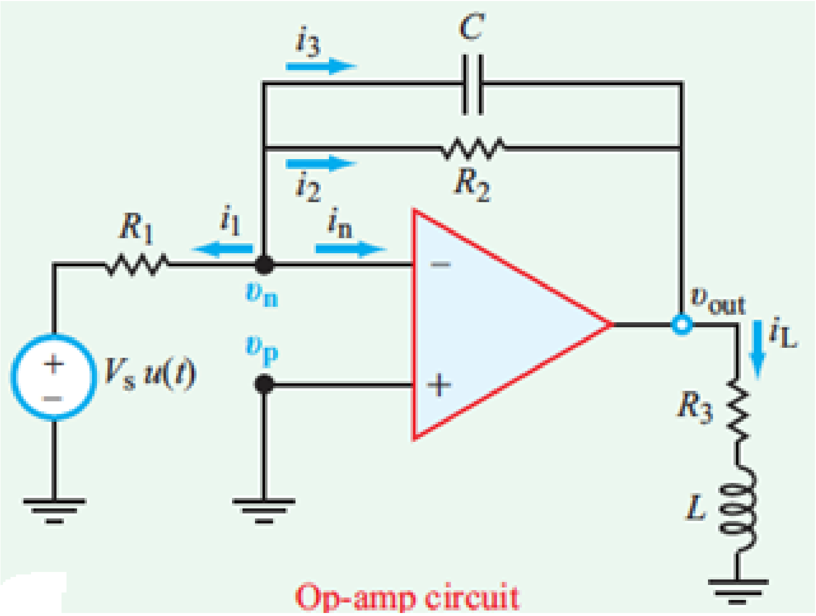
$$C_1\frac{dv_o}{dt} = \frac{v_1 - v_o}{R_2} \qquad and\ we\ have\ v_1 - v_2 = v_o$$

$$\Rightarrow \quad \frac{d^2v_o}{dt^2} + \left(\frac{1}{R_1C_2} + \frac{1}{R_2C_2}\right)\frac{dv_o}{dt} + \frac{v_o}{R_1R_2C_1C_2} = \frac{v_s}{R_1R_2C_1C_2}$$

Example-2
find current through the inductor for t>0



Example-2



$$i_L(0) = i_L(0^-) = 0, \quad i_L'(0) = \frac{1}{L}\,v_L(0) = 0.$$

$$\frac{R_3}{R_2}\,i_L + \left(\frac{L}{R_2} + R_3C\right)\frac{di_L}{dt} + LC\,\frac{d^2 i_L}{dt^2} = -\frac{V_s}{R_1}$$

Example-3

In the op amp circuit shown in Fig. 8.34, $v_s = 10u(t)$ V, find $v_o(t)$ for $t > 0$. Assume that $R_1 = R_2 = 10\ \text{k}\Omega$, $C_1 = 20\ \mu\text{F}$, and $C_2 = 100\ \mu\text{F}$.