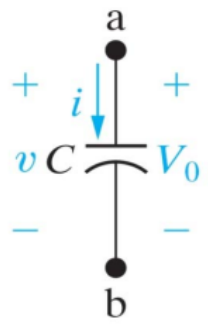
Lecture 14
-- Laplace Transform in Circuit Analysis

Introduction

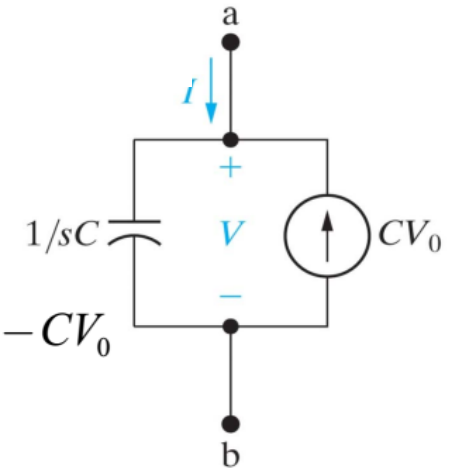
- In this lecture, we introduce the concept of modeling circuits in the *s* domain using the Laplace transform.

- The elegance of using the Laplace transform in circuit analysis lies in the automatic inclusion of initial conditions in the transformation process, thus providing a complete (transient and steady-state) solution.

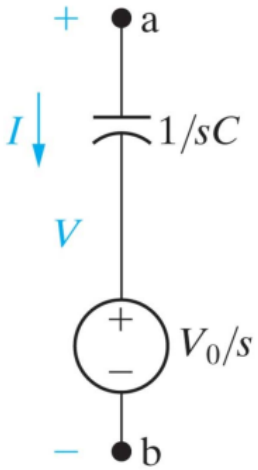
S-domain circuit models for a capacitor



$$i(t) = C\frac{dv(t)}{dt} \Rightarrow I(s) = sCV(s) - CV_0$$



$$V(s) = \frac{1}{sC}I(s) + \frac{V_0}{s}$$



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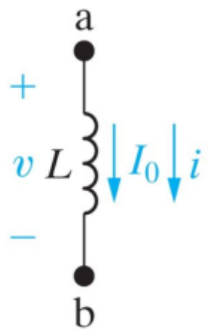
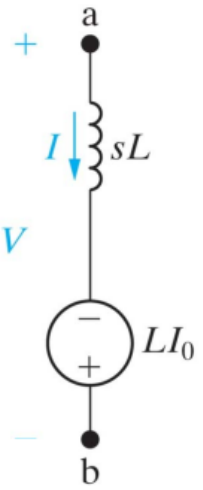
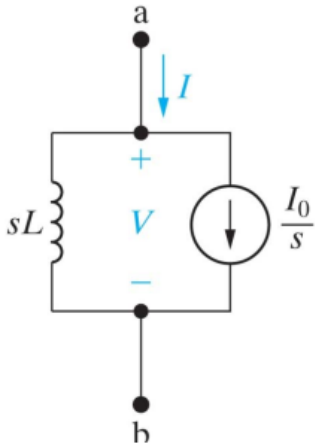
S-domain circuit models for an inductor



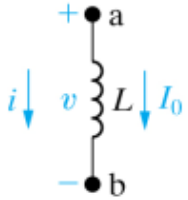
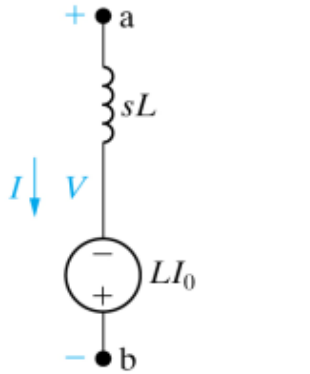
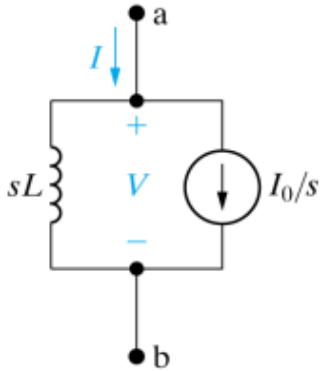
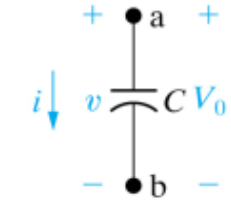
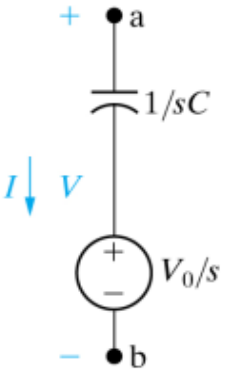
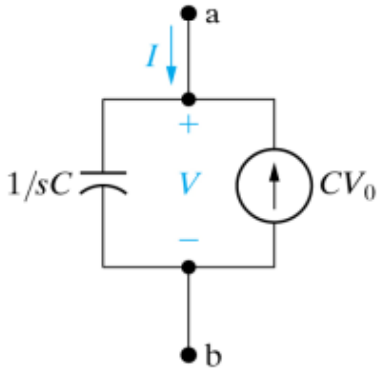
$$v(t) = L\frac{di(t)}{dt} \quad \Rightarrow \quad V(s) = sLI(s) - LI_0$$



$$I(s) = \frac{1}{sL}V(s) + \frac{I_0}{s}$$



Summary

Time domain	s-domain
 $v = Ri$	 $V = RI$
 $v = L\,di/dt,$ $i = \frac{1}{L}\int_{0^-}^{t} v\,dx + I_0$	 $V = sLI - LI_0$  $I = \frac{V}{sL} + \frac{I_0}{s}$
 $i = C\,dv/dt,$ $v = \frac{1}{C}\int_{0^-}^{t} i\,dx + V_0$	 $V = \frac{I}{sC} + \frac{V_0}{s}$  $I = sCV - CV_0$

Dependent Sources

- The models for dependent sources are easy to develop, drawing from the simple fact that if the Laplace transform of *f(t)* is *F(s)*, then the Laplace transform of *af*(*t*) is *aF*(*s*) — the linearity property.

$$\mathcal{L}[av(t)] = aV(s)$$

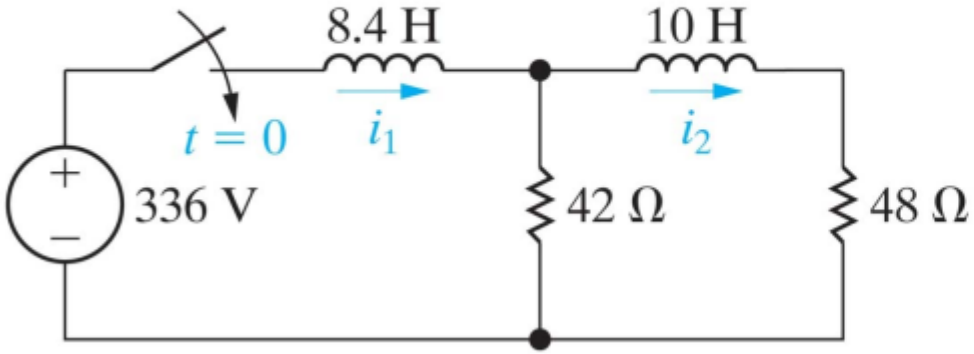
$$\mathcal{L}[ai(t)] = aI(s)$$

Steps in Applying the Laplace transform

- Transform the circuit from the time domain to the Laplace (*s*) domain, including possible initial conditions.
- Solve the circuit using nodal analysis, mesh analysis, source transformation, superposition, or any other analysis technique with which we are familiar.
- Take the inverse transform of the solution and thus obtain the solution in the time domain.

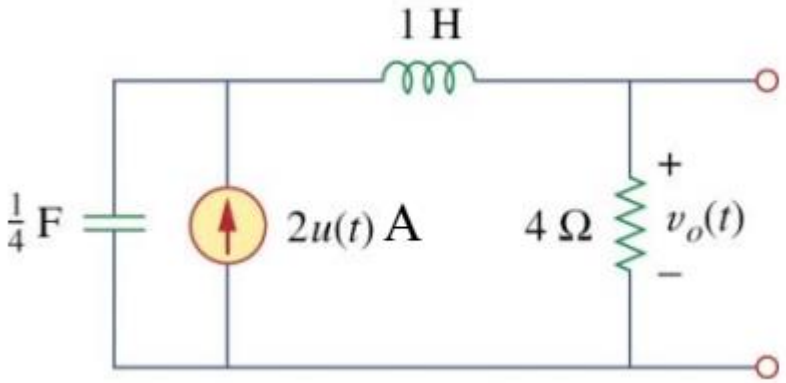
Example 1

Assuming no initial energy storage, find $i_1(t)$ and $i_2(t)$ for $t > 0$.



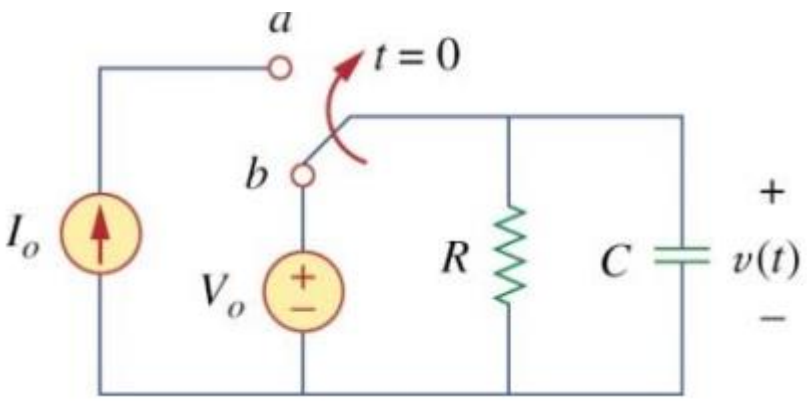
Example 2

Determine $v_0(t)$ *for* $t>0$ *assuming zero initial conditions:*



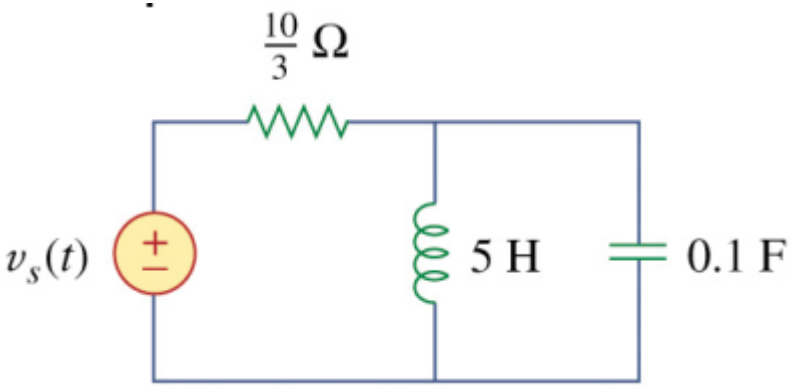
Example 3

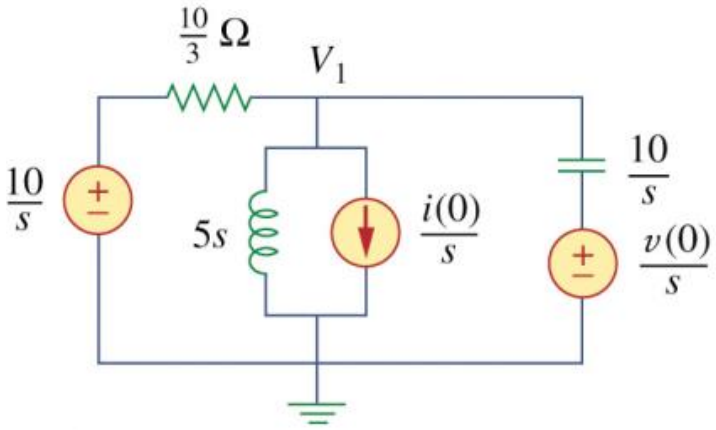
- The switch has been in position *b* for a long time. It is moved to position *a* at $t = 0$. Determine $v(t)$ for $t > 0$.



Example 4

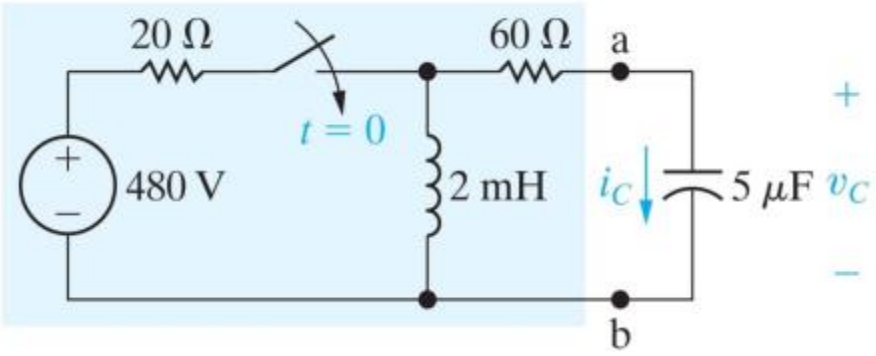
- Find the voltage across the capacitor assuming that *vs*(*t*) = 10*u*(*t*) V, and assume that at *t* = 0, *-1 A* flows through the inductor and +5 V is across the capacitor.





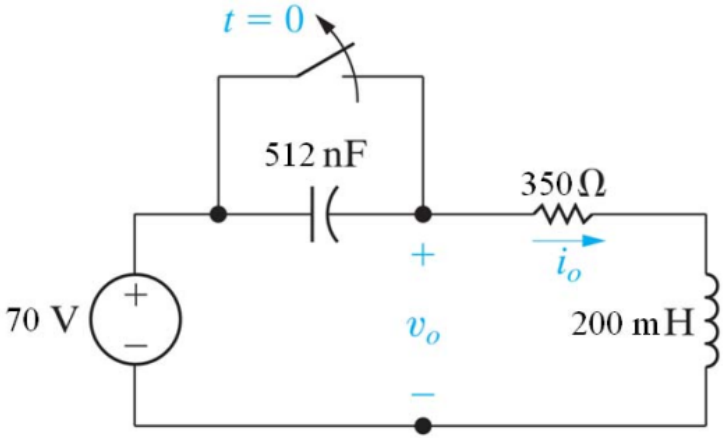
Example 5

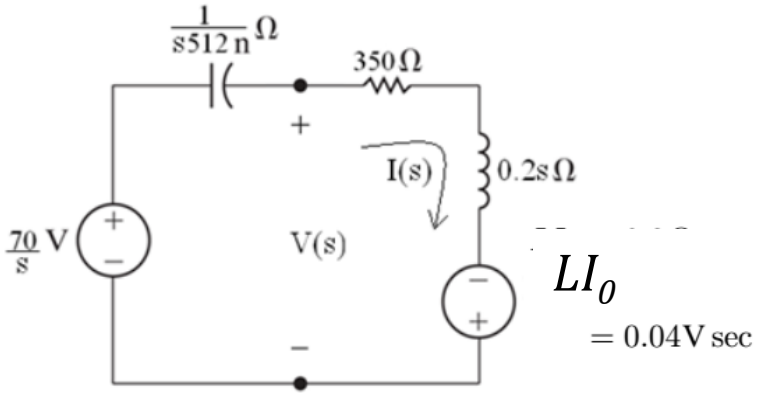
- Use Thevenin's equivalent circuit wrt terminals *a*-*b* to find current $i_C(t)$.



Example 6

Laplace transform the circuit and solve for $V_0(s)$.





$LI_0 = 0.04\text{V sec}$

Example 7

- There is no initial energy stored in this circuit. Find $i(t)$ if
- $v(t) = e^{-0.6t} \sin 0.8t$ V.