ShanghaiTech University

CS101 Algorithms and Data Structures

Fall 2020

Homework 20

Due date: 23:59, November 30, 2019

1. Please write your solutions in English.
2. Submit your solutions to gradescope.com.
3. Set your FULL Name to your Chinese name and your STUDENT ID correctly in Account Settings.
4. If you want to submit a handwritten version, scan it clearly. Camscanner is recommended.
5. When submitting, match your solutions to the according problem numbers correctly.
6. No late submission will be accepted.
7. Violations to any of the above may result in zero grade.

Problem 1: Multiple Choices

Multiple Choices: Each question has one or more correct answer(s). Select all the correct answer(s). For each question, you get 0 point if you select one or more wrong answers, but you get half points if you select a non-empty subset of the correct answers.
Note that you should write you answers of Problem 1 in the table below.

Q(1)	Q(2)	Q(3)
CD	BCD	AC

(1) Topological Sort can be carried out on what kinds of graphs:

(A) All weight acyclic graphs

(B) All directed graphs

(C) All (Directed) Trees

(D) All unweight directed acyclic graphs

(E) All cyclic directed graphs

(2) Which of the following statement about Dijkstra's algorithm is/are true?

(A) Dijkstra's algorithm could only find the shortest path on directed graph.

(B) Dijkstra's algorithm could only find the shortest path on non-negative weighted graph.

(C) If we add a constraint that each node can only pass once, then Dijkstra's algorithm could find the shortest path on non-negative weighted graph.

(D) If we add a constraint that each edge can only pass once, then Dijkstra's algorithm could find the shortest path on non-negative weighted graph.

(3) Which of the following statement about Dijkstra's algorithm is/are true?

(A) Dijkstra's algorithm with a binary heap could run in time $\Theta((|V| + |E|) \log |V|)$.

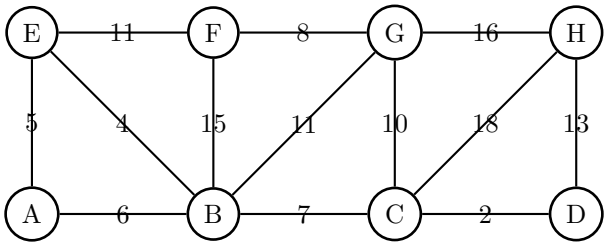
(B) Dijkstra's algorithm on a tree with a binary heap could run in time $\Theta(|E| + |V|)$.

(C) Dijkstra's algorithm on connected graphs with a binary heap could run in time $\Theta(|E| \log |V|)$.

(D) None of above.

Problem 2: Dijkstra's algorithm

Given a weighted graph below, please run Dijkstra's algorithm using vertex A as the source. Write down the vertices in the order which they are marked and the updated distances at each step. Please use ∞ to represent initial value.



Solution:

step	vertex
1	A
2	E
3	B
4	C
5	D
6	F
7	G
8	H

step	dist[A]	dist[B]	dist[C]	dist[D]	dist[E]	dist[F]	dist[G]	dist[H]
1	0	6	∞	∞	5	∞	∞	∞
2	0	6	∞	∞	5	16	∞	∞
3	0	6	13	∞	5	16	17	∞
4	0	6	13	15	5	16	17	31
5	0	6	13	15	5	16	17	28
6	0	6	13	15	5	16	17	28
7	0	6	13	15	5	16	17	28
8	0	6	13	15	5	16	17	28

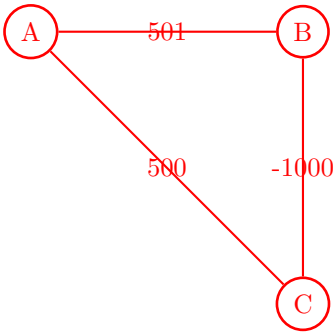
Problem 3: Providing Counterexample

For the following three statements, please provide counterexamples to illustrate that these three statements are pseudo-proposition.

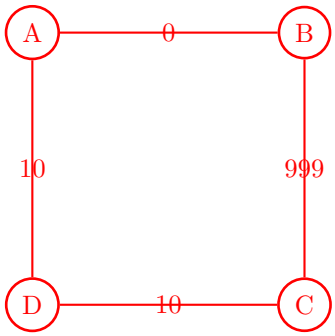
(a) A directed graph with |V| vertices and |V|-1 edges has a unique Topological sort result.

not connected

(b) If G is a connected and undirected graph without negative cycles, we can apply Dijkstra's algorithm to find the shortest path.

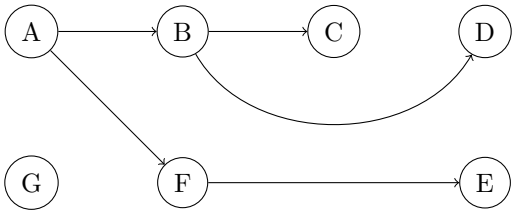


(c) Assume that we use the priority queue to implement Dijkstra's algorithm on a non-negative graph, then we can do the following operation. When the terminate vertex is pushed into the priority queue, we can stop the algorithm so that we can get the shortest path without running the algorithm until popping the terminate vertex from the priority queue



dist[C] 's final value is 20. But after expanding B and pushing C, dist[C]=999

Problem 4: Topological sort counting



(a) Show a topological sort result.

ABCDFEG

(The answer is not unique)

(b) Count the number of different topological sort result (show your process).

$10 \times 2 \times 7 = 140$