# Image alignment

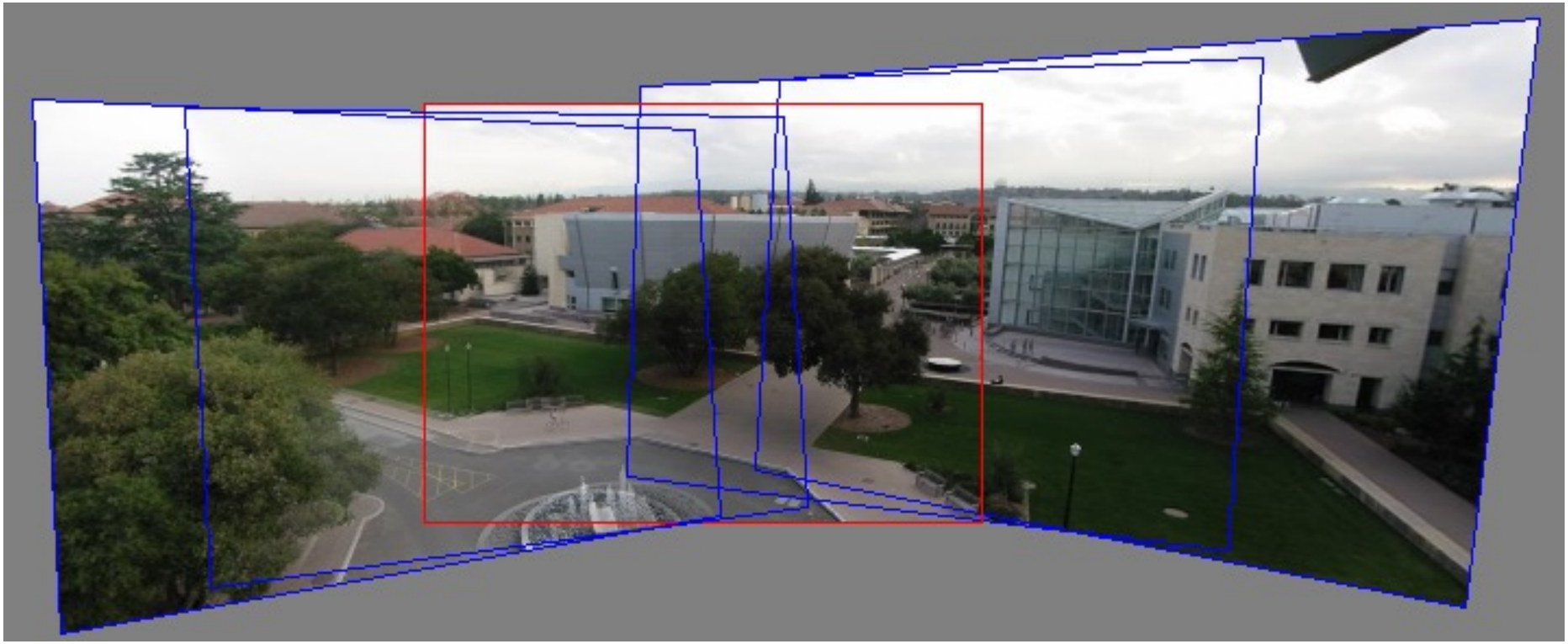
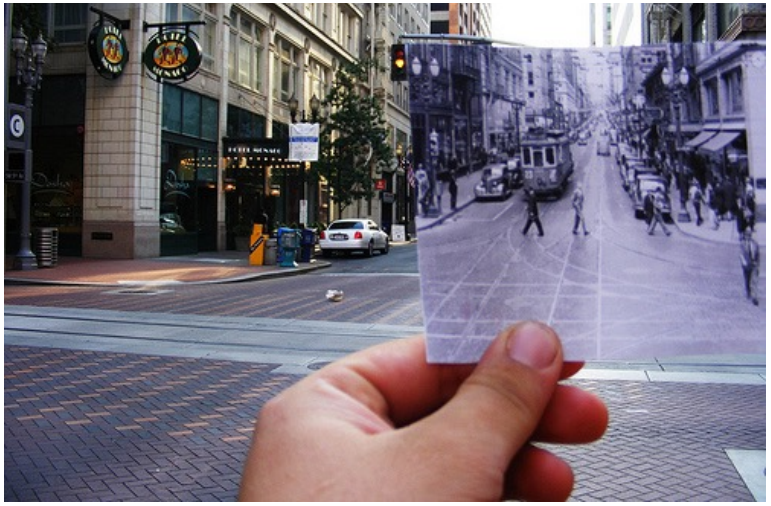
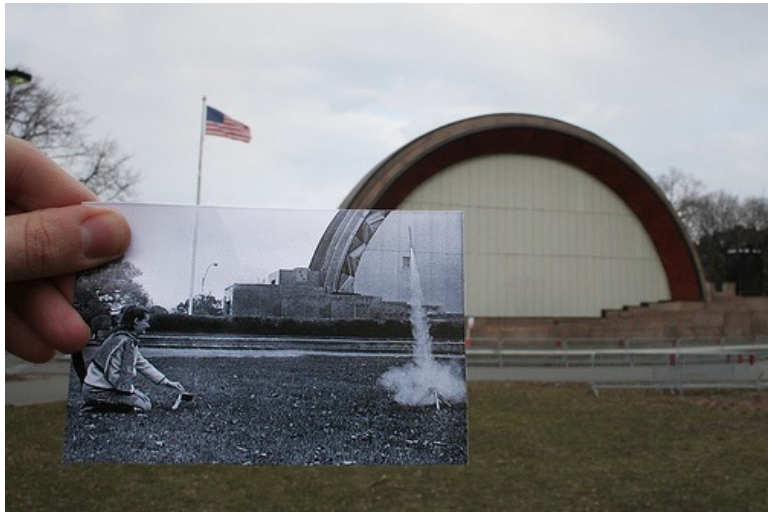


Image from http://graphics.cs.cmu.edu/courses/15-463/2010_fall/

# Alignment applications

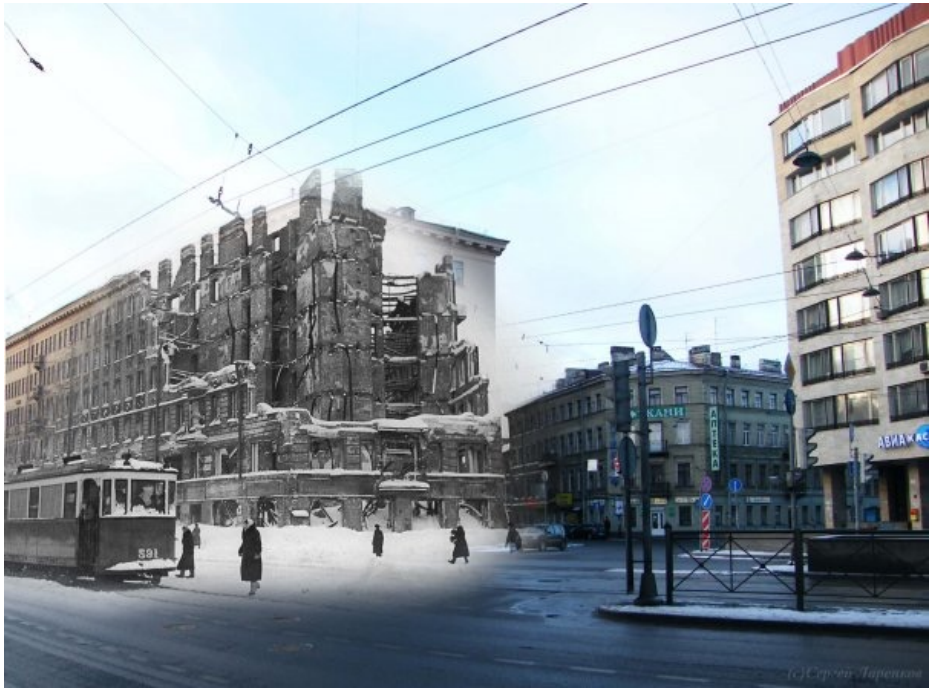
- A look into the past





# Alignment applications

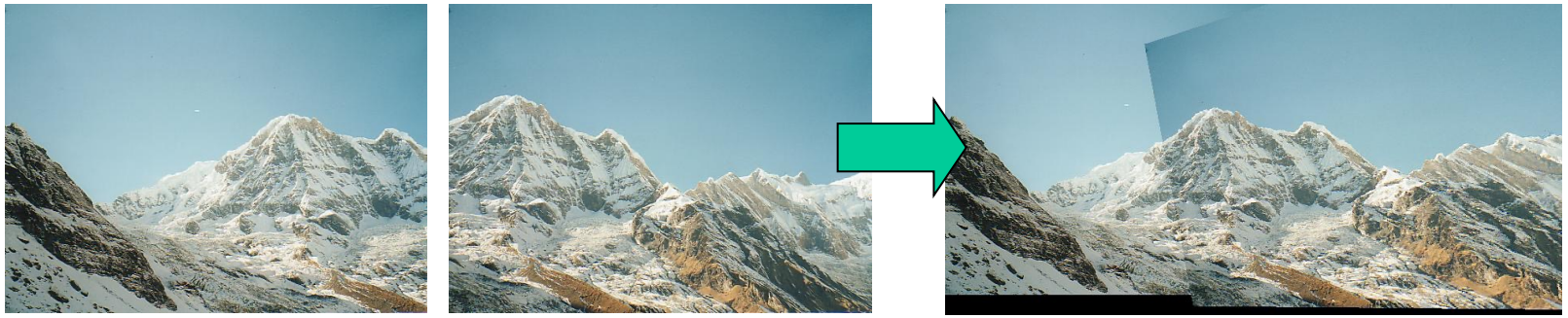
- A look into the past



# Alignment applications

- Streetside images

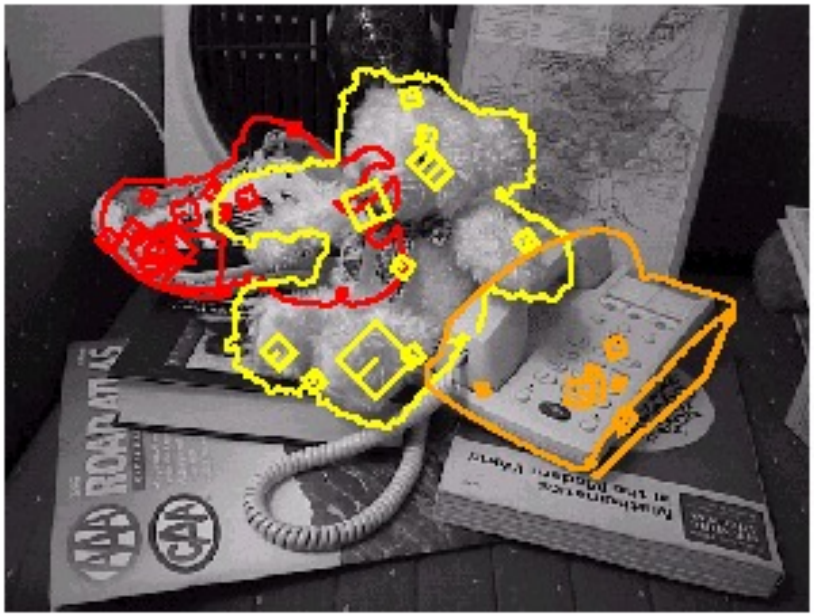

# Alignment applications



Panorama stitching

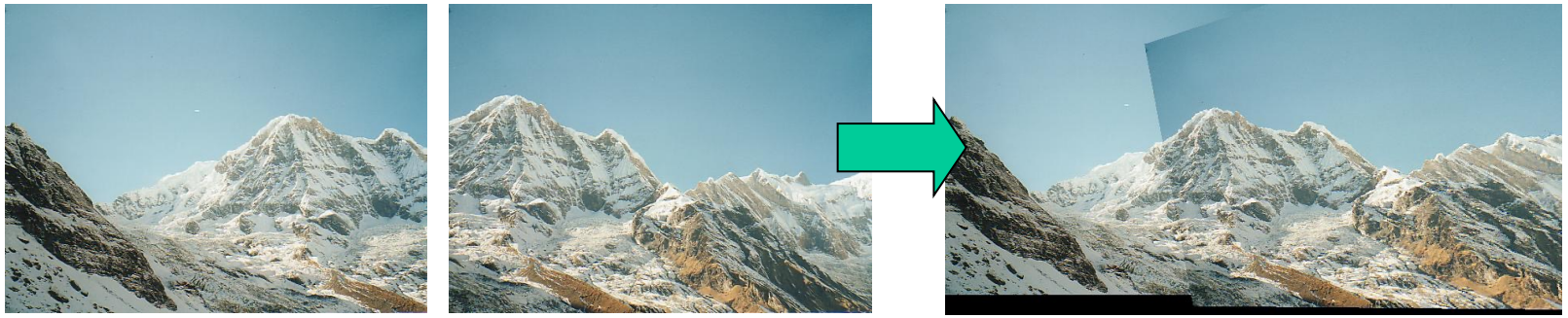


# Alignment applications

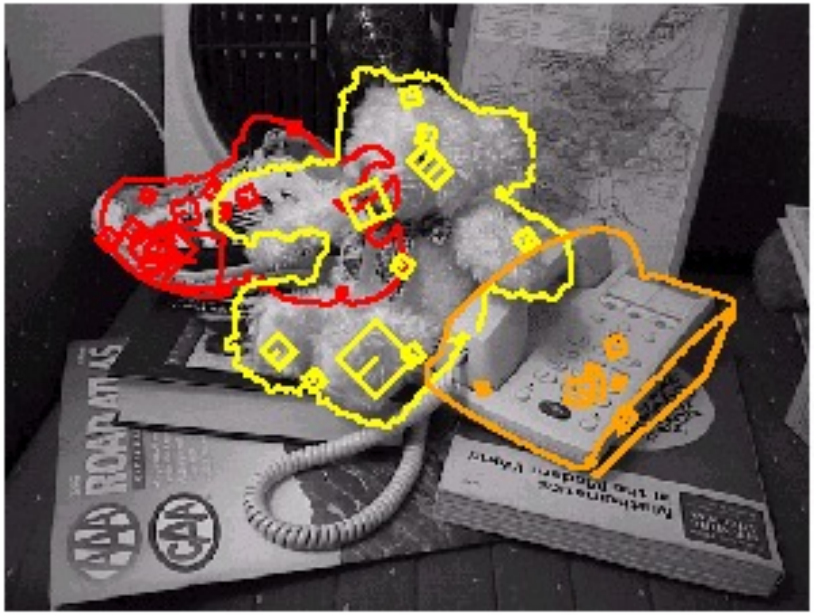


Recognition of object instances

# Alignment challenges



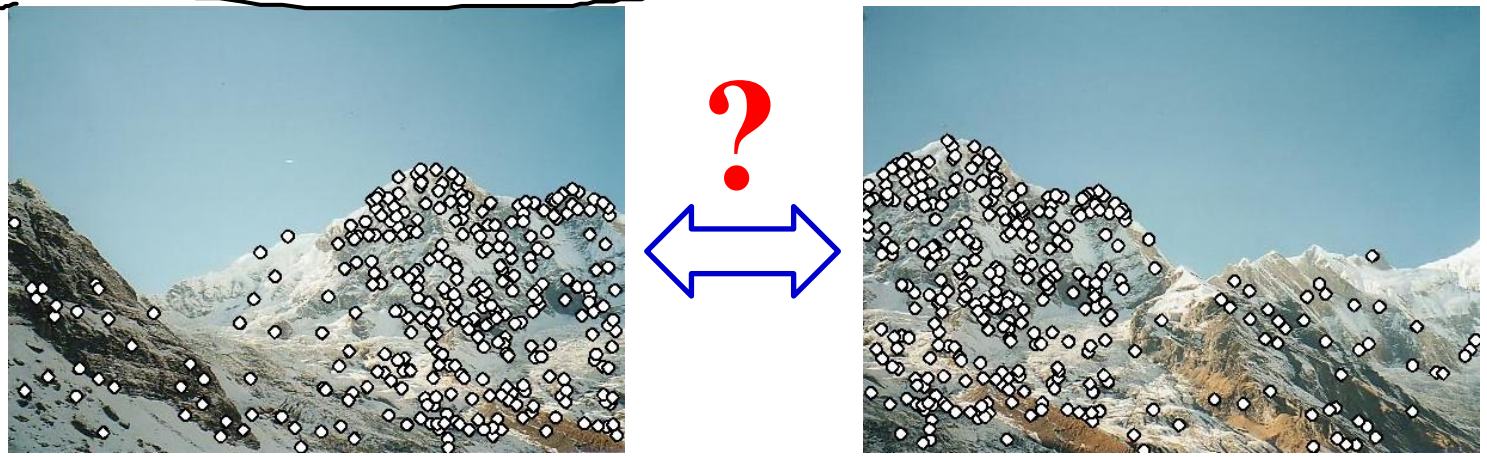
Small degree of overlap
Intensity changes



Occlusion, clutter

# Feature-based alignment

- Search sets of feature matches that agree in terms of:
  a) ~~Local appearance~~
  b) Geometric configuration

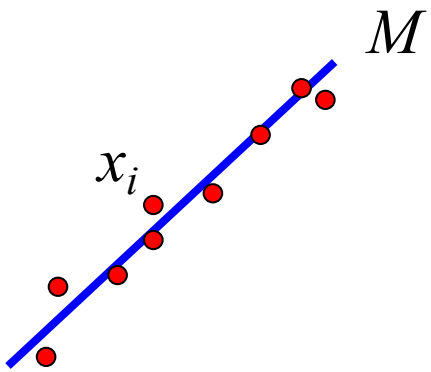



# Feature-based alignment: Overview

- Alignment as fitting
  - Affine transformations
  - Homographies
- Robust alignment
  - Descriptor-based feature matching
  - RANSAC

# Alignment as fitting

- Previous lectures: fitting a model to features in one image

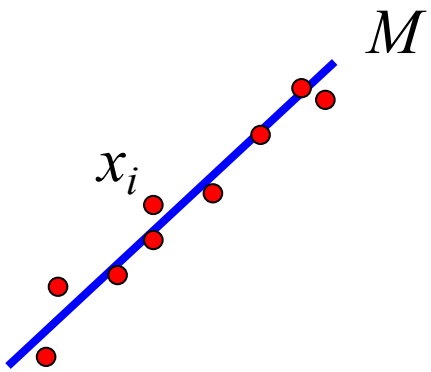



Find model *M* that minimizes

$$\sum_i \text{residual}(x_i, M)$$

# Alignment as fitting

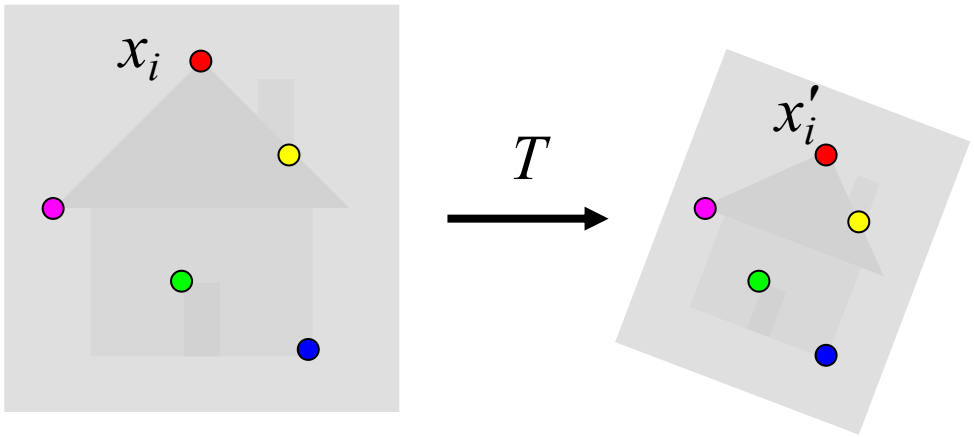
- Previous lectures: fitting a model to features in one image




Find model $M$ that minimizes

$$\sum_i \text{residual}(x_i, M)$$

- Alignment: fitting a model to a transformation between pairs of features (*matches*) in two images

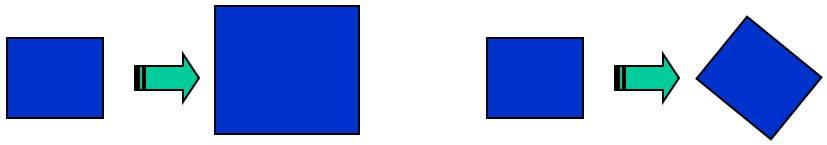



Find transformation $T$ that minimizes

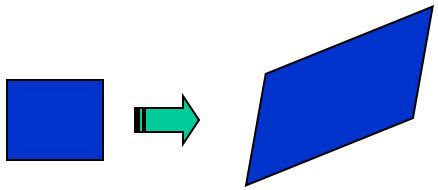
$$\sum_i \text{residual}(T(x_i), x_i')$$

# 2D transformation models

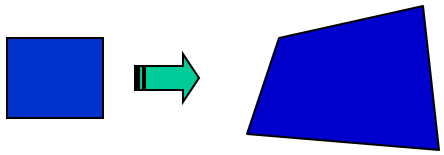
- Similarity (translation, scale, rotation)



- Affine

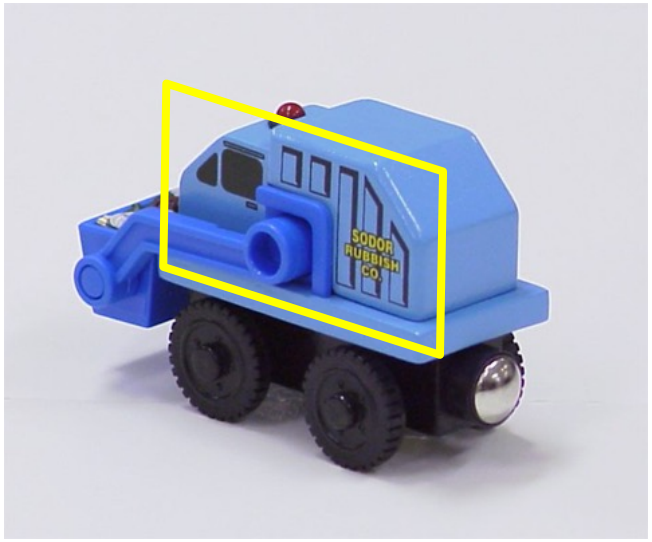


- Projective (homography)



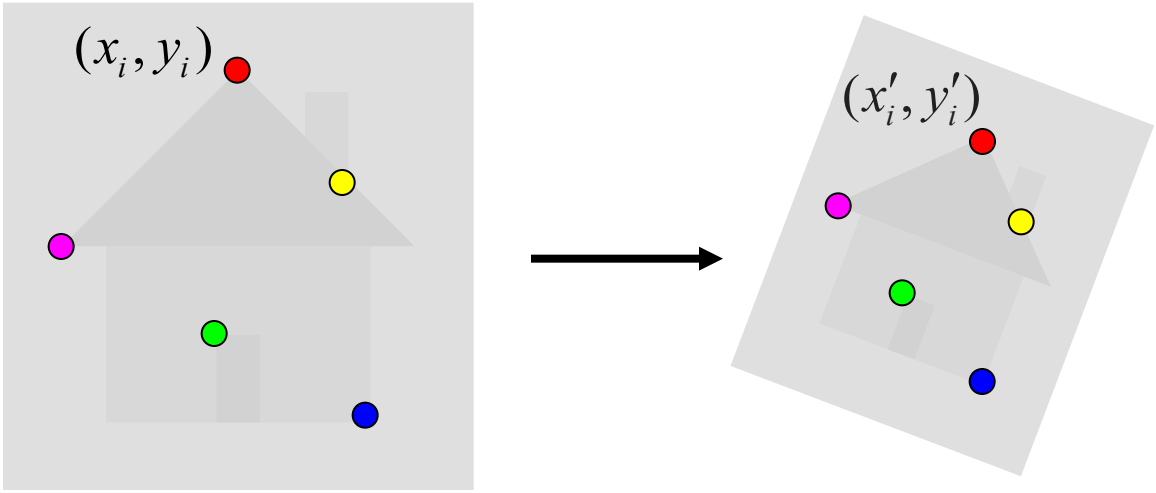
# Let's start with affine transformations

- Simple fitting procedure (linear least squares)
- Approximates viewpoint changes for roughly planar objects and roughly orthographic cameras
- Can be used to initialize fitting for more complex models

# Fitting an affine transformation

- Assume we know the correspondences, how do we get the transformation?




$$\begin{bmatrix} x_i' \\ y_i' \end{bmatrix} = \begin{bmatrix} m_1 & m_2 \\ m_3 & m_4 \end{bmatrix} \begin{bmatrix} x_i \\ y_i \end{bmatrix} + \begin{bmatrix} t_1 \\ t_2 \end{bmatrix}$$

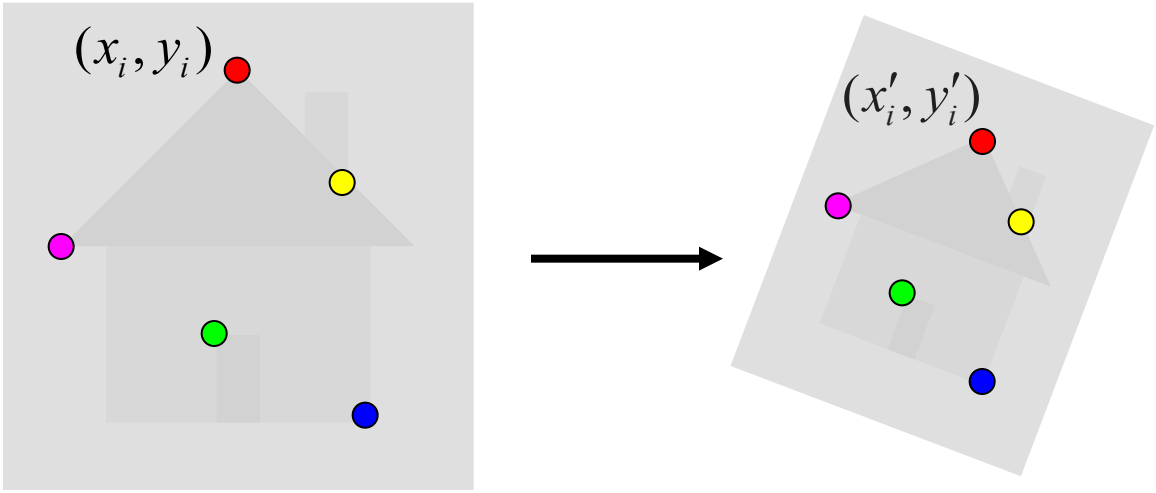
$$\mathbf{x}_i' = \mathbf{M}\mathbf{x}_i + \mathbf{t}$$

Want to find **M**, **t** to minimize

$$\sum_{i=1}^{n} || \mathbf{x}_i' - \mathbf{M}\mathbf{x}_i - \mathbf{t} ||^2$$

# Fitting an affine transformation

- Assume we know the correspondences, how do we get the transformation?

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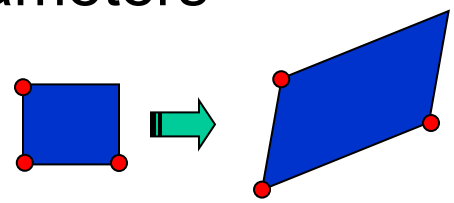
$$\begin{bmatrix} & & & & & \\ & & & & & \\ & & & & & \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \\ t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} \cdots \\ x_i' \\ y_i' \\ \cdots \end{bmatrix}$$

# Fitting an affine transformation

$$\begin{bmatrix} & & \cdots & & & \\ x_i & y_i & 0 & 0 & 1 & 0 \\ 0 & 0 & x_i & y_i & 0 & 1 \\ & & \cdots & & & \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \\ t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} \cdots \\ x_i' \\ y_i' \\ \cdots \end{bmatrix}$$

- Linear system with six unknowns
- Each match gives us two linearly independent equations: need at least three to solve for the transformation parameters



# How about the translation only?

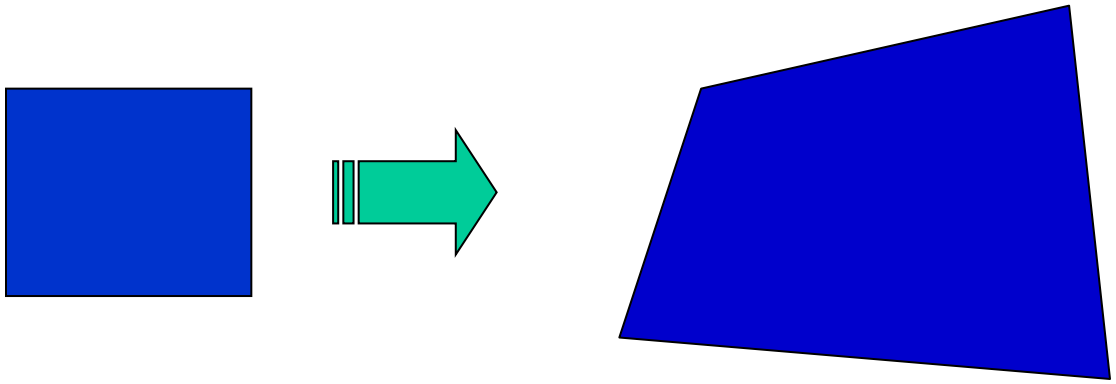
Similarity transform

M=I and t=?

What's the minimum number of points needed for optimizing the objective?

# Fitting a plane projective transformation

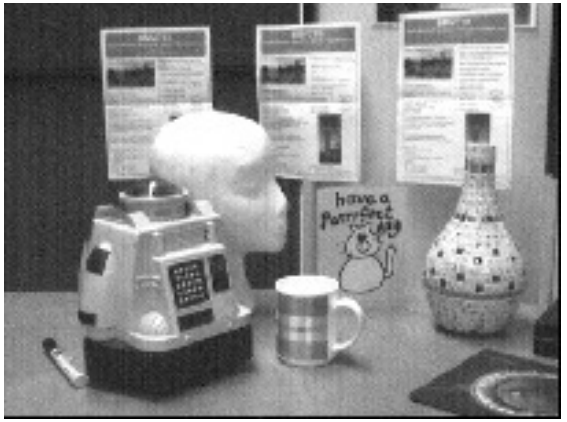
- **Homography:** plane projective transformation (transformation taking a quad to another arbitrary quad)



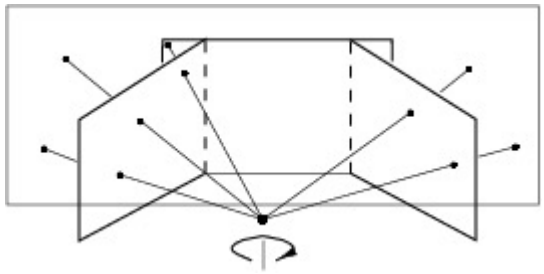
# Homography

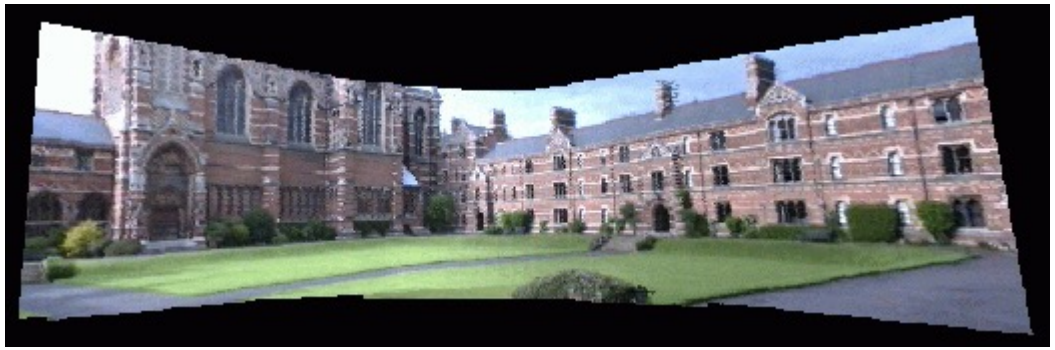
- The transformation between two views of a planar surface


- The transformation between images from two cameras that share the same center

# Application: Panorama stitching





Source: Hartley & Zisserman

# Fitting a homography

- Recall: homogeneous coordinates

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Converting *to* homogeneous image coordinates

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w)$$

Converting *from* homogeneous image coordinates

# Fitting a homography

- Recall: homogeneous coordinates

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Converting *to* homogeneous image coordinates

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w)$$

Converting *from* homogeneous image coordinates

- Equation for homography:

$$\lambda \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

# Fitting a homography

- Equation for homography:

$$\lambda \begin{bmatrix} x_i' \\ y_i' \\ 1 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix} \qquad \lambda \mathbf{x}_i' = \mathbf{H}\mathbf{x}_i$$

$$\mathbf{x}_i' \times \mathbf{H}\mathbf{x}_i = 0$$

$$\begin{bmatrix} x_i' \\ y_i' \\ 1 \end{bmatrix} \times \begin{bmatrix} \mathbf{h_1}^T \mathbf{x}_i \\ \mathbf{h_2}^T \mathbf{x}_i \\ \mathbf{h_3}^T \mathbf{x}_i \end{bmatrix} = \begin{bmatrix} y_i' \mathbf{h}_3^T \mathbf{x}_i - \mathbf{h}_2^T \mathbf{x}_i \\ \mathbf{h}_1^T \mathbf{x}_i - x_i' \mathbf{h}_3^T \mathbf{x}_i \\ x_i' \mathbf{h}_2^T \mathbf{x}_i - y_i' \mathbf{h}_1^T \mathbf{x}_i \end{bmatrix}$$

$$\begin{bmatrix} 0^T & -\mathbf{x}_i^T & y_i' \mathbf{x}_i^T \\ \mathbf{x}_i^T & 0^T & -x_i' \mathbf{x}_i^T \\ -y_i' \mathbf{x}_i^T & x_i' \mathbf{x}_i^T & 0^T \end{bmatrix} \begin{pmatrix} \mathbf{h}_1 \\ \mathbf{h}_2 \\ \mathbf{h}_3 \end{pmatrix} = 0$$

3 equations, only 2 linearly independent

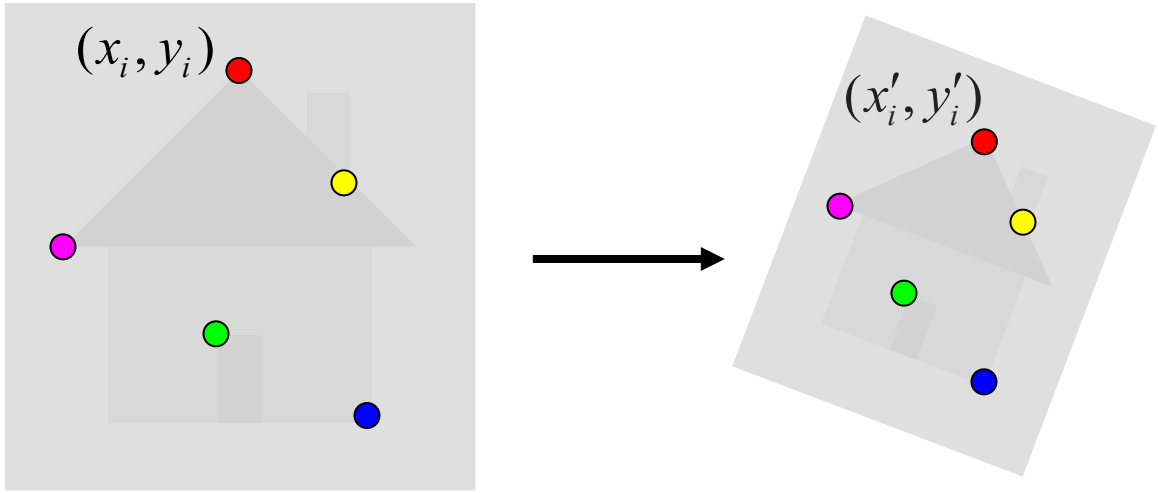
# Fitting a homography

$$\begin{bmatrix} 0^T & \mathbf{x}_1^T & -y_1' \mathbf{x}_1^T \\ \mathbf{x}_1^T & 0^T & -x_1' \mathbf{x}_1^T \\ \dots & \dots & \dots \\ 0^T & \mathbf{x}_n^T & -y_n' \mathbf{x}_n^T \\ \mathbf{x}_n^T & 0^T & -x_n' \mathbf{x}_n^T \end{bmatrix} \begin{pmatrix} \mathbf{h}_1 \\ \mathbf{h}_2 \\ \mathbf{h}_3 \end{pmatrix} = 0 \qquad \mathbf{A}\,\mathbf{h} = 0$$

- H has 8 degrees of freedom (9 parameters, but scale is arbitrary)
- One match gives us two linearly independent equations
- Homogeneous least squares: find $\mathbf{h}$ minimizing $||\mathbf{A}\mathbf{h}||^2$
  - Eigenvector of $\mathbf{A}^\mathbf{T}\mathbf{A}$ corresponding to smallest eigenvalue
  - Four matches needed for a minimal solution

# Robust feature-based alignment

- So far, we've assumed that we are given a set of "ground-truth" correspondences between the two images we want to align
- What if we don't know the correspondences?



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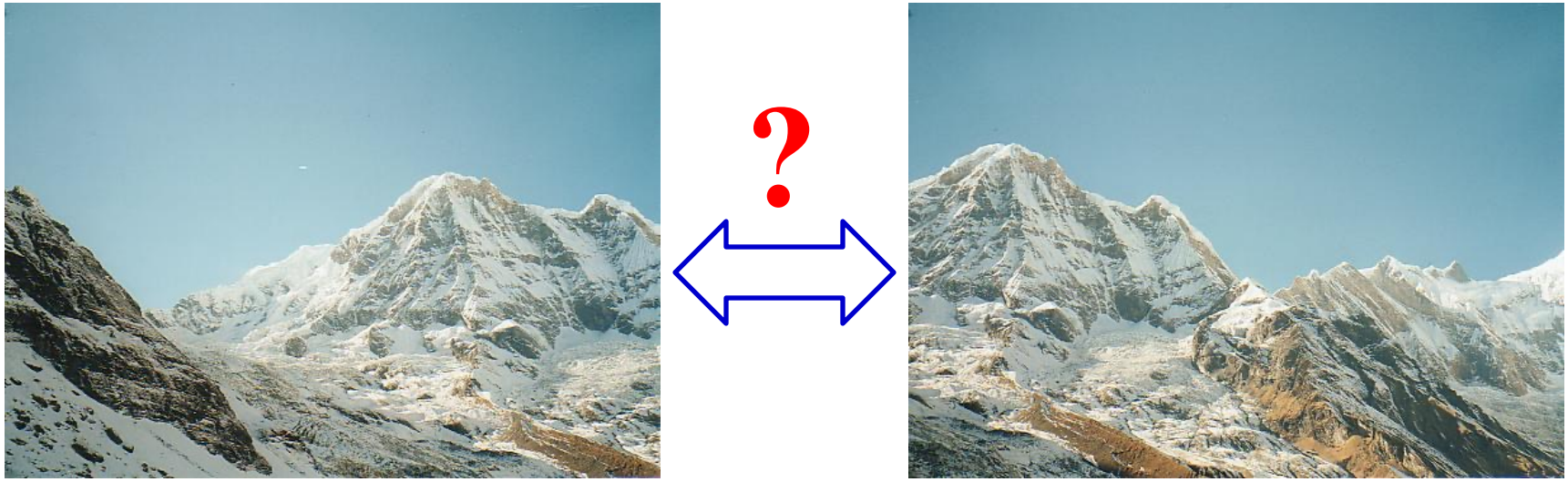

# Robust feature-based alignment

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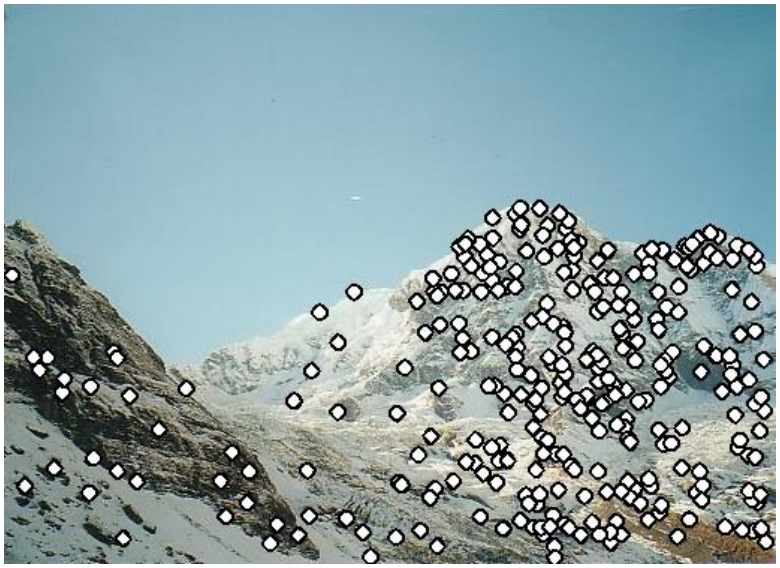
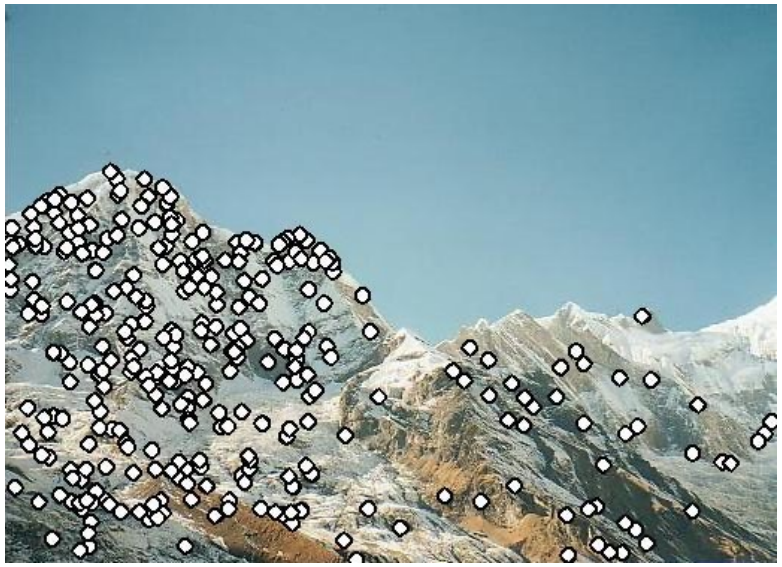


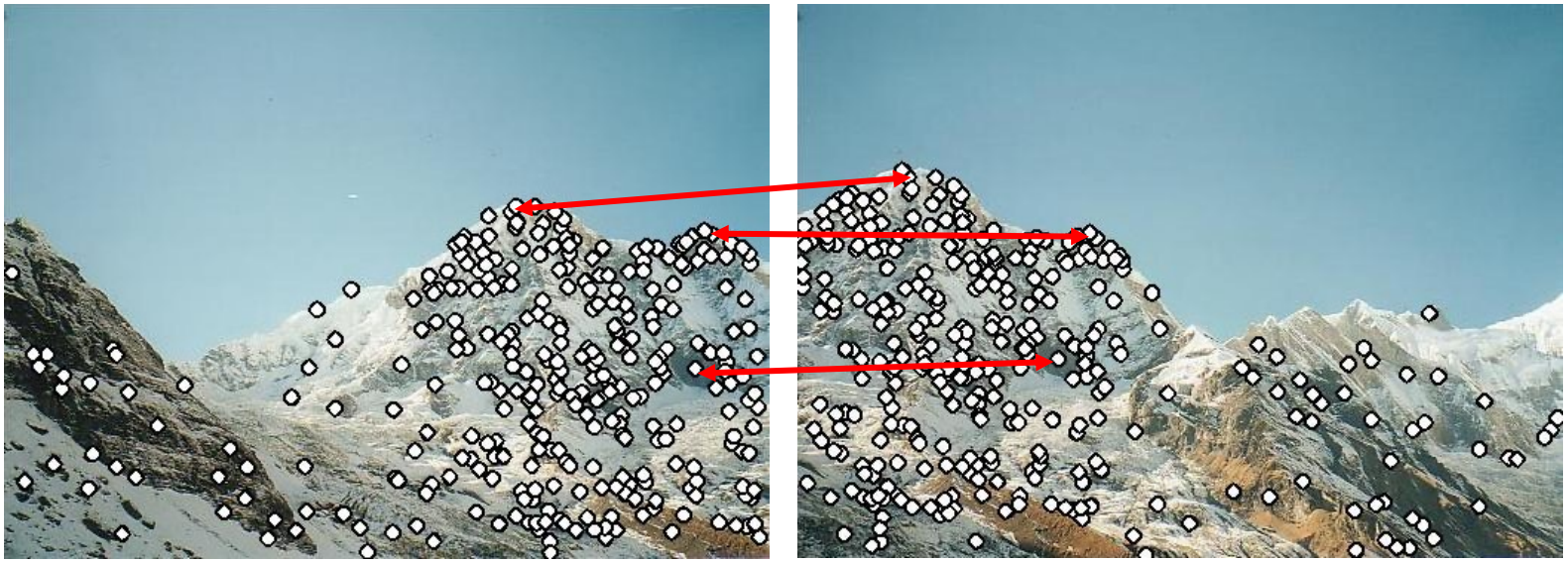


- Extract features

# Robust feature-based alignment

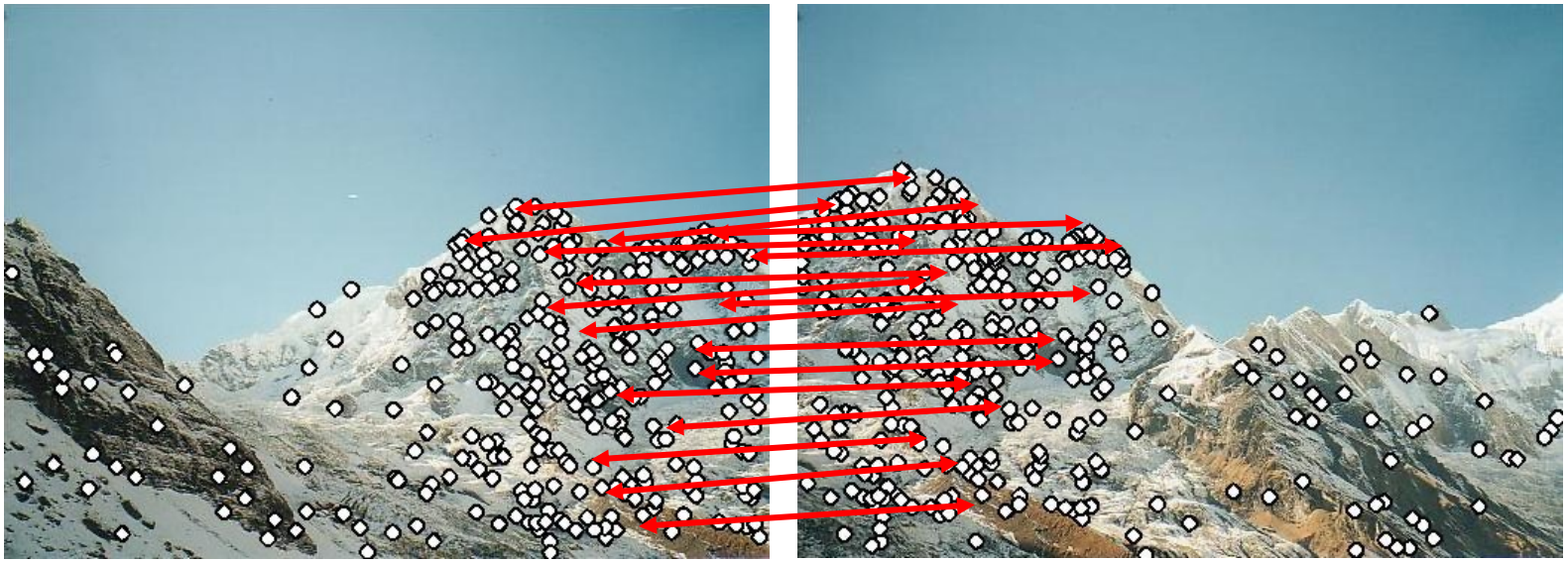
- Extract features
- Compute *putative matches*

# Robust feature-based alignment



- Extract features
- Compute *putative matches*
- Loop:
  - *Hypothesize* transformation *T*

# Robust feature-based alignment

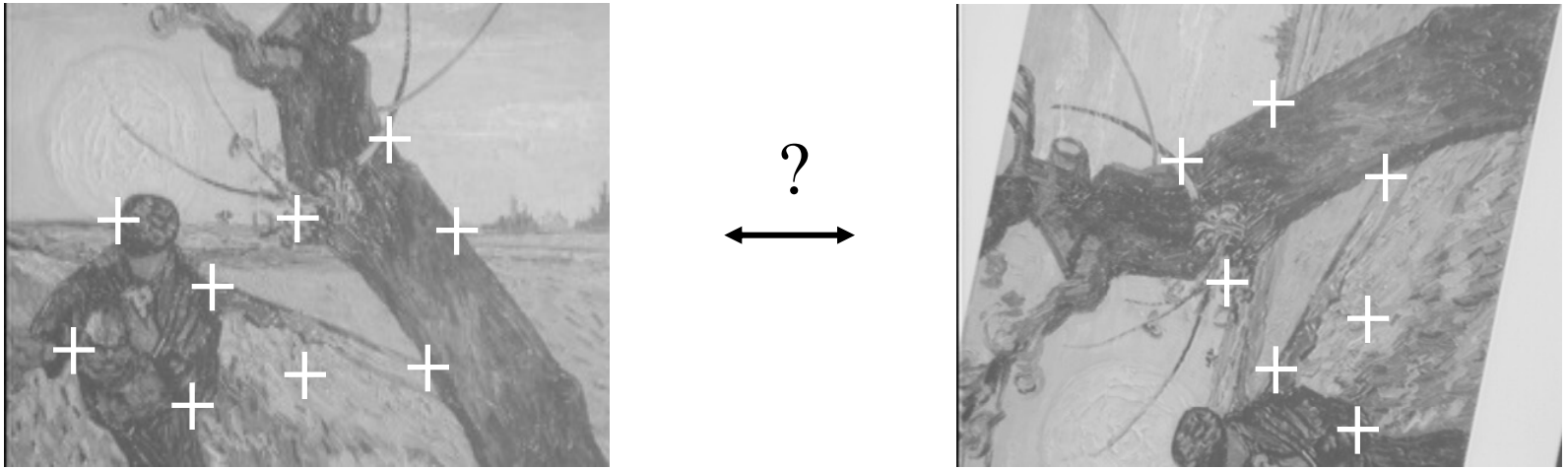


- Extract features
- Compute *putative matches*
- Loop:
  - *Hypothesize* transformation *T*
  - *Verify* transformation (search for other matches consistent with *T*)

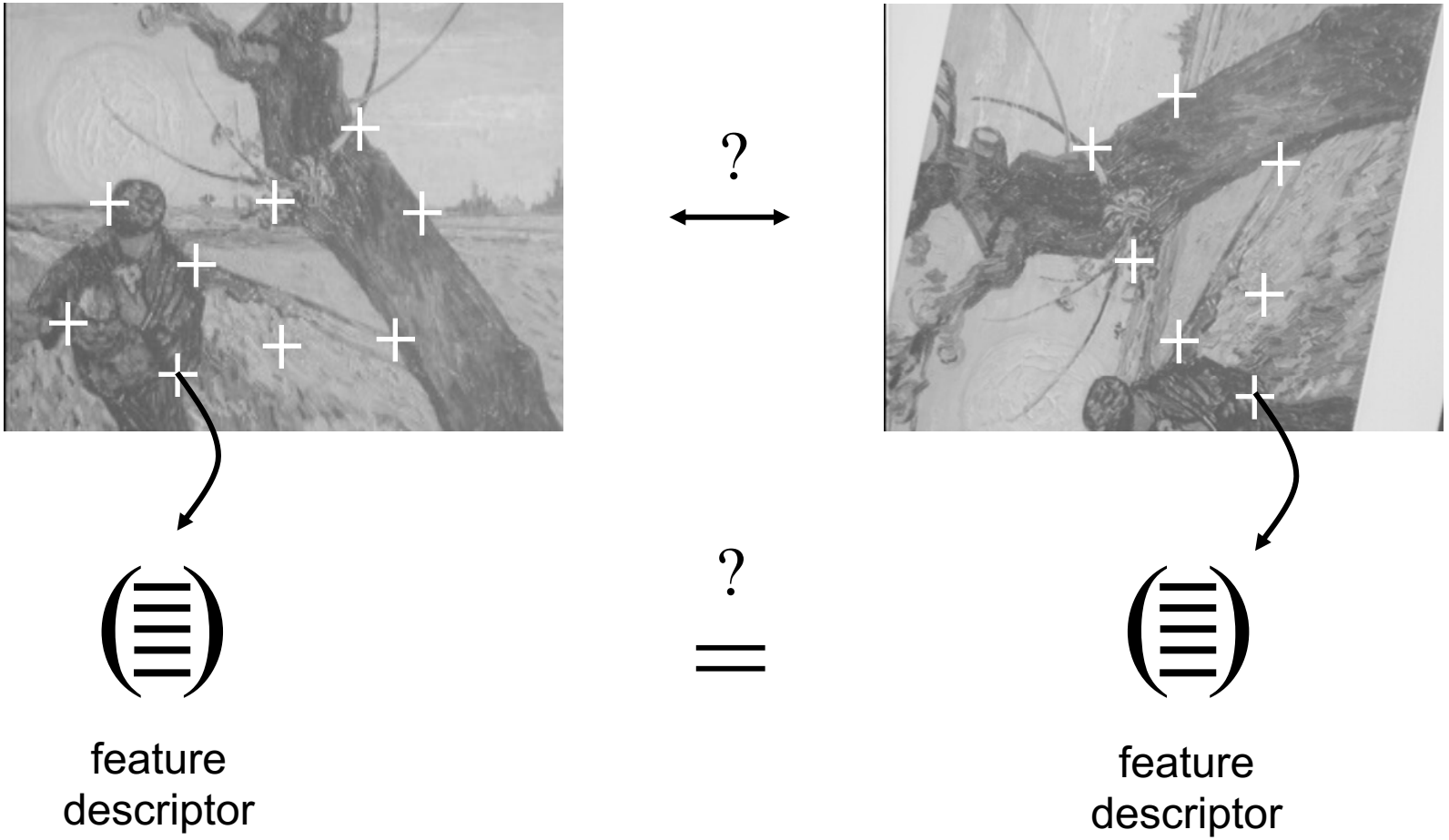
# Robust feature-based alignment

- Extract features
- Compute *putative matches*
- Loop:
  - *Hypothesize* transformation *T*
  - *Verify* transformation (search for other matches consistent with *T*)

# Generating putative correspondences



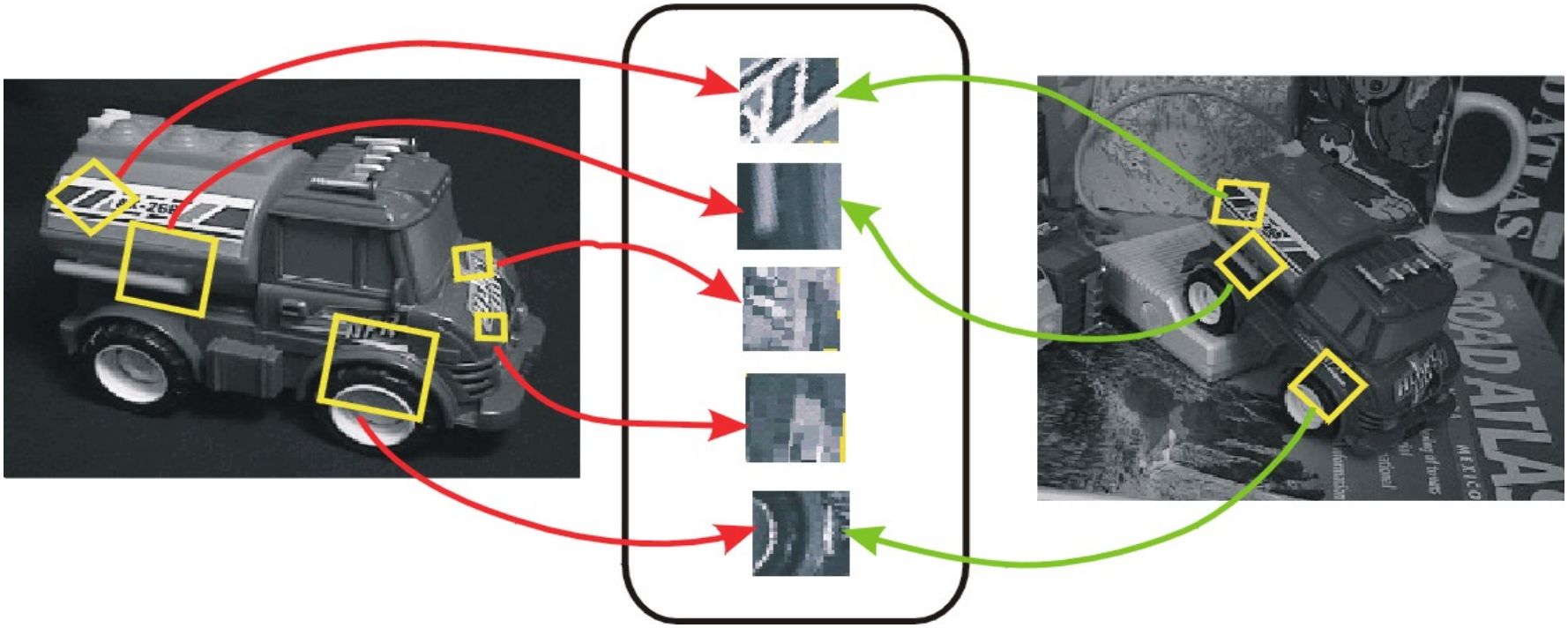
# Generating putative correspondences




- Need to compare *feature descriptors* of local patches surrounding interest points

# Feature descriptors

- Recall: feature detection and description



# Feature descriptors

- Simplest descriptor: vector of raw intensity values
- How to compare two such vectors?
  - Sum of squared differences (SSD)

$$\mathrm{SSD}(\mathbf{u},\mathbf{v}) = \sum_i \left(u_i - v_i\right)^2$$

    - Not invariant to intensity change

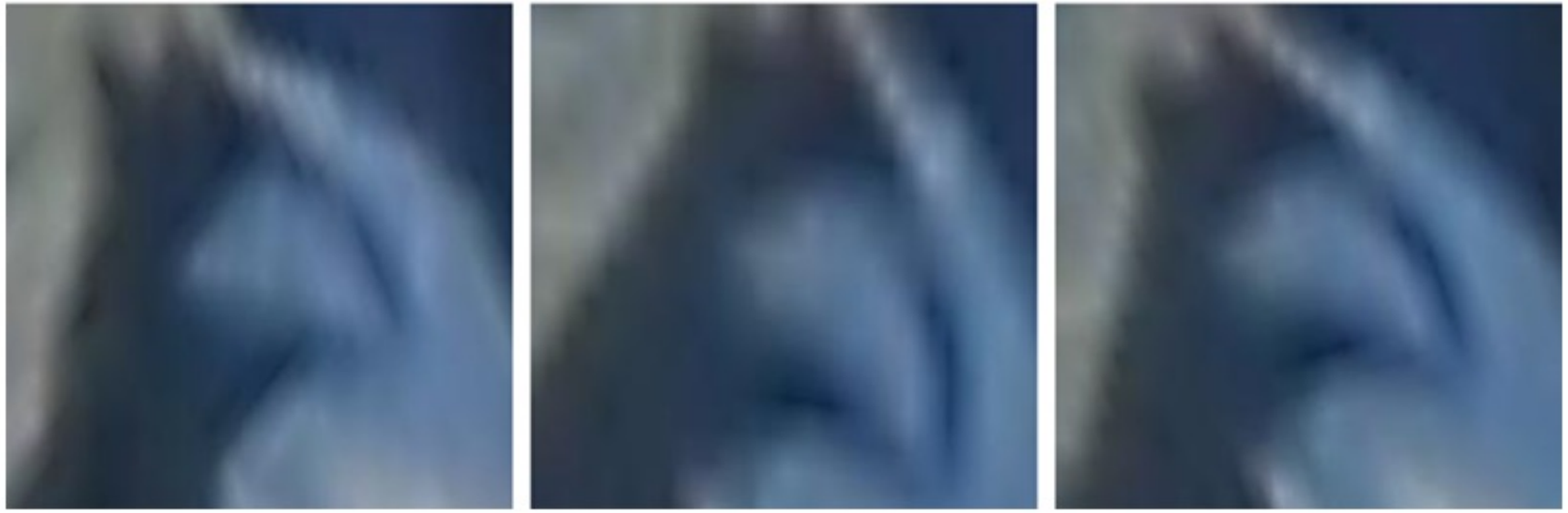
  - Normalized correlation

$$\rho(\mathbf{u},\mathbf{v}) = \frac{(\mathbf{u}-\bar{\mathbf{u}})}{\|\mathbf{u}-\bar{\mathbf{u}}\|} \cdot \frac{(\mathbf{v}-\bar{\mathbf{v}})}{\|\mathbf{v}-\bar{\mathbf{v}}\|} = \frac{\sum_i (u_i - \bar{\mathbf{u}})(v_i - \bar{\mathbf{v}})}{\sqrt{\left(\sum_j (u_j - \bar{\mathbf{u}})^2\right)\left(\sum_j (v_j - \bar{\mathbf{v}})^2\right)}}$$

    - Invariant to affine intensity change

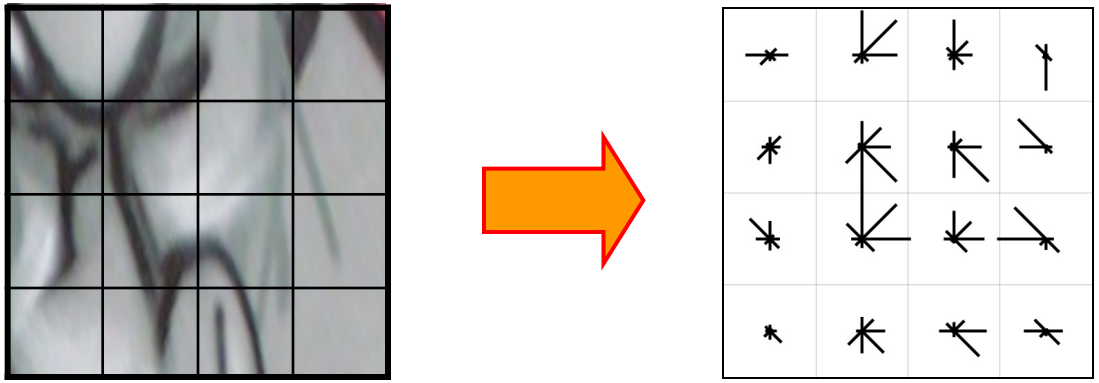
# Disadvantage of intensity vectors as descriptors

- Small deformations can affect the matching score a lot



# Recall: Feature descriptors: SIFT

- Descriptor computation:
  - Divide patch into 4x4 sub-patches
  - Compute histogram of gradient orientations (8 reference angles) inside each sub-patch
  - Resulting descriptor: 4x4x8 = 128 dimensions



David G. Lowe. "Distinctive image features from scale-invariant keypoints." *IJCV* 60 (2), pp. 91-110, 2004.

# Recall: Feature descriptors: SIFT

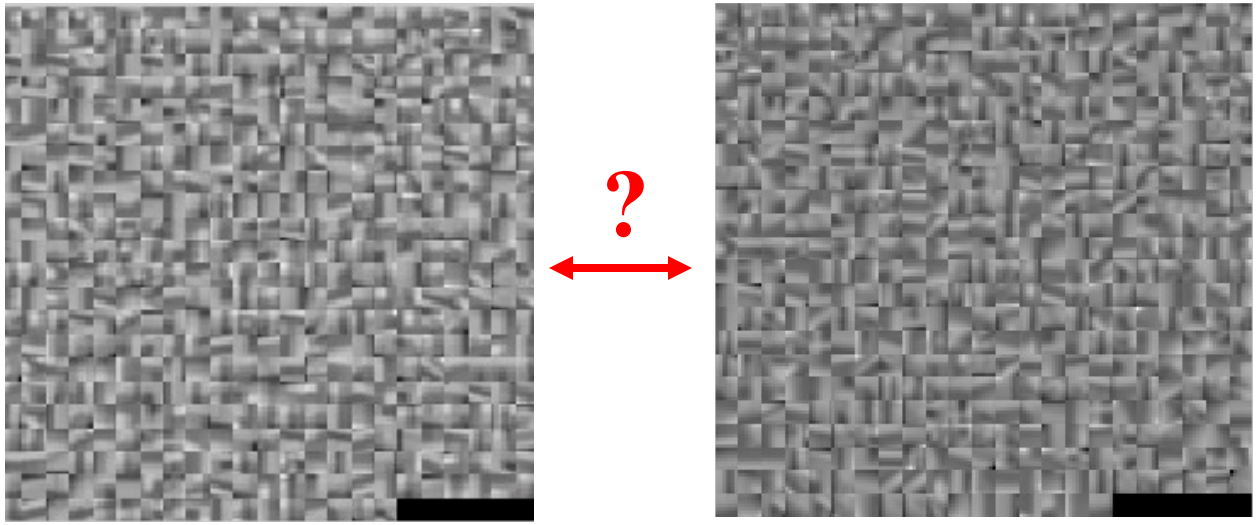
- Descriptor computation:
  - Divide patch into 4x4 sub-patches
  - Compute histogram of gradient orientations (8 reference angles) inside each sub-patch
  - Resulting descriptor: 4x4x8 = 128 dimensions

- Advantage over raw vectors of pixel values
  - Gradients less sensitive to illumination change
  - Pooling of gradients over the sub-patches achieves robustness to small shifts, but still preserves some spatial information

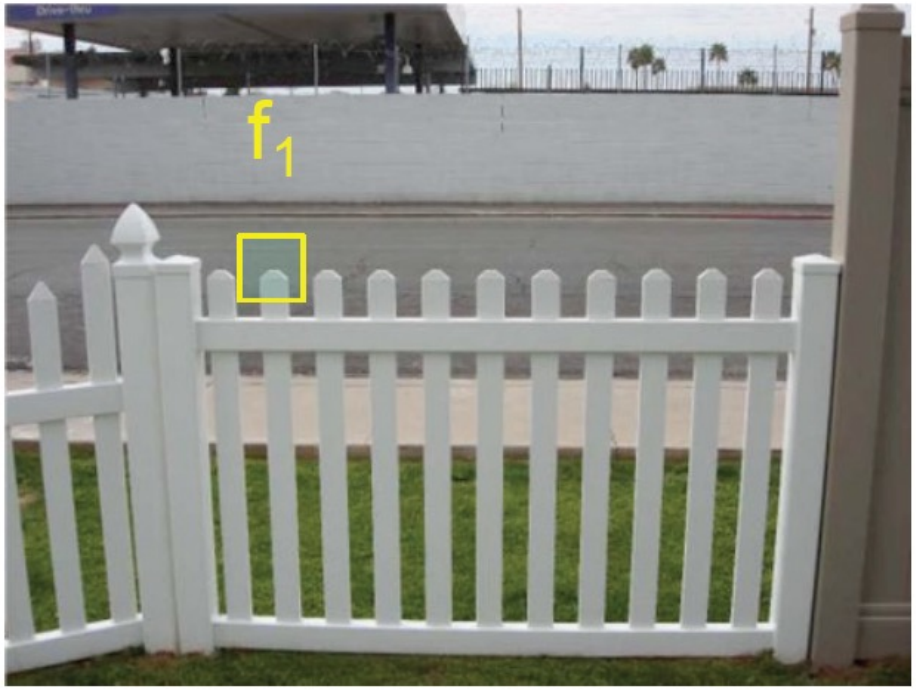
David G. Lowe. "Distinctive image features from scale-invariant keypoints." *IJCV* 60 (2), pp. 91-110, 2004.

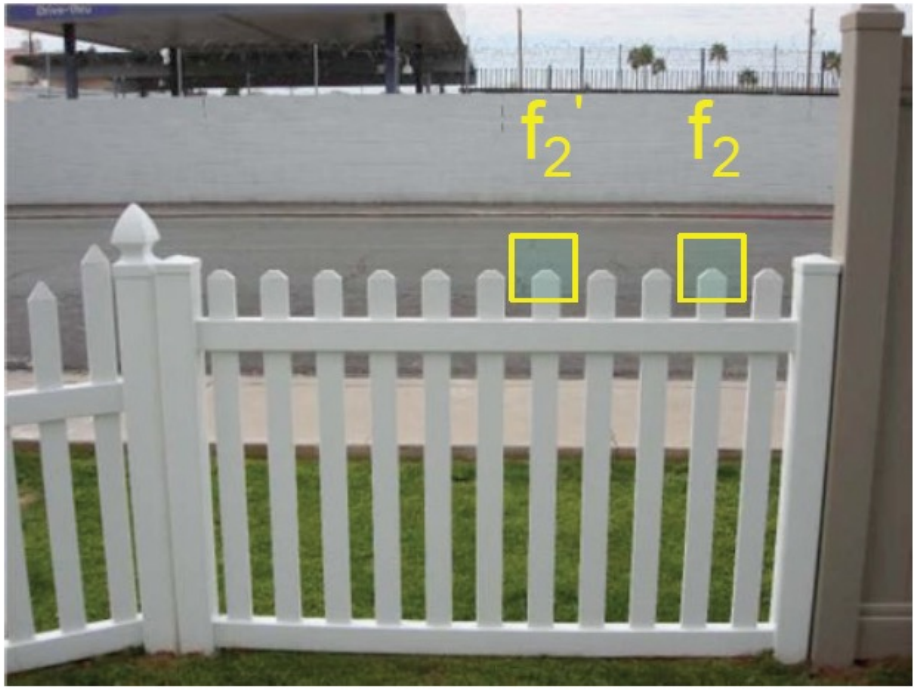
# Feature matching

- Generating *putative matches*: for each patch in one image, find a short list of patches in the other image that could match it based solely on appearance



# Problem: Ambiguous putative matches

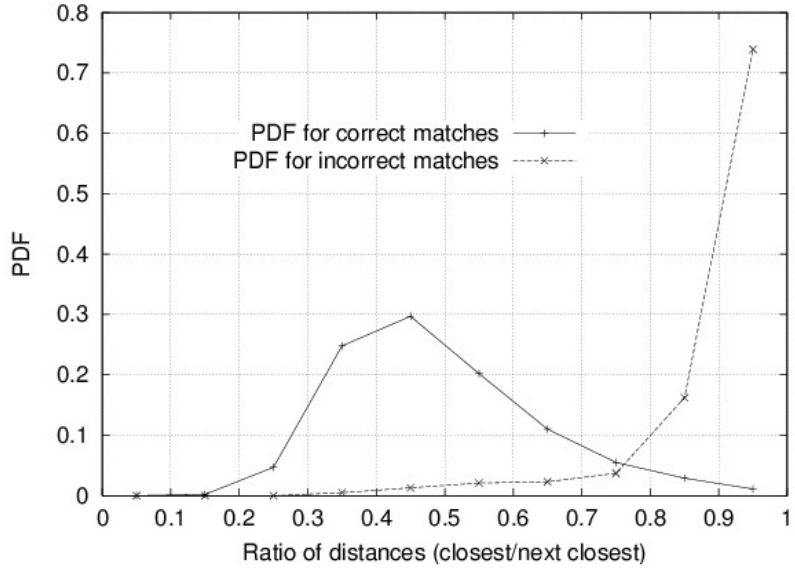



Source: Y. Furukawa

# Rejection of unreliable matches

- How can we tell which putative matches are more reliable?
- Heuristic: compare distance of **nearest** neighbor to that of **second** nearest neighbor
  - Ratio of closest distance to second-closest distance will be *high* for features that are *not* distinctive




**Threshold of 0.8 provides good separation**

David G. Lowe. "Distinctive image features from scale-invariant keypoints." *IJCV* 60 (2), pp. 91-110, 2004.

# RANSAC

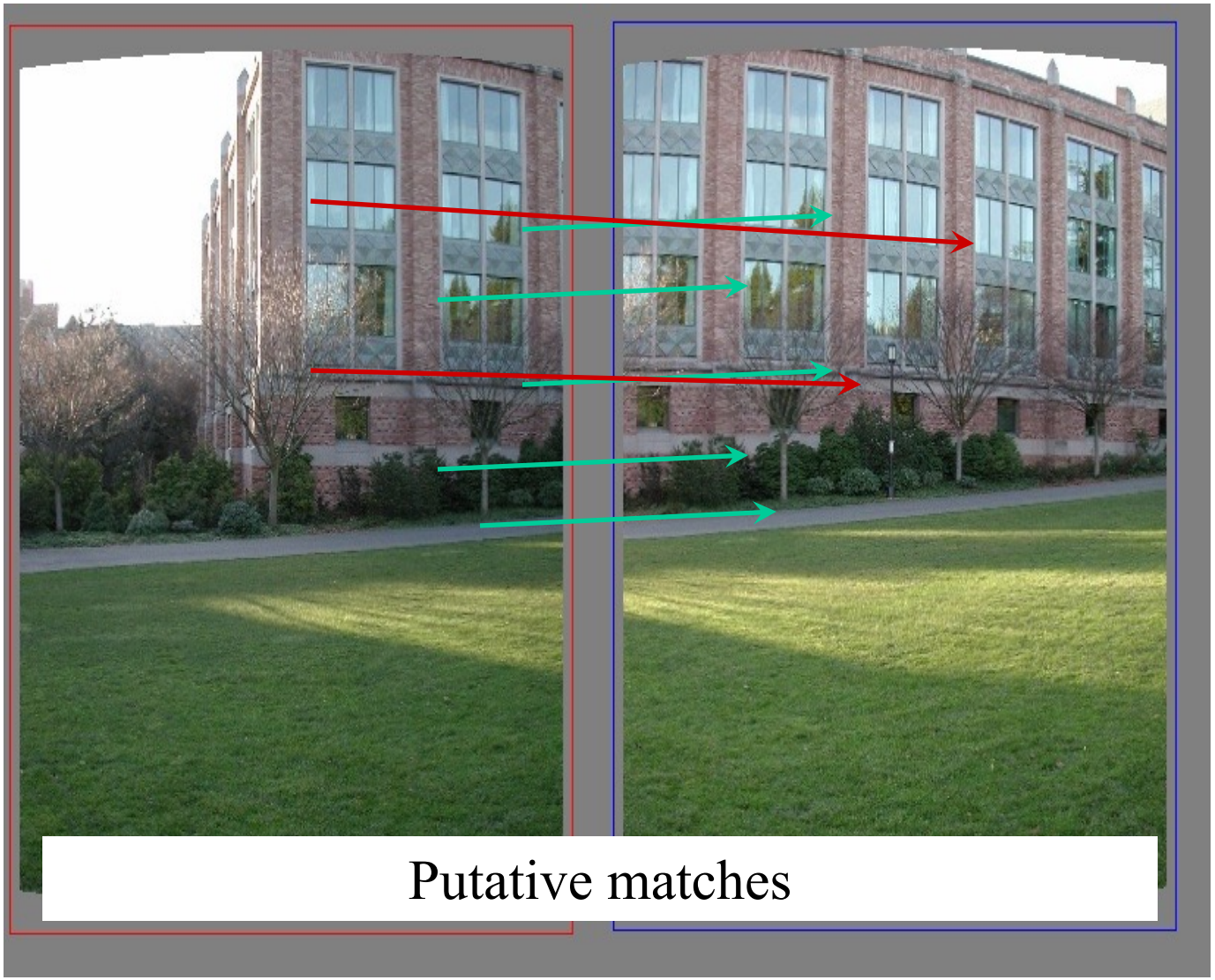
- The set of putative matches contains a very high percentage of outliers

**RANSAC loop:**

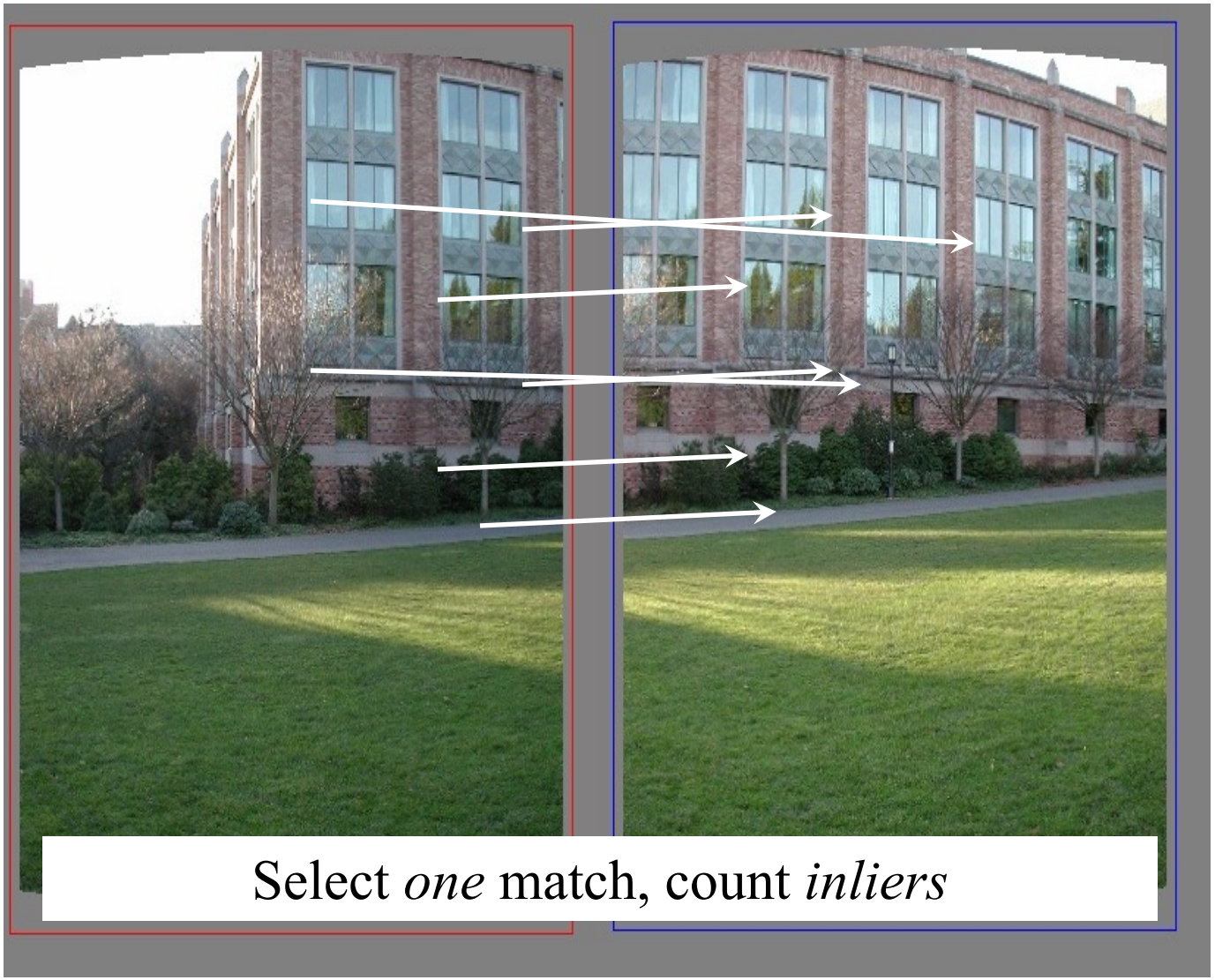
1. Randomly select a *seed group* of matches
2. Compute transformation from seed group
3. Find *inliers* to this transformation
4. If the number of inliers is sufficiently large, re-compute least-squares estimate of transformation on all of the inliers

Keep the transformation with the largest number of inliers

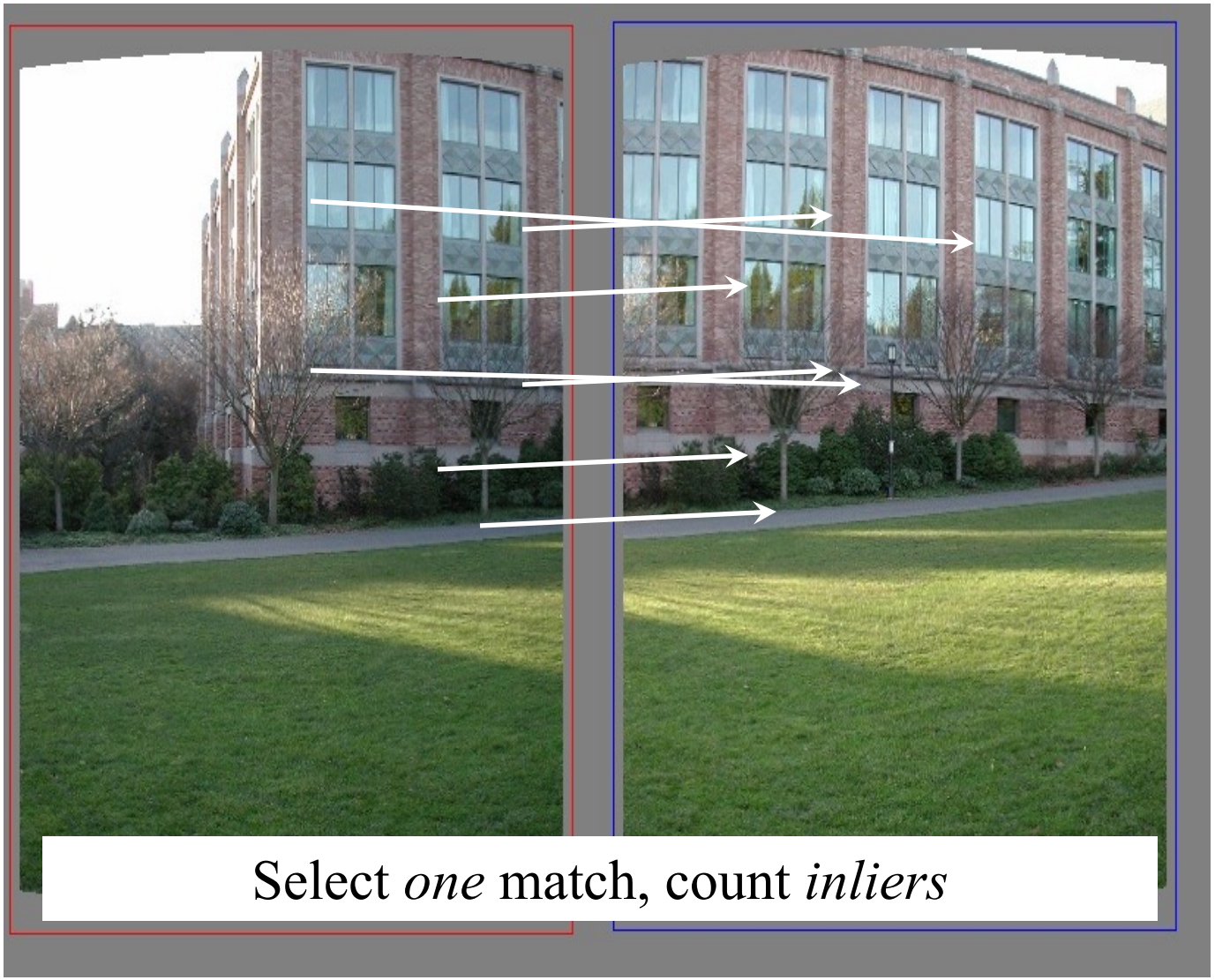
# RANSAC example: Translation



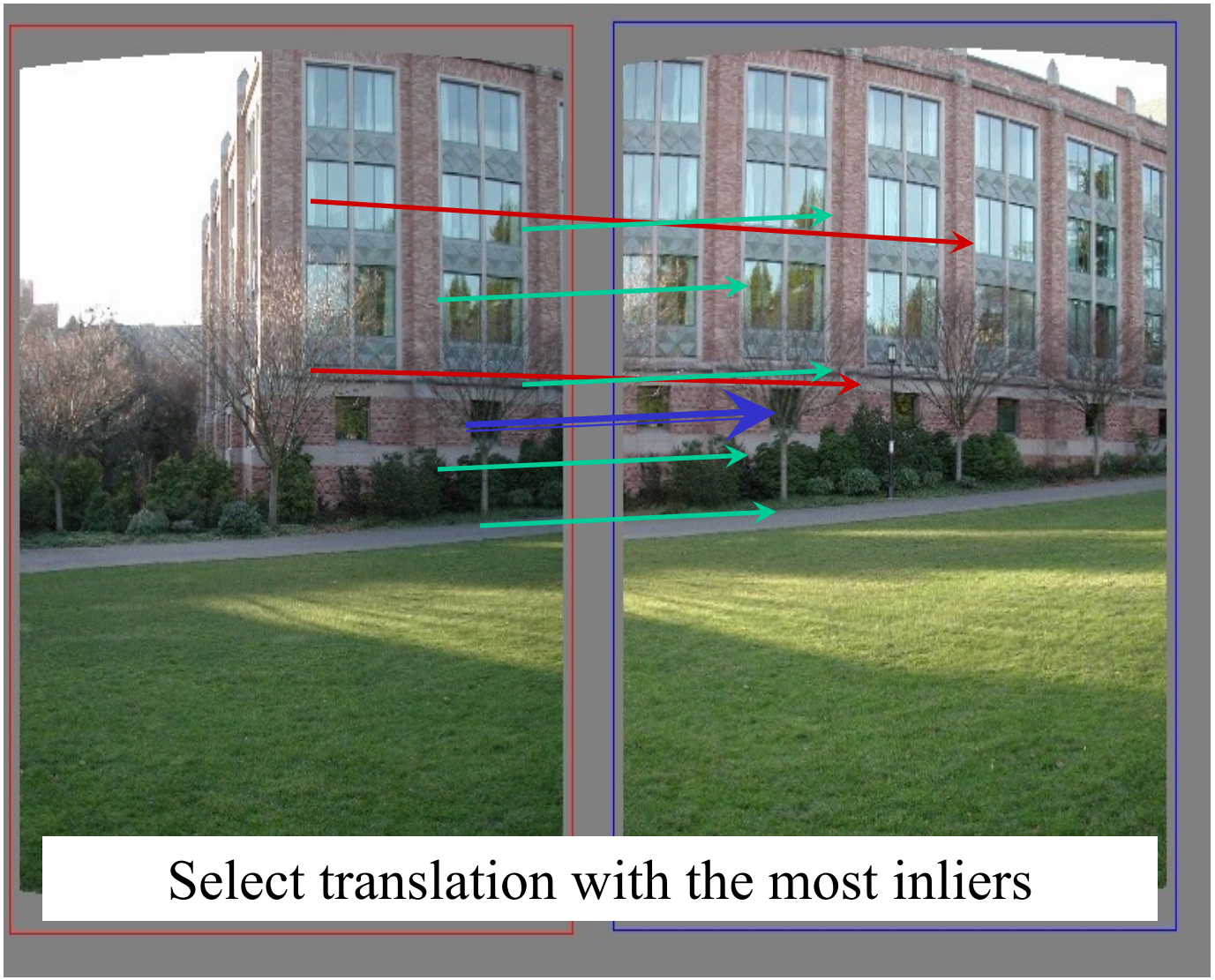
# RANSAC example: Translation



# RANSAC example: Translation



# RANSAC example: Translation



# Proj: Local Feature Extraction

interest point detector, SIFT-like local feature descriptor, ( done ！ ) VLfeat http://www.vlfeat.org/

simple matching algorithm.

Image stitching by using sift feature

**Data preparation： Take some image pairs of some buildings in ShanghaiTech campus**

**Codes+Report, including how to run codes, your results, and the problem your encountered and your solution. Your new findings? etc..**

**Please evaluate your algorithm with at least 3 pairs of images!!**

**Due date: Oct 15**