Signals & Systems: Homework #4

Problem 1

(15 points) Compute the Fourier transform of each of the following signals

(a)

$$\sum_{n=-\infty}^{+\infty} e^{-|t-2n|}$$

(b)

$$x(t) = [te^{-2t} sin(4t)]u(t)$$

(c)

$$x(t) = \begin{cases} 1 - t^2, & 0 < t < 1 \\ 0, & otherwise \end{cases}$$

Problem 2

(15 points) Consider a signal p(t) = $\sum_{k=-\infty}^{+\infty} \delta(t - kT)$ and a signal s(t) with spectrum S(jω), where 3Tω_1 = 2π

(a) Determine the FT of p(t)

(b) Dentermine and sketch the FT of r(t) = p(t)s(t)

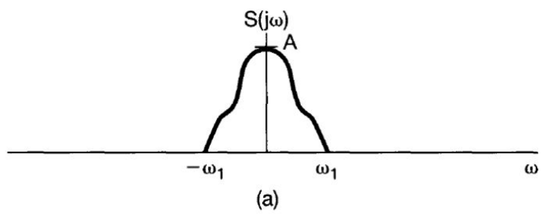

Figure 1 2(a)

Problem 3

(20 points) Calculate the Fourier Transform of the following signals:

(a) Calculate the Fourier Transform of x(t) = $\frac{2}{1+(t-5)^2}$

(b) Calculate the inverse Fourier Transform of $X(j\omega) = \frac{1}{(a+j(\omega-3))^2}$

Problem 4

(20 points) Frequency response of a Linear Time-Invariant system is shown below:

$$H(j\omega) = \frac{j\omega+5}{2-\omega^2+3j\omega}$$

(a) Write out the differential equation that associates system input x(t) with output y(t).

(b) Determine the impulse response h(t) of the system.

(c) Determine output of the system with input $x(t) = e^{-5t}u(t)$.

Problem 5

(30 points) Let x(t) and y(t) be two real signals. Then the cross-correlation function of x(t) and y(t) is defined as

$$\phi_{xy}(t) = \int_{-\infty}^{+\infty} x(t+\tau)y(\tau)d\tau \quad .$$

Similarly, we can define $\phi_{yx}(t), \phi_{xx}(t)$, and $\phi_{yy}(t)$. The last two of these are called the auto-correlation functions of the signals x(t) and y(t), respectively. Let $\Phi_{xy}(j\omega), \Phi_{yx}(j\omega), \Phi_{xx}(j\omega)$ and $\Phi_{yy}(j\omega)$ denote the Fourier transforms of $\phi_{xy}(t), \phi_{yx}(t), \phi_{xx}(t)$, and $\phi_{yy}(t)$, respectively.

(a) Determine the relationship between $\Phi_{xy}(j\omega)$ and $\Phi_{yx}(j\omega)$.
$Hint$: You may need to prove $\phi_{yx}(t) = \phi_{xy}(-t)$ firstly.

(b) Find an expression for $\Phi_{yx}(j\omega)$ in terms of $X(j\omega)$ and $Y(j\omega)$.

(c) Show that $\Phi_{yy}(j\omega)$ is real and non-negative for every ω.

(d) Suppose now that x(t) is the input to an LTI system with a real-valued impulse response and with frequency response $H(j\omega)$ and that y(t) is the output. Find expressions for $\Phi_{xy}(j\omega)$ and $\Phi_{yy}(j\omega)$ in terms of $\Phi_{xx}(j\omega)$ and $H(j\omega)$.

(e) Let x(t) be as is illustrated in Figure 1, and let the LTI system impulse response be $h(t) = e^{-at}u(t), a > 0$. Compute $\Phi_{xx}(j\omega), \Phi_{xy}(j\omega)$ and $\Phi_{yy}(j\omega)$ using the results of parts (a)-(d).

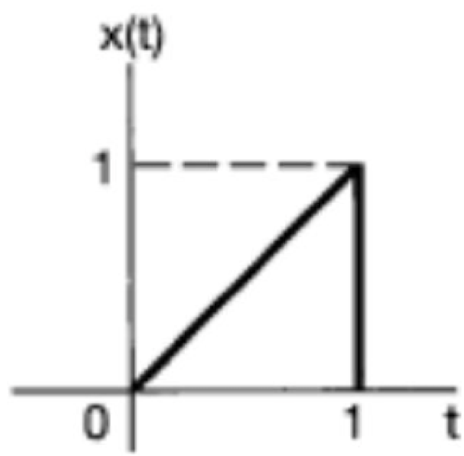

Figure 2 5(e)