

Lecture 17–Morphological Operations

(Chapter 9.1–9.5)

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Course piazza link:
piazza.com/shanghaitech.edu.cn/spring2021/cs270spring2021



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Outline

This lecture will cover:

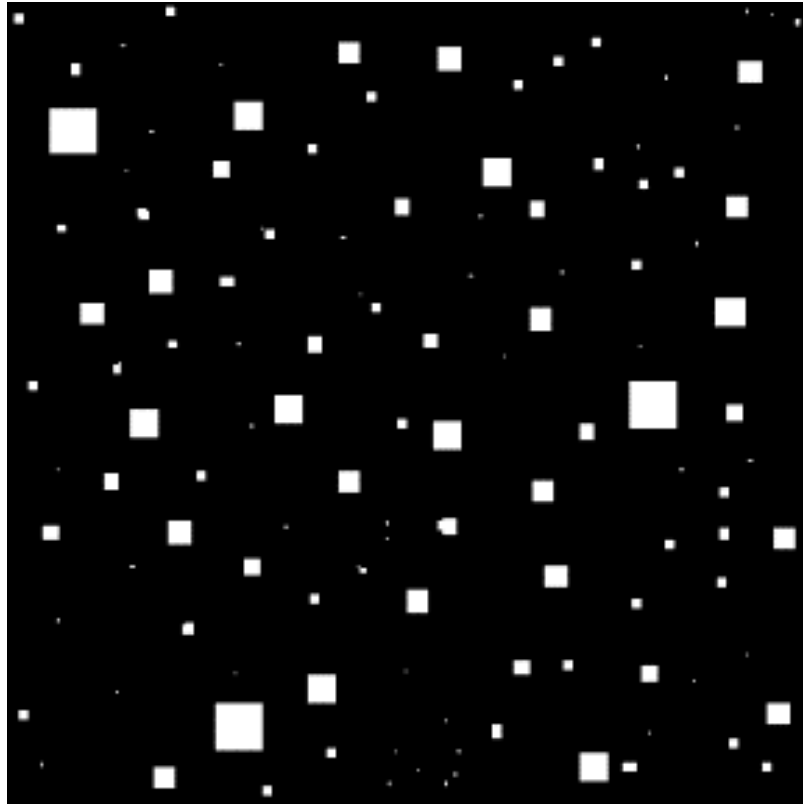
- Morphology Image Processing (形态学图像处理)
 - Morphological operation.
 - Morphological algorithms.



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Problem try to solve

- Imperfect from image segmentation.



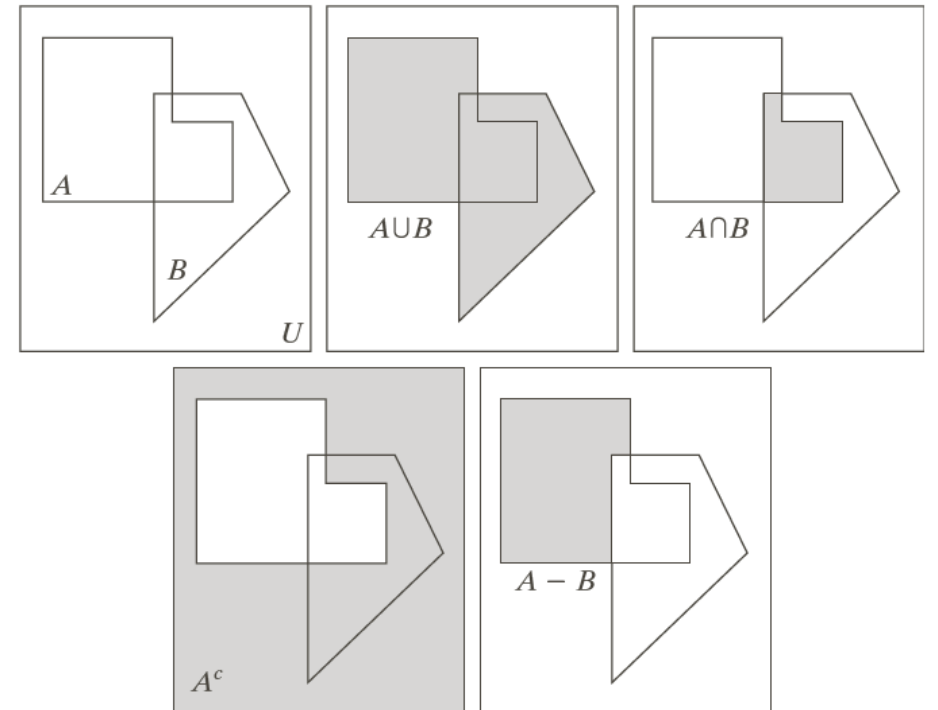
Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.



Preliminaries --- Set Operation

A digital image $f(x, y)$ can be considered as a set A , if $w(x, y)$ in 2D integer space Z^2 , then

- $w \in A$: w is an element of A .
- $w \notin A$: w is not an element of A .
- $B = \{w | \text{condition}\}$: all elements which meet the specific condition.
 - $A \cup B = \{w | w \in A \text{ or } w \in B\}$: union (并集)
 - $A \cap B = \{w | w \in A \text{ and } w \in B\}$: intersection (交集)
 - $A^c = \{w | w \notin A\}$: complement (补集)
 - $A - B = \{w | w \in A \text{ and } w \notin B\}$: difference (差集)



Structure element (结构元)

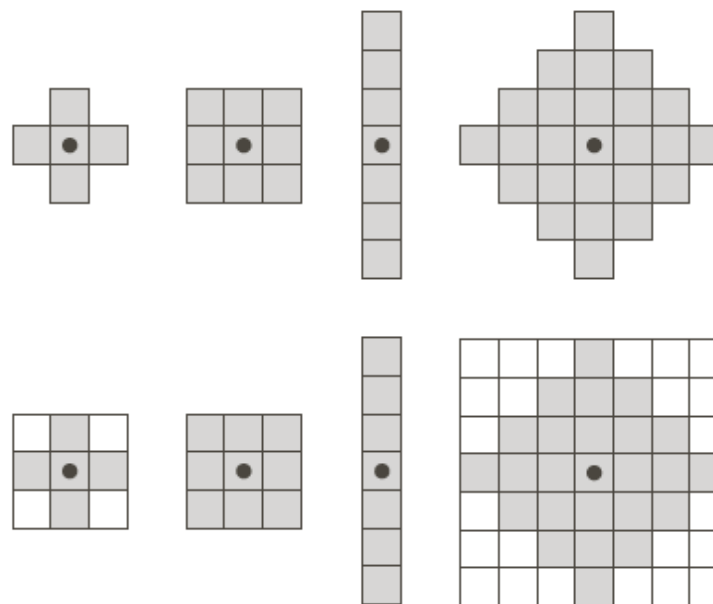
➤ **Structuring Element (SE)** --- small sets or sub-images used to probe an image under study for properties of interest.

➤ **SE Selection**

- Simpler than the image
- With boundary
- Convex

➤ **Structures**

- Origin
- Rectangular



Erosion (腐蚀)

➤ Definition:

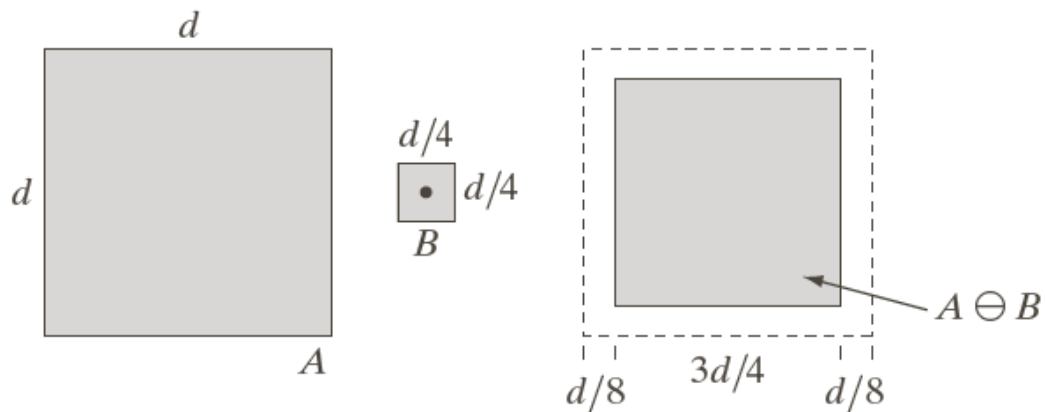
$$A \ominus B = \{z \mid (B)_z \subseteq A\}$$

or

$$A \ominus B = \{z \mid (B)_z \cap A^c = \emptyset\}$$

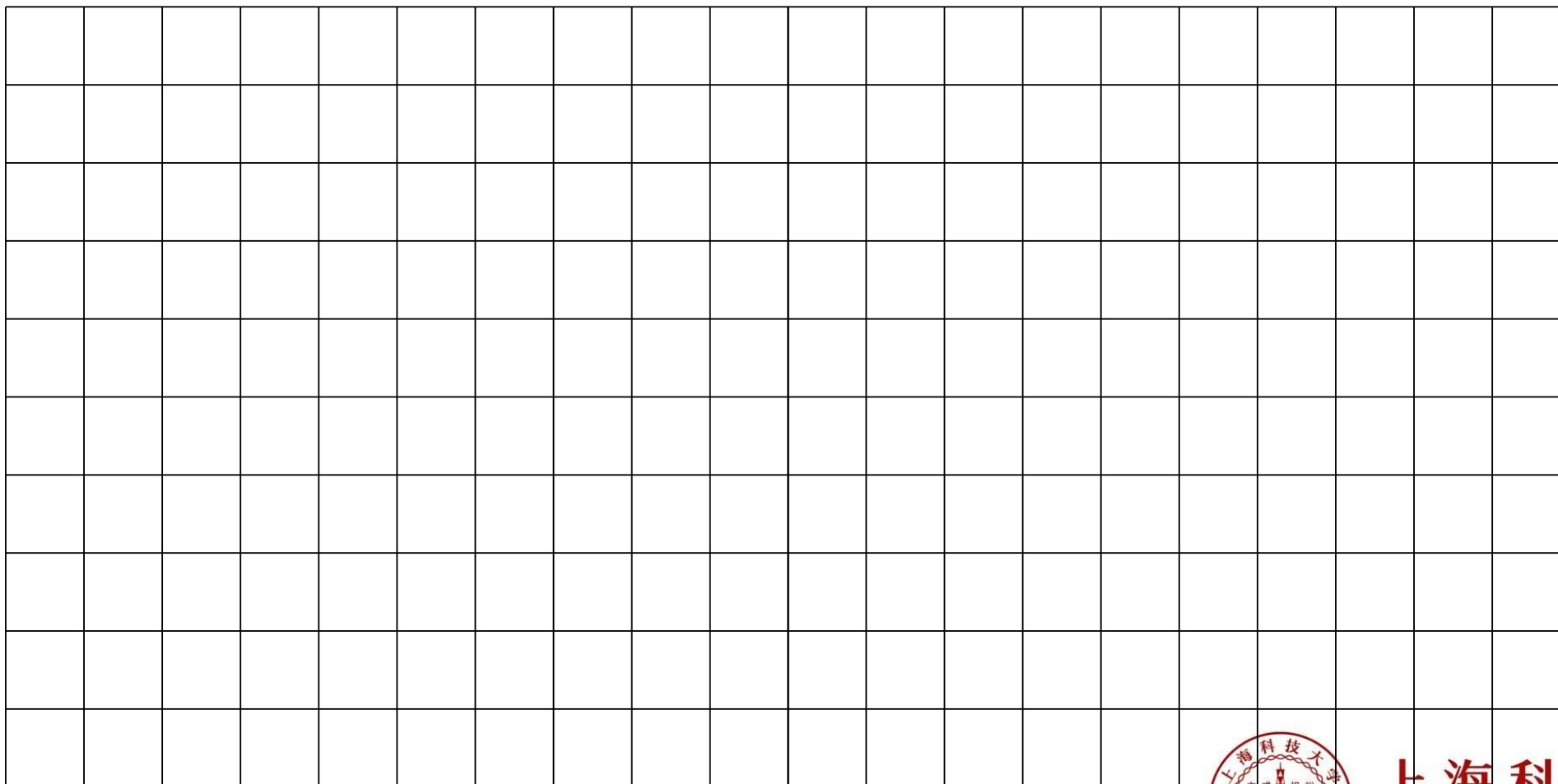
➤ Erosion will do:

- removes thin lines
- isolate dots
- leaves gross details
- “Peeling away” layers
- Is always a sub-set of A

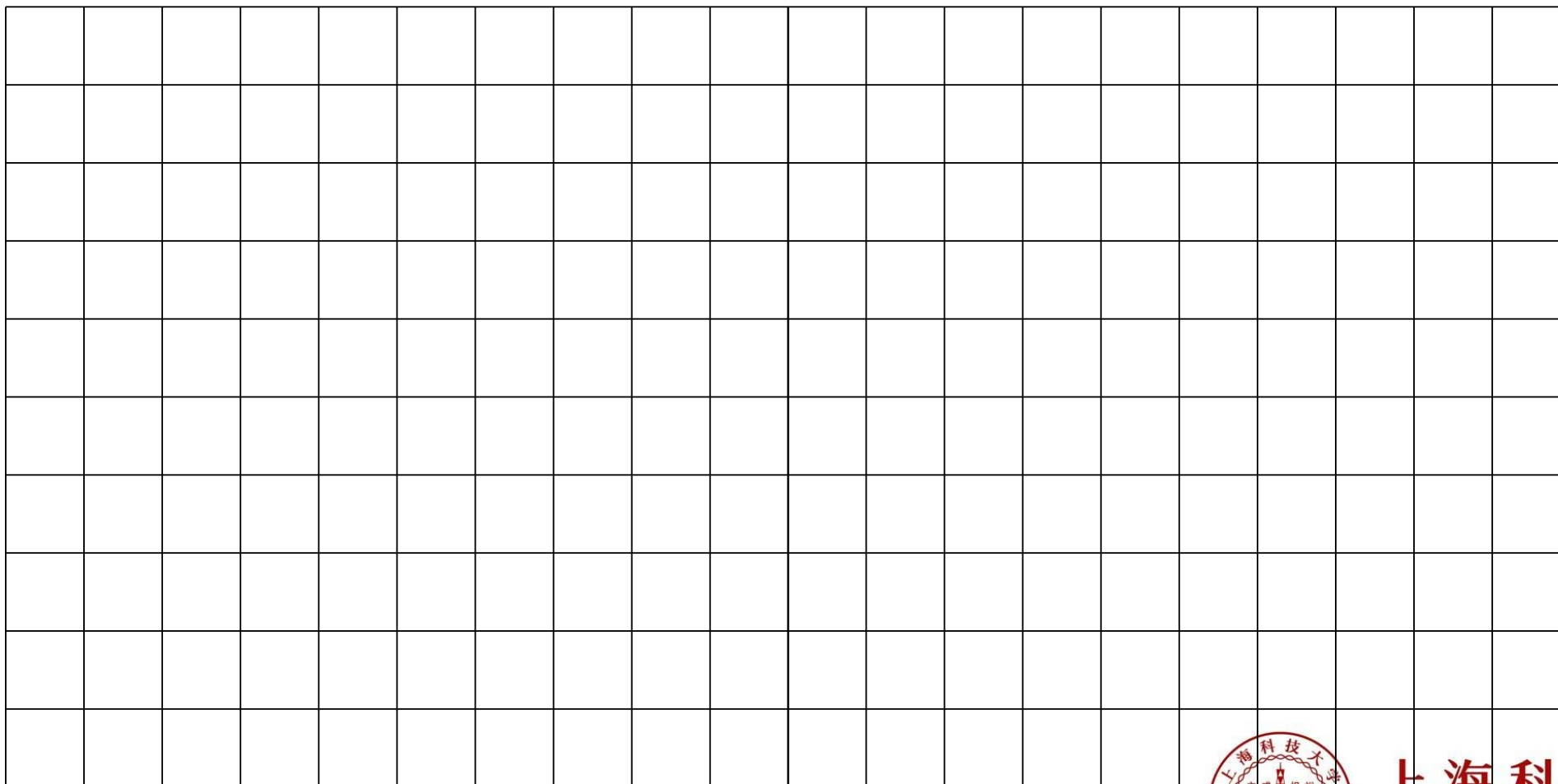


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Erosion (腐蚀)



Erosion (腐蚀)



Dilation (膨胀)

➤ Definition

$$A \oplus B = \{z | \widehat{B_z} \cap A \subseteq A\} \text{ or}$$

$$A \oplus B = \{z | \widehat{B_z} \cap A \subseteq \emptyset\}$$

Dilation will do:

Fatten up. Kind of opposite of

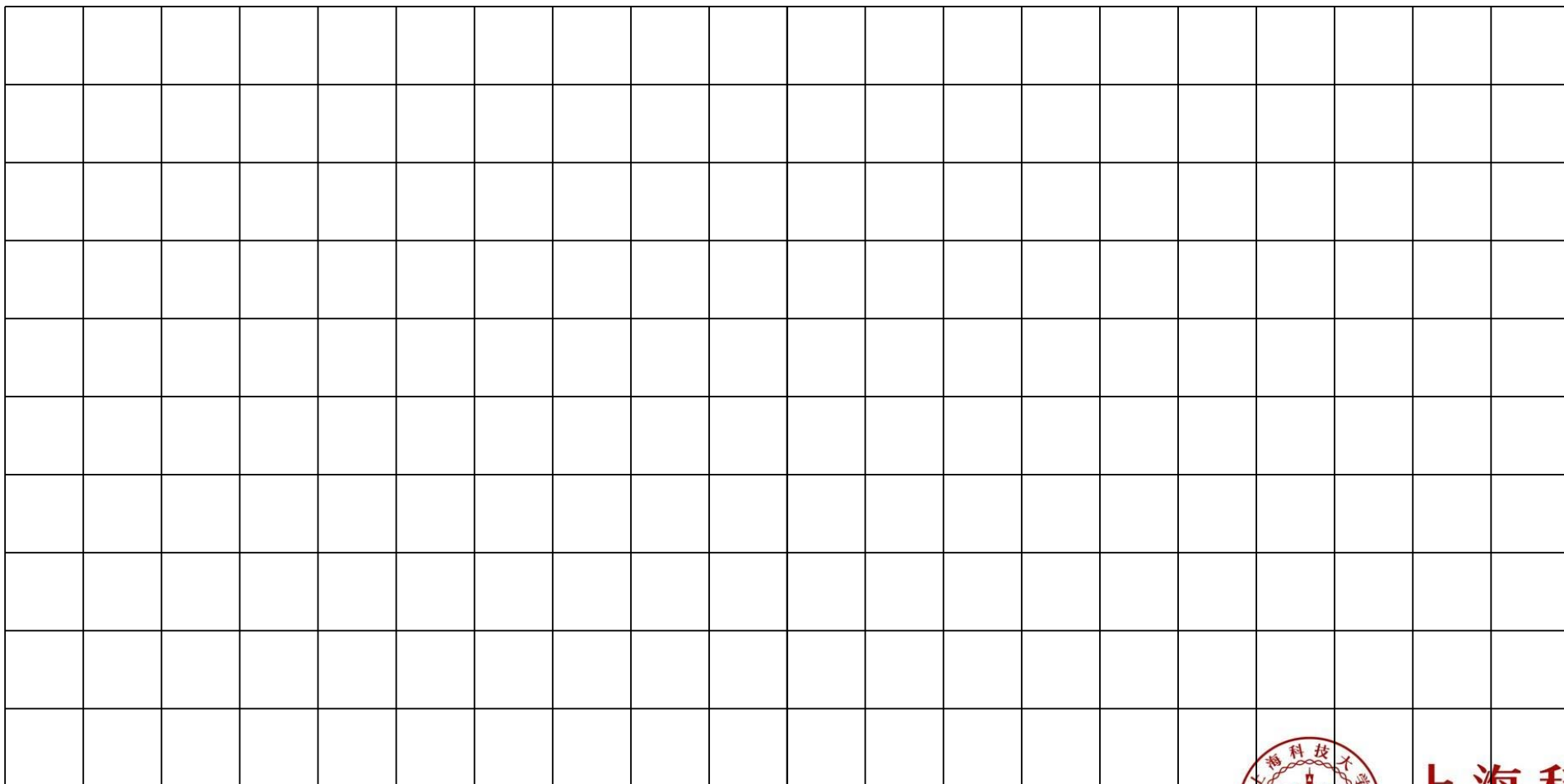
Erosion.

Bridge gaps, fill holes, without change overall size of object.



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Dilation (膨胀)



Opening (开操作)

➤ **Definition:**

$$A \circ B = (A \ominus B) \oplus B$$

or

$$A \circ B = \bigcup \{ (B)_z \mid (B)_z \subseteq A \}$$

➤ **Erode then dilate: break narrow bridges, eliminate thin structures**

➤ **Matlab Function:** `J = imopen(I,SE)`

➤ **Properties:**

- ① $A \circ B$ is a subset (subimage) of A
- ② If C is a subset of D , the $C \circ B$ is a subset of $D \circ B$
- ③ $(A \circ B) \circ B = A \circ B$



Closing (闭操作)

➤ Definition:

$$A \bullet B = (A \oplus B) \ominus B$$

➤ Dilate, then erode: fuse narrow breaks, eliminate small holes

➤ Matlab Function: $J = \text{imclose}(I, SE)$

➤ Properties:

- ① A is a subset (subimage) of $A \bullet B$
- ② If C is a subset of D , the $C \bullet B$ is a subset of $D \bullet B$
- ③ $(A \bullet B) \bullet B = A \bullet B$



Opening & Closing (开操作和闭操作)

➤ Opening (开操作)

- Smooth the contour of an object
- Break narrow bridges.
- Eliminate thin structures.

➤ Closing (闭操作)

- Smooth the contour of an object
- Fuse narrow breaks and long thin gulfs
- Eliminate small holes
- Fill gaps in the contour



The Hit-or-Miss Transformation (击中或击不中变换)

$$A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2)$$

```

0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 1 0 0 0 1 1 1 1 0 0 0 0 0 0
0 1 1 1 0 0 0 0 0 0 0 0 0 1 1 0
0 0 1 0 0 0 0 0 0 0 0 0 0 1 1 1
0 0 0 0 0 1 0 0 0 0 0 0 0 0 1 0
0 0 0 0 1 1 1 0 0 0 0 0 0 0 0 0
0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

```

B_1

```

      1
    1 1
    1

```

```

1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
1 1 0 1 1 1 1 1 1 1 1 1 1 1 1 1
1 1 0 1 1 1 0 0 0 0 1 1 1 1 1 1
1 0 0 0 1 1 1 1 1 1 1 1 0 0 1 1
1 1 0 1 1 1 1 1 1 1 1 1 0 0 0 1
1 1 1 1 1 0 1 1 1 1 1 1 1 0 1 1
1 1 1 1 0 0 0 1 1 1 1 1 1 1 1 1
1 1 1 1 1 0 1 1 1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1

```

B_2

```

      1 1
    1 1
    1 1

```

```

0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

```

```

1 0 1 0 1 1 1 1 1 1 1 1 1 1 1 1
1 0 1 0 1 0 0 0 0 0 0 1 1 1 1 1
0 0 0 0 0 1 1 1 1 1 1 0 0 0 0 1
1 0 1 0 1 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 1 0 1 1 1 1 0 0 0 0 1
1 0 1 0 0 0 0 0 1 1 1 0 0 0 0 0
1 1 1 1 0 1 0 1 1 1 1 1 0 1 0 1
1 1 1 0 0 0 0 0 1 1 1 1 1 1 1 1
1 1 1 1 0 1 0 1 1 1 1 1 1 1 1 1

```

```

0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

```



The Hit-or-Miss Transformation (击中或击不中变换)

$$A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2)$$

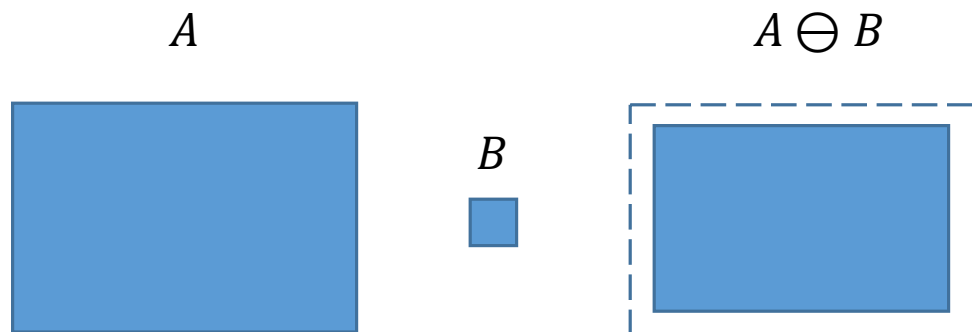
- A method to find the location of a shape B_1 in an image A .
 - Erosion of $A \ominus B_1$ gives all places where B_1 fits in A .
- So also require the boundary around the shape, B_2 to be empty.
 - Erosion of $A^c \ominus B_2$ gives all places where B_2 fits in empty places of A .
- Then take the intersection:

$$A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2)$$

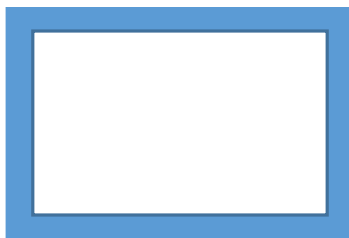


Boundary Extraction (边界提取)

Morphological algorithm: $\beta(A) = A - (A \ominus B)$



$$\beta(A) = A - (A \ominus B)$$



Hole Filling (孔洞填充)

- Let A be the set of 8-connected boundary points of a region
- Start with a point inside the region
- Repeatedly dilate
- At each step, the points corresponding to the region boundary are set to zero :

$$X_k = (X_{k-1} \oplus B) \cap A^c, k = 1, 2, 3, \dots$$

- Stop when no more changes

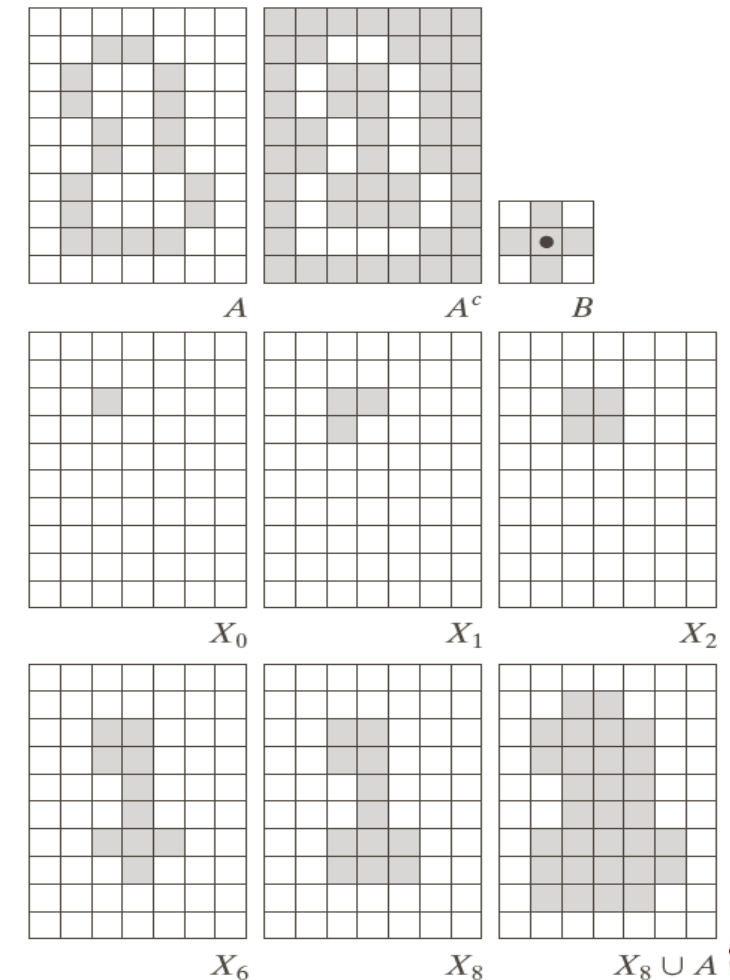


Figure 9.5 Region filling



Take home message

- **Morphological Language: Set theory (集合)**
- **Morphological operations take a set of pixels:**
 - Key element: “structure element”.
 - Insensitive to noise.
 - Smooth edge
 - **Key operations**
 - HMT (Hit or Miss Transformation)(击中与否变换)
 - Dilation (膨胀) and Erosion (腐蚀)

