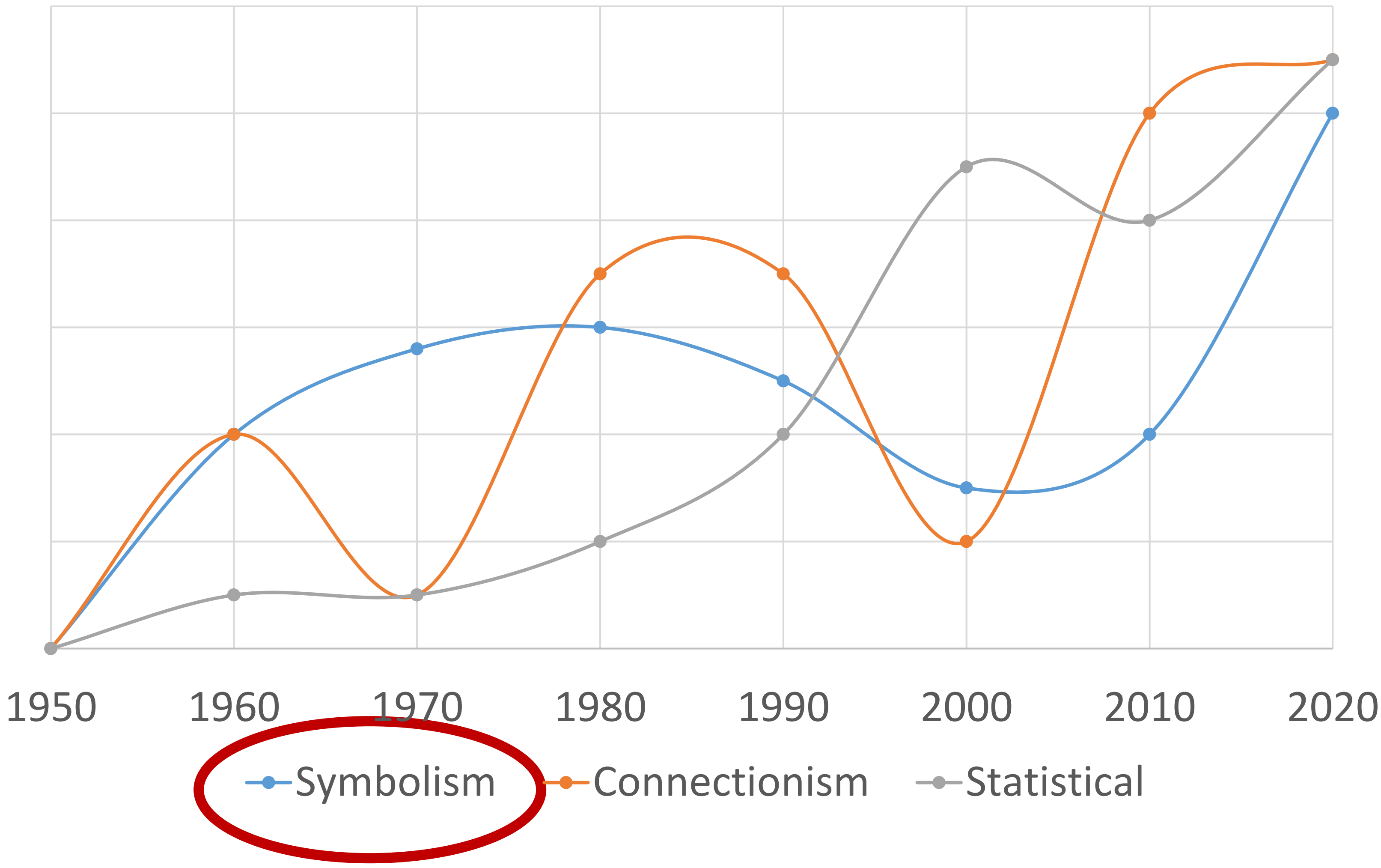
Three types of (strong) AI approaches



Propositional Logic

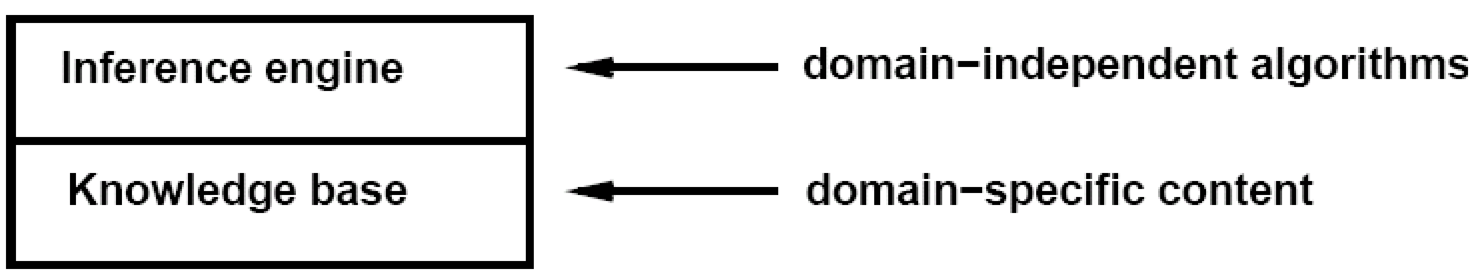
AIMA Chapter 7

Logic-based Symbolic AI

- Logic
  - Formal language in which knowledge can be expressed
  - A means of carrying out reasoning in the language

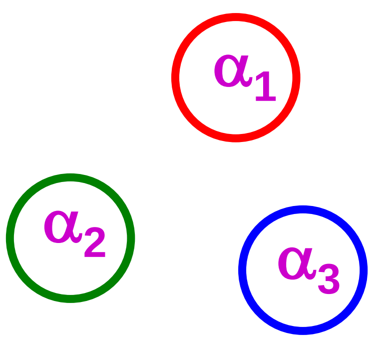
Logic-based Symbolic AI

- Logic (Knowledge-Based) AI
  - Knowledge base
    - set of sentences in a formal language to represent knowledge about the “world”
  - Inference engine
    - answers any answerable question following the knowledge base

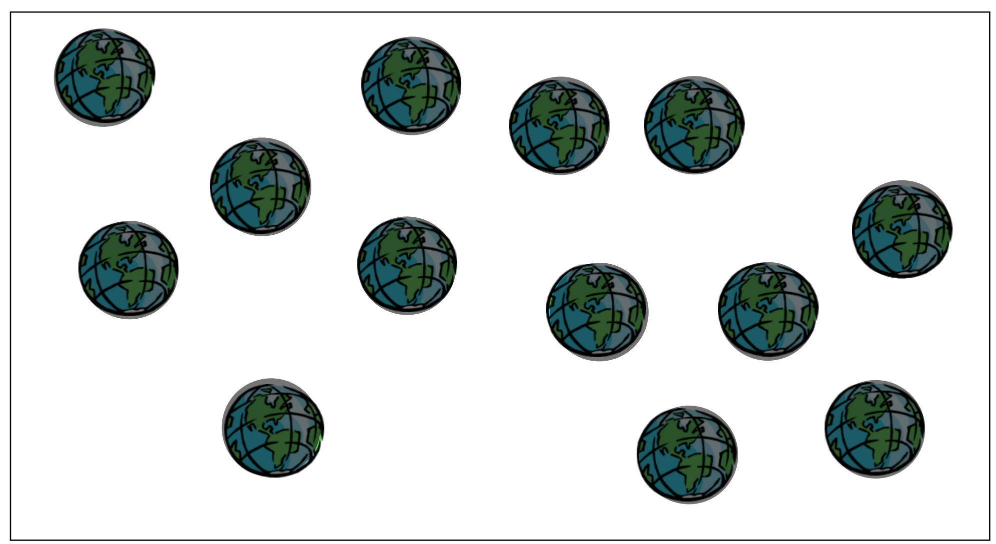


Formal Language

- Components of a formal language in a logic
  - Syntax: What sentences are allowed?
  - Semantics:
    - Which sentences are true/false in each model (possible world)?



Syntaxland



Semanticsland

Formal Language

- Example: the language of arithmetic
  - Syntax
    - $x+2 \geq y$ is a sentence
    - x2+y > {} is not a sentence
  - Semantics
    - $x+2 \geq y$ is true in a world where $x = 7$, $y = 1$
    - $x+2 \geq y$ is false in a world where $x = 0$, $y = 6$

Propositional Logic

Propositional logic: Syntax

- Propositional logic is the "simplest" logic
  - The proposition symbols P1, P2, etc. are sentences
  - If S is a sentence, $\neg$S is a sentence (negation)
  - If S1 and S2 are sentences, S1 $\wedge$ S2 is a sentence (conjunction)
  - If S1 and S2 are sentences, S1 $\vee$ S2 is a sentence (disjunction)
  - If S1 and S2 are sentences, S1 $\Rightarrow$ S2 is a sentence (implication)
  - If S1 and S2 are sentences, S1 $\Leftrightarrow$ S2 is a sentence (biconditional)

$\neg$, $\wedge$, $\vee$, $\Rightarrow$, $\Leftrightarrow$ are called logic connectives or operators

Sometimes $\rightarrow$ and $\leftrightarrow$ are used

Examples of PL sentences

- P means "It is hot."
- Q means "It is humid."
- R means "It is raining."
- $(P \wedge Q) \Rightarrow R$
  - "If it is hot and humid, then it is raining"
- $Q \Rightarrow P$
  - "If it is humid, then it is hot"

Propositional logic: Semantics

- Each model specifies true/false for each proposition symbol
  - E.g. P_1 P_2 P_3

P_1	P_2	P_3
false	true	false

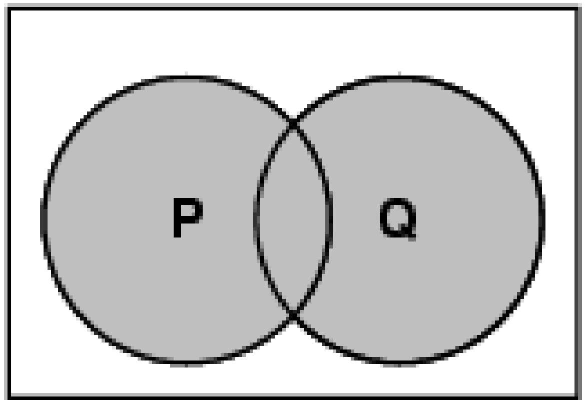
- Rules for evaluating truth with respect to a model m:
  - $\neg$S is true iff S is false
  - S1 $\wedge$ S2 is true iff S1 is true and S2 is true
  - S1 $\vee$ S2 is true iff S1 is true or S2 is true
  - S1 $\Rightarrow$ S2 is true iff S1 is false or S2 is true
  - S1 $\Leftrightarrow$ S2 is true iff S1$\Rightarrow$S2 is true and S2$\Rightarrow$S1 is true

Truth tables for connectives

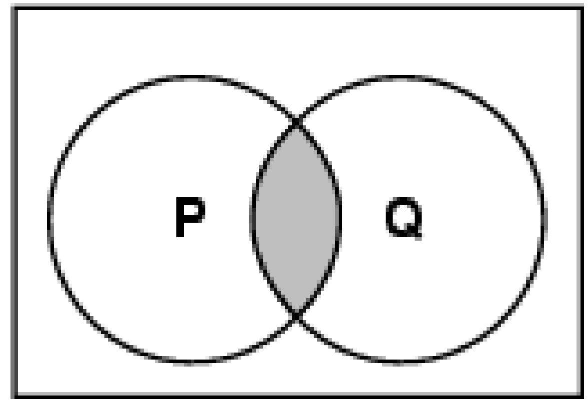
P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
$false$	$false$	$true$	$false$	$false$	$true$	$true$
$false$	$true$	$true$	$false$	$true$	$true$	$false$
$true$	$false$	$false$	$false$	$true$	$false$	$false$
$true$	$true$	$false$	$true$	$true$	$true$	$true$

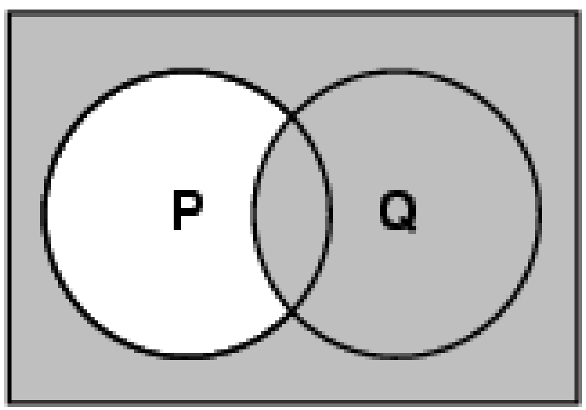
Venn Diagrams

$P \vee Q$

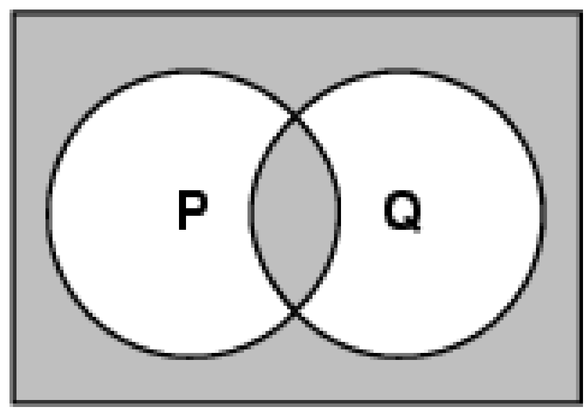


$P \wedge Q$





$P \Rightarrow Q$



$P \Leftrightarrow Q$

Material Implication

- S1 $\Rightarrow$ S2 is true iff S1 is false or S2 is true
- Given the following propositions, is “S1 $\Rightarrow$ S2” true?
  - S1 means “the moon is made of green cheese”
  - S2 means “the world is coming to an end”

- Material implication does not capture the meaning of “if... then”.
- See “Paradoxes of material implication” in Wikipedia

Logical equivalence

- Two sentences are logically equivalent iff true in same models

$$
\begin{aligned}
(\alpha \wedge \beta) &\equiv (\beta \wedge \alpha) \quad \text{commutativity of } \wedge \\
(\alpha \vee \beta) &\equiv (\beta \vee \alpha) \quad \text{commutativity of } \vee \\
((\alpha \wedge \beta) \wedge \gamma) &\equiv (\alpha \wedge (\beta \wedge \gamma)) \quad \text{associativity of } \wedge \\
((\alpha \vee \beta) \vee \gamma) &\equiv (\alpha \vee (\beta \vee \gamma)) \quad \text{associativity of } \vee \\
\neg(\neg\alpha) &\equiv \alpha \quad \text{double-negation elimination} \\
(\alpha \Rightarrow \beta) &\equiv (\neg\beta \Rightarrow \neg\alpha) \quad \text{contraposition} \\
(\alpha \Rightarrow \beta) &\equiv (\neg\alpha \vee \beta) \quad \text{implication elimination} \\
(\alpha \Leftrightarrow \beta) &\equiv ((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)) \quad \text{biconditional elimination} \\
\neg(\alpha \wedge \beta) &\equiv (\neg\alpha \vee \neg\beta) \quad \text{de Morgan} \\
\neg(\alpha \vee \beta) &\equiv (\neg\alpha \wedge \neg\beta) \quad \text{de Morgan} \\
(\alpha \wedge (\beta \vee \gamma)) &\equiv ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma)) \quad \text{distributivity of } \wedge \text{ over } \vee \\
(\alpha \vee (\beta \wedge \gamma)) &\equiv ((\alpha \vee \beta) \wedge (\alpha \vee \gamma)) \quad \text{distributivity of } \vee \text{ over } \wedge
\end{aligned}
$$

Validity and satisfiability

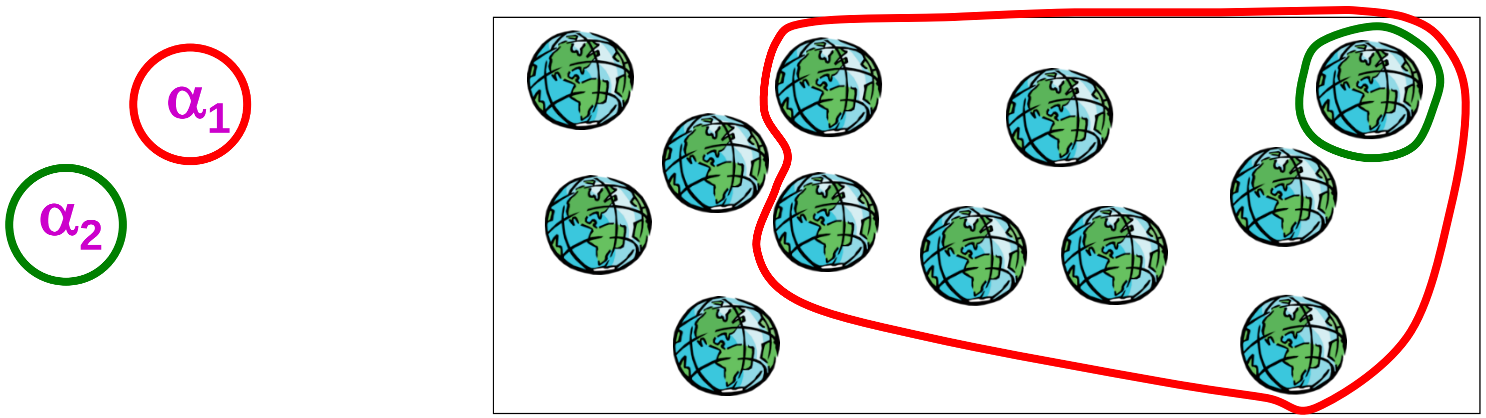
- A sentence is valid if it is true in all models
  - e.g., $A \vee \neg A$, $A \Rightarrow A$, $(A \wedge (A \Rightarrow B)) \Rightarrow B$

- A sentence is satisfiable if it is true in some model
  - e.g., $A \vee B$, C
- A sentence is unsatisfiable if it is true in no models
  - e.g., $A \wedge \neg A$

- Obviously, S is valid iff. $\neg S$ is unsatisfiable

Inference: entailment

- Entailment: $\alpha \models \beta$ ("α entails β" or "β follows from α") means in every world where α is true, β is also true
  - i.e., the α-worlds are a subset of the β-worlds [models(α) $\subseteq$ models(β)]
- In the example, $\alpha 2 \models \alpha 1$



Inference: proof

- A proof (α |- β) is a demonstration of entailment from α to β
  - Method 1: model checking
    - Truth table enumeration to check if models(α) $\subseteq$ models(β)
    - Time complexity always exponential in n ☹

P1	**P2**	**…**	**Pn**	$\boldsymbol{\alpha}$	$\boldsymbol{\beta}$
F	F	…	F	F	T
F	F	…	T	T	T
		……			
T	T	…	F	T	T
T	T	…	T	F	F

Inference: proof

- A proof (α |- β) is a demonstration of entailment from α to β
  - Method 2: application of inference rules
    - Search for a finite sequence of sentences each of which is an axiom or follows from the preceding sentences by a rule of inference
    - Axiom: a sentence known to be true
    - Rule of inference: a function that takes one or more sentences (premises) and returns a sentence (conclusion)

Inference: soundness & completeness

- Sound inference
  - everything that can be proved is in fact entailed
- Complete inference
  - everything that is entailed can be proved

- Method 1 (enumeration) is obviously sound and complete
- For method 2 (applying inference rules), it is much less obvious
  - Example: arithmetic is found to be not complete! (Gödel's theorem, 1931)

Quiz

- What’s the connection between complete inference algorithms and complete search algorithms?
- Answer 1: they both have the words “complete…algorithm”
- Answer 2: Formulate inference α |- β as a search problem
  - Initial state: KB contains α
  - Actions: apply any inference rule that matches KB, add conclusion
  - Goal test: KB contains β

  Hence any complete search algorithm can be used to produce a complete inference algorithm

Resolution: an inference rule in PL

- Conjunctive Normal Form (CNF)
  - conjunction of disjunctions of literals (clauses)
  - Ex
    - $(A \lor \neg B) \land (B \lor \neg C \lor \neg D)$
    - $(\neg B_{1,1} \lor P_{1,2} \lor P_{2,1}) \land (\neg P_{1,2} \lor B_{1,1}) \land (\neg P_{2,1} \lor B_{1,1})$

Conversion to CNF

$B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$

1. Eliminate $\Leftrightarrow$, replacing $\alpha \Leftrightarrow \beta$ with $(\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)$.

   $(B_{1,1} \Rightarrow (P_{1,2} \vee P_{2,1})) \wedge ((P_{1,2} \vee P_{2,1}) \Rightarrow B_{1,1})$

2. Eliminate $\Rightarrow$, replacing $\alpha \Rightarrow \beta$ with $\neg\alpha \vee \beta$.

   $(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg(P_{1,2} \vee P_{2,1}) \vee B_{1,1})$

3. Move $\neg$ inwards using de Morgan's rules and double-negation:

   $(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge ((\neg P_{1,2} \wedge \neg P_{2,1}) \vee B_{1,1})$

4. Apply distributivity law ($\wedge$ over $\vee$) and flatten:

   $(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg P_{1,2} \vee B_{1,1}) \wedge (\neg P_{2,1} \vee B_{1,1})$

Resolution: an inference rule in PL

- Resolution inference rule (for CNF):

  Suppose l_i is $\neg m_j$

$$\frac{l_1 \vee \ldots \vee l_k, \qquad m_1 \vee \ldots \vee m_n}{l_1 \vee \ldots \vee l_{i-1} \vee l_{i+1} \vee \ldots \vee l_k \vee m_1 \vee \ldots \vee m_{j-1} \vee m_{j+1} \vee \ldots \vee m_n}$$

  Examples:

$$\frac{P_{1,3} \vee P_{2,2}, \qquad P_{2,3} \vee \neg P_{2,2}}{P_{1,3} \vee P_{2,3}} \qquad \frac{P_1, \neg P_1}{\{\}}$$

- Resolution is sound and complete for propositional logic

Resolution algorithm

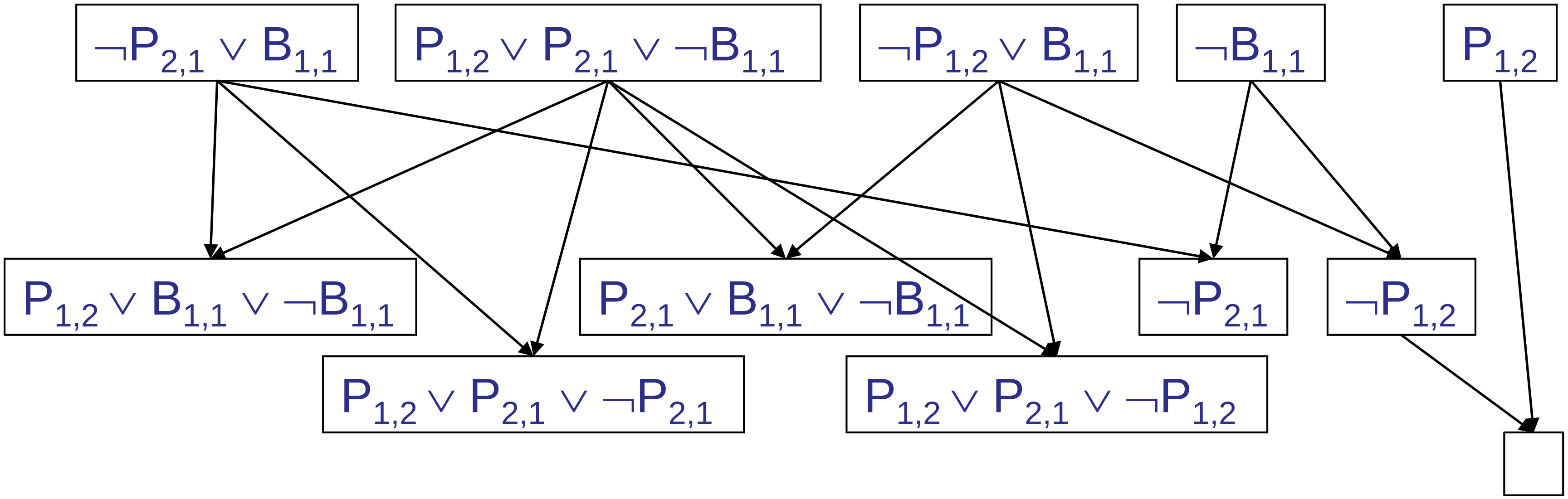
- The best way to prove $KB \models \alpha$?
  - Proof by contradiction, i.e., show $KB \land \neg \alpha$ is unsatisfiable

1. Convert $KB \land \neg \alpha$ to CNF
2. Repeatedly apply the resolution rule to add new clauses, until one of the two things happens
   a) Two clauses resolve to yield the empty clause, in which case *KB* entails α
   b) There is no new clause that can be added, in which case *KB* does not entail α

Resolution example

$KB = (B_{1,1} \Leftrightarrow (P_{1,2} \lor P_{2,1})) \land \neg B_{1,1}$

$\alpha = \neg P_{1,2}$



Horn Logic

Horn logic

- Inference in propositional logic is in general NP-complete!
- Solution: a subset of propositional logic that supports efficient inference

Expressiveness vs. Inference difficulty!!

- Horn logic: only (strict) Horn clauses are allowed
  - A Horn clause has the form:

    $P1 \wedge P2 \wedge P3 \ldots \wedge Pn \Rightarrow Q$

    or alternatively

    $\neg P1 \vee \neg P2 \vee \neg P3 \ldots \vee \neg Pn \vee Q$

    where Ps and Q are non-negated proposition symbols (atoms)
  - n can be zero, i.e., the clause contains a single atom

Inference in Horn logic

- Modus Ponens

$$\frac{\alpha 1, \ldots, \alpha n, \quad \overbrace{\alpha 1 \wedge \ldots \wedge \alpha n}^{\text{premises}} \Rightarrow \overset{\text{conclusion}}{\beta}}{\beta}$$

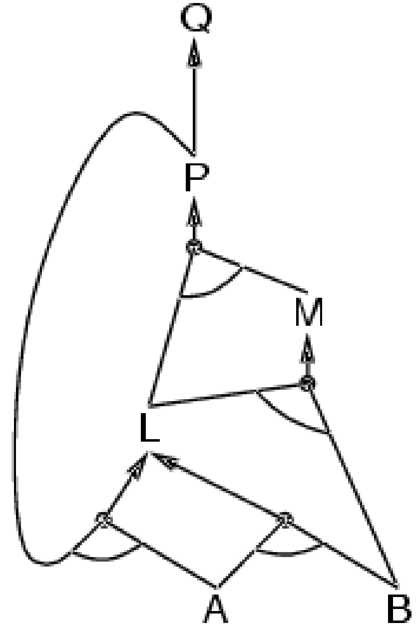
  - Modus Ponens is sound and complete for Horn logic

- Inference algorithms (for Horn logic)
  - Forward chaining, backward chaining
  - These algorithms are very natural and run in linear time

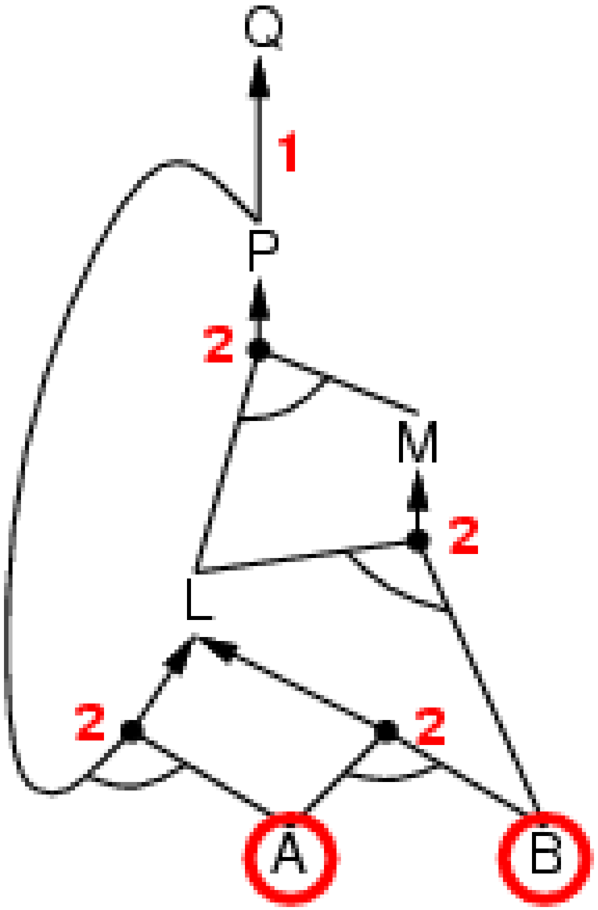
Forward chaining

- Idea: to prove KB |= Q
  - Add new clauses into the KB by applying Modus Ponens, until Q is added

$$P \Rightarrow Q$$
$$L \wedge M \Rightarrow P$$
$$B \wedge L \Rightarrow M$$
$$A \wedge P \Rightarrow L$$
$$A \wedge B \Rightarrow L$$
$$A$$
$$B$$



Forward chaining example



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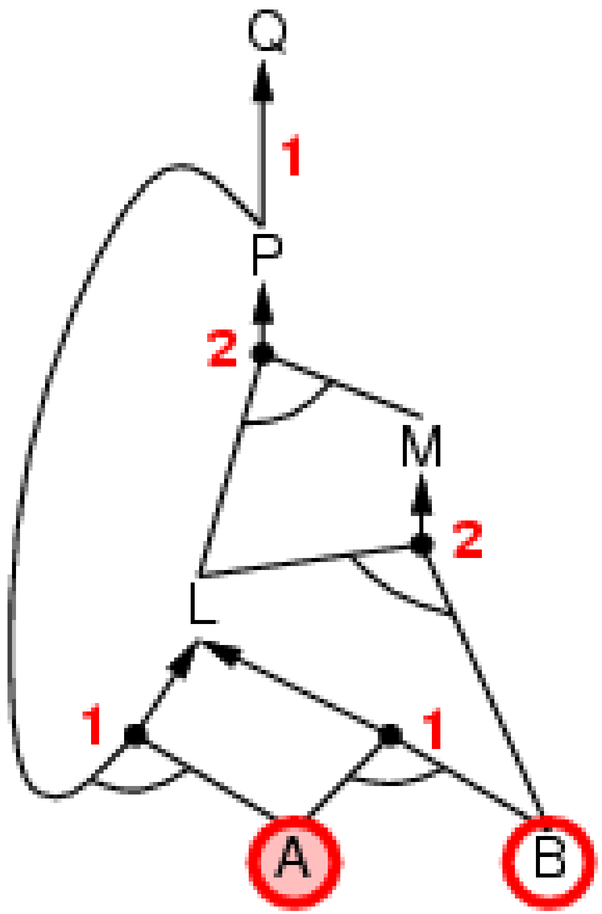

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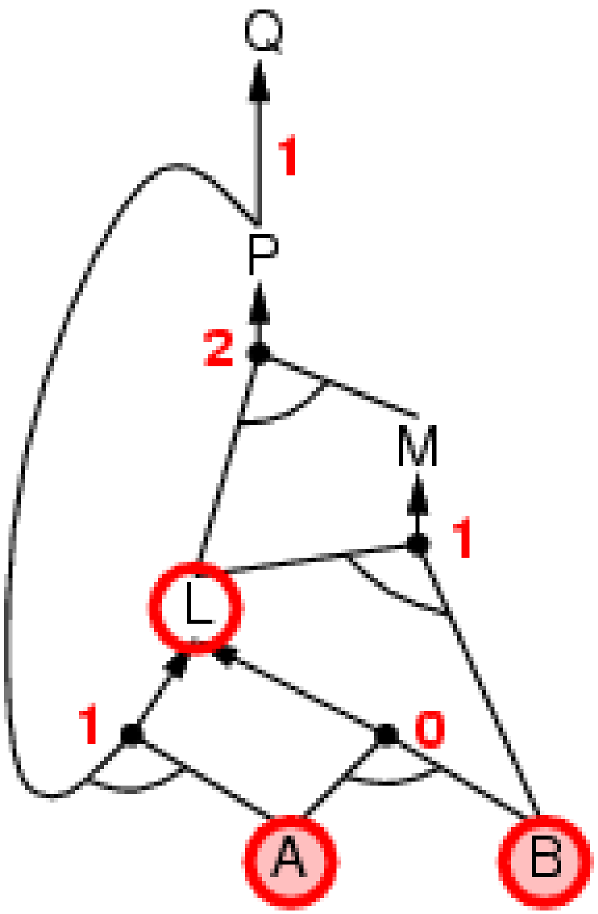

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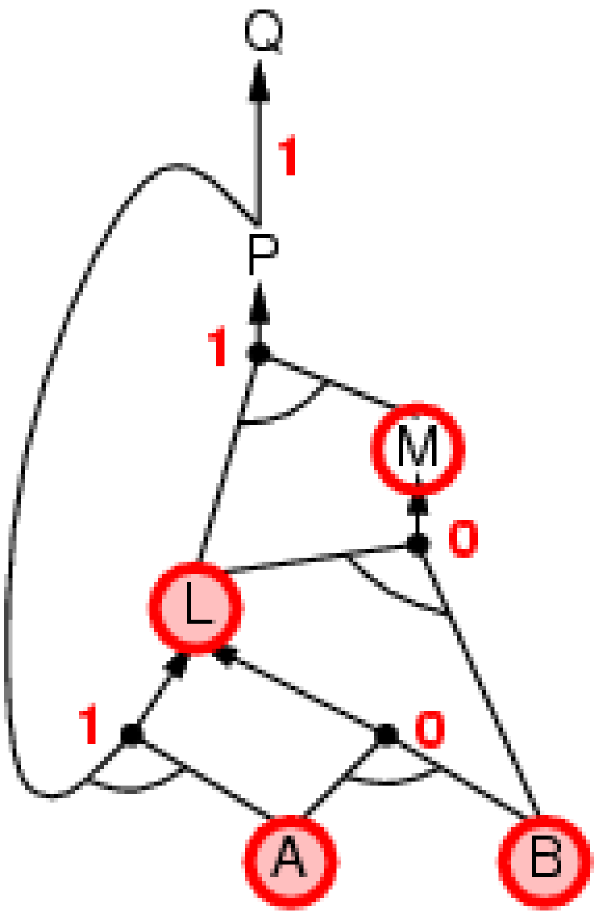

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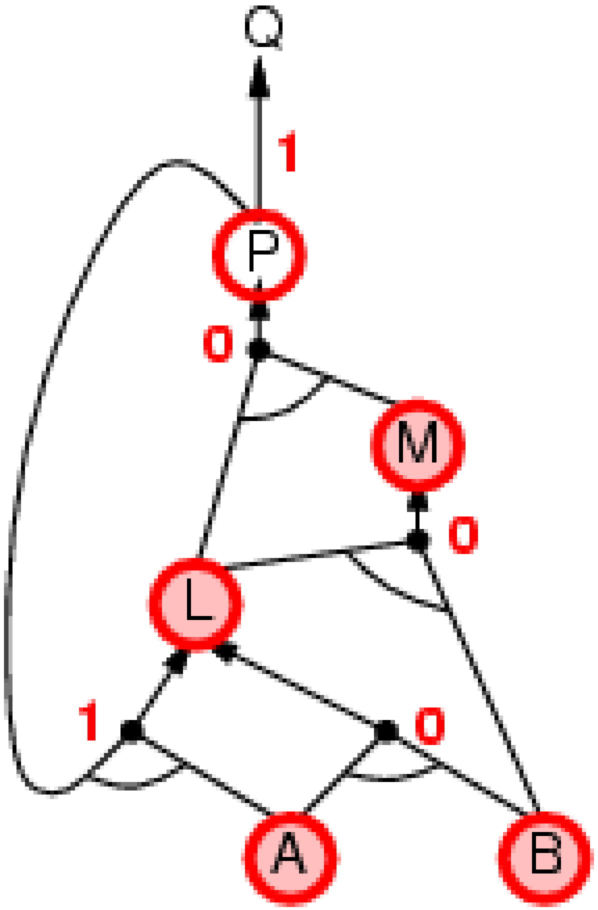

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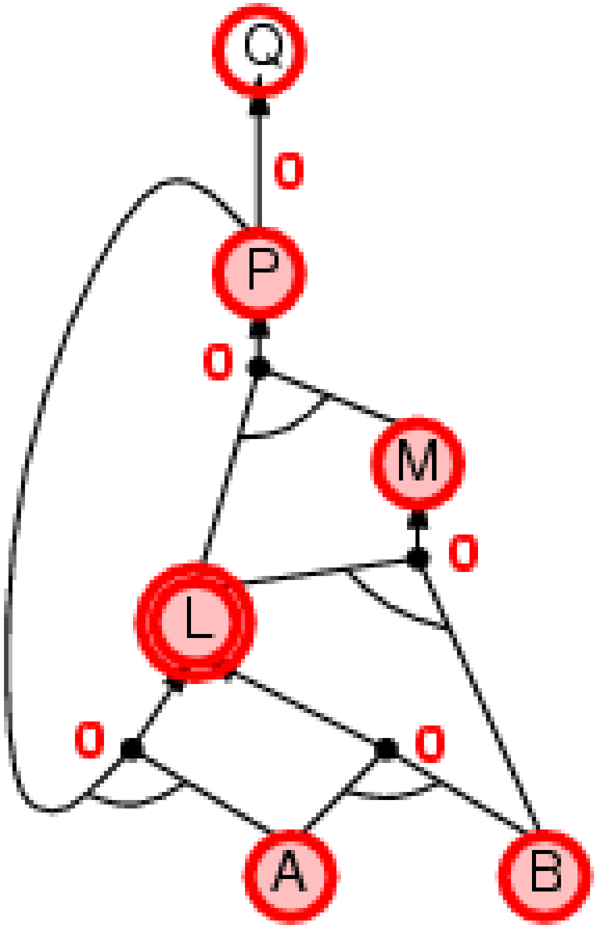

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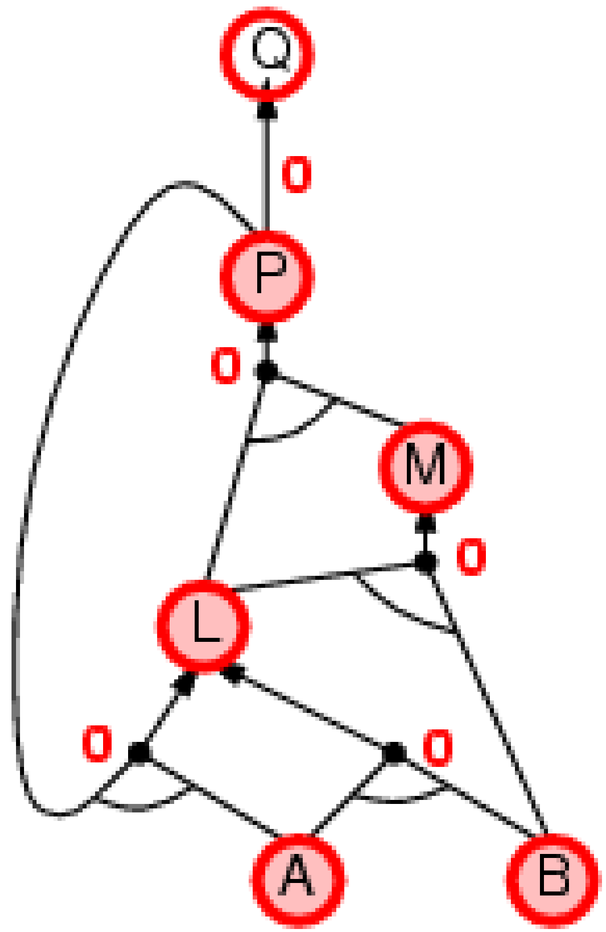

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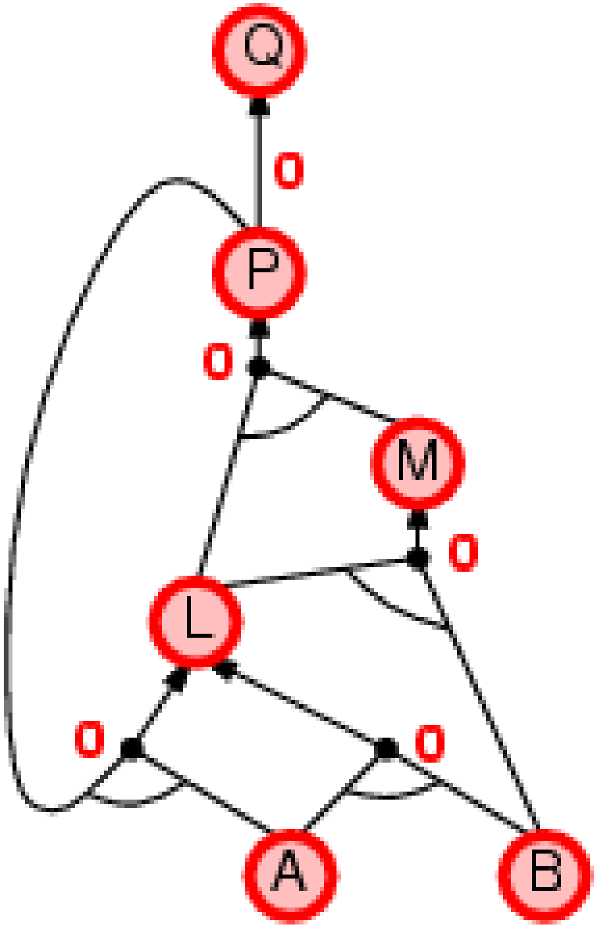
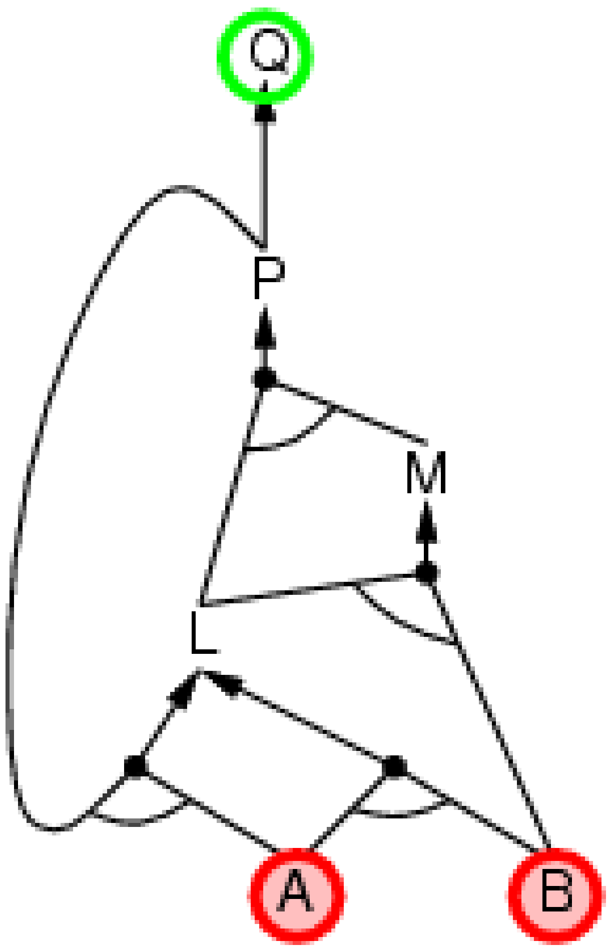


Backward chaining

- Idea: work backwards from the query q:
  - to prove q by BC,
    - check if q is known to be true already, or
    - prove by BC all premises of some rule concluding q
- Avoid loops: check if new subgoal is already on the goal stack
- Avoid repeated work: check if new subgoal
  1. has already been proved true, or
  2. has already failed

Backward chaining example



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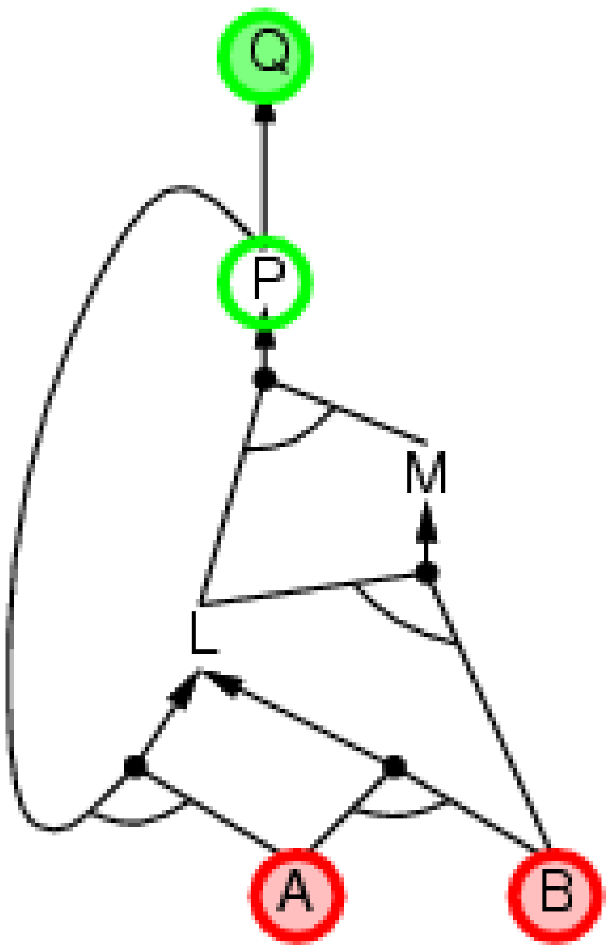

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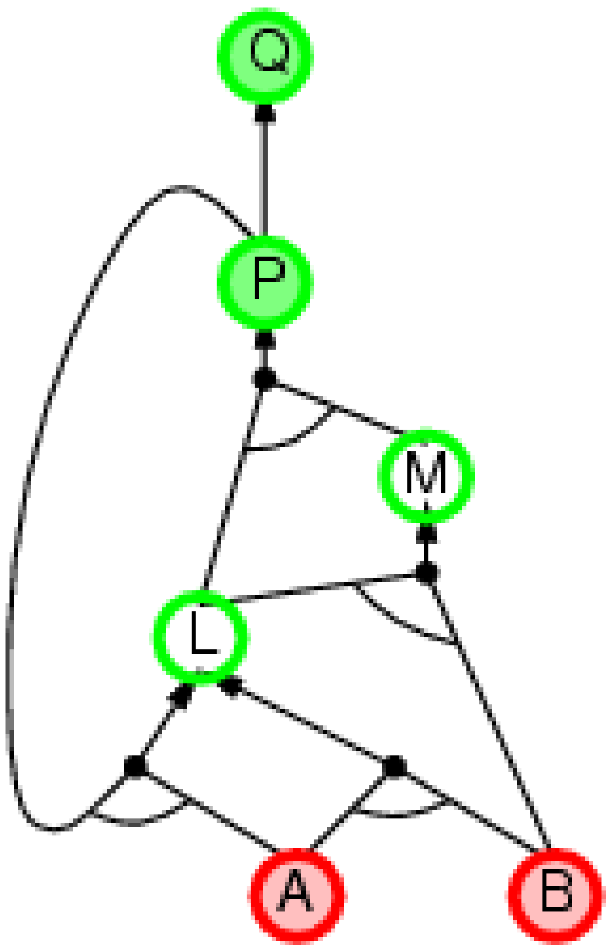

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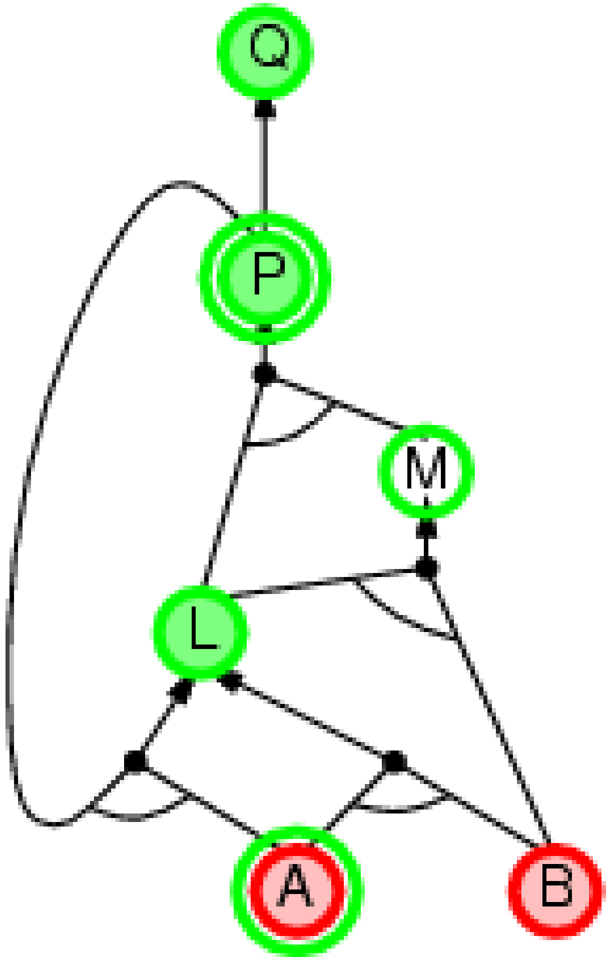

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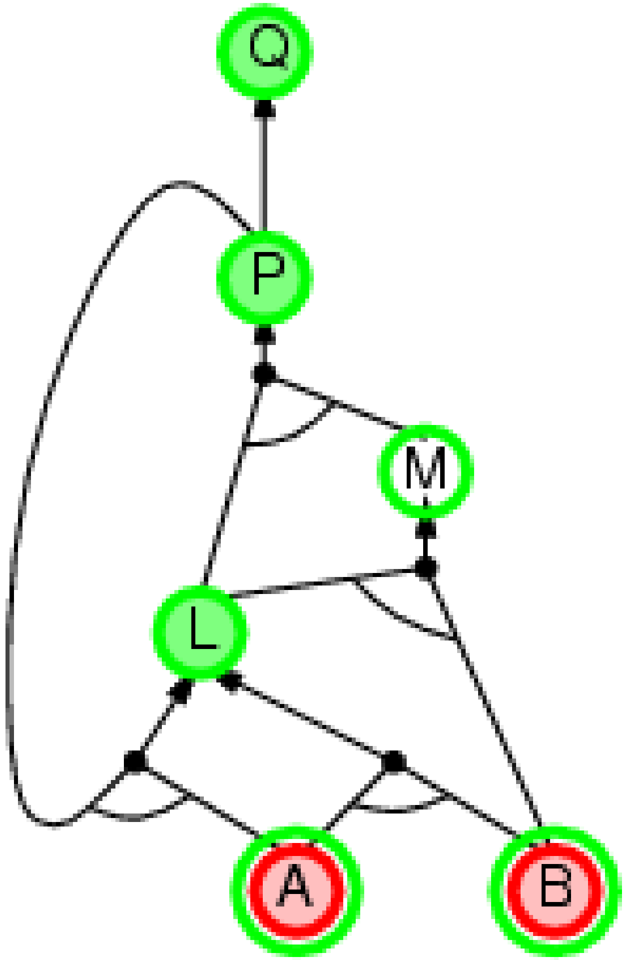

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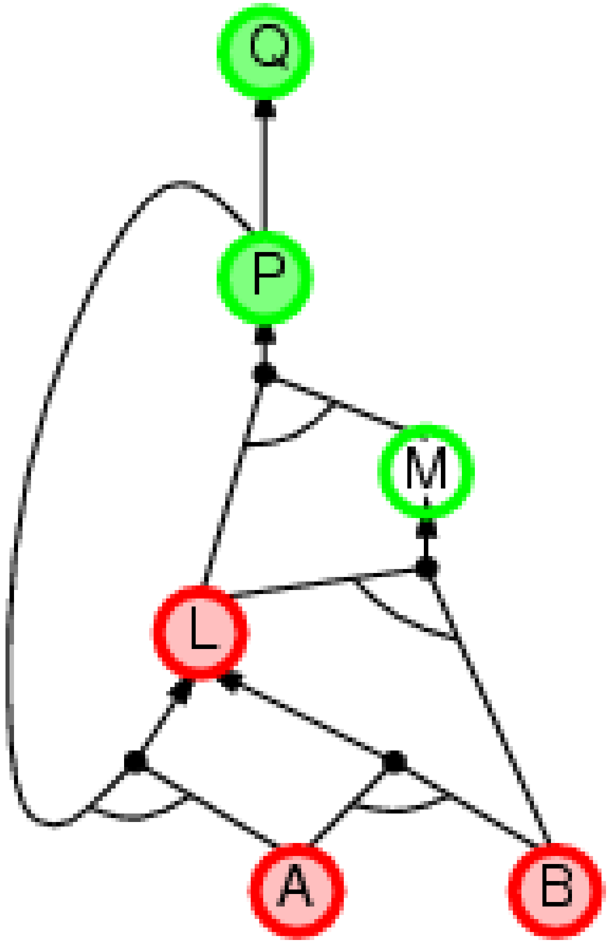

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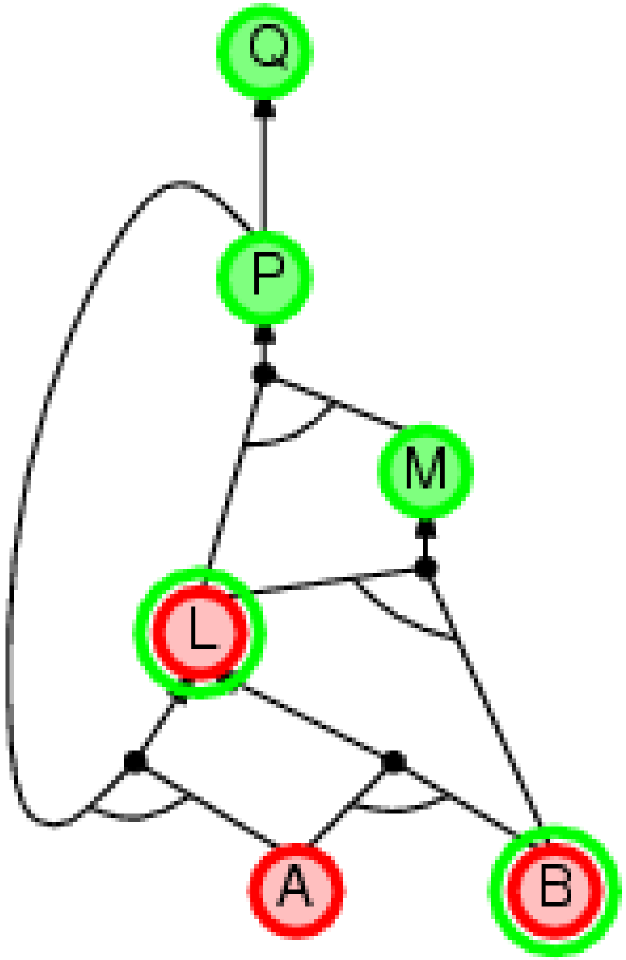

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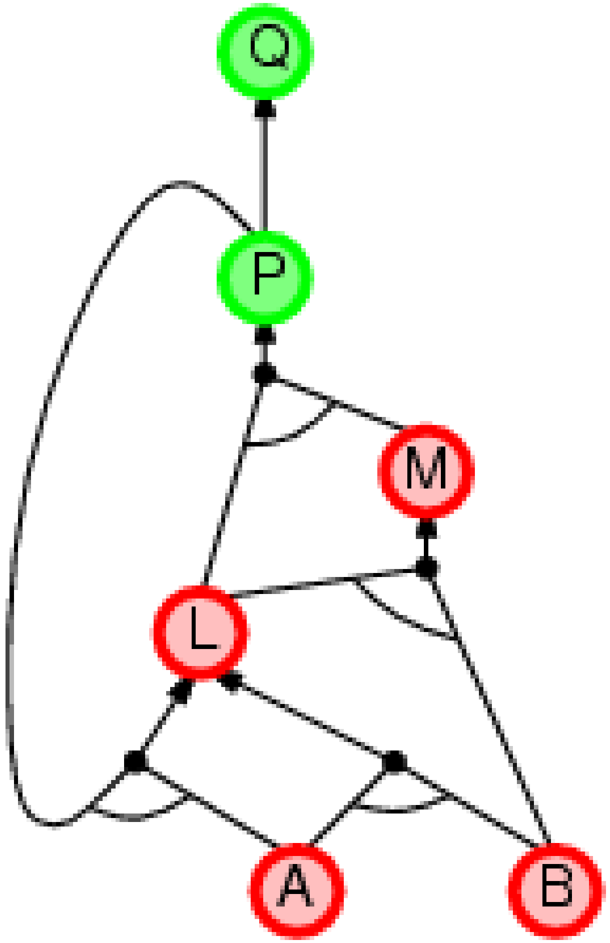

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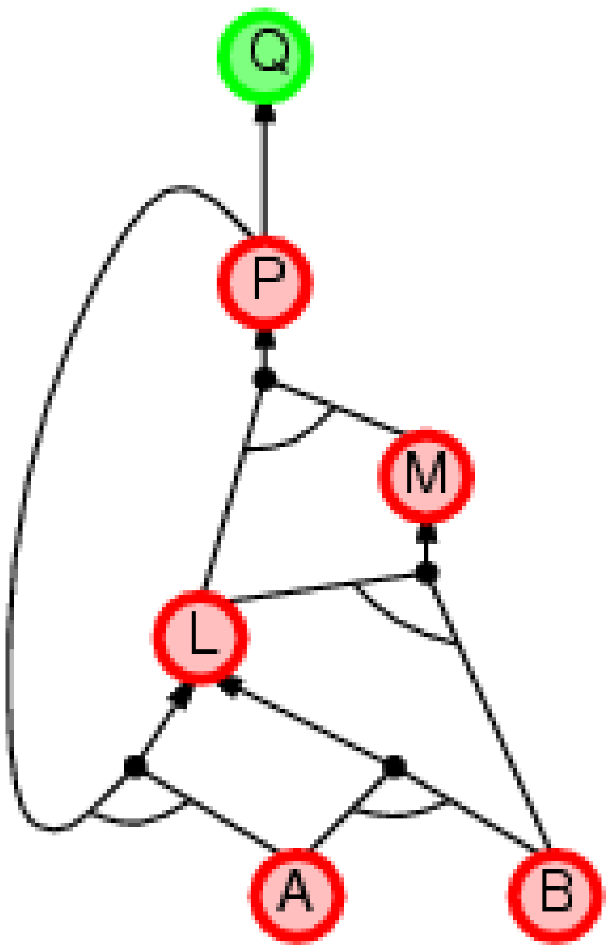

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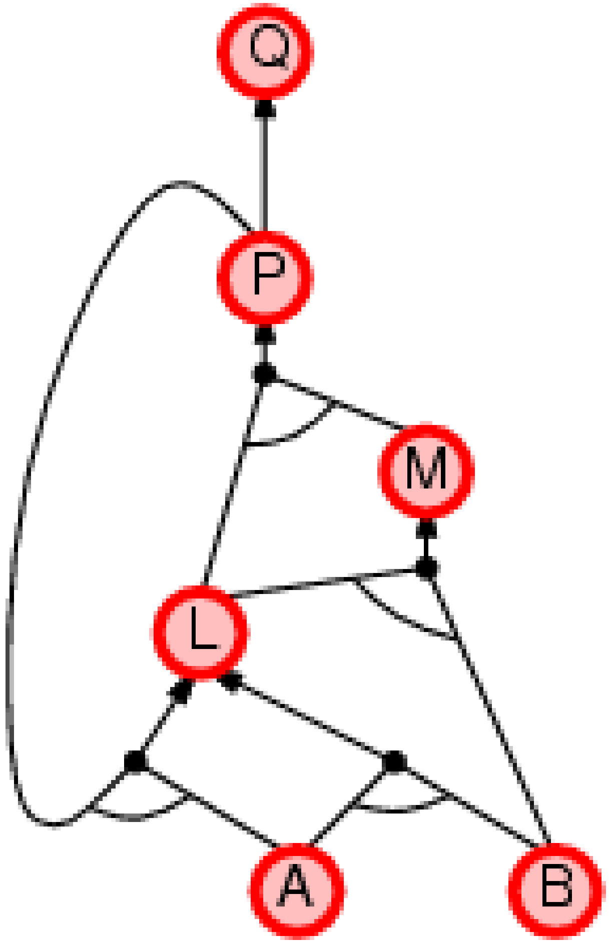

Forward vs. backward chaining

- FC is data-driven, automatic, unconscious processing,
  - e.g., object recognition, routine decisions
  - May do lots of work that is irrelevant to the goal
- BC is goal-driven, appropriate for problem-solving,
  - e.g., Where are my keys? How do I get into a PhD program?
  - Complexity of BC can be much less than linear in size of KB

A Logical Pacman

Partially observable Pacman

- Pacman perceives just the local walls/gaps
- Formulation: what proposition symbols do we need?
  - Pacman's location
    - At_1,1_0 (Pacman is at [1,1] at time 0)   At_3,3_1   etc
  - Wall locations
    - Wall_0,0   Wall_0,1   etc
  - Percepts
    - Blocked_W_0 (blocked by wall to my West at time 0) etc.
  - Actions
    - W_0 (Pacman moves West at time 0)   E_0   etc.
- NxN world for T time steps => $N^2T + N^2 + 4T + 4T = O(N^2T)$ symbols

Sensor model

- State facts about how Pacman's percepts arise…
- Pacman perceives a wall to the West at time t if he is in x,y and there is a wall at x-1,y

$$((\text{At\_1,1\_0} \wedge \text{Wall\_0,1}) \vee$$
$$(\text{At\_1,2\_0} \wedge \text{Wall\_0,2}) \vee$$
$$(\text{At\_1,3\_0} \wedge \text{Wall\_0,3}) \vee \ldots.) \Rightarrow \text{Blocked\_W\_0}$$

Sensor model

- State facts about how Pacman's percepts arise…
- Pacman perceives a wall to the West at time t ***if and only if*** he is in x,y and there is a wall at x-1,y

$$
\begin{aligned}
\text{Blocked\_W\_0} \Leftrightarrow ((&\text{At\_1,1\_0} \wedge \text{Wall\_0,1}) \vee \\
&(\text{At\_1,2\_0} \wedge \text{Wall\_0,2}) \vee \\
&(\text{At\_1,3\_0} \wedge \text{Wall\_0,3}) \vee \ldots)
\end{aligned}
$$

Transition model

- How does each state symbol at each time get its value?
  - E.g., should At_3,3_17 be T or F?
- A state symbol gets its value according to a successor-state axiom

$$X_t \Leftrightarrow [X_{t-1} \wedge \neg(\text{some action}_{t-1} \text{ made it false})] \vee$$
$$[\neg X_{t-1} \wedge (\text{some action}_{t-1} \text{ made it true})]$$

- For Pacman location:

At_3,3_17 ⇔
[At_3,3_16 ∧ ¬((¬Wall_3,4 ∧ N_16) ∨ (¬Wall_4,3 ∧ E_16) ∨ …)]
∨ [¬At_3,3_16 ∧ ((At_3,2_16 ∧ ¬Wall_3,3 ∧ N_16) ∨
(At_2,3_16 ∧ ¬Wall_3,3 ∧ E_16) ∨ …)]

Initial state

- The agent may know its initial location:

  At_1,1_0

- Or, it may not:

  At_1,1_0 $\vee$ At_1,2_0 $\vee$ At_1,3_0 $\vee$ … $\vee$ At_3,3_0

Domain constraint

- Pacman cannot be in two places at once!

$\neg$(At_1,1_0 $\wedge$ At_1,2_0) $\wedge$ $\neg$(At_1,1_0 $\wedge$ At_1,3_0) $\wedge$ …

$\neg$(At_1,1_1 $\wedge$ At_1,2_1) $\wedge$ $\neg$(At_1,1_1 $\wedge$ At_1,3_1) $\wedge$ …

…

- Pacman cannot take two actions at the same time!

$\neg$(E_0 $\wedge$ S_0) $\wedge$ $\neg$(E_0 $\wedge$ W_0) $\wedge$ …

$\neg$(E_1 $\wedge$ S_1) $\wedge$ $\neg$(E_1 $\wedge$ W_1) $\wedge$ …

…

- Pacman cannot go into a wall!

At_1,1_0 $\wedge$ N_0 $\Rightarrow$ $\neg$Wall_1,2

…

Planning as satisfiability

- SAT solver
  - Input: a logic expression
  - Output: a model (true/false assignments to symbols) that satisfies the expression if such a model exists
- Can we use it to make plans for Pacman (e.g., to move to a goal position)?
  - For T = 1 to infinity, set up the KB as follows and run SAT solver:
    - Initial state, domain constraints, sensor & transition model sentences up to time T
    - Goal is true at time T
  - If a model is returned, extract a plan from action assignment

Planning as satisfiability

- Q: Isn’t this a search problem? Any advantage of using logic?
- A: We can use logic to solve not only search problems, but any problems that are representable using the logic.

Logic programming

- Ordinary programming
  - Identify problem
  - Assemble information
  - Figure out solution
  - Encode solution
  - Encode problem instance as data
  - Apply program to data

- Logic programming
  - Identify problem
  - Assemble information
  - <coffee break> ☺
  - Encode info in KB
  - Encode problem instance as facts
  - Ask queries (run SAT solver)

Summary

- Logic
  - Logical AI applies inference to a knowledge base to derive new information
- Propositional logic
  - Syntax
  - Semantics
  - Inference (resolution)
- Horn logic
  - Inference (forward/backward chaining)
- Application of logic to Pacman