Quiz for lecture 21 and 22

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1 lecture 21

$$
\begin{aligned}
\mu &= \frac{1}{n}\sum_{1}^{n} x^i \\
\frac{1}{n}\sum_{1}^{n} \|x^i - c\|^2 &= \frac{1}{n}\sum_{1}^{n} \|x^i - \mu + \mu - c\|^2 \\
&= \frac{1}{n}\sum_{1}^{n} \|x^i - \mu\|^2 + \frac{1}{n}\sum_{1}^{n} \|\mu - c\|^2 + \frac{2}{n}\sum_{1}^{n} (x^i - \mu)^T(\mu - c) \\
&= \frac{1}{n}\sum_{1}^{n} \|x^i - \mu\|^2 + \|\mu - c\|^2 + 0^T(\mu - c) \\
&= \frac{1}{n}\sum_{1}^{n} \|x^i - \mu\|^2 + \|\mu - c\|^2
\end{aligned}
$$

take derivative w.r.t c and set it to 0, then we have the optimal $c = \mu$

2 lecture 22

Each image except the top left one forms an eigenvector, so there are 15 eigenvectors in total.
We can reconstruct the image by the following steps:

1. Reshape each image as a "long" vector $v_i,\ i \in \{1, \dots, 15\}$ and x
2. calculate the coefficient by projecting x onto each v_i, and we get $\langle x, v_i \rangle$
3. construct a linear combination $\hat{x} = \sum_1^{15} \langle x, v_i \rangle v_i$
4. reshape $\hat{x}$ back to the matrix shape