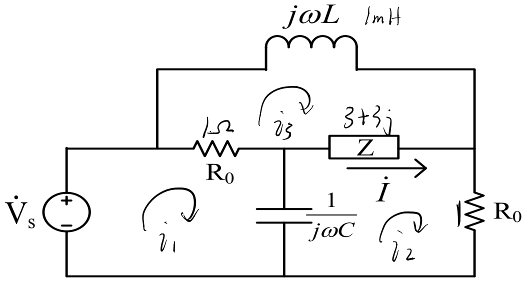
1. (10 points) Determine the value of C so that $\dot{I}=0$ A. Assume $v_s(t)$=cos(1000t) V, $R_0 = 1\ \Omega$, Z=(3+j3) Ω and L=1 mH.



(1)

$$\begin{cases} -1 + 1\cdot(i_1 - i_3) + \frac{1}{j\omega C}(i_1 - i_2) = 0 & -2 \\ \frac{1}{j\omega C}(i_2 - i_1) + (3+3j)(i_2 - i_3) + 1\cdot i_2 = 0 & -2 \\ 1(i_3 - i_1) + j\omega L\cdot i_3 + (3+3j)\cdot(i_3 - i_2) = 0 & -2 \end{cases}$$

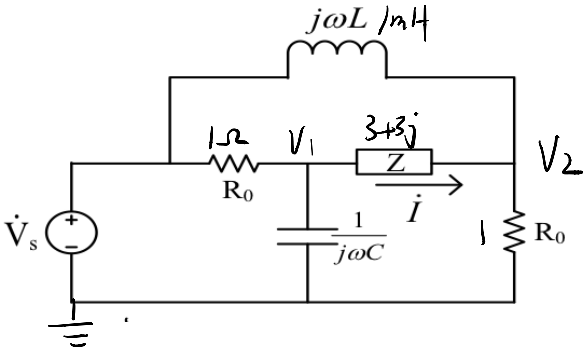
$$\Rightarrow \begin{cases} \left(1+\frac{1}{j\omega C}\right)i_1 - \frac{1}{j\omega C}i_2 - i_3 = 1 \\ -\frac{1}{j\omega C}\cdot i_1 + \left(\frac{1}{j\omega C} + 4 + 3j\right)i_2 - (3+3j)i_3 = 0 \\ -i_1 - (3+3j)i_2 + (4 + j\omega L + 3j)i_3 = 0 \end{cases}$$

$\because \dot{i} = \dot{i}_2 - \dot{i}_3 = 0$

$\therefore \dot{i}_2 = \dot{i}_3$ −2

$\Rightarrow C = \frac{1}{\omega} = 1mF$ −2

(2)



$$\begin{cases} \frac{V_1 - 1}{1} + V_1\cdot j\omega C + \frac{V_1 - V_2}{3+3j} = 0 & -3 \\ \frac{V_2 - 1}{j} + \frac{V_2 - V_1}{3+3j} + \frac{V_2}{1} = 0 & -3 \end{cases}$$

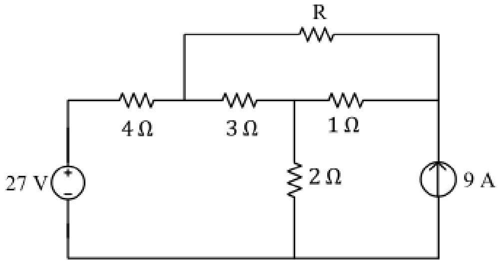
$\because i = 0$

$\therefore V_1 = V_2$ −2

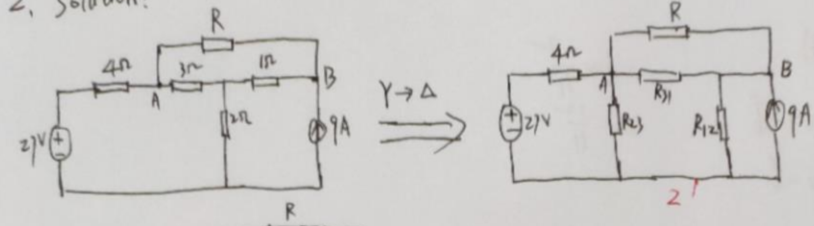
$\Rightarrow C = \frac{1}{\omega} = 1mF$ −2

2. (10 points) (1) Determine the value of R so that the maximum power can be delivered to the resistor R.

(2) Find the maximum power delivered to the resistor R.



2. Solution:

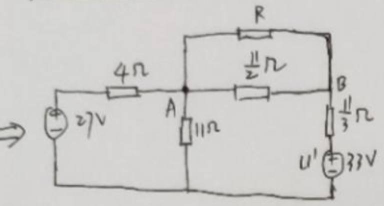


$R_{12} = 1+2+\frac{1\times 2}{3} = \frac{11}{3}\Omega$

$R_{23} = 3+2+\frac{3\times 2}{1} = 11\Omega$

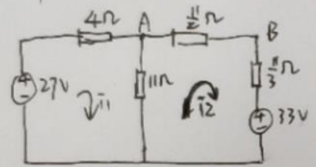
$R_{31} = 3+1+\frac{3\times 1}{2} = \frac{11}{2}\Omega$

$\Rightarrow$



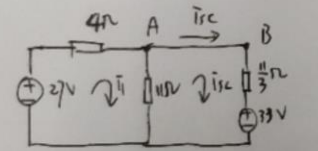
$u' = R_{12}\times 9 = \frac{11}{3}\times 9 = 33V$

Firstly calculate open-circuit voltage of AB, U_{oc}



$$\begin{cases} 4i_1 + 11(i_1 - i_2) - 27 = 0 \\ \frac{11}{2}i_2 + \frac{11}{3}i_2 + 33 + 11(i_2 - i_1) = 0 \\ U_{oc} = i_2 \cdot \frac{11}{2} \end{cases} \Rightarrow \begin{cases} i_1 = 1A \\ i_2 = -\frac{12}{11}A \\ U_{oc} = -6V \end{cases} \quad 2'$$

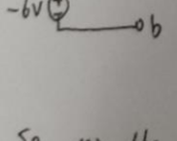
Secondly, Calculate short-circuit current of AB, I_{sc}



$$\begin{cases} 4i_1 + 11(i_1 - i_{sc}) - 27 = 0 \\ \frac{11}{3}i_{sc} + 33 + 11(i_{sc} - i_1) = 0 \end{cases} \Rightarrow \begin{cases} i_1 = \frac{1}{3}A \\ i_{sc} = -2A \end{cases}$$

$$R_{eq} = \frac{U_{oc}}{I_{sc}} = \frac{-6}{-2} = 3\Omega \quad \}2'$$

So the Thevenin equivalent circuit is:



when $R = R_{eq} = 3\Omega$, the Max Power is $2'$

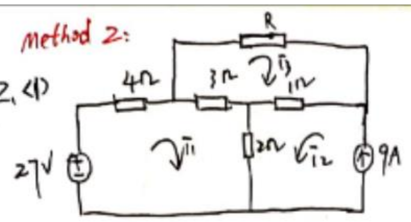
$$P_{max} = \frac{U_{oc}^2}{4R_{eq}} = \frac{36}{4\times 3} = 3W \quad 2'$$

So, (1) the value of R is 3Ω

(2) Maximum power $P_{max} = 3W$

method 2:

2.(1)



$i_2 = 9A$

$$\begin{cases} 4i_1 + 3(i_1 - i_3) + 2(i_1 + 9) - 27 = 0 \\ i_3 R + 1 \times (i_3 + 9) + 3(i_3 - i_1) = 0 \end{cases} \quad 4'$$

$$\Rightarrow i_3 = \frac{-18}{3R+9} = \frac{-6}{R+3}$$

$$P_R = i_3^2 R = \left(\frac{-6}{R+3}\right)^2 \cdot R = \frac{36R}{(R+3)^2}$$

$$P_R' = 36\,\frac{(R+3)^2 - 2R(R+3)}{(R+3)^4} = 36\,\frac{-R^2+9}{(R+3)^4}$$

let $P_R' = 0$ we can get $R_1 = -3$ $R_2 = 3$

We know $R \in [0, +\infty]$

when $R \in [0, 3]$, P_R is monotone ~~decreased~~ increased;

when $R \in [3, +\infty)$, P_R is monotone ~~increased~~ decreased. 2'

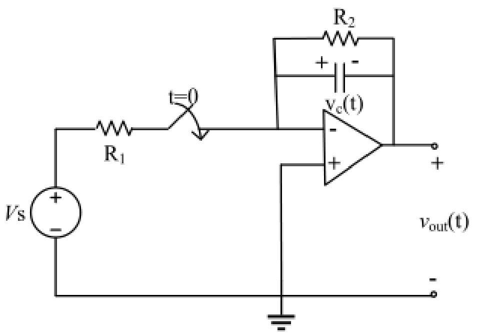
So when $R = 3\Omega$ 2', the maximum power is

$$\therefore P_{R\max} = \frac{36 \times 3}{(3+3)^2} = 3w \quad 2'$$

(1) the value of R is 3Ω.

(2) maximum power $P_{max} = 3w$

3. (10 points) In the following circuit, $V_s = 10$ V, $R_1 = 10$ kΩ, $R_2 = 5$ kΩ, $C = 50$ μF. The energy stored in the capacitor is 100 μJ at $t = 0_-$. The switch is closed at t=0. Find $v_{out}(t)$ for $t \geq 0$.

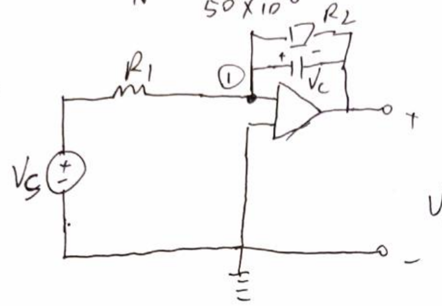


Problem 3. (方法一)

$\because W_c(0_-) = 100\,uJ = \frac{1}{2} \cdot C \cdot V_c^2(0_-)$

$\therefore V_c(0_-) = \pm \sqrt{\frac{2 \cdot 100 \cdot 10^{-6}}{50 \times 10^{-6}}} = \pm 2V.$ 2′

$t \geq 0$:



For Point ①: $\frac{V_S}{R_1} = \frac{V_c}{R_2} + C\frac{dV_c}{dt}$

$\therefore \frac{dV_c}{dt} + \frac{V_c}{R_2 C} = \frac{V_S}{R_1 C}$

$\Rightarrow \frac{dV_c}{dt} + 4V_c = 20$ 3′

$\therefore V_c = e^{-\int 4dt}\left[\int 20 e^{\int 4dt} dt + C_0\right)$

$= e^{-4t}\left[\int 20 e^{4t} dt + C_0\right)$

$= 5 + C_0 e^{-4t}\ V, t \geq 0$ 3′

Case A: $V_c(0_-) = 2V$, $C_0 = -3$, $V_c(t) = 5 - 3e^{-4t}\ V, t \geq 0$

$\therefore V_{out}(t) = -V_c(t) = 3e^{-4t} - 5\ V,\ t \geq 0.$ 1′

Case B: $V_c(0_-) = -2V$, $C_0 = -7$, $V_c(t) = 5 - 7e^{-4t}\ V, t \geq 0$

$\therefore V_{out}(t) = -V_c(t) = 7e^{-4t} - 5\ V,\ t \geq 0.$ 1′

Problem 3. (方法二).

$t=0_-$, $W_C(0_-)=\frac{1}{2}C V_C^2(0_-) = 100uJ$

$\therefore V_C(0_-)= \pm\sqrt{\frac{2\times100\times10^{-6}}{50\times10^{-6}}} = \pm 2V = V_C(0_+)$

$\therefore V_{out}(0_+) = -V_C(0_+) = \pm 2V$. 2'

$t=\infty$, $\cancel{V_{out}(\infty)=}$ $\frac{V_S}{R_1} + \frac{V_{out}(\infty)}{R_2} = 0$.

$\therefore V_{out}(\infty) = -\frac{R_2}{R_1} V_S = -5V$. 2'

$\tau = R_2 C = 5\times10^3 \times 50\times10^{-6} = 0.25S$. 2'

$\therefore V_{out}(t) = V_{out}(\infty) + (V_{out}(0) - V_{out}(\infty)) \cdot e^{-\frac{t}{\tau}}$

$= -5 + (2-(-5))e^{-4t}$

$= -5 + 7e^{-4t}\,V,\ t \geq 0$ 2'

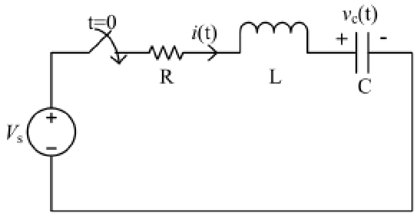
or $V_{out}(t) = -5 + (-2-(-5))e^{-4t}$

$= -5 + 3e^{-4t}\,V.\ t \geq 0$. 2'

4. (10 points) The switch is closed at $t = 0$ s. $V_s = 6$ V, R = 5 Ω, L=1H, C=0.25 F. $i(0_-) = 0$, $v_c(0_-) = 3\ V$. Please

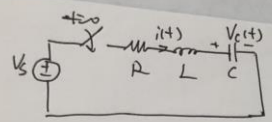
(1) write the differential equation that $v_c(t)$ should obey and solve $v_c(t)$ in time domain for $t \geq 0$,

(2) construct the S-domain circuit. Then solve $V_c(S)$ in S-domain, and use inverse Laplace transform to find $v_c(t)$ in time domain for $t \geq 0$,



E111 Homework 9 Sprin

4. (10 points)



$V_s=6V$, $R=5\Omega$, $L=1H$, $C=0.25F$. $i(0_-)=0$, $V_c(0_-)=3V$.

(1) write the differential equation that $V_c(t)$ should obey and solve $V_c(t)$.

(2) construct the S-domain circuit, solve $V_c(s)$ in S-domain, use inverse Laplace transform to find $V_c(t)$.

Solution:

(1)

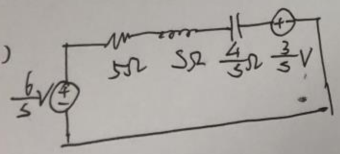
$$\begin{cases} LC\dfrac{d^2V_c(t)}{dt^2} + RC\dfrac{dV_c(t)}{dt} + V_c(t) = V_s & \cdots 1' \\ V_c(0_+) = 3V & \cdots 1' \\ \dfrac{dV_c(0_+)}{dt} = 0 & \cdots 1' \end{cases}$$

$$\Rightarrow V_c(t) = A_1e^{-t} + A_2e^{-4t} + 6, \quad \cdots 1'$$

$$A_1 = -4,\ A_2 = 1$$

$$\Rightarrow V_c(t) = -4e^{-t} + e^{-4t} + 6\ V\ (t \geq 0) \quad \cdots 1'$$

(2)



$\cdots 2'$

$$V_c(S) = \frac{\frac{6}{S} - \frac{3}{S}}{5 + S + \frac{4}{S}} \times \frac{4}{S} + \frac{3}{S}$$

$$= \frac{12}{(S+4)(S+1)S} + \frac{3}{S}$$

$$= \frac{K_1}{S} + \frac{K_2}{S+1} + \frac{K_3}{S+4} + \frac{3}{S} \quad \cdots 2'$$

$$\Rightarrow K_1 = 3,\ K_2 = -4,\ K_3 = 1$$

$$\Rightarrow V_c(S) = \frac{6}{S} - \frac{4}{S+1} + \frac{1}{S+4}$$

$$\Rightarrow V_c(t) = 6 - 4e^{-t} + e^{-4t}\ V\ (t \geq 0)$$

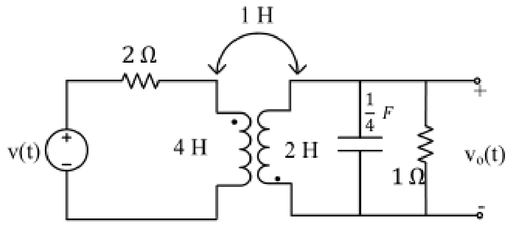
$\cdots 1'$

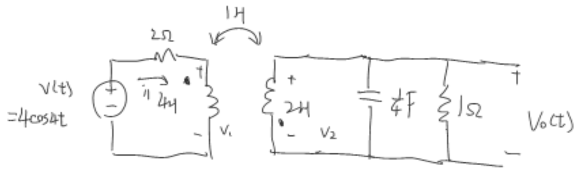
5. (12 points) Assume v(t)=4cos(4t) V, determine:

(1) the coupling coefficient,

(2) voltage $v_o(t)$ across the 1 Ω resistor,

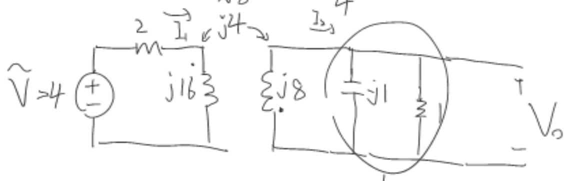
(3) the input impedance Z_{in} as seen from the voltage source.





2′ 1° $k=\frac{M}{\sqrt{L_1L_2}}=\frac{1}{\sqrt{8}}=\frac{\sqrt{2}}{4}$

2°



$Z_o=\frac{1}{\frac{1}{-j1}+1}=0.5-j0.5$

4′ 方程

① $I_1\cdot R+L_1\cdot j\omega I_1+M\cdot j\omega I_2=4$

② $M\cdot j\omega I_1+L_2\cdot j\omega I_2+I_2\cdot(0.5-j0.5)=0$

$\Rightarrow 2I_1+j16I_1+j4I_2=4$

$j4I_1+j8I_2+I_2(0.5-j0.5)=0$

3′结果 $\Rightarrow I_1(2+16j)+j4\cdot I_2=4$

$j4\cdot I_1=I_2\cdot(-0.5-j7.5)$

（中间步骤）

$\therefore V_o(t)=0.107\cos(4t+57.588^\circ)$ V

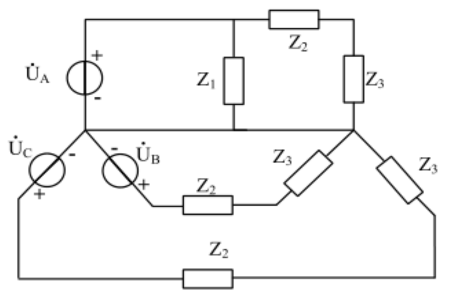
3′ ③ $Z_{in}=2+j16+\frac{\omega^2M^2}{j8+\frac{1-j}{2}}=2.14+j13.88\,\Omega$

6. (12 points) In the following three-phase circuit, $\dot{U}_{A_{rms}} = 220\angle 0^o$ V, $\dot{U}_{B_{rms}} = 220\angle 120^o$ V, $\dot{U}_{C_{rms}} = 220\angle 240^o$ V, $Z_1 = 50 + 50j\ \Omega$, $Z_2 = 20\ \Omega$, $Z_3 = 40 + 80j\ \Omega$. Find:

(1) the average power generated by $\dot{U}_A$,

(2) the power factor of $\dot{U}_A$,

(3) the total complex power generated by the three-phase voltage sources.



Pro6

Solution:

(1)

$$\dot{I}_{Arms} = \frac{\dot{U}_{Arms}}{Z_1} + \frac{\dot{U}_{Arms}}{Z_2 + Z_3} \qquad (1')$$

$$\begin{aligned} P_A &= U_{Arms} \times I_{Arms} \times \cos(\theta_{\dot{U}_{Arms}} - \theta_{\dot{I}_{Arms}}) \\ &= 220 \times \frac{220}{50\sqrt{2}} \times \cos(45^o) + 220 \times \frac{220}{100} \times \cos(53.1^o) \qquad (2') \\ &= 774.4W \end{aligned}$$

(2)

$$\begin{aligned} Q_A &= U_{Arms} \times I_{Arms} \times \sin(\theta_{\dot{U}_{Arms}} - \theta_{\dot{I}_{Arms}}) \\ &= 220 \times \frac{220}{50\sqrt{2}} \times \sin(45^o) + 220 \times \frac{220}{100} \times \sin(53.1^o) \\ &= 871.2 \text{ var} \qquad (2') \end{aligned}$$

$$pf_A = \frac{P_A}{\sqrt{P_A^2 + Q_A^2}} = \frac{774.4}{\sqrt{774.4^2 + 871.2^2}} = 0.664 \quad (2') \quad (lagging) \quad (1')$$

(2)

$$S_A = 774.4 + j871.2VA$$

$$\begin{aligned} S_B &= \dot{U}_{Brms} \times \dot{I}^*_{Brms} \\ &= \dot{U}_{Brms} \times \left(\frac{\dot{U}_{Brms}}{Z_2 + Z_3}\right)^* \\ &= 220\angle 120^o \times \left(\frac{220\angle 120^o}{20 + 40 + j80}\right)^* \\ &= 290.4 + j387.2VA \qquad (1') \end{aligned}$$

$$\begin{aligned} S_C &= \dot{U}_{Crms} \times \dot{I}^*_{Crms} \\ &= \dot{U}_{Crms} \times \left(\frac{\dot{U}_{Crms}}{Z_2 + Z_3}\right)^* \\ &= 220\angle 240^o \times \left(\frac{220\angle 240^o}{20 + 40 + j80}\right)^* \\ &= 290.4 + j387.2VA \qquad (1') \end{aligned}$$

$$\begin{aligned} S &= S_A + S_B + S_C \\ &= (774.4 + 290.4 + 290.4) + j(871.2 + 387.2 + 387.2)VA \\ &= 1355.2 + j1645.6VA \qquad (2') \end{aligned}$$

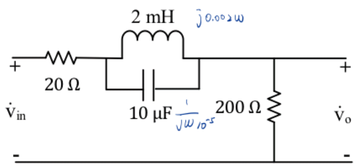
7. (12 points) The AC circuit is as shown below. Find:

(1) the transfer function $H(j\omega) = \frac{\dot{V}_o}{\dot{V}_{in}}$,

(2) the type of the filter,

(3) the center frequency ω_0,

(4) the bandwidth of the filter.



(1) $\frac{V_{in} - V_o}{R_1 + R_L \| R_C} = \frac{V_o}{R_2}$

$$H(jw) = \frac{200}{20 + R_L \| R_C + 200} = \frac{200}{220 + \frac{200}{0.002jw + \frac{10^5}{jw}}} = \frac{200}{220 + jw\frac{2\times10^{-3}}{1 - w^2 \times 2\times10^{-8}}} = \frac{10}{11 + \frac{10jw}{10^5 - 2\times10^{-3}w^2}}$$

2'

$$= \frac{200 - 4\times10^{-6}w^2}{220 - 4.4\times10^{-6}w^2 + 2\times10^{-3}jw} = \frac{10^7 - 0.2w^2}{1.1\times10^7 - 0.22w^2 + 100jw} = \frac{2\times10^{-3}jw + \frac{10^5}{jw}}{2.2\times10^{-3}jw + \frac{1.1\times10^5}{jw} + 1}$$

$$= \frac{5\times10^7 - w^2}{5.5\times10^7 - 1.1w^2 + 500jw} = \frac{200}{220 + \frac{jw10^5}{5\times10^7 - w^2}}$$

2'

以上形式及方便化简成以上形式的得4分，相差较远难以化简成以上形式的酌情扣1-2分。
未代入数据前都是正确的得2分，代入数据只有最后答案化错的得3分。
本小问共4分。

(2) $w \to 0 \quad H(jw) \to \frac{10}{11}$

$w \to \infty \quad H(jw) \to \frac{10}{11}$

$w \to w_0 \quad H(jw) \to 0$

1'

Bandreject Filter 1'

答案正确过程错误不扣分，中文不得答案分。

(3) $jw_0L + \frac{1}{jw_0C} = 0$ 1'

$w_0 = \sqrt{\frac{1}{LC}}$ 1'

$= 7071.07\ \text{rad/s}$ 1'

单位及未化成小数扣1分。

(4) $|H(jw)| = \frac{-200w^2 + 10^{10}}{\sqrt{(-220w^2 + 1.1\times10^{10})^2 + (10^5 w)^2}} = \frac{1}{\sqrt{2}}\frac{10}{11}$ 1'

① $w > 7071.07$ rad/s

$w_1 = -6847.446$ rad/s < 0

$w_2 = 7301.992$ rad/s

② $w < 7071.07$ rad/s

$w_1 = 6847.446$ rad/s

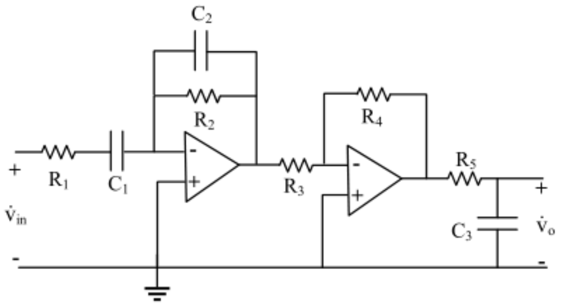
$w_2 = -7301.992$ rad/s < 0 1'

$B = w_2 - w_1 = 454.546$ rad/s 1'

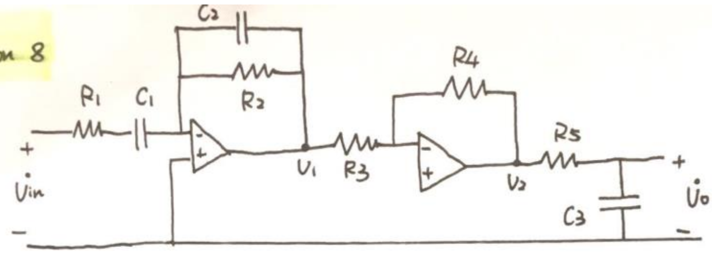
8. (12 points) In the following AC circuit, $R_1 = 10\ \text{k}\Omega$, $R_2 = 1\ \text{k}\Omega$, $R_3 = 10\ \text{k}\Omega$, $R_4 = 100\ \text{k}\Omega$, $R_5 = 5\ \text{k}\Omega$, $C_1 = 10\ \mu\text{F}$, $C_2 = 0.1\ \mu\text{F}$, $C_3 = 0.2\ \mu\text{F}$.

(1) find $H(j\omega) = \frac{\dot{V}_o}{\dot{V}_{in}}$,

(2) sketch the Bode plot of H, please label the corner frequencies, the gains, phases and slopes of the plot.



Question 8



第1问共4分，1) 公式2分，答案2分

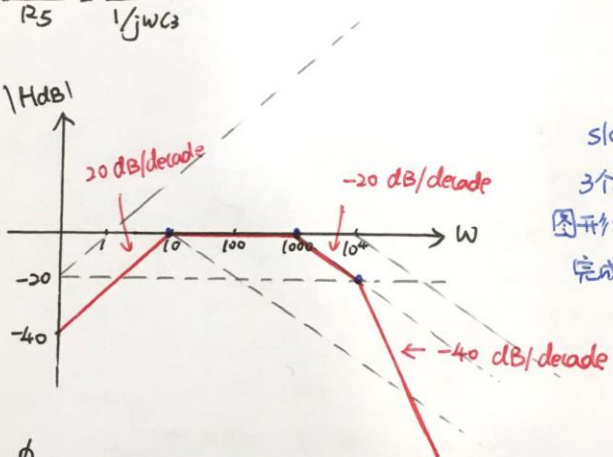
2'

$$\begin{cases} \dfrac{\dot{V}_{in}}{R_1 + 1/j\omega C_1} + \dfrac{V_1}{R_2} + \dfrac{V_1}{1/j\omega C_2} = 0 \\ \dfrac{V_1}{R_3} + \dfrac{V_2}{R_4} = 0 \\ \dfrac{V_2 - V_o}{R_5} = \dfrac{V_o}{1/j\omega C_3} \end{cases}$$

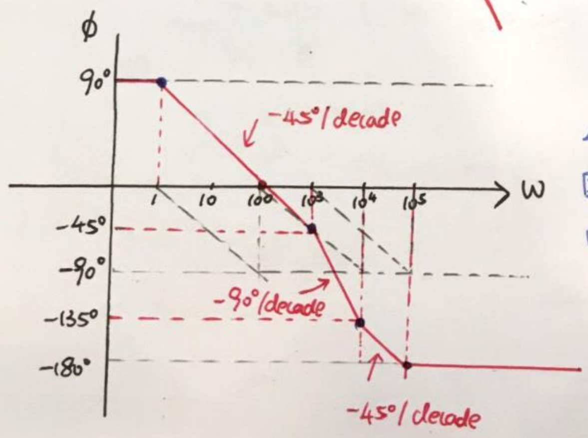
2'

$$\Rightarrow H(j\omega) = \frac{\dot{V}_o}{\dot{V}_{in}} = \frac{0.1\,j\omega}{(1+\frac{j\omega}{10})(1+\frac{j\omega}{10^3})(1+\frac{j\omega}{10^4})}$$

第2问共8分，2) 每张图4分



slope 1'
3个拐点，1'
图形状一致 1'
完成正确 1'



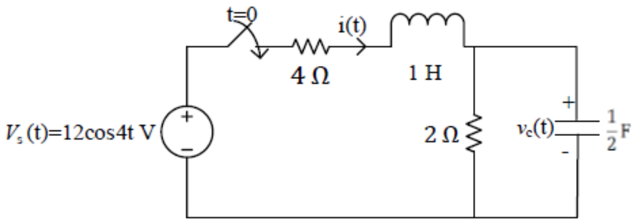
slope 1'
4个拐点，1'
图形状一致 1'
完全正确 1'

9. (12 points) Initially, $i(0_-) = 0$ A, $v_c(0_-) = 0$ V. The switch is closed at t=0 s.

4' (1) build the phasor domain circuit, solve $\dot{V}_C$ in phasor domain, and convert it back to time domain to find $v_c(t)$ for $t \geq 0$,

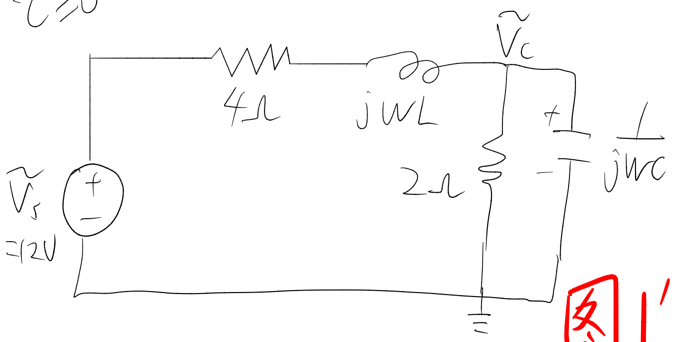
6' (2) construct the S-domain circuit, solve $V_c(S)$ in S-domain, and use inverse Laplace transform to find $v_c(t)$ for $t \geq 0$ in time domain,

2' (3) compare the solutions of $v_c(t)$ for $t \geq 0$ in (1) and (2) and explain the relationship.



(1) $\omega = 4$ rad/s $i(0^+) = i(0^-) = 0$ A $V_c(0^+) = V_c(0^-) = 0$ V

$t \geq 0$



$$\frac{\tilde{V}_c - 12}{4+j4} + \frac{\tilde{V}_c}{2} + \frac{\tilde{V}_c}{-j0.5} = 0$$

$\tilde{V}_c \doteq 1.073\angle -116.57^\circ = -0.48 - j0.96$ V 2'

图1'

$V_c(t) = 1.073\cos(4t - 116.57^\circ)$ V $(t \geq 0)$

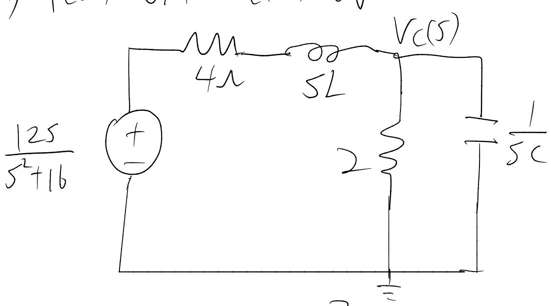
$= [1.073\cos(4t - 116.57^\circ)]u(t)$ V

$= -1.073\cos(4t + 63.43^\circ)$ V

1' (1、、4')

$j\omega L = j4\,\Omega$ $\frac{1}{j\omega C} = -j0.5\,\Omega$

(2) $i(0^+) = 0$ A $V_c(0^+) = 0$ V



$$\frac{V_c(s) - \frac{12s}{s^2+16}}{4+s} + \frac{V_c(s)}{2} + \frac{V_c(s)}{\frac{2}{s}} = 0$$

$sL = s$ $\frac{1}{sC} = \frac{2}{s}$

$$V_c(s) = \frac{24s}{(s^2+16)(s+2)(s+3)}$$ 1'

图1'

$$V_c(s) = \frac{As+B}{s^2+16} + \frac{C}{s+2} + \frac{D}{s+3}$$

$C = -2.4$ $D = 2.88$ 1' (ABCD共2')

$$(As+B)(s+2)(s+3)-2.4(s^2+16)(s+3)+2.88(s^2+16)(s+2)=24s$$

$$A=-0.48 \qquad B=3.84 \qquad 1'$$

$$V_C(s)=\frac{-0.48s+3.84}{s^2+16}+\frac{-2.4}{s+2}+\frac{2.88}{s+3}=\frac{-0.48s+0.96\cdot 4}{s^2+16}+\frac{-2.4}{s+2}+\frac{2.88}{s+3} \qquad 1'$$

$$V_C(t)=[-0.48\cos 4t+0.96\sin 4t-2.4e^{-2t}+2.88e^{-3t}]u(t)\ V \qquad 1'$$

$$=[1.073\cos(4t-116.57^\circ)-2.4e^{-2t}+2.88e^{-3t}]u(t)\ V \qquad (\cdots 10')$$

(3) $-0.48\cos 4t+0.96\sin 4t=1.073\cos(4t-116.57^\circ)$

The solutions of Vc(t) for t≥0 in (1) and (2) are the same at steady-state 2′

phasor → steady-state Laplace → Transient-state and steady-state

(2) is the absolutely correct way to solve this problem

1️⃣ 第三问强调了steady-state相同即给2分，如果直接说（1）与（2）问答案相等不得分

2️⃣ 第二问ABCD处，求对一个给0.5分，满分2分

3️⃣ 强调第一问和第二问V（t）最终答案单位，少一个扣一分，单位扣分上限1分

4️⃣ 答案写分数不额外扣分

5️⃣ 允许答案有小量误差