**CS 101 Fall 2021 - Quiz 6-B**

**11/23/2021 - 25 Minutes** **Name:** **ID number:**

---

Remember that your work is graded on the quality of your writing and explanation as well as the validity.

**Problem 1 Notes of discussion (1 pts)**:

I promise that I will complete this QUIZ independently, and will not use any electronic products or paper-based materials during the QUIZ, nor will I communicate with other students during this QUIZ.

True or False: I have read the notes and understood them.

| 1 |
|---|
| |

**Problem 2 True or False(5×1pts)**:

The following questions are True or False questions, you should judge whether each statement is true or false and write the answer(T/F) in the box below.

*Note: You should write those answers in the box below.* **2.1** The time complexity of both DFS and BFS is $\Theta(V+E)$

| Question 2.1 | Question 2.2 | Question 2.3 | Question 2.4 | Question 2.5 |
|---|---|---|---|---|
| T | T | F | T | T |

**2.2** BFS can be used to give a path to any node in the graph which has the smallest number of edges.

**2.3** We should use an adjacency matrix instead of adjacency list to permanently store the graph in which there are 100 vertices and 200 edges.

**2.4** We can use DFS to determine whether a graph is bipartite.

**2.5** In a directed graph, the maximal number of edges is $|V|(|V|-1)$.

**Problem 3 Disjoint Set(3 pts)**:

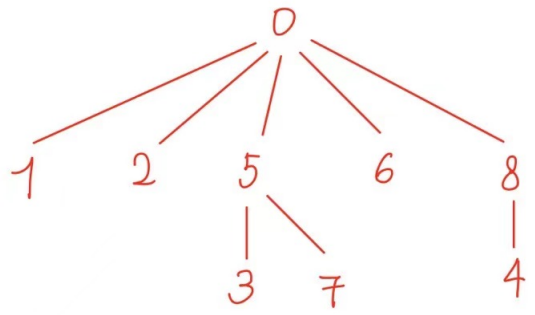
Consider a disjoint set with both path compression and union-by-size optimization. When two trees have the same height, the set specified first in the union will be the root of the merged set. The following operations are done:

union(2,3);union(1,2);union(5,7);union(8,4);union(7,2);

find(3);union(0,6);union(6,4);union(6,3);find(1);

Please write down the result for each *find* operation appeared in the above operations, and draw the disjoint set tree after all operations above is finished.

5, 0



**CS 101 Fall 2021 - Quiz 6-B**

**11/23/2021 - 25 Minutes** **Name:** **ID number:**

---

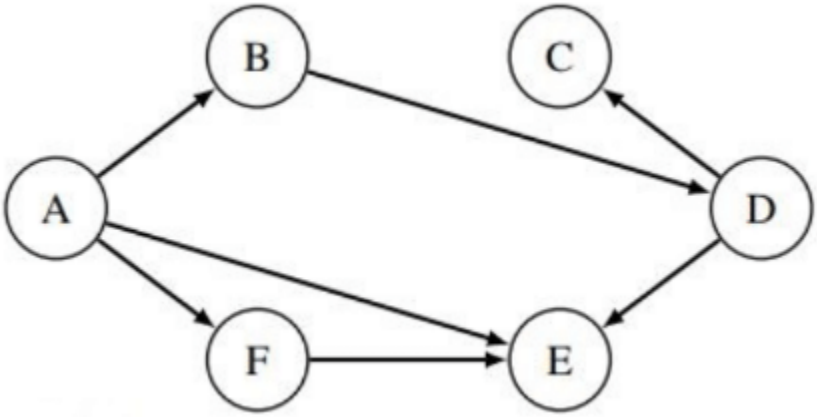
**Problem 3 Graph Algorithms(3+3 pts)**:

(1) Design an algorithm to solve the problem below efficiently and give a worst case runtime bound in $\Theta$ notation in terms of $V$ and $E$.

Problem: Determine whether or not a cycle exists in a directed graph.

Run DFS. If a node encountered is in the call stack (disjoint set can only be used on undirected graphs when finding cycles), there is a cycle. Runtime: $\Theta(|V| + |E|)$

(2) Run BFS on the following graph starting with vertex A. Please write down the result. Whenever there is a choice of which node to visit next, follow the alphabetical order. A B E F D C



**CS 101 Fall 2021 - Quiz 6-B**

**11/23/2021 - 25 Minutes** **Name:** **ID number:**

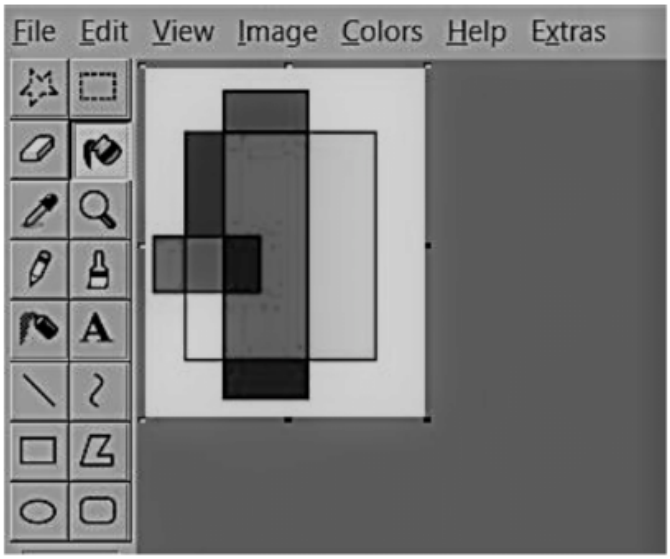
---

**Problem 4 MS Paint(5 pts)**:

There is an image of $m \times n$ pixels which is mono-color (only contains black or white). To make the image colorful, you decided to use the tool in **Microsoft Paint** to paint its **white** part. Note that **the black part should NOT be painted with this tool**.

This tool has the following property: if we use this tool to color one **white** pixel with position (x, y), its neighbour pixel (one or more of: (x-1, y), (x+1, y), (x, y-1), (x, y+1)) will also be colored **if they are originally white**; also, this property will keep executing on **every** pixel that is colored just now (i.e, the origin pixel, together with its neighbor pixels that are colored), until no more pixel can be colored.

Please design an algorithm to count how many different types of color can be used at most. Briefly explain your algorithm with natural language. For the provided example, the number is 10.




The input image can be seen as a undirected graph, each pixel as a vertex, and there are edges between each white pixel and each of its neighbor. Traverse this graph, each time we see a unvisited white pixel, do DFS or BFS starting with this vertex, until all nodes are visited. Finally count how many times DFS is done, and the number is exactly the answer.