CS 101 Fall 2021 - Quiz 4-A

10/25/2021 - 30 Minutes **Name:** **ID number:**

The Master Theorem for $T(n) = aT(\frac{n}{b}) + \Theta(n^d)$: If $\log_b a = d$ then $T(n) = O(n^d \log n)$ else $T(n) = O(n^{max(\log_b a, d)})$.

Problem 1 Notes of Discussion (5 pts)

I promise that I will complete this QUIZ independently, and will not use any electronic products or paper-based materials during the QUIZ, nor will I communicate with other students during this QUIZ.

True or False: I have read the notes and understood them.

Problem1

Problem 2 True or False (3×2 pts)

The following questions are True or False questions, you should judge whether each statement is true or false.

Note: You should write down your answers in the box below.

Problem 2.1	Problem 2.2	Problem 2.3

(1) Queue is the common data structure for implementation of Breadth First Traversal.

(2) The degree and the depth of the root node are both zero in all tress.

(3) If a is an ancestor of b, then there is exactly one unique path from a to b in the tree.

Problem 3 Recurrence and the Master Theorem (8pts)

Given the recurrence $T(n) = aT(n/b) + f(n)$ with $T(1) = 1$.

(1) If the recurrence indicates a divide and conquer algorithm,

a. the original problem of size n is divided into _______ subproblems and each subproblem has size _______(2pts);

(A) a (B) b (C) n/a (D) n/b (E) $f(n)$

b. $f(n)$ is the time complexity of ___________ (2pts)

(A) Divide and Conquer (B) Divide and Combine (C) Conquer and Combine

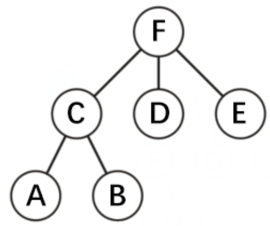
(2) a. If $(a, b, f(n)) = (2, 3, 3\sqrt{n})$, then the solution to this recurrence is $T(n) =$______________________. (2pts)

b. If $(a, b, f(n)) =$___________________, then the recurrence indicates the **Merge Sort** algorithm covered in our lecture. The solution to this recurrence is $T(n) =$____________________. (2pts)

Note: Write your answer for time complexity in asymptotic order form i.e. $T(n) = O(g(n))$.

Problem 4 Tree Traversal (6pts)

Run **Depth First Traversal** on the tree shown below.



Note:

1. Decide on an appropriate data structure to implement the traversal.

2. When you are pushing the children of a node into your data structure, please push them **in a reverse order** i.e. from right to left.

3. **Show all current elements in your data structure at each step** clearly . **Popping a node** or **pushing a sequence of children** can be considered as one single step.

4. **Write down your traversal sequence** i.e. the order that you pop elements out of the data structure. *Don't worry if you can't write the right answer at one chance. You can scratch in this paper but please* ***mark your final answer****.*

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Problem 5 Matrix Multiplication(10pts)

Recall that Strassen found an more efficient approach to calculate matrix multiplication $\mathbf{A} \times \mathbf{B}$, where $\mathbf{A}$ and $\mathbf{B}$ are both square matrices of size $n \times n$ ($n = 2^k$).

(1) Let $\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$, calculate $\mathbf{A} \times \mathbf{B}$. How many scalar multiplications do you perform?(2pts)

(2) Recall that Strassen's algorithm partitions matrices and computes $\mathbf{A} \times \mathbf{B} = \left[\begin{array}{c|c} \mathbf{A_{11}} & \mathbf{A_{12}} \\ \hline \mathbf{A_{21}} & \mathbf{A_{22}} \end{array}\right] \times \left[\begin{array}{c|c} \mathbf{B_{11}} & \mathbf{B_{12}} \\ \hline \mathbf{B_{21}} & \mathbf{B_{22}} \end{array}\right]$ in the following way($\mathbf{A_{ij}}$ and $\mathbf{B_{ij}}$ are $2^{k-1} \times 2^{k-1}$ submatrices of $\mathbf{A}$ and $\mathbf{B}$). Let

$$\mathbf{P_1} = \mathbf{A_{11}} \times (\mathbf{B_{12}} - \mathbf{B_{22}}) \qquad \mathbf{P_2} = (\mathbf{A_{11}} + \mathbf{A_{12}}) \times \mathbf{B_{22}} \qquad \mathbf{P_3} = (\mathbf{A_{21}} + \mathbf{A_{22}}) \times \mathbf{B_{11}}$$

$$\mathbf{P_4} = \mathbf{A_{22}} \times (\mathbf{B_{21}} - \mathbf{B_{11}}) \qquad \mathbf{P_5} = (\mathbf{A_{11}} + \mathbf{A_{22}}) \times (\mathbf{B_{11}} + \mathbf{B_{22}}) \qquad \mathbf{P_6} = (\mathbf{A_{12}} - \mathbf{A_{22}}) \times (\mathbf{B_{21}} + \mathbf{B_{22}})$$

$$\mathbf{P_7} = (\mathbf{A_{11}} - \mathbf{A_{21}}) \times (\mathbf{B_{11}} + \mathbf{B_{12}})$$

then we can obtain:

$$(\mathbf{A} \times \mathbf{B})_{\mathbf{11}} = \mathbf{P_5} + \mathbf{P_4} - \mathbf{P_2} + \mathbf{P_6} \qquad (\mathbf{A} \times \mathbf{B})_{\mathbf{12}} = \mathbf{P_x} + \mathbf{P_2}$$

$$(\mathbf{A} \times \mathbf{B})_{\mathbf{21}} = \mathbf{P_3} + \mathbf{P_y} \qquad (\mathbf{A} \times \mathbf{B})_{\mathbf{22}} = \mathbf{P_1} + \mathbf{P_5} - \mathbf{P_3} - \mathbf{P_7}$$

What value should x and y take? How many scalar multiplications do you perform if you apply this to (1)? (2pts)
Hint: Matrix multiplication still applies to partitioned matrices.

(3) Use Strassen's algorithm from (2) to come up with a divide-and-conquer algorithm to calculate the matrix multiplication $\mathbf{A} \times \mathbf{B}$ in more efficient than $\Theta(n^3)$ time. Write down your main idea briefly. (4pts)

(4) What is the time complexity of your algorithm? Write down the corresponding recurrence and solve it. You **are not required** to show your analysis or calculation. (2pts)
Note: You can assume that all the numbers involved are small enough so that basic arithmetic operations like scalar addition and scalar multiplication take $O(1)$ time.