1 Nash Equilibrium

Given a two player game where the action space of both players is $\{A, B\}$. Consider whether the following statements are true or false. If true, give the proof, otherwise give a counterexample.

1.1 (1pt)

Suppose (A, A) is the unique pure strategy Nash equilibrium, then action A is a dominant strategy for at least one of the players.

1.2 (1pt)

Suppose (A, A) is the unique Nash equilibrium, then action A is a dominant strategy for both players.

2 Myerson's Mechanism

Suppose there are n agents who bid for one single item. Their probability density functions of their valuation distributions are of the Pareto's form and same (i.i.d.):

$$f(x) = \frac{\alpha}{x^{\alpha+1}} \qquad x \geq 1$$

2.1 (1pt)

If $\alpha = 2$ and there are five bidders $\{A, B, C, D, E\}$ with bids $v_A = 20$, $v_B = 18$, $v_C = 16$, $v_D = 14$ and $v_E = 12$. Compute the allocation and payment of Myerson's mechanism.

2.2 (1pt)

If $\alpha = 1/2$, will the Myerson's Mechanism be truthful? Prove your statement.

3 Expected Revenue

Consider an auction for a single indivisible item where there are n buyers. Suppose all bidders have the same probability distribution of their valuations independently (i.i.d.) as uniform distribution on $[0, 1]$.

3.1 (1pt)

If the seller uses second price auction and $n = 3$, compute the expected revenue of the seller.

3.2 (1pt)

If the seller uses Myerson's mechanism and $n = k$ ($k > 0$), compute the expected revenue of the seller.

3.3 (2pt)

If the seller uses second price auction and $n = k + 1$ ($k > 0$), compute the expected revenue of the seller. Comparing the result with that in 3.2, what can you observe?