

Homework 5

Due date:

Apr.16th, 2018

Turn in your homework in class

Rules:

- Please try to work on your own. Discussion is permissible, but identical submissions are unacceptable!
- Please show all intermediate steps: a correct solution without an explanation will get zero credit.
- Please submit on time. No late submission will be accepted.
- Please prepare your submission in English only. No Chinese submission will be accepted.

(本题满分
6分)

1. The circuit contains two switches, both of which had been open for a long time before $t = 0$. Switch 1 closes at $t = 0$, and switch 2 follows suit at $t = 5$ s. Determine and plot $V_C(t)$ for $t \geq 0$ given that $V_0 = 24$ V, $R_1 = R_2 = 16$ k Ω , and $C = 250$ μ F. Assume $V_C(0) = 0$.

Solution:

Time Segment 1: $0 \leq t \leq 5$ s

(3分)

$$\tau_1 = R_1 C = 16 \times 10^3 \times 250 \times 10^{-6} = 4 \text{ s.} \quad (1分)$$

$$v_{C_1}(t) = v_{C_1}(\infty) + (v_{C_1}(t) - v_{C_1}(\infty))e^{-t/\tau_1} \quad (1分)$$

$$= V_0 + (0 - V_0)e^{-0.25t} = 24(1 - e^{-0.25t}), \quad \text{for } 0 \leq t \leq 5 \text{ s.} \quad (1分)$$

Time Segment 2: $t \geq 5$ s (3分)

Through source transformation, it is easy to see that R_1 and R_2 should be combined in parallel. Hence:

$$\textcircled{1} \quad \tau_2 = \left(\frac{R_1 R_2}{R_1 + R_2} \right) C = 8 \times 10^3 \times 250 \times 10^{-6} = 2 \text{ s.}$$

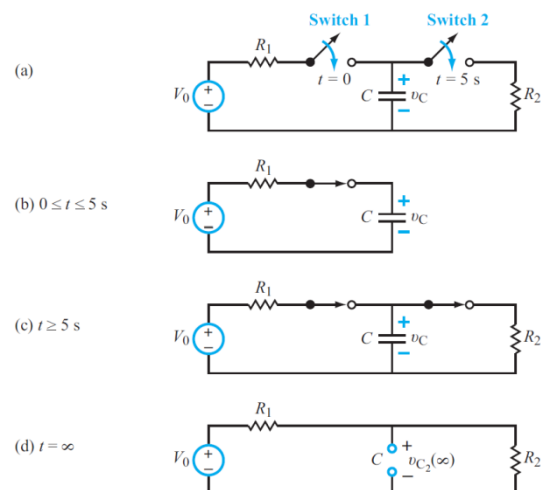
$$v_{C_2}(t) = v_{C_2}(\infty) + [v_{C_2}(5 \text{ s}) - v_{C_2}(\infty)]e^{-(t-5)/\tau_2} \quad (1分)$$

$$\textcircled{2} \quad v_{C_2}(\infty) = \frac{V_0 R_2}{R_1 + R_2} = \frac{24 \times 16}{16 + 16} = 12 \text{ V.}$$

$$\textcircled{3} \quad v_{C_2}(5 \text{ s}) = v_{C_1}(5 \text{ s}) = 24(1 - e^{-0.25 \times 5}) = 17.12 \text{ V}$$

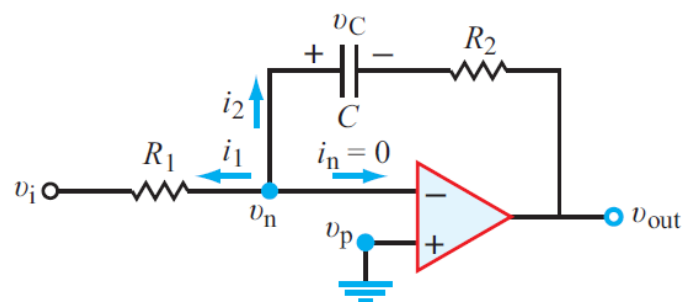
$$v_{C_2}(t) = 12 + [17.12 - 12]e^{-0.5(t-5)} = 12 + 5.12e^{-0.5(t-5)}, \quad \text{for } t \geq 5 \text{ s.} \quad (1分)$$

①-⑤②③ 相加 6分



2. Relate V_{out} to V_i in the circuit. Assume $V_c = 0$ at $t = 0$.
 (本题满分6分)

Solution:



Solution:

$$i_1 = \frac{v_n - v_i}{R_1}$$

$$V_c(0) = 0, i_2 = C \frac{dV_c}{dt} \quad (2 \text{分})$$

By applying integration from out, we have:

$$\int_0^t i_2 dt = \int_0^t C \frac{dV_c}{dt} dt = C [V_c(t) - V_c(0)] \quad (2 \text{分})$$

$$\Rightarrow V_c(t) = V_c(0) + \frac{1}{C} \int_0^t i_2 dt = \frac{1}{C} \int_0^t i_2 dt$$

Therefore:

$$v_n - v_{\text{out}} = i_2 R_2 + \frac{1}{C} \int_0^t i_2 dt$$

But $v_n = v_p = 0$, and

$$i_2 = -i_1 = \frac{v_i}{R_1},$$

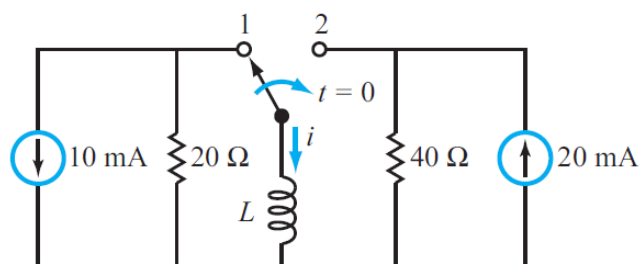
which leads to

$$v_{\text{out}} = - \left(\frac{R_2}{R_1} v_i + \frac{1}{R_1 C} \int_0^t v_i dt \right). \quad (2 \text{分})$$

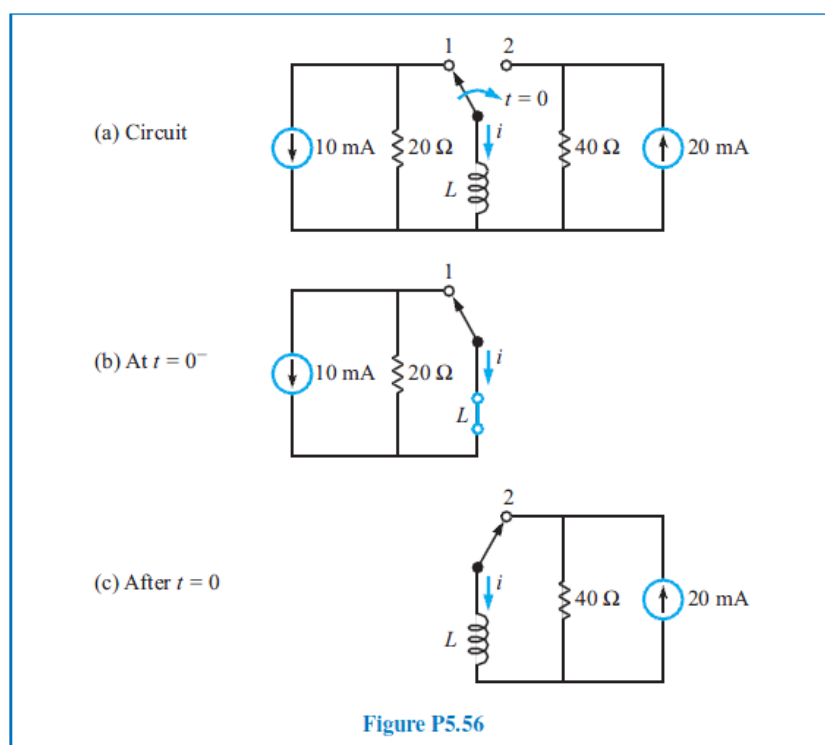
(本题满分6分)

3. The switch in the circuit was moved from position 1 to position 2 at $t = 0$, after it had been in position 1 for a long time. If $L = 80 \text{ mH}$, determine $i(t)$ for $t \geq 0$.

Solution:



Solution:



At $t = 0^-$ (Fig. P5.56(b)),

(2分) $i(0^-) = -10 \text{ mA}$ (current flows entirely through short circuit).

The circuit in Fig. P5.56(c) represents the circuit condition after $t = 0$.

$$i(\infty) = 20 \text{ mA.} \quad (1分)$$

$$\tau = \frac{L}{R} = \frac{80 \times 10^{-3}}{40} = 2 \times 10^{-3} \text{ s.}$$

$$\begin{aligned} i(t) &= [i(\infty) + [i(0) - i(\infty)]e^{-t/\tau}] \\ &= [20 + (-10 - 20)e^{-500t}] \text{ mA} \\ &= [20 - 30e^{-500t}] \text{ mA.} \end{aligned}$$

} \Rightarrow (3分)

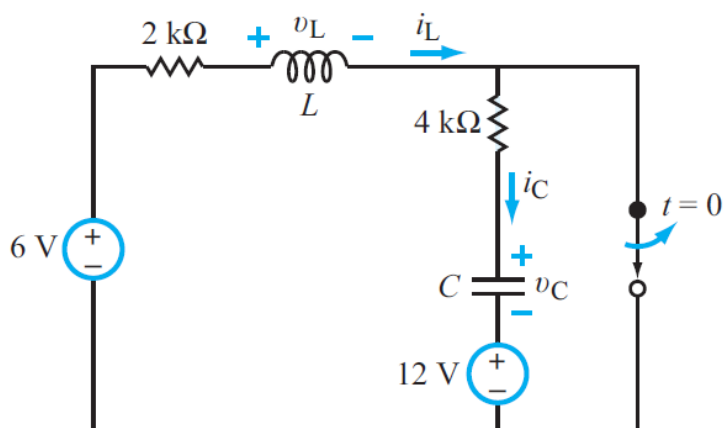
(本题满分12分)

(每小题对应的

那个值为2分)

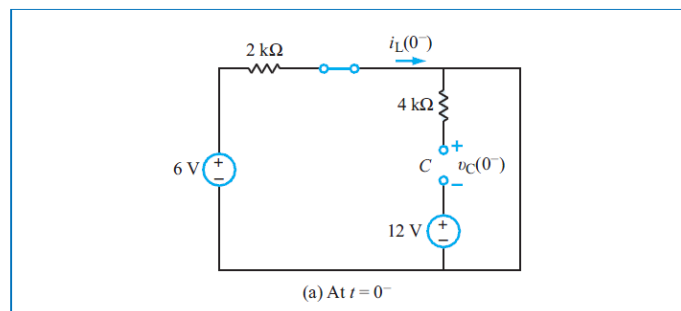
4. The switch in the circuit opens at $t = 0$ after it has been closed for a long time. Find(1) $V_C(0^+)$ and $i_L(0^+)$ (2) $i_C(0^+)$ and $V_L(0^+)$ (3) $V_C(\infty)$ and $i_L(\infty)$.

Solution:



Solution:

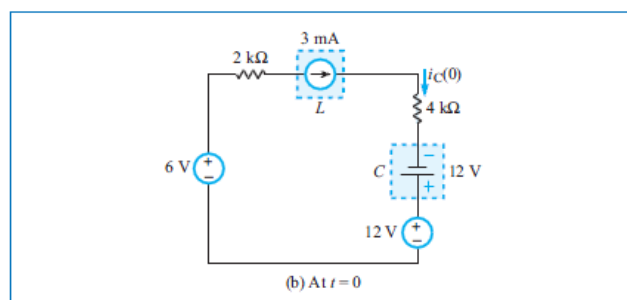
(a)



$$v_C(0) = v_C(0^-) = -12 \text{ V},$$

$$i_L(0) = i_L(0^-) = \frac{6}{2\text{k}} = 3 \text{ mA}.$$

(b)

At $t = 0$, L is replaced with 3-mA current source and C with -12 V voltage source.

$$i_C(0) = 3 \text{ mA}.$$

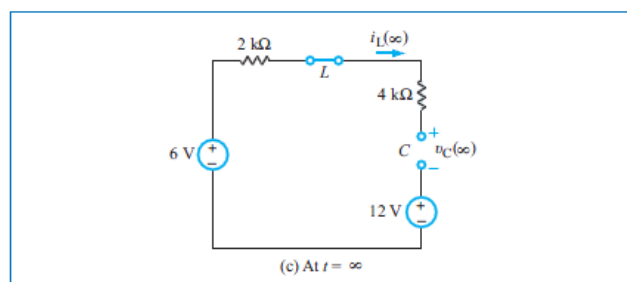
Application of KVL around the loop gives

$$-6 + 2 \text{ k}\Omega \times 3 \text{ mA} + v_L(0) + 3 \text{ mA} \times 4 \text{ k}\Omega - 12 + 12 = 0,$$

whose solution is

$$v_L(0) = -12 \text{ V}.$$

(c)

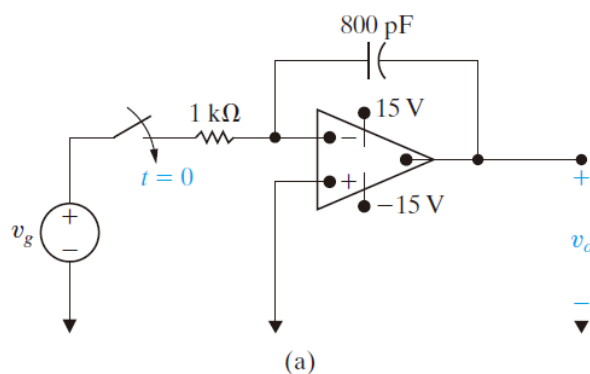
Absence of a closed circuit at $t = \infty$ leads to

$$v_C(\infty) = 6 - 12 = -6 \text{ V},$$

$$i_L(\infty) = 0.$$

(本题满分12分)

5. The voltage source in the circuit in the figure is generating the triangular waveform shown in figure. Assume that the energy stored in the capacity is zero at $t = 0$ and the Op-Amp is ideal.



(本题满分8分)

(1) Derive the numerical expressions for $V_o(t)$ for the following time in the following time intervals: $0 \leq t \leq 1\mu\text{s}$; $1 \leq t \leq 3\mu\text{s}$; $3 \leq t \leq 4\mu\text{s}$.

对应分值下面有)

(图2分, 曲线, 坐标)

(理由1分。其中看

到电压一样1分。解

释理由重新开始过程1分)

(2) Sketch the output waveform between 0 and $4\mu\text{s}$.
(3) If the triangular input voltage continues to repeat itself for $t > 4\mu\text{s}$, what would you expect the output voltage to be? Explain.

Solution:

$$[a] RC = (1000)(800 \times 10^{-12}) = 800 \times 10^{-9}; \quad \frac{1}{RC} = 1,250,000$$

$$0 \leq t \leq 1\mu\text{s}:$$

$$v_g = 2 \times 10^6 t$$

$$v_o = -1.25 \times 10^6 \int_0^t 2 \times 10^6 x dx + 0 \quad (\text{式子1分})$$

$$= -2.5 \times 10^{12} \frac{x^2}{2} \Big|_0^t = -125 \times 10^{10} t^2 \text{ V}, \quad 0 \leq t \leq 1\mu\text{s} \quad (\text{答案1分}) \quad [b]$$

$$v_o(1\mu\text{s}) = -125 \times 10^{10} (1 \times 10^{-6})^2 = -1.25 \text{ V} \quad (\text{初状态1分})$$

$$1\mu\text{s} \leq t \leq 3\mu\text{s}:$$

$$v_g = 4 - 2 \times 10^6 t$$

$$v_o = -125 \times 10^4 \int_{1 \times 10^{-6}}^t (4 - 2 \times 10^6 x) dx - 1.25 \quad (\text{式子1分})$$

$$= -125 \times 10^4 \left[4x \Big|_{1 \times 10^{-6}}^t - 2 \times 10^6 \frac{x^2}{2} \Big|_{1 \times 10^{-6}}^t \right] - 1.25$$

$$= -5 \times 10^6 t + 5 + 125 \times 10^{10} t^2 - 1.25 - 1.25$$

$$= 125 \times 10^{10} t^2 - 5 \times 10^6 t + 2.5 \text{ V}, \quad 1\mu\text{s} \leq t \leq 3\mu\text{s} \quad (\text{答案1分})$$

$$v_o(3\mu\text{s}) = 125 \times 10^{10} (3 \times 10^{-6})^2 - 5 \times 10^6 (3 \times 10^{-6}) + 2.5$$

$$= -1.25 \quad (\text{初状态1分})$$

$$3\mu\text{s} \leq t \leq 4\mu\text{s}:$$

$$v_g = -8 + 2 \times 10^6 t$$

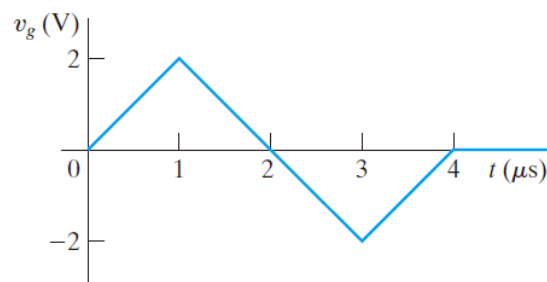
$$v_o = -125 \times 10^4 \int_{3 \times 10^{-6}}^t (-8 + 2 \times 10^6 x) dx - 1.25 \quad (\text{式子1分})$$

$$= -125 \times 10^4 \left[-8x \Big|_{3 \times 10^{-6}}^t + 2 \times 10^6 \frac{x^2}{2} \Big|_{3 \times 10^{-6}}^t \right] - 1.25$$

$$= 10^7 t - 30 - 125 \times 10^{10} t^2 + 11.25 - 1.25$$

$$= -125 \times 10^{10} t^2 + 10^7 t - 20 \text{ V}, \quad 3\mu\text{s} \leq t \leq 4\mu\text{s} \quad (\text{答案1分})$$

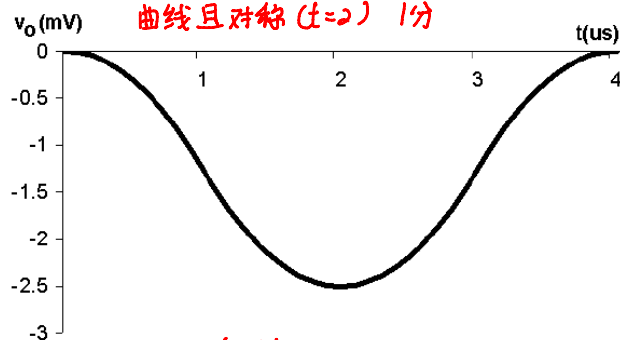
$$v_o(4\mu\text{s}) = -125 \times 10^{10} (4 \times 10^{-6})^2 + 10^7 (4 \times 10^{-6}) - 20 = 0$$



(b)

横纵坐标正确1分

曲线且对称(t=2) 1分



(1分)

[c] The output voltage will also repeat. This follows from the observation that at $t = 4\mu\text{s}$ the output voltage is zero, hence there is no energy stored in the capacitor. This means the circuit is in the same state at $t = 4\mu\text{s}$ as it was at $t = 0$, thus as v_g repeats itself, so will v_o .

(1分)

(本题满分 12 分)

6. The gap in the circuit seen in figure will arc over whenever the voltage across the gap reaches 30 kV.

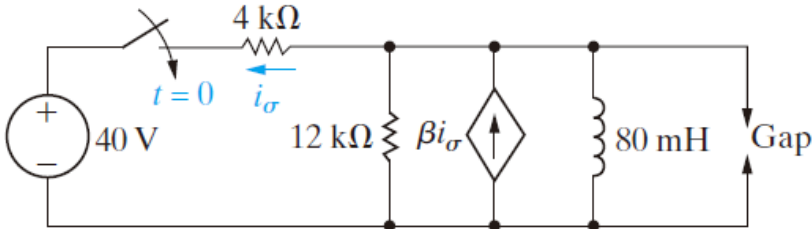
(第 1 题 6 分)

The initial current in the inductor is zero. The value of β is adjusted so the Thevenin resistance with respect to the terminals of the inductor when the switch is closed is $-4 \text{ k}\Omega$.

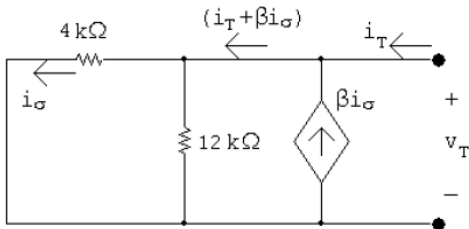
(第 2 题 6 分)

(1) What is the value of β ?

(2) How many microseconds after the switch has been closed will the gap arc over?



[a]



Using Ohm's law,

$$v_T = 4000i_\sigma \rightarrow (\text{Vth 得 1 分})$$

Using current division,

$$i_\sigma = \frac{12,000}{12,000 + 4000}(i_T + \beta i_\sigma) = 0.75i_T + 0.75\beta i_\sigma$$

Solve for i_σ :

$$i_\sigma(1 - 0.75\beta) = 0.75i_T$$

$$i_\sigma = \frac{0.75i_T}{1 - 0.75\beta}; \quad v_T = 4000i_\sigma = \frac{3000i_T}{(1 - 0.75\beta)}$$

Find β such that $R_{Th} = -4 \text{ k}\Omega$:

$$R_{Th} = \frac{v_T}{i_T} = \frac{3000}{1 - 0.75\beta} = -4000$$

$$1 - 0.75\beta = -0.75 \quad \therefore \beta = 2.33$$

$$\frac{V_{TH} - 40}{4k} = 16$$

$$1.33i_b = \frac{V_{TH}}{12k}$$

$$\frac{4}{3}(V_{TH} - 40) \times 3 = V_{TH}$$

$$4V_{TH} - 160 = V_{TH} \quad (1 \text{ 分})$$

$$V_{TH} = \frac{160}{3} \text{ V}$$

Write a KVL equation around the loop:

$$40 = -4000i - 0.08 \frac{di}{dt} \quad (1 \text{ 分})$$

Rearranging:

$$\frac{di}{dt} = -100 + 50,000i = 25,000(i + 0.01)$$

Separate the variables and integrate to find i :

$$\frac{di}{i + 0.01} = 50,000 dt$$

$$\int_0^i \frac{dx}{x + 0.01} = \int_0^t 50,000 dx$$

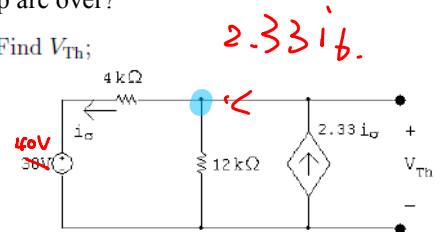
$$\therefore i = -0.01 + 10e^{50,000t} \text{ mA} \quad (1 \text{ 分})$$

$$\frac{di}{dt} = (10 \times 10^{-3})(50,000)e^{50,000t} = 500e^{50,000t}$$

Solve for the arc time:

$$v = 0.08 \frac{di}{dt} = 40e^{50,000t} = 30,000; \quad e^{50,000t} = 750 \quad (1 \text{ 分})$$

$$\therefore t = \frac{\ln 750}{50,000} = 132.4 \mu\text{s} \quad (1 \text{ 分})$$

[b] Find V_{Th} ;

Write a KCL equation at the top node:

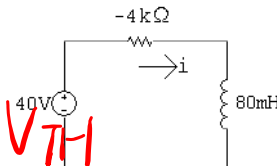
$$\frac{V_{Th} - 40}{4000} + \frac{V_{Th}}{12,000} - 2.33i_\sigma = 0 \quad (1 \text{ 分})$$

The constraint equation is:

$$i_\sigma = \frac{(V_{Th} - 40)}{4000} = 0$$

Solving,

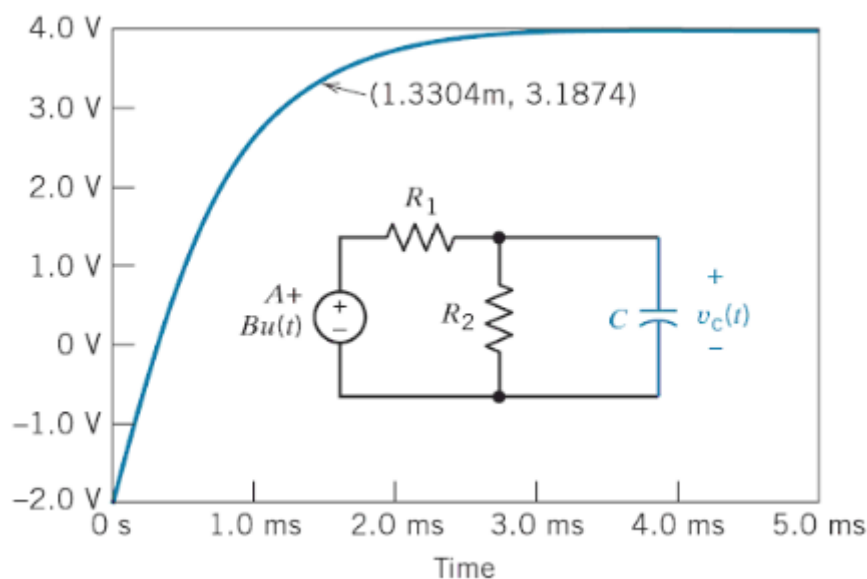
$$V_{Th} = 40 \text{ V} \quad (1 \text{ 分})$$



(本题 满分 10 分)

7. About one months ago, Wang Xiaoming went into the lab to find the principle of Thevenin circuit. Today, Keyi Yuan, who is an SIST student, is also interested in the principle of the circuit. He wanted to find how RL circuit works. With the help of Wang Xiaoming, he built a circuit like the one in figure.

In the figure shows the transient response of a first-order circuit. A point on this transient response has been labeled. The label indicates a time and the capacitor voltage at that time. Assume that this circuit has reached steady state before the time $t=0$. Placing the circuit diagram on the plot suggests that the plot corresponds to the circuit. Suppose $R_1 = 1k\Omega$, $R_2 = 2k\Omega$. Specify values of A, B and C that cause the voltage across the capacitor in this circuit to be accurately represented by this plot.



Step 1 of 8

$$u(t) = \begin{cases} 0 & \text{for } t < 0 \\ 1 & \text{for } t > 0 \end{cases}$$

$$A + Bu(t) = \begin{cases} A & \text{for } t < 0 \\ A + B & \text{for } t > 0 \end{cases}$$

Comment

By voltage division rule

$$v_c(0^-) = A \times \frac{R_2}{R_1 + R_2}$$

(1分)

In capacitor voltage can't change instantaneously

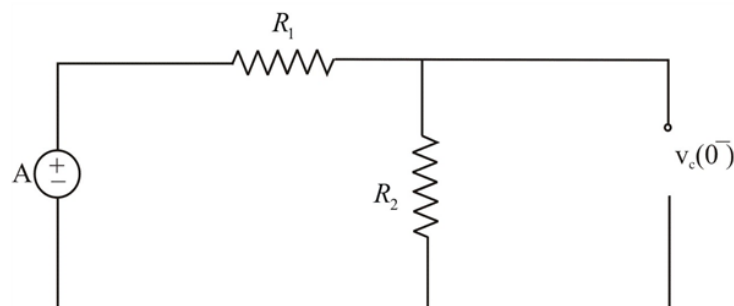
$$v_c(0^-) = v_c(0^+)$$

→ (1分)

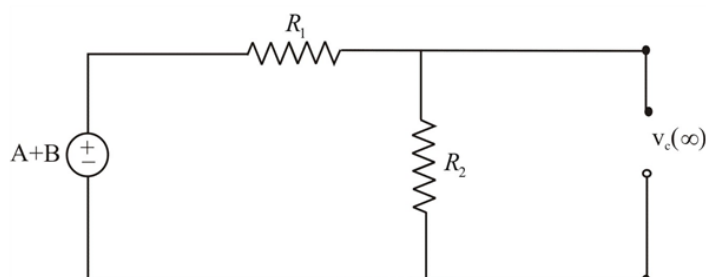
$$v_c(0^+) = \frac{AR_2}{R_1 + R_2} \quad \text{---(1)}$$

Comment

Step 2 of 8

For $t < 0$ circuit reached to steady state and capacitor becomes open circuited∴ For $t < 0$ circuit is

Step 4 of 8

For $t >> 0$ i.e. at $t = \infty$ again circuit reached to steady state and capacitor becomes open circuited. For $t >> 0$ circuit is

Step 5 of 8 ^

By voltage division rule

$$v_c(\infty) = \frac{(A+B)R_2}{R_1+R_2} \text{ ----- (2) } \quad (1 \text{分})$$

From the graph

$$v_c(0) = -2V \text{ and } v_c(\infty) = 4V$$

∴ Capacitor voltage equation is

$$v_c(t) = v_c(\infty) - [v_c(\infty) - v_c(0)]e^{-\frac{t}{\tau}} \quad (1 \text{分})$$

$$v_c(t) = 4 - (4+2)e^{-\frac{t}{\tau}}$$

$$v_c(t) = 4 - 6e^{-\frac{t}{\tau}} \text{ for } t > 0 \quad (1 \text{分})$$

Step 6 of 8 ^

From the plot it is given that point is (1.3304m, 3.1874)

$$3.1874 = 4 - 6e^{\frac{-1.3304 \times 10^{-3}}{\tau}}$$

$$0.1355 = e^{\frac{-1.3304 \times 10^{-3}}{\tau}}$$

$$-1.99 = \frac{-1.3304 \times 10^{-3}}{\tau}$$

$$\tau = 0.669\text{ms} \quad (1 \text{分})$$

From the circuit time constant $\tau = RC$

$$\tau = R_{eq}C$$

$$R_{eq} = R_1 \parallel R_2$$

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

$$\tau = \frac{R_1 R_2}{R_1 + R_2} C$$

Step 7 of 8 ^

$$0.669 \times 10^{-3} = \frac{R_1 R_2}{R_1 + R_2} C$$

From (1) and (2)

$$\begin{aligned} -2 &= \frac{AR_2}{R_1 + R_2} \\ 4 &= \frac{(A+B)R_2}{R_1 + R_2} \end{aligned} \quad \left. \begin{array}{l} \\ \end{array} \right\} \Rightarrow 1 \text{分}$$

Step 8 of 8 ^

Let $R_1 = 1k$ and $R_2 = 2k$

$$\therefore A = -3$$

$$A+B=6$$

$$\therefore B=9$$

And $0.669 \times 10^{-3} = 0.667 \times 10^{-3} \times C$

$$C = 1\mu F$$

$$R_1 = 1k$$

$$R_2 = 2k$$

$$A = -3$$

$$B = 9$$

$$C = 1\mu F$$

(本题满分12分)

8. Determine $v(t)$ for the circuit shown in figure a when the voltage source V_1 varies as shown in figure. The initial capacitor voltage is $V_s(0)=0$.

Sol: $V_1 = \begin{cases} 5t, & 0 \leq t \leq 2s \\ 10V, & t \geq 2s \end{cases}, i_c(t) = C \frac{dV_s(t)}{dt}$

First, for $0 \leq t \leq 2s$:

Using KCL on the top node, we can get that:

$$\begin{cases} \frac{V_s(t) - V_1}{2} + i_c(t) - i_x + i_x = 0 \\ V_1 = 5t \end{cases} \Rightarrow (1 \text{分})$$

Therefore, $\frac{dV_s(t)}{dt} + \frac{V_s(t)}{2} = \frac{5}{2}t$

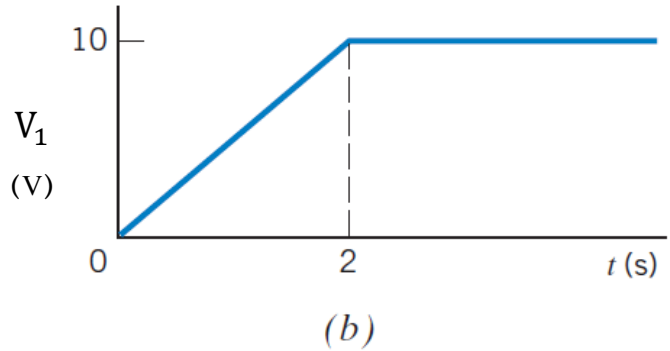
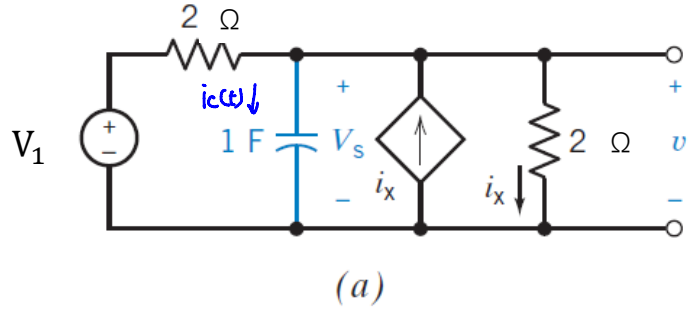
Solving the equation, we can get that:

$$\begin{aligned} V_s(t) &= e^{-\int \frac{1}{2} dt} \left(\int \frac{5}{2} t e^{\frac{1}{2} t} dt + C \right) \\ &= C e^{-\frac{1}{2} t} + \frac{5}{2} e^{-\frac{1}{2} t} \int t e^{\frac{1}{2} t} dt \\ &= C e^{-\frac{1}{2} t} + 5t - 10 \end{aligned} \Rightarrow (2 \text{分})$$

For the initial condition that $V_s(0)=0$

Therefore $C=10$ (1分)

So $V_s(t) = 10e^{-\frac{1}{2}t} + 5t - 10$ (1分)



Second, for $t > 2s$:

$V_s(2) = 3.679V, V_1 = 10V$ (1分)

Using KCL, we can get that:

$$\begin{cases} \frac{V_s(t) - V_1}{2} + i_c(t) + i_x - i_x = 0 \\ V_1 = 10 \end{cases} \Rightarrow (1 \text{分})$$

Therefore, $\frac{dV_s(t)}{dt} + \frac{V_s(t)}{2} = 5$

Solving the equation, we can get that:

$$\begin{aligned} V_s(t) &= e^{-\int \frac{1}{2} dt} \left(\int 5 e^{\frac{1}{2} t} dt + C \right) \\ &= C e^{-\frac{1}{2} t} + 10 \end{aligned} \Rightarrow (2 \text{分})$$

But because we change it at $t=2s$, thus it has $2s$ delay.

We need to replace t with $t-2$ to make that it will change at the time $\boxed{t-2=0}$.

$\therefore V_s(t') = C e^{-\frac{1}{2}(t'-2)} + 10$ (1分)

Because $V_s(2) = 3.679V$, thus:

$C = -6.321$ (1分)

$\therefore V_s(t) = 10 - 6.321 e^{-\frac{1}{2}(t-2)}$

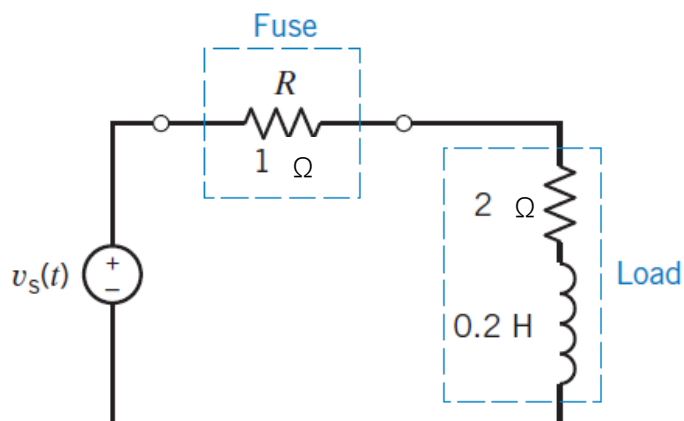
$\therefore V_s(t) = \begin{cases} 10e^{-\frac{1}{2}t} + 5t - 10, & t \in [0, 2] \\ -6.321e^{-\frac{1}{2}(t-2)} + 10, & t \in [2, \infty) \end{cases}$

(1分) 注意范围

此2分还包括2后 $t-2$ 的式子

(本题满分 12 分)

9. Fuses are used to open a circuit when excessive current flows. This circuit was designed by Wright in 1990. One fuse is designed to open when the power absorbed by R exceeds 10W for 0.5s. Consider the circuit shown in figure. The input is given by $V_s = A[u(t) - u(t - 0.75)]$. Assume that $i_L(0^-) = 0$. Determine the largest value of A that will not cause the fuse to open.



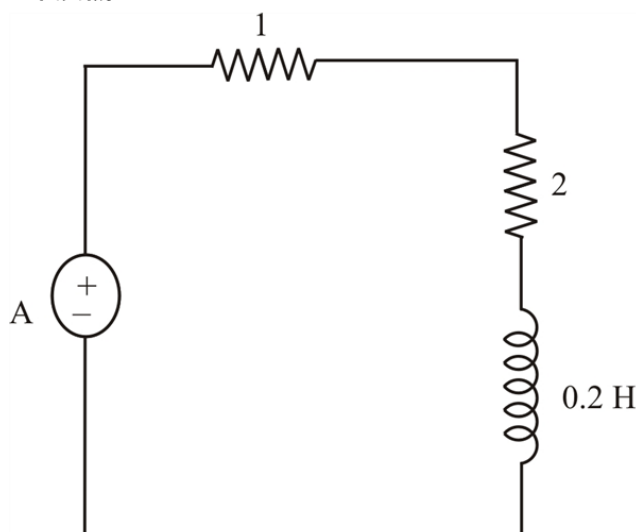
Step 2 of 10 ^

Given

$$v_s(t) = A[u(t) - u(t - 0.75)] \text{ V}$$

$$\therefore v_s(t) = \begin{cases} A & \text{for } 0 < t < 0.75 \\ 0 & \text{for } t > 0.75 \end{cases}$$

Step 3 of 10 ^

For $0 < t < 0.75$ circuit is

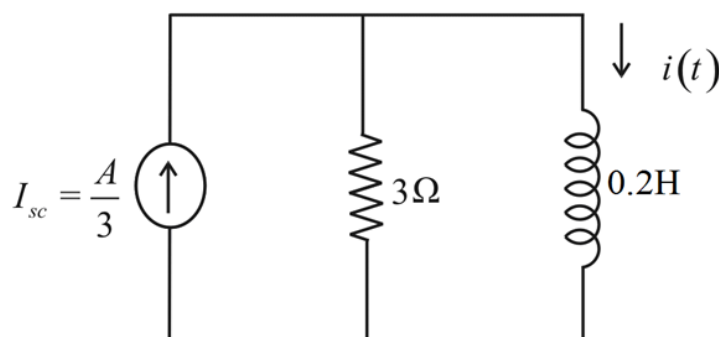
Step 4 of 10 ^

\therefore Generalized expression for inductor current is

$$i(t) = I_{sc} + (i(0) - I_{sc})e^{-\frac{t}{\tau}}$$

Step 5 of 10 ^

By source transformation



Step 6 of 10 ^

Given $i_L(0^-) = 0$

$$i_L(0^-) = i_L(0^+)$$

$$i_L(0) = 0$$

$$\therefore i(t) = \frac{A}{3} + \left(0 - \frac{A}{3}\right)e^{-\frac{t}{\tau}}$$

$$i(t) = \frac{A}{3} \left(1 - e^{-\frac{t}{\tau}}\right) \quad (2 \text{分})$$

Step 7 of 10

$$\tau = \frac{L}{R_{eq}}$$

$$R_{eq} = 3\Omega$$

$$\therefore \tau = \frac{0.2}{3} \quad (1分)$$

$$\tau = 0.0667 \text{ sec}$$



$$i(t) = \frac{A}{3}(1 - e^{-15t}) \text{ A} \quad (1分) \quad 0 < t < 0.75$$

Step 10 of 10 ^

We need to find the value of A such that maximum power through 1Ω should not exceed 10W for 0.5 sec

Current through 1Ω resistor = $i(t)$

$$i(t) = \frac{A}{3}(1 - e^{-15t})$$

Now we have to find A so that power through the resistor is greater than 10W during $0.25 < t < 0.75 \text{ sec}$

$$\therefore i^2(0.25)R_{fuse} = 10 \text{ W}$$

$$\left(\frac{A}{3}(1 - e^{-15 \times 0.25})\right)^2 \cdot 1 = 10 \quad (2分)$$

$$\left(\frac{A}{3} \cdot 0.97648\right)^2 = 10$$

$$0.1059A^2 = 10$$

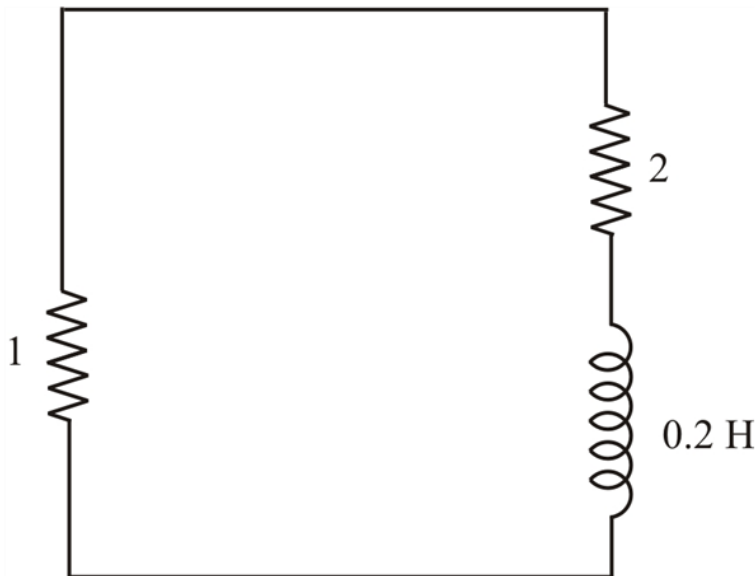
$$A^2 = 94.428$$

$$A = 9.715$$

→ (2分)

Step 8 of 10 ^

For $t > 0.75$ circuit is



Step 9 of 10 ^

$$I_{sc} = 0$$

$$i(0.75) = \frac{A}{3}(1 - e^{-15 \times 0.75})$$

$$i(0.75) = 0.333 \text{ A} \quad (1分)$$

$$\therefore i(t) = 0 + (0.333A - 0)e^{-\frac{(t-0.75)}{\tau}} \quad (2分) \quad t > 0.75$$

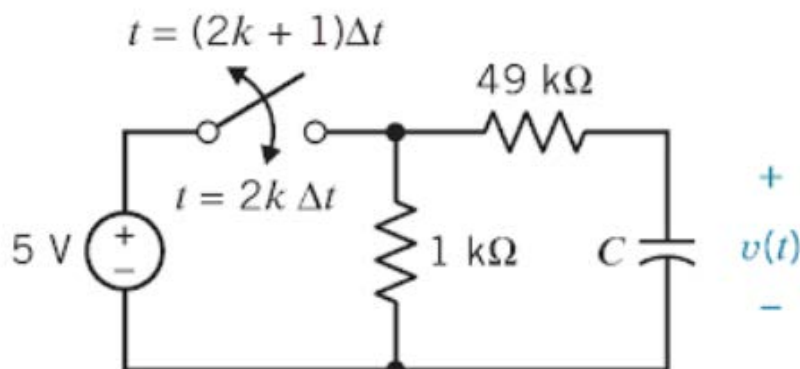
$$i(t) = 0.333Ae^{-15(t-0.75)} \quad t > 0.75$$

(本题满分12分)

(主体分7分)

10. The switch in Figure closes at time $0, 2\Delta t, 4\Delta t, \dots, 2k\Delta t$ and opens at times $\Delta t, 3\Delta t, 5\Delta t, \dots, (2k+1)\Delta t$. When the switch closes, $v(t)$ makes the transition from $v(t) = 0$ to $v(t) = 5V$. Conversely, when switch opens, $v(t)$ makes the transition from $v(t) = 5V$ to $v(t) = 0$. Suppose we require that one transition be 95 percent complete before the next one begins.

- (1) Determine the value of C required so that $\Delta t = 1\mu s$.
- (2) How large must Δt be when $C = 2\mu F$?



Step 1 of 7 ^

Generalized expression for capacitor voltage is

$$v(t) = v(\infty) + (v(0) - v(\infty))e^{-\frac{t}{\tau}}$$

When the switch closes $v(t)$ makes transition from $v(t) = 0$ to $5V$ in Δt sec

$$\begin{aligned} \therefore v(0) &= 0 \\ v(\infty) &= 5 \end{aligned} \quad \left. \vphantom{\begin{aligned} \therefore v(0) &= 0 \\ v(\infty) &= 5 \end{aligned}} \right\} \quad (1分)$$

$$\begin{aligned} v(t) &= 5 + (0 - 5)e^{-\frac{t}{\tau}} \\ v(t) &= 5 \left(1 - e^{-\frac{t}{\tau}} \right) \end{aligned} \quad (1分)$$

Step 2 of 7 ^

It is given that one transition is 95 percentage complete before the next one begins.

$$\begin{aligned} v(t) &= 0.95v(\infty) \\ v(t) &= 0.95 \times 5 \\ v(t) &= 4.75 \end{aligned} \quad \left. \vphantom{\begin{aligned} v(t) &= 0.95v(\infty) \\ v(t) &= 0.95 \times 5 \\ v(t) &= 4.75 \end{aligned}} \right\} \Rightarrow (1分)$$

$$4.75 = 5 \left(1 - e^{-\frac{\Delta t}{\tau}} \right) \quad (2分)$$

$$0.95 = 1 - e^{-\frac{\Delta t}{\tau}}$$

$$0.05 = e^{-\frac{\Delta t}{\tau}}$$

$$2.995 = \frac{\Delta t}{\tau} \quad (1分)$$

Step 3 of 7 ^

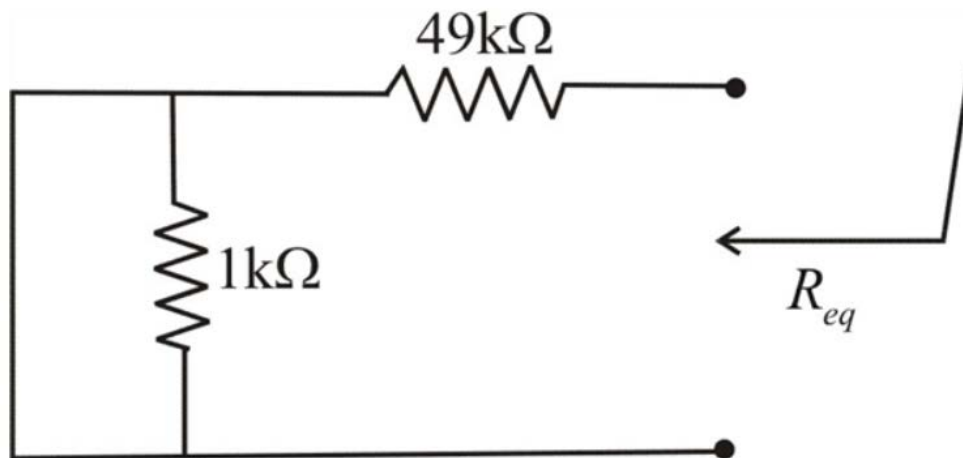
When the switch is opened the equivalent resistance is,

$$R_{eq} = 50k\Omega$$

When the switch closes time constant $\tau = R_{eq}C$

Step 4 of 7 ^

R_{eq} can be calculated by short circuiting the voltage source



Step 5 of 7 ^

$$R_{eq} = 49k\Omega$$

$$R_{eq} \square 50k\Omega \quad (1分)$$

$$\tau = 49 \times 10^3 \times C$$

Step 6 of 7 ^

(a) Given $\Delta t = 1\mu\text{sec}$

$$\therefore 2.995 = \frac{1 \times 10^{-6}}{50 \times 10^3 \times C}$$

$$\boxed{C = 6.67\text{PF}}$$

} $\Rightarrow 2分$

Step 7 of 7 ^

(b) Given $C = 2\mu\text{F}$

$$2.995 = \frac{\Delta t}{\tau}$$

$$\Delta t = 2.995 \times \tau$$

$$\Delta t = 2.995 \times R_{eq} C$$

$$\Delta t = 2.995 \times 50 \times 10^3 \times 2 \times 10^{-6}$$

$$\Delta t = 0.2995$$

$$\boxed{\Delta t = 0.3\text{s}}$$

} $\Rightarrow (3分)$