Prediction Markets: Economics, Computation, and Mechanism Design

a tutorial by

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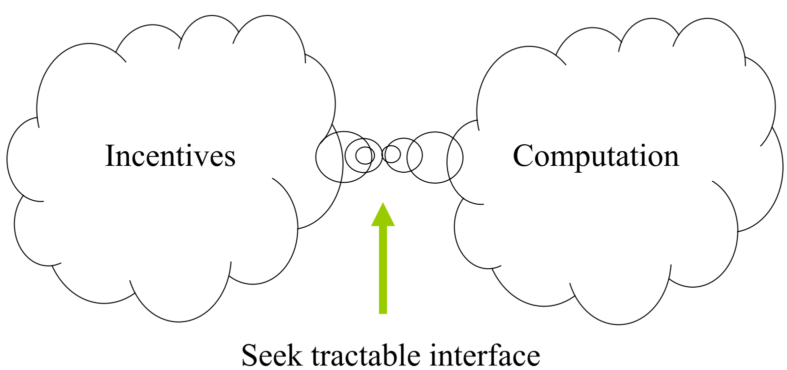
[Thanks: David Pennock]

Economics & Computer Science



[Source: Hanson 2002]

Outline

1. Introduction (15 min)
   - What is a prediction market?
   - Functions of markets
   - A list of prediction markets
2. Background (15 min)
   - Uncertainty, risk, and information
   - Decision making under uncertainty
   - Security markets

Outline

3. Instruments and Mechanisms (15 min)
   - Contracts in prediction markets
   - Prediction market mechanisms
     - Call market
     - Continuous double auction
     - Continuous double auction /w market maker
     - Pari-mutuel market
     - Bookmaker

Outline

4. Examples: Empirical Studies (25 min)
   - Iowa Electronic Markets: Political election
   - Tradesports: Effect of war
   - Hollywood Stock Exchange
   - Tech Buzz Game
   - Real money vs. Play Money
5. Theory and Lab Experiments (20 min)
   - Theory
     - Rational Expectations Equilibrium
     - Can't agree to disagree
     - Efficient Market Hypothesis
     - No Trade Theorem
   - Lab experiments on information aggregation

Outline

6. Computational Perspectives (60 min)

6A. Mechanism Design for Prediction Markets
   - Design criteria
   - Mechanisms for Prediction Markets
     - Combinatorial betting
       - Betting on permutations
       - Betting on Boolean expressions
     - Automated market makers
       - Market scoring rules
       - Dynamic pari-mutuel market
       - Utility-based market maker

6B. Distributed Market Computation

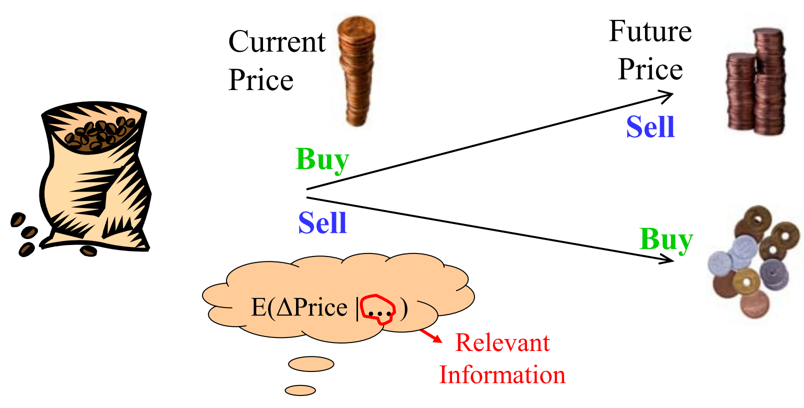
7. Legal Issues and Other (5 min)

1. Introduction

- What is a prediction market?
- Functions of markets
- A list of prediction markets

Markets

- Items to Trade: Products, Contracts, …
- Buy Low Sell High



Prediction Markets

- A prediction market is a financial market that is designed for information aggregation and prediction.
- Payoffs of the traded item is associated with outcomes of future events.

$f(x)

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Prediction Market 1, 2, 3

1. Turn an uncertain event of interest into a random variable
   - category 3 (or higher) hurricane make landfall in Florida in 2007? (Y/N) => 1/0 random var.
2. Create a financial contract, payoff = value of the random variable

$$\begin{cases} \$1 & \text{if category 3 (or higher) hurricane make landfall in Florida in 2007} \\ \$0 & \text{otherwise} \end{cases}$$

3. Open a market in the financial contract and attract traders to wager and speculate

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Terminology

- Contract, security, contingent claim, stock, derivatives (futures, options), bet, gamble, wager, lottery
  - Key aspect: payoff is uncertain
- Prediction markets, information markets, virtual stock markets, decision markets, betting markets, contingent claim markets
- Historically mixed reputation, but can serve important social roles

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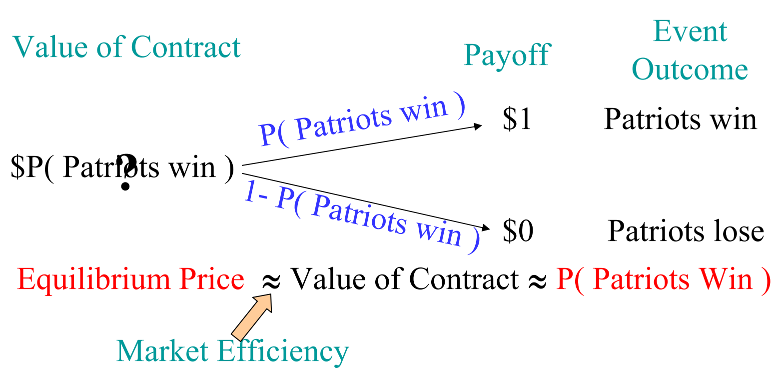
Function of Markets 1:
Get Information

- price ≈ expectation of r.v. | all information

(in theory, lab experiments, empirical studies, ...more later)

$1 if Patriots win, $0 otherwise



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Non-Market Alternatives vs. Markets

- Opinion poll
  - Sampling
  - No incentive to be truthful
  - Equally weighted information
  - Hard to be real-time
- Ask Experts
  - Identifying experts can be hard
  - Incentives
  - Combining opinions can be difficult
- Prediction Markets
  - Self-selection
  - Monetary incentive and more
  - Money-weighted information
  - Real-time
  - Self-organizing

Incentives for Experts: Proper Scoring Rules

- Report a probability estimate: $\mathbf{r}=(r_1,r_2, \ldots,r_n)$
- Get payment $s_i(\mathbf{r})$ if outcome ω_i happens
- Proper: incentive compatible

  A risk neutral agent should chose $r_i=\Pr(\omega_i)$

  to maximize the expected profit
- Proper scoring rules
  - Logarithmic: $s_i(\mathbf{r})=a + b \log(r_i)$ (b>0)
  - Quadratic: $s_i(\mathbf{r})=a + 2 b r_i - b\sum_j r_j^2$ (b>0)

Non-Market Alternatives vs. Markets

- Machine learning/Statistics
  - Historical data
  - Past and future are related
  - Hard to incorporate recent new information
- Prediction Markets
  - No need for data
  - No assumption on past and future
  - Immediately incorporate new information

Does it work?

- Yes, evidence from real markets, laboratory experiments, and theory
  - Racetrack odds beat track experts [Figlewski 1979]
  - Orange Juice futures improve weather forecast [Roll 1984]
  - I.E.M. beat political polls 451/596 [Forsythe 1992, 1999][Oliven 1995][Rietz 1998][Berg 2001][Pennock 2002]
  - HP market beat sales forecast 6/8 [Plott 2000]
  - Sports betting markets provide accurate forecasts of game outcomes [Gandar 1998][Thaler 1988][Debnath EC'03][Schmidt 2002]
  - Market games work [Servan-Schreiber 2004][Pennock 2001]
  - Laboratory experiments confirm information aggregation [Plott 1982;1988;1997][Forsythe 1990][Chen, EC'01]
  - Theory: "rational expectations" [Grossman 1981][Lucas 1972]
  - More later …

Function of Markets 2: Risk Management

- If  is terrible to me,

  I buy a bunch of

$1 if		$0 otherwise

- If my house is struck by lightening, I am compensated.

Risk Management Examples

- Insurance
  - I buy car insurance to hedge the risk of accident
- Futures
  - Farmers sell soybean futures to hedge the risk of price drop
- Options
  - Investors buy options to hedge the risk of stock price changes

Financial Markets vs. Prediction Markets

	Financial Markets	Prediction Markets
Primary	Social welfare (trade) Hedging risk	Information aggregation
Secondary	Information aggregation	Social welfare (trade) Hedging risk

An Incomplete List of Prediction Markets

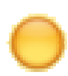
- Real Money
  - Iowa Electronic Markets (IEM), http://www.biz.uiowa.edu/iem/
  - TradeSports, http://www.tradesports.com
  - InTrade, http://www.intrade.com
  - Betfair, http://www.betfair.com/
  - Gambling markets? sports betting, horse racetrack …
- Play Money
  - Hollywood Stock Exchange (HXS), http://www.hsx.com/
  - NewsFutures, http://www.newsfutures.com
  - Yahoo!/O'REILLY Tech Buzz Game, http://buzz.research.yahoo.com
  - World Sports Exchange (WSE), http://www.wsex.com/
  - Foresight Exchange, http://www.ideosphere.com/
  - Inkling Markets http://inklingmarkets.com/
- Internal Prediction Markets
  - HP, Google, Microsoft, Eli-Lilly, Corning …

2. Background

- Uncertainty, risk, and information
- Decision making under uncertainty
- Security markets

Uncertainty, Risk, & Information

- Uncertainty

 or 

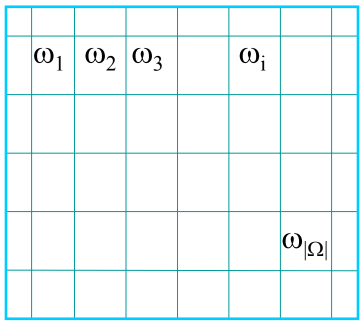
- Risk

Pr() Pr()

- Information

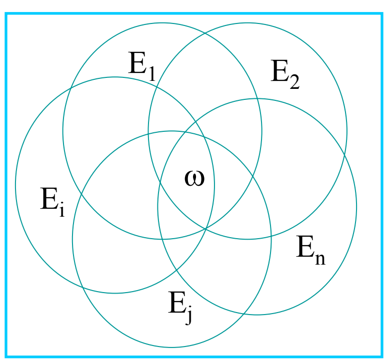
Pr( | info) Pr( | info)

Uncertainty & Risk, in General



- Ω: State Space
- ω are disjoint exhaustive *states of the world*
- ω_j: rain tomorrow & have umbrella & ...
- Pr(ω) ⟶

Uncertainty & Risk, in General

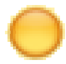


Alternatively,

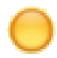
- Overlapping events
  - E1: rain tomorrow
  - E2: have umbrella
- $|\Omega|=2^n$

Preference and Utility

- Preference

 $\succ$  $\succ$  $\succ$ 

- Utility, u(ω)

u()=10 >

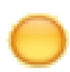
u()=8 >

u()=-4 >

u()=-10

Decision Making Under Uncertainty

- Maximize expected utility
  - E[u]= Σ_{ω} Pr(ω)u(ω)
- Decisions (actions) can affect Pr(ω) or u(ω)

					E[u]
Don't Take umbrella	0.5	0	0	0.5	.5*10+.5*(-10) =0
Take umbrella (but I may leave it at the library)	0.25	0.25	0.25	0.25	.25*10+.25*8+.25*(-4)+.25*(-10) =1

Should take umbrella!

Utility of Money and Risk Attitude

- Outcomes are $
- Risk attitude:
  - risk neutral: $u(x) \sim x$
  - risk averse (typical): u concave ($u''(x) < 0$ for all x), e.g. $u(x)=log(x)$
  - risk prone: u convex
- Absolute risk aversion:

$$r_u(x) = - u''(x) \,/\, u'(x)$$

Risk Attitude & Hedging

- I'm risk averse, u(x) = log (x), insurance company A is risk neutral, u(x)=x.
- I believe that my car might be stolen with prob. 0.01

ω_1: car stolen	ω_2: car not stolen
$u(\omega_1) = \log(10{,}000)$	$u(\omega_2) = \log(20{,}000)$

E[u]=.01 (4)+.99 (4.3) = 4.2980

$\wedge$

- I buy $10,000 insurance for $125

$u(\omega_1) = \log(19{,}875)$	$u(\omega_2) = \log(19{,}875)$

E[u]=.01 (4.2983)+.99 (4.2983) = 4.2983

- Insurance company A also believes Pr(car stolen)=0.01

$u(\omega_1) = -9{,}875$	$u(\omega_2) = 125$

E[u]=.01 (-9875)+.99 (125) = 25 >0

I am happy to buy insurance. Insurance company A is happy to sell it. The transaction allocates risk.

Probability and Speculating

- Suppose that I'm also risk neutral, $u(x)=x$.
- But I think that the probability for my car being stolen is much higher than 0.01, say 0.1.
- A \$10,000 car insurance is worth

  $$.1\ (10{,}000)+.9\ (0)=\$1{,}000$$

  to me, but the insurance company only asks for \$125.

  Too cheap!
- Buy the insurance, and I get \$825 on expectation.

I am speculating the insurance company.

Risk-Neutral Probability

- Subjective probability: an agent's personal judgment
  - Always mixes with the agent's utility (risk attitude)
- Risk neutral probability: the probability that a risk neutral agent has to have the same expected utility

  $$\Sigma_\omega \Pr^{RN}(\omega)\ u^{RN}(x_\omega) = \Sigma_\omega \Pr(\omega) u(x_\omega)$$
- Risk neutral probability is the normalized product of subjective probability and marginal utility

  $$\Pr^{RN}(\omega) \sim \Pr(\omega) u'(x_\omega)$$

Security Markets

- Note, the car insurance in fact a contract

$10,000 if Car Stolen, $0 otherwise

- Security markets generalize this to
  - arbitrary states
  - more than two parties
- Market mechanism to *allocate risk* and *allow speculation* among participants.

What is traded: Securities

- Securities: specify state-contingent *returns*, $r = (r_1,\ldots,r_{|\Omega|})$
- Examples:
  - $(1,\ldots,1)$ riskless numeraire ($1)
  - $(0,\ldots,0,1,0,\ldots,0)$ pays off $1 in designated state (Arrow-Debreu security)
  - $r_i = 1$ if $\omega_i \in E_1$, $r_i = 0$ otherwise $\boxed{\$1 \text{ if } \mathbf{E_1}}$

Terms of trade: Prices

- Price $p^{<E_i>}$ associated with security $\boxed{\$1 \text{ if } \mathbf{E_i}}$
  - *Relative prices* dictate *terms of exchange*
- Facilitate multilateral exchange via bilateral exchange:
  - defines a common scale of resource value
- Can significantly simplify a resource allocation mechanism
  - compresses all factors contributing to value into a single number

General Equilibrium

- General (*competitive*, *Walrasian*) equilibrium describes a simultaneous equilibrium of interconnected markets
- Definition: A price vector and allocation such that
  - all agents making optimal demand decisions (positive demand = buy; negative demand = sell)
  - all markets have zero aggregate demand (buy volume equals sell volume)

Complete securities market

- A set of securities is *complete* if rank of returns matrix = $|\Omega| - 1$
- For example, set of $|\Omega| - 1$ Arrow-Debreu securities
- Market with complete set of securities guarantees a Pareto optimal allocation of risk, under classical conditions
- An allocation is Pareto optimal iff there *does not exist* another solution that is
  - better for one agent *and*
  - no worse for all the rest.

Speculating and Hedging

- Speculating: Increase expected future wealth
  - Information aggregation
- Hedging: Reduce uncertainty
  - Allocate risk

Roles of Markets

- Typically mixed together, and inseparable

3. Instruments & Mechanisms

- Contracts in prediction markets
- Prediction market mechanisms
  - Call market
  - Continuous double auction
  - Continuous double auction /w market maker
  - Pari-mutuel market
  - Bookmaker

Contracts and Mechanisms

- What is being traded? the “good”
- Define:
  - Random variable
  - Payoff function
  - Payoff output
- How is it traded? the “mechanism”
  - Call market
  - Continuous double auction
  - Continuous double auction w/ market maker
  - Pari-mutuel market
  - Bookmaker
  - Combinatorial (later)
  - Automated market maker (later)

Contracts

- Random variables (Questions to ask)
  - Binary, Discrete: tomorrow  or 
  - Continuous: interest rate, temperature, vote share
  - Clarity: “Clinton wins”, “Saddam out”
- Payoff functions
  - Winner-takes-all, Arrow-Debreu — $1 if 
  - Index, continuous — $1 × vote share
  - Dividend, pari-mutuel, option: max[0, s-k], arbitrary function
- Payoff output
  - Real money, play money, prize, lottery

Call Market

- Stock market mechanism before 1800
- Batch order processing
  - Orders are collected over a period of time; collected orders are matched at end of period
  - Price is set such that demand=supply
  - Price determination
    - Mth price auction
    - M+1st price auction
    - k-double auction
- lim period→0: Continuous double auction

Call Market

$1 if  $0 if 

- Buy offers (N=4)
  - $0.15
  - $0.12
  - $0.09
  - $0.05
- Sell offers (M=5)
  - **$0.30**
  - **$0.17**
  - **$0.13**
  - **$0.11**
  - **$0.08**

Price	≤0.05	(0.05, 0.08)	[0.08, 0.09]	(0.09, 0.11)	[0.11, 0.12]	(0.12, 0.13)	[0.13, 0.15]	(0.15, 0.17)	[0.17, 0.30)	≥0.30
Demand	4	3	3	2	2	1	1	0	0	0
Supply	0	0	1	1	2	2	3	3	4	5

Price Determination: Mth Price Auction

$1 if  $0 if 

- Buy offers (N=4)
- Sell offers (M=5)

$0.30 1
$0.17 2
√ $0.15 3
$0.13 4
√ $0.12 5
$0.11 √
$0.09
$0.08 √
$0.05

price = $0.12

✓Matching buyers/sellers

Price Determination: M+1st Price Auction

$1 if [rain] $0 if [sun]

- Buy offers (N=4)
- Sell offers (M=5)

$0.30	1
$0.17	2
√ $0.15	3
$0.13	4
√ $0.12	5
$0.11 √	6
$0.09	
$0.08 √	
$0.05	

price = $0.11

✓Matching buyers/sellers

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Price Determination: k-Double Auction

$1 if [rain] $0 if [sun]

- Buy offers (N=4)
- Sell offers (M=5)

$0.30	1
$0.17	2
√ $0.15	3
$0.13	4
√ $0.12	5
$0.11 √	6
$0.09	
$0.08 √	
$0.05	

Mth price – M+1st Price

price = $0.11 + $0.01*k

0≤k≤1;

K=0: M+1st Price Auction

K=1: Mth Price Auction

✓Matching buyers/sellers

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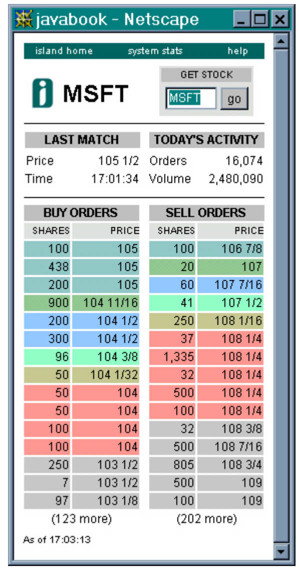
Continuous Double Auction (CDA)

- k-double auction repeated continuously
- Stock market mechanism
- Buy and sell orders continuously come in
- As soon as bid ≥ ask, a transaction occurs
- At any given time, there is a bid-ask spread
- IEM, TradeSports, NewsFutures



BUY ORDERS SHARES	PRICE	SELL ORDERS SHARES	PRICE
100	105	100	106 7/8
438	105	20	107
200	105	60	107 7/16
900	104 11/16	41	107 1/2
200	104 1/2	250	108 1/16
300	104 1/2	37	108 1/4
96	104 3/8	1,335	108 1/4
50	104 1/32	32	108 1/4
50	104	500	108 1/4
50	104	100	108 1/4
100	104	32	108 3/8
100	104	500	108 7/16
250	103 1/2	805	108 3/4
7	103 1/2	500	109
97	103 1/8	100	109
(123 more)		(202 more)	

As of 17:03:13

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CDA with Market Maker

- Same as CDA, but with a market maker
- A market maker is an extremely active, high volume trader (often institutionally affiliated) who is nearly always willing to buy at some price p and sell at some price q ≥ p
- Market maker essentially sets prices; others take it or leave it
- Market maker bears risk, increases liquidity
- HXS, WSE

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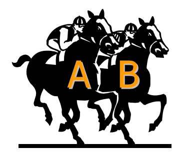
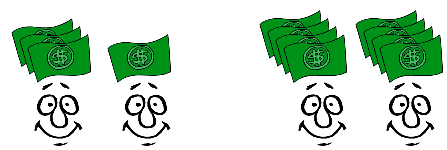
Pari-Mutuel Market



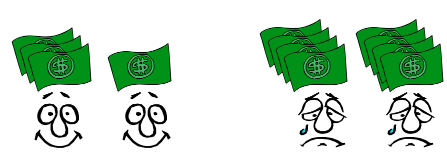
- horse racetrack style wagering
- Two outcomes: A B
- Wagers:



[Source: Pennock 2004]

Pari-Mutuel Market

- E.g. horse racetrack style wagering
- Two outcomes: A B
- Wagers:



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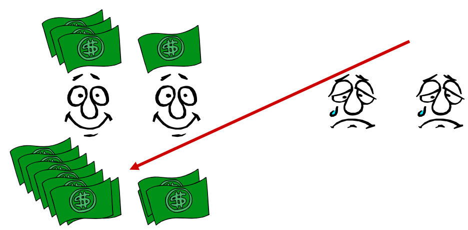


[Source: Pennock 2004]

Bookmaker

- Common in sports betting, e.g. Las Vegas
- Bookmaker is like a market maker in a CDA
- Bookmaker sets "money line", or the amount you have to risk to win $100 (favorites), or the amount you win by risking $100 (underdogs)
- Bookmaker makes adjustments considering amount bet on each side &/or subjective prob's
- Alternative: bookmaker sets "game line", or number of points the favored team has to win the game by in order for a bet on the favorite to win; line is set such that the bet is roughly a 50/50 proposition

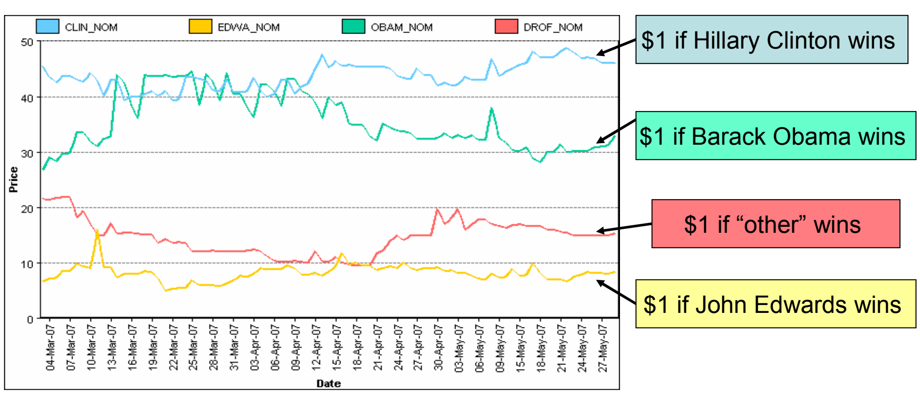
4. Examples: Empirical Studies

- Iowa Electronic Markets: Political election
- Tradesports: Effect of war
- Hollywood Stock Exchange
- Tech Buzz Game
- Real money vs. Play Money

Example: Iowa Electronic Markets (IEM)

IEM Iowa Electronic Markets http://www.biz.uiowa.edu/iem

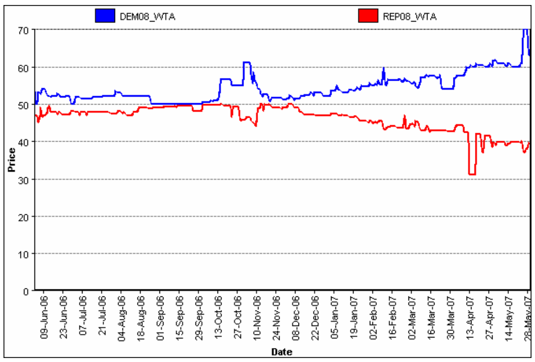
2008 U.S. Presidential Democratic Nomination Markets



[source: http://iemweb.biz.uiowa.edu/graphs/graph_DConv08.cfm, as of 5/30/07]

IEM Winner Takes All Market

2008 US Presidential Election WTA Market



$1 if Democrat votes > Repub

$1 if Republican votes > Dem

Quotes current as of **11:00:02 CST, Wednesday, May 30, 2007**.

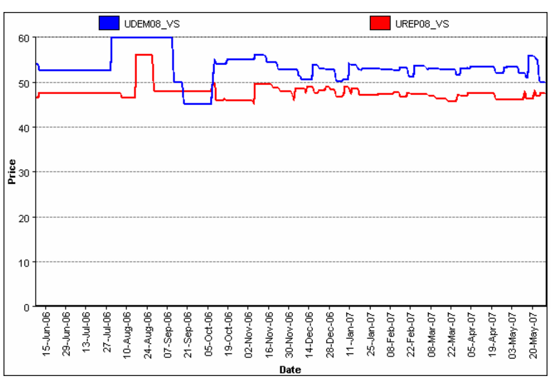
Symbol	Bid	Ask	Last	Low	High	Average
DEM08_WTA	0.605	0.637	0.630	---	---	---
REP08_WTA	0.350	0.389	0.395	---	---	---

price=E[R]=Pr(R)=0.395

[Source: http://www.biz.uiowa.edu/iem/, as of 5/30/07]

IEM Vote Share Market

2008 US Presidential Election Vote Share Market



$1 × vote share of Dem

$1 × vote share of Repub

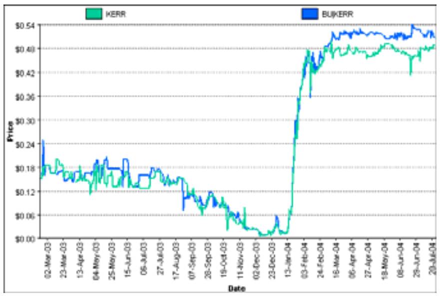
Quotes current as of **11:00:02 CST, Wednesday, May 30,**

Symbol	Bid	Ask	Last	Low	High	Ave
UDEM08_VS	0.520	0.537	0.500	---	---	---
UREP08_VS	0.469	0.470	0.470	0.470	0.470	0.47

price=E[VS of Repub]=47%

[Source: http://www.biz.uiowa.edu/iem/, as of 5/30/07]

IEM Vote Share Market

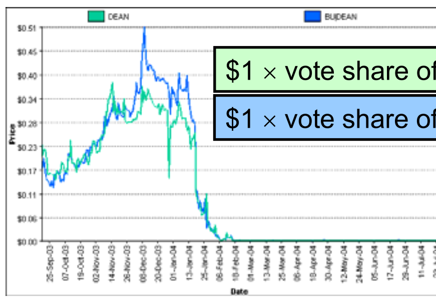


2004 US Pres. election vote share

$1 × vote share of Kerry

$1 × vote share of Bush v. Kerry

price=E[VS for B v. K]=0.508



$1 × vote share of Dean

$1 × vote share of Bush v. Dean

Symbol	Bid	Ask	Last
KERR	0.484	0.489	0.488
BU|KERR	0.507	0.519	0.508
CLRK	0.000	0.001	0.000
BU|CLRK	0.000	0.001	0.000
DEAN	0.000	0.001	0.000
BU|DEAN	0.000	0.001	0.000
EDWD	0.002	0.003	0.002
BU|EDWD	0.002	0.003	0.002
EB	0.000	0.001	0.001
BU|LIEB	0.000	0.001	0.000
ODEM	0.000	0.001	0.001
BU|ODEM	0.000	0.010	0.001
CLIN	0.001	0.009	0.007
BU|CLIN	0.001	0.009	0.008
GEPH	0.000	0.001	0.000
BU|GEPH	0.000	0.001	0.000

as of 7/25/2004

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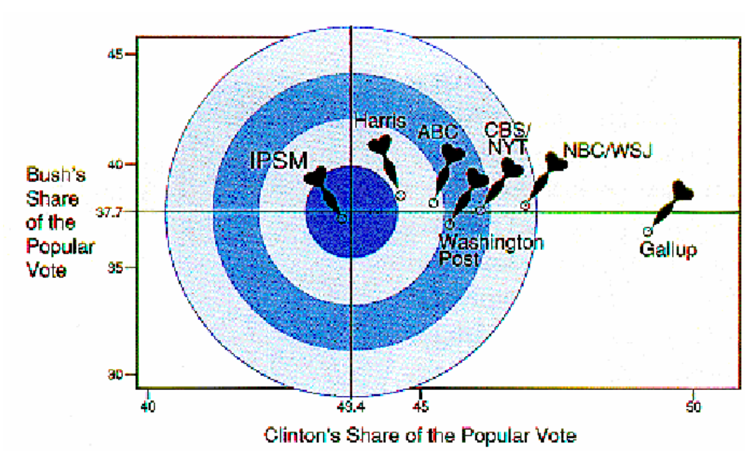
[Source: Berg, DARPA Workshop, 2002]

IEM 1992



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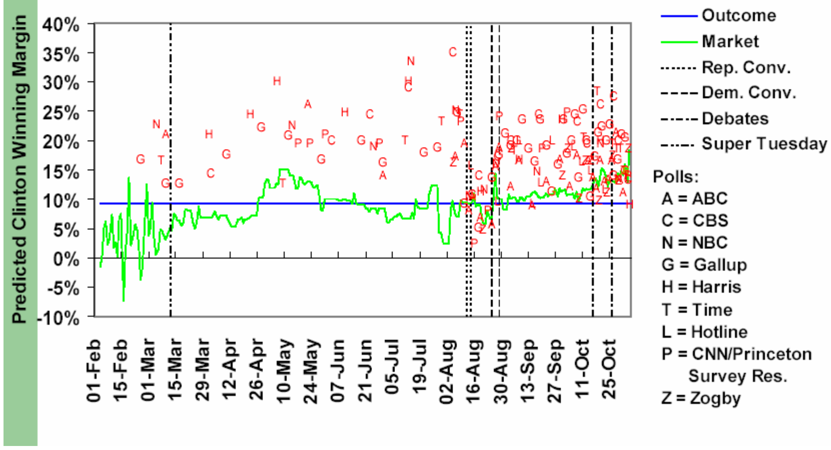
[Source: Berg, DARPA Workshop, 2002]

IEM versus Polls: 1996

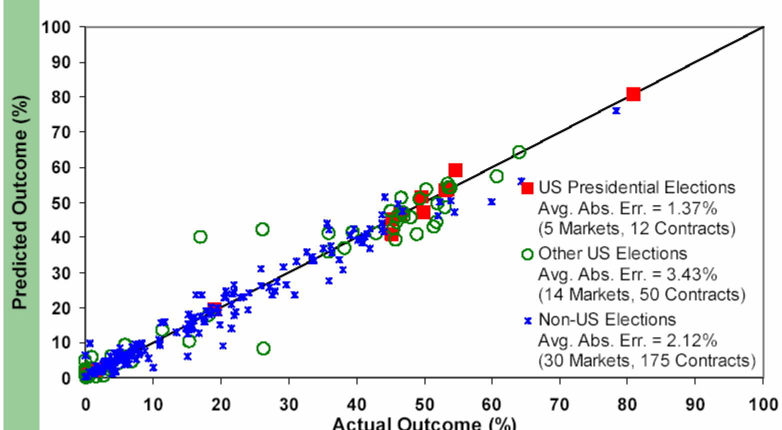
(Berg, Nelson and Rietz, 2001)



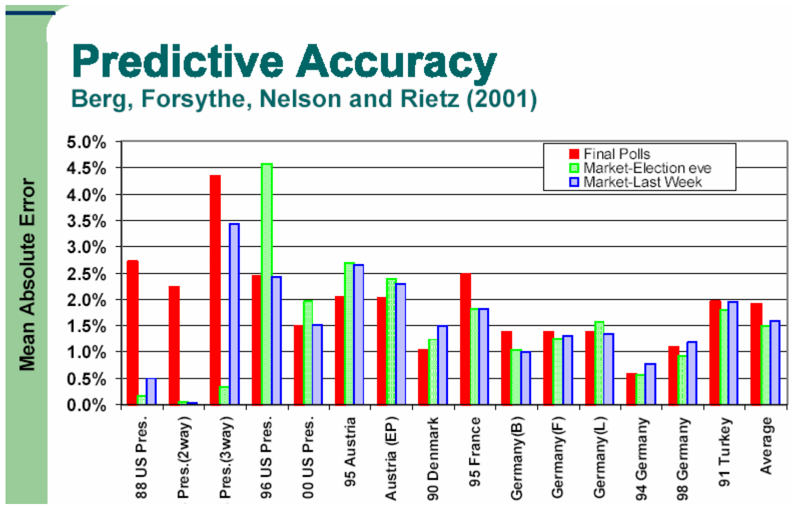
[Source: Berg, DARPA Workshop, 2002]

Predictive Accuracy

Berg, Forsythe, Nelson and Rietz (2001)

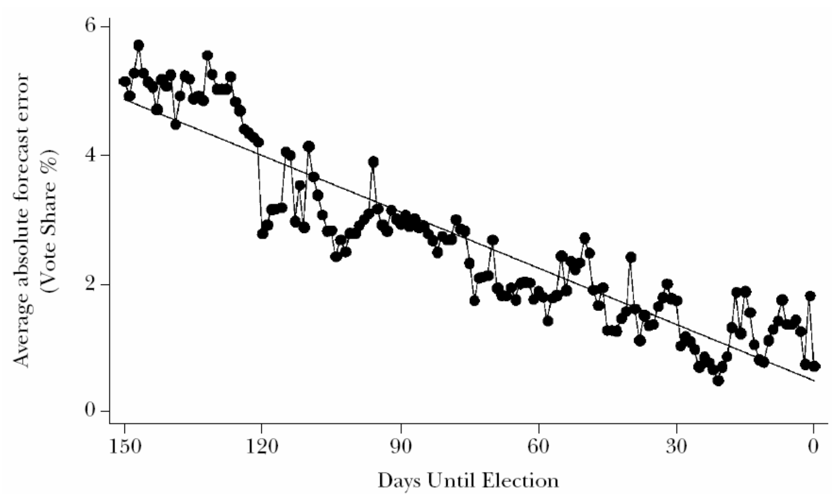


[Source: Berg, DARPA Workshop, 2002]



[Source: Wolfers & Zitzewitz,
J. of Economic Perspectives, 2004]

IEM Information Revelation Through Time



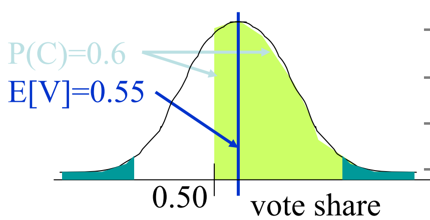
Accuracy and Forecast Std Error

[Berg, Nelson and Rietz, 2003]

- A good forecast for v:

  point estimate + confidence

- IEM Vote share market ⟹ E(v)
- IEM WTA market ⟹ Pr(v>0.5)
- Can we get the confidence (error bound)? Yes!

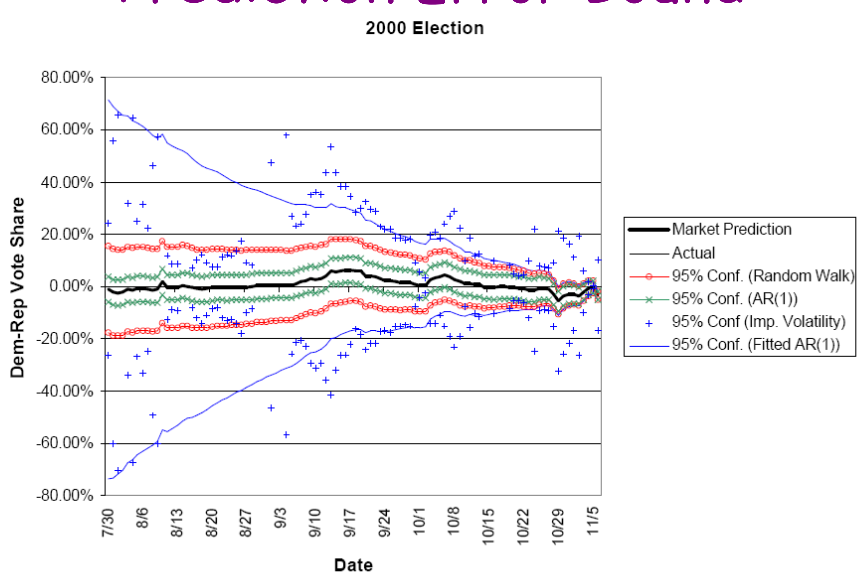


– Assume e.g. normal dist of votes
– Vote share gives mean of dist
– WTA gives P(C) = P(V>0.5)
– Report 95% confidence intervals = error bounds

[Source: Pennock 2004]

[Source: Berg, Nelson and Rietz, 2003]

Prediction Error Bound



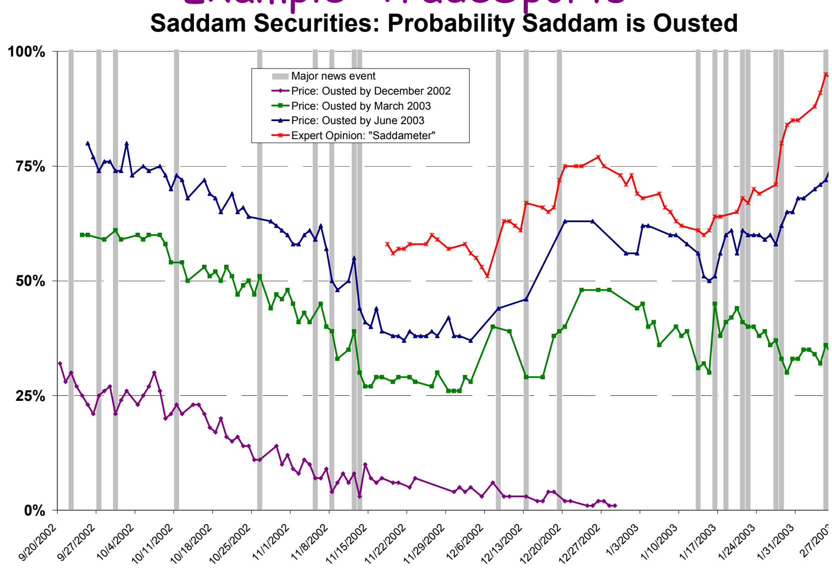
The Marginal Traders

[Forsythe 1992,1999; Oliven 1995; Rietz 1998]

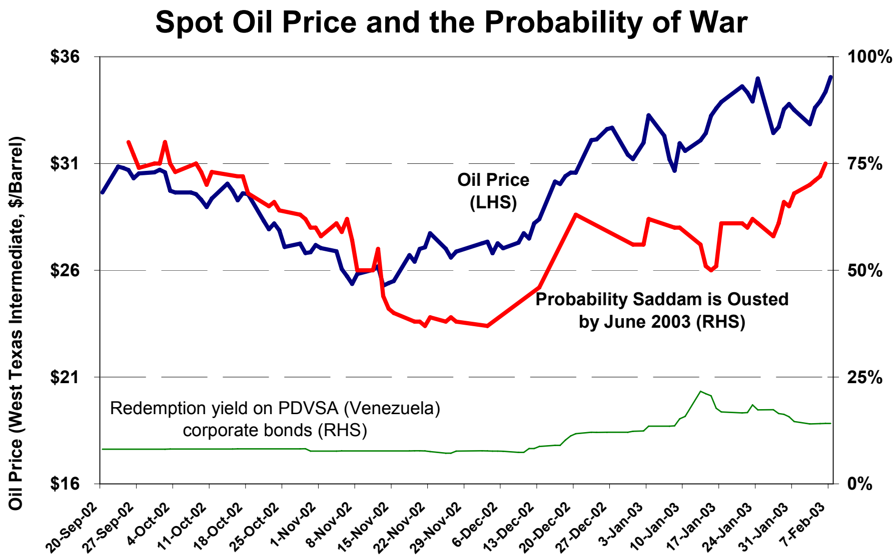
- Participants of IEM are non-representative
- They are error-prone, irrational
  - Leave arbitrage opportunities on the table
  - Not always pick the cheapest trade
  - Democrats buy too much Dem stocks
- Market prices are still accurate
- Because prices are set by marginal traders
  - Marginal traders are less biased and more active. They are better performers and price setters.

[Source: Wolfers 2004]

Example: TradeSports



[Source: Wolfers 2004]



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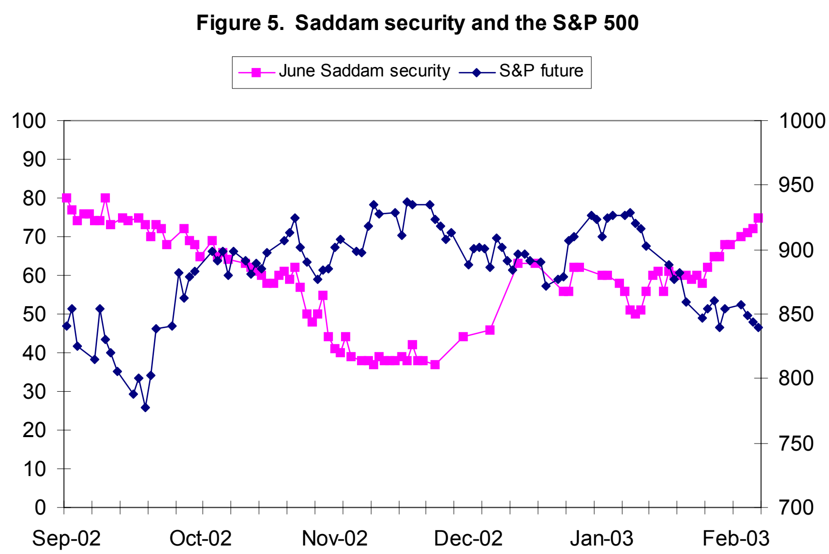
T1-65

[Source: Wolfers 2004]



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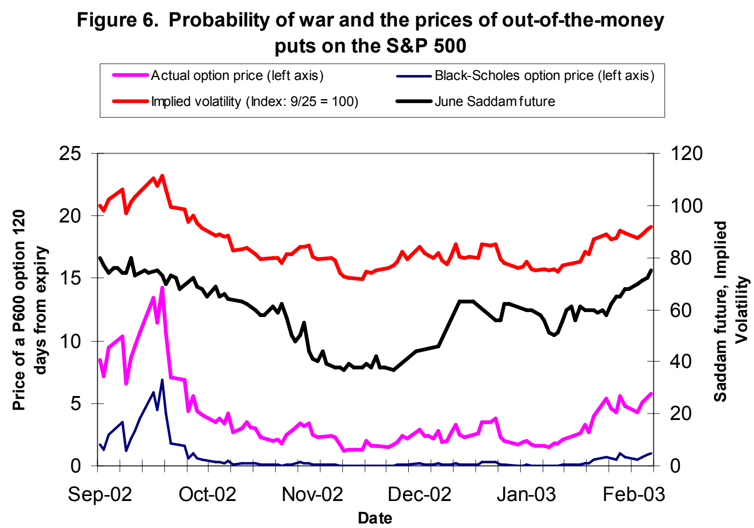
T1-66

[Source: Wolfers 2004]



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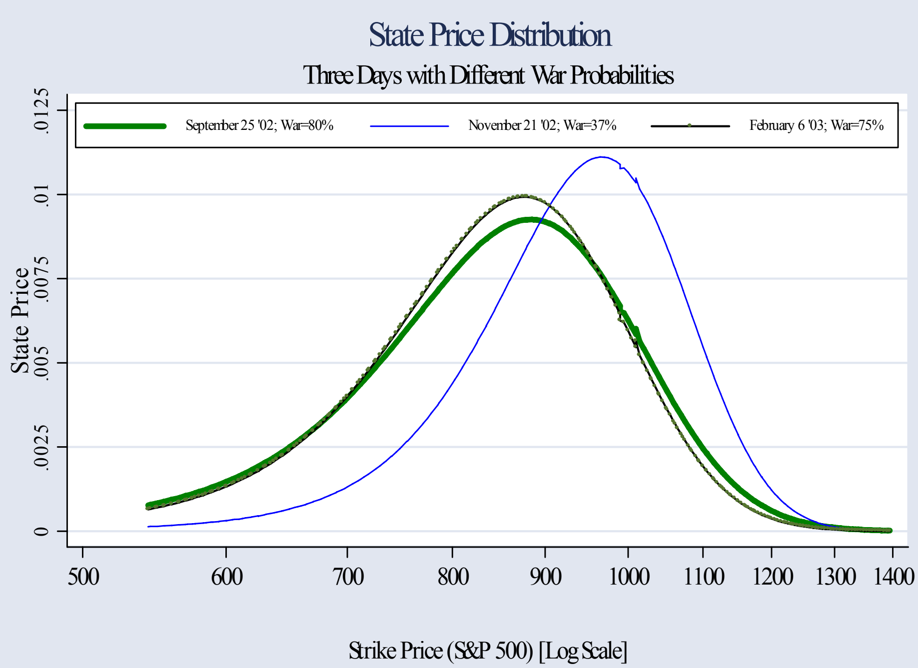
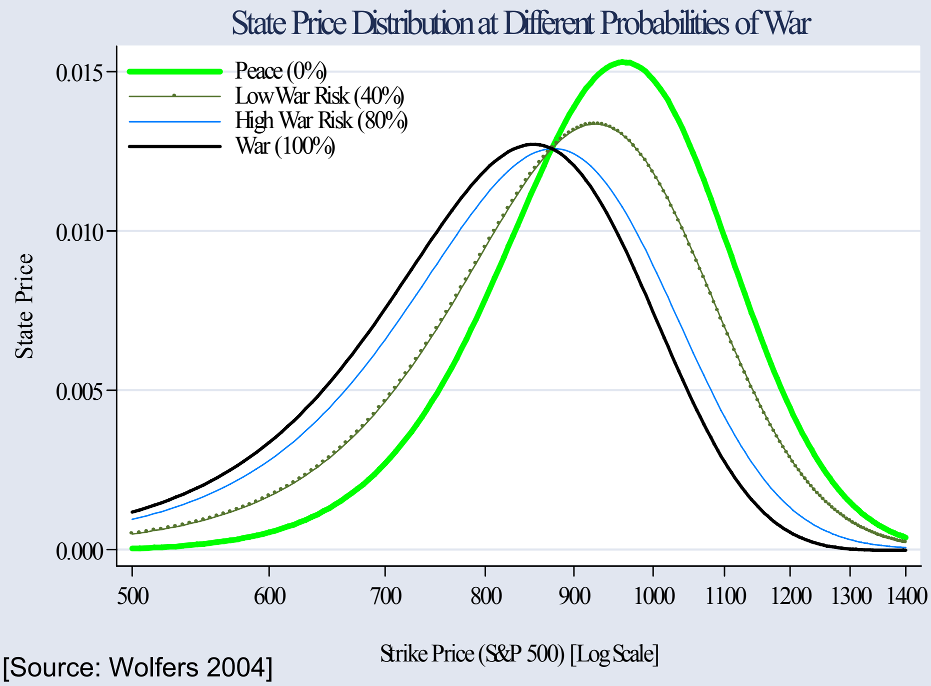
T1-67



[Source: Wolfers 2004]



[Source: Wolfers 2004]

Example: Hollywood Stock Exchange

HSX.COM

- MovieStock

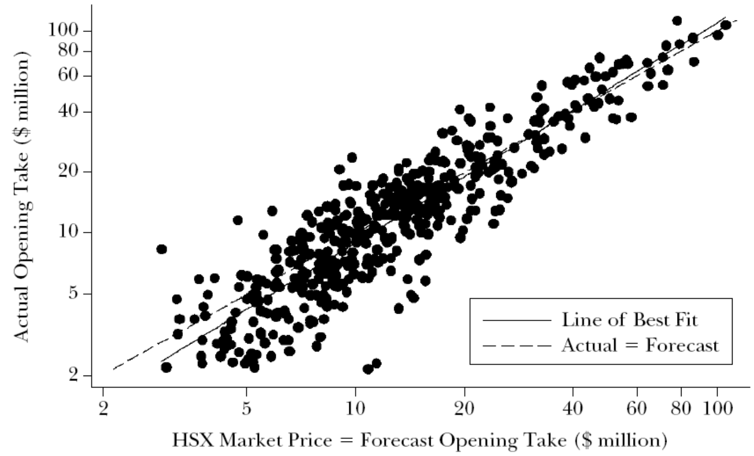
  $x if *Oceans Thirteen* makes *x* million box office proceeds in its first four weeks

- MovieStock option

  Oceans Thirteen $35 put option: A right to sell Oceans Thirteen MovieStock at price $35

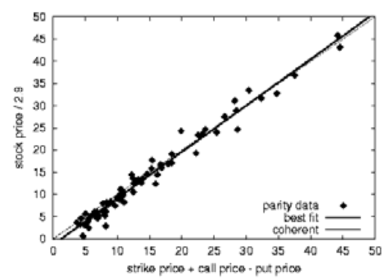
HSX Prediction Accuracy
[Source: Wolfers & Zitzewitz 2004]

Predicting Movie Success

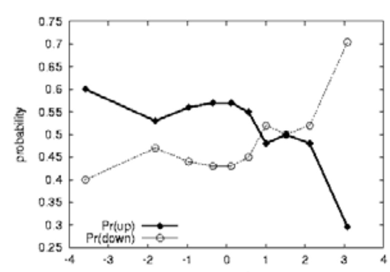


Internal Coherence: HSX
[Source: Pennock et. al. 2000]

- Prices of movie stocks and options adhere to put-call parity, as in real markets



- Arbitrage loopholes disappear over time, as in real markets



Example: Tech Buzz Game

YAHOO! O'REILLY
BUZZ GAME

http://buzz.research.yahoo.com

- Yahoo!,O'Reilly launched Buzz Game 3/05 @ETech
- Research testbed for investigating prediction markets
- Buy "stock" in hundreds of technologies

Market: Peer-to-Peer File Sharing
BitTorrent $13.48 ↓0.16 (1%)
Symbol: BITTORRENT
52-Week Range: $9.81 - $14.10

Market: Net Radio
Podcasting $10.47

Market: Web Application Frameworks
Ruby on Rails $14.79

Market: Web User Interface
Asynchronous Javascript & XML (AJAX) $14.29 ↓0.05 (0%)

Market: Digital Video Recorders
MythTV $9.16 ↓0.04 (0%)

Market: Linux Distributions
Ubuntu $16.99 ↓0.01 (0%)

- Earn dividend based on search "buzz" at Yahoo! Search

YAHOO! SEARCH podcasting Search

- Mechanism: dynamic pari-mutuel market (more later)

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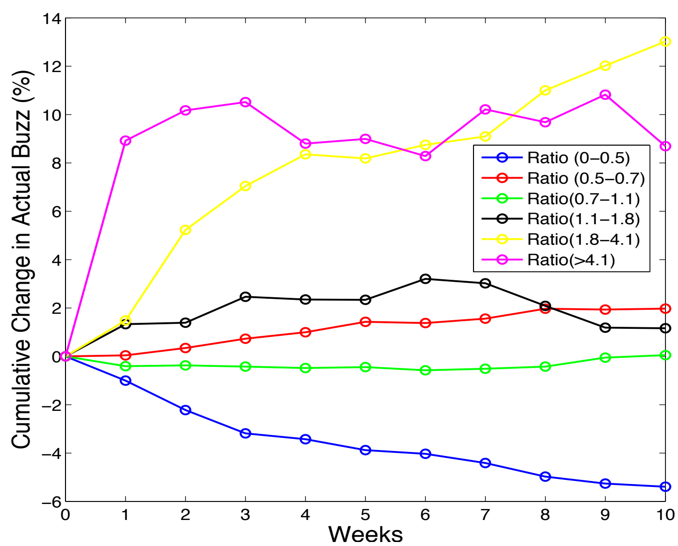
Tech Buzz Game Performance



Based on data from 9/29/05 to 1/27/06,
175 stocks in 44 markets

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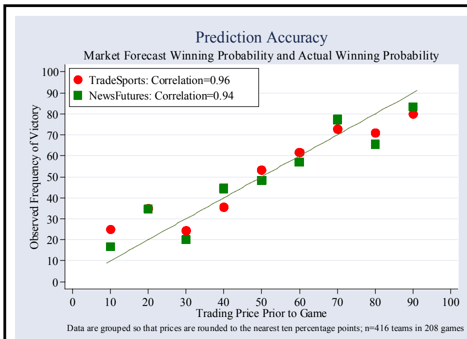
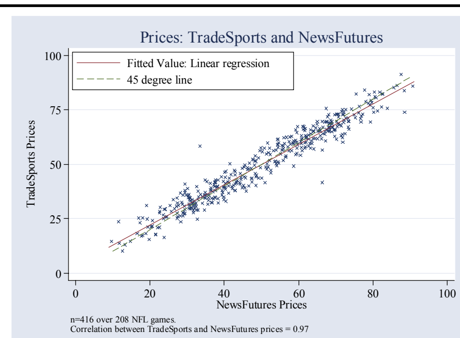
Does money matter?

[Servan-Schreiber et. al. 2004]

<u>Head to Head Comparison</u>
- 2003 NFL Season
- Football prediction markets
  - NewsFutures (play $)
  - Tradesports (real $)
- Online football forecasting competition
  - probabilityfootball.com
  - Contestants assess probabilities for each game
  - Quadratic scoring rule
  - ~2,000 “experts”

<u>Results:</u>
- Play money and real money performed similarly
  - 6th and 8th respectively
- Markets beat most of the ~2,000 contestants
  - Average of experts came 39th





[Source: Servan-Schreiber et. al. 2004]

Does money matter?
Play vs real, head to head

[Source: Servan-Schreiber et. al. 2004]

	Probability-Football Avg	TradeSports (real-money)	NewsFutures (play-money)	Difference TS - NF
Mean Absolute Error	0.443	0.439	**0.436**	0.003
= lose_price	(0.012)	(0.011)	(0.012)	(0.016)
[lower is better]				
Root Mean Squared Error	0.476	0.468	**0.467**	0.001
= ?Average(lose_price^2)	(0.025)	(0.023)	(0.024)	(0.033)
[lower is better]				
Average Quadratic Score	9.323	12.410	**12.427**	-0.017
= 100 - 400(lose_price^2)*	(4.75)	(4.37)	(4.57)	(6.32)
[higher is better]				
Average Logarithmic Score	-0.649	**-0.631**	**-0.631**	0.000
= Log(win_price)	(0.027)	(0.024)	(0.025)	(0.035)
[higher (less negative) is better]				

Statistically:
TS ~ NF
NF >> Avg
TS > Avg

5. Theory and Lab Experiments

- Theory
  - Rational Expectations Equilibrium
  - Can't agree to disagree
  - Efficient Market Hypothesis
  - No Trade Theorem
- Lab experiments on information aggregation

Rational Expectations Equilibrium

[Grossman 1981; Lucas 1972]

Competitive Equilibrium

- Symmetric information
- Demand & Supply reflects preferences, budgets
- Demand=Supply

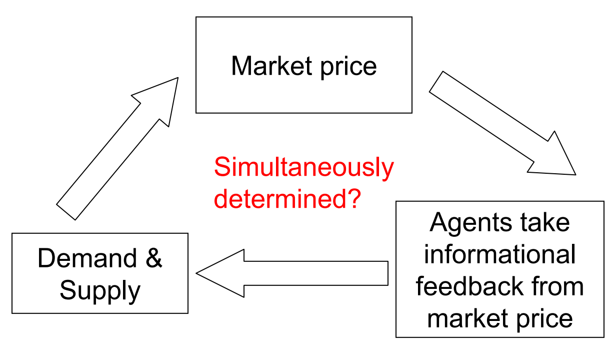
Rational Expectations Equilibrium

- Asymmetric information
- Demand & Supply reflects preferences, budgets, and private information
- Demand=Supply
- Equilibrium price provides informational feedback

➢ Fully Revealing Rational Expectations Equilibrium

At a fully revealing rational expectations equilibrium, the equilibrium price reveals all private information. Agents behave as if they know the pooled information of all agents.

Common Criticism of REE



➢ How can rational expectations equilibrium be reached?

Can't Agree to Disagree

[Auman 76; Mckelvey 86; Mckelvey 90; Nielsen 90; Hanson 98]

- Procedural explanation: agents learn from prices
  - Bayesian agents
  - Agents begin with common priors, different private information
  - Observe sufficient summary statistic (e.g., price)
  - Update beliefs
  - Converge to common posteriors

Efficient market hypotheses (EMH)

- **Internal coherence**
  prices are self-consistent or arbitrage-free
- **Weak form:** Internal unpredictability
  future prices unpredictable from past prices
- **Semi-strong form:** Unpredictability
  future prices unpredictable from all public info
- **Strong form:** Expert-level accuracy
  unpredictable from all public & private info;
  experts cannot outperform naïve traders
- More:`http://www.investorhome.com/emh.htm`

stronger
assump's

How efficient are markets?

- As many opinions as experts
- Cannot prove efficiency; can only detect inefficiency
- Generally, it is thought that large public markets are very efficient, smaller markets questionable
- Still, strong form is sometimes too strong:
  - There is betting on Oscars until winners are announced
  - Prices do not converge completely on eventual winners
  - Yet aggregating all private knowledge in the world (including Academy members' votes) would yield the precise winners with certainty

No Trade Theorems

[Milgrom & Stokey 1982]

- Why trade? These markets are zero-sum games (negative sum w/ transaction fees)
- For all money earned, there is an equal (greater) amount lost; am I smarter than average?
- Rational risk-neutral traders will *never* trade Informally:
  - Only those smarter than average should trade
  - But once below avg traders leave, avg goes up
  - Ad infinitum until no one is left
  - Or: If a rational trader is willing to trade with me, he or she must know something I don't know

But… Trade happens

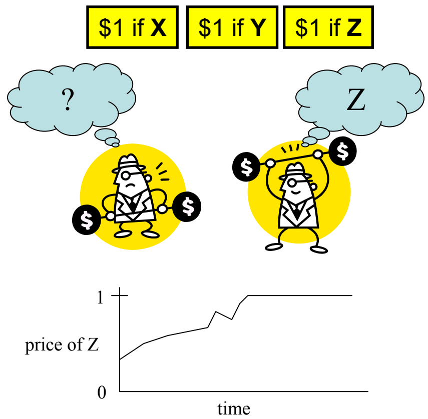
- Volume in financial markets, gambling is high
- Why do people trade?
  1. Different risk attitudes (insurance, hedging)
     Can't explain all volume
  2. Irrational (bounded rational) behavior
     - Rationality arguments require unrealistic computational abilities, including infinite precision Bayesian updating, infinite game-theoretic recursive reasoning
     - More than 1/2 of people think they're smarter than average
     - Biased beliefs, differing priors, inexperience, mistakes, etc.
- Note that it's rational to trade as long as *some* participants are irrational

Laboratory Experiments

- Experimental economics
- Controlled tests of information aggregation
- Participants are given information, asked to trade in market for real monetary stakes
- Equilibrium is examined for signs of information incorporation

Plott & Sunder 1982

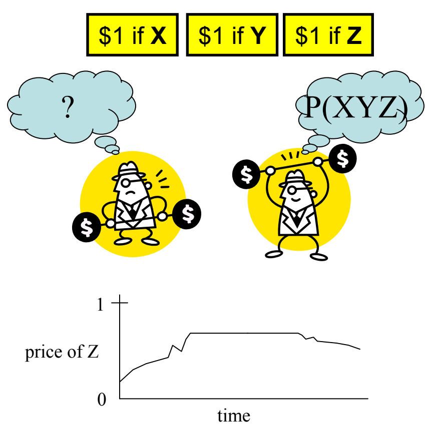
- Three disjoint exhaustive states X,Y,Z
- Three securities
- A few insiders know true state Z
- Market equilibrates according to rational expectations: as if everyone knew Z



[source: Pennock 2004]

Plott & Sunder 1982

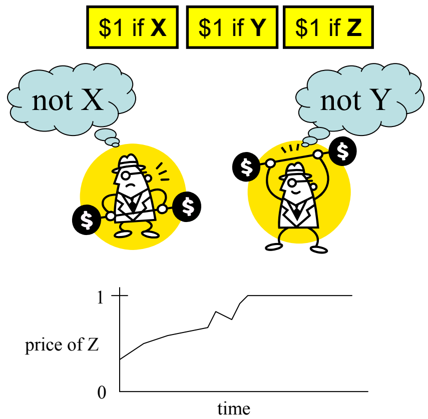
- Three disjoint exhaustive states X,Y,Z
- Three securities
- Some see samples of joint; can infer P(Z|samples)
- Results less definitive



[source: Pennock 2004]

Plott & Sunder 1988

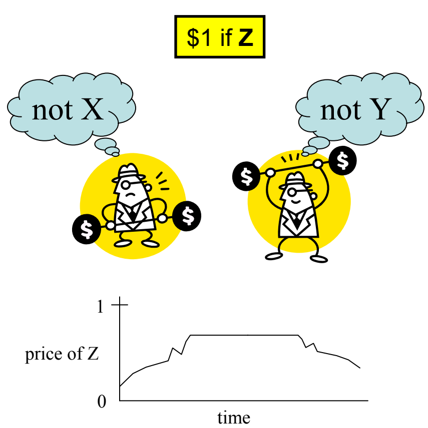
- Three disjoint exhaustive states X,Y,Z
- Three securities
- A few insiders know true state is *not* X
- A few insiders know true state is *not* Y
- Market equilibrates according to rational expectations: Z true



[source: Pennock 2004]

Plott & Sunder 1988

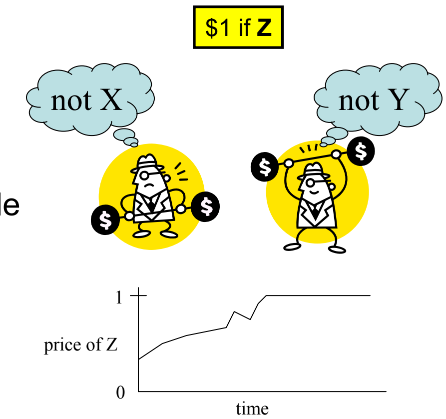
- Three disjoint exhaustive states X,Y,Z
- ***One*** security
- A few insiders know true state is *not* X
- A few insiders know true state is *not* Y
- Market *does not* equilibrate according to rational expectations



[source: Pennock 2004]

Forsythe and Lundholm 90

- Three disjoint exhaustive states X,Y,Z
- ***One*** security
- Some know *not* X
- Some know *not* Y
- As long as traders are sufficiently knowledgeable & experienced, market equilibrates according to rational expectations



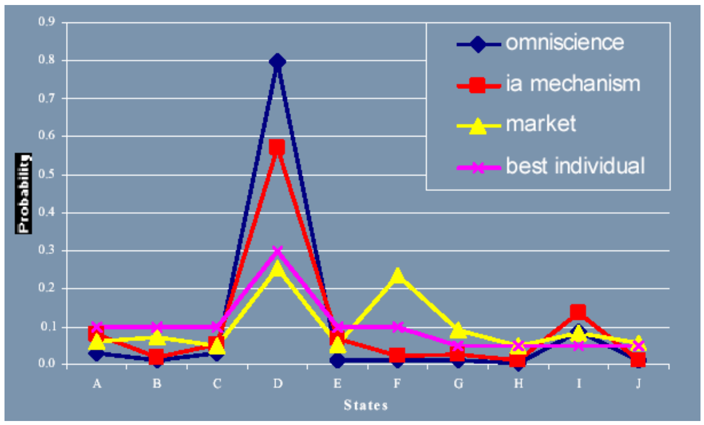
[source: Pennock 2004]

Small groups

- In small, illiquid markets, information aggregation can fail
- Chen, Fine, & Huberman [EC-2001] propose a two stage process
  1. Trade in a market to assess participants' risk attitude and predictive ability
  2. Query participants' probabilities using the log score; compute a weighted average of probabilities, with weights derived from step 1

Small groups

[Source: Fine DARPA Workshop, 2002]

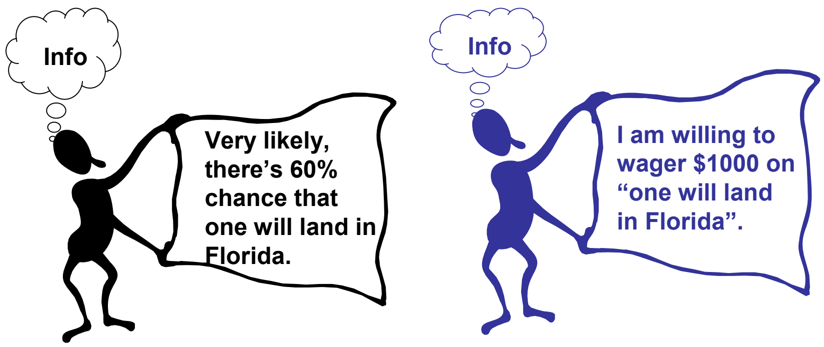


6A. Mechanism Design for Prediction Markets

- Design criteria
- Mechanisms for Prediction Markets
  - Combinatorial betting
    - Betting on permutations
    - Betting on Boolean expressions
  - Automated market makers
    - Market scoring rules
    - Dynamic pari-mutuel market
    - Utility-based market maker

Betting and Prediction

- Q: Will category 3 (or higher) hurricane make landfall in Florida in 2007?



What we care is the information!

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Mechanism Design for Prediction

- An uncertain event to be predicted
  - Q: Will category 3 (or higher) hurricane make landfall in Florida in 2007?
- Dispersed information/evidence
  - Residents of Florida, meteorologists, ocean scientists…
- Design goal: Generate a prediction that is based on information from all sources

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Design Criteria

- Standard Properties
  - Allocation efficiency
  - Budget balance
  - Revenue
  - Individual rationality
  - Computational complexity
- Prediction Market Properties
  - **Information efficiency**
  - Expressiveness
  - Liquidity
  - Bounded budget (loss)
  - Individual rationality
  - Computational complexity

Auctioneers for Combinatorial Betting

- Large outcome space
- Specify bidding languages
- Centralized auctioneer to improve liquidity and information aggregation
  - The auctioneer receives orders
  - The auctioneer risklessly matches orders (accept/reject)
  - Multilateral order matching

The Auctioneer Problem

- Auctioneer's Goal: Accept/Reject orders with non-negative profits
  - May optimize some objective, e.g. worst-case profit, trading volume
- Called the Matching Problem
- Formulated as a LP/IP problem
  - Divisible order – LP
  - Indivisible order – IP

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Tradeoff for Auctioneers

- We'd love to allow traders bet on any one of the possible outcomes

  (Expressiveness Yes)
- But
  - not natural and less interesting
  - thin market (Liquidity No)
  - High computational cost (Comp. Complexity No)

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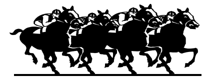
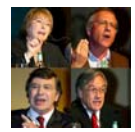
Predicting Permutations

- An event whose outcome is an ordering of a set of statistics
  - ❖Horse race finishing time 
  - ❖Political election vote share 
  - ❖Stock price changes
  - ❖Any ordinal predictions

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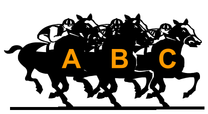
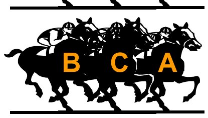
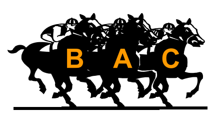
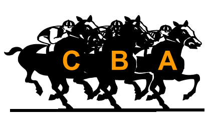
Market Combinatorics: Permutations

- 

0.1
- 

0.25
- 

0.2
- 

0.16
- 

0.15
- 

0.14

3 candidates – 6 outcomes (states)

4 candidates – 24 outcomes

N candidates – N! outcomes

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Betting on Permutations

[Chen, Fortnow, Nikolova, Pennock, EC'07]

- Bidding languages: Traders bet on properties of ordering, not explicitly on orderings
  - A will win
  - A, C, or D will finish the second
  - A will finish ahead of C
- Compromise some expressiveness, but more natural and interesting to traders and hopeful have better liquidity and comp. complexity.
- Supported to a limited extent at racetrack today, but each in different betting pools
  - Win, place, show
- Centralized auctioneer

Subset Betting

- Contracts can be created on the fly: specify a candidate and a subset of positions, or a subset of candidates and a position

$1 if	A finishes at {2, 3, or 5}	$0 Otherwise
$1 if	{A, B, or C} finishes at 2	$0 Otherwise

- Participants submit buy orders, specifying which contract to buy, the price of buying, and the desired quantity.
  - Buy 10 shares "A will finish at position {2, 3, or 5}" at price $0.80 per share.

Bilateral Matching for Subset Betting

- Only match opposite bets
  - Buy 1 share "A finishes at position 1 or 2" at price $0.6

    is matched with

    Buy 1 share "A will appear at position 3 or 4" at price $0.5
- But,

  very illiquid

Multilateral Matching

- 3 candidates (A, B, and C), 4 orders
  - O1: Buy 1 share "A finishes at 1" at $0.9
  - O2: Buy 1 share "B finishes at {1, 2}" at $0.7
  - O3: Buy 1 share "C finishes at {1, 3}" at $0.8
  - O4: Buy 1 share "{A, B} finishes at 3" at $0.7

Auctioneer's Profit

	A B C	A C B	B C A	B A C	C A B	C B A
O1	0.9	0.9	-0.1	0.9	0.9	-0.1
O2	-0.3	-0.3	0.7	0.7	-0.3	-0.3
O3	-0.2	0.8	0.8	-0.2	-0.2	-0.2
O4	-0.3	-0.3	-0.3	-0.3	0.7	0.7
O1+O2+O4	0.3	0.3	0.3	1.3	1.3	0.3
O1+O2+O3 +O4	0.1	1.1	1.1	1.1	1.1	0.1

The Matching Problem

- Solve a linear programming problem for the auctioneer.
  - Maximize worst-case profit
  - A constraint for each state

$$
\begin{aligned}
\max_{x_i, c} \quad & c \\
s.t. \quad & \textstyle\sum_i \big(b_i - I_i(s)\big) q_i x_i \geq c, \quad && \forall s \in \mathcal{S} \\
& 0 \leq x_i \leq 1, && \forall i \in \mathcal{O}.
\end{aligned}
$$

- However, brute-force method takes exponential time to solve it.

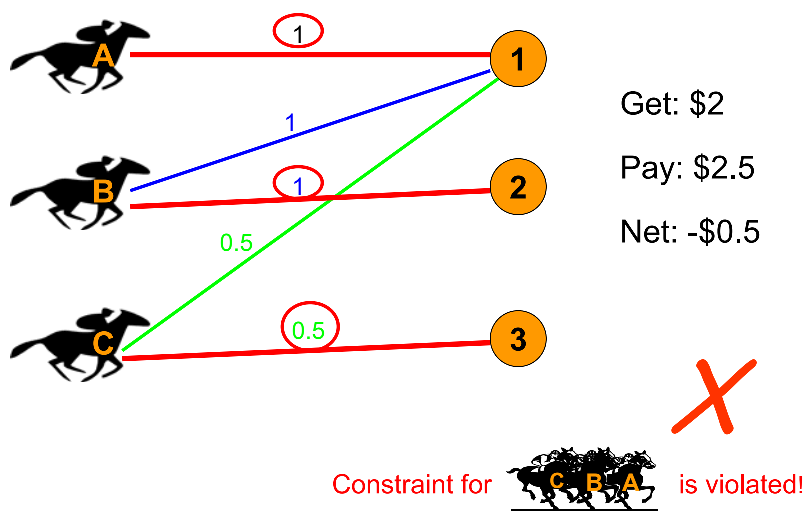
Matching is Easy for Subset Betting !

- Theorem: The auctioneer's matching problem for subset betting can be solved in polynomial time
- Ellipsoid method + maximum matching separation oracle
- Separation problem oracle: takes a set of order quantities as input, returns if they are feasible or otherwise returns a violated constraint.

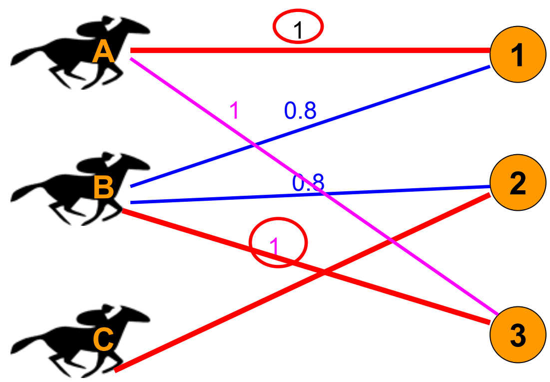
Separation Oracle

- Take advantage of the structure of the betting language
- Maximum weighted bipartite matching problem
  - A perfect matching where the sum of the values of the edges in the matching have a maximal value
  - Polynomial time algorithms are known

Accept O1: Buy 1 share "A finishes at 1" at \$0.9 in full
Accept O2: Buy 1 share "B finishes at {1, 2}" at \$0.7 in full
Accept O3: Buy 1 share "C finishes at {1, 3}" at \$0.8 in half



Accept O1: Buy 1 share “A finishes at 1” at \$0.9 in full
Accept 4/5 O2: Buy 1 share “B finishes at {1, 2}” at \$0.7
Accept O4: Buy 1 share “{A, B} finishes at 3” at \$0.7 in full



Get: \$2.16

Pay: \$2

Net: \$0.16

√

Pair Betting

- Contracts can be created for all ordered pairs, in the form of “A beats B”

\$1 if A > B	\$0 Otherwise

- Participants submit buy orders, specifying which contract to buy, the price of buying, and the desired quantity.
  - Buy 30 shares of A>B at price not exceeding \$0.80.

Pair Betting Matching

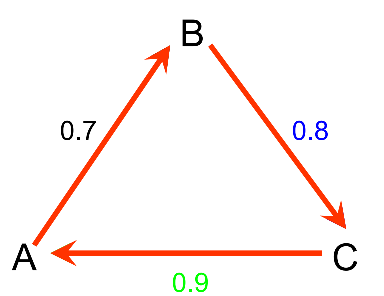
- Bilateral matching is very illiquid
- The matching problem (same as subset betting)
  - Solve a LP/IP problem for the auctioneer.
    - Maximize worst-case profit
    - A no-risk constraint for each state

$$
\begin{aligned}
\max_{x_i, c} \quad & c \\
s.t. \quad & \textstyle\sum_i \big(b_i - I_i(s)\big) q_i x_i \geq c, \quad && \forall s \in \mathcal{S} \\
& 0 \leq x_i \leq 1, && \forall i \in \mathcal{O}.
\end{aligned}
$$

EC'07 June 2007 T1-113

An Example: Pair Betting Match

- Example: 3 unit orders
  - O1: Buy 1 share "A>B" at price \$0.7
  - O2: Buy 1 share "B>C" at price \$0.8
  - O3: Buy 1 share "C>A" at price \$0.9



Get: \$2.4

Pay: \$2

Net: \$0.4

EC'07 June 2007 T1-114

Pair Betting Theorems

- Cycle with sum of prices > k-1 ==> Match
- Find best match cycle: Polynomial time
- Match =/=> Cycle with sum of prices > k-1
- The Matching Problem for Pair Betting is NP-hard (reduce from min feedback arc set problem)
- Greedy algorithm can give bad approximation

Predicting Compound Event

- Boolean combination of binary events
  - (Clinton wins Ohio) & (Clinton wins Florida)
  - (House struck by lightening) & (YHOO price goes up)
  - Any joint outcome of binary events

Market Combinatorics: Boolean

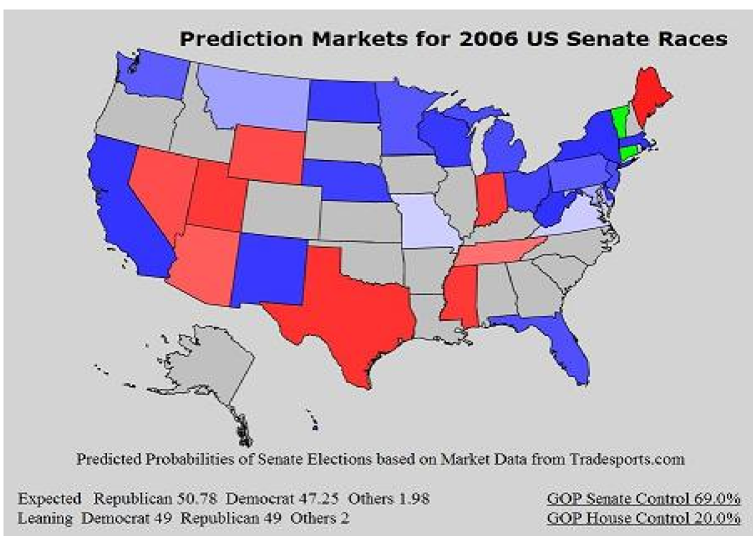
- A1&A2&A3 0.1
- A1&$\overline{A2}$&A3 0.1
- $\overline{A1}$&A2&A3 0.15
- $\overline{A1}$&$\overline{A2}$&A3 0.2
- A1&A2&$\overline{A3}$ 0.05
- A1&$\overline{A2}$&$\overline{A3}$ 0.1
- $\overline{A1}$&A2&$\overline{A3}$ 0.12
- $\overline{A1}$&$\overline{A2}$&$\overline{A3}$ 0.18

3 base events – 8 compound events

N base events – 2^N compound events

Betting on complete conjunctions is both unnatural and infeasible

Market Combinatorics: Boolean



[Fortnow's Election Map]

Betting Boolean-Style

[Fortnow, Kilian, Pennock, Wellman, 2004]

- Contracts: write your own logical expression

\$1 if	Boolean_exp | Boolean_exp	\$0 Otherwise

For example,

\$1 if	A1&A3&A5	\$0 Otherwise

\$1 if	(A1&$\overline{A5}$)||A3 | (A2&$\overline{A7}$)	\$0 Otherwise

- Participants submit buy/sell orders, specifying which contract to buy/sell, the price and quantity.
  - Sell 2 shares of “A1&A3” at price \$0.5 per share

The Matching Problem

- Solve a LP/IP problem for the auctioneer
  - Maximize trades
  - A no-risk constraint for each state
- Example match
  - O1: Sell 1 share “A1” at price \$0.6
  - O2: Buy 1 share “A1&A2” at price \$0.3
  - O3: Buy 1 share “A1&$\overline{A2}$” at price \$0.5

= Buy 1 share A1 at \$0.8

	A1&A2	A1&$\overline{A2}$	$\overline{A1}$&A2	$\overline{A1}$&$\overline{A2}$
O1	0.4	0.4	-0.6	-0.6
O2	-0.7	0.3	0.3	0.3
O3	0.5	-0.5	0.5	0.5
O1+O2+O3	0.2	0.2	0.2	0.2

Auctioneer’s Profit

Betting Boolean-Style Complexity Results

- Divisible orders: will accept any $q^* \leq q$
- Indivisible: will accept all or nothing

LP

reduction from X3C

# events	divisible	indivisible
O(log n)	polynomial	NP-complete
O(n)	co-NP-complete	Σ_2^p complete

reduction from SAT

reduction from T∃∀BF

- Natural algorithms
  - divisible: linear programming
  - indivisible: integer programming; logical reduction?

Automated Market Makers

- A market maker (a.k.a. bookmaker) is a firm or person who is almost always willing to accept both buy and sell orders at some prices
- Why an institutional market maker? Liquidity!
  - Without market makers, the more expressive the betting mechanism is the less liquid the market is (few exact matches)
  - Illiquidity discourages trading: Chicken and egg
  - Subsidizes information gathering and aggregation: Circumvents no-trade theorems
- Market makers, unlike auctioneers, bear risk. Thus, we desire mechanisms that can bound the loss of market makers

Automated Market Makers

- n disjoint and exhaustive outcomes
- Market maker maintain vector Q of outstanding shares
- Market maker maintains a cost function C(Q) recording total amount spent by traders
- To buy ΔQ shares trader pays C(Q+ ΔQ) – C(Q) to the market maker; Negative "payment" = receive money
- Instantaneous price functions are $p_i(Q) = \frac{\partial C(Q)}{\partial q_i}$
- At the beginning of the market, the market maker sets the initial Q^0, hence subsidizes the market with $C(Q^0)$.
- At the end of the market, $C(Q^f)$ is the total money collected in the market. It is the maximum amount that the MM will pay out.

EC'07 June 2007 T1-123

Proper Scoring Rules

- Report a probability estimate: $\mathbf{r}=(r_1,r_2, \ldots,r_n)$
- Get payment $s_i(\mathbf{r})$ if outcome ω_i happens
- Proper: incentive compatible

  A risk neutral agent should chose $r_i=\Pr(\omega_i)$

  to maximize the expected profit
- Proper scoring rules
  - Logarithmic: $s_i(\mathbf{r})=a + b \log(r_i)$ (b>0)
  - Quadratic: $s_i(\mathbf{r})=a + 2 b r_i - b\sum_j r_j^2$ (b>0)

EC'07 June 2007 T1-124

Market Scoring Rules (MSR)

[Hanson 2002, 2003, 2006]

- Use a proper scoring rule
- A trader can change the current probability estimate to a new estimate
- The trader pays the scoring rule payment according to the old probability estimate
- The trader receives the scoring rule payment according to the new probability estimate

An Example MSR Transaction

	$A1A2$	$A1\overline{A2}$	$\overline{A1}A2$	$\overline{A1}\,\overline{A2}$
current probabilities:	0.25	0.25	0.25	0.25
Trader can change to:	0.20	0.20	0.30	0.30
Trader gets $$ in state:	100+5log(.2)	100+5log(.2)	100+5log(.3)	100+5log(.3)
Trader pays $$ in state:	100+5log(.25)	100+5log(.25)	100+5log(.25)	100+5log(.25)
total transaction:	5log(.2) - 5log(.25)	5log(.2) - 5 log(.25)	5 log(.3) - 5log(.25)	5 log(.3) - 5 log(.25)

$$s_i(\mathbf{r}) = 100 + 5\log(r_i)$$

An Example MSR Market

$$s_i(\mathbf{r}^t) = 100 + 5\log(r_i^t)$$

5log(0.9)-5log(0.5)	~~5log(0.6)~~- 5log(0.5)	~~5log(0.8)~~- ~~5log(0.6)~~	~~5log(0.4)~~- ~~5log(0.8)~~	5log(0.9)- ~~5log(0.4)~~
5log(0.1)-5log(0.5)	~~5log(0.4)~~- 5log(0.5)	~~5log(0.2)~~- ~~5log(0.4)~~	~~5log(0.6)~~- ~~5log(0.2)~~	5log(0.1)- ~~5log(0.6)~~

Requires a "patron", though only pays final trader

Bounded Budget

- From a trader's point of view, every transaction goes through a market maker
- The market maker is the patron who subsidizes the market: pays the last trader
- Market maker's loss

$$l = s_{true}(\mathbf{r}^f) - s_{true}(\mathbf{r}^0)$$

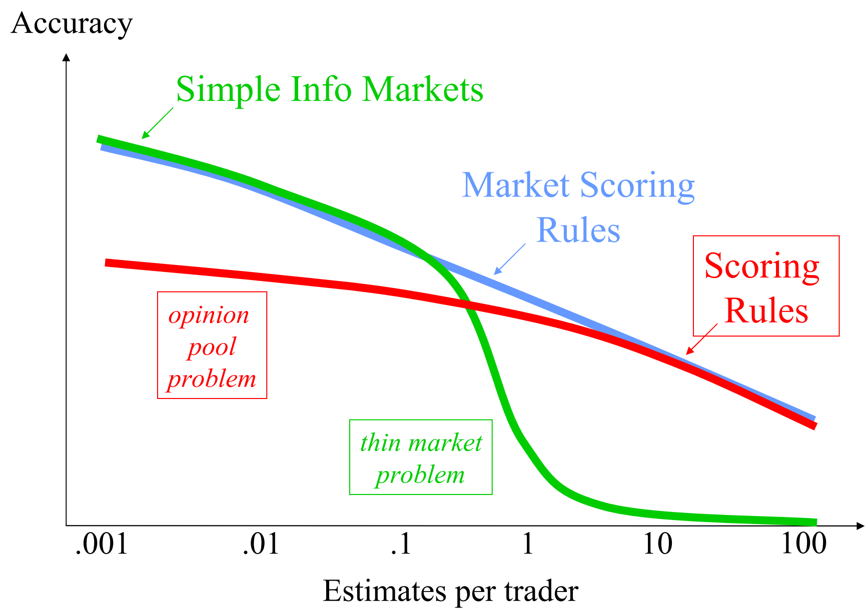
$\mathbf{r}^0$ uniform

$$l^{\log} \le b\log(1) - b\log(r^0_{true}) = b\log n$$

$$l^{quad} \le b - (2br^0_{true} - b\sum_j (r_j^0)^2) = b\frac{n-1}{n}$$

- Higher b $\Longrightarrow$ more risk, more "liquidity"

[Source: Hanson, 2002]



MSR Cost Fn Formulation

- Report probabilities ⟺ Change prices

$C(q_1,q_2) = \log(e^{q_1} + e^{q_2})$

$p_1 = e^{q_1}/(e^{q_1} + e^{q_2})$

$p_2 = e^{q_2}/(e^{q_1} + e^{q_2})$

p_1	p_2	q_1	q_2
$1 if	$1 if	$1 if	$1 if
0.5	0.5	10	10
0.8	0.2	11.4	10

Buy 1.4 shares $1 if , pay C(11.4, 10) – C(10, 10) = 0.93.

Trader Profit

	MSR $s_i(r) = \log(r_i)$	Above market
	log(.8) – log(.5) =**0.47**	1.4 – 0.93 =**0.47**
	log(.2) – log(.5) = **-0.93**	0 – 0.93 = **-0.93**

MSR Market Maker

Logarithmic Market Scoring Rule

- n mutually exclusive outcomes
- Shares pay $1 iff outcome occurs
- Cost Function

$$C(Q) = b \times \log\left(\sum_{i=1}^{n} e^{\frac{q_i}{b}}\right)$$

- Price Function

$$p_i(Q) = \frac{e^{\frac{q_i}{b}}}{\sum_{j=1}^{n} e^{\frac{q_j}{b}}}$$

MSR Market Maker

Quadratic Market Scoring Rule

- Cost Function

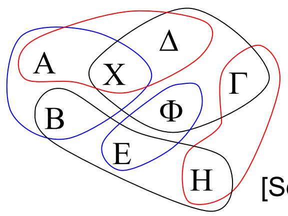
$$C(Q) = \frac{\sum_{i=1}^{n} q_i}{n} + \frac{\sum_{i=1}^{n} q_i^2}{4b} + \frac{(\sum_{i=1}^{n} q_i)^2}{4b} - \frac{b}{n}$$

- Price Function

$$p_i(Q) = \frac{1}{n} + \frac{q_i}{2b} - \frac{\sum_{j=1}^{n} q_j}{2nb}$$

Computational Issues of MSR

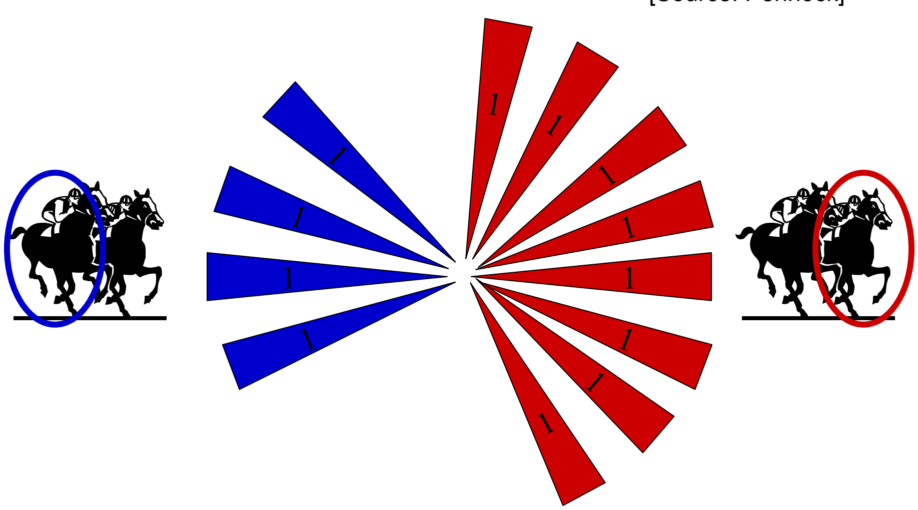
- Straightforward approach requires exponential space for prices, holdings, portfolios
- Could use multiple overlapping patrons, each with bounded loss. Limited arbitrage could be obtained by smart traders exploiting inconsistencies between patrons



[Source: Hanson, 2002]

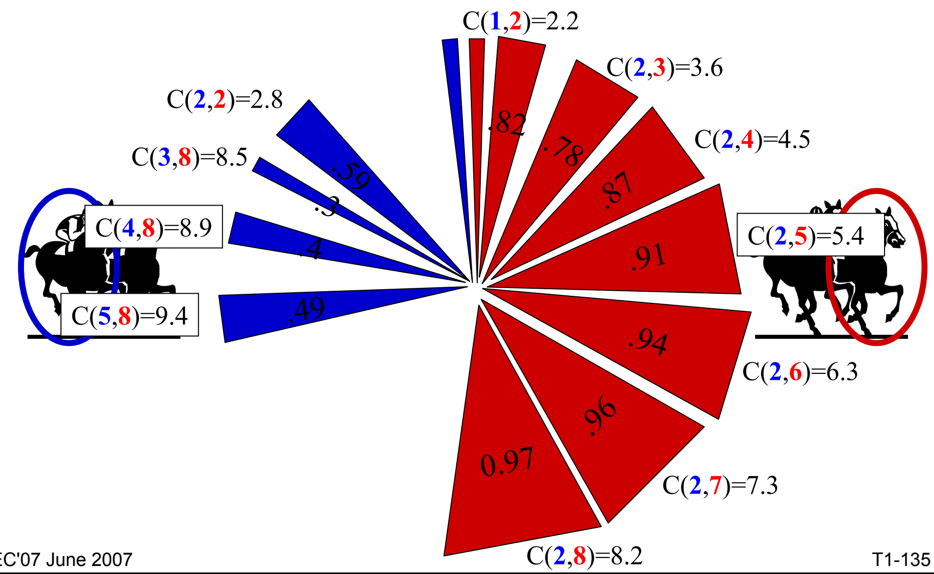
Pari-Mutuel Market

[Source: Pennock]



Dynamic Parimutuel Market (DPM)

[Pennock, EC'04] [Source: Pennock]



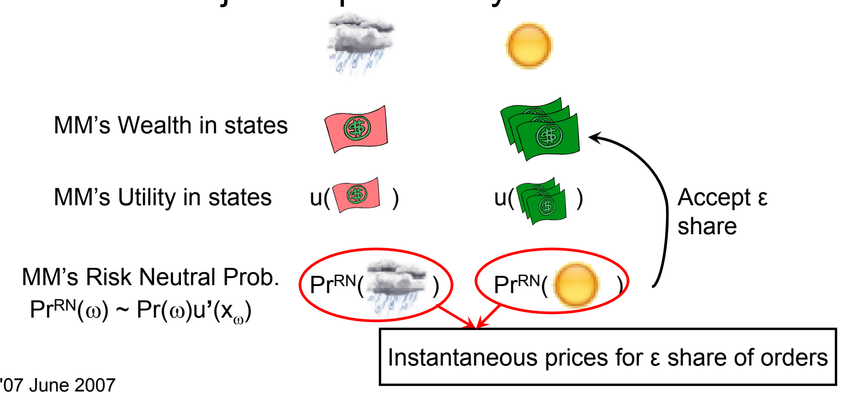
DPM: Share-Ratio Price function

- One can view DPM as a market maker
- Cost Function: $C(Q) = \sqrt{\sum_{i=1}^{n} q_i^2}$
- Price Function: $p_i(Q) = \frac{q_i}{\sqrt{\sum_{j=1}^{n} q_j^2}}$
- Properties
  - No arbitrage
  - $price_i/price_j = q_i/q_j$
  - $price_i < \$1$
  - payoff if right = $C(Q_{final})/q_o > \$1$

Utility-Based Market Maker

[Chen & Pennock, UAI 2007]

- Market maker has a utility function of money, and a subjective probability estimate



Utility-Based Market Maker

- Keep expected utility constant
- Cost function is determined by

$$\sum_i \Pr(\omega_i)(C(Q) - q_i) = k$$

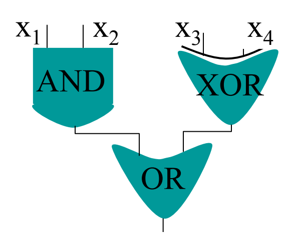
- Bounded budget if utility function satisfy some regularity conditions
- For many utility functions, it's equivalent to MSR
  - ❖E.g. Negative exponential utility market maker is equivalent to logarithmic MSR

6B. Distributed Market Computation

- A market along with its participants can be viewed as a computing device
  - Input: private information
  - Output: equilibrium price (function value)
- Questions of interest
  - What can a market compute?
  - How fast? (time complexity)

Feigenbaum et. al. EC-2003

- General formulation
  - Set up the market to compute some function $f(x_1,x_2,\ldots,x_n)$ of the information x_i available to each market participant (e.g., we want the market to compute future interest rates given other economic variables)
  - Represent f(**x**) as a *circuit* →



$f(x_1,x_2,x_3,x_4)= (x_1 \wedge x_2) \vee (x_3 \oplus x_4)$

Market Model: Security

- Each participant has some bit of information x_i
- The market aims at predicting the value of a Boolean function, $f(\mathbf{x}): \{0, 1\}^n \rightarrow \{0,1\}$.
- One security is traded in the market. It pays:

$$\begin{cases} \$1 & \text{if } f(\mathbf{x}) = 1 \\ \$0 & \text{if } f(\mathbf{x}) = 0. \end{cases}$$

EC'07 June 2007 T1-141

Market Model: Mechanism

Restricted Shapley-Shubik Market Game

- Market proceeds in rounds until equilibrium is reached.
- Each trader puts 1 share of the security for sale in each round.
- Trader *i* submit bid b_i, which is the money that trader *i* wants to spend on buying the security.
- No restriction on credit.
- Market clearing price is

$$p = (\sum_{i=1}^{n} b_i) / n$$

EC'07 June 2007 T1-142

Theorems

[Feigenbaum et. al. EC-2003]

- For *any* prior distribution on $\mathbf{x}$, if $f(\mathbf{x})$ takes the form of a *weighted threshold function* (i.e., $f(\mathbf{x}) = 1$ iff $\Sigma_i w_i x_i > 1$ for some weights w_i), then the market price *will ultimately converge* to the true value of $f(\mathbf{x})$ in *at most n rounds*
  - E.g. majority function: $f(\mathbf{x}) = 1$ if $\Sigma_i x_i > n/2$
- If $f(\mathbf{x})$ *cannot* be expressed as a weighted threshold function (i.e., $f(\mathbf{x})$ is not linearly separable), then there is some prior on $\mathbf{x}$ for which the price *does not reveal* the true value of $f(\mathbf{x})$
  - E.g. parity function: $f(\mathbf{x}) = x_1 \oplus x_2 \oplus x_3 \ldots \oplus x_n$

7. Legal Issues and Other

- IEM has "no action" letter from Commodity Futures Trading Commission (CFTC)
- Setting up markets for hedging risks is legal, but setting up markets for information aggregation may be gambling.
  - Trading options ⇔ betting on Oscars ⇔ Sports betting ⇔ Horse racetrack?

Legal Issues

- Gambling in US
  - Legal in some form in 48 states (lotteries, bingo, Indian reservations, riverboat)
  - Illegal in many forms in all states
    - Sports betting legal only in Las Vegas
    - Federal Wire Act: “bans the use of telephones to accept wagers on sporting events.”
  - “Law prohibits U.S. financial institutions from processing payments to online gambling sites.

[Source: Hanson, 2002]

RIP Policy Analysis Market

Real combinatorial markets in Middle East issues

- DARPA, Net Exchange, Caltech, GMU
- Two year field test, starts 2003
- Open to public, real-money markets
- ~20 nations, 8 quarters, ~5 variables each:
  - Economic, political, military, US actions
- Want many combos (> 2^{500} states)
- Legal: “DARPA & its agents not under CFTC’s regulatory umbrella” (paraphrased)
- **http://www.policyanalysismarket.org**

Some Open Questions

- 5 open questions in prediction markets
  [Wolfers & Zitzewitz 2006]
  - How to attract uninformed trader?
  - How to tradeoff interest and contractability?
  - How to limit manipulation?
  - Are markets well calibrated on small probability?
  - How to separate correlation from causation?

Some Open Questions

- Computational aspect
  - Are there natural, useful, expressive bidding languages that admit polynomial time matching for combinatorial prediction markets?
  - Are there good heuristic matching algorithms?
  - Does there exist polynomial time market makers?
  - For every bidding language with polynomial time matching, does there exist a polynomial time market maker?
  - The automated market maker algorithms are online algorithms: Are there other online market maker algorithms that trade more for same loss bound?