Lecture 11

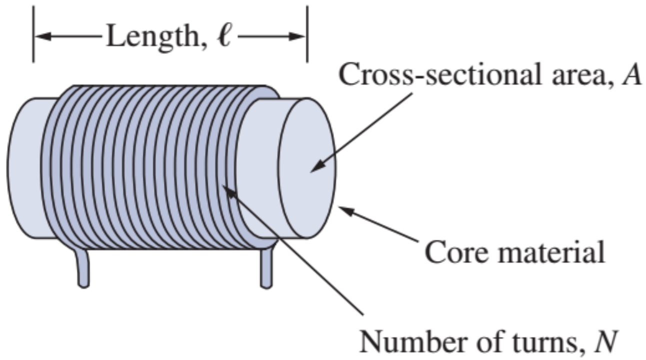
- Magnetically Coupled Circuits

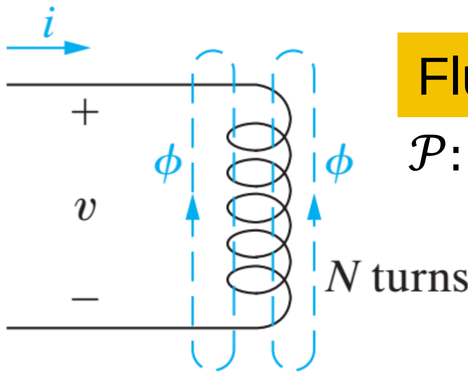
Outline

- Mutual inductance
- Transformers

Review: Self Inductance

- Self inductance: reaction of the inductor to the change in current through itself.





Flux $\phi = \mathcal{P}Ni$

$\mathcal{P}$: permeance

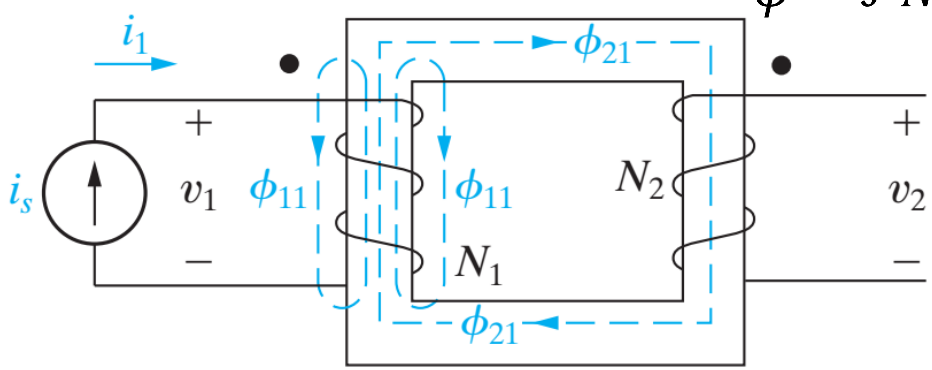
$$v = N\frac{d\phi}{dt} = L\frac{di}{dt}$$

$$where\ L = N\frac{d\phi}{di}$$

Mutual Inductance

- Mutual inductance: reaction of the inductor to change in current through another inductor.

$$\phi = \mathcal{P}Ni$$



$$\phi_1 = \phi_{11} + \phi_{21}$$

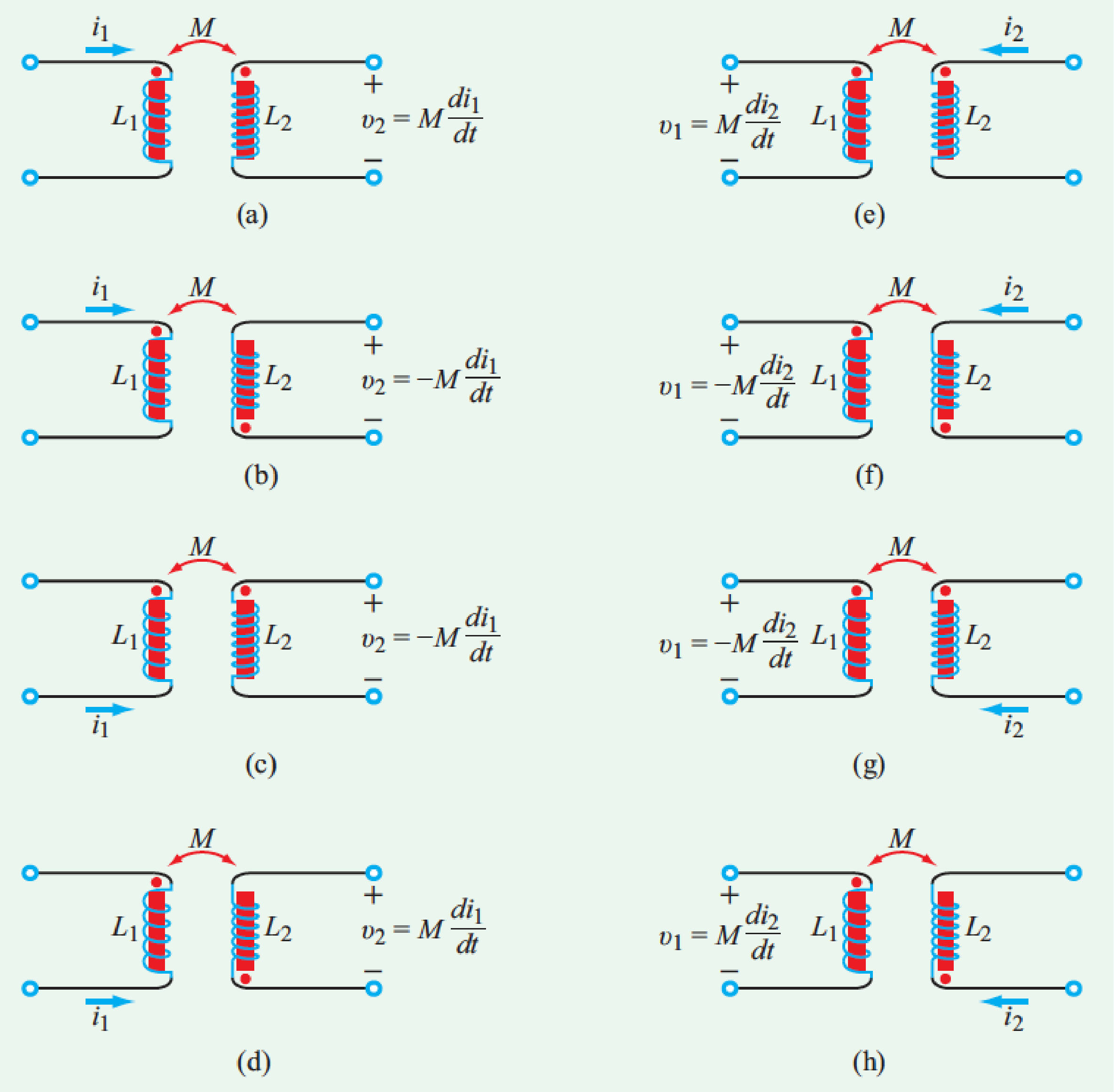
$$\phi_{11} = \mathcal{P}_{11}N_1 i_1, \quad \phi_{21} = \mathcal{P}_{21}N_1 i_1,$$

$$\phi_1 = \mathcal{P}_1 N_1 i_1$$

$$v_1 = \frac{d(N_1\phi_1)}{dt} = N_1\frac{d}{dt}(\phi_{11} + \phi_{21})$$
$$= N_1^2(\mathcal{P}_{11} + \mathcal{P}_{21})\frac{di_1}{dt} = N_1^2\mathcal{P}_1\frac{di_1}{dt}$$

$$v_2 = \frac{d(N_2\phi_{21})}{dt} = N_2\frac{d}{dt}(\mathcal{P}_{21}N_1 i_1)$$
$$= N_2 N_1 \mathcal{P}_{21}\frac{di_1}{dt}$$
$$\boxed{= M_{21}\frac{di_1}{dt}}$$

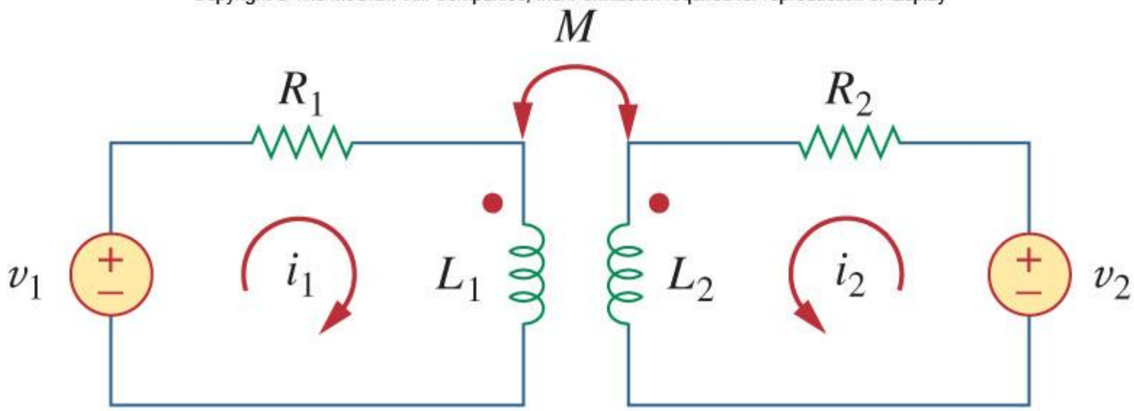
Dot Convention: Defines Directions of Windings



[Source: Berkeley]

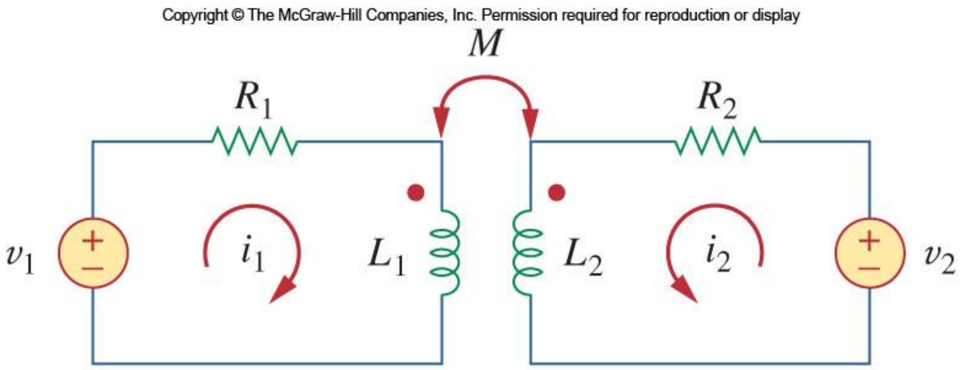
Magnetically Coupled Circuits

- L_1, L_2: self-inductances
- M: mutual inductance
- Dots: indicating polarity of mutually induced voltages.



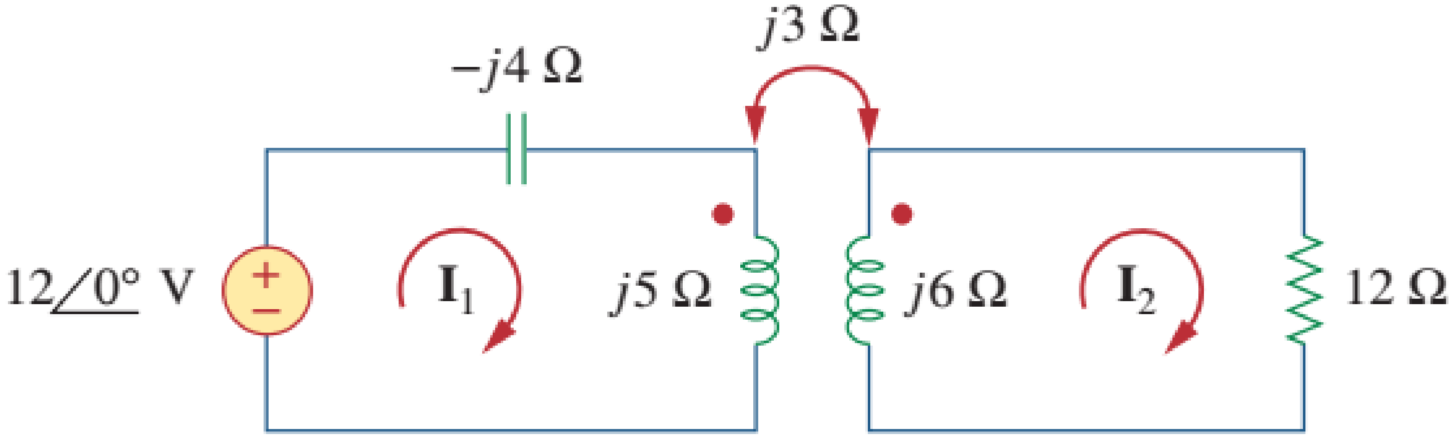
Analysis

- Relate v_1, v_2 with $i_1$and i_2.
  - In time domain
  - In phasor domain



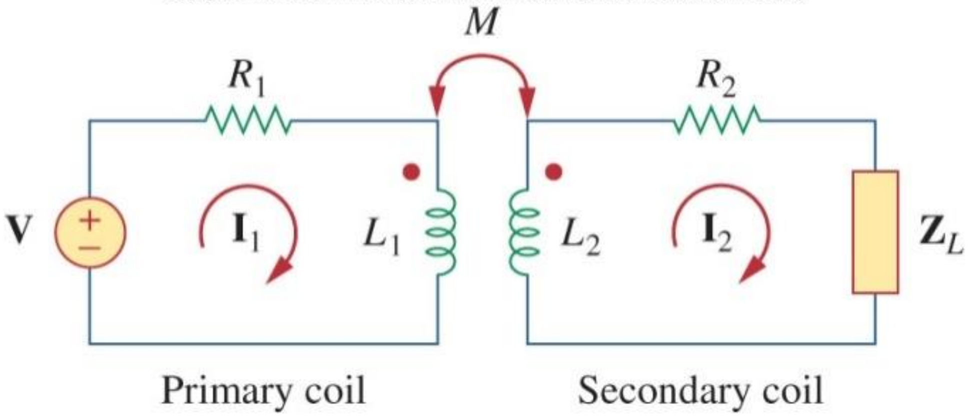
Exercise

- Calculate the phasor currents $\mathbf{I_1}$, and $\mathbf{I_2}$



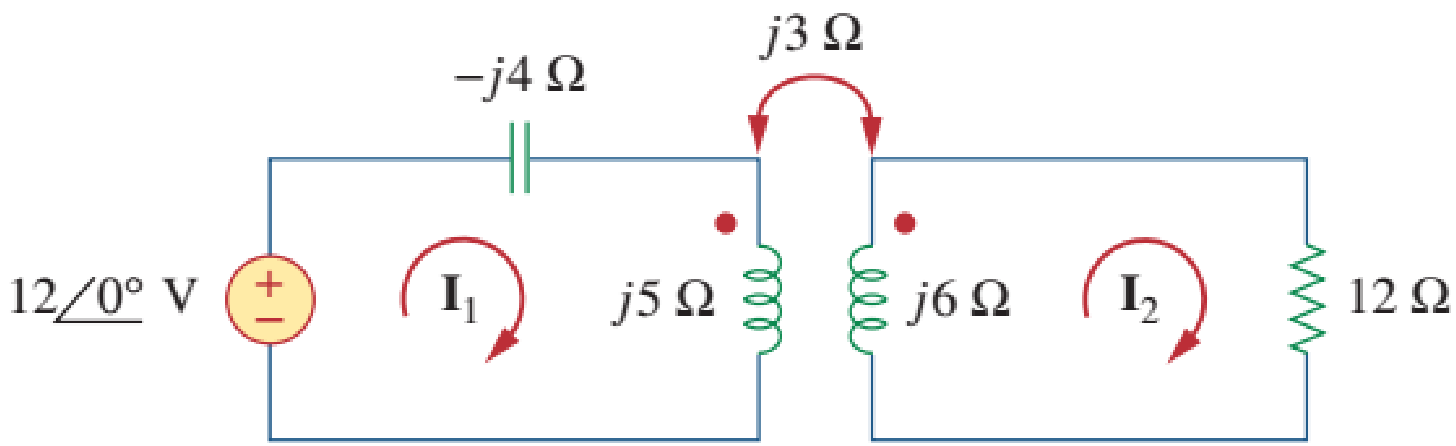
Linear Transformers

- A transformer is a magnetic device that takes advantage of mutual inductance.
  - Called linear if the coils are wound on a magnetically linear material, i.e. permeability μ is constant.



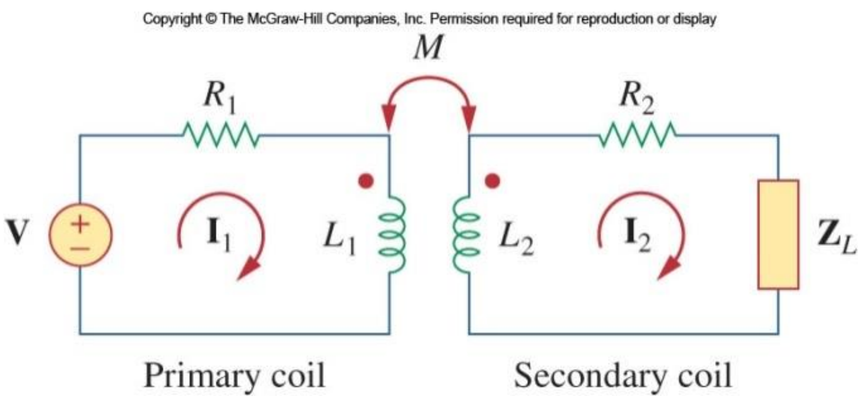
Revisit exercise

- Calculate the phasor voltages $\mathbf{V}_1$, and $\mathbf{V}_2$ across the inductors



Transformer Impedance

- An important parameter to know for a transformer is how the input impedance Z_{in} is seen from the source.
  - Z_{in} is important because it governs the behavior of the primary circuit.

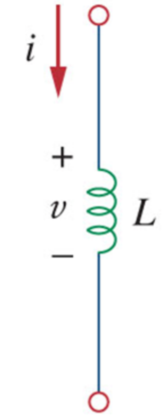


$$\mathbf{Z}_{\text{in}} = \frac{\mathbf{V}}{\mathbf{I}_1} = R_1 + j\omega L_1 + \frac{\omega^2 M^2}{R_2 + j\omega L_2 + \mathbf{Z}_L}$$

Reflected impedance from secondary to primary

Energy in a Coupled Circuit

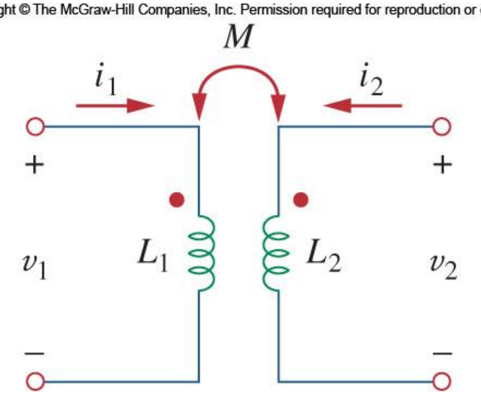
- The energy stored in an inductor is



- For coupled inductors, the total energy stored is

$$w = \frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 \pm M i_1 i_2$$

  - The positive sign is selected when the currents both enter or leave the dotted terminals.



Coupling Coefficient *k*

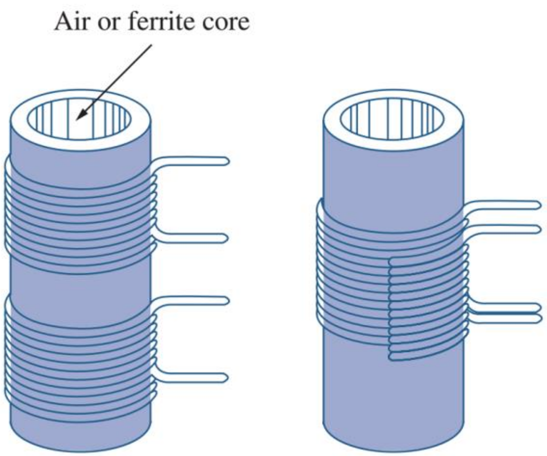
- The system cannot have negative energy

$$\frac{1}{2}L_1 i_1^{\,2} + \frac{1}{2}L_2 i_2^{\,2} - M i_1 i_2 \geq 0 \Rightarrow \boxed{M \leq \sqrt{L_1 L_2}}$$

- Define a parameter describes how closely M approaches upper limit.

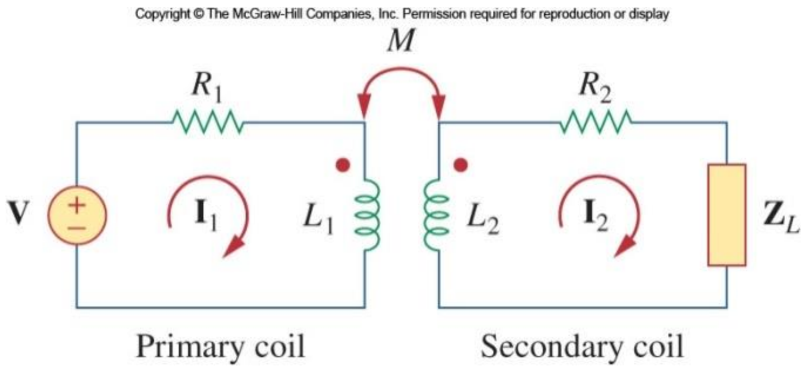
$$\boxed{k = \frac{M}{\sqrt{L_1 L_2}}}$$

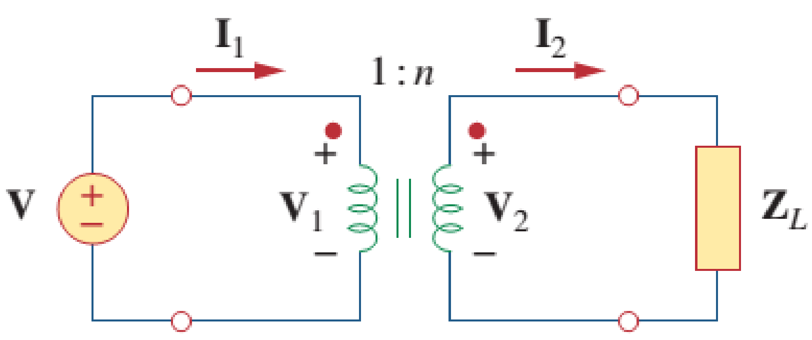
  - Coupling coefficient, $0 \leq k \leq 1$.
  - determined by the physical configuration of the coils.



Ideal Transformers

- The ideal transformer has:
  - Coils with very large reactance
    $(L_1, L_2, M \rightarrow \infty)$
  - Coupling coefficient k=1.
  - Primary and secondary coils are lossless, $R_1 = R_2 = 0$.

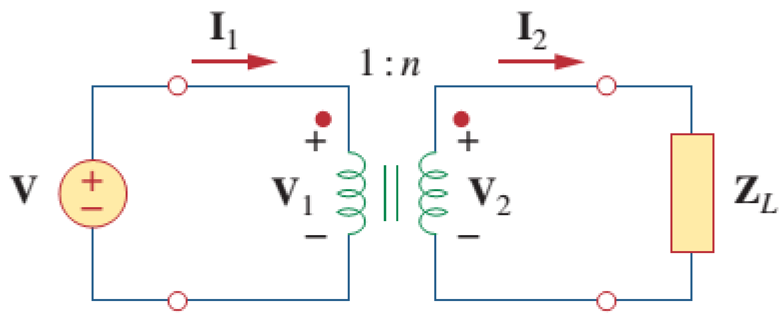




$$\frac{\mathbf{V_2}}{\mathbf{V_1}} = \frac{N_2}{N_1} = n$$

Ideal Transformers

$$\frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{N_2}{N_1} = n$$

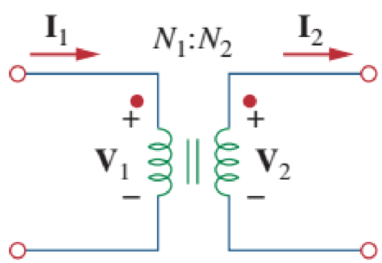


- The current is related as:

- Reflected impedance

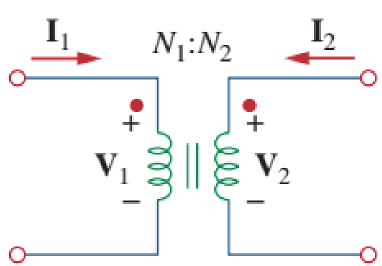
$$\mathbf{Z}_{\text{in}} = \frac{\mathbf{V}_1}{\mathbf{I}_1} =$$

1. If $\mathbf{V}_1$ and $\mathbf{V}_2$ are *both* positive or both negative at the dotted terminals, use $+n$ in Eq. (13.52). Otherwise, use $-n$.
2. If $\mathbf{I}_1$ and $\mathbf{I}_2$ *both* enter into or both leave the dotted terminals, use $-n$ in Eq. (13.55). Otherwise, use $+n$.



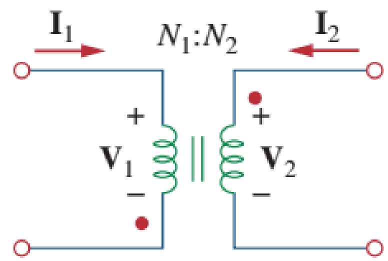
$$\frac{\mathbf{V}_2}{\mathbf{V}_1}=\frac{N_2}{N_1} \qquad \frac{\mathbf{I}_2}{\mathbf{I}_1}=\frac{N_1}{N_2}$$

(a)



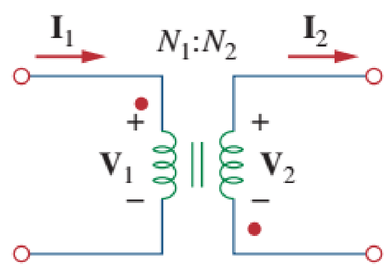
$$\frac{\mathbf{V}_2}{\mathbf{V}_1}=\frac{N_2}{N_1} \qquad \frac{\mathbf{I}_2}{\mathbf{I}_1}=-\frac{N_1}{N_2}$$

(b)



$$\frac{\mathbf{V}_2}{\mathbf{V}_1}=-\frac{N_2}{N_1} \qquad \frac{\mathbf{I}_2}{\mathbf{I}_1}=\frac{N_1}{N_2}$$

(c)

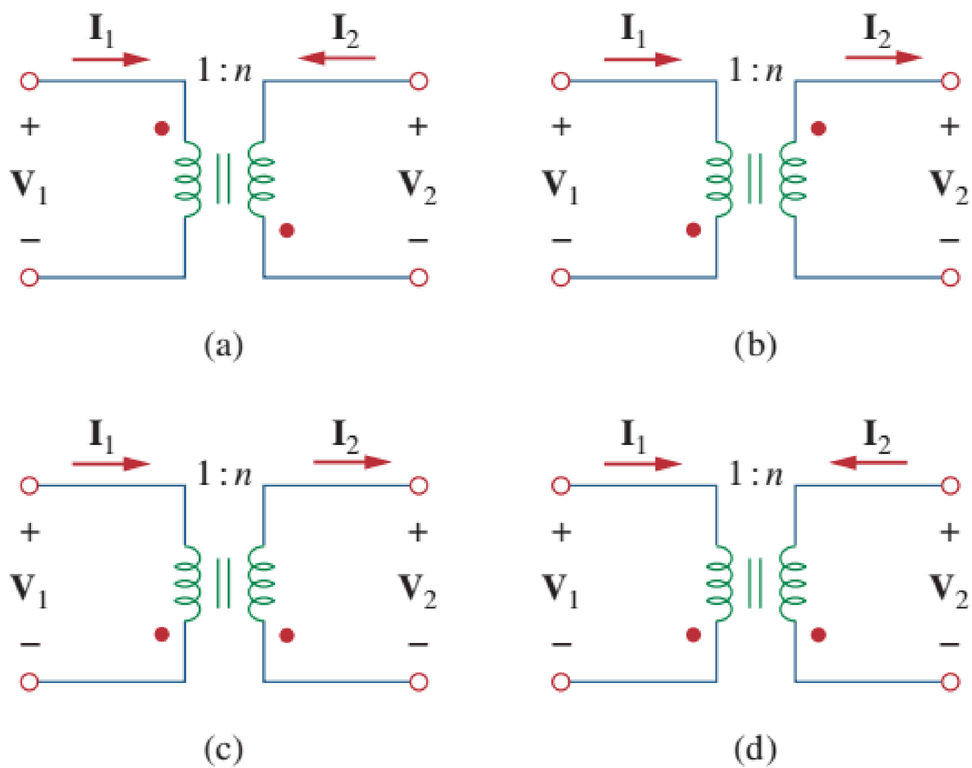


$$\frac{\mathbf{V}_2}{\mathbf{V}_1}=-\frac{N_2}{N_1} \qquad \frac{\mathbf{I}_2}{\mathbf{I}_1}=-\frac{N_1}{N_2}$$

(d)

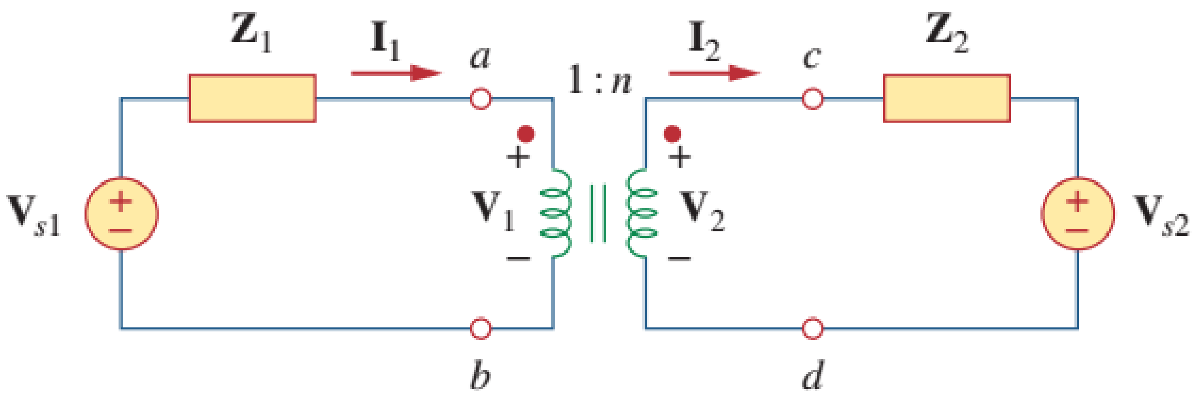
Practice

13.36 As done in Fig. 13.32, obtain the relationships between terminal voltages and currents for each of the ideal transformers in Fig. 13.105.



Ideal Transformers

$$\frac{\mathbf{V_2}}{\mathbf{V_1}} = \frac{N_2}{N_1} = n$$

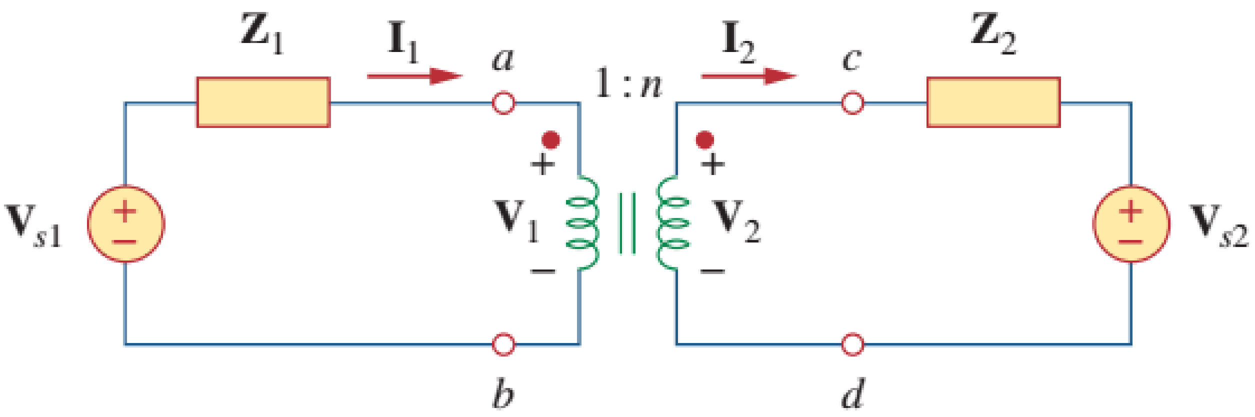


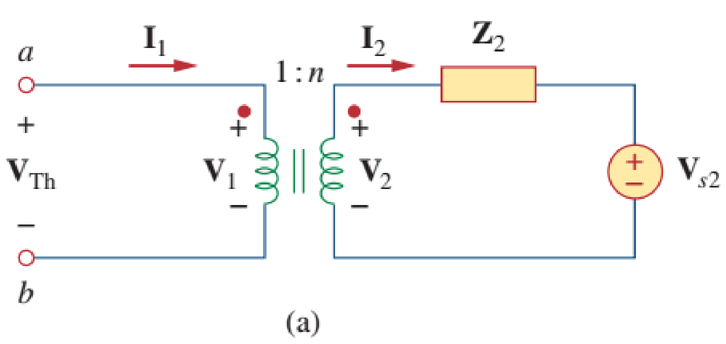
- The current is related as:

Ideal Transformers

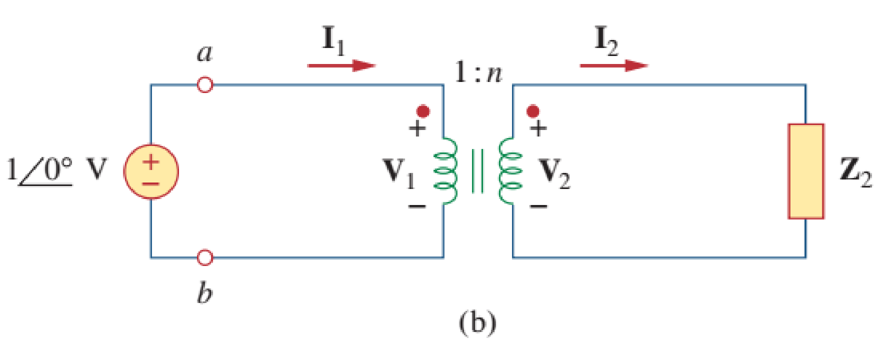
$$\frac{\mathbf{V_2}}{\mathbf{V_1}} = \frac{N_2}{N_1} = n$$

- Reflected impedance and source

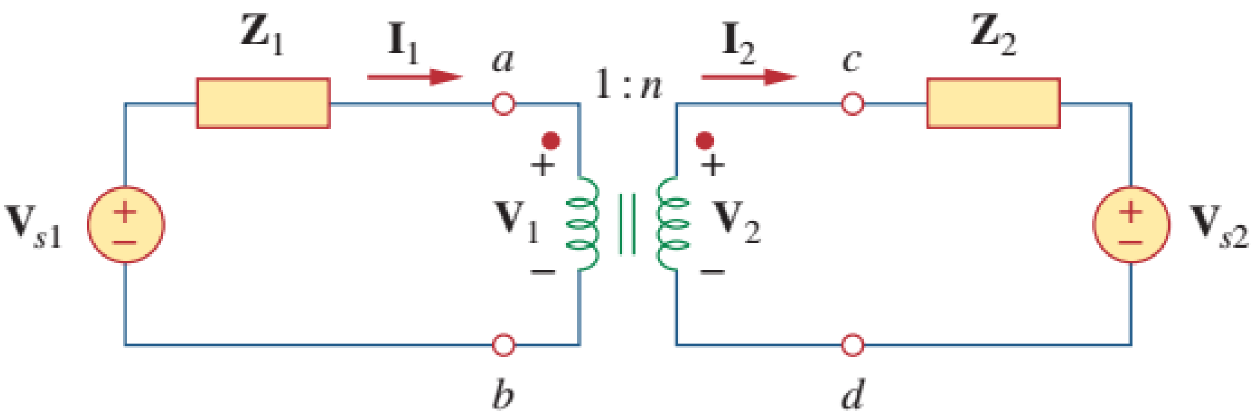


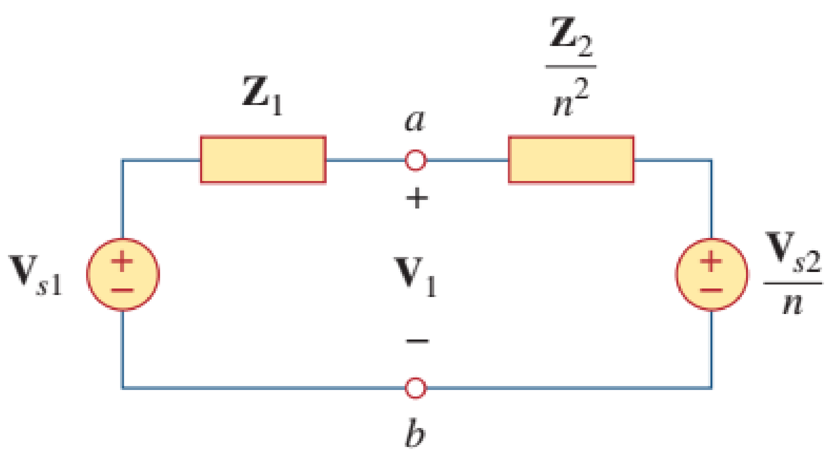


(a)



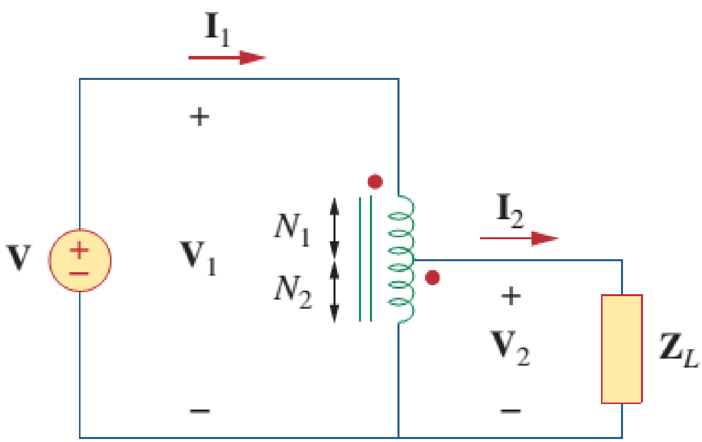
(b)

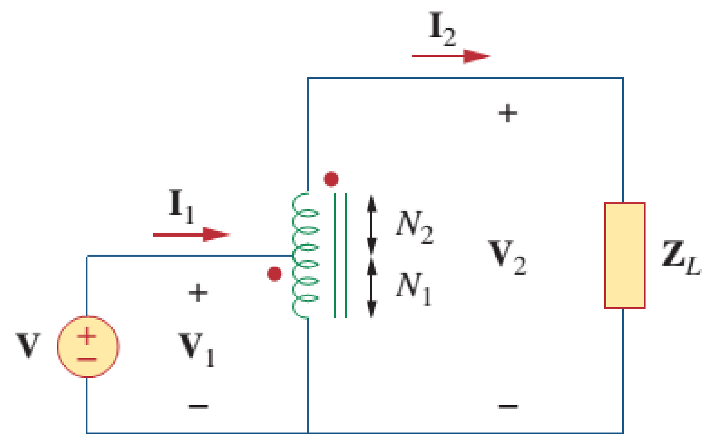

Z1
I1
a
1 : n
I2
c
Z2
+
+
Vs1
V1
V2
Vs2
−
−
b
d


Z2
n2
Z1
a
+
Vs1
V1
Vs2
n
−
b

Ideal Autotransformer

- Autotransformer uses one winding for primary & secondary
  - It does not offer isolation!





$$\frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{N_1 + N_2}{N_2} = 1 + \frac{N_1}{N_2}$$

$$\frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{N_1}{N_1 + N_2}$$

$$\frac{\mathbf{I}_1}{\mathbf{I}_2} = \frac{N_2}{N_1 + N_2}$$

$$\frac{\mathbf{I}_1}{\mathbf{I}_2} = \frac{N_1 + N_2}{N_1} = 1 + \frac{N_2}{N_1}$$

$$\mathbf{S}_1 = \mathbf{V}_1\mathbf{I}_1^* = \mathbf{S}_2 = \mathbf{V}_2\mathbf{I}_2^*$$

Practice

- Find reflected impedance and $\mathbf{I_1}$