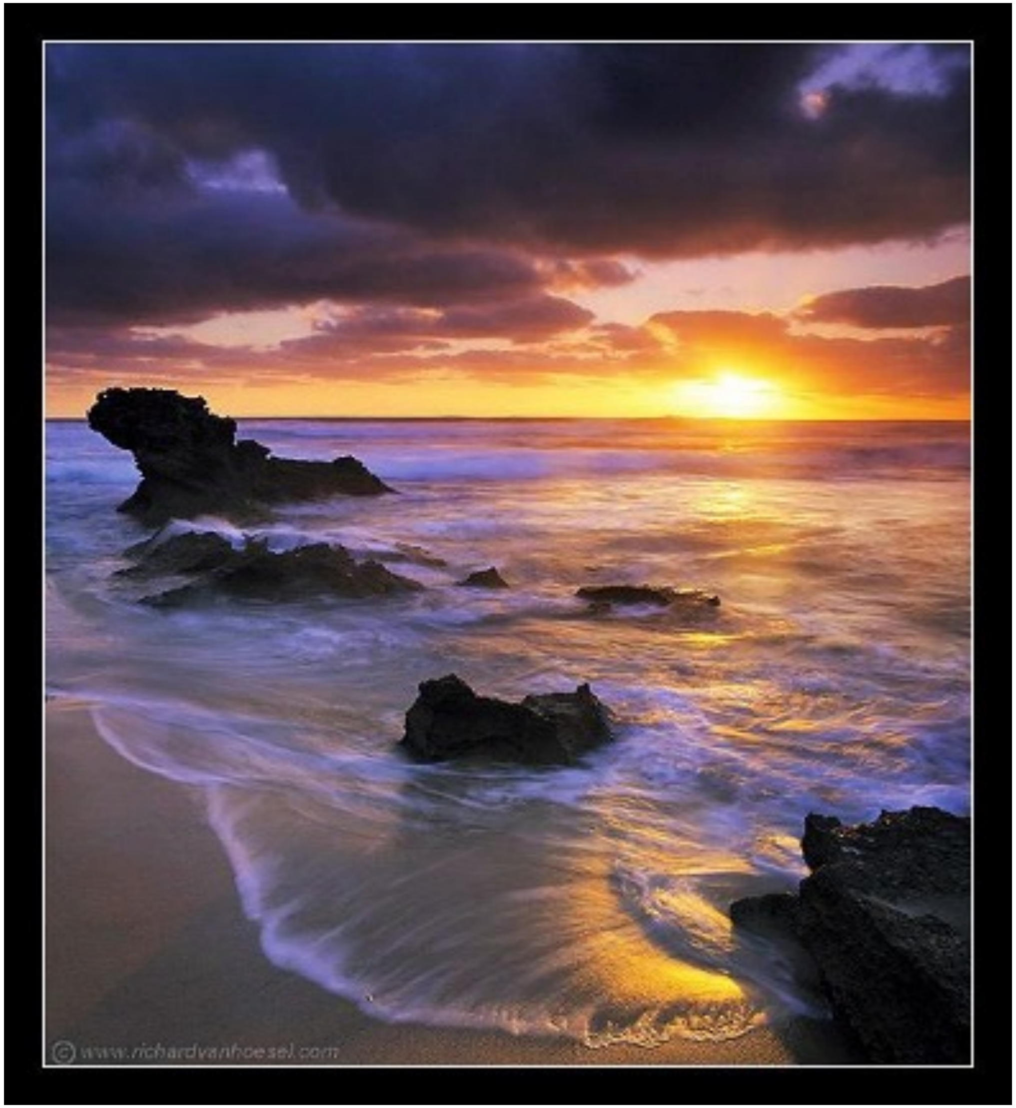
Light and shading



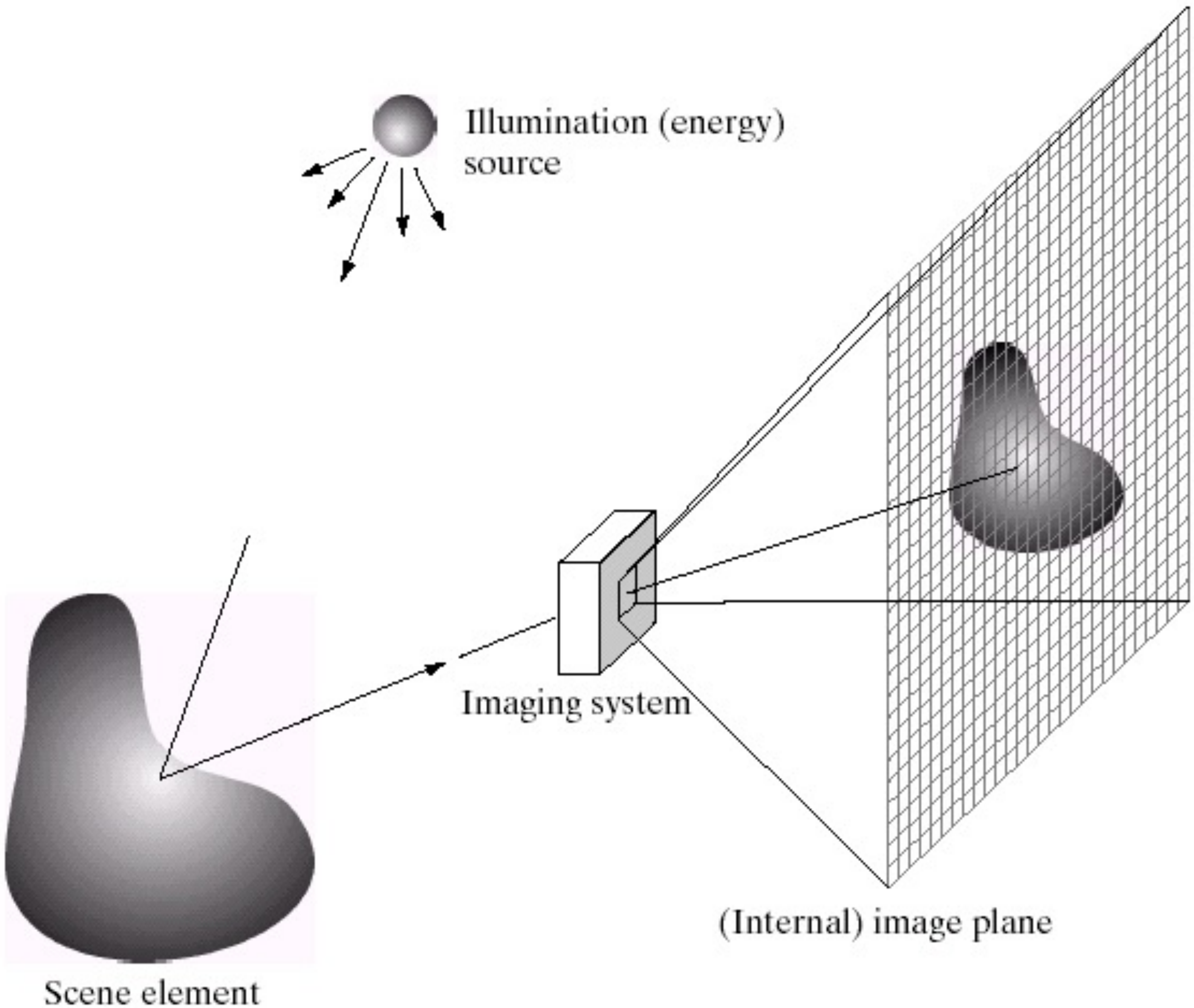
What determines a pixel's intensity?

What can we infer about the scene from pixel intensities?

Image Source: A. Efros

Some slides credited to James Hays@Brown University, Kristen Grauman@UT Austin and Svatalana Lazebnik@UIUC

How light is recorded



Digital camera

A digital camera replaces film with a sensor array

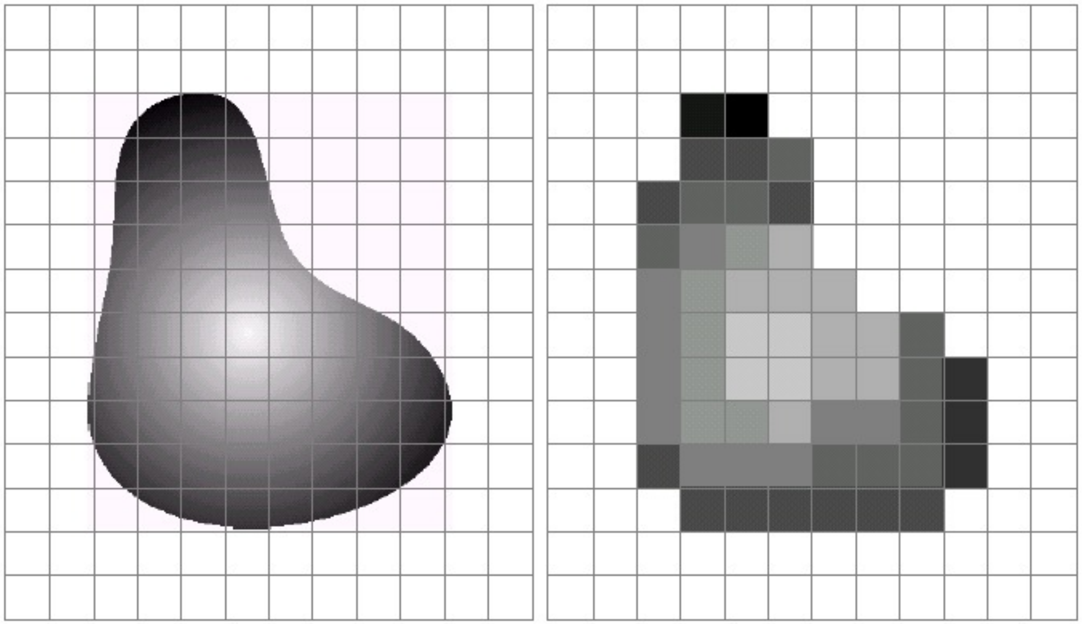
Each cell in the array is light-sensitive diode that converts photons to electrons

Two common types: Charge Coupled Device (CCD) and CMOS

http://electronics.howstuffworks.com/digital-camera.htm

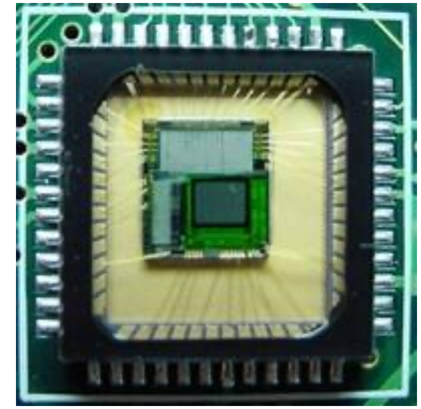
Slide by Steve Seitz

Sensor Array

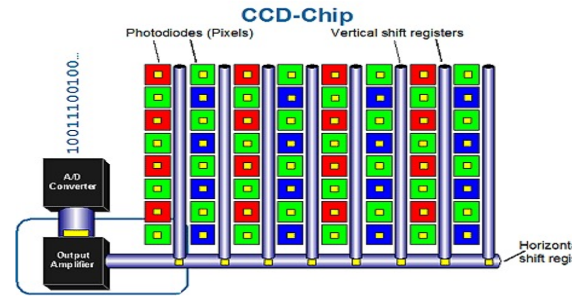


a b

FIGURE 2.17 (a) Continuos image projected onto a sensor array. (b) Result of image sampling and quantization.



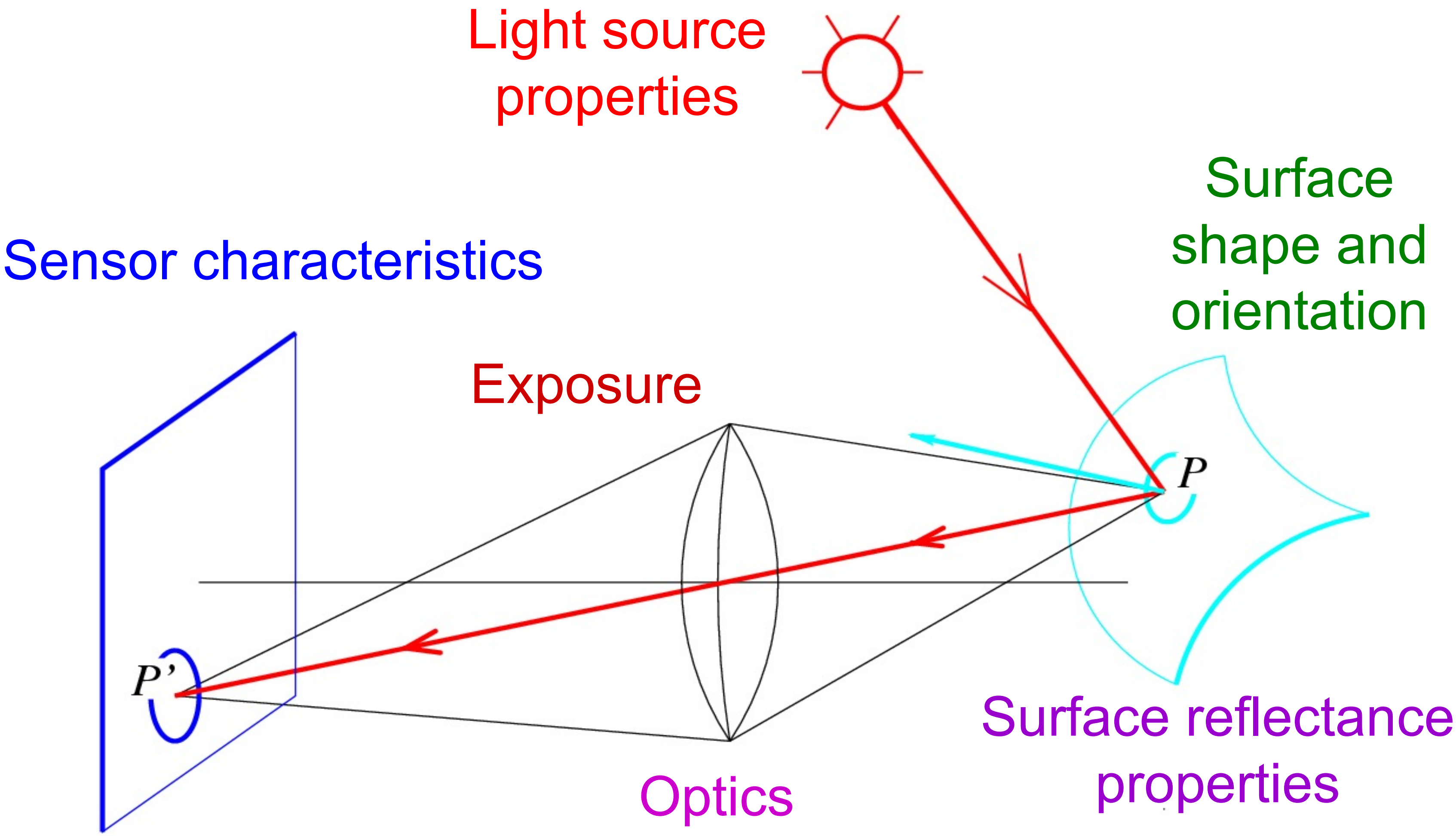
CMOS sensor



Each sensor cell record a small range of orientations
amount of light coming in

Image formation

What determines the brightness of an image pixel?



Slide by L. Fei-Fei

Intensity and Surface Orientation

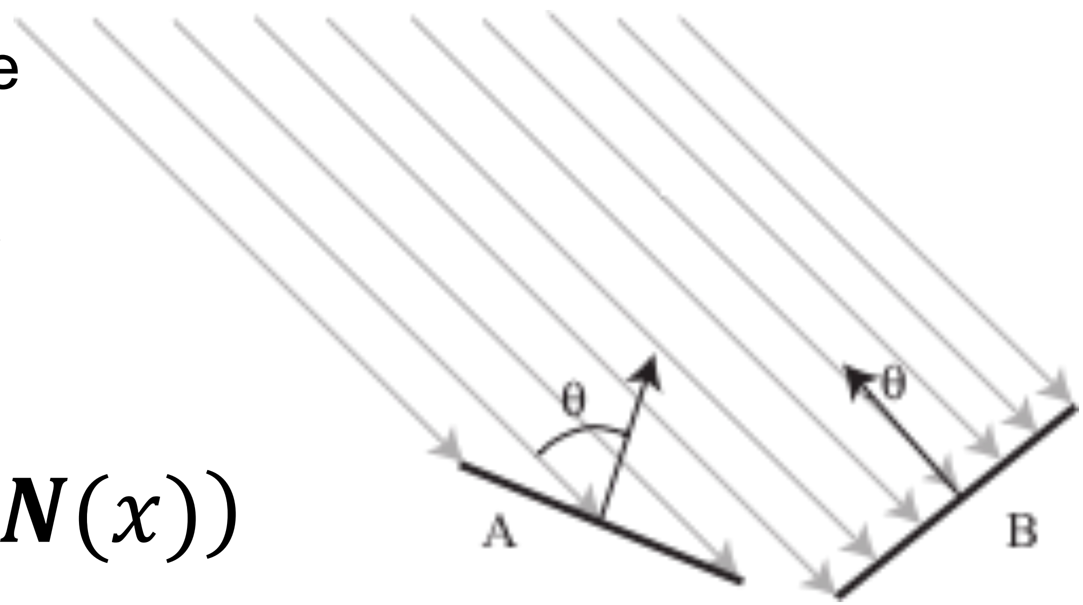
Intensity depends on illumination angle because less light comes in at oblique angles.

$\rho =$ albedo

$\boldsymbol{S} =$ directional source

$\boldsymbol{N} =$ surface normal

$\mathrm{I} =$ reflected intensity



$$I(x) = \rho(x)(\boldsymbol{S} \cdot \boldsymbol{N}(x))$$

Slide: Forsyth

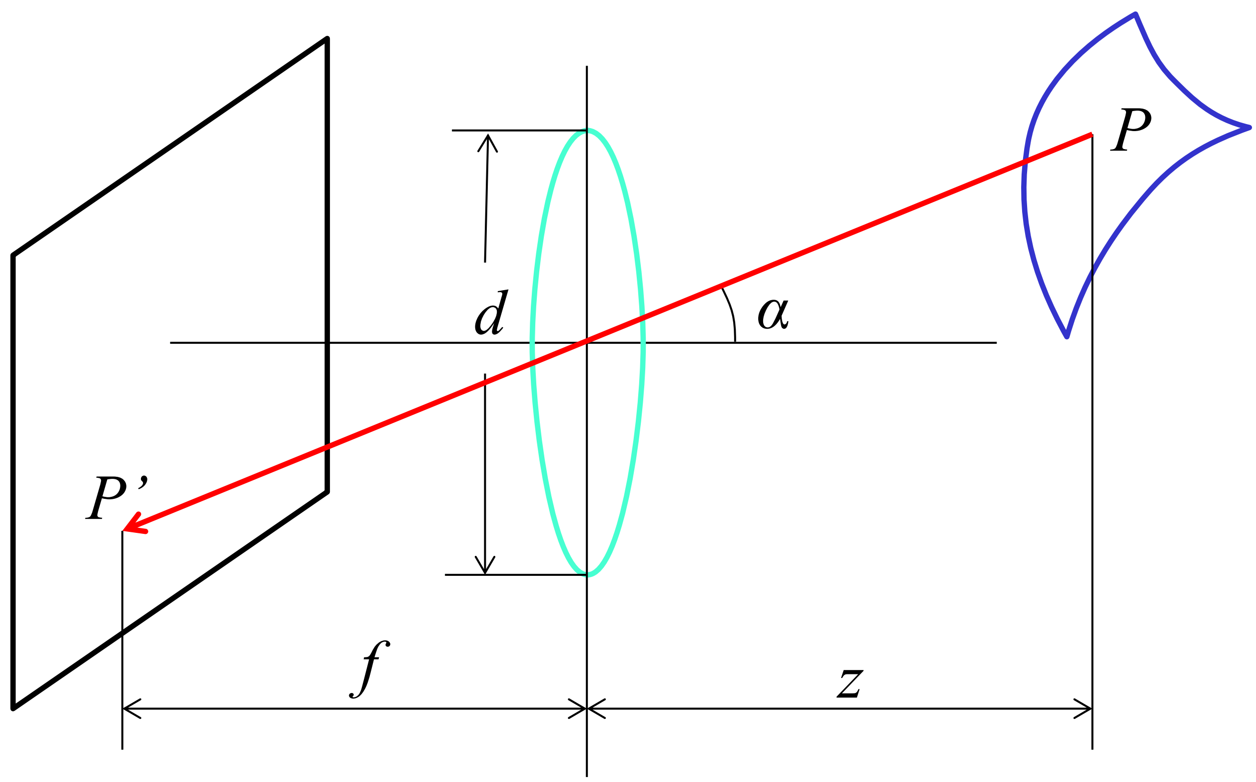
Fundamental radiometric relation

L: *Radiance* emitted from *P* toward *P*' (辐射度)

- Energy carried by a ray (Watts per sq. meter per steradian)

E: *Irradiance* falling on *P*' from the lens (辐照度)

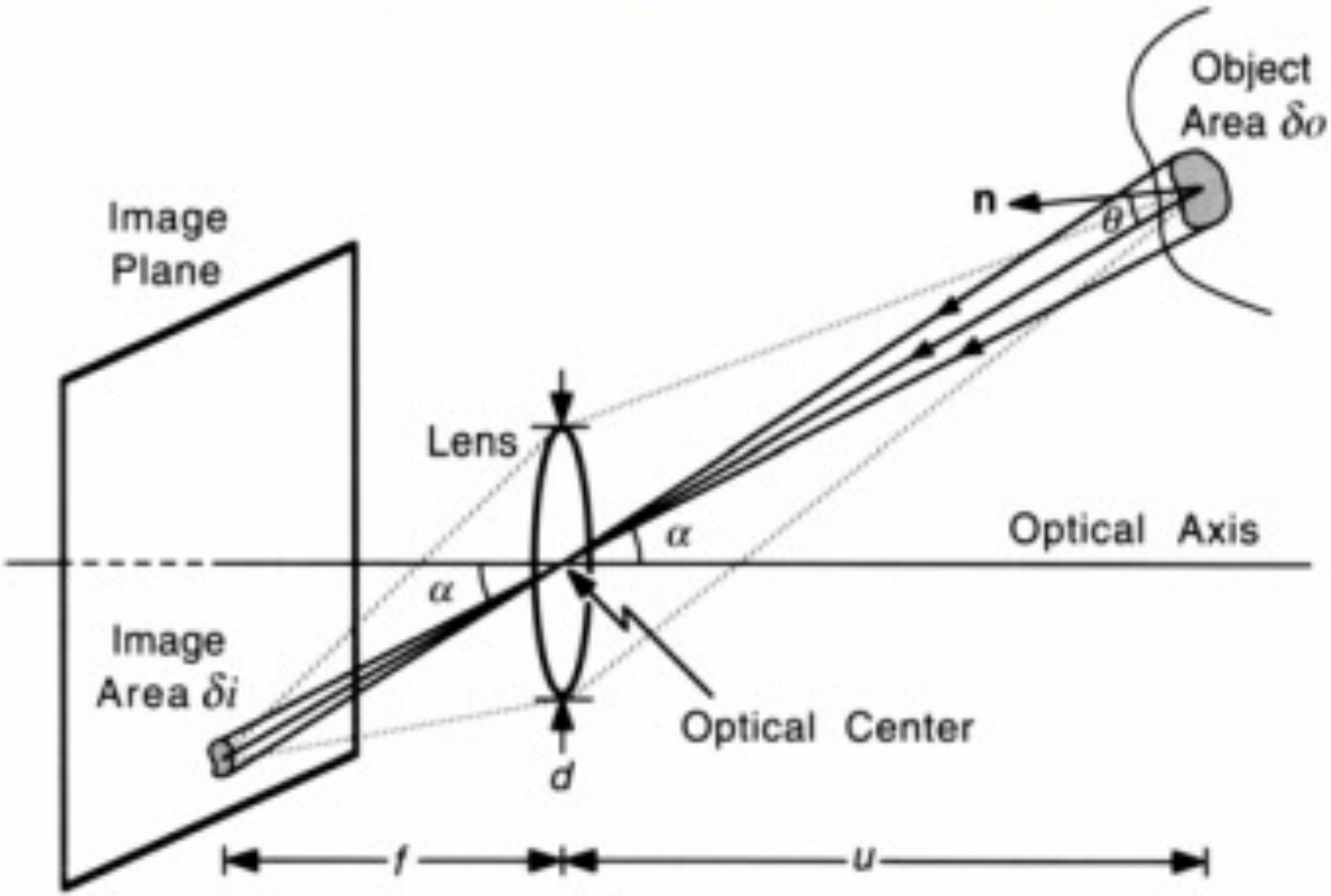
- Energy arriving at a surface (Watts per sq. meter)



What is the relationship between *E* and *L*?

Szeliski 2.2.3

Image formation by thin lens



Power captured by the lens

- Power captured by the lens from the surface patch is the product of the following three terms
  - The radiance L of the surface patch in the direction of the lens
  - The foreshortened area $(\delta o \cos\theta)$ of the patch in the direction of the lens
  - The **solid angle** subtended by the aperture of the lens at the surface patch

Foreshortened area of the aperture in the direction of the patch

$$\boxed{\frac{\pi(d/2)^2\cos\alpha}{(u/\cos\alpha)^2}}$$

Square of the distance of the aperture from the patch

Fundamental Radiometric Relation

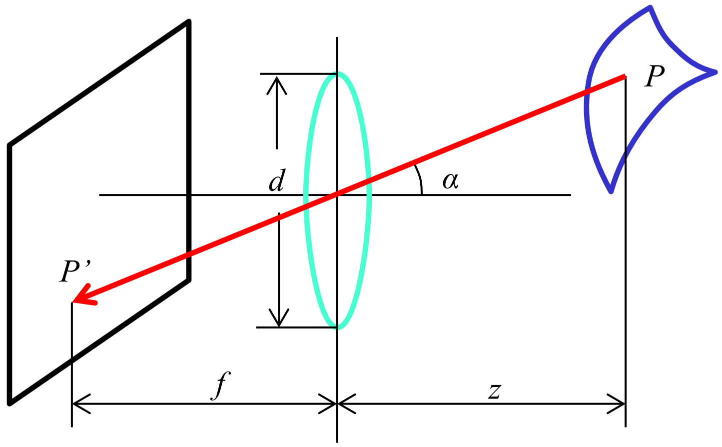
$$\delta P = L(\delta o \cos\theta)\frac{\pi (d/2)^2 \cos\alpha}{(u/\cos\alpha)^2}$$

$$\frac{\delta o \cos\theta}{(u/\cos\alpha)^2} = \frac{(\delta i \cos\alpha)}{(f/\cos\alpha)^2} = \texttt{solid angle at the lens}$$

$$\longrightarrow \quad \delta P = L\,\pi\,(d/2)^2\,\cos\alpha\,\frac{(\delta i \cos\alpha)}{(f/\cos\alpha)^2}$$

Irradiance $E = \dfrac{\delta P}{\delta i} = L\,\dfrac{\pi}{4}\left(\dfrac{d}{f}\right)^2 \cos^4\alpha$

Fundamental radiometric relation

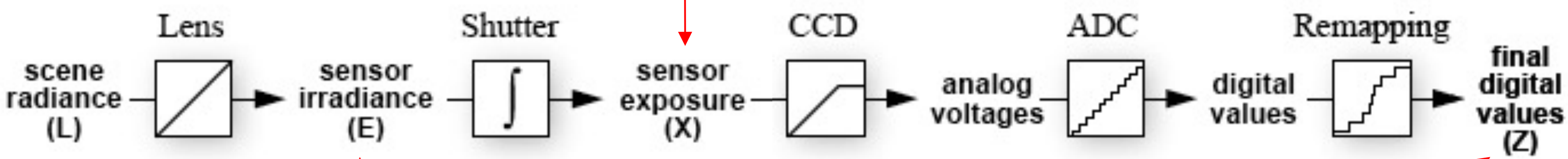


$$E = \left[\frac{\pi}{4} \left(\frac{d}{f} \right)^2 \cos^4 \alpha \right] L$$

- Image irradiance is linearly related to scene radiance
- Irradiance is proportional to the area of the lens and inversely proportional to the squared distance between the lens and the image plane
- The irradiance falls off as the angle between the viewing ray and the optical axis increases (natural vignetting)

From light rays to pixel values

$$\boxed{X = E \cdot \Delta t}$$



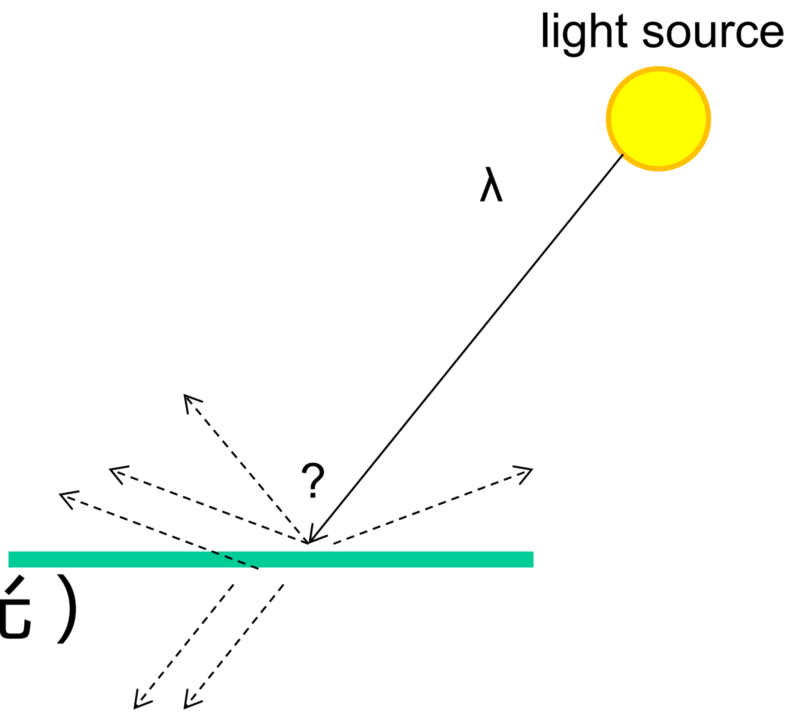
$$\boxed{E = \left[\frac{\pi}{4} \left(\frac{d}{f} \right)^2 \cos^4 \alpha \right] L}$$

$$\boxed{Z = f(E \cdot \Delta t)}$$

- Camera response function: the mapping f from irradiance to pixel values
  - Useful if we want to estimate material properties
  - Enables us to create high dynamic range images
  - For more info: P. E. Debevec and J. Malik, [Recovering High Dynamic Range Radiance Maps from Photographs](), *SIGGRAPH 97*

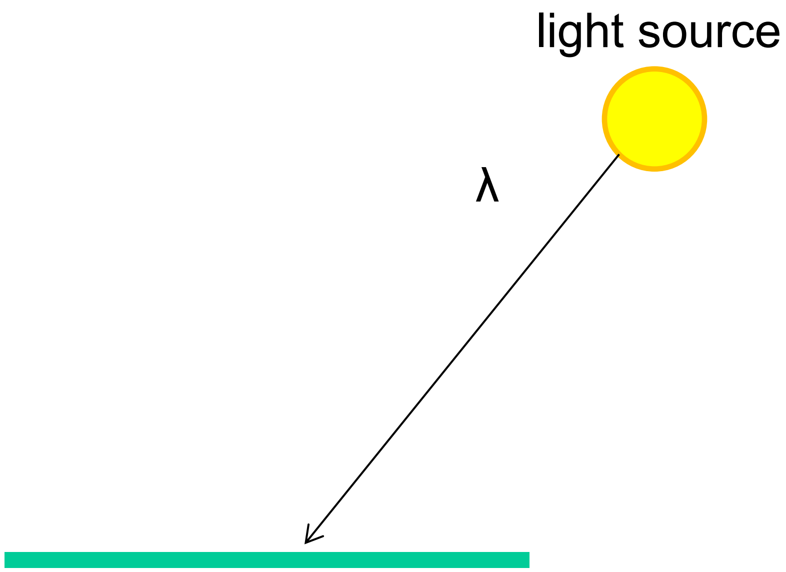
A photon’s life choices

- Absorption
- Diffusion
- Reflection
- Transparency
- Refraction
- Fluorescence（荧光）
- Subsurface scattering
- Phosphorescence（磷光）
- Interreflection



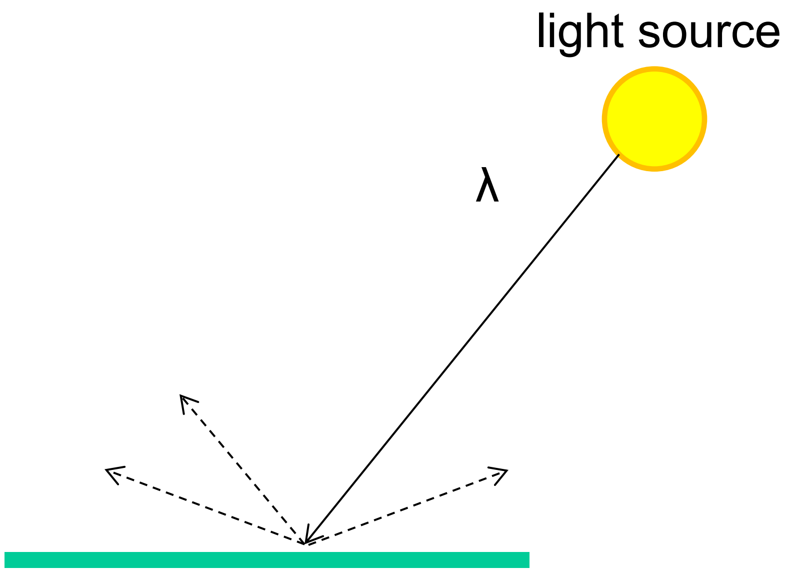
A photon's life choices

- **Absorption**
- Diffusion
- Reflection
- Transparency
- Refraction
- Fluorescence
- Subsurface scattering
- Phosphorescence
- Interreflection



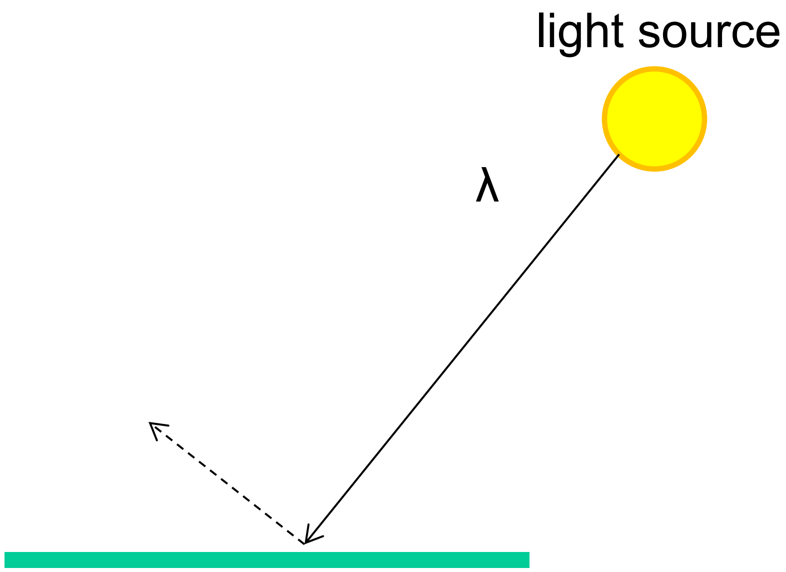
A photon’s life choices

- Absorption
- **Diffuse Reflection**
- Reflection
- Transparency
- Refraction
- Fluorescence
- Subsurface scattering
- Phosphorescence
- Interreflection



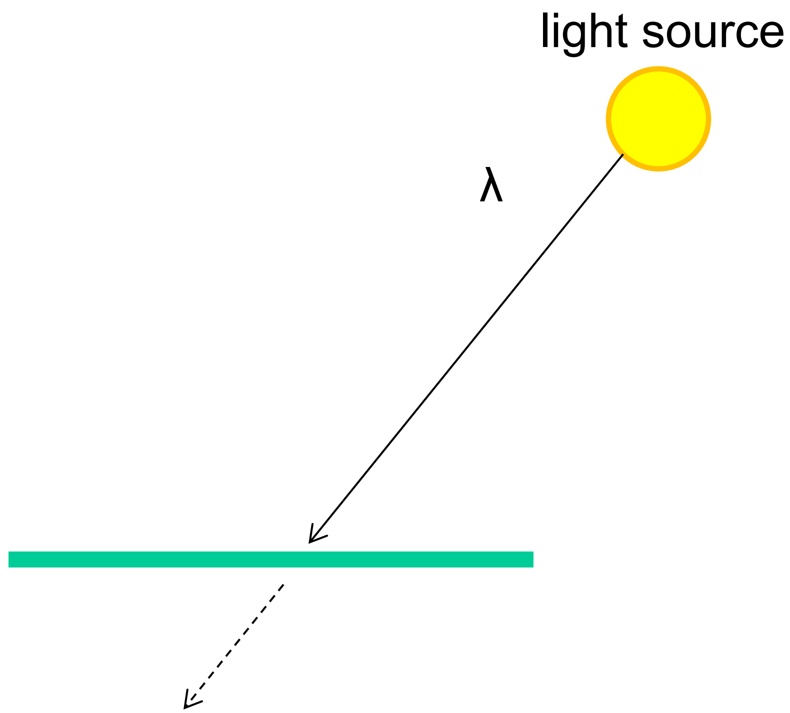
A photon's life choices

- Absorption
- Diffusion
- **Specular Reflection**
- Transparency
- Refraction
- Fluorescence
- Subsurface scattering
- Phosphorescence
- Interreflection



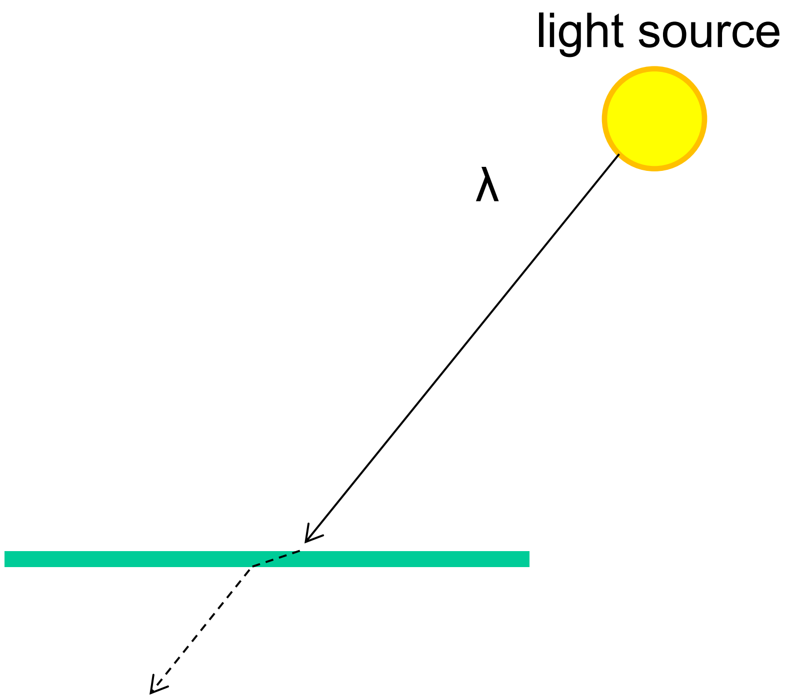
A photon's life choices

- Absorption
- Diffusion
- Reflection
- **Transparency**
- Refraction
- Fluorescence
- Subsurface scattering
- Phosphorescence
- Interreflection



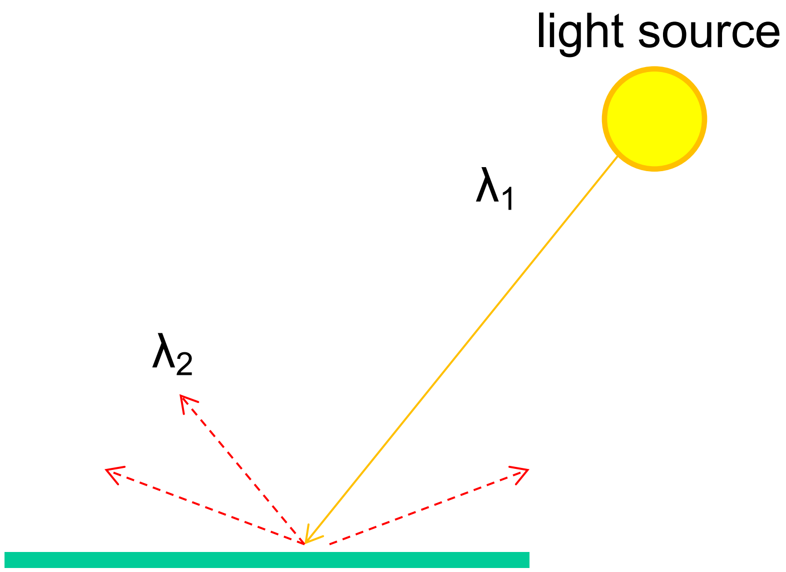
A photon's life choices

- Absorption
- Diffusion
- Reflection
- Transparency
- **Refraction**
- Fluorescence
- Subsurface scattering
- Phosphorescence
- Interreflection



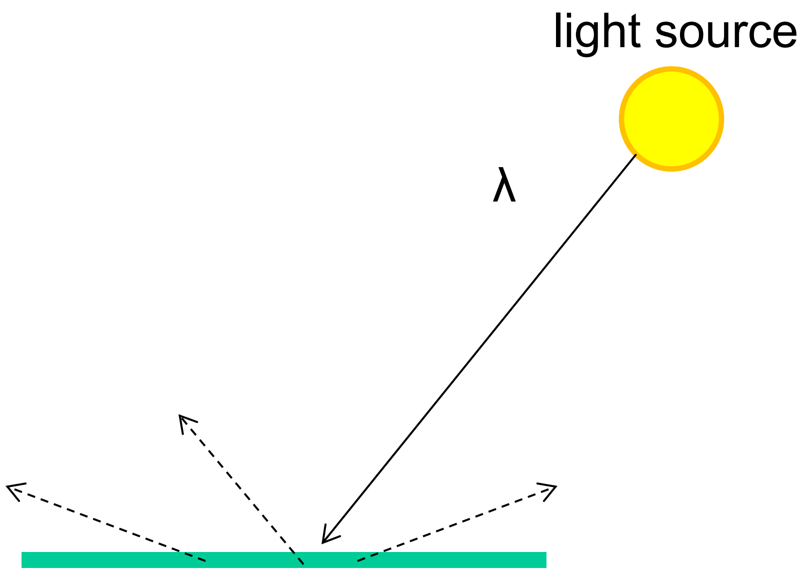
A photon's life choices

- Absorption
- Diffusion
- Reflection
- Transparency
- Refraction
- **Fluorescence**
- Subsurface scattering
- Phosphorescence
- Interreflection



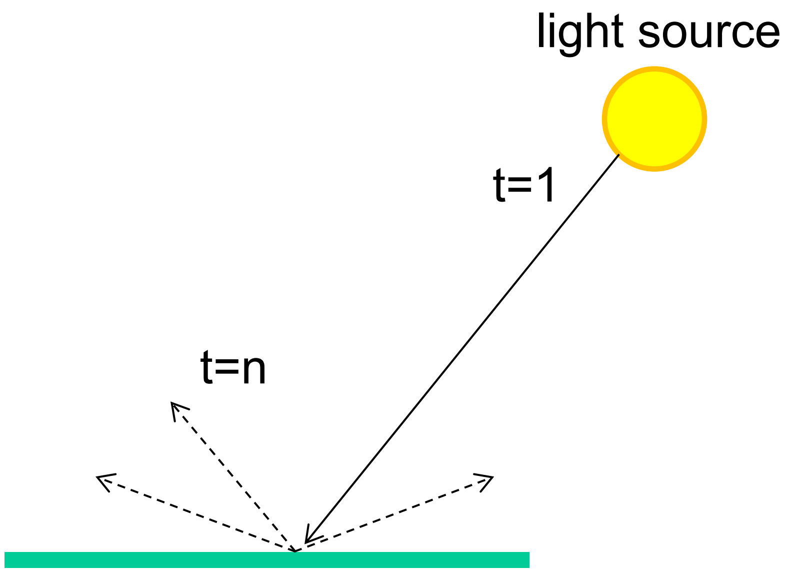
A photon's life choices

- Absorption
- Diffusion
- Reflection
- Transparency
- Refraction
- Fluorescence
- **Subsurface scattering**
- Phosphorescence
- Interreflection



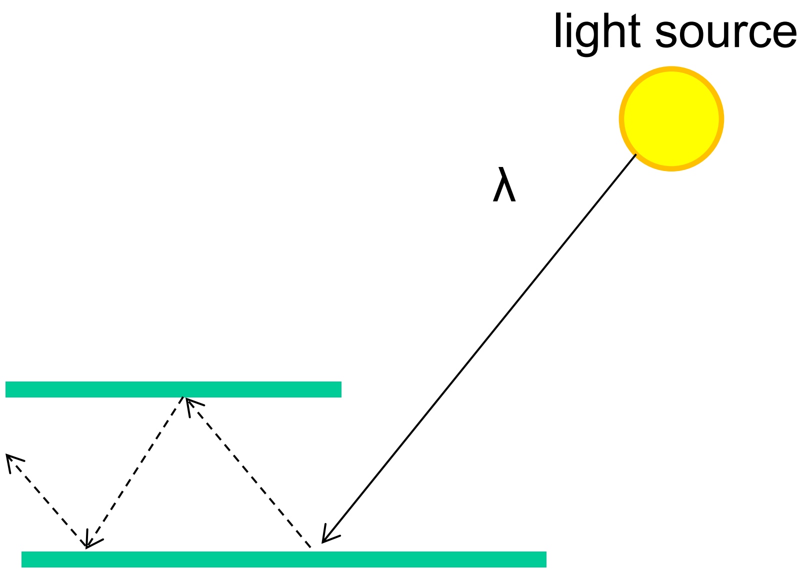
A photon's life choices

- Absorption
- Diffusion
- Reflection
- Transparency
- Refraction
- Fluorescence
- Subsurface scattering
- **Phosphorescence**
- Interreflection



A photon's life choices

- Absorption
- Diffusion
- Reflection
- Transparency
- Refraction
- Fluorescence
- Subsurface scattering
- Phosphorescence
- **Interreflection**

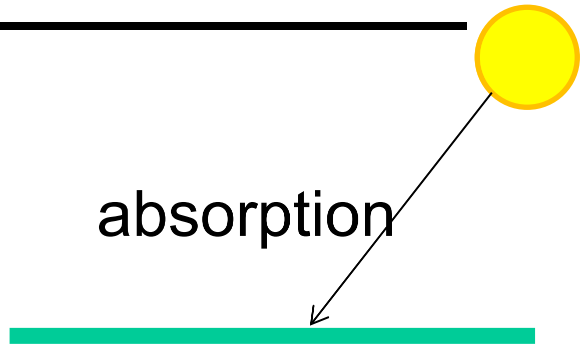


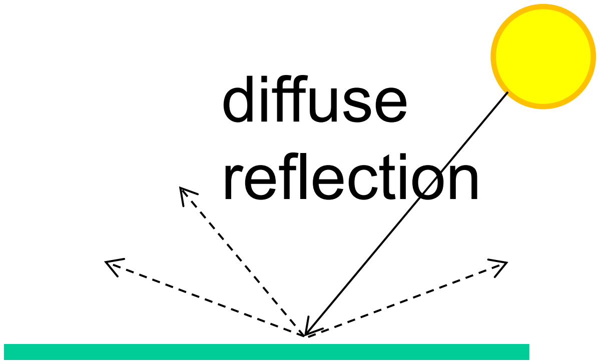
(Specular Interreflection)

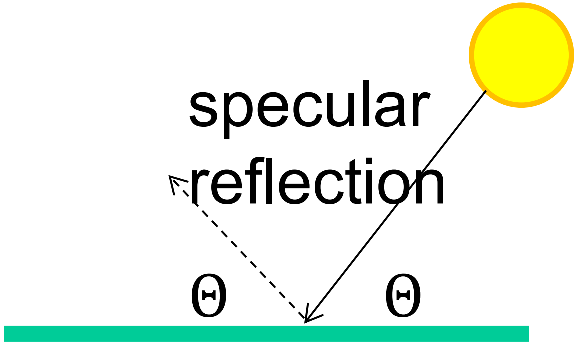
Some common effects

When light hits a typical surface

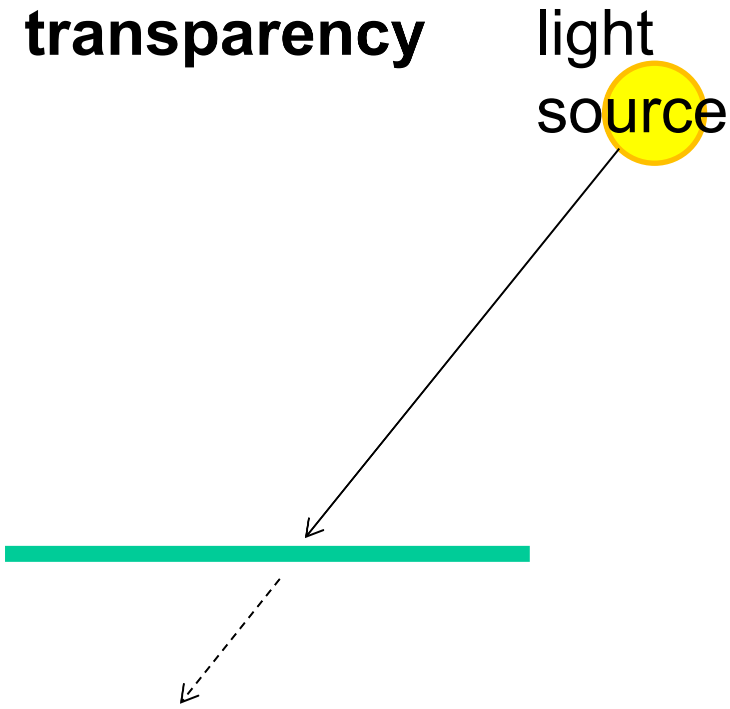
- Some light is absorbed ($1-\rho$)
  - More absorbed for low albedos
- Some light is reflected diffusely
  - Independent of viewing direction
- Some light is reflected specularly
  - Light bounces off (like a mirror), depends on viewing direction

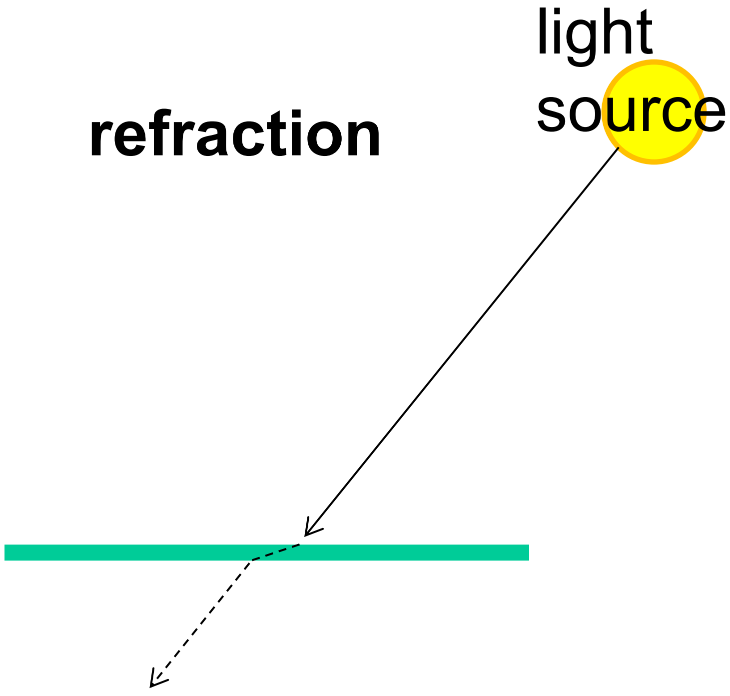




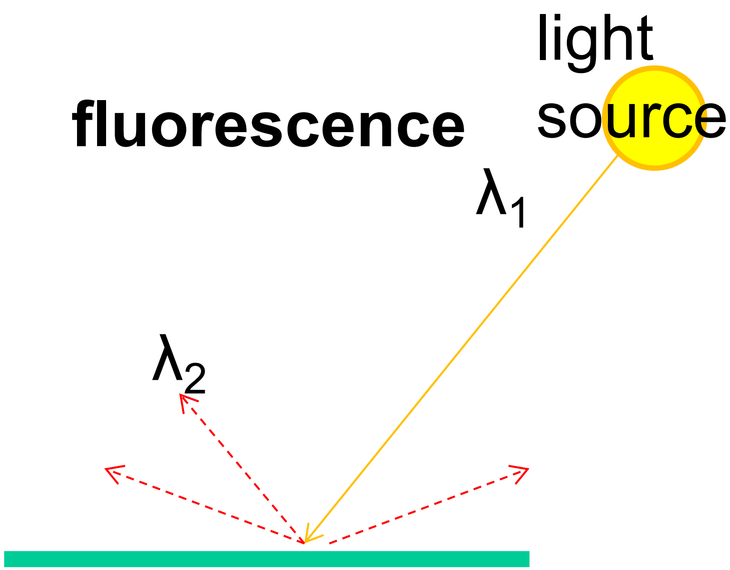


Other possible effects

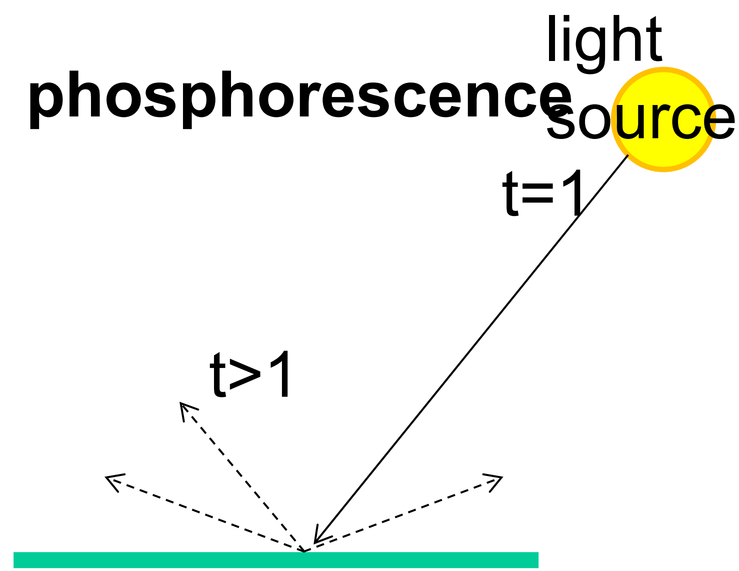


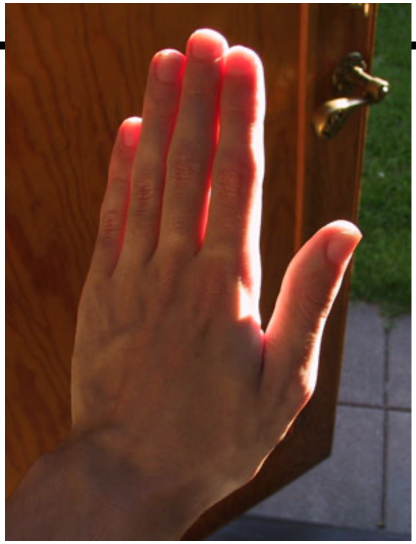


fluorescence

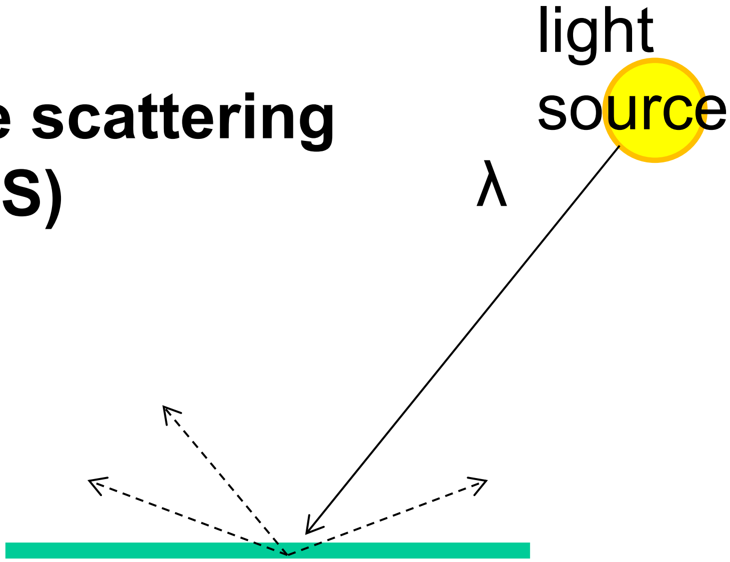


phosphorescence



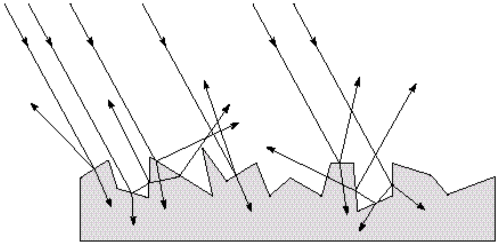


subsurface scattering (3S)

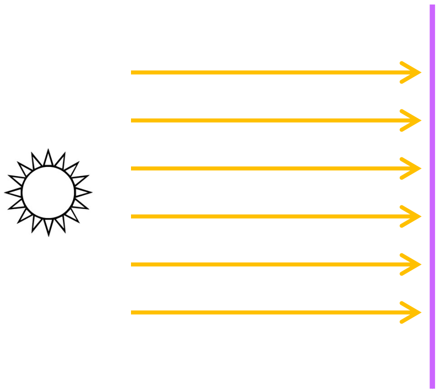


Diffuse reflection

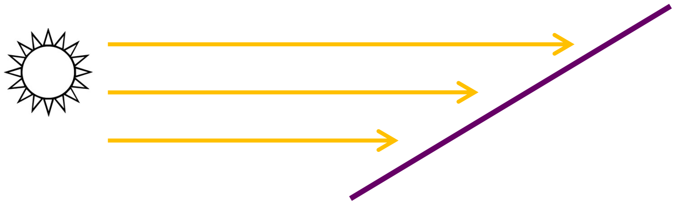
- Light is reflected equally in all directions
  - Dull, matte surfaces like chalk or latex paint
  - Microfacets scatter incoming light randomly
  - Effect is that light is reflected equally in all directions



- Brightness of the surface depends on the incidence of illumination

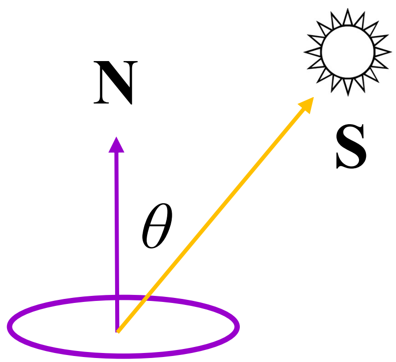


brighter

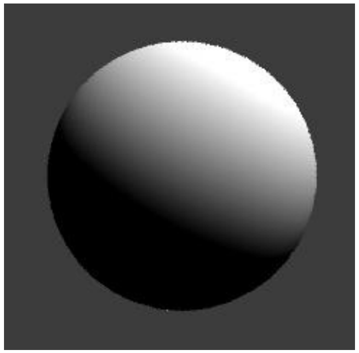


darker

Diffuse reflection: Lambert's law



$$B = \rho(\mathbf{N} \cdot \mathbf{S})$$
$$= \rho \|\mathbf{S}\| \cos\theta$$



B: radiosity (total power leaving the surface per unit area)
ρ: albedo (反射率fraction of incident irradiance reflected by the surface)
N: unit normal
S: source vector (magnitude proportional to intensity of the source)

Photometric stereo (shape from shading)

- Can we reconstruct the shape of an object based on shading cues?

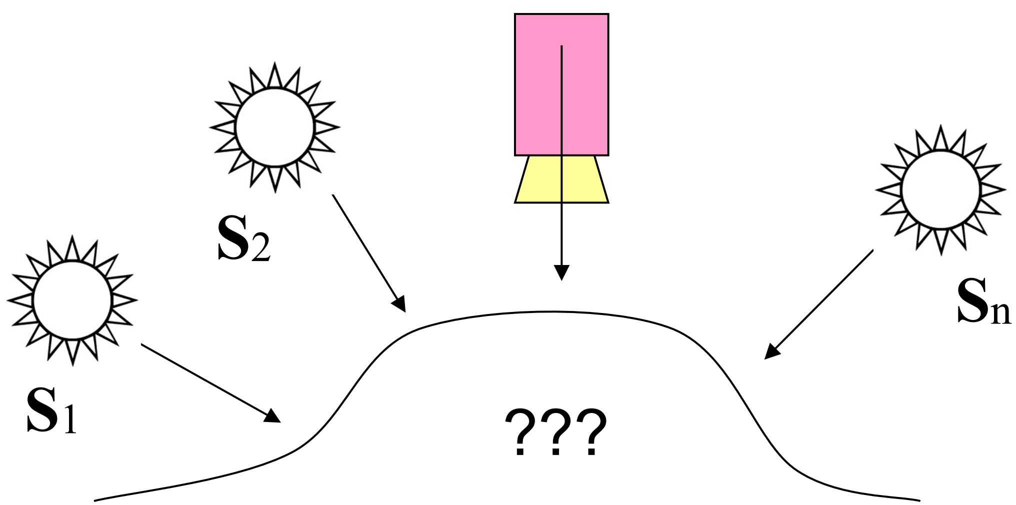
Luca della Robbia,
Cantoria, 1438

Photometric stereo

Assume:

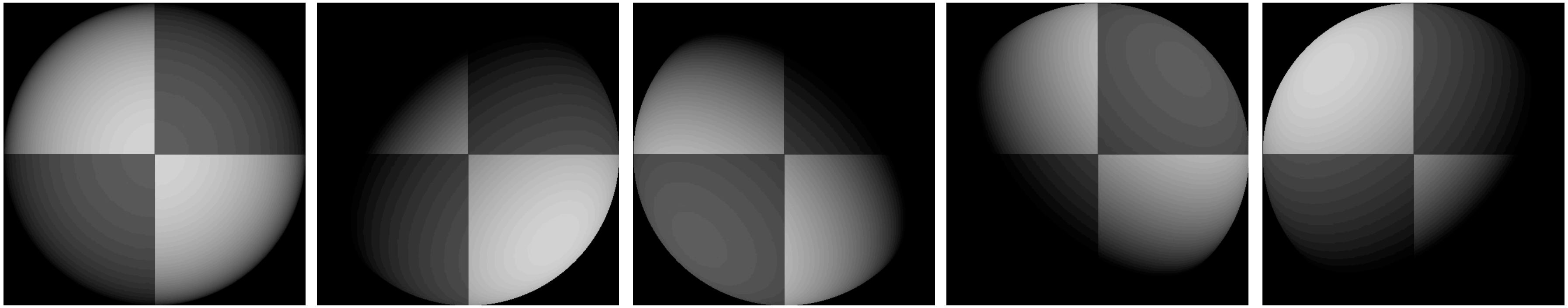
- A Lambertian object
- A *local shading model* (each point on a surface receives light only from sources visible at that point)
- A set of *known* light source directions
- A set of pictures of an object, obtained in exactly the same camera/object configuration but using different sources

Goal: reconstruct object shape and albedo

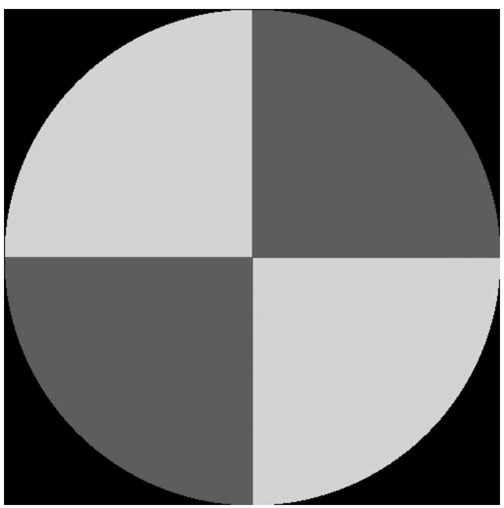


F&P 2nd ed., sec. 2.2.4

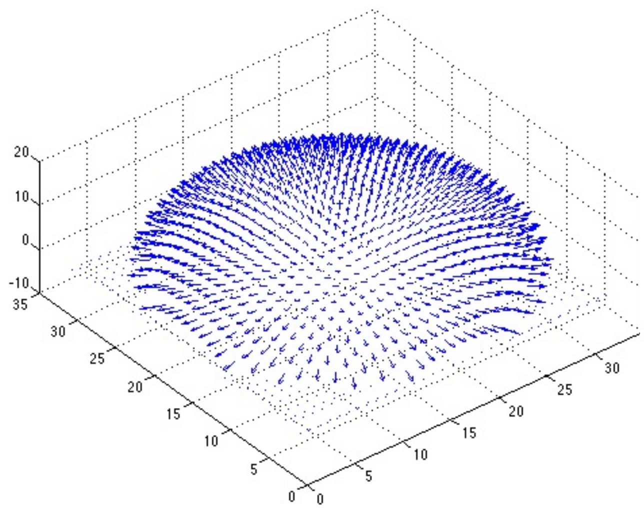
Example 1



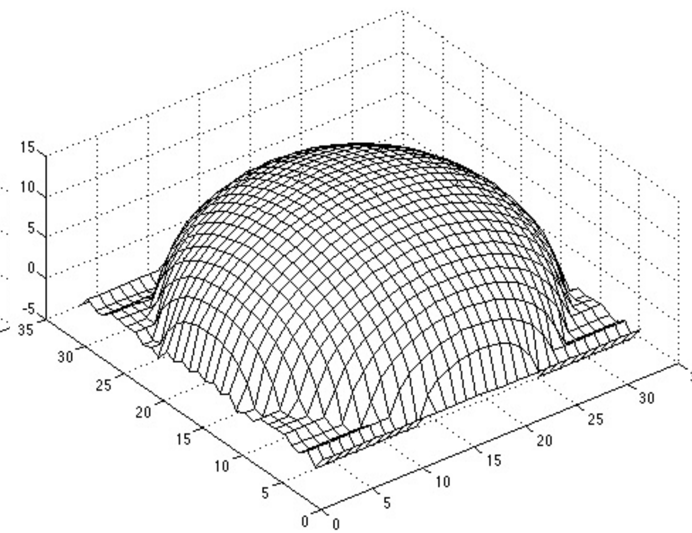
Recovered albedo



Recovered normal field



Recovered surface model



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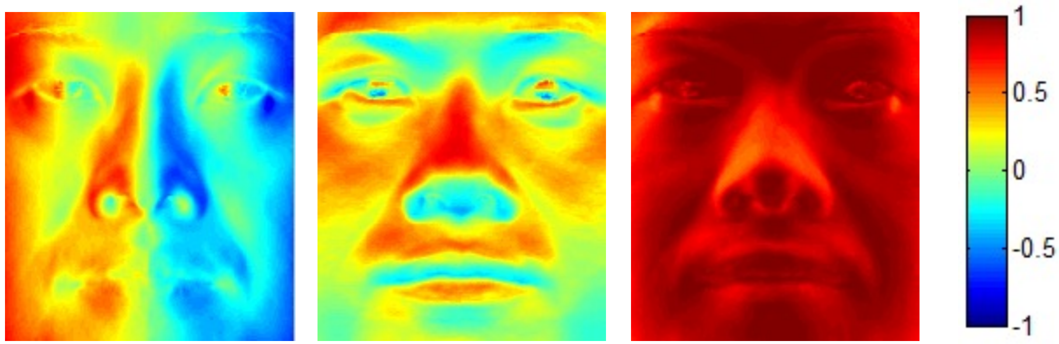
Example 2

Input

Recovered albedo

Recovered normal field



x y z

Recovered surface model

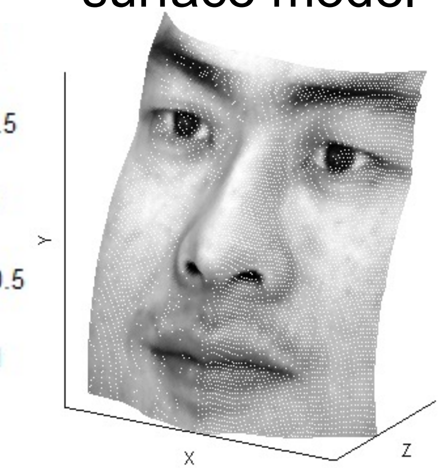


Image model

- **Known:** source vectors $\mathbf{S}_j$ and pixel values $I_j(x,y)$
- **Unknown:** surface normal $\mathbf{N}(x,y)$ and albedo $\rho(x,y)$

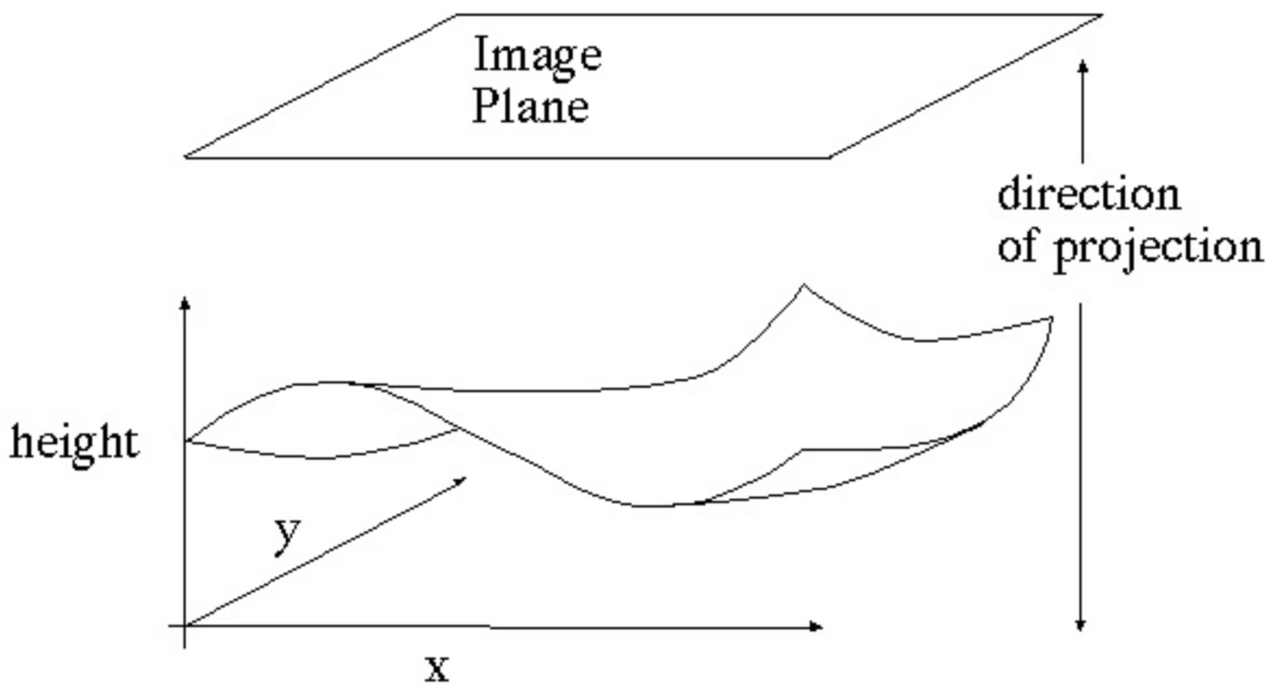


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Image model

- **Known:** source vectors $\mathbf{S}_j$ and pixel values $I_j(x,y)$
- **Unknown:** surface normal $\mathbf{N}(x,y)$ and albedo $\rho(x,y)$
- Assume that the response function of the camera is a linear scaling by a factor of k
- Lambert's law:

$$
\begin{aligned}
I_j(x,y) &= k\,\rho(x,y)\big(\mathbf{N}(x,y)\cdot\mathbf{S}_j\big) \\
&= \big(\rho(x,y)\mathbf{N}(x,y)\big)\cdot(k\mathbf{S}_j) \\
&= \mathbf{g}(x,y)\cdot\mathbf{V}_j
\end{aligned}
$$

F&P 2nd ed., sec. 2.2.4

Least squares problem

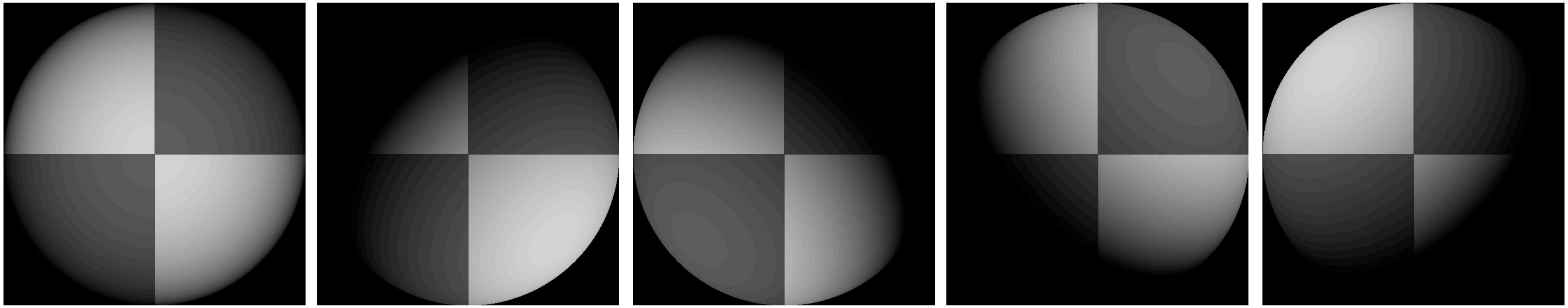
- For each pixel, set up a linear system:

$$\underbrace{\begin{bmatrix} I_1(x,y) \\ I_2(x,y) \\ \vdots \\ I_n(x,y) \end{bmatrix}}_{\substack{(n \times 1) \\ \text{known}}} = \underbrace{\begin{bmatrix} \mathbf{V}_1^T \\ \mathbf{V}_2^T \\ \vdots \\ \mathbf{V}_n^T \end{bmatrix}}_{\substack{(n \times 3) \\ \text{known}}} \underbrace{\mathbf{g}(x,y)}_{\substack{(3 \times 1) \\ \text{unknown}}}$$

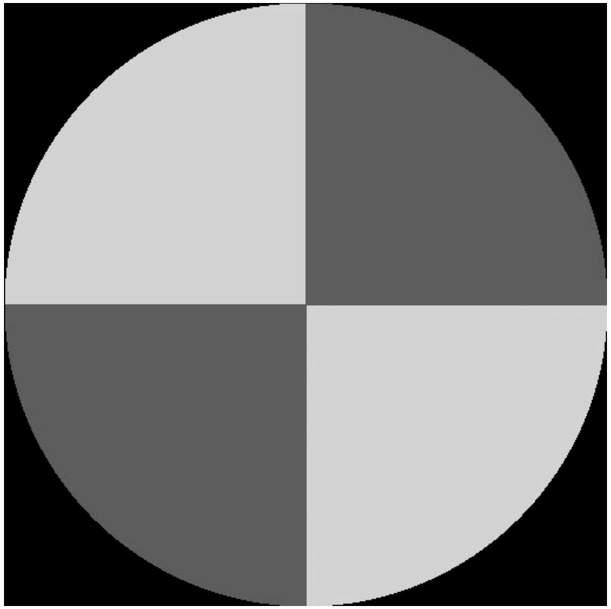
- Obtain least-squares solution for $\mathbf{g}(x,y)$ (which we defined as $\mathbf{N}(x,y)\ \rho(x,y)$)
- Since $\mathbf{N}(x,y)$ is the unit normal, $\rho(x,y)$ is given by the magnitude of $\mathbf{g}(x,y)$
- Finally, $\mathbf{N}(x,y) = \mathbf{g}(x,y) \,/\, \rho(x,y)$

F&P 2nd ed., sec. 2.2.4

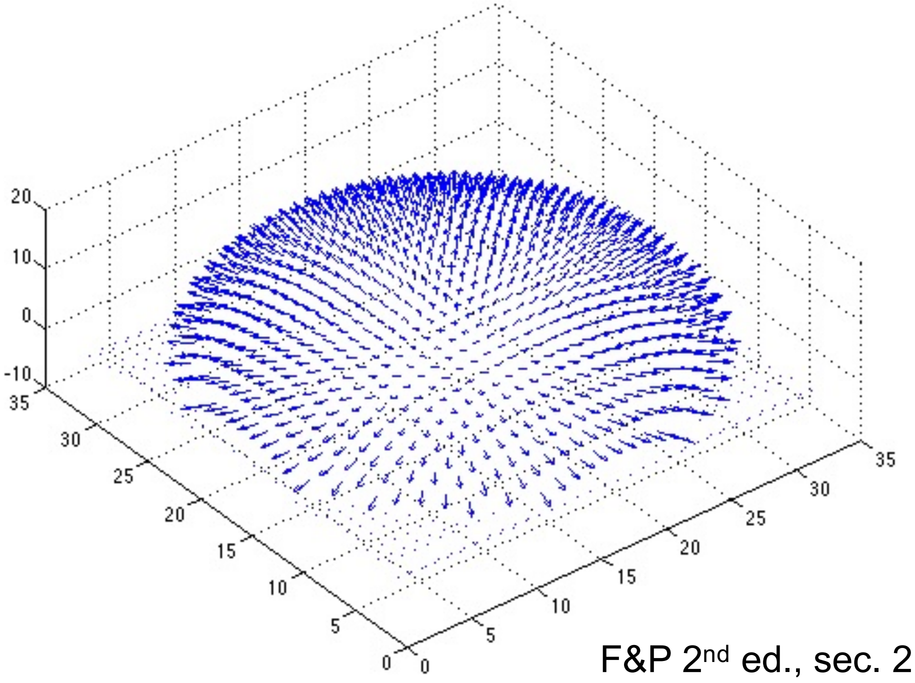
Synthetic example



Recovered albedo



Recovered normal field



F&P 2nd ed., sec. 2.2.4

Recovering a surface from normals

Recall the surface is written as

$$(x, y, f(x, y))$$

This means the normal has the form:

$$\mathbf{N}(x, y) = \frac{1}{\sqrt{f_x^2 + f_y^2 + 1}} \begin{pmatrix} f_x \\ f_y \\ 1 \end{pmatrix}$$

If we write the estimated vector *g* as

$$\mathbf{g}(x, y) = \begin{pmatrix} g_1(x, y) \\ g_2(x, y) \\ g_3(x, y) \end{pmatrix}$$

Then we obtain values for the partial derivatives of the surface:

$$f_x(x, y) = g_1(x, y) / g_3(x, y)$$

$$f_y(x, y) = g_2(x, y) / g_3(x, y)$$

F&P 2nd ed., sec. 2.2.4

Recovering a surface from normals

We can now recover the surface height at any point by integration along some path, e.g.

$$f(x,y) = \int_0^x f_x(s,0)\,ds + \int_0^y f_y(x,t)\,dt + C$$

(for robustness, should take integrals over many different paths and average the results)

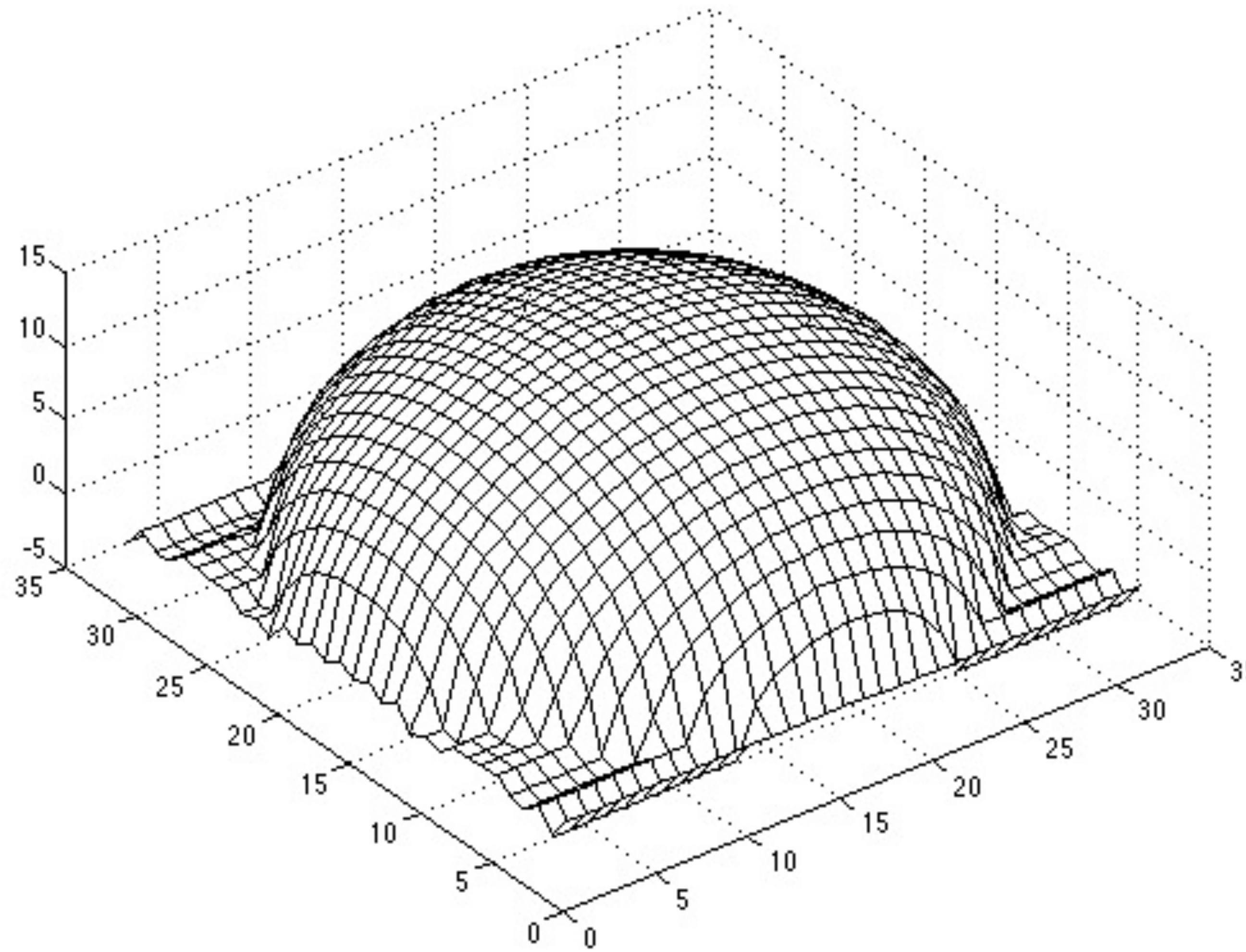
Integrability: for the surface f to exist, the mixed second partial derivatives must be equal:

$$\frac{\partial}{\partial y}(g_1(x,y)/g_3(x,y)) = \frac{\partial}{\partial x}(g_2(x,y)/g_3(x,y))$$

(in practice, they should at least be similar)

F&P 2nd ed., sec. 2.2.4

Surface recovered by integration



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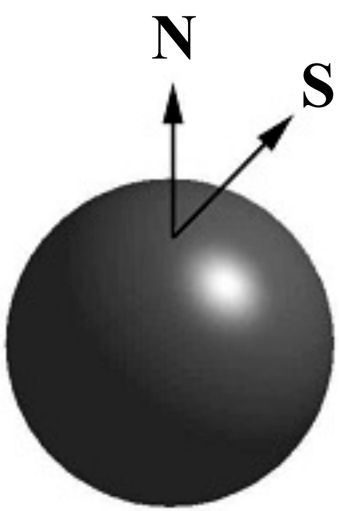
Limitations

- Simplistic reflectance and lighting model
- No shadows
- No interreflections
- No missing data
- Integration is tricky

Finding the direction of the light source

$$\boxed{I(x,y) = \mathbf{N}(x,y) \cdot \mathbf{S}(x,y)}$$



Full 3D case:

$$\begin{pmatrix} N_x(x_1,y_1) & N_y(x_1,y_1) & N_z(x_1,y_1) \\ N_x(x_2,y_2) & N_y(x_2,y_2) & N_z(x_2,y_2) \\ \vdots & \vdots & \vdots \\ N_x(x_n,y_n) & N_y(x_n,y_n) & N_z(x_n,y_n) \end{pmatrix} \begin{pmatrix} S_x \\ S_y \\ S_z \end{pmatrix} = \begin{pmatrix} I(x_1,y_1) \\ I(x_2,y_2) \\ \vdots \\ I(x_n,y_n) \end{pmatrix}$$

For points on the *occluding contour*:

$$\begin{pmatrix} N_x(x_1,y_1) & N_y(x_1,y_1) \\ N_x(x_2,y_2) & N_y(x_2,y_2) \\ \vdots & \vdots \\ N_x(x_n,y_n) & N_y(x_n,y_n) \end{pmatrix} \begin{pmatrix} S_x \\ S_y \end{pmatrix} = \begin{pmatrix} I(x_1,y_1) \\ I(x_2,y_2) \\ \vdots \\ I(x_n,y_n) \end{pmatrix}$$

P. Nillius and J.-O. Eklundh, “Automatic estimation of the projected light source direction,” CVPR 2001

Finding the direction of the light source

P. Nillius and J.-O. Eklundh, “Automatic estimation of the projected light source direction,” CVPR 2001

Application: Detecting composite photos

Fake photo

Real photo

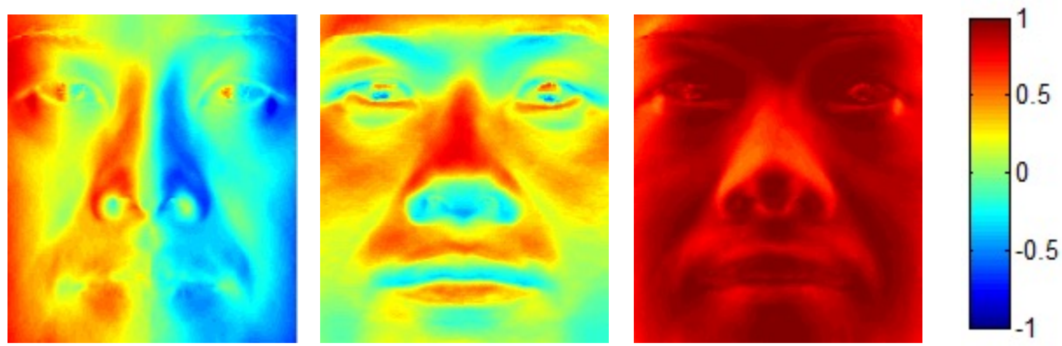
M. K. Johnson and H. Farid, [Exposing Digital Forgeries by Detecting Inconsistencies in Lighting](), ACM Multimedia and Security Workshop, 2005.

Exercise (optional, due date: Oct 8)

Input

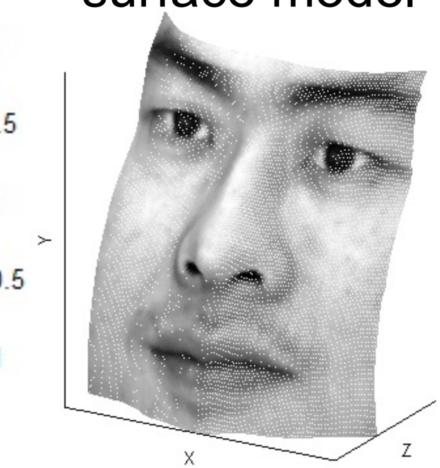
Recovered albedo

Recovered normal field



x y z

Recovered surface model



Use the Extended Yale B dataset and pick one subject randomly