Signals and Systems Homework 11

Due Time: 23:59 June 1^{st}, 2018
Submitted in-class on Thu (May 31),
or to the box in front of SIST 1C 403E (the instructor's office).

1. (5) Consider the siganl $x[n] = (\frac{1}{5})^n u[n-3]$ and evaluate the z-transform of this signal, then specify the region of convergence.

2. (10) A cuasal LTI system is described by the difference equation.

$$y[n] = y[n-1] + y[n-2] + x[n-1]$$

   (a) Find the system function $H(z) = \frac{Y(z)}{X(z)}$ for the system. Plot the ploes and zeros of $H(z)$ and indicate the region of convergence.

   (b) Find the unit sample response of the system.

   (c) You should have found the system to be unstable. Find a stable(noncausal) unit sample response that satisfies the difference equation.

3. (5) Consider the linear discrete, shift-invariant system with input $x[n]$ and output $y[n]$ for which

$$y[n-1] - \frac{10}{3}y[n] + y[n+1] = x[n]$$

The system is stable. Determine the unit sample response.

4. (20) A causal LTI system with input $x[n]$ and output $y[n]$ has the following block-diagram representation

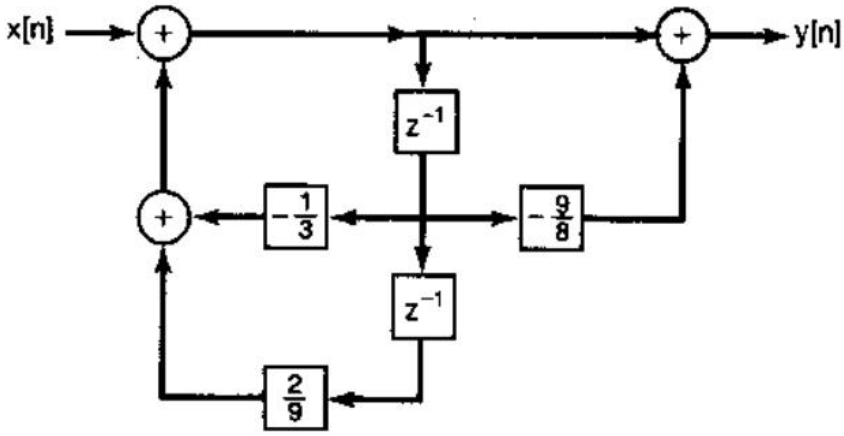

Figure 1: Block-diagram

(a) Determine a difference equation relating $y[n]$ and $x[n]$.

(b) Is the system stable?

5. (20) The following is known about a discrete-time system with input $x[n]$ and output $y[n]$:
   a. If $x[n] = (-2)^n$ for all n, then $y[n] = 0$ for all n;
   b. If $x[n] = (\frac{1}{2})^n u[n]$ for all n, then $y[n]$ for all n is of the form:

$$y[n] = \delta[n] + a(\frac{1}{4})^n u[n]$$

   a is a constant.

   (a) Determine the value of the constant a.

   (b) Determine the response $y[n]$ if the input $x[n]$ is

$$x[n] = 1$$

   for all n

6. (20) By using the power-series expansion:

$$log(1-w) = -\sum_{i=1}^{\infty} \frac{w^i}{i}$$

and $|w| < 1$ determin the inverse of each of the following two z-transform:

(a) $X(z) = log(1-2z)$, $|z| < \frac{1}{2}$

(b) $X(z) = log(1-\frac{1}{2}z^{-1})$, $|z| > \frac{1}{2}$

7. (20) Use the unilateral z-transform

$$F(z) = \sum_{n=0}^{\infty} f[n] z^{-n}$$

to compute the value of the n-th term of the Fibonacci sequence:

$$f(n+2) = f(n+1) + f(n), \quad n \geq 0$$

with $f(0) = 1$, $f(1) = 1$. (Hint: Be careful about using the properties of ZT since the above equality just holds for $n \geq 0$.)