# Signals and Systems Homework 7

**Due Time: 21:59 May 4, 2018**
**Submitted in-class on Thu (May 4),**
**or to the box in front of SIST 1C 403E (the instructor's office).**

1. (20 points) The following are discrete-time signals and Fourier transforms. Determine the signal/FT for each one.

   (a) $x_1[n] = (\frac{1}{2})^{|n-1|}$

   (b) $\sin(\frac{\pi}{3}n+\frac{\pi}{4})$ (Determine the Fourier transform for $-\pi \leq \omega < \pi$ . Hint: It's the Fourier transform for periodic signals).

   (c) $X_1(jw) = \frac{e^{-jw}-\frac{1}{5}}{1-\frac{1}{5}e^{-jw}}$

   (d) $X_2(jw) = \sum_{k=-\infty}^{+\infty}(-1)^k\delta(\omega - \frac{\pi}{2}k)$

2. (15 points) Given that $x[n]$ has Fourier transform $X(jw)$ , express the Fourier transforms of the following signals in the terms of $X(jw)$ .

   (a) $x_1[n] = x[1-n] + x[-1-n]$ .

   (b) $x_2[n] = \frac{x^*[-n]+x[n]}{2}$ .

   (c) $x_3[n] = (n-1)^2 x[n]$

3. (15 points) Let

$$y[n] = (\frac{\sin \frac{\pi}{4} n}{\pi n})^2 * (\frac{\sin \omega_c n}{\pi n})$$

where $*$ denotes convolution and $\mid \omega_c n \mid \leq \pi$ . Determine a stricter constraint on $\omega_c n$, which ensures that

$$y[n] = (\frac{\sin \frac{\pi}{4} n}{\pi n})^2$$

4. (15 points) Let $x_1[n]$ be the discrete-time signal whose Fourier transform $X_1(jw)$ is depicted in Figure 1. Consider the signal $x_2[n]$ with Fourier transform $X_2(jw)$ , as illustrated in Figure 2. Please express $x_2[n]$ in terms of $x_1[n]$.

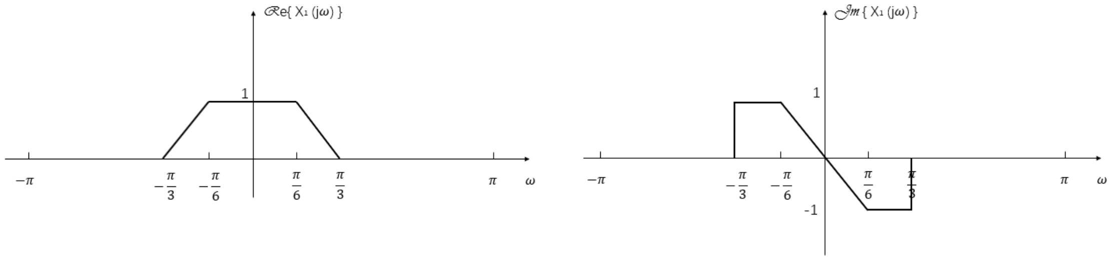


Figure 1: The real and imaginary parts of the Fourier transform $X_1(jw)$

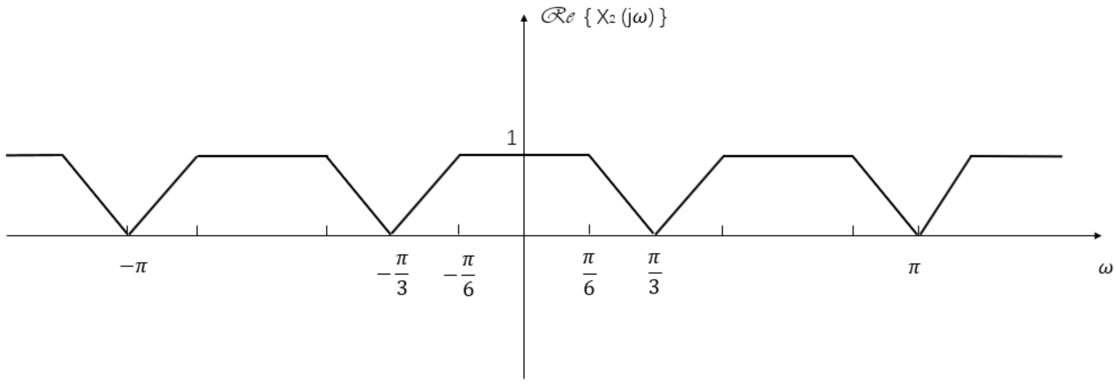


Figure 2: the Fourier transform $X_2(jw)$

5. (15 points) Let $x[n] = e^{jwn}$ for $0 \leq n < N$ and let $X[k]$ be the DFT of $x[n]$.

   (a) Calculate a simplified expression for $X[k]$ that is correct for any value of $\omega$.

   (b) Calculate a simplified expression for $X[k]$ when $\omega = 2\pi m/N$ where m is an integer. And sketch a plot of $|\ X[k]\ |$

6. (20 points) Let $x[n]$ be a signal of finite duration, that is, there is an integer $N$ so that

$$x[n] = 0 \qquad \text{outside the interval } 0 \leq n \leq N-1$$

The DFT of $x[n]$ is denoted by $X[k]$, and $X(j\omega)$ denote the Fourier transform of $x[n]$.

(a) Show that

$$X[k] = \frac{1}{N} X(j(2\pi k/N))$$

(b) Let us consider samples of $X(j\omega)$ taken every $\frac{2\pi}{M}$ , where $M < N$. These samples correspond to more than one sequence of duration $N$. To illustrate this, consider the two signals $x_1[n]$ and $x_2[n]$ depicted in Figure 3. Show that if we choose $M = 4$, we have

$$X_1(2\pi k/4) = X_2(j(2\pi k/4))$$

for all values of k.

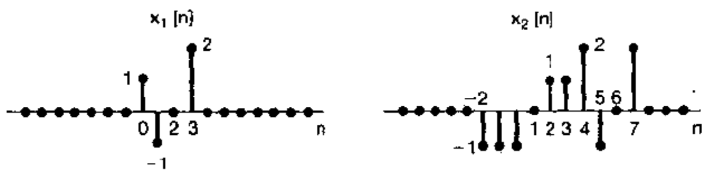


Figure 3: $x_1[n]$ and $x_2[n]$