Homework 9 Solution

Due date:
Jun. 4th, 2018
Turn in your homework in class

1. Determine the resonant frequency of the circuit shown below.

   Given that R= 1kΩ, L = 10 mH, and C = 10nF.

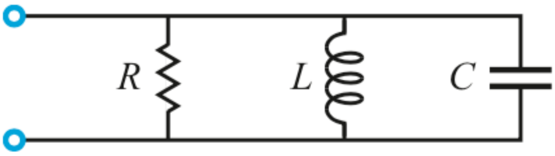


Fig.1

Solution:

$$
\begin{aligned}
\mathbf{Z}_i &= R \parallel (j\omega L) \parallel (-j/\omega C) \\
&= R \parallel \left(\frac{L/C}{j(\omega L - \frac{1}{\omega C})} \right) \\
&= R \parallel \left(\frac{-j\omega L}{\omega^2 LC - 1} \right) \\
&= \left(\frac{-j\omega RL}{\omega^2 LC - 1} \right) \Big/ \left(R - \frac{j\omega L}{\omega^2 LC - 1} \right) \\
&= \frac{-j\omega RL}{\omega^2 LC - 1} \cdot \frac{\omega^2 LC - 1}{R(\omega^2 LC - 1) - j\omega L} \\
&= \frac{-j\omega RL}{\cancel{\omega^2 LC - 1}} \cdot \frac{\cancel{\omega^2 LC - 1}}{R(\omega^2 LC - 1) - j\omega L} \\
&= \frac{-j\omega RL}{R(\omega^2 LC - 1) - j\omega L} \cdot \frac{R(\omega^2 LC - 1) + j\omega L}{R(\omega^2 LC - 1) + j\omega L} \\
&= \frac{\omega^2 RL^2 - j\omega R^2 L(\omega^2 LC - 1)}{R^2 L(\omega^2 LC - 1)^2 + \omega^2 L^2} .
\end{aligned}
$$

At resonance, $\mathbf{Z}_i$ is purely real. This occurs when

$$\omega R^2 L(\omega^2 LC - 1) = 0,$$

which is satisfied when

$$
\begin{aligned}
\omega &= 0 \text{ (trivial resonance), or} \\
\omega &= \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-2} \times 10^{-8}}} = 10^5 \text{ rad/s}.
\end{aligned}
$$

2. For the circuit shown below, determine the transform function $\mathbf{H} = \mathbf{V}_o/\mathbf{V}_i$,and determine the frequency w at which H is purely real.

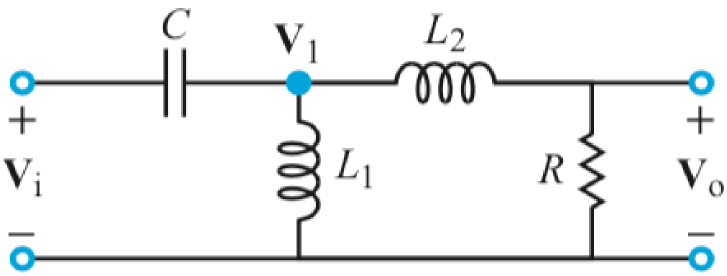


Fig.2

Solution:

(a) KCL at node $\mathbf{V}_1$ gives:

$$\frac{\mathbf{V}_1 - \mathbf{V}_i}{\mathbf{Z}_C} + \frac{\mathbf{V}_1}{\mathbf{Z}_{L_1}} + \frac{\mathbf{V}_1}{R + \mathbf{Z}_{L_2}} = 0,$$

where $\mathbf{Z}_C = 1/j\omega C$, $\mathbf{Z}_{L_1} = j\omega L_1$, and $\mathbf{Z}_{L_2} = j\omega L_2$.

Also, voltage division gives

$$\mathbf{V}_o = \frac{\mathbf{V}_1 R}{R + j\omega L_2}.$$

Solving for the transfer function gives

$$\mathbf{H} = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{-\omega^2 R L_1 C}{R(1 - \omega^2 L_1 C) + j\omega(L_1 + L_2 - \omega^2 L_1 L_2 C)}.$$

(b) We need to rationalize the expression for $\mathbf{H}$:

$$\begin{aligned}\mathbf{H} &= \frac{-\omega^2 R L_1 C}{R(1 - \omega^2 L_1 C) + j\omega(L_1 + L_2 - \omega^2 L_1 L_2 C)} \\ &\quad \times \frac{R(1 - \omega^2 L_1 C) - j\omega(L_1 + L_2 - \omega^2 L_1 L_2 C)}{R(1 - \omega^2 L_1 C) - j\omega(L_1 + L_2 - \omega^2 L_1 L_2 C)} \\ &= \frac{-\omega^2 R^2 L_1 C(1 - \omega^2 L_1 C) + j\omega^3 R L_1 C(L_1 + L_2 - \omega^2 L_1 L_2 C)}{R^2(1 - \omega^2 L_1 C)^2 + \omega^2(L_1 + L_2 - \omega^2 L_1 L_2 C)^2}.\end{aligned}$$

The imaginary part of $\mathbf{H}$ is zero if $\omega = 0$ (trivial solution) or if

$$\omega_0 = \sqrt{\frac{L_1 + L_2}{L_1 L_2 C}}.$$

3. Generate Bode magnitude and phase plots for the following voltage transfer functions (with some necessary approximation for straight-line in drawing) (more space for drawing and enough annotation is necessary):

(a) $\mathbf{H}(\omega) = \dfrac{4\times 10^4(60+j6\omega)}{(4+j2\omega)(100+j2\omega)(400+j4\omega)}$

(b) $\mathbf{H}(\omega) = \dfrac{(1+j0.2\omega)^2(100+j2\omega)^2}{(j\omega)^3(500+j\omega)}$

(c) $\mathbf{H}(\omega) = \dfrac{8\times 10^{-2}(10+j10\omega)}{j\omega(16-\omega^2+j4\omega)}$

(d) $\mathbf{H}(\omega) = \dfrac{4\times 10^4\omega^2(100-\omega^2+j50\omega)}{(5+j5\omega)(200+j2\omega)^3}$

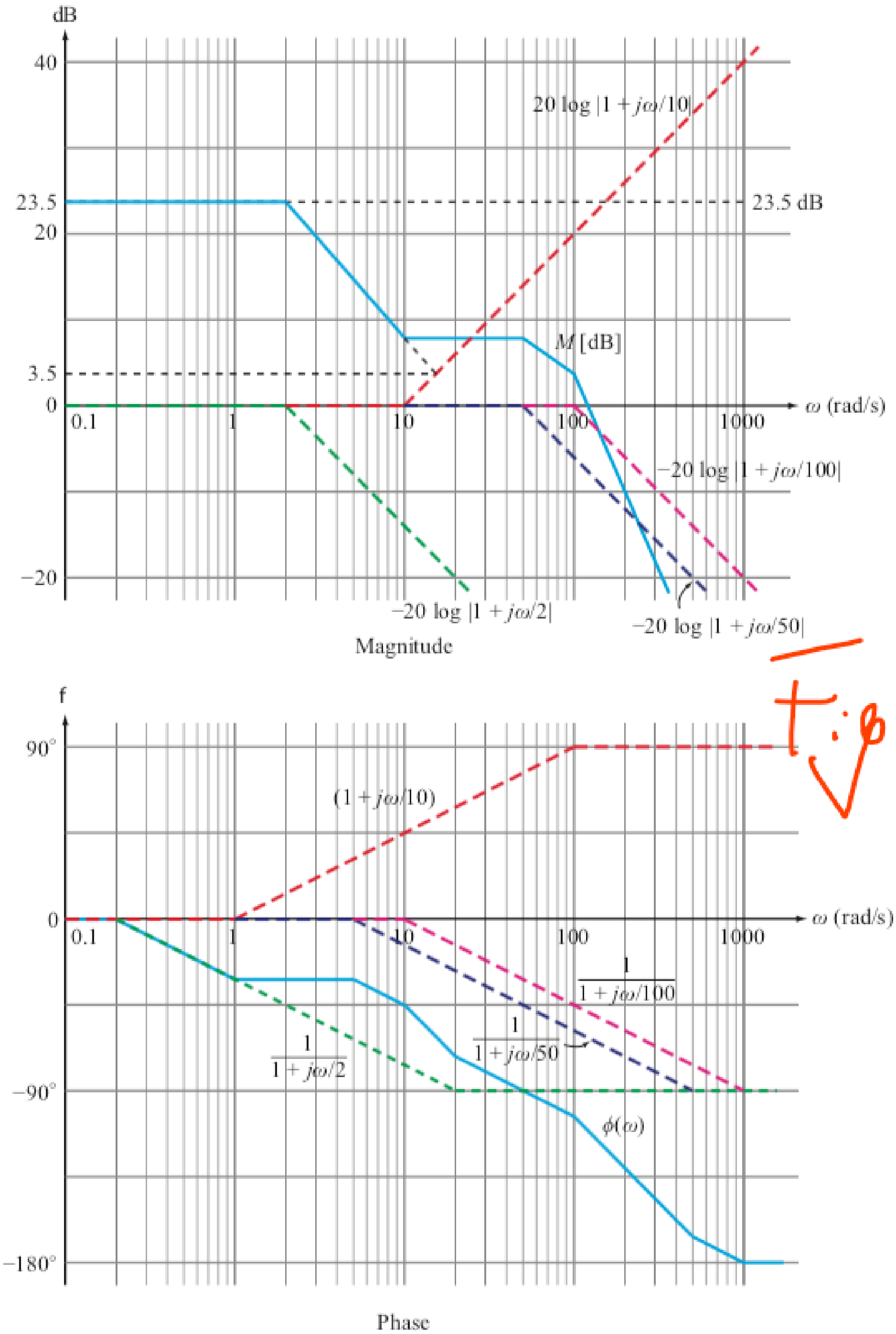

dB
40
23.5
20
3.5
0
−20
0.1
1
10
100
1000
ω (rad/s)
20 log |1 + jω/10|
23.5 dB
M [dB]
−20 log |1 + jω/100|
−20 log |1 + jω/2|
−20 log |1 + jω/50|
Magnitude
f
90°
0
−90°
−180°
(1 + jω/10)
1/(1 + jω/100)
1/(1 + jω/50)
1/(1 + jω/2)
φ(ω)
Phase

Fig. 1

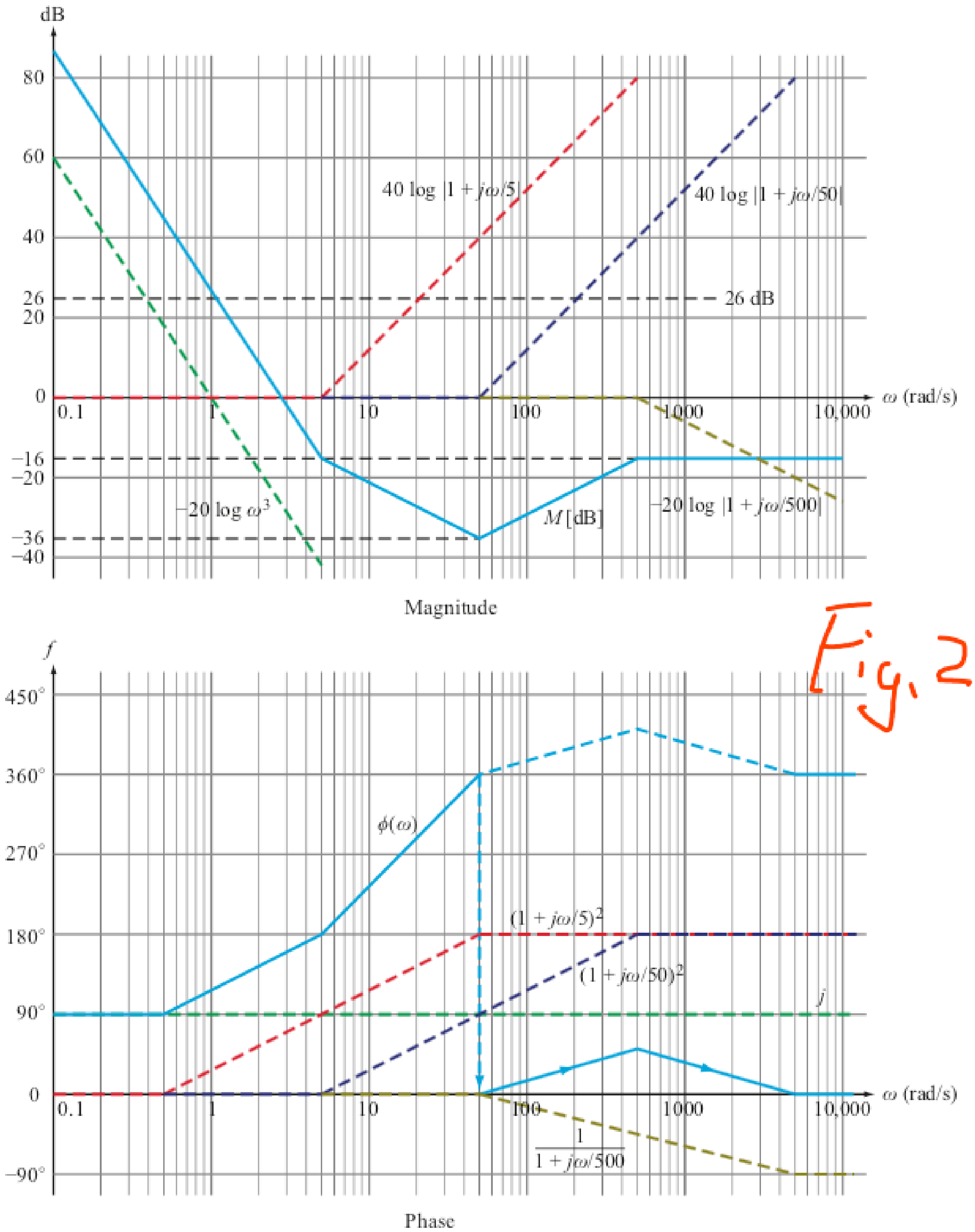

dB
80
60
40
26
20
0
−16
−20
−36
−40
0.1
1
10
100
1000
10,000
ω (rad/s)
40 log |1 + jω/5|
40 log |1 + jω/50|
26 dB
−20 log ω³
M [dB]
−20 log |1 + jω/500|
Magnitude
f
450°
360°
270°
180°
90°
0
−90°
φ(ω)
(1 + jω/5)²
(1 + jω/50)²
j
1/(1 + jω/500)
Phase

Fig. 2

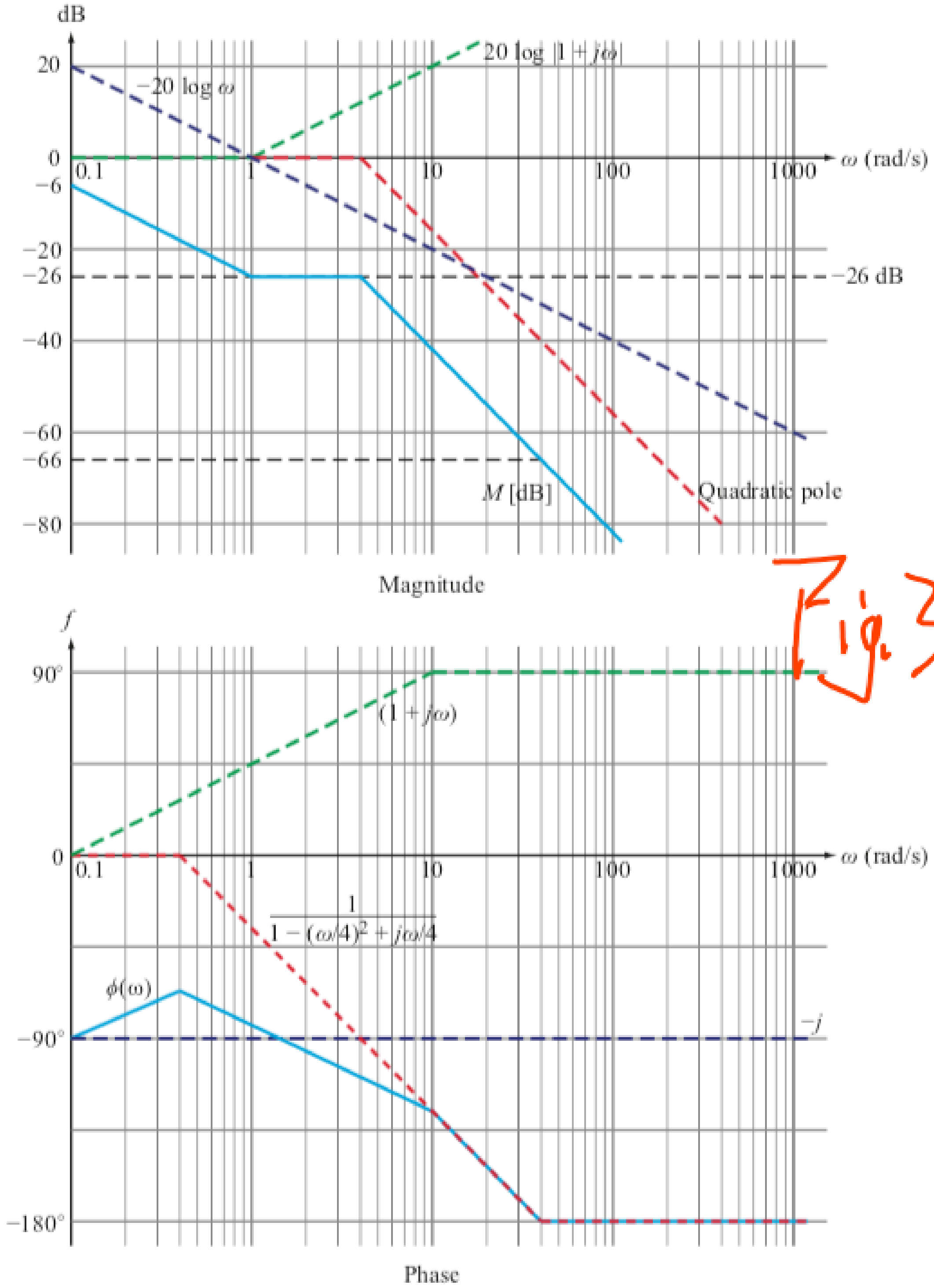

dB
20
0
−6
−20
−26
−40
−60
−66
−80
0.1
1
10
100
1000
ω (rad/s)
20 log |1 + jω|
−20 log ω
−26 dB
M [dB]
Quadratic pole
Magnitude
f
90°
(1 + jω)
$\frac{1}{1 - (\omega/4)^2 + j\omega/4}$
φ(ω)
−90°
−j
−180°
Phase

Fig. 3

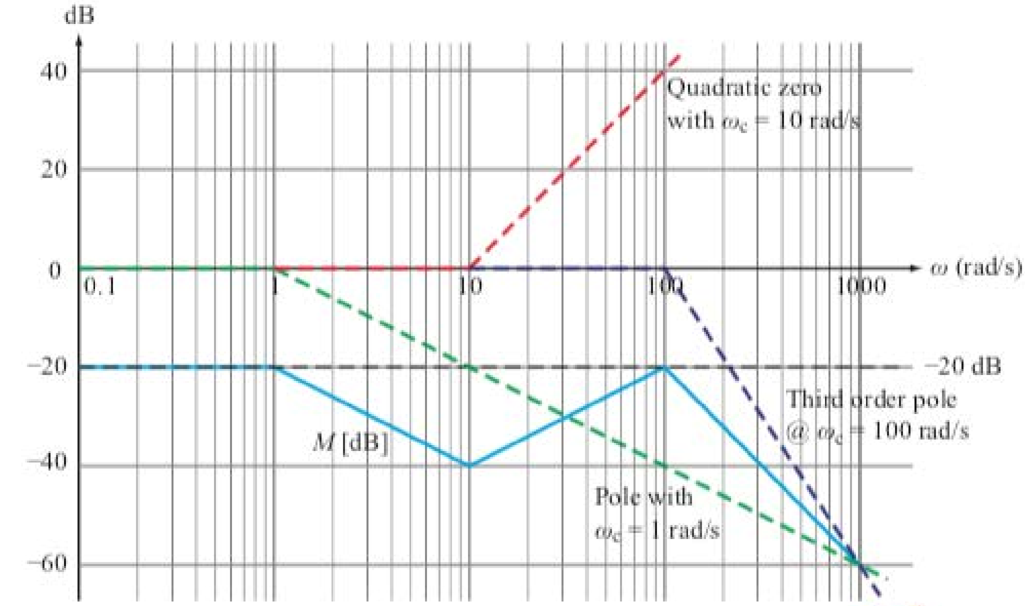
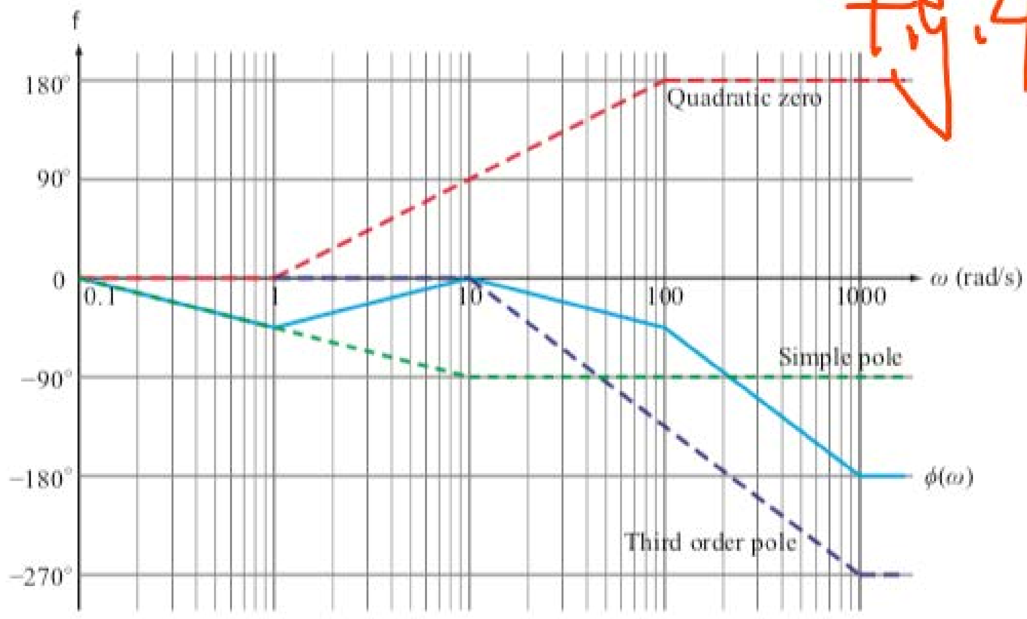

dB
40
20
0
-20
-40
-60
0.1
1
10
100
1000
ω (rad/s)
Quadratic zero
with ω_c = 10 rad/s
-20 dB
Third order pole
@ ω_c = 100 rad/s
M [dB]
Pole with
ω_c = 1 rad/s
Magnitude

Fig.4


f
180°
90°
0
-90°
-180°
-270°
0.1
1
10
100
1000
ω (rad/s)
Quadratic zero
Simple pole
φ(ω)
Third order pole
Phase

4. Determine the voltage transfer function **H(** ω **)** corresponding to the Bode magnitude plot shown below and corresponding information provided below:

(a) The phase of **H(ω)** is 90° at $\omega = 0$ in Fig 4.1.

(b) The phase of **H(ω)** is -90° at $\omega = 0$ in Fig 4.2.

(c) The phase of **H(ω)** is 0° at $\omega = 0$ in Fig 4.3.

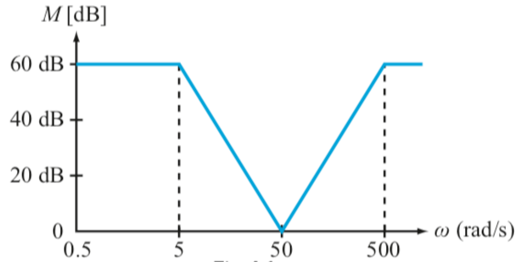


Fig 4.1

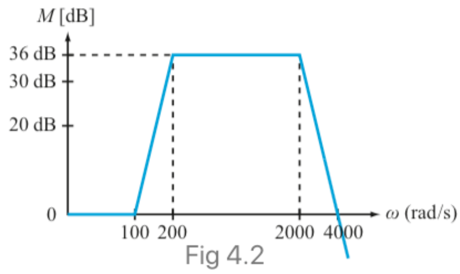


Fig 4.2

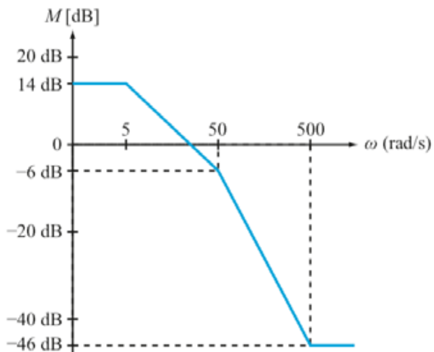


Fig 4.3

Solution: $\mathbf{H}(\omega)$ consists of:

(1) A constant term K whose dB value is 60 dB, or

$$K = 10^{60/20} = 1000.$$

(2) A simple pole of order 3 with $\omega_c = 5$ rad/s (slope $= -60$ dB/decade)

(3) A simple zero of order 6 with $\omega_c = 50$ rad/s (slope reverses from -60 dB/decade to $+60$ dB/decade)

(4) A simple pole of order 3 with $\omega_c = 500$ rad/s (slope changes to 0 dB at $\omega_c = 500$ rad/s).

Hence,

$$\mathbf{H}(\omega) = \frac{(j)^N 1000(1 + j\omega/50)^6}{(1 + j\omega/5)^3(1 + j\omega/500)^3} = \frac{j1000(50 + j\omega)^6}{(5 + j\omega)^3(500 + j\omega)^3}.$$

Given that the phase of $\mathbf{H}(\omega)$ is 90° at $\omega = 0$, it follows that $N = 1$.

Solution: The transfer function consists of:

(1) A simple zero of order N with $\omega_c = 100$ rad/s

(2) A simple pole of order N with $\omega_c = 200$ rad/s

(3) A simple pole of order N with $\omega_c = 2000$ rad/s

(4) A factor $-j$ (phase at $\omega = 0$ is $-90°$).

Hence,

$$\mathbf{H}(\omega) = \frac{-j(1+j\omega/100)^N}{(1+j\omega/200)^N(1+j\omega/2000)^N}.$$

To determine N, we note that the first term reaches 36 dB at $\omega = 200$ rad/s. That is, the straight-line approximation

$$20\log|1+\omega/100|^N \simeq 20N\log\frac{\omega}{100}\bigg|_{\omega=200} = 36\text{ dB},$$

or

$$20N\log 2 = 36\text{ dB} \qquad \Longrightarrow \qquad N = 6.$$

Hence,

$$\begin{aligned}\mathbf{H}(\omega) &= \frac{-j(200)^6(2000)^6(100+j\omega)^6}{(100)^6(200+j\omega)^6(2000+j\omega)^6}\\ &= \frac{-j4.096\times 10^{21}(100+j\omega)^6}{(200+j\omega)^6(2000+j\omega)^6}.\end{aligned}$$

Solution: The transfer function consists of:

(1) A constant K whose magnitude in dB is 14 dB.

(2) A simple pole factor with $\omega_c = 5$ rad/s.

(3) A simple pole factor with $\omega_c = 50$ rad/s (slope changes from -20 dB/decade to -40 dB/decade at $\omega = 50$ rad/s).

(4) A simple zero of order 2 at $\omega_c = 500$ rad/s (slope changes to zero).

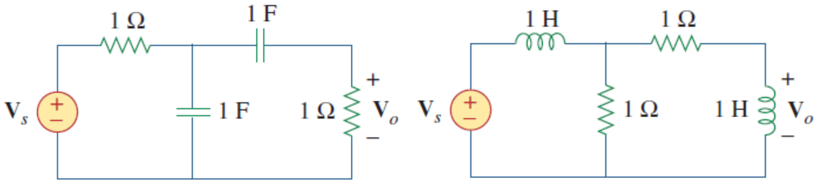
Hence,

$$K = 10^{14/20} = 5,$$

and

$$\begin{aligned}\mathbf{H}(\omega) &= \frac{5(1+j\omega/500)^2}{(1+j\omega/5)(1+j\omega/50)}\\ &= \frac{(500+j\omega)^2}{200(5+j\omega)(50+j\omega)}.\end{aligned}$$

(10分)5. Determine the center frequency and bandwidth of the bandpass filters in Fig.5



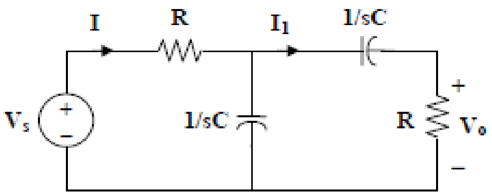
(a) (b)

注：由于本周才学Laplace变换，
这道题按s=jw计算，结果是等价的

Figure 5

Solution

(a) Consider the circuit below.



$$\mathbf{Z}(s) = R + \frac{1}{sC} \parallel \left(R + \frac{1}{sC}\right) = R + \frac{\frac{1}{sC}\left(R + \frac{1}{sC}\right)}{R + \frac{2}{sC}}$$

$$\mathbf{Z}(s) = R + \frac{1 + sRC}{sC(2 + sRC)}$$

$$\mathbf{Z}(s) = \frac{1 + 3sRC + s^2R^2C^2}{sC(2 + sRC)}$$ 1分

$$\mathbf{I} = \frac{\mathbf{V}_s}{\mathbf{Z}}$$

$$\mathbf{I}_1 = \frac{1/sC}{2/sC + R}\mathbf{I} = \frac{\mathbf{V}_s}{\mathbf{Z}(2 + sRC)}$$

$$\mathbf{V}_o = \mathbf{I}_1 R = \frac{R\mathbf{V}_s}{2 + sRC} \cdot \frac{sC(2 + sRC)}{1 + 3sRC + s^2R^2C^2}$$

$$\mathbf{H}(s) = \frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{sRC}{1 + 3sRC + s^2R^2C^2}$$

$$\mathbf{H}(s) = \frac{1}{3}\left[\frac{\frac{3}{RC}s}{s^2 + \frac{3}{RC}s + \frac{1}{R^2C^2}}\right]$$ 或 $$\mathrm{H}(\omega) = \frac{R}{3R + j(\omega CR^2 - \frac{1}{\omega C})}$$ 2分

Thus, $\omega_0^2 = \dfrac{1}{R^2C^2}$ or $\omega_0 = \dfrac{1}{RC} = 1\,\mathbf{rad/s}$ 1分

$B = \dfrac{3}{RC} = 3\,\mathbf{rad/s}$ 1分

每个结果无单位扣1分

(b) Similarly,

$$\mathbf{Z}(s) = sL + R \parallel (R + sL) = sL + \frac{R(R + sL)}{2R + sL}$$

$$\mathbf{Z}(s) = \frac{R^2 + 3sRL + s^2L^2}{2R + sL}$$ 1分

$$\mathbf{I} = \frac{\mathbf{V}_s}{\mathbf{Z}}, \qquad \mathbf{I}_1 = \frac{R}{2R + sL}\mathbf{I} = \frac{R\mathbf{V}_s}{\mathbf{Z}(2R + sL)}$$

$$\mathbf{V}_o = \mathbf{I}_1 \cdot sL = \frac{sLR\mathbf{V}_s}{2R + sL} \cdot \frac{2R + sL}{R^2 + 3sRL + s^2L^2}$$

$$\mathbf{H}(s) = \frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{sRL}{R^2 + 3sRL + s^2L^2} = \frac{\frac{1}{3}\left(\frac{3R}{L}s\right)}{s^2 + \frac{3R}{L}s + \frac{R^2}{L^2}}$$ 2分

或 $$\mathrm{H}(s) = \frac{R}{3R + j(\omega L - \frac{R^2}{\omega L})}$$

Thus, $\omega_0 = \dfrac{R}{L} = 1\,\mathbf{rad/s}$ 1分

$B = \dfrac{3R}{L} = 3\,\mathbf{rad/s}$ 1分

11分6. For the op-amp circuit of Fig. 6:

(a) Obtain an expression for H (ω) = Vo/V_i in standard form.

(b) Generate spectral plots for the magnitude and phase of H (ω), given that $R_1 = R_2$ =100Ω, and C_1 = 10μF, C_2 = 0.4μF.

(c) What type of filter is it? What is its maximum gain?

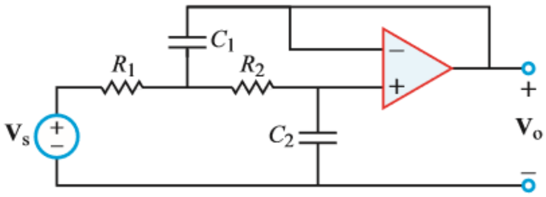
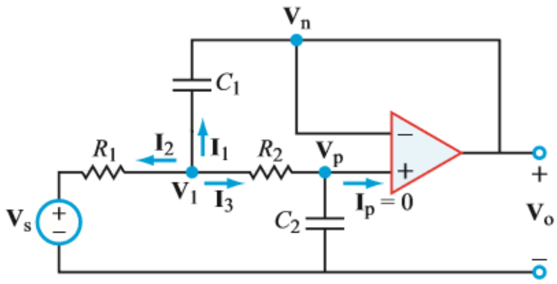


Figure 6



(a) At node $\mathbf{V}_1$:

$$\mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3 = 0,$$

or equivalently

$$\frac{\mathbf{V}_1 - \mathbf{V}_o}{1/j\omega C_1} + \frac{\mathbf{V}_1 - \mathbf{V}_s}{R_1} + \frac{\mathbf{V}_1}{R_2 + 1/j\omega C_2} = 0.$$

Also,

$$\mathbf{V}_p = \mathbf{V}_n = \mathbf{V}_o,$$

and by voltage division

$$\mathbf{V}_p = \frac{\mathbf{V}_1/j\omega C_2}{R_2 + 1/j\omega C_2}.$$ 1分

Simultaneous solution leads to:

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{1}{1 + j\omega(R_1 + R_2)C_2 + (j\omega\sqrt{R_1 R_2 C_1 C_2})^2}$$

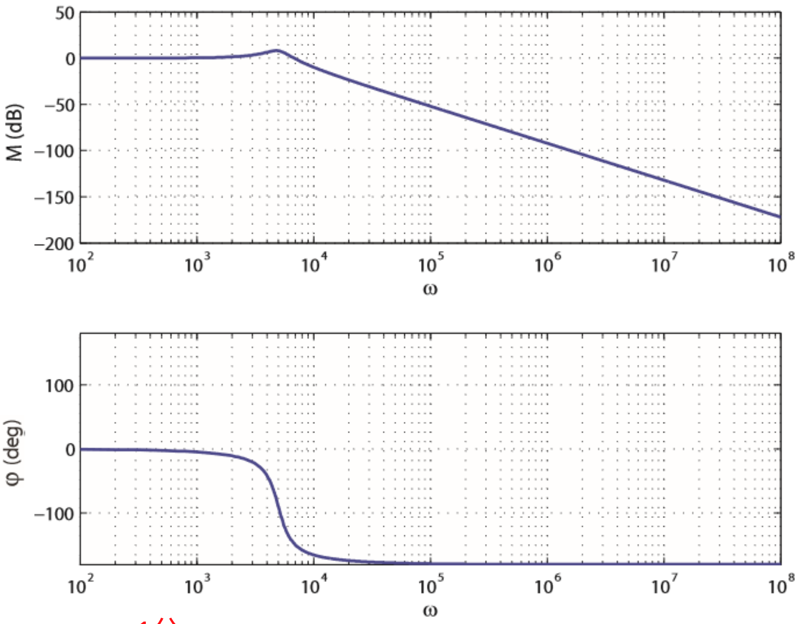
$$= \frac{1}{1 + j2\xi\,\omega/\omega_c + (j\omega/\omega_c)^2},$$ 2分，这一问没有标明数值，写到这一步或上一步即可

with

$$\omega_c = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}} = \frac{1}{\sqrt{100 \times 100 \times 10^{-5} \times 0.4 \times 10^{-6}}} = 5000 \text{ rad/s},$$

$$\xi = \frac{(R_1 + R_2)C_2\omega_c}{2} = 100 \times 0.4 \times 10^{-6} \times 5000 = 0.2.$$

(b) Spectral plots of the transfer function are shown in Fig. P9.39(b) and (c).



3分。趋势正确:1分；标出坐标轴单位:1分；标出斜率：1分

2分。趋势正确:1分；标出坐标轴单位:1分

1分

(c) This is a low pass filter with a slope of −40 dB/decade at frequencies much greater than $\omega_c = 5000$ rad/s. Maximum gain (at dc) is 0 dB.

1分

8分 7. Design the filter in Fig. 7 to meet the following requirements, given R = 10k Ω:
- It must attenuate a signal at 2 kHz by 3 dB compared with its value at 10 MHz.
- It must provide a steady-state output of $v_o(t) = 10\sin(2\pi \times 10^8 t + 180^\circ)\,V$ for an input $v_s(t) = 10\sin(2\pi \times 10^8 t)V$.

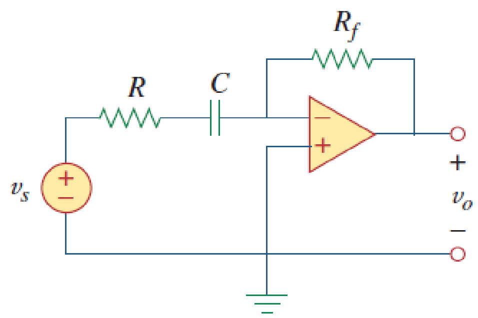


Figure 7

Solution

This is a highpass filter with $f_c = 2$ kHz. 3分

$$\omega_c = 2\pi f_c = \frac{1}{RC}$$

$$RC = \frac{1}{2\pi f_c} = \frac{1}{4\pi \times 10^3}$$

10^8 Hz may be regarded as high frequency. Hence the high-frequency gain is

$$\frac{-R_f}{R} = \frac{-10}{4} \qquad \text{or} \qquad R_f = 2.5R$$

2分，比值应为-1

1分，R_f=R=10kΩ　2分

If we let $R = \mathbf{10\ k\Omega}$, then $R_f = \mathbf{25\ k\Omega}$, and $C = \dfrac{1}{4000\pi \times 10^4} = \mathbf{7.96\ nF}$.

虽然是设计题，但参数R给定，答案基本固定，如有不同于参考答案的参数设计，酌情给分

11分 8. In the circuit shown below, find current I_0.

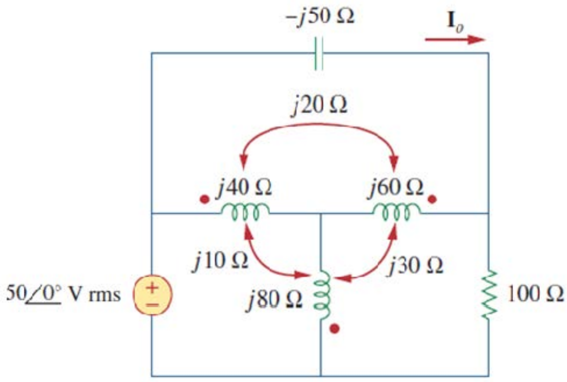
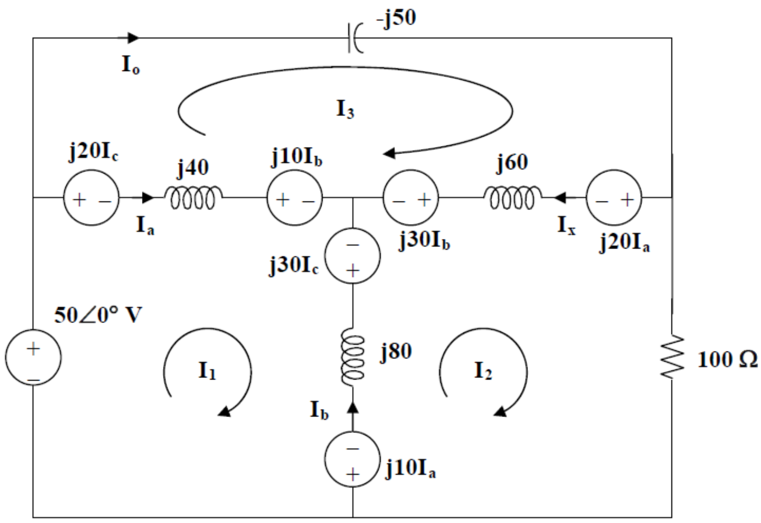


Figure 8

Solution:



Note the following,

$$Ia = I_1 - I_3$$

$$Ib = I_2 - I_1$$

$$Ic = I_3 - I_2$$

And Io = I_3.

Loop # 1,

1. $-50 + j20(I_3 - I_2)\ j\,40(I_1 - I_3) + j10(I_2 - I_1) - j30(I_3 - I_2) + j80(I_1 - I_2) - j10(I_1 - I_3) = 0$

2. $j100I_1 - j60I_2 - j40I_3 = 50$ 3分

Multiplying everything by (1/j10) yields :

$10I_1 - 6I_2 - 4I_3 = -\,j5$

Loop # 2,

$j10(I_1 - I_3) + j80(I_2 - I_1) + j30(I_3 - I_2) - j30(I_2 - I_1) + j60(I_2 - I_3) - j20(I_1 - I_3) + 100I_2 = 0$

$-j60I_1 + (100 + j80) I_2 - j20I_3 = 0$ (2) 3分

Loop # 3,

$-j50I_3 + j20(I_1 - I_3) + j60(I_3 - I_2) + j30(I_2 - I_1) - j10(I_2 - I_1) + j40(I_3 - I_1) - j20(I_3 - I_2) = 0$

$-j40I_1 - j20I_2 + j10I_3 = 0$ 3分

Multiplying by (1/j10) yields,

$-4I_1 - 2I_2 + I_3 = 0$ (3)

Multiplying (2) by (1/j20) yields

$-3I_1 + (4 - j5) I_2 - I_3 = 0$ (4)

Multiplying (3) by (1/4) yields

$-I_1 - 0.5I_2 - 0.25I_3 = 0$ (5)

Multiplying (4) by (-1/3) yields

$I_1 - ((4/3) - j(5/3)) I_2 + (1/3) I_3 = -j0.5$ (7)

Multiplying [(6)+(5)] by 12 yields

$(-22 + j20) I_2 + 7I_3 = 0$ (8)

Multiplying [(5)+(7)] by 20 yields

$-22I_2 - 3I_3 = -j10$ (9)

(8) leads to

$I_2 = -7I_3/(-22 + j20) = 0.2355\angle\mathbf{42.3}^{\circ} = (0.17418+j0.15849) I_3$ (10)

(9) leads to

$I_3 = (j10 - 22I_2)/3$, substituting (1) into this equation produces,

$I_3 = j3.333 + (-1.2273 - j1.1623) I_3$

So $I_3 = I_0 = \mathbf{1.3040\angle 63^{\circ}}$ **amp.** 2分，中间过程酌情给分

10分 Electric Circuit, Spring 2018 Homework 9 Due: Jun. 4th

10分 9. In the ideal transformer circuit shown below, determine the average power delivered to the load (the 20-j40Ω resistance).

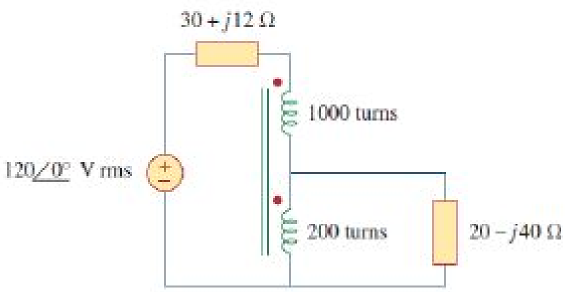


Figure 9

Solution

$$I_1/I_2 = N_2/(N_1 + N_2) = 200/1200 = 1/6, \text{ or } I_1 = I_2/6 \quad (1)$$ 2分 2分 1)

$$v_1/v_2 = (N_1 + N_2)/N_2 = 6, \text{ or } v_1 = 6v_2 \quad (2)$$ 2分

For the primary loop, $120 = (30 + j12)I_1 + v_1$ (3) 2分 2分 2)

For the secondary loop, $v_2 = (20 - j40)I_2$ (4) 1分 2分 3)

Substituting (1) and (2) into (3), 1分 4)

$$120 = (30 + j12)(I_2/6) + 6v_2$$

and substituting (4) into this yields

$$\cancel{120 = (49 - j38)I_2 \text{ or } I_2 = 1.935\angle 37.79°}$$ 1分

$$p = |I_2|^2(20) = \cancel{74.9 \text{ watts}}$$ 2分，无单位扣1分

120=(125−j238) I2 1分

3.985W

2分，无单位扣1分

I2=0.446∠63°