CS121 Parallel Computing
Problem Set 9

1) Show that Gaussian elimination on an $n \times n$ matrix 2D partitioned on an $n \times n$ logical mesh of processes is not cost-optimal if the $2n$ one-to-all broadcasts are performed synchronously.

2) Prove that the isoefficiency function of the asynchronous version of Gaussian elimination with 2D mapping is $\Theta\left(p^{3/2}\right)$.

3) In a p processor CREW PRAM architecture (for $p \geq 1$), a set of p parallel processors are connected to a shared memory. The processors repeatedly perform a series of basic cycles, where in each cycle all the processors in parallel perform a logical (e.g. an *if*) or arithmetic operation, then in parallel perform a memory access (read or write) to an arbitrary address; different processors may perform different operations and access different addresses. CREW stands for *concurrent read exclusive write*, so that processors are allowed to read the same memory location in a cycle, but not write to the same location. Assume that a logical or arithmetic operation takes t_c time, and a memory access takes t_m time, so that each basic cycle takes $t_c + t_m$ parallel time.
   Consider the parallel algorithm shown below for multiplying two $n \times n$ matrices A and B to obtain a product matrix C. Determine the parallel running time for this algorithm on an n^2 processor CREW PRAM. Is this parallel algorithm cost-optimal?

```
1.   procedure MAT_MULT_CREW_PRAM (A, B, C, n)
2.   begin
3.      Organize the n^2 processes into a logical mesh of n x n;
4.      for each process P_{i,j} do
5.      begin
6.         C[i, j] := 0;
7.         for k := 0 to n - 1 do
8.             C[i, j] := C[i, j] + A[i, k] x B[k, j];
9.      endfor;
10.  end MAT_MULT_CREW_PRAM
```

4) Consider the Kernighan-Lin partitioning algorithm described in lecture. Design pseudocode for the algorithm, and analyze the time complexity of your code.