1. Determine the Laplace transform and the associated region of convergence and pole-zero plot for each of the following functions of time:

(a) $x(t) = e^{-2t}u(t) + e^{-3t}u(t)$

(b) $x(t) = e^{-4t}u(t) + e^{-5t}(sin5t)u(t)$

(c) $x(t) = \delta(t) + u(t)$

(d) $x(t) = te^{-2|t|}$

2. Consider a continuous-time LTI system for which the input x(t) and output y(t) are related by the differential equation

$$\frac{d^2y(t)}{dt^2} - \frac{dy(t)}{dt} - 2y(t) = x(t).$$

Let X(s) and Y(s) denote Laplace transforms of x(t) and y(t), respectively, and let H(s) denote the Laplace transform of h(t), the system impulse response.

(a) Determine H(s) as a ratio of two polynomials in s. Sketch the pole-zero pattern of H(s).

(b) Determine h(t) for each of the following cases:

1. The system is stable.
2. The system is causal.
3. The system is neither stable nor causal.

3. Suppose we are given the following information about a causal and stable LTI system S with impulse response $h(t)$ and a rational system function $H(s)$:

1. $H(1) = 0.2$.
2. When the input is $u(t)$, the output is absolutely integrable.
3. When the input is $tu(t)$, the output is not absolutely integrable.
4. The signal $d^2h(t)/dt^2 + 2\,dh(t)/dt + 2h(t)$ is of finite duration.
5. $H(s)$ has exactly one zero at infinity.

Determine $H(s)$ and its region of convergence.

9.36. In this problem, we consider the construction of various types of block diagram representations for a causal LTI system S with input $x(t)$, output $y(t)$, and system function

$$H(s) = \frac{2s^2 + 4s - 6}{s^2 + 3s + 2}.$$

To derive the direct-form block diagram representation of S, we first consider a causal LTI system S_1 that has the same input $x(t)$ as S, but whose system function is

$$H_1(s) = \frac{1}{s^2 + 3s + 2}.$$

With the output of S_1 denoted by $y_1(t)$, the direct-form block diagram representation of S_1 is shown in Figure P9.36. The signals $e(t)$ and $f(t)$ indicated in the figure represent respective inputs into the two integrators.

(a) Express $y(t)$ (the output of S) as a linear combination of $y_1(t)$, $dy_1(t)/dt$, and $d^2y_1(t)/dt^2$.

(b) How is $dy_1(t)/dt$ related to $f(t)$?

(c) How is $d^2y_1(t)/dt^2$ related to $e(t)$?

(d) Express $y(t)$ as a linear combination of $e(t)$, $f(t)$, and $y_1(t)$.

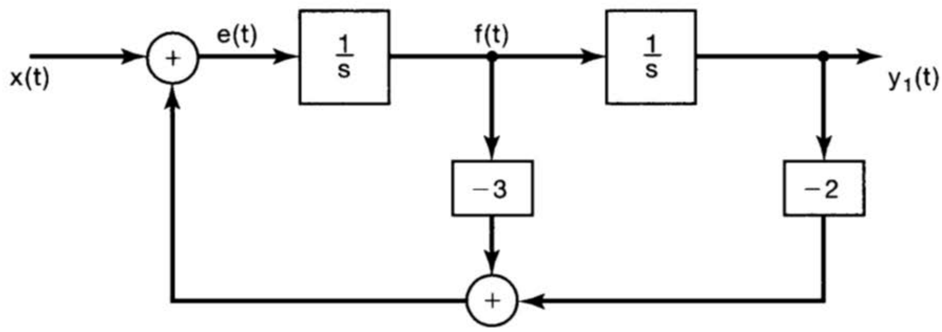

Figure P9.36

5. Consider the system S characterized by the differential equation

$$\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 6y(t) = x(t)$$

(a) Determine the zero-state response of this system for the input $x(t) = e^{-4t}\,u(t)$.

(b) Determine the zero-input response of the system for $t > 0^-$, given that

$$y(0^-) = 1, \qquad \left.\frac{dy(t)}{dt}\right|_{t=0^-} = -2$$

(c) Determine the output of S when the input is $x(t) = e^{-4t}\,u(t)$ and the initial conditions are the same as those specified in part (b).