Signals and Systems Homework 7

Due Time: 21:59 May 4, 2018
Submitted in-class on Thu (May 4),
or to the box in front of SIST 1C 403E (the instructor's office).

1. (20 points) The following are discrete-time signals and Fourier transforms. Determine the signal/FT for each one.
   (a) $x_1[n] = (\frac{1}{2})^{|n-1|}$
   (b) $\sin(\frac{\pi}{3}n + \frac{\pi}{4})$ (Determine the Fourier transform for $-\pi \leq \omega < \pi$. Hint: It's the Fourier transform for periodic signals).
   (c) $X_1(jw) = \frac{e^{-jw} - \frac{1}{5}}{1 - \frac{1}{5}e^{-jw}}$
   (d) $X_2(jw) = \sum_{k=-\infty}^{+\infty}(-1)^k \delta(\omega - \frac{\pi}{2}k)$

2. (15 points) Given that $x[n]$ has Fourier transform $X(jw)$, express the Fourier transforms of the following signals in the terms of $X(jw)$.
   (a) $x_1[n] = x[1-n] + x[-1-n]$.
   (b) $x_2[n] = \frac{x^*[-n] + x[n]}{2}$.
   (c) $x_3[n] = (n-1)^2 x[n]$

3. (15 points) Let
$$y[n] = (\frac{\sin\frac{\pi}{4}n}{\pi n})^2 * (\frac{\sin \omega_c n}{\pi n})$$
   where $*$ denotes convolution and $\mid \omega_c n \mid \leq \pi$. Determine a stricter constraint on $\omega_c n$, which ensures that
$$y[n] = (\frac{\sin\frac{\pi}{4}n}{\pi n})^2$$

4. (15 points) Let $x_1[n]$ be the discrete-time signal whose Fourier transform $X_1(jw)$ is depicted in Figure 1. Consider the signal $x_2[n]$ with Fourier transform $X_2(jw)$, as illustrated in Figure 2. Please express $x_2[n]$ in terms of $x_1[n]$.

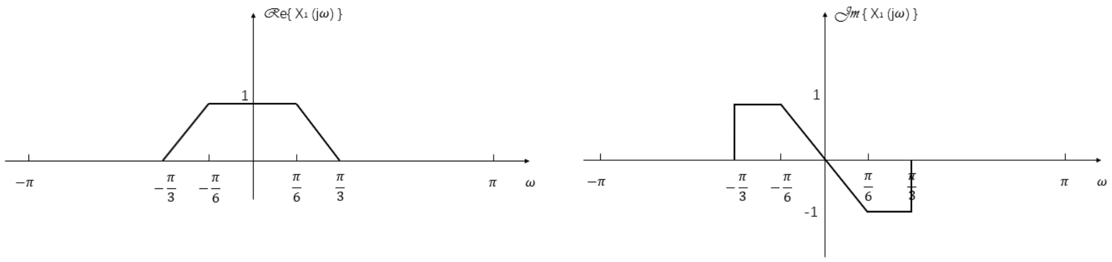

Figure 1: The real and imaginary parts of the Fourier transform $X_1(jw)$

5. (15 points) Let $x[n] = e^{jwn}$ for $0 \leq n < N$ and let $X[k]$ be the DFT of $x[n]$.

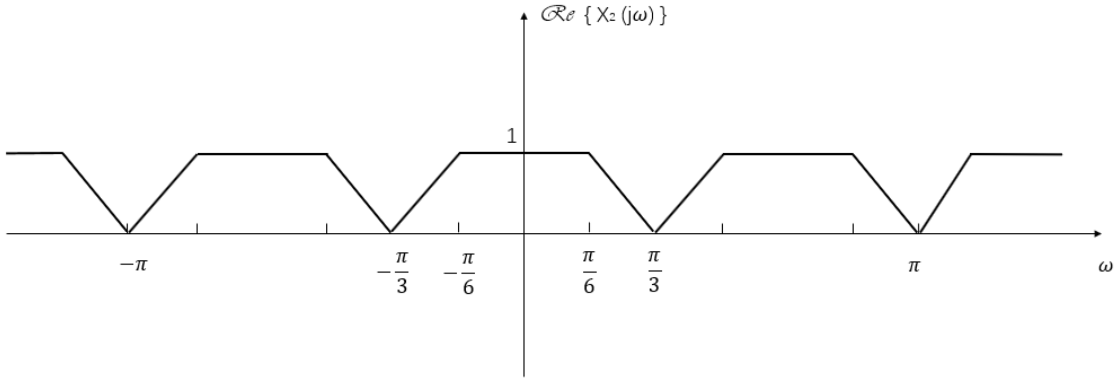

Figure 2: the Fourier transform $X_2(jw)$

(a) Calculate a simplified expression for $X[k]$ that is correct for any value of ω.

(b) Calculate a simplified expression for $X[k]$ when $\omega = 2\pi m/N$ where m is an integer. And sketch a plot of $\mid X[k] \mid$

6. (20 points) Let $x[n]$ be a signal of finite duration, that is, there is an integer N so that

$$x[n] = 0 \qquad \text{outside the interval } 0 \leq n \leq N-1$$

The DFT of $x[n]$ is denoted by $X[k]$, and $X(j\omega)$ denote the Fourier transform of $x[n]$.

(a) Show that

$$X[k] = \frac{1}{N} X(j(2\pi k/N))$$

(b) Let us consider samples of $X(j\omega)$ taken every $\frac{2\pi}{M}$, where $M < N$. These samples correspond to more than one sequence of duration N. To illustrate this, consider the two signals $x_1[n]$ and $x_2[n]$ depicted in Figure 3. Show that if we choose $M = 4$, we have

$$X_1(2\pi k/4) = X_2(j(2\pi k/4))$$

for all values of k.

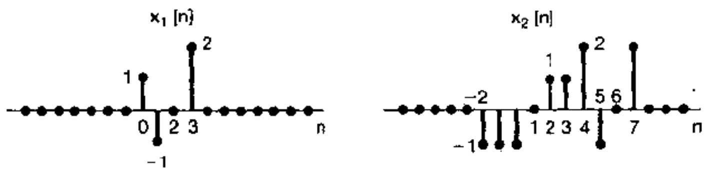

Figure 3: $x_1[n]$ and $x_2[n]$