Lecture 6-1 Parametric Transform and Scattered Data Interpolation

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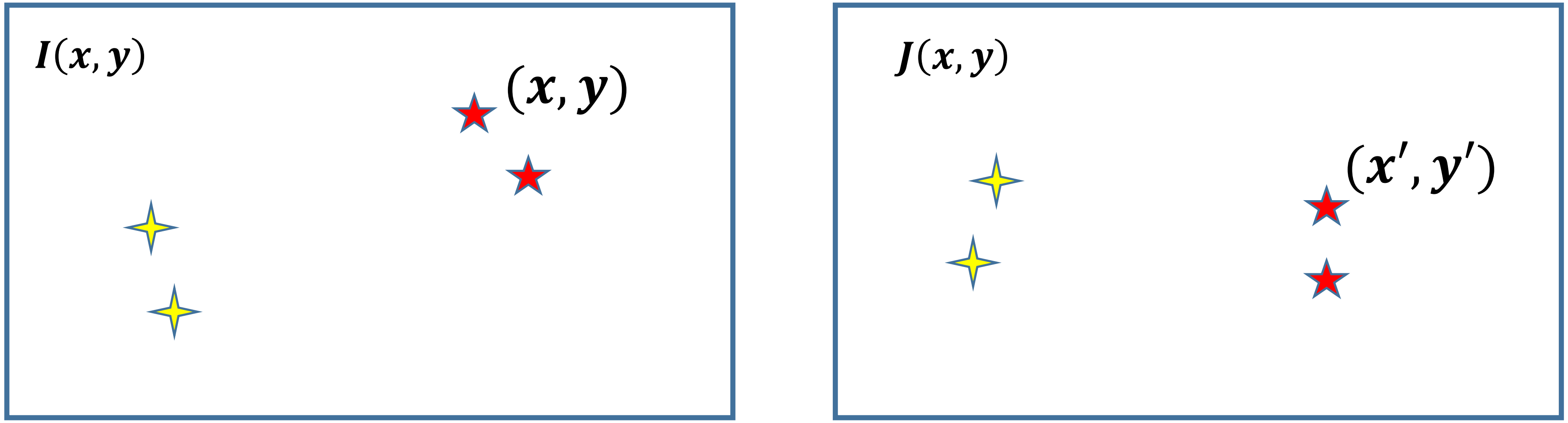
Course piazza link： piazza.com/shanghaitech.edu.cn/spring2021/cs270spring2021

Outline

- Dense correspondence
  - -Simplest Image Registration
- Scattered data interpolation
  - -Thin Plate Spline Interpolation

Dense correspondence

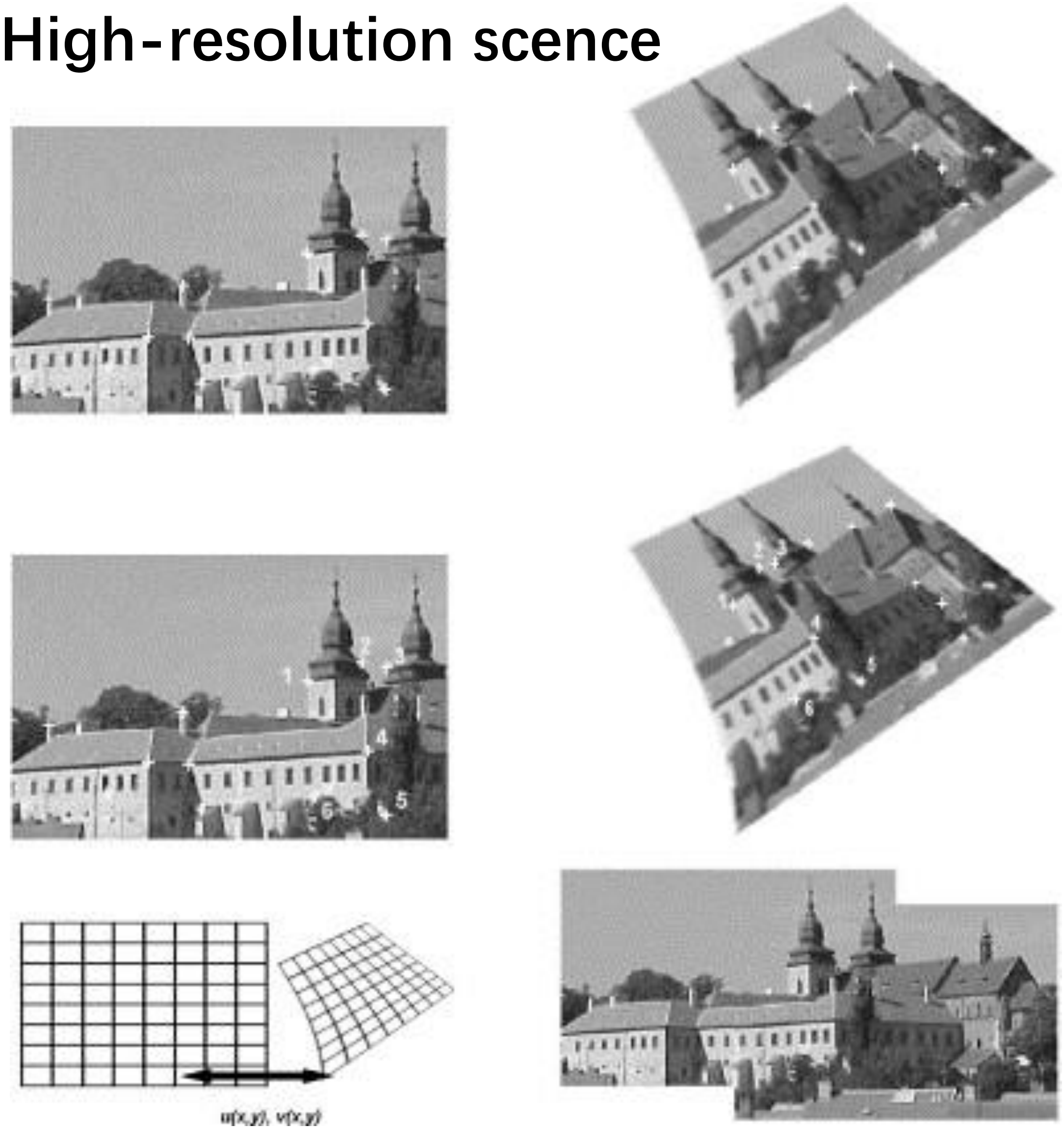


- For every pixel (x, y) we want to find a motion vector (u, v) so that $I(x, y) = J(x + v, y + u) = J(x', y')$.

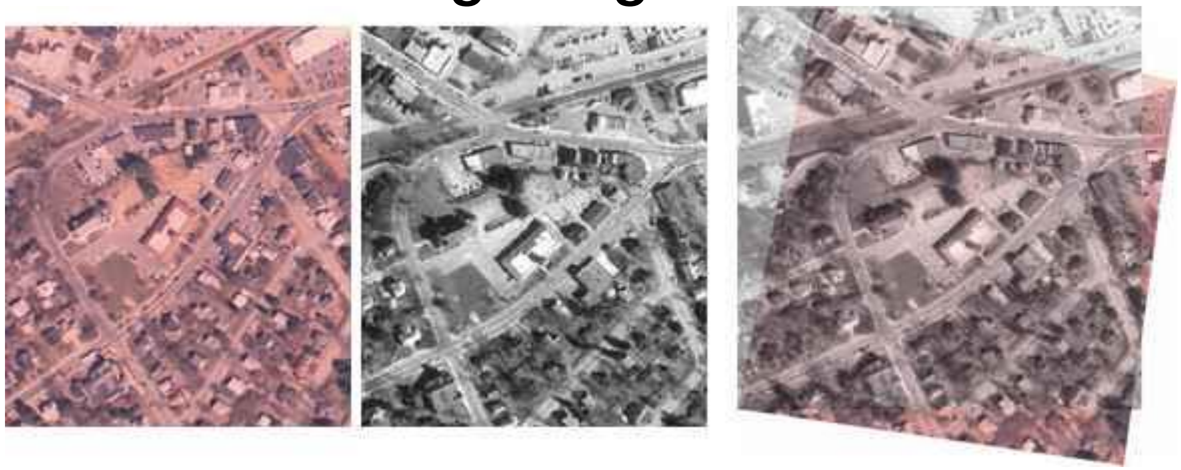
- Where u and v are easy functions of (x, y).

Application examples

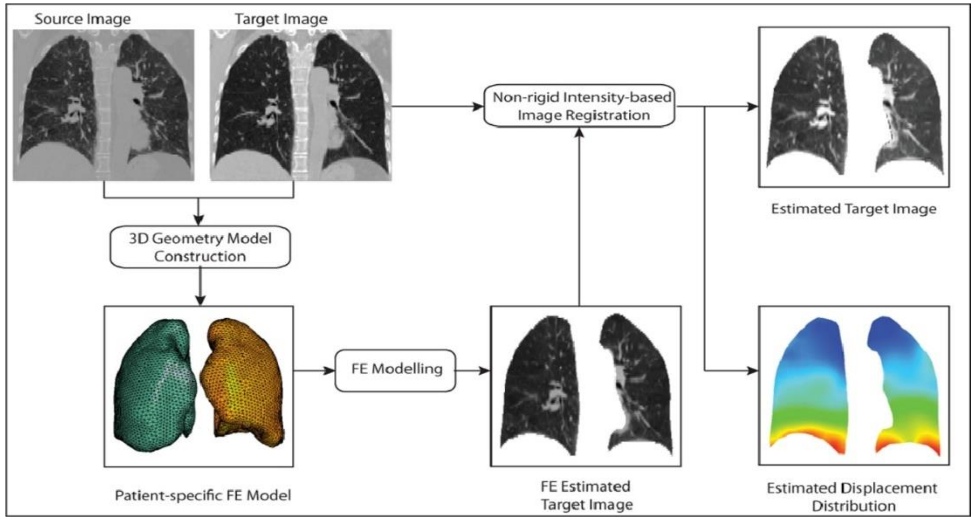
High-resolution scence



Remote sensing image

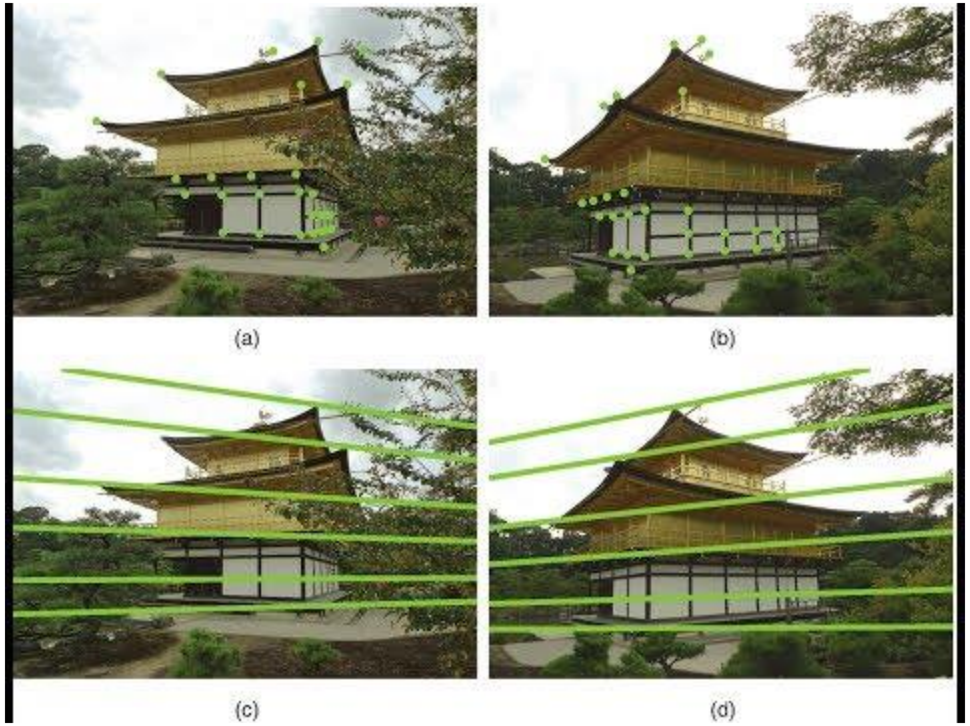


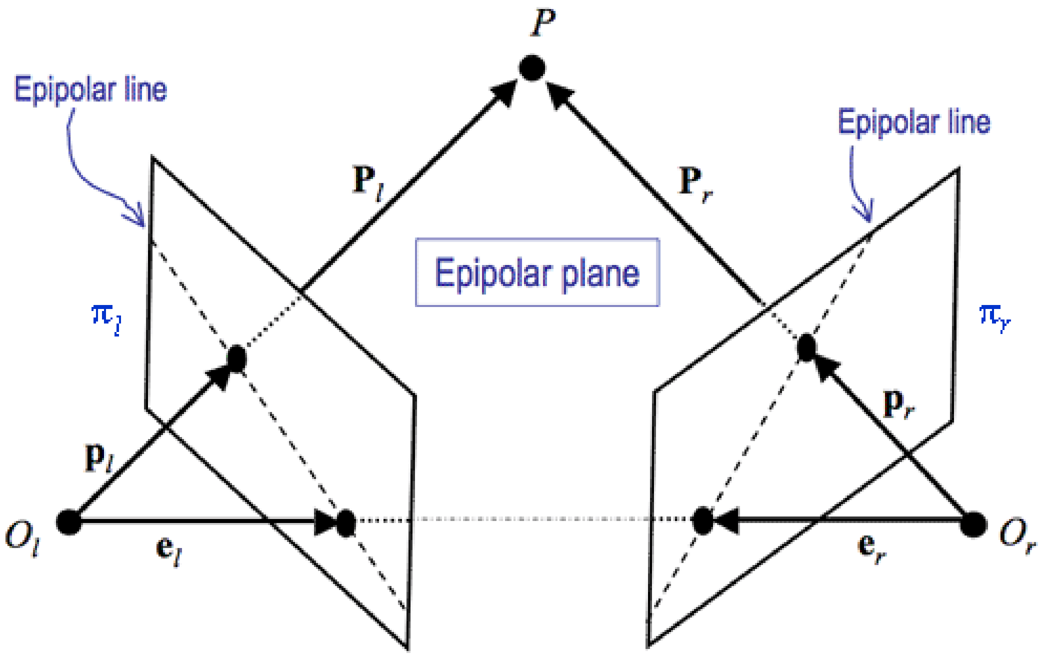
Medical image



Other extensions

Epipolar geometry





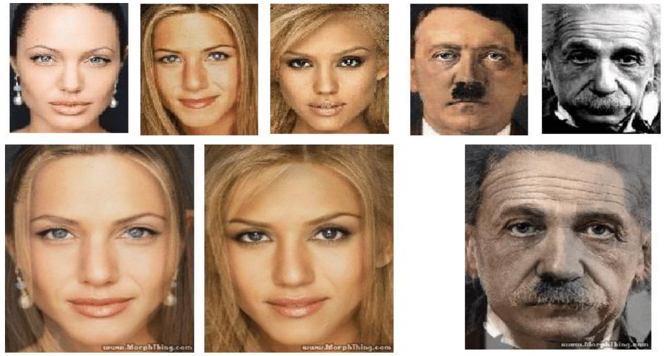


Photo morphing

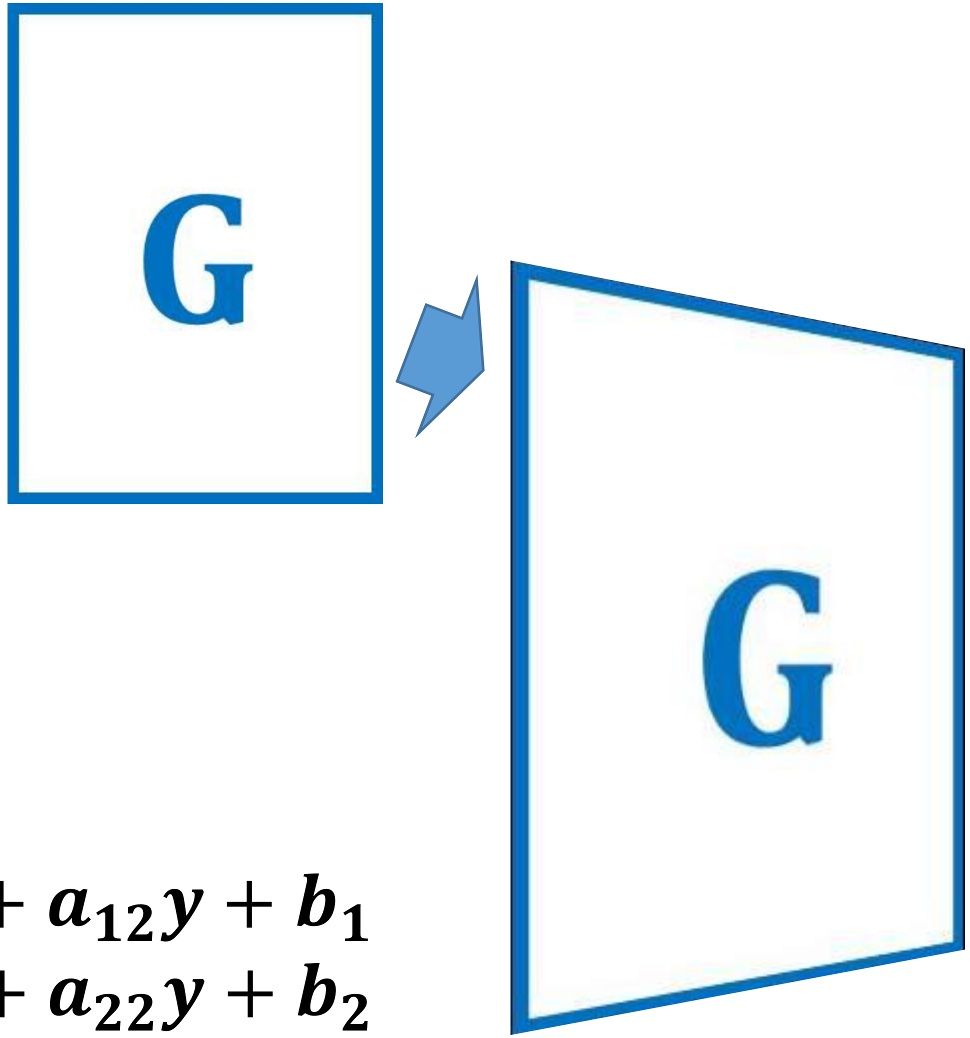
Projective transformation

- $$\begin{bmatrix} \tilde{x}' \\ \tilde{y}' \\ \tilde{z}' \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ c_1 & c_2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- $$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \dfrac{a_{11}x+a_{12}y+b_1}{c_1x+c_2y+1} \\ \dfrac{a_{21}x+a_{22}y+b_2}{c_1x+c_2y+1} \end{bmatrix} = \begin{bmatrix} \dfrac{\tilde{x}'}{\tilde{z}'} \\ \dfrac{\tilde{y}'}{\tilde{z}'} \end{bmatrix}$$

- Then we have:

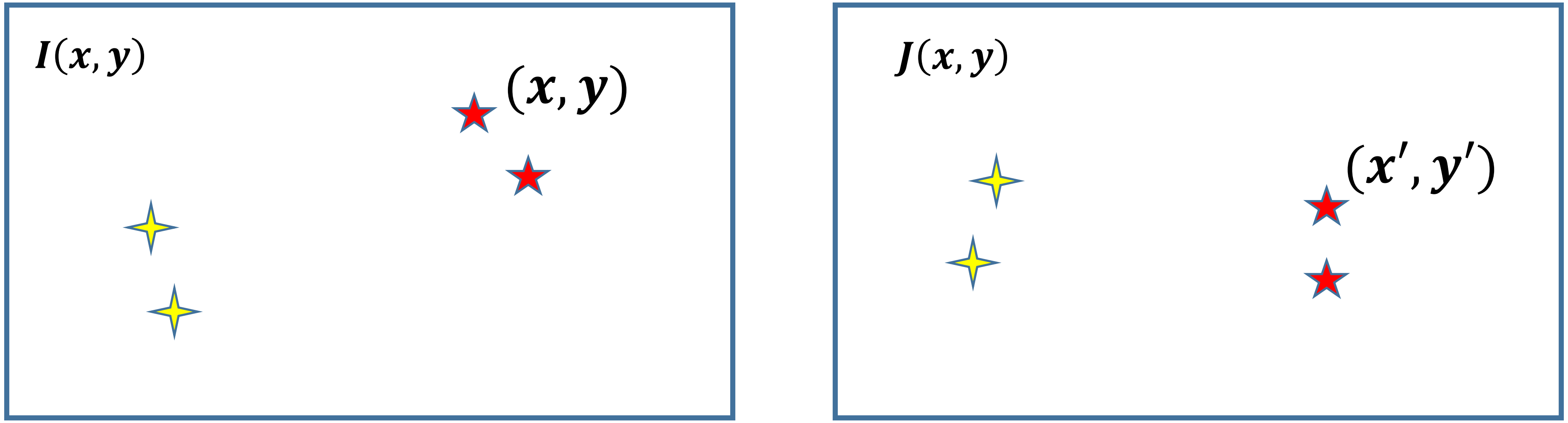
$$(c_1x + c_2y + 1)x' = a_{11}x + a_{12}y + b_1$$
$$(c_1x + c_2y + 1)y' = a_{21}x + a_{22}y + b_2$$



Linear equations

$$(c_1x + c_2y + 1)x' = a_{11}x + a_{12}y + b_1$$
$$(c_1x + c_2y + 1)y' = a_{21}x + a_{22}y + b_2$$

- We already know: x', y', x, y
- We don't know: a_{11}, a_{12}, a_{21}, a_{22}, b_1, b_2, c_1, c_2



Linear equations

$$(c_1 x_i + c_2 y_i + 1)x_i{}' = a_{11}x_i + a_{12}y_i + b_1$$

$$(c_1 x_i + c_2 y_i + 1)y_i{}' = a_{21}x_i + a_{22}y_i + b_2$$

$$a_{11}x_i + a_{12}y_i + b_1 - c_1 x_i x_i{}' - c_2 y_i x_i{}' = x_i{}'$$

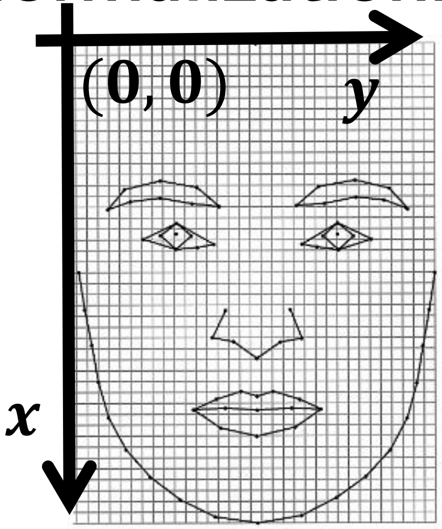
$$a_{21}x_i + a_{22}y_i + b_2 - c_1 x_i y_i{}' - c_2 y_i y_i{}' = y_i{}'$$

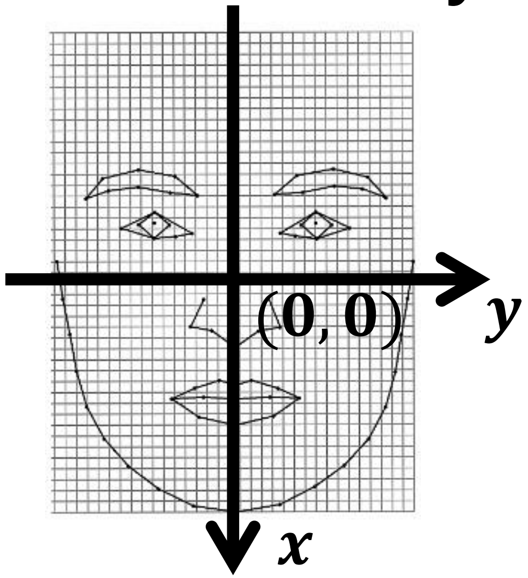
$$\begin{bmatrix} x_i & y_i & 0 & 0 & 1 & 0 & -x_i\, x_i' & -y_i\, x_i' \\ 0 & 0 & x_i & y_i & 0 & 1 & -x_i\, y_i' & -y_i\, y_i' \end{bmatrix} \begin{bmatrix} a_{11} \\ a_{12} \\ a_{21} \\ a_{22} \\ b_1 \\ b_2 \\ c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} x_i' \\ y_i' \end{bmatrix}$$

Every 4 pairs of landmarks determine 8 parameters

Some tips

- Pre-normalization: numerical safer way





- Outlier rejection: ex. RANSAC

Outline

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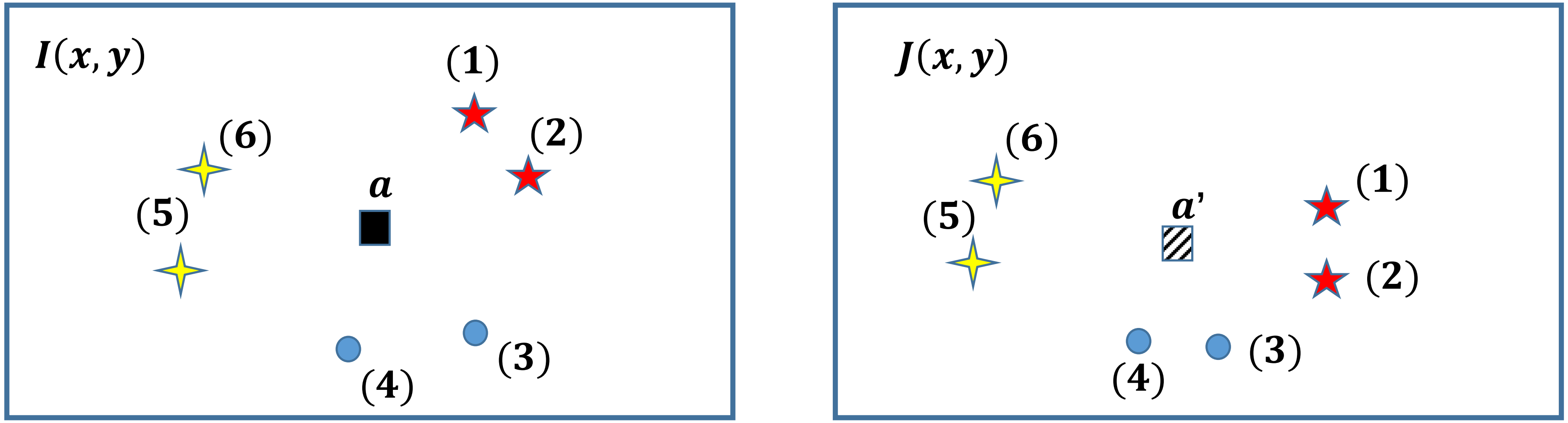
- Scattered data interpolation
  - -Thin Plate Spline Interpolation

Scattered Data interpolation



- What we know: pixel (1) (2) (3) (4) (5) (6)
- What we don't know: pixel a and a'
- Key idea: image interpolation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38
38

Thin plate spline interpolation

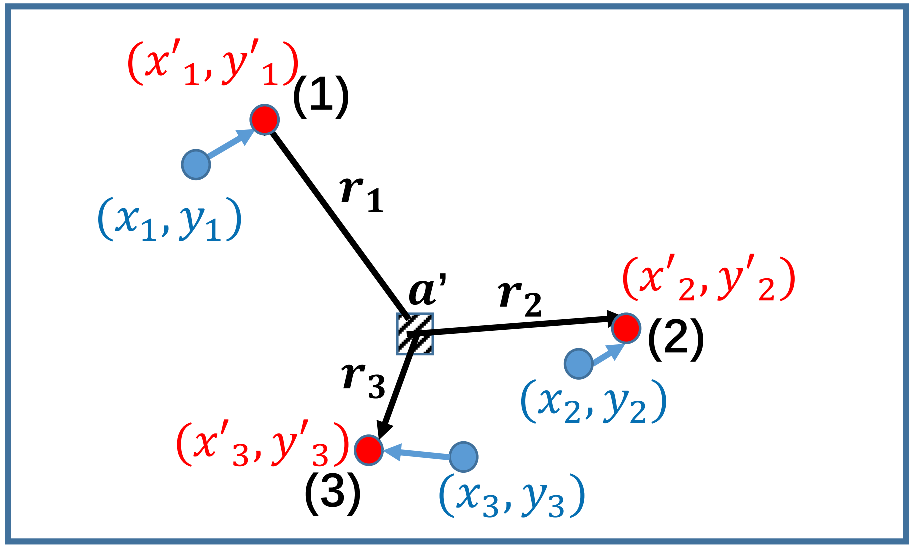
- Suggested for image registration by Ardi Goshtasby in 1988 [IEEE Trans. Geosci. and Remote Sensing, vol 26, no.1, 1988]
- Based on an analogy to the approximate shape of thin metal plates deflected by normal forces at discrete points.

Thin plate spline interpolation

- To describe the unknown pixel $\boldsymbol{a'}$
- Find its distance to each of the $\boldsymbol{N}$ localized points.

$$\boldsymbol{x' = \sum_{i=1}^{N} \omega_{1i}\, \varphi(r_i) + a_{11}x + a_{12}y + b_1}$$

$$\boldsymbol{y' = \sum_{i=1}^{N} \omega_{2i}\, \varphi(r_i) + a_{12}x + a_{22}y + b_2}$$



Radio-basis function

$$\varphi(r_i) = {r_i}^2 log r_i$$

$${r_i}^2 = \|(x, y) - (x_i - y_i)\|_2$$

Thin plate spline interpolation

- For N pairs of $(x, y) \longrightarrow (x', y')$
- Find the $6 + 2N$ coefficients a_{11}, a_{12}, a_{21}, a_{22}, b_1, b_2,ω_{1i}, ω_{2i} that satisfy

$$x' = \sum_{i=1}^{N} \omega_{1i}\, \varphi(r_i) + a_{11}x + a_{12}y + b_1$$

$$y' = \sum_{i=1}^{N} \omega_{2i}\, \varphi(r_i) + a_{12}x + a_{22}y + b_2$$

for all N pairs ($2N$ equations) and also satisfy $\cdots$

Thin plate spline interpolation

- $\cdots$ these 6 equations:

$$\sum_{i=1}^{N} \omega_{1i} = 0 \qquad \sum_{i=1}^{N} \omega_{2i} = 0$$

$$\sum_{i=1}^{N} \omega_{1i} x_i = 0 \qquad \sum_{i=1}^{N} \omega_{2i} x_i = 0$$

$$\sum_{i=1}^{N} \omega_{1i} y_i = 0 \qquad \sum_{i=1}^{N} \omega_{2i} y_i = 0$$

- Use all these $2N + 6$ equations to compute (x, y) for every point (x', y') in the image.

Linear equations

$$x' = \sum_{i=1}^{N} \omega_{1i}\,\varphi(r_i) + a_{11}x + a_{12}y + b_1$$

$$y' = \sum_{i=1}^{N} \omega_{2i}\,\varphi(r_i) + a_{12}x + a_{22}y + b_2$$

$$\sum_{i=1}^{N} \omega_{1i} = 0 \qquad \sum_{i=1}^{N} \omega_{2i} = 0$$

$$\sum_{i=1}^{N} \omega_{1i}x_i = 0 \qquad \sum_{i=1}^{N} \omega_{2i}x_i = 0$$

$$\sum_{i=1}^{N} \omega_{1i}y_i = 0 \qquad \sum_{i=1}^{N} \omega_{2i}y_i = 0$$

$$\begin{bmatrix} \varphi(r_{11})\ \varphi(r_{12}) & \ldots\ldots\varphi(r_{1n}) & x_1 & y_1 & 1 \\ & \ldots\ldots & & & \\ & \ldots\ldots & & & \\ \varphi(r_{n1})\ \varphi(r_{n2}) & \ldots\ldots\varphi(r_{nn}) & x_n & y_n & 1 \\ x_1 \quad \ldots\ldots & x_n & 0 & 0 & 0 \\ y_1 \quad \ldots\ldots & y_n & 0 & 0 & 0 \\ 1 \quad \ldots\ldots & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \omega_{11} & \omega_{21} \\ \omega_{12} & \omega_{22} \\ \vdots & \vdots \\ \omega_{1n} & \omega_{2n} \\ a_{11} & a_{21} \\ a_{12} & a_{22} \\ b_1 & b_2 \end{bmatrix} = \begin{bmatrix} x'_1 & y'_1 \\ x'_2 & y'_2 \\ \vdots & \vdots \\ x'_n & y'_n \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Why does TPS behave well?

- As (x, y) moves away from the N fiducial points, the terms in the sum
$$\sum_{i=1}^{N} \omega_{1i}\,\varphi(r_i) \qquad \sum_{i=1}^{N} \omega_{2i}\,\varphi(r_i)$$
begin to cancel out. When the sum $\longrightarrow$ 0.

$$x' \longrightarrow a_{11}x + a_{12}y + b_1$$
$$y' \longrightarrow a_{21}x + a_{22}y + b_2$$

Results

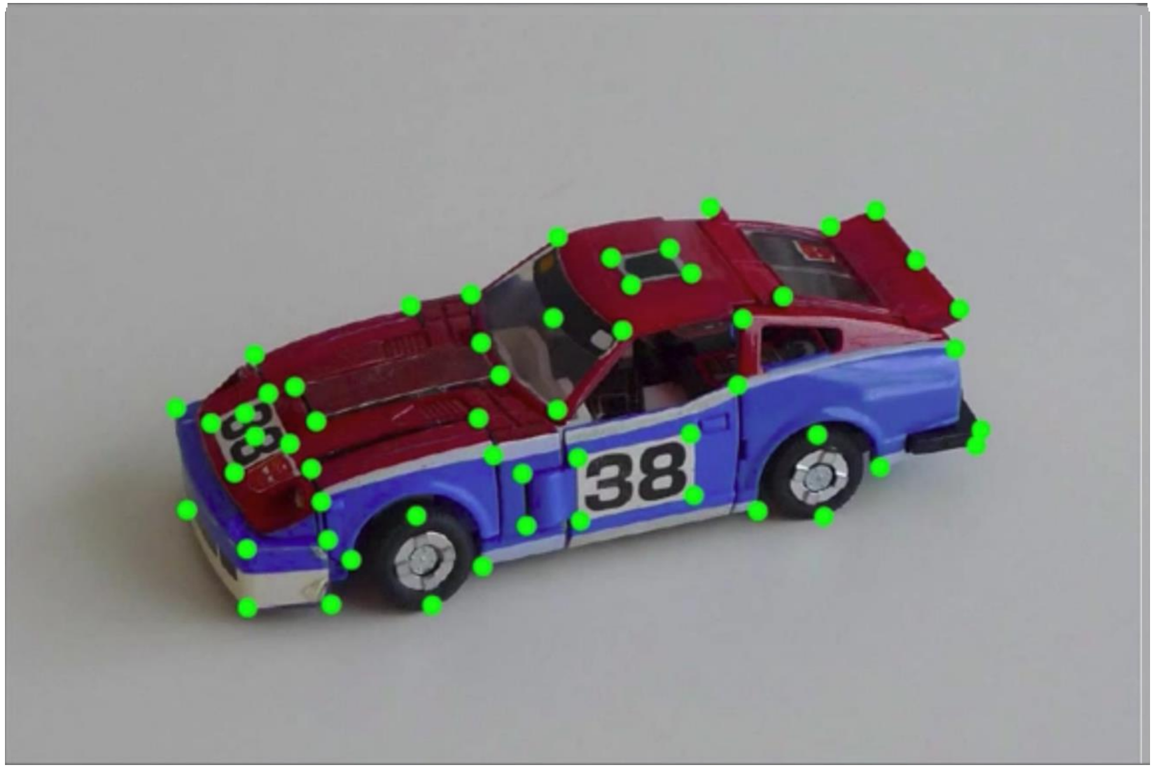
Image 1

Results

Image 2

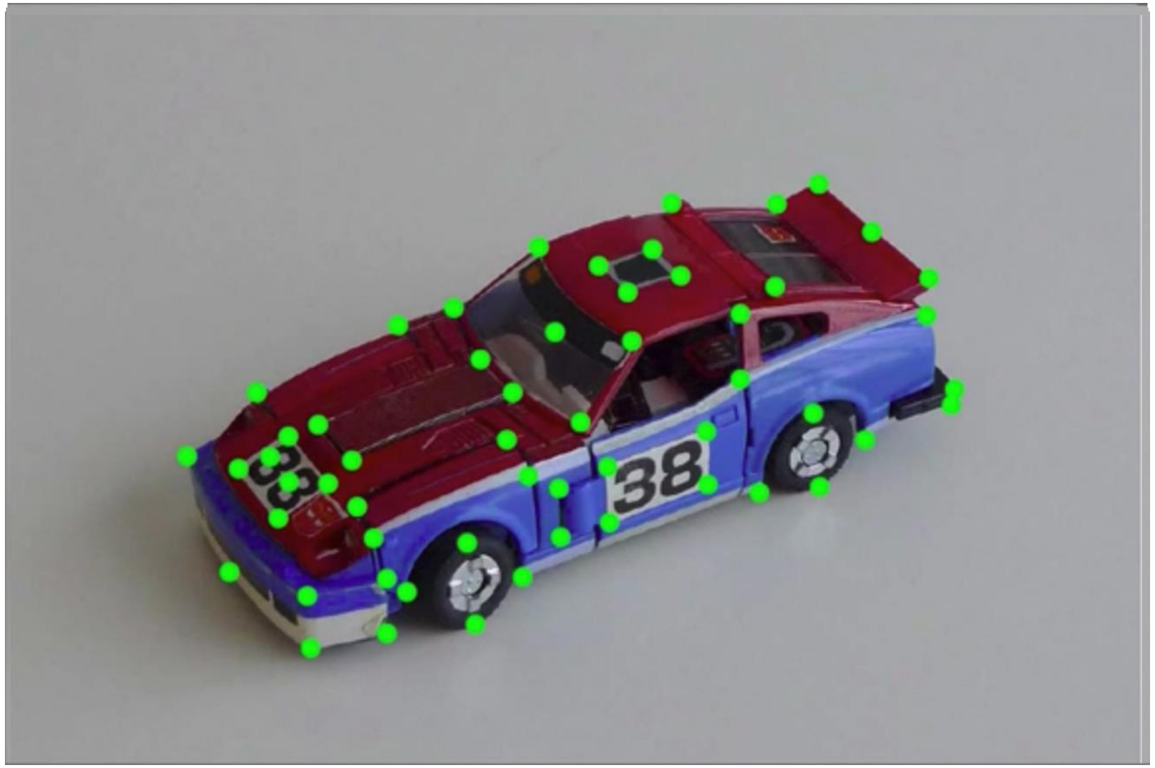
Results

Landmark selected on Image 2



Results

Landmark selected on Image 1

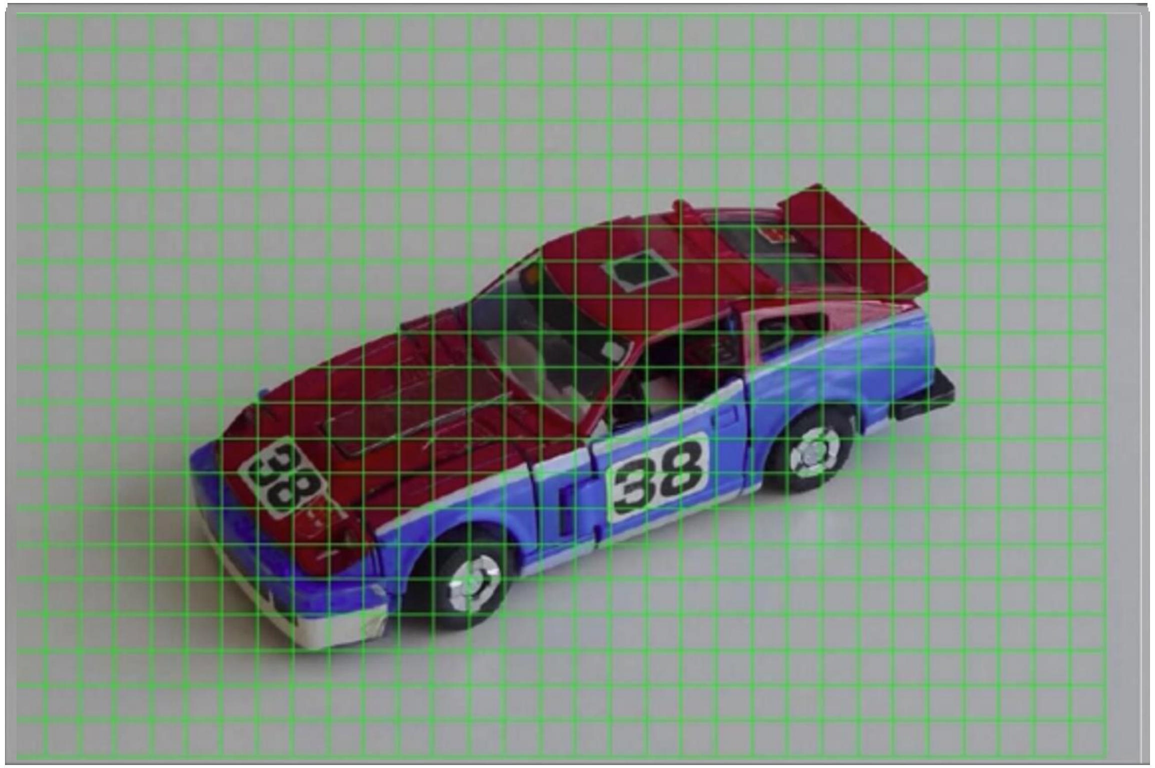


Results

Registration result from Image 1 to Image 2

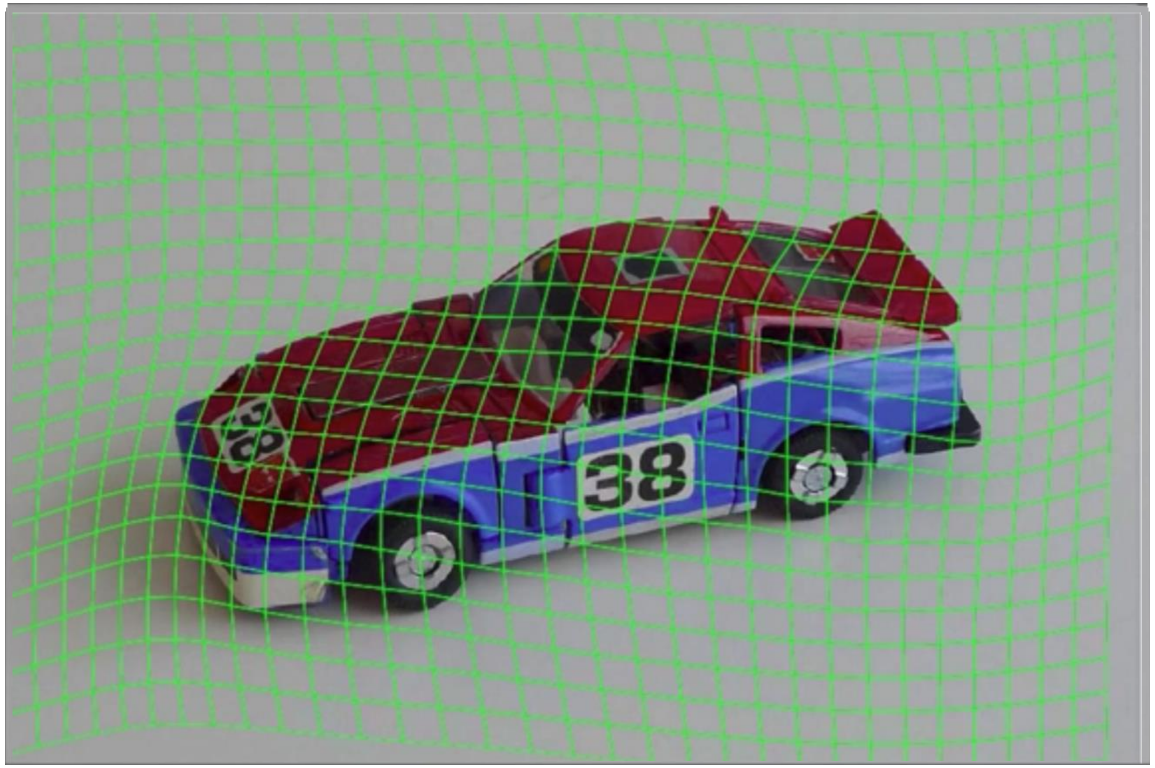
Results

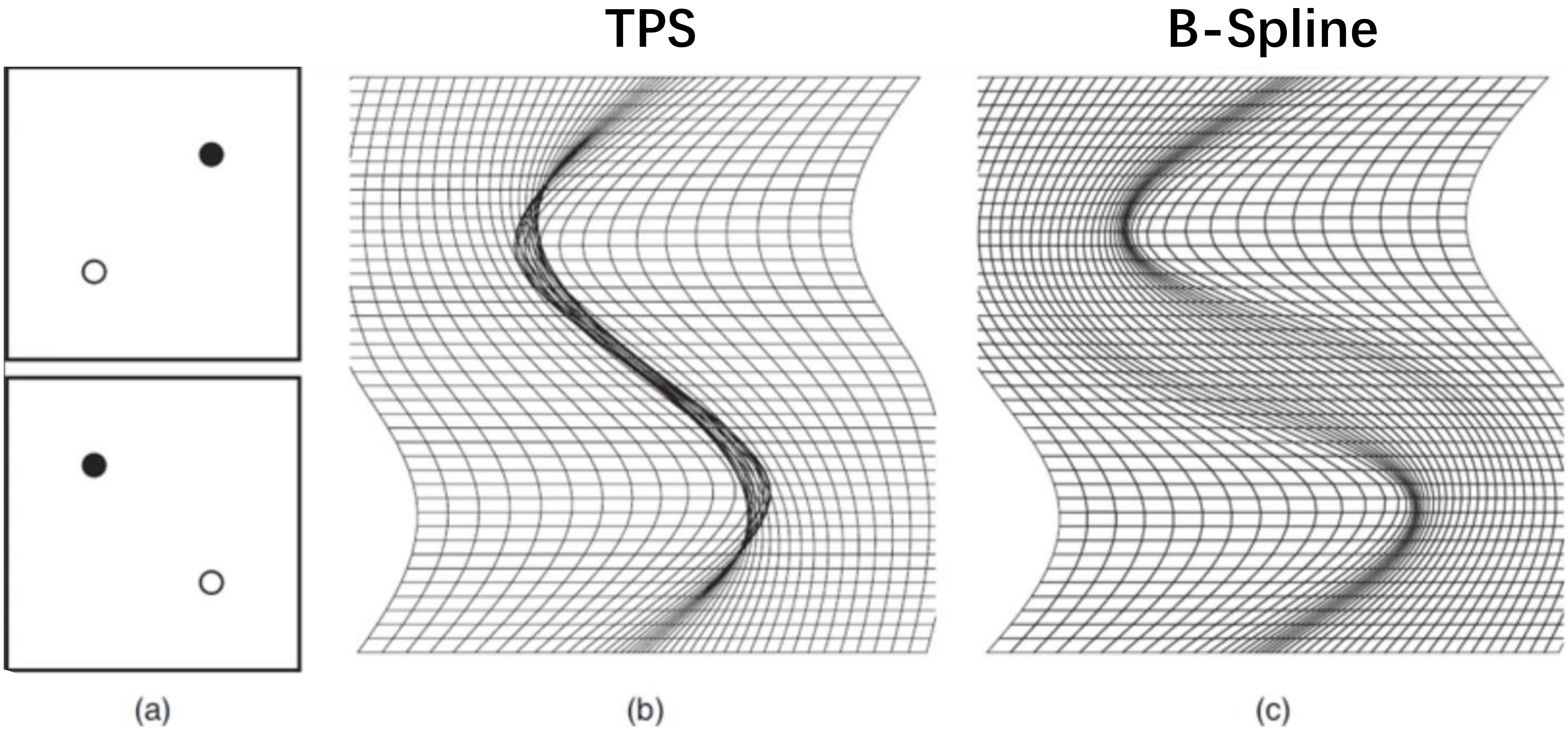
We put a grid to show the transform



Results

The deformation field from Image 1 to Image 2




TPS
B-Spline
(a)
(b)
(c)

Take home message

- Not invertible transform.

  -Enforce diffeomorphic transformations using PDE-based fluid flow method.

- Transformations depends on all the correspondences.

  - (B-spline).

- Straight lines generally don’t stay straight
- Image intensities don’t play a role!

  -Optical flow

- Active Shape Models … find landmarks