Relational Query Optimization II: Costing and Searching

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What is needed for query optimization?

- Given: A closed set of operators
  - Relational ops (table in, table out)
  - Physical implementations (of those ops and a few more)

1. **Plan space**
   - Based on relational equivalences, different implementations
2. **Cost Estimation** based on
   - Cost formulas
   - Size estimation, in turn based on
     - Catalog information on base tables
     - Selectivity (Reduction Factor) estimation
3. **A search algorithm**
   - To sift through the plan space and find lowest cost option!

Reminder

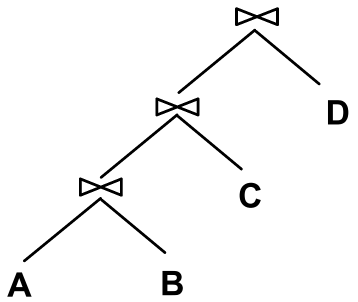
- We’re focusing on “System R” (“Selinger”) optimizers
  - Remarkably comprehensive framework
  - Many of the details have been refined over time
  - We’ll see some refinements today
  - This remains an area of ongoing research!

Big Picture of System R Optimizer

- Works well for up to 10-15 joins.
- **Plan Space**: Too large, must be pruned.
  - Algorithmic insight:
    - Many plans could have the same “overpriced” subtree
    - Ignore all those plans
  - Common heuristic: consider only left-deep plans
  - Common heuristic: avoid Cartesian products
- Cost estimation
  - Very inexact, but works ok in practice.
  - Stats in system catalogs used to estimate sizes & costs
  - Considers combination of CPU and I/O costs.
  - System R’s scheme has been improved since that time.
- Search Algorithm: Dynamic Programming

Query Blocks: Units of Optimization

- Break query into query blocks
- Optimize one block at a time
- Uncorrelated nested blocks computed once
- Correlated nested blocks are like function calls
  - But sometimes can be "decorrelated"
  - Beyond the scope of CS186!




```
SELECT S.sname
  FROM Sailors S
 WHERE S.age IN
```

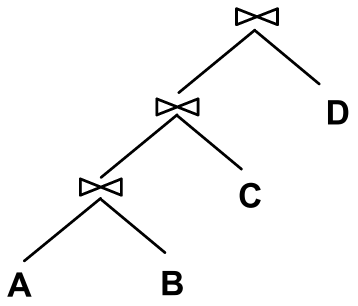
Outer block

```
  (SELECT MAX (S2.age)
      FROM  Sailors S2
GROUP BY  S2.rating)
```

Nested block

Query Blocks: Units of Optimization Pt 2

- For each block, the plans considered are:
  - All relevant access methods, for each relation in FROM clause.
  - All left-deep join trees
    - right branch always a base table
    - consider all join orders and join methods

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Nested block

Schema for Examples

```
Sailors (sid: integer, sname: text, rating: integer,
         age: float)
Reserves (sid: integer, bid: integer, day: date,
          rname: text)
```

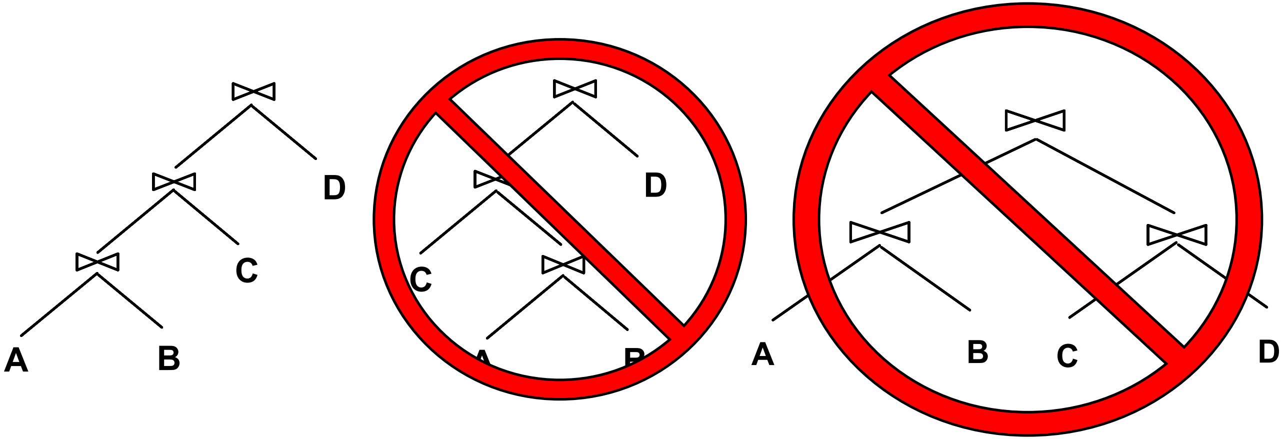
- Reserves:
  - Each tuple is 40 bytes long, 100 tuples per page, 1000 pages.
  - 100 distinct bids.
- Sailors:
  - Each tuple is 50 bytes long,
  - 80 tuples per page, 500 pages.
  - 10 ratings, 40,000 sids.

“Physical” Properties

- Two common “physical” properties of an output:
  - Sort order
  - Hash Grouping
- Certain operators produce these properties in output
  - E.g. Index scan (result is sorted)
  - E.g. Sort (result is sorted)
  - E.g. Hash (result is grouped)
- Certain operators require these properties at input
  - E.g. MergeJoin requires sorted input
- Certain operators preserve these properties from inputs
  - E.g. MergeJoin preserves sort order of inputs
  - E.g. INLJ preserves sort order of outer (left) input

Queries Over Multiple Relations

- A System R heuristic: only left-deep join trees considered.
  - Restricts the search space
  - Left-deep trees allow us to generate all fully pipelined plans.
    - Intermediate results not written to temporary files.
    - Not all left-deep trees are fully pipelined (e.g., SM join).



Plan Space Review

- For a SQL query, full plan space:
  - All equivalent relational algebra expressions
    - Based on the equivalence rules we learned
  - All mixes of physical implementations of those algebra expressions
- We might prune this space:
  - Selection/Projection pushdown
  - Left-deep trees only
  - Avoid cartesian products
- Along the way we may care about physical properties like sorting
  - Because downstream ops may depend on them
  - And enforcing them later may be expensive

Query Optimization: Cost Estimation

1. Plan Space
2. **Cost Estimation**
3. Search Algorithm

Cost Estimation

- For each plan considered, must estimate total cost:
  - Must estimate ***cost*** of each operation in plan tree.
    - Depends on input cardinalities.
    - We've already discussed this for various operators
      - sequential scan, index scan, joins, etc.
  - Must estimate ***size of result*** for each operation in tree!
    - Because it determines downstream input cardinalities!
    - Use information about the input relations.
    - For selections and joins, assume independence of predicates.
- In System R, cost is boiled down to a single number consisting of #I/O + ***CPU-factor*** * #tuples

Statistics and Catalogs

- Need info on relations and indexes involved.
- **Catalogs** typically contain at least:

Statistic	Meaning
NTuples	# of tuples in a table (cardinality)
NPages	# of disk pages in a table
Low/High	min/max value in a column
Nkeys	# of distinct values in a column
IHeight	the height of an index
INPages	# of disk pages in an index

- Catalogs updated periodically.
  - Too expensive to do continuously
  - Lots of approximation anyway, so a little slop here is ok.
- Modern systems do more
  - Esp. keep more detailed statistical information on data values
    - e.g., histograms

Size Estimation and Selectivity

- Max output cardinality = product of input cardinalities
- **Selectivity (sel)** associated with each **term**
  - reflects the impact of the term in reducing result size.
  - selectivity = |output| / |input|
  - Book calls selectivity "Reduction Factor" (RF)
- Avoid confusion:
  - "highly selective" in common English is opposite of a high selectivity value (|output|/|input| high!)

```
SELECT  attribute list
  FROM  relation list
 WHERE  term1 AND ... AND termk
```

Result Size Estimation

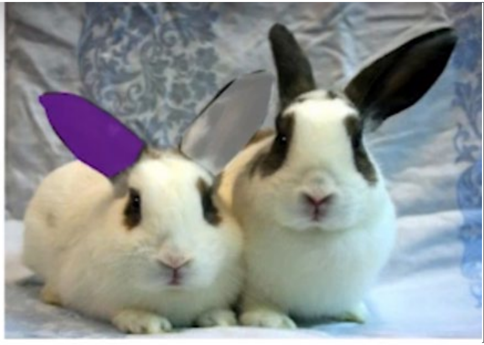
- Result cardinality = Max # tuples * **product** of all selectivities.
- Term col=value (given Nkeys(I) on col)
  - sel = 1/NKeys(I)
- Term col1=col2 (handy for joins too…)
  - sel = 1/MAX(NKeys(I1), NKeys(I2))
  - Why MAX? See bunnies in 2 slides…
- Term col>value
  - sel = (High(I)-value)/(High(I)-Low(I) + 1)
- Note, if missing the needed stats, assume 1/10!!!

Let’s dig into selectivity estimation more deeply

- Clarify how some of these estimates came to be
- Refine our stored statistics
- Expose our statistical assumptions

P(leftEar = rightEar)

- 100 bunnies
- 2 distinct LeftEar colors
  - {C1, C2}
- 10 distinct RightEar colors
  - {C1..C10}
- Independent ears
- What's the probability of matching ears?



$$
\begin{aligned}
&P(L = R) \\
&= \sum_i P(C_i, C_i) \\
&= P(C_1, C_1) + P(C_2, C_2) + P(C_3, C_3) + \ldots \\
&= (\frac{1}{2} \cdot \frac{1}{10}) + (\frac{1}{2} \cdot \frac{1}{10}) + (0 \cdot \frac{1}{10}) + \ldots \\
&= 1/10 \\
&= 1/\mathrm{MAX}(2, 10)
\end{aligned}
$$

Postgres 10.0: src/include/utils/selfuncs.h

```
/* default selectivity estimate for equalities such as "A = b" */
    #define DEFAULT_EQ_SEL  0.005

    /* default selectivity estimate for inequalities such as "A < b" */
    #define DEFAULT_INEQ_SEL  0.3333333333333333

    /* default selectivity estimate for range inequalities "A > b AND A < c" */
    #define DEFAULT_RANGE_INEQ_SEL  0.005

/* default selectivity estimate for pattern-match operators such as LIKE */
    #define DEFAULT_MATCH_SEL 0.005

    /* default number of distinct values in a table */
    #define DEFAULT_NUM_DISTINCT  200

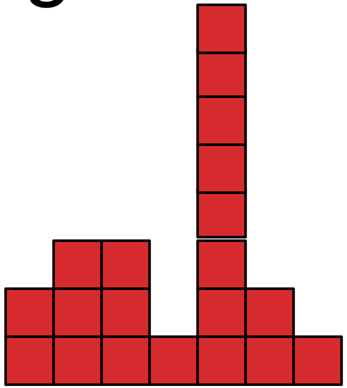
    /* default selectivity estimate for boolean and null test nodes */
    #define DEFAULT_UNK_SEL     0.005
    #define DEFAULT_NOT_UNK_SEL   (1.0 - DEFAULT_UNK_SEL)
```

Reduction Factors & Histograms

- For better estimation, use a histogram

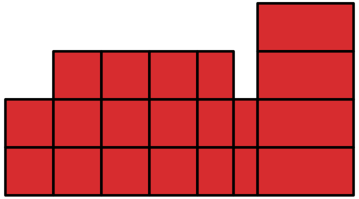
equiwidth

No. of Values	2	3	3	1	8	2	1
Value	0-.99	1-1.99	2-2.99	3-3.99	4-4.99	5-5.99	6-6.99



equidepth

No. of Values	2	3	3	3	3	2	4
Value	0-.99	1-1.99	2-2.99	3-4.05	4.06-4.67	4.68-4.99	5-6.99



Note: 10-bucket equidepth histogram divides the data into *deciles*

- akin to quantiles, median, etc.

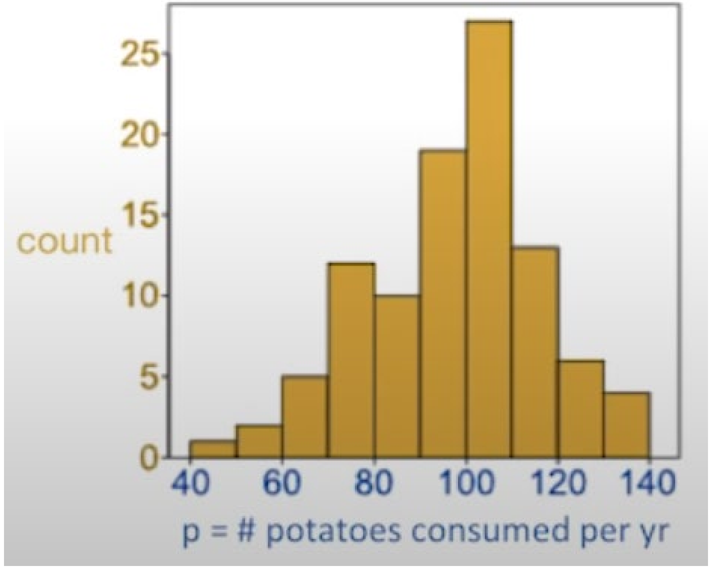
Common trick: “end-biased” histogram

- very frequent values in their own buckets

See also V-Optimal histograms on Wikipedia

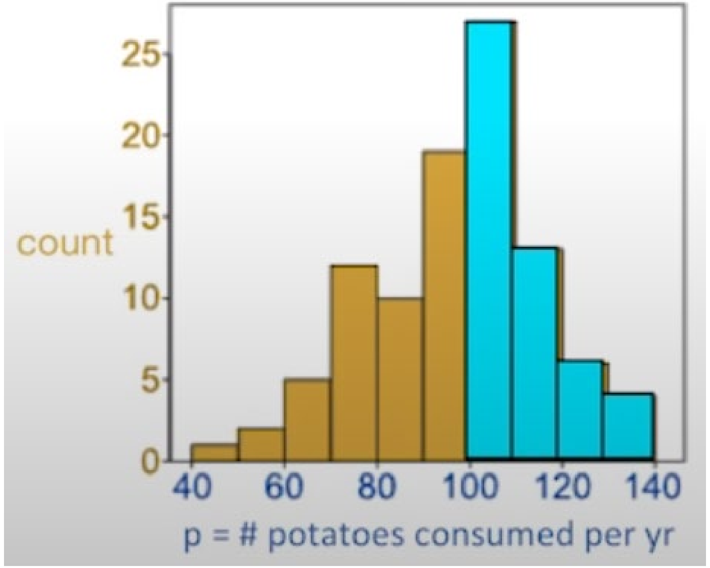
Computing selectivity with histograms

- 100 rows
- $\sigma_{p > 99}$?



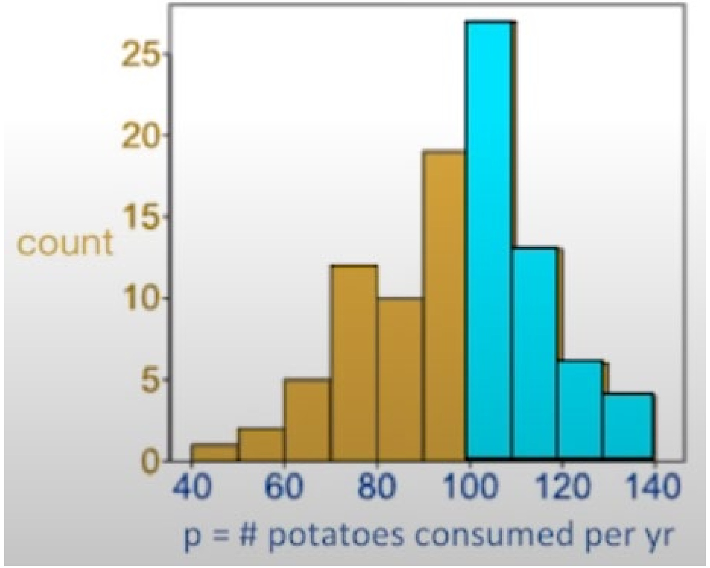
Computing selectivity with histograms, Pt 2

- 100 rows
- $\sigma_{p > 99}$?



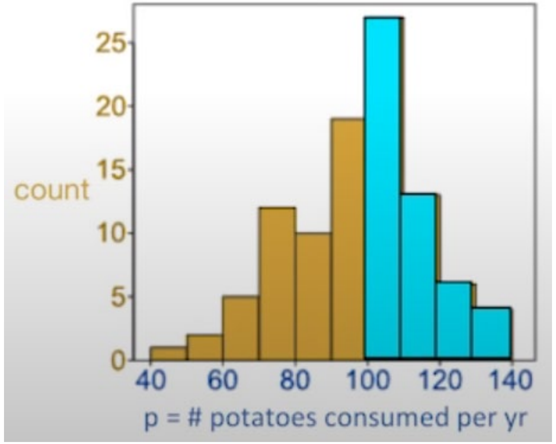
Computing selectivity with histograms, Pt 3

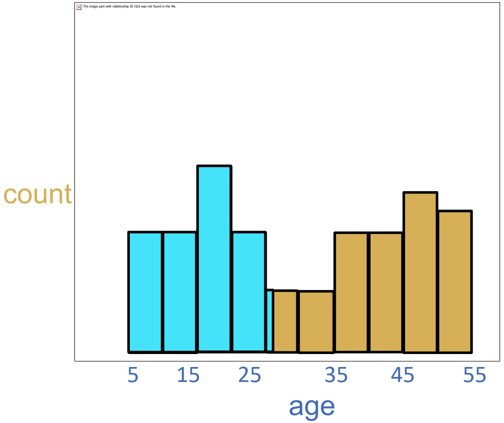
- 100 rows
- $\sigma_{p > 99}$? 50/100 = 50%.



Computing selectivity with histograms, Pt 4

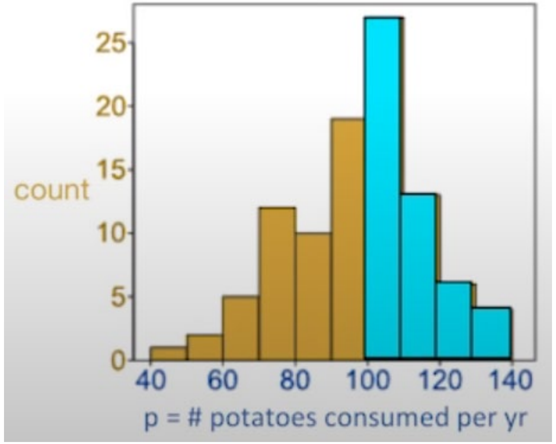
- 100 rows
- $\sigma_{age < 26}$?

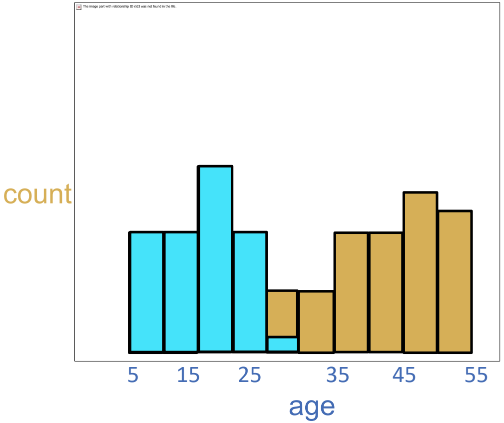




Computing selectivity with histograms, Pt 5

- 100 rows
- $\sigma_{age < 26}$?

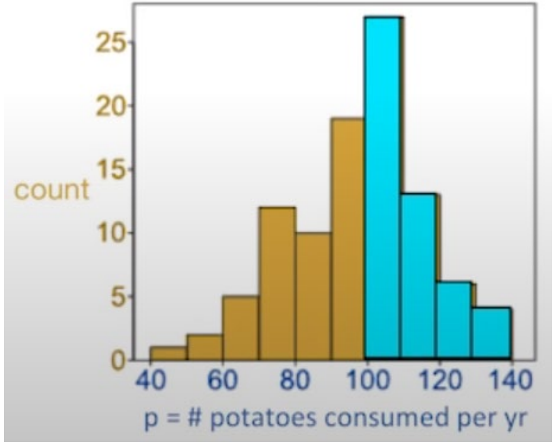


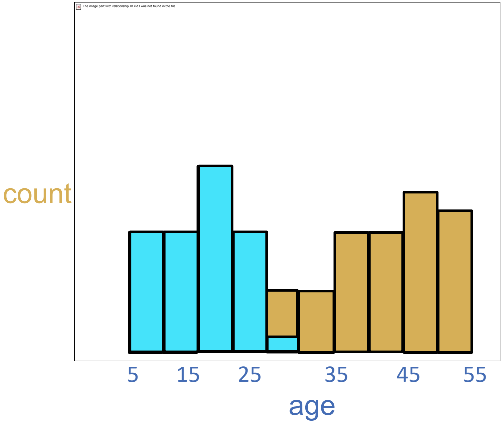


Computing selectivity with histograms, Part 6

- 100 rows
- $\sigma_{age < 26}$?

- **Uniformity assumption:**
  - Uniform distribution within each bin
    - Each vertical slice the same
  - Hence ⅕ of the population of bin [25,30)
    - has age < 26.
  - 10 + 10 + 15 + 10 + (⅕ * 5) = 46/100 = **46%**

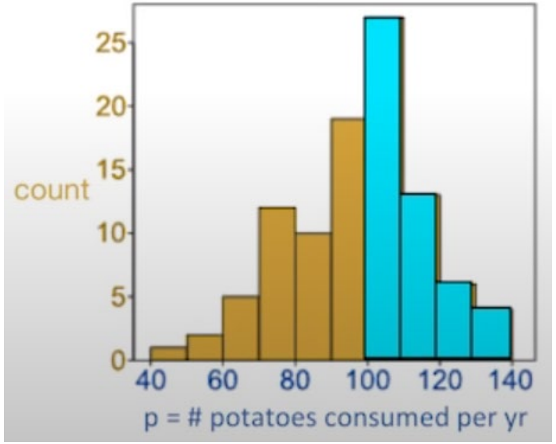


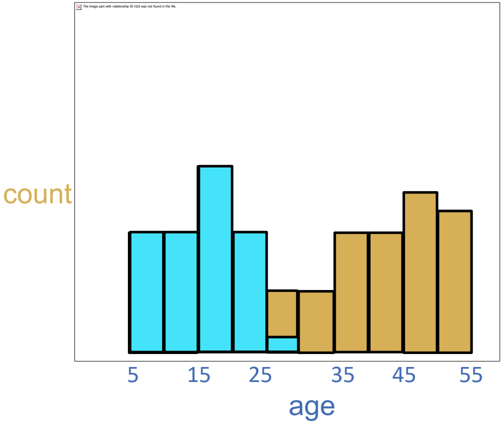


Selectivity of Conjunction

- 100 rows
- $\sigma_{p > 99 \wedge age < 26}$?

  50% 46%





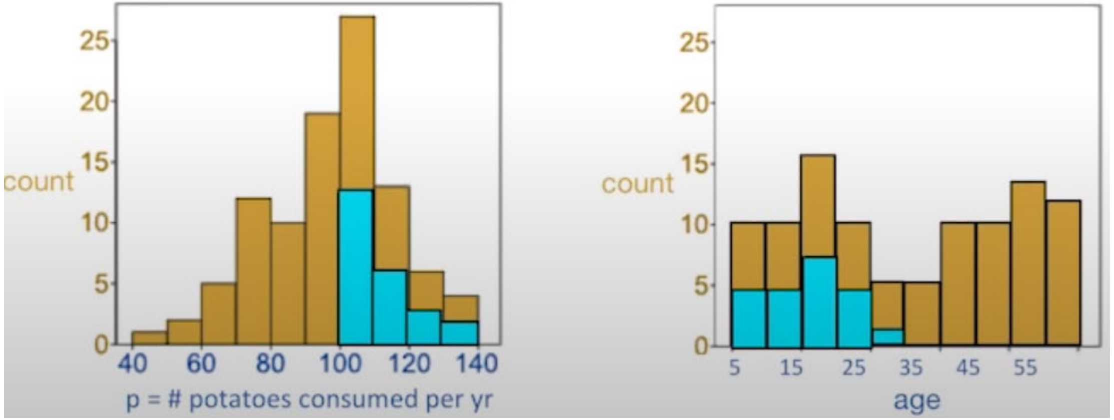
Selectivity of Conjunction, cont

- 100 rows
- $\sigma_{p\ >\ 99\ \wedge\ age\ <\ 26}$?

  50% 46%

- **Independence assumption:**
  - Age and potato consumption are independent
  - Hence p bins all shrink by 46%.
  - Hence age bins all shrink by 50%.

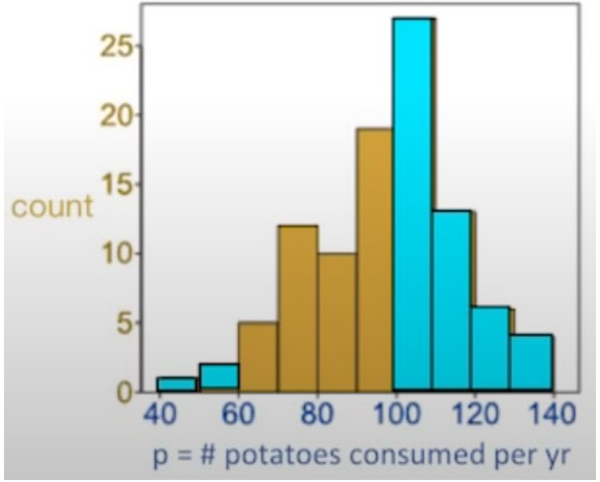
Selectivity: 50% × 46% = **23%**



Selectivity of Disjunction

- 100 rows
- $\sigma_{p\,>\,99\ \vee\ p\,<\,60}$?

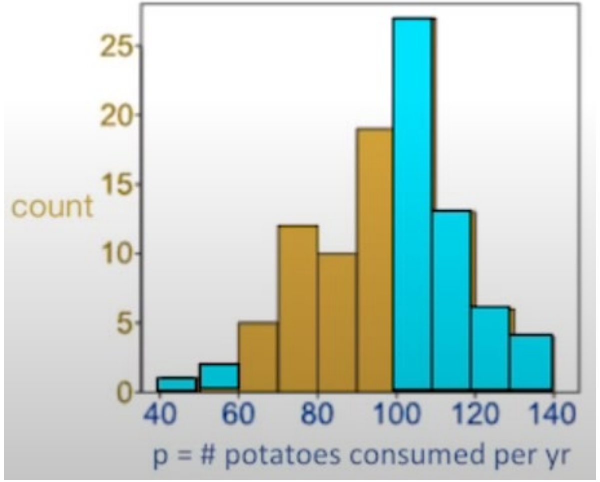
  50%   3%



Selectivity of Disjunction, Part 2

- 100 rows
- $\sigma_{p > 99 \vee p < 60}$?

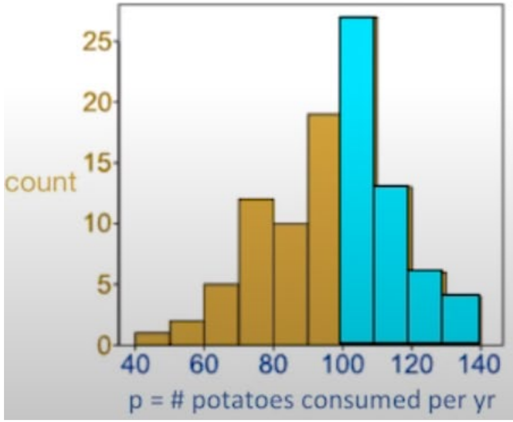
  50% 3%
- Selectivity: 50% + 3% = **53%**

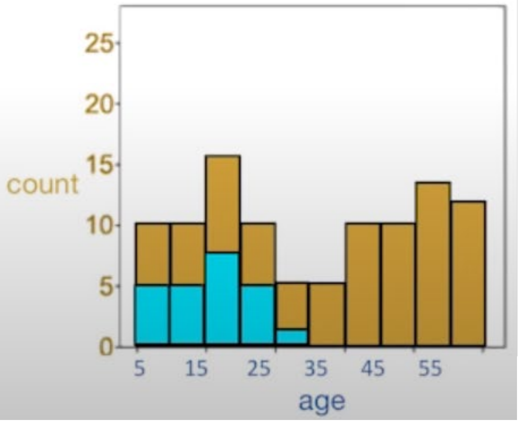


Selectivity of Disjunction, Part 3

- 100 rows
- $\sigma_{p\,>\,99\ \vee\ age\,<\,26}$?

  50% 46%





- Answer tuples satisfy one or both predicates
- By independence assumption:
  - Satisfy the first predicate: 50%
  - Satisfy the second predicate: 46%
  - Satisfy both: 50% × 46%
    - **Don't double-count!**

Selectivity:
50% + 46% - (50% × 46%) = **73%**

Selectivity for more complicated queries?

- $R \bowtie_p \sigma_q(S)$
  - Selectivity of join predicate p is s_p
  - Selectivity of selection predicate q is s_q
  - How to think about overall selectivity?

Join Selectivity

- Recall from algebraic equivalences: $R \bowtie_p S \equiv \sigma_p(R \times S)$
- Hence join selectivity is “just” selectivity s_p
  - Over a big input: $|R| \times |S|$!
- Total rows: $s_p \times |R| \times |S|$

Selectivity for our earlier query?

- Recall from algebraic equivalences

  $$R \bowtie_p \sigma_q(S) \equiv \sigma_p(R \times \sigma_q(S)) \equiv \sigma_{p \wedge q}(R \times S))$$

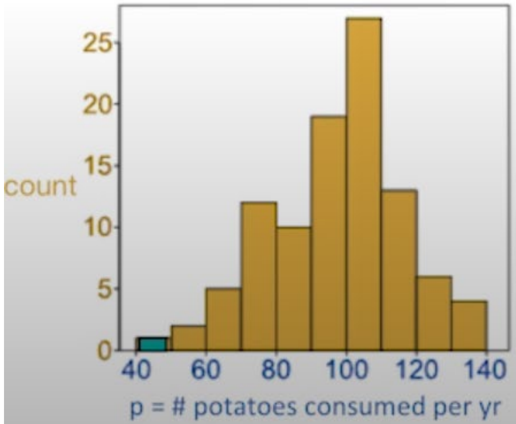
  - Hence selectivity just $s_p s_q$
    - Applied to $|R| \times |S|$!

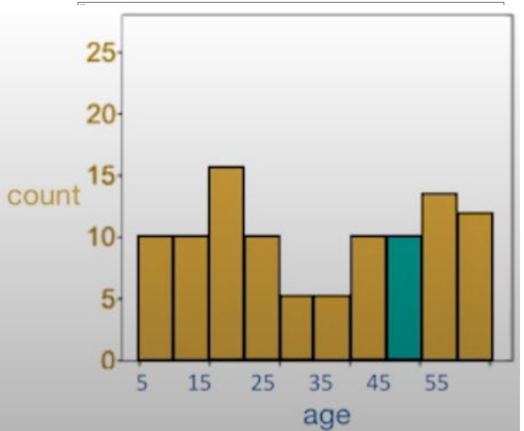
- Total rows: $s_p s_q |R||S|$

Column Equality?

T.p = T.age ??
Intuition: similar to bunny ears, but weighted by the histogram bins.

```
s = 0
For each value v covered in either histogram:
    // uniformity assumption within bins:
    // P(T.p = v) = height(binp(v))/n * 1/width(binp(v))
    // P(T.age = v) = height(binage(v))/n * 1/width(binage(v))
```





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    // P(T.age = v) = height(binage(v))/n * 1/width(binage(v))

    // independence assumption across columns:
    // P(T.p = v ∧ T.age = v)
    // = P(T.p = v) * P(T.age = v)
    s += height(binp(v))/(n*width(binp(v)))
              * height(binage(v))/(n*width(binage(v)))
```

Challenge: make this more efficient by iterating over bin boundaries rather than values!

Upshot

- Know how to compute selectivities for basic predicates
  - The original Selinger version
  - The histogram version

- Assumption 1: uniform distribution within histogram bins
  - Within a bin, fraction of range = fraction of count

- Assumption 2: independent predicates
  - Selectivity of AND = product of selectivities of predicates
  - Selectivity of OR = sum of selectivities of predicates - product of selectivities of predicates
  - Selectivity of NOT = 1 – selectivity of predicates

- Joins are not a special case
  - Simply compute the selectivity of all predicates
  - And multiply by the product of the table sizes

Query Optimization

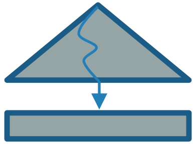
1. Plan Space
2. Cost Estimation
3. Search Algorithm

Enumeration of Alternative Plans

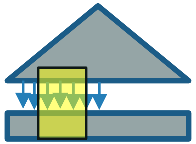
- There are two main cases:
  - **Single-table plans (base case)**
  - **Multiple-table plans (induction)**
- Single-table queries include selects, projects, and groupBy/agg:
  - Consider each available access path (file scan / index)
    - Choose the one with the least estimated cost
  - Selection/Projection done on the fly
  - Result pipelined into grouping/aggregation

Cost Estimates for Single-Relation Plans

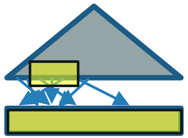
- Index I on primary key matches selection:
  - Cost is (Height(I) + 1) + 1 for a B+ tree.



- Clustered index I matching selection:
  - (NPages(I)+**NPages**(R)) * selectivity.



- Non-clustered index I matching selection:
  - (NPages(I)+**NTuples**(R)) * selectivity.



- Sequential scan of file:
  - NPages(R).
- Recall: Must also charge for duplicate elimination if required

Example

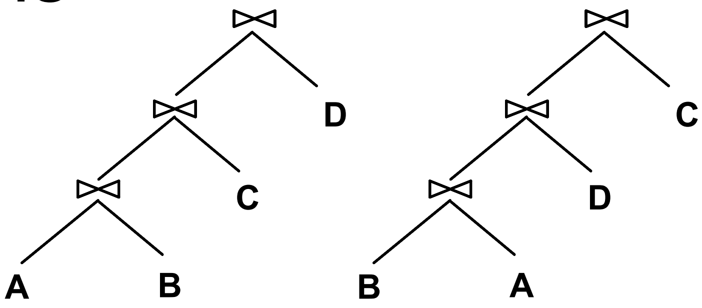
```
SELECT S.sid
  FROM Sailors S
 WHERE S.rating=8
```

- If we have an index on rating:
  - **Cardinality** = (1/NKeys(I)) * NTuples(R) = (1/10) * 40000 tuples
  - **Clustered index:** (1/NKeys(I)) * (NPages(I)+NPages(R))
    = (1/10) * (50+500) = **55 pages are retrieved**. (This is the cost.)
  - **Unclustered index**: (1/NKeys(I)) * (NPages(I)+NTuples(R))
    = (1/10) * (50+40000) = **4005 pages are retrieved**.

- If we have an index on sid:
  - Would have to retrieve all tuples/pages. With a clustered index, the cost is 50+500, with unclustered index, 50+40000.

- Doing a file scan:
  - We retrieve all file pages (500).

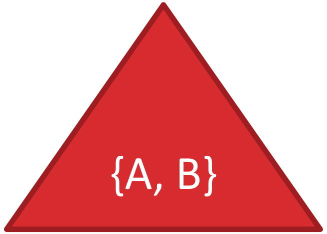
Enumeration of Left-Deep Plans



- Left-deep plans differ in
  - the order of relations
  - the access method for each leaf operator
  - the join method for each join operator
- Enumerated using N passes (if N relations joined):
  - **Pass 1:** Find best 1-relation plan for each relation
  - **Pass i:** Find best way to join result of an (i -1)-relation plan (as outer) to the i' th relation. (i between 2 and N.)
- For each subset of relations, retain only:
  - Cheapest plan overall, plus
  - Cheapest plan for each *interesting order* of the tuples.

The Principle of Optimality

- Richard Bellman (slightly adapted to our setting)
- The best overall plan is composed of best decisions on the subplans
  - Optimal result has optimal substructure
- For example, the best left-deep plan to join tables A, B, C is either:
  - (The best plan for joining A, B) ⋈ C
  - (The best plan for joining A, C) ⋈ B
  - (The best plan for joining B, C) ⋈ A
- This is great!
  - When optimizing a subplan (e.g. A ⋈ B), we don't have to think about how it will be used later (e.g. when dealing with C)!
  - When optimizing a higher-level plan (e.g. A ⋈ B ⋈ C) we can reuse the best results of subroutines (e.g. A ⋈ B)!



Dynamic Programming Algorithm for System R

- Principle of optimality allows us to build best subplans “bottom up”
    - Pass 1: Find best plans of height 1 (base table accesses), and record them in a table
    - Pass 2: Find best plans of height 2 (joins of base tables) by combining plans of height 1, record them in a table
    - …
    - Pass i: Find best plans of height i by combining plans of height i - 1 with plans of height 1, record them in a table
    - …
    - Pass *n:* Find best plan overall by combining plans of height *n-1* with plans of height 1.

The Basic Dynamic Programming Table

Table keyed on 1st column

Subset of tables in FROM clause	Best plan	Cost
{R, S}	hashjoin(R,S)	1000
{R, T}	mergejoin(R,T)	700

A Wrinkle: Interesting Orders

- Physical properties can break the principle of optimality
  - For example, consider a suboptimal plan p for A ⋈ B that is ordered on column x
  - Suppose we need to join with table C on column x
  - Sort-merge of p with C might be the best overall plan
    - The best plan for A ⋈ B requires us to sort for Sort-Merge join
    - But the suboptimal plan p doesn't require us to sort A ⋈ B
- Solution: expand our definition of "optimal substructure"
  - The structure will include both the set of tables and the physical properties (order)
  - But not all orders are "interesting"! We can prune further

A Note on "Interesting Orders"

- Physical property: Order.
  When should we care? When is it "interesting"?

- An intermediate result has an "interesting order" if it is sorted by anything we can use later in the query ("downstream" the arrows):
  - ORDER BY attributes
  - GROUP BY attributes
  - Join attributes of yet-to-be-added joins
    - subsequent merge join might be good

The Dynamic Programming Table

Table keyed on concatenation of 1st two columns

Subset of tables in FROM clause	Interesting-order columns	Best plan	Cost
{R, S}	<none>	hashjoin(R,S)	1000
{R, S}	<R.a, S.b>	sortmerge(R,S)	1500

Enumeration of Plans (Contd.)

- First figure out the scans and joins (select-project-join) using D.P.
  - **Avoid Cartesian Products** in dynamic programming as follows:
    When matching an i -1 way subplan with another table, only consider it if
    - There is a join condition between them, **or**
    - All predicates in WHERE have been "used up" in the i -1 way subplan.

- Then handle ORDER BY, GROUP BY, aggregates etc. as a post-processing step
  - Via "interestingly ordered" plan if chosen (free!)
  - Or via an additional sort/hash operator

- Despite pruning, this System R D.P. algorithm is
  exponential in #tables.

Example

```
SELECT S.sid, COUNT(*) AS number
FROM Sailors S, Reserves R, Boats B
WHERE S.sid = R.sid
AND R.bid = B.bid
AND B.color = “red”
GROUP BY S.sid
```

<u>Sailors:</u>
Hash, B+ tree indexes on *sid*
<u>Reserves:</u>
Clustered B+ tree on *bid*
B+ on *sid*
<u>Boats</u>
B+ on *color*

Pass 1: Best plan(s) for each relation

- Sailors, Reserves: File Scan
- Also B+ tree on Reserves.bid as interesting order
- Also B+ tree on Sailors.sid as interesting order
- Boats: B+ tree on color

Subset of tables in FROM clause	Interesting-order columns	Best plan	Cost
{Sailors}	--	filescan	
{Reserves}	--	Filescan	
{Boats}	--	B-tree on color	
{Reserves}	(bid)	B-tree on bid	
{Sailors}	(sid)	B-tree on sid	

Pass 2

```
// for each left-deep logical plan
for each plan P in pass 1
  for each FROM table T not in P
      // for each physical plan
      for each access method M on T
         for each join method
            generate P ⋈ M(T)
```

- File Scan Reserves (outer) with Boats (inner)
- File Scan Reserves (outer) with Sailors (inner)
- Reserves Btree on bid (outer) with Boats (inner)
- Reserves Btree on bid (outer) with Sailors (inner)
- File Scan Sailors (outer) with Boats (inner)
- File Scan Sailors (outer) with Reserves (inner)
- Boats Btree on color with Sailors (inner)
- Boats Btree on color with Reserves (inner)

- Retain cheapest plan for each (pair of relations, order)

Subset of tables in FROM clause	Interesting-order columns	Best plan	Cost
{Sailors}	--	filescan	
{Reserves}	--	Filescan	
{Boats}	--	B-tree on color	
{Reserves}	(bid)	B-tree on bid	
{Sailors}	(sid)	B-tree on sid	
{Boats, Reserves}	(B.bid) (R.bid)	SortMerge(B-tree on Boats.color, filescan Reserves)	
Etc...			

Pass 3 and beyond

- Using Pass 2 plans as outer relations, generate plans for the next join in the same way as Pass 2
  - E.g. {SortMerge(B-tree on Boats.color, filescan Reserves)} (outer) | with Sailors (B-tree sid) (inner)
- Then, add cost for groupby/aggregate:
  - This is the cost to sort the result by sid, *unless it has already been sorted by a previous operator.*
- Then, choose the cheapest plan

```
SELECT S.sid, COUNT(*) AS number
FROM Sailors S, Reserves R, Boats B
WHERE S.sid = R.sid
AND R.bid = B.bid
AND B.color = “red”
GROUP BY S.sid
```

Now you understand the optimizer!

So what?!

- Benefit #1: You could build one.
  - And you will!
- Benefit #2: You can influence one
  - People who write non-trivial SQL often get frustrated with the optimizer
    - It picked a crummy plan!
    - It didn't use the index I built!
    - Etc.
  - Understanding the optimizer can lead you to:
    - Design your DB & Indexes better
    - Avoid "weak spots" in your optimizer's implementation
    - Coax your optimizer to do what you want