Problem 1 (6 points) – Boost converter

In Lab9, we learned a boost converter circuit as shown in Fig.1. For this circuit,

1) Find the ratio of T_{on}/T_{off}.
2) Find the value of inductance L when $\Delta I_L = 2\text{mA}$ and $T = 3\mu\text{s}$.

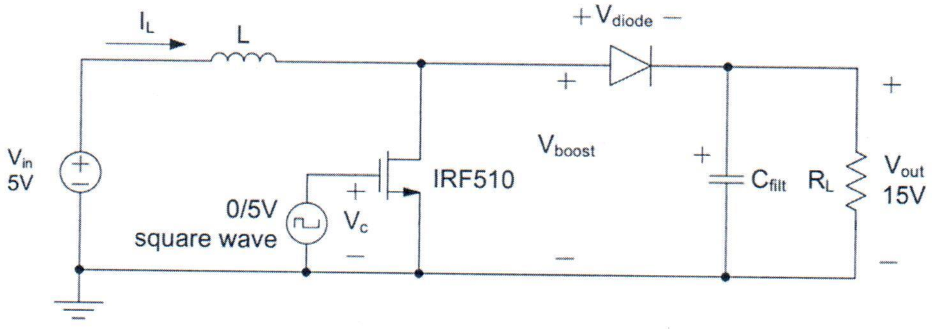


Fig. 1 (a) Boost Converter used in Lab 9.

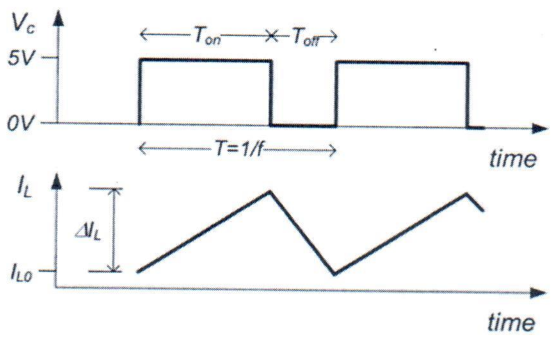


Fig. 1 (b) Timing diagram of the boost converter shown in Fig. 1(a).

Your answer:

① $V = L\frac{di}{dt} \Rightarrow \Delta i = \frac{V}{L}\cdot\Delta t$ ✓

$\Delta I_{on} = \Delta I_{off}$

$\frac{5V}{L}\cdot T_{on} = \frac{15-5}{L}\cdot T_{off} \Rightarrow \frac{T_{on}}{T_{off}} = 2:1$ 3 $\cancel{3}$

② $\frac{5V}{L}\cdot T_{on} = 2mA$, $T_{on} = \frac{2}{3}T = 2\mu s$

$L = \frac{5V\cdot 2\mu s}{2mA} = 5\,mH$ $\cancel{3}$ 3

3+3=6

Problem 2 (20 points) – First-order circuit analysis

You must show your work to get full credit.

Determine the current $i_o(t)$ in the circuit shown in Fig. 2.

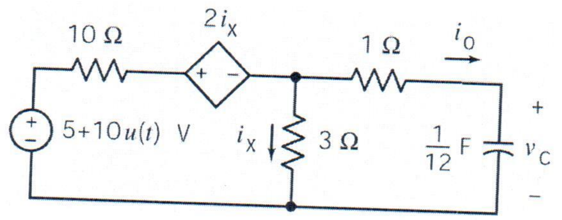


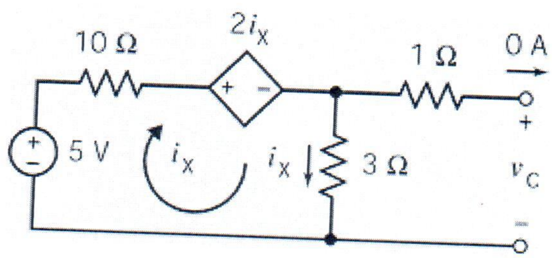
Fig. 2

Your answer:

Consider the circuit for time $t < 0$.

Step 1: Determine the initial capacitor voltage.

$t < 0$, at steady state:



The circuit will be at steady state before the source voltage changes abruptly at time $t = 0$.

The source voltage will be 5 V, a constant.

The capacitor will act like an open circuit.

Apply KVL to the mesh to get:

$$(10+2+3)i_x - 5 = 0 \;\Rightarrow\; i_x = \frac{1}{3}\text{ A}$$

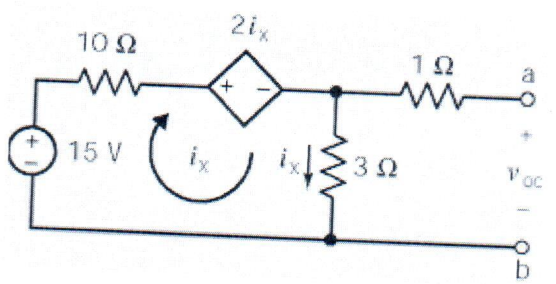
3

Then

$$v_C(0) = 3i_x = 1\text{ V}$$

Consider the circuit for time $t > 0$.

Step 2. The circuit will not be at steady state immediately after the source voltage changes abruptly at time $t = 0$. Determine the Thevenin equivalent circuit for the part of the circuit connected to the capacitor. First, determine the open circuit voltage, v_{oc}:



Apply KVL to the mesh to get:

$$(10+2+3)i_x - 15 = 0 \;\Rightarrow\; i_x = 1\text{ A}$$

Then

$$v_{oc} = 3i_x = 3\text{ V}$$

3

Next, determine the short circuit current, i_{sc}:

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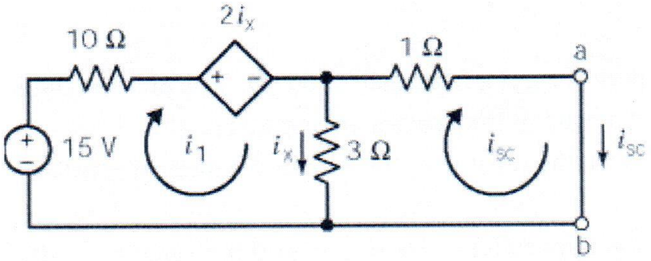


Express the controlling current of the CCVS in terms of the mesh currents:

$$i_x = i_1 - i_{sc}$$

The mesh equations are

$$10\,i_1 + 2\left(i_1 - i_{sc}\right) + 3\left(i_1 - i_{sc}\right) - 15 = 0 \;\Rightarrow\; 15\,i_1 - 5\,i_{sc} = 15$$

And

$$i_{sc} - 3\left(i_1 - i_{sc}\right) = 0 \;\Rightarrow\; i_1 = \frac{4}{3}\,i_{sc}$$

so

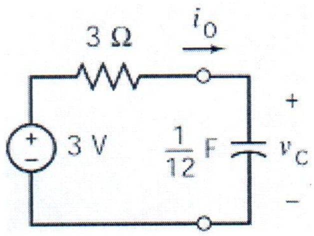
$$15\left(\frac{4}{3}i_{sc}\right) - 5\,i_{sc} = 15 \;\Rightarrow\; i_{sc} = 1\text{ A}$$

The Thevenin resistance is

$$R_t = \frac{3}{1} = 3\ \Omega$$

5

Step 3. The time constant of a first order circuit containing an capacitor is given by



$$\tau = R_t\,C$$

Consequently

$$\tau = R_t\,C = 3\left(\frac{1}{12}\right) = 0.25\text{ s and } a = \frac{1}{\tau} = 4\ \frac{1}{\text{s}}$$

Step 4. The capacitor voltage is given by:

$$v_C(t) = v_{oc} + \left(v_C(0) - v_{oc}\right)e^{-at} = 3 + (1-3)e^{-4t} = 3 - 2e^{-4t} \quad \text{for } t \ge 0$$

5

Step 5. Express the output current as a function of the source voltage and the capacitor voltage.

$$i_o(t) = C\frac{d}{dt}v_C(t) = \frac{1}{12}\frac{d}{dt}v_C(t)$$

Step 6. The output current is given by

$$i_o(t) = \frac{1}{12}\frac{d}{dt}\left(3 - 2e^{-4t}\right) = \frac{1}{12}(-2)(-4)e^{-4t} = \frac{2}{3}e^{-4t} \quad \text{for } t \ge 0$$

4

20 = 3 + 3 + 5 + 5 + 4

Problem 3 (15 points) – Bandstop filter

You must show your work to get full credit.

It is very common to see interference caused by the power lines, at a frequency of 60 Hz. This problem outlines the design of a notch filter, shown in Fig. 3(a), to reject a band of frequencies around 60 Hz.

a) Write the impedance function for the filter of Fig. 3(a) (the resistor r_L represents the internal resistance of a practical inductor).

b) For what value of C will the center frequency of the filter equal 60 Hz if $L = 100\ mH$ and $r_L = 5\Omega$?

c) Assume that the filter is used to eliminate the 60-Hz noise from a signal generator with output frequency of 1 kHz in Fig. 3(b). Evaluate the frequency response $\boldsymbol{V_L}(j\omega)/\boldsymbol{V_{in}}(j\omega)$ at both frequencies (60Hz and 1 kHz) if:

$$V_g(t) = \sin(2\pi 1000t)\ \text{V}\ ,\ r_g = 50\Omega$$

$$V_n(t) = 3\sin(2\pi 60t)\ \text{V},\ R_L = 300\Omega$$

and if L and C are as in part (b).

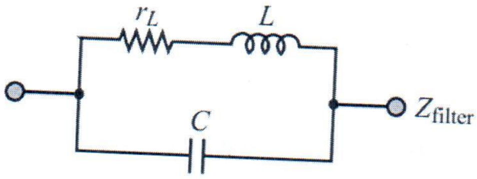


Fig. 3(a)

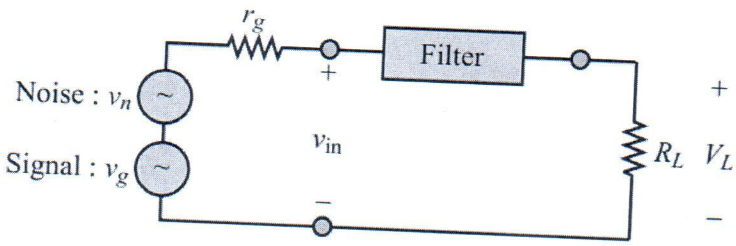


Fig. 3(b)

Your answer:

3

a) $Z_{filter} = (r_L + sL)||\frac{1}{sC} = \frac{1}{\frac{1}{r_L+sL}+sC} = \frac{r_L+sL}{(r_L+sL)sC+1}$, $s = j\omega$

$\frac{r_L + j\omega L}{j\omega r_L C \cancel{+} \omega^2 LC + 1}$

b)

Z

$\frac{I}{V} = Y = \frac{1}{Z_{filter}} = \frac{1}{r_L+j\omega L} + j\omega C = \frac{r_L}{r_L^2+\omega^2L^2} + j(\omega C - \frac{\omega L}{r_L^2+\omega^2L^2})$

5

At resonance, $\omega_0 C - \frac{\omega_0 L}{r_L^2+\omega_0{}^2L^2} = 0$

Which leads $C = \frac{L}{r_L^2+\omega_0{}^2L^2} = \frac{0.1}{25+(2\pi*60)^2 0.1^2} = 7 \times 10^{-5} C.$

6.94×10^{-5}

$\frac{L}{r^2+\omega_0^2L^2} = C$

$L = Cr^2 + \omega_0^2 L^2 C$

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c)

$$\frac{V_L(j\omega)}{V_{in}(j\omega)} = \frac{R_L}{Z_{filter}+R_L} = \frac{YR_L}{1+YR_L}.$$ 3

When $\omega = 2\pi 1000$,

$$Y = \frac{5}{25+(2\pi 1000)^2 0.1^2} + j\left(0.14\pi - \frac{2\pi 1000 \times 0.1}{25+(2\pi 1000)^2 0.1^2}\right) \approx 0.14\pi j \ or \ 0.44j$$

Then

$$\frac{V_L(2\pi 1000)}{V_{in}(2\pi 1000)} = \frac{j132}{1+j132}$$ 2

When $\omega = 2\pi 60$,

$$Y = \frac{5}{25+(2\pi 60)^2 0.1^2} = 3.45 \times 10^{-3}$$

Then

$$\frac{V_L(2\pi 1000)}{V_{in}(2\pi 1000)} = \frac{3.45\times10^{-3}\times300}{1+3.45\times10^{-3}\times30} = 0.51.$$ 2

15 = 3 + 5 + 3 + 2 + 2

Problem 4 (33 points) – Second-order circuit analysis

You must show your work to get full credit.

For the circuit in Fig. 4,

(a) Find the t-domain differential equations for $t > 0$ and solve for $v(t)$ in time domain.

(b) Construct the s-domain equivalent circuit for $t > 0$, then find $v(t)$ in s-domain.

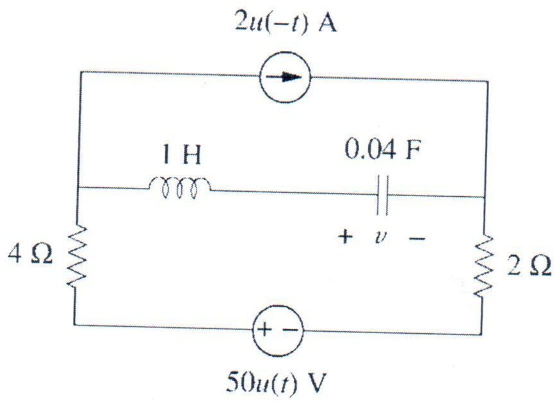
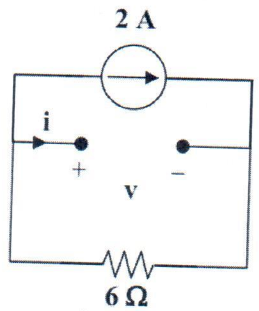


Fig. 4

Your answer:

a) For $t = 0-$, the equivalent circuit is shown below.



$$i(0-) = 0, v(0-) = -2 \times 6 = -12\text{V} \leftarrow 2$$

For $t > 0$, we have a series RLC circuit with a step input.

$$\alpha = \frac{R}{2L} = \frac{6}{2} = 3, \quad \omega_0 = \frac{1}{\sqrt{RC}} = \frac{1}{\sqrt{0.04}}, \quad s = -3 \pm \sqrt{9-25} = -3 \pm j4 \leftarrow 3$$

Thus, $v(t) = V_\infty + [(A\cos 4t + B\sin 4t)e^{-3t}] \leftarrow 2$

where $V_\infty = 50\text{V} \leftarrow 1$

$$v(t) = 50 + [(A\cos 4t + B\sin 4t)e^{-3t}]$$

$v(0) = -12 = 50 + A$ which gives $A = -62$. $\leftarrow 2$

$i(0) = 0 = C\frac{dv}{dt}|_{t=0}$

$\frac{dv}{dt}|_{t=0} = [-3(A\cos 4t + B\sin 4t)e^{-3t}] + [4(-A\sin 4t + B\cos 4t)e^{-3t}]|_{t=0} = -3A + 4B = 0$ 4

Thus $B = -46.5$

$$v(t) = \{50 + [(-62\cos 4t - 46.5\sin 4t)e^{-3t}]\}\text{V} \leftarrow 2$$

$2+3+2+1+2+4+2 = 16$

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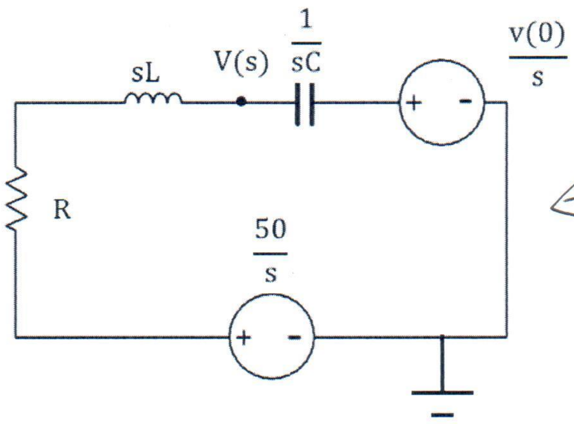
b) In s-domain the equivalent circuit is shown below.



← 5

where R = 6Ω, L = 1H, C = 0.04F

$$V(s) = \frac{v(0)}{s} + \frac{\frac{1}{sC}}{R + sL + \frac{1}{sC}}\left[\frac{50}{s} - \frac{v(0)}{s}\right] = -\frac{12}{s} + \frac{1550}{(s^2+6s+25)s}$$ ← 3

Let $\frac{1}{(s^2+6s+25)s} = \mathrm{F}(s)$, so let

$$F(s) = \frac{A}{s} + \frac{B(s+3)+C}{(s+3)^2+4^2} = \frac{(A+C)s^2 + (3A+B+6C)s + 25C}{(s^2+6s+25)s} = \frac{1}{(s^2+6s+25)s}$$

So

$$\begin{cases} A + C = 0 \\ 3A + B + 6C = 0 \\ 25C = 1 \end{cases} \xrightarrow{\text{yields}} \begin{cases} A = -\frac{1}{25} \\ B = -\frac{3}{25} \\ C = \frac{1}{25} \end{cases}$$ 2×3 = 6

So

$$V(s) = -62\frac{(s+3)+3}{(s+3)^2+4^2} + \frac{50}{s}$$ 3

↓ 3

$$v(t) = \left[-62\left(\mathrm{e}^{-3t}\cos 4t + \frac{3}{4}\mathrm{e}^{-3t}\sin 4t\right) + 50\mathrm{u(t)}\right]\mathrm{V}$$

5+3+6+3 = 17

16+17=33

Problem 5 (26 points) – Fourier series

You must show your work to get full credit.

The transfer function for the narrowband band-pass filter circuit in Fig. 5(a) is

$$H(s)=\frac{-K_0\beta s}{s^2+\beta s+\omega_0^2}$$

a) Find $K_0, \beta\ and\ \omega_0^2$ as functions of the circuit parameters $R_1, R_2, R_3, C_1\ and\ C_2$.

b) Write the first three terms in the Fourier series that represents v_0 if v_g is the periodic voltage in Fig. 5(b).

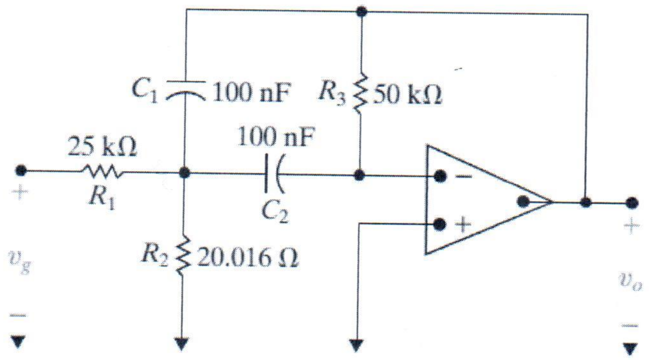


Fig. 5(a)

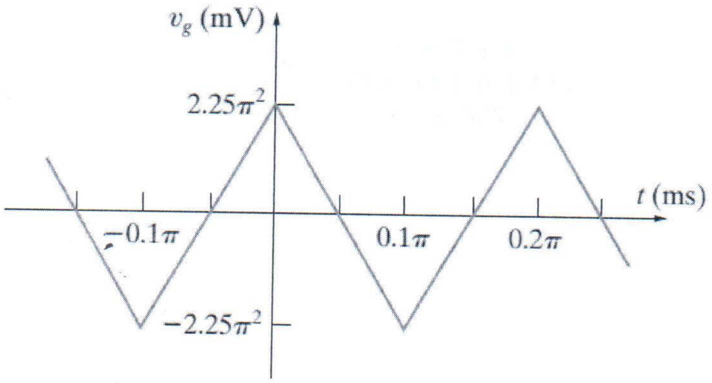


Fig. 5(b)

Your answer:

Let V_a represent the node voltage across R_2, then the node-voltage equations are

$$\frac{V_a-V_g}{R_1}+\frac{V_a}{R_2}+V_asC_2+(V_a-V_0)sC_1=0$$ 2

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$$(0 - V_a)sC_2 + \frac{0 - V_o}{R_3} = 0$$ ← 2

Solving for V_o in terms of V_g yields

$$\frac{V_o}{V_g} = H(s) = \frac{\frac{-1}{R_1C_1}s}{s^2 + \frac{1}{R_3}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)s + \frac{R_1+R_2}{R_1R_2R_3C_1C_2}}$$

It follows that

$$\omega_o^2 = \frac{R_1 + R_2}{R_1R_2R_3C_1C_2}$$ ← 2

$$\beta = \frac{1}{R_3}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$ ← 2

$$K_o = \frac{R_3}{R_1}\left(\frac{C_2}{C_1 + C_2}\right)$$ ← 2

Note that

$$H(s) = \frac{-\frac{R_3}{R_1}\left(\frac{C_2}{C_1+C_2}\right)\frac{1}{R_3}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)s}{s^2 + \frac{1}{R_3}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)s + \left(\frac{R_1+R_2}{R_1R_2R_3C_1C_2}\right)}$$

[b] For the given values of R_1, R_2, R_3, C_1, and C_2 we have

$$H(s) = \frac{-400s}{s^2 + 400s + 10^8}$$

$$v_g = \frac{(8)(2.25\pi^2)}{\pi^2}\sum_{n=1,3,5,}^{\infty}\frac{1}{n^2}\cos n\omega_o t$$

$$= 18\left[\cos\omega_o t + \frac{1}{9}\cos 3\omega_o t + \frac{1}{25}\cos 5\omega_o t + \cdots\right] \text{mV}$$

$$= [18\cos\omega_o t + 2\cos 3\omega_o t + 0.72\cos 5\omega_o t + \cdots]\,\text{mV}$$

8

$$\omega_o = \frac{2\pi}{0.2\pi} \times 10^3 = 10^4 \text{ rad/s}$$

$$H(jk10^4) = \frac{-400jk10^4}{10^8 - k^2 10^8 + j400k10^4} = \frac{-jk}{25(1-k^2) + jk}$$

$$H_1 = -1 = 1\underline{/180^\circ}$$ ← 1

$$H_3 = \frac{-j3}{-200 + j3} = 0.015\underline{/90.86^\circ}$$ ← 2

$$H_5 = \frac{-j5}{-600 + j5} = 0.0083\underline{/90.48^\circ}$$ ← 2

$$\cancel{v_o = -18\cos\omega_o t + 0.03\cos(3\omega_o t + 90.86^\circ)}$$

$$\cancel{H_5 = \frac{-j5}{-600 + j5} = 0.0083\underline{/90.48^\circ}}$$

$$v_o = -18\cos\omega_o t + 0.03\cos(3\omega_o t + 90.86^\circ)$$ ← 3

$$+ 0.006\cos(5\omega_o t + 90.48^\circ) + \cdots \text{ mV}$$

[c] The fundamental frequency component dominates the output, so we expect the quality factor Q to be quite high.

[d] $\omega_o = 10^4$ rad/s and $\beta = 400$ rad/s. Therefore, $Q = 10{,}000/400 = 25$. We expect the output voltage to be dominated by the fundamental frequency component since the bandpass filter is tuned to this frequency!

$$\underbrace{2+2+2+2+2}_{10} + \underbrace{8+1+2+2+3}_{16} = 26$$