Problem 1

(20 points)

(a) Determine the Fourier series coefficients a_k for $x_1(t)$ shown below.

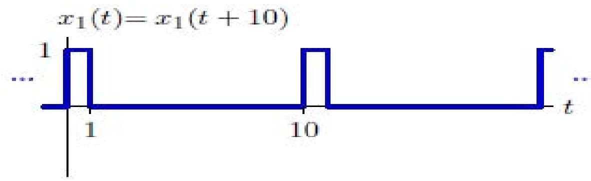

Figure 1: Problem 1(a)

(b) Determine the Fourier series coefficients b_k for $x_2(t)$ shown below.

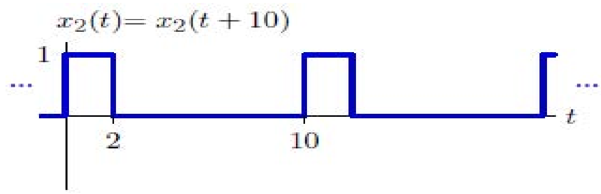

Figure 2: Problem 1(b)

(c) Determine the Fourier series coefficients c_k for $x_3(t)$ shown below.

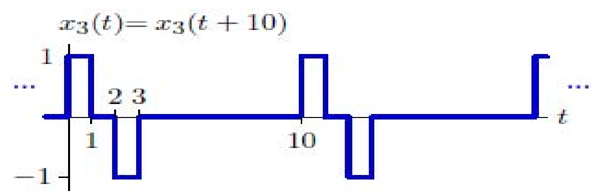

Figure 3: Problem 1(c)

Problem 2

(20 points) Suppose that we are given the following information about a signal $x[n]$

1. $x[n]$ is a real and even signal.

2. $x[n]$ has a period $N = 10$ and Fourier coefficients a_k.

3. $a_{11} = 5$.

4. $\frac{1}{10}\sum_{n=0}^{9}|x[n]|^2 = 50$.

Show that $x[n] = Acos(Bn + C)$, and specify numerical values for the constants $A, BandC$.

Problem 3

(20 points) Consider the following three continuous-time signals with a fundamental period of $T = \frac{1}{2}$:

$$\begin{aligned} x(t) &= cos(4\pi t) \\ y(t) &= sin(4\pi t) \\ z(t) &= x(t)y(t) \end{aligned} \tag{1}$$

(a) Determine the Fourier series coefficients of $x(t)$.

(b) Determine the Fourier series coefficients of $y(t)$.

(c) Use the result of part(a) and (b), along with the multiplication property of the continuous-time Fourier series, to determine the Fourier series coefficients of $z(t) = x(t)y(t)$.

(d) Determine the Fourier series coefficients of $z(t)$ through direct expansion of $z(t)$ in trigonometric form, and compare your result with that of part(c).

Problem 4

(20 points)

(a) Draw the Fourier series coefficients of $x_1(t)$ and give explanation.

$$x_1(t) = 2 - 2cos(\frac{2\pi}{3}t) \tag{2}$$

(b) Draw the Fourier series coefficients of $x_2(t)$ and give explanation.

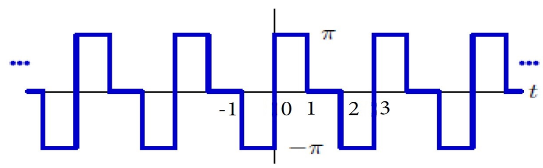

Figure 4: $\mathrm{x}_2(t)$

Hint: When you graph, just draw the case where $k \in [-6, 6]$. And make sure to write their Fourier series coefficients' expressions.

Problem 5

(20 points)

(1) Consider a continuous-time ideal lowpass filter $h(t)$ whose frequency response is

$$H(j\omega) = \begin{cases} 1, & |\omega| \leq 100 \\ 0, & |\omega| > 100 \end{cases}$$

When the input to this filter is a signal $x(t)$ with fundamental period $T = \pi/6$ and Fourier series coefficients a_k, it is found that

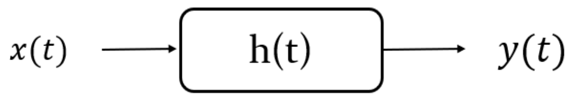

Figure 5: y(t)

Where $y(t) = x(t)$, and for what values of k is it guaranteed that $a_k = 0$?

(2) Consider a discrete-time LTI system with impulse response

$$h[n] = \begin{cases} 1, & 0 \leq n \leq 2 \\ -1, & -2 \leq n \leq -1 \\ 0, & \text{otherwise} \end{cases}$$

Given that the input to this system is

$$x[n] = \sum_{k=-\infty}^{+\infty} \delta[n - 4k],$$

determine the Fourier series coefficients of the output $y[n]$.