Homework 3

Due date: Apr. 7th, 2020, Tuesday

Submit online before 23:59

Rules:

- Work on your own. Discussion is permissible, but extremely similar submissions will judged as plagiarism.
- Please show all intermediate steps: a correct solution without an explanation will get zero credit.
- Please submit on time.
- Please prepare your submission in English only. No Chinese submission will be accepted.

1. Find the voltage across the capacitors in the circuit of Fig.1 under dc conditions.

(5 points)

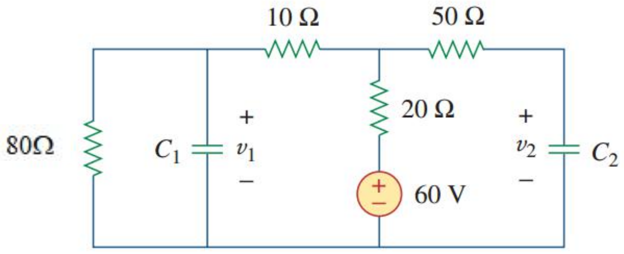


Figure 1.

As DC conditions apply,set each capacitor in the circuit as an open circuit.

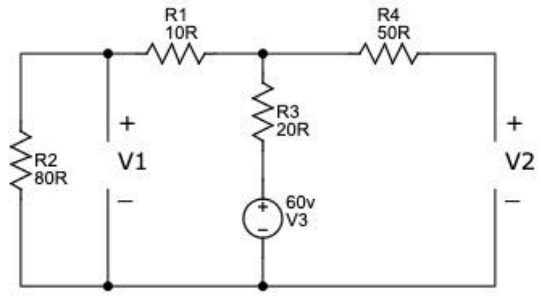


Figure 1a

Redraw the resultant circuit after removing the off-components

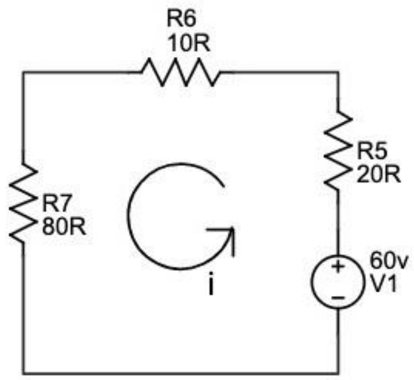


Figure 1b

Find the value of the current i, by applying KVL,

$$20i + 10i + 80i - 60 = 0$$

$$i = \frac{6}{11} A$$

Finding v_1&v_2

$$V_1 = \frac{6}{11} \times 80 = 43.64V$$

$$V_{20\Omega} = \frac{6}{11} \times 20 = \frac{120}{11}V$$

$$V_2 = V_{Voltage\ Source} - V_{20\Omega} = 49.09V$$

2. The current in a 50 mH inductor is known to be

$i = 60\text{mA}, \qquad t \le 0;$

$i = A_1 e^{-500t} + A_2 e^{-2000t}\,\text{A}, \qquad t \ge 0.$

The voltage across the inductor (passive sign convention) is 3 V at $t = 0$.

a) Find the expression for the voltage across the inductor for $t > 0$.

b) Find the time, greater than zero, when the power at the terminals of the inductor is zero.

(10 points)

(a)

$$i(0) = A_1 e^0 + A_2 e^0 = 0.06A$$

$$\frac{di}{dt} = -500A_1 e^{-500t} - 2000A_2 e^{-2000t}$$

$$v = L\frac{di}{dt} = -25A_1 e^{-500t} - 100A_2 e^{-2000t}$$

$$v(0) = -25A_1 e^0 - 100A_2 e^0 = -25A_1 - 100A_2 = 3V$$

$$A_1 + A_2 = 0.06$$

$$-25A_1 - 100A_2 = 3$$

$$A_1 = 0.12$$
$$A_2 = -0.06$$

$$v(t) = -25(0.12)e^{-500t} - 100(-0.06)e^{-2000t}$$

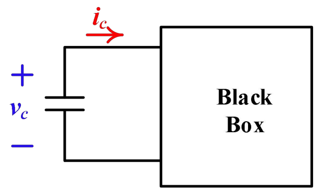
$$v(t) = -3e^{-500t} + 6e^{-2000t}V$$

(b)

$$v(t) = -3e^{-500t} + 6e^{-2000t}$$
$$v(t) = 0 \Rightarrow$$
$$3e^{-500t} = 6e^{-2000t}$$
$$t = \frac{\ln 2}{1500} = 4.62 \times 10^{-4} s$$

3. The voltage across a 5uF capacitor is known to be

$v_c = 400te^{-2500t}\text{V}$ for $t \geq 0.$



a) Find the current flow out of the capacitor (i_c) for $t > 0$.
b) Find the instant power delivered from the capacitor when t = 100us.
c) Is the capacitor absorbing or delivering power at t = 100us ?
d) Find the energy stored in the capacitor at t = 100us.
e) Find the maximum energy stored in the capacitors and the time when the maximum occurs.

(10 points)

(a)

$$i_c = -C\frac{dv}{dt} = -C\frac{d}{dt}(400te^{-2500t}) = -5\times10^{-6}\times400(e^{-2500t} - 2500te^{-2500t})$$
$$i_c = -0.002e^{-2500t} + 5te^{-2500t}\text{A}$$

(b)

$$p = v(t = 100\mu s) \times (-i_c)(t = 100\mu s) = 3.64 \times 10^{-5}W$$

So the capacitor delivered -3.639*10^{-5}W power

(c)

The capacitor is absorbing power because the power is positive.

(d)

$$W = C\int_{t0}^{t} v(t)\frac{dv(t)}{dt}dt = 2.43\times10^{-9}J$$

(e)

$$W = 0.5CV^2 = 0.5(5\times10^{-6})\times160000t^2e^{-5000t} = 0.4t^2e^{-5000t}$$

$$W' = 0.8te^{-5000t} - 2000t^2e^{-5000t} = 0$$

$$t = 4\times10^{-4}s$$

$$W = 8.66\times10^{-9}J$$

4. Consider the circuit in Fig. 2. Given that $v(t) = 12e^{-3t}\text{mV}$ for $t > 0$ and $i_1(0) = -10$ mA, $i_2(0) = -20$ mA. find: $i_1(t)$ and $i_2(t)$.(5 points)

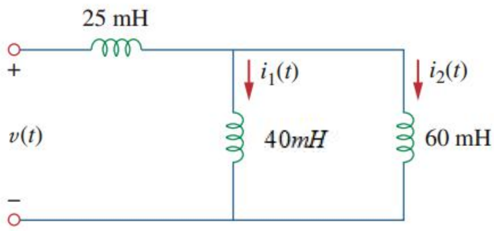


Figure 2.

$$L_{eq1} = 40//60$$
$$L_{eq} = 25 + L_{eq1}$$

Thus,

$$L_{eq1} = \frac{40 \times 60}{40 + 60} = 24mH$$
$$L_{eq} = 25 + 24 = 49mH$$

$$\begin{cases} v(t) = L_{eq}\dfrac{di(t)}{dt} \\ i(0) = i_1(0) + i_2(0) = -30mA \end{cases}$$
$$\Rightarrow i(t) = -\frac{4}{49}e^{-3t} + \frac{253}{4900}A$$

$$v(t) - L_{25mH}\frac{di_0(t)}{dt} = L_{40mH}\frac{di_1(t)}{dt} = L_{60mH}\frac{di_2(t)}{dt}$$
$$\Rightarrow i_1(t) = -\frac{12}{245}e^{-3t} + \frac{955}{24500}A = -0.049e^{-3t} + 0.039A$$
$$\Rightarrow i_2(t) = -\frac{8}{245}e^{-3t} + \frac{31}{2450}A = -0.033e^{-3t} + 0.013A$$

5. For the circuit in Fig. 3, if

$$v = 20e^{-4t}\text{V} \quad \text{and} \quad i = 0.25e^{-4t}\text{A}, \quad t \geq 0$$

(a) Find R and C.
(b) Determine the time constant.
(c) Calculate the initial energy in the capacitor.
(d) Obtain the time it takes to dissipate 50 percent of the initial energy.

(10 points)

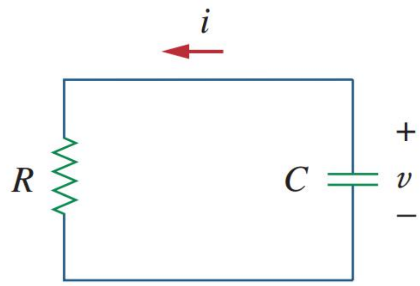


Figure 3.

(a)

The voltage across the capacitor is given by,

$$v(t) = v_0 e^{\frac{-t}{\tau}}$$

$$\frac{1}{\tau} = 4$$

$$\tau = \frac{1}{4}$$

But,

$$\tau = RC$$

So,

$$RC = 1/4$$

And,

$$i = C\frac{dv}{dt}$$
$$-0.25e^{-4t} = C\frac{d(20e^{-4t})}{dt}$$
$$C = \frac{1}{320} = 0.003125F$$
$$R = 80\Omega$$

(b)

$$\tau = 0.25s$$

(c)

The initial energy in the capacitor is given by,

$$w_c(0) = \frac{1}{2}Cv^2(0)$$
$$v(0) = 20V$$
$$w_c(0) = 0.5\times 0.003125\times 400 = 0.625J$$

(d)

$$w_c(t) = 0.5Cv^2(t)$$

To find the time it takes to dissipate 50% of the initial energy,

$$w_c(t) = \frac{w_c(0)}{2}$$

Substitute,

$$w_c(t) = \frac{w_c(0)}{2}$$
$$0.3125 = 0.5\times 0.003125\times (20e^{-4t})^2$$

$$t = 8.66\times 10^{-2}s$$

6. The switch in Fig. 4 has been closed for a long time before opening at $t = 0$. Find

   a) $i_L(t),\quad t \geq 0$.
   b) $v_L(t),\quad t \geq 0^+$.
   c) $i_\Delta(t),\quad t \geq 0^+$.

   (10 points)

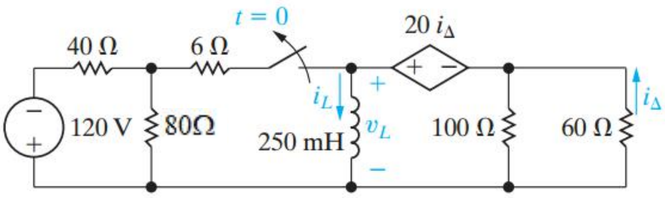


Figure 4.

(a)

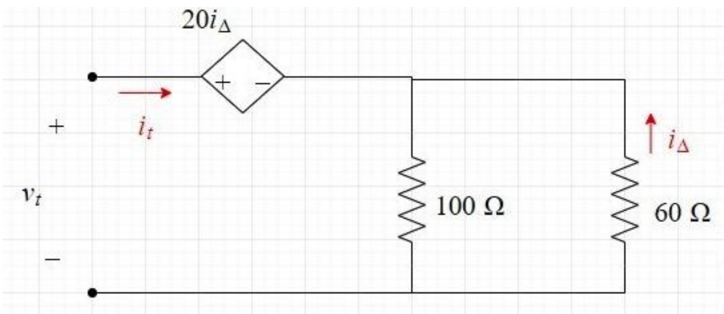
$$i_{total}(0) = \frac{-120}{40 + (80//6)} = -2.63A$$

$$i_L(0) = \frac{80}{80+6} i_{total}(0) = -2.45A$$

$$i_\Delta = \frac{-100}{100+60} i_t = -0.625 i_t$$

By applying KVL in the left mesh,

$$v_t = 20 i_\Delta + i_t \frac{100 \times 60}{100+60} = 20 i_\Delta + 37.5 i_t V$$



$$v_t = 25 i_t$$

$$R_{th} = \frac{v_t}{i_t} = 25\Omega$$

$$i_L(t) = i_L(0)e^{-\frac{R}{L}t} = -2.45e^{-\frac{25}{0.25}t} = -2.45e^{-100t}A$$

(b)

$$v_L(t) = L\frac{di_L(t)}{dt} = 0.25(-2.45)(-100)e^{-100t} = 61.25e^{-100t}V$$

(c)

$$i_\Delta(t) = \frac{100}{100+60}i_L(t) = -1.53125e^{-100t}A$$

7. Express $v(t)$ in Fig. 5 in terms of step functions.(5 points)

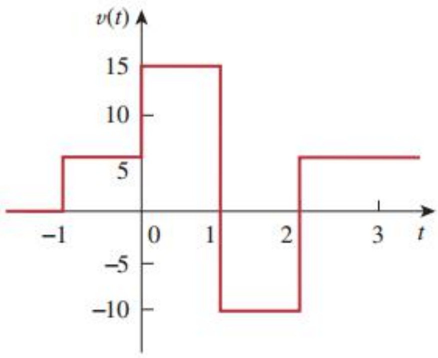


Figure 5.

For-1< t <0:

The amplitude of the voltage increases by 5 voltage units with respect to the previous time range amplitude,thus,

$$v_1(t) = 5u(t-(-1))$$
$$v_1(t) = 5u(t+1)$$

For 0< t < 1:

The amplitude of the voltage increases by 10 voltage units with respect to the previous time range amplitude,thus,

$$v_2(t) = 5u(t-0)$$
$$v_2(t) = 10u(t)$$

For 1< t <2:

The amplitude of the voltage decreases by 25 voltage units with respect to the previous time range amplitude,thus,

$$v_3(t) = -25u(t-1)$$

For $t > 2$:

The amplitude of the voltage increases by 15 voltage units with respect to the previous time range amplitude, thus,

$$v_4(t) = 15u(t-2)$$

The expression for the voltage $v(t)$ is given by,

$$v(t) = v_1(t) + v_2(t) + v_3(t) + v_4(t)$$

$$v(t) = 5u(t+1) + 10u(t) - 25u(t-1) + 15u(t-2)$$

8. (a) If the switch in Fig. 6 has been open for a long time and is closed at $t = 0$, find $v_o(t)$.
   (b) Suppose that the switch has been closed for a long time and is opened at $t = 0$. Find $v_o(t)$.

   (5 points)

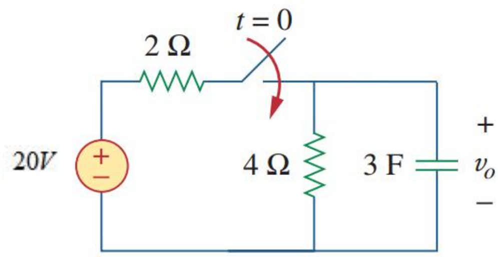


Figure 6.

(a)

For $t \le 0$

$$v_0(t) = 0V$$

For $t > 0$,

$$\mathrm{Re}\,q = 2//4 = \frac{4}{3}\Omega$$

When $t \to \infty$,

$$v_0(\infty) = v_{4\Omega}$$

$$v_0(\infty) = v_{4\Omega} = 20 \times \frac{4}{4+2} = \frac{40}{3}V = 13.33V$$

$$\tau = RC = \frac{4}{3} \times 3 = 4s$$

For $t > 0$,

$$v_0(t) = 13.33 - 13.33e^{-\frac{t}{4}}\,\mathrm{V}$$

(b)

For $t < 0$

$$v_0(t) = v_{4\Omega} = 20 \times \frac{4}{4+2} = 13.33V$$

For $t > 0$

$$v_0(\infty) = 0$$

$$\tau = RC = 4 \times 3 = 12s$$

For $t > 0$,

$$v_0(t) = 13.33e^{-\frac{t}{12}} \text{ V}$$

9. The switch in the circuit seen in Fig. 7 has been in position x for a long time. At $t = 0$, the switch moves instantaneously to position y.
   a) Find α so that the time constant for $t > 0$ is 40 ms.

   b) For the α found in (a), find v_Δ

   (10 points)

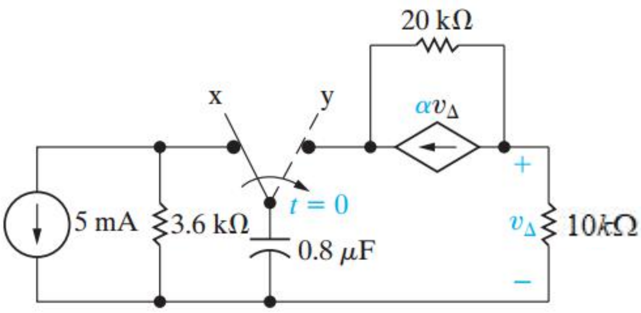


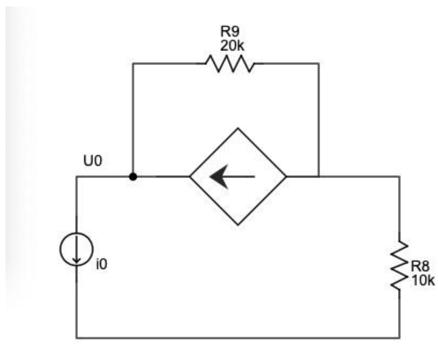
Figure 7.

(a)

For $t = 0$,

$$v_{3.6k\Omega} = -5mA \times 3.6k\Omega = -18V$$
$$\tau = RC = 40ms$$
$$R = 5 \times 10^4 \Omega$$

According to the Thevenin's theorem



$$u_0 = 10k \times i_0 + (i_0 + 10k\alpha i_0)20k$$
$$R = \frac{u_0}{i_0} = 5 \times 10^4 \Omega$$
$$\alpha = 1 \times 10^{-4}$$

(b)

$$i(t) = C\frac{dv(t)}{dt}$$
$$v(t) = i(t)5 \times 10^4 \Omega$$
$$\Rightarrow i(t) = 3.6 \times 10^{-4} e^{-25t} A$$
$$\Rightarrow v_\Delta = -3.6e^{-25t} V$$

10. For the circuit in Fig. 8, calculate $i(t)$ if $i(0) = 0$. (10 points)

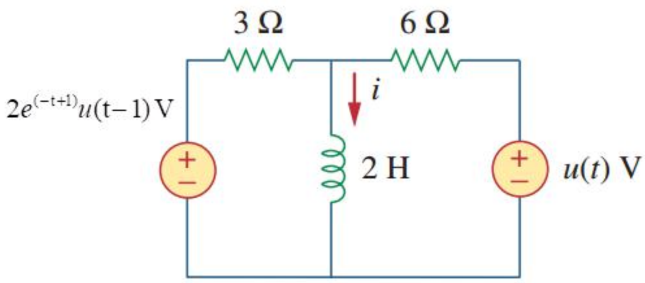


Figure 8.

The unit step function $u(t)$ is given by,

$$u(t) = \begin{cases} 0, & t < 0 \\ 1, & t \geq 0 \end{cases}$$

Thus,

$$v_R(t) = \begin{cases} 0\text{V}, & t < 0 \\ 1\text{V}, & t \geq 0 \end{cases}$$

Where v_R is the voltage value of the right-hand side voltage source.

The unit step function $2e^{(-t+1)}u(t-1)$ is given by,

$$u(t) = \begin{cases} 0, & t < 1 \\ 2e^{(-t+1)}, & t \geq 1 \end{cases}$$

Thus,

$$v_L(t) = \begin{cases} 0\text{V}, & t < 1 \\ 2e^{(-t+1)}\text{V}, & t \geq 1 \end{cases}$$

Where v_L is the voltage value of the left-hand side voltage source.

For $0 < t < 1$

When the steady state conditions are reached in this time range, the inductor acts as a short circuit and the resultant circuit is shown in Figure 8a,

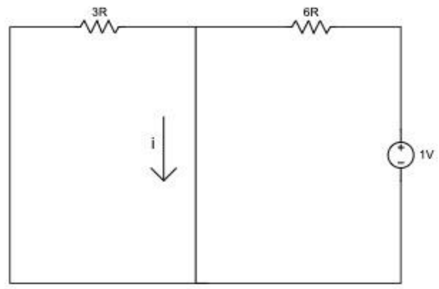


Figure 8a

From Figure 8a, no current flows in the 3 Ω resistor, thus the current through the inductor branch is,

$$i_{ss} = \frac{1}{6}\text{A}$$

i_{ss} is the current through the inductor branch in the steady state.

The equivalent resistance from the inductor terminal is the parallel connection of the 3 Ω and the 6 Ω resistors, thus,

$$R_{eq} = \frac{3 \times 6}{3+6} = 2\Omega$$

The time constant is given by,

$$\tau = \frac{L}{R_{eq}} = \frac{2}{2} = 1\,\text{sec}$$

The current through the inductor for this time range (0 < t <1) is given by,

$$i(t) = i_{ss} + \left(i(0) + i_{ss}\right)e^{-\frac{t}{\tau}}$$

Substitute,

$$i(t) = \frac{1}{6} + \left(0 - \frac{1}{6}\right)e^{-\frac{t}{1}}$$

$$i(t) = \frac{1}{6}\left(1 - e^{-t}\right)\text{A}, \qquad 0 < t < 1 \ldots (1)$$

For $t > 1$

The initial current through the inductor is obtained by substituting $t = 1$ in eq(1),

$$i(t)=\frac{1}{6}\left(1-e^{-1}\right)=0.105\text{A}$$

When $t>1$, the left- and right-hand side sources exist, the resultant circuit is shown in Figure 8b,

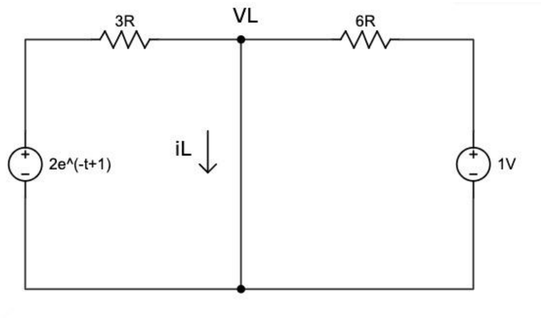


Figure 8b

$$\begin{cases} \dfrac{2e^{(-t+1)}-v(t)}{3\Omega}+\dfrac{1\text{V}-v(t)}{6\Omega}=i(t) \\ v(t)=L\dfrac{di(t)}{dt} \end{cases}$$

$$\Rightarrow i(t)=\frac{2}{3}(t-1)e^{-t+1}+\frac{1}{6}-\frac{1}{6}e^{-t}\text{A}$$

Result

$$i(t)=\begin{cases} 0\text{A}, & t\le 0 \\ \dfrac{1}{6}(1-e^{-t})\text{A}, & 0<t<1 \\ \dfrac{2}{3}(t-1)e^{-t+1}+\dfrac{1}{6}-\dfrac{1}{6}e^{-t}\text{A}, & \text{t}\ge 1 \end{cases}$$

11. The switch in the circuit in Fig. 9 has been open a long time before closing at $t = 0$.

Find $v_o(t)$ for $t \geq 0^+$.(10 points)

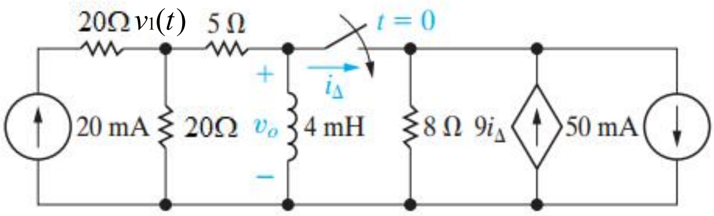


Figure 9.

$$i_0(0^-) = \frac{20}{20+5} \times 0.02 = 0.016A = i_0(0^+)$$

For $t > 0$,apply KCL

$$\begin{cases} 0.02 = \dfrac{v_1(t)-0}{20} + \dfrac{v_1(t)-v_0(t)}{5} \\ \dfrac{v_1(t)-v_0(t)}{5} = i_0(t) + i_\Delta(t) \\ i_\Delta(t) = \dfrac{v_0(t)-0}{8} - 9i_\Delta(t) + 0.05 \\ v_0(t) = L\dfrac{di_0(t)}{dt} \end{cases}$$

$$\Rightarrow \frac{di_0(t)}{dt} + \frac{1}{21} \times 10^5 i_0(t) = \frac{1100}{21}, p = \frac{1}{21} \times 10^5, q = \frac{1100}{21}$$

$$\Rightarrow \tau = \frac{1}{p} = 21 \times 10^{-5}, i_0(\infty) = 0.011$$

$$\Rightarrow i_0(t) = 0.011 + 0.005e^{-4761.905t}\text{A}$$

$$\Rightarrow v_0(t) = -0.0952e^{-4761.905t}\text{V}$$

12. In the circuit of Fig. 10, find v_o and i_o, given that $v_s = 4u(t)$ V and $v(0) = 1$ V.

(10 points)

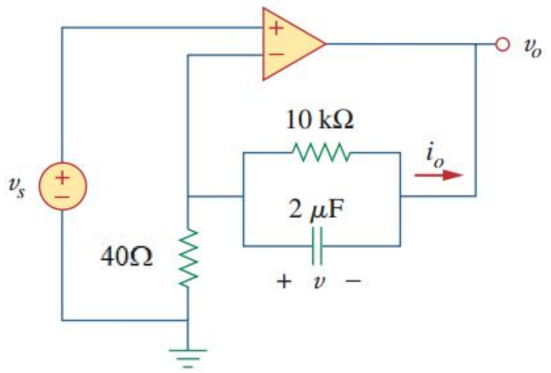


Figure 10.

$$\frac{0 - v_{40\Omega}}{40} = \frac{v_{40\Omega} - v_0}{10k}$$

$$v_{40\Omega} = vs = 4V$$

$$v_0(\infty) = 1004V$$

$$v_{40\Omega}(\infty) = 4V$$

$$v(\infty) = v_{40\Omega}(\infty) - v_0(\infty) = -1000V$$

$$\text{Req} = 10k\Omega$$

$$\tau = \text{Re}\,qC = 0.02s$$

$$v(t) = v(\infty) + (v(0) - v(\infty))e^{-\frac{t}{\tau}} = -1000 + 1001e^{-50t}V$$

$$v_0(t) = v_{40\Omega}(t) - v(t) = 1004 - 1001e^{-50t}V$$

$$i_0(t) = -\frac{v_{40\Omega}}{40} = -0.1A$$