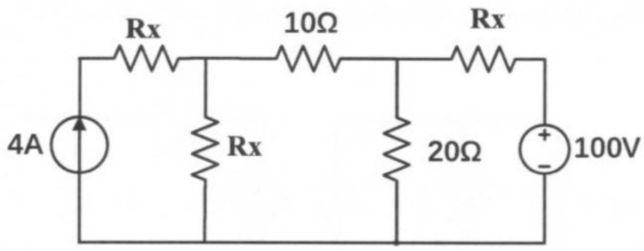
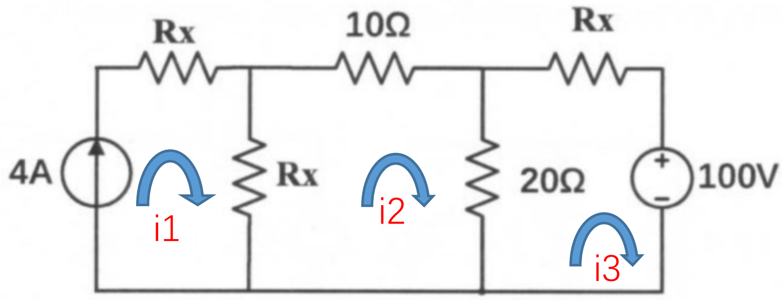
1. For the circuit below, determine the value of Rx such that NO current flows through the 10 Ω resistor in the circuit. (12pt)



Mesh analysis:



Mesh1 : $i_1 = 4\text{A}$ ....①(2')

Mesh2 : $R_\text{x}(i_2 - i_1) + 10i_2 + 20(i_2 - i_3) = 0$ ....②(2')

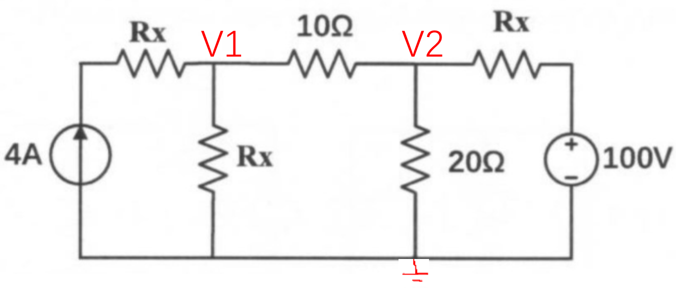
Mesh3 : $20(i_3 - i_2) + R_\text{x}i_3 + 100 = 0$ ....③(2')

Additional equation(No current flows the 10Ω resistor) : $i_2 = 0\text{A}$ ....④(2')

We can get: $R_x^2 + 20R_\text{x} - 500 = 0$

and the value of Rx is : $R_\text{x} = 10(\sqrt{6} - 1) = 14.49\Omega$ ....(4')

Nodal analysis:



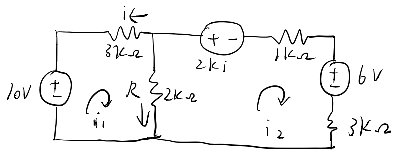
Node 1 : $-4 + \frac{V_1}{R_x} + \frac{V_1 - V_2}{10} = 0$ ....①(3')

Node 2 : $\frac{V_2 - V_1}{10} + \frac{V_2}{20} + \frac{V_2 - 100}{R_x} = 0$ ....②(3')

Additional equation(No current flows the 10Ω resistor) : $V_1 = V_2$ .... ③(2')

We can get: $R_x^2 + 20R_\text{x} - 500 = 0$

and the value of Rx is : $R_\text{x} = 10(\sqrt{6} - 1) = 14.49\Omega$ ....(4')



mesh

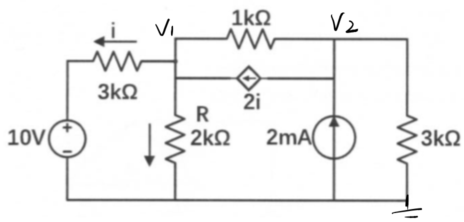
$$\begin{cases} \text{mesh1: } -10 + 3k\,i_1 + 2k(i_1 - i_2) = 0 \quad \cdots\ 2 \\ \text{mesh2: } 6k\,i_2 - 2k\,i_1 + 2ki + 6 = 0 \quad \cdots\ 2 \\ i = -i_1 \quad \cdots\ 2 \end{cases}$$

方程共6分 按实际情况给分

$$\Rightarrow \begin{cases} i_1 = \frac{24}{11000}\ A \quad \cdots\ 2 \\ i_2 = \frac{1}{2200}\ A \quad \cdots\ 2 \end{cases}$$

$$i_R = i_1 - i_2 \approx 0.0017\,A = 1.7\,mA \quad \cdots\ 2$$

nodal



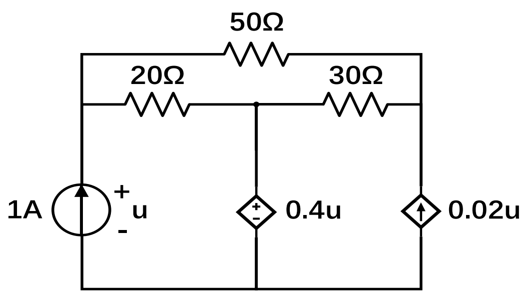
$$\begin{cases} \text{nodal 1: } \frac{V_1 - 10}{3k} + \frac{V_1}{2k} + \frac{V_1 - V_2}{1k} = 2i \quad \cdots\ 2 \\ \text{nodal 2: } \frac{V_2 - V_1}{1k} + \frac{V_2}{3k} + 2i = 0.002 \quad \cdots\ 2 \\ i = \frac{V_1 - 10}{3k} \quad \cdots\ 2 \end{cases}$$

方程共6分 按实际情况给分

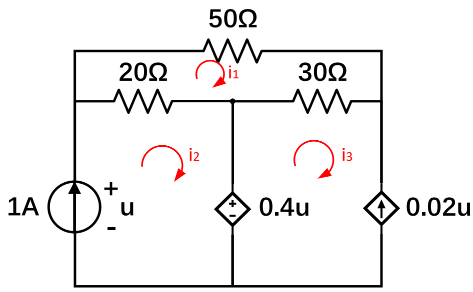
$$\begin{cases} V_1 = \frac{38}{11}\ V \quad \cdots\ 3 \\ V_2 = \frac{81}{11}\ V. \quad \cdots\ 1 \end{cases}$$

$$i_R = \frac{V_1}{2k} = \frac{19}{11}\,mA \approx 1.73\,mA. \quad \cdots\ 2$$

3.Use Mesh Analysis method to calculate the power supplied by the 1A current source.(12’)



Mesh analysis:



Mesh 1:$50i_1 + 30(i_1 - i_3) + 20(i_1 - i_2) = 0$ ………….①(2’)

Mesh2: $20(i_2 - i_3) + 0.4u - u = 0$ ………….②(2’)

Mesh3: $i_3 = -0.02u$ ………….③(2’)

Additional equation: $i_2 = 1$ ….………..④(1’)

We can get: $u = \frac{100}{3} = 33.33v$ …………. …(2’)

and the power is $P = -ui_2 = -\frac{100}{3} = -33.33W$ ……………..(2’)

It represents the 1A current source is supplying power of 33.33W…….(1’)

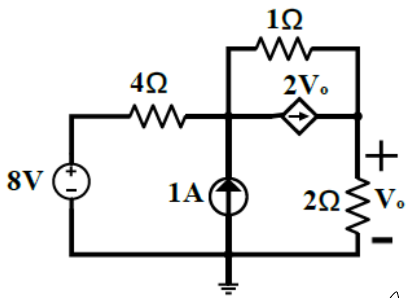
Hint: if students used the nodal analysis method, they would not get grades.

4. Linear Property& Superposition (12pt)

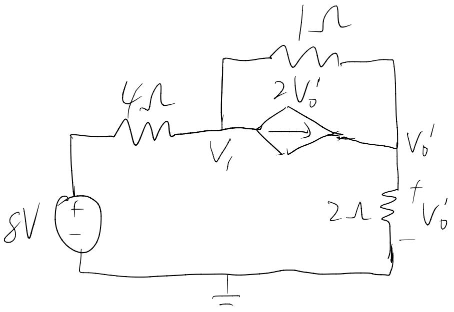
(1) For the two independent sources below, if keeping the independent voltage source only (turn off the independent current source), find Vo and the power absorbed by the 2Ω resistor.

(2) For the two independent sources below, if keeping the independent current source only (turn off the independent voltage source), find Vo and the power absorbed by the 2Ω resistor.

(3) From the results in (1) and (2), find the power absorbed by the 2Ω resistor in the following figure.



(1)



$$\begin{cases} \dfrac{V_1-8V}{4\Omega}+2V_0'+\dfrac{V_1-V_0'}{1\Omega}=0 & \sim 1' \\ \dfrac{V_0'-V_1}{1\Omega}-2V_0'+\dfrac{V_0'}{2\Omega}=0 & \sim 1' \end{cases}$$

$V_0'=\frac{16}{3}V=5.33V$ (5.33V±0.01V 都给分) ~1'

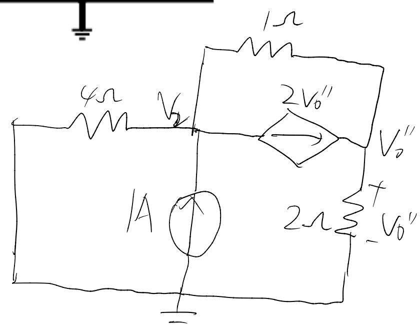
$P_1=\frac{V_0'^2}{2\Omega}=\frac{128}{9}W=14.22W$ ~1'

(14.22W±0.05W 都给分)

* (1)中V_0', P_1 (2)中V_0'', P_2 (3)中P 五个答案分数小数均给分，但单位少1个扣1分

整个大题因单位扣的分≤2分

(2)



$$\begin{cases} \dfrac{V_2}{4\Omega}-1A+2V_0''+\dfrac{V_2-V_0''}{1\Omega}=0 & \sim 1' \\ \dfrac{V_0''-V_2}{1\Omega}-2V_0''+\dfrac{V_0''}{2\Omega}=0 & \sim 1' \end{cases}$$

$V_0''=\frac{8}{3}V=2.67V$ (2.67V±0.01V 都给分) ~1'

$P_2=\frac{V_0''^2}{2\Omega}=\frac{32}{9}W=3.56W$ ~1'

(3.56W±0.05W 都给分)

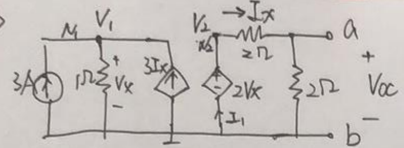
(3) $V_o=V_0'+V_0''=8V$ (8V±0.05V 都给分) ~2'

$P=\frac{V_0^2}{2\Omega}=32W$ ~2'

(32W±1W 都给分)

* 错误解答: $P=P_1+P_2=17.78W$

5)



Solution: 1° Find V_{oc}.

For N_1: $3 + \frac{0-V_1}{1} + 3I_x = 0$

For N_2: $I_1 + \frac{0-V_2}{4} = 0$ ... 2′

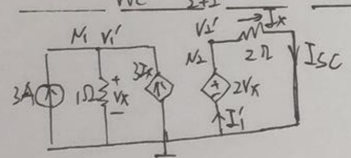
And, $V_2 = 2V_x$

$V_x = V_1$

$I_x = \frac{V_2 - 0}{4}$

$\Rightarrow \begin{cases} V_1 = -6V \\ V_2 = -12V \end{cases}$

$V_{oc} = \frac{2}{2+2} \times V_2 = -6V.$ ... 3′



2° Find I_{sc}.

For N_1: $3 + \frac{0-V_1'}{1} + 3I_x = 0$

For N_2: $I_1' + \frac{0-V_2'}{2} = 0$ ... 2′

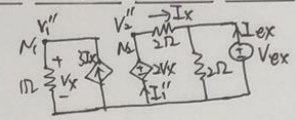
And, $V_2' = 2V_x$

$V_x = V_1'$

$I_x = \frac{V_2' - 0}{2}$

$\Rightarrow \begin{cases} V_1' = -\frac{3}{2}V \\ V_2' = -3V \end{cases}$

$I_{sc} = \frac{V_2'}{2} = -\frac{3}{2}A$ ... 3′



3° Find R_{eq}.

For N_1: $\frac{0-V_1''}{1} + 3I_x = 0.$

For N_2: $I_1'' + \frac{V_{ex} - V_2''}{2} = 0$ ... 2′

And, $V_2'' = 2V_x$

$V_x = V_1''$

$I_x = \frac{V_2'' - V_{ex}}{2}$

$\Rightarrow \begin{cases} V_1'' = \frac{3}{4}V_{ex} \\ V_2'' = \frac{3}{2}V_{ex} \end{cases}$

$I_{ex} = \frac{V_{ex} - V_2''}{2} + \frac{V_{ex} - 0}{2} = \frac{1}{4}V_{ex}$

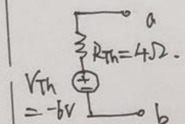
$R_{eq} = \frac{V_{ex}}{I_{ex}} = 4\Omega.$ ... 3′

1°, 2°, 3° 任意两个部分组合为10分过程分。其中，每个部分的网孔/节点方程2分，算出正确结果5分。另外，3°可取V_{ex}的特殊值计算，评分标准相同。

(1) Thevenin equivalent circuit.

$V_{Th} = V_{oc} = -6V$

$R_{Th} = R_{eq} = \frac{V_{oc}}{I_{sc}} = 4\Omega.$

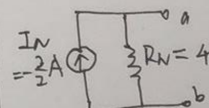


... 2′

(2) Norton equivalent circuit

$I_N = I_{sc} = -\frac{3}{2}A$

$R_N = R_{eq} = \frac{V_{oc}}{I_{sc}} = 4\Omega$

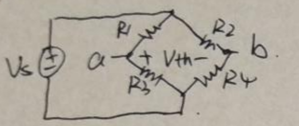


... 2′

等效电路图每个2分。其中，电源可反向标记，电压、电流取正值。

6. Max power Transfer.

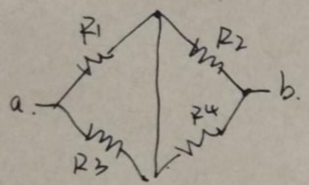
(1). ① 断路求 V_{th} — 10分



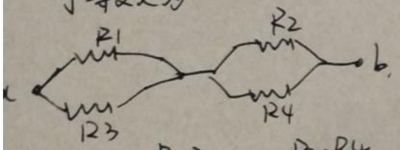
$$V_{th} = \left(\frac{R_3}{R_1+R_3} - \frac{R_4}{R_2+R_4}\right)V_s$$ — 4分

② 求 R_{th}

法 ①. 直接求等效电阻.

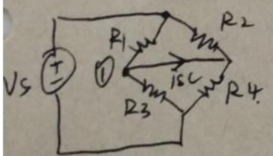


可等效为



$$R_{th} = \frac{R_1 \cdot R_3}{R_1+R_3} + \frac{R_2 \cdot R_4}{R_2+R_4}$$ — 4分

法② 求短路求 i_{sc}.



assume at Node ①, voltage is v.

use KCL:

$$\frac{V-V_s}{R_1} + \frac{V-V_s}{R_2} + \frac{V-0}{R_3} + \frac{V-0}{R_4} = 0.$$ — 1分

$$V = \frac{\frac{1}{R_1}+\frac{1}{R_2}}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}} V_s$$ — 1分

$$i_{sc} = \frac{V_s - V}{R_1} + \frac{0-V}{R_3}$$ — 1分

$$= \frac{1}{R_1}V_s - \frac{\left(\frac{1}{R_1}+\frac{1}{R_3}\right)\left(\frac{1}{R_1}+\frac{1}{R_2}\right)}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}} \cdot V_s$$

$$R_{th} = \frac{V_{th}}{i_{sc}}$$

$$= \frac{R_1R_2R_3 + R_1R_3R_4 + R_2R_3R_4 + R_1R_2R_4}{(R_1+R_3)(R_2+R_4)}$$

$$= \frac{R_2R_4(R_1+R_3) + R_1R_3(R_2+R_4)}{(R_1+R_3)(R_2+R_4)}$$

$$= \frac{R_2R_4}{R_2+R_4} + \frac{R_1R_3}{R_1+R_3}$$ — 1分

③ R_L与R_{th}关系

When power delivered to R_L is max

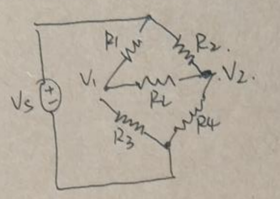
$$R_L = R_{th}.$$ — 2分

(2) $$P_{max} = \frac{\left(\frac{1}{2}V_{th}\right)^2}{R_L}$$ — 2分

$$= \frac{1}{4}V_s^2 \cdot \frac{(R_2R_3 - R_1R_4)^2}{R_2R_4(R_1+R_3)^2(R_2+R_4) + R_1R_3(R_1+R_3)(R_2\ldots}$$

— 2分

使用R_L式子 求导求解答案. 14分.



$$\begin{cases} \dfrac{V_1-V_s}{R_1}+\dfrac{V_1-V_2}{R_L}+\dfrac{V_1-0}{R_3}=0. \\ \dfrac{V_2-V_s}{R_2}+\dfrac{V_2-V_1}{R_L}+\dfrac{V_2-0}{R_4}=0. \end{cases}$$ — 2~~4~~3′

$\Downarrow$

$$V_1=\frac{\left[\left(\frac{1}{R_2}+\frac{1}{R_4}+\frac{1}{R_L}\right)\cdot\frac{R_L}{R_1}+\frac{1}{R_2}\right]\cdot V_s}{\left(\frac{1}{R_2}+\frac{1}{R_4}+\frac{1}{R_L}\right)\cdot\left(1+\frac{R_L}{R_1}+\frac{R_L}{R_3}\right)-\frac{1}{R_L}}$$ — 2′

$$V_2=\frac{\left[\left(\frac{1}{R_1}+\frac{1}{R_3}+\frac{1}{R_L}\right)\cdot\frac{R_L}{R_2}+\frac{1}{R_1}\right]\cdot V_s}{\left(\frac{1}{R_1}+\frac{1}{R_3}+\frac{1}{R_L}\right)\cdot\left(1+\frac{R_L}{R_2}+\frac{R_L}{R_4}\right)-\frac{1}{R_L}}$$ — 2′

令$V_{RL}=V_1-V_2$

V_{RL}为R_L的函数.

~~$\frac{dV_{RL}}{dR_L}=0 \Rightarrow$ 解出R_L为何值时~~

V_{RL} ~~最~~

$$P=\cancel{V_{RL}}\frac{(V_{RL})^2}{R_L}.$$

令$\dfrac{dP}{dR_L}=0$. 取$R_L>0$时的值

可解出R_L为何值时.

P最大.

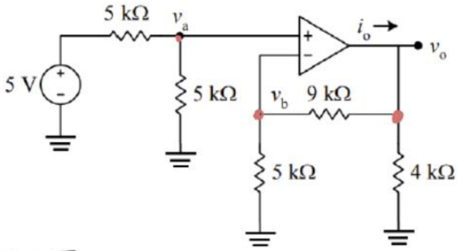
$$R_L=\frac{R_1\cdot R_3}{R_1+R_3}+\frac{R_2\cdot R_4}{R_2+R_4}$$ — 4′

代入P.

可得 $$P=\frac{1}{4}V_s^2\cdot\frac{(R_2R_3-R_1R_4)^2}{R_2R_4(R_2+R_4)(R_1+R_3)^2+R_1R_3(R_1+R_3)(R_2+R_4)^2}.$$ —4′

7. Assume that the ideal operational amplifier is operating in its linear range. (12pt)

(1) Solve for v_b. (2) Solve for v_o. (3) Solve for i_o.



4' 1) $\frac{U_a-5}{5k}+\frac{U_a}{5k}=0$ 2'

$\Rightarrow U_a=2.5V$

$U_b=U_a=2.5V$ 2'

4' 2) $\frac{U_o-U_b}{9k}=\frac{U_b}{5k}$ 2'

$5U_o-5U_b=9U_b$

$U_o=\frac{14}{5}U_b=7V$ 2'

4' 3) $\frac{U_o-U_b}{9k}+\frac{U_o}{4k}=i_o$ 2'

$i_o=2.25mA$ 2'

* 关于 3) 问

公式部分 1) 分母为 9,4 而非 9k,4k ⇒ 给 1 分
2) 分母 4k, 9k 写对, 但方向写反 ⇒ 给 1 分
3) 其余错误不得分

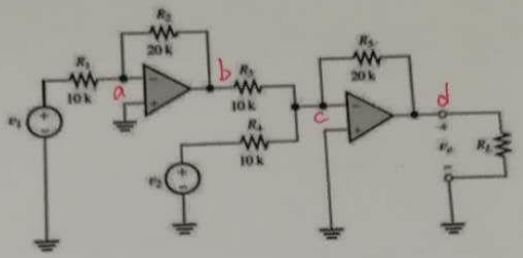
答案部分：写 2.25A 而非 2.25mA，一律不得分
(属于答案错误而非单位错误)

* 关于 1). 2). 3) 问, 只要式子列对, 答案错仍给过程分

* 前 2 小问求错, 3) 问答案对不给分 (已验证你无法得出该答案)

* 未在对应小问给出对应答案 (写在别处), 此次未扣分

8. Assume that the ideal operational amplifiers in following circuit are both operating in linear range. Please calculate the output voltage v_o given $v_1 = 4\text{V}$ and $v_2 = 6\text{V}$. (12pt)



1)

$$\begin{cases} V_a = 0V & (2') \\ V_c = 0V \\ \dfrac{V_1 - V_a}{10k} + \dfrac{V_b - V_a}{20k} = 0 & (4') \\ \dfrac{V_2 - V_c}{10k} + \dfrac{V_b - V_c}{10k} + \dfrac{V_d - V_c}{20k} = 0 & (4') \end{cases}$$

$V_d = V_o$

$$\begin{cases} V_b = -8V \\ V_o = 4V \end{cases} \quad (2')$$

2) $V_b = -\frac{R_2}{R_1} V_1 = -8V$ (4')

$V_d = -\left(\frac{R5}{R_3} V_b + \frac{R5}{R4} V_2\right)$ (4')

$V_o = V_d = 4V$ (4')