EE 111 Homework 9

Due date: Jun. 5^{th}, 2019

Turn in your homework in class

Rule:

- Work on your own. Discussion is permissible, but similar submissions will be judged as plagiarism.
- Please show all intermediate steps: a correct solution without an explanation will get zero credit.
- Please submit on time. No late submission will be accepted.
- Please prepare your submission in English only. No Chinese submisson will be accepted.

16' (4×4') 1. Determine the Laplace transforms of these functions:

(a) $f(t) = 5e^{-5t}u(t-5)$

(b) $g(t) = 5\cos(2t-1)\,u(t)$

(c) $h(t) = \sin(2t)\,u(t-\tau)$

(d) $p(t) = \begin{cases} 5t & 0 < t < 1 \\ -5t & 1 < t < 2 \\ 0 & otherwise \end{cases}$

f(t) 变形 2'

F(t) 2'

a) $f(t) = 5\cdot e^{-5(t-5)}u(t-5)\cdot e^{-25}$ 2'

$F(s) = \dfrac{5\cdot e^{-5s}}{e^{25}(s+5)}$ 2'

b) $g(t) = 5\cos(2t-1)\,u(t)$

$= 5\cos 2t\cos(1)\,u(t) + 5\sin(2t)\sin(1)\,u(t)$ 2'

$G(s) = 5\cos(1)\cdot\dfrac{s}{s^2+4} + 5\sin(1)\dfrac{2}{s^2+4}$ 2'

OR

$G(s) = \dfrac{2.702s}{s^2+4} + \dfrac{8.415}{s^2+4}$

c) $\sin 2t = \sin[2(t-\tau) + 2\tau]$

$= \sin 2(t-\tau)\cos 2\tau + \cos 2(t-\tau)\sin 2\tau$

$f(t) = \cos 2\tau\sin 2(t-\tau)\,u(t-\tau) + \sin 2\tau\cos 2(t-\tau)\,u(t-\tau)$ 2'

$F(s) = \cos 2\tau\, e^{-\tau s}\dfrac{2}{s^2+4} + \sin 2\tau\, e^{-\tau s}\dfrac{s}{s^2+4}$ 2'

d) $f(t) = 5t[u(t) - u(t-1)] - 5t[u(t-1) - u(t-2)]$

$= 5[tu(t) - 2tu(t-1) + tu(t-2)]$

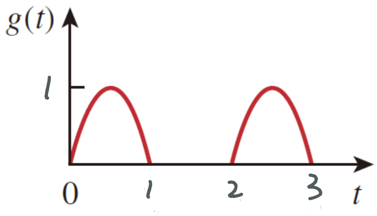
$= 5[tu(t) - 2(t-1)u(t-1) - 2u(t-1) + (t-2)u(t-2) + 2u(t-2)]$ 2'

$F(s) = 5[\frac{1}{s^2} - \frac{2e^{-s}}{s^2} - \frac{2e^{-s}}{s} + \frac{e^{-2s}}{s^2} + \frac{2e^{-2s}}{s}]$ 2'

$= 5\cdot\dfrac{1-2e^{-s}+e^{-2s}}{s^2} + 5\cdot\dfrac{2e^{-2s}-2e^{-s}}{s}$

8' (2×4') 2. Determine the Laplace transform of the **period function** in the following Figures

b) tips: sin(t)



$$g(t)=\begin{cases}\sin\pi t & 0<t<1\\ 0 & 1<t<2\end{cases}$$ 1'

$g_1(t)=\sin(\pi t) \quad 0<t<1$

$=\sin(\pi t)\,[u(t)-u(t-1)]$

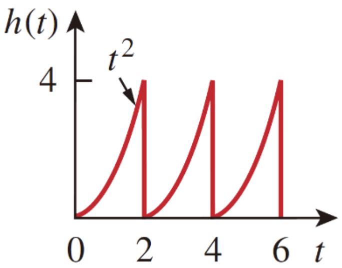
$=\sin(\pi t)\,u(t)-\sin(\pi t)\,u(t-1)$ 1'

$=\sin(\pi t)\,u(t)+\sin(\pi(t-1))\,u(t-1)$

$G_1(s)=\frac{\pi}{s^2+\pi^2}(1+e^{-s})$

$G(s)=\frac{G_1(s)}{1-e^{-2s}}=\frac{\pi(1+e^{-s})}{(s^2+\pi^2)\cdot(1-e^{-2s})}$ 2'

c)



$h_1(t)=t^2[u(t)-u(t-2)]$

$=t^2u(t)-t^2u(t-2)$

$=t^2u(t)-(t-2)^2u(t-2)-4(t-2)u(t-2)-4u(t-2)$ 1'

$\therefore H_1(s)=\frac{2}{s^3}(1-e^{-2s})-\frac{4}{s^2}e^{-2s}-\frac{4}{s}e^{-2s}$ 1'

$H(s)=\frac{H_1(s)}{(1-e^{-Ts})}$, $T=2$

$H(s)=\frac{2(1-e^{-2s})-4s\,e^{-2s}(1+s)}{s^3(1-e^{-2s})}$ 2'

12' (3x4')

3. Find the inverse Laplace transform of:

(a) H(s) = $\frac{s+8}{s(s+4)}$

(b) G(s) = $\frac{4-e^{-2s}}{s^2+5s+4}$

(c) D(s) = $\frac{10s}{(s^2+1)(s^2+4)}$

求系数 1'

F(s) 拆解 1'

求 f(t) 2'

a) $H(s) = \frac{s+6}{s(s+4)} = \frac{A}{s} + \frac{B}{s+4}$

$\Rightarrow A=2,\ B=-1$ 1'

$\therefore H(s) = \frac{2}{s} - \frac{1}{s+4}$ 1'

$h(t) = (2 - e^{-4t})u(t)$ 2'

b) Let $H(s) = \frac{1}{(s+1)(s+4)} = \frac{A}{s+1} + \frac{B}{s+4}$

$\Rightarrow A = \frac{1}{3},\ B = -\frac{1}{3}$ 1'

$H(s) = \frac{1}{3(s+1)} - \frac{1}{3(s+4)}$

$h(t) = \frac{1}{3}[e^{-t} - e^{-4t}]$ 1'

$G(s) = 4H(s) - e^{-2s}H(s)$

$\therefore g(t) = 4h(t)u(t) - h(t-2)u(t-2)$

$= \frac{4}{3}[e^{-t} - e^{-4t}]u(t) - \frac{1}{3}[e^{-(t-2)} - e^{-4(t-2)}]u(t-2)$ 2'

c) $D(s) = \frac{10s}{(s^2+1)(s^2+4)} = \frac{As+B}{s^2+1} + \frac{Cs+D}{s^2+4}$

$10s = (s^2+4)(As+B) + (s^2+1)(Cs+D)$

$\Rightarrow A = \frac{10}{3},\ B=0,\ C=-\frac{10}{3},\ D=0$ 1'

$\therefore D(s) = \frac{10s/3}{s^2+1} - \frac{10s/3}{s^2+4}$ 1'

$d(t) = \left(\frac{10}{3}\cos t - \frac{10}{3}\cos 2t\right)u(t)$ 2'

4. (10pt) There is no energy stored in the circuit shown in Fig.4 at the time the switch is opened.

   (a) Derive the integrodifferential equations that govern the behavior of the node voltages v_1 and v_2.

   (b) Show that

$$V_2(s) = \frac{sI_g(s)}{C[s^2 + (R/L)s + (1/LC)]}.$$

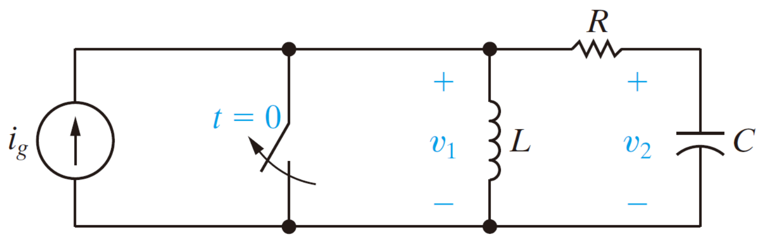


Fig. 4

Solution:

(a)

$$\frac{1}{L}\int_0^t v_1 d\tau + \frac{v_1 - v_2}{R} = i_g u(t) \qquad (3')$$

$$C\frac{dv_2}{dt} + \frac{v_2}{R} - \frac{v_1}{R} = 0 \qquad (3')$$

(b)

$$\frac{V_1}{sL} + \frac{V_1 - V_2}{R} = I_g \qquad (2')$$

$$\frac{V_2 - V_1}{R} + sCV_2 = 0 \qquad (2')$$

Solving,

$$V_2(s) = \frac{sI_g(s)}{C[s^2 + (R/L)s + (1/LC)]}$$

5. (12pt) (C=1.5625mF)The switch in the circuit shown in Fig.5 has been in position x for a long time. At $t = 0$, the switch moves instantaneously to position y.

   (a) Construct an s-domain circuit for $t > 0$.

   (b) Find $I_o(s)$.

   (c) Find $i_o(t)$.

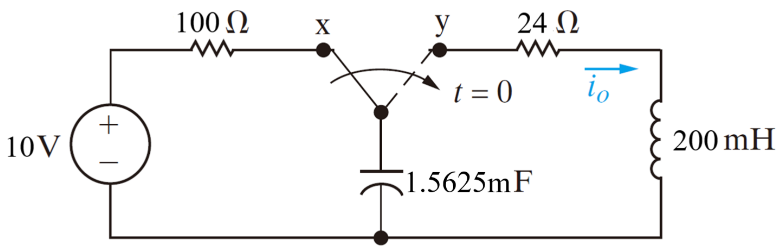
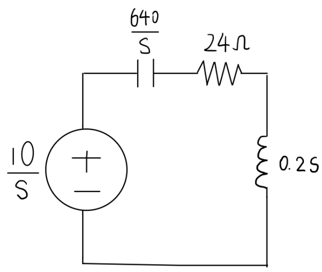


Fig. 5

Solution:

(a)

(4′)



(b)

$$\frac{640}{s}I_o + 24I_o + 0.2sI_o - \frac{10}{s} = 0 \qquad (2')$$

$$\Rightarrow I_o(s) = \frac{\frac{10}{s}}{\frac{640}{s} + 24 + 0.2s} = \frac{50}{s^2 + 120s + 3200} \qquad (2')$$

(c)

$$I_o(s) = \frac{50}{(s+40)(s+80)} = \frac{K_1}{s+40} + \frac{K_2}{s+80}$$

$$K_1 = \frac{50}{s+80}|_{s=-40} = 1.25 \qquad (1')$$

$$K_2 = \frac{50}{s+40}|_{s=-80} = -1.25 \qquad (1')$$

$$I_o(s) = \frac{1.25}{s+40} - \frac{1.25}{s+80}$$

$$i_o(t) = (1.25e^{-40t} - 1.25e^{-80t})u(t)A \qquad (2')$$

5. (12pt) (C=1.565mF)The switch in the circuit shown in Fig.5 has been in position x for a long time. At $t = 0$, the switch moves instantaneously to position y.

   (a) Construct an s-domain circuit for $t > 0$.

   (b) Find $I_o(s)$.

   (c) Find $i_o(t)$.

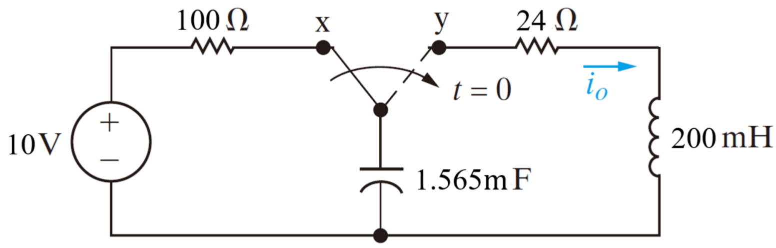
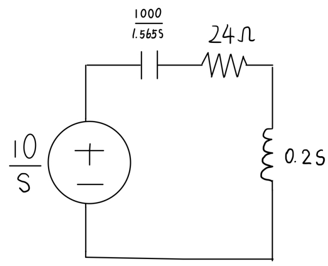


Fig. 5

Solution:

(a)

(4′)



(b)

$$\frac{1000}{1.565s}I_o + 24I_o + 0.2sI_o - \frac{10}{s} = 0 \qquad (2')$$

$$\Rightarrow I_o(s) = \frac{\frac{10}{s}}{\frac{1000}{1.565s} + 24 + 0.2s} = \frac{50}{s^2 + 120s + 3194.888} \qquad (2')$$

(c)

$$I_o(s) = \frac{50}{(s + 39.873)(s + 80.127)} = \frac{K_1}{s + 39.873} + \frac{K_2}{s + 80.127}$$

$$K_1 = \frac{50}{s + 80.127}|_{s=-39.873} = 1.242 \qquad (1')$$

$$K_2 = \frac{50}{s + 39.873}|_{s=-80.127} = -1.242 \qquad (1')$$

$$I_o(s) = \frac{1.242}{s + 39.873} - \frac{1.242}{s + 80.127}$$

$$i_o(t) = (1.242e^{-39.873t} - 1.242e^{-80.127t})u(t)A \qquad (2')$$

6. (16pt) There is no energy stored in the circuit in Fig.6 at the time $t(0^-)$, and $v_g(t) = 325u(t)$ V.

   (a) Find $V_o(s)$ and $I_o(s)$.

   (b) Find $v_o(t)$ and $i_o(t)$.

   (c) Do the solutions for $v_o(t)$ and $i_o(t)$ make sense in terms of known circuit behavior? Explain.

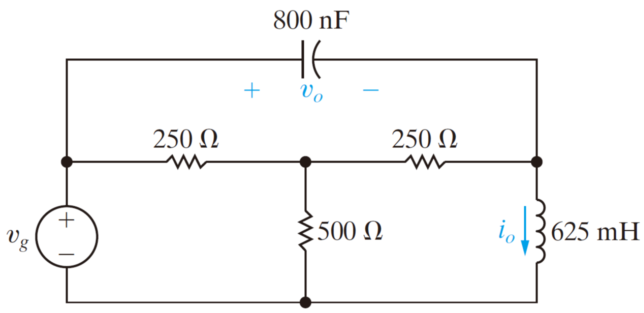
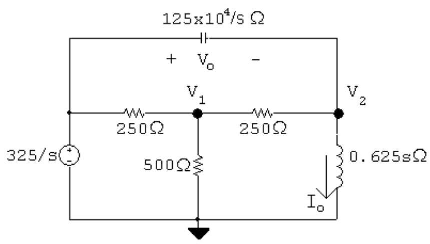


Fig. 6

Solution:



(a)

$$\frac{V_1 - 325/s}{250} + \frac{V_1}{500} + \frac{V_1 - V_2}{250} = 0 \qquad (1')$$

$$\frac{V_2}{0.625s} + \frac{V_2 - V_1}{250} + \frac{(V_2 - 325/s)s}{1250000} = 0 \qquad (1')$$

Simplify,

$$5V_1 - 2V_2 = \frac{650}{s}$$

$$-5000sV_1 + (s^2 + 5000s + 2000000)V_2 = 325s$$

Then we have

$$V_2 = \frac{325}{s + 1000} \qquad (2')$$

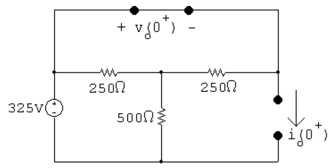
$$V_o(s) = \frac{325}{s} - \frac{325}{s + 1000} \qquad (2')$$

$$I_o(s) = \frac{V_2}{0.625s} = \frac{0.52}{s} - \frac{0.52}{s + 1000} \qquad (2')$$

(b)

$$v_o(t) = (325 - 325e^{-1000t})u(t)V \qquad (2')$$

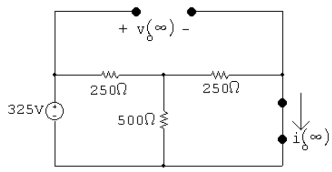
$$i_o(t) = (0.52 - 0.5e^{-1000t})u(t)A \qquad (2')$$



(c) At $t = 0^+$ the circuit is

$$v_o(0^+) = 0 \quad i_o(0^+) = 0 \quad \text{Checks} \tag{2'}$$

At $t = \infty$ the circuit is



$$v_o(\infty) = 325V \quad i_o(\infty) = \frac{325}{250 + (500||250)} \cdot \frac{500}{750} = 0.52A \quad \text{Checks} \tag{2'}$$

7. (12pt) (C=3.33mF)The op amp in the circuit shown in Fig.7 is ideal. There is no energy stored in the capacitors at the instant the circuit is energized.

   (a) Find $v_o(t)$ if $v_{g1}(t) = 40u(t)$ V and $v_{g2}(t) = 16u(t)$ V.

   (b) How many milliseconds after the two voltage sources are turned on does the op amp saturate?

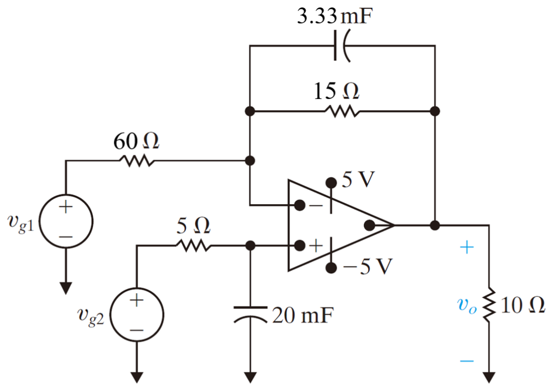


Fig. 7

Solution:

(a)

$$\frac{V_p - 16/s}{5} + \frac{V_p}{50/s} = 0 \quad \Rightarrow V_p = \frac{160}{s(s+10)} \qquad (2')$$

$$\frac{V_p - 40/s}{60} + \frac{V_p - V_o}{15} + \frac{V_p - V_o}{300/s} = 0 \qquad (2')$$

$$\begin{aligned} V_o &= \frac{-40s + 2000}{s(s+10)(s+20)} \\ &= \frac{10}{s} - \frac{24}{s+10} + \frac{14}{s+20} \end{aligned} \qquad (2')$$

$$v_o(t) = (10 - 24e^{-10t} + 14e^{-20t})u(t)V \qquad (2')$$

(b)

$$10 - 24e^{-10t} + 14e^{-20t} = 5 \qquad (2')$$

$$e^{-10t} = 0 \quad \text{or} \quad 0.2427$$

$$t = 141.60ms \qquad (2')$$

7. (12pt) (C=30mF)The op amp in the circuit shown in Fig.7 is ideal. There is no energy stored in the capacitors at the instant the circuit is energized.

   (a) Find $v_o(t)$ if $v_{g1}(t) = 40u(t)$ V and $v_{g2}(t) = 16u(t)$ V.

   (b) How many milliseconds after the two voltage sources are turned on does the op amp saturate?

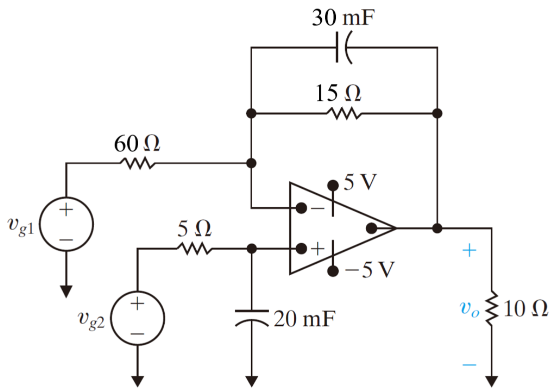


Fig. 8

Solution:

(a)

$$\frac{V_p - 16/s}{5} + \frac{V_p}{50/s} = 0 \quad \Rightarrow V_p = \frac{160}{s(s+10)} \qquad (2')$$

$$\frac{V_p - 40/s}{60} + \frac{V_p - V_o}{15} + \frac{V_p - V_o}{100/3s} = 0 \qquad (2')$$

$$\begin{aligned} V_o &= \frac{1240s + 2000}{s(s+10)(9s+20)} \\ &= \frac{10}{s} - \frac{104}{7(s+10)} + \frac{34}{7(s+\frac{20}{9})} \end{aligned} \qquad (2')$$

$$v_o(t) = (10 - 14.857e^{-10t} + 4.87e^{-2.22t})u(t)V \qquad (2')$$

(b)

$$10 - \frac{104}{7}e^{-10t} + \frac{34}{7}e^{-\frac{20}{9}t} = 5 \qquad (2')$$

$$t = 45.92ms \qquad (2')$$

8. (14pt) The switch in the circuit seen in Fig.8 has been in position a for a long time. At $t = 0$, it moves instantaneously to position b. Find $i_o(t)$ for $t \geq 0$.

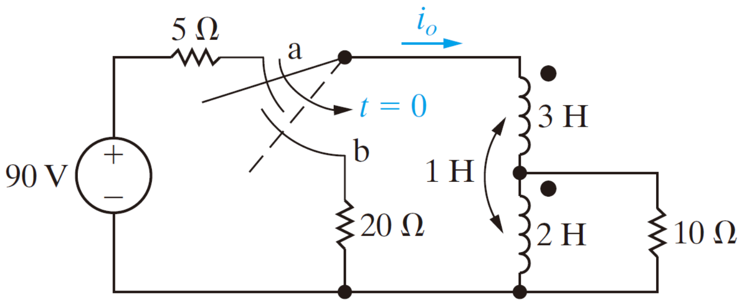
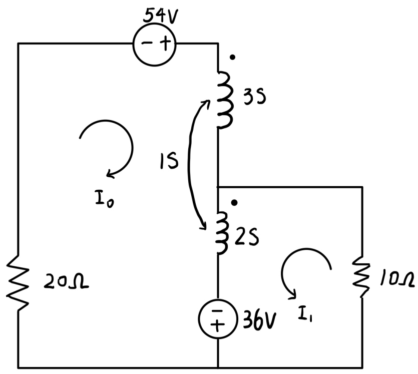


Fig. 9

Solution:



$$\begin{cases} -54 + 3sI_o + s(I_o + I_1) - 18 + 2s(I_o + I_1) + sI_o - 18 - 36 + 20I_o = 0 \\ 2s(I_1 + I_o) + sI_o - 18 - 36 + 10I_o = 0 \end{cases} \qquad (4')$$

Solving,

$$I_o = \frac{90s + 1260}{5s^2 + 110s + 200} = \frac{18s + 252}{(s+2)(s+20)} = \frac{K_1}{s+2} + \frac{K_2}{s+20} \qquad (4')$$

$$K_1 = \left.\frac{18s + 252}{s+20}\right|_{s=-2} = 12 \qquad (2')$$

$$K_2 = \left.\frac{18s + 252}{s+2}\right|_{s=-20} = 6 \qquad (2')$$

$$\Rightarrow I_o = \frac{12}{s+2} + \frac{6}{s+20}$$

$$i_o(t) = (12e^{-2t} + 6e^{-20t})u(t)A \qquad (2')$$