

- (1) (5 Points) Consider Knapsack problem without repetition with n items with values v_i and weight w_i . We have following defined sub-problems:

$K[w, i, 1]$ = maximum value of a collection of items with total weight w that contains item i

$K[w, i, 0]$ = maximum value of a collection of items with total weight w that does not contain item i

Is it possible to define a recurrence relation to solve above sub-problems? If possible, give the recurrence formula; otherwise provide the reason why it is not possible.

- (2) (10 Points) In this problem, we want to figure out the number of structurally unique BSTs (binary search trees) that stores the given values.
- (5 Points) Given values 1, 2, 3, draw all the structurally unique BSTs that stores these values. How many structurally unique BSTs can you draw?
 - (5 Points) Given values $1, \dots, n$, design an algorithm with dynamic programming that figures out how many structurally unique BSTs can you draw. Give the explanation of your algorithm and the time complexity.