CS 101 Fall 2020 - Quiz 2-A

9/29/2020 - 20 Minutes **Name:** **ID number:**

---

(1) (8 Points) Here is a sorting algorithm in the following.

```
Procedure Sort(A):
  for j = 2 to A.length:
      key = A[j]
      i = j - 1
      while i > 0 and A[i] > key:
          A[i+1] = A[i]
          i = i - 1
      A[i+1] = key
      // Mark
```

- (3 Points) Which sorting algorithm does it describe?
- (5 Points) Given a list as `[31, 4, 59, 26, 41, 58]`, we use the above procedure to sort it. Write down what will the list be like each time when the procedure meets the `Mark`.

- Insertion Sort.
- `[4, 31, 59, 26, 41, 58]`
  `[4, 31, 59, 26, 41, 58]`
  `[4, 26, 31, 59, 41, 58]`
  `[4, 26, 31, 41, 59, 58]`
  `[4, 26, 31, 41, 58, 59]`
  or if misunderstanding "`2 to A.length`",
  `[31, 4, 59, 26, 41, 58]`
  `[31, 4, 26, 59, 41, 58]`
  `[31, 4, 26, 41, 59, 58]`
  `[31, 4, 26, 41, 58, 59]`

CS 101 Fall 2020 - Quiz 2-A

9/29/2020 - 20 Minutes **Name:** **ID number:**

---

(2) (7 Points) A hash table of size m is used to store n items, with $n \leq m/2$. Open addressing is used for collision resolution.

- (3 Points) Assuming uniform hashing, show that for $i = 1, 2, \ldots, n$, the probability that the ith insertion requires strictly more than k probes is at most 2^{-k}.
- (4 Points) Show that for $i = 1, 2, \ldots, n$, the probability that the kth insertion requires more than $2 \log n$ probes is at most $1/n^2$. (You can use the conclusion in the above question directly.)

- Define X to be the number of probes made in a search and A_k to be the event that there is an kth probe and it is to an occupied slot. Then

$$\begin{aligned}
\mathsf{Pr}\{X \geq k\} &= \mathsf{Pr}\{A_1 \cap A_2 \cap \cdots \cap A_k\} \\
&= \mathsf{Pr}\{A_1\} \cdot \mathsf{Pr}\{A_2 \mid A_1\} \cdot \mathsf{Pr}\{A_3 \mid A_1 \cap A_2\} \cdot \cdots \cdot \mathsf{Pr}\{A_k \mid A_1 \cap A_2 \cap \cdots \cap A_{k-1}\} \\
&= \frac{n}{m} \cdot \frac{n-1}{m-1} \cdot \cdots \cdot \frac{n-(k-1)}{m-(k-1)} \\
&\leq \left(\frac{n}{m}\right)^k \\
&\leq \left(\frac{1}{2}\right)^k
\end{aligned}$$

- With the conclusion above, we have

$$\mathsf{Pr}\{X \geq 2 \log n\} \leq 2^{-2 \log n} = 1/n^2$$