CS 101 Fall 2021 - Quiz 4-B

10/26/2021 - 30 Minutes **Name:** **ID number:**

The Master Theorem for $T(n) = aT(\frac{n}{b})+\Theta(n^d)$: If $\log_b a = d$ then $T(n) = O(n^d \log n)$ else $T(n) = O(n^{max(\log_b a, d)})$.

Problem 1 Notes of Discussion (5 pts)

I promise that I will complete this QUIZ independently, and will not use any electronic products or paper-based materials during the QUIZ, nor will I communicate with other students during this QUIZ.

True or False: I have read the notes and understood them.

Problem1

Problem 2 True or False (3×2 pts)

The following questions are True or False questions, you should judge whether each statement is true or false.

Note: You should write down your answers in the box below.

Problem 2.1	Problem 2.2	Problem 2.3

(1) Queue is the common data structure for implementation of Depth First Traversal.

(2) There exists at least one non-leaf node in a tree whose depth is the height of the tree.

(3) If b is a descendant of a, then there is exactly one unique path from a to b in the tree.

Problem 3 Recurrence and the Master Theorem (8pts)

Given the recurrence $T(n) = aT(n/b) + cn^d$ with $T(1) = 1$.

(1) If the recurrence indicates a divide and conquer algorithm,

a. the original problem of size n is divided into _____ subproblems and each subproblem has size _____(2pts);

(A) a (B) b (C) c (D) n/a (E) n/b (F) n^d

b. cn^d is the time complexity of _____. *Note: This question has one or more correct answer(s).* (2pts)

(A) Dividing the original problem into several subproblems

(B) Recurring for all subproblems

(C) Merging solutions to subproblems into the overall one

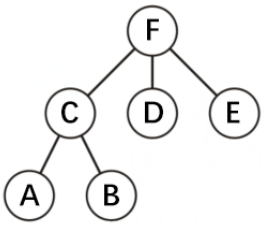
(2) a. If $(a, b, c, d) = (2, 3, \frac{1}{2}, \frac{3}{2})$, then the solution to this recurrence is $T(n) =$_____.(2pts)

b. If the recurrence indicates the **Strassen's algorithm** covered in our lecture which multiplies two 2-by-2 partitioned matrices via only 7 matrix multiplications, then $(a, b, d) =$_____ and the solution to this recurrence is $T(n) =$_____. (2pts)

Note: Write your answer for time complexity in asymptotic order form i.e. $T(n) = O(f(n))$.

Problem 4 Tree Traversal (6pts)

Run **Breadth First Traversal** on the tree shown below.



Note:

1. Decide on an appropriate data structure to implement the traversal.

2. When you are pushing the children of a node into your data structure, please push them **alphabetically** i.e. from left to right.

3. **Show all current elements in your data structure at each step** clearly . **Popping a node** or **pushing a sequence of children** can be considered as one single step.

4. **Write down your traversal sequence** i.e. the order that you pop elements out of the data structure. *Don't worry if you can't write the right answer at one chance. You can scratch in this paper but please* ***mark your final answer.***

CS 101 Fall 2021 - Quiz 4-B

10/26/2021 - 30 Minutes **Name:** **ID number:**

Problem 5 Magical Matrix (10pts)

Let's consider such a special square matrix of size $n \times n$ ($n = 2^k$) named **Magical Matrix $\mathbf{H_k}$**, which satisfies the following properties:

(a) $\mathbf{H_0} = \left[c\right]$, where c is a 1×1 constant.

(b) For $k \geq 1$, define $\mathbf{H_k} = \left[\begin{array}{c|c} \mathbf{H_{k-1}} & \mathbf{H_{k-1}} \\ \hline \mathbf{H_{k-1}} & -\mathbf{H_{k-1}} \end{array}\right]$, where $\mathbf{H_k}$ is a $2^k \times 2^k$ matrix and $\mathbf{H_{k-1}}$ is a $2^{k-1} \times 2^{k-1}$ matrix.

Let $\mathbf{v} = \begin{bmatrix} \mathbf{v_1} \\ \mathbf{v_2} \end{bmatrix}$ be a column vector of length $n = 2^k$, where $\mathbf{v_1}$ is the upper half of $\mathbf{v}$ of length $\frac{n}{2} = 2^{k-1}$ and $\mathbf{v_2}$ is the bottom half of $\mathbf{v}$ also of length $\frac{n}{2} = 2^{k-1}$. Now, we are going to develop a faster approach to calculate the matrix-vector product $\mathbf{H_k v}$.

(1) When $c = 1$ and $\mathbf{v_1} = \mathbf{v_2} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, write down $\mathbf{H_2}$ and calculate $\mathbf{H_2 v}$ according to the definition above. (2pts)

(2) Write the matrix-vector product $\mathbf{H_k v}$ in terms of $\mathbf{H_{k-1}}$, $\mathbf{v_1}$ and $\mathbf{v_2}$. (2pts)
Hint: Matrix multiplication still applies to partitioned matrices.

(3) Use your result from (2) to come up with a divide-and-conquer algorithm to calculate the matrix-vector product $\mathbf{H_k v}$ in more efficient than $\Theta(n^2)$ time. Write your main idea briefly. (4pts)

(4) What is the time complexity of your algorithm? Write down the corresponding recurrence and solve it. You **are not required** to show your analysis or calculation. (2pts)
Note: You can assume that all the numbers involved are small enough so that basic arithmetic operations like scalar addition and scalar multiplication take $O(1)$ time.