Discussion 1

9/22/2016

Outline

- Review
  - the slides last week
- Extension
  - Something new
- Q & A
  - Homework
  - Something that you are interested in

What we have learned?

- Introduction
- Circuit Analysis: An Overview
  - Voltage and Current
  - The Ideal Basic Circuit Element
    - I-V characteristics of circuit elements
  - Power and Energy
- Circuit Elements
  - Voltage and Current Sources
  - Electrical Resistance (Ohm's Law)
- Kirchhoff's Laws (KCL and KVL)
- Simple Resistive Circuits
  - Resistors in Series and Parallel
  - Voltage Division and Current Division

Extension Contents

- Dependent sources (Example: Transistor)
- Ohm’s Law: Nonlinearity
- Voltmeter/Ammeter
- Wye-Delta Transformations

Electric Current Voltage and Power

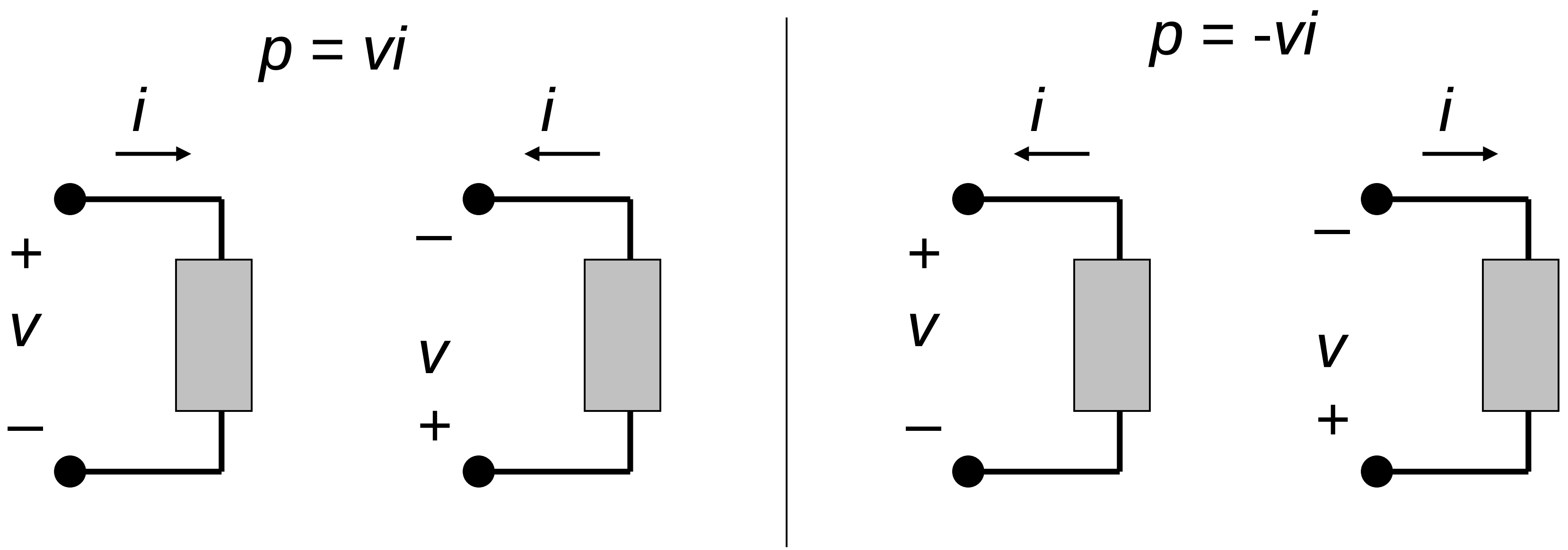
- $$I = \frac{\textit{Net}\ \text{Charge crossing surface in time}\ \Delta t}{\Delta t} = \frac{dq}{dt}$$

$$v = \frac{dw}{dq}$$

$$E = \Delta q \cdot V_{AB}, \qquad V_{AB} \equiv V_A - V_B$$

$$p \triangleq \frac{dw}{dt} = \frac{dw}{dq} \cdot \frac{dq}{dt} = v \cdot i$$

Passive Sign Convention (for Power)

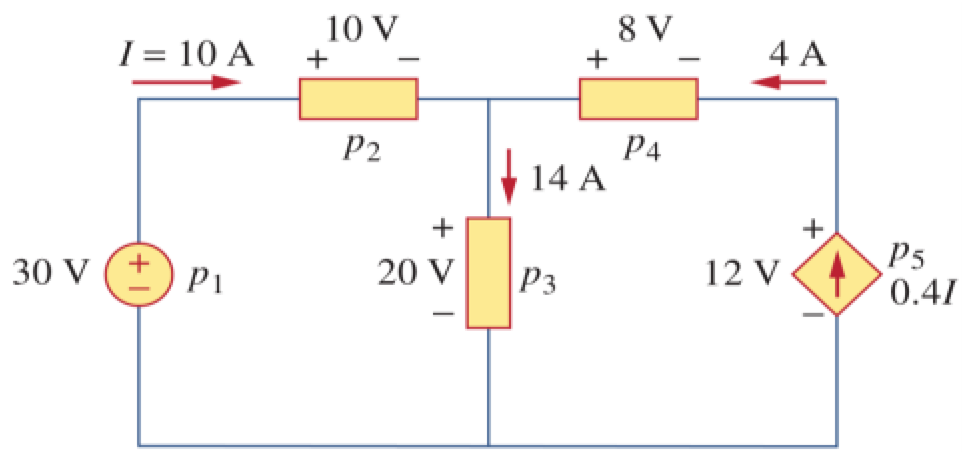


- If p > 0, power is absorbed by the element.
  - electrical energy into heat (resistors in toasters), light (light bulbs), or acoustic energy (speakers); by storing energy (charging a battery).
- If p < 0, power is extracted from the element.

Example

- Find the power absorbed by each of the elements

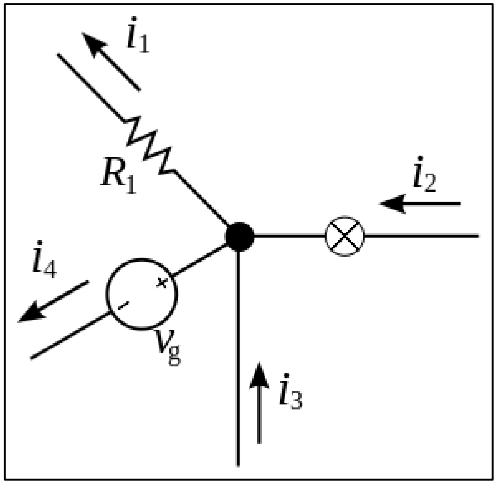
$$p_1 = 30\text{ V} * (-10\text{ A}) = -300\text{ W}$$
$$p_2 = 10\text{ V} * 10\text{ A} = 100\text{ W}$$
$$p_3 = 20\text{ V} * 14\text{ A} = 280\text{ W}$$
$$p_4 = 8\text{ V} * (-4\text{ A}) = -32\text{ W}$$
$$p_5 = 12\text{ V} * (-0.4 * 10\text{ A}) = -48\text{ W}$$



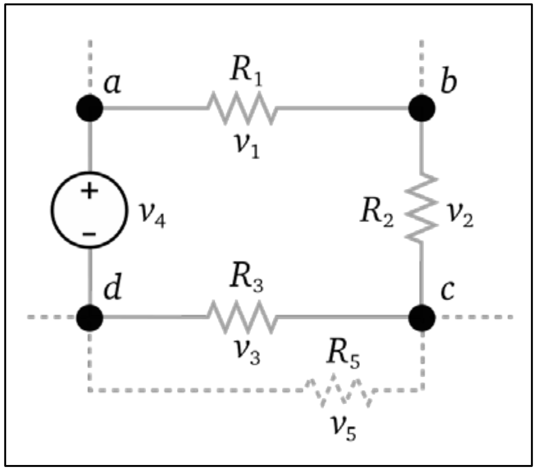
Kirchhoff's laws (KVL/KCL)

- KCL and KVL

$$\sum_{n=1}^{N} i_n = 0$$

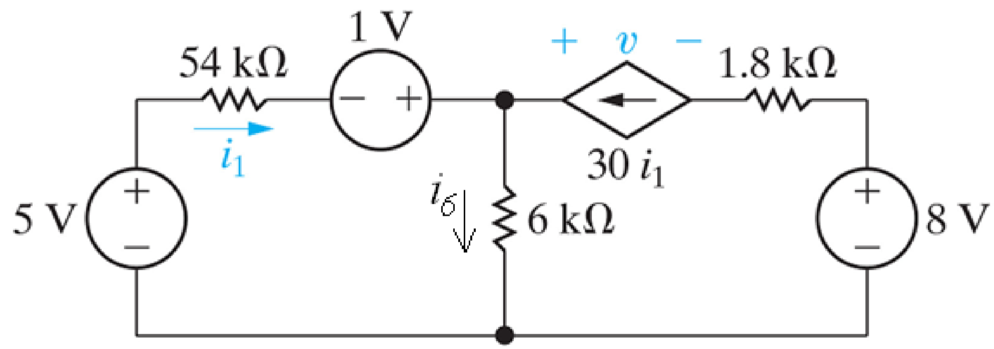


$$\sum_{m=1}^{M} v_m = 0$$



Example

Find i_l, v, and show that the power balances.



$$\text{KVL left}: -5 + 54{,}000 i_1 - 1 + 6000 i_6 = 0$$

$$\text{KCL top}: i_1 + 30 i_1 = i_6 = 31 i_1$$

$$\Rightarrow \quad -5 + 54{,}000 i_1 - 1 + 6000(31 i_1) = 0$$

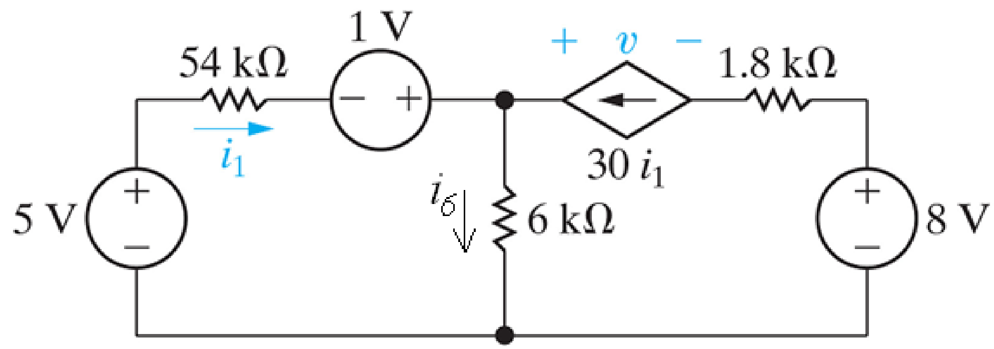
$$\Rightarrow \quad [54{,}000 + (6000)(31)] i_1 = 6 \quad \therefore \quad i_1 = 25\mu\text{A}$$

$$\text{KVL right}: -v + (6000)(31 i_1) - 8 + 1800(30 i_1) = 0$$

$$\Rightarrow \quad v = (6000)(31)(25\mu) - 8 + (1800)(30)(25\mu) = -2\text{ V}$$

Example

Find i_l, v, and show that the power balances.



$$p_{5V} = -(5)(25\mu) = -120\mu\text{W} \qquad p_{d.s.} = (2)(750\mu) = 1500\mu\text{W}$$

$$p_{54k} = (55{,}000)(25\mu)^2 = 33.75\mu\text{W} \qquad p_{1.8k} = (1800)(750\mu)^2 = 1012.5\mu\text{W}$$

$$p_{1V} = -(1)(25\mu) = -25\mu\text{W} \qquad p_{8V} = -(8)(750\mu) = -6000\mu\text{W}$$

$$p_{6k} = (6000)(775\mu)^2 = 3603.75\mu\text{W}$$

The sum of the power is
– 120 + 1500 + 33.75 + 1012.5 – 25 – 6000 + 3603.75 = 0!

Parallel/Series Resistors and Current/Voltage division

- Series Resistors and Voltage divison

$$v = v_1 + v_2 = i\left(R_1 + R_2\right)$$

$$R_{eq} = R_1 + R_2$$

$$v_1 = \frac{R_1}{R_1 + R_2} v \quad v_2 = \frac{R_2}{R_1 + R_2} v$$

- Parallel Resistors and Current division

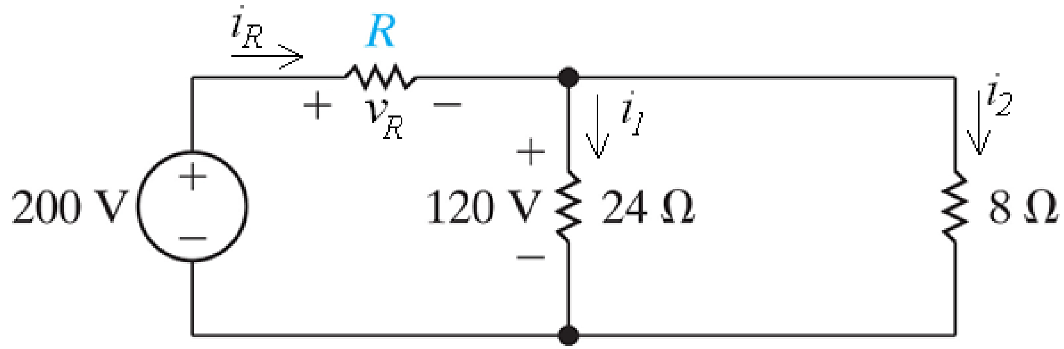
$$v = i_1 R_1 = i_2 R_2$$

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

$$i_1 = \frac{i R_2}{R_1 + R_2} \quad i_2 = \frac{i R_1}{R_1 + R_2}$$

Example

Find R



To find R we need to find the voltage drop across R and the current through R:

$$\text{KVL left}: -200 + v_R + 120 = 0 \quad \Rightarrow \quad v_R = 200 - 120 = 80\text{ V}$$

$$\text{KCL top}: i_R = i_1 + i_2$$

$$\text{KVL right}: -120 + v_2 = 0 \quad \Rightarrow \quad v_2 = 120\text{ V}$$

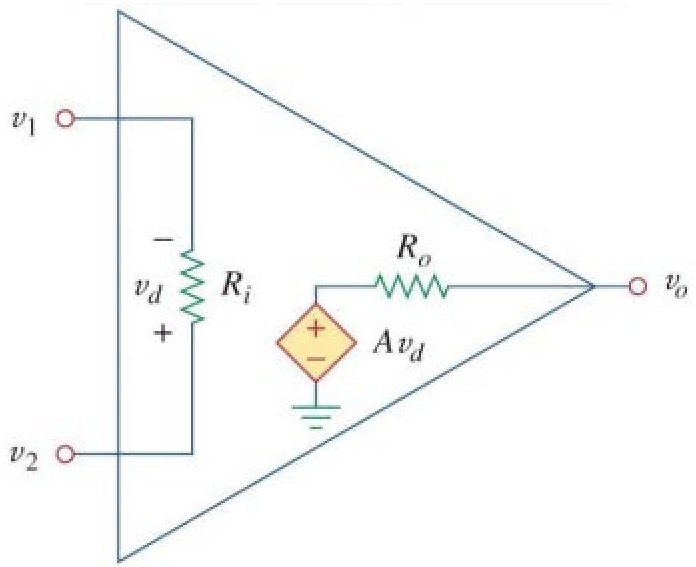
$$\therefore \quad i_R = \frac{120\text{ V}}{24\Omega} + \frac{120\text{ V}}{8\Omega} = 5 + 15 = 20\text{ A}$$

$$\Rightarrow \quad R = \frac{v_R}{i_R} = \frac{80}{20} = 4\Omega$$

Or we can use equivalent resistance of series-connected and parallel-connected resistors

Extension 1：Dependent Sources

- Dependent sources are good models for some common circuit elements:
  - ✦Transistors: In certain modes of operation, transistors take either a voltage or current input to one terminal and cause a current that is somehow proportional to the input to appear at two other terminals.
  - ✦Operational Amplifiers: Not covered yet, but the basic concept is they take an input voltage and generate an output voltage that is proportional to that.



Transistors

- There are two basic types of transistors: *bipolar junction transistors* (BJTs) and *field-effect transistors* (FETs).
- There are two types of BJTs: *npn* and *pnp*, with their circuit symbols as shown in Fig. 3.38. Each type has three terminals, designated as emitter (E), base (B), and collector (C).

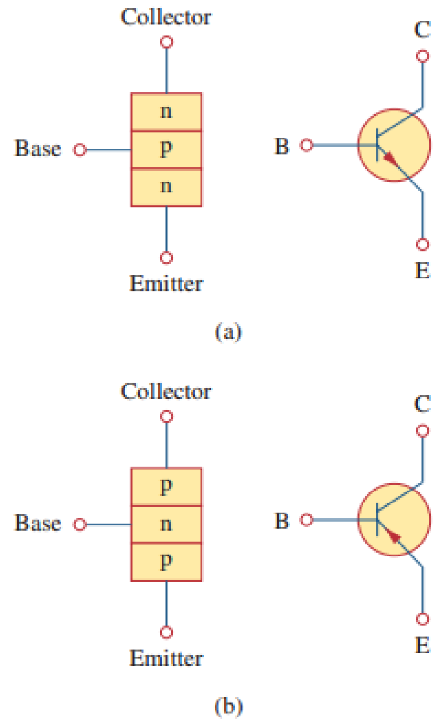


Figure 3.38
Two types of BJTs and their circuit symbols: (a) *npn*, (b) *pnp*.

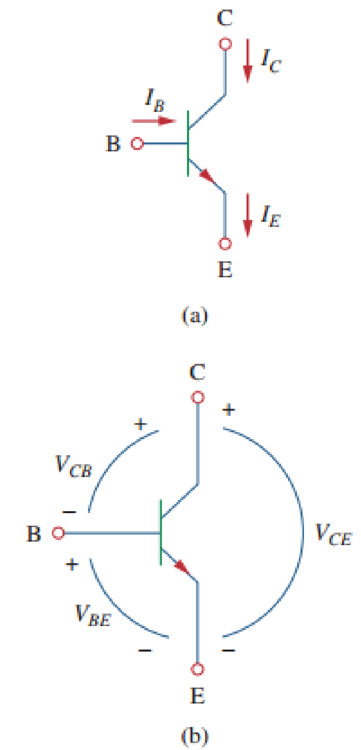


Figure 3.39
The terminal variables of an *npn* transistor: (a) currents, (b) voltages.

What we finally get

- The equivalent model of a npn transistor.

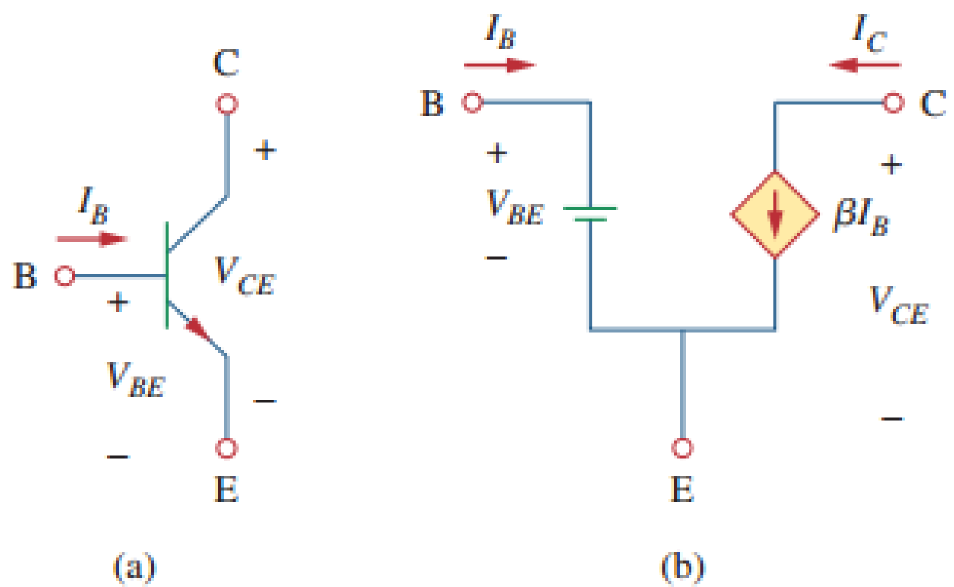
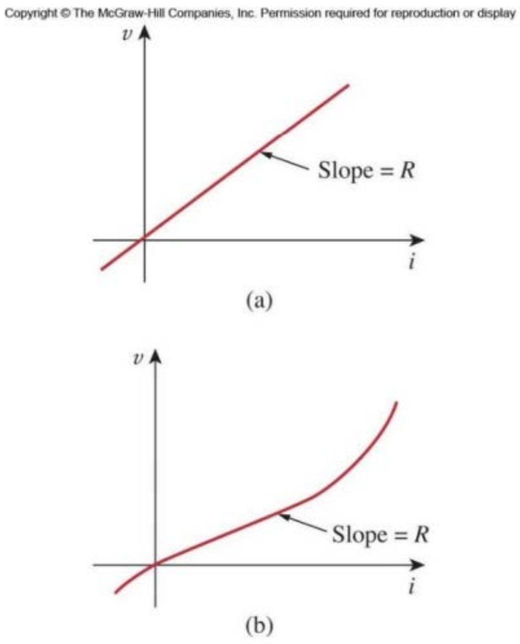
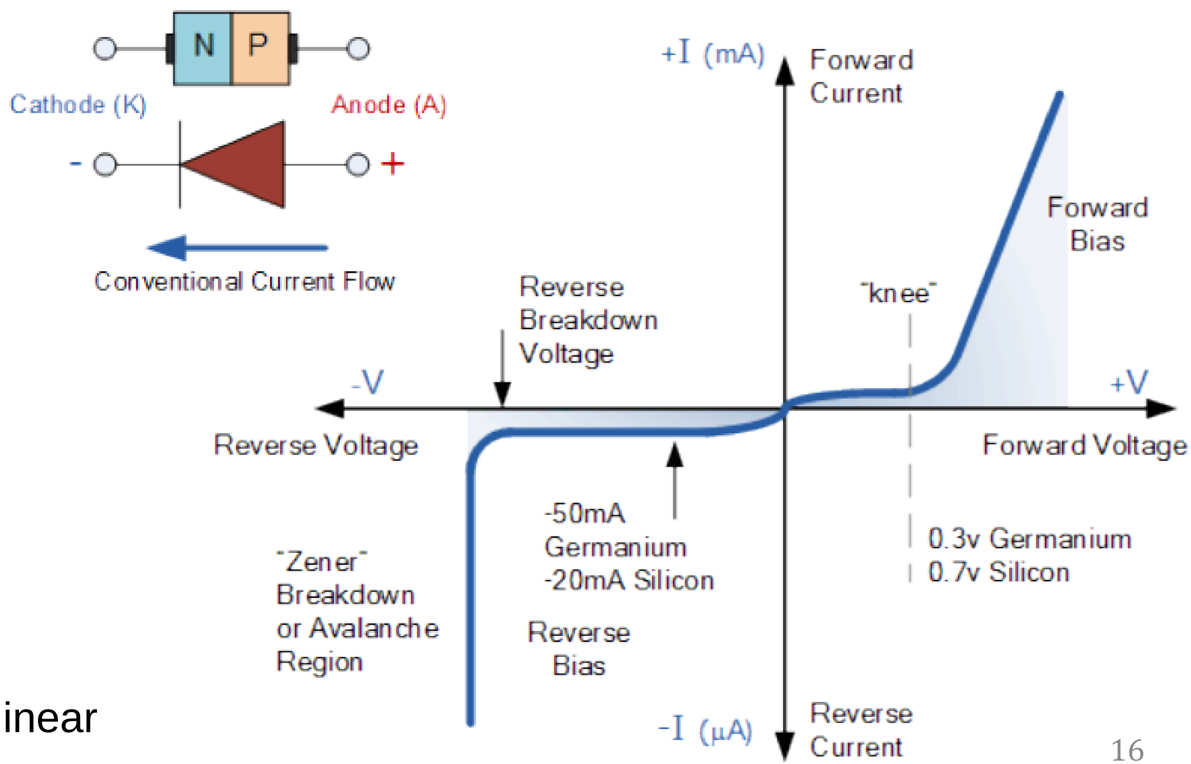


Figure 3.40
(a) An *npn* transistor, (b) its dc equivalent model.

Extension 2： Linearity vs Nonlinearity

- Not all resistors obey Ohm's Law
  - ✦Resistors that do are called linear resistors because their current voltage relationship is always linearly proportional.
  - ✦Diodes and light bulbs are examples of non-linear elements





I-V characteristics linear and nonlinear

- In the circuit world, we have i-v graphs. Therefore, we classify a circuit as linear or non-linear by examining its i-v graph. If the i-v graph of the circuit is a straight line, then the circuit is classified as linear. Note that the definition can be extended even to circuit elements. For instance, a resistor's i-v graph is a straight line, hence it is a linear device.

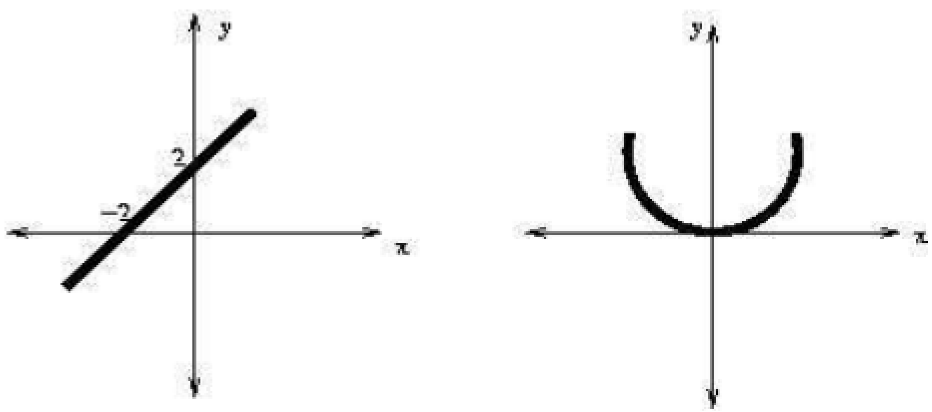


Figure 1. A linear versus nonlinear function

Quick glance at negative resistance converter

- According to some analyze and calculation, we obtain following equations

$$i = \frac{-R_a}{(R_b R_f)} \times v$$

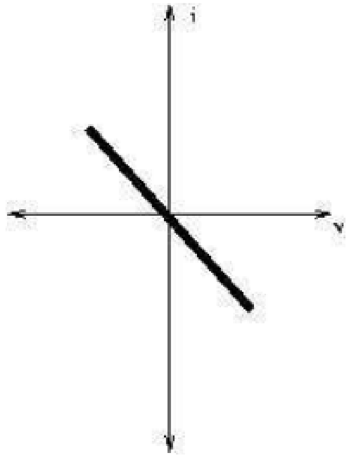
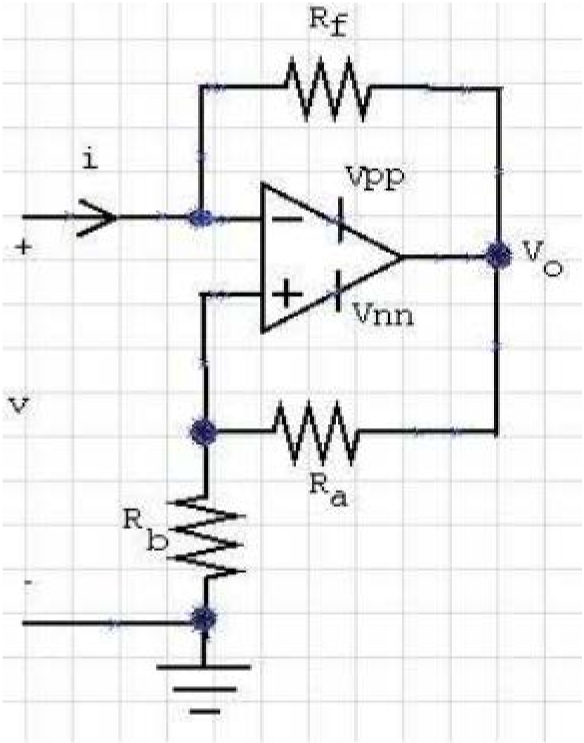


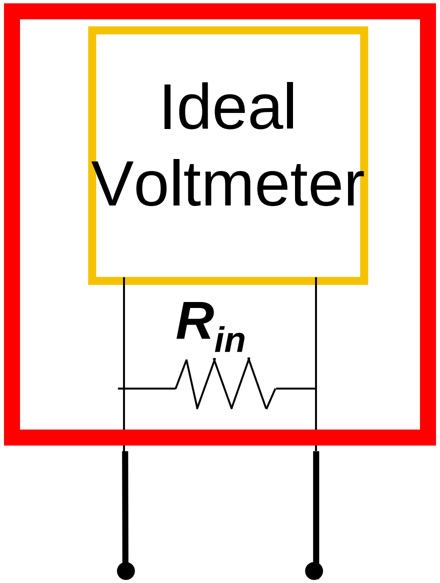
Figure 5. The i-v graph of the circuit when the op-amp is in the linear region.



Extension 3： Measuring Voltage (Voltmeter)

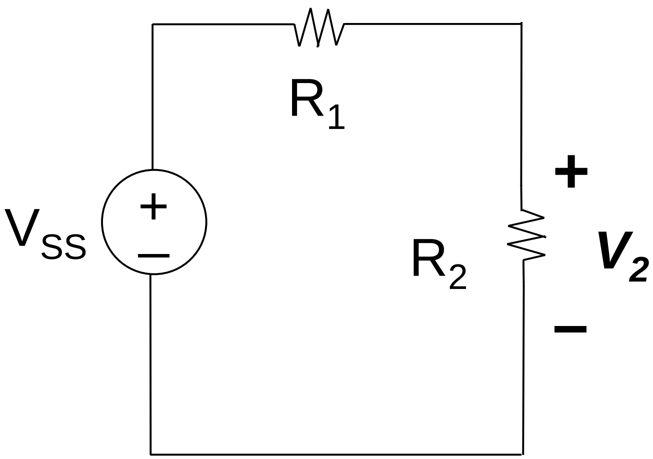
To measure the voltage drop across an element in a real circuit, insert a voltmeter (digital multimeter in voltage mode) **in parallel** with the element.

Voltmeters are characterized by their “voltmeter input resistance” ($\boldsymbol{R_{in}}$). Ideally, this should be very high (typical value 10 MΩ)

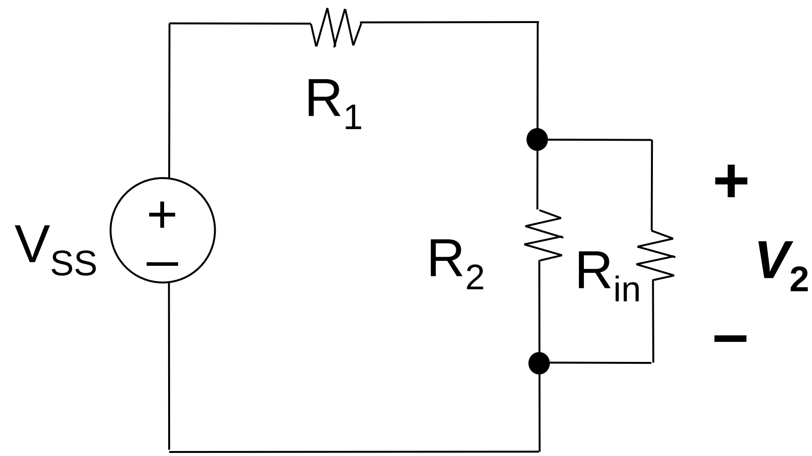


Effect of Voltmeter

undisturbed circuit



circuit with voltmeter inserted



$$V_2 = V_{SS}\left[\frac{R_2}{R_1 + R_2}\right]$$

$$V_2' = V_{SS}\left[\frac{R_2 \,||\, R_{in}}{R_2 \,||\, R_{in} + R_1}\right]$$

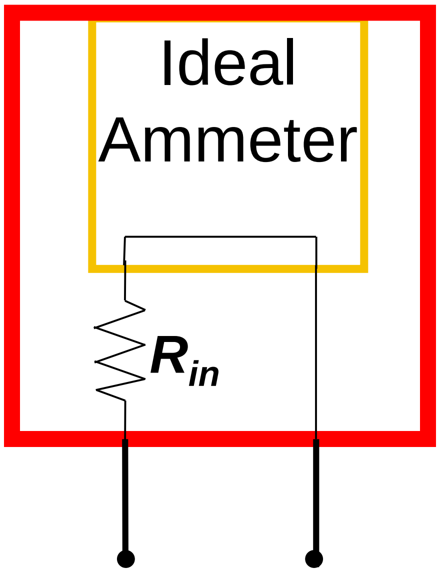
Example: $V_{SS} = 10V, R_2 = 100K, R_1 = 900K \Rightarrow V_2 = 1V$

$R_{in} = 10M, \; V_2' = ?$

Extention 3： Measuring Current (Ammeter)

To measure the current flowing through an element in a real circuit, insert an ammeter (digital multimeter in current mode) **in series** with the element.

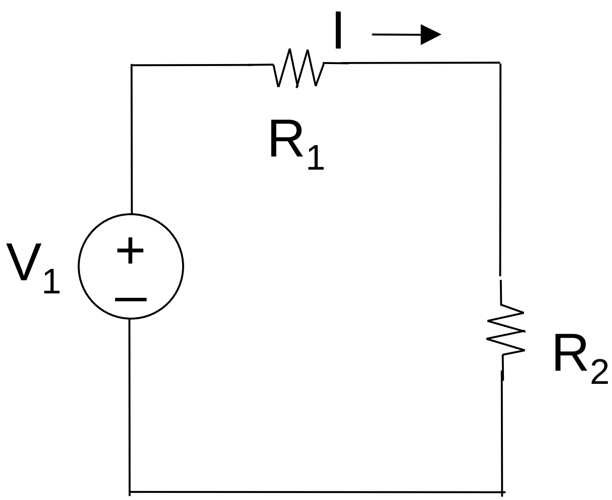
Ammeters are characterized by their “ammeter input resistance” (R_{in}). Ideally, this should be very low (typical value 1∧).



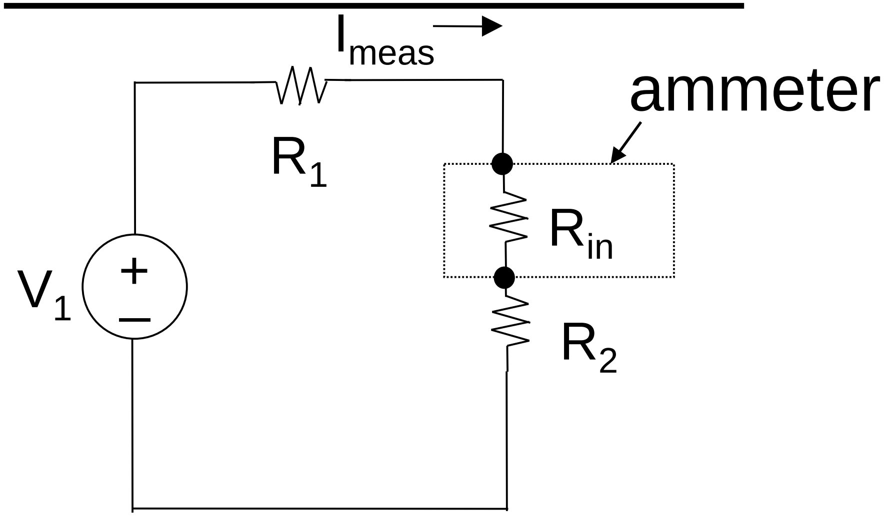
Effect of Ammeter

Measurement error due to non-zero input resistance:

undisturbed circuit



circuit with ammeter inserted



$$I = \frac{V_1}{R_1 + R_2}$$

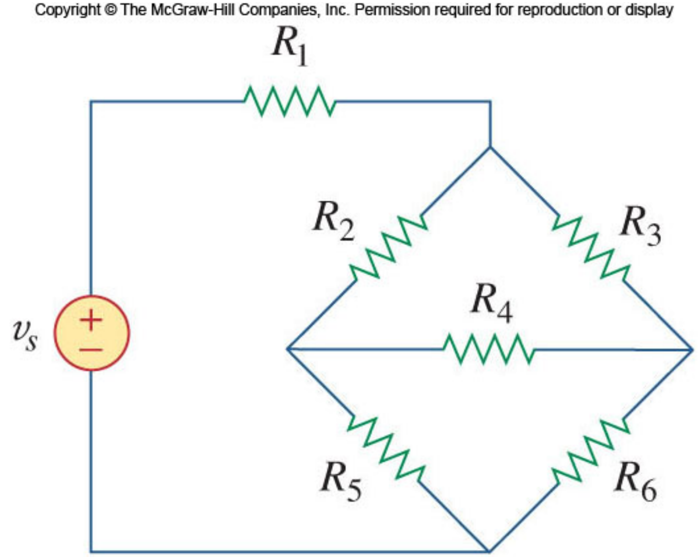
$$I_{meas} = \frac{V_1}{R_1 + R_2 + R_{in}}$$

Example: $V_1 = 1$ V, $R_1 = R_2 = 500$ ∧, $R_{in} = 1$∧

$$I = \frac{1V}{500\Omega + 500\Omega} = 1mA, \quad I_{meas} = ?$$

Extention 4： Wye-Delta Transformations

- There are cases where resistors are neither parallel nor series.
- Consider the bridge circuit shown here. This circuit can be simplified to a three-terminal equivalent



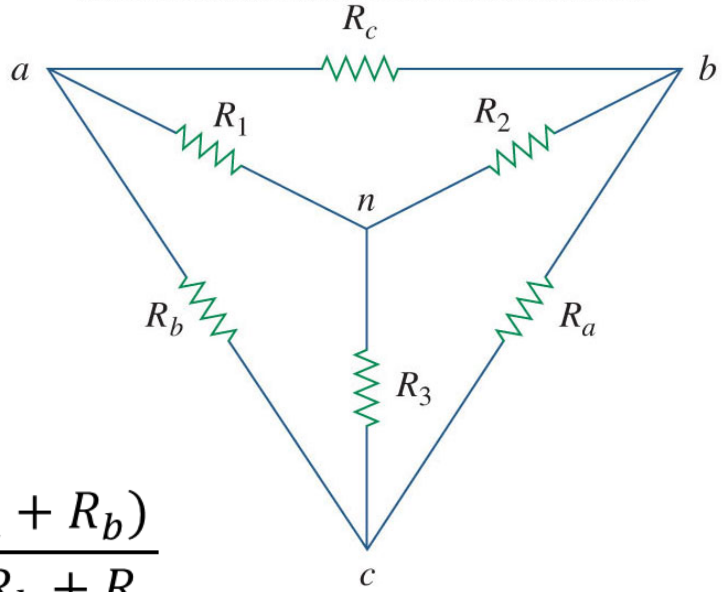
Delta to Wye

- The conversion formula for a delta to wye transformation are:

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$



$$R_{ab}(Y) = R_{ab}(\Delta) \Rightarrow R_1 + R_2 = \frac{R_c(R_a + R_b)}{R_a + R_b + R_c}$$

$$R_{ac}(Y) = R_{ac}(\Delta) \Rightarrow R_1 + R_3 = \frac{R_b(R_a + R_c)}{R_a + R_b + R_c}$$

$$R_{bc}(Y) = R_{bc}(\Delta) \Rightarrow R_2 + R_3 = \frac{R_a(R_b + R_c)}{R_a + R_b + R_c}$$

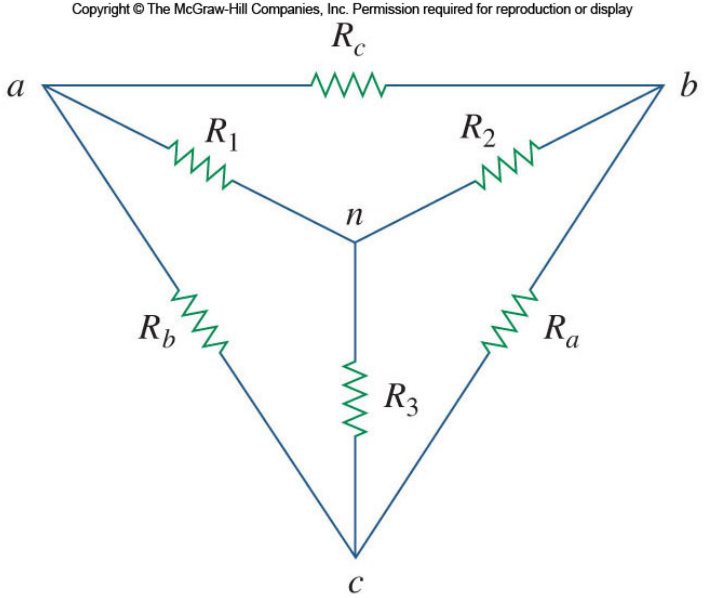
Wye to Delta

- The conversion formula for a wye to delta transformation are:

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

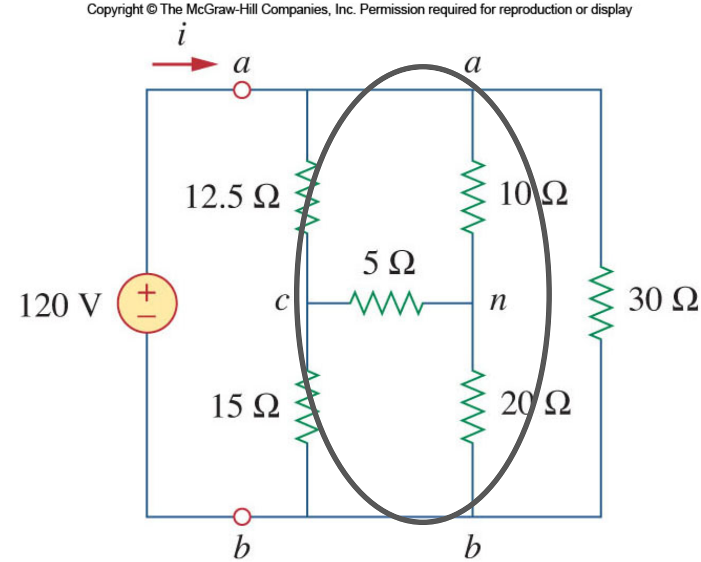


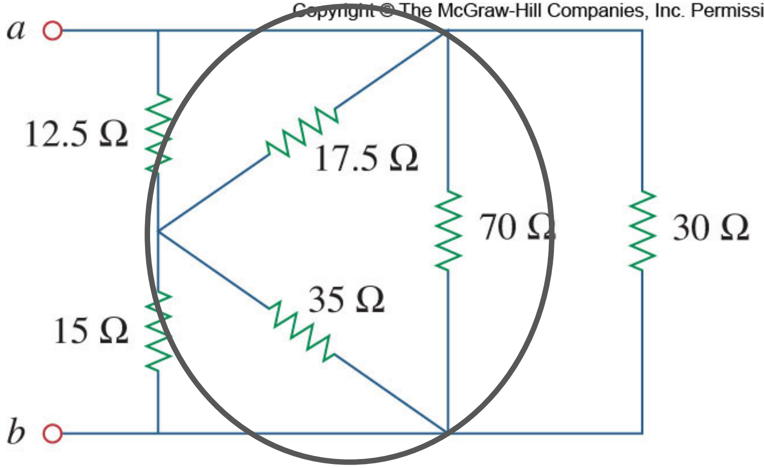
$$R_1R_2 + R_1R_3 + R_2R_3 = \frac{R_aR_bR_c}{R_a + R_b + R_c}$$

Example

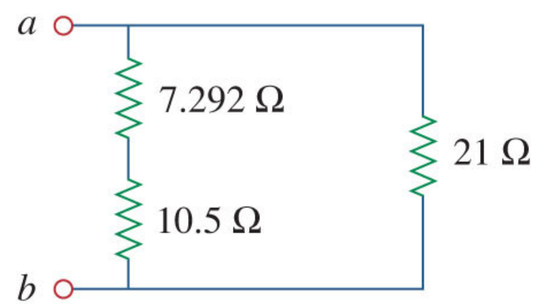
- Find the equivalent resistance

Wye -> Delta





(a)



(b)

Q & A

- Any question will be welcomed