Problem 1

(30')

Solution:

(a) $X(e^{jw}) = j\pi\left\{\delta\left(w+\frac{\pi}{2}\right) - \delta\left(w-\frac{\pi}{2}\right)\right\} + \pi\{\delta(w+1)+\delta(w-1)\}, -\pi < w < \pi$

Or $X(e^{jw}) = j\pi\sum_{l=-\infty}^{\infty}\{\delta\left(w+\frac{\pi}{2}-2\pi l\right) - \delta\left(w-\frac{\pi}{2}-2\pi l\right)\} + \pi\sum_{l=-\infty}^{\infty}\{\delta(w+1-2\pi l)+\delta(w-1-2\pi l)\}$

(b) $\left(\frac{1}{3}\right)^{|n|} \overset{F}{\leftrightarrow} \frac{4}{5-3\cos(w)}$

$$n\left(\frac{1}{3}\right)^{|n|} \overset{F}{\leftrightarrow} j\frac{d}{dw}\left(\frac{4}{5-3\cos(w)}\right) = \frac{-12j\sin w}{(5-3\cos w)^2}$$

$$X(e^{jw}) = \frac{-12j\sin w}{(5-3\cos w)^2} - \frac{4}{5-3\cos(w)}$$

(c) $x[n] = \frac{\sin\left(\frac{\pi(n-2)}{2}\right)}{\pi(n-2)} = \frac{1}{2}sinc\left(\frac{n-2}{2}\pi\right)$

$$x_1[n] = \frac{1}{2}sinc\left(\frac{n}{2}\pi\right) \overset{F}{\leftrightarrow} X_1(e^{jw}) = \begin{cases} 1, & 0 \le |w| \le \frac{\pi}{2} \\ 0, & \frac{\pi}{2} < |w| \le \pi \end{cases}, X_1(e^{jw}) \text{ is periodic with period } 2\pi$$

$$x[n] = x_1[n-2]$$

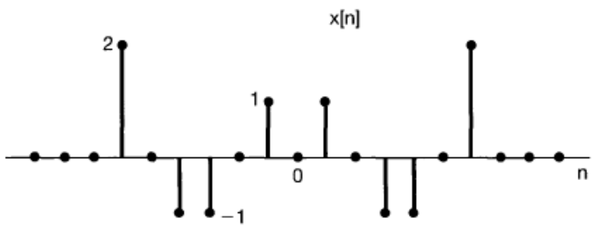
$$X(e^{jw}) = e^{-jw2}X_1(e^{jw}) = \begin{cases} e^{-jw2}, & 0 \le |w| \le \frac{\pi}{2} \\ 0, & \frac{\pi}{2} < |w| \le \pi \end{cases}, X(e^{jw}) \text{ is periodic with period } 2\pi$$

Problem 2

(20')

Solution:

a.



Correct:2,3,4,5

b. $x[n] = \delta[n-1] - \delta[n+1]$

Correct:1,4,5,6

1: signal real and odd.

2: signal real and even.

3: signal has to be symmetric about α.

4: $\int_{-\pi}^{\pi} X(e^{jw})dw = 2\pi x[0]$→x[n]=0.

5: $X(e^{jw})$ is always periodic with T=2π.

6: $X(e^{j0}) = \sum_{n=-\infty}^{\infty} x[n]$→the samples of the signal add up to zero.

Problem 3

(20')

Solution:

(a) Since the two systems are cascaded, the frequency response of the overall system is $\mathrm{H}(e^{jw}) = H_1(e^{jw})H_2(e^{jw}) = \frac{2-e^{-jw}}{1+\frac{1}{8}e^{-j3w}}$

Therefore, the Fourier transform of the input and output of the overall system are related by $\frac{Y(e^{jw})}{X(e^{jw})} = \frac{2-e^{-jw}}{1+\frac{1}{8}e^{-3jw}}$

Cross-multiplying and taking the inverse Fourier transform, we get

$$\mathrm{y[n]} + \frac{1}{8}y[n-3] = 2x[n] - x[n-1]$$

(b) We may rewrite the overall frequency response as

$$\begin{aligned}
\mathrm{H}(e^{jw}) &= \frac{2-e^{-jw}}{\left(1+\frac{1}{2}e^{-jw}\right)\left[1-\left(\frac{1}{4}-\frac{\sqrt{3}}{4}j\right)e^{-jw}\right]\left[1-\left(\frac{1}{4}+\frac{\sqrt{3}}{4}j\right)e^{-jw}\right]} \\
&= \frac{2-e^{-jw}}{\left(1+\frac{1}{2}e^{-jw}\right)\left[1-\frac{1}{2}e^{-j\frac{\pi}{3}}e^{-jw}\right]\left[1-\frac{1}{2}e^{j\frac{\pi}{3}}e^{-jw}\right]} \\
&= \frac{4/3}{1+\frac{1}{2}e^{-jw}} + \frac{(1-j\sqrt{3})/3}{1-\frac{1}{2}e^{-j\frac{\pi}{3}}e^{-jw}} + \frac{(1+j\sqrt{3})/3}{1-\frac{1}{2}e^{j\frac{\pi}{3}}e^{-jw}}
\end{aligned}$$

Taking the inverse Fourier transform, we get

$$\mathrm{h[n]} = \frac{4}{3}\left(-\frac{1}{2}\right)^n u[n] + \frac{1-j\sqrt{3}}{3}\left(\frac{1}{2}e^{-j\frac{\pi}{3}}\right)^n u[n] + \frac{1+j\sqrt{3}}{3}\left(\frac{1}{2}e^{j\frac{\pi}{3}}\right)^n u[n]$$

Problem 4

(30')

Solution:

(a) $H(e^{jw}) = \frac{b+e^{-jw}}{1-ae^{-jw}}$

$\because |H(e^{jw})| = 1, \therefore |b + e^{-jw}| = |1 - ae^{-jw}|$

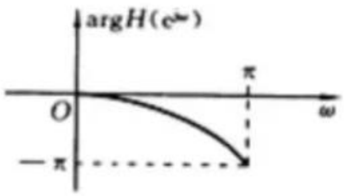
$\therefore 1 + b^2 + 2bcosw = 1 + a^2 - 2acosw, for\ all\ w$

→b=−a

(b) $a = -\frac{1}{2} \rightarrow b = \frac{1}{2}$

$$H(e^{jw}) = \frac{\frac{1}{2} + e^{-jw}}{1 + \frac{1}{2}e^{-jw}} = e^{-jw}\frac{1 + \frac{1}{2}e^{jw}}{1 + \frac{1}{2}e^{-jw}} = e^{-jw}\frac{1 + \frac{1}{2}cosw + j\frac{1}{2}sinw}{1 + \frac{1}{2}cosw - j\frac{1}{2}sinw}$$

$$arg\left(H(e^{jw})\right) = -w + 2\,arctan\left(\frac{\frac{1}{2}sinw}{1 + \frac{1}{2}cosw}\right)$$



(c) $X(e^{jw}) = \frac{1}{1-\frac{1}{2}e^{-jw}}$

$$Y(e^{jw}) = X(e^{jw}) \cdot H(e^{jw}) = \frac{\frac{1}{2} + e^{-jw}}{\left(1 - \frac{1}{2}e^{-jw}\right)\left(1 + \frac{1}{2}e^{-jw}\right)} = \frac{\frac{5}{4}}{1 - \frac{1}{2}e^{-jw}} - \frac{\frac{3}{4}}{1 + \frac{1}{2}e^{-jw}}$$

$$y[n] = \left[\frac{5}{4}\left(\frac{1}{2}\right)^n - \frac{3}{4}\left(-\frac{1}{2}\right)^n\right]u[n]$$