You should submit your homework in Gradescope. (Code: RWVZVN)

1 HEAD or TAIL

Alice and Bob are playing a game. They both have a coin. The game requires Alice and Bob to choose one side (HEAD or TAIL) of the coin. Then they show their choices at the same time, and the result is decided according to the following rules.

a) If they show different sides, Bob wins \$3 from Alice.

b) If they both show HEAD, Alice wins \$1 from Bob. If they both show TAIL, Alice wins \$5 from Bob.

1.1 Mixed Strategies (1pt)

Suppose Alice's strategy is showing HEAD with probability p and Bob's strategy is showing HEAD with probability q. Calculate Bob's expected payoff in term of p and q.

1.2 Best Response (1pt)

If Alice choose her side uniformly randomly ($p = 0.5$), then which strategy is Bob's best response?

1.3 Nash Equilibrium (1pt)

Calculate the Nash equilibrium of this game. What are the expected payoffs of Alice and Bob in the equilibrium? Do you think the game is fair?

2 Aladdin Lamp

Tom and Jerry have found Aladdin Lamp which can give them power but the lamp only has 10 units of power. Thus the lamp has made a rule. They should show how much power they wish to have at the same time. If the sum of their wishes exceeds 10, the lamp will only satisfy the lower power (with random tie-breaking). If the sum of their wishes is within 10, the lamp will satisfy both of them.

2.1 Action Spaces (1pt)

Write down the action spaces of Tom and Jerry.

2.2 Nash Equilibrium (1pt)

Find a Nash equilibrium for this game.

2.3 A New Rule (1pt)

Suppose the lamp has made an alternative rule. If the sum of wishes exceeds 10, the lamp will satisfy the lower power first and give the rest of the power to the other. If the sum of their wishes is within 10, the lamp will still satisfy both of them. Now, in this new rule, is the strategy you have found in 2.2 still a Nash equilibrium? If yes, is it the unique Nash equilibrium? If it is no longer a Nash equilibrium, find a Nash equilibrium for the new rule.

3 Second Price Auction with Budget (2pt)

Consider a second price auction for a single indivisible item. Suppose each bidder i has a value $v_i > 0$ and a budget $c_i > 0$. If a bidder wins the object and has to pay higher than the budget, the bidder will simply drop out from the auction but is charged with a small penalty $\epsilon > 0$. Compute a bid in the auction for each player i which will be a weakly dominant strategy for the player.

4 Vickrey-Clarke-Groves Mechanism (2pt)

A mechanism $(f, p_1, \ldots, p_n)$ is called Vickrey-Clark-Groves (VCG) mechanism if

- $f(v_1, \ldots, v_n) \in \arg\max_{a \in A} \sum_i v_i(a)$; that is, f maximizes the social welfare, and
- for some functions $h_1, \ldots, h_n$, where $h_i : V_{-i} \to \mathbb{R}$ (i.e., h_i does not depend on v_i), we have that for all $v_1 \in V_1, \ldots, v_n \in V_n$, $p_i(v_1, \ldots, v_n) = h_i(v_{-i}) - \sum_{j \neq i} v_j(f(v_1, \ldots, v_n))$.

Prove that VCG mechanism is incentive compatible.