Homework 4

Due date: Apr.9th, 2018

Turn in your homework in class

Rules:

- Work on your own. Discussion is permissible, but extremely similar submissions will be judged as plagiarism.
- Please show all intermediate steps: a correct solution without an explanation will get zero credit.
- Please submit on time. No late submission will be accepted.
- Please prepare your submission in English only. No Chinese submission will be accepted.

1. The current in a $150\mu H$ inductor is known to be:

$$i_L = 25te^{-500t}\text{A for } t \geq 0$$

(a) Find the voltage across the inductor for $t > 0$.

(b) Find the power (in microwatts) at the terminals of the inductor when $t = 5ms$.

[a] $v = L\dfrac{di}{dt}$

$$= (150 \times 10^{-6})(25)[e^{-500t} - 500te^{-500t}] = 3.75e^{-500t}(1 - 500t)\,\text{mV}$$

[b] $i(5\,\text{ms}) = 25(0.005)(e^{-2.5}) = 10.26\,\text{mA}$

$$v(5\,\text{ms}) = 0.00375(e^{-2.5})(1 - 2.5) = -461.73\,\mu\text{V}$$

$$p(5\,\text{ms}) = vi = (10.26 \times 10^{-3})(-461.73 \times 10^{-6}) = -4.74\,\mu\text{W}$$

2. Assume that the initial energy stored in the inductors of Fig. 1 is zero. Find the equivalent inductance with respect to the terminals a, b.

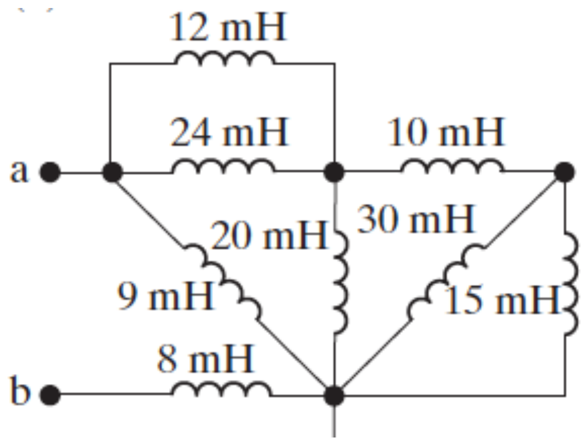


Fig. 1

[a] $15 \| 30 = 10\,\text{mH}$

$$10 + 10 = 20\,\text{mH}$$

$$20 \| 20 = 10\,\text{mH}$$

$$12 \| 24 = 8\,\text{mH}$$

$$10 + 8 = 18\,\text{mH}$$

$$18 \| 9 = 6\,\text{mH}$$

$$L_{\text{ab}} = 6 + 8 = 14\,\text{mH}$$

3. The capacitance and associated voltage for each capacitor is given in Fig. 2. Find the equivalent capacitance and the associated voltage with respect to the terminals a, b for the circuit shown in Fig. 2.

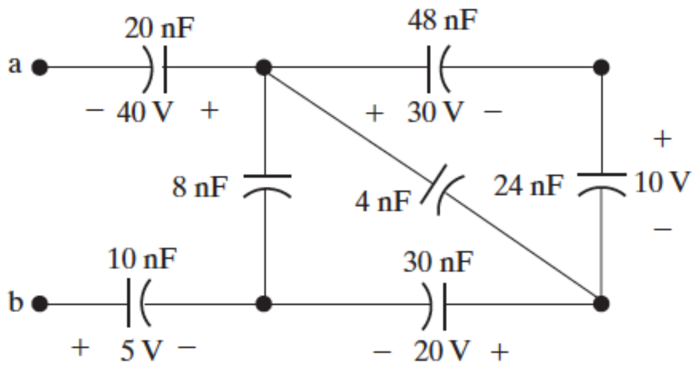
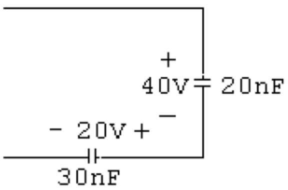


Fig. 2

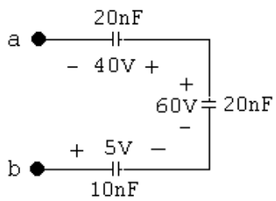
[a] $\frac{1}{C_1} = \frac{1}{48} + \frac{1}{24} = \frac{1}{16}$; $\quad C_1 = 16\,\text{nF}$

$C_2 = 4 + 16 = 20\,\text{nF}$



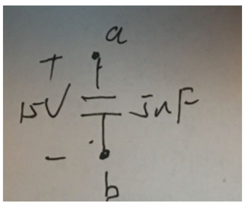
$\frac{1}{C_3} = \frac{1}{20} + \frac{1}{30} = \frac{1}{12}$; $\quad C_3 = 12\,\text{nF}$

$C_4 = 12 + 8 = 20\,\text{nF}$



$\frac{1}{C_5} = \frac{1}{20} + \frac{1}{20} + \frac{1}{10} = \frac{1}{5}$; $\quad C_5 = 5\,\text{nF}$

Equivalent capacitance is $5\,\text{nF}$ with an initial voltage drop of +15 V.



4. The switch in the circuit in Fig. 3 has been open for a long time. At $t = 0$ the switch is closed.
   (a) Determine $i_0(0)$ and $i_0(\infty)$.
   (b) Determine $i_0(t)$ for $t \geq 0$.

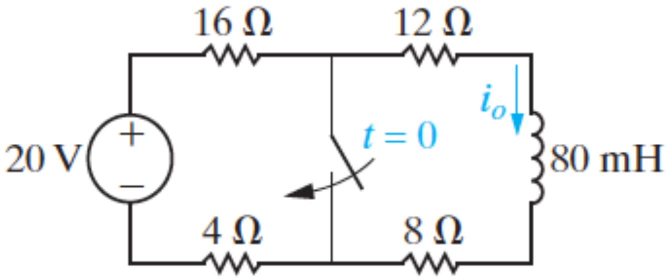


Fig. 3

$$[\mathbf{a}]\quad i_o(0) = \frac{20}{16+12+4+8} = \frac{20}{40} = 0.5\,\text{A}$$

$$i_o(\infty) = 0\,\text{A}$$

$$[\mathbf{b}]\quad i_o = 0.5e^{-t/\tau}; \qquad \tau = \frac{L}{R} = \frac{80\times 10^{-3}}{12+8} = 4\,\text{ms}$$

$$i_o = 0.5e^{-250t}\,\text{A}, \quad t \geq 0$$

5. The switch shown in Fig. 4 has been open for a long time before closing at $t = 0$. Write the expression for the capacitor voltage, $v(t)$, for $t \geq 0$.

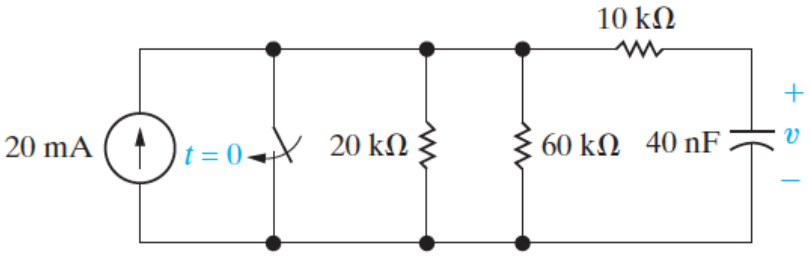
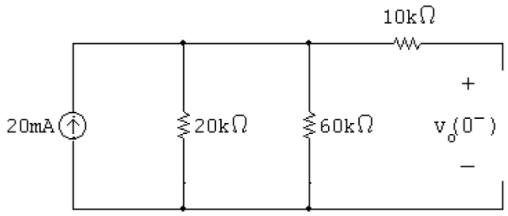


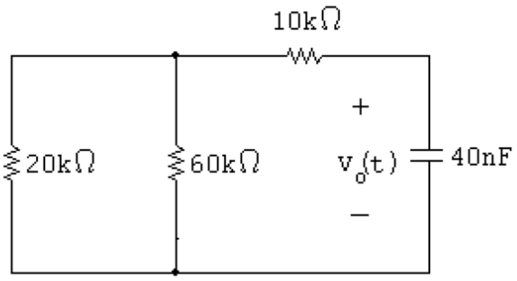
Fig. 4

For $t < 0$:



$$V_o = (20{,}000 \| 60{,}000)(20 \times 10^{-3}) = 300\text{ V}$$

For $t \geq 0$:



$$R_{\text{eq}} = 10{,}000 + (20{,}000 \| 60{,}000) = 25\text{ k}\Omega$$

$$\tau = R_{\text{eq}}C = (25{,}000)(40 \times 10^{-9}) = 1\text{ ms}$$

$$v(t) = V_o e^{-t/\tau} = 300e^{-1000t}\text{ V} \qquad t \geq 0$$