Problem 1 – Short Answers (25 points)

Provide your short answers to these questions. Use proper units if needed.

A phasor voltage V is a function of time. True or false? [1 point]

Solution: False.

1) Name a component or device that turns a steady current into a steady voltage. [1 point]

   Solution: resistor (DC), inductor (AC).

2) Name two components or devices that can increase the amplitude of an AC voltage. [2 points]

   Solution: Op Amp, transformer.

3) Name two components or devices that can block a DC signal, but pass an AC signal. [2 points]

   Solution: capacitor, transformer.

4) For the circuit shown below

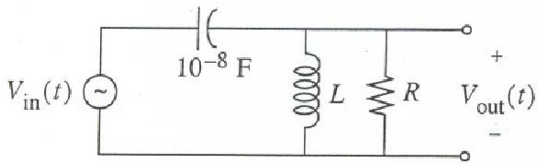


Figure 1

Where $V_{in}(t) = \cos(\omega t)$, $\mathrm{L} = 2 \times 10^{-4}\mathrm{H}$ and $\mathrm{R} = 200\Omega$.

   a) What is $|V_{out}(t)|$ for $\omega = 0$? [1 point]

      Solution: 0 (capacitor open)

   b) What is $|V_{out}(t)|$ for $\omega \longrightarrow \infty$? [1 point]

      Solution: 1 (capacitor shorted)

      Note that you should represent $V_{out}(t)$ as its phasor. Here $|\cdot|$ means that amplitude of a phasor.

   c) What is $|V_{out}(t)|$ for $\omega = 10^6$? [4 points]

      Solution: $\sqrt{2}$

5) You measure an AC voltage across a $1\mathrm{k}\Omega$ resistor. The digital voltmeter that you use measures in RMS (like all voltmeters), and report that the voltage is 1V.

   a) What is the peak-to-peak voltage? [2 points]

      Solution: 2.83 or $2\sqrt{2}$V

   b) What is the max power dissipated in the resistor? [1 point]

      Solution: 2mW

c) What is the average power dissipated in the resistor? [1 point]

Solution: 1mW

6) Find the value of Z in the circuit below, if $V_g = 100 - j50V, I_g = 30 + j20\ A$, and $V_1 = 140 + j30V$. [4 points]

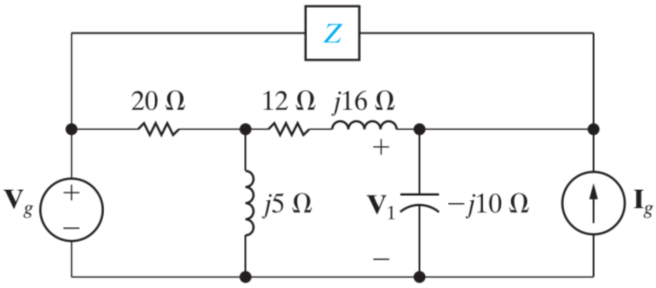
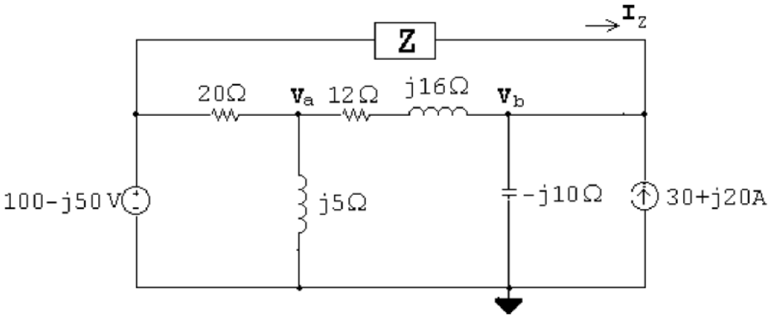


Figure 2

Solution: 2 + j2.



$$\boldsymbol{V}_b = \boldsymbol{V}_1 = 140 + j30\text{ V}$$

KCL at node a:

$$\frac{\boldsymbol{V}_a - \boldsymbol{V}_g}{20\ \Omega} + \frac{\boldsymbol{V}_a}{j5\ \Omega} + \frac{\boldsymbol{V}_a - \boldsymbol{V}_b}{12 + j16\ \Omega} = 0$$

KCL at node b:

$$\boldsymbol{I}_z + \boldsymbol{I}_g - \frac{\boldsymbol{V}_b}{-j10\ \Omega} + \frac{\boldsymbol{V}_a - \boldsymbol{V}_b}{12 + j16\ \Omega} = 0$$

$$\boldsymbol{I}_z Z = \boldsymbol{V}_a - \boldsymbol{V}_b$$

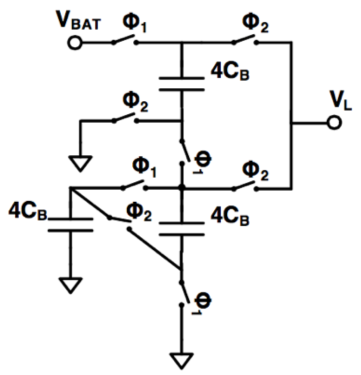
Solving equations above, we get

$$\mathbf{V}_\text{a} = 40 + j30\text{ V}$$

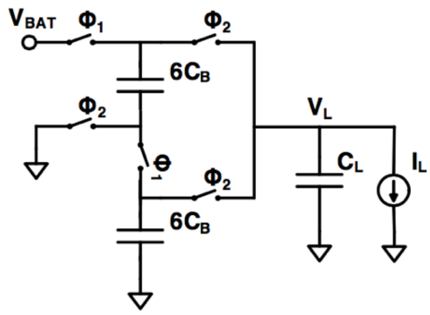
$$\mathbf{I}_Z = -30 - j10\text{ A}$$

$$Z = \frac{(100 - j50) - (140 + j30)}{-30 - j10} = 2 + j2\,\Omega$$

7) Figure 3 shows two Switched Capacitor (SC) Converter circuits which convert input DC voltage V_{BAT} to output voltage V_L. The conversion ratio is defined as n = $\frac{V_{BAT}}{V_L}$. All the switches in these circuits are controlled by a periodic square wave with 50% duty cycle: During high voltage phase, the ϕ_1 switches are turned on, meanwhile the ϕ_2 switches are turned off; during low voltage phase, the ϕ_2 switches are turned on, but the ϕ_1 switches are turned off. Assuming that the capacitors can be fully charged in a half cycle. **Find the conversion ratios n_1 and n_2 for the two SC converters shown in Figure 3.** [5 points]



(a) Solution: $n_1 = 3/2$



(b) **Solution: $n_2 = 2/1$**

Figure 3

Problem 2 (12 points)

You must show your work to get full credit.

The neon bulb in the circuit shown in Figure 4 has the following behavior: The bulb remains off and acts as an open circuit until the bulb voltage v reaches a threshold voltage V_T = 65 V. Once v reaches V_T, a discharge occurs and the bulb acts like a simple resistor of value R_N = 1 kΩ; the discharge is maintained as long as the bulb current i remains above the value I_S = 10 mA needed to sustain the discharge (even if the voltage v drops below V_T). As soon as i drops below 10 mA, the bulb again becomes an open circuit.

a) Find the waveform of $v(t)$ and sketch it, showing only the first and second charging intervals. Assuming that at t = 0, the capacitor voltage is 0V.

b) Estimate the flashing rate.

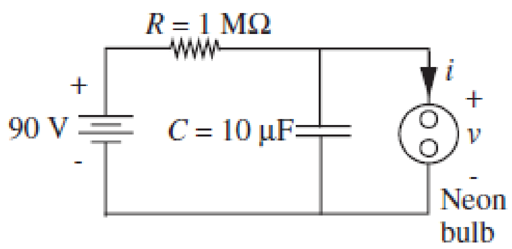


Figure 4

Solution:

a) In first circle:

Charging (0< v < 65 V):

$\tau_c = RC = 1M\Omega \cdot 10\mu F = 10s$

$v_{charge} = 90(1 - e^{-t/\tau_c})$

Note in first charging circle when charging v approaches 65 V, charging time

$t_{c1} = -\tau_c \ln\frac{(90-65)}{90} \approx 17.92s$

Discharging (i > 10mA):

$\tau_d = R_{eq}C = (R||R_N)\mathrm{C} = \frac{1M\Omega \cdot 1k\Omega}{1M\Omega + 1k\Omega} 10\mu F \approx 10ms$

$v_{discharge} = 65e^{-t/\tau_d}$

The minimum v when discharging is $v_{min} = i_{min} \cdot R_N = 10\ V$

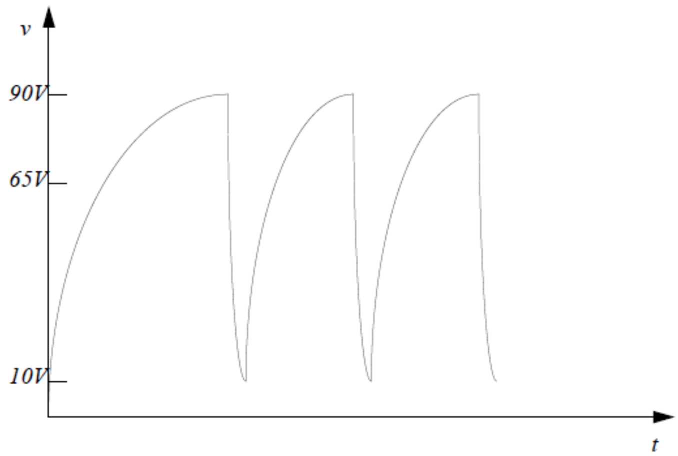
As a result in next charging circles (10 V< v < 65 V):

$v_{charge} = 90 + (10 - 90)e^{-t/\tau_c}$

Charging time $t_c = -\tau_c \ln\frac{(90-65)}{80} \approx 11.63s$

Also note $\tau_c \gg \tau_d$, so the charging time is much longer than the discharging time.

Draw the figure as follow:



(Note that this figure is not correct: At t=0, v should be 0! Also The peak of v is 65V, not 90V!)

b) Since the discharge time is so small in comparison to the charge time, we will only consider the charge time.
$T \approx t_c = 11.63s$

Therefore the flashing rate is $f = \frac{1}{T} = \frac{1}{11.63} = 0.086\ Hz.$

Problem 3 (18 points)

You must show your work to get full credit.

The voltage signal of Figure 5(a) is applied to the cascaded integrating amplifiers shown in Figure 5(b). There is no energy stored in the capacitors at the instant when the signal is applied。

a) Derive the numerical expressions for $v_{o1}(t)$ and $v_o(t)$ for the time intervals

$$0 \le t \le 0.5\,s \text{ and } 0.5\,s \le t \le t_{sat}.$$

b) Compute the value of t_{sat}.

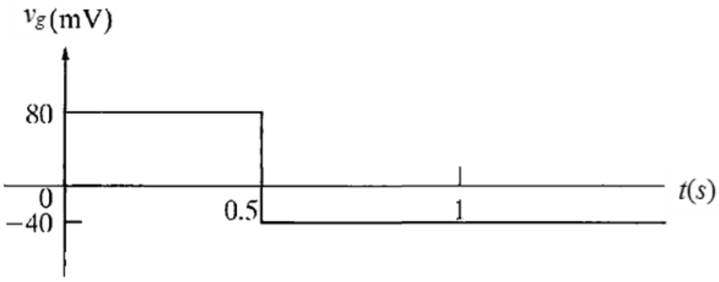


Figure 5 (a)

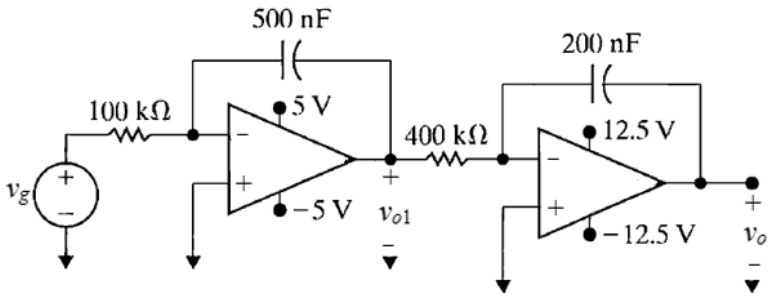


Figure 5 (b)

Solution:

a) v_{o1} is the output of the first integrator, while v_o is the output of the second integrator.

$$\frac{dv_{o1}}{dt} = -\frac{1}{R_1C_1}v_g = -20v_g$$

$$\frac{d^2v_o}{dt^2} = \frac{1}{R_1C_1R_2C_2}v_g = \frac{10^6}{100\times 400\times 0.5\times 0.1}v_g = 250v_g$$

When $0 \le t \le 0.5^-$:

$v_g = 80\,mV$

$$\frac{dv_{o1}}{dt} = -20v_g = -1.6$$

$$\frac{d^2v_o}{dt^2} = 250v_g = 20$$

And initial condition: $v_{o1}(0) = 0$, $v_o(0) = 0$

So

$$v_{o1}(t) = \int -1.6dt = -1.6t$$

$$v_o(t) = \iint 20dt = 10t^2$$

When $\mathbf{0.5^-} \le \boldsymbol{t} \le \boldsymbol{t_{sat}}$:

$v_g = -40\ mV$

$$\frac{dv_{o1}}{dt} = -20v_g = 0.8$$

$$\frac{d^2v_o}{dt^2} = 250v_g = -10$$

Initial condition:

$$v_{o1}(0.5^+) = v_{o1}(0.5^-) = -1.6 \times 0.5 = -0.8\ V$$

$$v_{o1}(t) = \int_{0.5^+}^{t} 0.8\,dt + \boldsymbol{v_{o1}(0.5^+)} = 0.8(t - 0.5) - 0.8 = 0.8t - 1.2\ V$$

Similarly,

$$v_o(0.5^+) = v_o(0.5^-) = 10 \times 0.25 = 2.5\ V$$

And

$$\frac{dv_o}{dt}(0.5^+) = \frac{dv_o}{dt}(0.5^-) = 20 \times 0.5 = 10,$$

So,

$$\frac{dv_o}{dt}(t) - \frac{dv_o}{dt}(0.5^+) = \int_{0.5^+}^{t} -10dt = -10t + 5$$

$$v_o(t) = \int_{0.5^+}^{t}(-10t + 5 + 10)\,dt + v_o(0.5^+)$$

$$= -5t^2 + 15t - (-5 \times 0.25 + 15 \times 0.5) + 2.5 = -5t^2 + 15t - 3.75\ V$$

Summary:

$0 \le t \le 0.5^-$:

$$v_{o1}(t) = -1.6t\ V, \qquad v_o(t) = 10t^2 V$$

$0.5^- \le t \le t_{sat}$:

$$v_{o1}(t) = 0.8t - 1.2\ V, \qquad v_o(t) = -5t^2 + 15t - 3.75\ V$$

b)

$$v_o(t_{sat}) = -12.5\ V$$

Solving, $t_{sat} = 3.5\ s$

Check $v_{o1}(3.5) = 1.6\ V < 5V$, So the answer is available.

Problem 4 (10 points)

You must show your work to get full credit.

Find the current i for the circuit shown in Figure 6 when $L = 0.1\text{H}$, $C = 25\text{mF}$, $R_1 = 3\Omega$, $R_2 = 3\Omega$, $R_3 = 1\Omega$ and

$$V_s(t) = -4\,cos(10t + 30°)\text{V}$$

$$I_s(t) = 2\,sin(40t + 30°)\text{A}$$

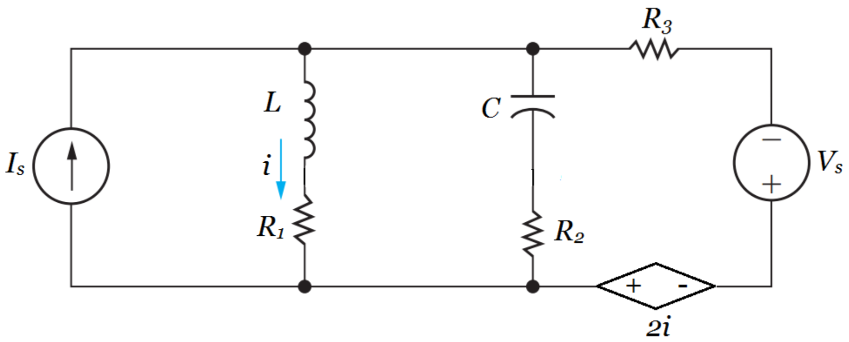
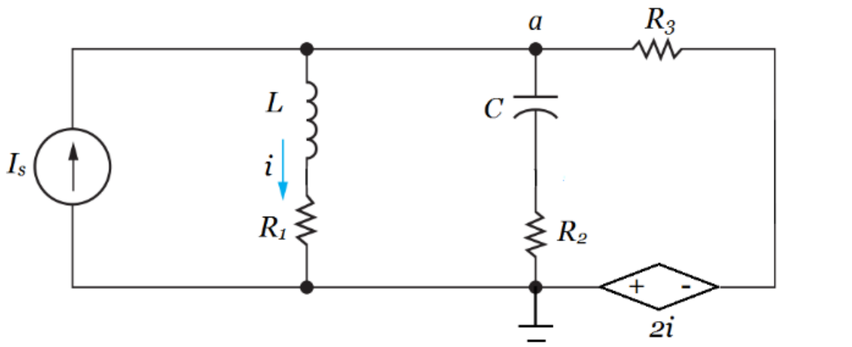


Figure 6

Solution:

1) Firstly we turn off the voltage source:



$$I_s(t) = 2\,sin(40t + 30°)\text{ A} = 2\,cos(40t - 60°)\text{A} = 2\angle - 60°\text{A} = 1 - j\sqrt{3}\text{ A}$$

Thus

$$\omega_1 = 40\text{rad/s},\ Z_{L1} = j4\Omega,\ Z_{C1} = -j\Omega,$$

Apply KCL at node a:

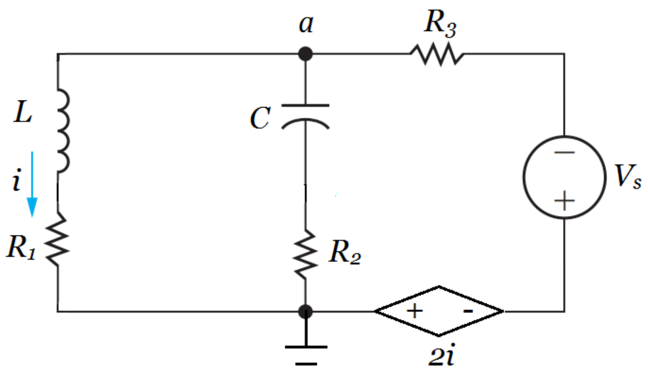
$$-I_s + i_1 + \frac{V_a}{3 - j} + \frac{V_a - (-2i_1)}{1} = 0 \qquad ①$$

$$i_1 = \frac{V_a}{3 + j4} \qquad ②$$

Solving ① and ②， we get

$$i_1 = -0.0416 - j0.231\text{A} = -0.235\angle -79.8°\text{A} = -0.235cos(40t - 79.8°)\ \text{A}$$

2) Then we turn off the current source:



$V_s(t) = -4\,cos(10t + 30°)\text{V} = \ 4\angle 30°\text{V} = -2\sqrt{3} - j2\ \text{V}$

Thus ω_2 =10rad/s, $Z_{L2} = j\Omega$, $Z_{C2} = -j4\Omega$

Apply KCL at node a:

$$i_2 + \frac{V_a}{3-4j} + \frac{V_a - (-2i_2 - V_s)}{1} = 0 \qquad ③$$

$$i_2 = \frac{V_a}{3+j} \qquad ④$$

Solving ③ and ④， we get

$$i_2 = 0.601 + j0.167\text{A} = 0.62\angle 15.5°\text{A} = 0.62cos(10t + 15.5°)\ \text{A}$$

According to superposition theory,

$$i = i_1 + i_2 = [-0.235cos(40t - 79.8°) \ + 0.62cos(10t + 15.5°)]\ \text{A}$$

Problem 5 (15 points)

You must show your work to get full credit.

The impedance Z_L in the circuit shown in Figure 7 is adjusted for maximum average power transfer to Z_L. The internal impedance of the sinusoidal voltage source is $4 + j7\ \Omega$.

a) What is the maximum average power delivered to Z_L?

b) What percentage of the average power delivered to the linear transformer is delivered to Z_L?

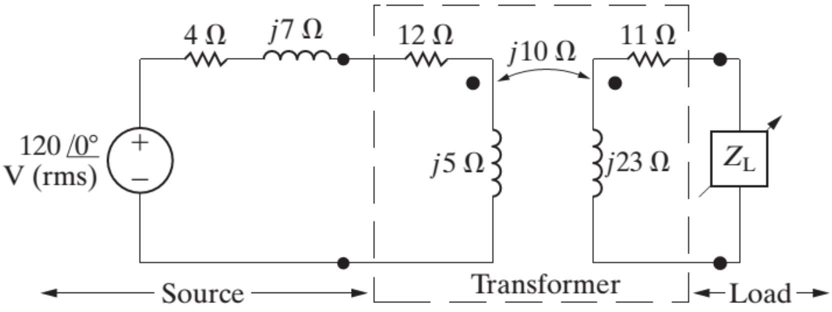
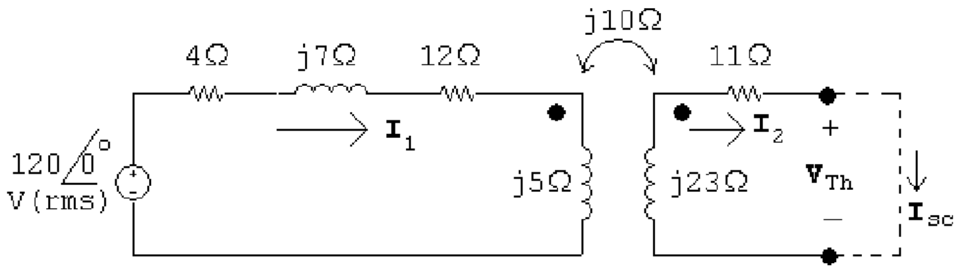


Figure 7

Solution:

First, find the Thevenin Equivalent circuit.



Open circuit:

$$\mathbf{V}_{\text{Th}} = \frac{120}{16 + j12}(j10) = 36 + j48\,\text{V}$$

Short circuit:

$$(16 + j12)\mathbf{I}_1 - j10\mathbf{I}_{\text{sc}} = 120$$

$$-j10\mathbf{I}_1 + (11 + j23)\mathbf{I}_{\text{sc}} = 0$$

Solving,

$$\mathbf{I}_{\text{sc}} = 2.4\,\text{A}$$

$$Z_{\text{Th}} = \frac{36 + j48}{2.4} = 15 + j20\,\Omega$$

4

$$\therefore \quad Z_{\text{L}} = Z_{\text{Th}}^* = 15 - j20\,\Omega$$

$$\mathbf{I}_{\text{L}} = \frac{\mathbf{V}_{\text{Th}}}{Z_{\text{Th}} + Z_L} = \frac{36 + j48}{30} = 1.2 + j1.6\,\text{A(rms)}$$

$$P_{\text{L}} = |\mathbf{I}_{\text{L}}|^2(15) = 60\,\text{W}$$

[b] $\mathbf{I}_1 = \dfrac{Z_{22}\mathbf{I}_2}{j\omega M} = \dfrac{26+j3}{j10}(1.2+j1.6) = 5.23\underline{/-30.29^\circ}\,\text{A (rms)}$

$$P_{\text{transformer}} = (120)(5.23)\cos(-30.29^\circ) - (5.23)^2(4) = 432.8\,\text{W}$$

$$\%\text{ delivered } = \frac{60}{432.8}(100) = 13.86\%$$

Problem 6 (20 points)

You must show your work to get full credit.

Consider the circuit in Figure 8. The ideal op-amp's power supplies of ±V volts limit the range of the output voltage. Note that this circuit has positive feedback. Assuming that $v_A(0^-) = v_1(0^-) = -\frac{V}{2}$, express and sketch the voltages $v_A(t)$ and $v_B(t)$ as functions of time. Assuming that the response time of the op-amp's output to the changes at its inputs can be ignored.

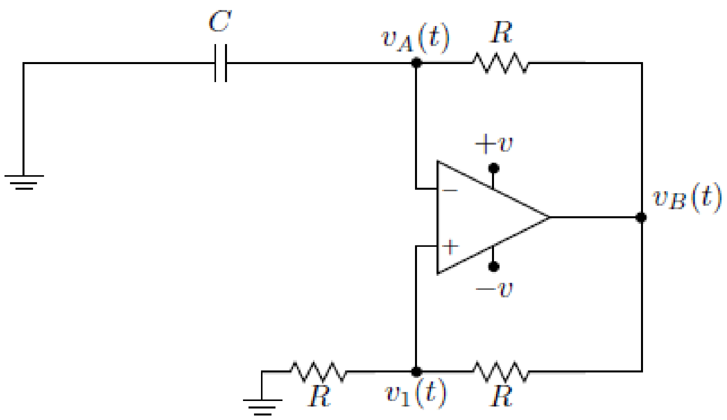


Figure 8

Solution:

The op-amp is ideal, but its output is limited to the range [−V, V] by the supply voltages. $v_B(t)$ will drop across the series combination of resistors, so

$$v_1(t) = v_B(t)/2$$

Analysis of the circuit:

1) Initially, $v_A(t) = -\frac{V}{2} > v_B(t) = -V$, capacitor starts to discharge, and $v_A(t)$ drop below $-\frac{V}{2}$.

2) When $v_A(t) < v_1(t) = -\frac{V}{2}$, we have

$$v_B(t) = +V$$

   This means the capacitor voltage $v_A(t)$ (referred to ground) will begin to climb towards V.

3) At some point it will cross V/2 which will make $v_B(t)$ switch to −V , taking $v_1(t)$ to −V/2. Now the voltage across the capacitor will discharge towards −V.
4) When it crosses −V/2 we have $v_B(t)$ switching again to +V (and $v_1(t)$ to V/2) so the cycle repeats.

For $v_A(0^+) = v_A(0^-) = -\frac{V}{2}$, so $v_B(0^-) = -V$, C will discharge, when $v_A(0^+) < -\frac{V}{2}$, then $v_B(0^+) = V$, and $v_1(0^+) = \frac{V}{2}$. The capacitor starts to charge, we have the equation

$$v_A(t) + RC\frac{dv_A(t)}{dt} = V$$

with solution

$$v_A(t) = -\frac{V}{2} + \frac{3V}{2}(1 - e^{-\frac{t}{RC}})$$

But it will only continue till $v_A(t)$ reaches V/2, then $v_B(t)$ is going to switch to -V, so the time capacitor charging is $RC\ln 3$.

Similar to next discharging process, initial condition $v_A(RC\ln 3) = \frac{V}{2}$, we have

$$v_A(t) + RC\frac{dv_A(t)}{dt} = -V$$

with solution

$$v_A(t) = \frac{V}{2} - \frac{3V}{2}\left(1 - e^{-\frac{t-RC\ln 3}{RC}}\right), \qquad RC\ln 3 \le t \le 2RC\ln 3$$

Then the capacitor will run the circle again.

Above all, we can express

$$v_A(t) = \begin{cases} -\frac{V}{2} + \frac{3V}{2}\left(1 - e^{-\frac{t-2k\sigma}{RC}}\right), & 2k\sigma \le t \le (2k+1)\sigma \\ \frac{V}{2} - \frac{3V}{2}\left(1 - e^{-\frac{t-(2k+1)\sigma}{RC}}\right), & (2k+1)\sigma \le t \le (2k+2)\sigma \end{cases}$$

$$v_B(t) = \begin{cases} V, & 2k\sigma \le t \le (2k+1)\sigma \\ -V, & (2k+1)\sigma \le t \le (2k+2)\sigma \end{cases}$$

Where $\sigma = RC\ln 3$, $k = 0, 1, 2, 3, \cdots$

And the graphs of $v_A(t)$ and $v_B(t)$ are