Homework 1

Due date:
Mar.12th, 2018
Turn in your homework in class

Rules:

- Please try to work on your own. Discussion is permissible, but identical submissions are unacceptable!
- Please show all intermediate steps: a correct solution without an explanation will get zero credit.
- Please submit on time. No late submission will be accepted.
- Please prepare your submission in English only. No Chinese submission will be accepted.

1. (16%=4*4%) Determine the current flowing through an element if the charge flow is given by (a)(b). Also, find the charge flowing through an element if the current is given by (c)(d).
   (a) $q(t) = 1.7t(1 - e^{-1.2t})\ nC$
   (b) $q(t) = 0.2t\sin(120\pi t) + \cos(2e^{-\sin t})\ mC$
   (c) $i(t) = 4e^{-t} - 3e^{-2t}\ mA, q(0) = 0.2A$
   (d) $i(t) = 12e^{-3t}\cos(40\pi t)\ nA, q(0) = 2.28$ pA

Solutions:

(a) $\mathbf{q(t) = 1.7t(1 - e^{-1.2t})\ nC}$

$$i(t) = \frac{dq(t)}{dt} = \frac{d}{dt}[1.7t(1 - e^{-1.2t})] * 10^{-9}\ A$$
$$= [1.7(1 - e^{-1.2t}) + 1.7t * (-1.2) * (1 - e^{-1.2t})] * 10^{-9}\ \mathrm{A}$$
$$= 1.7(1 - e^{-1.2t} + 1.2te^{-1.2t})\ nA$$

(b) $\mathbf{q(t) = 0.2t\sin(120\pi t) + \cos(2e^{-\sin t})\ mC}$

$$i(t) = \frac{dq(t)}{dt} = \frac{d}{dt}[0.2t\sin(120\pi t)] + \frac{d}{dt}[\cos(2e^{-\sin t})]\ mA$$
$$= 0.2\sin(120\pi t) + 0.2t * 120\pi\cos(120\pi t) + (-\sin(2e^{-\sin t}))\frac{d}{dt}(2e^{-\sin t})\ mA$$
$$= 0.2\sin(120\pi t) + 75.4t\cos(120\pi t) + 2\cos(t) * e^{-\sin t} * \sin(2e^{-\sin t})\ mA$$

(c) $\mathbf{i(t) = 4e^{-t} - 3e^{-2t}\ mA, q(0) = 0.2\ C}$

$$q(t) = \int_0^t i(t)dt + q(0) = \int_0^t (4e^{-t} - 3e^{-2t})dt + 0.2 * 10^3\ mC$$
$$= \left(-4e^{-t} + \frac{3}{2}e^{-2t}\right)\Big|_0^t + 200\ \mathrm{mC} = -4e^{-t} + \frac{3}{2}e^{-2t} + 202.5\ mC$$

(d) $\mathbf{i(t) = 12e^{-3t}\cos(40\pi t)\ nA, q(0) = 2.28\ pC}$

$$q(t) = \int_0^t i(t)dt + q(0) = \int_0^t 12e^{-3t}\cos(40\pi t)\,dt + 2.28 * 10^{-3}\ nC$$
$$= 12e^{-3t} * \frac{-3\cos(40\pi t) + 40\pi\sin(40\pi t)}{9 + (40\pi)^2}\Big|_0^t + 2.28 * 10^{-3}\ nC$$
$$= 12e^{-3t} * \frac{-3\cos(40\pi t) + 40\pi\sin(40\pi t)}{9 + (40\pi)^2} + 4.56 * 10^{-3}\ \mathrm{nC}$$

2. (14%) Find the current i_1 and i_2 shown in Figure 1.

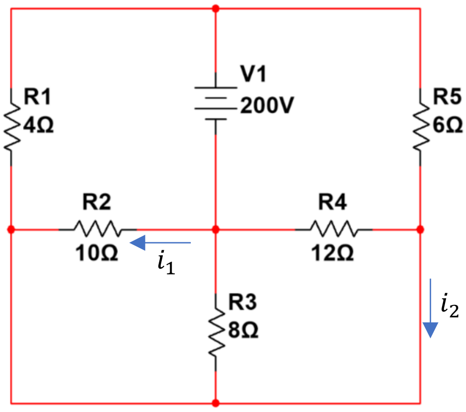
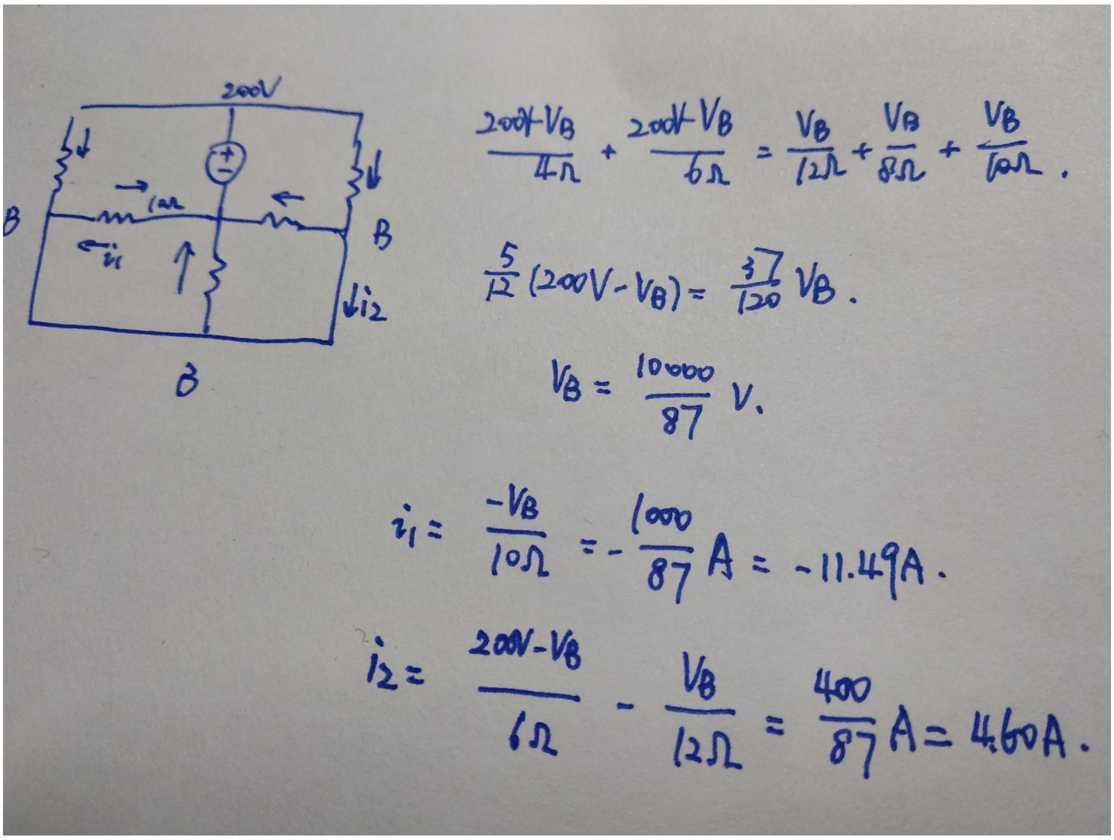


Figure 1



$$\frac{200V - V_B}{4\Omega} + \frac{200V - V_B}{6\Omega} = \frac{V_B}{12\Omega} + \frac{V_B}{8\Omega} + \frac{V_B}{10\Omega}.$$

$$\frac{5}{12}(200V - V_B) = \frac{37}{120} V_B.$$

$$V_B = \frac{10000}{87} V.$$

$$i_1 = \frac{-V_B}{10\Omega} = -\frac{1000}{87} A = -11.49A.$$

$$i_2 = \frac{200V - V_B}{6\Omega} - \frac{V_B}{12\Omega} = \frac{400}{87} A = 4.60A.$$

3. (14%) Find the power absorbed by each of the elements from p_1 to p_5 with the following circuit shown in Figure 2.

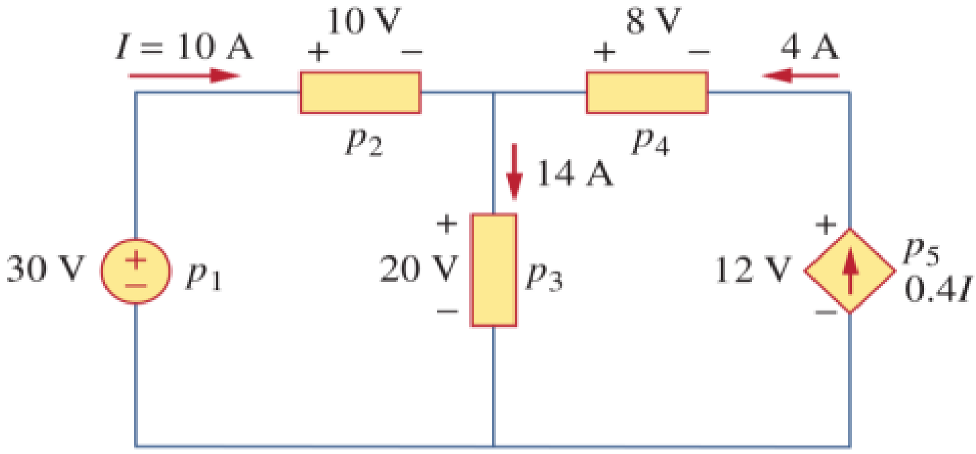


Figure 2

$$p_1 = 30\text{ V} * (-10\text{ A}) = -300\text{ W}$$
$$p_2 = 10\text{ V} * 10\text{ A} = 100\text{ W}$$
$$p_3 = 20\text{ V} * 14\text{ A} = 280\text{ W}$$
$$p_4 = 8\text{ V} * (-4\text{ A}) = -32\text{ W}$$
$$p_5 = 12\text{ V} * (-0.4 * 10\text{ A}) = -48\text{ W}$$

4. (14%) Find i_1, i_6, v $and\ power\ on\ the\ voltage\ source\ of$ $8V$ with the circuit shown in the Figure 3.

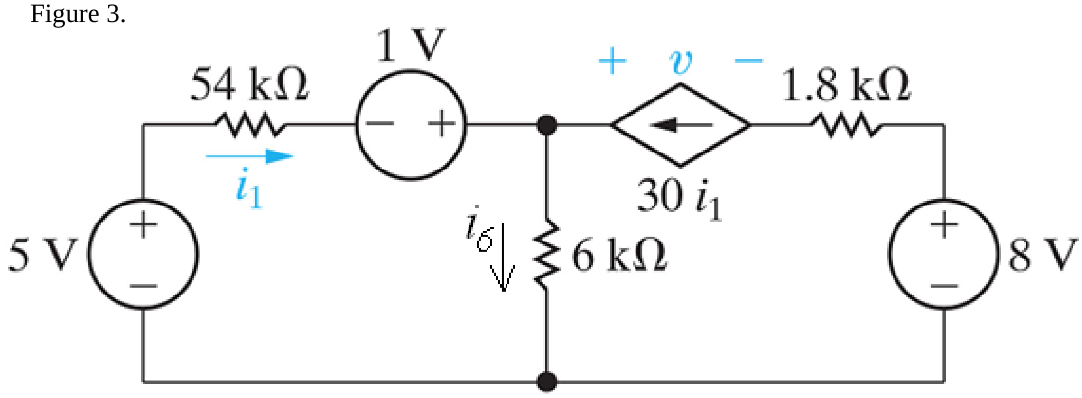


Figure 3

$$\text{KVL left}: -5 + 54{,}000 i_1 - 1 + 6000 i_6 = 0$$

$$\text{KCL top}: i_1 + 30 i_1 = i_6 = 31 i_1$$

$$\Rightarrow \qquad -5 + 54{,}000 i_1 - 1 + 6000(31 i_1) = 0$$

$$\Rightarrow \qquad [54{,}000 + (6000)(31)] i_1 = 6 \qquad \therefore \qquad i_1 = 25\mu\text{A}$$

$$\text{KVL right}: -v + (6000)(31 i_1) - 8 + 1800(30 i_1) = 0$$

$$\Rightarrow \qquad v = (6000)(31)(25\mu) - 8 + (1800)(30)(25\mu) = -2\text{ V}$$

$$i_1 = 25\mu A, i_6 = 31 i_1 = 775\mu A, v = -2\text{ V}$$

$$\text{P}_{8\text{V}} = -8 * 750\mu W = -6\ mW$$

5. (14%) Use nodal analysis to find V_0 in the circuit of Figure 4.

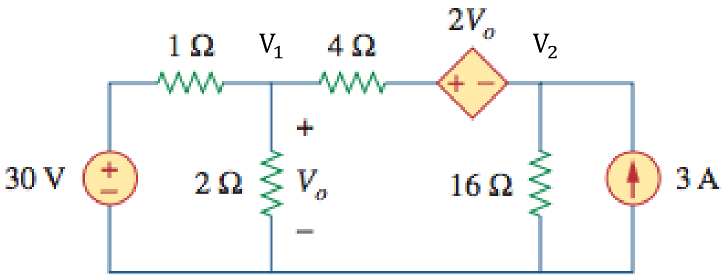


Figure 4

$$\begin{cases} \text{Node 1: } \dfrac{30 - V_1}{1} + \dfrac{V_2 + 2V_0 - V_1}{4} + \dfrac{0 - V_1}{2} = 0 \\ \text{Node 2: } \quad 3 + \dfrac{0 - V_2}{16} + \dfrac{V_1 - (V_2 + 2V_0)}{4} = 0 \\ V_1 = V_0 \end{cases}$$

$$=> V_0 = 22.34\,V$$

6. (14%) Apply mesh analysis to find I_X in the circuit of Figure 6.

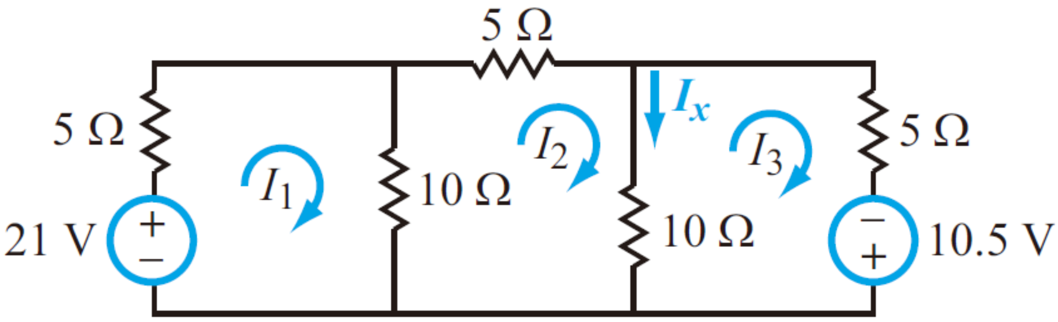


Figure 6

Solution:

Mesh 1: $-21+5I_1+10(I_1-I_2)=0$

Mesh 2: $10(I_2-I_1)+5I_2+10(I_2-I_3)=0$

Mesh 3: $10(I_3-I_2)+5I_3-10.5=0$

Solution is:

$$I_1=\frac{13}{5}\text{ A},\qquad I_2=\frac{9}{5}\text{ A},\qquad I_3=\frac{19}{10}\text{ A},$$

and

$$I_x=I_2-I_3=\frac{9}{5}-\frac{19}{10}=-\frac{1}{10}=-0.1\text{ A}.$$

7. (14%) Determine **A** if $V_{out}/V_S = 9$ in the circuit of Figure 7.

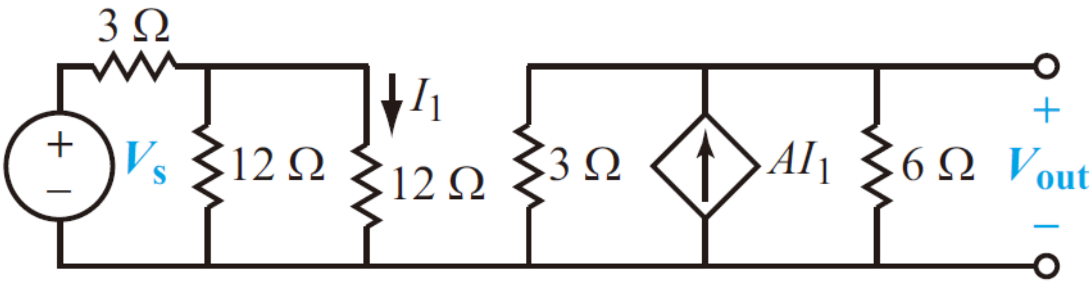
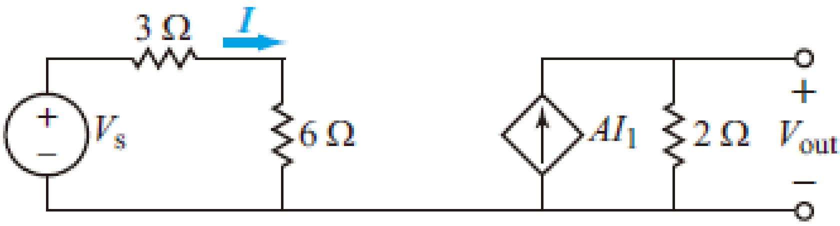


Figure 7



$$I = \frac{V_s}{9}$$

$$I_1 = \frac{I}{2} = \frac{V_s}{18}$$

$$V_{out} = AI_1 \times 2 = \frac{AV_s}{18} \times 2 = \frac{AV_s}{9}$$

$$\frac{V_{out}}{V_s} = \frac{A}{9} = 9.$$

Hence

$$A = 81.$$