CS290: Introduction to Algorithmic Game Theory

Week 6.2, Cake Cutting (Dengji ZHAO)

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Recap: The General Setting of Mechanism Design

- A set of n participants/players, denoted by N.
- A mechanism needs to choose some alternative from A (allocation space), and to decide a payment for each player.
- Each player $i \in N$ has a private valuation function $v_i : A \to \mathbb{R}$, let V_i denote all possible valuation functions for i.
- Let $v = (v_1, \cdots, v_n)$, $v_{-i} = (v_1, \cdots, v_{i-1}, v_{i+1}, \cdots, v_n)$.
- Let $V = V_1 \times \cdots \times V_n$, $V_{-i} = V_1 \times \cdots V_{i-1} \times V_{i+1} \times \cdots \times V_n$.

Recap: Social Choice

- A set of n players/voters.
- A set of alternatives A (the candidates).
- Let L be the set of all linear orders on A.
- Each voter i has a preference $\succ_i \in L$, a total order on A (antisymmetric, transitive). $a \succ_i b$ means i prefers a to b.

Cake Cutting

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Cardinal Preferences

- A divisible resource C, say a cake.
- A set of n players to share/divide.
- Each player has valuation function v_i, which gives a value for each subset of C.

Question

How to divide the resource fairly?

Fairness

Proportionality Each player receives a piece that he values as at least $1/n$ of the value of the entire cake.

Envy-freeness Each player receives a piece that he values at least as much as every other piece.

A Cake Cutting Procedure: Divide and Choose

- Two person share one cake.
- One person (the cutter) cuts the cake into two pieces.
- The other person chooses one (the chooser).

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Does it satisfy *proportionality*?

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Does it satisfy *proportionality*? Yes!

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Quiz

Does it satisfy *envy-freeness*?

A Cake Cutting Procedure: Last Diminisher

Question

How to extend Divide and Choose to more than two person settings?

- The players being ranged A, B, C, ... N.
- A cuts from the cake an arbitrary part.
- B has now the right, but is not obliged, to diminish the slice cut off.
- Whatever B does, C has the right (without obligation) to diminish still the already diminished (or not diminished) slice, and so on up to N.
- The rule obliges the "last diminisher" to take as his part the slice he was the last to touch.

A Cake Cutting Procedure: Last Diminisher

Quiz

- Does Last Diminisher satisfy *proportionality*?
- Does Last Diminisher satisfy *envy-freeness*?

Advanced Reading

- *Computational Social Choice* by F. Brandt, V. Conitzer and U. Endriss