Lecture 5 – Image Segmentation （图像分割）

This lecture will cover:

– Morphological Image Processing （形态学图像处理）

- Morphological operation
- **Morphological algorithm**

– Image Segmentation（图像分割）

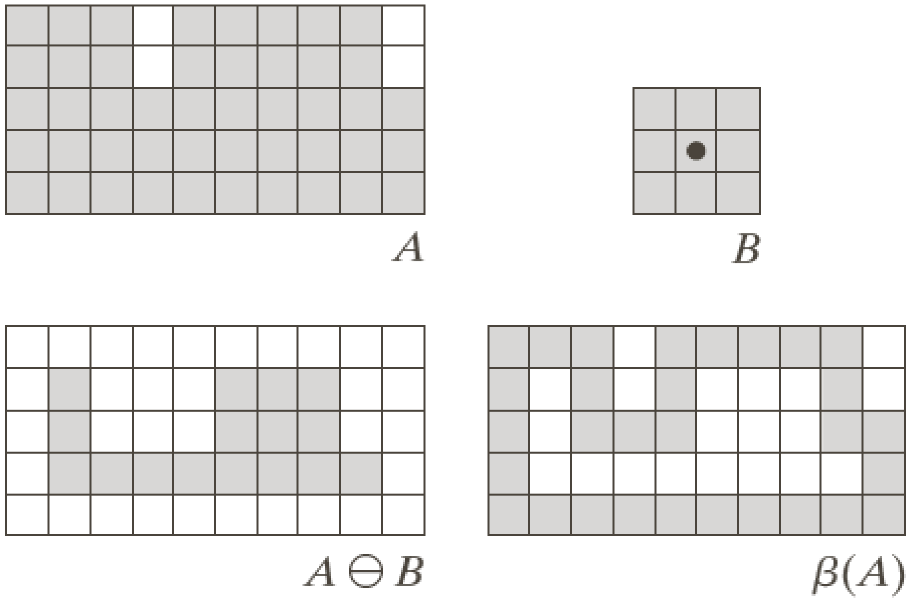
- Point, Line and Edge Detection（点、线和边缘检测）
- Thresholding（阈值处理）
- Segmentation using Morphological Watersheds（形态学分水岭分割）

Basic Morphologic Algorithms

- Boundary Extraction (边界提取)
- Hole Filling (孔洞填充)
- Extraction of Connected components (连通分量提取)
- Convex Hull (凸壳)
- Thinning (细化)
- Thickening (粗化)
- Skeleton (骨架)
- Pruning (裁剪)

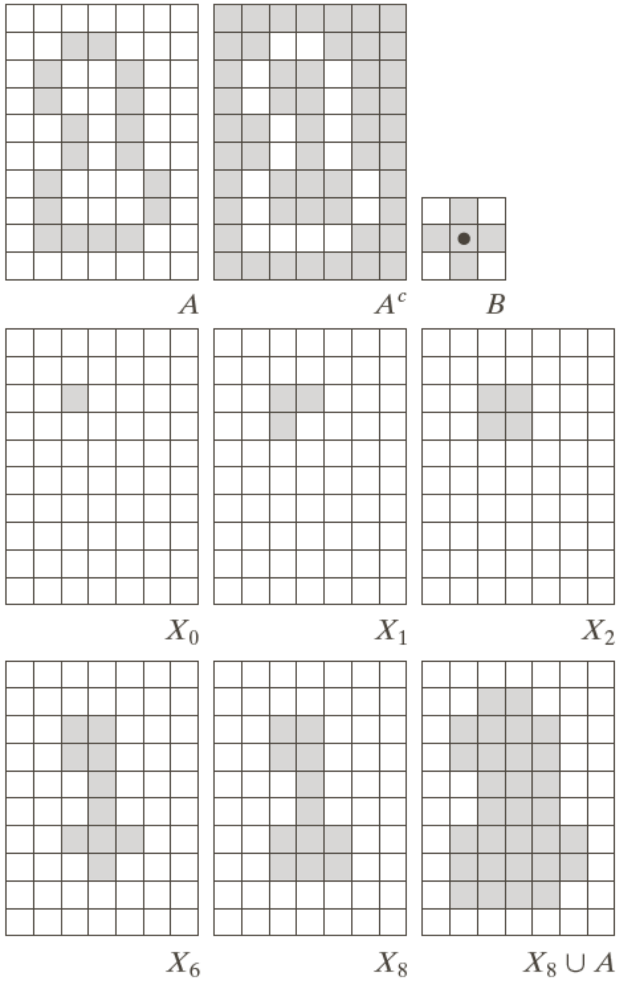
Boundary Extraction (边界提取)

Morphological algorithm: $\beta(A) = A - (A \ominus B)$



Hole Filling (孔洞填充)

Morphological algorithm: $X_k = (X_{k-1} \oplus B) \cap A^c$



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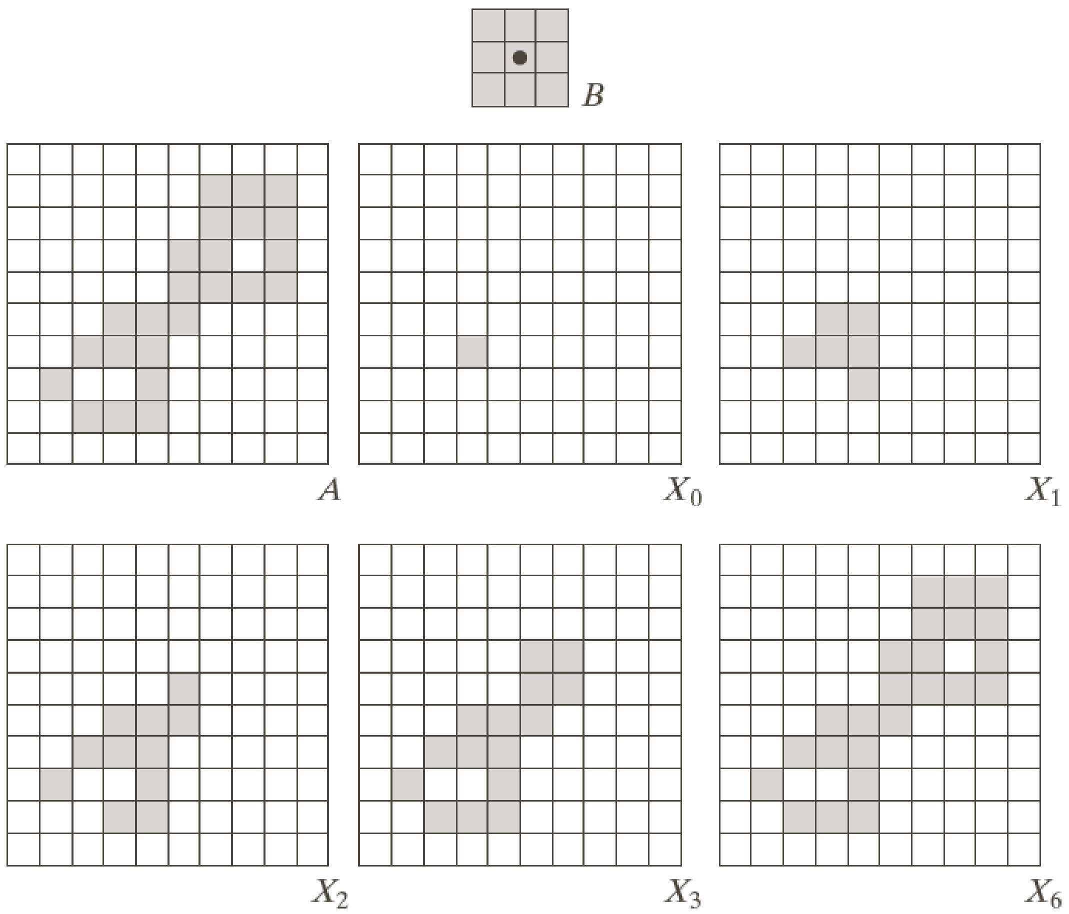
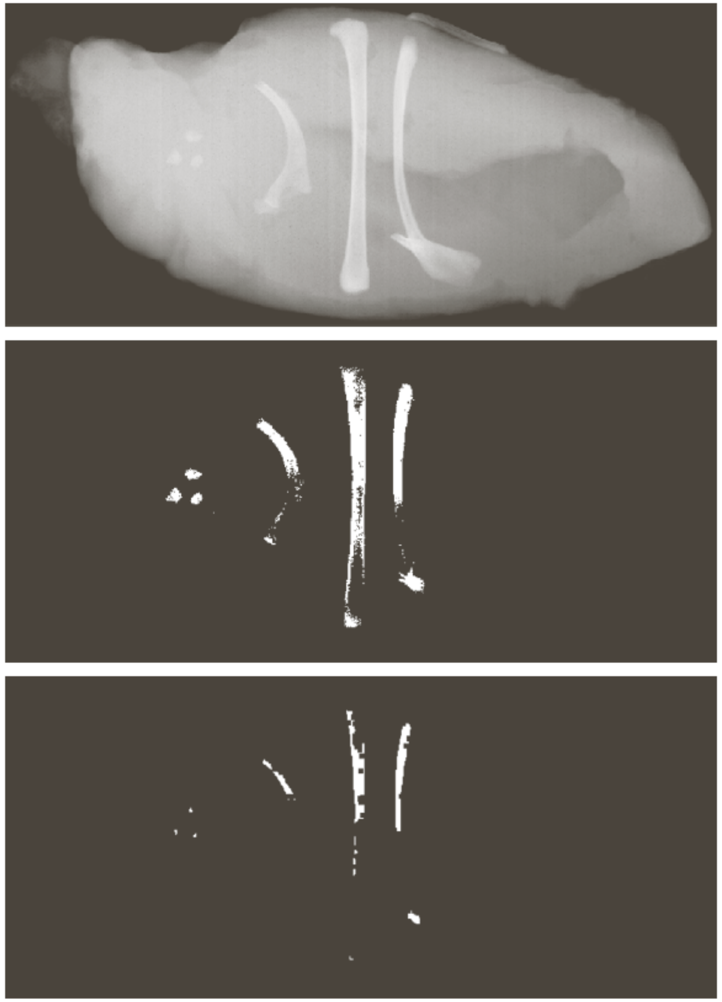
Extraction of Connected components (连通分量提取)

Morphological algorithm: $X_k = (X_{k-1} \oplus B) \cap A$



Extraction of Connected components



Connected component	No. of pixels in connected comp
01	11
02	9
03	9
04	39
05	133
06	1
07	1
08	743
09	7
10	11
11	11
12	9
13	9
14	674
15	85

Convex Hull (凸壳)

Morphological algorithm:

$$C(A) = \bigcup_{i=1}^{4} D^i$$

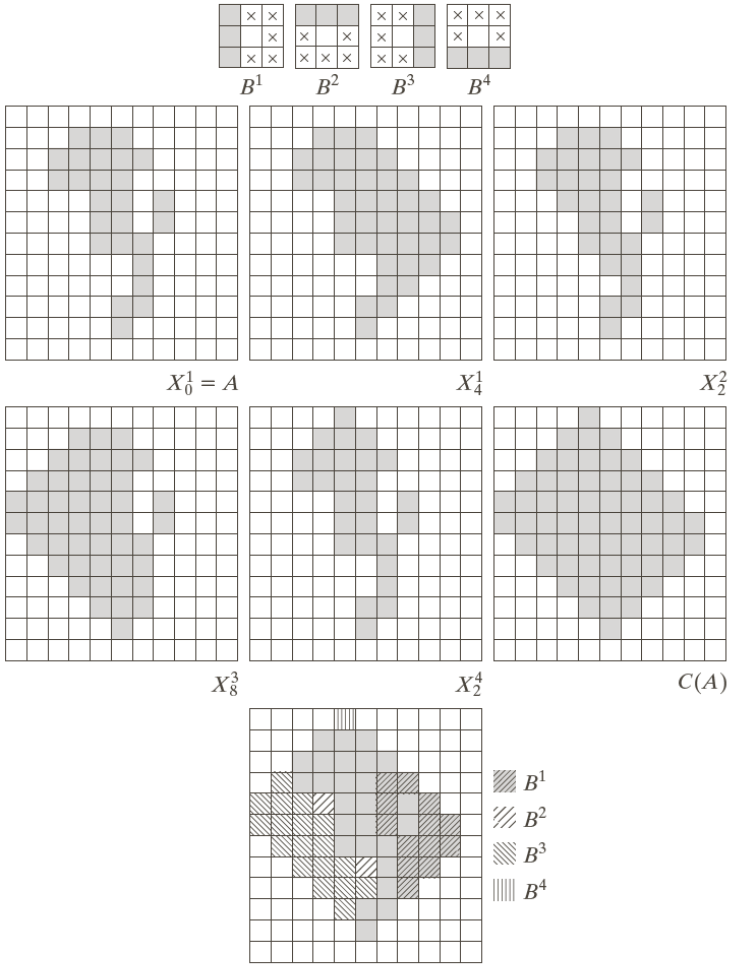
Where

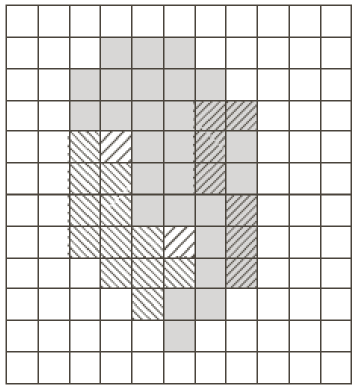
B^i: structuring elements

$X_0^i = A$

$X_k^i = \left(X_{k-1}^i \circledast B^i\right) \cup A$

$D^i = X_k^i$ when $X_k^i = X_{k-1}^i$





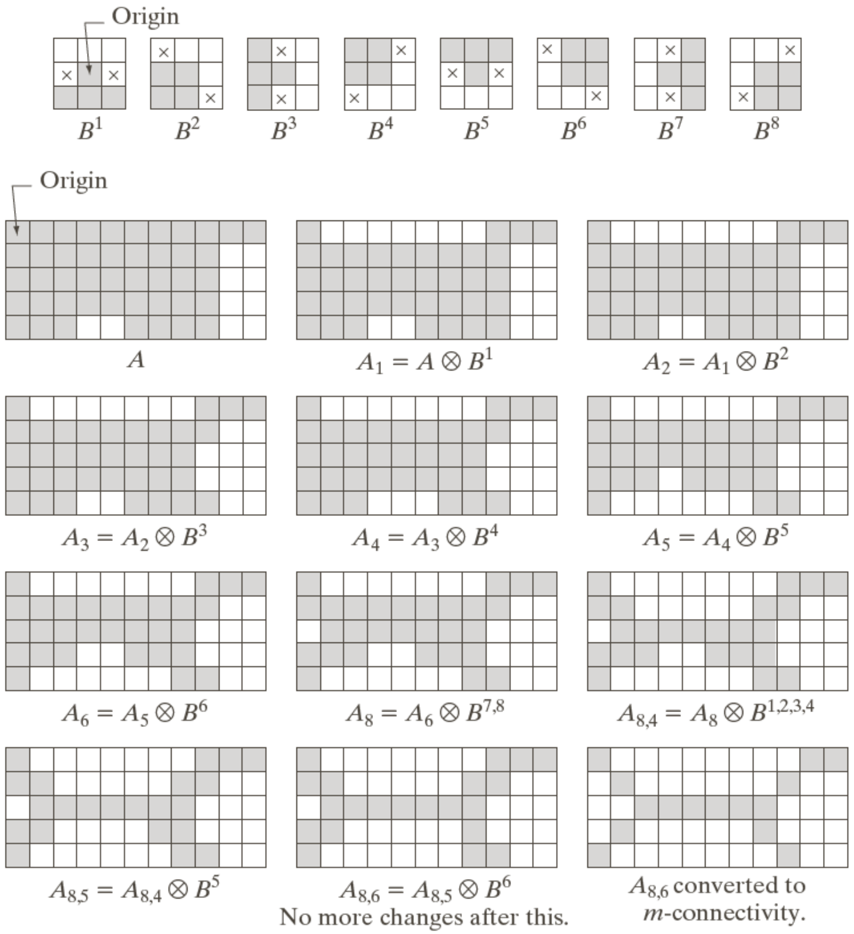
Thinning (细化)

Morphological algorithm:

$$A \otimes B = A - (A \circledast B)$$
$$= A \cap (A \circledast B)^c$$

Let $B = \{B^1, B^2, B^3 \cdots, B^n\}$

$$A \otimes \{B\} = ((\cdots((A \otimes B^1) \otimes B^2) \cdots) \otimes B^n)$$



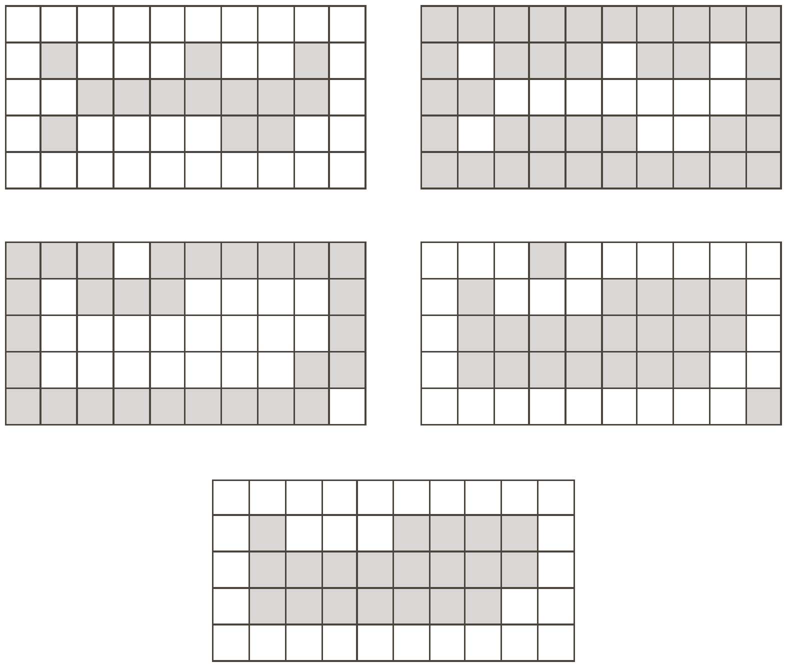
Thickening (粗化)

Morphological algorithm:

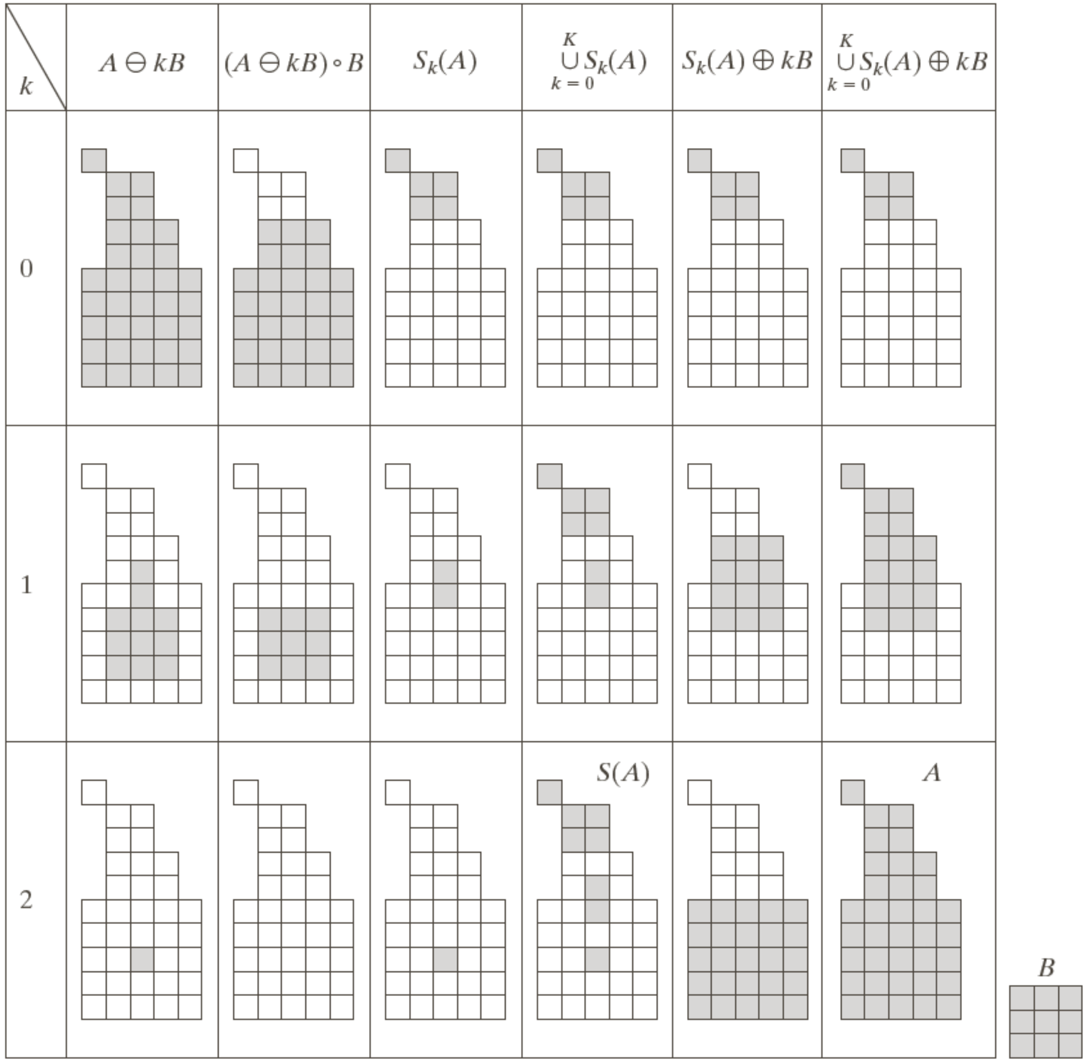
$$A \odot B = A \cup (A \circledast B)$$

Let $B = \{B^1, B^2, B^3 \cdots, B^n\}$

$$A \odot \{B\} = ((\cdots((A \odot B^1) \odot B^2) \cdots) \odot B^n)$$



Skeleton (骨架)



Morphological algorithm:

$$S(A) = \bigcup_{k=0}^{K} S_k(A)$$

Where

$$S_k(A) = (A \ominus kB) - (A \ominus kB) \circ B$$

$$(A \ominus kB) = ((\cdots((A \ominus B) \ominus B)\cdots) \ominus B)$$

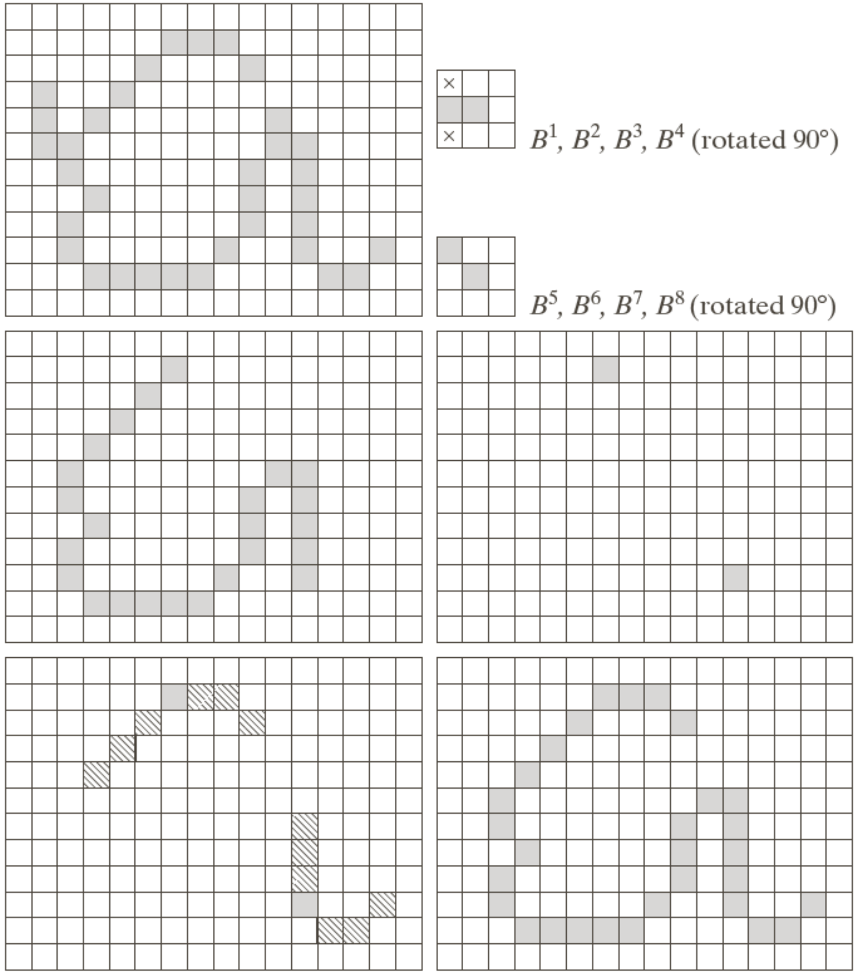
$$K = \max\{k | A \ominus kB \neq \emptyset\}$$

A can be reconstructed by

$$A = \bigcup_{k=0}^{K} (S_k(A) \oplus kB)$$

Where $(A \oplus kB) = ((\cdots((A \oplus B) \oplus B)\cdots) \oplus B)$

Pruning (裁剪)



Morphological algorithm:

1. Thinning: $X_1 = A \otimes \{B\}$
2. Finding endpoints: $X_2 = \bigcup_{k=1}^{8}(X_1 \circledast B^k)$
3. Dilating from endpoints: $X_3 = (X_2 \oplus H) \cap A$
4. $P = X_3 \cup X_1$

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Function for Morphological Algorithms

Operation	Description
`bothat`	"Bottom-hat" operation using a 3×3 structuring element; use `imbothat` (see Section 9.6.2) for other structuring elements.
`bridge`	Connect pixels separated by single-pixel gaps.
`clean`	Remove isolated foreground pixels.
`close`	Closing using a 3×3 structuring element of 1s; use `imclose` for other structuring elements.
`diag`	Fill in around diagonally-connected foreground pixels.
`dilate`	Dilation using a 3×3 structuring element of 1s; use `imdilate` for other structuring elements.
`erode`	Erosion using a 3×3 structuring element of 1s; use `imerode` for other structuring elements.
`fill`	Fill in single-pixel "holes" (background pixels surrounded by foreground pixels); use `imfill` (see Section 10.1.2) to fill in larger holes.
`hbreak`	Remove H-connected foreground pixels.
`majority`	Make pixel p a foreground pixel if at least five pixels in $N_8(p)$ (see Section 9.4) are foreground pixels; otherwise make p a background pixel.
`open`	Opening using a 3×3 structuring element of 1s; use function `imopen` for other structuring elements.
`remove`	Remove "interior" pixels (foreground pixels that have no background neighbors).
`shrink`	Shrink objects with no holes to points; shrink objects with holes to rings.
`skel`	Skeletonize an image.
`spur`	Remove spur pixels.
`thicken`	Thicken objects without joining disconnected 1s.
`thin`	Thin objects without holes to minimally-connected strokes; thin objects with holes to rings.
`tophat`	"Top-hat" operation using a 3×3 structuring element of 1s; use `imtophat` (see Section 9.6.2) for other structuring elements.

- **Matlab Function:**

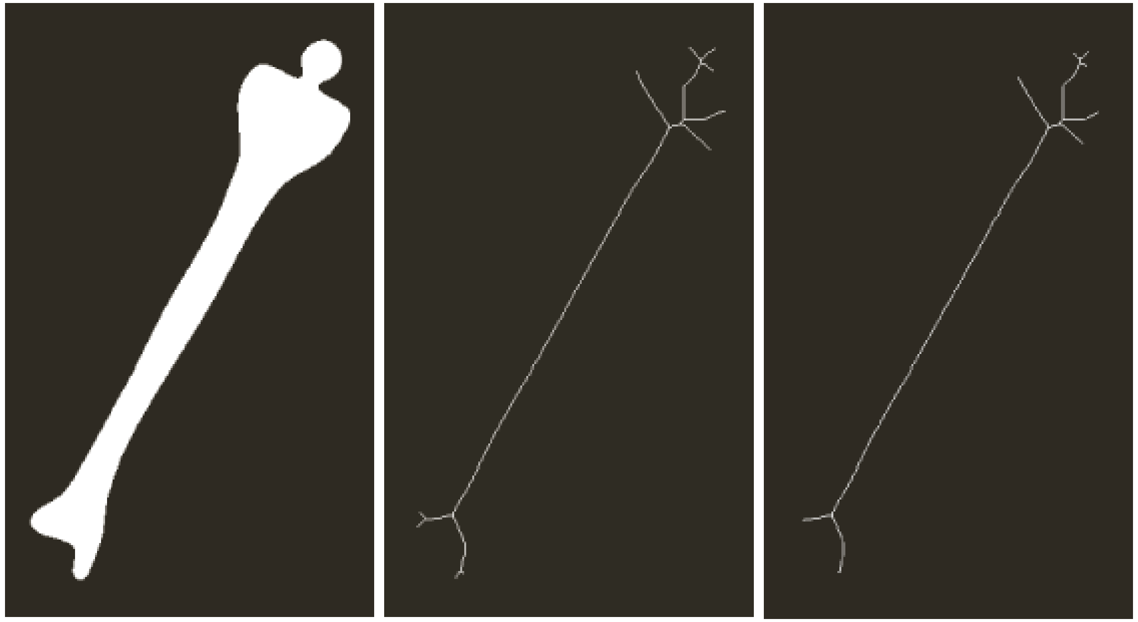
BW2 = bwmorph(BW,operation,n)

Example (Thinning)

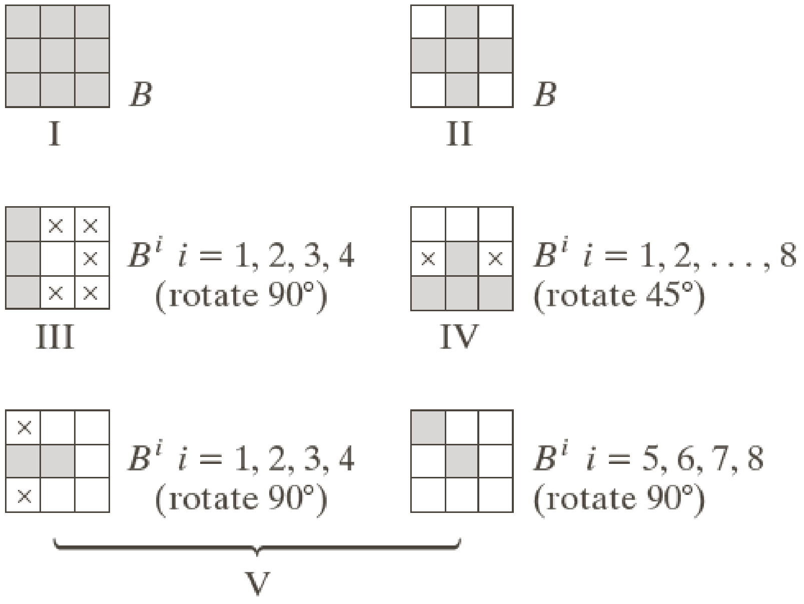
```
>> g1 = bwmorph(f, 'thin', 1);
>> g2 = bwmorph(f, 'thin', 2);
>> ginf = bwmorph(f, 'thin', Inf);
```

Example (Skeleton)

```
>> fs = bwmorph(f, 'skel', Inf);
```



Summary



B
I
B
II
B^i $i = 1, 2, 3, 4$
(rotate 90°)
III
B^i $i = 1, 2, \ldots, 8$
(rotate 45°)
IV
B^i $i = 1, 2, 3, 4$
(rotate 90°)
B^i $i = 5, 6, 7, 8$
(rotate 90°)
V

Summary

Operation	Equation	Comments (The Roman numerals refer to the structuring elements in Fig. 9.33.)
Translation	$(B)_z = \{w \mid w = b + z, \text{ for } b \in B\}$	Translates the origin of B to point z.
Reflection	$\hat{B} = \{w \mid w = -b, \quad \text{for } b \in B\}$	Reflects all elements of B about the origin of this set.
Complement	$A^c = \{w \mid w \notin A\}$	Set of points not in A.
Difference	$A - B = \{w \mid w \in A, w \notin B\} = A \cap B^c$	Set of points that belong to A but not to B.
Dilation	$A \oplus B = \left\{z \mid (\hat{B}_z) \cap A \neq \varnothing\right\}$	"Expands" the boundary of A. (I)
Erosion	$A \ominus B = \left\{z \mid (B)_z \subseteq A\right\}$	"Contracts" the boundary of A. (I)
Opening	$A \circ B = (A \ominus B) \oplus B$	Smoothes contours, breaks narrow isthmuses, and eliminates small islands and sharp peaks. (I)

(*Continued*)

Operation	Equation	Comments (The Roman numerals refer to the structuring elements in Fig. 9.33.)
Closing	$A \bullet B = (A \oplus B) \ominus B$	Smoothes contours, fuses narrow breaks and long thin gulfs, and eliminates small holes. (I)
Hit-or-miss transform	$A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2)$ $= (A \ominus B_1) - (A \oplus \hat{B}_2)$	The set of points (coordinates) at which, simultaneously, B_1 found a match ("hit") in A and B_2 found a match in A^c
Boundary extraction	$\beta(A) = A - (A \ominus B)$	Set of points on the boundary of set A. (I)
Hole filling	$X_k = (X_{k-1} \oplus B) \cap A^c$; $k = 1, 2, 3, \ldots$	Fills holes in A; X_0 = array of 0s with a 1 in each hole. (II)
Connected components	$X_k = (X_{k-1} \oplus B) \cap A$; $k = 1, 2, 3, \ldots$	Finds connected components in A; X_0 = array of 0s with a 1 in each connected component. (I)
Convex hull	$X_k^i = (X_{k-1}^i \circledast B^i) \cup A$; $i = 1, 2, 3, 4$; $k = 1, 2, 3, \ldots$; $X_0^i = A$; and $D^i = X^i_{\text{conv}}$	Finds the convex hull $C(A)$ of set A, where "conv" indicates convergence in the sense that $X_k^i = X_{k-1}^i$. (III)
Thinning	$A \otimes B = A - (A \circledast B)$ $= A \cap (A \circledast B)^c$ $A \otimes \{B\} =$ $((\ldots((A \otimes B^1) \otimes B^2) \ldots) \otimes B^n)$ $\{B\} = \{B^1, B^2, B^3, \ldots, B^n\}$	Thins set A. The first two equations give the basic definition of thinning. The last equations denote thinning by a sequence of structuring elements. This method is normally used in practice. (IV)
Thickening	$A \odot B = A \cup (A \circledast B)$ $A \odot \{B\} =$ $((\ldots(A \odot B^1) \odot B^2 \ldots) \odot B^n)$	Thickens set A. (See preceding comments on sequences of structuring elements.) Uses IV with 0s and 1s reversed.
Skeletons	$S(A) = \bigcup_{k=0}^{K} S_k(A)$ $S_k(A) = \bigcup_{k=0}^{K} \{(A \ominus kB) - [(A \ominus kB) \circ B]\}$ Reconstruction of A: $A = \bigcup_{k=0}^{K} (S_k(A) \oplus kB)$	Finds the skeleton $S(A)$ of set A. The last equation indicates that A can be reconstructed from its skeleton subsets $S_k(A)$. In all three equations, K is the value of the iterative step after which the set A erodes to the empty set. The notation $(A \ominus kB)$ denotes the kth iteration of successive erosions of A by B. (I)

(Continued)

Operation	Equation	Comments (The Roman numerals refer to the structuring elements in Fig. 9.33.)
Pruning	$X_1 = A \otimes \{B\}$ $X_2 = \bigcup_{k=1}^{8} (X_1 \circledast B^k)$ $X_3 = (X_2 \oplus H) \cap A$ $X_4 = X_1 \cup X_3$	X_4 is the result of pruning set A. The number of times that the first equation is applied to obtain X_1 must be specified. Structuring elements V are used for the first two equations. In the third equation H denotes structuring element I.
Geodesic dilation of size 1	$D_G^{(1)}(F) = (F \oplus B) \cap G$	F and G are called the *marker* and *mask* images, respectively.
Geodesic dilation of size n	$D_G^{(n)}(F) = D_G^{(1)}[D_G^{(n-1)}(F)]$; $D_G^{(0)}(F) = F$	
Geodesic erosion of size 1	$E_G^{(1)}(F) = (F \ominus B) \cup G$	
Geodesic erosion of size n	$E_G^{(n)}(F) = E_G^{(1)}[E_G^{(n-1)}(F)]$; $E_G^{(0)}(F) = F$	
Morphological reconstruction by dilation	$R_G^D(F) = D_G^{(k)}(F)$	k is such that $D_G^{(k)}(F) = D_G^{(k+1)}(F)$
Morphological reconstruction by erosion	$R_G^E(F) = E_G^{(k)}(F)$	k is such that $E_G^{(k)}(F) = E_G^{(k+1)}(F)$
Opening by reconstruction	$O_R^{(n)}(F) = R_F^D[(F \ominus nB)]$	$(F \ominus nB)$ indicates n erosions of F by B.
Closing by reconstruction	$C_R^{(n)}(F) = R_F^E[(F \oplus nB)]$	$(F \oplus nB)$ indicates n dilations of F by B.
Hole filling	$H = \left[R_{I^c}^D(F)\right]^c$	H is equal to the input image I, but with all holes filled. See Eq. (9.5-28) for the definition of the marker image F.
Border clearing	$X = I - R_I^D(F)$	X is equal to the input image I, but with all objects that touch (are connected to) the boundary removed. See Eq. (9.5-30) for the definition of the marker image F.