EE150 Signals and Systems
– Part 1: Overview

$\downarrow$ Week 1, Tue, 20180227

Introduction to Signals

Signal:

a function of one or more independent variables; typically contains information about the behaviour or nature of some physical phenomena.

e.g. Voice (audio), TV (audio + video), light, voltage, current, stock price, etc.

Introductions to Systems

System:

responds to a particular signal input by producing another signal (output).

e.g. Biological sensory system, electronic circuits, automobile, etc.

$$\text{Input signal} \longrightarrow \boxed{\text{System}} \longrightarrow \text{Output signal}$$

Objectives

System characterization

how it responds to input signal
(e.g. human auditory system)

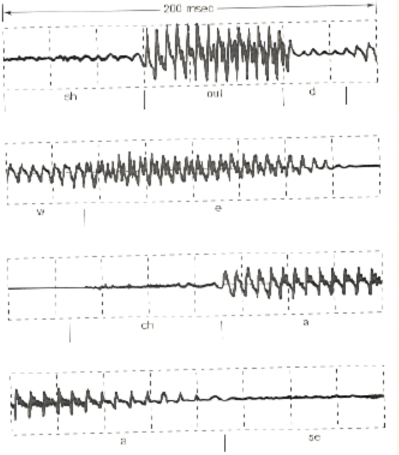
System design

to process signal in a particular way
(e.g. signal restoration, signal identification, image processing)

Examples of Signals

Audio (intensity vs. time)

characteristics: volume, rhythm, pitch

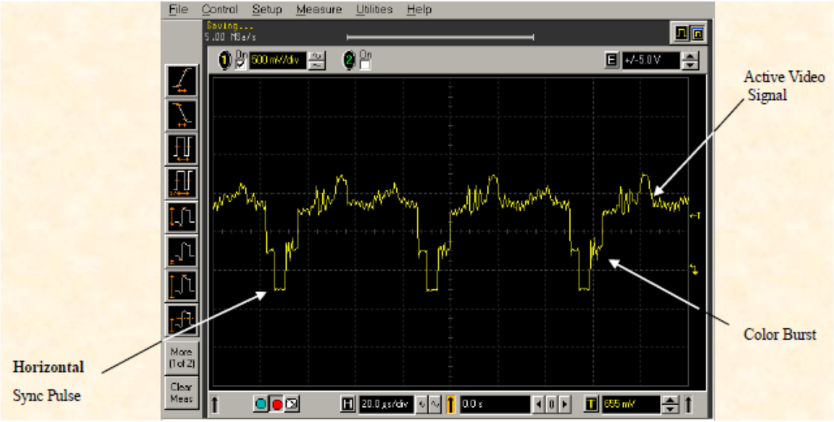


Examples of Signals cont.

TV signal (voltage vs. time)

modulated picture signal + audio signal; carrier signal;

system involves: antenna, tuner, CRT



Examples of Signals cont.

Biomedical signal (voltage vs. time)
  e.g. Electrocardiogram

Traffic flow (quantity vs. time)
  volume, composition, pattern, mobility;
  system involved: traffic lights, roads, junctions

Network throughput
  # of packets/sec., pattern

Examples of Signals cont.

Stock price (index vs. time; $ vs. time)

Examples of Signals cont.

Picture (intensity vs. space)

- on film – continuous space, continuous tone
- on newspaper – half-tone
- on computer – discrete, quantized

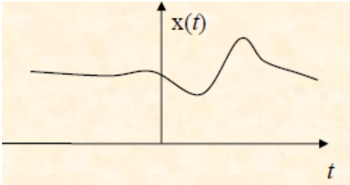
Image (measurement over space)

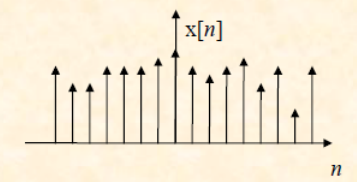
- medical images – X-ray, CT, ultrasound, MRI
- satellite images – weather map
- seismic image – earthquake

Continuous vs. Discrete Signal

Continuous-time Signal
(independent variable: t)

Discrete-time Signal
(independent variable: n)



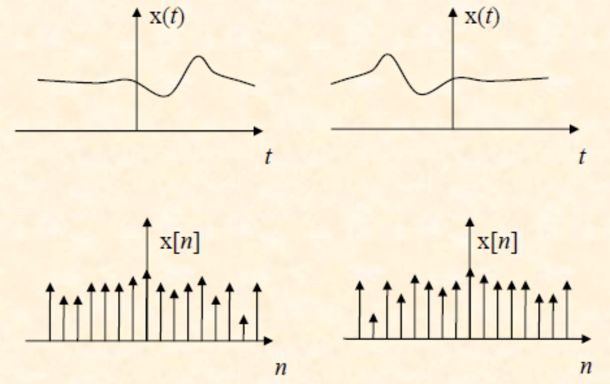


$x[n]$ is also referred as a sequence. Any particular one in $x[n]$ is called a sample.

Transformation of independent variable

(1). Reflection

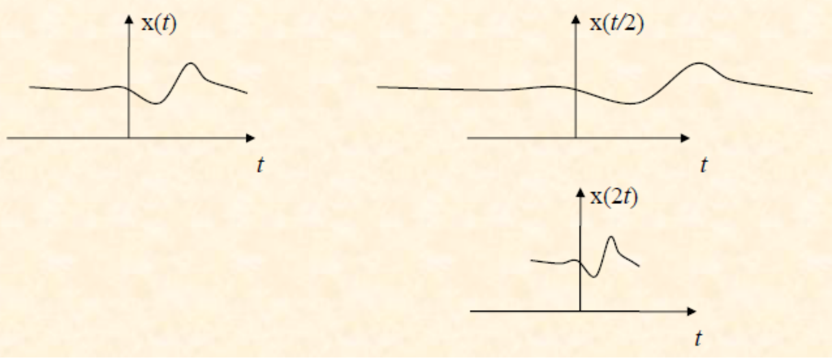
$$x(t) \longleftrightarrow x(-t)$$
$$x[n] \longleftrightarrow x[-n]$$



Transformation of independent variable cont.

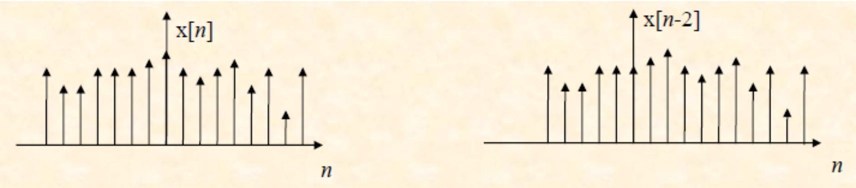
(2). Scaling

$$x(t) \longleftrightarrow x(ct)$$



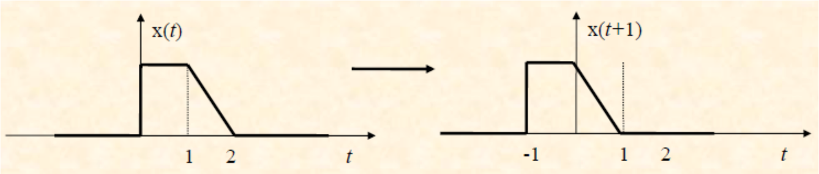
(3). Time-shift

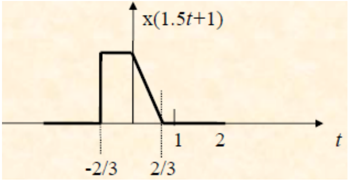
$$x(t) \longleftrightarrow x(t - t_0)$$
$$x[n] \longleftrightarrow x[n - n_0]$$



Transformation of independent variable cont.

To perform transformation $x(t) \to x(\alpha t + \beta)$,
You have to do time-shifting then scaling.





Why this order?

$y(t) = x(1.5t + 1)$
Work out a few points:
$y(0) = x(1)$
$y(1) = x(2.5)$
$y(2) = x(4)$

To get from y to x, we first scale t, then shift.

Therefore, to get from x to y, we first shift and then scale.

Q: What if we first scale and then shift?

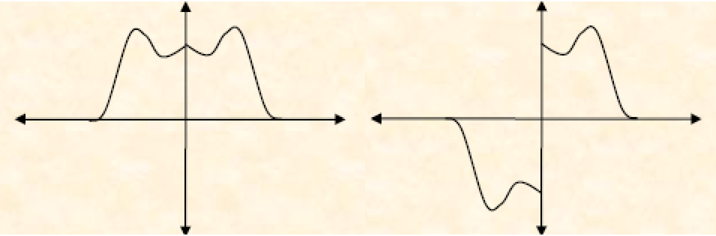
Even and Odd functions

A signal is called an even signal (function) if

$$x(t) = x(-t), \quad x[n] = x[-n].$$

A signal is called an odd signal (function) if

$$x(t) = -x(-t), \quad x[n] = -x[-n].$$



Even and Odd functions cont.

Any signal can be broken into sum of one even and one odd signal.

$$x(t) = Evenx(t) + Oddx(t)$$

How?

Even and Odd functions cont.

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How?

$$\begin{aligned} x(t) &= Evenx(t) + Oddx(t), \\ x(-t) &= Evenx(-t) + Oddx(-t) = Evenx(t) - Oddx(t), \end{aligned}$$

Even and Odd functions cont.

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How?

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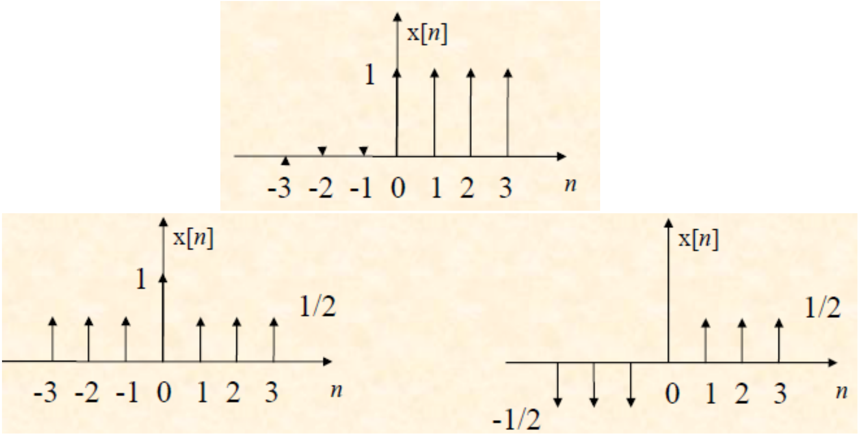
$$\begin{aligned} \implies Evenx(t) &= \frac{1}{2}(x(t) + x(-t)), \\ Oddx(t) &= \frac{1}{2}(x(t) - x(-t)). \end{aligned}$$

Even and Odd functions cont.

Example:



Periodic and Aperiodic Signal

IF $x(t)$ is periodic with period T, then
$x(t) = x(t + mT)$; m any integer

IF $x[n]$ is periodic with period N, then
$x[n] = x[n + mN]$; m any integer

Fundamental period (T_0 or N_0):
the <u>smallest positive</u> value of (T or N) for which the above equation holds.

Aperiodic is also called Non-periodic

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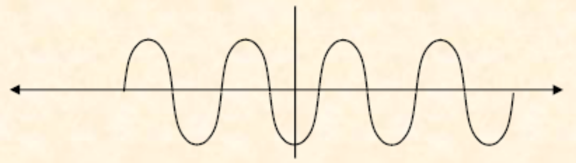
Periodic and Aperiodic Signal cont.

Q: What is the fundamental period of
(1) a constant function
(2) Dirichlet function (defined on $\mathbb{R}$)

Real exponential: $x(t) = ce^{at}$

positive a: negative a:

Imaginary exponential: $x(t) = e^{j(\omega_0 t + \Phi)}$



Periodic and Sinusoidal Signal

Euler's Formula:

$$e^{j\omega_0 t} = \cos(\omega_0 t) + j \cdot \sin(\omega_0 t)$$

Is $x(t) = e^{j\omega_0 t}$ periodic? Suppose the period is T, then

$$x(t) = x(t + T), \implies e^{j\omega_0 t} = e^{j\omega_0 (t+T)}.$$

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Hence $e^{j\omega_0 T} = 1$.
Fundamental period:

$$T_0 = 2\pi/|\omega_0|.$$

Periodic and Sinusoidal Signal cont.

Sinusoidal signal: $x(t) = A\cos(\omega_0 t + \phi)$

unit ω_0: radians/sec; ϕ: radians

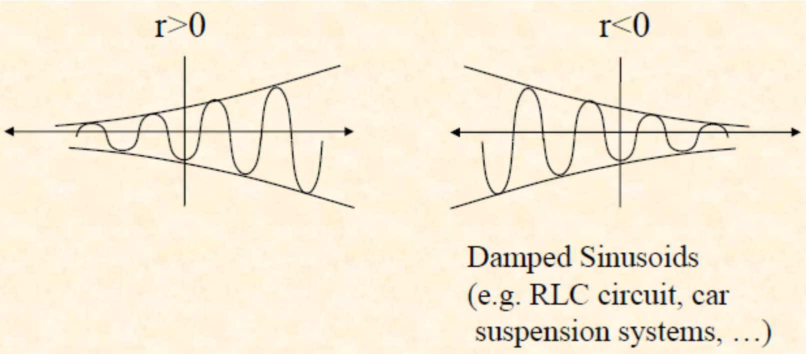
phase: $\omega_0 t + \phi$

General Complex Exponential

$$x(t) = c \cdot e^{(r+j\omega_0)t},$$

$$c = |c|e^{j\theta}, \quad r, \omega_0 \in \mathbb{R}$$

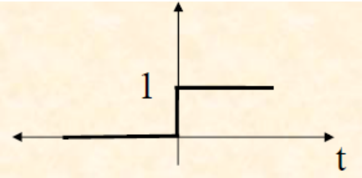
$$\implies x(t) = |c|e^{rt}e^{j(\omega_0 t+\theta)}$$

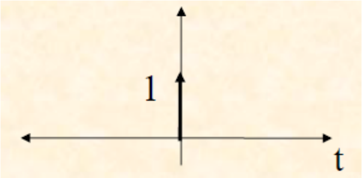


Unit Step and Unit Impulse function

Unit Step function $u(t)$, and Unit Impulse function $\delta(t)$:

$$u(t) = \begin{cases} 0, & t < 0 \\ 1, & t > 0 \end{cases}, \qquad \delta(t) = \frac{d}{dt}u(t).$$





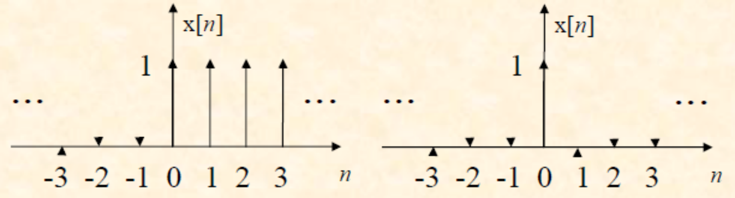
$$u(t) = \int_{-\infty}^{t} \delta(s)ds.$$

A system can be characterized by its unit step response or unit impulse response.

Discrete Time Unit Step and Unit Impulse Sequence

Unit Step function $u[n]$, and Unit Impulse function $\delta[n]$:

$$u[n] = \begin{cases} 0, & n < 0 \\ 1, & n \geq 0 \end{cases}, \qquad \delta[n] = \begin{cases} 0, & n \neq 0 \\ 1, & n = 0 \end{cases}$$



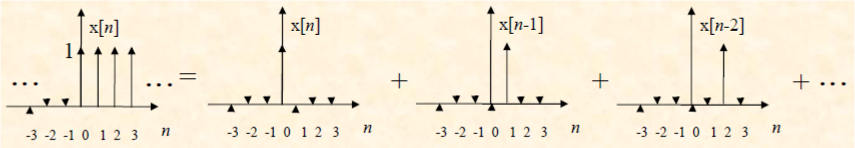
Note: $u[n]$ at $n = 0$ is defined.

Discrete Time Unit Step and Unit Impulse Sequence cont.

$\delta[n] = u[n] - u[n-1]$: first difference of unit step

running sum of unit sample:

$$u[n] = \sum_{m=-\infty}^{n} \delta[m] = \sum_{k=0}^{+\infty} \delta[n-k]$$



Periodic Properties of Discrete-time Complex Exponential

For continuous-time complex exponential $x(t) = e^{j\omega_0 t}$

(1) the larger the ω_0, the higher the rate of oscillation

(2) $e^{j\omega_0 t}$ is periodic for any value of ω_0

Are the above two statements still valid for the discrete case $x[n] = e^{j\Omega_0 n}$?

(1) oscillation

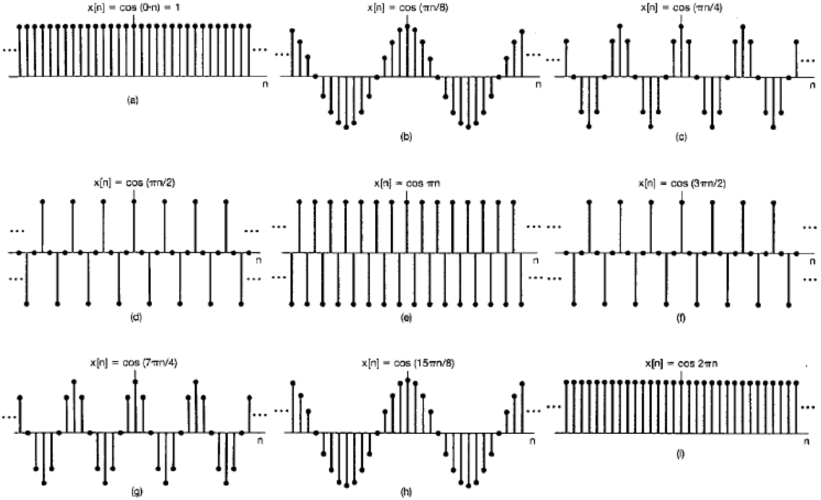


Figure 1.27 Discrete-time sinusoidal sequences for several different frequencies.

Periodic Properties of Discrete-time Complex Exponential

(1) For Ω_0 within an interval $0 \leq \Omega_0 \leq 2\pi$,
the frequency $\uparrow$ as $\Omega_0 \uparrow$ for $0 \leq \Omega_0 \leq \pi$,
the frequency $\downarrow$ as $\Omega_0 \uparrow$ for $\pi \leq \Omega_0 \leq \pi$.

(2) $e^{j\Omega_0 n}$ might be non-periodic:

$$e^{j\Omega_0(n+N)} = e^{j\Omega_0 n}$$

implies $e^{j\Omega_0 N} = 1$, hence $\Omega_0 N$ needs to be a multiple of 2π:

$$\frac{\Omega_0}{2\pi} = \frac{m}{N} \quad \text{is rational.}$$

Overview of System

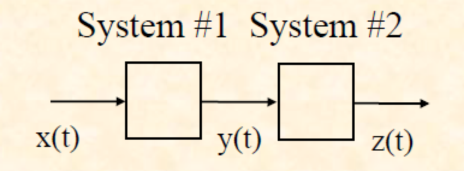
System: any process that results in the transformation of signal

A system is continuous-time if both input and output are continuous-time.

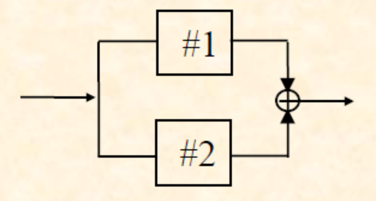
A system is discrete-time if both input and output are discrete-time.

Interaction of Multiple Systems

(1) Cascade (Series)

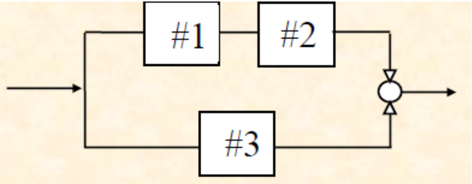


(2) Parallel

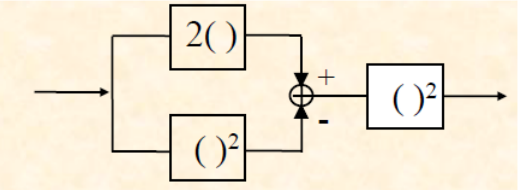


Interaction of Multiple Systems

(3) Series/Parallel

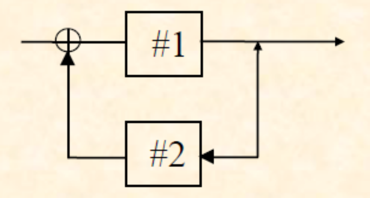


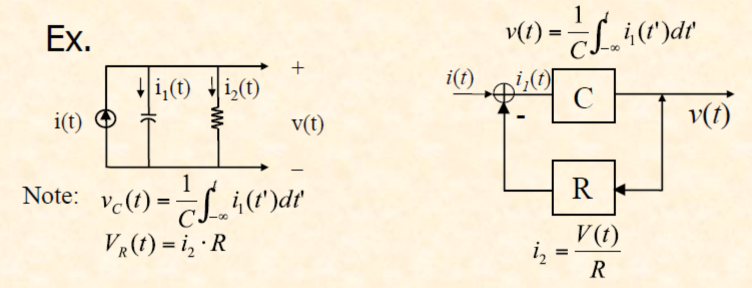
Ex. $y[n] = (2x[n] - x[n]^2)^2$



Interaction of Multiple Systems cont.

(4) Feedback





Properties of System

Properties of System

1. Memory/Memoryless
2. Invertibility and Inverse System
3. Causality
4. Stability
5. Time-invariance
6. Linearity

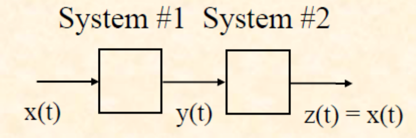
(1). Memory and Memoryless

A system is memoryless if the output only depends on input at the same time.

Ex. (i) Resister is a memoryless component: $v(t) = Ri(t)$.
(ii) With memory: e.g. $y[n] = \sum_{k=-\infty}^{n} x[k]$.
Capacitor has memory (why?)

(2). Invertibility and Inverse System

A system is invertible if distinct inputs lead to distinct outputs.



If $z(t) = x(t)$, then system $\#2$ is the inverse system of system $\#1$.

E.g. $y(t) = 2x(t)$, then $z(t) = 0.5y(t)$
$y[n] = \sum_{k=-\infty}^{n} x[k]$, then $z[n] = y[n] - y[n-1]$

(3). Causality

A system is causal if the output at any time only depends on the input at the present time and before.

E.g. $y[n] = x[n] - x[n-1]$: causal
$y(t) = x(t+1)$: non-causal

Q: Causal and Memoryless

(3). Causality

A system is causal if the output at any time only depends on the input at the present time and before.

E.g. $y[n] = x[n] - x[n-1]$: causal
$y(t) = x(t+1)$: non-causal

Q: Causal and Memoryless

Causal property is more important for real-time processing.

But for some applications, such as image-processing, no need to process the data causally.

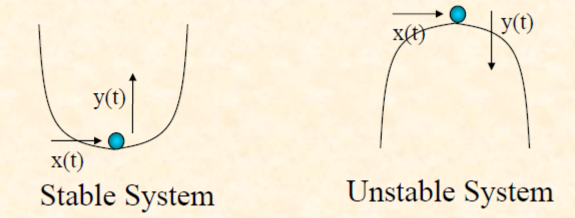
$$y[n] = \frac{1}{2M+1} \sum_{k=-M}^{M} x[n-k].$$

(4). Stability

A system is stable if bounded input gives bounded output.

BIBO stable

E.g. $x(t)$: the horizontal force; $y(t)$: vertical displacement



$\uparrow$ Week 2, Thu, 20180301

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(5). Time-invariance

A system is time-invariant if a time shift in the input only causes a time shift in the output.

i.e. If $x[n] \to y[n]$, then $x[n - n_0] \to y[n - n_0]$

Ex. 1 $y(t) = \sin(x(t))$

Let $y_1(t) = sin(x_1(t))$, $x_2(t) = x_1(t - t_0)$
Then $y_2(t) = sin(x_2(t)) = sin(x_1(t - t_0)) = y_1(t - t_0)$

Hence time-invariant (T.I.)

(5). Time-invariance cont.

Ex. 2 $y[n] = nx[n]$

Let $y_1[n] = nx_1[n]$, $x_2[n] = x_1[n - n_0]$
Then $y_2[n] = nx_2[n] = nx_1[n - n_0]$
However $y_1[n - n_0] = (n - n_0)x_1[n - n_0]$
$\neq y_2[n]$

Hence not time-invariant (T.I.)

(6). Linearity

A system is linear if

1. the response to $x_1(t) + x_2(t)$ *is* $y_1(t) + y_2(t)$

— *additivity*

2. the response to $a \cdot x_1(t)$ *is* $a \cdot y_1(t)$, *where* a *is any complex constant.*

— *scaling*

Combine the above two properties, we can conclude

$$ax_1(t) + bx_2(t) \implies ay_1(t) + by_2(t)$$

— superposition property

For discrete-time: $ax_1[n] + bx_2[n] \implies ay_1[n] + by_2[n]$

(6). Linearity cont.

If linear, zero input gives zero output.

Q: Is $y[n] = 2x[n] + 3$ linear?
A: No, because it violates zero-in zero-out property.

However, this system is an "incremental linear system": difference of output is a linear function of difference of input.

$$y_1[n] - y_2[n] = 2x_1[n] + 3 - (2x_2[n] + 3) = 2(x_1[n] - x_2[n])$$

Exercise

(1). $y[n] = x^2[n]$
non-linear; time-invariant

(2). $y[n] = nx[n]$
memoryless; causal; linear; not time-invariant; not stable

(3). $y(t) = x(\frac{t}{3})$
(4). $y(t) = \frac{d}{dt}x(t)$
(5). $y(t) = \int_{-\infty}^{2t} x(\tau)d\tau$
(6). $y(t) = x(t-2) + x(2-t)$

Exercise cont.

(7). $y(t) = x(t)\cos(8t)$
(8). $y(t) = x[n-3] - 8x[n-5]$
(9). $y[n] = x[3n]$
(10). $y(t) = \sin(x(t))$

Why LTI is so important?

So we can represent any input by a summation of basic signal ($\delta(t)$ or $\delta[n]$) and its time-shifted version. Then by using LTI property, the output can be found from the summation of the individual output of each basic signal and its times-shifted.