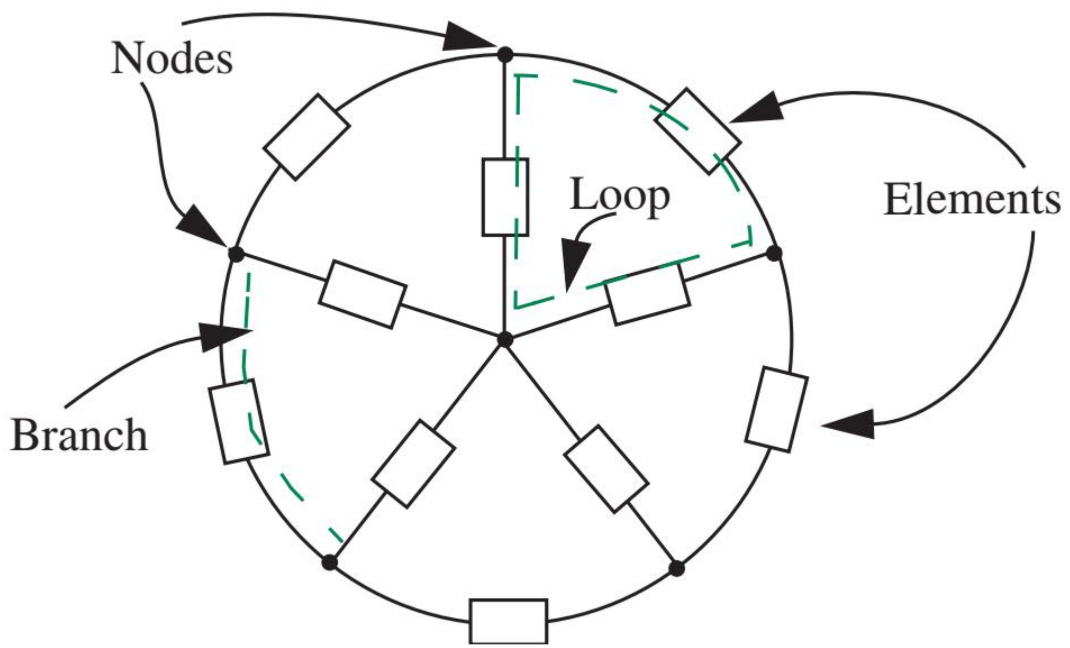
Lecture 2
Basic Laws & Circuit Analysis

Outline

- Terminology: Branches, Nodes, and Loops
- Basic Laws
  - Ohm’s Law
  - Kirchhoff’s Laws -- KCL,KVL
- Circuit Analysis
  - Nodal Analysis
  - Mesh Analysis

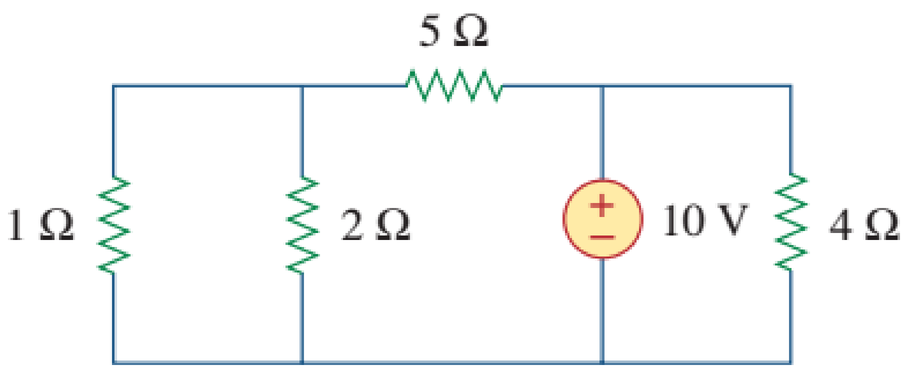
Terminology: Branch, Node, and Loop

- **Branch**: represents a single element;
- **Node**: a point of connection between two or more branches;
- **Loop**: **Any** closed path in a circuit.



[Source: MIT]

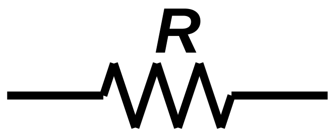
Example



- b – number of branches
- n – number of nodes
- l – number of loops

Ohm’s Law

- Resistance: the ratio of voltage drop and current. The circuit element used to model this behavior is the resistor.

<u>Circuit symbol</u>: 

- The current flowing in the resistor is proportional to the voltage across the resistor:

$$\boxed{V = I*R}$$ (Ohm’s Law)

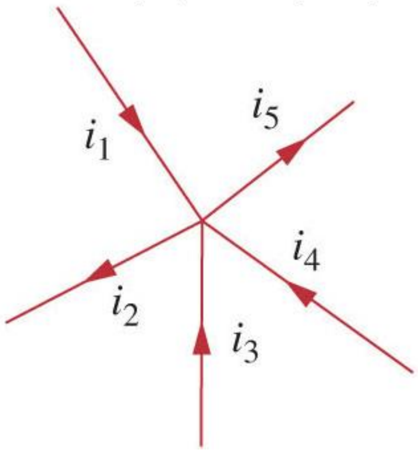
- **Conductance** is the reciprocal of resistance

$$G = \frac{1}{R} = \frac{I}{V}$$

Werner von Siemens
1816-1892

Kirchhoff's Laws

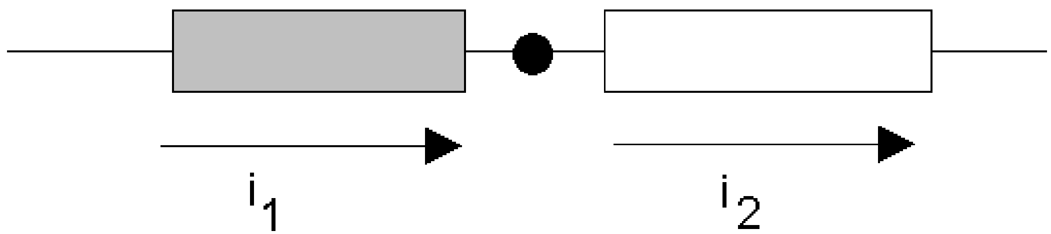
- Kirchhoff's Current Law (KCL):
  - The algebraic sum of all the **currents** entering any **node** in a circuit equals zero.
  - Why?



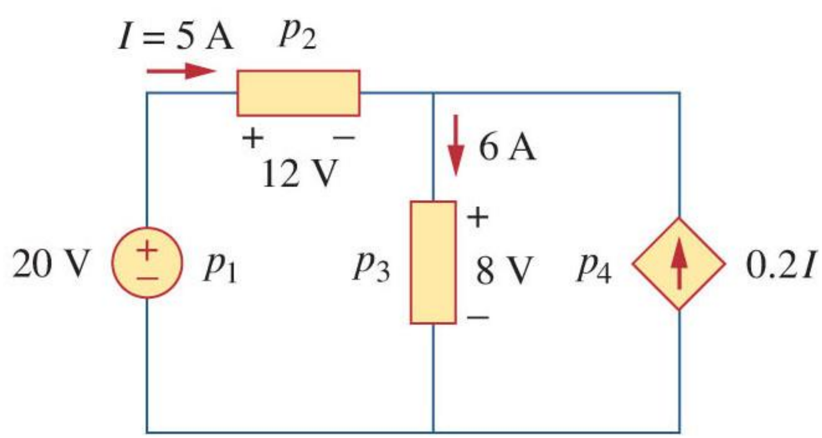
Gustav Robert Kirchhoff
1824-1887

KCL

- KCL tells us that **all of the elements that are connected *in series* carry the same current**.



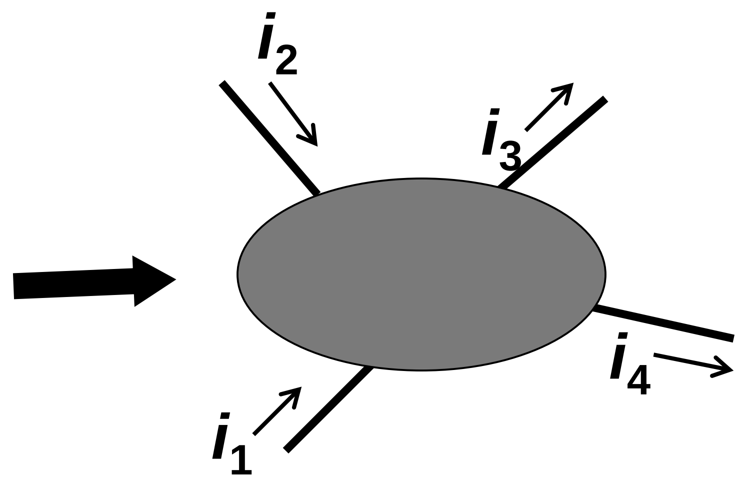
Current entering node = Current leaving node



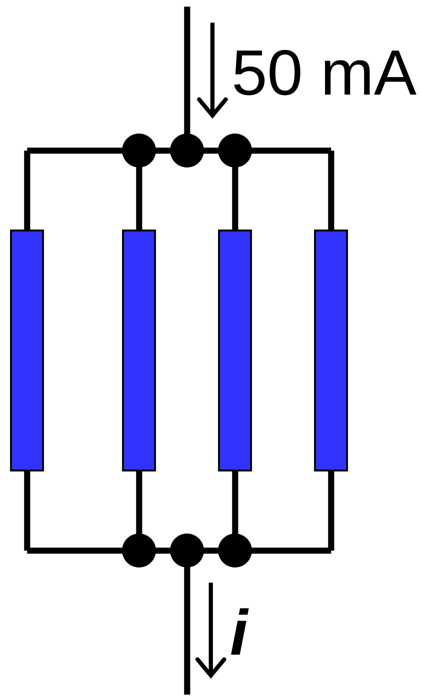
Generalization of KCL

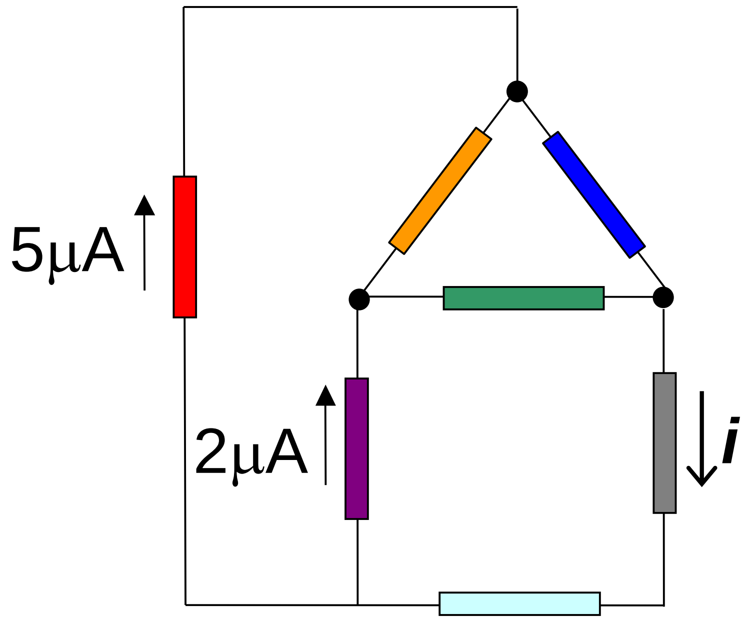
- The sum of currents entering/leaving a **closed surface** is zero.
  - Circuit branches can be inside this surface, *i.e.* the surface can enclose more than one node!

This could be a big chunk of a circuit, *e.g.* a “black box”



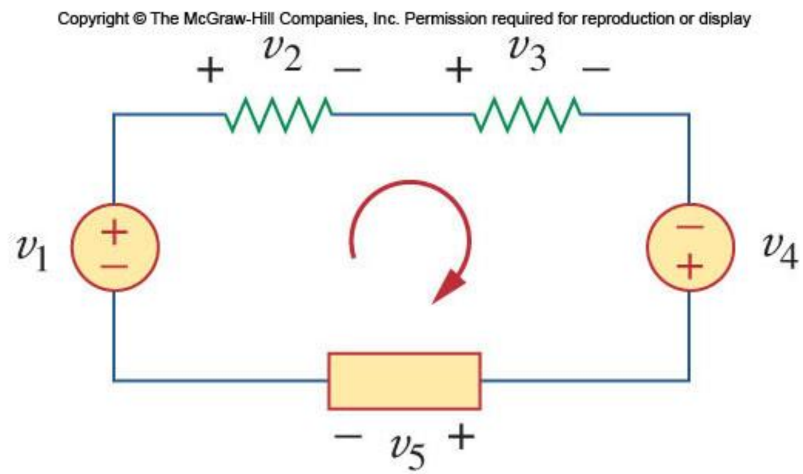
Generalized KCL Examples





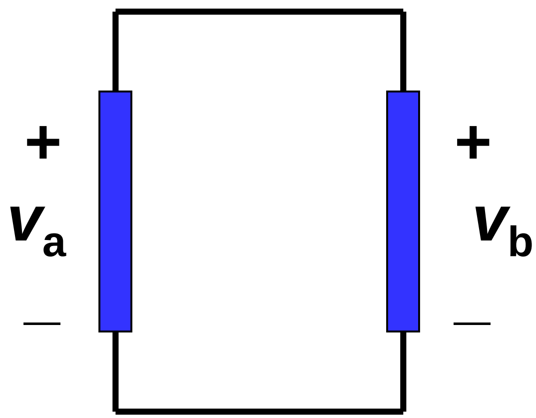
Kirchhoff's Voltage Law (KVL)

- The algebraic sum of all the **voltages** around any **loop** in a circuit equals zero.
- Why?



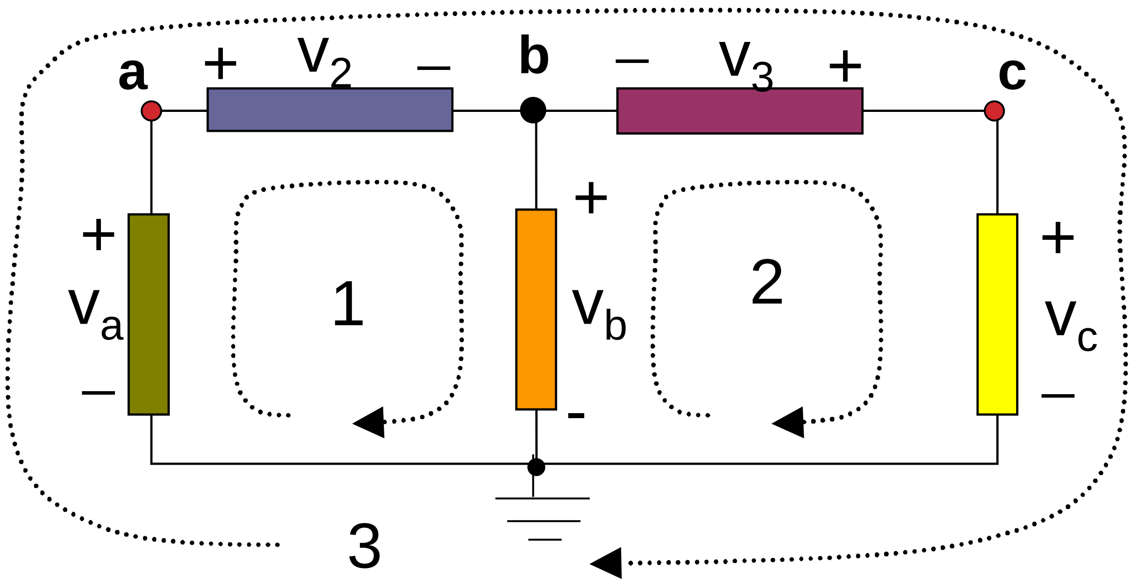
KVL

- KVL tells us that **any set of elements which are connected at both ends carry the same voltage**.
- We say these elements are connected **in parallel**.



KVL Example

Three closed paths:

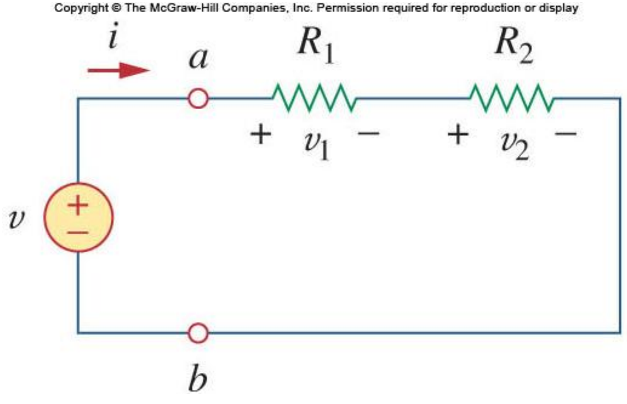


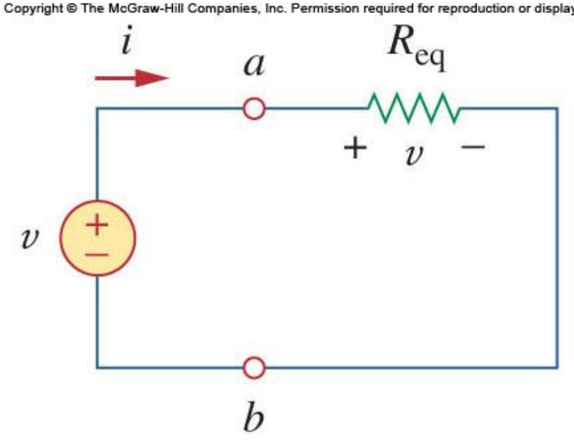
Path 1:

Path 2:

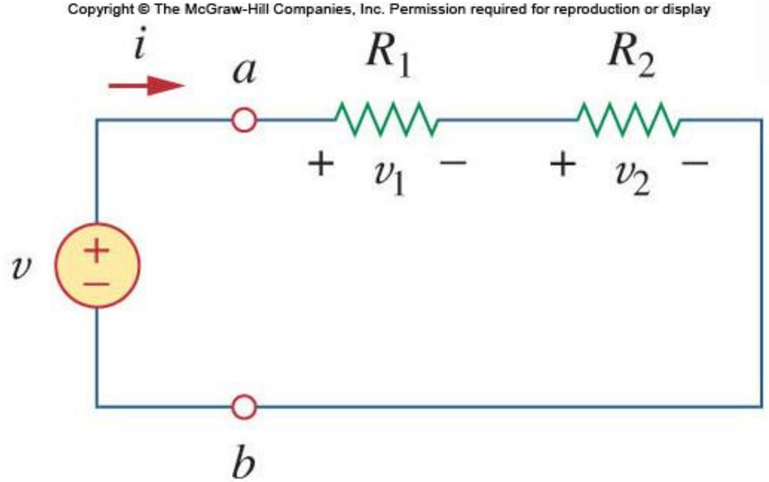
Path 3:

Series Resistors



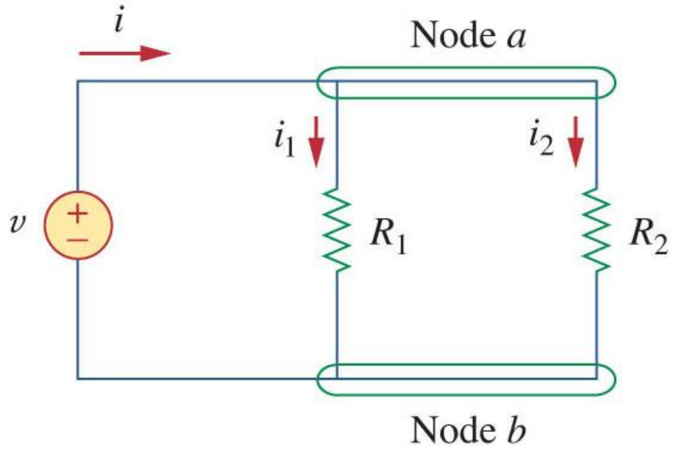


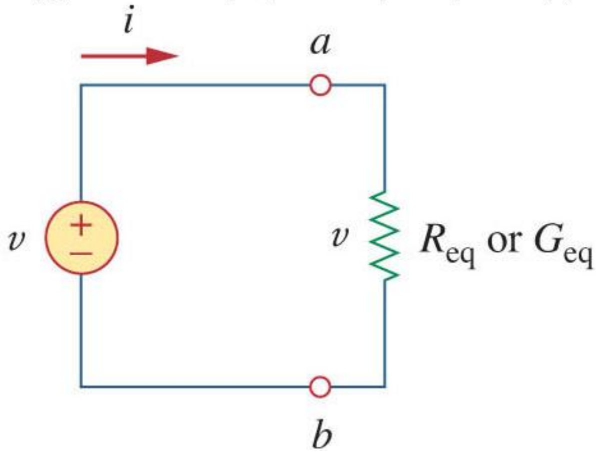
Voltage Division



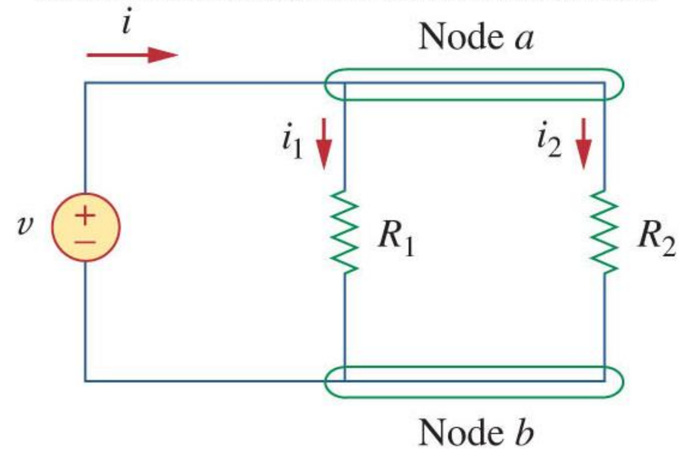
Three-terminal rheostat

Parallel Resistors





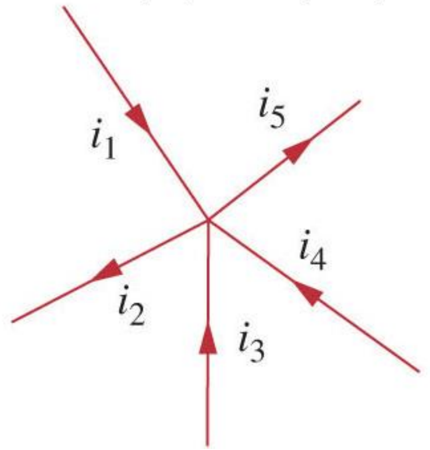
Current Division

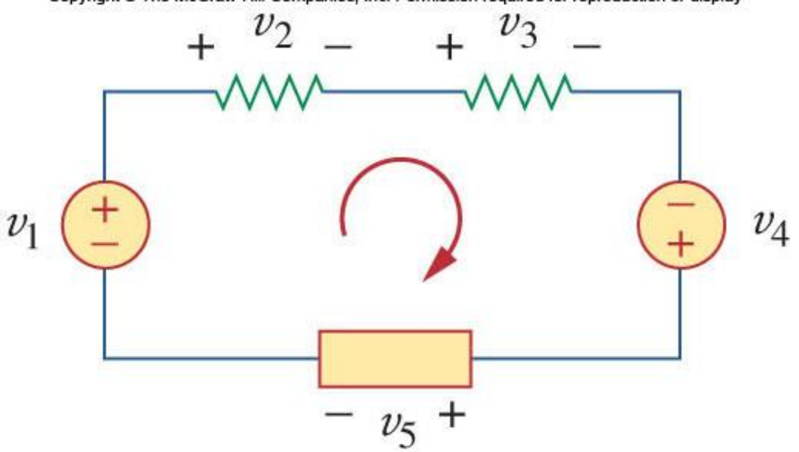


Summary-1

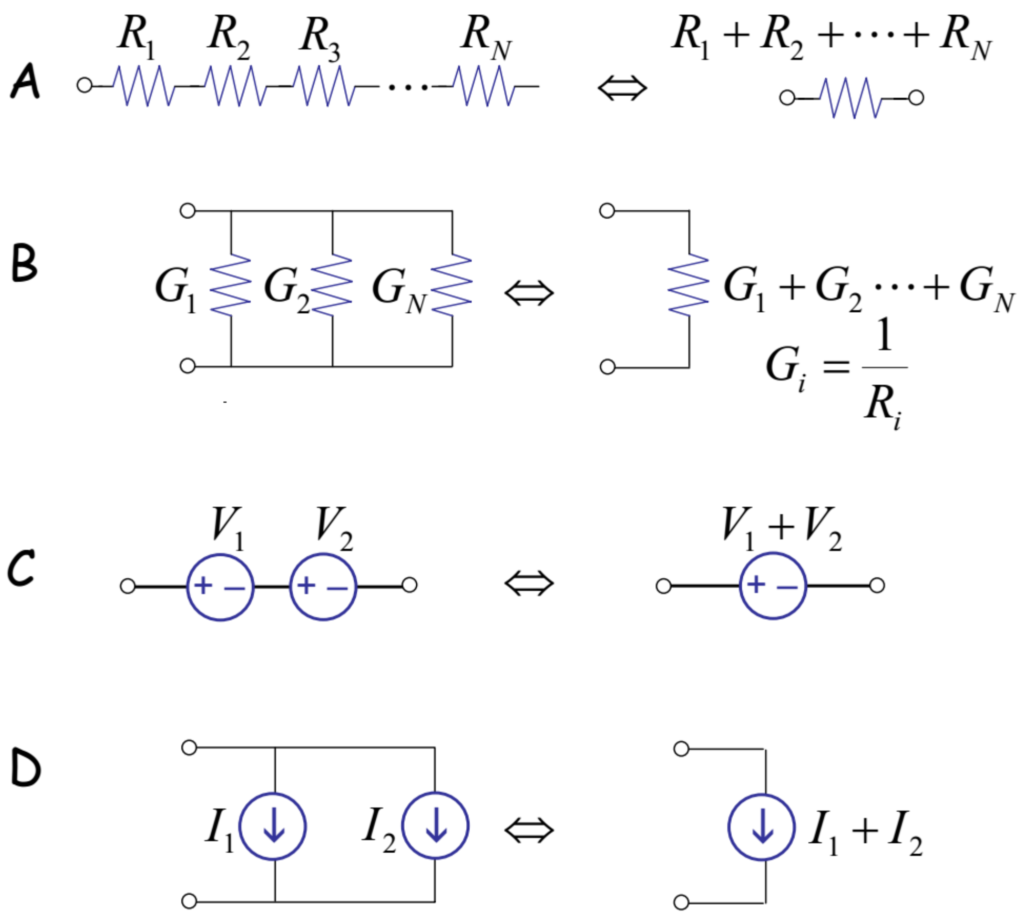
- KCL and KVL

$$\sum_{n=1}^{N} i_n = 0 \qquad \sum_{m=1}^{M} v_m = 0$$



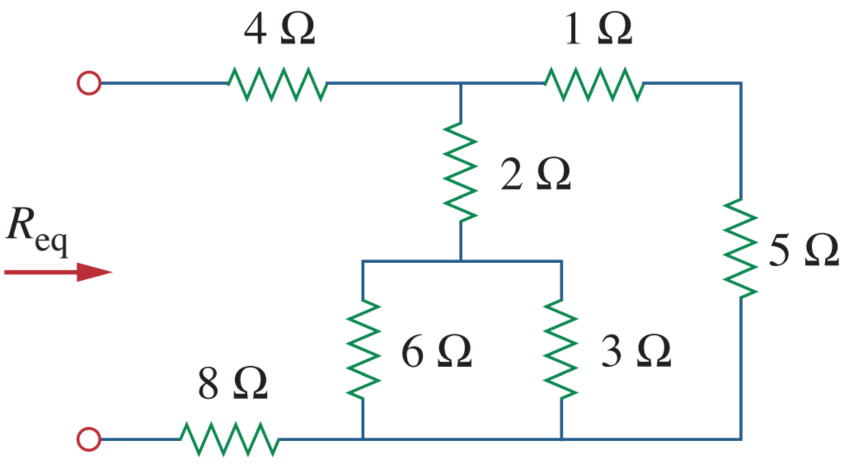


Summary-2

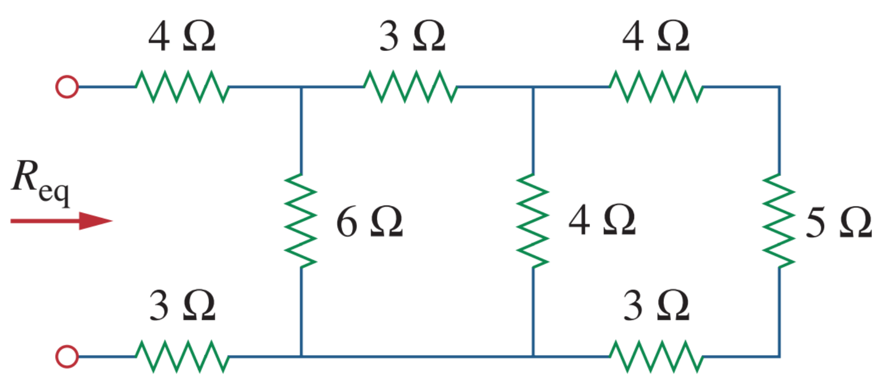


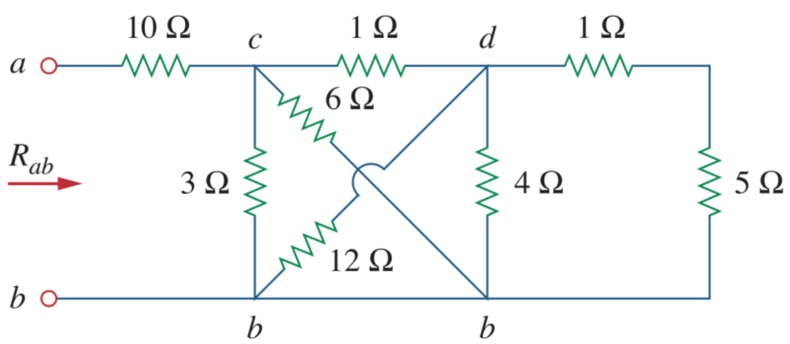
[Source: MIT]

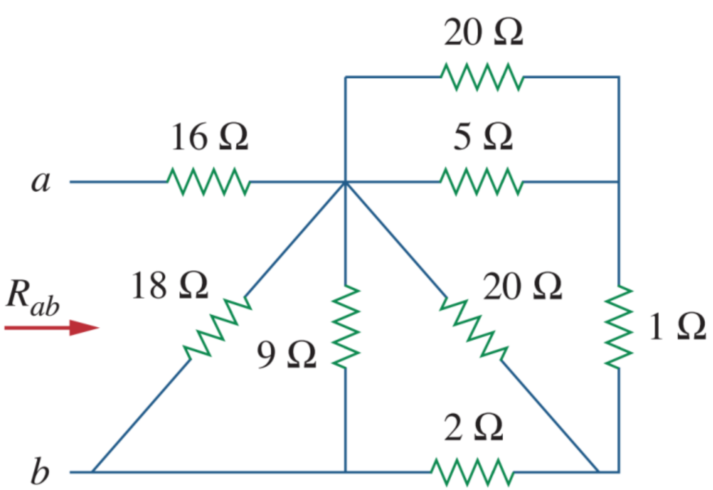
Example

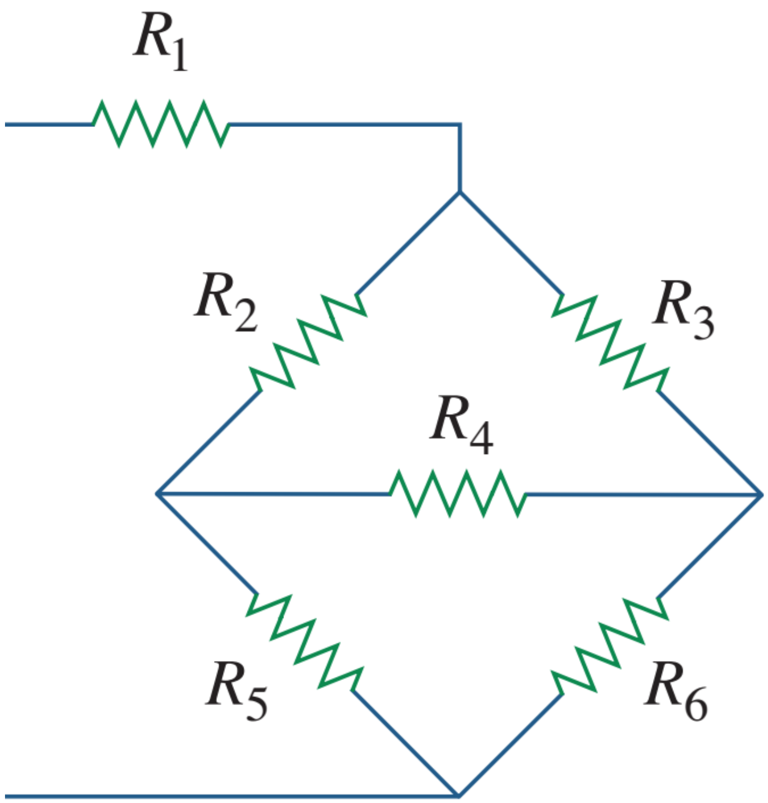


Practice

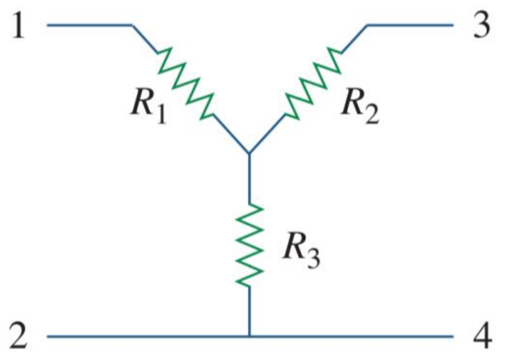

4 Ω
3 Ω
4 Ω
R_{eq}
6 Ω
4 Ω
5 Ω
3 Ω
3 Ω

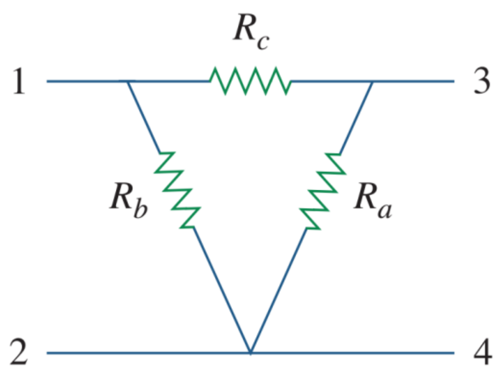

10 Ω
c
1 Ω
d
1 Ω
a
6 Ω
R_{ab}
3 Ω
4 Ω
5 Ω
12 Ω
b
b
b


20 Ω
16 Ω
5 Ω
a
18 Ω
20 Ω
R_{ab}
9 Ω
1 Ω
2 Ω
b


R_1
R_2
R_3
R_4
R_5
R_6

Delta-wye conversion





$$R_{12}(\mathrm{Y}) = R_1 + R_3 \tag{2.46}$$

$$R_{12}(\Delta) = R_b \parallel (R_a + R_c)$$

Setting $R_{12}(\mathrm{Y}) = R_{12}(\Delta)$ gives

$$R_{12} = R_1 + R_3 = \frac{R_b(R_a + R_c)}{R_a + R_b + R_c} \tag{2.47a}$$

Similarly,

$$R_{13} = R_1 + R_2 = \frac{R_c(R_a + R_b)}{R_a + R_b + R_c} \tag{2.47b}$$

$$R_{34} = R_2 + R_3 = \frac{R_a(R_b + R_c)}{R_a + R_b + R_c} \tag{2.47c}$$

Subtracting Eq. (2.47c) from Eq. (2.47a), we get

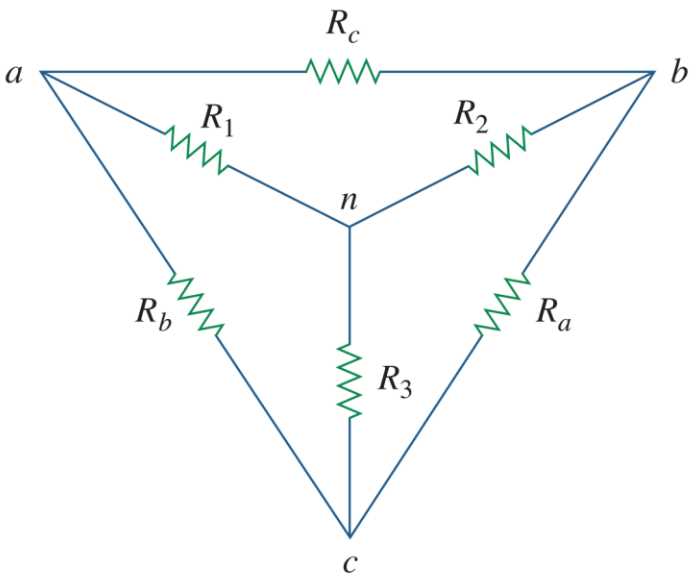
$$R_1 - R_2 = \frac{R_c(R_b - R_a)}{R_a + R_b + R_c} \tag{2.48}$$

Adding Eqs. (2.47b) and (2.48) gives

$$\boxed{R_1 = \frac{R_b R_c}{R_a + R_b + R_c}} \tag{2.49}$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c} \qquad R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

Wye-delta conversion



$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

The Y and Δ networks are said to be *balanced* when

$$R_1 = R_2 = R_3 = R_Y, \qquad R_a = R_b = R_c = R_\Delta \tag{2.56}$$

Under these conditions, conversion formulas become

$$R_Y = \frac{R_\Delta}{3} \qquad \text{or} \qquad R_\Delta = 3R_Y \tag{2.57}$$

Outline

- Basic Laws
  - Ohm’s Law
  - Kirchhoff’s Laws -- KCL,KVL

- Circuit Analysis
  - Nodal Analysis
  - Mesh Analysis

Circuit Analysis

- Two techniques will be presented in this part:
  - Nodal analysis, which is based on **KCL**
  - Mesh analysis, which is based on **KVL**

Nodal Analysis – Three Steps

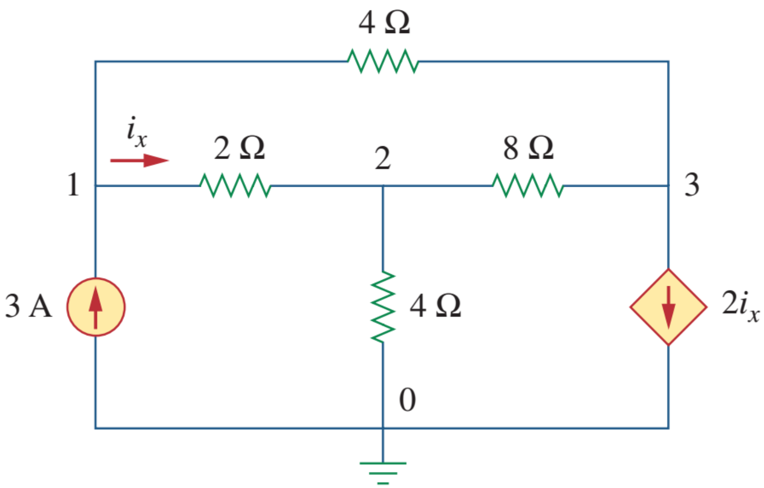
- Given a circuit with ***n*** nodes, the nodal analysis is accomplished via three steps:
  1. Select a node as the reference (i.e., ground) node. Assign the node voltages to the remaining ***(n-1)*** nodes. Voltages are relative to the reference node.

  2. Apply KCL to the ***(n-1)*** nodes, expressing branch current in terms of the node voltages (using the *I*-*V* relationships of branch elements).

  3. Solve the resulting simultaneous equations to obtain the unknown node voltages.

[Source: Berkeley]

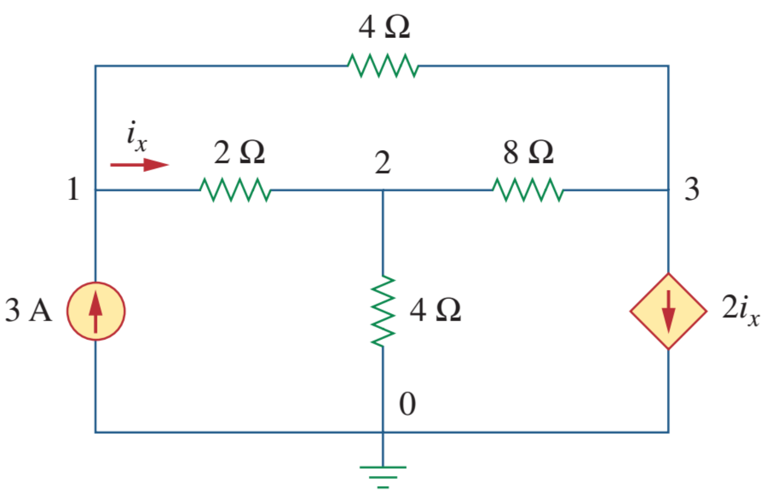
Nodal Analysis: Example #1



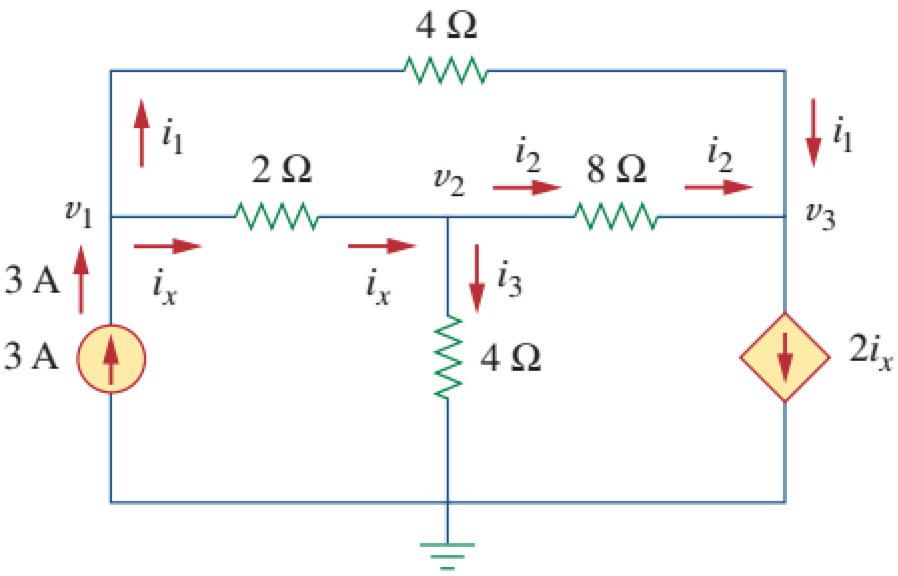
(a)

[Source: Berkeley]

Nodal Analysis: Example #1



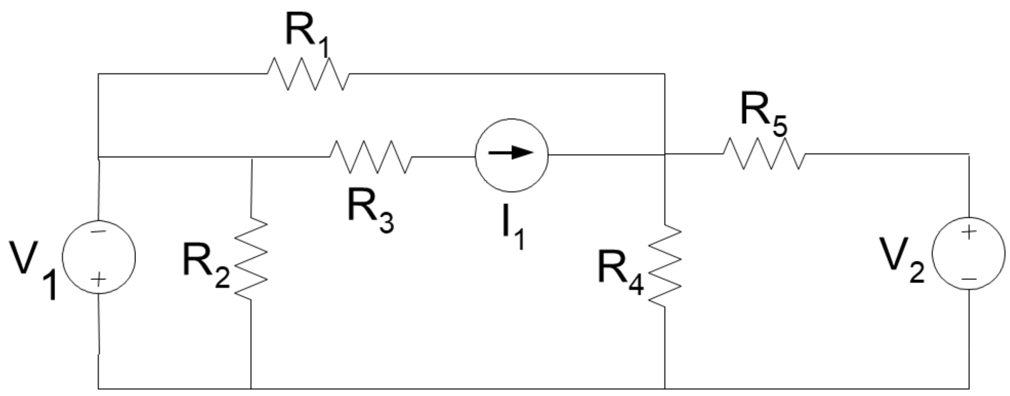
(a)



[Source: Berkeley]

Nodal Analysis with Voltage Sources

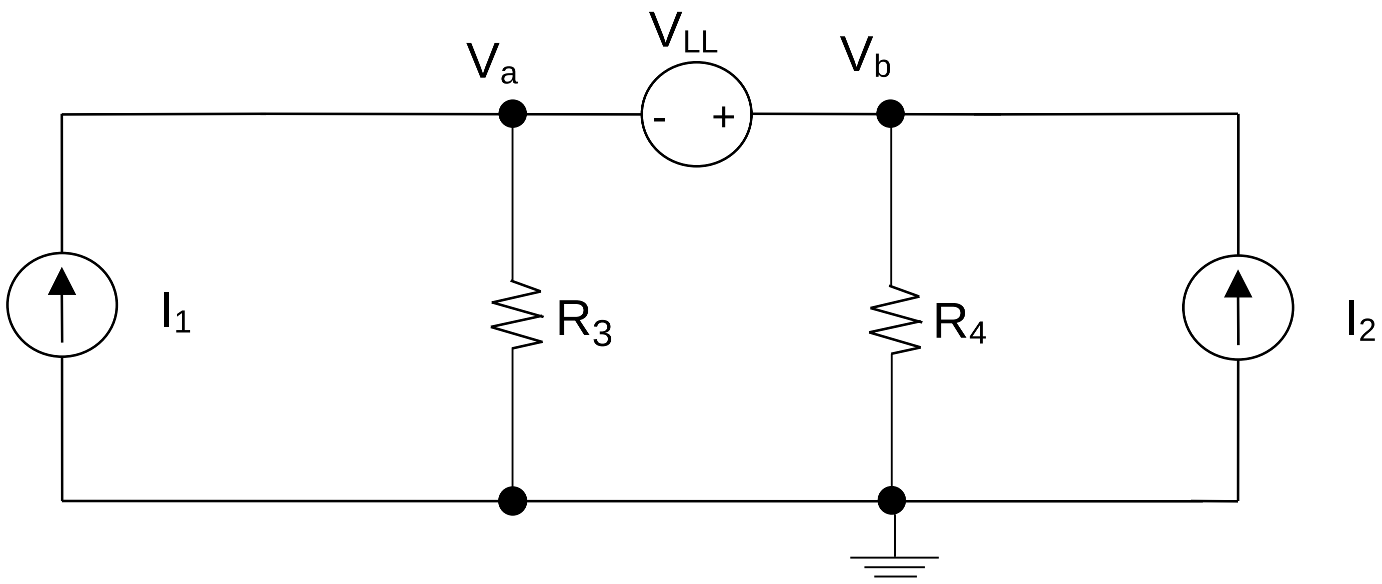
Case I:



Nodal Analysis: Supernode

Case II

A "floating" voltage source is one for which neither side is connected to the reference node, e.g. V_{LL} in the circuit below:

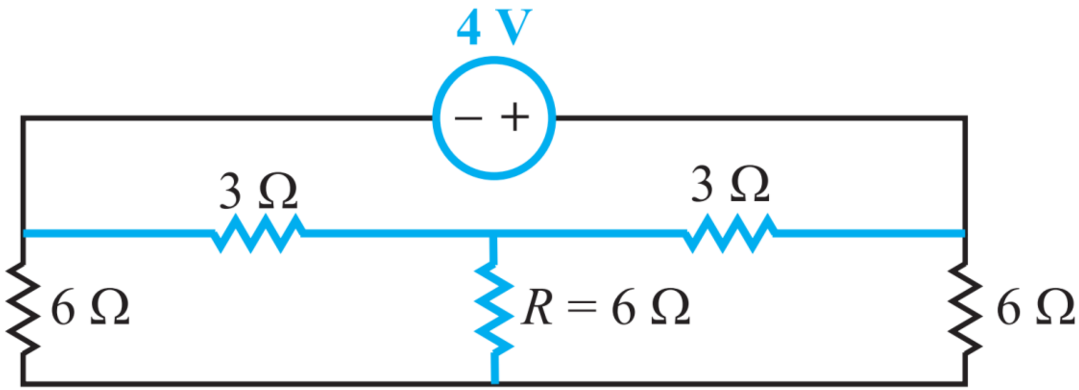


A supernode is formed by enclosing a (dependent or independent) voltage source connected between two nonreference nodes and any elements connected in parallel with it.

[Source: Berkeley]

Exercise

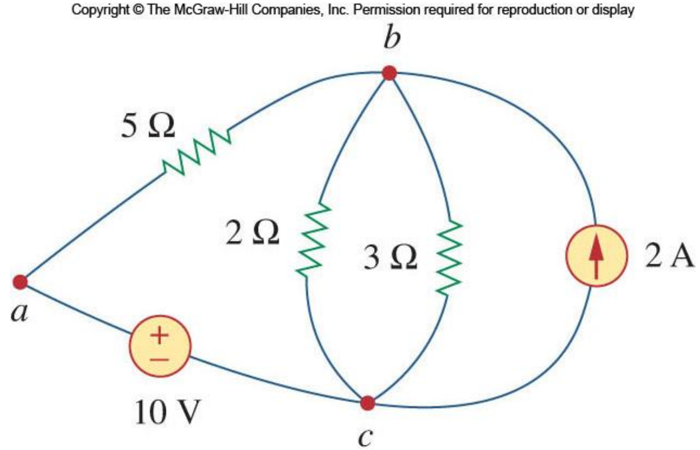
- Find the power supplied by the voltage source.



[Source: Berkeley]

Mesh Analysis--Loop, Independent Loop, Mesh

- A loop is a closed path.
- A loop is independent if it contains at least one branch which is not a part of any other independent loop.
- A mesh is a loop that does not contain any other loop within it.



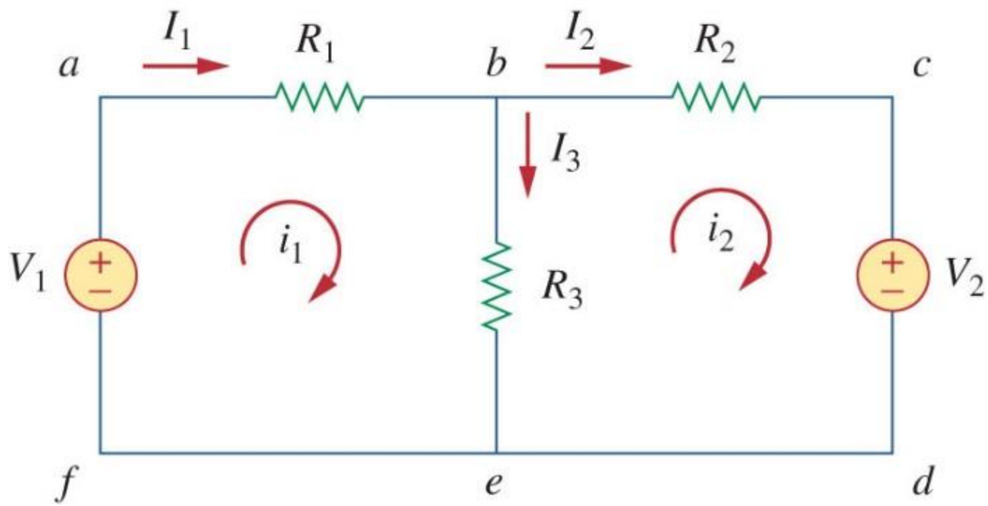
- b – number of branches
- n – number of nodes
- l_{ind} – number of ind. loops

Mesh = Independent loop?

$$l_{ind} = b - (n - 1)$$

Mesh Analysis

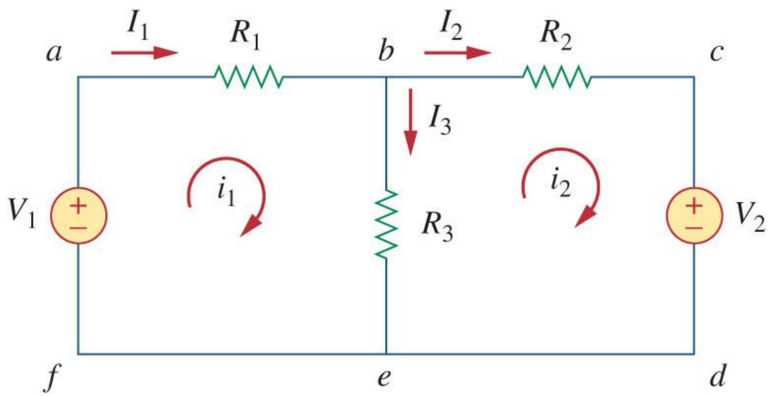
- Another general procedure for analyzing circuits is to use the mesh currents as the circuit variables.



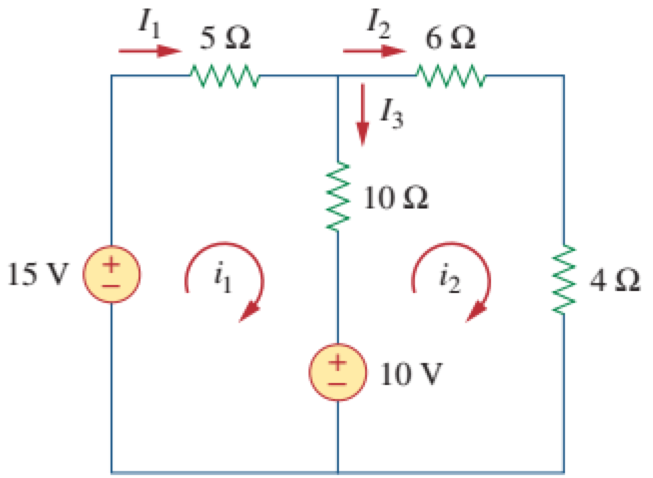
- Mesh analysis uses KVL to find unknown currents.

Mesh Analysis Steps

- Mesh analysis follows these steps:
  1. Assign mesh currents $i_1, i_2, \ldots i_x$ to the x meshes
  2. Apply KVL to each of the x mesh currents.
  3. Solve the resulting x simultaneous equations to get the mesh currents.

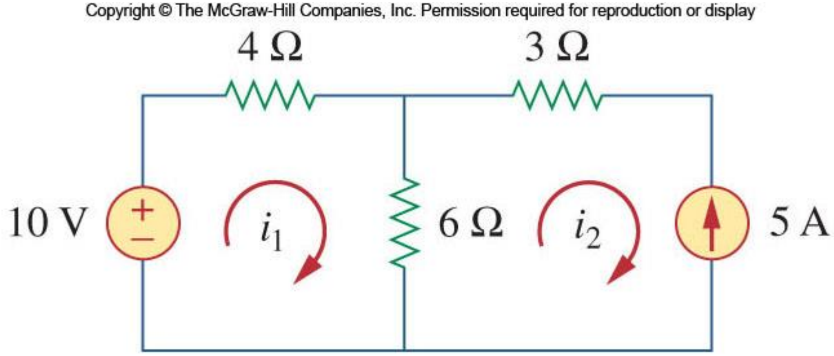


Example

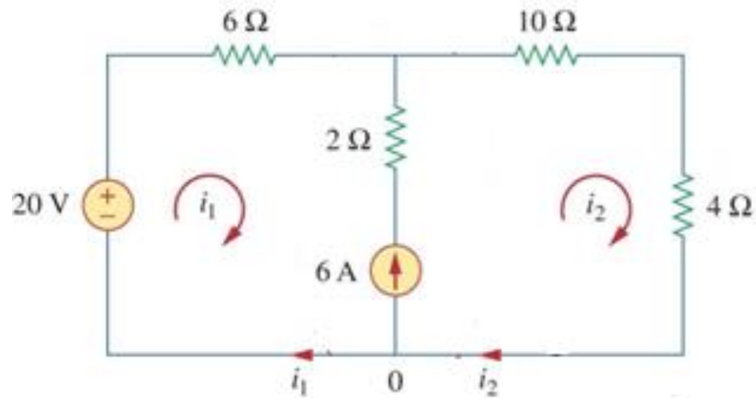


Mesh Analysis with Current Sources

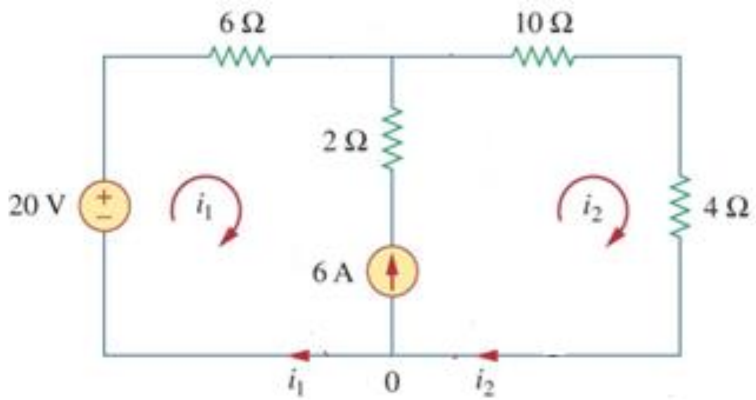
- The presence of a current source makes the mesh analysis simpler in that it reduces the number of equations.
  - If the current source is located on only one mesh, the current for that mesh is defined by the source. For example:

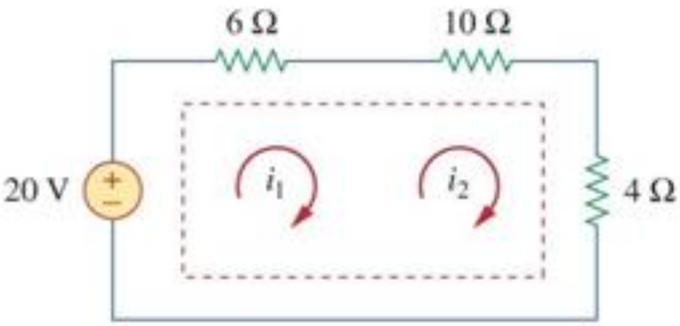


If the current source is located…



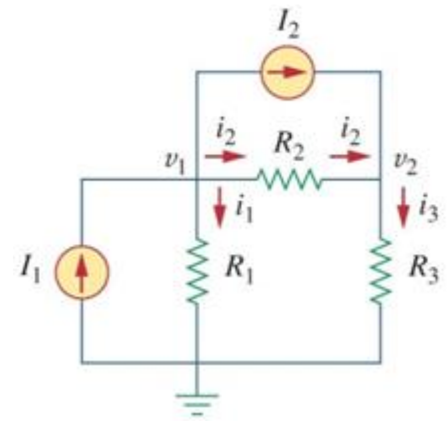
Supermesh





Summary

- Node Analysis
  - Node voltage is the unknown
  - Solve by KCL
  - Special case: Floating voltage source



- Mesh Analysis
  - Loop current is the unknown
  - Solve by KVL
  - Special case: Current source