Siganls & System: Homework #8

Due on January 6, 2022 at 23:59

Problem 1

(3 * 5 points) Determine the z-transform for each of the following wequences. Sketch the pole zero plot and indicate the ROC.

(a) $2^n u[-n] + (\frac{1}{2}^n u[n-1])$

(b) $4^n cos[\frac{\pi}{3}n + \frac{\pi}{4}]u[-n-1]$

(c) $n(\frac{1}{2}^{|n|})$

Problem 2

(2 * 5 points) Suppose we are given the following facts about a paorticular LTI system S with impluse response h[n] and z-transform H(z).

- h[n] is real.
- h[n] is right-sided.
- $\lim_{z \to +\infty} H(z) = 0$.
- H(z) has two zeros.
- H(z) has one of its poles at a non-real location on the circle defined by $|z| = \frac{3}{4}$

Determine the correctness of the following statements. Correct them if they are incorrect and give reasons:

(a) Since $\lim_{z \to +\infty} H(z) = 0$, H(z) has no poles at infinity. Furthermore, since h[n] is right sided, h[n] has to be casual.

(b) Since h[n] is causal, the numerator and denominator polynomials of H(z) have the same order. Since H(z) is given to have two zeros, we may conclude that it also has two poles. Since h[n] is real, the poles must occur in conjugate pairs. Also, it is given that one of the poles lies on the circle defined by $|z| = \frac{3}{4}$. Therefore, the other pole also lies on this circle.From above analysis, we can conclude that ROC of H(z) will be of form $|z| > \frac{3}{4}$, which include the unit circle. As a result, the system is stable.

Problem 3

(3 * 5 points) A causal LTI discrete-time system is described by the difference equation

$$y[n] = 0.4y[n-1] + 0.05y[n-2] + 3x[n]$$

where x[n] and y[n] are the input and output sequences of the system, respectively.

(a) Determine the transfer function H(z) of the system.

(b) Determine the impulse response h[n] of the system.

(c) Determine the step response s[n] of the system.

Problem 4

(3 * 10 points) Consider the system function corresponding to casual LTI systems:

$$H(z) = \frac{1}{(1 - z^{-1} + \frac{1}{4}z^{-2})(1 - \frac{2}{3}z^{-1} + \frac{1}{9}z^{-2})}$$

(a) Draw a direct-form block diagram.

(b) Draw a block diagram that corresponds to the cascade connection of two second-order block diagrams.

(c) Determine whether there exists a block diagram which is the cascade of four first-order block diagrams with the constraint that all teh coefficient multipliers must be real. If false, state the reason. If true, draw the diagram.

Problem 5

(3 * 10 points) Consider a system whose input x[n] and output y[n] are related by

$$y[n-2] + 5y[n-1] + 6y[n] = x[n].$$

(a) Determine the zero input reponse of this system if y[-2] = 6 and y[-1] = 0.

(b) Determine the zero state reponse of this system to the input $x[n] = -6\delta[n]$.

(c) Determine the output of this system for n$\geq$0 when $x[n] = -6\delta[n]$, y[-2]=6 and y[-1]=0.