CS243: Introduction to Algorithmic Game Theory

Redistribution (Dengji ZHAO)

SIST, ShanghaiTech University, China

Recap: The General Setting of Mechanism Design

- A set of n participants/players, denoted by N.
- A mechanism needs to choose some alternative from A (allocation space), and to decide a payment for each player.
- Each player $i \in N$ has a private valuation function $v_i : A \to \mathbb{R}$, let V_i denote all possible valuation functions for i.
- Let $v = (v_1, \cdots, v_n)$, $v_{-i} = (v_1, \cdots, v_{i-1}, v_{i+1}, \cdots, v_n)$.
- Let $V = V_1 \times \cdots \times V_n$, $V_{-i} = V_1 \times \cdots V_{i-1} \times V_{i+1} \times \cdots \times V_n$.

Recap: Myerson's Optimal Auction

- Given the bids **b** and the distribution of agents' valuations **F**, compute virtual bids $b'_i = \phi_i(b_i) = b_i - \frac{1-F_i(b_i)}{f_i(b_i)}$.
- Run VCG on the virtual bids $\mathbf{b}'$ to get allocation $\mathbf{x}'$ and payment $\mathbf{p}'$.
- Output $\mathbf{x} = \mathbf{x}'$ and $\mathbf{p}$ with $p_i = \phi_i^{-1}(p'_i)$.

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Profit maximisation

Myerson's Optimal Auction maximises the seller's profit.

Recap: Auctions

- Truthful Mechanisms
  - Second-price auction
  - Generalization: VCG auctions
  - Optimal: Myerson's mechanism
- On social networks
  - Incentive diffusion mechanism (IDM)

Outline

Alternative Objective in Auctions

- Previously we focus on seller's revenue.
- What if the seller is not keen on revenue (e.g., an external agent or the government)?
- We now want to return the surplus to the agents.

Redistribution

Seeks to minimize net transfers from agents to an external body by return of VCG surplus to the agents.

Requirements

incentive compatibility Each agent will truthfully report her valuation v_i.

individual rationality Each agent will not suffer loss when she report her true valuation.

budget balanced The amount of extracted wealth that cannot be redistributed among the agents is 0.

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A First Attempt

Question

What if we uniformly return the VCG surplus, i.e., for each agent, we return v_2/n, where v_2 is the second highest bid among all agents?

Impossibility

Myerson-Satterthwaite Theorem

No mechanism is capable of achieving incentive compatibility, individual rationality, efficiency and budget balance at the same time.

Requirements

incentive compatibility: Each agent will truthfully report her valuation v_i.

individual rationality: Each agent will not suffer loss when she report her true valuation.

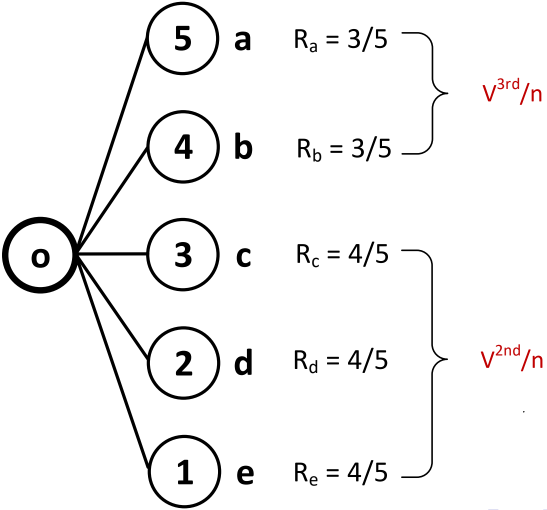
asymptotically budget balanced: As the number of participating agents goes to infinity, the amount of extracted wealth that cannot be redistributed among the agents goes to 0.

Cavallo's Method

- Suppose agents $a_1, a_2, \ldots, a_n$ has bids $v'_1 \geq v'_2 \geq \cdots \geq v'_n$.
- Let a_1 be the winner and pays v'_2. (VCG)
- Return the surplus v'_2 back to agents as follows

$$r_i = \begin{cases} v'_3/n & \text{for } i = a_1, a_2 \\ v'_2/n & \text{for } i = a_3, \ldots, a_n \end{cases}$$

Cavallo's Method



Cavallo's Method

The amount not redistributed is

$$r_c = v_2' - \sum r_i = \frac{2}{n}\left(v_2' - v_3'\right)$$

Cavallo's Method

Theorem

Cavallo's Method is incentive compatible, individually rational, efficient and asymptotically budget balanced.

Advanced Reading

- *Optimal DecisionMaking With Minimal Waste: Strategyproof Redistribution of VCG Payments* by Ruggiero Cavallo (AAMAS 2006)