Signals and Systems Homework 5

Due Time: 23:59 April 20, 2018

1. (15)Suppose $g(t) = x(t)cos(t)$ and the Fourier transform of the $g(t)$ is

$$G(jw) = \begin{cases} 1, |w| \leq 2 \\ 0, else \end{cases}$$

   (a) (5)Determine $x(t)$ Draw the frequency domain.

   (b) (10)Specify the Fourier transform $X_1(jw)$ of a signal $x_1(t)$,

$$g(t) = x_1(t)cos(\frac{2}{3}t)$$

Solution:
(a) We have that

$$w(t) = cos(t) \Leftrightarrow_{FT} W(jw) = \pi[\delta(w-1) + \delta(w+1)]$$

$$g(t) = x(t)cost \Leftrightarrow G(jw) = \frac{1}{2\pi}[X(jw) * W(jw)]$$

Therefore $G(jw) = \frac{1}{2}X(j(w-1)) + \frac{1}{2}X(j(w+1))$
is the first picture and (a) is the second.

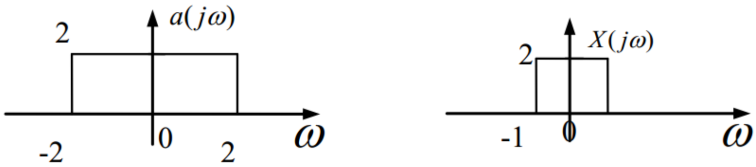

Figure 1: solution

(b)$G(jw) = \frac{1}{2\pi}X_1(jw) * \pi[\delta(w+\frac{2}{3}) + \delta(w-\frac{2}{3})] = \frac{1}{2}[X_1(j(w+\frac{2}{3})) + X_1(j(w-\frac{2}{3}))]$
Do a shift on frequency domain:
$1/2(X_1(jw) + X_1(w+\frac{4}{3})) = G(j(2+\frac{2}{3}))$ and
$1/2(X_1(j(w+\frac{4}{3}) + X_1(w+\frac{8}{3})) = G(j(2+\frac{2}{3}+\frac{4}{3}))$
In this way we can get the solution:

$$X_1(jw) = 2 \times [G(j(w+\frac{2}{3})) - G(j(w+\frac{2}{3}+\frac{4}{3})) + G(j(w+\frac{2}{3}+\frac{4}{3}\times 2))...]$$
$$= \sum_{n=0}^{\infty} 2 \times (-1)^{n+1} G(j(w+\frac{2}{3}+\frac{4}{3}\times n))$$

2. (15)Consier a LTI system whose response to the input $x(t) = [e^{-t} + e^{-3t}]u(t)$ is $y(t) = [2e^{-t} - 2e^{-4t}]u(t)$

   (a) (5)Find the frequence response of this system.

   (b) (5)Determine the impulse response of the system.

   (c) (5)Find the differential equation of the system.

   Solution:
   (a) $H(jw) = \frac{Y(iw)}{X(jw)} = \frac{3(3+jw)}{(4+iw)(2+jw)}$
   (b) The inverse of (a) is the solution $h(t) = \frac{3}{2}[e^{-4t} + e^{-2t}]u(t)$
   (c) We have $\frac{Y(jw)}{X(jw)} = \frac{9+3jw}{8+6jw-w^2}$
   The inverse of it is the solutuon $-\frac{d^2y(t)}{dt^2} + 6\frac{dy(t)}{dt} + 8y(t) = 3\frac{dx(t)}{dt} + 9x(t)$

3. (10)Consider a causal LTI system with frequence response

$$H(jw) = \frac{1}{jw + 3}$$

For an input

$$y(t) = [e^{-3t} - e^{-4t}]u(t)$$

determine $x(t)$ solution:
$H(jw) = \frac{Y(jw)}{X(jw)}$, as we know $y(t) = e^{-3t} - e^{-4t}u(t)$,we can get $Y(jw) = \frac{1}{3+jw} - \frac{1}{4+jw} = \frac{1}{(3+jw)(4+jw)}$,
and$H(jw) = \frac{1}{3+jw}$, we can get $x(jw) = \frac{1}{4+jw}$
so $x(t) = e^{-4t}u(t)$

4. (20)Ideal low pass filter frequency response is shown. Draw the spectrum of the output signal when input is the following function.

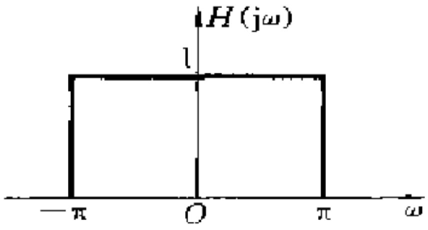

Figure 2: Lowpass Fliter

(a) $f(t) = \frac{sin(\pi t)}{\pi t}$

(b)

$$f(t) = \begin{cases} 1, |t| \leq 1 \\ 0, |t| > 1 \end{cases}$$

solution:
(a) If $f(t) = \frac{sin(\pi t)}{\pi t}$, we have $F(jw) = g_{2\pi}(w)$ so the spectrum of output signal $Y(jw) = H(jw)H(jw) = g_{2\pi}(w) \times g_{2\pi}(w) = g_{2\pi}(w)$
(b) $F(jw) = 2Sa(w)$, so $Y(jw) = H(jw)F(jw) = 2Sa(w) \times g_{2\pi}(w)$
The spectrum

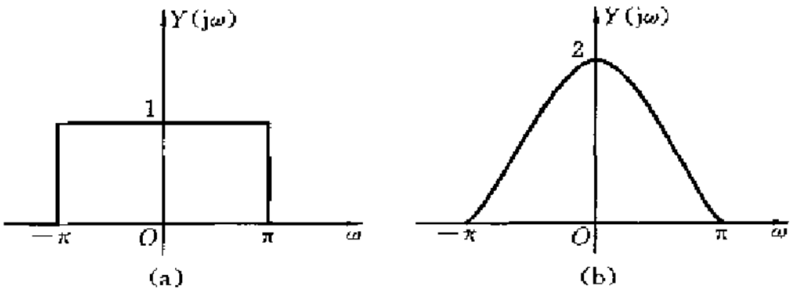

Figure 3: spectrum

5. (20)The spectrum of input band-limited signals is shown in figure a. The highest angular frequency is w_m and $w_b > w_m$, the cutoff frequency of figure b(HP) is w_b,

$$H_1(jw) = \begin{cases} K_1, |w| > w_b \\ 0, |w| < w_b \end{cases}$$

LP is

$$H_2(jw) = \begin{cases} K_2, |w| < w_b \\ 0, |w| > w_b \end{cases}$$

draw the spectrum of $x(t)$ and $y(t)$.

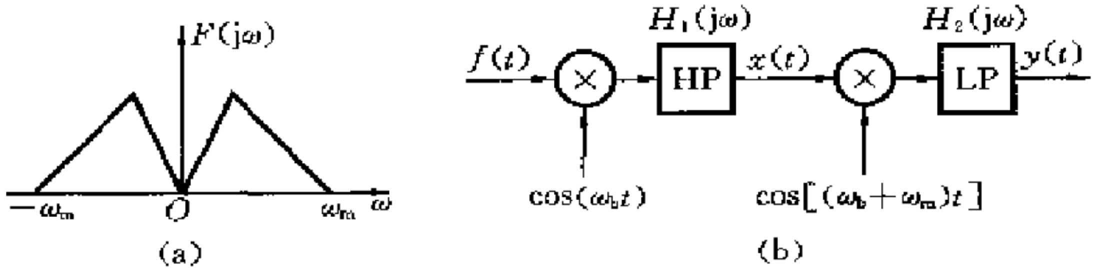

Figure 4: Singal and System

solution :
Let $p(t) = f(t)cos(w_b t)$, form the frequency domain convolution theorem $P(jw) = \frac{1}{2\pi}F(jw) * \pi[\delta(w + w_b) + \delta(w - w_b)] = \frac{1}{2}[F(w + w_b) + F(w - w_b)]$
Because $w_b > w_m$, $P(jw)$ is shown in (a).$p(t)$ after the filter of w at the Angle of w_b is filtered out, and the component that is higher than that is multiplied by the K_1 weight.
$x(t)$ is shown in (b)
$Q(jw)$ and $Y(jw)$ is shown:

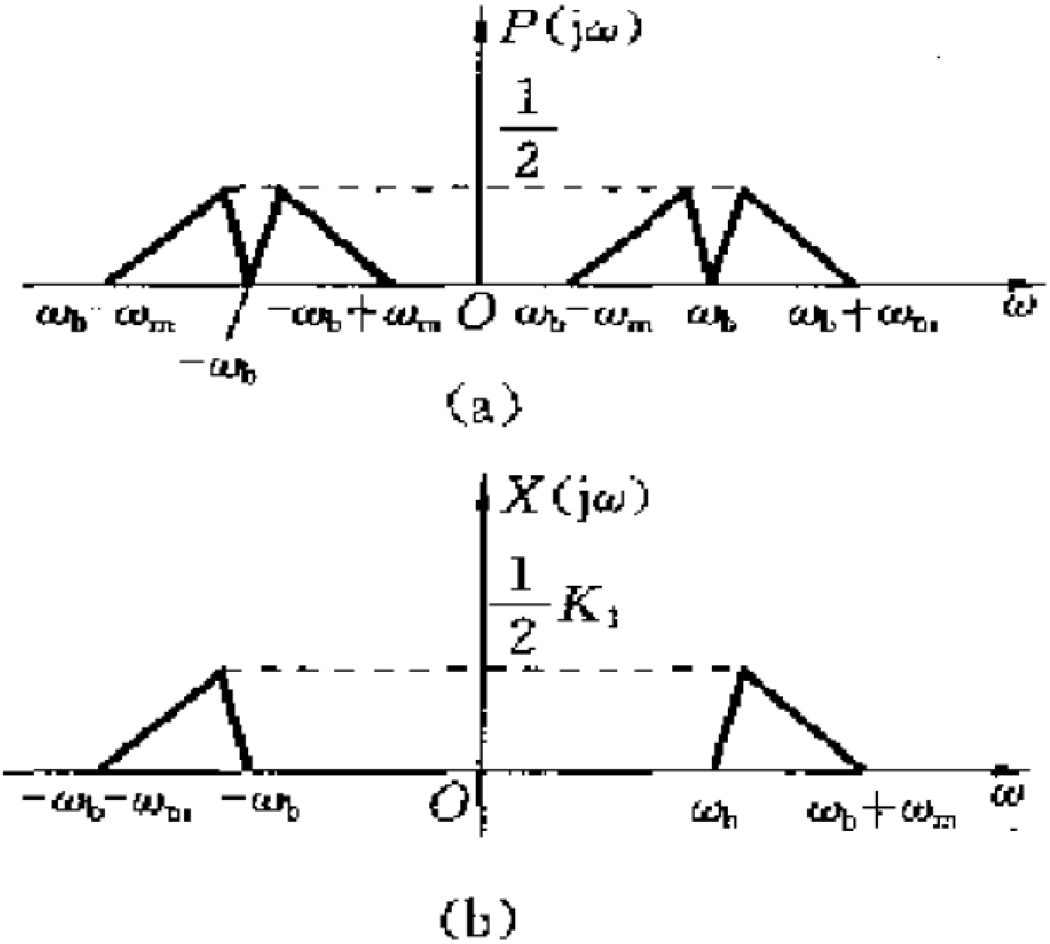

Figure 5: $P(jw)$ and $X(jw)$

(c) Let $q(t) = x(t)cos[(w_b + w_m)t]$, we have $Q(jw) = \frac{1}{2}[X(w + w_b + w_m) + X(w - w_b - w_m)]$
$q(t)$ after the low-pass filter at the angular frequency w_m is filtered out, and the component of this frequency is multiplied by the K_2 weight.
(d)$Q(jw)$ and $Y(jw)$ is shown:

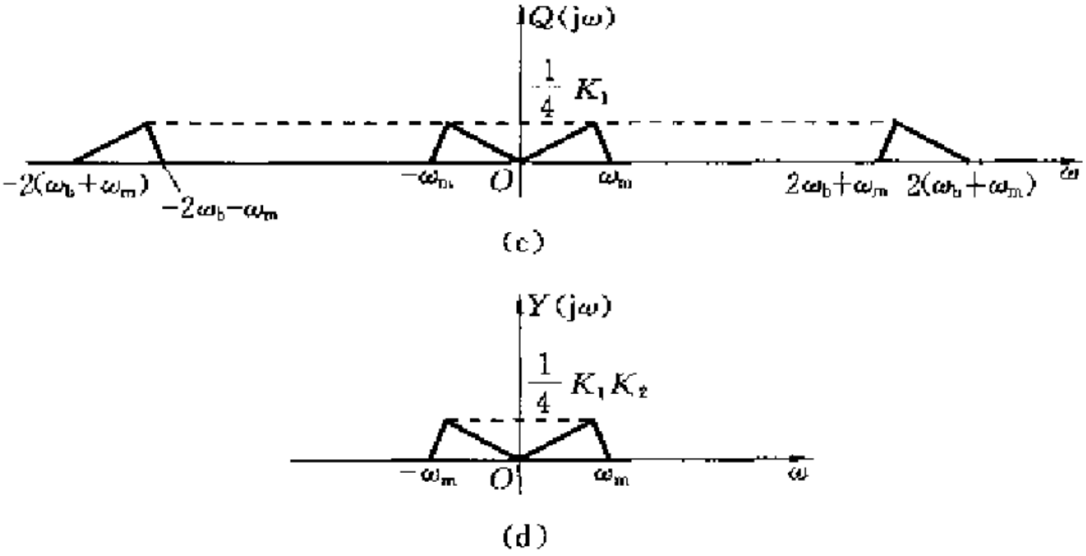

Figure 6: $Q(jw)$ and $Y(jw)$

6. (20)The bandpass filter responds to the figure.

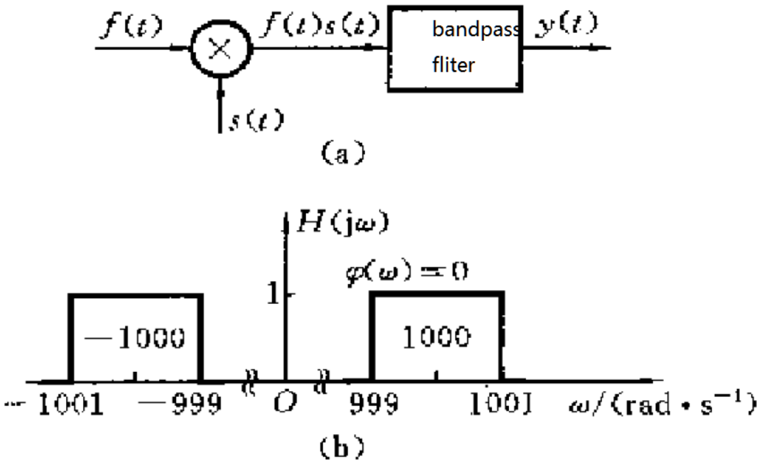

Figure 7: Singal and System

The inputs are $f(t) = \frac{sin(2*\pi t)}{2\pi t}$, $s(t) = cos(1000t)$. Determin the output signal $y(t)$
Solution: $f(t) = \frac{sin(2\pi t)}{2\pi t} \Leftrightarrow F(jw) = \frac{1}{2}g_{4\pi(w)}$

$$s(t) = cos(1000t) \Leftrightarrow S(jw) = \pi[\delta(w+1000) + \delta(w-1000)]$$

so

$$f(t)s(t) \Leftrightarrow \frac{1}{2\pi}F(jw) * S(jw) = \frac{1}{4}[g_{4\pi}(w+1000) + g_{4\pi}(w-1000)]$$

let $x(t) = f(t)s(t)$, then

$$X(jw) = \frac{1}{4}[g_{4\pi}(w+1000) + g_{4\pi}(w-1000)]$$

the output signal

$$Y(iw) = X(jw)H(jw) = \frac{1}{4}[g_2(w+1000) + g_2(w-1000)]$$

because $\frac{1}{\pi}Sa(t) \Leftrightarrow g_2(w)$, in conclusion

$$y(t) = \frac{1}{4\pi}Sa(t)e^{-j1000t} + \frac{1}{4\pi}Sa(t)e^{j1000t} = \frac{1}{2\pi}Sa(t)cos(1000t)$$

$$= \frac{sint}{2\pi t}cos(1000t)$$