Tutorial 3

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Assignment 2 is released

- **[must]** You are required to implement the basic iterative de Casteljau Bézier vertex evaluation algorithm. [30%]
- **[must]** You are required to construct Bézier surfaces with the normal evaluation at each mesh vertex. [40%]
- **[must]** You are required to render the Bézier surfaces based on the vertex array. [10%]
- **[must]** You are required to create more complex meshes by stitching multiple Bézier surface patches together. [20%]
- **[optional]** You may construct the B-Spline/NURBS surfaces. [15%]
- **[optional]** You may support the interactive editing (by selection) of control points. [10%]
- **[optional]** You may implement the adaptive mesh construction based on the curvature estimation. [15%]

Agenda

- Bézier iterative algorithm
- Bézier surface construction algorithm
- Stitching multiple Bézier surfaces
- Skeleton code
- Demo
- Report

Bézier iterative algorithm

Bézier curve

- Explicit definition

$$\mathbf{B}(t) = \sum_{i=0}^{n} \binom{n}{i} (1-t)^{n-i} t^i \mathbf{P}_i$$

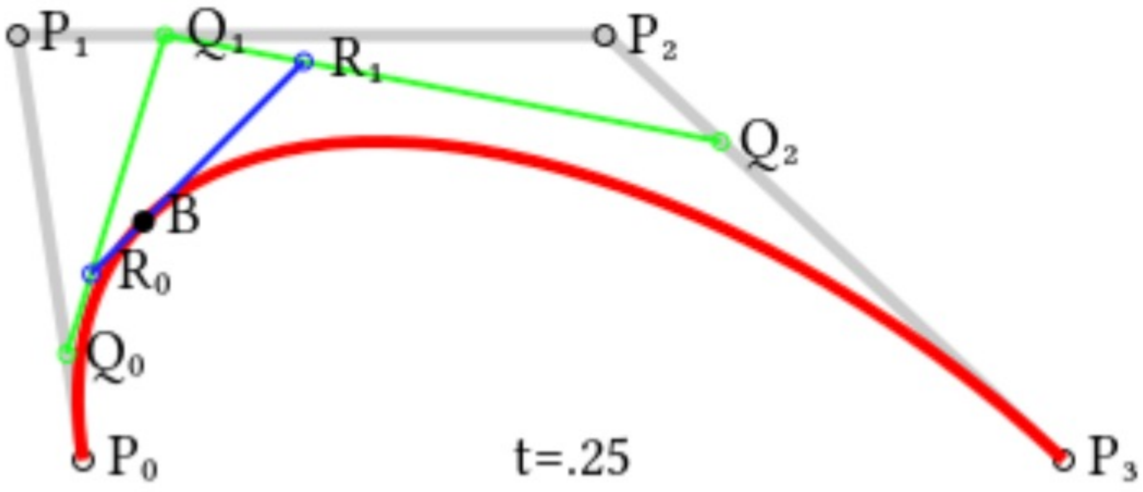
- Recursive definition

$$\mathbf{B}_{\mathbf{P}_0}(t) = \mathbf{P}_0, \text{ and}$$

$$\mathbf{B}(t) = \mathbf{B}_{\mathbf{P}_0\mathbf{P}_1\ldots\mathbf{P}_n}(t) = (1-t)\mathbf{B}_{\mathbf{P}_0\mathbf{P}_1\ldots\mathbf{P}_{n-1}}(t) + t\mathbf{B}_{\mathbf{P}_1\mathbf{P}_2\ldots\mathbf{P}_n}(t)$$

Evaluation of Bézier curve

- A simple way: Compute each term in the explicit defition
  - Not numerically stable
  - It could introduce numerical errors while evaluating the Bernstein polynomials
- De Casteljau's Algorithm



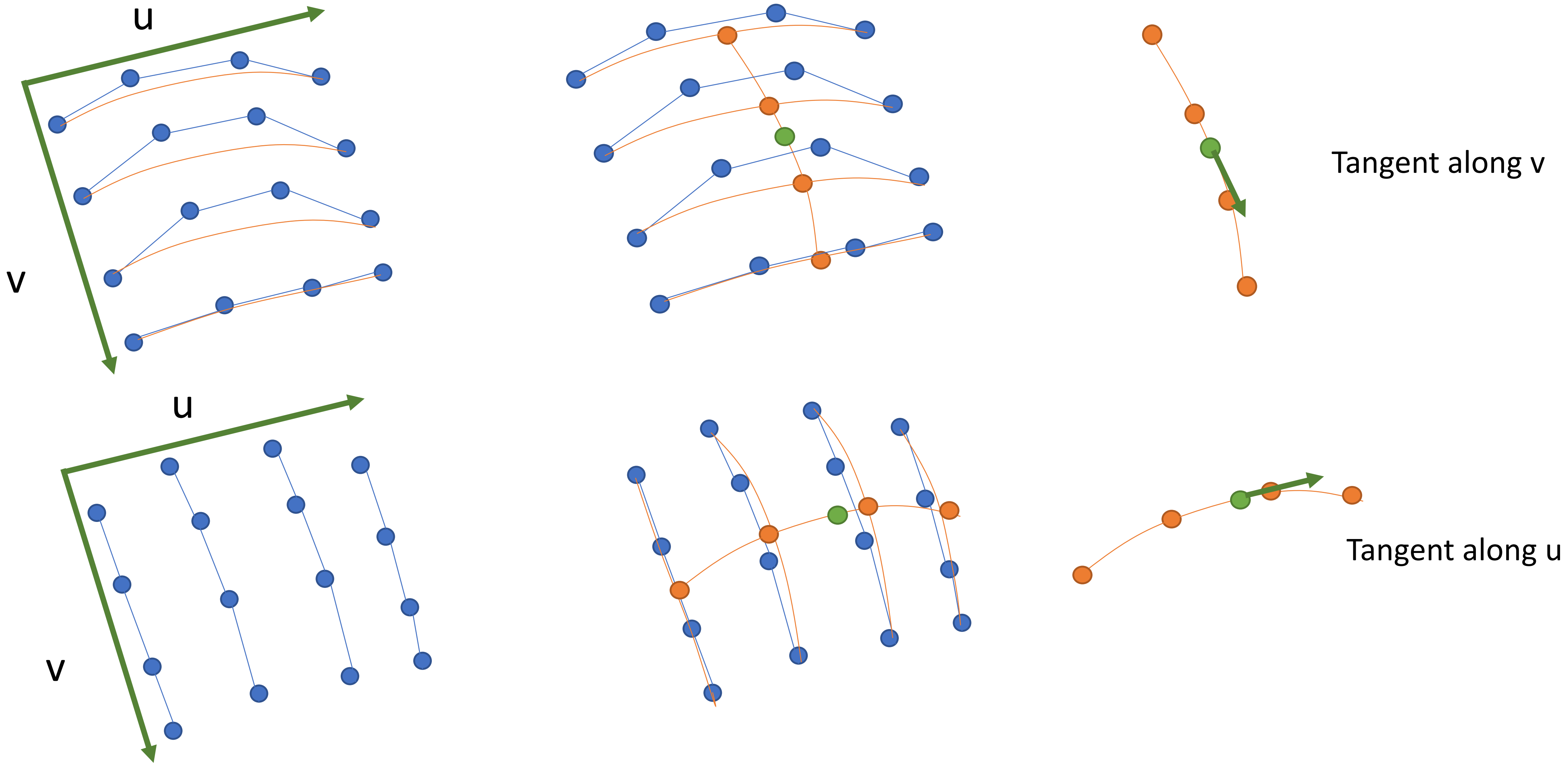
Bézier surface construction algorithm

Bézier surface

- A tenso rproduct of 1D Bézier curve

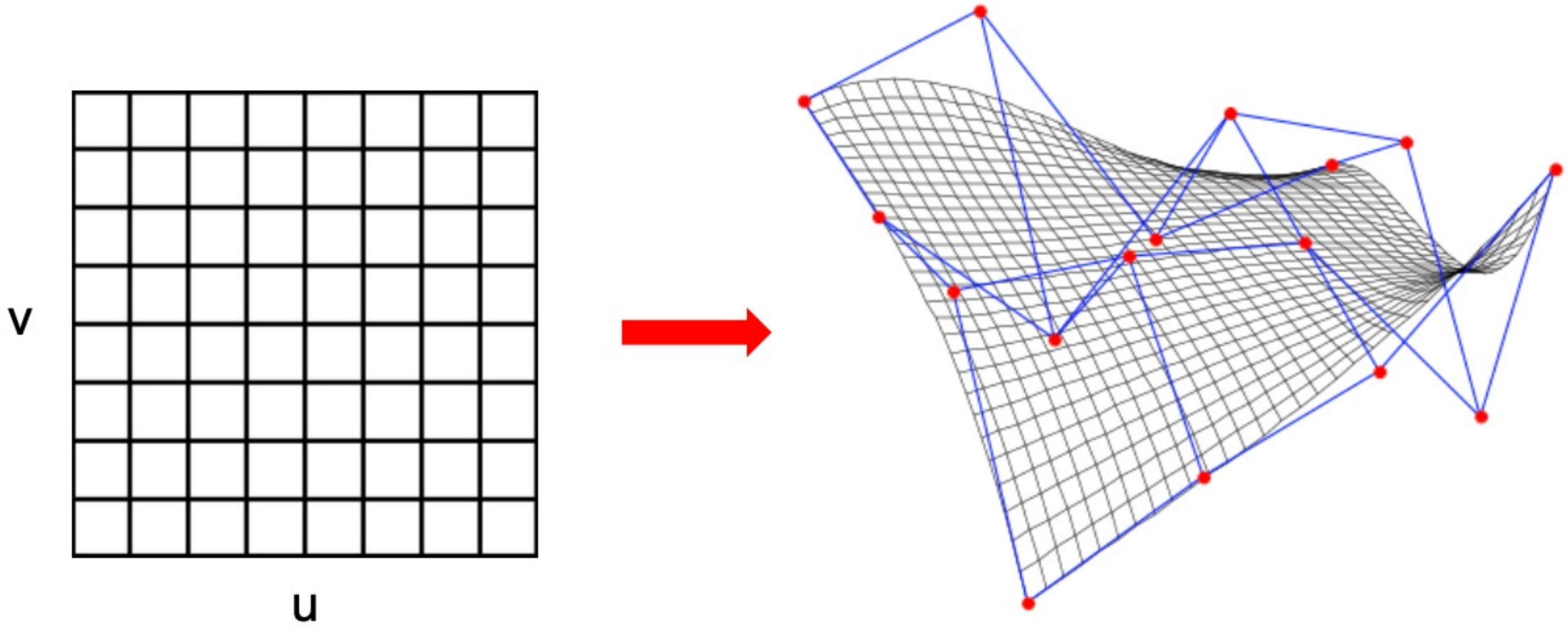
$$\mathbf{p}(u,v) = \sum_{i=0}^{n} \sum_{j=0}^{m} B_i^n(u)\; B_j^m(v)\; \mathbf{k}_{i,j}$$

Evaluation



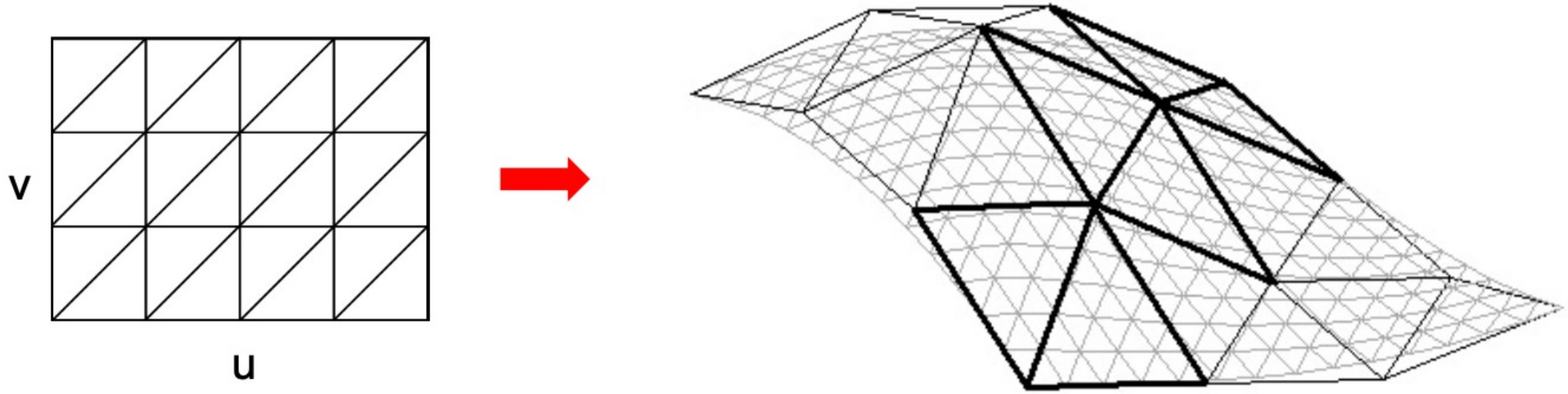
Meshing in parameter space

- **Meshing in parameter space**
  - gridding in u,v parameter space



Meshing in parameter space

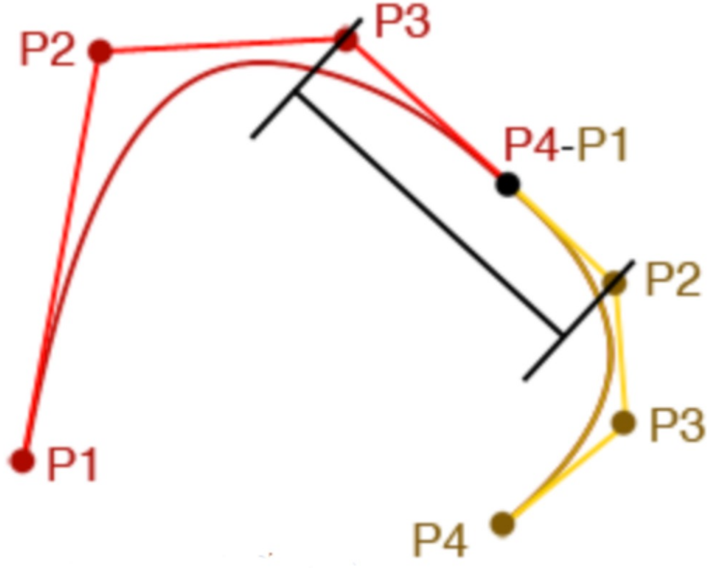
- **Meshing in parameter space**
  - triangulation in u,v parameter space



Stitching multiple Bézier surfaces

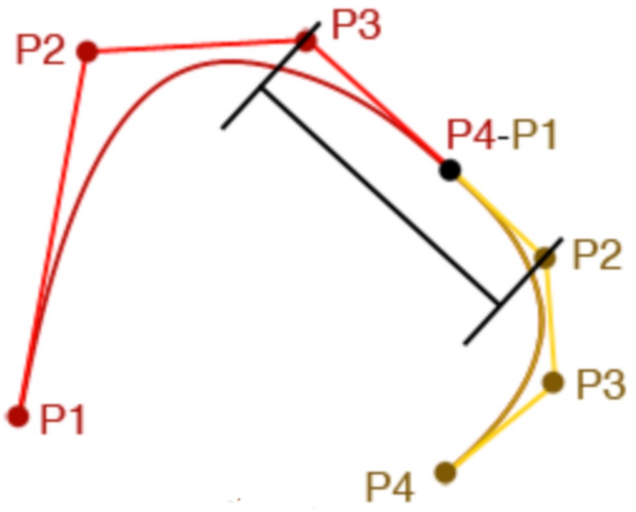
Continuity

- 0-th order(L0)
  - The curves/surfaces are connected only
- 1-st order(L1)
  - First derivatives are continous at the joint



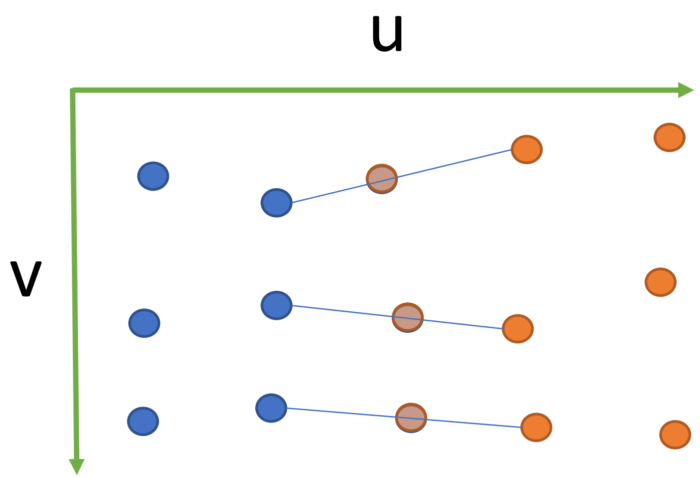
Bézier Curves with L1 Continuity

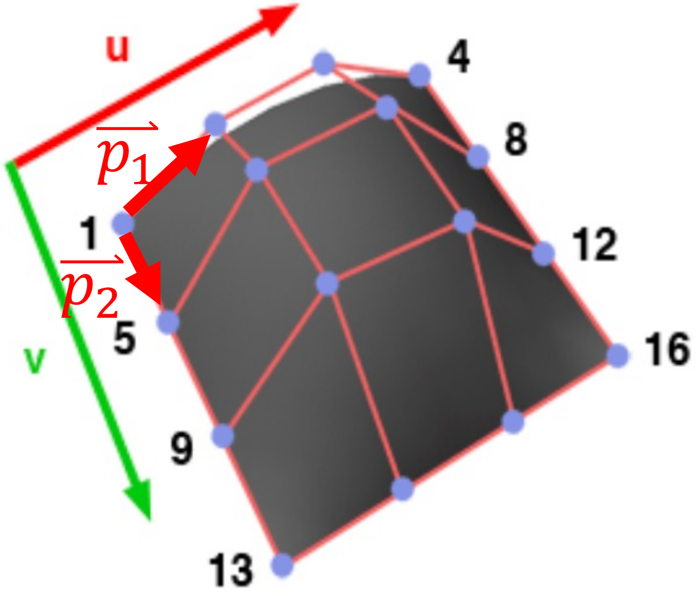
- The tangents are continous at the joint
  ⇒ Red P3P4 and yellow P1P2 are colinear
- Why?
  - Tangent of the red P4 is P4-P3
  - Tangent of the yellow P1 is P2-P1



Bézier Surfaces with L1 Continuity

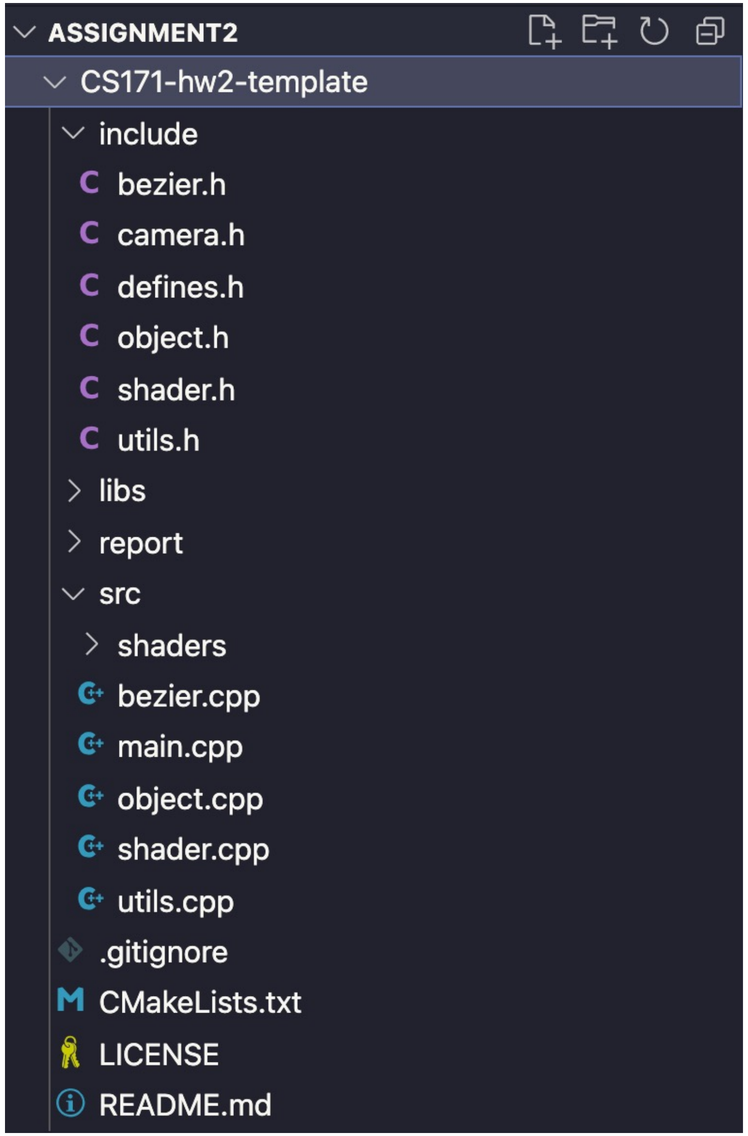
- The normals are continous at the joint
  - Tangents along u are continous
  - Tangents along v are continous
- Control points should be colinear with the points on both sides.
- Why?





Skeleton Code

Files



- bezier.h
  - Evaluation of bezier curves and bezier surfaces
  - Generating objects to render
- object.h
  - Functions to render objects

Demo

Report

Pay attention to the format!

- Do NOT use Chinese.
- Do NOT handwrite.

Good Luck!