Lecture 7-2 Spatial filtering 2

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Course piazza link： piazza.com/shanghaitech.edu.cn/spring2021/cs270spring2021

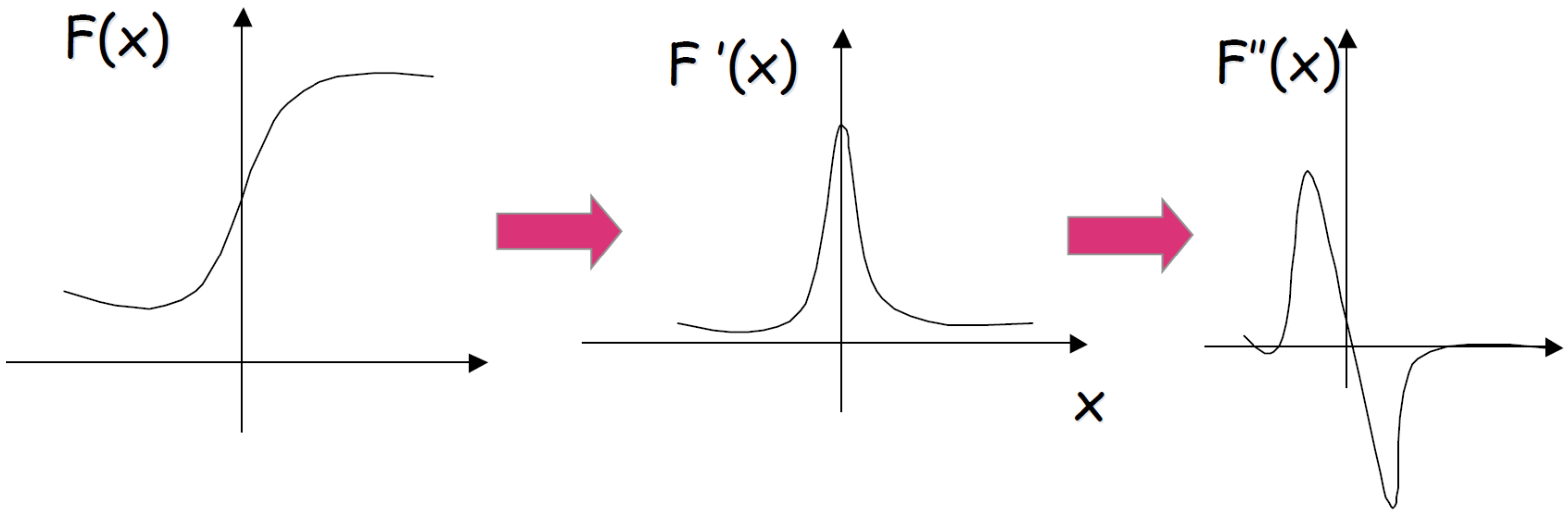
Outline

- **Sobel Filter**
- **Unsharpen Filter (非锐化掩蔽)**
- **LoG Filter**
  - useful for finding edges
  - also useful for finding blobs

Recall: First & Second-Derivative filters

- **Sharp changes in gray level of the input image corresponds to "peaks or valleys" of the first-derivative of the input signal.**

- **Peaks or valleys of the first derivative of the input signal, correspond to "zero-crossings" of the second-derivative of the input signal.**



Laplacian(拉普拉斯算子)

For an image function $\boldsymbol{f(x,y)}$,

X direction： $\dfrac{\partial^2 f}{\partial x^2} = f(x+1,y) + f(x-1,y) - 2f(x,y)$

Y direction： $\dfrac{\partial^2 f}{\partial y^2} = f(x,y+1) + f(x,y-1) - 2f(x,y)$

$$\nabla^2 f(x,y) = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

$$= f(x,y+1) + f(x,y-1) + f(x+1,y) + f(x-1,y) - 4f(x,y)$$

Laplacian Filter Masks

$$\nabla^2 f(x,y) = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = f(x,y+1) + f(x,y-1) + f(x+1,y) + f(x-1,y) - 4f(x,y)$$

0	1	0
1	−4	1
0	1	0

1	1	1
1	−8	1
1	1	1

0	−1	0
−1	4	−1
0	−1	0

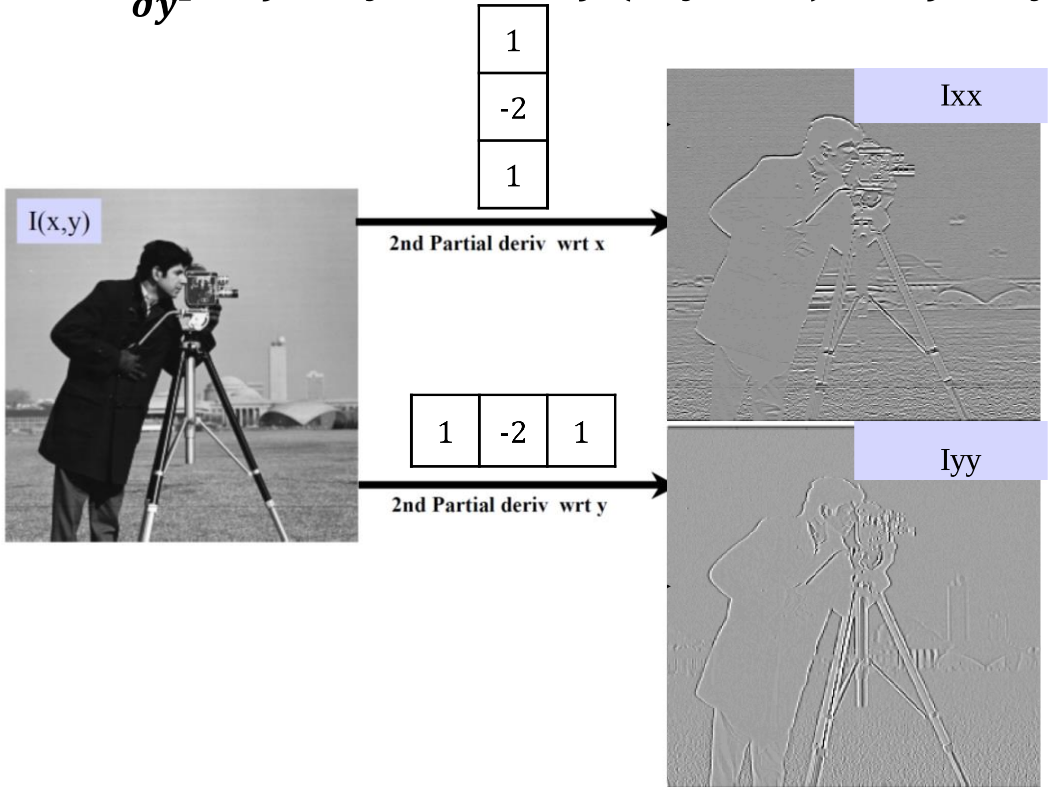
−1	−1	−1
−1	8	−1
−1	−1	−1

Laplacian(拉普拉斯算子)

For an image function $f(x, y)$,

X direction： $\frac{\partial^2 f}{\partial x^2} = f(x+1, y) + f(x-1, y) - 2f(x, y)$

Y direction： $\frac{\partial^2 f}{\partial y^2} = f(x, y+1) + f(x, y-1) - 2f(x, y)$



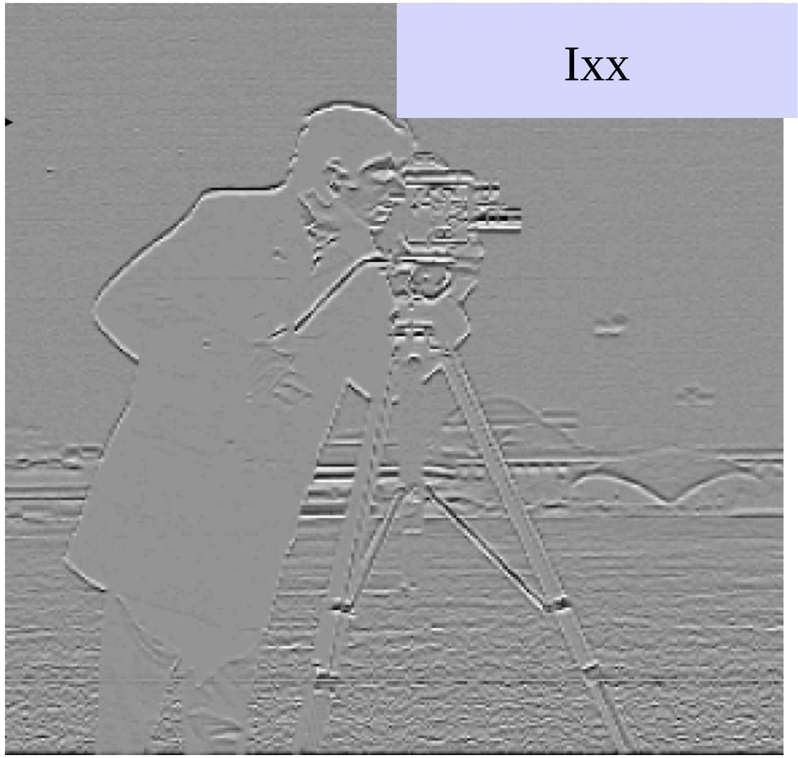
I_{yy}

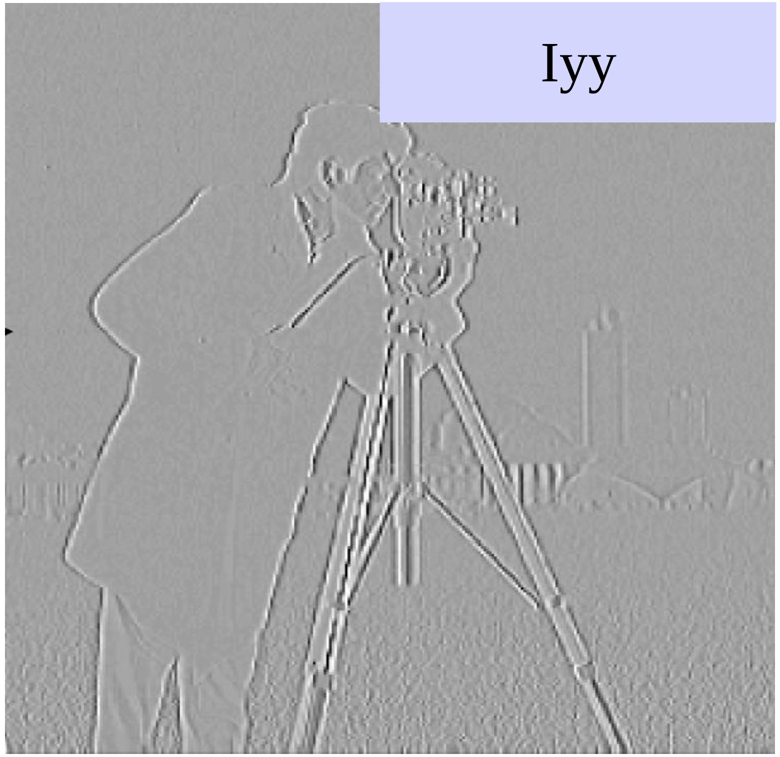
1	-2	1

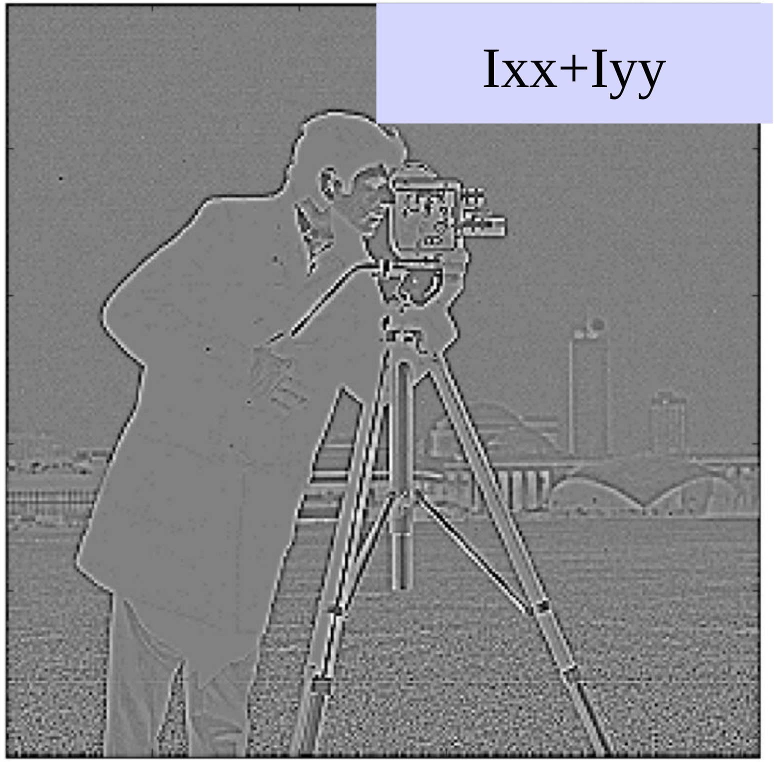
1
-2
1

I_{xx}

Laplacian(拉普拉斯算子)







Gradient(梯度)

The first-order derivative of $f(x,y)$: $\nabla \boldsymbol{f} \equiv \mathbf{grad}(\boldsymbol{f}) \equiv \begin{cases} \boldsymbol{g_x} \\ \boldsymbol{g_y} \end{cases} = \begin{cases} \dfrac{\partial \boldsymbol{f}}{\partial \boldsymbol{x}} \\ \dfrac{\partial \boldsymbol{f}}{\partial \boldsymbol{y}} \end{cases}$

The amplitude： $\boldsymbol{M(x,y)} = \mathbf{mag}(\nabla \boldsymbol{f}) = \sqrt{\boldsymbol{g_x^2} + \boldsymbol{g_y^2}}$

$$\boldsymbol{M(x,y)} \approx |\boldsymbol{g_x}| + |\boldsymbol{g_y}|$$

Gradient(梯度)

➢ **Roberts cross-gradient operator（罗伯特交叉梯度算子）**

$$\boldsymbol{M(x,y) \approx |g_x| + |g_y|}$$

$$\boldsymbol{= |z_9 - z_5| + |z_8 - z_6|}$$

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

-1	0
0	1

0	-1
1	0

Gradient(梯度)

➢ **Sobel operator （Sobel算子）**

$$M(x,y) = |(z_7+2z_8+z_9)-(z_1+2z_2+z_3)| + |(z_3+2z_6+z_9)-(z_1+2z_4+z_7)|$$

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

−1	−2	−1
0	0	0
1	2	1

−1	0	1
−2	0	2
−1	0	1

Sobel operator

More Sobel operators

−1	−1	−1
2	2	2
−1	−1	−1

Horizontal

2	−1	−1
−1	2	−1
−1	−1	2

+45°

−1	2	−1
−1	2	−1
−1	2	−1

Vertical

−1	−1	2
−1	2	−1
2	−1	−1

−45°

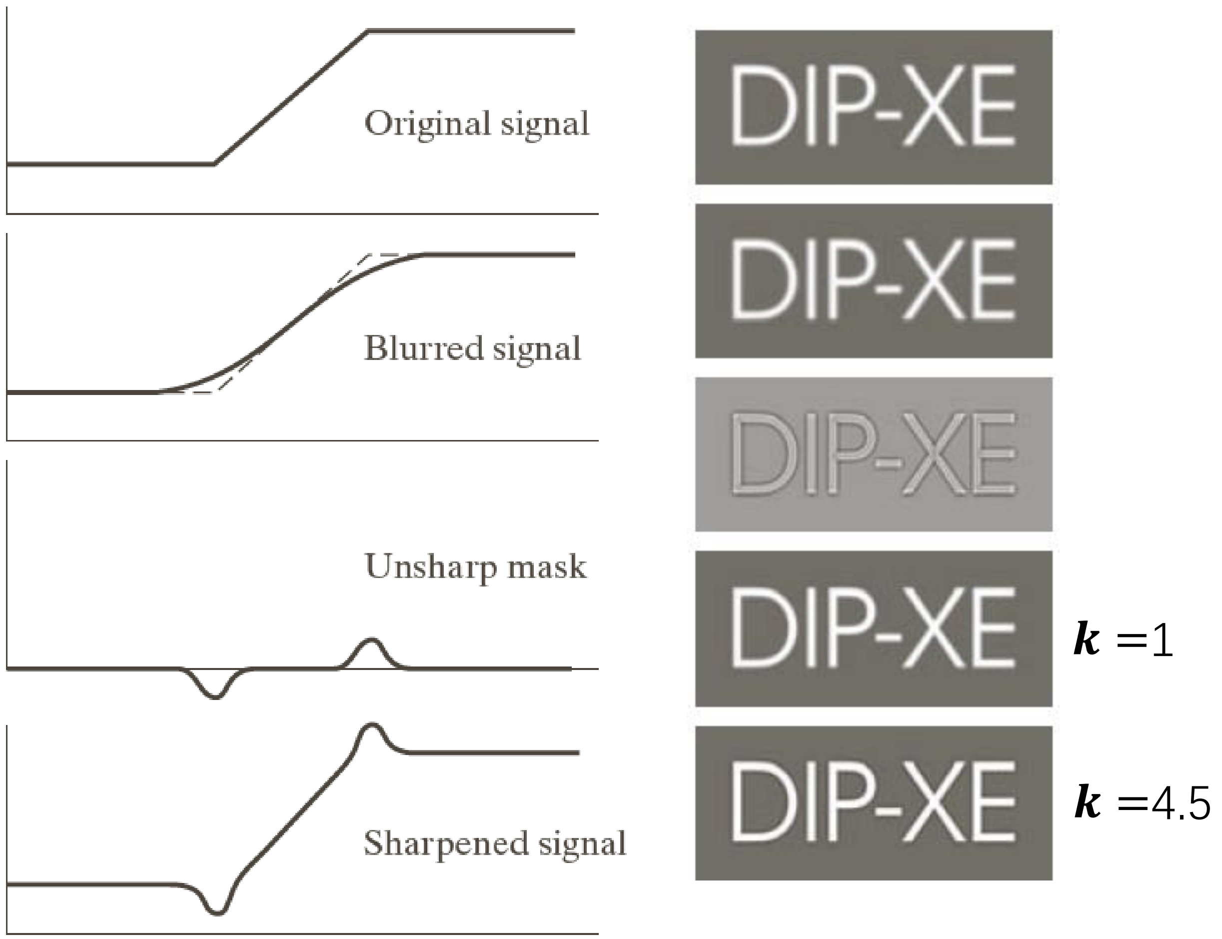
The Notes about the Laplacian

- $\nabla^2 I(x, y)$ is a SCALAR
  - – ↑ Can be found using a SINGLE mask
  - – ↓ Orientation information is lost
- $\nabla^2 I(x, y)$ is the sum of SECOND-order derivatives
  - – But taking derivatives increases noise.
  - – Very noise sensitive!
- It is always combined with a smoothing operation.

Unsharpen Mask(非锐化掩蔽)

$$g_{\text{mask}}(x,y) = f(x,y) - \overline{f(x,y)}$$

$$g(x,y) = f(x,y) + k * g_{\text{mask}}(x,y)$$



Laplacian of Gaussian (LoG) Filter

- **First smooth (Gaussian filter),**
- **Then, find zero-crossings (Laplacian filter):**

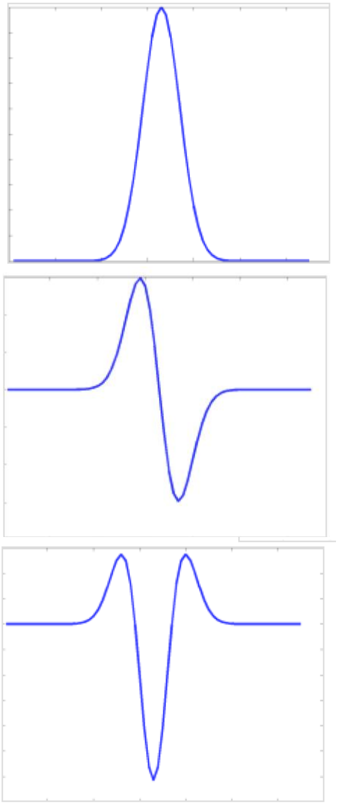
$$\nabla^2\,(G(x,y))$$

$$G(x,y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

$$G_x(x,y) = -\frac{1}{2\pi\sigma^4} x e^{-\frac{x^2+y^2}{2\sigma^2}},\ G_y(x,y) = -\frac{1}{2\pi\sigma^4} y e^{-\frac{x^2+y^2}{2\sigma^2}},$$

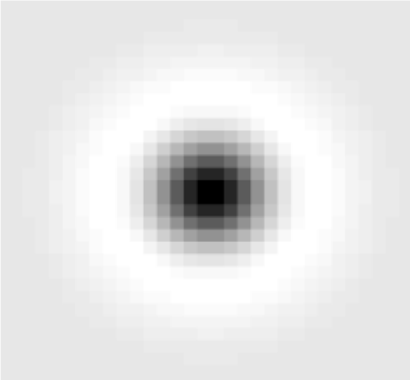
$$G_{xx}(x,y) = -\frac{1}{2\pi\sigma^4}\left(1-\frac{x^2}{\sigma^2}\right) e^{-\frac{x^2+y^2}{2\sigma^2}},\ G_{yy}(x,y) = -\frac{1}{2\pi\sigma^4}\left(1-\frac{y^2}{\sigma^2}\right) e^{-\frac{x^2+y^2}{2\sigma^2}}$$

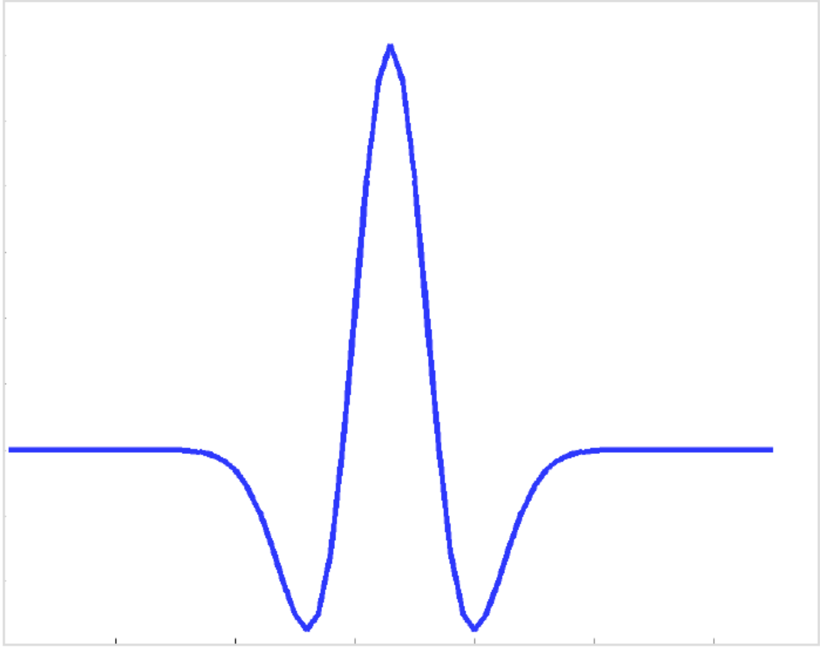
$$\nabla^2\,(G(x,y)) = -\frac{1}{\pi\sigma^4}\left(1-\frac{x^2+y^2}{2\sigma^2}\right) e^{-\frac{x^2+y^2}{2\sigma^2}}$$



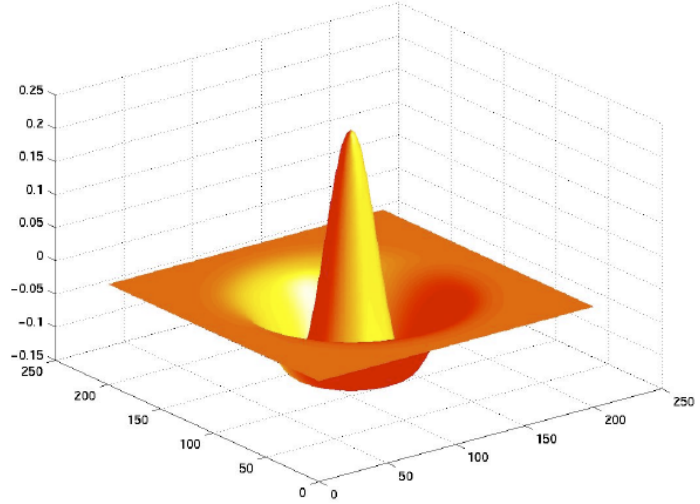
Second derivative of a Gaussian

$$\nabla^2\,(G(x,y)) = -\frac{1}{\pi\sigma^4}\left(1-\frac{x^2+y^2}{2\sigma^2}\right)e^{-\frac{x^2+y^2}{2\sigma^2}}$$





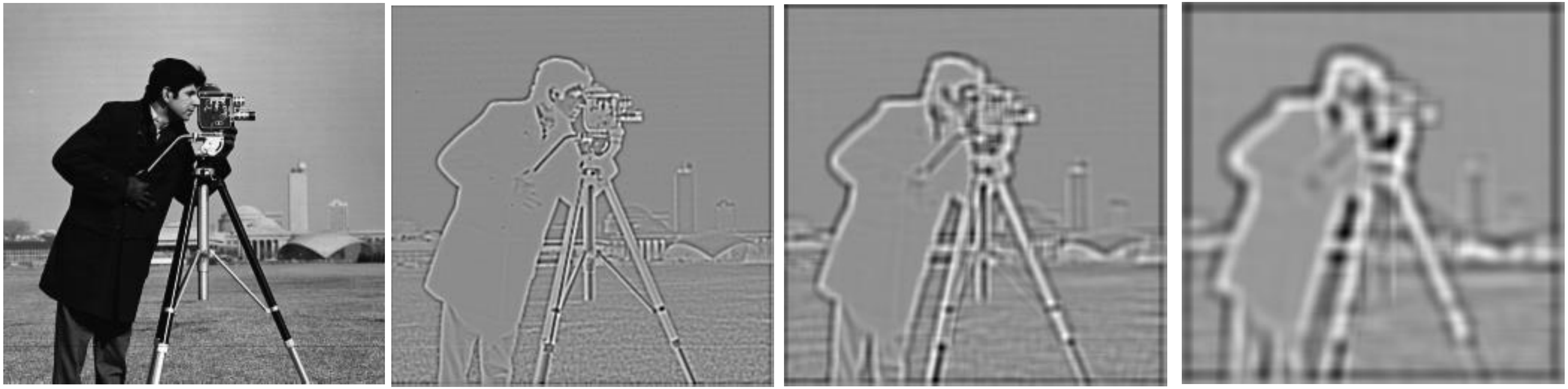
2D analog



LoG "Mexican Hat"

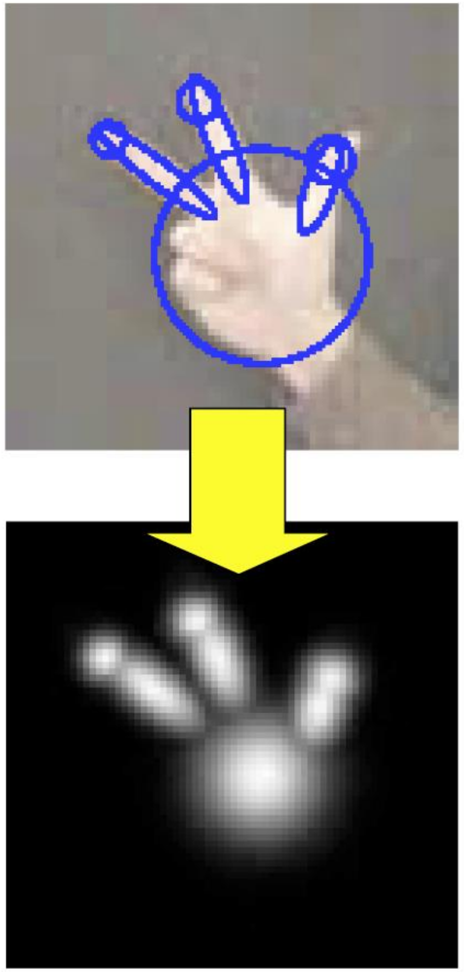
Effect of LoG Filter

Sigma = 1 Sigma = 4 Sigma = 10



Band-Pass Filter (suppresses both high and low frequencies)

Application of LoG Filter

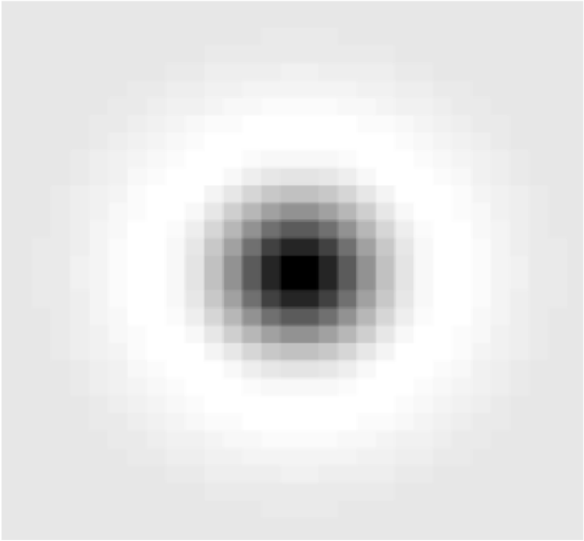


Gesture recognition for
the ultimate couch potato

Take home message

- **Key idea: Cross correlation with a filter can be viewed as comparing a little "picture" of what you want to find against all local regions in the image.**

LoG



Derivative of Gaussian