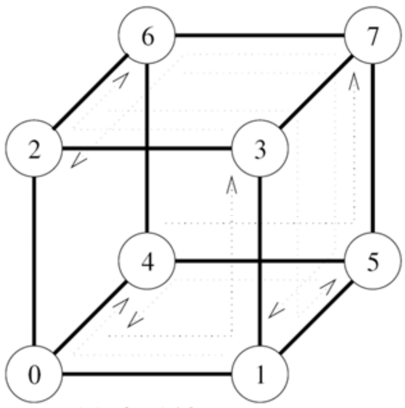
CS121 Problem Set 3

Due: 23:59, April 13, 2021

1. Submit your solutions to Gradescope (www.gradescope.com).
2. In "Account Settings" in Gradescope, set FULL NAME to your Chinese name and enter your STUDENT ID.
3. If you submit handwritten solutions, write neatly and submit a clear scan.
4. When submitting your homework, be sure to match each of your solutions to the corresponding problem number.

1) Consider the problem of performing a circular q-shift in a p-node hypercube, for $0 < q < p$, using *E-cube routing*. Recall that E-cube routing from a node x to y consists of writing x and y in binary, and routing along the dimensions in which x and y differ, in order of increasing dimension.
Show that if links are bidirectional, then for any q, all p data paths in a circular q-shift are congestion-free when E-cube routing is used. For example, Figure Q1 shows a congestion-free routing of a 3-shift in an 8 node hypercube. Conclude that any q-shift can be performed in $t_s + t_w m$ time when all messages have size m.



(c) 3-shift

Figure Q1

2) Consider the following sequential *rank sort* algorithm for *n* values assuming no duplicate values:

```
for (i = 0; i < n; i++) {
    x = 0;
    for (j = 0; j < n; j++)
        if (a[i] > a[j]) x++;
    b[x] = a[i];
}
```

(a) Rewrite this as a parallel algorithm using OpenMP assuming that $p < n$ threads are used. Indicate clearly the use of private and shared variables and the schedule type.

(b) Modify the OpenMP code in part (a) to handle duplicates in the list of values, i.e. to sort into non-decreasing order. For example, the list of values [3, 5, 7, 5, 7, 9, 2, 3, 6, 7, 8, 1] should give the sorted list [1, 2, 3, 3, 5, 5, 6, 7, 7, 7, 8, 9].

3) The following sequential code calculates a triangular matrix using a function **`calc(i,j)`**, which has no data dependences and requires a constant (but large) amount of computation.

```
for (i = 0; i < n; i++)
   for (j = 0; j <= i; j++)
      a[i,j] = calc(i,j);
```

By inserting OpenMP directives into the sequential code, show how the following schemes for assigning work to threads may be implemented on a shared memory parallel architecture, commenting on the efficiency of each scheme:

a) A static block assignment of contiguous rows to threads.

b) A static cyclic assignment of single rows to threads.

c) A dynamic assignment of single rows to threads.

4) The *Back Substitution* algorithm solves a set of linear equations in upper (or lower) triangular form, as shown in Figure Q4a. A sequential algorithm to solve such a set of linear equations is given in Figure Q4b. Design a parallel algorithm for a shared memory architecture and express it using OpenMP assuming that p < n threads are used. What schedule type would you use?

$$
\begin{array}{llll}
a_{n-1,0}x_0 + a_{n-1,1}x_1 + a_{n-1,2}x_2 & \ldots & + a_{n-1,n-1}x_{n-1} & = b_{n-1} \\
 & . & & \\
 & . & & \\
a_{2,0}x_0 + a_{2,1}x_1 + a_{2,2}x_2 & & & = b_2 \\
a_{1,0}x_0 + a_{1,1}x_1 & & & = b_1 \\
a_{0,0}x_0 & & & = b_0
\end{array}
$$

Figure Q4a

```
/* Back Substitution */
for (i = 0; i < n; i++) {
    x[i] = b[i]/a[i][i];
    for (j = i+1; j < n; j++) {
        b[j] = b[j] - a[j][i]*x[i];
        a[j,i] = 0;
    }
```

```
}
```

Figure Q4b