Problem 1

(15 points)

Determine the Laplace transform by definition and the associated ROC and pole-zero plot for each of the following functions of time.

(a) $x(t) = e^{2t}u(-t) + e^{3t}u(-t)$

(b) $x(t) = \delta(3t) + u(3t)$

(c) $x(t) = |t|e^{-2|t|}$

Problem 2

(15 points)
A causal LTI system with impulse response h(t) has the following properties:
1. When the input to the system is $x(t) = e^{2t}$ for all t, the output is $y(t) = \frac{1}{6}e^{2t}$ for all t.
2. The impulse response h(t) satisfies the differential equation:

$$\frac{dh(t)}{dt} + 2h(t) = e^{-4t}u(t) + bu(t),$$

where b is an unknown constant.
Determine the system function H (s) of the system, consistent with the information above. There should be no unknown constants in your answer; that is, the constant b should not appear in the answer.

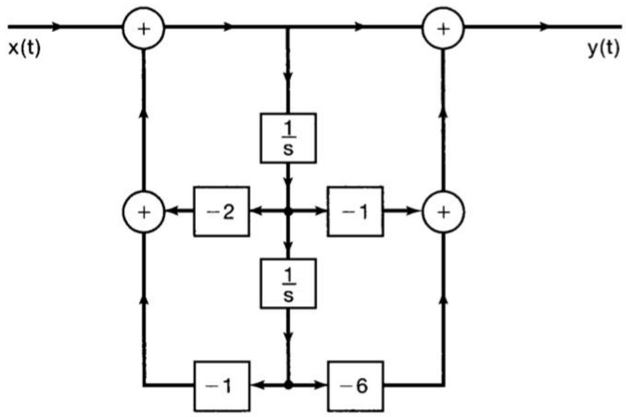
Problem 3

(20 points)
We are given the following five facts about a real signal x(t) with Laplace transform X(s):
1. X(s) has exactly two poles.
2. X(s) has no zeros in the finite s-plane.
3. X(s) has a pole at s = -1 + j.
4. $e^{2t}x(t)$ is not absolutely integrable.
5. X(0) = 8.

Determine X(s) and specify its region of convergence.

Problem 4

(25 points)
The input x(t) and output y(t) of a causal LTI system are related through the block- diagram representation shown below.



(a) Determine a differential equation relating y(t) and x(t).

(b) Is this system stable?

Problem 5

(25 points)
Consider the system S characterized by the differential equation

$$\frac{d^3y(t)}{dt^3} + 6\frac{d^2y(t)}{dt^2} + 11\frac{dy(t)}{dt} + 6y(t) = x(t)$$

(a) Determine the zero-state response of this system for the input $x(t) = e^{-4t}u(t)$.

(b) Determine the zero-input response of the system for $t > 0^-$, given that

$$y(0^-) = 1,\quad \lim_{t=0^-}\frac{dy(t)}{dt} = -1,\quad \lim_{t=0^-}\frac{d^2y(t)}{dt^2} = 1$$

(c) Determine the output of S when the input is $x(t) = e^{-4t}u(t)$ and the initial conditions are the same as those specified in part (b).