

Homework 5

Rules:

- 1、Work on your own. Discussion is permissible, but extremely similar submissions will be judged as plagiarism.
- 2、Please show all intermediate steps: a correct solution without an explanation will get zero credit.
- 3、Please submit on time. No late submission will be accepted.
- 4、Please prepare your submission in English only. No Chinese submission will be accepted.

1、Simplify the following expressions and give the answer with exponential form.

$$(a) \frac{j*(-5+3j)-5}{\frac{8-3j}{6+j}-(1-6j)*(8-9j)}$$

$$(b) (7-8j)^2 \sqrt{(10-j) * (4+7j)}$$

$$(c) \frac{(3\angle 75^\circ + 10\angle 108^\circ) * 7j}{(3-9j) * (-1-4j) * 8\angle 10^\circ}$$

$$(a) 0.13e^{j161.99}$$

$$(b) 1.02*10^3 * e^{-j70.35} \text{ or } 1.02*10^3 * e^{j109.65}$$

$$(c) 0.28e^{-j3.84}$$

每小问4分，一共12分

2、Simplify the following expressions by using phasors

- (a) $i_1(t) = 30\sin(\omega t - 97^\circ) + 65\sin(\omega t + 56^\circ)$ A
 (b) $i_2(t) = 106\sin(4t + 348^\circ) - 43\cos(4t - 134^\circ)$ mA
 (c) $i_3(t) = 79\cos(45t - 36^\circ) + 54\sin(45t)$ μ A
 (d) $v_1(t) = 256\sin(\omega t + 1^\circ) + 156\cos(\omega t - 173^\circ)$ V
 (e) $v_2(t) = 13\cos(45t - 60^\circ) + 55\cos(45t + 30^\circ)$ mV
 (f) $v_3(t) = 30\sin(99t) - 65\cos(99t)$ μ V

$$(a) \quad I_1 = 30 \angle -187^\circ + 65 \angle -34^\circ = 24.11 - j32.69 = 40.62 \angle -53.59^\circ \quad A$$
~~$$i_1(t) = 40.62 \cos(\omega t - 53.59^\circ)$$~~

$$i_1(t) = 40.62 \cos(\omega t - 53.59^\circ) \quad A$$

$$(b) \quad I_2 = 106 \angle 58^\circ - 43 \angle -134^\circ = 7.83 - j72.75 = 73.17 \angle -83.86^\circ \quad mA$$

$$i_2(t) = 73.17 \cos(4t - 83.86^\circ) \quad mA$$

$$(c) \quad I_3 = 79 \angle -36^\circ + 54 \angle -90^\circ = 63.91 - j100.44 = 119.05 \angle -57.53^\circ \quad \mu A$$

$$i_3(t) = 119.05 \cos(45t - 57.53^\circ) \quad \mu A$$

$$(d) \quad V_1 = 256 \angle -89^\circ + 156 \angle -173^\circ = -150.37 - j274.97 = 313.40 \angle -118.67^\circ \quad V$$

$$v_1(t) = 313.40 \cos(\omega t - 118.67^\circ) \quad V$$

$$(e) \quad V_2 = 13 \angle -60^\circ + 55 \angle 30^\circ = 54.13 + j16.24 = 56.51 \angle 16.70^\circ \quad mV$$

$$v_2(t) = 56.51 \cos(45t + 16.70^\circ) \quad mV$$

$$(f) \quad V_3 = 30 \angle -90^\circ - 65 \angle 0^\circ = -65 - j30 = 71.59 \angle -155.22^\circ \quad \mu V$$

$$v_3(t) = 71.59 \cos(99t - 155.22^\circ) \quad \mu V$$

每小问2分，一共12分

3、 Find steady state solution of $v(t)$ or $i(t)$ in the following differential equations using the phasor approach:

(a) $i(t) - 5 \frac{di(t)}{dt} = 48\sin(4t)$

(b) $6 \int i(t) dt - 19i(t) + 8 \frac{di(t)}{dt} = 59\cos(10t + 70^\circ)$

(c) $9v(t) - 13 \int v(t) dt = 32\sin(15t + 10^\circ)$

(d) $7 \int v(t) dt - 24v(t) + 3 \frac{dv(t)}{dt} = 108\cos(46t - 9^\circ)$

(a) $I - 5j\omega I = 48\angle -90^\circ \quad \omega = 4$
 $I = \frac{48\angle -90^\circ}{1 - j20} = 2.40\angle -2.86^\circ$
 $i(t) = 2.40\cos(4t - 2.86^\circ)$

(b) $\frac{6I}{j\omega} - 19I + 8j\omega I = 59\angle 70^\circ \quad \omega = 10$
 ~~$I = \frac{59\angle 70^\circ}{-19 + j79.4}$~~
 $I = \frac{59\angle 70^\circ}{-19 + j79.4} = 0.72\angle -33.46^\circ$

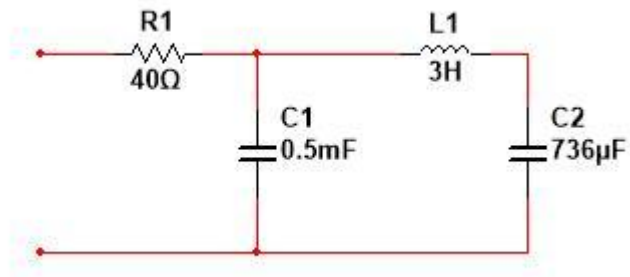
(c) $9V - \frac{13V}{j\omega} = 32\angle -80^\circ \quad \omega = 15$
 $V = \frac{32\angle -80^\circ}{9 - j0.87} = 3.54\angle -74.48^\circ$
 $v(t) = 3.54\cos(15t - 74.48^\circ)$

(d) $\frac{7V}{j\omega} - 24V + 3j\omega V = 108\angle -9^\circ \quad \omega = 46$
 $V = \frac{108\angle -9^\circ}{-24 + j137.85} = 0.77\angle -108.88^\circ$
 $v(t) = 0.77\cos(46t - 108.88^\circ)$

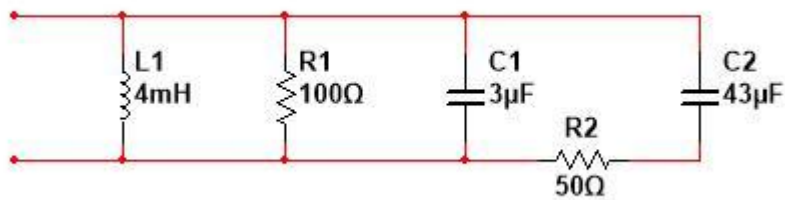
每小问2分，一共8分

4、 Determine the equivalent impedance:

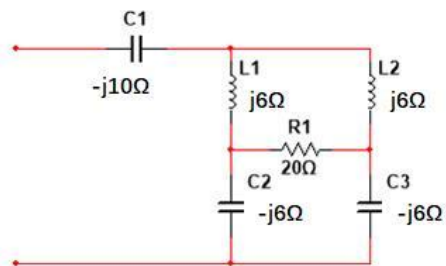
(a) $\omega=500\text{rad/s}$



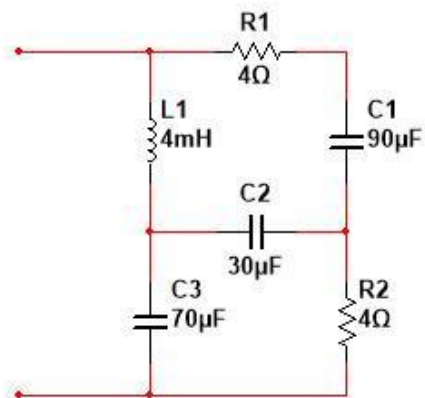
(b) $f=2000\text{Hz}$

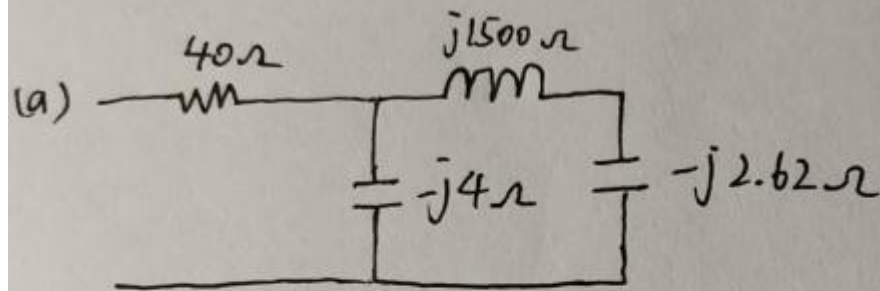


(c)



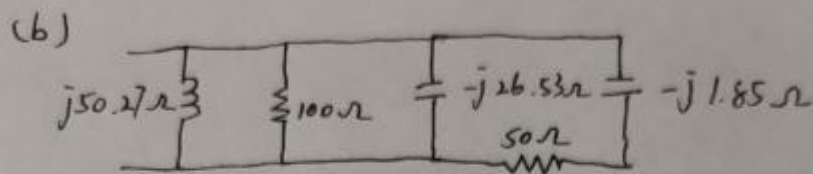
(d) $\omega=50000\text{rad/s}$





$$Z_{eq} = 40 + (-j4) // (j1500 - j2.62)$$

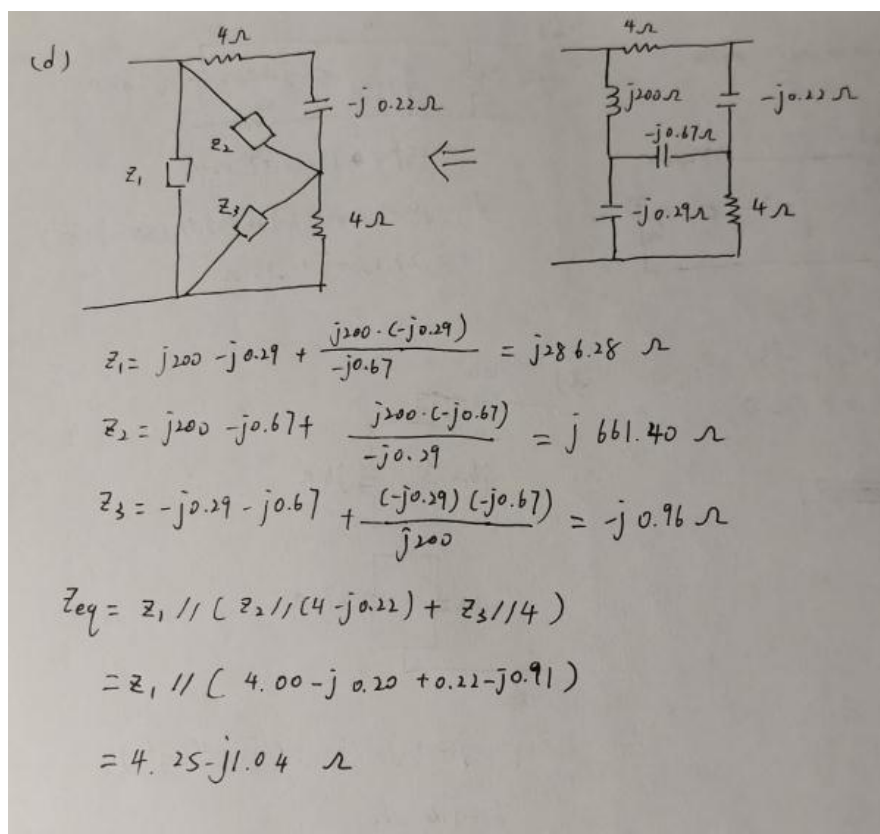
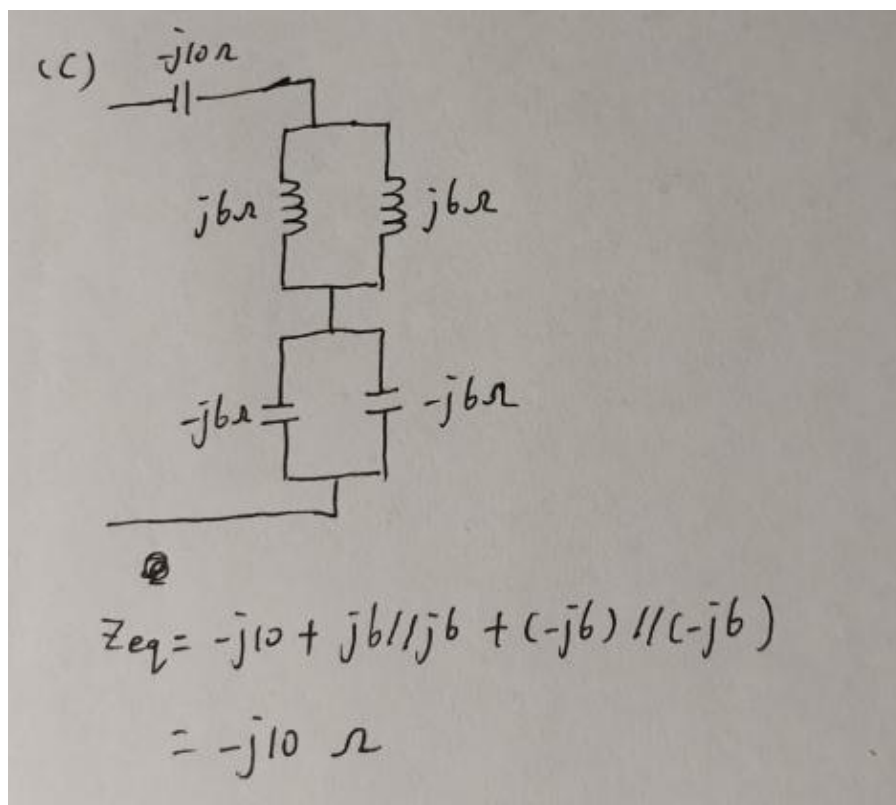
$$= 40 - j4.01 \Omega$$



$$\omega = 2\pi f = 12566.37 \text{ rad/s}$$

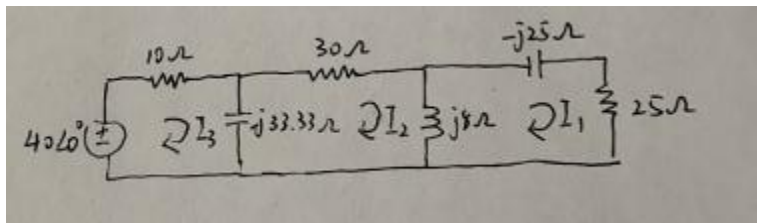
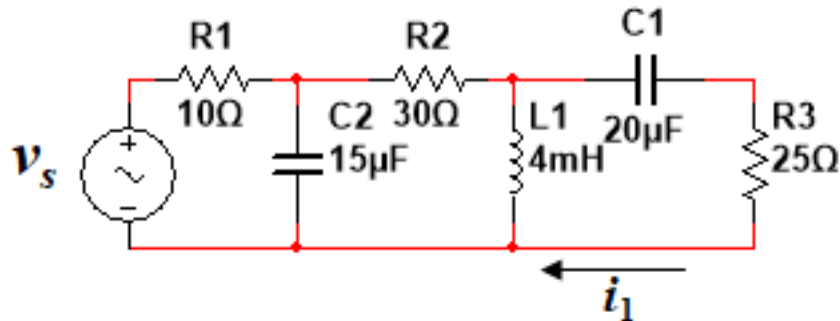
$$Z_{eq} = j50.27 // 100 // (-j26.53) // (50 - j1.85)$$

$$= 24.13 - j14.93 \Omega$$



每小问2分，一共8分

5、 The circuit is operating in the sinusoidal steady state. Find $i_1(t)$ if $v_s(t) = 40\cos(2000t)$ V



$$\begin{cases} -40\angle 0^\circ + I_3 10 - (I_3 - I_2) j33.33 = 0 \\ -j33.33(I_2 - I_3) + 30I_2 + j8(I_2 - I_1) = 0 \\ 25I_1 + j8(I_1 - I_2) - j25I_1 = 0 \end{cases}$$

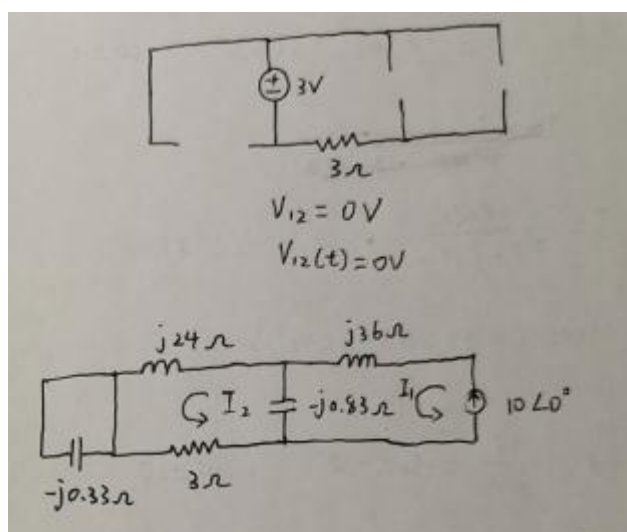
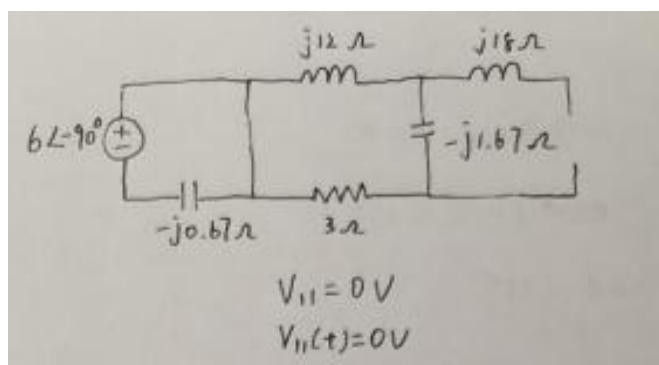
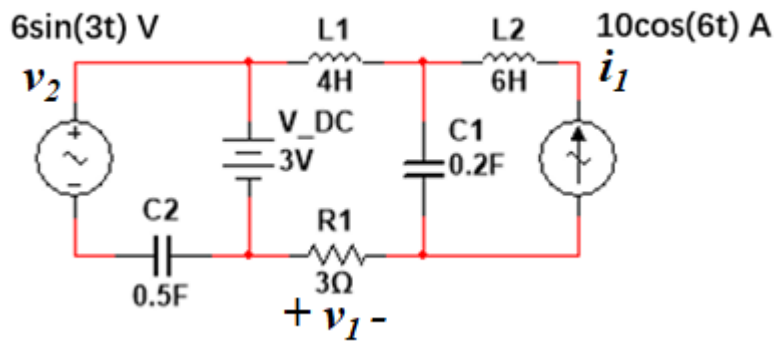
$$I_1 = 0.24\angle 98.57^\circ \text{ A}$$

$$\hat{i}_2(t) = 0.24 \cos(2000t + 98.57^\circ) \text{ A}$$

图片中最后一行写错，应将 $i_2(t)$ 改为 $i_1(t)$ 。

列出方程组3分，解出 I_1 4分，最后换成 $i_1(t)$ 3分，一共10分

6、 The circuit is operating in the sinusoidal steady state. Find v_1 in the circuit using superposition.



$$\begin{cases} I_2 \cdot j24 + I_2 \cdot 3 + (I_2 - I_1) \cdot (-j0.83) = 0 \\ I_1 = 10 \angle 0^\circ \end{cases}$$

$$I_2 = 0.36 \angle -172.62^\circ \text{ A}$$

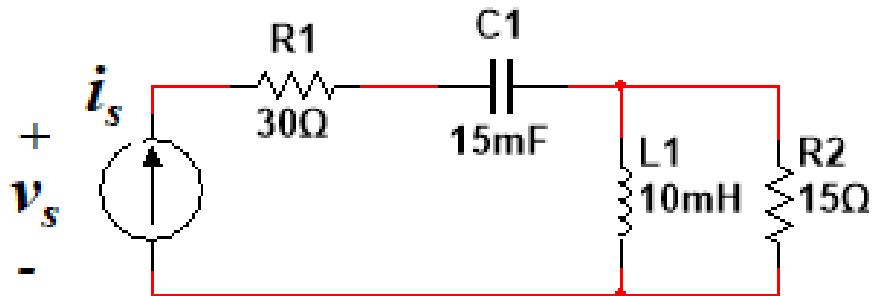
$$V_3 = I_2 R = 1.08 \angle -172.62^\circ \text{ V}$$

$$V_3(t) = 1.08 \cos(6t - 172.62^\circ) \text{ V}$$

$$V_1(t) = V_{11}(t) + V_{12}(t) + V_{13}(t) = 1.08 \cos(6t - 172.62^\circ) \text{ V}$$

三个电源分别计算出的 V_1 各4分，最后得出总 V_1 3分，一共15分

- 7、 Find the value of ω at which $v_s(t)$ and $i_s(t)$ in the circuit are in-phase (in-phase means that there is no imaginary part in the total Z_{eq}).

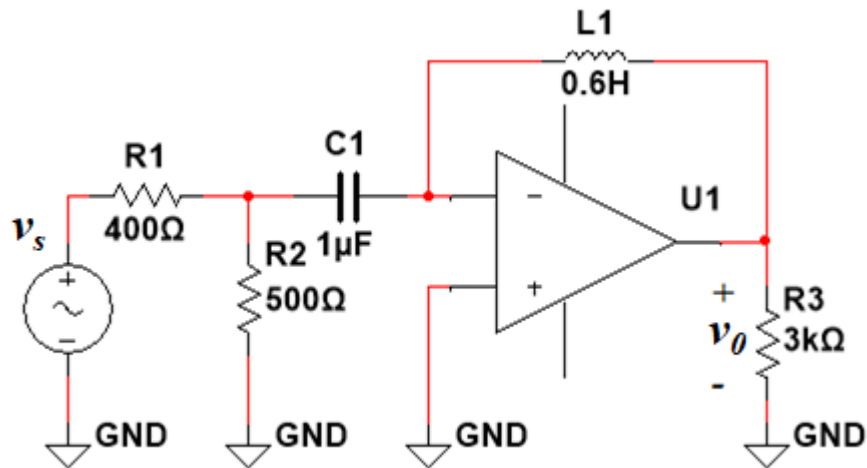


$$\begin{aligned}
 Z_{eq} &= 30 + \frac{1}{j\omega 0.015} + j\omega 0.01 \parallel 15 \\
 &= 30 + \frac{1}{j\omega 0.015} + \frac{j\omega 0.01 \times 15}{15 + j\omega 0.01} \\
 &= 30 - j \frac{1}{\omega 0.015} + \frac{j\omega 225 \times 0.01 - \omega^2 1.5 \times 10^{-5}}{225 + \omega^2 10^{-4}} \\
 &= 30 - \frac{\omega^2 1.5 \times 10^{-5}}{225 + \omega^2 10^{-4}} + j \left(\frac{\omega 2.25}{225 + \omega^2 10^{-4}} - \frac{1}{0.015\omega} \right) \\
 \omega 2.25 \times 0.015\omega - 225 \omega^2 10^{-4} &= 0 \\
 \omega &= 81.77 \text{ rad/s}
 \end{aligned}$$

算出总的等效电阻的式子3分，列出方程3分，得到 ω 4分，一共10分。

8、 The sinusoidal voltage source in the circuit is generating the voltage. If the op amp is ideal, what is the steady-state expression for $v_o(t)$?

$$v_s(t) = 2\cos(1000t) \text{ V}$$



Handwritten solution showing the circuit diagram and the following equations:

$$\frac{2\angle 0^\circ - V_1}{400} = \frac{V_1}{500} + \frac{V_1 - V_-}{j1000}$$

$$\frac{V_1 - V_-}{j1000} + \frac{V_0 - V_-}{j600} = 0$$

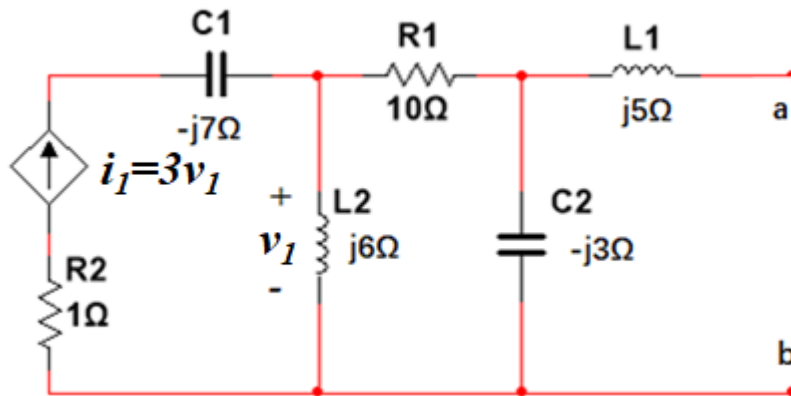
$$V_- = V_+ = 0$$

$$V_0 = 0.43 \angle 12.53^\circ \text{ V}$$

$$v_o(t) = 0.43 \cos(1000t + 12.53^\circ) \text{ V}$$

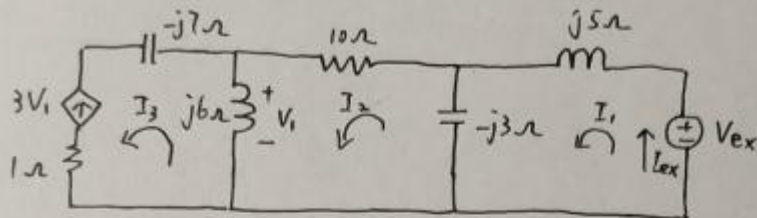
列出方程组3分，解出 V_0 4分，最后换成 $v_o(t)$ 3分，一共10分

9、 The circuit is in the phasor domain. Determine and plot its Thevenin equivalent circuit at terminals (a,b).



There is no independent source

$$V_{Th} = 0 \text{ V}$$



$$\begin{cases} I_1 = I_{ex} \\ -V_{ex} + I_1 \cdot j5 + (I_1 - I_2) \cdot (-j3) = 0 \\ (-j3) \cdot (I_2 - I_1) + I_2 \cdot 10 + V_1 = 0 \\ V_1 = (I_2 - I_3) \cdot j6 \\ \cancel{(I_2 - I_3) \cdot j6 + I_3 \cdot (-j7) = 0} \\ I_3 = -3V_1 \\ V_{ex} = 1 \end{cases}$$

$$I_{ex} = 0.41 \angle -69.40^\circ \text{ A}$$

$$= 0.105 - j0.39 \text{ A}$$

$$Z_{eq} = \frac{V_{ex}}{I_{ex}} = \frac{1}{0.15 - j0.39}$$

$$= 0.86 + j2.23 \text{ } \Omega$$



写出 V_{th} 3分，列出方程3分，解出流经外加电源的电流3分，算出等效阻抗3分，画图3分，一共15分。