Homework 10

Due date: Jun.13th, 2018
Turn in your homework in class

Rules:

- Work on your own. Discussion is permissible, but extremely similar submissions will be judged as plagiarism.
- Please show all intermediate steps: a correct solution without an explanation will get zero credit.
- Please submit on time. No late submission will be accepted.
- Please prepare your submission in English only. No Chinese submission will be accepted.

1. (20%)Find the Laplace transform of each of the following functions:
   (a). $f(t) = 20e^{-500(t-10)}u(t-10)$.
   (b). $f(t) = (5t+20)[u(t+4) - u(t+2)] - 5t[u(t+2) - u(t-2)] - 10u(t-2)$.

(a)

1. $\int_0^\infty 20 e^{-500(t-10)} u(t-10)\, e^{-st} dt.$

$= \int_0^\infty 20\, e^{-10s} [\, e^{-500(t-10)} u(t-10)\, e^{-s(t-10)} d(t-10)].$

$= \int 20\, e^{-10s} \int_0^\infty e^{-500t} \cdot e^{-st} dt$

$\Rightarrow F(s) = \frac{20e^{-10s}}{s+500}$

(b). $f(t) = (5t+20)\, u(t+4) - (10t+20)\, u(t+2) + (5t-10)\, u(t-2) + \cancel{5t\, u(t-2)}.$

$= 5(t+4)\, u(t+4) - 10(t+2)\, u(t+2) + 5(t-2)\, u(t-2)$

$\Rightarrow F(s) = \frac{5(e^{4s} - 2e^{2s} + e^{-2s})}{s^2}$

2. (20%)Find $f(t)$ for each of the following functions:

(a). $F(s) = \frac{6(s+10)}{(s+5)(s+8)}$

(b). $F(s) = \frac{320}{s^2(s+8)}$

(a)

$$F(s) = \frac{K_1}{s+5} + \frac{K_2}{s+8}$$

$$K_1 = \frac{6(s+10)}{(s+8)}\bigg|_{s=-5} = 10$$

$$K_2 = \frac{6(s+10)}{(s+5)}\bigg|_{s=-8} = -4$$

$$f(t) = [10e^{-5t} - 4e^{-8t}]u(t)$$

(b)

$$F(s) = \frac{K_1}{s^2} + \frac{K_2}{s} + \frac{K_3}{s+8}$$

$$K_1 = \frac{320}{s+8}\bigg|_{s=0} = 40$$

$$K_2 = \frac{d}{ds}\left[\frac{320}{s+8}\right] = \left[\frac{-320}{(s+8)^2}\right]_{s=0} = -5$$

$$K_3 = \frac{320}{s^2}\bigg|_{s=-8} = 5$$

$$f(t) = [40t - 5 + 5e^{-8t}]u(t)$$

（c）

[d] $F(s) = \frac{K_1}{s+5-j3} + \frac{K_1^*}{s+5+j3} + \frac{K_2}{s+4-j2} + \frac{K_2^*}{s+4+j2}$

$$K_1 = \frac{8(s+1)^2}{(s+5+j3)(s^2+8s+20)}\bigg|_{s=-5+j3} = 4.62\underline{/-40.04^\circ}$$

$$K_2 = \frac{8(s+1)^2}{(s+4+j2)(s^2+10s+34)}\bigg|_{s=-4+j2} = 3.61\underline{/168.93^\circ}$$

$$f(t) = [9.25e^{-5t}\cos(3t - 40.05^\circ) + 7.21e^{-4t}\cos(2t + 168.93^\circ)]u(t)$$

（d）

[d] $F(s) = \frac{K_1}{s^2} + \frac{K_2}{s} + \frac{K_3}{(s+5)^2} + \frac{K_4}{s+5}$

$$K_1 = \frac{25(s+4)^2}{(s+5)^2}\bigg|_{s=0} = 16$$

$$K_2 = \frac{d}{ds}\left[\frac{25(s+4)^2}{(s+5)^2}\right] = \left[\frac{25(2)(s+4)}{(s+5)^2} - \frac{25(2)(s+4)^2}{(s+5)^3}\right]_{s=0} = 1.6$$

$$K_3 = \frac{25(s+4)^2}{s^2}\bigg|_{s=-5} = 1$$

$$K_4 = \frac{d}{ds}\left[\frac{25(s+4)^2}{s^2}\right] = \left[\frac{25(2)(s+4)}{s^2} - \frac{25(2)(s+4)^2}{s^3}\right]_{s=-5} = -1.6$$

$$f(t) = [16t + 1.6 + te^{-5t} - 1.6e^{-5t}]u(t)$$

3. (20%)Find V_0(s domain) and v_0(time domain) in the circuit shown in Fig.1 if the initial energy is zero and the switch is closed at $t = 0$.

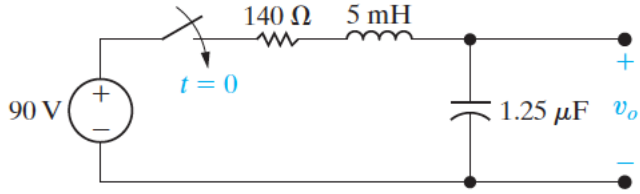
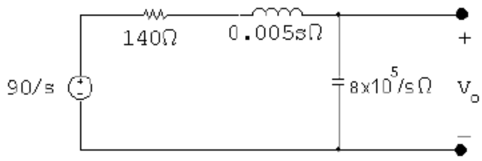


Fig.1



$$V_o = \frac{(90/s)(8 \times 10^5/s)}{140 + 0.005s + (8 \times 10^5/s)}$$

$$= \frac{144 \times 10^8}{s(s^2 + 28{,}000s + 16 \times 10^7)}$$

$$= \frac{144 \times 10^8}{s(s + 8000)(s + 20{,}000)}$$

$$= \frac{K_1}{s} + \frac{K_2}{s + 8000} + \frac{K_3}{s + 20{,}000}$$

$$K_1 = \frac{144 \times 10^8}{16 \times 10^7} = 90$$

$$K_2 = \frac{144 \times 10^8}{(-8000)(12{,}000)} = -150$$

$$K_3 = \frac{144 \times 10^8}{(-12{,}000)(-20{,}000)} = 60$$

$$V_o = \frac{90}{s} - \frac{150}{s + 8000} + \frac{60}{s + 20{,}000}$$

$$v_o(t) = [90 - 150e^{-8000t} + 60e^{-20{,}000t}]u(t)\ \mathrm{V}$$

4. The switch in the circuit in Fig.2 has been closed for a long time. At $t = 0$ the switch is opened.
(a). Find i_0 for $t \geq 0$.
(b). Find v_0 for $t \geq 0$.

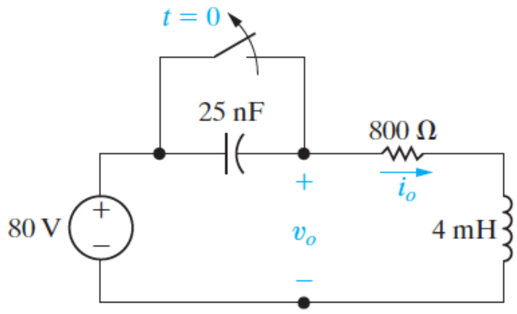
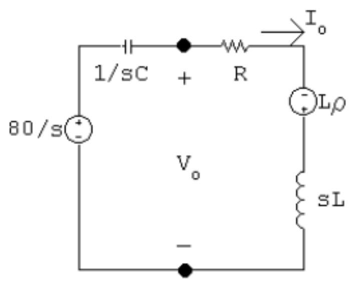


Fig.2

[a] $i_o(0^-) = \dfrac{20}{4000} = 5\,\text{mA}$



$$I_o = \frac{80/s + L\rho}{R + sL + 1/sC} = \frac{sC(80/s + L\rho)}{s^2LC + RsC + 1}$$

$$= \frac{80/L + s\rho}{s^2 + sR/L + 1/LC} = \frac{20{,}000 + s(0.1)}{s^2 + 200{,}000s + 10^{10}}$$

$$= \frac{0.1(s + 200{,}000)}{s^2 + 200{,}000s + 10^{10}} = \frac{K_1}{(s + 100{,}000)^2} + \frac{K_2}{s + 100{,}000}$$

$K_1 = 10{,}000; \qquad K_2 = 0.1$

$i_o(t) = [10{,}000te^{-100{,}000t} + 0.1e^{-100{,}000t}]u(t)\,\text{A}$

[b] $V_o = (R + sL)I_o - L\rho = \dfrac{(800 + 0.004s)(0.1s + 20{,}000)}{s^2 + 200{,}000s + 10^{10}} - 4 \times 10^{-4}$

$$= \frac{80(s + 150{,}000)}{(s + 100{,}000)^2} = \frac{K_1}{(s + 100{,}000)^2} + \frac{K_2}{s + 100{,}000}$$

$K_1 = 4 \times 10^6 \qquad K_2 = 80$

$v_o(t) = [4 \times 10^6te^{-100{,}000t} + 80e^{-100{,}000t}]u(t)\,\text{A}$

5. The switch in the circuit in Fig.3 has been closed for a long time before opening at $t = 0$. Find v_0 for $t \geq 0$.

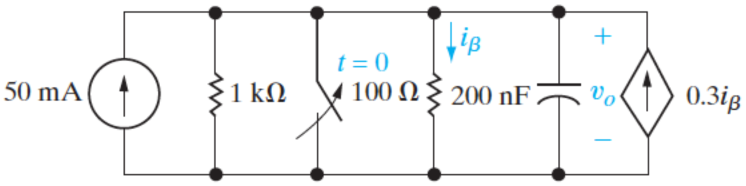
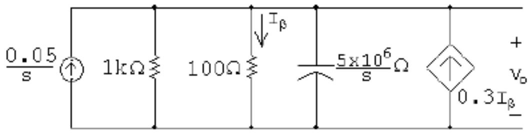


Fig.3

$$v_C(0^-) = v_C(0^+) = 0$$



$$\frac{0.05}{s} = \frac{V_o}{1000} + \frac{V_o}{100} + \frac{V_o s}{5 \times 10^6} - \frac{0.3V_o}{100}$$

$$\frac{250 \times 10^3}{s} = (5000 + 50{,}000 + s - 15{,}000)V_o$$

$$V_o = \frac{250 \times 10^3}{s(s + 40{,}000)} = \frac{K_1}{s} + \frac{K_2}{s + 40{,}000}$$

$$= \frac{6.25}{s} - \frac{6.25}{s + 40{,}000}$$

$$v_o(t) = [6.25 - 6.25e^{-40{,}000t}]u(t)\text{ V}$$