Lecture 2
Basic Laws & Circuit Analysis

Outline

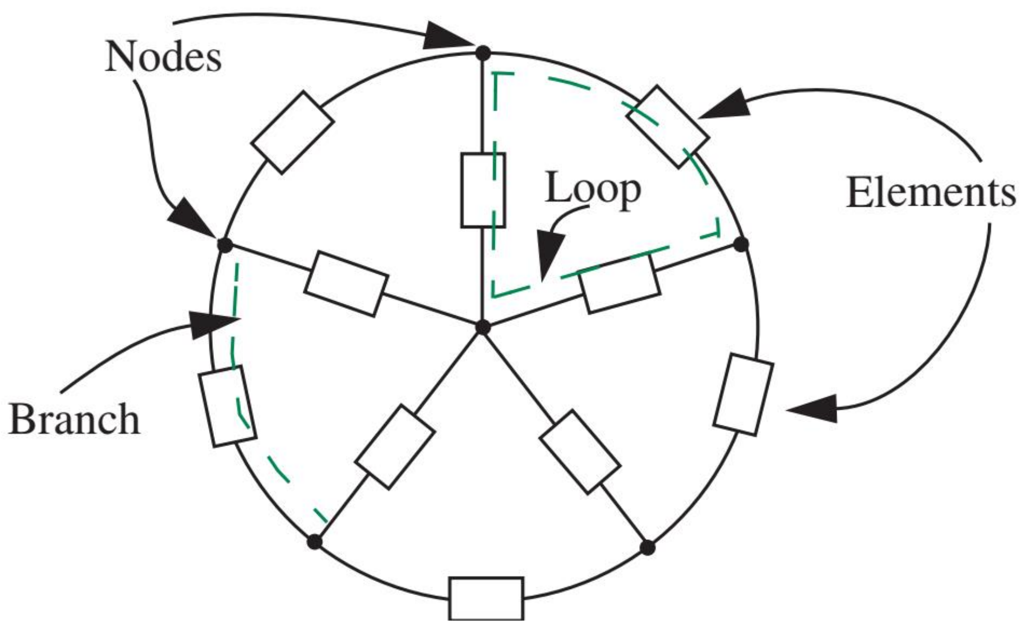
- Terminology: Branches, Nodes, and Loops

- Basic Laws
  - Ohm’s Law
  - Kirchhoff’s Laws -- KCL,KVL

- Circuit Analysis
  - Nodal Analysis
  - Mesh Analysis

Terminology: Branches, Nodes, and Loops

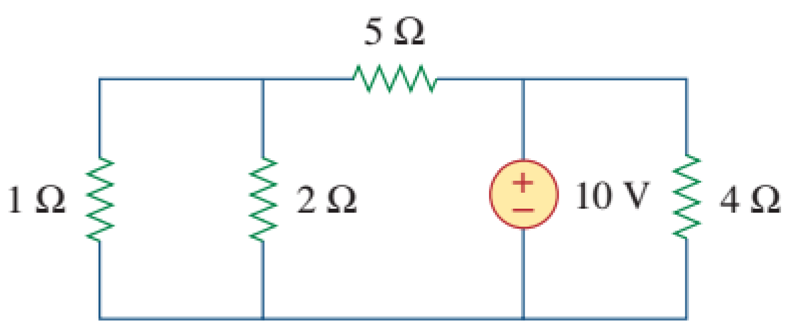
- **Branch**: represents a single element;
- **Node**: a point of connection between two or more branches;
- **Loop**: **Any** closed path in a circuit.



[Source: MIT]

Loop, Independent Loop, Mesh

- A loop is a closed path with no node passed more than once.
- A loop is independent if it contains at least one branch which is not a part of any other independent loop.
- A mesh is a loop that does not contain any other loop within it.



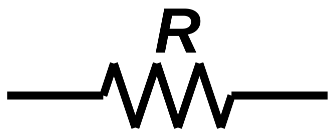
- b – number of branches
- n – number of nodes
- l_{ind} – number of ind. loops

$$l_{ind} = b - (n - 1)$$

Mesh = Independent loop?

Ohm’s Law

- Resistance: the ratio of voltage drop and current. The circuit element used to model this behavior is the resistor.

Circuit symbol: 

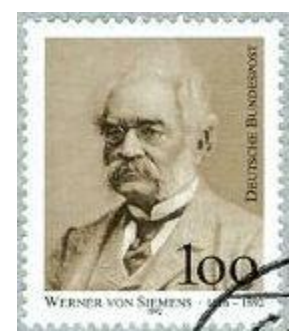
- The current flowing in the resistor is proportional to the voltage across the resistor:

$$\boxed{v = i\,R}$$

(Ohm’s Law)

- **Conductance** is the reciprocal of resistance

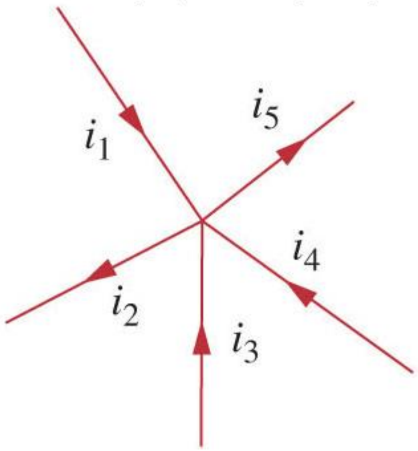
$$G = \frac{1}{R} = \frac{i}{v}$$



Werner von Siemens
1816-1892

Kirchhoff's Laws

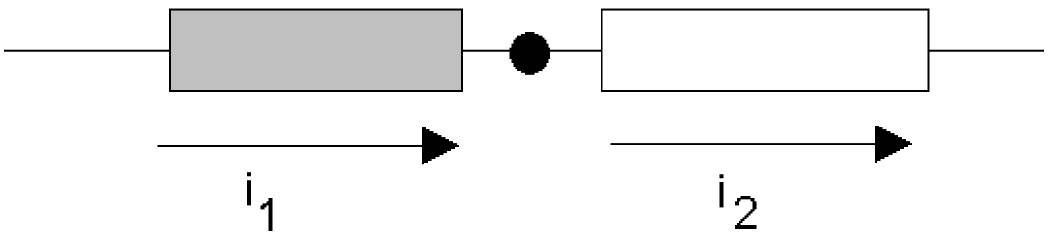
- Kirchhoff's Current Law (KCL):
  - The algebraic sum of all the **currents** entering any **node** in a circuit equals zero.
  - Why?



Gustav Robert Kirchhoff
1824-1887

A Major Implication of KCL

- KCL tells us that **all of the elements that are connected *in series* carry the same current**.

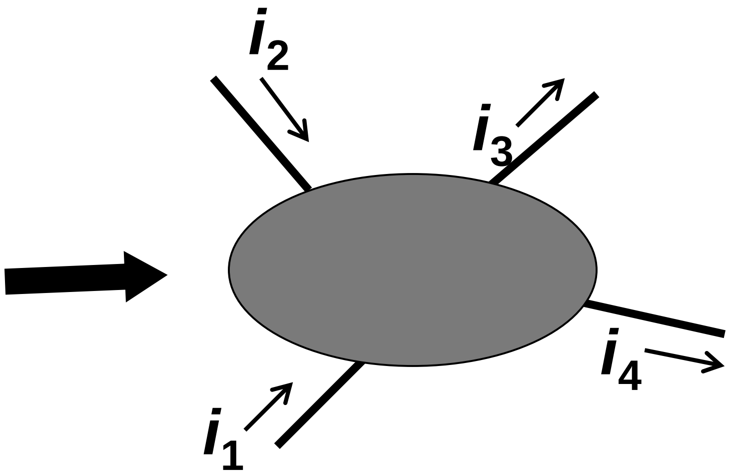


Current entering node = Current leaving node

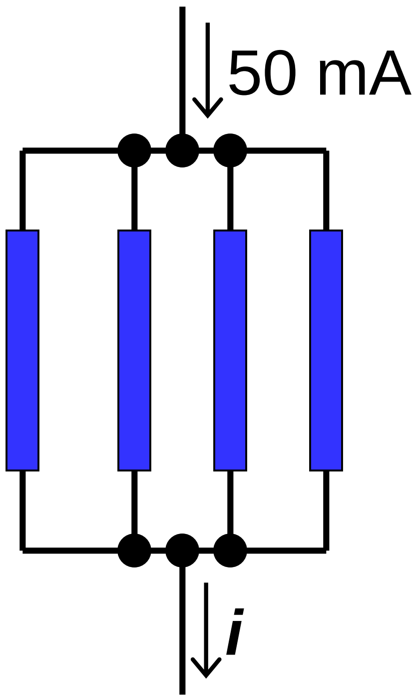
Generalization of KCL

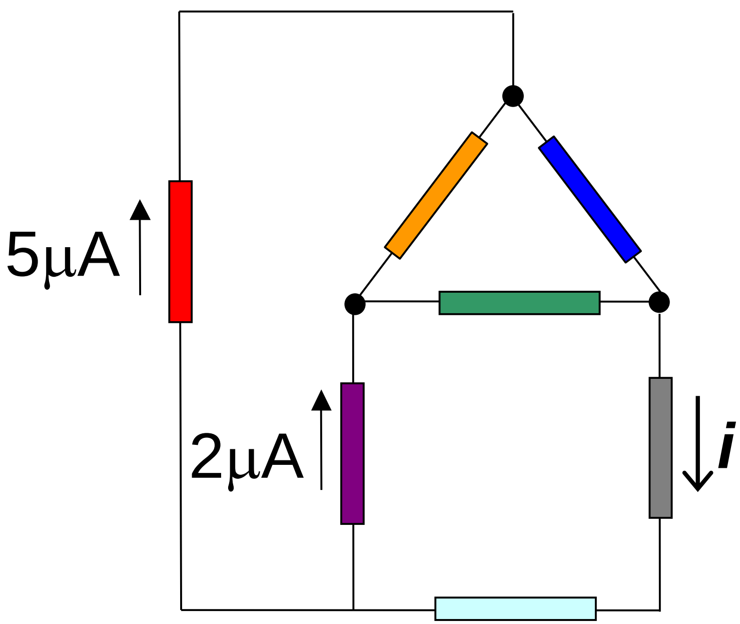
- The sum of currents entering/leaving a **closed surface** is zero.
  - Circuit branches can be inside this surface, *i.e.* the surface can enclose more than one node!

This could be a big chunk of a circuit, *e.g.* a "black box"

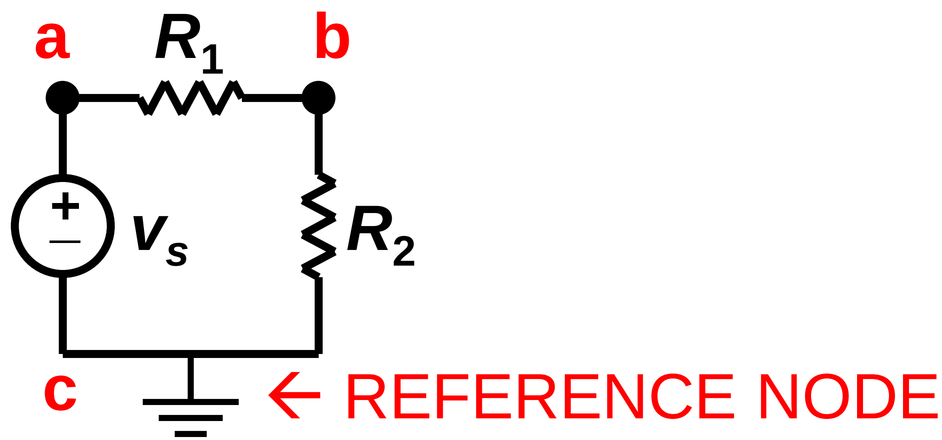


Generalized KCL Examples





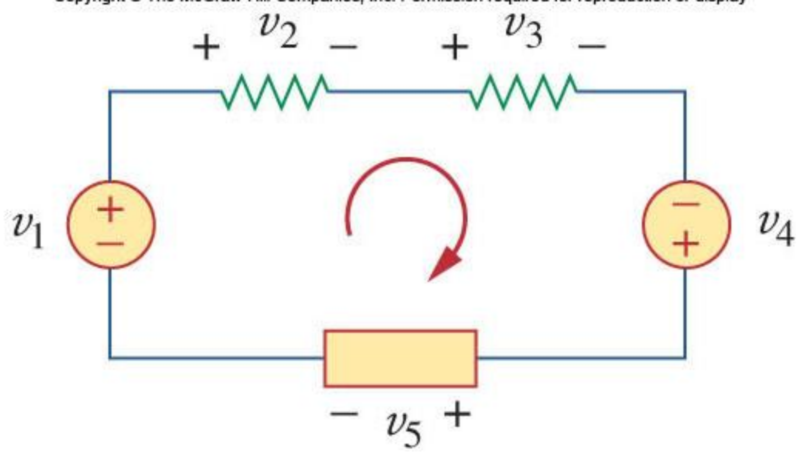
Notation: Node and Branch Voltages



- Use one node as the reference (the "common" or "ground" node) – label it with a symbol.
- The voltage drop from node ***x*** to the reference node is called the **node voltage** V_x.
- The voltage across a circuit element is defined as the difference between the node voltages at its terminals.

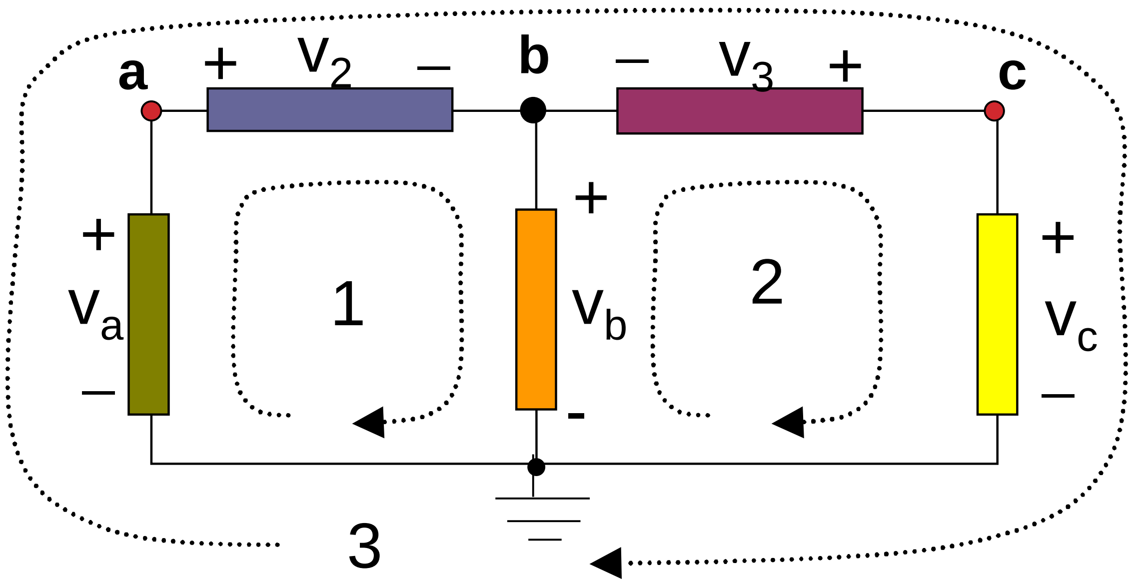
Kirchhoff's Voltage Law (KVL)

- The algebraic sum of all the **voltages** around any **loop** in a circuit equals zero.
- Why?



KVL Example

Three closed paths:



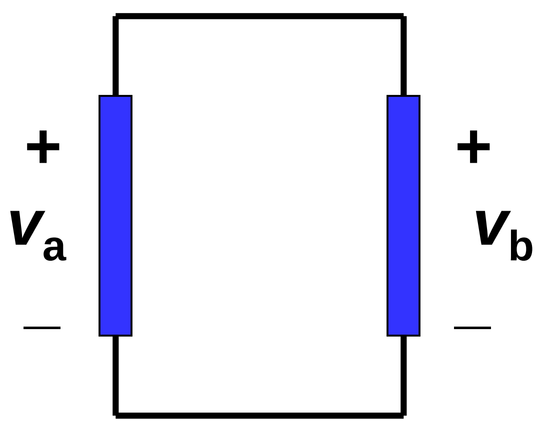
Path 1:

Path 2:

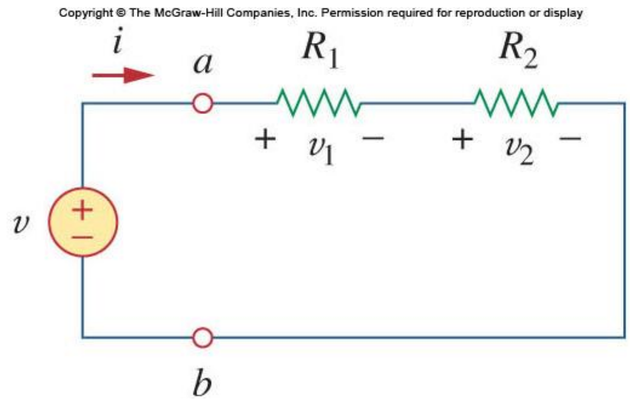
Path 3:

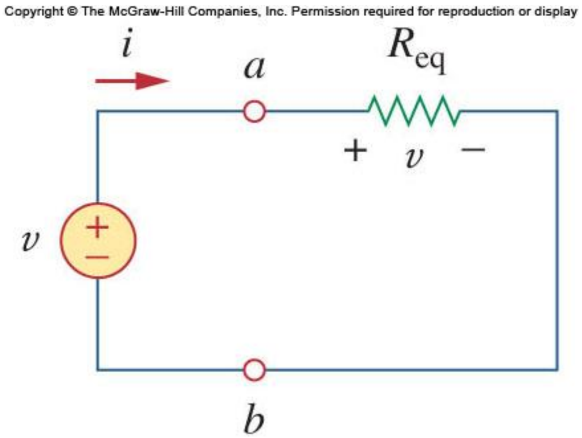
A Major Implication of KVL

- KVL tells us that **any set of elements which are connected at both ends carry the same voltage**.
- We say these elements are connected **in parallel**.

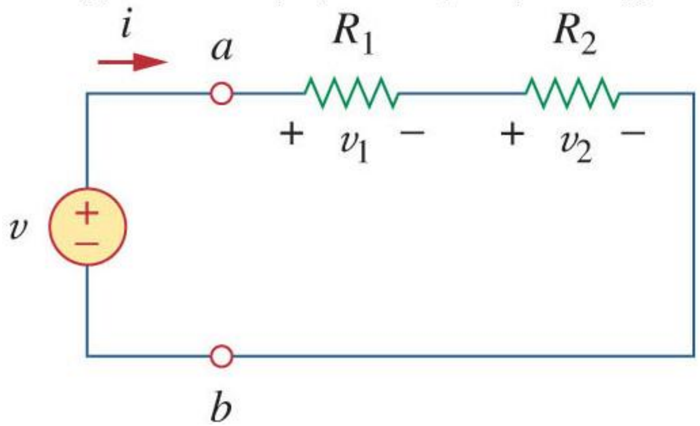


Series Resistors



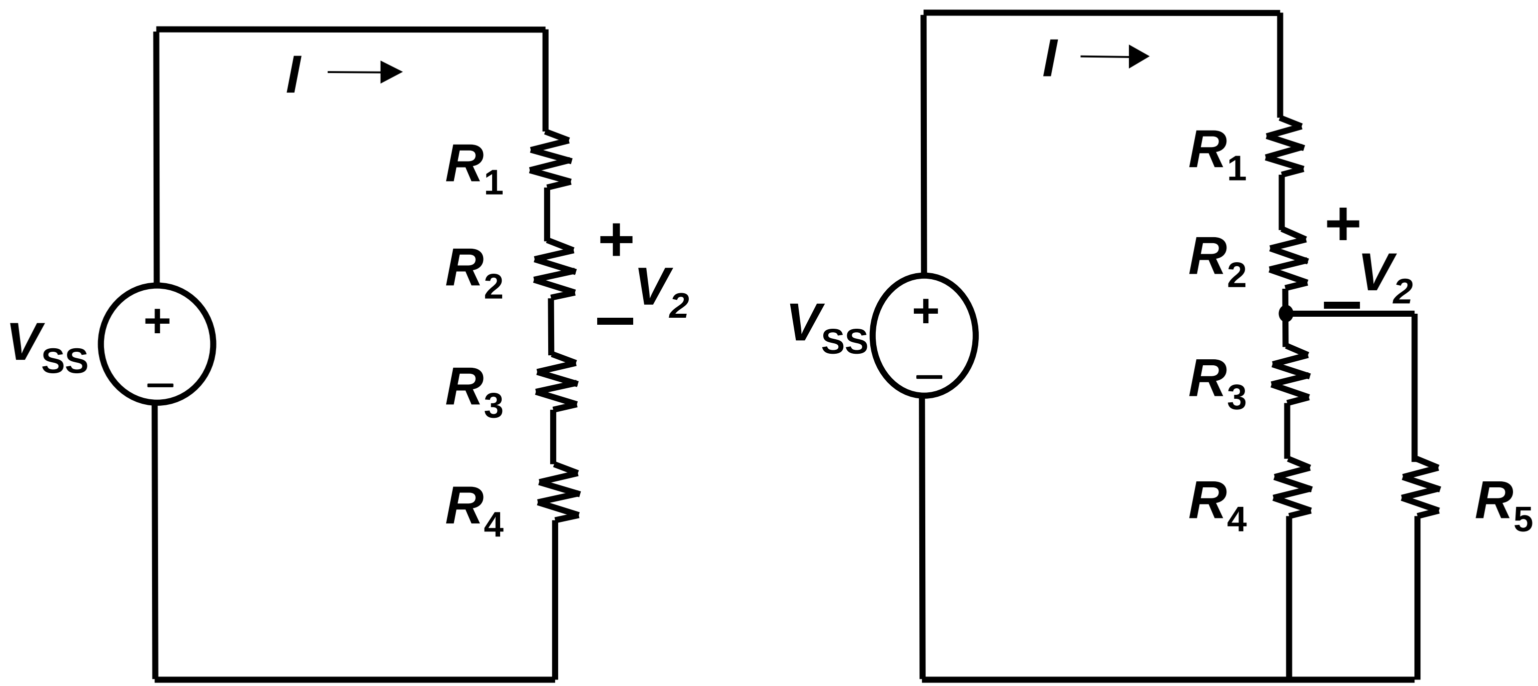


Voltage Division

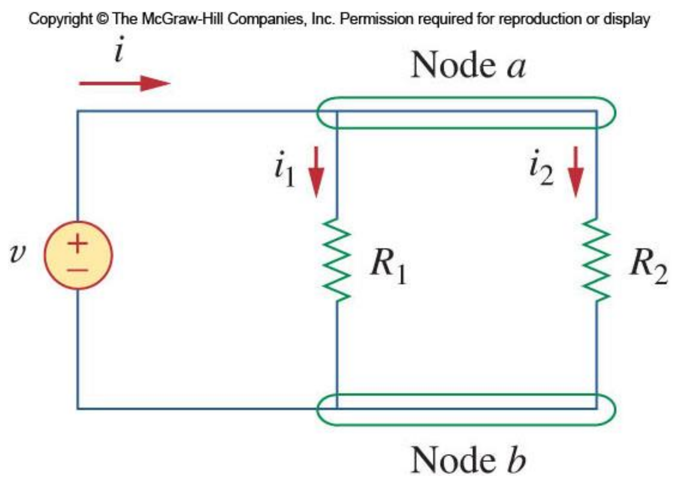


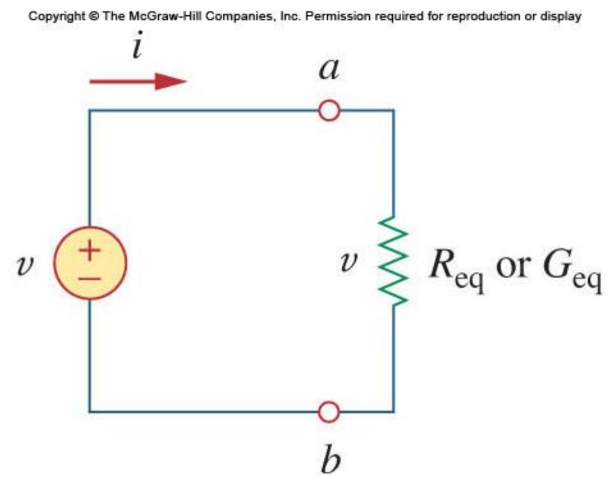
Three-terminal rheostat

Voltage Divider

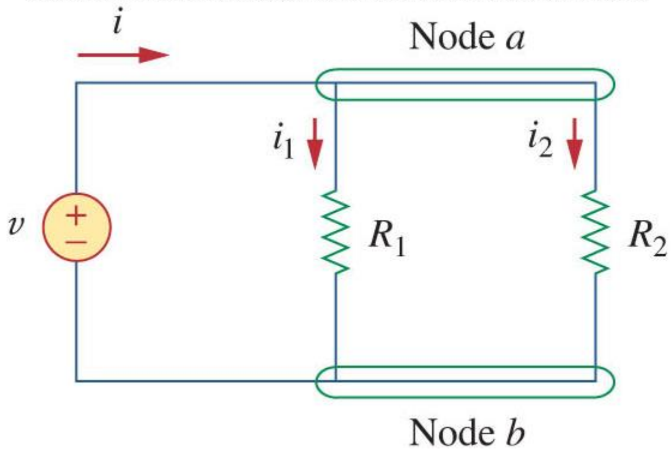


Parallel Resistors





Current Division

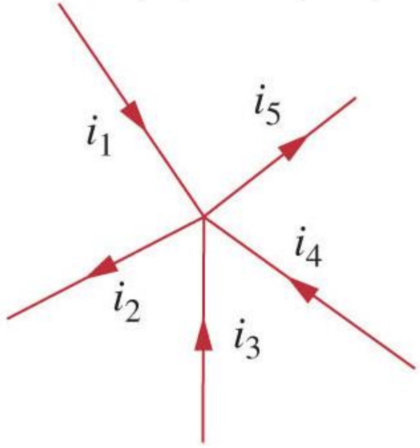


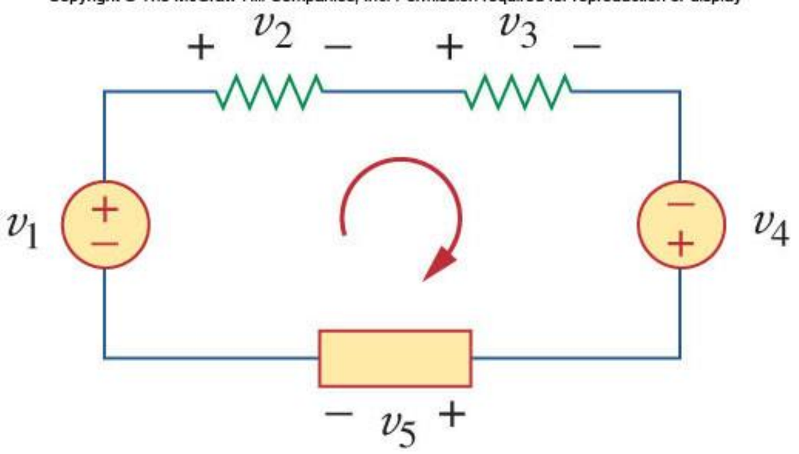
Summary

- KCL and KVL

$$\sum_{n=1}^{N} i_n = 0$$

$$\sum_{m=1}^{M} v_m = 0$$





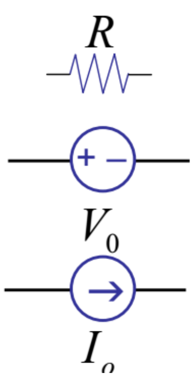
What you have learned

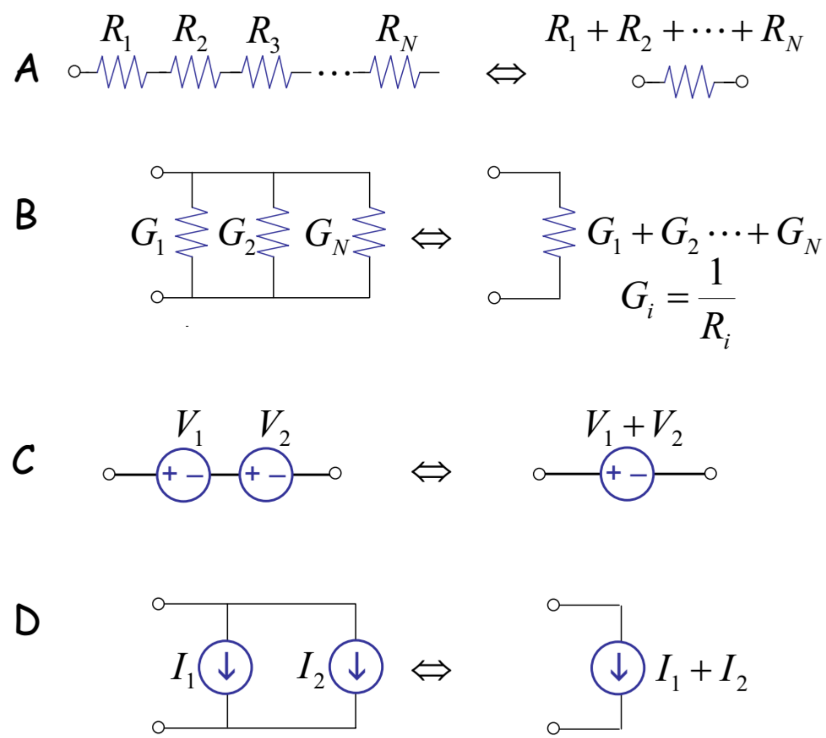
- KCL
- KVL
- Element relationships

For R, $V = IR$

For voltage source, $V = V_0$

For current source, $I = I_0$





[Source: MIT]

Outline

- Basic Laws
  - Ohm’s Law
  - Kirchhoff’s Laws -- KCL,KVL

- Circuit Analysis
  - Nodal Analysis
  - Mesh Analysis

Circuit Analysis

- Two techniques will be presented in this part:
  - Nodal analysis, which is based on **KCL**
    - Used in SPICE, the internal engine of circuit simulators.
  - Mesh analysis, which is based on **KVL**

- The analysis will result in a set of simultaneous equations which may be solved by Cramer's rule or computationally (using MATLAB for example)

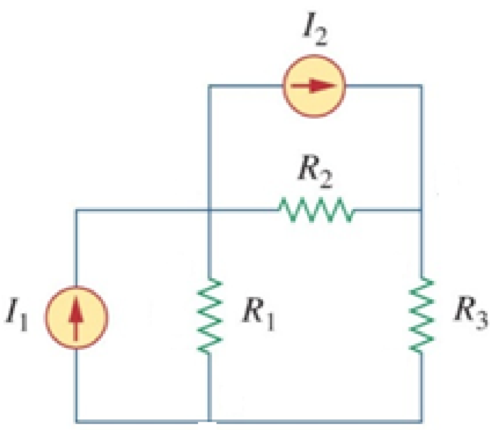
http://bwrcs.eecs.berkeley.edu/Classes/IcBook/SPICE/
http://www.ni.com/white-paper/5413/zhs/

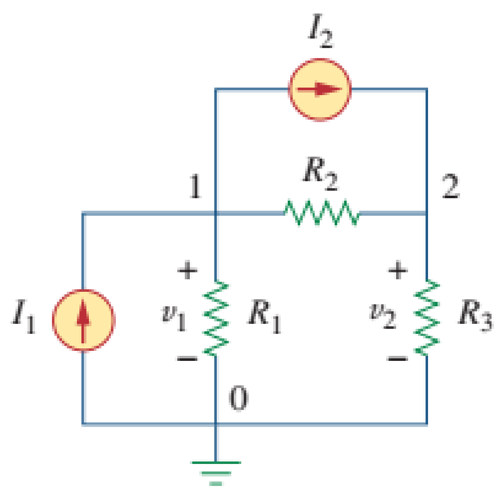
Nodal Analysis – Three Steps

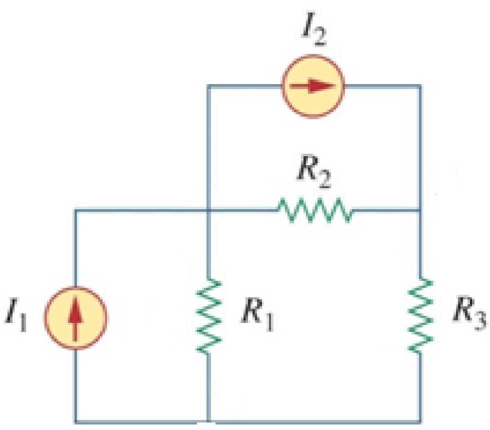
- Given a circuit with *n* nodes, the nodal analysis is accomplished via three steps:
  1. Select a node as the reference (i.e., ground) node. Assign the node voltages to the remaining ***(n-1)*** nodes. Voltages are relative to the reference node.
  2. Apply KCL to the ***(n-1)*** nodes, expressing branch current in terms of the node voltages (using the *I*-*V* relationships of branch elements).
  3. Solve the resulting simultaneous equations to obtain the unknown node voltages.

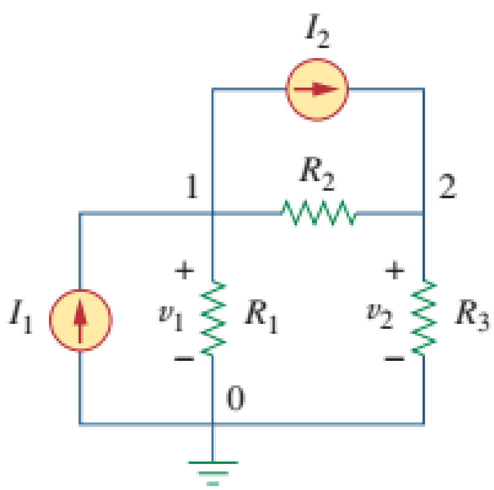
[Source: Berkeley]

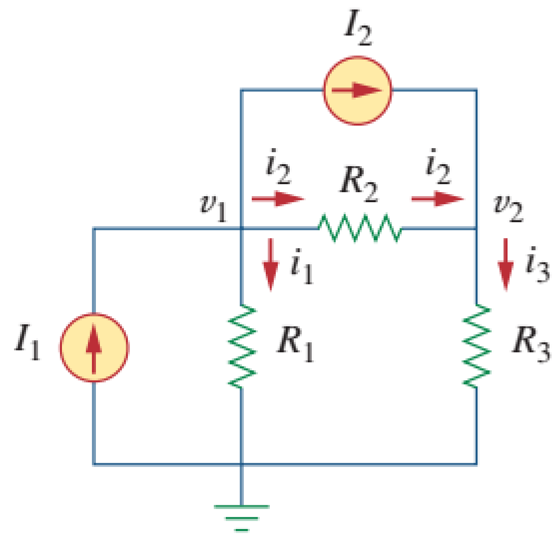
Nodal Analysis Example #1



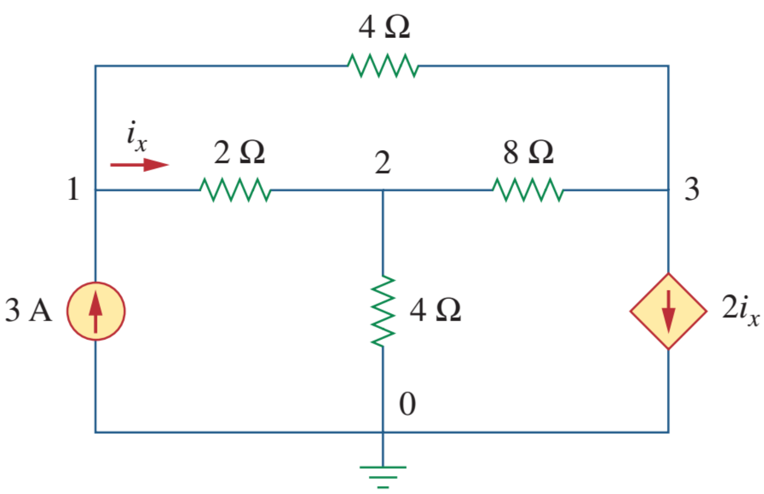



I_2
R_2
I_1
R_1
R_3


I_2
1
R_2
2
I_1
+
v_1
−
R_1
+
v_2
−
R_3
0


I_2
i_2
R_2
i_2
v_1
v_2
i_1
i_3
I_1
R_1
R_3

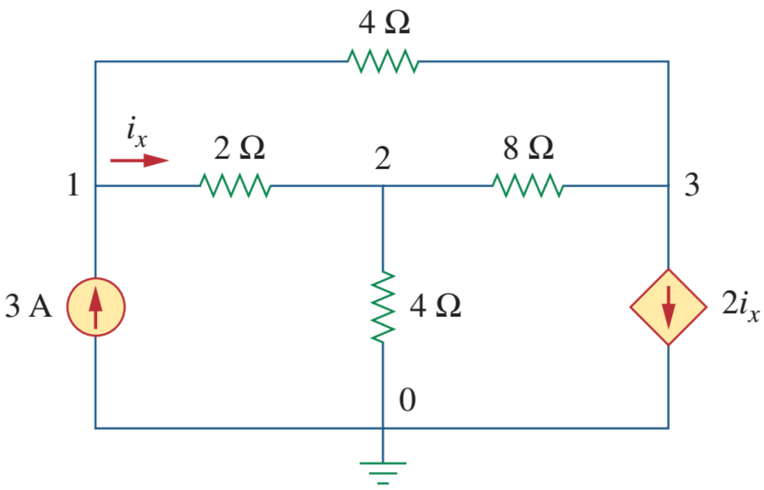
Nodal Analysis: Example #2



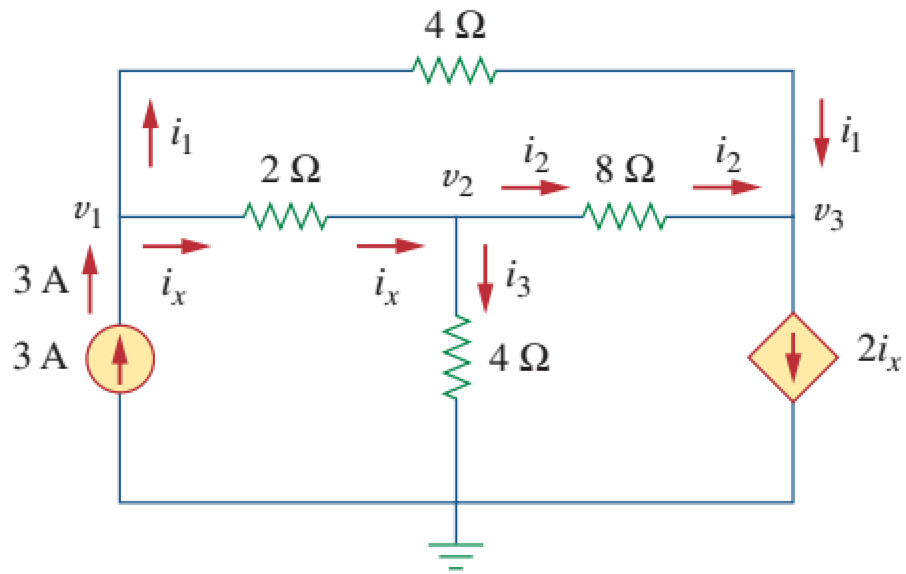
(a)

[Source: Berkeley]

Nodal Analysis: Example #2

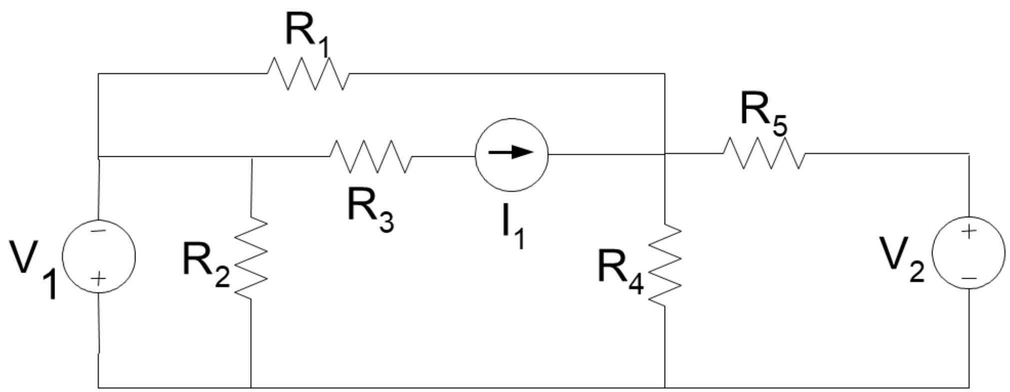


(a)



Nodal Analysis with Voltage Sources

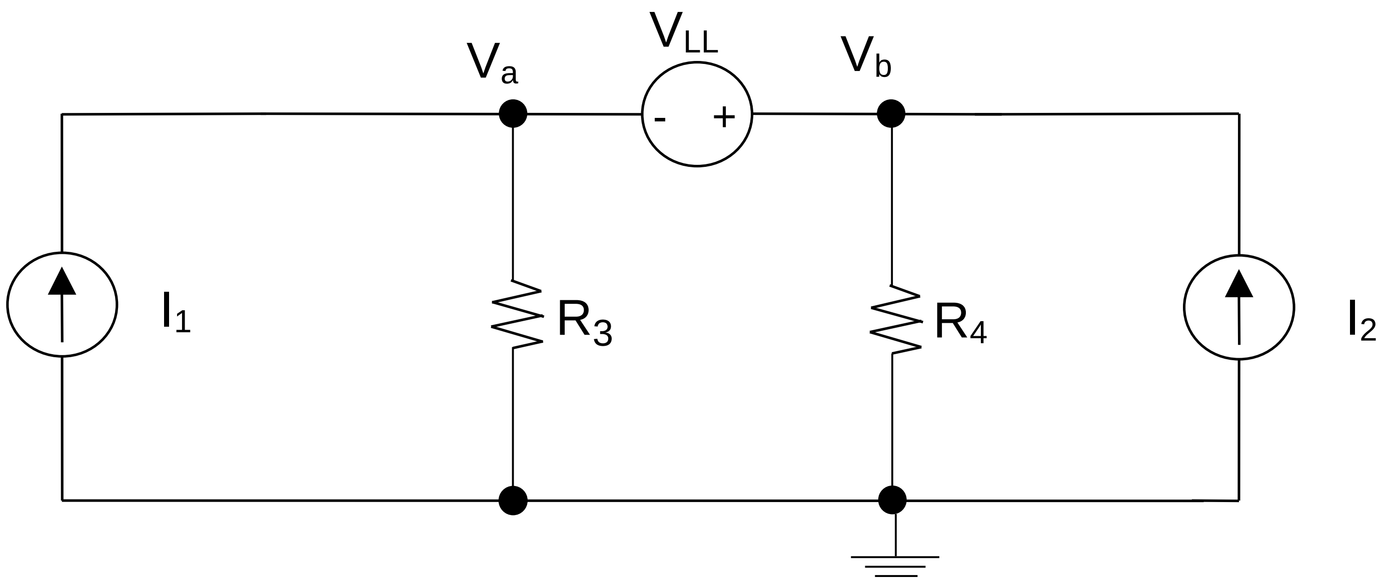
Case I:



Nodal Analysis: Supernode

Case II

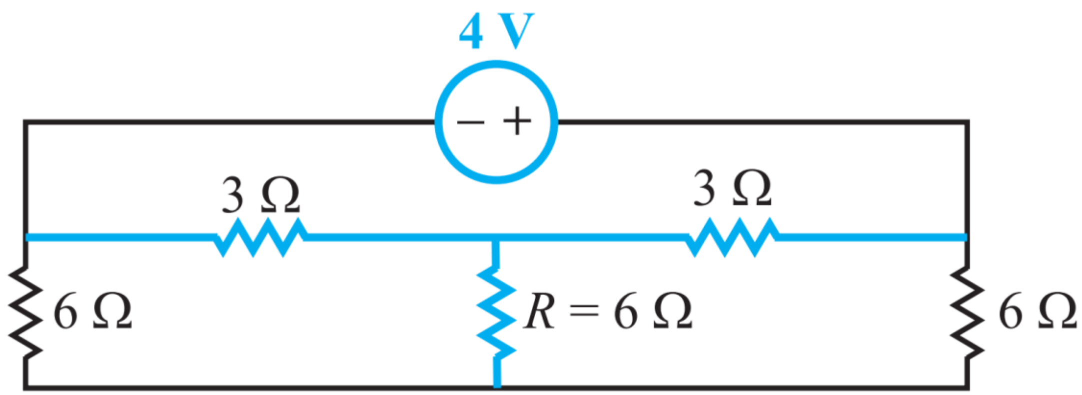
A “floating” voltage source is one for which neither side is connected to the reference node, e.g. V_{LL} in the circuit below:



A supernode is formed by enclosing a (dependent or independent) voltage source connected between two nonreference nodes and any elements connected in parallel with it.

Exercise

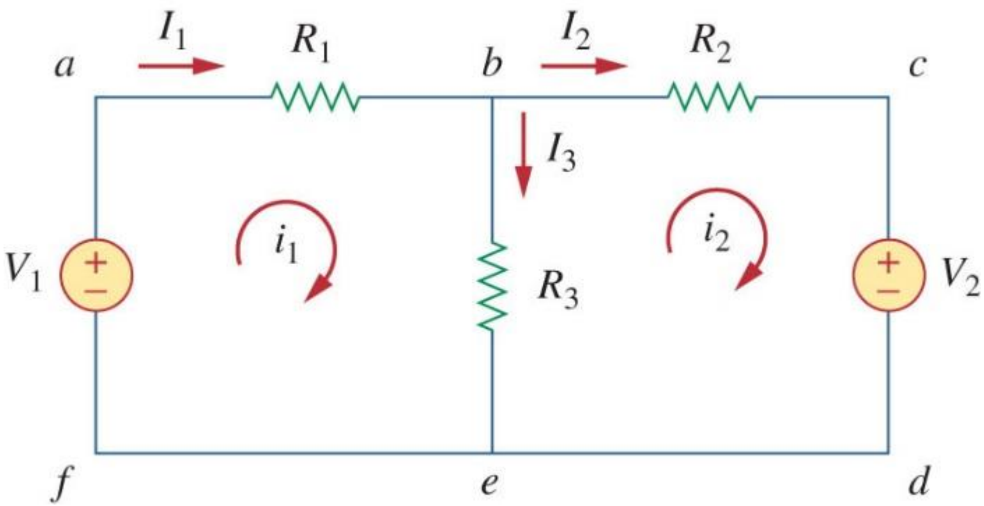
- Find the power supplied by the voltage source.



[Source: Berkeley]

Mesh Analysis

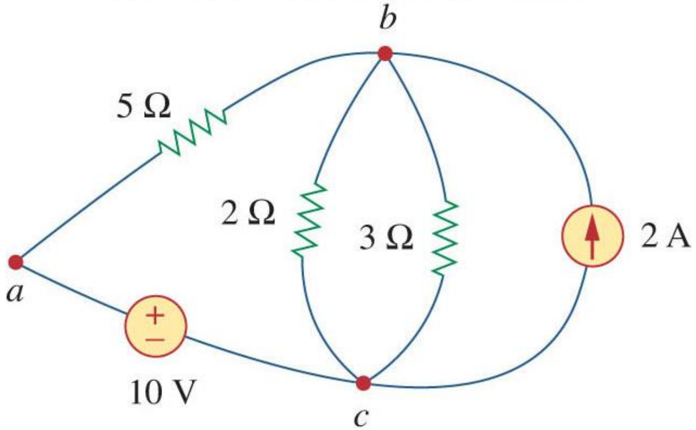
- Another general procedure for analyzing circuits is to use the mesh currents as the circuit variables.



- Mesh analysis uses KVL to find unknown currents.

Loop, Independent Loop, Mesh

- A loop is a closed path with no node passed more than once.
- A loop is independent if it contains at least one branch which is not a part of any other independent loop.
- A mesh is a loop that does not contain any other loop within it.



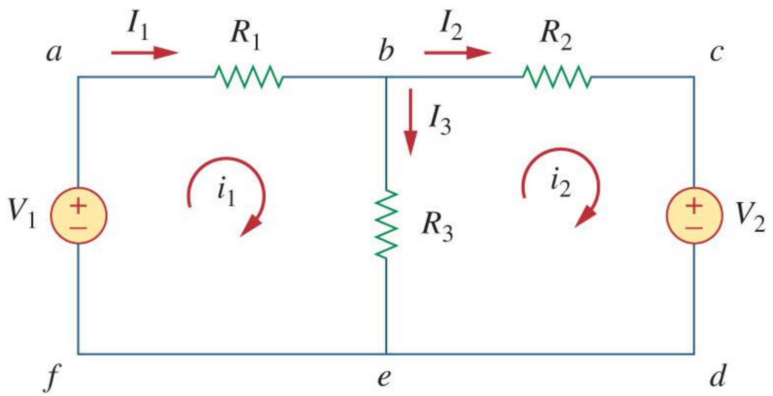
- b – number of branches
- n – number of nodes
- l_{ind} – number of ind. loops

Mesh = Independent loop?

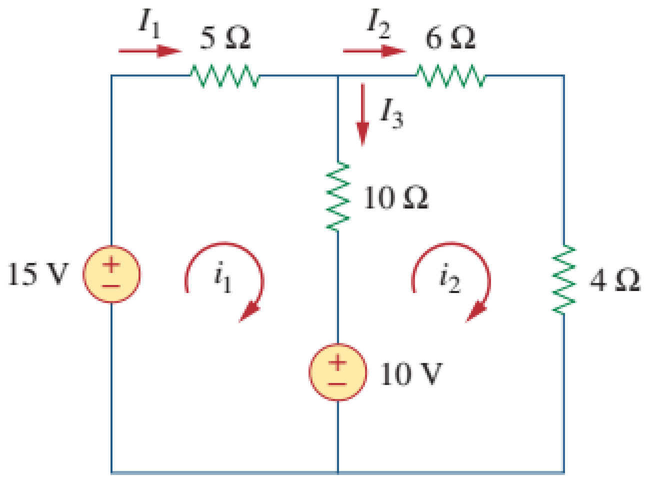
$$l_{ind} = b - (n - 1)$$

Mesh Analysis Steps

- Mesh analysis follows these steps:
  1. Assign mesh currents $i_1, i_2, \ldots i_x$ to the x meshes
  2. Apply KVL to each of the x mesh currents.
  3. Solve the resulting x simultaneous equations to get the mesh currents.

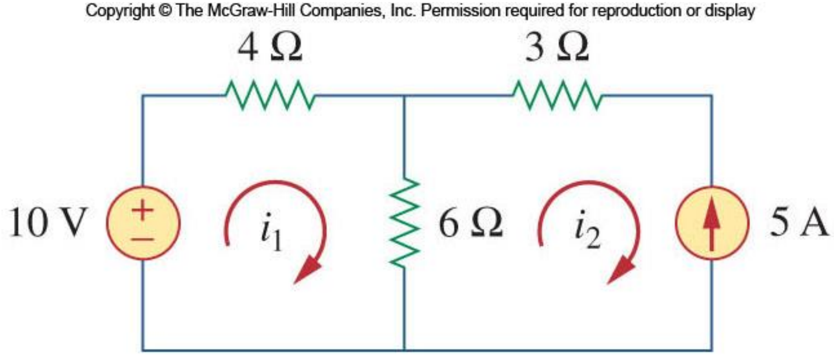


Example

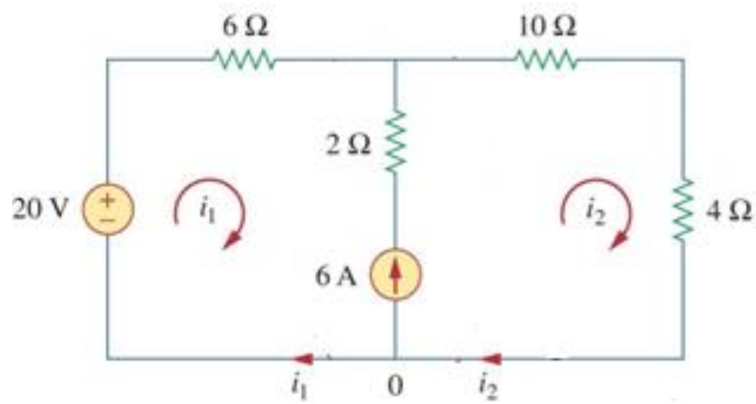


Mesh Analysis with Current Sources

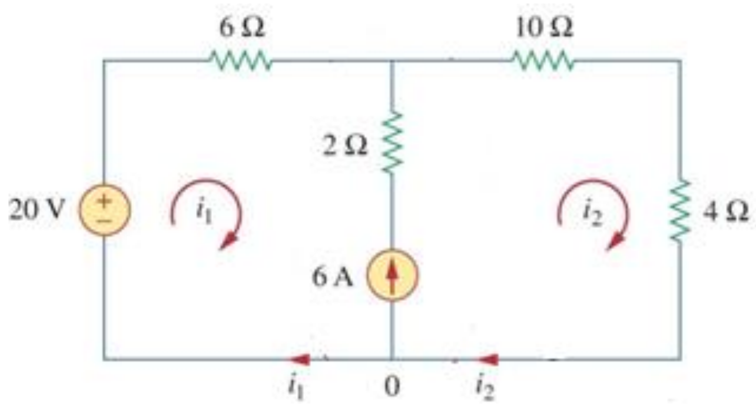
- The presence of a current source makes the mesh analysis simpler in that it reduces the number of equations.
  - If the current source is located on only one mesh, the current for that mesh is defined by the source. For example:

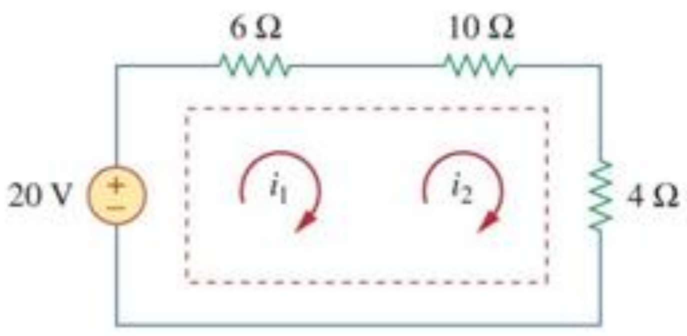


If the current source is located…



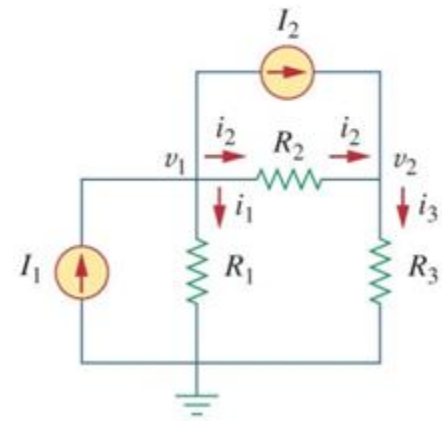
Supermesh



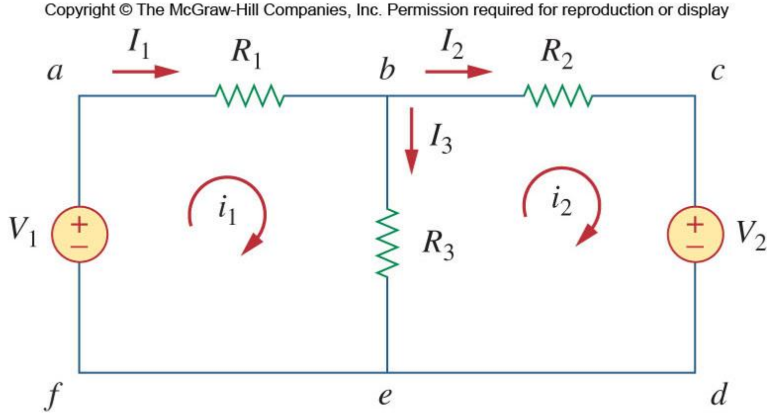


Summary

- Node Analysis
  - Node voltage is the unknown
  - Solve by KCL
  - Special case: Floating voltage source

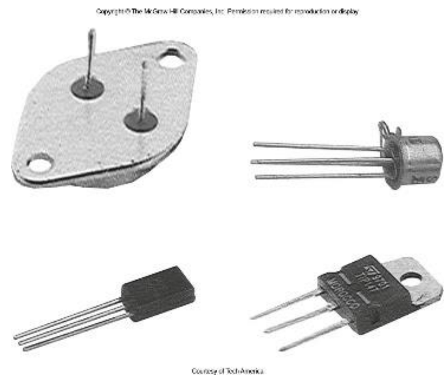


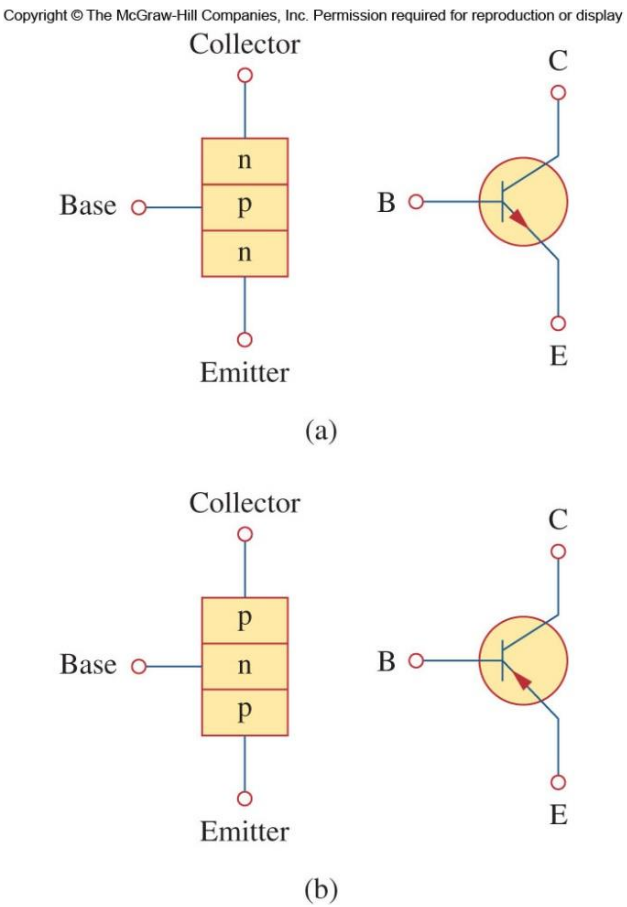
- Mesh Analysis
  - Loop current is the unknown
  - Solve by KVL
  - Special case: Current source



Application: DC Transistor Circuit

- In general, there are two types of transistors commonly used: Field Effect (FET) and Bipolar Junction (BJT). Here we will use the approaches learned in this lecture to analyze a BJT circuit.





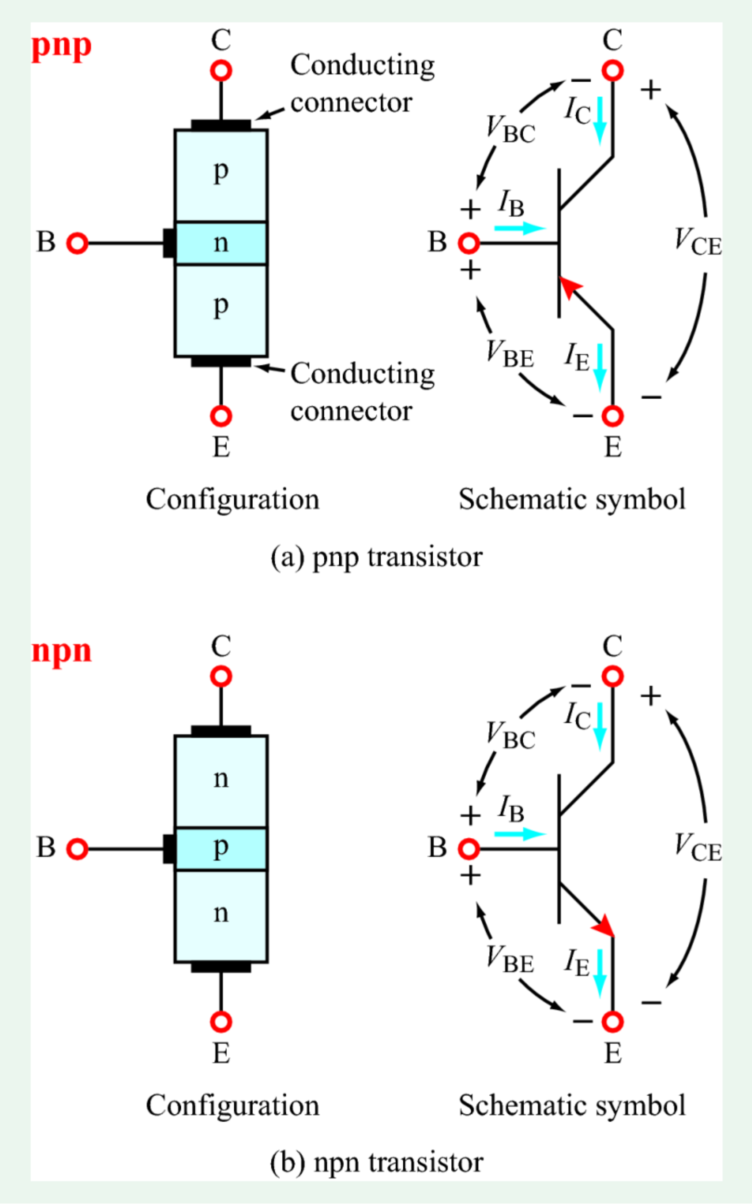
- A BJT is a three terminal device, where
  - The input current into one terminal (the base) affects the current flowing out of a second terminal (the collector).
  - The third terminal (the emitter) is the common terminal for both currents.

KCL and KVL for a BJT

- The currents from each terminal can be related to each other as follows:
- The base and collector current can be related to each other by the parameter β, which can range from 50-1000

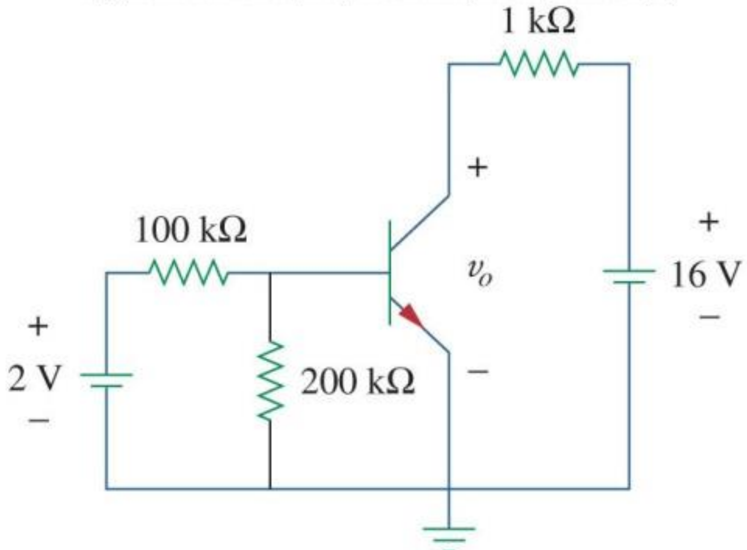
$$I_C = \beta I_B$$

- Applying KVL to the BJT gives:



Analysis of a BJT Circuit

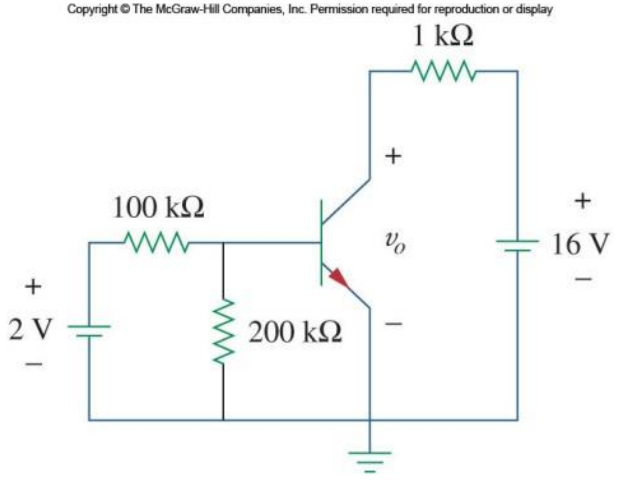
- A transistor has a few operating modes depending on the applied voltages/currents. In this problem, we will be interested in the operation in “active mode”
  - the mode used for amplifying signals.



$\beta = 150 \qquad v_{BE} = 0.7V$

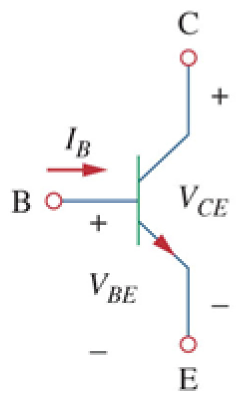
Mesh Analysis？

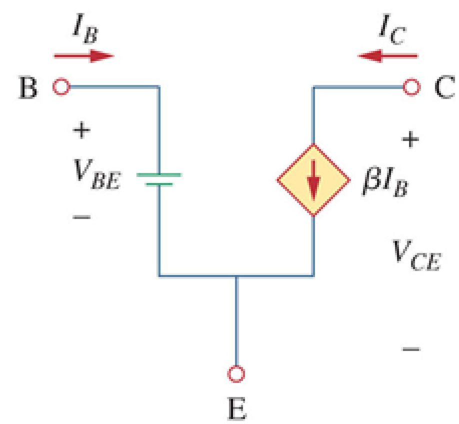
$\beta = 150 \quad v_{BE} = 0.7V$



DC model of a BJT

- The figure below shows the equivalent DC model for a BJT in active mode.





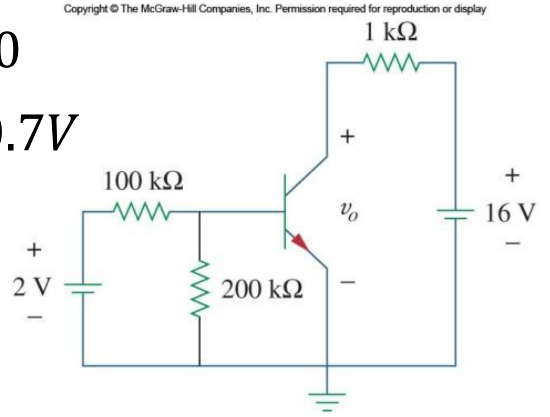
$$I_C = \beta I_B$$

Note that nodal analysis can be applied after using this model.

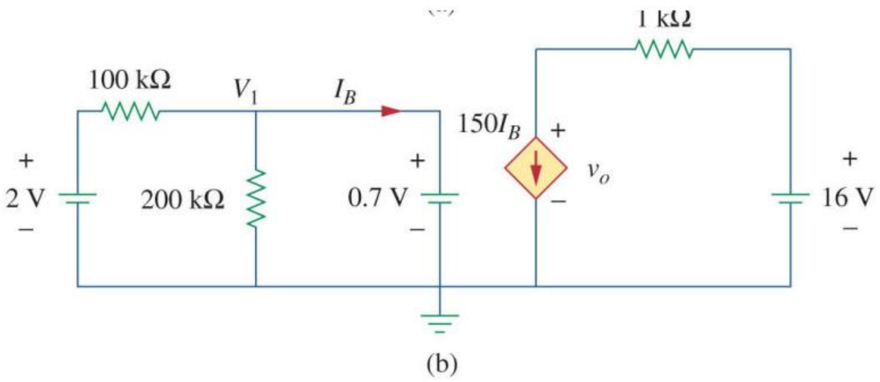
Setting up a BJT circuit

$\beta = 150$

$v_{BE} = 0.7V$



Original circuit



Circuit for nodal analysis