Signals and Systems Homework 11

Due Time: 23:59 June 1^{st}, 2018

1. (5)Consider the siganl $x[n] = (\frac{1}{5})^n u[n-3]$ and evaluate the z-transform of this signal, then specify the region of convergence.
   Solution:

$$X(z) = \sum_{n=-\infty}^{\infty} (\frac{1}{5})^n u[n-3] z^{-n} = \sum_{n=3}^{\infty} (\frac{1}{5})^n z^{-n}$$

$$= [\frac{z^{-3}}{125}] \sum_{n=0}^{\infty} (\frac{1}{5})^n z^{-n} = [\frac{z^{-3}}{125}] \frac{1}{1-\frac{1}{5}z^{-1}}$$

,$|z| > \frac{1}{5}$

2. (10)

(a) (10)Taking the z-transform lf both sides of the give difference equation and simplifying, we get

$$H(z) = \frac{Y(z)}{X(z)} = \frac{Z^{-1}}{1 - Z^{-1} - Z^{-2}}$$

The poles of $H(z)$ are $z = (\frac{1}{2} \pm (\frac{\sqrt{5}}{2}))$. $H(z)$ has a zero at $z = 0$. The pole-zero plot for $H(z)$ as shown in Figure 1:

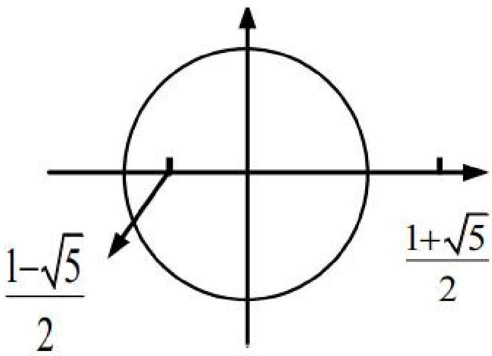

Figure 1: pole-zero plot

since $h[n]$ is causal, ROC for $H(z)$ has to be $|z| > (\frac{1}{2} + \frac{\sqrt{5}}{2})$

(b) The partial fraction expansion of $H(z)$ is

$$H(z) = \frac{\frac{1}{\sqrt{5}}}{1 - (\frac{1+\sqrt{5}}{2} z^{-1})} - \frac{\frac{1}{\sqrt{5}}}{1 - (\frac{1-\sqrt{5}}{2} z^{-1})}$$

Therefore,

$$h(n) = \frac{1}{\sqrt{5}}(\frac{1+\sqrt{5}}{2})^n u[n] - \frac{1}{\sqrt{5}}(\frac{1-\sqrt{5}}{2})^n u[n]$$

(c) Now assuming that the ROC is $(\sqrt{5}/2) - \frac{1}{2} < |z| < \frac{1}{2} + (\sqrt{5}/2)$ we get

$$h[n] = -\frac{1}{\sqrt{5}}(\frac{1+\sqrt{5}}{2})^n u[-n-1] - \frac{1}{\sqrt{5}}(\frac{1-\sqrt{5}}{2})^n u[n]$$

3. (5)Taking the z-transform of both side of the given difference equation and simplifying, we get

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{z^{-1} - \frac{10}{3} + z} = \frac{z^{-1}}{1 - \frac{10}{3}z^{-1} + z^{-2}}$$

The partial fraction expansion of H(z) is

$$H(z) = \frac{\frac{3}{8}}{1 - \frac{1}{3}z^{-1}} + \frac{\frac{3}{8}}{1 - 3z^{-1}}$$

Since $H(z)$corresponds to a stable system, the ROC has ro be $\frac{1}{3} < |z| < 3$ Therefore,

$$h(n) = -\frac{3}{8}(\frac{1}{3})^n u[n] - \frac{3}{8}(3)^n u[-n-1]$$

4. (20)

solution:

(a) The block-diagram may be redrawn as show in part (a) of the figure below . This may be treated as a cascade of the two systems shown within the dotted lines in Figure 2:

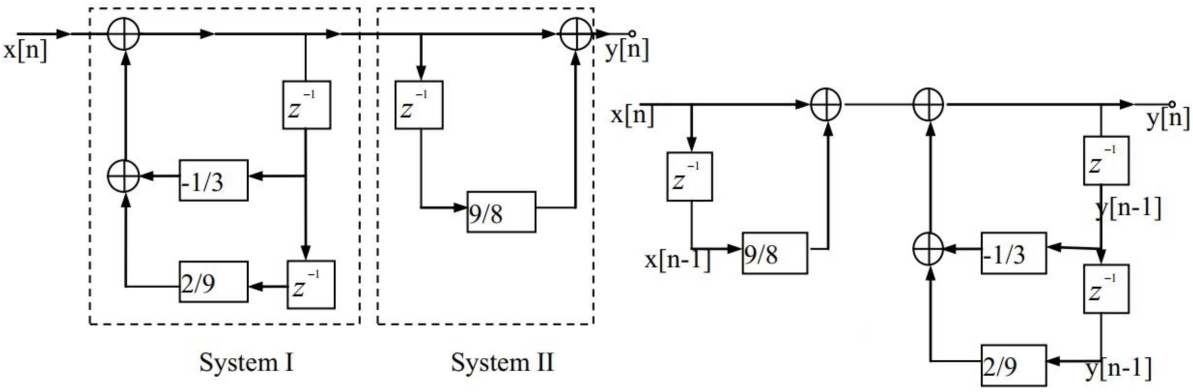

Figure 2: z-system

$y[n] = x[n] + \frac{9}{8}x[n-1] - \frac{1}{3}y[n-1] + \frac{2}{9}y[n-2]$

(b) Taking the z-transform of the above difference equation and simplifying , we get

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 - \frac{9}{8}z^{-1}}{1 + \frac{1}{3}z^{-1} - \frac{2}{9}z^{-2}} = \frac{1 + \frac{9}{8}z^{-1}}{(1 + \frac{2}{3}z^{-1})(1 - \frac{1}{3}z^{-1})}$$

$H(z)$ has poles at $z = \frac{1}{3}$ and $-\frac{2}{3}$. The ROC has to be $|z| > 2/3$. The ROC includes the unit circle and hence the system is stable.

5. (20) solution :
(a).we have $H(-2)=0$.We know that when $X(z)=\frac{1}{1-\frac{1}{2}z^{-1}}$, $|z|>\frac{1}{2}$, we have

$$Y(z)=1+\frac{a}{1-\frac{1}{4}z^{-1}}$$

, $|z|>\frac{1}{4}$, therefore

$$H(z)=\frac{Y(z)}{X(z)}=\frac{(1+a-\frac{1}{4}z^{-1})(1-\frac{1}{2}z^{-1})}{1-\frac{1}{4}z^{-1}}$$

, $|z|>\frac{1}{4}$
Substituting $z=-2$ in the above equation and nothing that $N(-2)=0$, we get $\alpha=-\frac{9}{8}$
(b). The response to the signal $x[n]=1=1^n$. Therefore

$$y[n]=H(1)=-\frac{1}{4}$$

6. (20)(a)Since the ROC is $|z| < 1/2$, the sequence is left-sided. Using the power-series expansion ,we get

$$log(1-2z) = -\sum_{n=1}^{\infty} \frac{2^n z^n}{n} = -\sum_{n=-\infty}^{-1} -\frac{2^{-n} z^{-n}}{n}$$

therefore

$$x[n] = \frac{2^{-n}}{n} u[-n-1]$$

(b) Since the ROC is $|z| > 1/2$, the sequence is right-sided. Using the power-series expansion ,we get

$$log(1-\frac{1}{2}z^{-1}) = -\sum_{n=1}^{\infty} \frac{(\frac{1}{2})^n z^{-n}}{n}$$

therefore,

$$x[n] = -\frac{2^{-n}}{n} u[n-1]$$

7. (20)

$$f(n+2) = f(n+1) + f(n)$$

We have two ways to solve the problem by z-transform
(1):Forward differential
We do a Z-transform of the function :

$$Z^2F(z) - f(0)z^2 - f(1)z = [ZF(z) - f(0)z] + F(z)$$

here $f(0) = 1$ and $f(1) = 1$

$$F(z) = \frac{z^2}{z^2 - z - 1} = \frac{z^2}{(z - \frac{1+\sqrt{5}}{2})(z - -\frac{1-\sqrt{5}}{2})} = \frac{z}{\sqrt{5}}(\frac{z}{z - \frac{1+\sqrt{5}}{2}} - \frac{z}{z - \frac{1-\sqrt{5}}{2}})$$

Use inverse Z-transform and get the solution:

$$f(n) = \frac{1}{\sqrt{5}}[(\frac{1+\sqrt{5}}{2})^{n+1} - (\frac{1-\sqrt{5}}{2})^{n+1}] \qquad (n \geq 0)$$

(2):backward difference
We do a Z-transform of the function :

$$F(z) = [z^{-1}F(z) + f(-1)] + [z^{-2}F(z) + f(-2) + f(-1)z^1]$$

$f(-2) = 1$ and $f(-1) = 0$.

$$F(z) = \frac{z^2}{z^2 - z - 1} = \frac{z^2}{(z - \frac{1+\sqrt{5}}{2})(z - -\frac{1-\sqrt{5}}{2})} = \frac{z}{\sqrt{5}}(\frac{z}{z - \frac{1+\sqrt{5}}{2}} - \frac{z}{z - \frac{1-\sqrt{5}}{2}})$$

Use inverse Z-transform and get the solution:

$$f(n) = \frac{1}{\sqrt{5}}[(\frac{1+\sqrt{5}}{2})^{n+1} - (\frac{1-\sqrt{5}}{2})^{n+1}] \qquad (n \geq 0)$$