

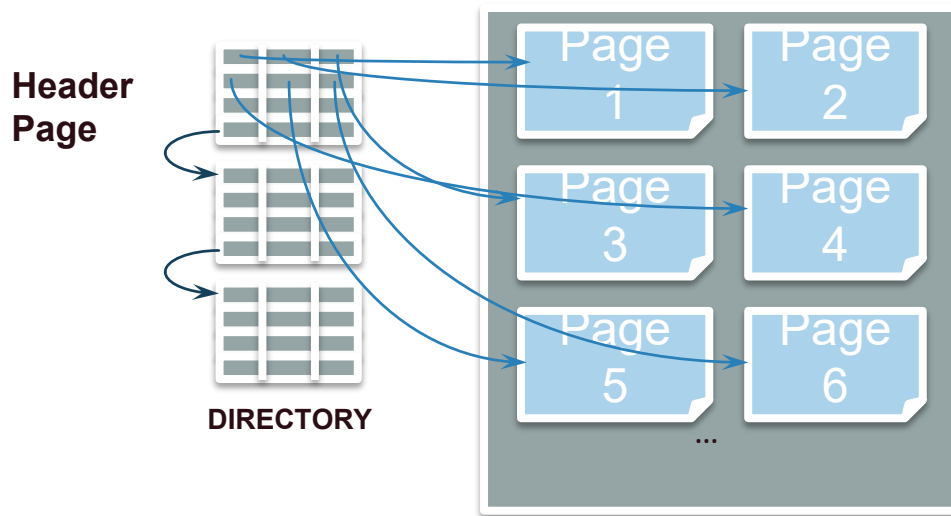
Tree Indexes



R & G - Chapter 10

Reminder on Heap Files

- Two access APIs:
 - fetch by recordId (pageId, slotId)
 - scan (starting from some page)

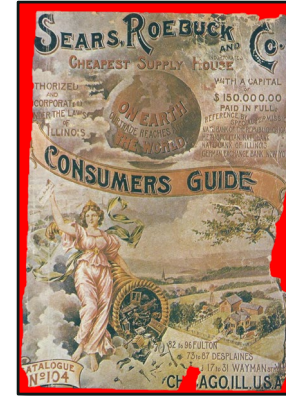


Wouldn't it be nice...

- ...if we could look things up by value?
- Toward a Declarative access API
- But ... efficiency?

“If you don't find it in the index, look very carefully through the entire catalog. ”

—Sears, Roebuck, and Co., Consumers' Guide, 1897



We've seen this before

- Data structures ... in RAM:
 - Search trees (Binary, AVL, Red-Black, ...)
 - Hash tables
- Needed: disk-based data structures
 - “paginated”: made up of disk pages!

Index

An **index** is data structure that enables fast **lookup** and **modification** of **data entries** by **search key**

- **Lookup:** may support many different operations
 - **Equality**, 1-d range, 2-d region, ...
- **Search Key:** any subset of columns in the relation
 - Do not need to be unique
 - —e.g. (firstname) or (firstname, lastname)

Index Part 2

An **index** is data structure that enables fast **lookup** and **modification** of **data entries** by **search key**

- **Data Entries:** items stored in the index
 - Assume for today: a pair (**k**, recordId) ...
 - Pointers to records in Heap Files!
 - Easy to generalize later
- **Modification:** want to support fast insert and delete

Many Types of indexes exist: B+-Tree, Hash, R-Tree, GiST, ...

Simple Idea?

Input
Heap
File

3, 4, 5

1, 2, 7

8, 6, 9

10, _, _

- **Step 1:** Sort heap file & leave some space
 - Pages physically stored in logical order (sequential access)
 - Do we need “next” pointers to link pages?
 - No. Pages are physically sorted in logical order

1, 2, _

3, 4, _

5, 6, _

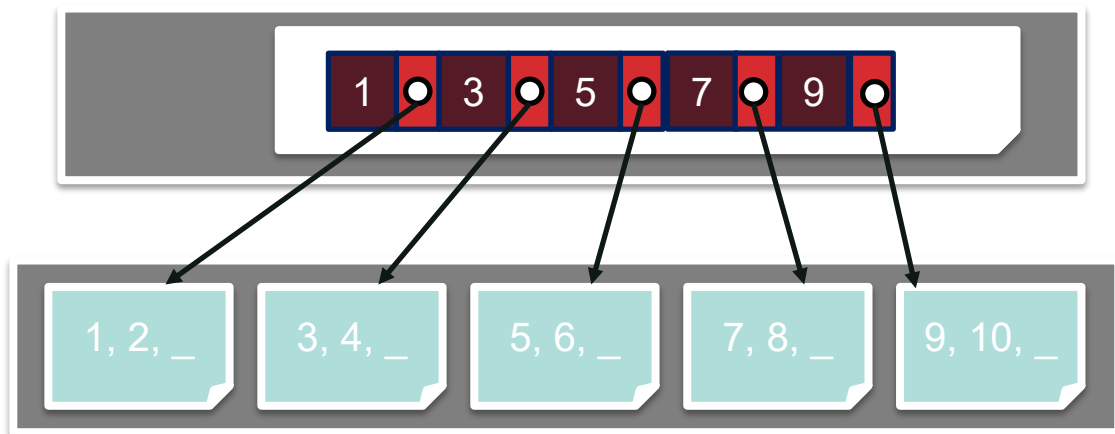
7, 8, _

9, 10, _

- **Step 2:** Build the index data structure over this...
 - Why not just use binary search in this heap file?
 - Fan-out of 2 → deep tree → lots of I/Os
 - Examine entire records just to read key during search

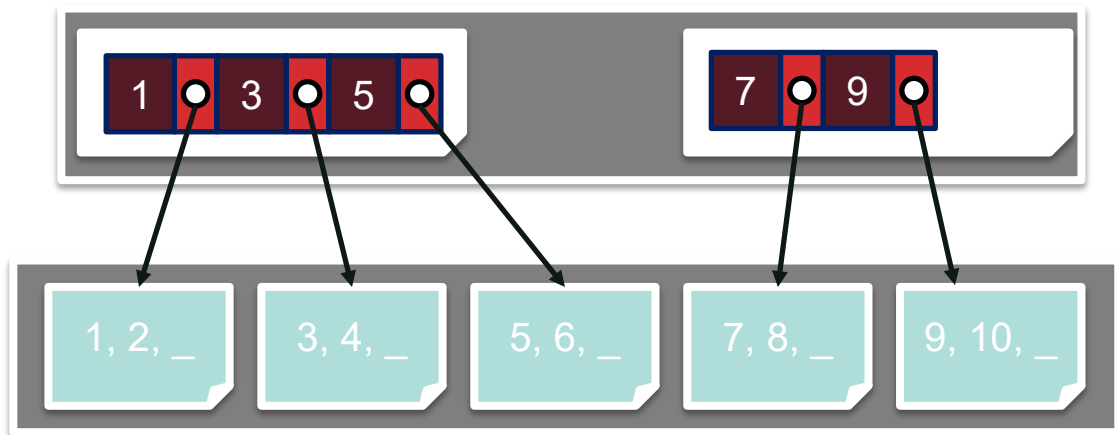
Build a high fan-out search tree

- Start simple: *Sorted (key, page id) file*
 - No record data
 - Binary search in the key file. Better!
 - **Forgot:** Need to break across pages!



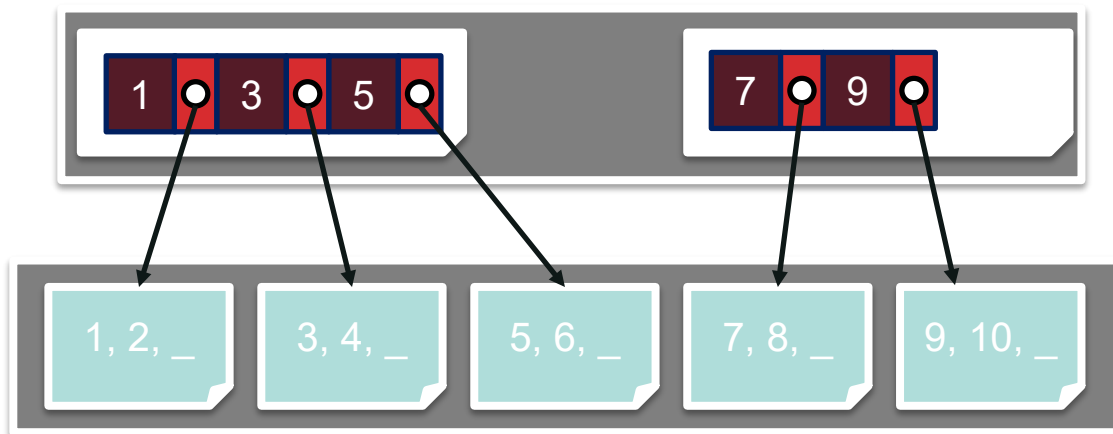
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- Start simple: *Sorted (key, page id) file*
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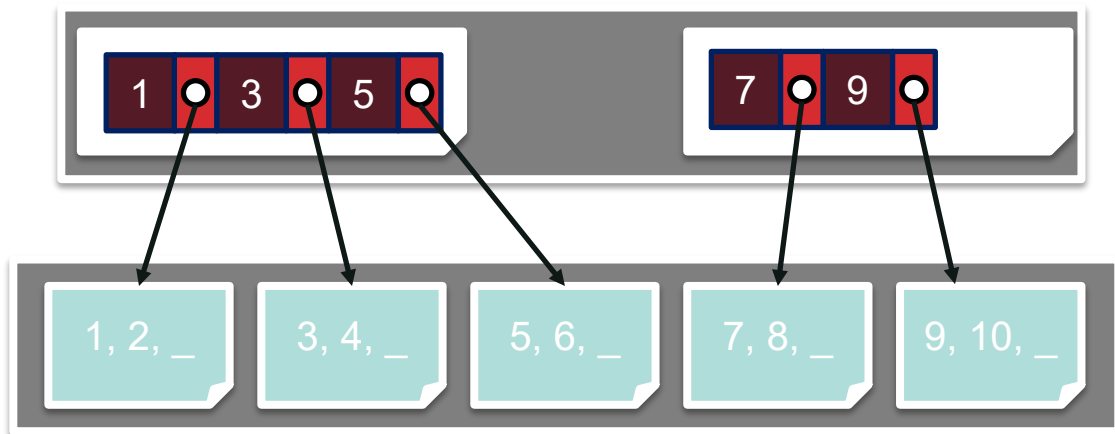
Build a high fan-out search tree Part 2

- Start simple: *Sorted (key, page id) file*
 - No record data
 - Binary search in the key file. Better!
 - **Complexity?**



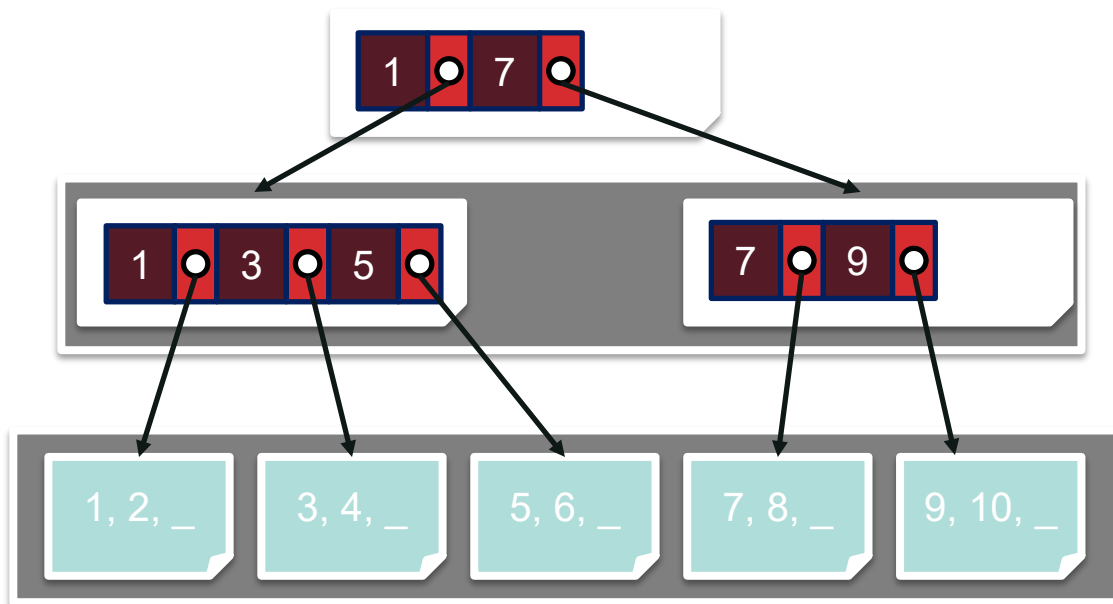
Build a high fan-out search tree Part 3

- Start simple: *Sorted (key, page id) file*
 - No record data
 - Binary search in the key file. Better!
 - **Complexity**: Still binary search, just a constant factor smaller input



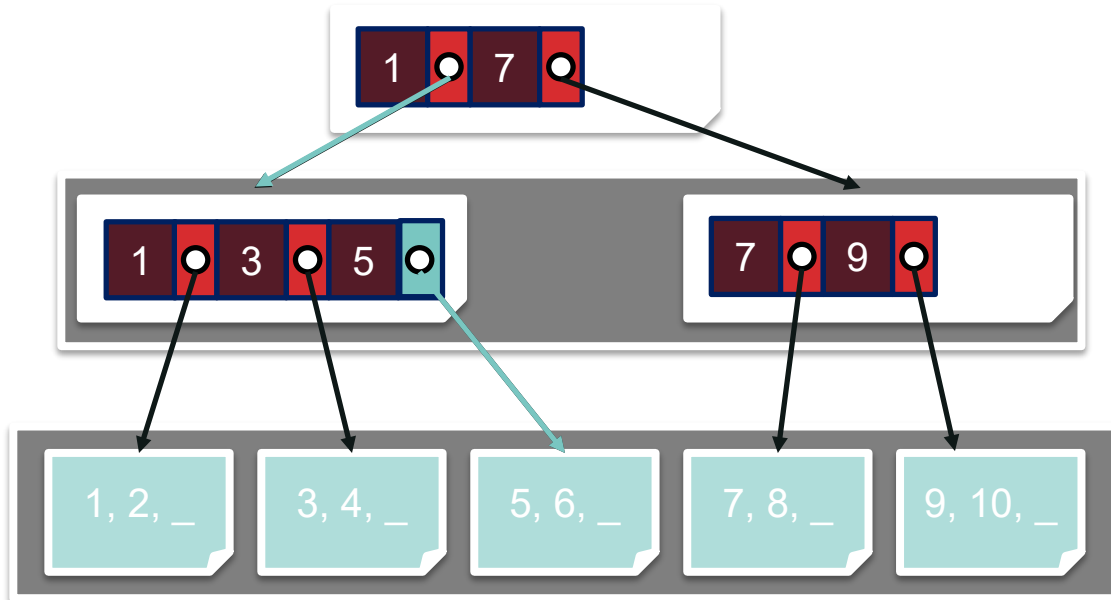
Build a high fan-out search tree Part 4

- Recursively “index” key file
- **Key Invariant:**
 - Node $[..., (K_L, P_L), (K_R, P_R), ...]$ \rightarrow All tuples in range $K_L \leq K < K_R$ are in tree P_L



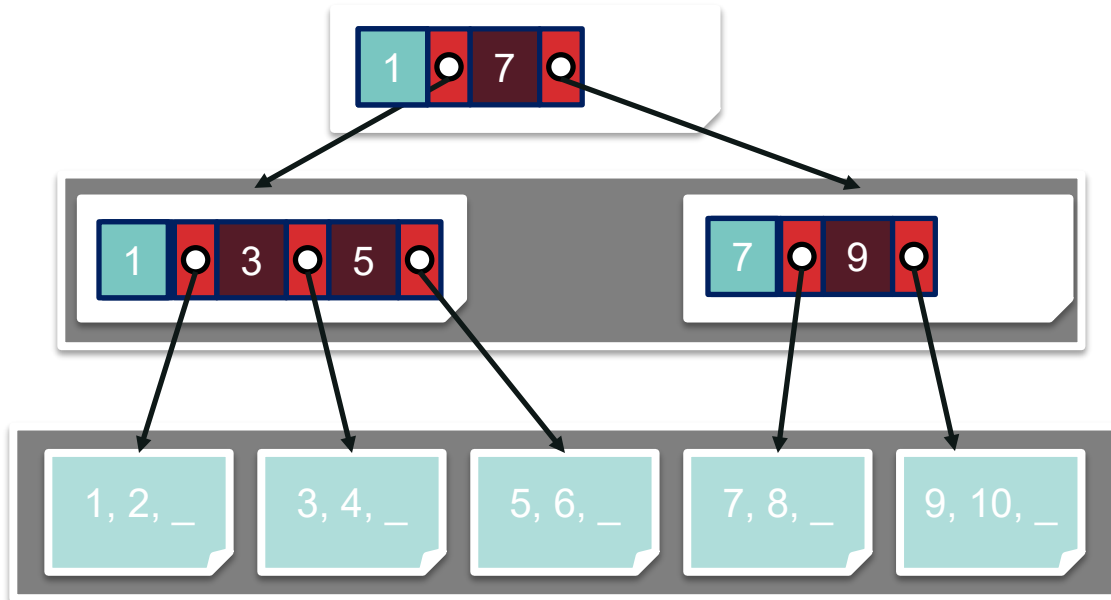
Search a high fan-out search tree

- Searching for **5**?
 - Binary Search each node (page) starting at root
 - Follow pointers to next level of search tree
- Complexity? $O(\log_F(\#Pages))$



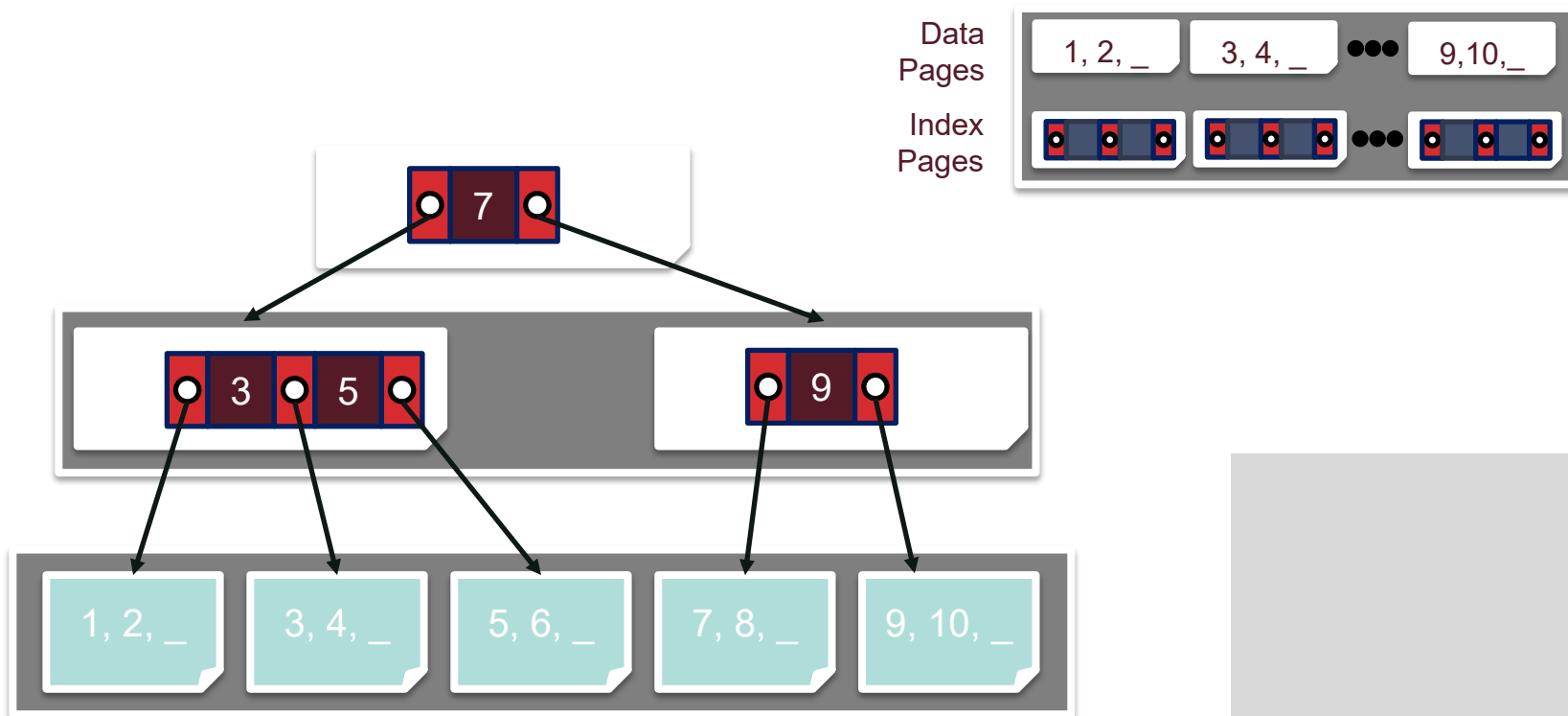
Left Key Optimization?

- Optimization
 - Do we need the left most key?



Build a high fan-out search tree

- Disk Layout? All in a single file, Data Pages first.



Status Check

- Some design goals:
 - Fast sequential access?
 - High Fan-out
 - Support insertions

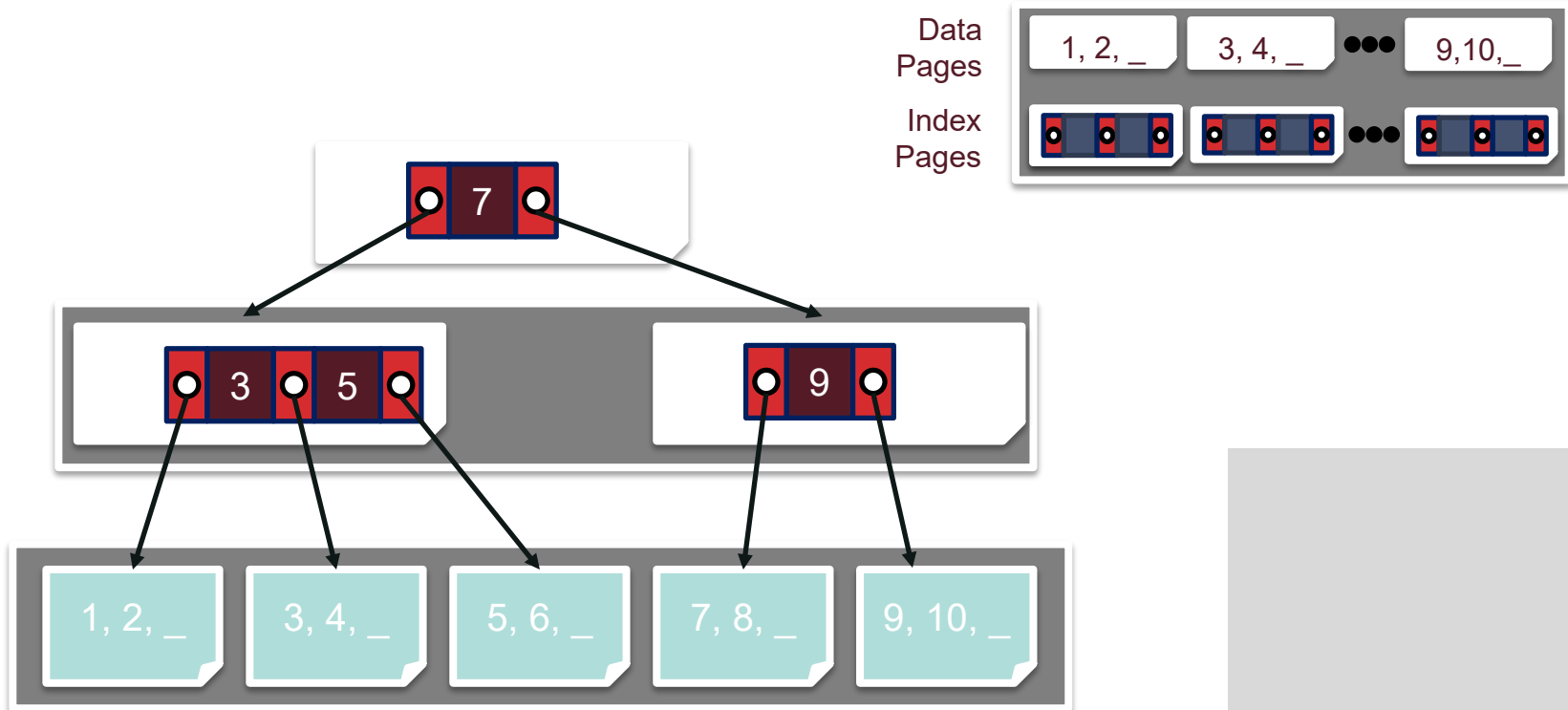
Indexed File

ISAM Indexed Sequential Access Method

(Early IBM Indexing Technology)

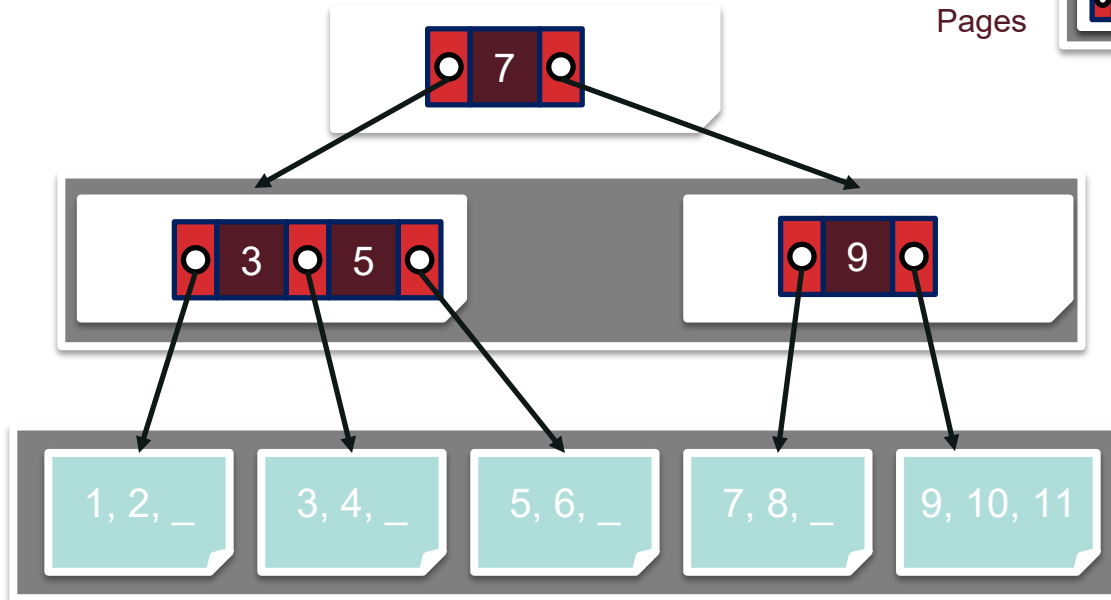


Insert 11, Before



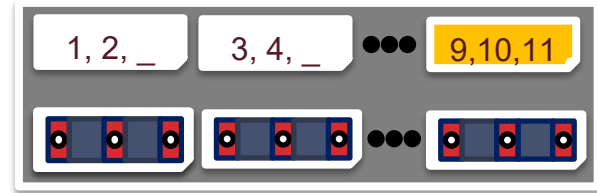
Insert 11, After

- Find location
- Place in data page
 - Re-sort page ...



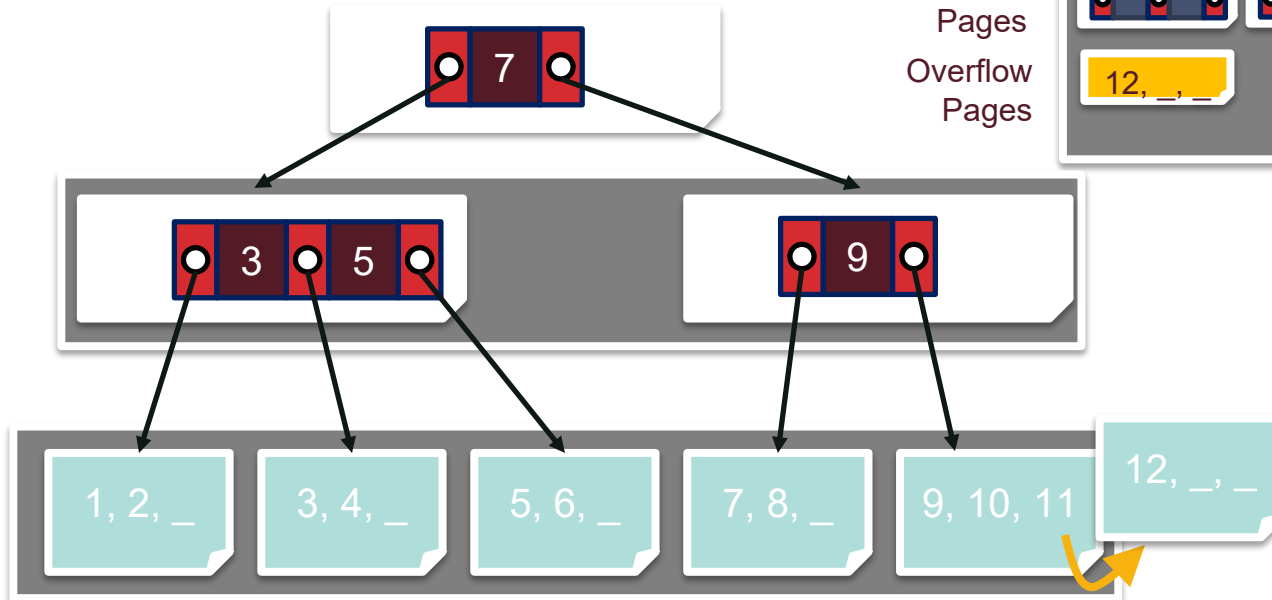
Data
Pages

Index
Pages



Insert 12?

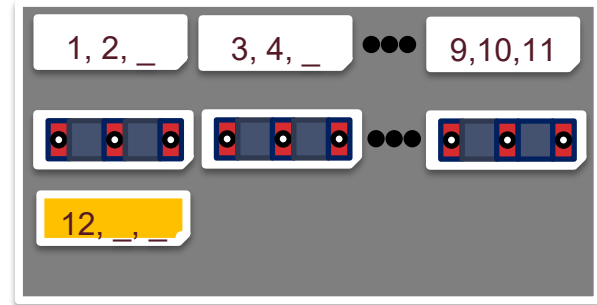
- Find location
- Place in data page
- Add overflow page if necessary ...



Data
Pages

Index
Pages

Overflow
Pages



Recap: ISAM

- Data entries in sorted heap file
- High fan-out static tree index
- Fast search + good locality
 - Assuming nothing changes
- Insert into overflow pages

A Note of Caution

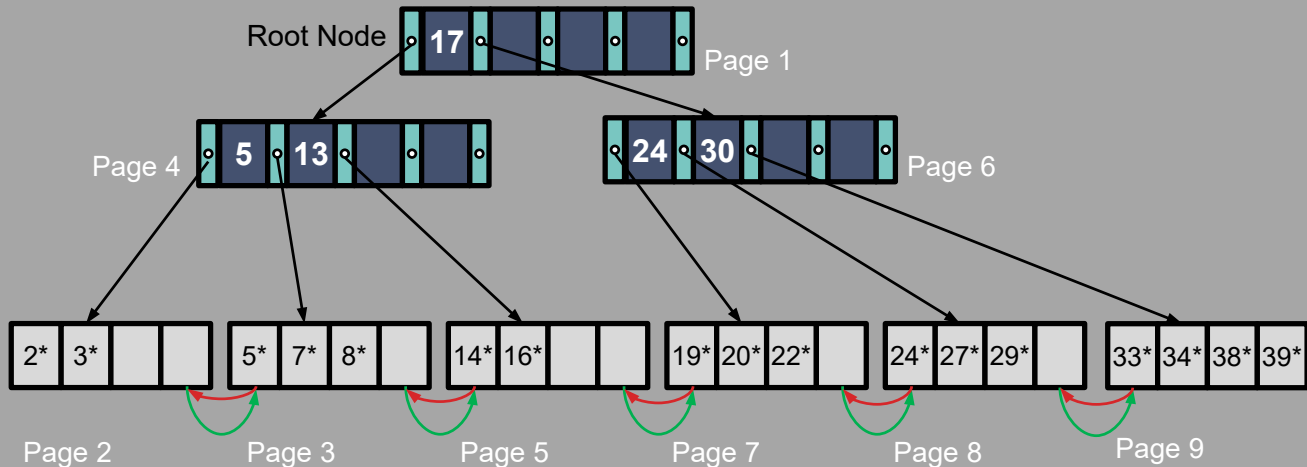
- ISAM is an old-fashioned idea
 - Introduced by IBM in 1960s
 - B+ trees are usually better, as we'll see
 - Though not always (← we'll come back to this)
- But, it's a good place to start
 - Simpler than B+ tree, many of the same ideas
- Upshot
 - Don't brag about ISAM on your resume
 - Do understand ISAM, and tradeoffs with B+ trees

B+-TREE

Enter the B+ Tree

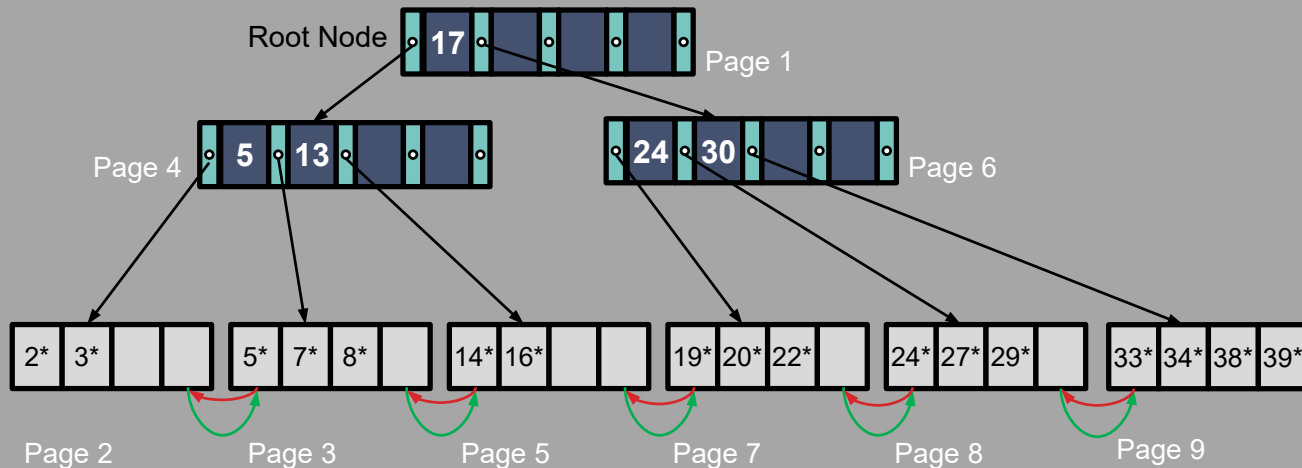
- Similar to ISAM
 - Same interior node structure
 - $\langle \text{Key}, \text{Page Ptr} \rangle$ pairs with same key invariant
 - Same search routine as before
- **Dynamic Tree Index**
 - Always Balanced
 - Support efficient insertion & deletion
 - Grows at root not leaves!
- “+”? B-tree that stores data entries in leaves only

Example of a B+ Tree



- Occupancy Invariant
 - Each interior node is at least partially full:
 - $d \leq \text{\#entries} \leq 2d$
 - d : order of the tree (max fan-out = $2d + 1$)
- Data pages at bottom need not be stored in logical order
 - Next and prev pointers

Sanity Check



What is the value of d?

2

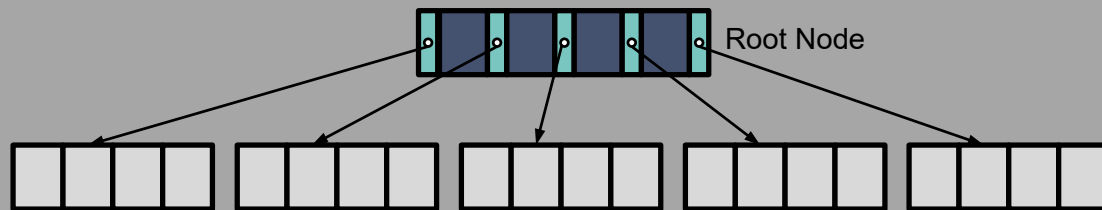
What about the root?

The root is special

Why not in sequential order?

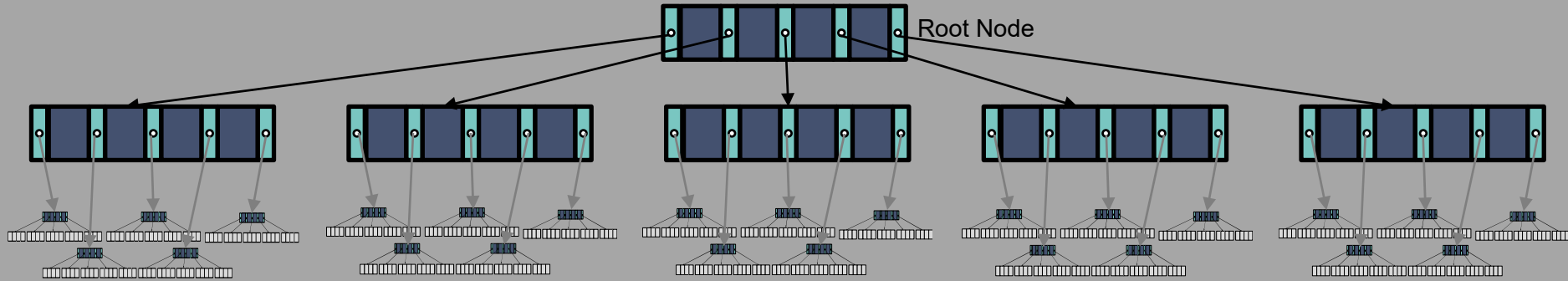
Data pages allocated dynamically

B+ Trees and Scale



- How big is a height 1 B+ tree
 - $d = 2 \rightarrow$ Fan-out?
 - Fan-out = $2d + 1 = 5$
 - **Height 1:** $5 \times 4 = 20$ Records

B+ Trees and Scale Part 2

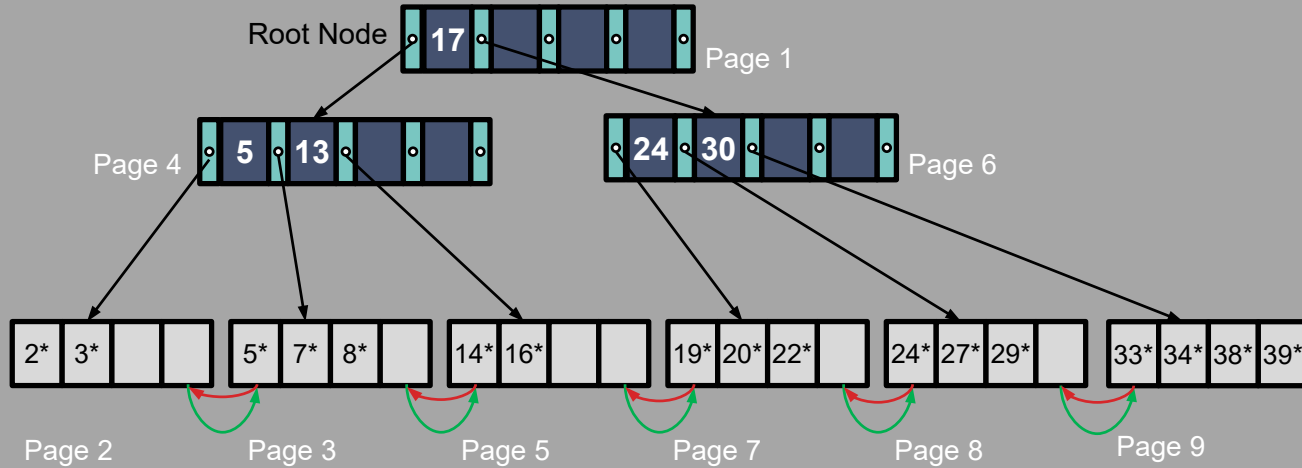


- How big is a height **3** B+ tree
 - $d = 2 \rightarrow$ Fan-out?
 - Fan-out = $2d + 1 = 5$
 - **Height 3:** $5^3 \times 4 = 500$ Records

B+ Trees in Practice

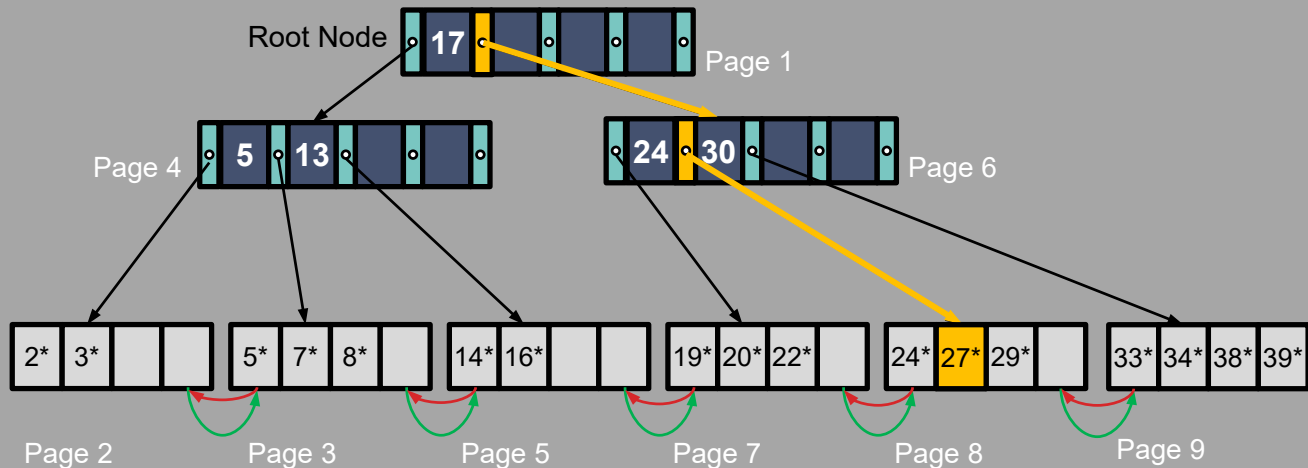
- Typical order: 1600. Typical fill-factor: 67%.
 - average fan-out = 2144
 - (assuming 128 Kbytes pages at 40Bytes per record)
- At typical capacities
 - Height 1: $2144^2 = \mathbf{4,596,736 \text{ records}}$
 - Height 2: $2144^3 = \mathbf{9,855,401,984 \text{ records}}$

Searching the B+ Tree



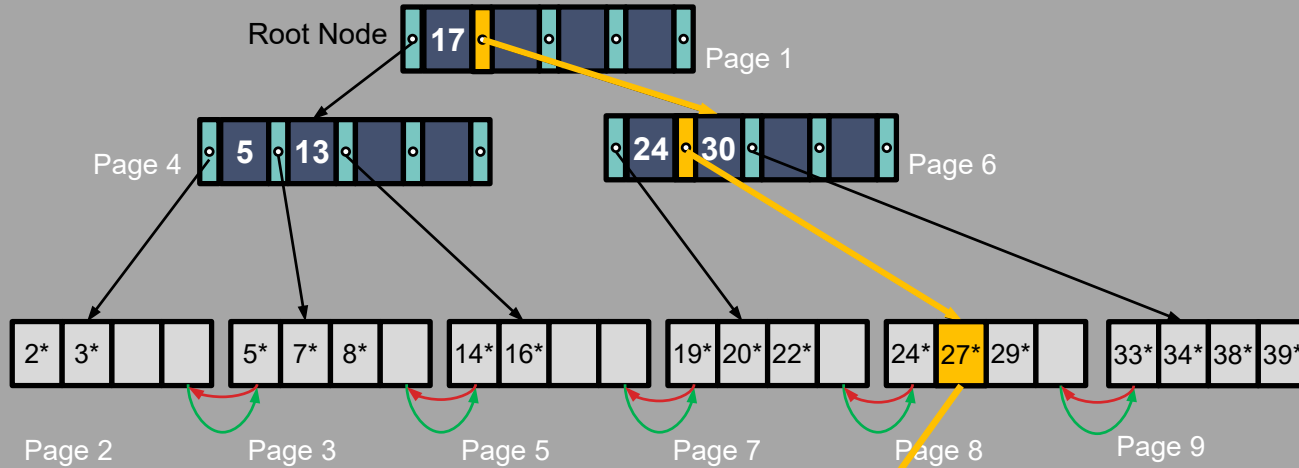
- Same as ISAM
- Find key = 27
 - Find split on each node (Binary Search)
 - Follow pointer to next node

Searching the B+ Tree: Find 27

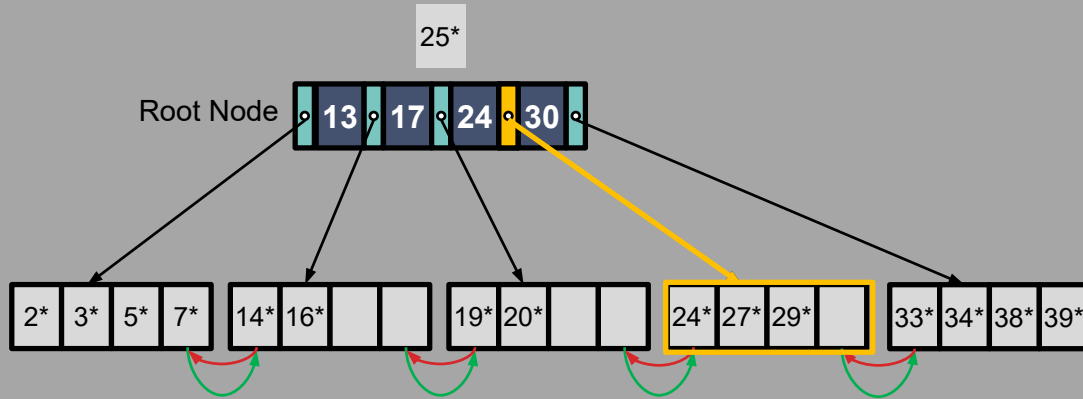


- Same as ISAM
- Find key = 27
 - Find split on each node (Binary Search)
 - Follow pointer to next node

Searching the B+ Tree: Fetch Data

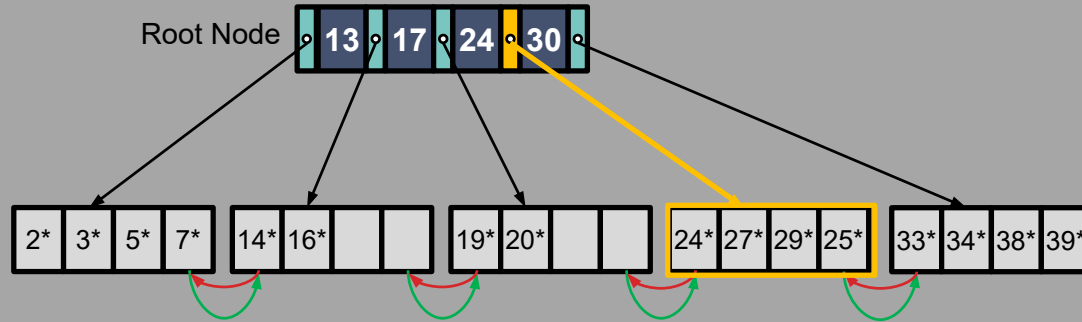


Inserting 25* into a B+ Tree Part 1



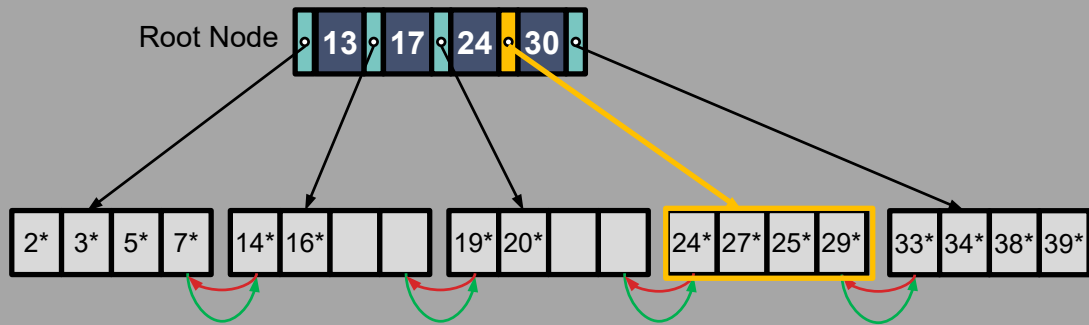
- Find the correct leaf

Inserting 25* into a B+ Tree Part 2



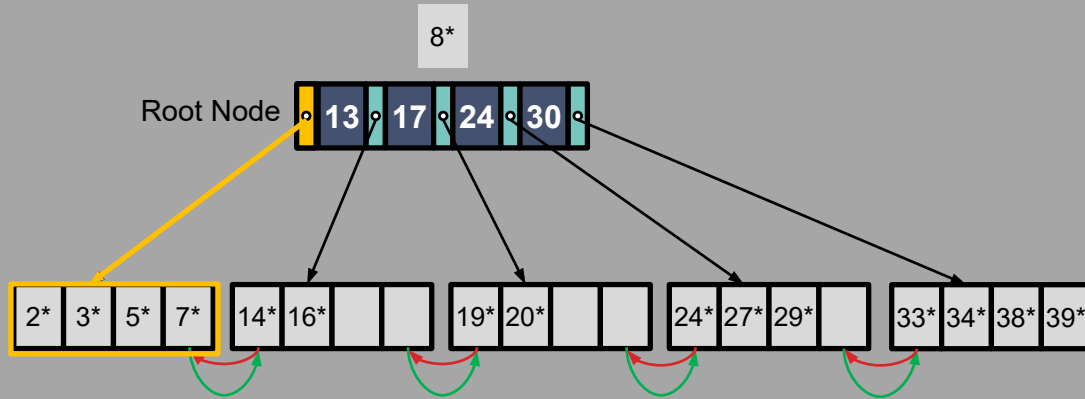
- Find the correct leaf
- If there is room in the leaf just add the entry

Inserting 25* into a B+ Tree Part 3



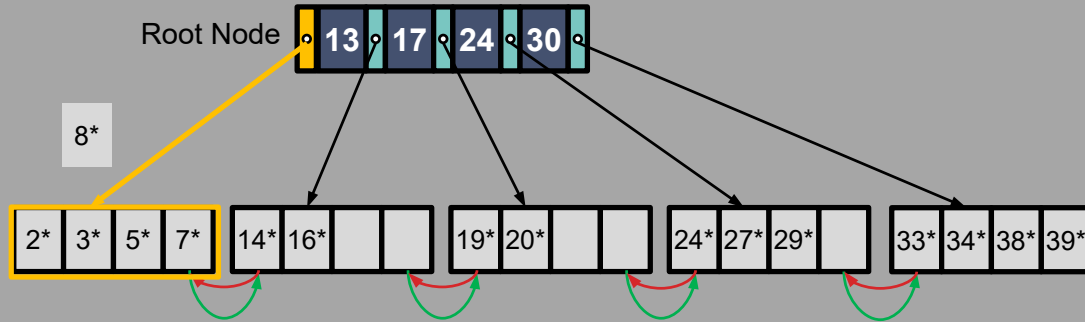
- Find the correct leaf
- If there is room in the leaf just add the entry
 - Sort the leaf page by key

Inserting 8* into a B+ Tree: Find Leaf



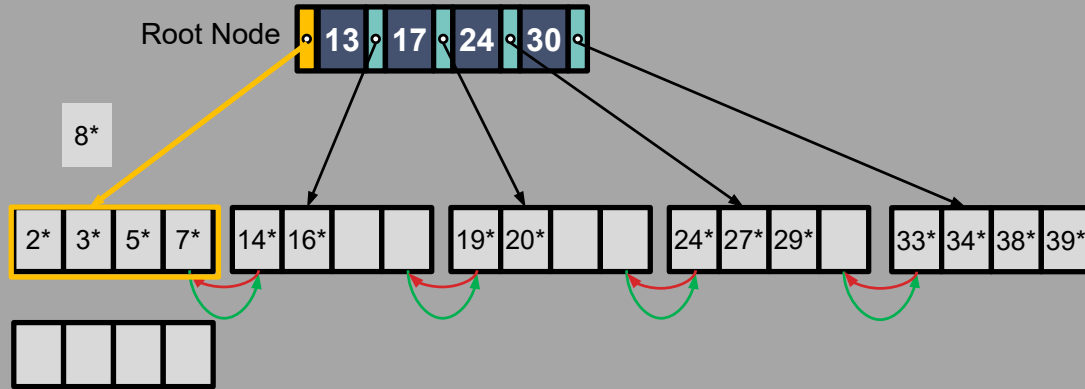
- Find the correct leaf

Inserting 8* into a B+ Tree: Insert



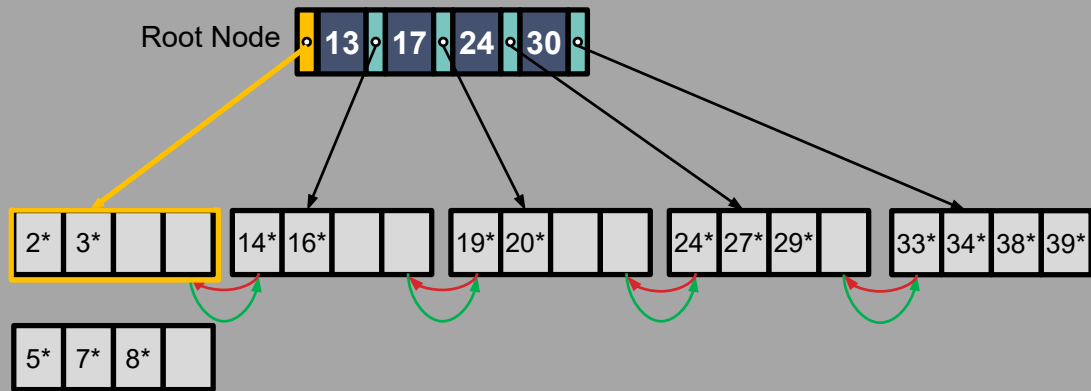
- Find the correct leaf
 - Split leaf if there is not enough room

Inserting 8* into a B+ Tree: Split Leaf



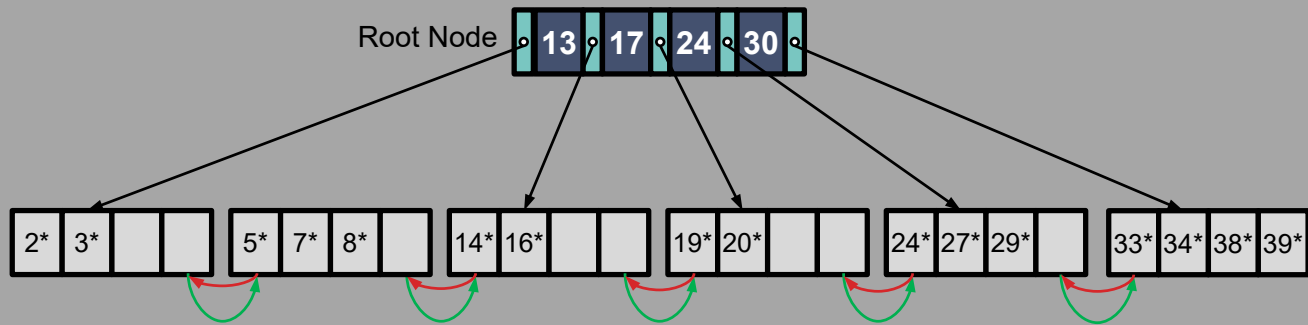
- Find the correct leaf
 - Split leaf if there is not enough room
 - Redistribute entries evenly

Inserting 8* into a B+ Tree: Split Leaf, cont



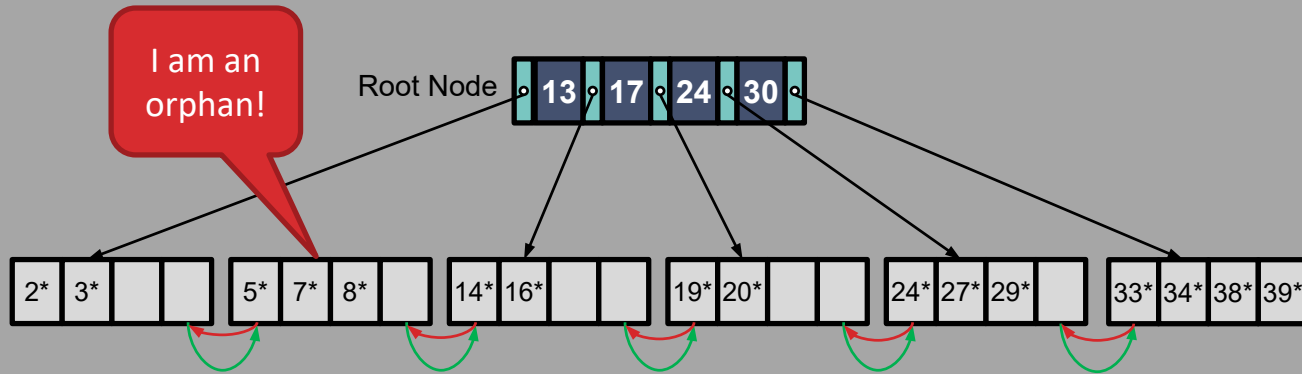
- Find the correct leaf
 - Split leaf if there is not enough room
 - Redistribute entries evenly
 - Fix next/prev pointers

Inserting 8* into a B+ Tree: Fix Pointers



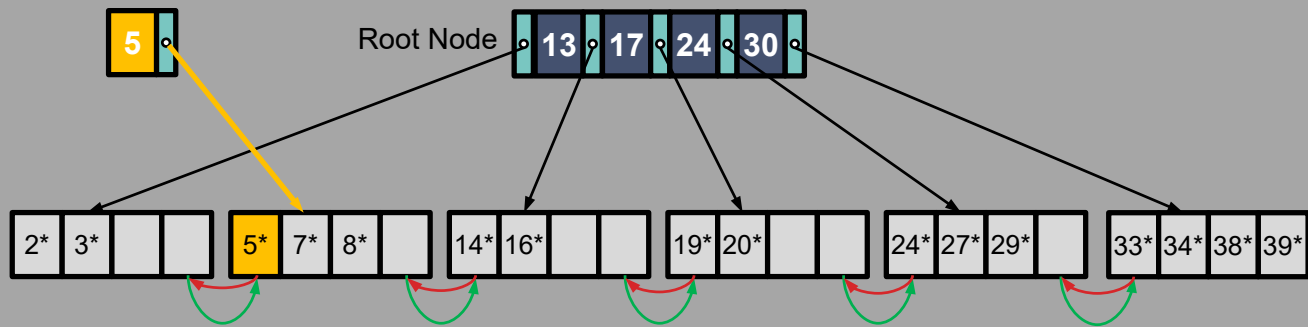
- Find the correct leaf
 - Split leaf if there is not enough room
 - Redistribute entries evenly
 - Fix next/prev pointers

Inserting 8* into a B+ Tree: Mid-Flight



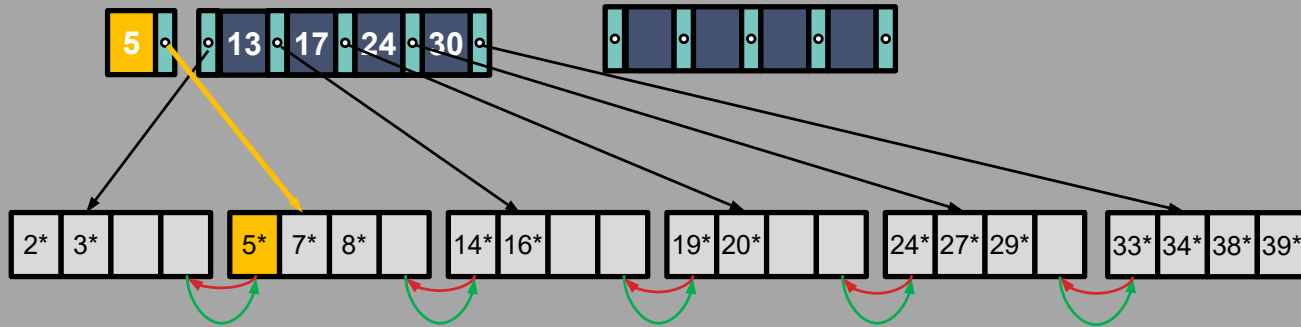
- Something is still wrong!

Inserting 8* into a B+ Tree: Copy Middle Key



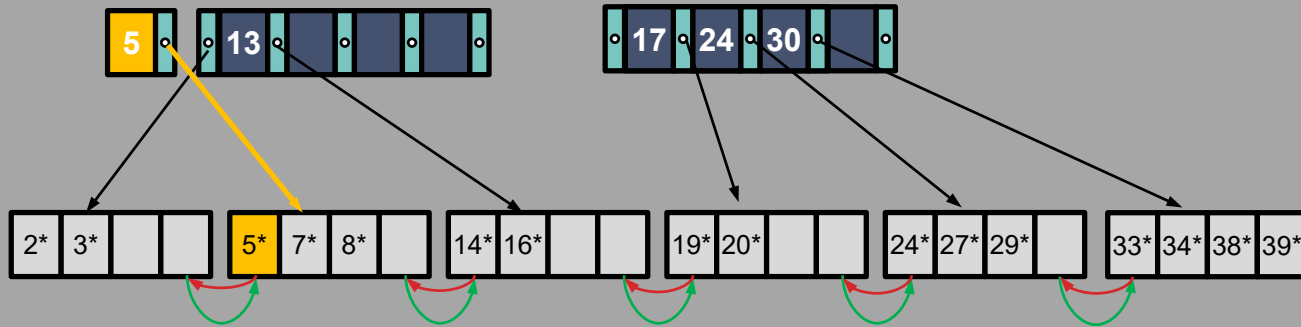
- **Copy up from leaf** the middle key
- No room in parent? Recursively split index nodes

Inserting 8* into a B+ Tree: Split Parent, Part 1



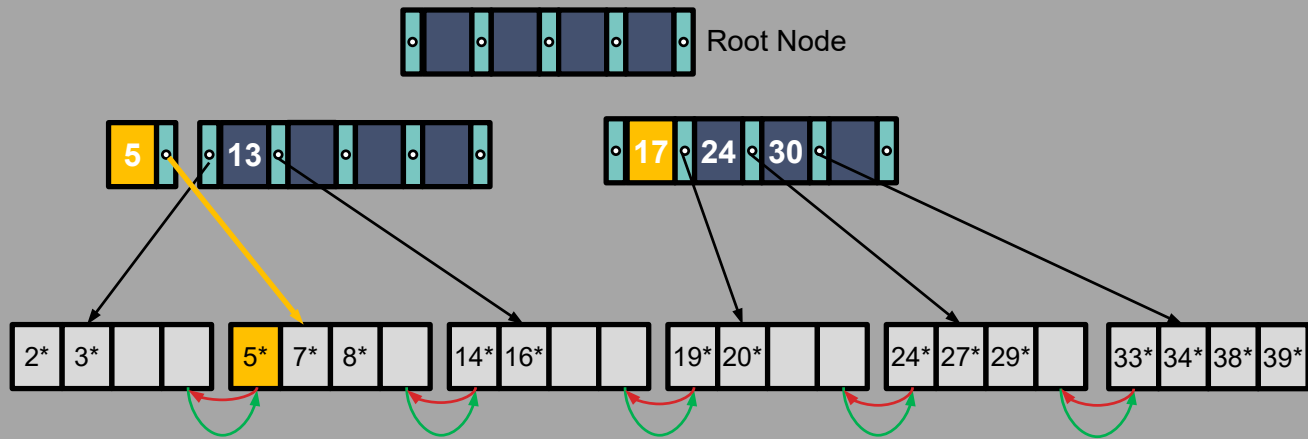
- **Copy up from leaf** the middle key
- No room in parent? Recursively split index nodes
 - Redistribute the rightmost d keys

Inserting 8^* into a B+ Tree: Split Parent, Part 2



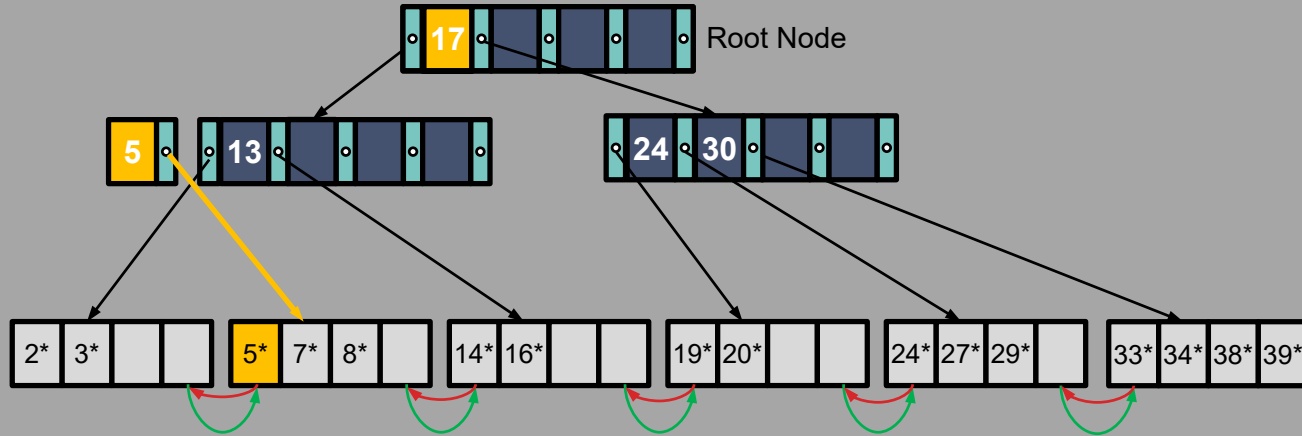
- **Copy up from leaf** the middle key
- No room in parent? Recursively split index nodes
 - Redistribute the rightmost d keys

Inserting 8* into a B+ Tree: Root Grows Up



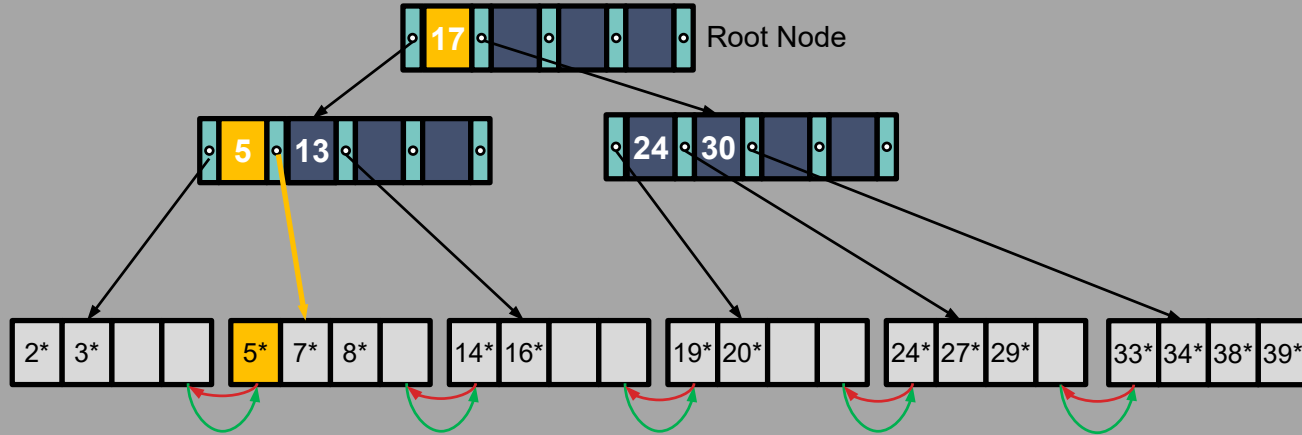
- **Push up from interior node** the middle key
 - Now the last key on left
- No room in parent? Recursively split index nodes
 - Redistribute the rightmost d keys

Inserting 8* into a B+ Tree: Root Grows Up, Pt 2



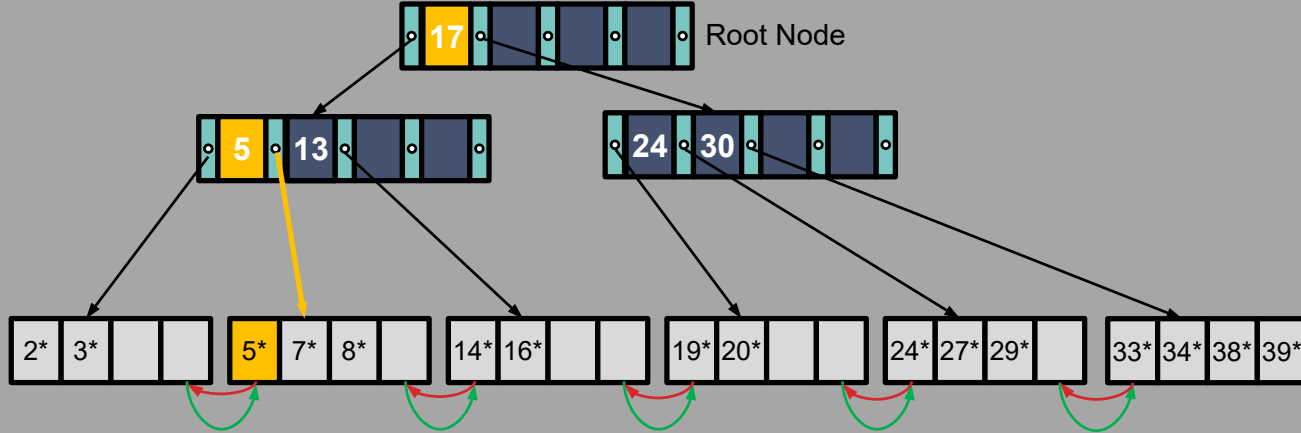
- Recursively split index nodes
 - Redistribute right d keys
 - **Push** up middle key

Inserting 8* into a B+ Tree: Root Grows Up, Pt 3



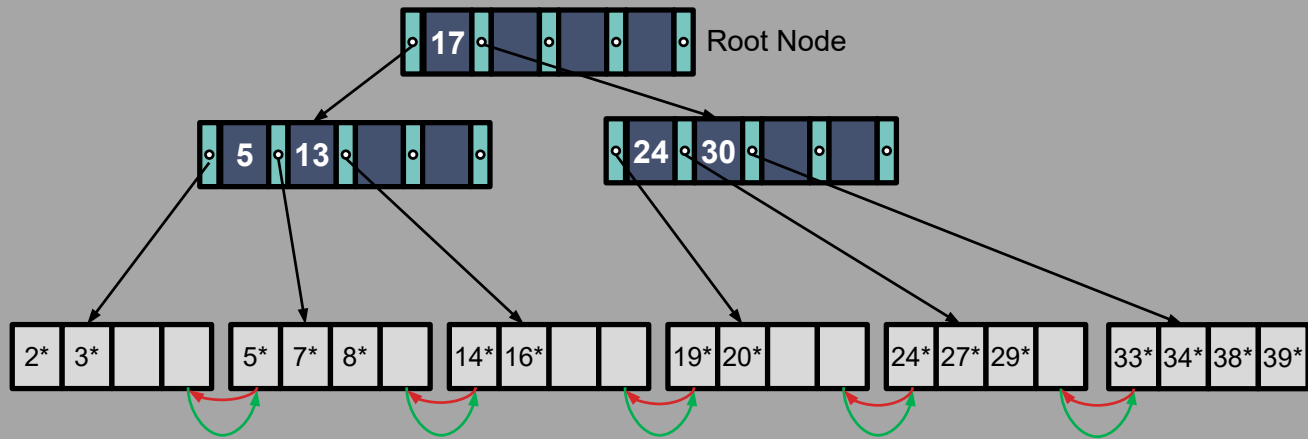
- Recursively split index nodes
 - Redistribute right d keys
 - **Push** up middle key

Copy up vs Push up!



- Notice:
 - The **leaf** entry (5) was **copied** up
 - The **index** entry (17) was **pushed** up

Inserting 8* into a B+ Tree: Final



- Check invariants
- **Key Invariant:**
 - $\text{Node}[\dots, (K_L, P_L), \dots] \rightarrow K_L \leq K$ for all K in P_L Sub-tree
- **Occupancy Invariant:**
 - $d \leq \# \text{ entries} \leq 2d$

B+ Tree Insert: Algorithm Sketch

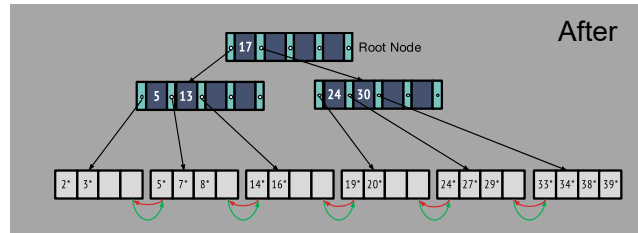
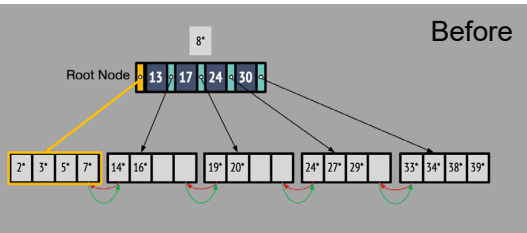
1. Find the correct leaf L.
2. Put data entry onto L.
 - If L has enough space, done!
 - Else, must split L (into L and a new node L2)
 - Redistribute entries evenly, copy up middle key
 - Insert index entry pointing to L2 into parent of L.

B+ Tree Insert: Algorithm Sketch Part 2

- Step 2 can happen recursively
 - To split index node, redistribute entries evenly, but push up middle key. (Contrast with leaf splits)
- Splits “grow” tree; root split increases height.
 - Tree growth: gets wider or one level taller at top.

Before and After Observations

- Notice that the root was split to increase the height
 - Grow from the root not the leaves
 - All paths from root to leaves are equal lengths
- Does the occupancy invariant hold?
 - Yes! All nodes (except root) are at least half full
 - Proof?

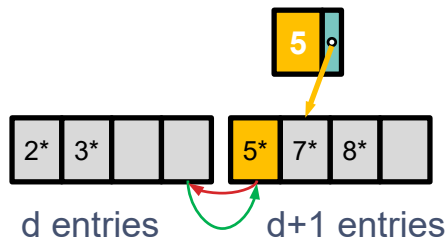


Splitting a Leaf

- Start with full leaf ($2d$) entries (let $d = 2$)
 - Add a $2d + 1$ entry (8^*)



- Split into leaves with $(d, d+1)$ entries
 - Copy key up to parent
- Why copy key and not push key up to parent?



Splitting an Inner Node

- Start with full interior node ($2d$) entries: (let $d = 2$)
 - Add a $2d + 1$ entry



- Split into nodes with $(d, d+1)$ entries
 - Push** key up to parent



d entries



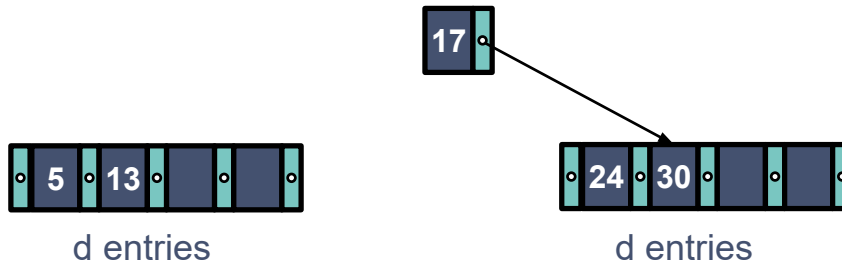
$d+1$ entries

Splitting an Inner Node Pt 2

- Start with full interior node ($2d$) entries: (let $d = 2$)
 - Add a $2d + 1$ entry



- Split into nodes with (d, d) entries
 - Push** key up to parent

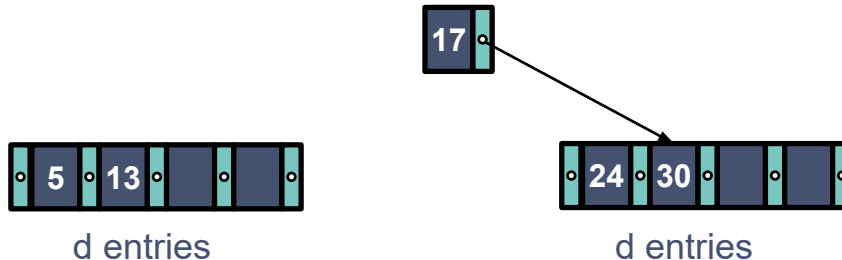


Splitting an Inner Node Pt 3

- Start with full interior node ($2d$) entries: (let $d = 2$)
 - Add a $2d + 1$ entry



- Split into nodes with (d, d) entries
 - Push** key up to parent



Why push not copy?

- Routing key not needed in child

Occupancy invariant holds after split

Nice Animation Online

- [Great animation online of B+ Trees](#)
- One small difference to note
 - Upon deletion of leftmost value in a node, it updates the parent index entry
 - Incurs unnecessary extra writes

B+-TREE DELETION

We will skip deletion

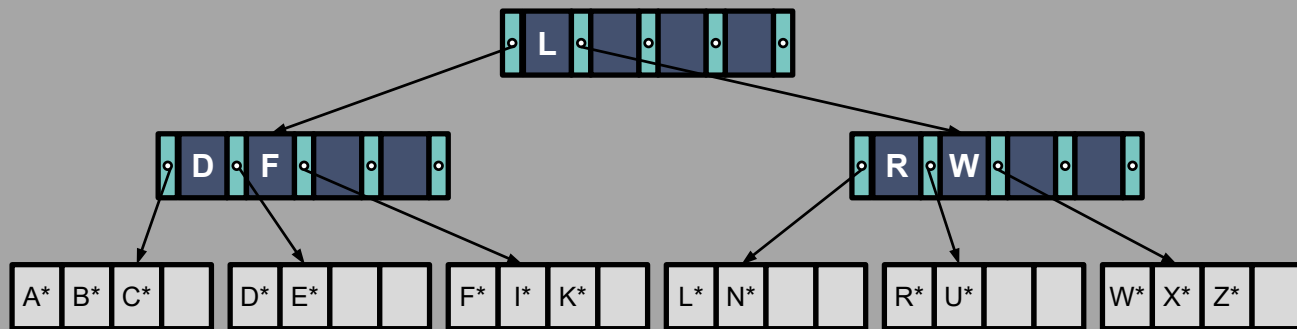
- In practice, occupancy invariant often not enforced
- Just delete leaf entries and leave space
- If new inserts come, great
 - This is common
- If page becomes completely empty, can delete
 - Parent may become underfull
 - That's OK too
- Guarantees still attractive: $\log_F(\text{max size of tree})$

BULK LOADING B+-TREES

Bulk Loading of B+ Tree Part 1

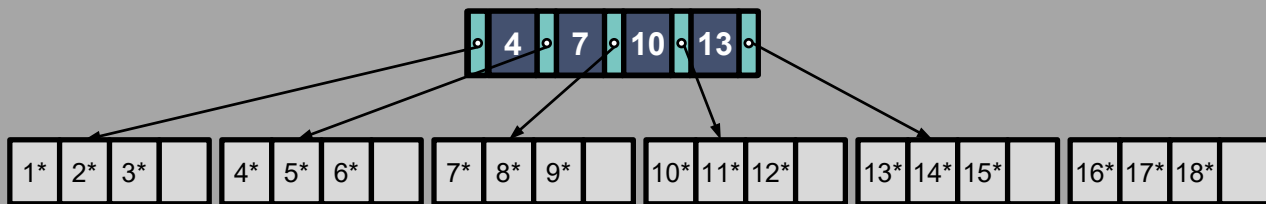
- Suppose we want to build an index on a large table
- Would it be efficient to just call insert repeatedly
 - No ... Why not?
 - Random Order: CLZARNDXEKFWIUB. Order 2.
 - Try it: [Interactive demo](#)

Bulk Loading of B+ Tree Part 2



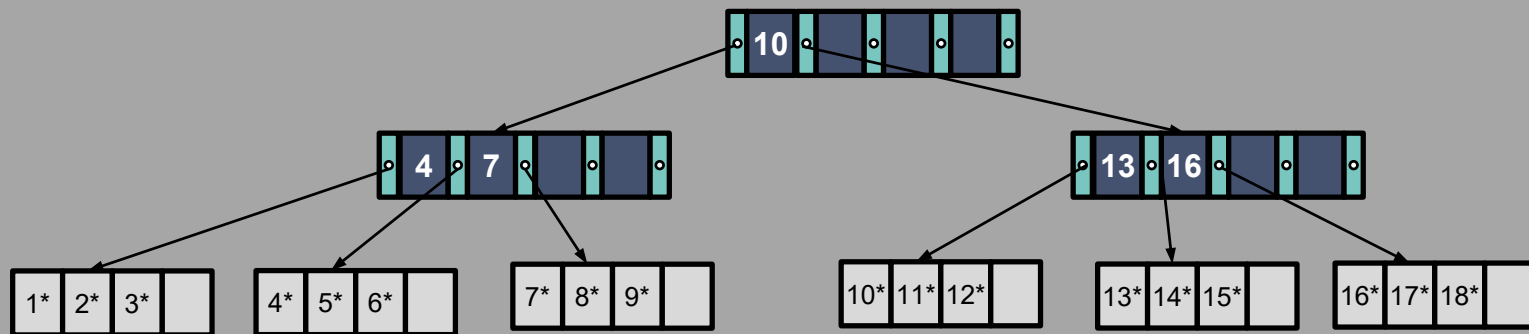
- Constantly need to search from root
- Leaves and internal nodes mostly half-empty
- **Modifying random pages:
poor cache efficiency**

Smarter Bulk Loading a B+ Tree



- Sort the input records by key:
 - 1*, 2*, 3*, 4*, ...
 - We'll learn a good disk-based sort algorithm soon!
- Fill leaf pages to some fill factor (e.g. $\frac{3}{4}$)
 - Updating parent until full

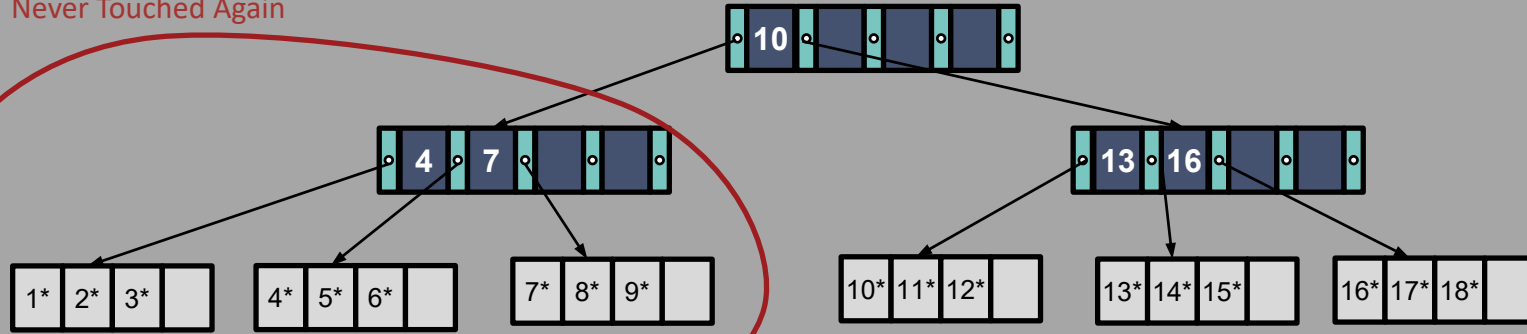
Smarter Bulk Loading a B+ Tree Part 2



- Sort the input records by key:
 - 1*, 2*, 3*, 4*, ...
- Fill leaf pages to some fill factor (e.g. $\frac{3}{4}$)
 - Update parent until full
 - Then split parent (50/50) and copy to sibling

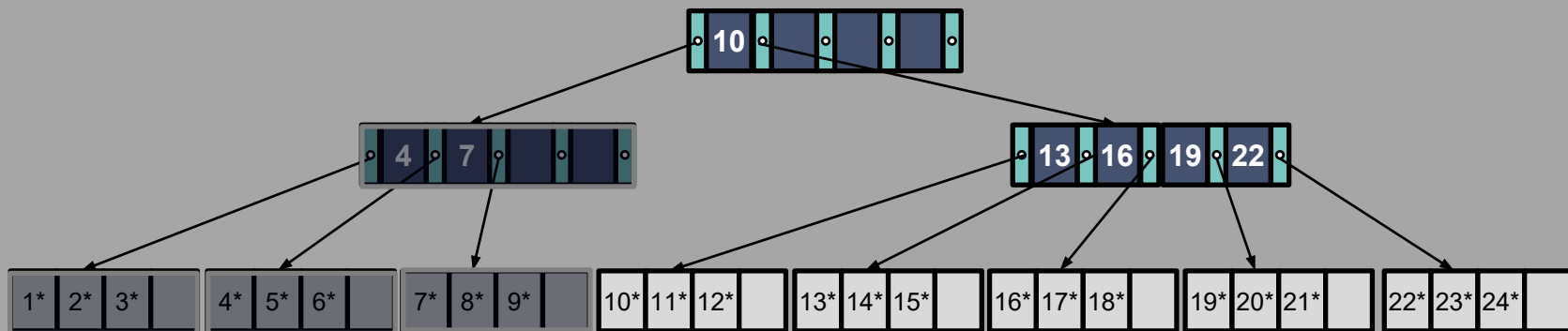
Smarter Bulk Loading a B+ Tree Part 3

Never Touched Again



- Lower left part of the tree is never touched again
- Occupancy invariant maintained

Smarter Bulk Loading a B+ Tree Part 4



- Sort the input records by key:
 - 1*, 2*, 3*, 4*, ...
- Fill leaf pages to some fill factor (e.g. $\frac{3}{4}$)
 - Update parent until full
 - Then split parent

Summary of Bulk Loading

- Option 1: Multiple inserts
 - **Slow**
 - Does not give sequential storage of leaves
- Option 2: Bulk Loading
 - Fewer I/Os during build. (Why?)
 - Leaves will be stored sequentially (and linked, of course)
 - Can control “fill factor” on pages.

Summary

- ISAM is a static structure
 - **Only leaf pages modified**; overflow pages needed
 - Overflow **chains can degrade performance** unless size of data set and data distribution stay constant
- **B+ Tree is a dynamic structure**
 - Inserts/deletes leave tree height-balanced; $\log_F N$ cost
 - High fanout (F) means depth rarely more than 3 or 4.
 - Almost always better than maintaining a sorted file.
 - Typically, 67% occupancy on average
 - Usually preferable to ISAM; adjusts to growth gracefully.

Summary Cont.

- Bulk loading can be much faster than repeated inserts for creating a B+ tree on a large data set.
- B+ tree widely used because of its versatility
 - One of the most optimized components of a DBMS.
 - Concurrent Updates
 - In-memory efficiency