1. The circuit is shown in Figure 1. When t<0, both of the switches are open and no energy is stored in the capacitance. Switch 1 closes at t = 0 and then switch 2 closes at t = 5 s. Determine v_C for $t \geq 0$, given that V_0 = 24 V, R_1 =16kΩ, R_2 =24kΩ ,and C=250μF.

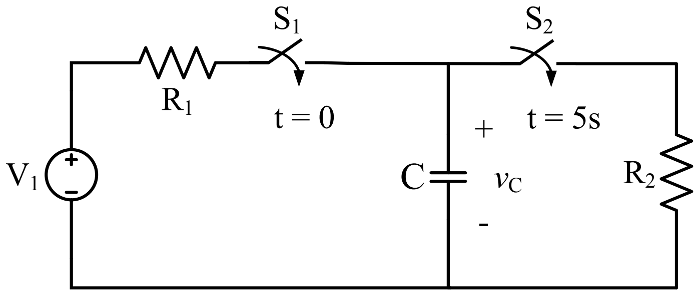
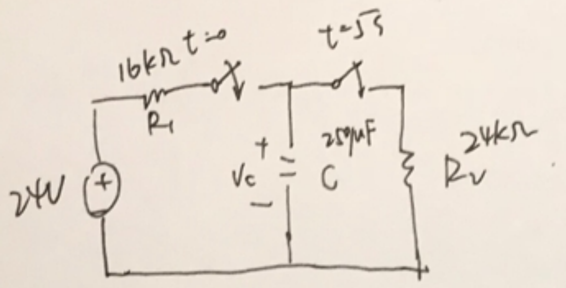


Figure 1



第一阶段共4分，那个奇怪的t求出得一分，解出一阶状态表达式共计三分（需要单位和定义域，我没写单位我检讨×）
第二阶段共6分，初状态即t=5s时的状态共1分，末状态电压共一分，第二阶段表达式共4分（注意单位与定义域，注意衰变方程解需要（t–5））。

$0 \le t \le 5s$

$$\tau_1 = R_1C = 16\times10^3 \times 250\times10^{-6} = 4\,s$$

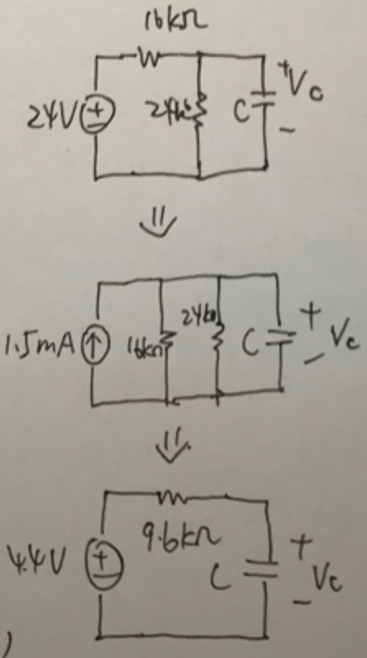
$$V_{C_1}(t) = V_{C_1}(\infty) + (V_{C_1}(t) - V_{C_1}(\infty))\, e^{-\frac{t}{\tau_1}}$$

$$= V_0 + (0 - V_0)\, e^{-0.25t}$$

$$= 24(1 - e^{-0.25t}) \quad t \in [0, 5]$$

$t \ge 5s$

$$V_{C_1}(t=5s) = 24(1 - e^{-1.25}) = 17.1239\,V$$



$$\tau_2 = R_2'C = 9.6\times10^3 \times 250\times10^{-6} = 2.4\,s = \frac{12}{5}\,s$$

$$V_{C_2}(\infty) = \frac{24k\Omega}{(16+24)k\Omega} \times 24 = 14.4\,V$$

$$V_{C_2}(t) = V_{C_2}(\infty) + (V_{C_2}(5s) - V_{C_2}(\infty)) \cdot e^{-\frac{5}{12}(t-5)}$$

$$= 14.4 + 2.7239\, e^{-\frac{5}{12}(t-5)} \ (V) \quad t \ge 5$$

2. In the sinusoidal steady state circuit shown in Figure 2, $\mathbf{V} = 120\angle 0^\circ$ V, $\mathbf{I} = 0.3\angle 30^\circ$ A, $\omega = 1000$ rad/s, $R_1 = 200\ \Omega$, $R_2 = 200\ \Omega$, $R_3 = 1.2\text{k}\Omega$, L = 0.2 H, and C = 10μF. Determine $\mathbf{I}_1$.

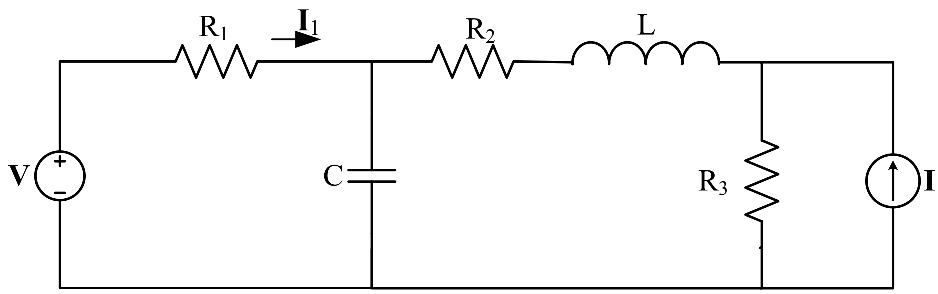


Figure 2

Solution

$$\mathbf{V}_1\left(\frac{1}{R_1}+\frac{1}{-j/\omega C}+\frac{1}{R_2+j\omega L}\right)-\mathbf{V}_2\left(\frac{1}{R_2+j\omega L}\right)=\frac{\mathbf{V}}{R_1} \quad \text{3分}$$

$$-\mathbf{V}_1\left(\frac{1}{R_2+j\omega L}\right)+\mathbf{V}_2\left(\frac{1}{R_3}+\frac{1}{R_2+j\omega L}\right)=\mathbf{I} \quad \text{3分}$$

Solution gives

$$\mathbf{V}_1 = 73.67e^{-j53.6^\circ}\ \text{V}, \ \text{1分}$$

$$\mathbf{V}_2 = 58.96e^{j10.9^\circ}\ \text{V}. \ \text{1分}$$

so

$$\mathbf{I}_1 = \frac{\mathbf{V}-\mathbf{V}_1}{\text{R}} = 0.48\angle 37.8^\circ\text{A} \quad \text{2分}$$

3. The circuit is as shown in Figure3, where $R_1 = 3k\Omega, R_2 = 4k\Omega, R_3 = 6k\Omega, V_2 = 2000I_x, V_1 = 15V$. R_L is a resistor, the value of which is variable. Determine the maximum power that can be extracted by R_L.

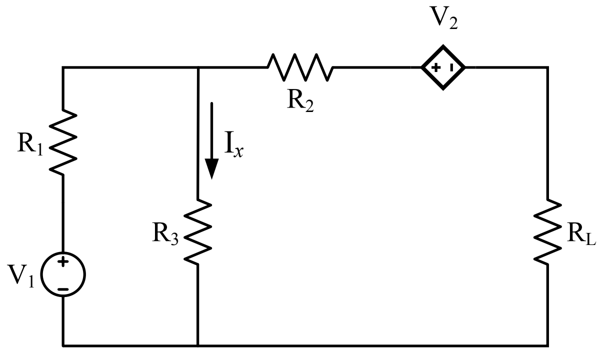
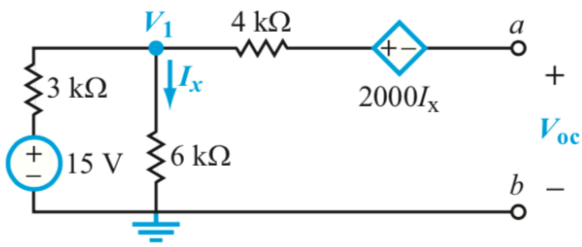


Figure 3

Solution: To find the Thévenin equivalent circuit, we start by determining $V_{Th} = V_{oc}$.



等效电压或电流共两分，等效电阻共两分。等效电阻与负载电阻相同时功率最大不能直接用需要说明（可以简单说明但是不能直接划等号），说明两分，得出最终最大功率大小为4分，注意功率单位。

Voltage division:

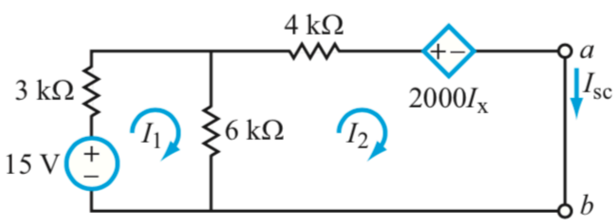
$$V_1 = \frac{15}{(3+6)\text{k}} \times 6\text{k} = 10\ \text{V}$$

$$I_x = \frac{V_1}{6\text{k}} = \frac{10}{6}\ \text{mA}.$$

The dependent voltage source is:

$$2000I_x = 2 \times \frac{10}{6} \times 10^3 \times 10^{-3} = \frac{20}{6}\ \text{V}.$$

With (a, b) an open circuit, no current flows through the 4-kΩ resistor. Hence, there is no voltage drop across it.



Next, we find I_{sc}:

$$-15 + 3\text{k}I_1 + 6\text{k}(I_1 - I_2) = 0$$

$$6\text{k}(I_2 - I_1) + 4\text{k}I_2 + 2000I_x = 0$$

Also,

$$I_x = I_1 - I_2$$

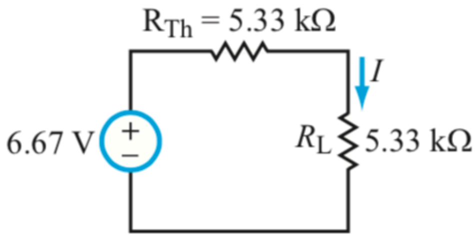
Solution yields:

$$I_1 = 2.5 \text{ mA}, \qquad I_2 = 1.25 \text{ mA}.$$

$$I_{sc} = I_2 = 1.25 \text{ mA}.$$

$$R_{Th} = \frac{V_{oc}}{I_{sc}} = \frac{6.67}{1.25 \times 10^{-3}} = 5.33 \text{ k}\Omega.$$

Hence, $R_L = 5.33$ kΩ extracts maximum power.



$$I = \frac{6.67}{2 \times 5.33} = 0.625 \text{ mA}$$

$$P_{max} = I^2 R_L = (0.625 \times 10^{-3})^2 \times 5.33 \times 10^3 = 2.09 \qquad \text{(mW)}.$$

4. In a three-phase balanced power system with positive sequence voltage sources ($\mathbf{U}_{AN} = \mathbf{U}_{BN}e^{j120°} = \mathbf{U}_{CN}e^{j240°}$), a three-phase balanced load is shown in Figure 4. The line voltage $\mathbf{U}_{AB}$=380∠0° V (phasor correlated to amplitude, not RMS value), frequency $f = 50$ Hz, R=30Ω, L=0.29H, M=0.12H.
   (1) Find the phase current $\mathbf{I}_A$.
   (2) Determine the total complex power of the three-phase loads.

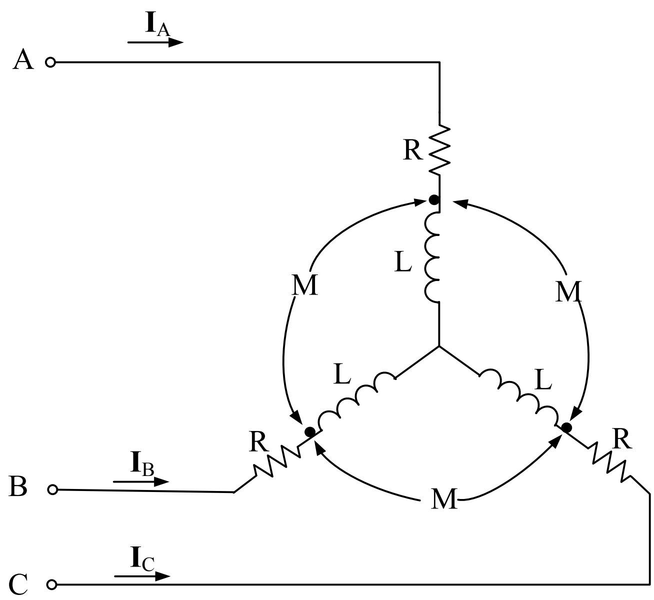


Figure 4

Solution

(1)Decouple three loads:

$$\mathbf{U}_A = (R + j\omega L)\mathbf{I}_A + j\omega M\mathbf{I}_B + j\omega M\mathbf{I}_C$$
$$= (R + j\omega L - j\omega M)\mathbf{I}_A \quad 2分$$

Then

$$\mathbf{U}_{AN} = \frac{\mathbf{U}_{AB}}{\sqrt{3}}\angle\text{-}30° = 220\angle\text{-}30°\,\text{V} \quad 2分$$

$$\mathbf{I}_A = \frac{\mathbf{U}_{AN}}{R + j\omega(L - M)} = \frac{220\angle\text{-}30°}{30 + j314\times(0.29 - 0.12)}$$
$$= 3.593\angle -90.66°\text{A} \quad 2分$$

(2)The total power is

2分

$$S_\Sigma = 3\mathbf{U}_{AN}\mathbf{I}_A^* = 3\times 220\angle\text{-}30°\times 3.593\angle 90.66°$$
$$= 2371.38\angle 60.66° = 1161.96 + j2067.20\,\text{VA} \quad 2分$$

(幅值+角度/实部+虚部)

$$S_\Sigma' = \frac{3}{2}\mathbf{U}_{AN}\mathbf{I}_A^*$$
$$= 1185.69\angle 60.66° = 580.98 + j1033.60\,\text{VA}$$

5. Consider a circuit with an ideal transformer shown in Figure 5, find $\mathbf{I}_o$, $\mathbf{V}_1$ and $\mathbf{V}_2$.

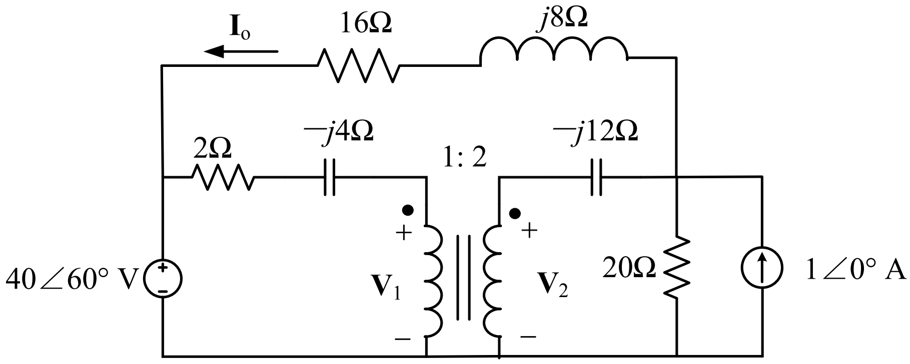


Figure 5

Solution

Transform the current right into a voltage source, then we have

$\mathbf{I}_1 = 2\mathbf{I}_2$ (1) 1分

$\mathbf{V}_2 = 2\mathbf{V}_1$(2) 1分

Applying mesh analysis gives

$40\angle 60° = (2 - j4)\mathbf{I}_1 + \mathbf{V}_1$ (3)

$\mathbf{V}_2 = -j12\mathbf{I}_2 + 20(\mathbf{I}_2 - \mathbf{I}_o) + 20\angle 0°$ (4)

$(16 + j8)\mathbf{I}_o + 40\angle 60° - 20\angle 0° + 20(\mathbf{I}_o - \mathbf{I}_2) = 0$ (5) 1分×3

Solve equation (1)-(5), we obtain $\mathbf{I}_o = 0.617\angle\text{-}172.81°\text{A}$, $\mathbf{V}_1 = 24.696\angle 56.86°$ V

$\mathbf{V}_2 = 49.392\angle 56.86°\text{V}$. 1分 × 3

过程酌情给分，优先看答案

6. In the circuit shown in Figure 6, all the operational amplifiers are ideal and work in linear region.
   (1) Derive the transfer function **H**(ω)=**V**$_o$/**V**$_i$.
   (2) Given that R=100 Ω, R_i=1k Ω, R_f=10k Ω, C_1=20μF, C_2=5μF, determine the cutoff frequencies of the filter.

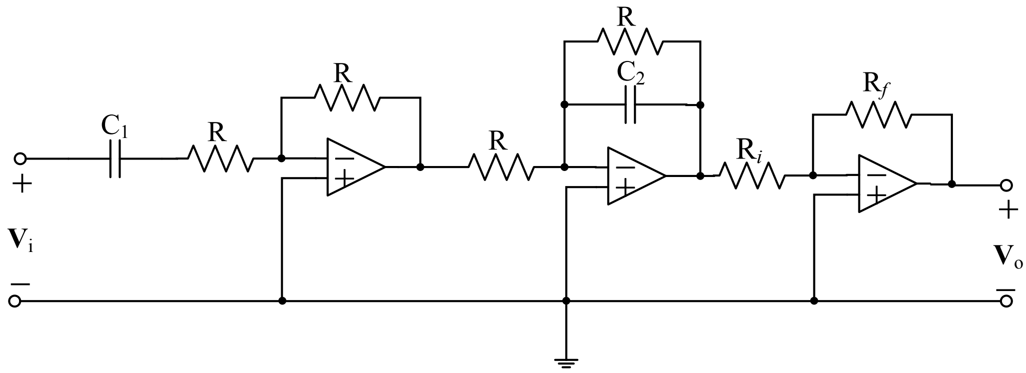


Figure 6

Solution

(1) This is a cascade filter

$$H(\omega)=\frac{\mathbf{V}_o}{\mathbf{V}_i}=\left(-\frac{j\omega C_1 R}{1+j\omega C_1 R}\right)\left(-\frac{1}{1+j\omega C_2 R}\right)\left(-\frac{R_f}{R_i}\right)$$

$$=-\frac{R_f}{R_i}\frac{j\omega C_1 R}{1+j\omega C_1 R}\frac{1}{1+j\omega C_2 R}$$ 4分

(2) Method1:

Since this is a bandpass filter, there are two cutoff frequencies in total:

The first cutoff frequency determined by the highpass part is

$$\omega_1=\frac{1}{RC_1}=500\ \text{ rad/s}$$ 3分，公式2分+答案1分

The second cutoff frequency determined by the lowpass part is

$$\omega_2=\frac{1}{RC_2}=2000\ \text{ rad/s}$$ 3分，同上

Method2:

Let

$$|H(\omega)|=\frac{R_f}{R_i}\frac{\omega C_1 R}{\sqrt{1+(\omega C_1 R)^2}}\frac{1}{\sqrt{1+(\omega C_2 R)^2}}=\frac{1000}{\sqrt{2}}$$ 4分，过程酌情给分

Then we could obtain that $\omega_1\approx 350$ rad/s, $\omega_2\approx 2.82\times10^3$ rad/s.
1分 1分

7. For sinusoid steady state circuits working at frequency ω, given that $\frac{1}{\omega C} = 3.75\Omega, R = 4\Omega$, calculate the equivalent impedance:

   (1) with respect to the terminals A,B for the circuit in Figure 7(a);

   (2) with respect to the terminals A,B for circuit in Figure 7(b), where the number of meshes approach infinite ($n \to \infty$).

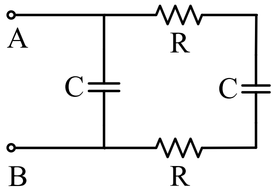


Figure 7(a)

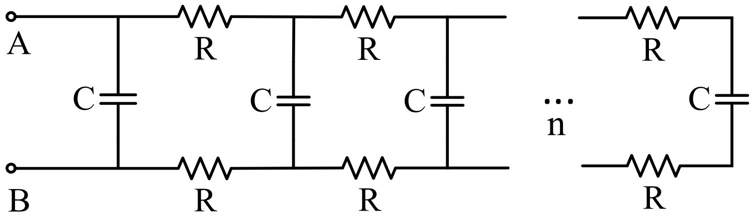


Figure 7(b)

(1) $Z_{AB} = -3.75j||(8-3.75j) = 0.9356 - 2.8729j$

(2) $\frac{1}{Z_{AB}} = \frac{1}{-3.75j} + \frac{1}{8+Z_{AB}}$

$Z_{AB}{}^2 + 8Z_{AB} + 30j = 0$

$(Z_{AB}+4)^2 = (5-3j)^2$

so $Z_{AB} = -4 \pm (5-3j)$

As the real component of Z_{AB} should be positive and the imaginary component should be negative,

$$Z_{AB} = 1 - 3j$$

（1）等效电路列式1分，解出等效值3分，共计4分。

（2）列出等效关系式及方程得4分，解出双解并舍一解得四分（需要理由，不然扣一分），最终等效阻抗需要单位，若仅对实部或虚部，解共4分给1分。

8. For the circuit shown in Figure 8, $R_1 = 4\Omega, R_2 = R_4 = 2\Omega, R_3 = 12\Omega, L = 4H, C = 0.64F, V_1 = 30\ V$. The switch has been closed for a long time when t<0 and is turned off at t=0. We intend to determine the voltage of the capacitance $v_C(t)$.
(1) Construct the *s*-domain circuit that can be used to solve $V_C(s)$, here $V_C(s)$ is the Laplace transform of $v_C(t)$.
(2) Derive $V_C(s)$ and find $v_C(t)$ by Laplace inverse transform of $V_C(s)$.
(3) Find the *t*-domain differential equation for $v_C(t)$ and solve it in time domain.
(Note: An abbreviated list of Laplace transform pairs and a list of operational transforms are attached at the end of this test paper.)

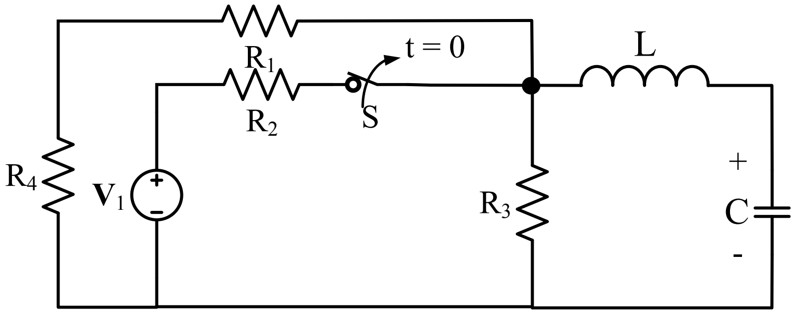
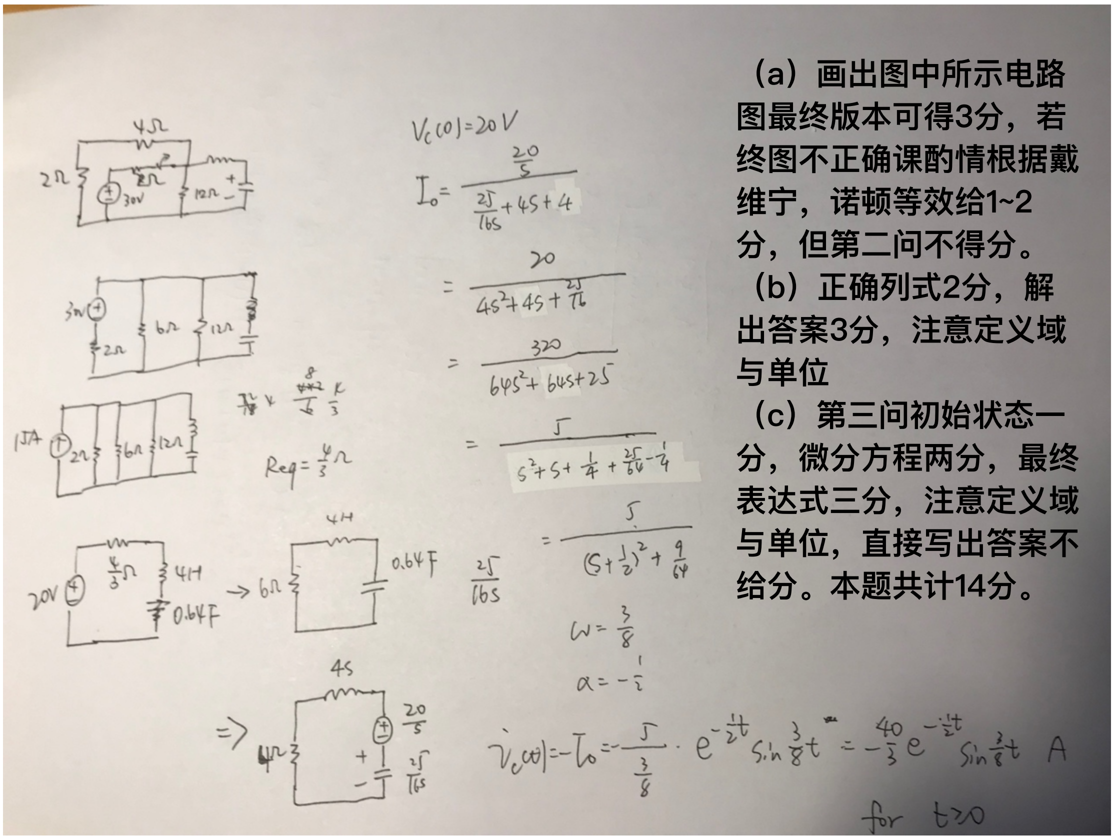


Figure 8



(3)

At $t = 0^-$, we apply nodal analysis to determine V_1, which is the same as $\upsilon_C(0^-)$:

$$\frac{V_1}{6} + \frac{V_1 - 30}{2} + \frac{V_1}{12} = 0,$$

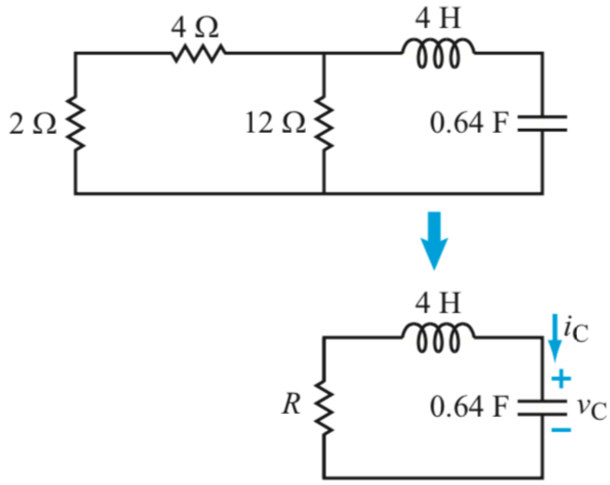
which gives

$$V_1 = 20 \text{ V}.$$

Hence,

$$\upsilon_C(0) = \upsilon_C(0^-) = V_1 = 20 \text{ V}, \tag{1}$$

$$i_C(0) = i_L(0) = i_L(0^-) = 0. \tag{2}$$



(b) At $t > 0$

At $t > 0$, the circuit contains three resistors, which when combined, we end up with a series RLC circuit with

$$R = (2+4) \parallel 12 = 4\ \Omega,$$
$$\alpha = \frac{R}{2L} = \frac{4}{2\times 4} = 0.5\ \text{Np/s},$$
$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{4\times 0.64}} = 0.625\ \text{rad/s}.$$

Since $\omega_0 > \alpha$, the response is underdamped with

$$\upsilon_C(t) = e^{-\alpha t}[D_1 \cos \omega_d t + D_2 \sin \omega_d t],$$
$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = 0.375\ \text{rad/s}.$$

Initial condition $\upsilon_C(0) = 20$ V leads to:

$$D_1 = 20\ \text{V}.$$

Initial condition $i_C(0) = 0$ leads to:

$$\left\{-\alpha e^{-\alpha t}[D_1 \cos \omega_d t + D_2 \sin \omega_d t] + e^{-\alpha t}[-\omega_d D_1 \sin \omega_d t + \omega_d D_2 \cos \omega_d t]\right\}_{t=0} = 0,$$

or

$$-\alpha D_1 + \omega_d D_2 = 0,$$

which results in

$$D_2 = \frac{\alpha D_1}{\omega_d} = \frac{0.5}{0.375} \times 20 = \frac{80}{3}\ \text{V}.$$

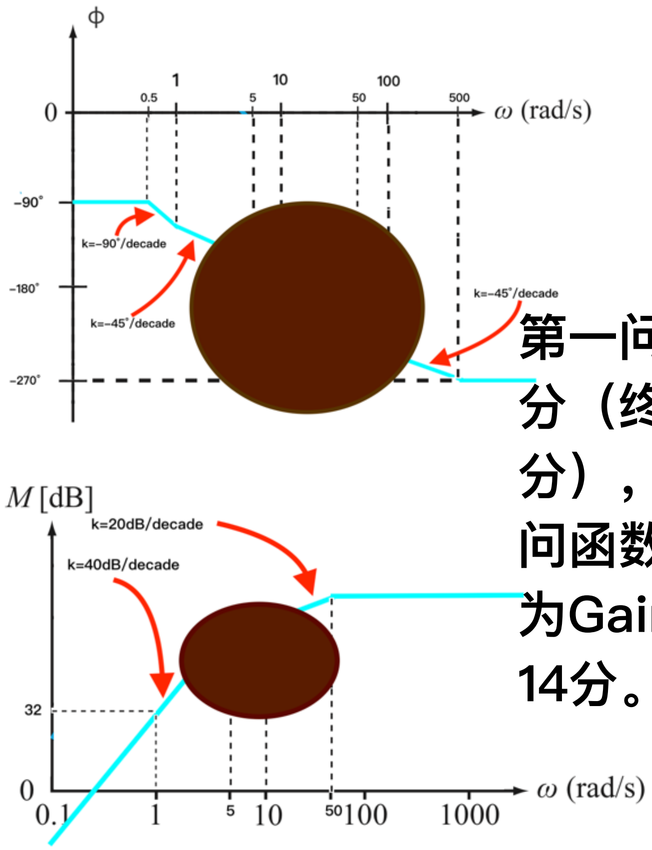
Hence,

$$\upsilon_C(t) = e^{-0.5t}\left[20\cos 0.375t + \frac{80}{3}\sin 0.375t\right] \qquad \text{(V)},$$

and the corresponding current is

$$\begin{aligned} i_C(t) &= C\,\upsilon_C'(t) \\ &= -\left[\frac{40}{3}\,e^{-0.5t}\sin 0.375t\right] \qquad \text{(A)}, \qquad \text{for } t \geq 0. \end{aligned}$$

9. A student studying in SIST building incautiously poured his coffee on his homework, which incidentally covered the transfer function and the part of the Bode plots. Could you help him repair the homework?
(1) Rewrite the transfer function according to the plots with the information left.
(2) Find the Maximum gain according to the plots and function you have achieved in previous problem.
(Note: typical Bode plots are presented at the end of the test paper.)



第一问abcdef推导各一分（终式对可直接给分），终式4分，第二问函数最大值2分，转为Gain正确2分，共计14分。

Figure 9

(1)
(a) for $M(1)=32\,dB$ $K=40$
(b) for $\phi(0)=-90^\circ$ $(j)^N$: $N=3$
(c) for $k[M(1)] = 40\,dB/decade$ $(j\omega)^N$: $N=2$
(d) for $\Delta k\,[M(50)] = -20\,dB/decade$
$(1+\frac{j\omega}{50})^{-1}$
(e) for $\Delta k\,[\phi(0.5)] = -90^\circ/decade$
$(1+\frac{j\omega}{5})^{-2}$
(f) for $\Delta k\,[\phi(10)] = 45^\circ/decade$
$(1+\frac{j\omega}{10})$

$$\Rightarrow H(\omega) = \frac{-40j\,\omega^2\cdot(1+\frac{j\omega}{10})}{(1+\frac{j\omega}{50})\,(1+\frac{j\omega}{5})^2}$$

(2) $32 \approx 20\lg 40$
$20\lg 40 + (\lg 5 - \lg 1)\times 40 + (\lg 50 - \lg 10)\times 20$
$= 60 + 20\lg 5 \approx 74\,dB = 20\lg G$
Gain = 5000