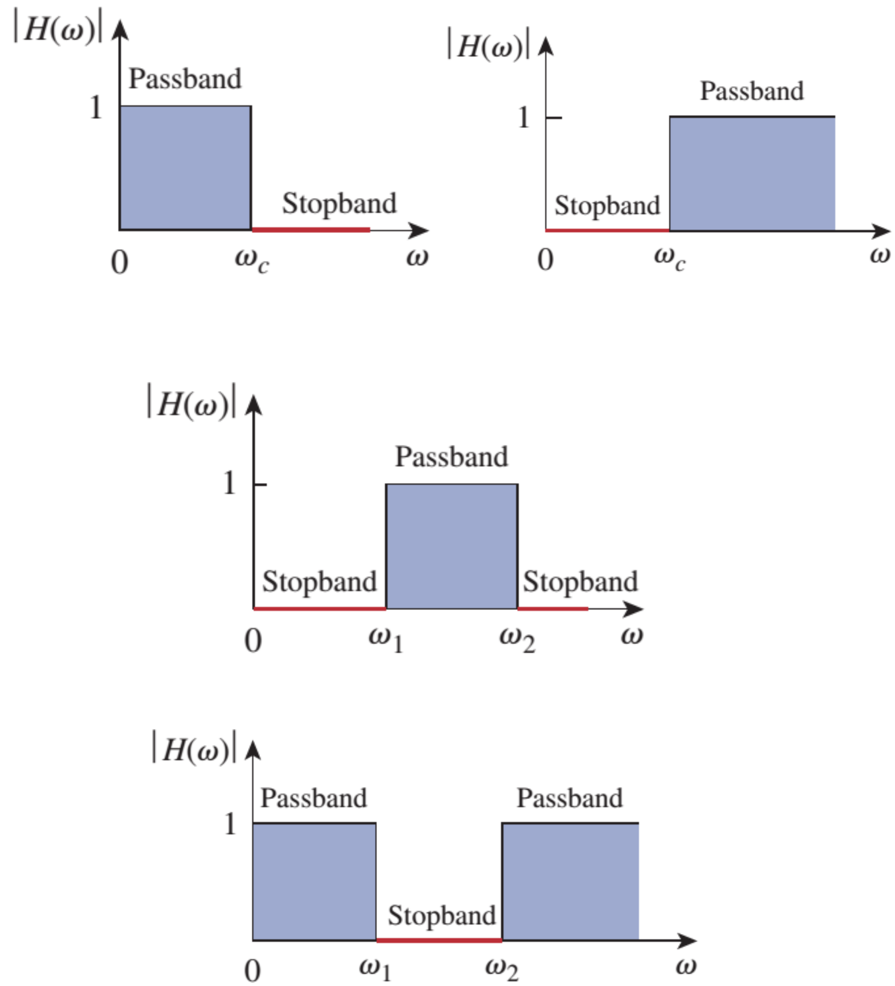
Lecture 13

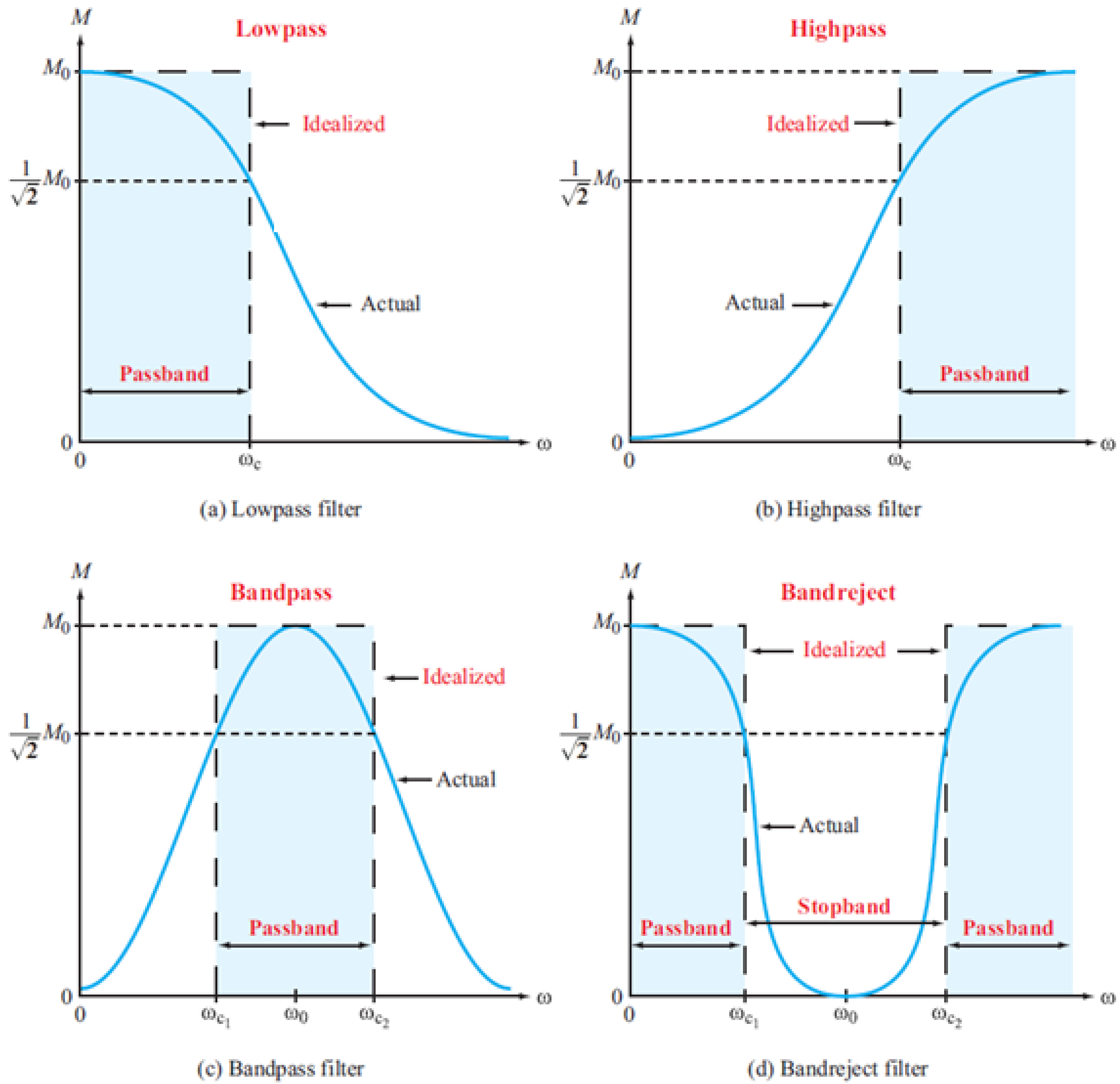
- Filters

Filters

- Circuit designed to retain certain frequency range but discard or attenuate others
  - *Low-pass*: pass low frequencies and reject high frequencies
  - *High-pass*: pass high frequencies and reject low frequencies
  - *Band-pass*: pass some particular range of frequencies, reject other frequencies outside that band
  - *Band-reject (notch)*: reject a range of frequencies and pass all other frequencies



Filters – Realistic Curves



[Source: Berkeley]

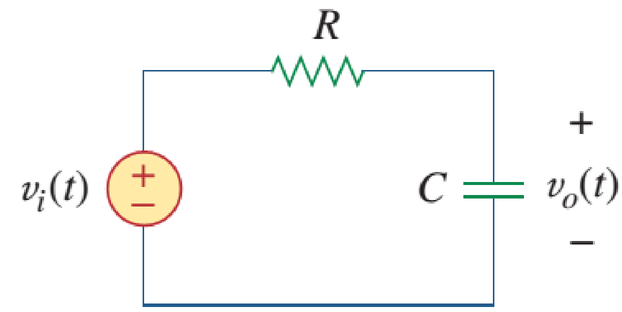
Passive Filters

- A filter is passive if it consists only of <u>passive</u> elements
  - R, L and C

- *LC* filters have been used in practical applications for more than eight decades
  - Very important circuits
    - many technological advances would not have been possible without the development of filters
  - *LC* filter technology feeds many areas
    - equalizers, impedance-matching networks, transformers, shaping networks, power dividers, attenuators, and directional couplers …

First-Order RC Lowpass Filter

- A typical lowpass filter is formed when the output of a RC circuit is taken off the capacitor.

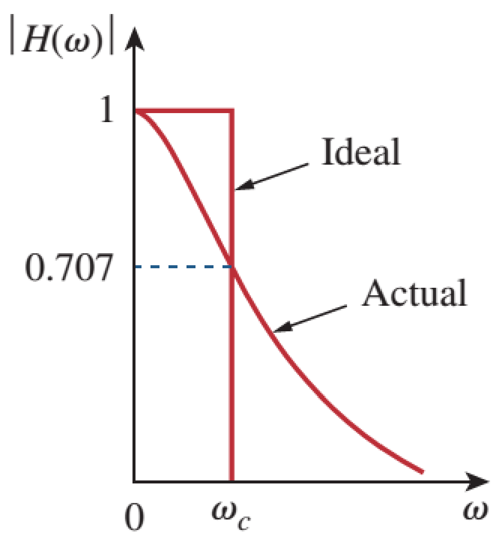


$$\mathbf{H}(\omega) =$$

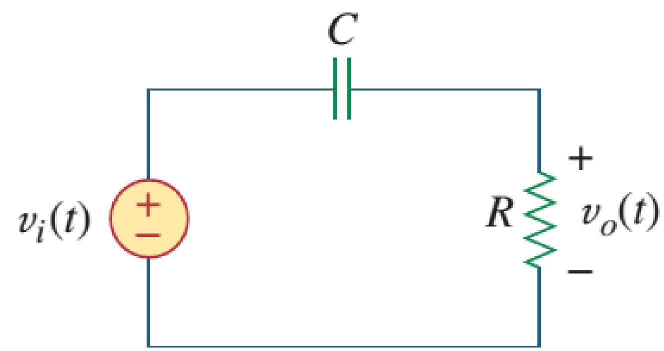
- The cutoff frequency is:

$$\omega_c =$$

  - Filter is designed to pass from DC up to ω_c.



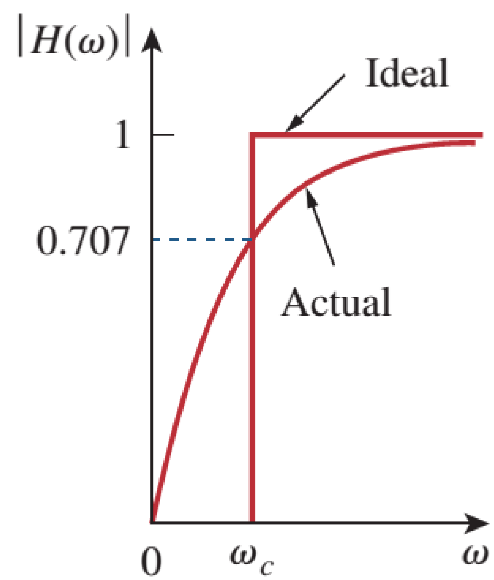
First-Order RC Highpass Filter



$$\mathbf{H}(\omega) =$$

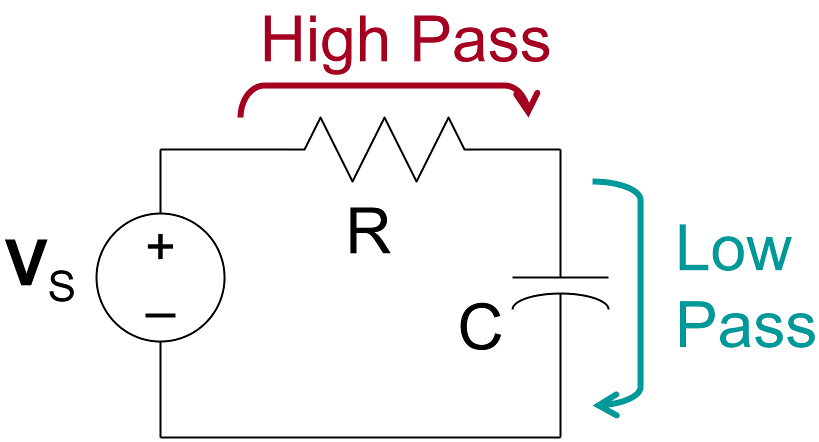
- The cutoff frequency.

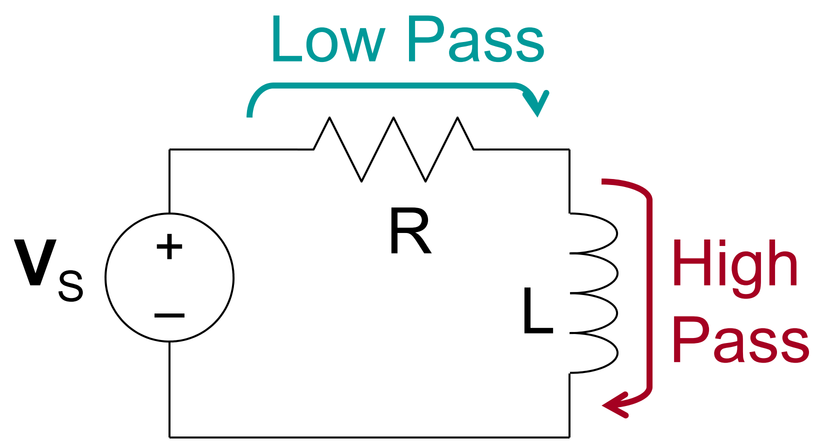
$$\omega_c =$$



- The difference being that the frequencies passed go from ω_c to infinity.

How about RL Circuits?





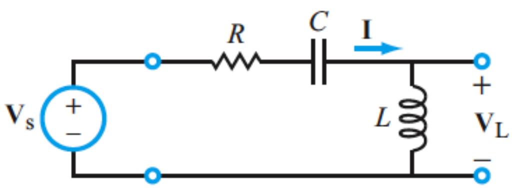
H_R = R / (R + 1/$j\omega$C)

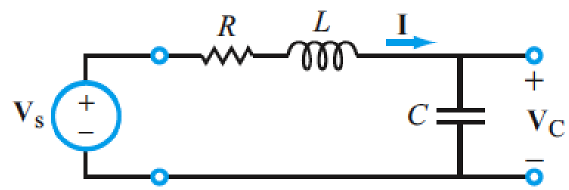
H_C = (1/$j\omega$C) / (R + 1/$j\omega$C)

H_R = R / (R + $j\omega$L)

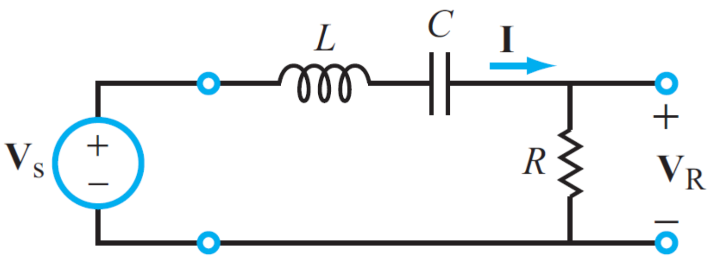
H_L = $j\omega$L / (R + $j\omega$L)

How about RLC Circuits?





Bandpass RLC Filter

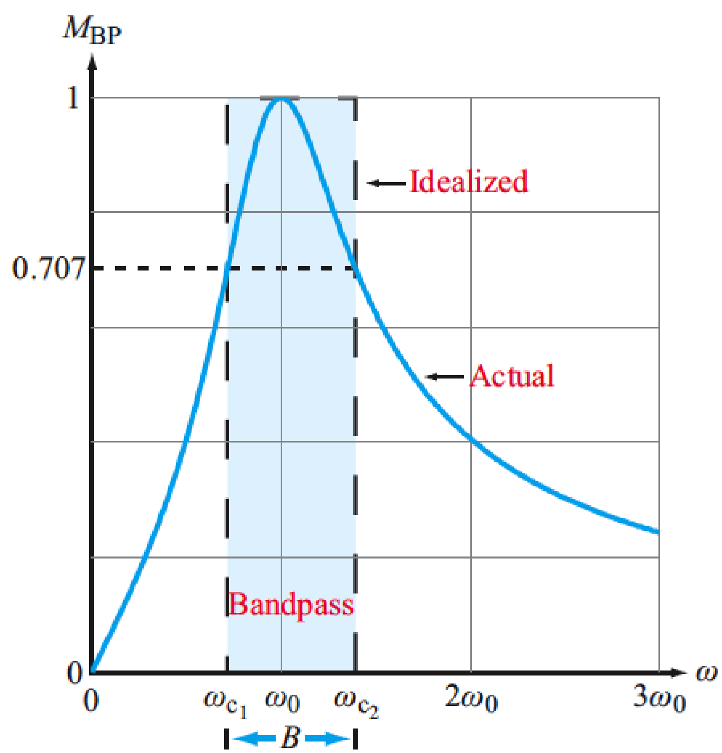


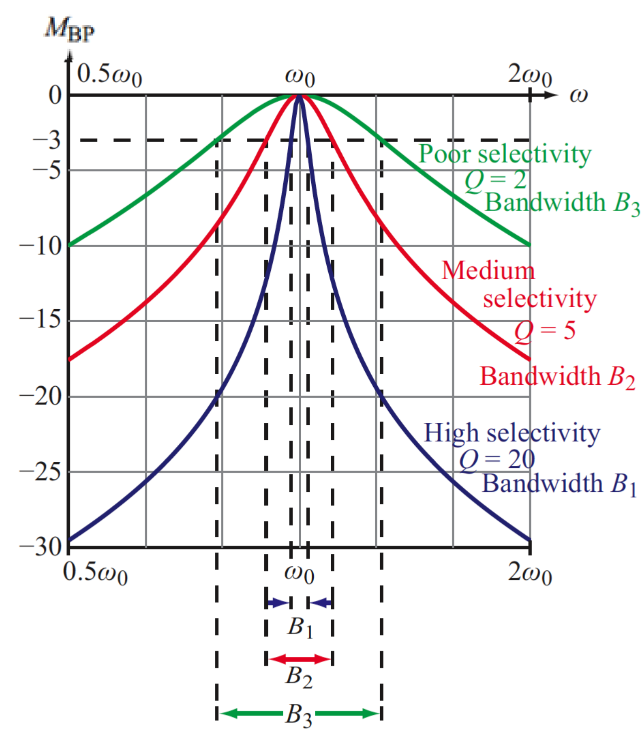
$$\mathbf{H}_{\mathrm{BP}}(\omega) = \frac{\mathbf{V}_{\mathrm{R}}}{\mathbf{V}_{\mathrm{s}}} =$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$\omega_{c_1} = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}},$$

$$\omega_{c_2} = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}.$$

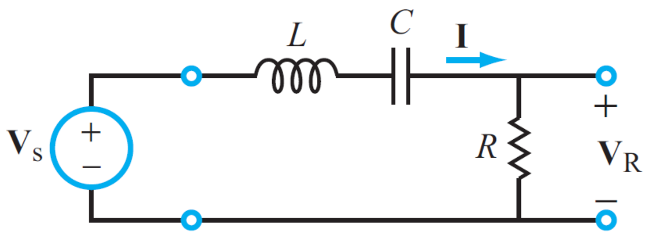


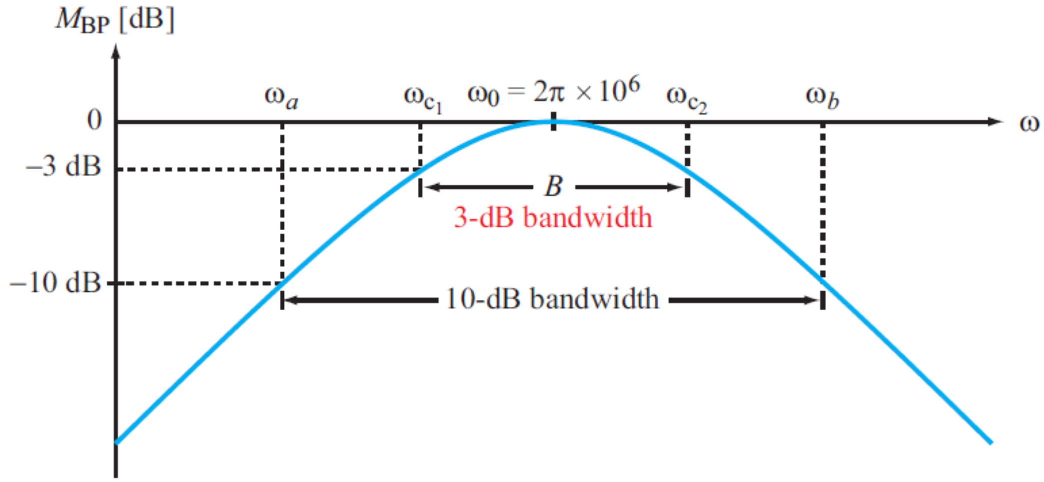


Example

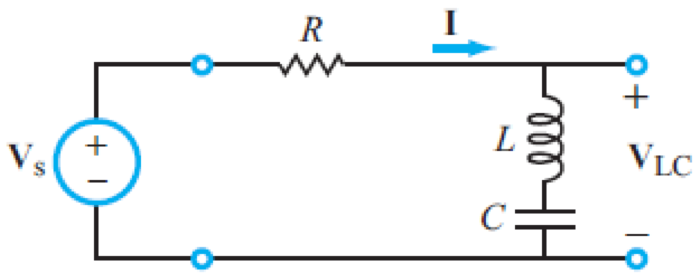
(a) Design a series RLC bandpass filter with a center frequency $f_0 = 1$ MHz and a quality factor $Q = 20$, given that $L = 0.1$ mH.

(b) Determine the 10-dB bandwidth of the filter, which is defined as the bandwidth between frequencies at which the power level is 10 dB below the peak value.

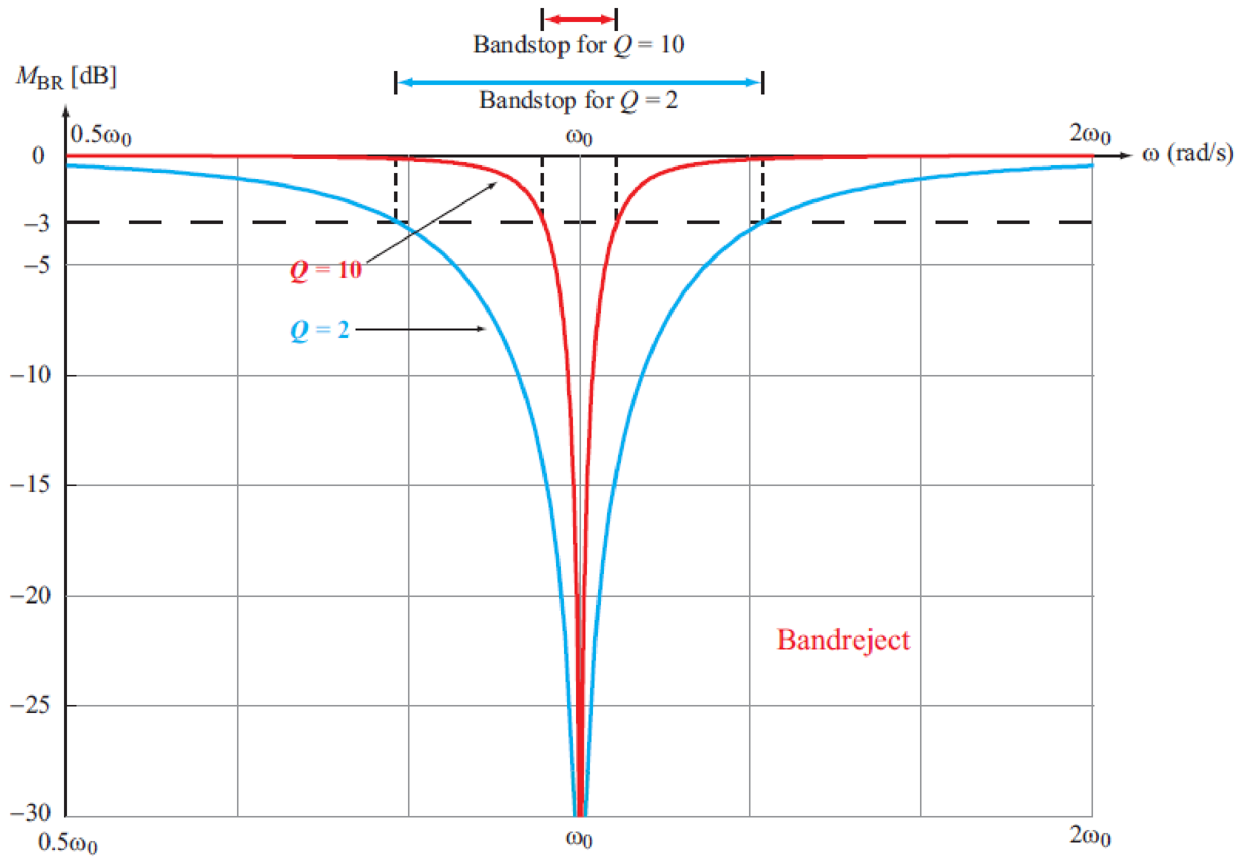




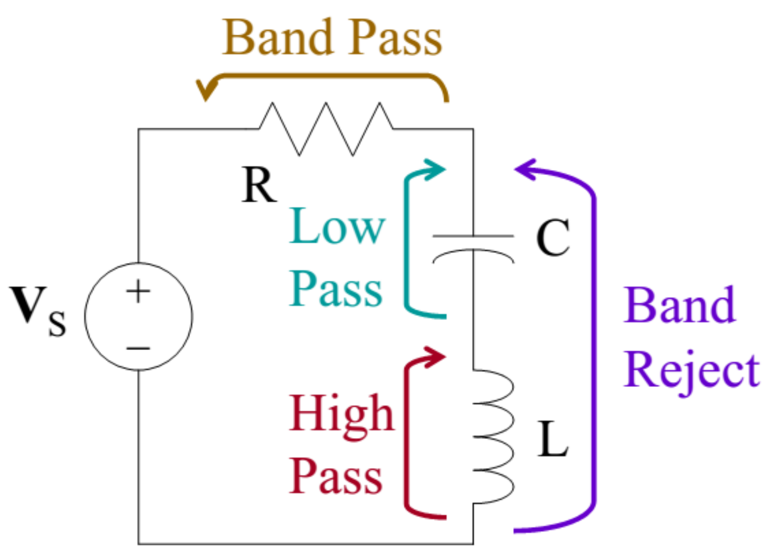
Bandstop (Bandreject) Filter



$$H_{BR} = V_{LC} / V_s$$


Bandstop for $Q = 10$
Bandstop for $Q = 2$
M_{BR} [dB]
0.5ω0
ω0
2ω0
ω (rad/s)
0
−3
−5
−10
−15
−20
−25
−30
$Q = 10$
$Q = 2$
Bandreject

Second-Order RLC Filter Circuits



$$\mathbf{Z} = R + 1/j\omega C + j\omega L$$

$$\mathbf{H}_{BP} = R / \mathbf{Z}$$

$$\mathbf{H}_{LP} = (1/j\omega C) / \mathbf{Z}$$

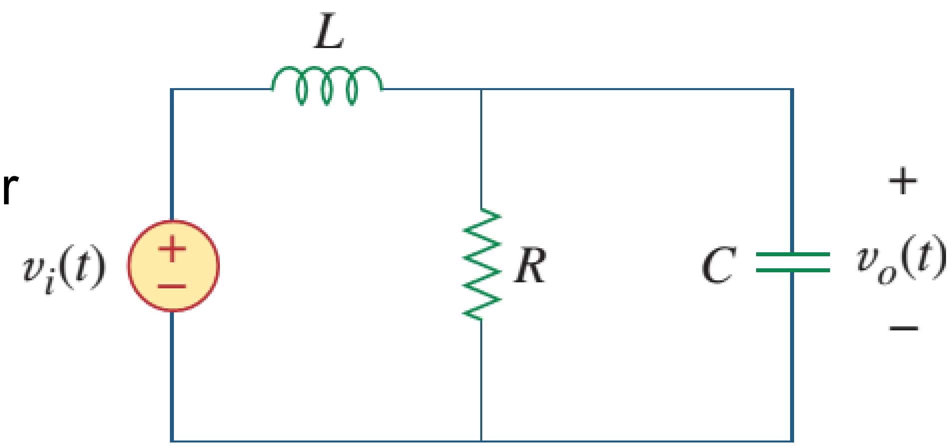
$$\mathbf{H}_{HP} = j\omega L / \mathbf{Z}$$

$$\mathbf{H}_{BR} = \mathbf{H}_{LP} + \mathbf{H}_{HP}$$

Example

- Determine what type of filter is shown below. Calculate half-power frequency.

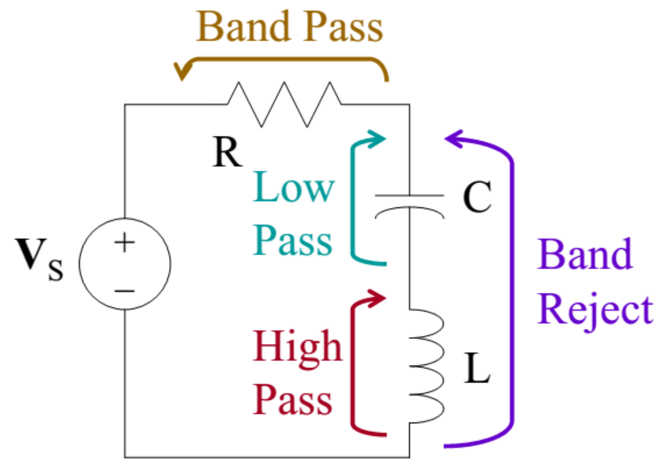
  $R = 2\text{k}\Omega$, $L = 2\text{H}$ and $C = 2\mu\text{F}$.



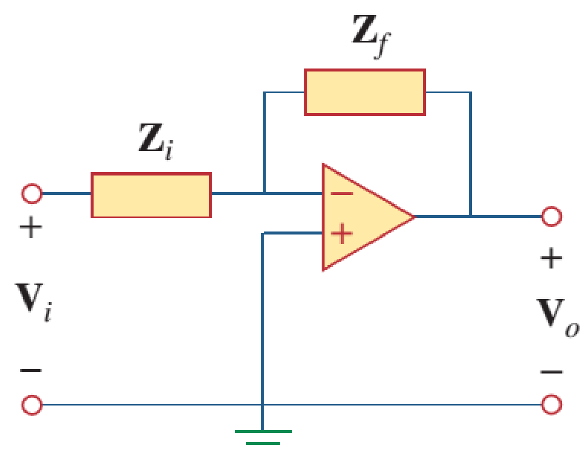
$$\mathbf{H}(s) = \frac{\mathbf{V}_o}{\mathbf{V}_i} =$$

Active Filters

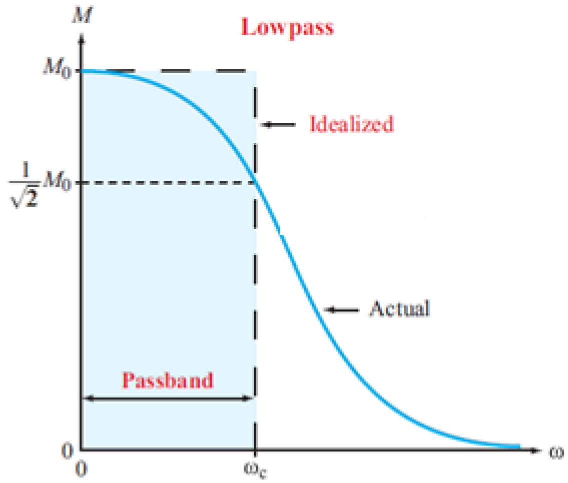
- Passive filters have a few drawbacks.
  - Generally, they cannot create gain greater than 1.
  - They require inductors, which tend to be bulky and more expensive than other components.



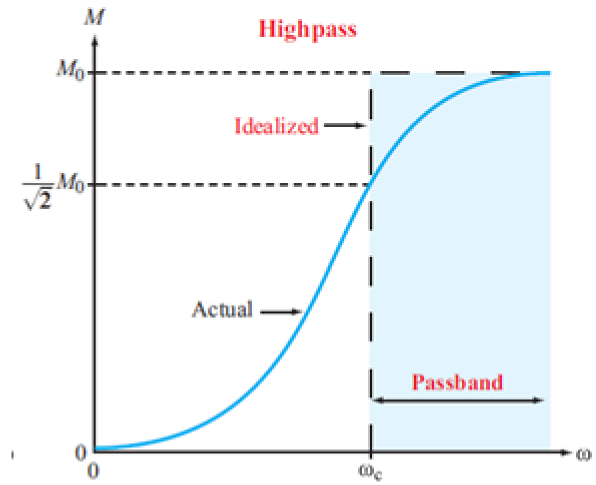
- It is possible, using <u>op-amps, together with resistors and capacitors</u>, to create all the common filters.
  - Their ability to isolate input and output also makes them very desirable.
  - Limited to frequency less than 1MHz.



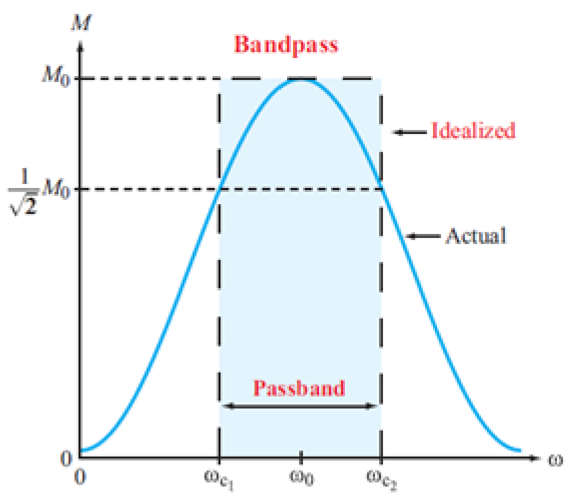
Four Types of Filters



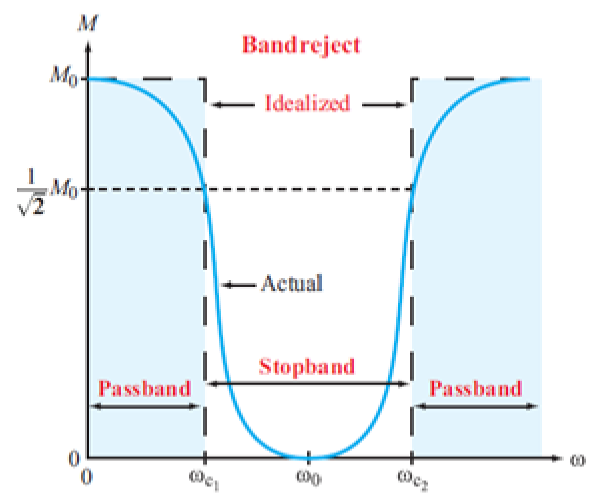
(a) Lowpass filter



(b) Highpass filter

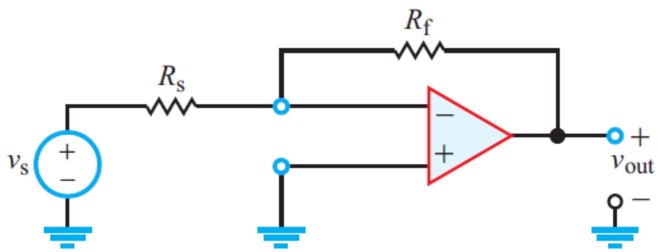


(c) Bandpass filter

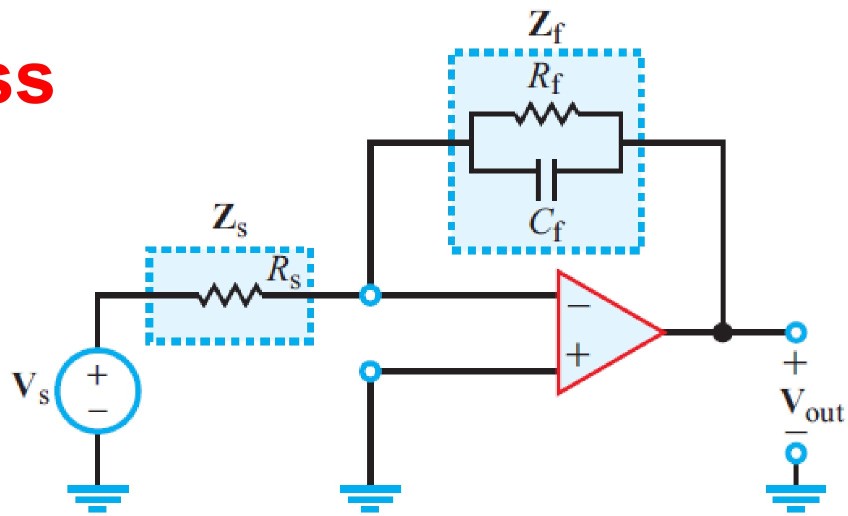


(d) Bandreject filter

Active Filters – Lowpass

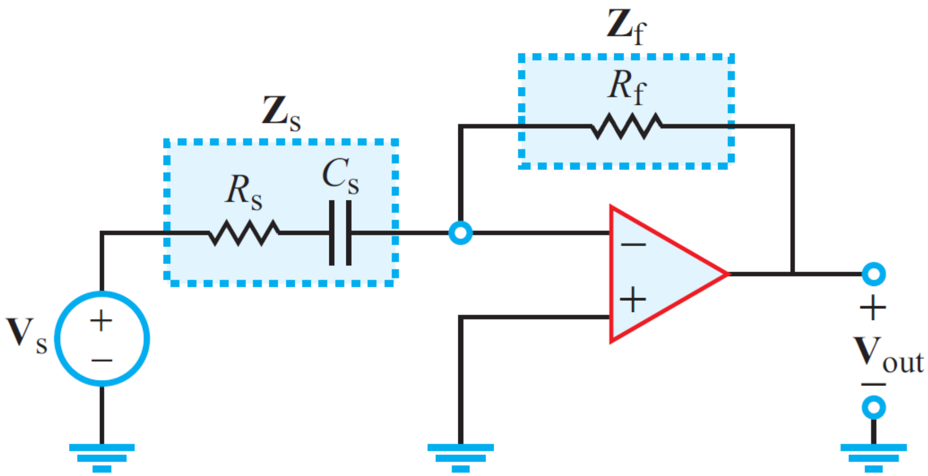


(a) Inverting amplifier



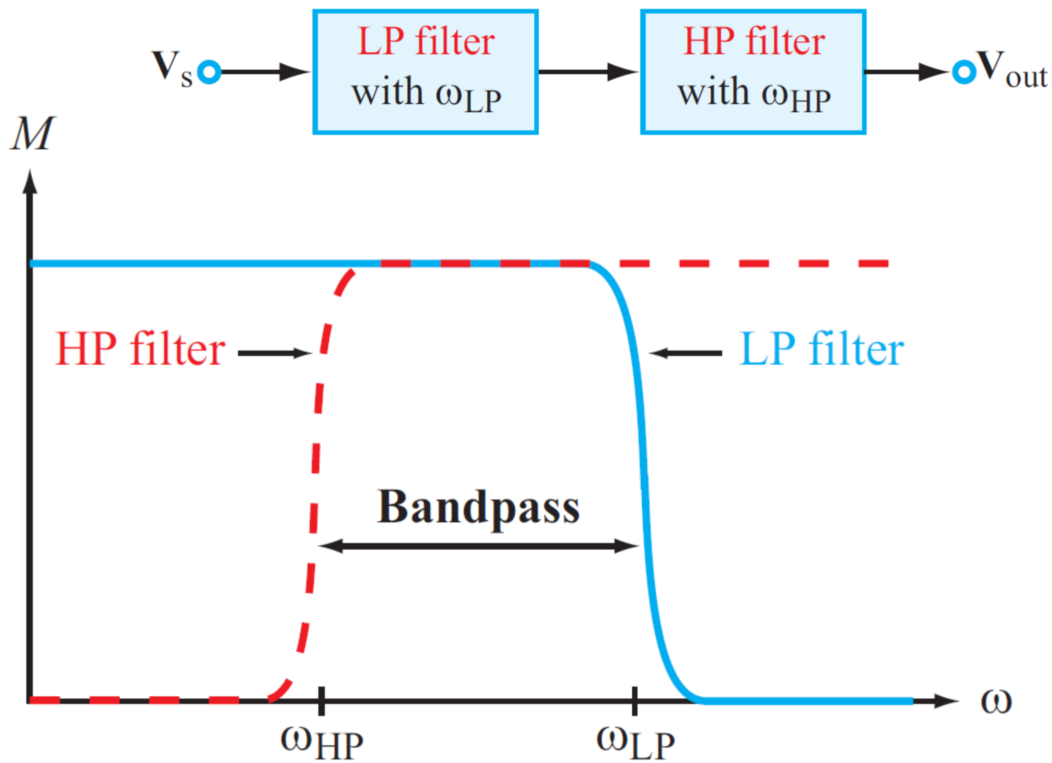
(b) Phasor domain with impedances

Active Filters – Highpass

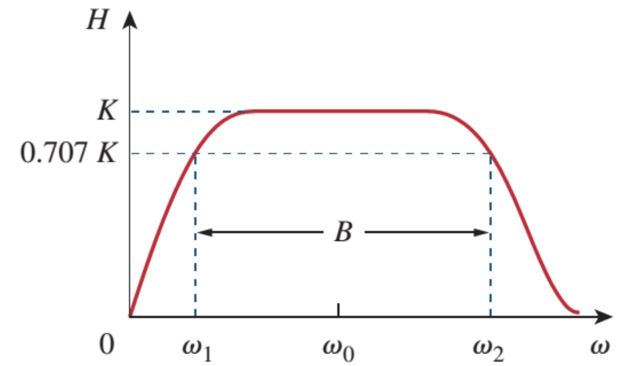


$$\mathbf{H}_{\mathrm{HP}}(\omega) = \frac{\mathbf{V}_{\mathrm{out}}}{\mathbf{V}_{\mathrm{s}}} =$$

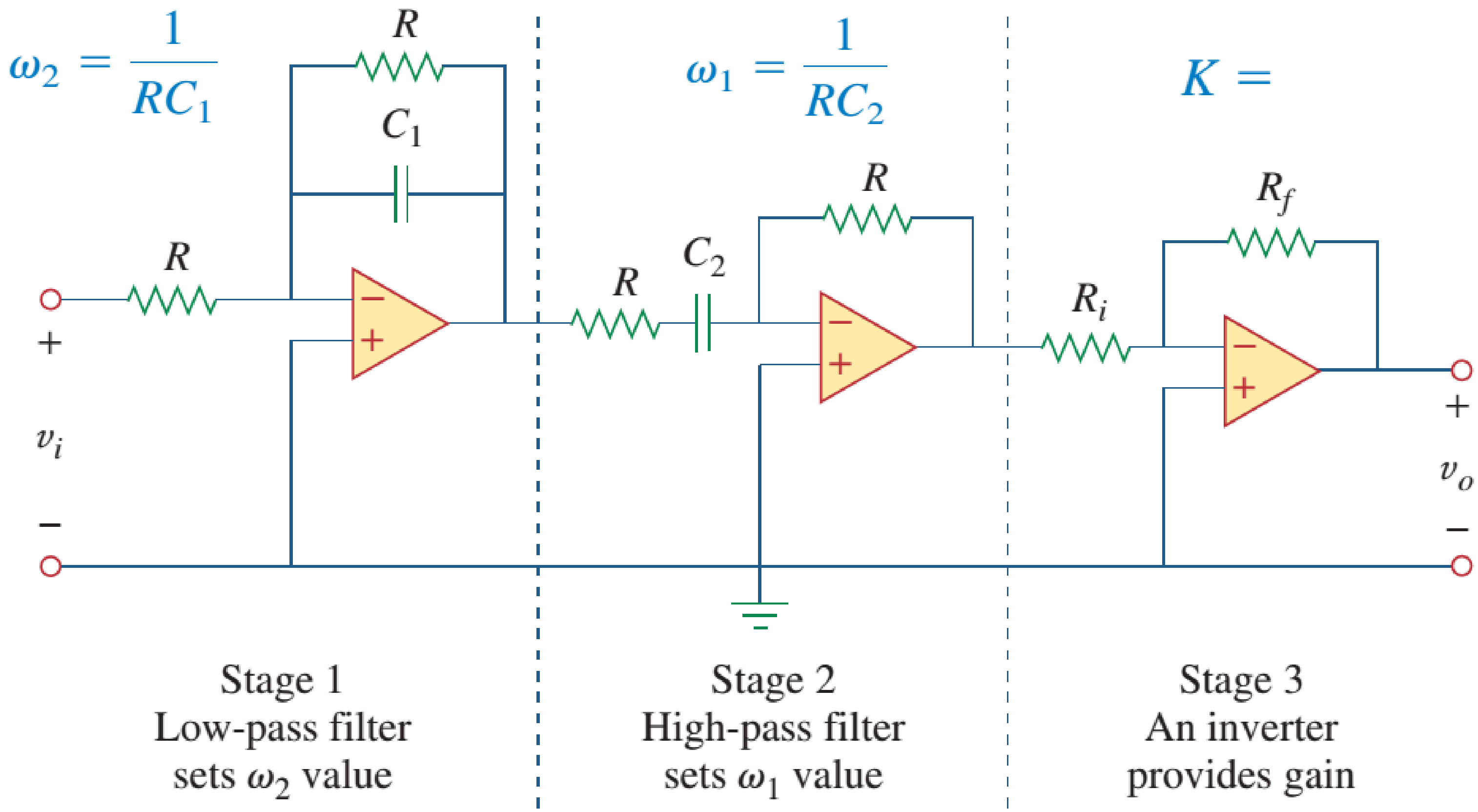
Bandpass



(a) Bandpass filter

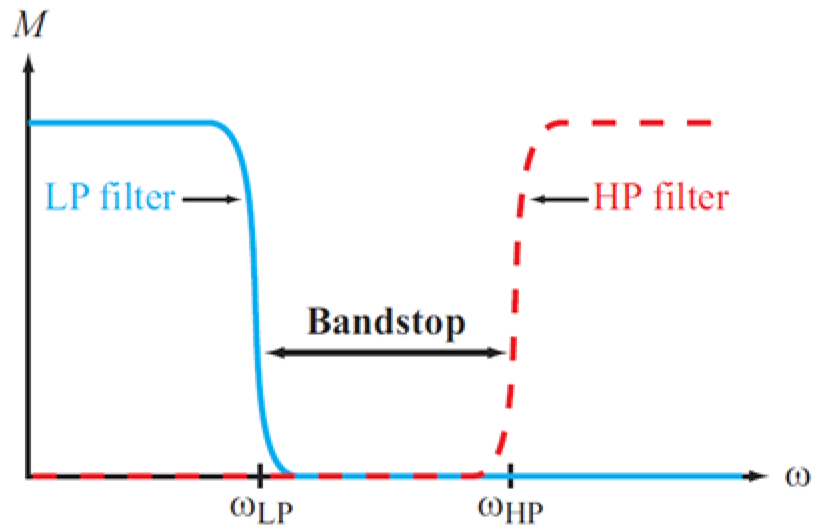


Active Bandpass Filter

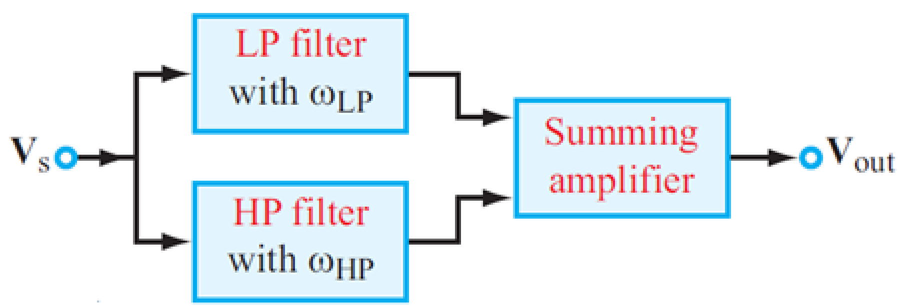


$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \left(-\frac{1}{1 + j\omega C_1 R}\right)\left(-\frac{j\omega C_2 R}{1 + j\omega C_2 R}\right)\left(-\frac{R_f}{R_i}\right)$$

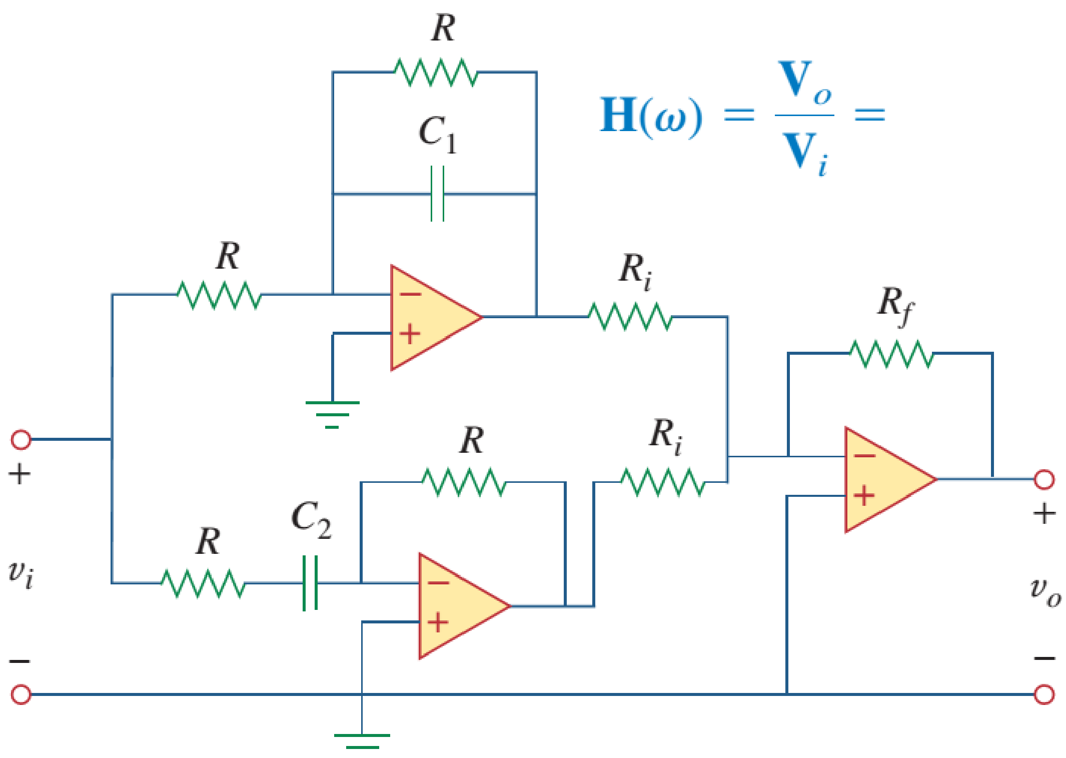
Active Bandreject Filter



(b) Bandreject filter



Active Bandreject Filter



$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} =$$