

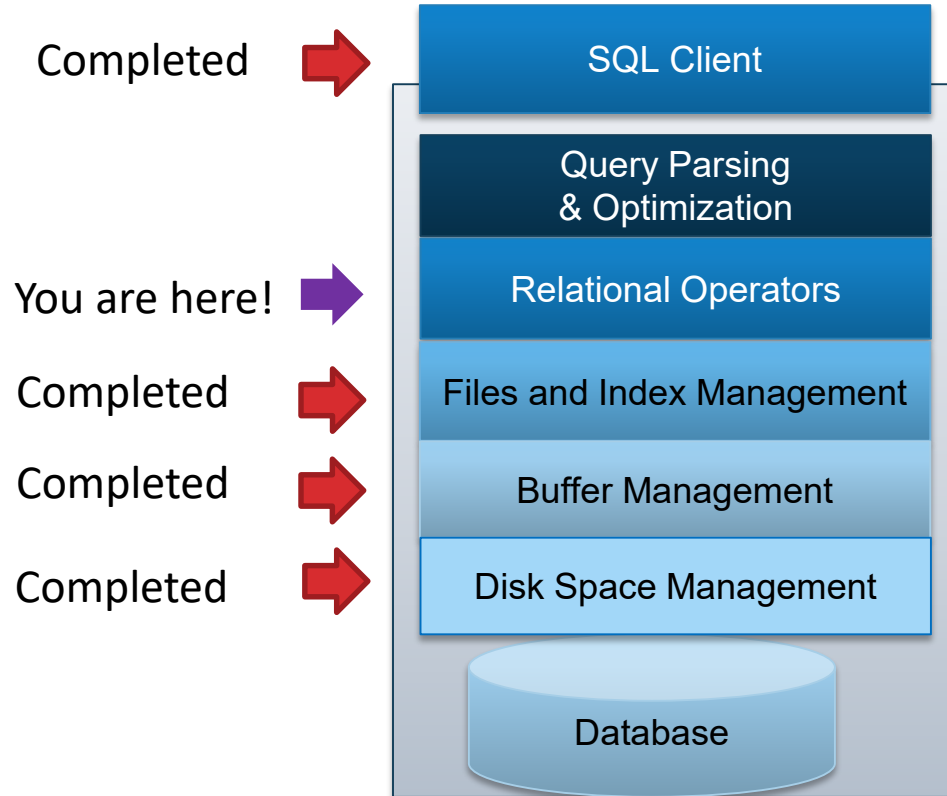
Relational Algebra

R & G, Chapters 4.1 - 4.2





Architecture of a DBMS: What we've learned



Today: *definitions* of the relational operators.

Coming soon:
implementations

An Overview of the Layer Above

SQL Query

```
SELECT S.name
FROM Reserves R, Sailors S
WHERE R.sid = S.sid
AND R.bid = 100
AND S.rating > 5
```

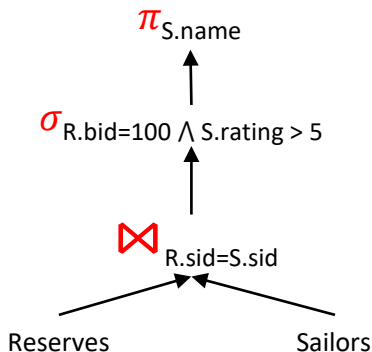
Query Parser
& Optimizer

Relational Algebra

$$\pi_{S.name}(\sigma_{bid=100 \wedge rating > 5}(\text{Reserves} \bowtie_{R.sid=S.sid} \text{Sailors}))$$

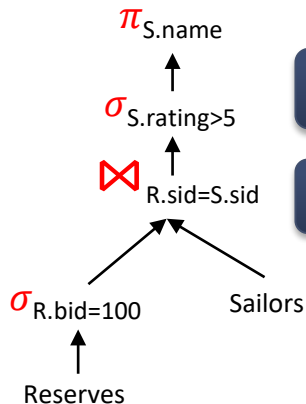
Equivalent to...

(Logical) Query Plan:



But actually will
produce...

Optimized (Physical) Query Plan:



On-the-fly
Project Iterator

On-the-fly
Select Iterator

Indexed Nested
Loop Join Iterator

Heap Scan
Iterator

SQL vs Relational Algebra

SQL Query

```
SELECT S.name
FROM Reserves R, Sailors S
WHERE R.sid = S.sid
AND R.bid = 100
AND S.rating > 5
```

Relational Algebra

$$\pi_{S.name}(\sigma_{bid=100 \wedge rating > 5}(\text{Reserves} \bowtie_{R.sid=S.sid} \text{Sailors}))$$

Query Parser
& Optimizer

Equivalent to...

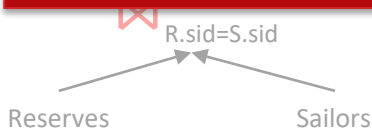
SQL

A **declarative** expression
of the query result

Relational Algebra

Operational description of
a computation.

But actually will
produce...



Operator Code

B+-Tree
Indexed
Iterator

Systems execute relational algebra
query plan.



SQL (Structured Query Language)

```
SELECT S.name  
FROM Reserves R, Sailors S  
WHERE R.sid = S.sid  
AND R.bid = 100  
AND S.rating > 5
```

- Key **System** Features: *Why do we like SQL*
 - Declarative:
 - Say **what** you want, not **how** to get it
 - Enables system to optimize the **how**
- Foundation in formal Query Languages
 - **Relational Calculus**



History: Formal Relational QL's

- **Relational Calculus:** (Basis for SQL)

- Describe the result of computation
- Based on first order logic
- Tuple Relational Calculus (**TRC**)
 - $\{S \mid S \in \text{Sailors} \exists R \in \text{Reserves} (R.\text{sid} = S.\text{sid} \wedge R.\text{bid} = 103)\}$

Are these equivalent?

Can we go from one to the other?

- **Relational Algebra:**

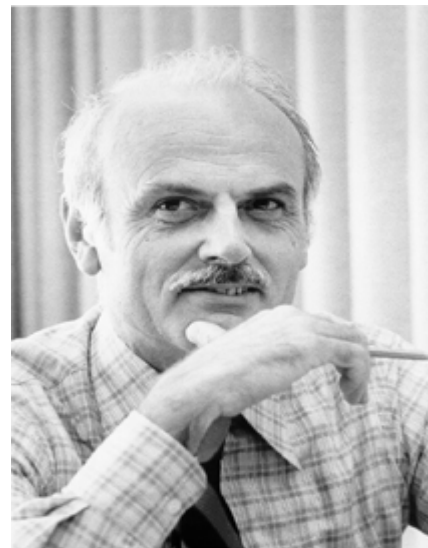
- Algebra on sets
- Operational description of transformations





Codd's Theorem

- Established equivalence in expressivity between :
 - Relational Calculus
 - Relational Algebra
- Why an important result?
 - Connects declarative representation of queries with operational description
 - Constructive: we can compile SQL into relational algebra



Edgar F. “Ted” Codd
(1923 - 2003)
Turing Award 1981



Relational Algebra Preliminaries

- Algebra of operators on relation instances
- $\pi_{S.name}(\sigma_{R.bid=100 \wedge S.rating>5}(R \bowtie_{R.sid=S.sid} S))$
 - Closed: result is also a relation instance
 - Enables rich composition!
 - Typed: input schema determines output
 - Can statically check whether queries are legal.



Relational Algebra and Sets

- Pure relational algebra has set semantics
 - No duplicate tuples in a relation instance
 - vs. SQL, which has multiset (bag) semantics
 - We will switch to multiset in the system discussion



Relational Algebra Operators: Unary

- Unary Operators: on **single relation**
- **Projection** (π): Retains only desired columns (vertical)
- **Selection** (σ): Selects a subset of rows (horizontal)
- **Renaming** (ρ): Rename attributes and relations.



Relational Algebra Operators: Binary

- Binary Operators: on **pairs of relations**
- **Union** (\cup): Tuples in r1 or in r2.
- **Set-difference** ($-$): Tuples in r1, but not in r2.
- **Cross-product** (\times): Allows us to combine two relations.



Relational Algebra Operators: Compound

- Compound Operators: common “*macros*” for the above
- **Intersection** (\cap): Tuples in r1 and in r2.
- **Joins** (\bowtie_{θ} , \bowtie): Combine relations that satisfy predicates



Projection (π)

- Corresponds to the SELECT list in SQL
- Schema determined by schema of attribute list
 - Names and types correspond to input attributes
- *Selects a subset of columns (vertical)*

$$\pi_{\text{sname, age}}(S2)$$

Relational Instance S2

<u>sid</u>	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

List of Attributes



sname	age
yuppy	35.0
lubber	55.5
guppy	35.0
rusty	35.0



Projection (π), cont.

- Set semantics \rightarrow results in fewer rows
 - Real systems don't automatically remove duplicates
 - Why? (Semantics and Performance reasons)
 - *Selects a subset of columns (vertical)*

$$\pi_{\text{age}}(S2)$$

Relational Instance S2

<u>sid</u>	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

Multiset

age
35.0
55.5
35.0
35.0

Set

age
35.0
55.5

Selection(σ)

- Corresponds to the WHERE clause in SQL
- Output schema same as input
- Duplicate Elimination? Not needed.
- *Selects a subset of rows (horizontal)*

$$\sigma_{\text{rating} > 8}(\text{S2})$$

Relational Instance S2

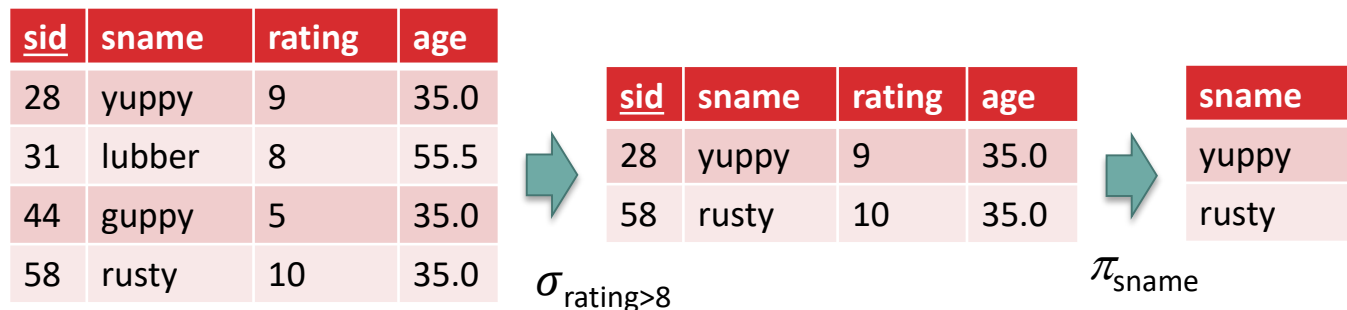
<u>sid</u>	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

Selection Condition (Boolean Expression)

<u>sid</u>	sname	rating	age
28	yuppy	9	35.0
58	rusty	10	35.0

Composing Select and Project

- Names of sailors with rating > 8: $\pi_{\text{sname}}(\sigma_{\text{rating}>8}(S2))$



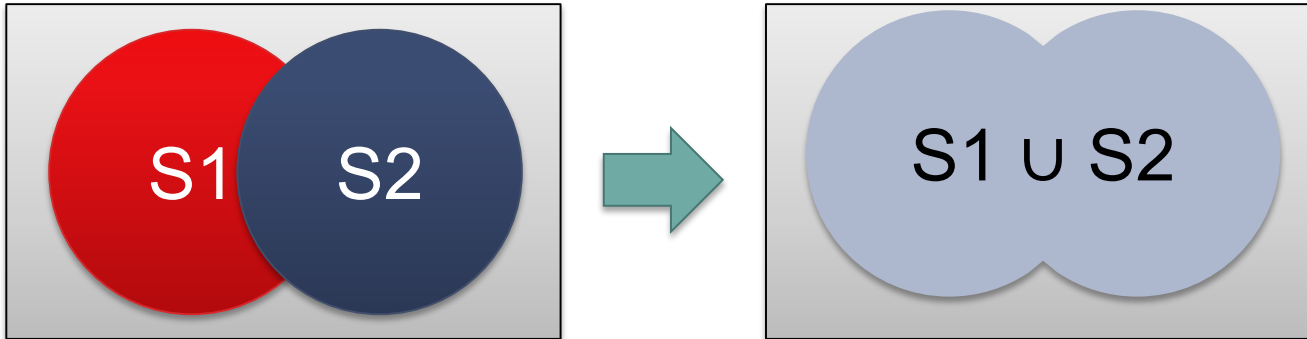
- What about: $\sigma_{\text{rating}>8}(\pi_{\text{sname}}(S2))$
 - Invalid types. Input to $\sigma_{\text{rating}>8}$ does not contain rating.*



Union (\cup)

- Two input relations, must be *compatible*:
 - Same number of fields
 - Fields in corresponding positions have same **type**
- SQL Expression: UNION

$S1 \cup S2$





Union (\cup) VS Union ALL

- Duplicate elimination in practice?
 - SQL's UNION vs UNION ALL

Relational Instance **S1**

<u>sid</u>	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

Relational Instance **S2**

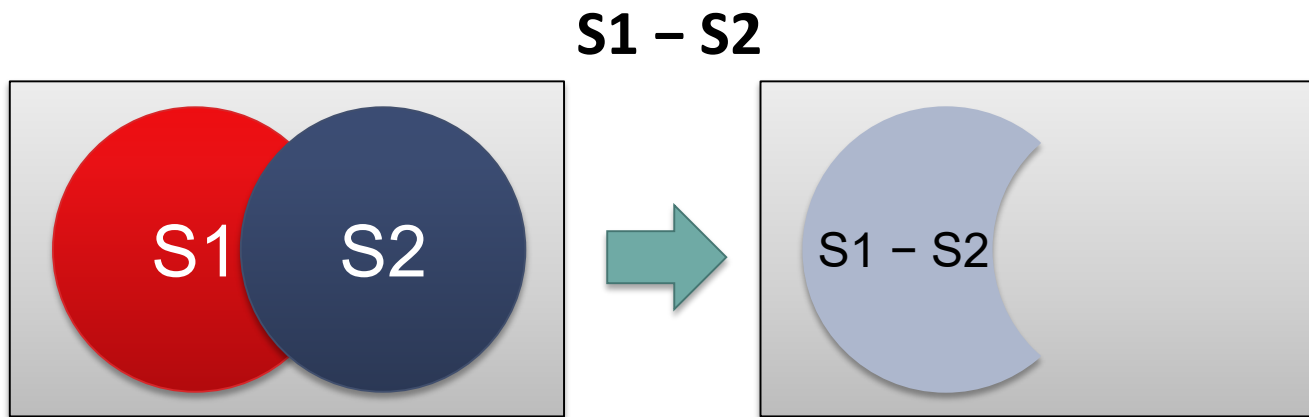
<u>sid</u>	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

S1 \cup S2

<u>sid</u>	sname	rating	age
22	dustin	7	45
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

Set Difference (-)

- Same as with union, both input relations must be *compatible*.
- SQL Expression: EXCEPT





Set Difference (–), cont.

- Duplicate elimination?
 - Not required
- EXCEPT vs EXCEPT ALL

Relational Instance **S1**

<u>sid</u>	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

Relational Instance **S2**

<u>sid</u>	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

S1 – S2

<u>sid</u>	sname	rating	age
22	dustin	7	45

S2 – S1

<u>sid</u>	sname	rating	age
28	yuppy	9	35.0
44	guppy	5	35.0



Cross-Product (\times)

- **$R1 \times S1$** : *Each row of $R1$ paired with each row of $S1$*

R1:

<u>sid</u>	<u>bid</u>	<u>day</u>
22	101	10/10/96
58	103	11/12/96

\times

S1:

<u>sid</u>	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

$=$

sid	bid	day	sid	sname	rating	age
22	101	10/10/96	22	dustin	7	45.0
22	101	10/10/96	31	lubber	8	55.5
22	101	10/10/96	58	rusty	10	35.0
58	103	11/12/96	22	dustin	7	45.0
58	103	11/12/96	31	lubber	8	55.5
58	103	11/12/96	58	rusty	10	35.0

- ***How many rows in result? $|R1| \times |R2|$***
- ***Schema compatibility? Not needed.***
- ***Duplicates? None generated.***

Renaming ($\rho = \text{"rho"}$)

- *Renames relations and their attributes:*
- Note that relational algebra doesn't require names.
 - We could just use positional arguments.

$$\rho(\underbrace{\text{Temp1}}_{\text{Output Relation Name}}(\underbrace{1 \rightarrow \text{sid1}, 4 \rightarrow \text{sid2}}_{\substack{\text{Renaming List} \\ \text{position} \rightarrow \text{New Name}}}, \underbrace{R1 \times S1}_{\text{Input Relation}})$$

$R1 \times S1$

sid	bid	day	sid	sname	rating	age
22	101	10/10/96	22	dustin	7	45.0
22	101	10/10/96	31	lubber	8	55.5
22	101	10/10/96	58	rusty	10	35.0
58	103	11/12/96	22	dustin	7	45.0
58	103	11/12/96	31	lubber	8	55.5
58	103	11/12/96	58	rusty	10	35.0



Temp1

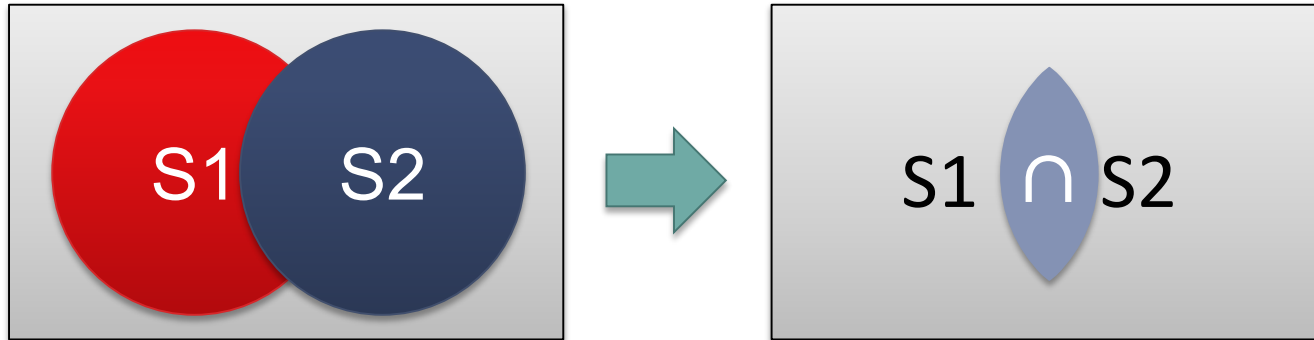
sid1	bid	day	sid2	sname	rating	age
22	101	10/10/96	22	dustin	7	45.0
22	101	10/10/96	31	lubber	8	55.5
22	101	10/10/96	58	rusty	10	35.0
58	103	11/12/96	22	dustin	7	45.0
58	103	11/12/96	31	lubber	8	55.5
58	103	11/12/96	58	rusty	10	35.0



Compound Operator: Intersection

- Same as with union, both input relations must be *compatible*.
- SQL Expression: INTERSECT

$$S1 \cap S2$$





Intersection (\cap)

- Equivalent to:
 $S1 \text{ --- } (S1 \text{ --- } S2)$

$S1 \cap S2$

<u>sid</u>	sname	rating	age
31	lubber	8	55.5
58	rusty	10	35.0

Relational Instance **S1**

<u>sid</u>	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

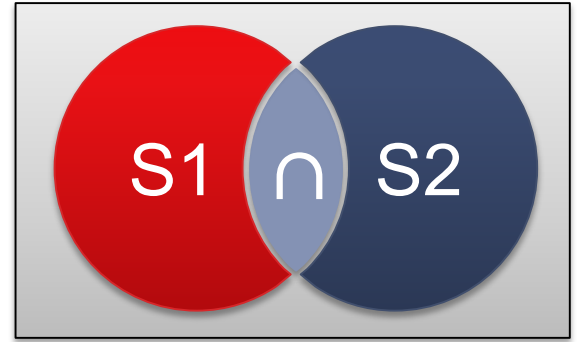
Relational Instance **S2**

<u>sid</u>	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0



Intersection (\cap), Pt 2

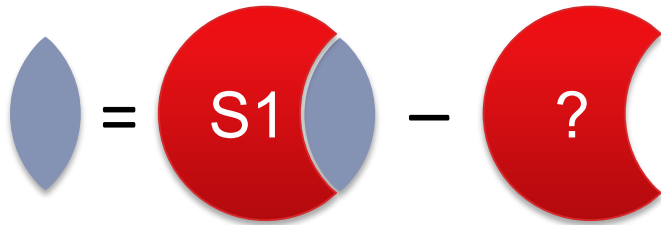
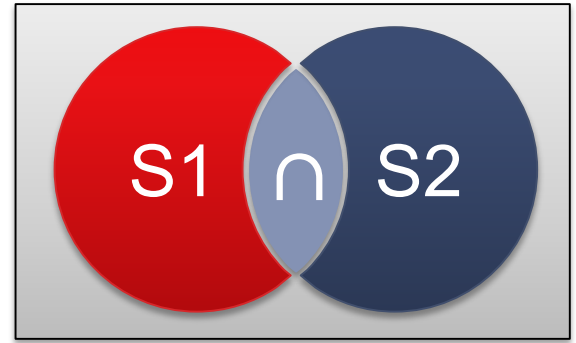
- $S1 \cap S2 = ?$





Intersection (\cap), Pt 3

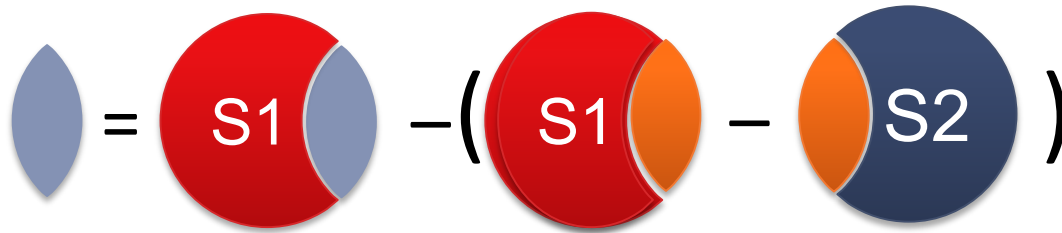
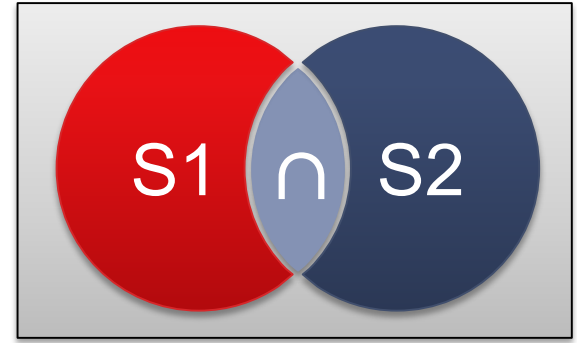
- $S1 \cap S2 = S1 - ?$





Intersection (\cap), Pt 4

- $S1 \cap S2 = S1 - (S1 - S2)$



- Is intersection monotonic?

$$R_1 \subseteq R_2 \Rightarrow S \cap R_1 \subseteq S \cap R_2$$



Compound Operator: Join

- Joins are compound operators (like intersection):
 - Generally, $\sigma_{\theta}(R \times S)$
- Hierarchy of common kinds:
 - **Theta Join** (\bowtie_{θ}): join on logical expression θ
 - **Equi-Join**: theta join with theta being a conjunction of equalities
 - **Natural Join** (\bowtie): equi-join on all matching column names

Note: we will need to learn a good join algorithm.

Avoid cross-product if we can!!



Theta Join (\bowtie_{θ}) Example

- $R1 \bowtie_{\text{sid}=\text{sid}} S1$

R1:

<u>sid</u>	<u>bid</u>	<u>day</u>
22	101	10/10/96
58	103	11/12/96

S1:

<u>sid</u>	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

$\bowtie_{\text{sid}=\text{sid}}$

Result:

sid	bid	day	sid	sname	rating	age
22	101	10/10/96	22	dustin	7	45.0
58	103	11/12/96	58	rusty	10	35.0

- Note that output needs a rename operator!

Another Theta Join (\bowtie_{θ}) Example

- $R \bowtie_{\theta} S = \sigma_{\theta}(R \times S)$
- **Example:** *More senior sailors for each sailor.*
- $S1 \bowtie_{f4 < f8} S1$

f1	f2	f3	f4
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

S1				S1			
f1	f2	f3	f4	f5	f6	f7	f8
22	dustin	7	45.0	22	dustin	7	45.0
22	dustin	7	45.0	31	lubber	8	55.5
22	dustin	7	45.0	58	rusty	10	35.0
31	lubber	8	55.5	22	dustin	7	45.0
31	lubber	8	55.5	31	lubber	8	55.5
31	lubber	8	55.5	58	rusty	10	35.0
58	rusty	10	35.0	22	dustin	7	45.0
58	rusty	10	35.0	31	lubber	8	55.5
58	rusty	10	35.0	58	rusty	10	35.0

Another Theta Join (\bowtie_{θ}), Pt 2

- $R \bowtie_{\theta} S = \sigma_{\theta}(R \times S)$
- **Example:** *More senior sailors for each sailor.*
- $S1 \bowtie_{\text{age} < \text{age2}} S1$

S1:

f1	f2	f3	f4
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

S1				S1			
f1	f2	f3	f4	f5	f6	f7	f8
22	dustin	7	45.0	22	dustin	7	45.0
22	dustin	7	45.0	31	lubber	8	55.5
22	dustin	7	45.0	58	rusty	10	35.0
31	lubber	8	55.5	22	dustin	7	45.0
31	lubber	8	55.5	31	lubber	8	55.5
31	lubber	8	55.5	58	rusty	10	35.0
58	rusty	10	35.0	22	dustin	7	45.0
58	rusty	10	35.0	31	lubber	8	55.5
58	rusty	10	35.0	58	rusty	10	35.0

Another Theta Join (\bowtie_{θ}), Pt 3

- $R \bowtie_{\theta} S = \sigma_{\theta}(R \times S)$
- **Example:** *More senior sailors for each sailor.*
- $S1 \bowtie_{f4 < f8} S1$

S1:

<u>sid</u>	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

S1				S1			
sid	sname	rating	age	sid	sname	rating	age2
22	dustin	7	45.0	31	lubber	8	55.5
58	rusty	10	35.0	22	dustin	7	45.0
58	rusty	10	35.0	31	lubber	8	55.5

- Result *schema* same as that of cross-product.
- Special Case:
 - **Equi-Join:** *theta join with AND of = predicates*
 - Special special case **Natural Join** ...



Natural Join (\bowtie)

- Special case of equi-join in which equalities are specified for all matching fields and duplicate fields are projected away

$$\mathbf{R} \bowtie \mathbf{S} = \pi_{\text{unique fld.}} \sigma_{\text{eq. matching fld.}} (\mathbf{R} \times \mathbf{S})$$

- Compute $\mathbf{R} \times \mathbf{S}$
- Select rows where fields appearing in both relations have equal values
- Project onto the set of all unique fields.



Natural Join (\bowtie) Pt 2

- $R \bowtie S = \pi_{\text{unique fld.}} \sigma_{\text{eq. matching fld.}} (R \times S)$

$R1 \bowtie S1$

sid	bid	day	sid	sname	rating	age
22	101	10/10/96	22	dustin	7	45.0
22	101	10/10/96	31	lubber	8	55.5
22	101	10/10/96	58	rusty	10	35.0
58	103	11/12/96	22	dustin	7	45.0
58	103	11/12/96	31	lubber	8	55.5
58	103	11/12/96	58	rusty	10	35.0

R1:

sid	bid	day
22	101	10/10/96
58	103	11/12/96

S1:

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0



Natural Join (\bowtie), Pt 3

- $R \bowtie S = \pi_{\text{unique fld.}} \sigma_{\text{eq. matching fld.}} (R \times S)$

$R1 \bowtie S1$

sid	bid	day	sname	rating	age
22	101	10/10/96	dustin	7	45.0
58	103	11/12/96	rusty	10	35.0

R1:

sid	bid	day
22	101	10/10/96
58	103	11/12/96

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

- Commonly used for foreign key joins (as above).



Extended Relational Algebra

- Group By / Aggregation Operator (γ):
 - $\gamma_{\text{age, AVG(rating)}}(\text{Sailors})$
 - With selection (HAVING clause):
 - $\gamma_{\text{age, AVG(rating), COUNT(*) > 2}}(\text{Sailors})$
- Textbook uses two operators:
 - GROUP BY age, AVG(rating) (Sailors)
 - HAVING COUNT(*) > 2
(GROUP BY age, AVG(rating)(Sailors))



Summary

- Relational Algebra: a small set of operators mapping relations to relations
 - Operational, in the sense that you specify the explicit order of operations
 - A closed set of operators! Mix and match.
- Basic ops include: σ , π , \times , \cup , $-$
- Important compound ops: \cap , \bowtie