Reputation and Imperfect Information

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Quantifying "Reputation"

- Definition
  - **Reputation** is defined as the probability of being a certain type.
- Establishing reputation
  - Observing actions.
- Influence of reputation
  - Good reputation such as *honesty* and *high quality of products* can bring huge benefits.
- Problem
  - How to identify *hypocrites*?

Entry Deterrence

Players

Two firms, Entrant and Incumbent.

The Order of Play

1. The entrant decides whether to *Enter* or *Stay Out*.
2. If the entrant enters, the incumbent can *Collude* with him, or *Fight* by cutting the price drastically.

Payoffs

Market profits are 40 at the monopoly price and 0 at the fighting price. The profits will be split between E and I.

Entry Deterrence

Entry Deterrence

Chainstore Paradox

Chainstore problem

- A Chainstore has outlets in 20 markets.
- It repeats Entry Deterrence 20 times.
- The "histories" of earlier periods are observed by later entrants.
- Should the chainstore fight the first entrant to deter the next 19?

SPE of the Chainstore Problem

Using **backward induction**, the unique **subgame perfect equilibrium** of the chainstore problem is the repetition of the SPE in Entry Deterrence.

Kreps and Wilson (1982)

Resolve the Chainstore Paradox

The contradiction between the Chainstore Paradox and what many people think of as real world behavior has been most successfully resolved by *adding incomplete information to the model.*

The reputation of being "tough"

The entrants may not be certain about the payoffs to the incumbent.

- The incumbent could *Fight* in response to *Enter*.

Mechanism

The incumbent may choose to fight early entrants to sustain or enhance his reputation of being tough, so as to deter subsequent challengers.

Two States $(0 < b < 1,\ a > 1)$

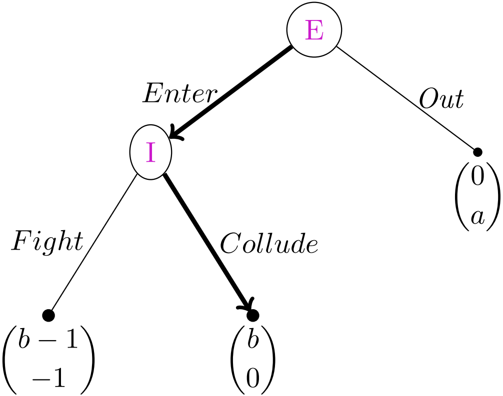


Figure: Weak Incumbent

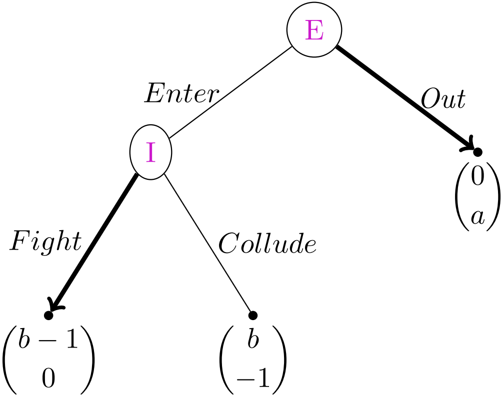


Figure: Tough Incumbent

Payoff: (Entrant, Incumbent)

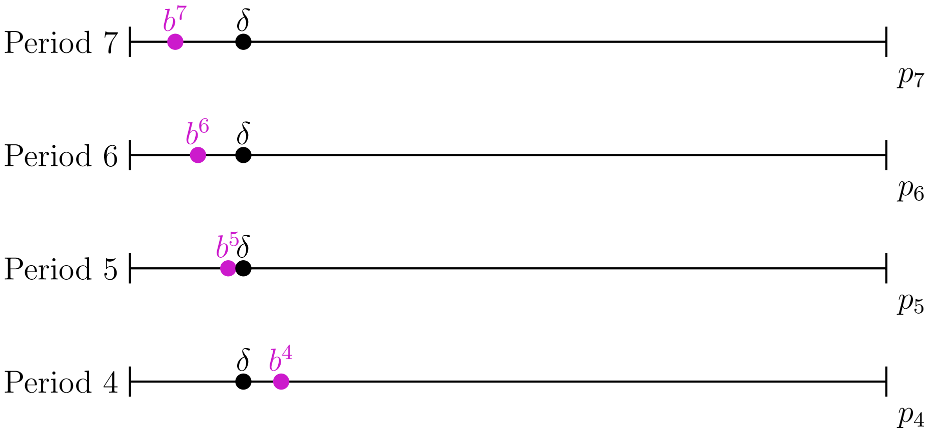
Repeated Entry Deterrence

Players
One Incumbent and N Entrants.
The Order of Play

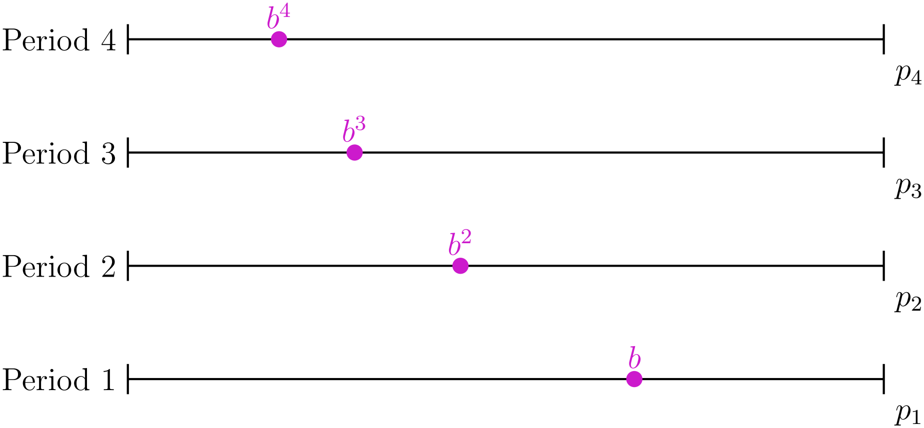
- Nature picks the state of the world with $Prob(tough) = \delta$.
- I is informed of the state. But not E's.
- Period N: I plays Entry Deterrence with E_N
- Period $N-1$: I plays Entry Deterrence with E_{N-1}
- $\ldots$
- Period 1: I plays Entry Deterrence with E_1

Payoffs
I: The sum of payoffs in each period.
E_n: The payoff in period n.

Latent Variable ($b = \frac{2}{3}$, $\delta = 0.15$)



Latent Variable - cont.



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Belief Evolution

History

Let h_n denote what has happened (history) up to period n, i.e., the moves in periods $N, N-1, \dots, n+1$.

Updated beliefs

Let $p_n(h_n)$ denote $Prob(tough)$ in history h_n.

- $p_N = \delta$

Belief Evolution - continued

Algorithm

- If there is no entry in period $n+1$, then $p_n = p_{n+1}$.
- If there is entry in period $n+1$, this entry is fought, and $p_{n+1} > 0$, then $p_n = \max(b^n, p_{n+1})$.
- If there is entry in period $n+1$ and this entry is met by *Collude*, then $p_n = 0$.
- If $p_{n+1} = 0$, then $p_n = 0$.

Perfect Equilibrium

Strategy of the Incumbent

- If *tough*: always fight entry.
- If *weak*:
  - If $n = 1$, $Colludes$,
  - If $n > 1$ and $p_n \geq b^{n-1}$, $Fight$,
  - If $n > 1$ and $p_n < b^{n-1}$, $Fight$ with prob. x and $Collude$ with prob. $1 - x$, where

$$x = \frac{(1 - b^{n-1})p_n}{(1 - p_n)b^{n-1}} \tag{1}$$

Note: When $p_n = 0$, $x = 0$. When $p_n = b^{n-1}$, $x = 1$.

Perfect Equilibrium - continued

Strategies of the Entrants

- If $p_n > b^n$, E_n stays *Out*.
- If $p_n < b^n$, E_n *Enter*.
- If $p_n = b^n$, E_n stays *Out* with prob $\frac{1}{a}$, *Enter* with $1 - \frac{1}{a}$.

Why it is an Equilibrium?

Two things to verify

- The beliefs are updated by Bayes rule.
- No player has incentive to deviate in any period.

Belief Consistency

- If no entry in period $n+1$, belief does not change, $p_n = p_{n+1}$.
- If there is entry in period $n+1$:
  - If $p_{n+1} \geq b^n$, both the weak and tough I fight entry, belief does not change, $p_n = p_{n+1}$.
  - If $0 < p_{n+1} < b^n$, the tough I fights and the weak I fights with prob x, the belief updates to $p_n = b^n$ when $Fight$ and $p_n = 0$ when $Collude$. (Calculation in the next slide)
  - If $p_{n+1} = 0$, I is 100% weak, $p_n = p_{n+1} = 0$.

Bayesian Updating When $0 < p_{n+1} < b^n$

$$
\begin{aligned}
p_n &= Prob(\ I \text{ is tough}|I\ Fight) \\
&= \frac{Prob(\ Fight|Tough)Prob(Tough)}{Prob(\ Fight|Tough)Prob(Tough) + Prob(\ Fight|Weak)Prob(Weak)} \\
&= \frac{1 \cdot p_{n+1}}{1 \cdot p_{n+1} + x \cdot (1 - p_{n+1})} \\
&= b^n
\end{aligned}
$$

To verify, substitute $x(p_{n+1})$ from Eq. (1).

Optimization of Entrant n, for each n

- When $p_n \geq b^{n-1}$
  - I fights entry, E_n stays out.
- When $p_n \in (b^n, b^{n-1})$
  - I fights with prob. more than b, E_n stays out.
- When $p_n = b^n$
  - I fights with prob. b, E_n is indifferent, so he can randomize.
- When $p_n < b^n$
  - I fights with prob. less than b, E_n enters.

Calculation

$$\begin{aligned} Prob(Fight) =& Prob(Tough) + Prob(Fight|Weak) \cdot Prob(Weak) \\ =& p_n + x(p_n)(1-p_n) = \frac{p_n}{b^{n-1}} \end{aligned}$$

Optimization of Incumbent

For tough I

- In the short-run, $Fight$ is better.
- In the long-run, $Fight$ deters entries.
- **Always** $Fight$.

For weak I

Reasoning inductively from back.

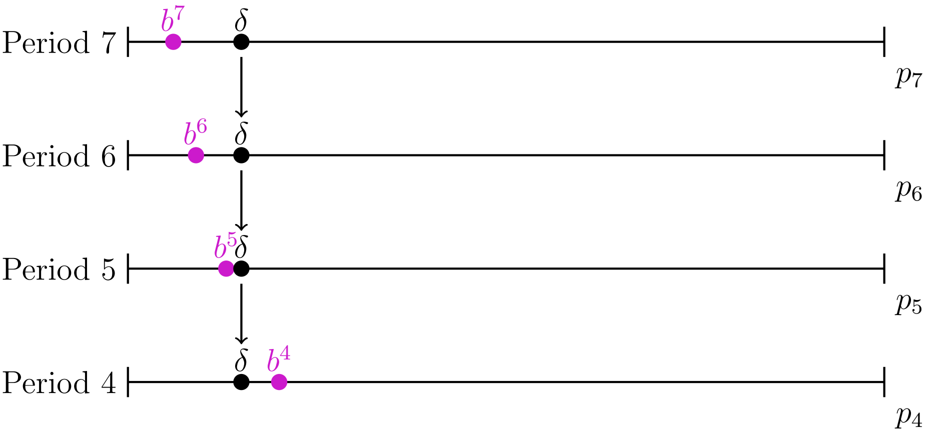
Period 1 Collude

Period 2
- Fight entry: -1 now, 1 next. (E_1 stays out with prob. $\frac{1}{a}$)
- Collude: 0 forever.
- Indifferent, randomize.

$\cdots$

Period N $Fight$ to deter early entries.

Illustration of the Play ($b = \frac{2}{3}$, $\delta = 0.15$)



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