SI252 Reinforcement Learning	2021/04/28

Homework 4

Professor: Ziyu Shao — *Due: 2021/05/14 11:59am*

1. **Binary Sequences with No Adjacent 1s**: Implement a MCMC algorithm to compute the expected number of 1s in a good sequence if all good sequences are equal likely (m=100)?

2. **Power-Law Distribution**: Implement a Metropolis-Hastings algorithm to simulate from the power-law distributions shown in the lecture.

3. **Knapsack Problem**: m=5, w=10, vector of worth $\{g_j\}$=(6,3,5,4,6), vector of weight $\{w_j\}$=(2,2,6,5,4). (a) Find the maximization of the total worth of the treasure by using MCMC method. (b) Find the maximization of the total worth of the treasure by using dynamic programming (DP) method. (c) Discuss the pros and cons of both MCMC and DP methods. (d) Please design a large-scale numerical experiments (m=3000, you have the freedom to set other parameters as long as it is not trivial) to confirm your conclusions in (c).

4. **Standard Normal Distribution**: Implement a Metropolis-Hastings algorithm to generate samples from the standard normal distribution. Then compare the MCMC method with the Box-Muller Method.

5. **Beta Simulation**: Implement a Metropolis-Hastings algorithm to generate samples from the Beta-distribution Beta(5, 5).

6. **Normal-Normal Conjugacy**: Implement a Metropolis-Hastings algorithm to find the posterior mean and variance of θ after observing the value of $Y = y$. The parameter setting: $y = 3$,$\mu = 0$,$\sigma^2 = 1$,$\tau^2 = 4$, and d=0.1, 0.5, 1, 5, 10, 100.

7. **Bivariate Standard Normal Distribution**: Implement a Gibbs sampler to generate samples from a bivariate standard normal distribution with correlation $\rho = 0.6, -0.6$.

8. **Chicken-Egg with Unknown Parameters**: The parameter setting: $\lambda = 10$,$a = b = 1$,$x = 7$. (a) Implement a Gibbs sampler to find the posterior mean and the variance of p after observing x hatched eggs. (b) Implement a Metropolis-Hastings algorithm to find the posterior mean and variance of p after observing x hatched eggs. (c) Compare such two methods of MCMC.

9. **Three-dimensional Joint Distribution**: Implement a Gibbs sampler to generate samples from the three-dimensional joint distribution shown in the lecture.

10. **Markov Chain Monte Carlo for Wireless Networks**. Given a wireless network with 24 links and $0-1$ interference model, i.e., any two links are either interfere with each other or not. To describe the interference relationship between wireless links, we introduce the conflict graph model. In such model, the vertex of the conflict graph represents the wireless link. An edge between two vertices means corresponding two links interfere with each other. The following Figure shows the corresponding conflict graph for 24-link wireless network. You are required to find the maximum independent set of the conflict graph, i.e., the largest set of wireless links that can simultaneously transmit without interferences. Design the algorithm by MCMC method and evaluate your algorithm.

- Show your MCMC Design with a discrete Markov chain. Use both theory and simulation results to justify your algorithms.
- Show your MCMC Design with a continuous Markov chain. Use both theory and simulation results to justify your algorithms.
- Discuss the pros and cons for such chains.