

# CS243: Introduction to Algorithmic Game Theory

Week 1.2, Basic Concepts (Dengji ZHAO)

SIST, ShanghaiTech University, China

# Outline

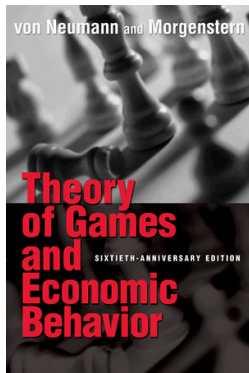
- 1 Recap
- 2 Basic Concepts

# Announcements

- 1 Piazza is online!

# Recap: What is Game Theory







- **Game theory** is the study of mathematical models of **conflict** and **cooperation** between intelligent rational decision-makers [von Neumann and Morgenstern 1944].



- **Extensive form**: Go, poker
- **Normal form**: rock-paper-scissors
- **Cooperative game**: coordination games

# Recap: What is Game Theory

- **Game theory** is the study of mathematical models of **conflict** and **cooperation** between intelligent rational decision-makers [von Neumann and Morgenstern 1944].

	Participants	Game	Outcome
Game Theory			
Mechanism Design			

# Mechanism Design (Reverse Game Theory)

Mechanism Design is to answer...

## Question

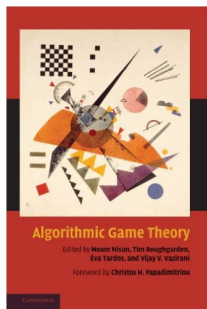
How to **design** a mechanism/game, toward desired objectives, in strategic settings?



- **Roger B. Myerson** (born March 29, 1951, University of Chicago, US)
  - **Nobel Prize** for economics (2007), for "having laid the foundations of **mechanism design theory**."
  - ***Eleven game-theorists** have won the economics Nobel Prize.*

# When Game Theory Meets CS?

- **Algorithmic Game Theory** is an area in the intersection of **game theory** and **algorithm design**, whose objective is to design algorithms in strategic environments [Nisan et al. 2007].



- *Computing in Games*: algorithms for computing equilibria
- *Algorithmic Mechanism Design*: design games that have both good game-theoretical and algorithmic properties
- ...

# When Game Theory Meets CS?

- **Algorithmic Game Theory** is an area in the intersection of **game theory** and **algorithm design**, whose objective is to design algorithms in strategic environments [Nisan et al. 2007].

It is multidisciplinary:

- Artificial Intelligence → Multi-agent Systems → Algorithmic Game Theory
- Economics
- Theoretical Computer Science



# Algorithmic Game Theory in Artificial Intelligence

- Algorithmic Game Theory research in AI (multi-agent systems):
  - **Game Playing**: computation challenge, AlphaGo, poker
  - **Social Choice**: preferences aggregation, voting, prediction
  - **Mechanism Design**: the allocation of scarce resources (security games), Ad auctions, online auctions, false-name-proof mechanisms (**Makoto Yokoo**)
- IJCAI Computers and Thought Award: **5 out of the 12 winners (1999-2017) had worked on AGT**, **Nick Jennings** (1999), Tuomas Sandholm (2003), Peter Stone (2007), Vice Conitzer (2011), Ariel Procaccia (2015).

# Outline

- 1 Recap
- 2 Basic Concepts
  - Classical Games
  - Solution Concepts

# Prisoners' Dilemma

- Two players: P1 and P2
- Strategies: Confess, Silent
- Outcomes: number of years in prison

		P2	
		Confess	Silent
P1	Confess	4 4	5 1
	Silent	1 5	2 2

# Battle of the Sexes

- Two players: Girl, Boy
- Strategies: Baseball (B), Softball (S)
- Outcomes: payoffs/benefits/utilities

		Boy	
		B	S
Girl	B	6 5	1 1
	S	2 2	5 6

# Simultaneous Move Game

- A set of  $n$  players
- Each player  $i$  has a set of strategies  $S_i$
- Let  $s = (s_1, \dots, s_n)$  be the vector of strategies selected by the  $n$  players. Also let  $\mathbf{s} = (s_i, \mathbf{s}_{-i})$ .
- Let  $S = \prod_i S_i$  be the strategy vector space of all players.
- Each  $s \in S$  determines the outcome for each player, denote  $u_i(s)$  the utility of player  $i$  under  $s$ .

# Dominant Strategy

## Definition

A strategy  $s_i \in S_i$  is a **dominant strategy** for player  $i$ , if for all  $s' \in S$ , we have that  $u_i(s_i, s'_{-i}) \geq u_i(s'_i, s'_{-i})$

## Definition

A strategy vector  $s \in S$  is a **dominant strategy equilibrium**, if for each player  $i$ , and each alternate strategy vector  $s' \in S$ , we have that  $u_i(s_i, s'_{-i}) \geq u_i(s'_i, s'_{-i})$

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Any dominant strategy equilibrium in Prisoners' Dilemma?

		P2	
		Confess	Silent
P1	Confess	4, 4	5, 1
	Silent	1, 5	2, 2

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		B	S
Girl	B	5, 6	1, 1
	S	2, 2	5, 6



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- What is the **difference** between *Dominant Strategy Equilibrium* and *Nash Equilibrium*?

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Any Nash equilibrium in Battle of the Sexes?

		Boy	
		B	S
Girl	B	6 5	1 1
	S	2 2	5 6

# Another Game: Matching Pennies

- Two players: 1, 2
- Strategies: Head (H), Tail (T)
- Outcomes: the row player (1) wins if the two pennies match, while the column player wins if they do not match

		2	
		H	T
1	H	-1 1	1 -1
	T	1 -1	-1 1

# Another Game: Matching Pennies

- Two players: 1, 2
- Strategies: Head (H), Tail (T)
- Outcomes: the row player (1) wins if the two pennies match, while the column player wins if they do not match
- Any dominant strategy or Nash equilibrium?

		2	
		H	T
1	H	-1 1	1 -1
	T	1 -1	-1 1

# Mixed Strategies

## Definition

Each player  $i$  picks a probability distribution  $p_i$  over his set of possible strategies  $S_i$ , such a choice is called a **mixed strategy**.

- Given a player  $i$ 's probability distribution choice  $p_i$  over  $S_i$ , let  $p_i(s_i)$  be the probability to choose strategy  $s_i$ , we have  $\sum_{s_i \in S_i} p_i(s_i) = 1$ .
- Assume that players are **risk-neutral**; that is, they act to maximize the **expected payoff**.

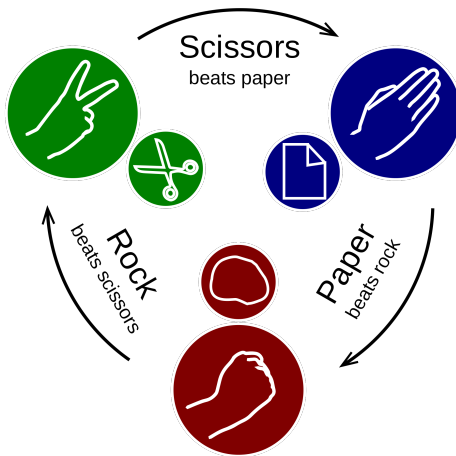


# Mixed Strategy Nash Equilibrium

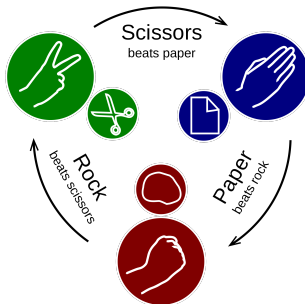
- Two players: 1, 2
- Strategies: Head (H), Tail (T)
- Outcomes: the row player (1) wins if the two pennies match, while the column player wins if they do not match
- If player 1 uses mixed strategy  
 $p_1(H) = p_1(T) = 0.5$ , what is the best strategy for player 2?

		2	
		H	T
1	H	-1 1	1 -1
	T	1 -1	-1 1

# Mixed Strategy Nash Equilibrium



# Mixed Strategy Nash Equilibrium



## Quiz

If one player can only choose Rock and Paper, what is the best strategy for the other player?



# Advanced Reading

- Games with no Nash equilibria [AGT Chapter 1.3.5]
- Correlated Equilibrium [AGT Chapter 1.3.6]