CS 101 Fall 2020 - Quiz 11-B

12/31/2020 - 25 Minutes **Name:** **ID number:**

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(1) (5 Points) Consider Knapsack problem with repetition with n items with values v_i and weight w_i. We have following defined sub-problems:

$$K[v] = \text{weight of the lightest collection with total value at least } v, \text{ repetitions allowed}$$

Is it possible to define a recurrence relation to solve above sub-problems? If possible, give the recurrence formula; otherwise provide the reason why it is not possible.

Possible. The idea is to consider all elements and find the lightest collection of other elements that, upon adding a new element, will reach the value v. More formally:

$$K[v] = \min_i (K[v - v_i] + w_i)$$

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(2) (10 Points) You are given some identical eggs and a building. You need to figure out the maximum floor l that you can drop them from without breaking them. Each egg will break if dropped from a floor greater than or equal to l, and will never break when dropped from a floor less then l. Note that once an egg breaks, you cannot use it any more.

- (4 Points) If you are given only one egg and a 100-story building, what is the strategy to figure out what l is, no matter what value l is? What is the maximum number of drops in the strategy?
- (3 Points) If you are given two eggs and a 10-story building, what is the strategy to figure out what l is, which has the minimum number of drops in worst cases?
- (3 Points) If you are given k eggs and a n-story building, give the algorithm to figure out the method that figures out what l is with minimum number of drops in worst cases and the minimum number of drops in worst cases using dynamic programming.

(a) Drop from the first floor until the egg is broken.

(b) First drop at floor 4, if the egg is not broken, then drop at floor 7, then floor 9. If the egg is broken, drop from the lowest floor we did not whether the egg will be broken.

(c) Let $A[k,n]$ be the minimum number of drops in worst cases when there are k eggs and n floors. Let $P[k,n]$ be the number of floor we should firstly test when there are k eggs and n floors. Then the base cases are $A[k,0]=0$, $A[k,1]=1$ and $A[1,n]=n$ for any k and n. The induction step is following.

$$A[k,n] = \min_{1\le j\le n}\{\max\{A[k-1,j-1], A[k,n-j]\}\}+1$$

$$P[k,n] = \arg\min_{1\le j\le n}\{\max\{A[k-1,j-1], A[k,n-j]\}+1\}$$