

# Active Learning

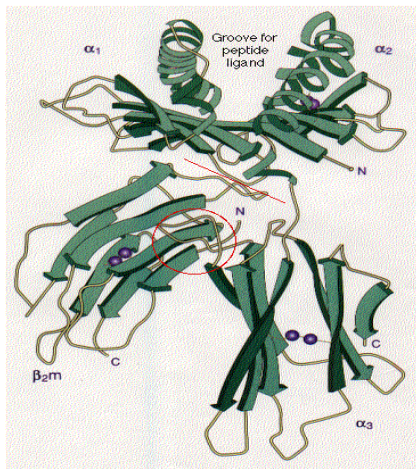
Maria-Florina Balcan

04/01/2015

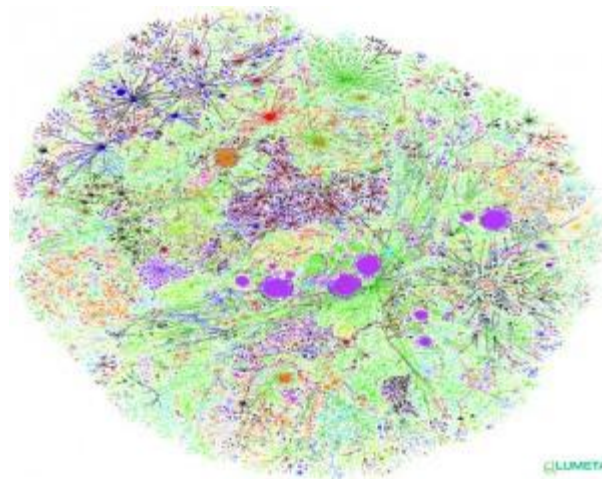
# Classic Fully Supervised Learning Paradigm Insufficient Nowadays

Modern applications: **massive amounts** of raw data.

Only **a tiny fraction** can be annotated by human experts.



Protein sequences



Billions of webpages



Images

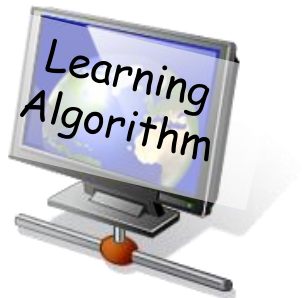
# Modern ML: New Learning Approaches

Modern applications: **massive amounts** of raw data.

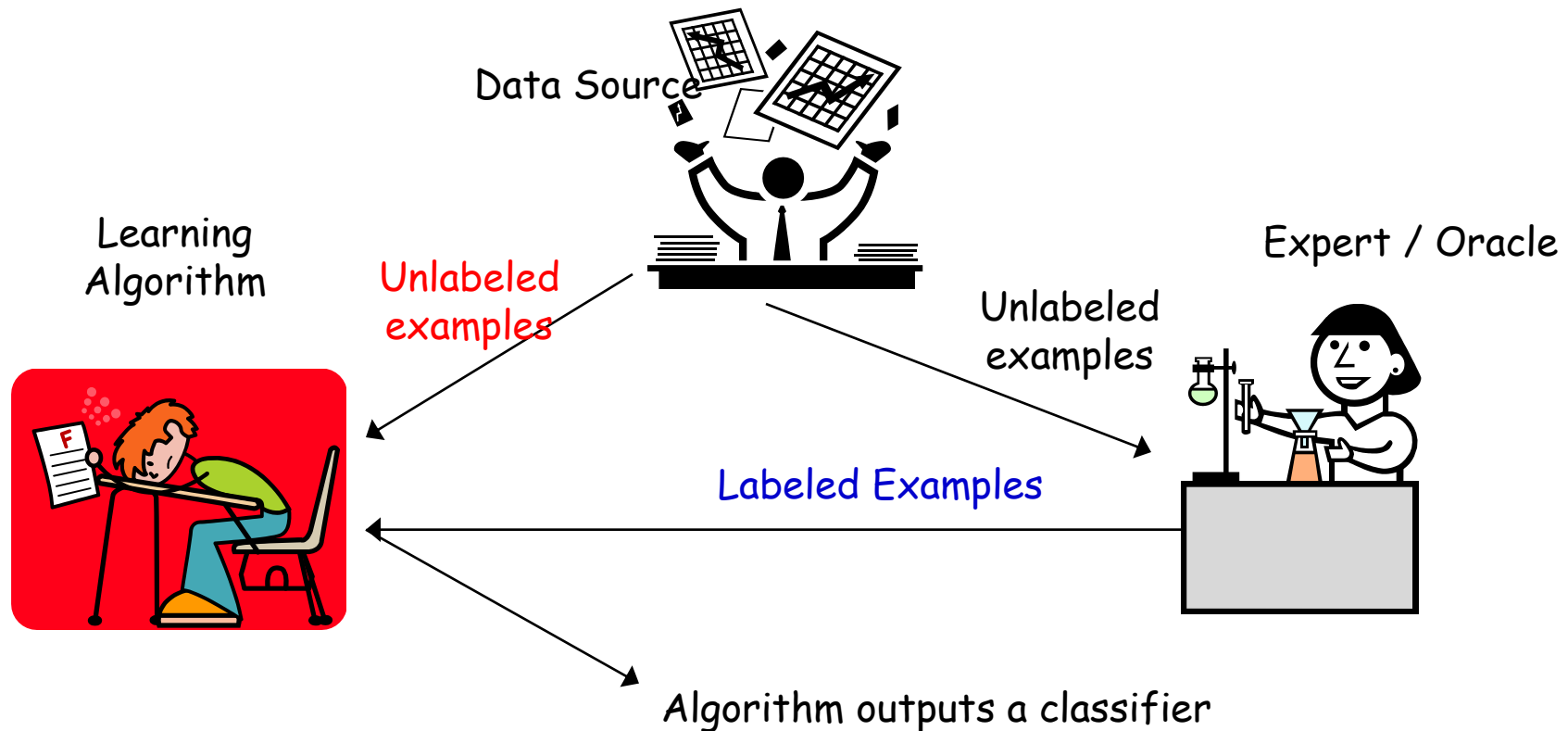
Techniques that best utilize data, **minimizing need for expert/human intervention.**

Paradigms where there has been great progress.

- Semi-supervised Learning, (Inter)active Learning.



# Semi-Supervised Learning



$$S_l = \{(x_1, y_1), \dots, (x_{m_l}, y_{m_l})\}$$

$x_i$  drawn i.i.d from  $\mathcal{D}$ ,  $y_i = c^*(x_i)$

$S_u = \{x_1, \dots, x_{m_u}\}$  drawn i.i.d from  $\mathcal{D}$

**Goal:**  $h$  has small error over  $\mathcal{D}$ .

$$\text{err}_{\mathcal{D}}(h) = \Pr_{x \sim \mathcal{D}} (h(x) \neq c^*(x))$$

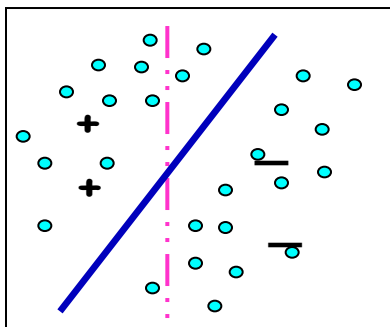
# Semi-supervised Learning



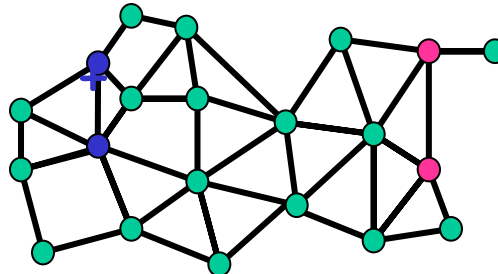
## Key Insight/Underlying Fundamental Principle

Unlabeled data useful if we have a bias/belief not only about the form of the target, but also about its relationship with the underlying data distribution.

E.g., "large margin separator"  
[Joachims '99]

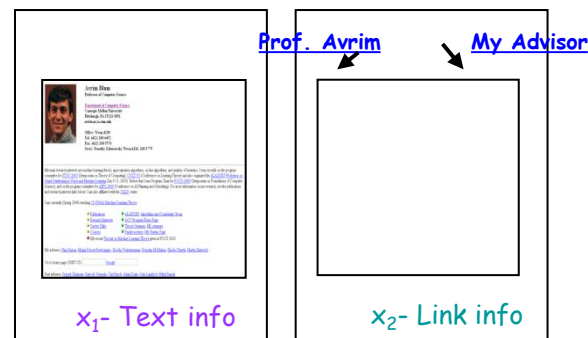


Similarity based  
("small cut")  
[B&C01], [ZGL03]



"self-consistent rules" [Blum & Mitchell '98]

$$\mathbf{x} = \langle \mathbf{x}_1, \mathbf{x}_2 \rangle \quad h_1(\mathbf{x}_1) = h_2(\mathbf{x}_2)$$



- Unlabeled data can help reduce search space or re-order the fns in the search space according to our belief, biasing the search towards fns satisfying the belief (which becomes concrete once we see unlabeled data).

# A General Discriminative Model for SSL

[BalcanBlum, COLT 2005; JACM 2010]

As in PAC/SLT, discuss algorithmic and sample complexity issues.

Analyze fundamental sample complexity aspects:

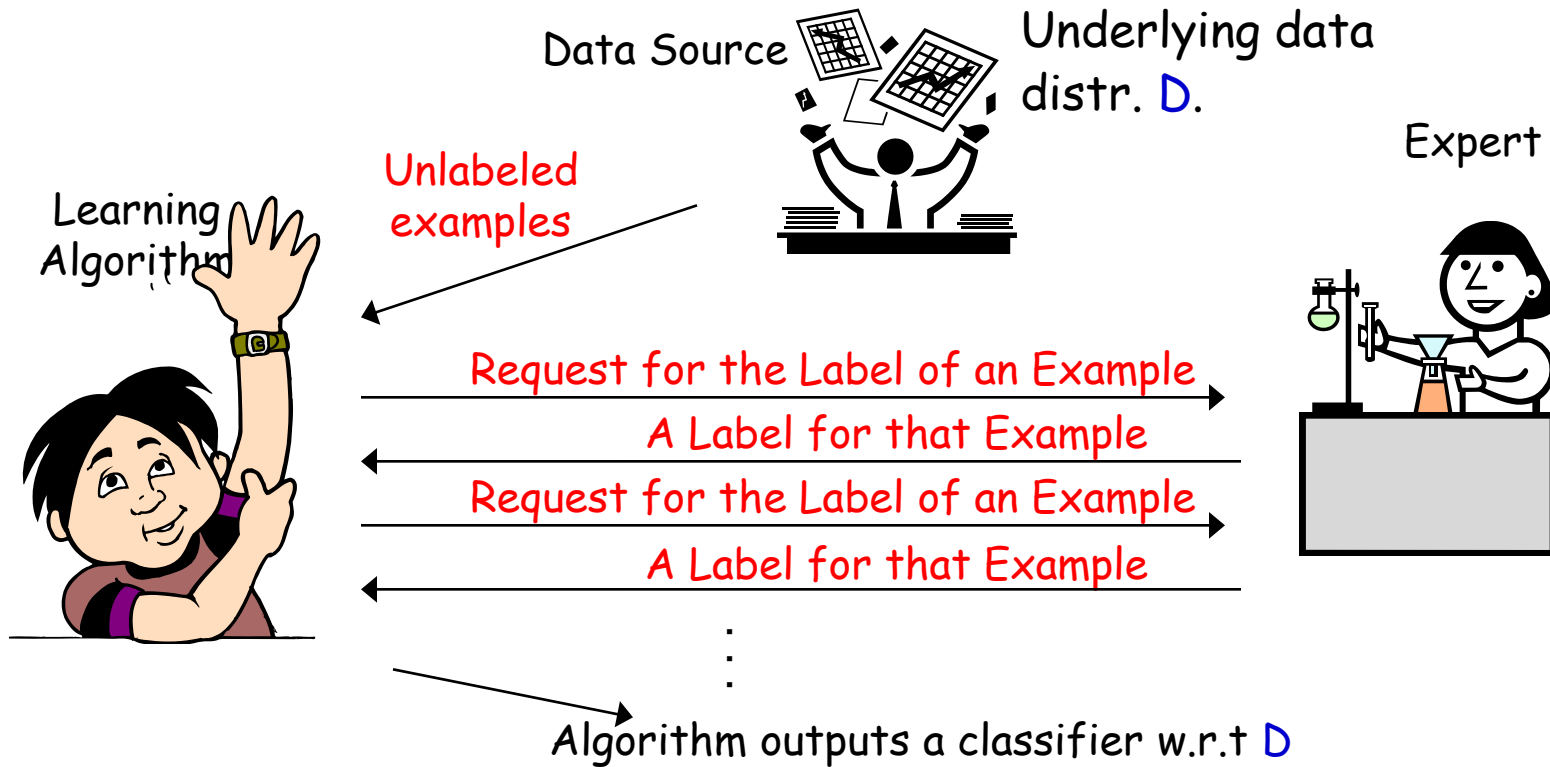
- How much unlabeled data is needed.
  - depends both on complexity of  $H$  and of compatibility notion.
- Ability of unlabeled data to reduce #of labeled examples.
  - compatibility of the target, helpfulness of the distrib.
- Survey on "Semi-Supervised Learning" (Jerry Zhu, 2010) explains the SSL techniques from this point of view.
- Note: the mixture method that Tom talked about on Feb 25th can be explained from this point of view too. See the Zhu survey.

# Active Learning

## Additional resources:

- Two faces of active learning. Sanjoy Dasgupta. 2011.
- Active Learning. Bur Settles. 2012.
- Active Learning. Balcan-Urner. Encyclopedia of Algorithms. 2015

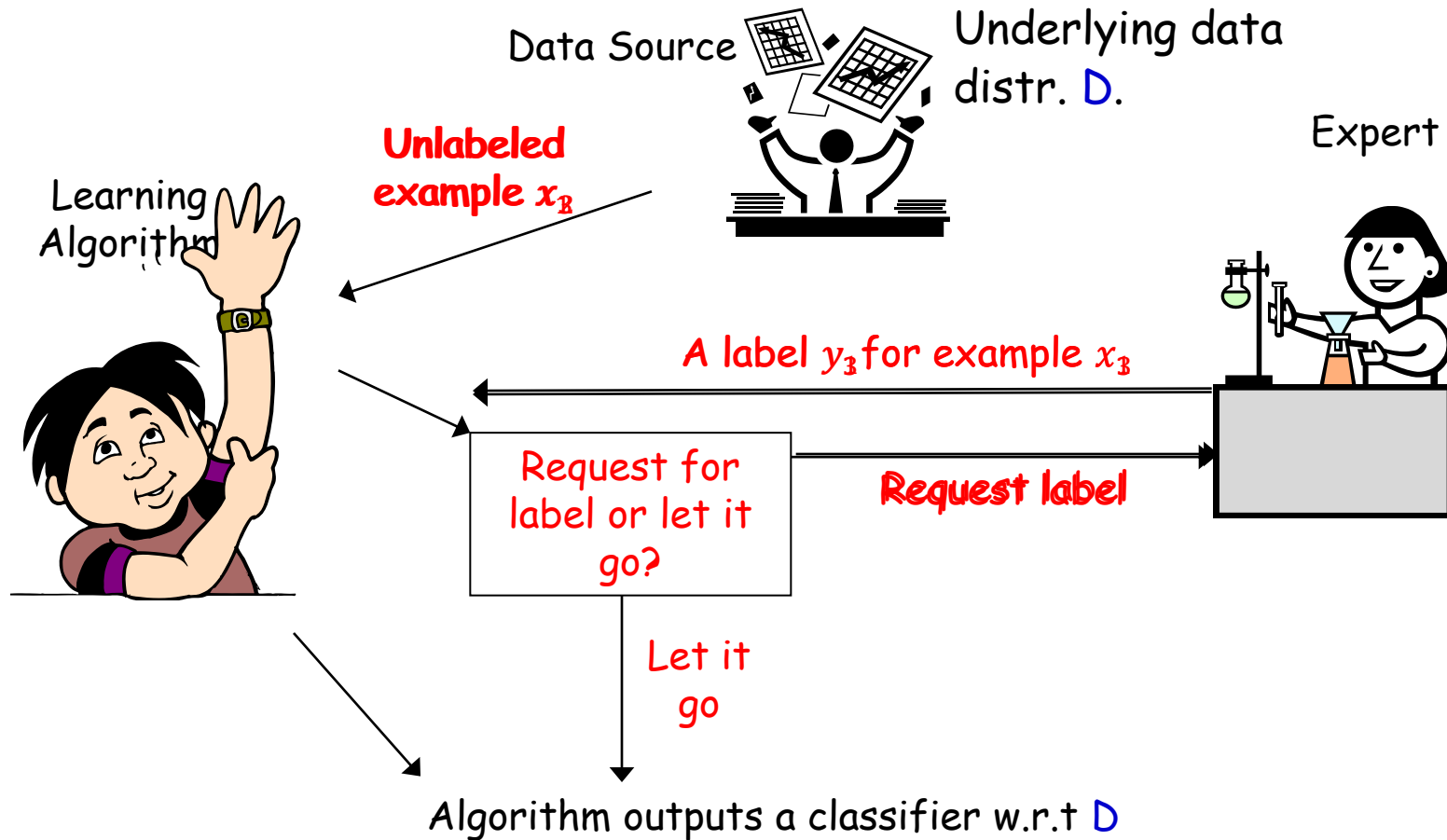
# Batch Active Learning



- Learner can choose specific examples to be labeled.
- Goal: use fewer labeled examples [pick **informative** examples to be labeled].



# Selective Sampling Active Learning



- **Selective sampling AL (Online AL)**: stream of unlabeled examples, when each arrives make a decision to ask for label or not.
- **Goal**: use fewer labeled examples [pick **informative** examples to be labeled].

# What Makes a Good Active Learning Algorithm?

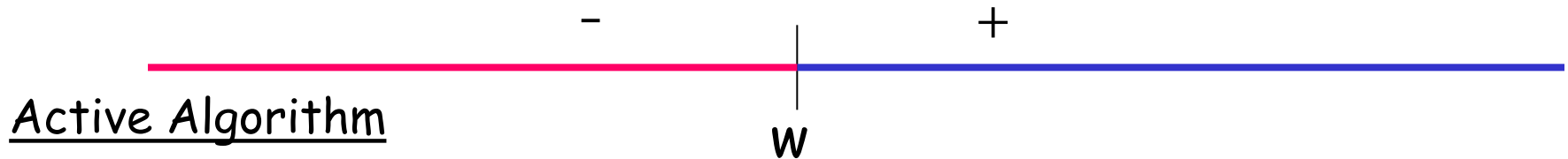
- Guaranteed to output a relatively good classifier for most learning problems.
- Doesn't make too many label requests.  
Hopefully a lot less than passive learning and SSL.
- Need to choose the label requests carefully, to get **informative** labels.

# Can adaptive querying really do better than passive/random sampling?

- YES! (sometimes)
- We often need far fewer labels for active learning than for passive.
- This is predicted by theory and has been observed in practice.

# Can adaptive querying help? [CAL92, Dasgupta04]

- Threshold fns on the real line:  $h_w(x) = 1(x \geq w)$ ,  $C = \{h_w: w \in \mathbb{R}\}$



## Active Algorithm

- Get  $N$  unlabeled examples
- How can we recover the correct labels with  $\ll N$  queries?
- Do binary search! Just need  $O(\log N)$  labels!



- Output a classifier consistent with the  $N$  inferred labels.

- $N = O(1/\epsilon)$  we are guaranteed to get a classifier of error  $\leq \epsilon$ .

Passive supervised:  $\Omega(1/\epsilon)$  labels to find an  $\epsilon$ -accurate threshold.

Active: only  $O(\log 1/\epsilon)$  labels. Exponential improvement.



# Common Technique in Practice

Uncertainty sampling in SVMs common and quite useful in practice. E.g., [Tong & Koller, ICML 2000; Jain, Vijayanarasimhan & Grauman, NIPS 2010; Schohn Cohn, ICML 2000]

## Active SVM Algorithm

- At any time during the alg., we have a “current guess”  $w_t$  of the separator: the max-margin separator of all labeled points so far.
- Request the label of the example closest to the current separator.

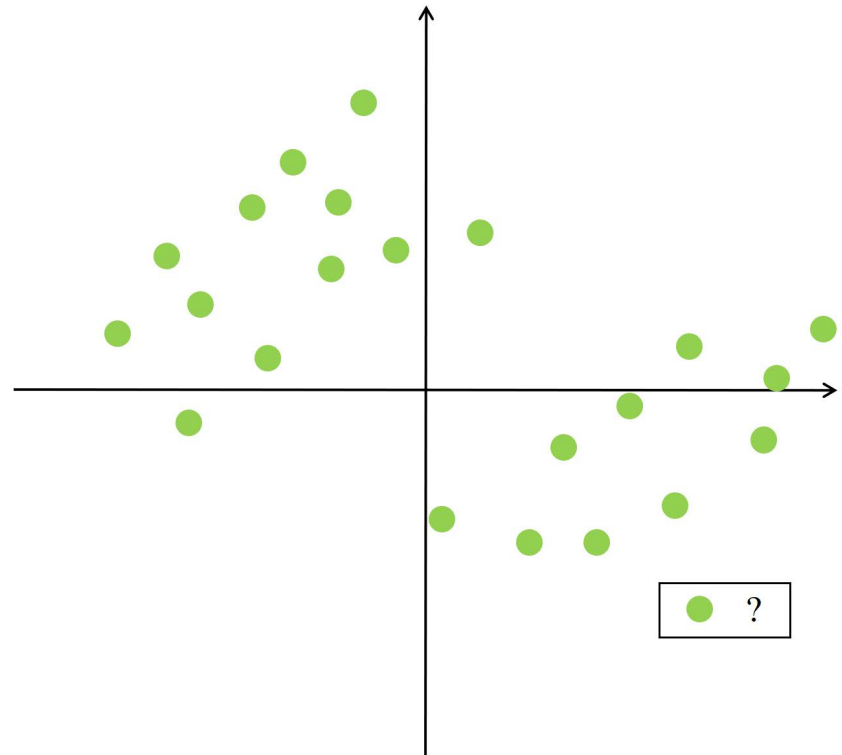
# Common Technique in Practice

**Active SVM** seems to be quite useful in practice.

[Tong & Koller, ICML 2000; Jain, Vijayanarasimhan & Grauman, NIPS 2010]

## Algorithm (batch version)

Input  $S_u = \{x_1, \dots, x_{m_u}\}$  drawn i.i.d from the underlying source  $D$



# Common Technique in Practice

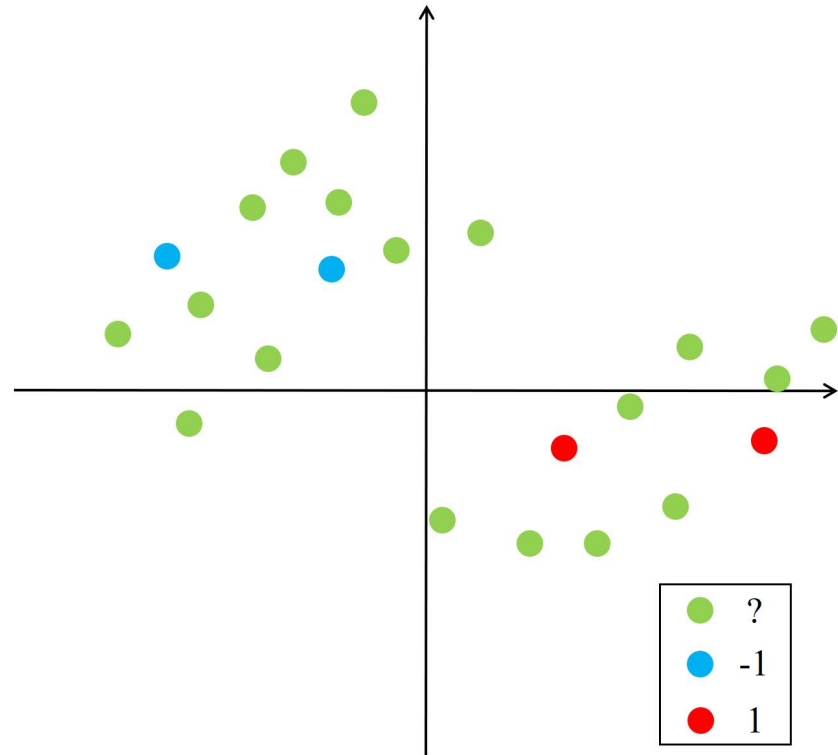
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Input  $S_u = \{x_1, \dots, x_{m_u}\}$  drawn i.i.d from the underlying source  $D$

Start: query for the labels of a few random  $x_i$ s.



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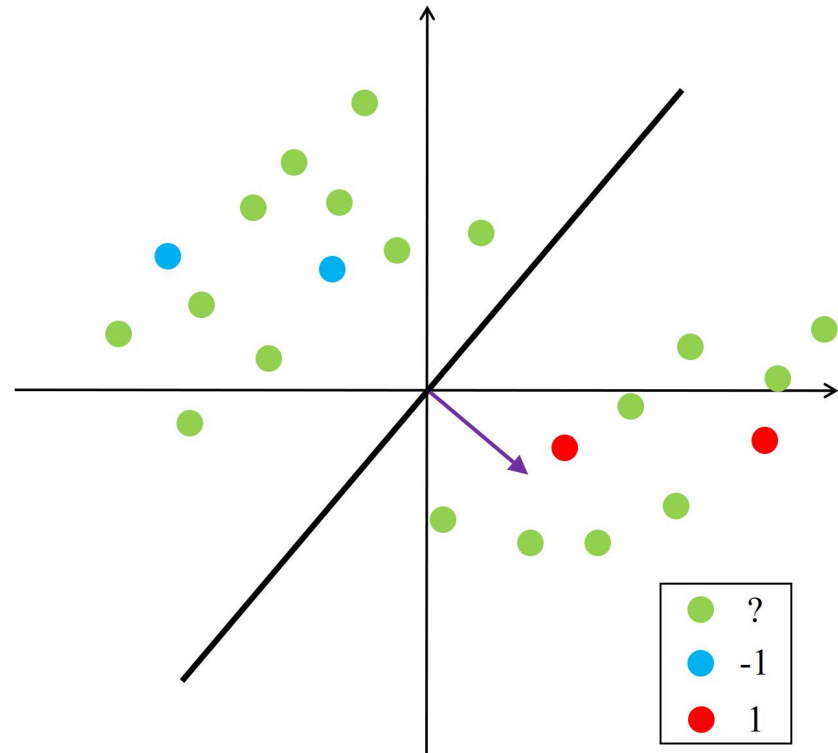
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Start: query for the labels of a few random  $x_i$ s.

For  $t = 1, \dots,$

- Find  $w_t$  the max-margin separator of all labeled points so far.





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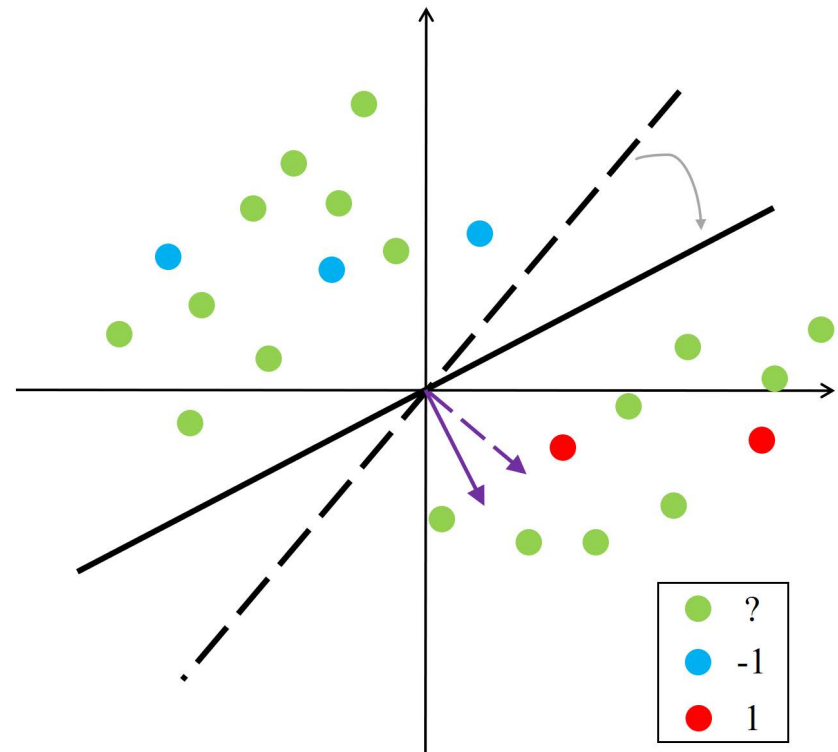
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Start: query for the labels of a few random  $x_i$ s.

For  $t = 1, \dots,$

- Find  $w_t$  the max-margin separator of all labeled points so far.
- Request the label of the example closest to the current separator: minimizing  $|x_i \cdot w_t|$ .

(highest uncertainty)

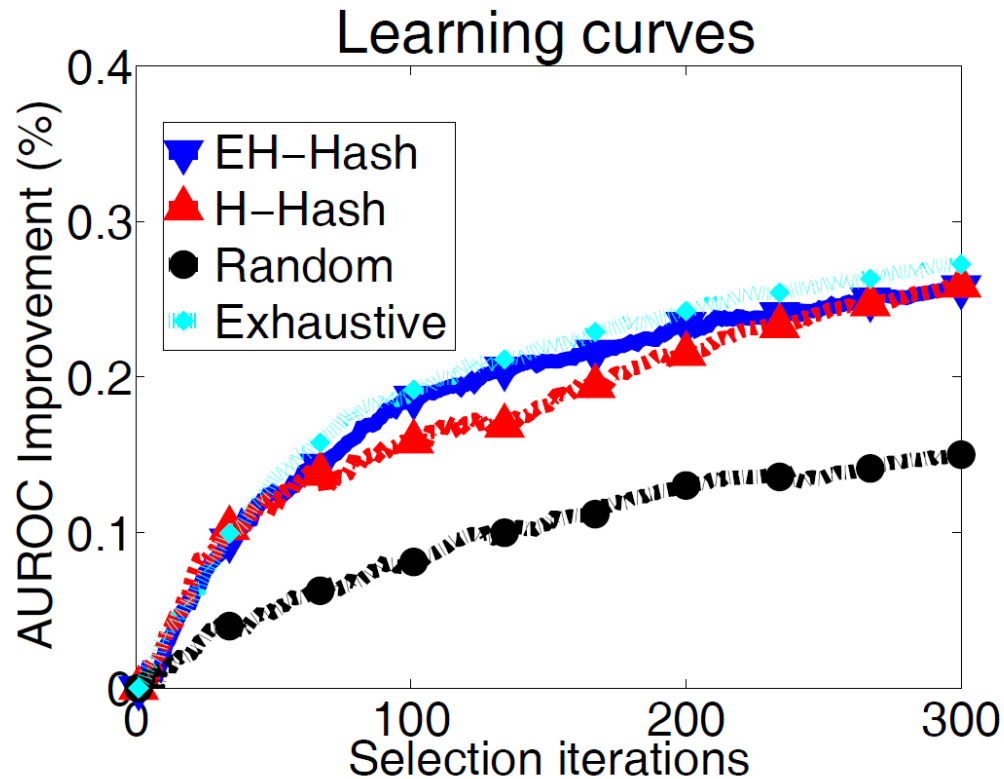


# Common Technique in Practice

Active SVM seems to be quite useful in practice.

E.g., Jain, Vijayanarasimhan & Grauman, NIPS 2010

Newsgroups dataset (20.000 documents from 20 categories)

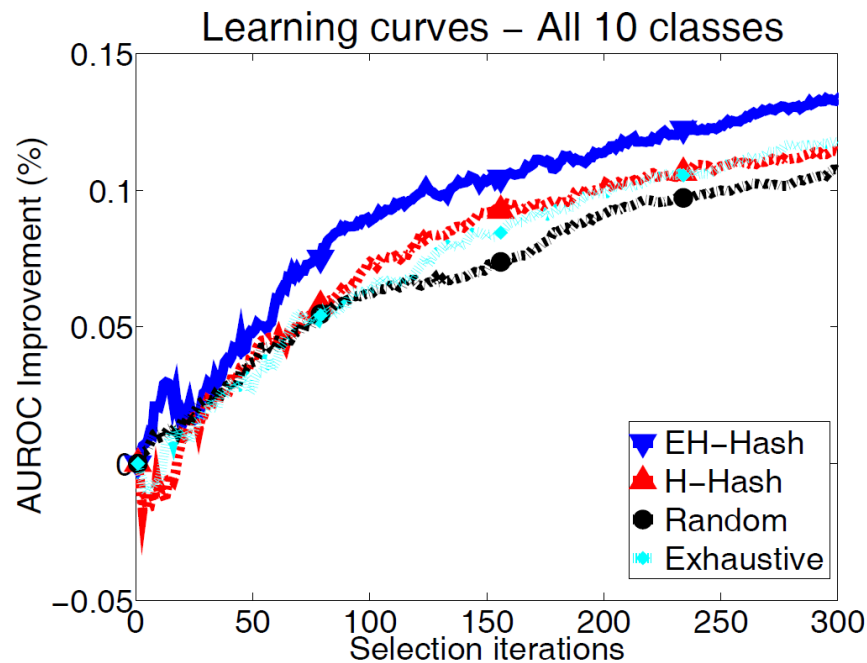


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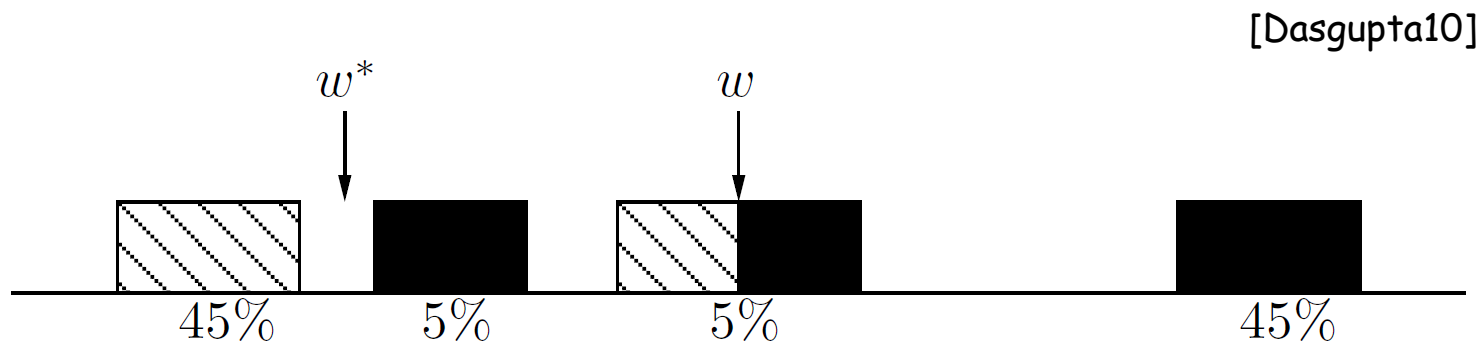
E.g., Jain, Vijayanarasimhan & Grauman, NIPS 2010

CIFAR-10 image dataset (60.000 images from 10 categories)



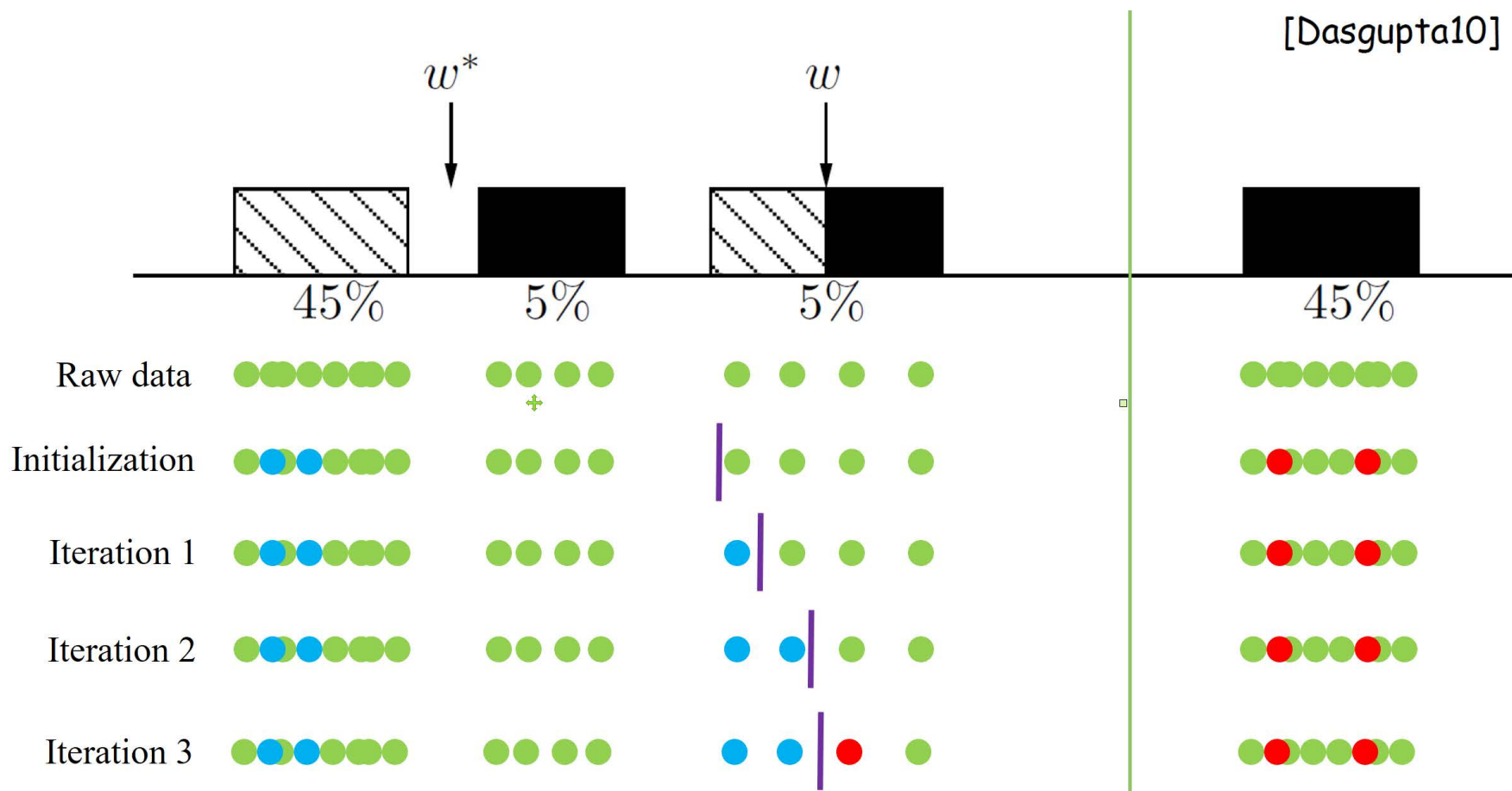
# Active SVM/Uncertainty Sampling

- Works sometimes....
- However, we need to be very very very careful!!!
  - Myopic, greedy technique can suffer from **sampling bias**.
  - A bias created because of the querying strategy; as time goes on the sample is less and less representative of the true data source.



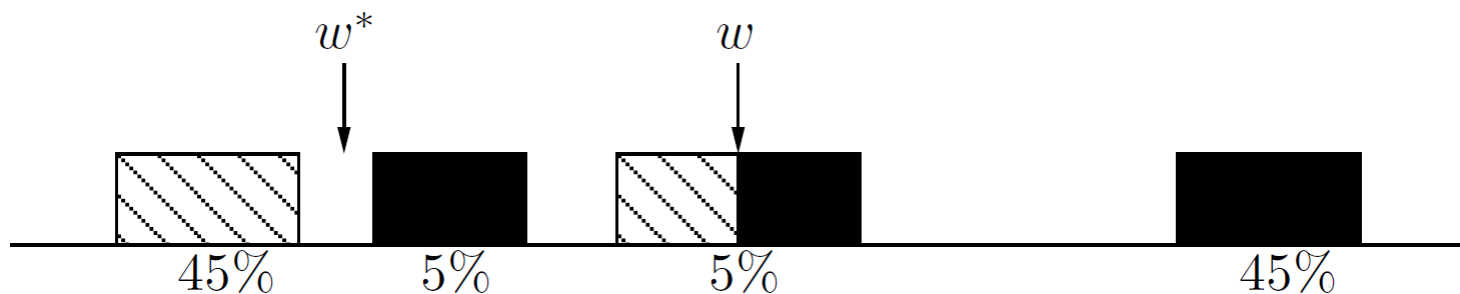
# Active SVM/Uncertainty Sampling

- Works sometimes....
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# Active SVM/Uncertainty Sampling

- Works sometimes....
- **However, we need to be very very careful!!!**
  - Myopic, greedy technique can suffer from **sampling bias**.
  - Bias created because of the querying strategy; as time goes on the sample is less and less representative of the true source.
  - Observed in practice too!!!!
- **Main tension:** want to choose informative points, but also want to guarantee that the classifier we output does well on true random examples from the underlying distribution.



# Safe Active Learning Schemes

Disagreement Based Active Learning

Hypothesis Space Search

[CAL92] [BBL06]

[Hanneke'07, DHM'07, Wang'09, Fridman'09, Kolt10, BHW'08, BHLZ'10, H'10, Ailon'12, ...]

# Version Spaces

- $X$  - feature/instance space; distr.  $D$  over  $X$ ;  $c^*$  target fnc
- Fix hypothesis space  $H$ .

**Definition (Mitchell'82)** Assume realizable case:  $c^* \in H$ .

Given a set of labeled examples  $(x_1, y_1), \dots, (x_{m_1}, y_{m_1}), y_i = c^*(x_i)$

**Version space of  $H$ :** part of  $H$  consistent with labels so far.

I.e.,  $h \in VS(H)$  iff  $h(x_i) = c^*(x_i) \forall i \in \{1, \dots, m_1\}$ .



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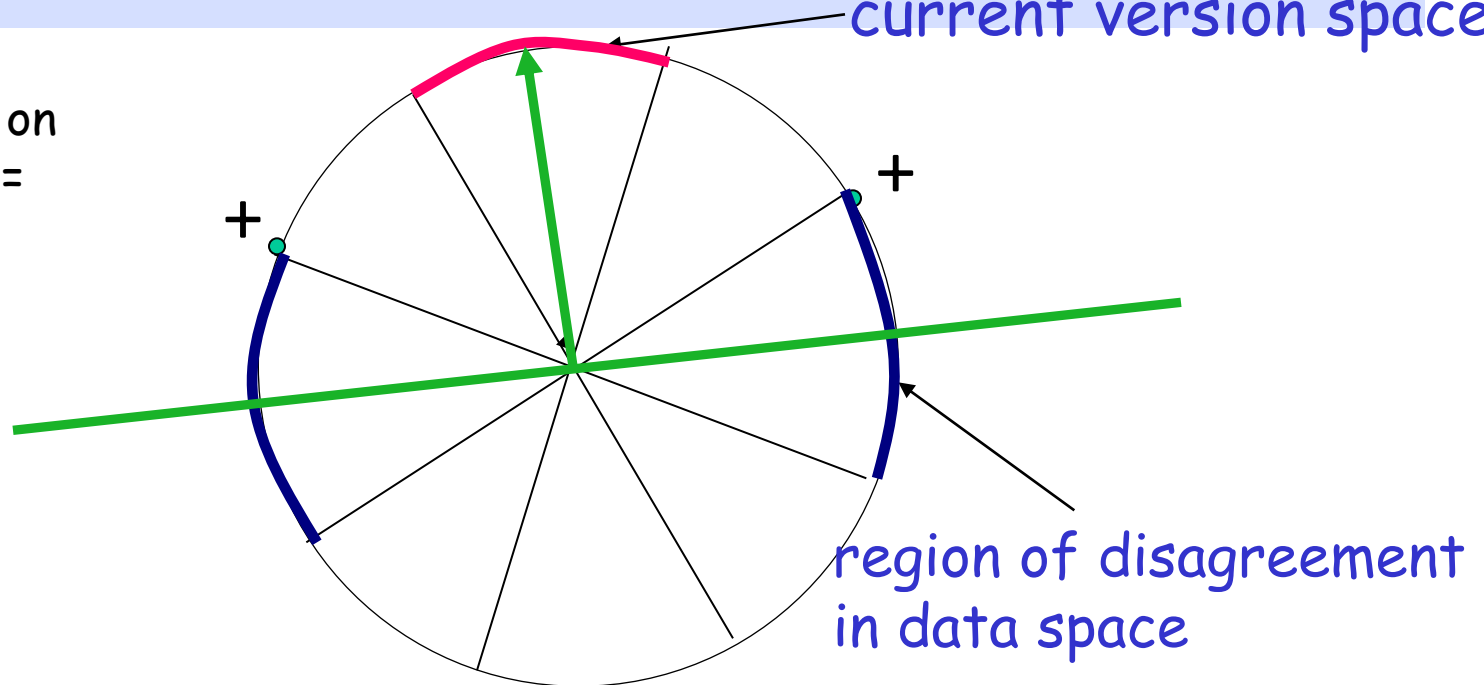
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**Version space of  $H$ :** part of  $H$  consistent with labels so far.

current version space

E.g.: data lies on circle in  $\mathbb{R}^2$ ,  $H$  = homogeneous linear seps.



# Version Spaces. Region of Disagreement

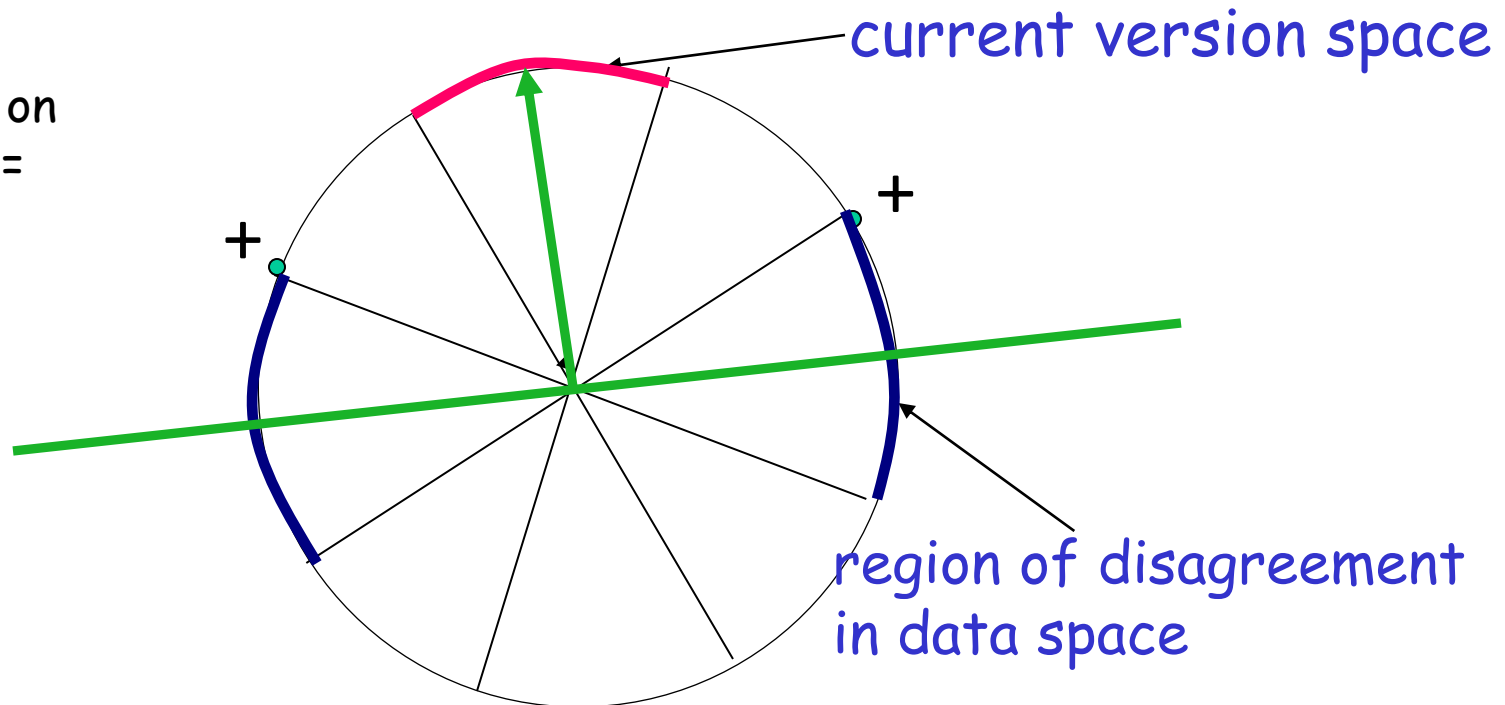
## Definition (CAL'92)

**Version space**: part of  $H$  consistent with labels so far.

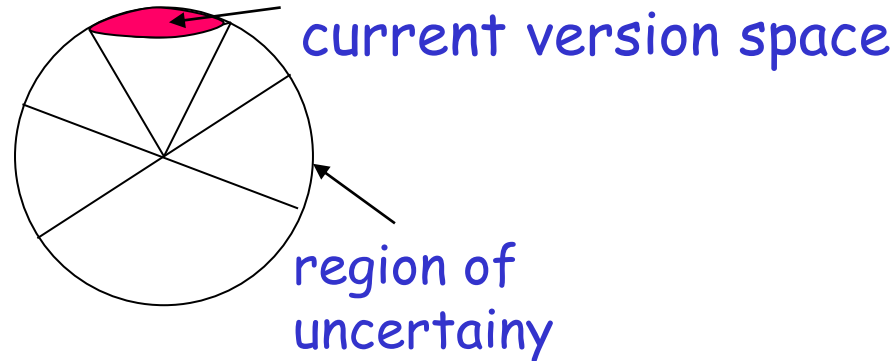
**Region of disagreement** = part of data space about which there is still some uncertainty (i.e. disagreement within version space)

$x \in X, x \in \text{DIS}(\text{VS}(H))$  iff  $\exists h_1, h_2 \in \text{VS}(H), h_1(x) \neq h_2(x)$

E.g.: data lies on circle in  $\mathbb{R}^2$ ,  $H$  = homogeneous linear sep.



# Disagreement Based Active Learning [CAL92]



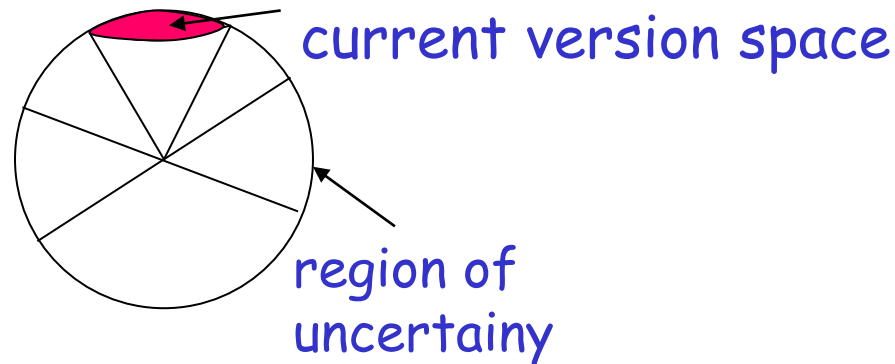
## Algorithm:

Pick a few points at random from the current region of uncertainty and query their labels.

Stop when region of uncertainty is small.

**Note:** it is active since we do not waste labels by querying in regions of space we are certain about the labels.

# Disagreement Based Active Learning [CAL92]



## Algorithm:

Query for the labels of a few random  $x_i$ s.

Let  $H_1$  be the current version space.

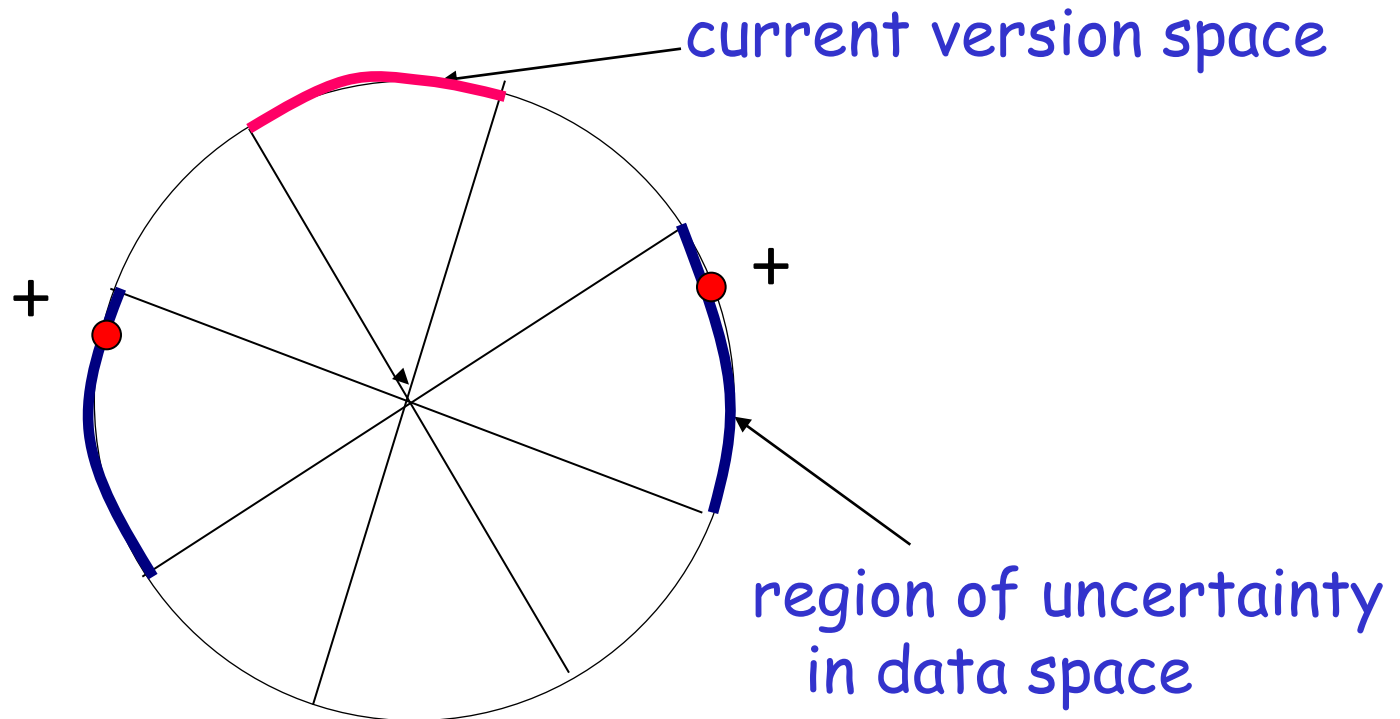
For  $t = 1, \dots,$

Pick a few points at random from the current region of disagreement  $\text{DIS}(H_t)$  and query their labels.

Let  $H_{t+1}$  be the new version space.

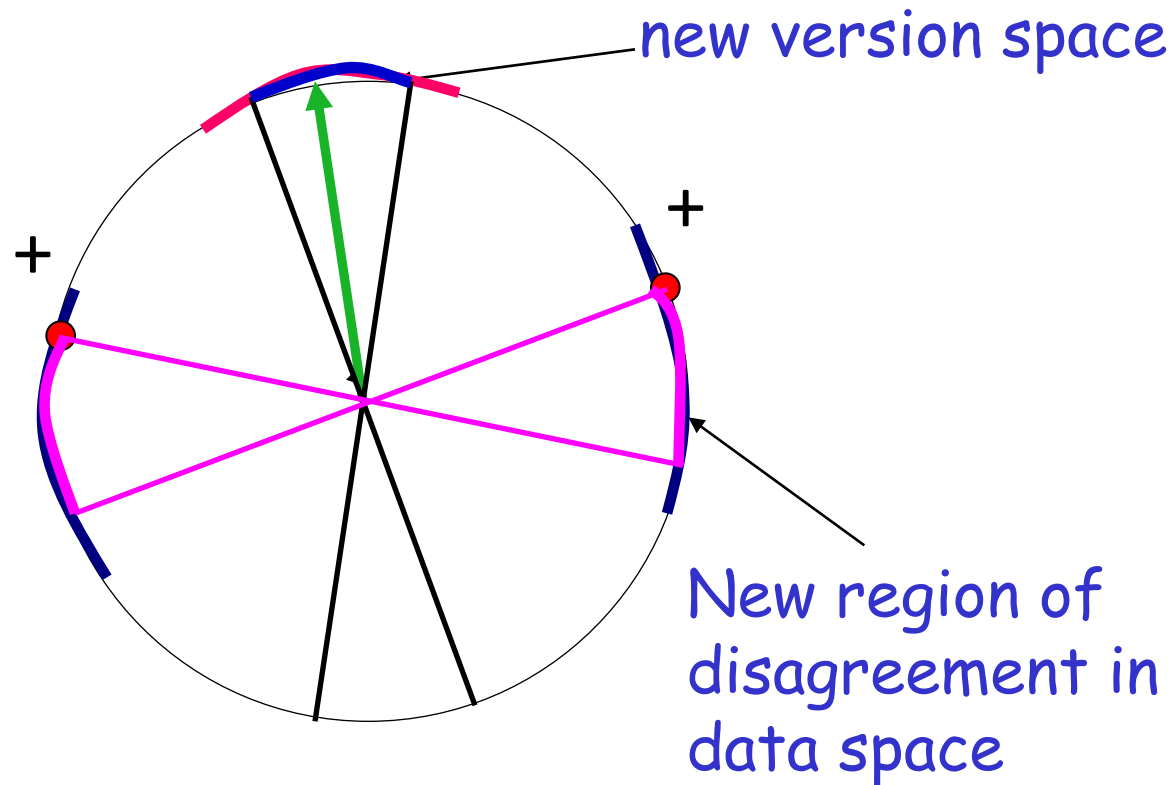
# Region of uncertainty [CAL92]

- Current **version space**: part of  $C$  consistent with labels so far.
- "**Region of uncertainty**" = part of data space about which there is still some uncertainty (i.e. disagreement within version space)



# Region of uncertainty [CAL92]

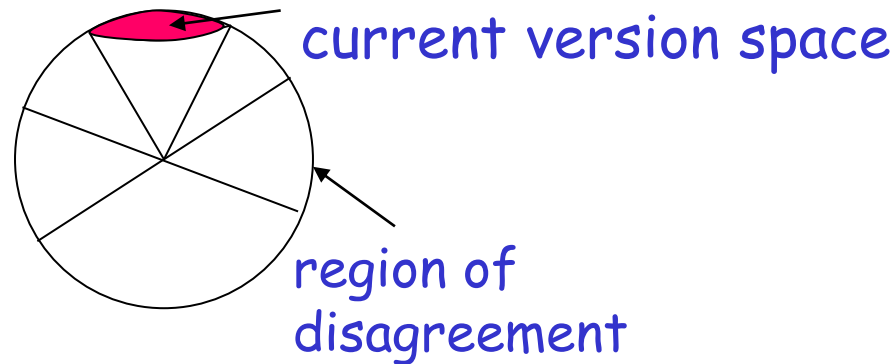
- Current **version space**: part of  $C$  consistent with labels so far.
- "**Region of uncertainty**" = part of data space about which there is still some uncertainty (i.e. disagreement within version space)





How about the agnostic case  
where the target might not  
belong the  $H$ ?

# $A^2$ Agnostic Active Learner [BBL'06]



## Algorithm:

Let  $H_1 = H$ .

For  $t = 1, \dots,$

- Pick a few points at random from the current region of disagreement  $\text{DIS}(H_t)$  and query their labels.
- Throw out hypothesis if you are statistically confident they are suboptimal.

Careful use of generalization bounds;  
Avoid the sampling bias!!!!



# The DHN Agnostic Active Learner [DHN'07]

$S = \emptyset$  (points with inferred labels)  
 $T = \emptyset$  (points with queried labels)  
For  $t = 1, 2, \dots$ :  
    Receive  $x_t$   
    If  $(h_{+1} = \text{learn}(S \cup \{(x_t, +1)\}, T))$  fails: Add  $(x_t, -1)$  to  $S$  and break  
    If  $(h_{-1} = \text{learn}(S \cup \{(x_t, -1)\}, T))$  fails: Add  $(x_t, +1)$  to  $S$  and break  
    If  $\text{err}(h_{-1}, S \cup T) - \text{err}(h_{+1}, S \cup T) > \Delta_t$ : Add  $(x_t, +1)$  to  $S$  and break  
    If  $\text{err}(h_{+1}, S \cup T) - \text{err}(h_{-1}, S \cup T) > \Delta_t$ : Add  $(x_t, -1)$  to  $S$  and break  
    Request  $y_t$  and add  $(x_t, y_t)$  to  $T$

Figure 16: The DHM selective sampling algorithm. Here,  $\text{err}(h, A) = (1/|A|) \sum_{(x,y) \in A} 1(h(x) \neq y)$ . A possible setting for  $\Delta_t$  is shown in Equation 1. At any time, the current hypothesis is  $\text{learn}(S, T)$ .

$\text{learn}(A, B)$  returns a hypothesis  $h \in \mathcal{H}$  consistent with  $A$ , and with minimum error on  $B$ . If there is no hypothesis consistent with  $A$ , a failure flag is returned.

$$\Delta_t = \beta_t^2 + \beta_t \left( \sqrt{\text{err}(h_{+1}, S \cup T)} + \sqrt{\text{err}(h_{-1}, S \cup T)} \right), \quad \beta_t = C \sqrt{\frac{d \log t + \log(1/\delta)}{t}}$$

# When Active Learning Helps. Agnostic case

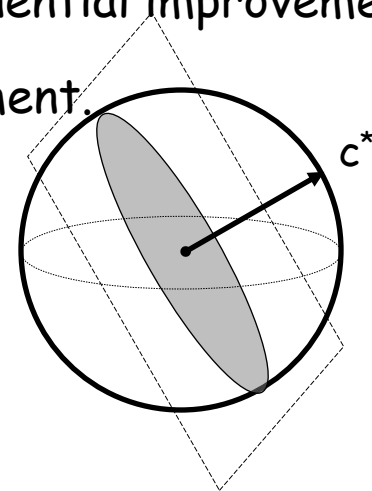
$A^2$  the first algorithm which is robust to noise.

[Balcan, Beygelzimer, Langford, ICML'06] [Balcan, Beygelzimer, Langford, JCSS'08]

"Region of disagreement" style: Pick a few points at random from the current region of disagreement, query their labels, throw out hypothesis if you are statistically confident they are suboptimal.

Guarantees for  $A^2$  [BBL'06,'08]:

- It is **safe** (never worse than passive learning) & exponential improvements.
- $C$  - thresholds, low noise, exponential improvement.
- $C$  - homogeneous linear separators in  $\mathbb{R}^d$ ,  
 $D$  - uniform, low noise, only  $d^2 \log(1/\epsilon)$  labels.



A lot of subsequent work.

[Hanneke'07, DHM'07, Wang'09, Fridman'09, Kolt10, BHW'08, BHLZ'10, H'10, Ailon'12, ...]

# General guarantees for $A^2$ Agnostic Active Learner

"Disagreement based": Pick a few points at random from the current region of uncertainty, query their labels, throw out hypothesis if you are statistically confident they are suboptimal. [BBL'06]

How quickly the region of disagreement collapses as we get closer and closer to optimal classifier

Guarantees for  $A^2$  [Hanneke'07]:

Disagreement coefficient  $\theta_{c^*} = \sup_{r \geq \eta + \epsilon} \frac{\Pr(DIS(B(c^*, r)))}{r}$

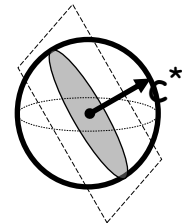
Theorem

$$m = \left(1 + \frac{\eta^2}{\epsilon^2}\right) VCdim(C) \theta_{c^*}^2 \log\left(\frac{1}{\epsilon}\right)$$

labels are sufficient s.t. with prob.  $\geq 1 - \delta$  output  $h$  with  $err(h) \leq \eta + \epsilon$ .

Realizable case:  $m = VCdim(C) \theta_{c^*} \log\left(\frac{1}{\epsilon}\right)$

Linear Separators, uniform distr.:  $\theta_{c^*} = \sqrt{d}$



# Disagreement Based Active Learning

"Disagreement based " algos: query points from current region of disagreement, throw out hypotheses when statistically confident they are suboptimal.

- Generic (any class), adversarial label noise.
- Computationally efficient for classes of small VC-dimension

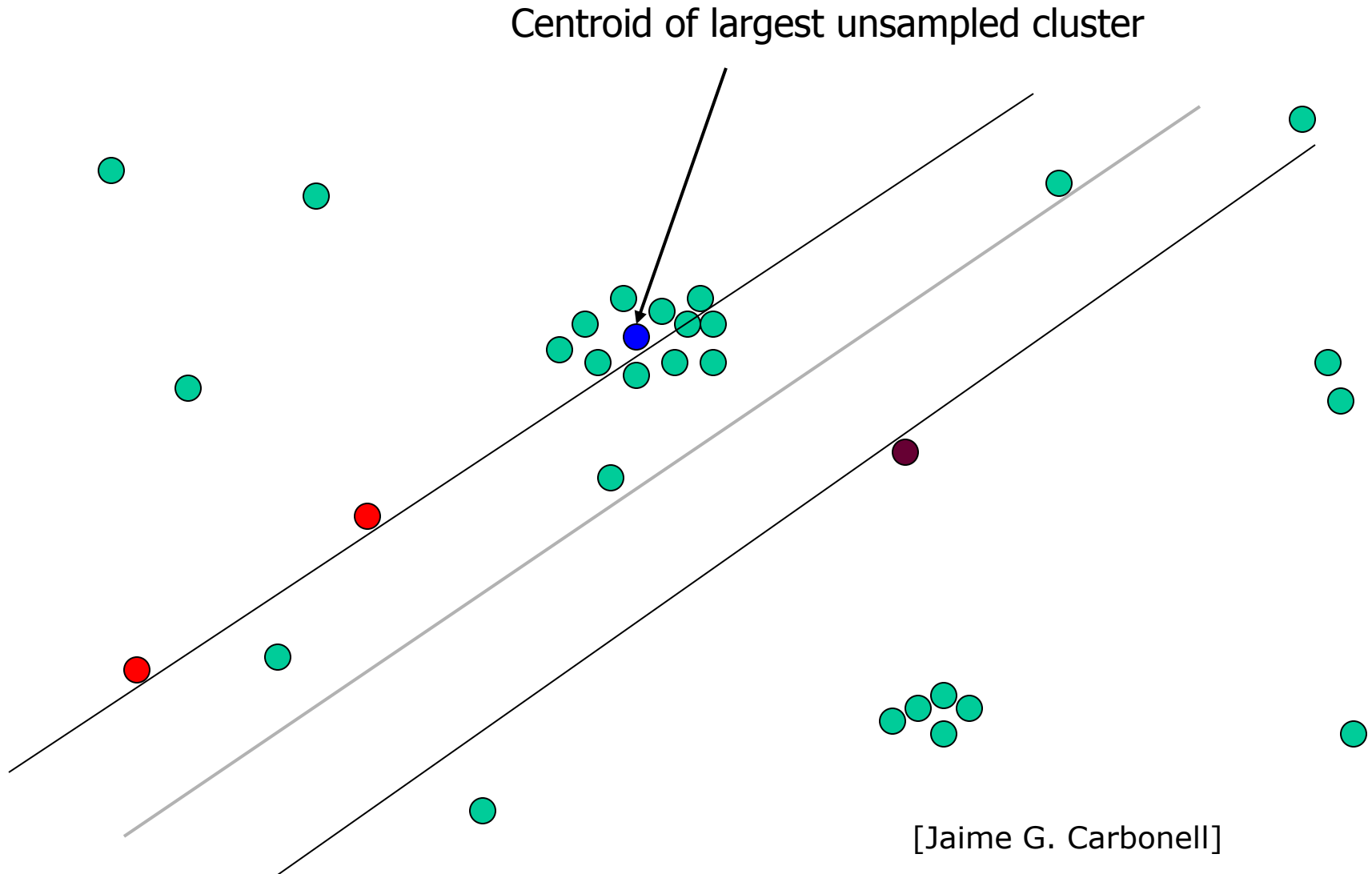
Still, could be suboptimal in label complex & computationally inefficient in general.

Lots of subsequent work trying to make is more efficient computationally and more aggressive too: [Hanneke07, DasguptaHsuMontleoni'07, Wang'09 , Fridman'09, Koltchinskii10, BHW'08, BeygelzimerHsuLangfordZhang'10, Hsu'10, Ailon'12, ...]

## Other Interesting AL Techniques used in Practice

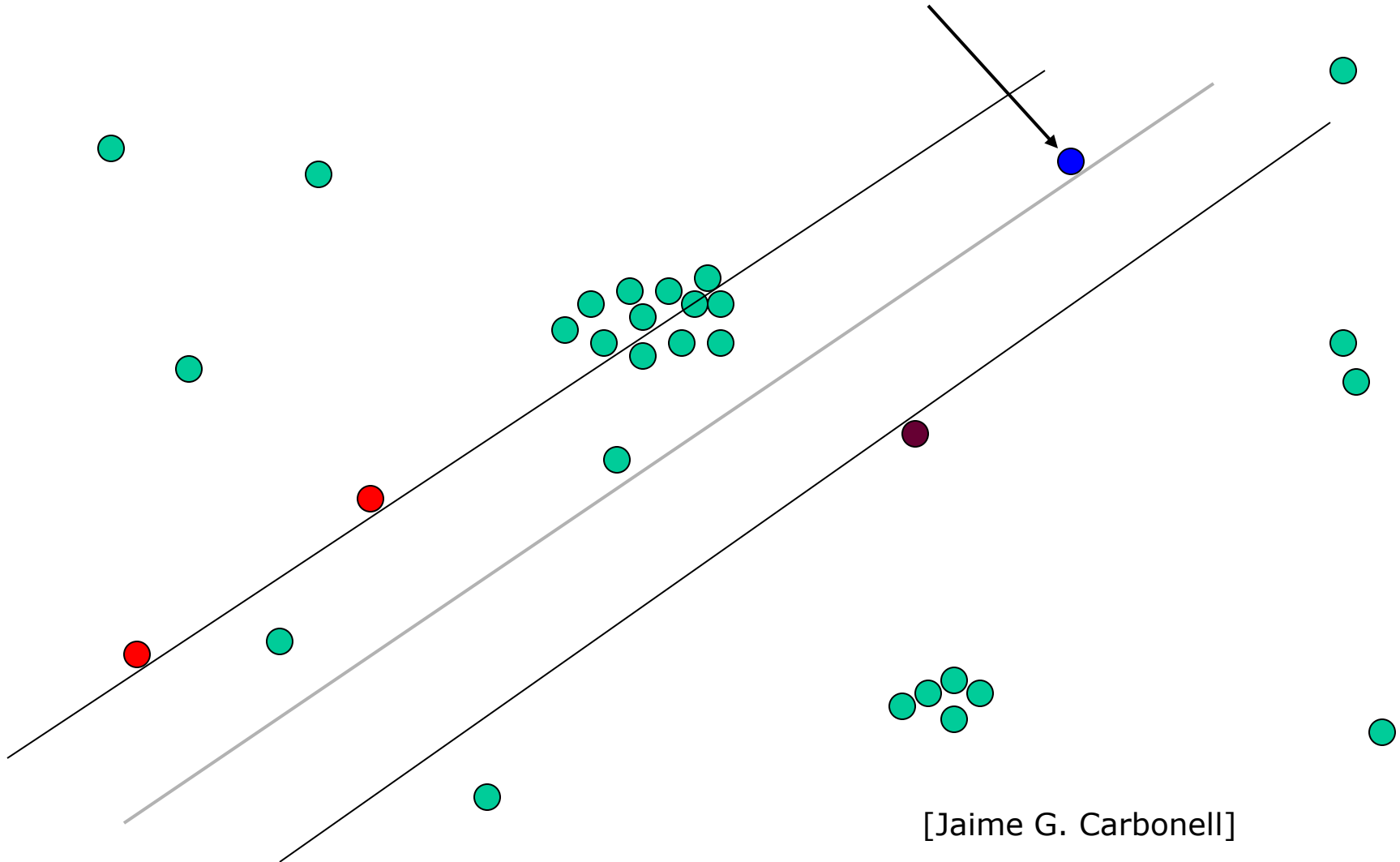
Interesting open question to analyze  
under what conditions they are successful.

# Density-Based Sampling



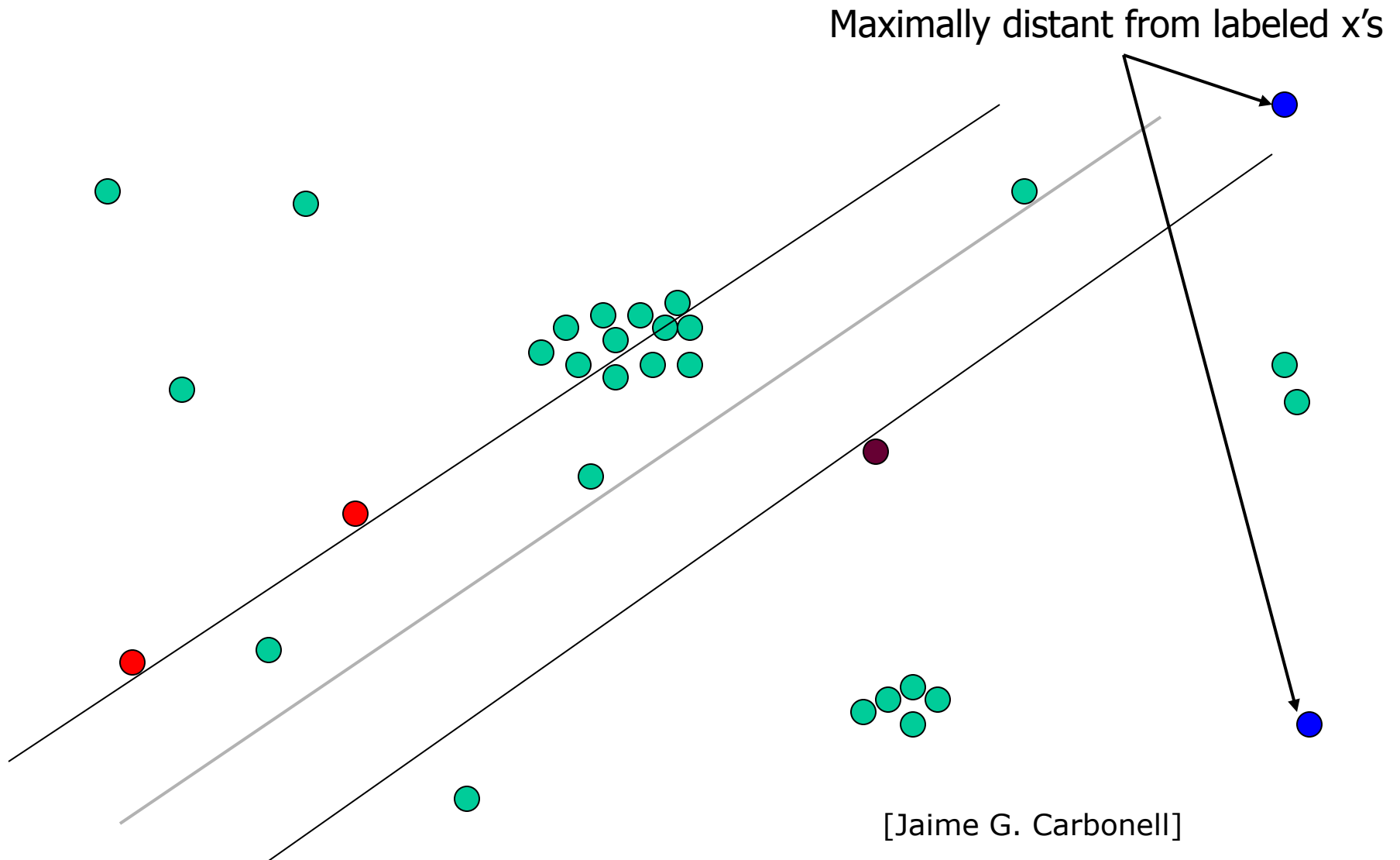
# Uncertainty Sampling

Closest to decision boundary (Active SVM)



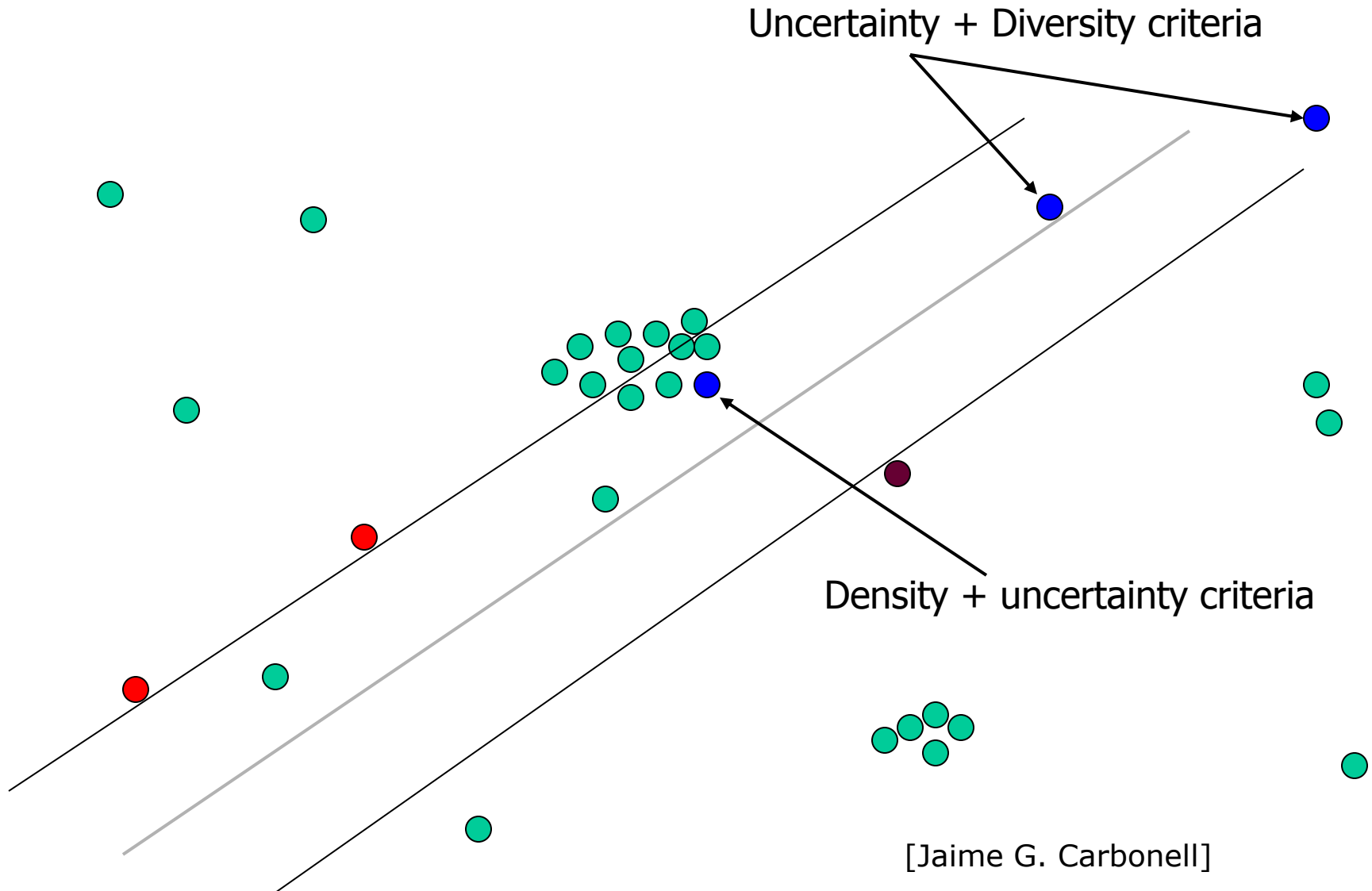
[Jaime G. Carbonell]

# Maximal Diversity Sampling





# Ensemble-Based Possibilities



# Graph-based Active and Semi-Supervised Methods

# Graph-based Methods

- Assume we are given a pairwise similarity fnc and that very similar examples probably have the same label.
- If we have a lot of labeled data, this suggests a Nearest-Neighbor type of algorithm.
- If you have a lot of **unlabeled** data, perhaps can use them as “stepping stones”.



not similar

E.g., handwritten digits [Zhu07]:

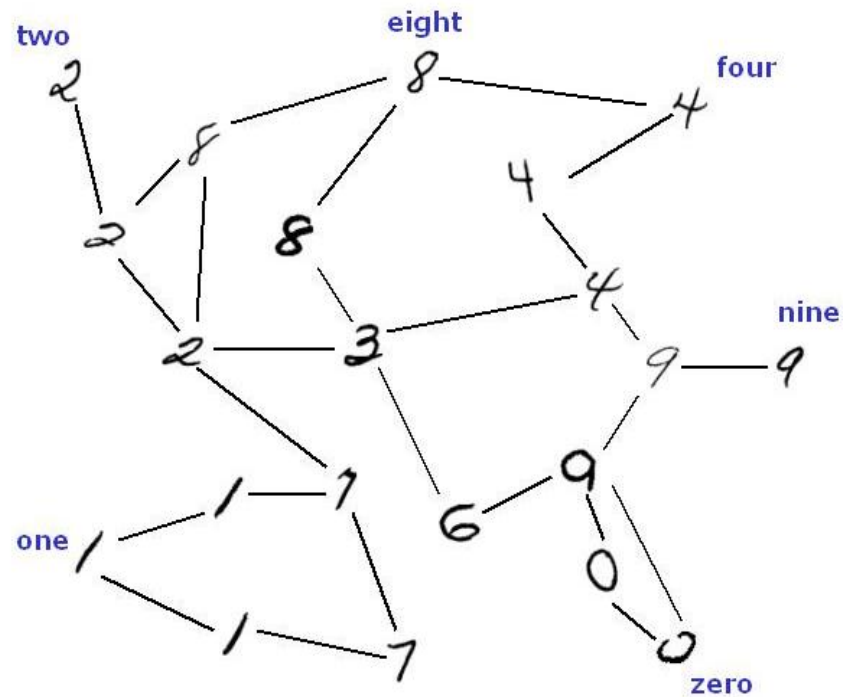


‘indirectly’ similar  
with stepping stones

# Graph-based Methods

**Idea:** construct a graph with edges between very similar examples.

Unlabeled data can help “glue” the objects of the same class together.

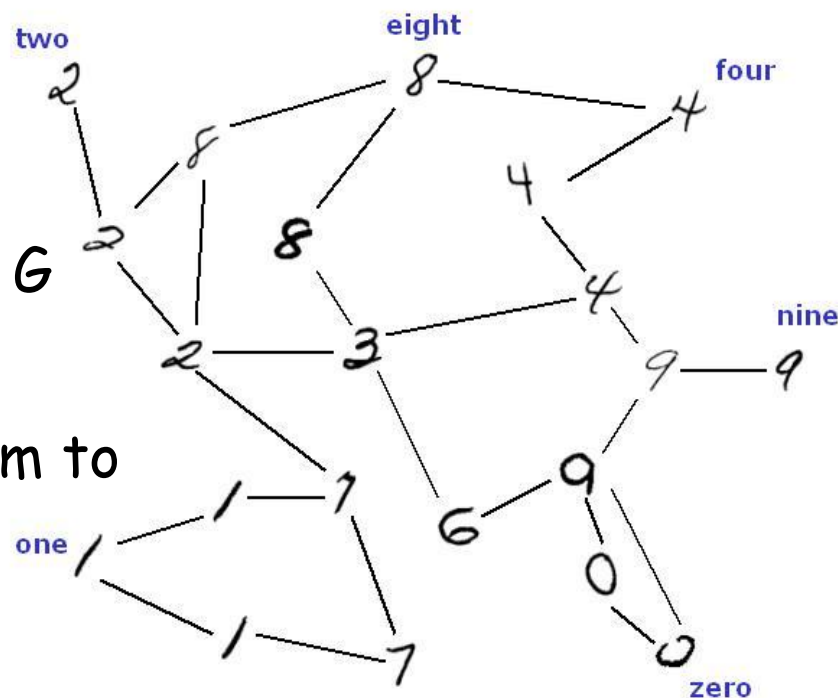


# Graph-based Methods

Often, **transductive approach**. (Given  $L + U$ , output predictions on  $U$ ). Are allowed to output any labeling of  $L \cup U$ .

## Main Idea:

- Construct graph  $G$  with edges between very similar examples.
- Might have also glued together in  $G$  examples of different classes.
- Run a graph partitioning algorithm to separate the graph into pieces.



Several methods:

- Minimum/Multiway cut [Blum&Chawla01]
- Minimum "soft-cut" [ZhuGhahramaniLafferty'03]
- Spectral partitioning
- ...

# SSL using soft cuts

[ZhuGhahramaniLafferty'03]

Solve for label function  $f(x) \in [0,1]$  to minimize:

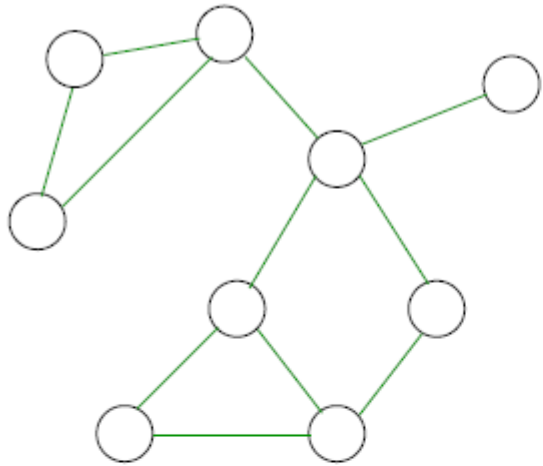
$$J(f) = \underbrace{\sum_{edges(i,j)} w_{ij} (f(x_i) - f(x_j))^2}_{\text{Similar nodes get similar labels (weighted similarity)}} + \underbrace{\sum_{x_i \in L} \lambda (f(x_i) - y_i)^2}_{\text{Agreement with labels (agreement not strictly enforces)}}$$

Similar nodes get  
similar labels  
(weighted similarity)

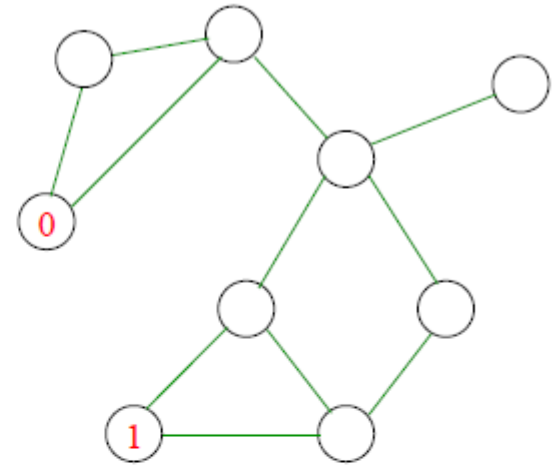
Agreement with labels  
(agreement not strictly enforces)

# Active learning with label propagation

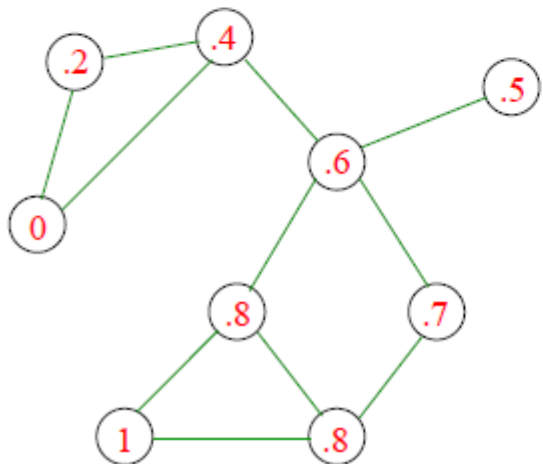
(1) Build neighborhood graph



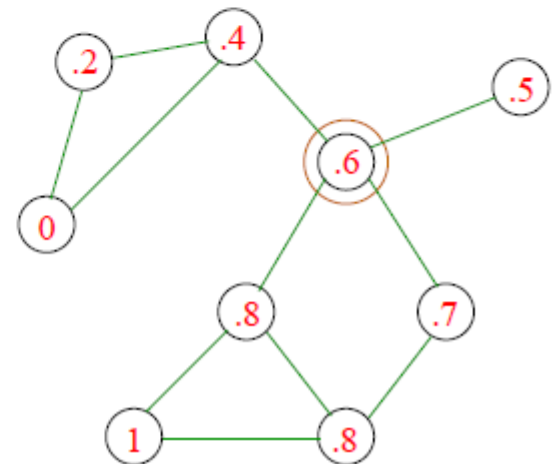
(2) Query some random points



(3) Propagate labels (using soft-cuts)



(4) Make query and go to (3)



How to choose  
which node to  
query?

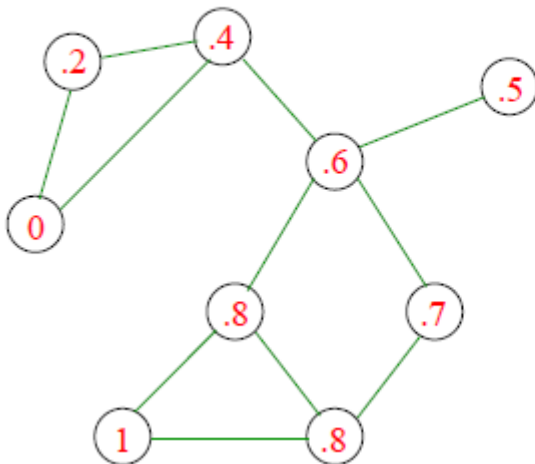
# Active learning with label propagation

One natural idea: query the most uncertain point.

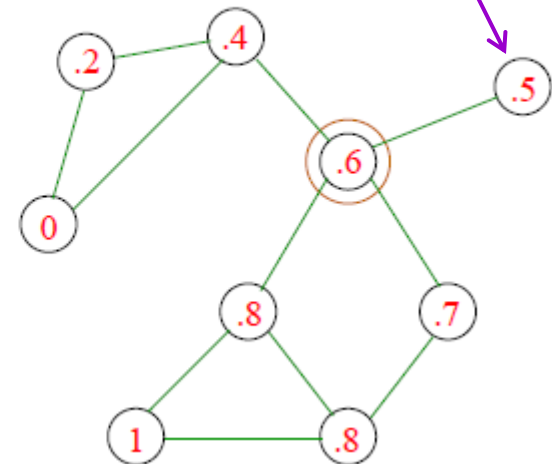
But this has only one edge. Query won't have much impact!

(even worse: a completely isolated node)

(3) Propagate labels (using soft-cuts)



(4) Make query and go to (3)



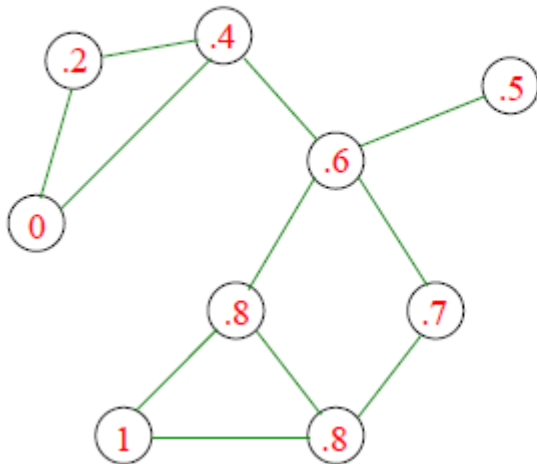


# Active learning with label propagation

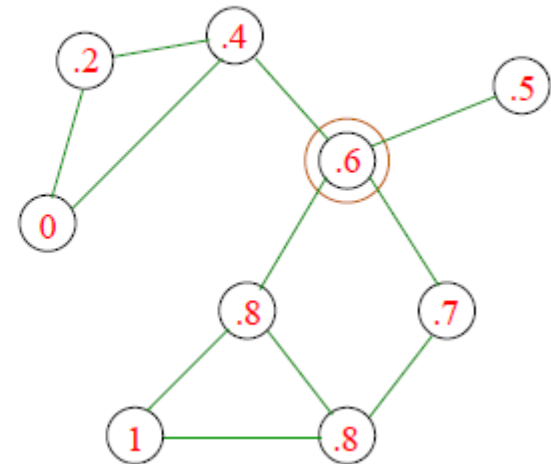
Instead, use a 1-step-lookahead heuristic:

- For a node with label  $p$ , assume that querying will have prob  $p$  of returning answer 1,  $1 - p$  of returning answer 0.
- Compute "average confidence" after running soft-cut in each case:
$$p \frac{1}{n} \sum x_i \max(f_1(x_i), 1 - f_1(x_i)) + (1 - p) \frac{1}{n} \sum x_i \max(f_0(x_i), 1 - f_0(x_i))$$
- Query node s.t. this quantity is highest (you want to be more confident on average).

(3) Propagate labels (using soft-cuts)



(4) Make query and go to (3)



# Active Learning with Label Propagation in Practice

- Does well for Video Segmentation (Fathi-Balcan-Ren-Regh, BMVC 11).



# What You Should Know

- Active learning could be really helpful, could provide exponential improvements in label complexity (both theoretically and practically)!
- Common heuristics (e.g., those based on uncertainty sampling). Need to be very careful due to sampling bias.
- Safe Disagreement Based Active Learning Schemes.
  - Understand how they operate precisely in noise free scenarios.