# Homework 8

Due date:
May.21st, 2018
Turn in your homework in class

Rules:

- Please try to work on your own. Discussion is permissible, but identical submissions are unacceptable!
- Please show all intermediate steps: a correct solution without an explanation will get zero credit.
- Please submit on time. No late submission will be accepted.
- Please prepare your submission in English only. No Chinese submission will be accepted.

1. A load consisting of a $480\ \Omega$ resistor in parallel with a $\frac{5}{9}\ \mu\text{F}$ capacitor is connected across the terminals of a sinusoidal voltage source $v_g$, where $v_g = 240\cos 5000t$ V.

(1) What is the peak value of the instantaneous power delivered by the source?
(2) What is the peak value of the instantaneous power absorbed by the source?
(3) What is the average power delivered to the load?
(4) What is the reactive power delivered to the load?
(5) Does the load absorb or generate magnetizing vars?
(6) What is the power factor of the load?

Solution:

(1) $P = \frac{1}{2}\frac{(240)^2}{480} = 60\ \text{W}$

$$-\frac{1}{\omega C} = \frac{-9\times 10^6}{(5000)(5)} = -360\ \Omega$$

$$Q = \frac{1}{2}\frac{(240)^2}{(-360)} = -80\ \text{VAR}$$

$$p_{\max} = P + \sqrt{P^2 + Q^2} = 60 + \sqrt{(60)^2 + (80)^2} = 160\ \text{W (del)}$$

(2) $p_{\min} = 60 - \sqrt{60^2 + 80^2} = -40\ \text{W (abs)}$

(3) $P = 60\ \text{W}$ from (a)

(4) $Q = -80\ \text{VAR}$ from (a)

(5) generates, because $Q < 0$

(6) pf $= \cos(\theta_v - \theta_i)$

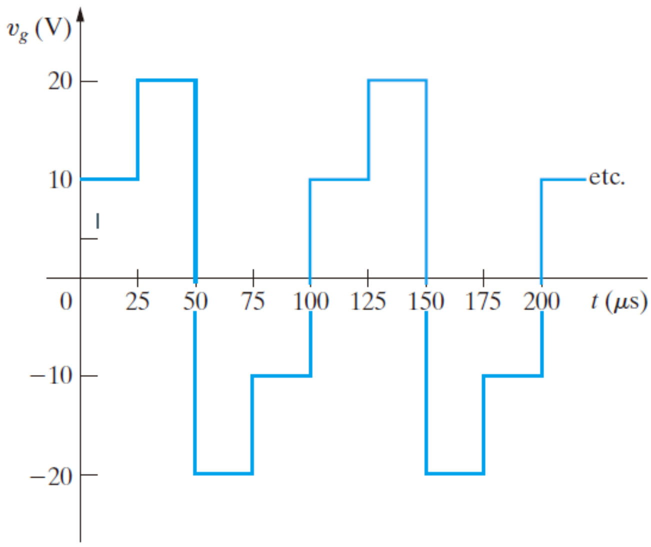
$$\mathbf{I} = \frac{240}{480} + \frac{240}{-j360} = 0.5 + j0.67 = 0.83\underline{/53.13^\circ}\ \text{A}$$

$$\therefore\ \text{pf} = \cos(0 - 53.13^\circ) = 0.6\ \text{leading}$$

2.

(1) Find the rms value of the periodic voltage shown in the figure below.

(2) If this voltage is applied to the terminals of a $4\Omega$ resistor, what is the average power dissipated in the resistor?




Solution:

(1) Area under one cycle of $v_g^2$:

$$A = (100)(25\times10^{-6}) + 400(25\times10^{-6}) + 400(25\times10^{-6}) + 100(25\times10^{-6})$$
$$= 1000(25\times10^{-6})$$

Mean value of $v_g^2$:

$$\text{M.V.} = \frac{A}{100\times10^{-6}} = \frac{1000(25\times10^{-6})}{100\times10^{-6}} = 250$$

$$\therefore\quad V_{\text{rms}} = \sqrt{250} = 15.81\,\text{V (rms)}$$

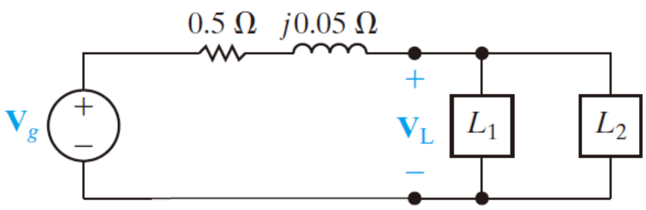
(2) $$P = \frac{V_{\text{rms}}^2}{R} = \frac{250}{4} = 62.5\,\text{W}$$

3. The two loads shown in the figure below can be described as follows: Load 1 absorbs an average power of $10\text{kW}$ and delivers $4\ \text{kVAR}$ of reactive power; Load 2 has an impedance of $(60+j80)\Omega$. The voltage at the terminals of the load is

   $1000\sqrt{2}\cos 100\pi t\ \text{V}$.

(1) Find the rms value of the source voltage.
(2) By how many microseconds is the load voltage out of phase with the source voltage?
(3) Does the load voltage lead or lag the source voltage? Explain your answer by calculation.

Solution:




(1) $$S_1 = 10{,}000 - j4000\,\text{VA}$$

$$S_2 = \frac{|\mathbf{V}_\text{L}|^2}{Z_2^*} = \frac{(1000)^2}{60 - j80} = 6 + j8\,\text{kVA}$$

$$S_1 + S_2 = 16 + j4\,\text{kVA}$$

$$1000\mathbf{I}_\text{L}^* = 16{,}000 + j4000; \qquad \therefore \quad \mathbf{I}_\text{L} = 16 - j4\,\text{A(rms)}$$

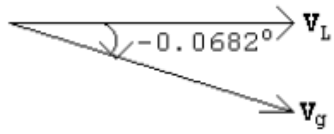
$$\mathbf{V}_g = \mathbf{V}_\text{L} + \mathbf{I}_\text{L}(0.5 + j0.05) = 1000 + (16 - j4)(0.5 + j0.05)$$

$$= 1008.2 - j1.2 = 1008.2\underline{/-0.0682^\circ}\ \text{Vrms}$$

(2) $$T = \frac{1}{f} = \frac{1}{50} = 20\,\text{ms}$$

$$\frac{-0.0682^\circ}{360^\circ} = \frac{t}{20\ \text{ms}}; \qquad \therefore \quad t = -3.79\,\mu\text{s}$$

(3) $\mathbf{V}_\text{L}$ leads $\mathbf{V}_g$ by $0.0682^\circ$ or $3.79\,\mu\text{s}$



4. A factory has an electrical load of $1600\mathrm{kW}$ at a lagging power factor of $0.8$. An additional variable power factor load is to be added to the factory. The new load will add $320\mathrm{kW}$ to the real power load of the factory. The power factor of added load is to be adjusted so that the overall power factor of the factory is $0.96$ lagging.

(1) Specify the reactive power associated with the added load.

(2) Does the added load absorb or deliver magnetizing vars? Explain your answer by calculation.

(3) What is the power factor of the additional load?

(4) Assume that the voltage at the input to the factory is $2400\ \mathrm{V\ (rms)}$. What is the rms magnitude of the current into the factory before the variable power factor load is added?

(5) What is the rms magnitude of the current into the factory after the variable power factor load has been added?

(6) Assume the factory is fed from a line having an impedance of $0.25 + j0.1\Omega$. The voltage at the factory is maintained at $2400\ \mathrm{V\ (rms)}$. Find the average power loss in the line before and after the load is added.

(7) Assume the factory is fed from a line having an impedance of $0.25 + j0.1\Omega$. The voltage at the factory is maintained at $2400\ \mathrm{V\ (rms)}$. Find the magnitude of the voltage at the sending end of the line before and after the load is added.

Solution:

1) $$S_o = \text{ original load } = 1600 + j\frac{1600}{0.8}(0.6) = 1600 + j1200\,\mathrm{kVA}$$

$$S_f = \text{ final load } = 1920 + j\frac{1920}{0.96}(0.28) = 1920 + j560\,\mathrm{kVA}$$

$$\therefore\ Q_{\text{added}} = 560 - 1200 = -640\,\mathrm{kVAR}$$

2) deliver

3) $$S_{\mathrm{a}} = \text{ added load } = 320 - j640 = 715.54\underline{/-63.43^\circ}\,\mathrm{kVA}$$

$$\mathrm{pf} = \cos(-63.43) = 0.447 \text{ leading}$$

4) $$\mathbf{I}_{\mathrm{L}}^* = \frac{(1600 + j1200) \times 10^3}{2400} = 666.67 + j500\,\mathrm{A}$$

$$\mathbf{I}_{\mathrm{L}} = 666.67 - j500 = 833.33\underline{/-36.87^\circ}\,\mathrm{A(rms)}$$

$$|\mathbf{I}_{\mathrm{L}}| = 833.33\,\mathrm{A\,(rms)}$$

5) $$\mathbf{I}_{\mathrm{L}}^* = \frac{(1920 + j560) \times 10^3}{2400} = 800 + j233.33$$

$$\mathbf{I}_{\mathrm{L}} = 800 - j233.33 = 833.33\underline{/-16.26^\circ}\,\mathrm{A(rms)}$$

$$|\mathbf{I}_{\mathrm{L}}| = 833.33\,\mathrm{A\,(rms)}$$

6) $$P_{\text{before}} = P_{\text{after}} = (833.33)^2(0.25) = 173.611\,\mathrm{kW}$$

7) $$\mathbf{V}_s(\text{before}) = 2400 + (666.67 - j500)(0.25 + j0.1)$$

$$= 2616.67 - j58.33 = 2617.32\underline{/-1.28^\circ}\,\mathrm{V(rms)}$$

$$|\mathbf{V}_s(\text{before})| = 2617.32\,\mathrm{V\,(rms)}$$

$$\mathbf{V}_s(\text{after}) = 2400 + (800 - j233.33)(0.25 + j0.1)$$

$$= 2623.33 + j21.67 = 2623.42\underline{/0.47^\circ}\,\mathrm{V(rms)}$$

$$|\mathbf{V}_s(\text{after})| = 2623.42\,\mathrm{V\,(rms)}$$

5. The peak amplitude of the sinusoidal voltage source in the circuit shown in the figure below is $180\ \text{V}$, and its frequency is $5000\ \text{rad/s}$. The load resistor can be varied from 0 to $4000\Omega$, and the load capacitor can be varied from $0.1\mu\text{F}$ to $0.5\mu\text{F}$.

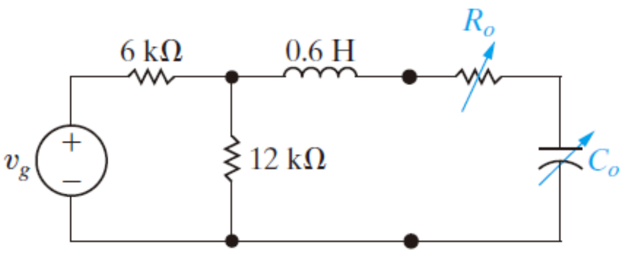
(1) Calculate the average power delivered to the load when $R_0 = 2000\Omega$ and $C_0 = 0.2\mu\text{F}$.

(2) Determine the settings of $R_0$ and $C_0$ that will result in the most average power being transferred to $R_0$.

(3) What is the average power in (2)?

(4) If there are no constraints on $R_0$ and $C_0$, what is the maximum average power that can be delivered to a load?

(5) What are the values of $R_0$ and $C_0$ for the condition of (4)?



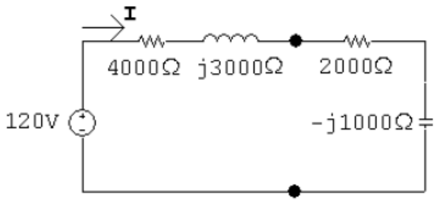

1) First find the Thévenin equivalent:

$$j\omega L = j3000\,\Omega$$

$$Z_{\text{Th}} = 6000\|12{,}000 + j3000 = 4000 + j3000\,\Omega$$

$$\mathbf{V}_{\text{Th}} = \frac{12{,}000}{6000 + 12{,}000}(180) = 120\,\text{V}$$

$$\frac{-j}{\omega C} = -j1000\,\Omega$$




$$\mathbf{I} = \frac{120}{6000 + j2000} = 18 - j6\,\text{mA}$$

$$P = \frac{1}{2}|\mathbf{I}|^2(2000) = 360\,\text{mW}$$

2) Set $C_o = 0.1\,\mu\text{F}$ so $-j/\omega C = -j2000\,\Omega$

Set $R_o$ as close as possible to

$$R_o = \sqrt{4000^2 + (3000 - 2000)^2} = 4123.1\,\Omega$$

$$\therefore\quad R_o = 4000\,\Omega$$

3) $$\mathbf{I} = \frac{120}{8000 + j1000} = 14.77 - j1.85\,\text{mA}$$

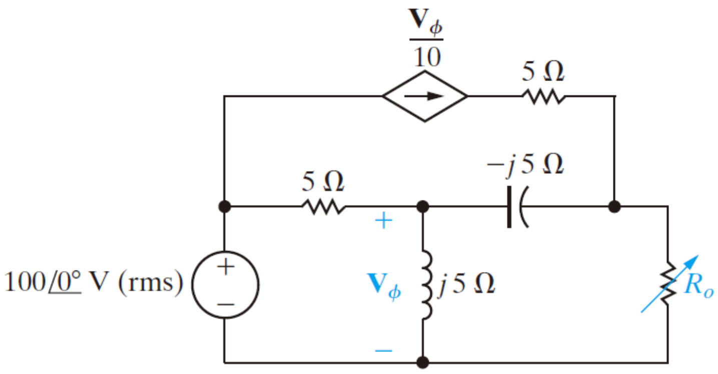
$$P = \frac{1}{2}|\mathbf{I}|^2(4000) = 443.1\,\text{mW}$$

4) $$\mathbf{I} = \frac{120}{8000} = 15\,\text{mA}$$

$$P = \frac{1}{2}(0.015)^2(4000) = 450\,\text{mW}$$

5) $R_o = 4000\,\Omega;\qquad C_o = 66.67\,\text{nF}$

6. The variable resistor $R_0$ in the circuit shown in the figure below is adjusted until maximum average power is delivered to $R_0$.




(1) What is the value of $R_0$ ?

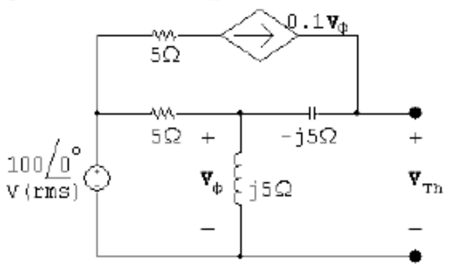
(2) Calculate the average power delivered to $R_0$.

(3) If $R_0$ is replaced with a variable impedance $Z_0$, what is the maximum average power that can be delivered to $Z_0$ ?

(4) In (3), what percentage of the circuit's developed power is delivered to the load $Z_0$ ?

Solution:

1) Open circuit voltage:

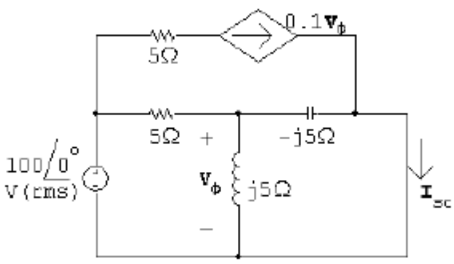



$$\frac{\mathbf{V}_\phi - 100}{5} + \frac{\mathbf{V}_\phi}{j5} - 0.1\mathbf{V}_\phi = 0$$

$$\therefore \quad \mathbf{V}_\phi = 40 + j80\,\mathrm{V(rms)}$$

$$\mathbf{V}_{\mathrm{Th}} = \mathbf{V}_\phi + 0.1\mathbf{V}_\phi(-j5) = \mathbf{V}_\phi(1 - j0.5) = 80 + j60\,\mathrm{V(rms)}$$

Short circuit current:




$$\mathbf{I}_{\mathrm{sc}} = 0.1\mathbf{V}_\phi + \frac{\mathbf{V}_\phi}{-j5} = (0.1 + j0.2)\mathbf{V}_\phi$$

$$\frac{\mathbf{V}_\phi - 100}{5} + \frac{\mathbf{V}_\phi}{j5} + \frac{\mathbf{V}_\phi}{-j5} = 0$$

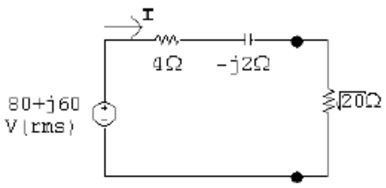
$$\therefore \quad \mathbf{V}_\phi = 100\,\mathrm{V(rms)}$$

$$\mathbf{I}_{\mathrm{sc}} = (0.1 + j0.2)(100) = 10 + j20\,\mathrm{A(rms)}$$

$$Z_{\mathrm{Th}} = \frac{\mathbf{V}_{\mathrm{Th}}}{\mathbf{I}_{\mathrm{sc}}} = \frac{80 + j60}{10 + j20} = 4 - j2\,\Omega$$

$$\therefore \quad R_o = |Z_{\mathrm{Th}}| = 4.47\,\Omega$$

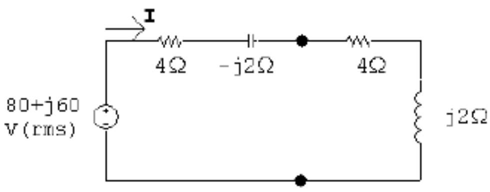
2)




$$\mathbf{I} = \frac{80 + j60}{4 + \sqrt{20} - j2} = 7.36 + j8.82\,\mathrm{A\ (rms)}$$

$$P = (11.49)^2(\sqrt{20}) = 590.17\,\mathrm{W}$$

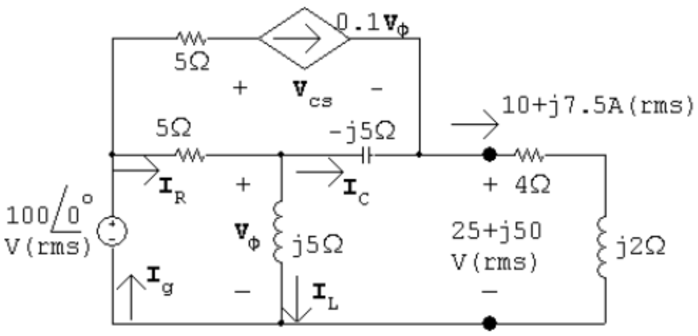
3)




$$\mathbf{I} = \frac{80 + j60}{8} = 10 + j7.5\,\mathrm{A\ (rms)}$$

$$P = (10^2 + 7.5^2)(4) = 625\,\mathrm{W}$$

4)




$$\frac{\mathbf{V}_\phi - 100}{5} + \frac{\mathbf{V}_\phi}{j5} + \frac{\mathbf{V}_o - (25+j50)}{-j5} = 0$$

$\mathbf{V}_\phi = 50 + j25\,\text{V (rms)}$

$0.1\mathbf{V}_\phi = 5 + j2.5\,\text{V (rms)}$

$5 + j2.5 + \mathbf{I}_C = 10 + j7.5$

$\mathbf{I}_C = 5 + j5\,\text{A (rms)}$

$\mathbf{I}_L = \dfrac{\mathbf{V}_\phi}{j5} = 5 - j10\,\text{A (rms)}$

$\mathbf{I}_R = \mathbf{I}_C + \mathbf{I}_L = 10 - j5\,\text{A (rms)}$

$\mathbf{I}_g = \mathbf{I}_R + 0.1\mathbf{V}_\phi = 15 - j2.5\,\text{A (rms)}$

$S_g = -100\mathbf{I}_g^* = -1500 - j250\,\text{VA}$

$100 = 5(5 + j2.5) + \mathbf{V}_{\text{cs}} + 25 + j50 \quad \therefore \quad \mathbf{V}_{\text{cs}} = 50 - j62.5\,\text{V (rms)}$

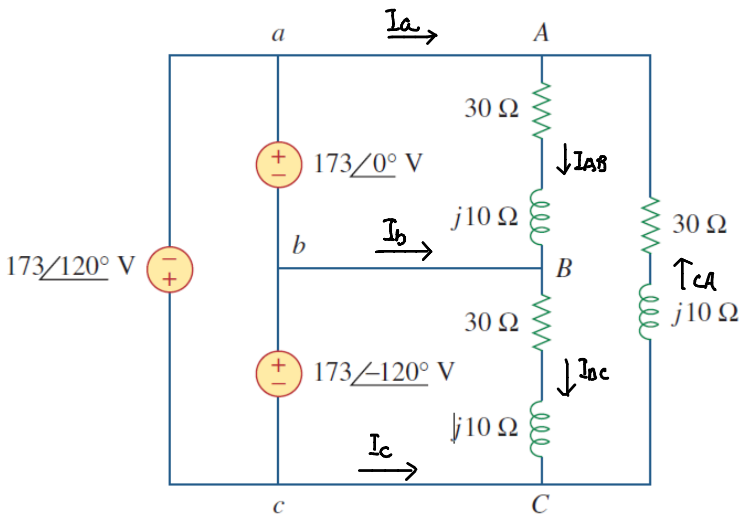
$S_{\text{cs}} = (50 - j62.5)(5 - j2.5) = 93.75 - j437.5\,\text{VA}$

Thus,

$\sum P_{\text{dev}} = 1500$

$\%\text{ delivered to } R_o = \dfrac{625}{1500}(100) = 41.67\%$

7. For the circuit below, calculate the phase currents $\mathbf{I}_{AB}, \mathbf{I}_{BC}, \mathbf{I}_{CA}$ and line currents $I_a, I_b, I_c$. Notice that in this figure, phasor expression corresponds to the rms value of the voltage.




Solution：

$$\mathbf{Z}_{\Delta} = 30 + j10 = 31.62\angle 18.43^{\circ}$$

The phase currents are

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{ab}}{\mathbf{Z}_{\Delta}} = \frac{173\angle 0^{\circ}}{31.62\angle 18.43^{\circ}} = \mathbf{5.47\angle -18.43^{\circ}\ A}$$

$$\mathbf{I}_{BC} = \mathbf{I}_{AB}\angle -120^{\circ} = \mathbf{5.47\angle -138.43^{\circ}\ A}$$

$$\mathbf{I}_{CA} = \mathbf{I}_{AB}\angle 120^{\circ} = \mathbf{5.47\angle 101.57^{\circ}\ A}$$

The line currents are

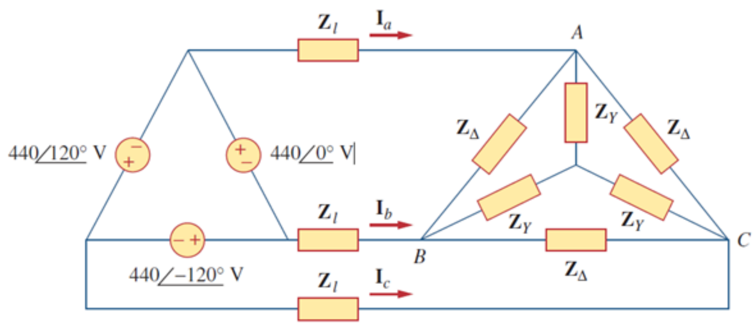
$$\mathbf{I}_{a} = \mathbf{I}_{AB} - \mathbf{I}_{CA} = \mathbf{I}_{AB}\sqrt{3}\angle -30^{\circ}$$

$$\mathbf{I}_{a} = 5.47\sqrt{3}\angle -48.43^{\circ} = \mathbf{9.474\angle -48.43^{\circ}\ A}$$

$$\mathbf{I}_{b} = \mathbf{I}_{a}\angle -120^{\circ} = \mathbf{9.474\angle -168.43^{\circ}\ A}$$

$$\mathbf{I}_{c} = \mathbf{I}_{a}\angle 120^{\circ} = \mathbf{9.474\angle 71.57^{\circ}\ A}$$

8. Find the line currents $\mathbf{I}_a, \mathbf{I}_b$ and $\mathbf{I}_c$ in the three-phase network below. Take $Z_\Delta = 12 - j15\Omega$, $Z_Y = 4 + j6\Omega$, $Z_l = 2\Omega$. Notice that in this figure, phasor expression corresponds to the rms value of the voltage.




Solution:

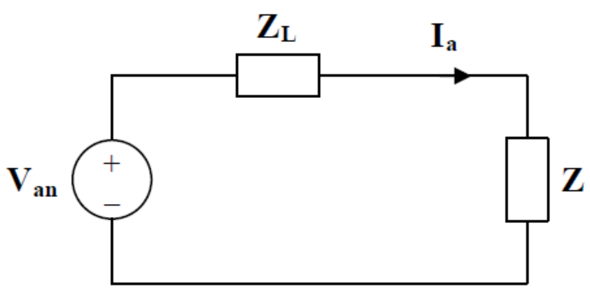
Convert the Δ-connected source to a Y-connected source.

$$\mathbf{V}_{an} = \frac{V_p}{\sqrt{3}}\angle -30^\circ = \frac{440}{\sqrt{3}}\angle -30^\circ = 254\angle -30^\circ$$

Convert the Δ-connected load to a Y-connected load.

$$\mathbf{Z} = \mathbf{Z}_Y \,||\, \frac{\mathbf{Z}_\Delta}{3} = (4 + j6) \,||\, (4 - j5) = \frac{(4 + j6)(4 - j5)}{8 + j}$$

$$\mathbf{Z} = 5.723 - j0.2153$$

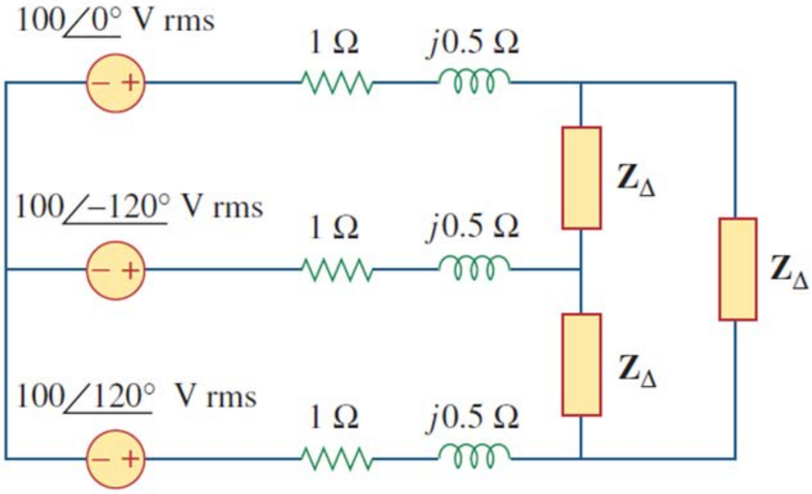



$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_L + \mathbf{Z}} = \frac{254\angle -30^\circ}{7.723 - j0.2153} = \mathbf{32.88\angle{-28.4}^\circ\ A}$$

$$\mathbf{I}_b = \mathbf{I}_a \angle -120^\circ = \mathbf{32.88\angle{-148.4}^\circ\ A}$$

$$\mathbf{I}_c = \mathbf{I}_a \angle 120^\circ = \mathbf{32.88\angle 91.6^\circ\ A}$$

9. For the three-phase circuit below, find the average power absorbed by the delta-connected load with $Z_\Delta = 21 + j24\Omega$.




Solution:

Transform the delta-connected load to its wye equivalent.

$$\mathbf{Z}_Y = \frac{\mathbf{Z}_\Delta}{3} = 7 + j8$$

Using the per-phase equivalent circuit above,

$$\mathbf{I}_a = \frac{100\angle 0°}{(1 + j0.5) + (7 + j8)} = 8.567\angle -46.75°$$

For a wye-connected load,

$$I_p = I_a = \left|\mathbf{I}_a\right| = 8.567$$

$$\mathbf{S} = 3\left|\mathbf{I}_p\right|^2 \mathbf{Z}_p = (3)(8.567)^2(7 + j8)$$

$$P = \mathrm{Re}(\mathbf{S}) = (3)(8.567)^2(7) = \mathbf{1.541\ kW}$$

10. In an AC power system, the total power delivered to a balanced three-phase load when operating at a line voltage of $2500\sqrt{3}$ V (rms) is 900 kW at a lagging power factor of 0.6. The impedance of the distribution line supplying the load is $1+j3\Omega$ per phase. Under these operating conditions, the drop in the magnitude of the line voltage between the sending end and the load end of the line is excessive. To compensate, a bank of Y-connected capacitors is placed in parallel with the load. The capacitor bank is designed to generate 1125 kVAR reactive power when operated at a line voltage of $2500\sqrt{3}$ V (rms).

(1) What is the magnitude of the voltage at the sending end of the line when the load is operating at a line voltage of $2500\sqrt{3}$ V (rms) and the capacitor bank is disconnected?

(2) Repeat (1) with the capacitor bank is connected.

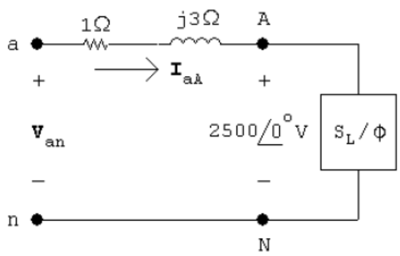
(3) What is the average power efficiency of the line in (1)?

(4) What is the average power efficiency in (2)?

(5) If the system is operating at a frequency of 60Hz, what is the size of each capacitor in microfarads?

Solution:

1)




$$S_{L/\phi} = \frac{1}{3}\left[900 + j\frac{900}{0.6}(0.8)\right]10^3 = 300{,}000 + j400{,}000\,\text{VA}$$

$$\mathbf{I}_{aA}^* = \frac{300{,}000 + j400{,}000}{2500} = 120 + j160\,\text{A (rms)}$$

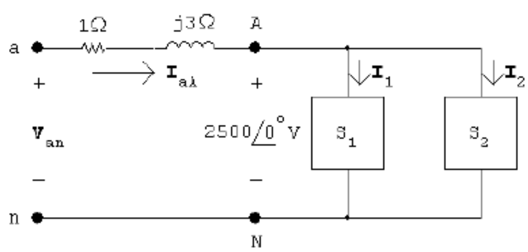
$$\mathbf{I}_{aA} = 120 - j160\,\text{A (rms)}$$

$$\mathbf{V}_{an} = 2500 + (1 + j3)(120 - j160)$$

$$= 3100 + j200 = 3106.44\underline{/3.69^\circ}\,\text{V (rms)}$$

$$|\mathbf{V}_{ab}| = \sqrt{3}(3106.44) = 5380.5\,\text{V (rms)}$$

2)




$$\mathbf{I}_1 = 120 - j160\,\text{A} \qquad \text{(from part [a])}$$

$$S_2 = 0 - j\frac{1}{3}(1125)\times 10^3 = -j375{,}000\,\text{VAR}$$

$$\mathbf{I}_2^* = \frac{-j375{,}000}{2500} = -j150\,\text{A (rms)}$$

$$\therefore\ \mathbf{I}_2 = j150\,\text{A (rms)}$$

$$\mathbf{I}_{aA} = 120 - j160 + j150 = 120 - j10\,\text{A (rms)}$$

$$\mathbf{V}_{an} = 2500 + (120 - j10)(1 + j3)$$

$$= 2650 + j350 = 2673.01\underline{/7.52^\circ}\,\text{V (rms)}$$

$$|\mathbf{V}_{ab}| = \sqrt{3}(2673.01) = 4629.8\,\text{V (rms)}$$

3)

$$|\mathbf{I}_{aA}| = 200\,\text{A (rms)}$$

$$P_{\text{loss}/\phi} = (200)^2(1) = 40\,\text{kW}$$

$$P_{g/\phi} = 300{,}000 + 40{,}000 = 340\,\text{kW}$$

$$\%\,\eta = \frac{300}{340}(100) = 88.2\%$$

4)

$$|\mathbf{I}_{aA}| = 120.416\,\text{A (rms)}$$

$$P_{\ell/\phi} = (120.416)^2(1) = 14{,}500\,\text{W}$$

$$\%\,\eta = \frac{300{,}000}{314{,}500}(100) = 95.4\%$$

5)

$$Z_{\text{cap/Y}} = \frac{2500^2}{j375{,}000} = -j16.67\,\Omega$$

$$\therefore\ \frac{1}{\omega C} = 16.67; \qquad C = \frac{1}{(16.67)(120\pi)} = 159.155\,\mu\text{F}$$