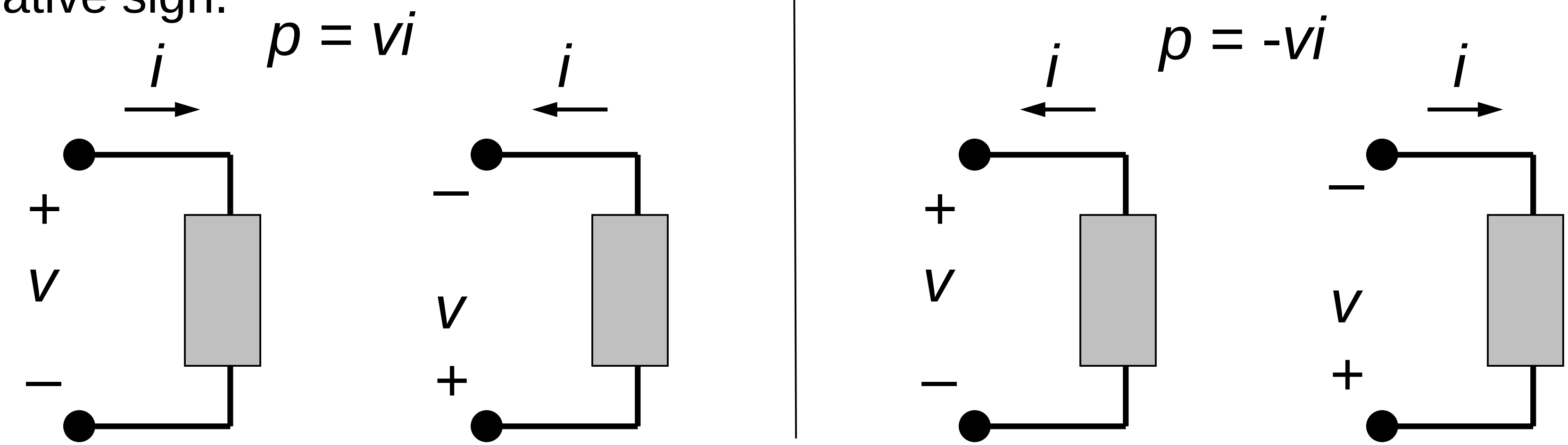
Lecture 15
--Review

Outline

- Circuit Basics
- Temporal Analysis
- AC circuits
- Laplace Transform

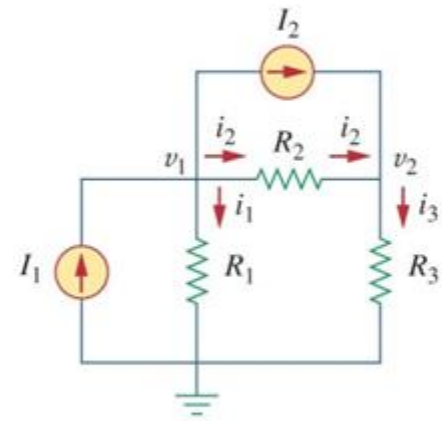
Passive Sign Convention

Whenever the reference direction for the current in an element is in the direction of the reference voltage drop across the element, use positive sign in any expression that relates the voltage to the current. Otherwise, use a negative sign.

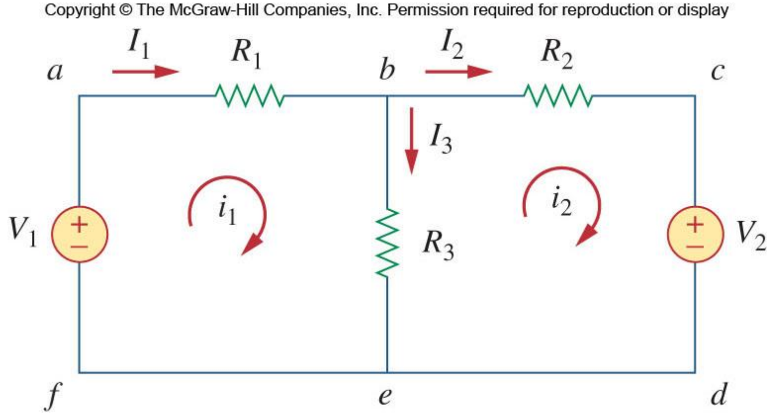


- If p > 0, power is absorbed by the element.
  - electrical energy into heat (resistors in toasters), light (light bulbs), or acoustic energy (speakers); by storing energy (charging a battery).
- If p < 0, power is extracted from the element.

- Node Analysis
  - Node voltage is the unknown
  - Solve by KCL
  - Special case: Floating voltage source



- Mesh Analysis
  - Loop current is the unknown
  - Solve by KVL
  - Special case: Current source

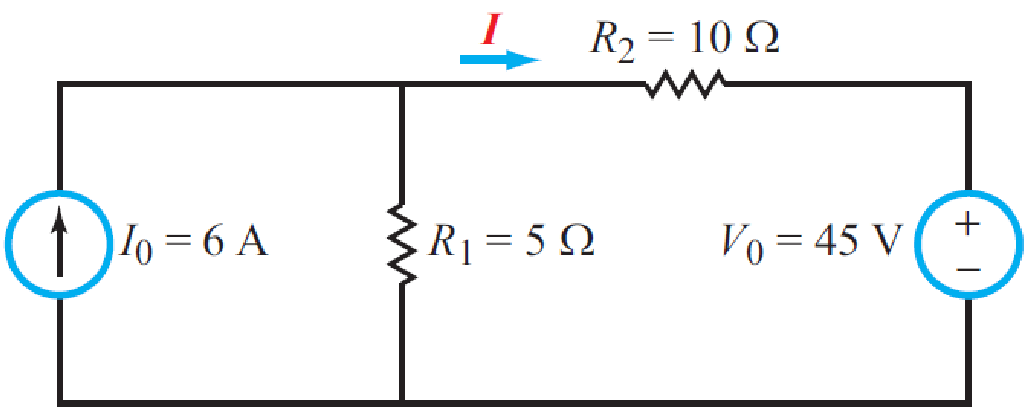


Circuit theorem

- Linearity property
- Superposition
- Thevenin’s theorem
- Source transformation
- Norton’s theorem

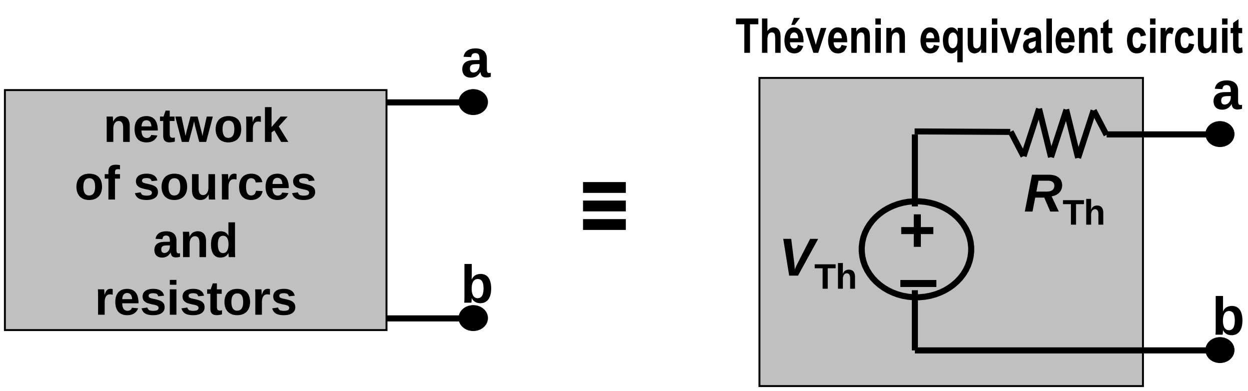
Superposition

- The superposition principle states that the voltage across (or current through) an element in *a linear circuit* is the algebraic sum of the voltages across (or currents through) that element due to each independent source acting alone.



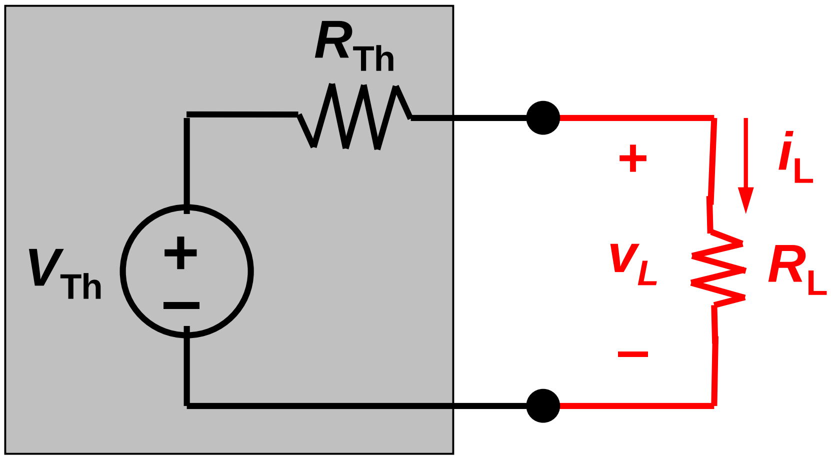
Thevenin's Theorem

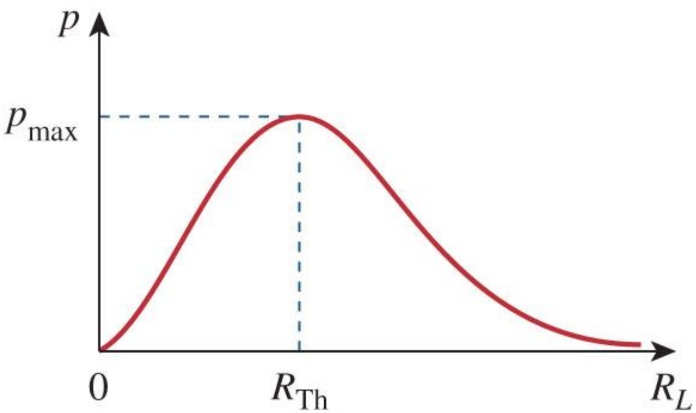
- In many circuits, one element will be variable (called *the load*), while others are fixed.
  - An example is the household outlet: many different appliances may be plugged into the outlet, each presenting a different resistance.
  - Ordinarily one has to re-analyze the circuit for load change.
  - This problem can be avoided by circuit theorem (e.g. Thevenin's theorem), which provides a technique to replace the fixed part of the circuit with an **equivalent** circuit.



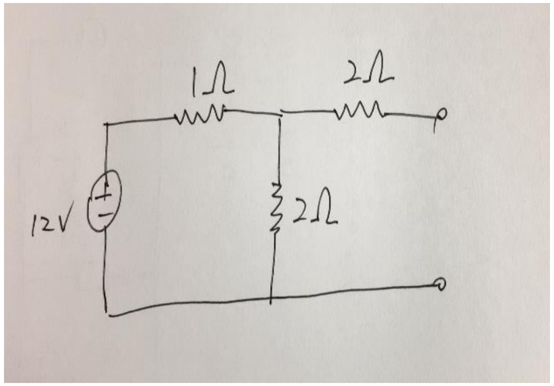
3 methods

Max Power Transfer

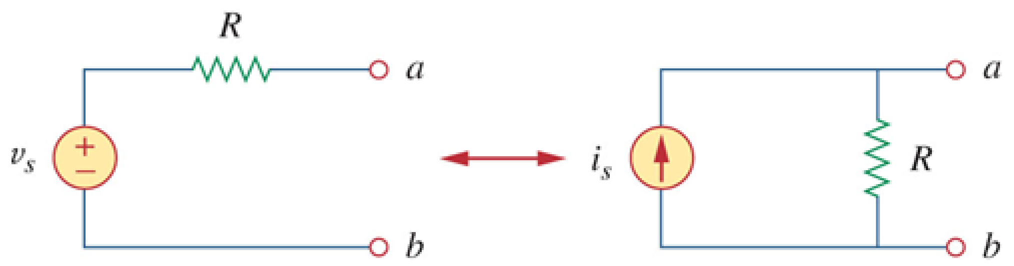




Percentage?



Source Transformation

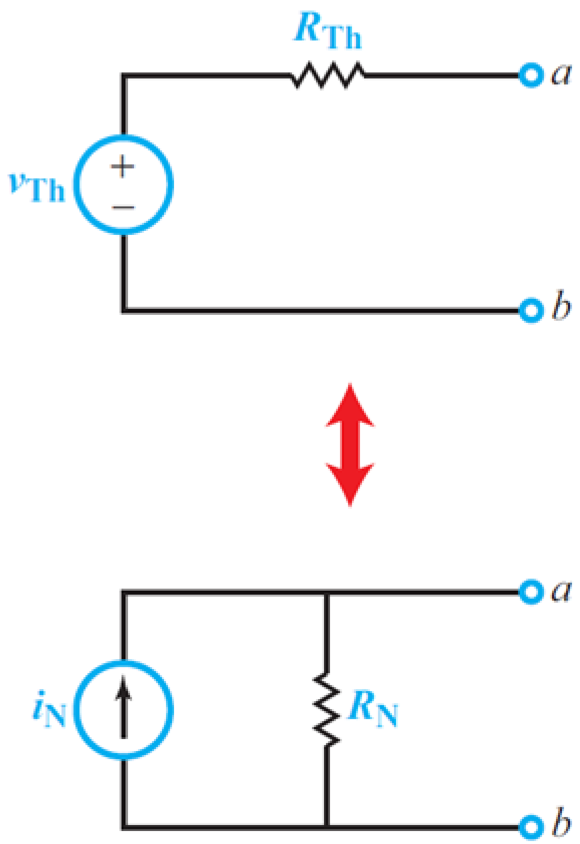


- A source transformation is the process of replacing a voltage source v_s in series with a resistor R by a current source i_s in parallel with a resistor R, or vice versa.
- These transformations work because the two sources have equivalent behavior at their terminals:
  - If the sources are turned off, resistance at the terminals are both R
  - If the terminals are short circuited, the currents need to be the same.

Norton's Theorem

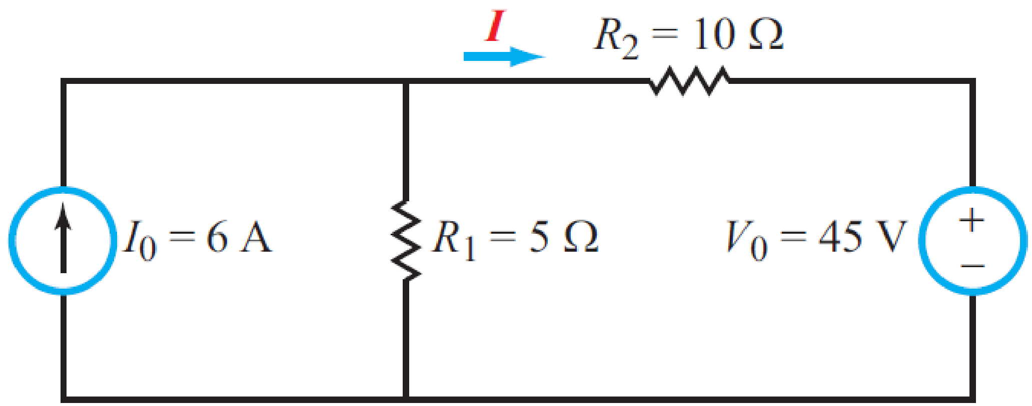
Thévenin equivalent circuit

Norton equivalent circuit

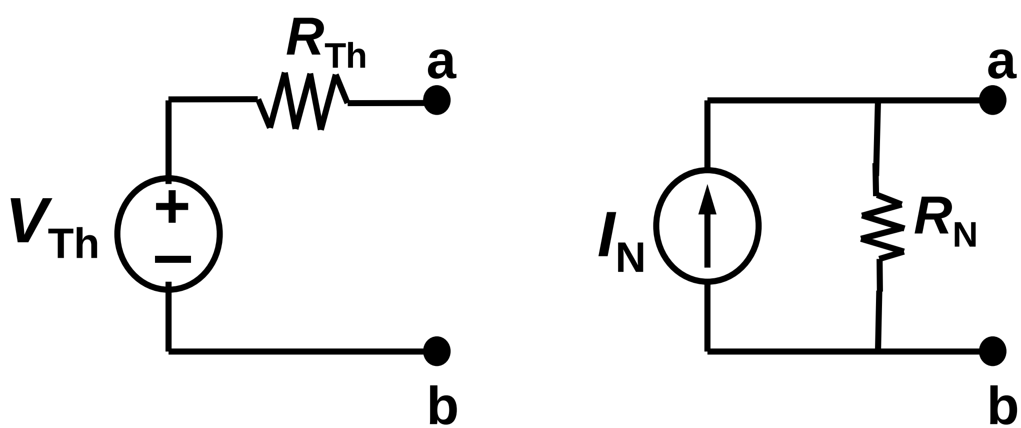


[Source: Berkeley]

Summary



- Superposition
  - Voltage off → SC
  - Current off → OC
- Thevenin and Norton Equivalent Circuits
  - Solve for OC voltage
  - Solve for SC current

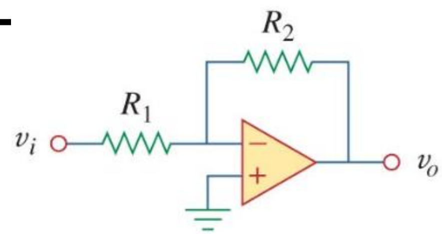


$$I_N = \frac{V_{Th}}{R_{Th}}$$

$$R_{\mathrm{N}} = R_{\mathrm{Th}}$$

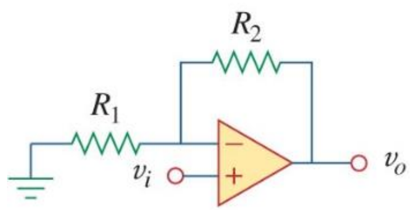
OA

Op amp circuit	Name/output-input relationship



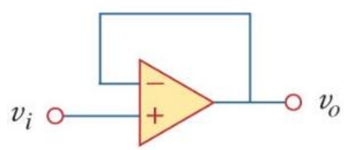
Inverting amplifier

$$v_o = -\frac{R_2}{R_1} v_i$$



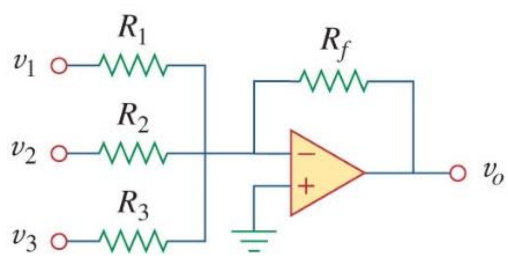
Noninverting amplifier

$$v_o = \left(1 + \frac{R_2}{R_1}\right) v_i$$



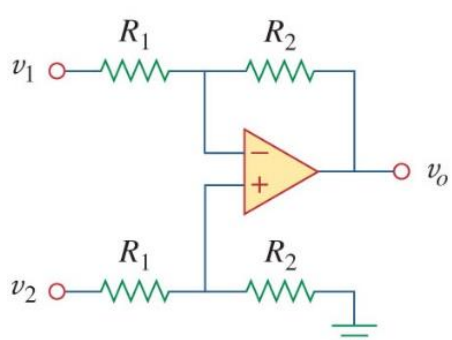
Voltage follower

$$v_o = v_i$$



Summer

$$v_o = -\left(\frac{R_f}{R_1} v_1 + \frac{R_f}{R_2} v_2 + \frac{R_f}{R_3} v_3\right)$$



Difference amplifier

$$v_o = \frac{R_2}{R_1}(v_2 - v_1)$$

Part 2 Temporal Analysis

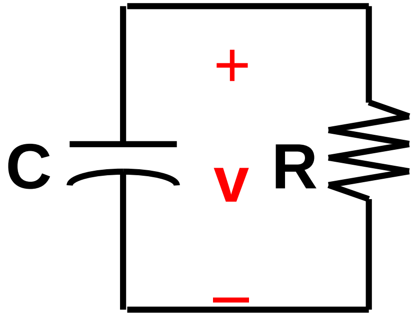
Summary of Capacitors and Inductors

Table 5-4: Basic properties of R, L, and C.

Property	R	L	C
i–υ relation	$i = \frac{\upsilon}{R}$	$i = \frac{1}{L}\int_{t_0}^{t} \upsilon\, dt' + i(t_0)$	$i = C\,\frac{d\upsilon}{dt}$
υ-i relation	$\upsilon = iR$	$\upsilon = L\,\frac{di}{dt}$	$\upsilon = \frac{1}{C}\int_{t_0}^{t} i\, dt' + \upsilon(t_0)$
p (power transfer in)	$p = i^2 R$	$p = Li\,\frac{di}{dt}$	$p = C\upsilon\,\frac{d\upsilon}{dt}$
w (stored energy)	0	$w = \frac{1}{2}Li^2$	$w = \frac{1}{2}C\upsilon^2$
Series combination	$R_{eq} = R_1 + R_2$	$L_{eq} = L_1 + L_2$	$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$
Parallel combination	$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$	$\frac{1}{L_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$	$C_{eq} = C_1 + C_2$
dc behavior	no change	short circuit	open circuit
Can υ change instantaneously?	yes	yes	no
Can i change instantaneously?	yes	no	yes

Natural Response Summary

RC Circuit



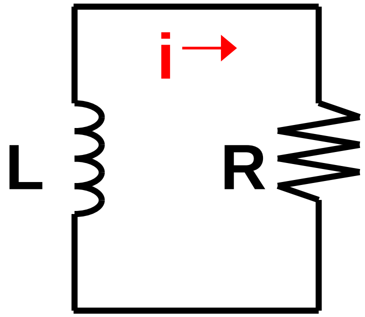
- Capacitor voltage cannot change instantaneously

$$v(0^-) = v(0^+)$$

$$v(t) = v(0)e^{-t/\tau}$$

- time constant $\tau = RC$

RL Circuit



- Inductor current cannot change instantaneously

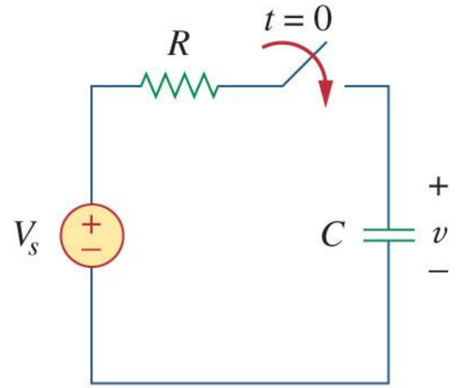
$$i(0^-) = i(0^+)$$

$$i(t) = i(0)e^{-t/\tau}$$

- time constant $\tau = \frac{L}{R}$

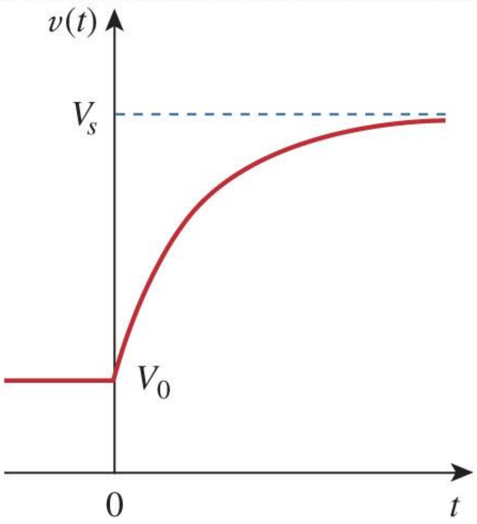
[Source: Berkeley]

Step Response of the RC Circuit



$$v(0^-) = v(0^+) = v_0$$

$$v(t) = \begin{cases} V_0, & t < 0 \\ V_s + (V_0 - V_s)e^{-t/\tau} & t > 0 \end{cases}$$



- This is known as the complete response, or total response.

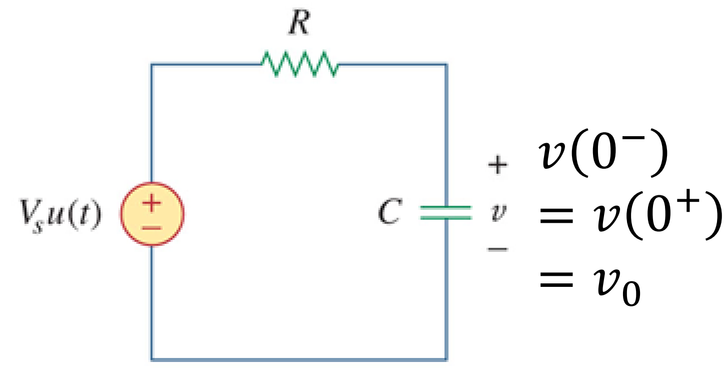
Forced Response

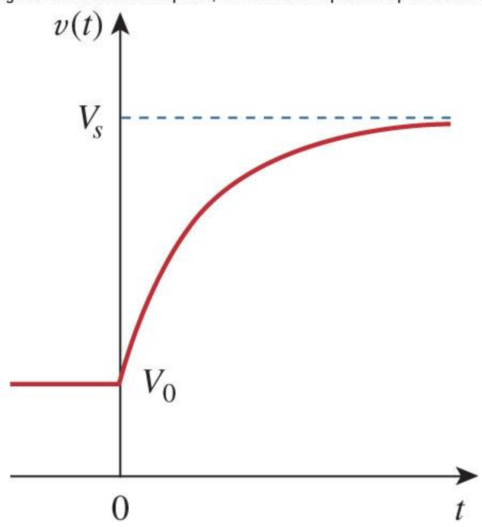
- The complete response

$$v(t)=\begin{cases} V_0, & t<0 \\ V_s+(V_0-V_s)e^{-t/\tau} & t>0 \end{cases}$$

can be written as:

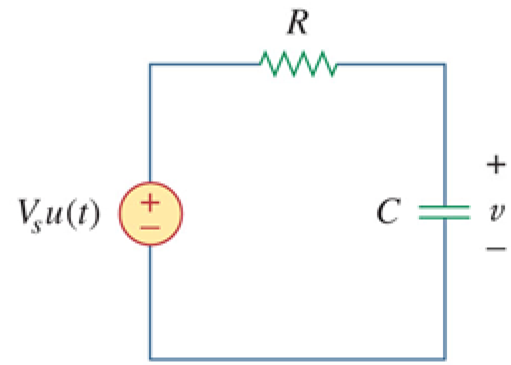
$$\boxed{v=v_n+v_f}$$





Another Perspective

$$v(t)=\begin{cases} V_0, & t<0 \\ V_s+(V_0-V_s)e^{-t/\tau} & t>0 \end{cases}$$



- Another way to look at the response is to break it up into the transient response and the steady state response:

$$v(t)= \underbrace{v(\infty)}_{\text{steady } v_{ss}} + \underbrace{[v(0)-v(\infty)]e^{-t/\tau}}_{\text{transient } v_t}$$

Can be extended as a “three-elements” method

General Procedure for Finding RC/RL Response

1. **Identify the variable of interest**
   - For RL circuits, it is usually the inductor current $i_L(t)$.
   - For RC circuits, it is usually the capacitor voltage $v_c(t)$.
2. **Determine the initial value (at $t = t_0^-$ and t_0^+) of the variable**
   - Recall that $i_L(t)$ and $v_c(t)$ are continuous variables:

     $$i_L(t_0^+) = i_L(t_0^-) \quad \text{and} \quad v_c(t_0^+) = v_c(t_0^-)$$

   - Assuming that the circuit reached steady state before t_0, use the fact that an inductor behaves like a short circuit in steady state or that a capacitor behaves like an open circuit in steady state.

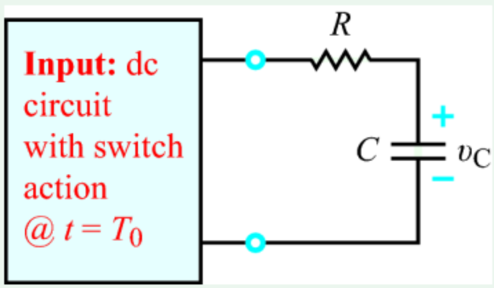
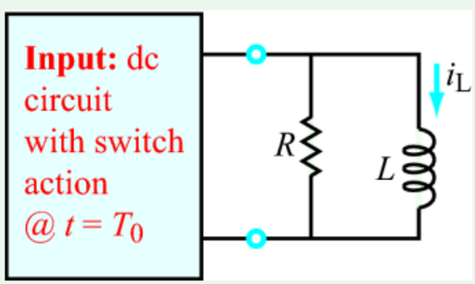
[Source: Berkeley]

Procedure (cont'd)

3. **Calculate the final value of the variable** (its value as ***t*** → ∞)
   - Again, make use of the fact that an inductor behaves like a short circuit in steady state (*t* → ∞) or that a capacitor behaves like an open circuit in steady state (*t* → ∞).

4. **Calculate the time constant for the circuit**
   - $\tau = L/R$ for an RL circuit, where ***R*** is the Thévenin equivalent resistance "seen" by the inductor.
   - $\tau = RC$ for an RC circuit where ***R*** is the Thévenin equivalent resistance "seen" by the capacitor.

Response Form of Basic First-Order Circuits

Circuit	Diagram	Response
RC		$v_C(t) = \left\{ v_C(\infty) + [v_C(T_0) - v_C(\infty)]e^{-(t-T_0)/\tau} \right\} u(t-T_0)$ $(\tau = RC)$
RL		$i_L(t) = \left\{ i_L(\infty) + [i_L(T_0) - i_L(\infty)]e^{-(t-T_0)/\tau} \right\} u(t-T_0)$ $(\tau = L/R)$

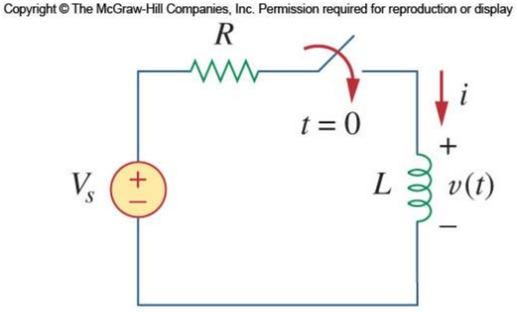
Step Response of RL Circuit

- This yields an overall response of:

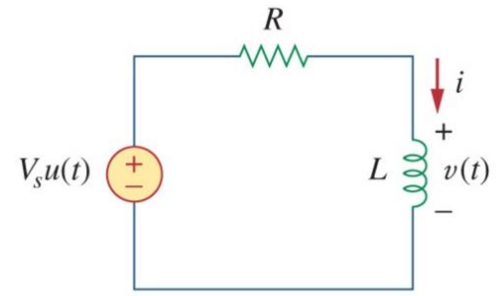
$$i = Ae^{-t/\tau} + \frac{V_s}{R}$$

$$i\left(0^+\right) = i\left(0^-\right) = I_0 \qquad A = I_0 - \frac{V_s}{R}$$

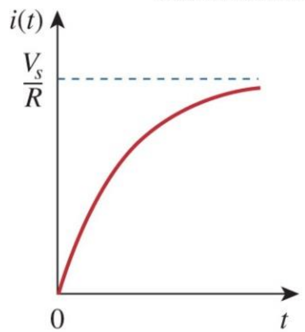
$$i\left(t\right) = \frac{V_s}{R} + \left(I_0 - \frac{V_s}{R}\right)e^{-t/\tau}$$



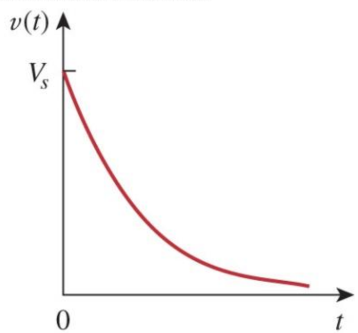
(a)



(b)

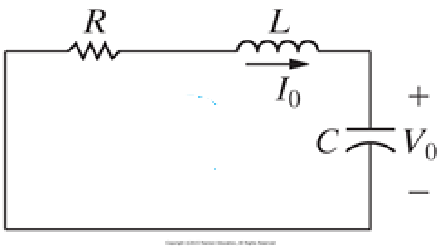


(a)



(b)

Source-Free Series RLC



$$\frac{d^2v}{d^2t} + \frac{R}{L}\frac{dv}{dt} + \frac{1}{\mathrm{LC}}v = 0$$

$$\boxed{s^2 + \frac{R}{L}s + \frac{1}{LC} = 0}$$

$$s = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

Three Damping Cases

$$\frac{d^2v}{dt^2}+\frac{1}{RC}\frac{dv}{dt}+\frac{1}{LC}v=0$$

$$s_{1,2}=-\alpha\pm\sqrt{\alpha^2-\omega_0^2}$$

$$\alpha=\frac{1}{2RC}\qquad \omega_0=\frac{1}{\sqrt{LC}}$$

$$\omega_d=\sqrt{\omega_0^2-\alpha^2}$$

- For the overdamped case, the roots are real and negative,

$$v(t)=A_1e^{s_1t}+A_2e^{s_2t}$$

- For critically damped, the roots are real and equal

$$v(t)=(A_2+A_1t)e^{-\alpha t}$$

- In the underdamped case, the roots are complex

$$v(t)=e^{-\alpha t}\left(A_1\cos\omega_d t+A_2\sin\omega_d t\right)$$

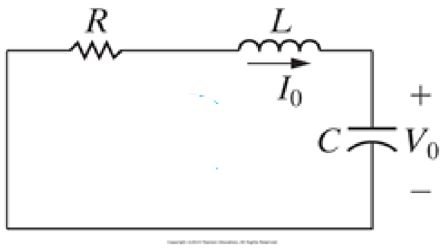
Series vs. Parallel (Source-Free RLC Network)

- Series

$$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

$$v(t) = (A_1 t + A_2) e^{-\alpha t}$$

$$\mathrm{v}(t) = e^{-\alpha t}(B_1 \cos \omega_d t + B_2 \sin \omega_d t)$$

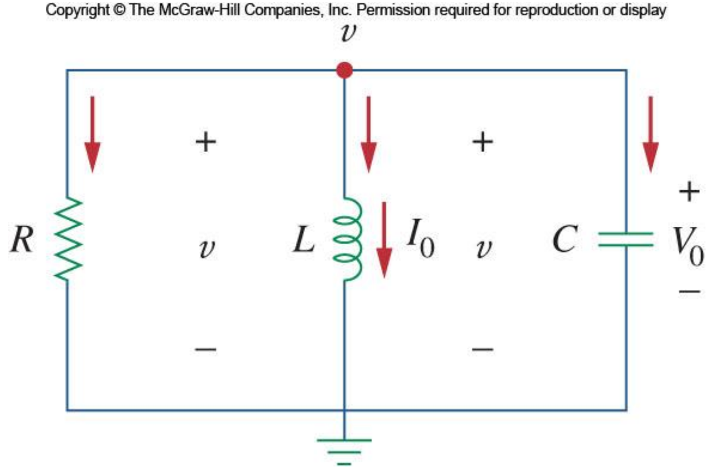


- Parallel

$$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

$$v(t) = (A_2 + A_1 t) e^{-\alpha t}$$

$$v(t) = e^{-\alpha t}(A_1 \cos \omega_d t + A_2 \sin \omega_d t)$$



General Second-Order Circuits

- The principles of the approach to solving the series and parallel forms of RLC circuits can be applied to general second order circuits, by taking the following four steps:
  1. First determine the initial conditions, *x(0)* and *dx(0)/dt*.
  2. Turn off the independent sources and find the form of the transient response by applying KVL and KCL.
     - Depending on the damping found, the unknown constants will be found.
  3. We obtain the steady-state response as:

$$x_{ss}(t) = x(\infty)$$

     where $x(\infty)$ is the final value of *x* obtained in step 1.
  4. The total response = transient response + steady-state response.

$$x(t) = x_t(t) + x_{ss}(t)$$

General solution for second-order circuits for $t \geq 0$.

$x(t)$ = unknown variable (voltage or current)

Differential equation: $x'' + ax' + bx = c$

Initial conditions: $x(0)$ and $x'(0)$

Final condition: $x(\infty) = \dfrac{c}{b}$

$$\alpha = \frac{a}{2} \qquad \omega_0 = \sqrt{b}$$

Overdamped Response $\alpha > \omega_0$

$$x(t) = [A_1 e^{s_1 t} + A_2 e^{s_2 t} + x(\infty)]\, u(t)$$

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} \qquad s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$

$$A_1 = \frac{x'(0) - s_2[x(0) - x(\infty)]}{s_1 - s_2} \qquad A_2 = -\left[\frac{x'(0) - s_1[x(0) - x(\infty)]}{s_1 - s_2}\right]$$

Critically Damped $\alpha = \omega_0$

$$x(t) = [(B_1 + B_2 t)e^{-\alpha t} + x(\infty)]\, u(t)$$

$$B_1 = x(0) - x(\infty) \qquad B_2 = x'(0) + \alpha[x(0) - x(\infty)]$$

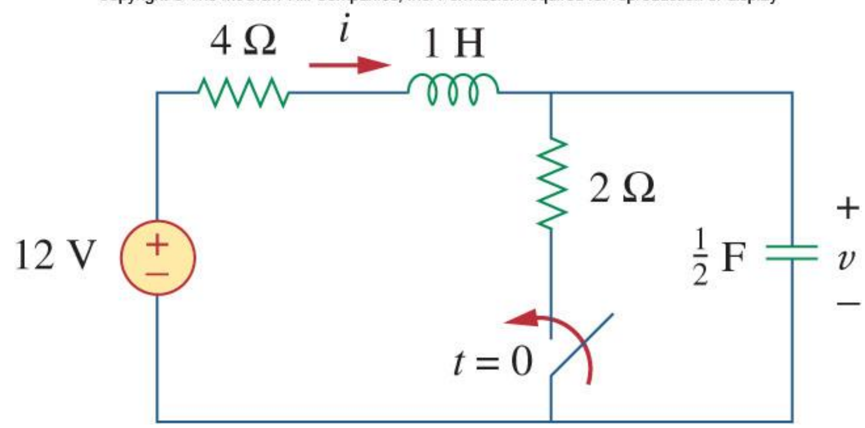
Underdamped $\alpha < \omega_0$

$$x(t) = [D_1 \cos\omega_d t + D_2 \sin\omega_d t + x(\infty)]e^{-\alpha t}\, u(t)$$

$$D_1 = x(0) - x(\infty) \qquad D_2 = \frac{x'(0) + \alpha[x(0) - x(\infty)]}{\omega_d}$$

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$$

General RLC Circuits

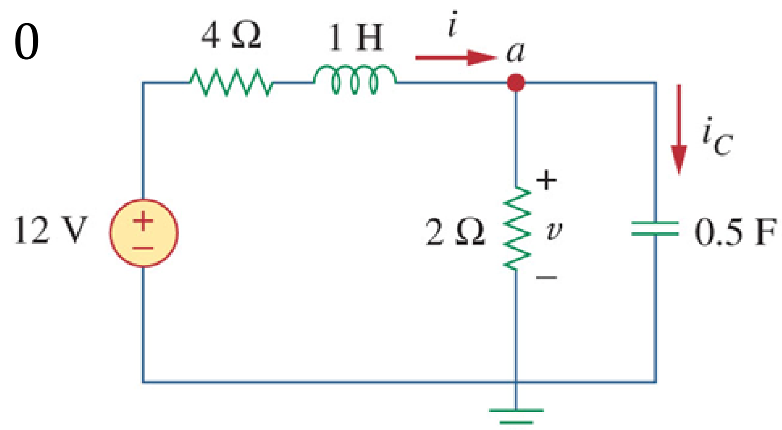


- Find the complete response v for $t > 0$ in the circuit.

1. Initial conditions

$$v(0^+) = v(0^-) = 12V,\ i(0^+) = i(0^-) = 0$$

$$\frac{dv(0^+)}{dt} = \frac{i_C(0^+)}{C} = \frac{-12/2}{0.5} = -12\ V/s$$



2. Transient response

KCL at node *a*: $i = \frac{v}{2} + 0.5\frac{dv}{dt}$

KVL on left mesh: $4i + 1\frac{di}{dt} + v = 0$

$$\Rightarrow \frac{d^2v}{dt^2} + 5\frac{dv}{dt} + 6v = 0 \Rightarrow v_t(t) = A_1e^{-2t} + A_2e^{-3t} = 12e^{-2t} - 4e^{-3t}$$

3. Steady-state response $v_{ss}(t) = 4V$

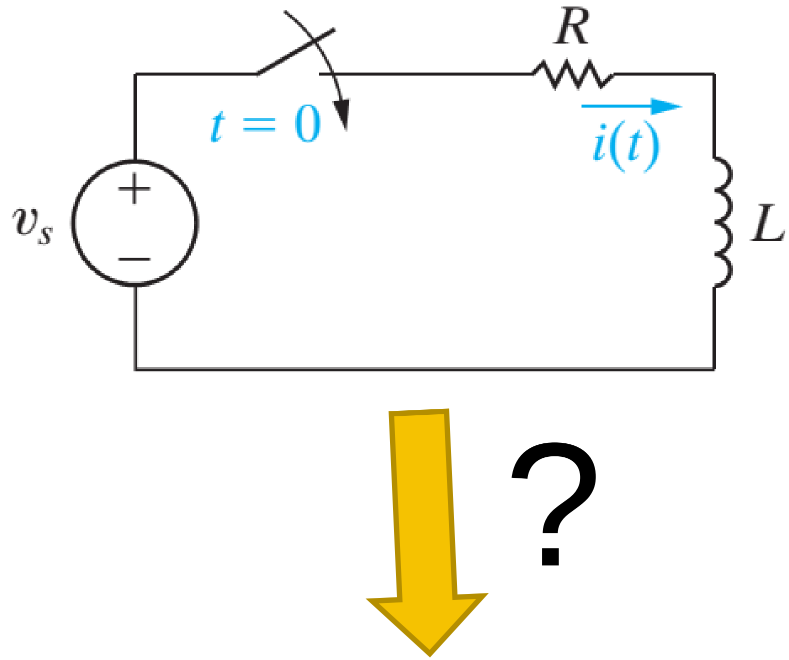
4. Combine together

How about Laplace transform in solving this circuit？

AC analysis-Phasor

AC Steady-State Analysis by Phasor Method

$$v_s = V_m \cos(\omega t + \phi)$$



$$i = \boxed{\frac{-V_m}{\sqrt{R^2 + \omega^2 L^2}} \cos(\phi - \theta) e^{-(R/L)t}} + \boxed{\frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \cos(\omega t + \phi - \theta)}$$

Transient response

Steady-state response

Sinusoid-Phasor Transformation

$$v(t) = V_m \cos(\omega t + \phi) \quad \Leftrightarrow \quad \mathbf{V} = V_m \angle \phi$$

(Time-domain representation) (Phasor-domain representation)

- Applying a derivative to a phasor yields:

$$\frac{dv}{dt} \Leftrightarrow j\omega V$$

(Time domain) (Phasor domain)

- Applying an integral to a phasor yields:

$$\int v dt \Leftrightarrow \frac{V}{j\omega}$$

(Time domain) (Phasor domain)

Review: Impedance and Admittance

Resistor	$\mathbf{Z} = R$	$\mathbf{Y} = 1/R$
Inductor	$\mathbf{Z} = j\omega L$	$\mathbf{Y} = 1/j\omega L$
Capacitor	$\mathbf{Z} = 1/j\omega C$	$\mathbf{Y} = j\omega C$

Impedance is voltage/current

$$\mathbf{Z} = R + jX$$

R = resistance = Re(Z)

X = reactance = Im(Z)

Admittance is current/voltage

$$\mathbf{Y} = \frac{1}{\mathbf{Z}} = G + jB$$

G = conductance = Re(Y)

B = susceptance = Im(Y)

AC Phasor Analysis General Procedure

Step 1: Adopt cosine reference

$$v_s(t) = 12\sin(\omega t - 45^\circ)$$
$$= 12\cos(\omega t - 45^\circ - 90^\circ) = 12\cos(\omega t - 135^\circ)\ \text{V}.$$

$$\mathbf{V}_s = 12e^{-j135^\circ}\ \text{V}.$$

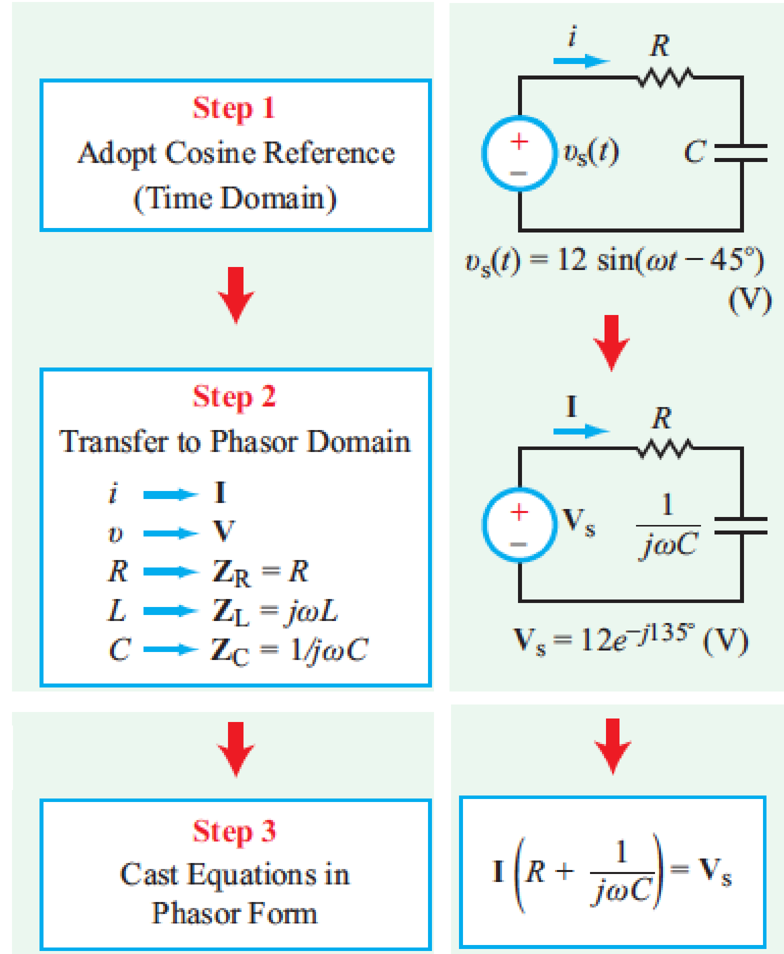
Step 2: Transform circuit to phasor domain

Step 3: Cast KCL and/or KVL equations in phasor domain

$$\mathbf{Z}_R\mathbf{I} + \mathbf{Z}_C\mathbf{I} = \mathbf{V}_s,$$

which is equivalent to

$$\left(R + \frac{1}{j\omega C}\right)\mathbf{I} = 12e^{-j135^\circ}.$$



AC Phasor Analysis General Procedure

Step 4: Solve for unknown variable

$$\mathbf{I} = \frac{12e^{-j135^\circ}}{R + \frac{1}{j\omega C}} = \frac{j12\omega C e^{-j135^\circ}}{1 + j\omega RC}.$$

Using the specified values, namely $R = \sqrt{3}\ \mathrm{k\Omega}$, $C = 1\ \mu\mathrm{F}$, and $\omega = 10^3$ rad/s,

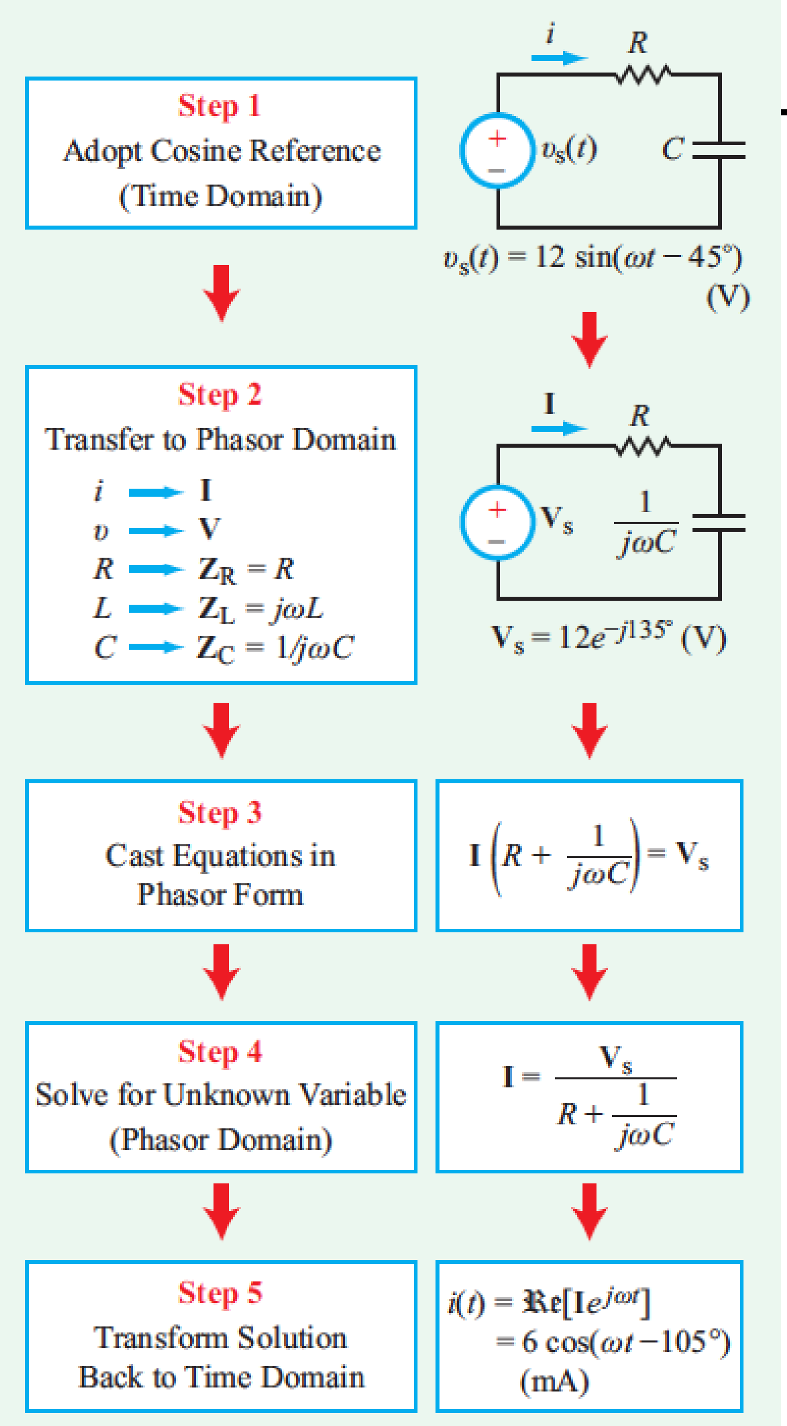
$$\mathbf{I} = \frac{j12 \times 10^3 \times 10^{-6} e^{-j135^\circ}}{1 + j10^3 \times \sqrt{3} \times 10^3 \times 10^{-6}} = \frac{j12e^{-j135^\circ}}{1 + j\sqrt{3}} \text{ mA.}$$

$$\mathbf{I} = \frac{12e^{-j135^\circ} \cdot e^{j90^\circ}}{2e^{j60^\circ}} = 6e^{j(-135^\circ + 90^\circ - 60^\circ)} = 6e^{-j105^\circ} \text{ mA.}$$

Step 5: Transform solution back to time domain

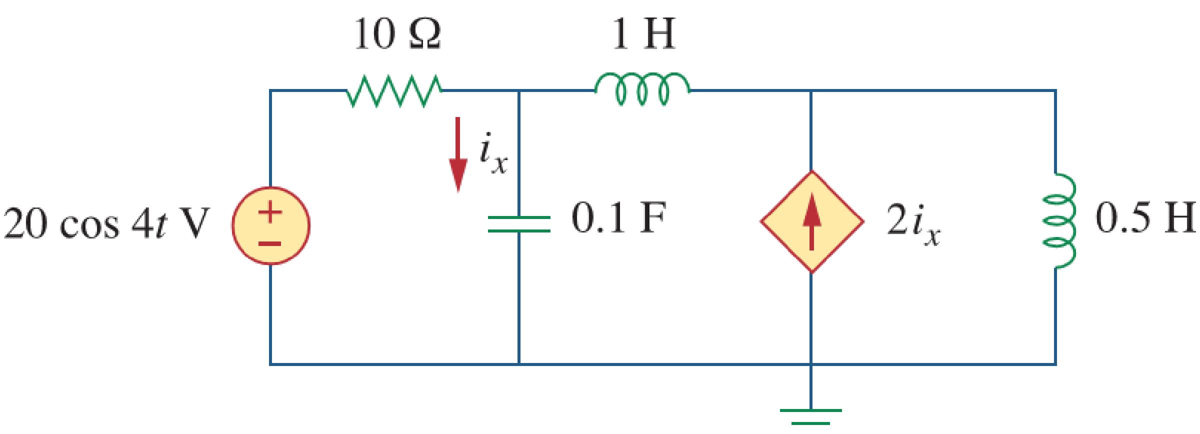
To return to the time domain, we apply the fundamental relation between a sinusoidal function and its phasor counterpart, namely

$$i(t) = \mathfrak{Re}[\mathbf{I}e^{j\omega t}] = \mathfrak{Re}[6e^{-j105^\circ} e^{j\omega t}] = 6\cos(\omega t - 105^\circ) \text{ mA.}$$

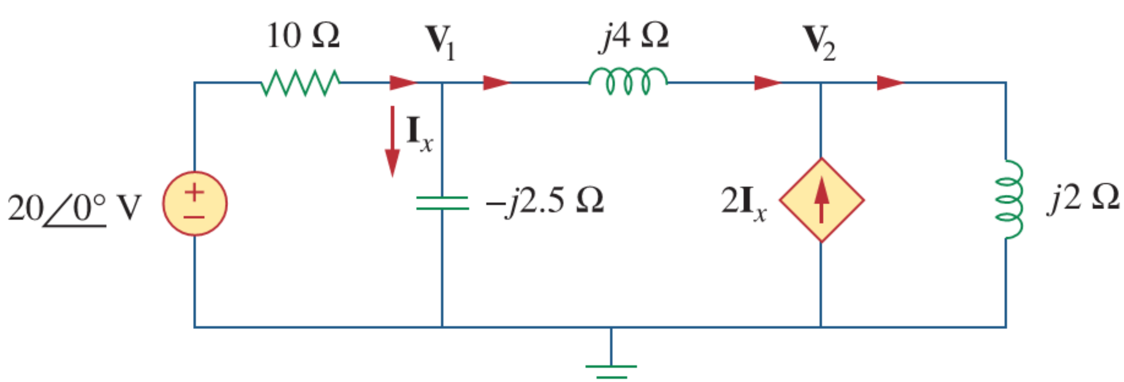


Example-Nodal Analysis

- Note that AC sources appear as DC sources with their values expressed as their amplitude.



Find i_x



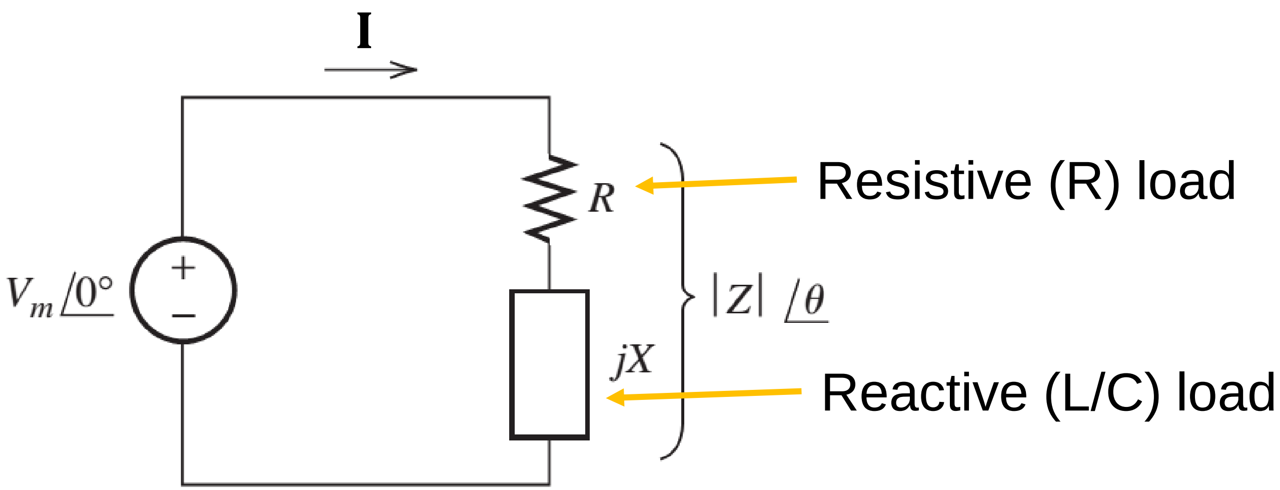
$$\frac{20 - \mathbf{V}_1}{10} = \frac{\mathbf{V}_1}{-j2.5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4}$$

$$2\mathbf{I}_x + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4} = \frac{\mathbf{V}_2}{j2}$$

$$i_x = 7.59 \cos(4t + 108.4^\circ) \text{ A}$$

Power in AC Circuits

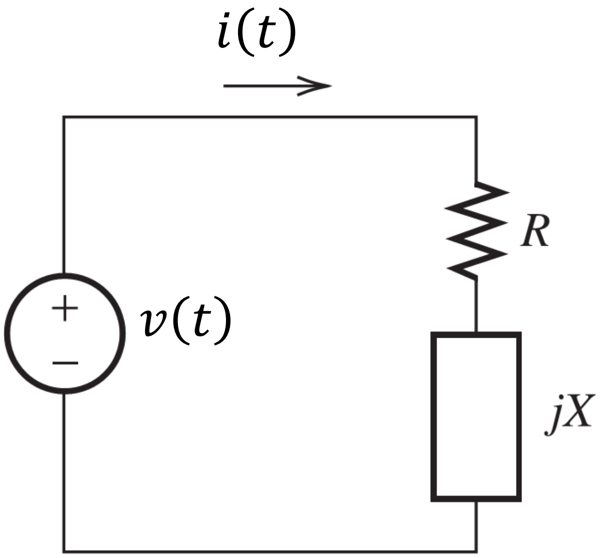
- Consider the situation shown below: A voltage $v(t) = V_m \cos(\omega t)$ is applied to an RLC network.



Quick Summary – Power Calculation

$$v(t) = V_m \cos(\omega t + \theta_v) \Longrightarrow \mathbf{V} = V_m \angle \theta_v$$

$$i(t) = I_m \cos(\omega t + \theta_i) \Longrightarrow \mathbf{I} = I_m \angle \theta_i$$



$$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) = V_{rms} I_{rms} \cos(\theta_v - \theta_i)$$

$$Q = \frac{1}{2} V_m I_m \sin(\theta_v - \theta_i) = V_{rms} I_{rms} \sin(\theta_v - \theta_i)$$

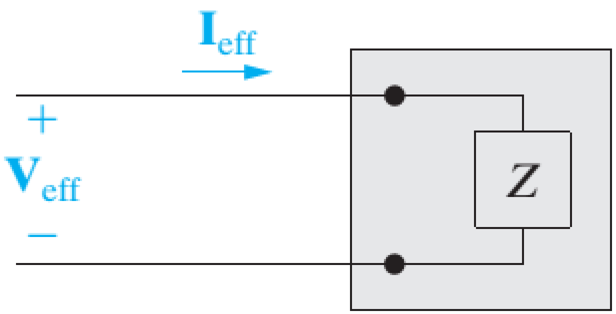
$$S = \frac{1}{2} V_m I_m = V_{rms} I_{rms}$$

$$\boxed{\mathbf{S} = S\angle(\theta_v - \theta_i) = P + jQ}$$

Another Way to Calculate Complex Power

$$\mathbf{S} = \mathbf{V}_{\mathrm{rms}}\mathbf{I}^*_{\mathrm{rms}}$$
$$= \mathbf{V}_{\mathrm{rms}}\left(\frac{\mathbf{V}_{\mathrm{rms}}}{Z}\right)^*$$
$$= \frac{|\mathbf{V}_{\mathrm{rms}}|^2}{Z^*}$$

$$\mathbf{S} = \mathbf{V}_{\mathrm{rms}}\mathbf{I}^*_{\mathrm{rms}}$$
$$= \mathbf{I}_{\mathrm{rms}}Z\mathbf{I}^*_{\mathrm{rms}}$$
$$= |\mathbf{I}_{\mathrm{rms}}|^2 Z$$
$$= |\mathbf{I}_{\mathrm{rms}}|^2 (R + jX)$$
$$= |\mathbf{I}_{\mathrm{rms}}|^2 R + j|\mathbf{I}_{\mathrm{rms}}|^2 X$$
$$= I_{rms}^2 R + jI_{rms}^2 X$$

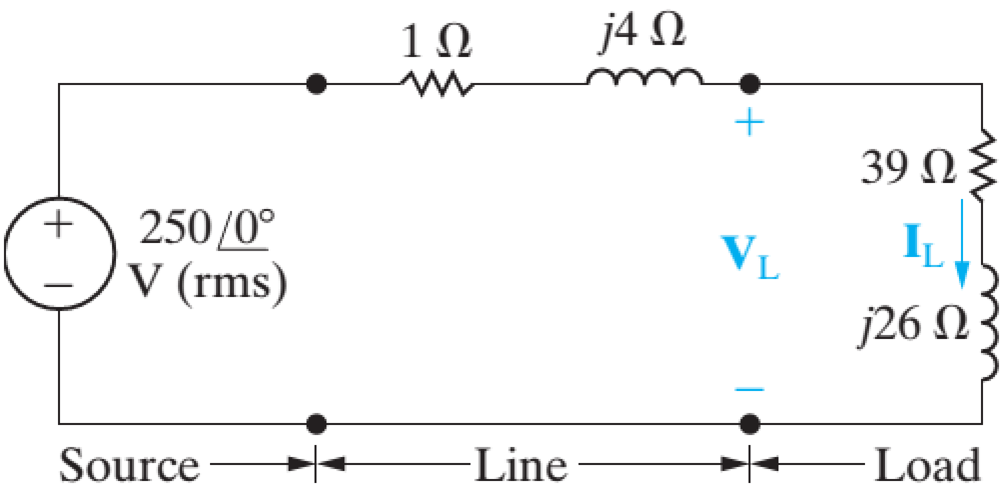


$$\mathbf{V}_{\mathrm{rms}} = \mathbf{I}_{\mathrm{rms}} Z$$

$$P = \mathrm{Re}(\mathbf{S}) = I^2_{\mathrm{rms}} R$$
$$Q = \mathrm{Im}(\mathbf{S}) = I^2_{\mathrm{rms}} X$$

$$\boxed{\mathbf{S} = I^2_{\mathrm{rms}}\mathbf{Z} = \frac{V^2_{\mathrm{rms}}}{\mathbf{Z}^*} = \mathbf{V}_{\mathrm{rms}}\mathbf{I}^*_{\mathrm{rms}}}$$

Example



- Find $\mathbf{V}_\mathrm{L}$ and $\mathbf{I}_\mathrm{L}$.
- Find the average and reactive power
  - Delivered to the load
  - Delivered to the line
  - Supplied by the source

$$\mathbf{I}_\mathrm{L} = \frac{250\angle 0^\circ}{40+j30} = 4 - j3$$
$$= 5\angle -36.87^\circ \text{ (rms)}$$

$$\mathbf{V}_\mathrm{L} = \mathbf{I}_\mathrm{L}(39 + j26)$$
$$= 234 - j13$$
$$= 234.36\angle -3.18^\circ$$

Load:

$$\mathbf{V}_\mathrm{L}\mathbf{I}_\mathrm{L}^* = 975 + \mathrm{j}650 \text{ VA}$$

Line:

$$P = (5)^2(1) = 25 \text{ W}$$

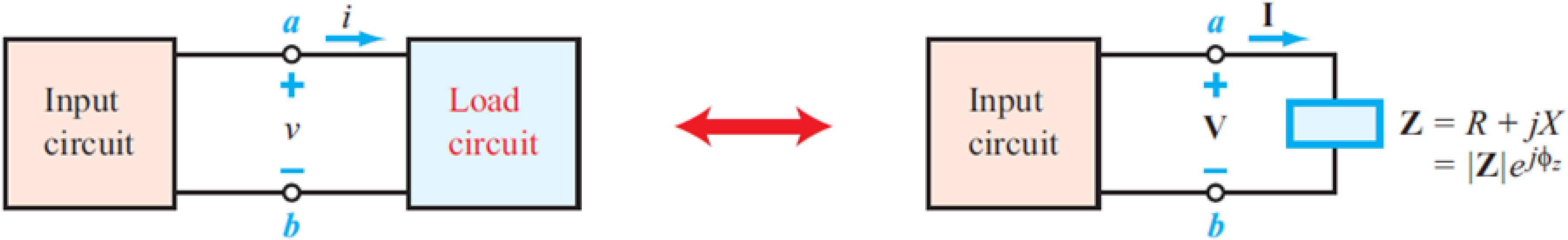
$$Q = (5)^2(4) = 100 \text{ VAR}$$

Source:

$$250\angle 0^\circ \mathbf{I}_\mathrm{L}^* = 1000 + \mathrm{j}750 \text{ VA}$$

Complex Power

Time Domain | Phasor Domain



$$v(t) = V_m \cos(\omega t + \phi_v) \longleftrightarrow \mathbf{V} = V_m e^{j\phi_v}$$

$$i(t) = I_m \cos(\omega t + \phi_i) \longleftrightarrow \mathbf{I} = I_m e^{j\phi_i}$$

$$V_{rms} = V_m/\sqrt{2} \longleftrightarrow \mathbf{V}_{rms} = V_{rms} e^{j\phi_v}$$

$$I_{rms} = I_m/\sqrt{2} \longleftrightarrow \mathbf{I}_{rms} = I_{rms} e^{j\phi_i}$$

Complex Power

$$\mathbf{S} = \frac{1}{2}\mathbf{V}\mathbf{I}^* = \mathbf{V}_{rms}\mathbf{I}^*_{rms} = P + jQ$$

Real Average Power

$$P = \Re\mathfrak{e}\,[\mathbf{S}] = V_{rms} I_{rms}\cos(\phi_v - \phi_i) = I^2_{rms} R$$

Reactive Power

$$Q = \Im\mathfrak{m}\,[\mathbf{S}] = V_{rms} I_{rms}\sin(\phi_v - \phi_i) = I^2_{rms} X$$

Apparent Power

$$S = |\mathbf{S}| = \sqrt{P^2 + Q^2} = V_{rms} I_{rms} = I^2_{rms}|\mathbf{Z}|$$

$$\mathbf{S} = S e^{j\phi_s}$$

$$\phi_s = \phi_v - \phi_i = \phi_z$$

Power Factor

$$pf = \frac{P}{S} = \cos(\phi_v - \phi_i) = \cos\phi_z$$

Example

- A series-connected load draws a current

$$i(t) = 4\cos(100\pi t + 10°)\text{A}$$

when the applied voltage is

$$v(t) = 120\cos(100\pi t - 20°)\text{V}$$

  - Find the apparent power and the power factor of the load.
  - Determine the values that form the series-connected load.

$$V_{\text{rms}} I_{\text{rms}} = 240 \text{ VA}$$

$$\text{pf} = \cos(\theta_v - \theta_i) = 0.866 \qquad \text{(leading)}$$

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = 25.98 - j15\ \Omega$$

$$C = \frac{1}{15\omega} = \frac{1}{15 \times 100\pi} = 212.2\ \mu\text{F}$$

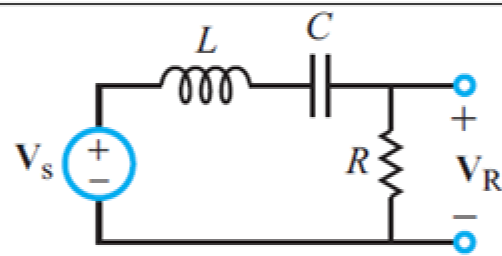
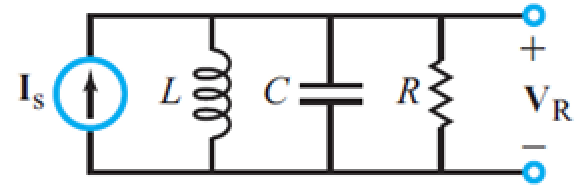
Exercise

- The voltage across a load is $v(t) = 60\cos(\omega t - 10°)\text{V}$, and the current through the load is $i(t) = 1.5\cos(\omega t + 50°)$. Find
  - The complex and apparent powers.
  - The real and reactive powers.
  - The power factor and the load impedance.

$$\mathbf{S} = \mathbf{V}_{\text{rms}}\mathbf{I}^*_{\text{rms}} = 45\underline{/-60°}\ \text{VA}$$

$$\text{pf} = 0.5\ \text{(leading)}$$

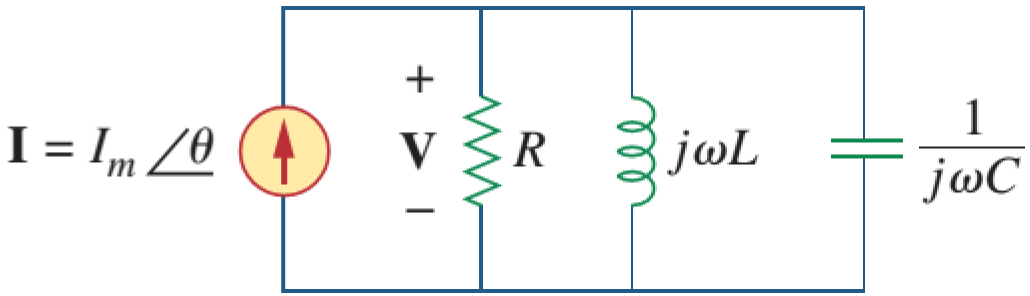
$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = 40\underline{/-60°}\ \Omega$$

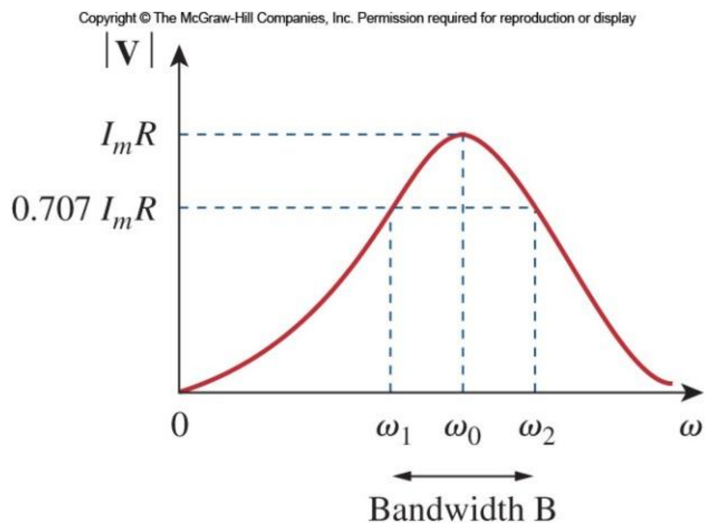
	Series	Parallel
RLC Circuit		
Transfer Function	$\mathbf{H} = \dfrac{\mathbf{V}_R}{\mathbf{V}_s}$	$\mathbf{H} = \dfrac{\mathbf{V}_R}{\mathbf{I}_s}$
Resonant Frequency, ω_0	$\dfrac{1}{\sqrt{LC}}$	$\dfrac{1}{\sqrt{LC}}$
Bandwidth, B	$\dfrac{R}{L}$	$\dfrac{1}{RC}$
Quality Factor, Q	$\dfrac{\omega_0}{B} = \dfrac{\omega_0 L}{R}$	$\dfrac{\omega_0}{B} = \dfrac{R}{\omega_0 L}$
Lower Half-Power Frequency, ω_1	$\left[-\dfrac{1}{2Q} + \sqrt{1 + \dfrac{1}{4Q^2}}\right]\omega_0$	$\left[-\dfrac{1}{2Q} + \sqrt{1 + \dfrac{1}{4Q^2}}\right]\omega_0$
Upper Half-Power Frequency, ω_2	$\left[\dfrac{1}{2Q} + \sqrt{1 + \dfrac{1}{4Q^2}}\right]\omega_0$	$\left[\dfrac{1}{2Q} + \sqrt{1 + \dfrac{1}{4Q^2}}\right]\omega_0$

Notes: (1) The expression for Q of the series RLC circuit is the inverse of that for Q of the parallel circuit. (2) For $Q \geq 10$, $\omega_1 \simeq \omega_0 - \frac{B}{2}$, and $\omega_2 \simeq \omega_0 + \frac{B}{2}$.

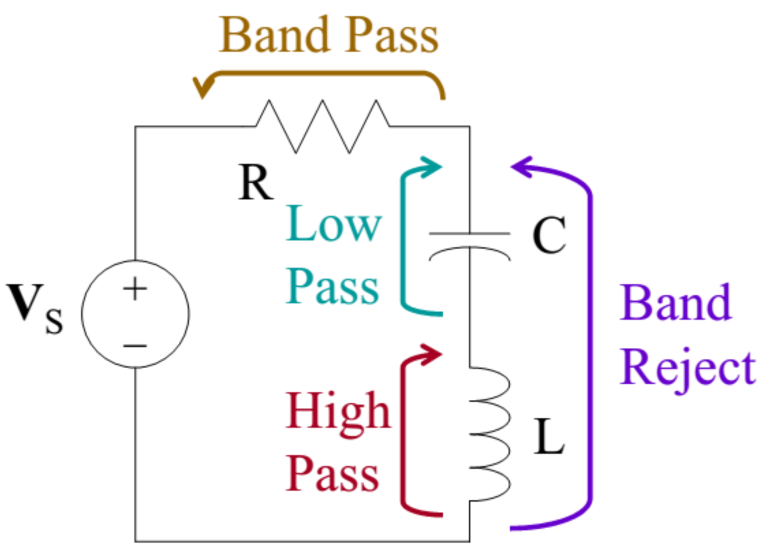
[Source: Berkeley]

Parallel resonance





Second-Order RLC Filter Circuits



$$\mathbf{Z} = R + 1/j\omega C + j\omega L$$

$$\mathbf{H}_{BP} = R / \mathbf{Z}$$

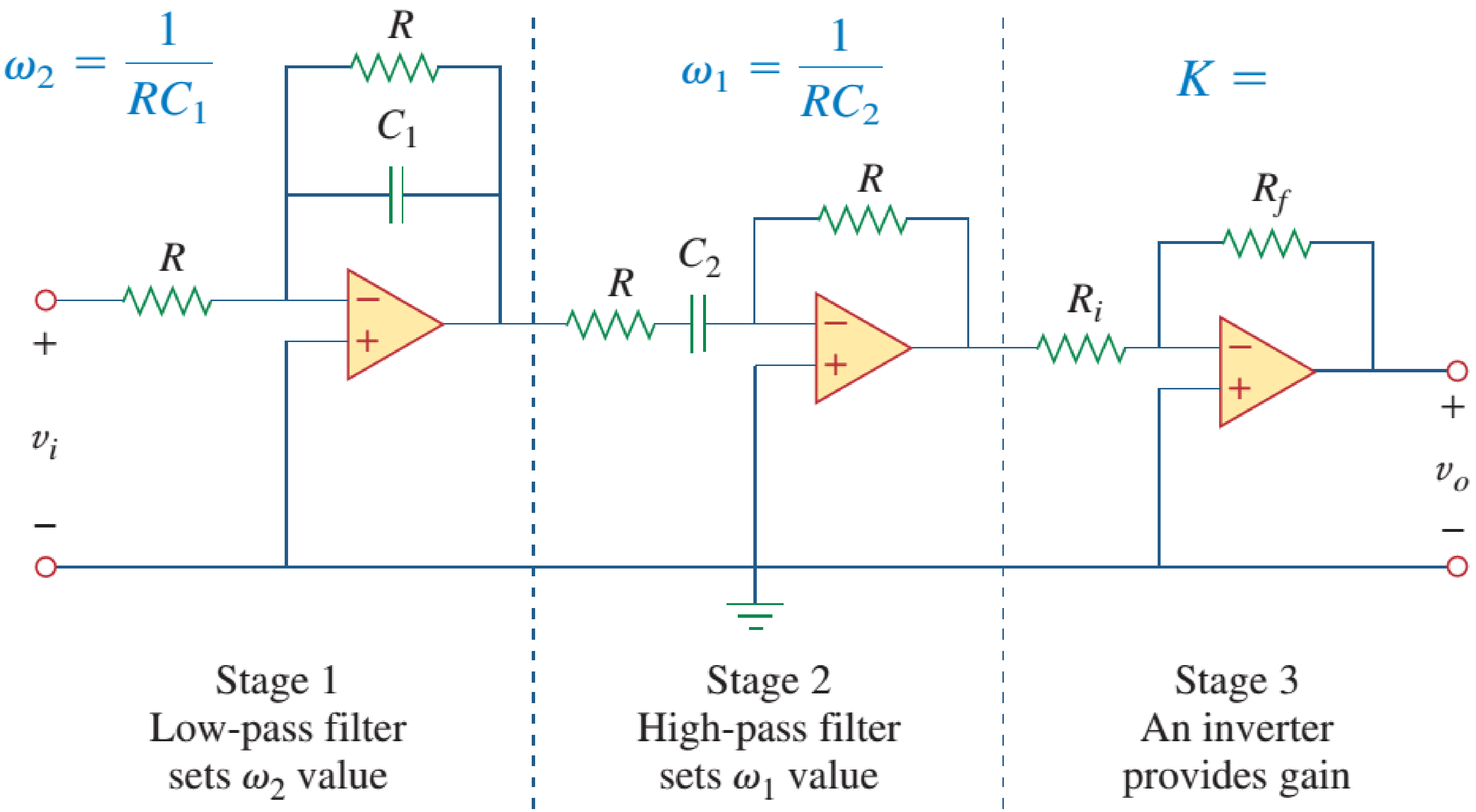
$$\mathbf{H}_{LP} = (1/j\omega C) / \mathbf{Z}$$

$$\mathbf{H}_{HP} = j\omega L / \mathbf{Z}$$

$$\mathbf{H}_{BR} = \mathbf{H}_{LP} + \mathbf{H}_{HP}$$

[Source: Berkeley]

Active Bandpass Filter



$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \left(-\frac{1}{1 + j\omega C_1 R}\right)\left(-\frac{j\omega C_2 R}{1 + j\omega C_2 R}\right)\left(-\frac{R_f}{R_i}\right)$$