EE 111 Homework 8

Due date: May. 29th, 2019

Turn in your homework in class

Rules:

- Work on your own. Discussion is permissible, but similar submissions will be judged as plagiarism.
- Please show all intermediate steps: a correct solution without an explanation will get zero credit.
- Please submit on time. No late submission will be accepted.
- Please prepare your submission in English only. No Chinese submission will be accepted.

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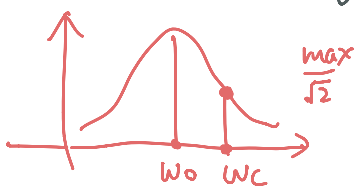
① transfer function $H(\omega)$

② type of fliter

③ cutoff frequency ω_c

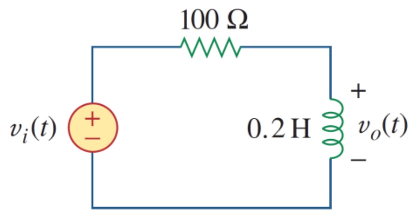
④ center frequency ω_0



$B = \omega_2 - \omega_1$

1. First Order Passive Filter (10 points)

An example filter in the following figure has the output of $v_o(t)$ and the input of $v_i(t)$. ① Determine the type of filter in the following figure and ② calculate the cutoff frequency f_c.



高通
低通
带阻滤波器
带通滤波器

$$H(\omega) = \frac{j\omega \cdot 0.2}{100 + j\omega \cdot 0.2} = \frac{2j\omega}{1000 + 2j\omega}$$

$$H(0) = 0 \quad , \quad H(\infty) = 1 \quad \Rightarrow \quad \text{High-pass filter} \; ①$$

$$|H(\omega)| = \frac{2\omega}{\sqrt{1000^2 + 4\omega^2}} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad \omega = 500$$

$$\therefore f_c = \frac{\omega}{2\pi} = \frac{500}{2\pi} = 79.58 \text{ Hz} \; ②$$

☆ 化小数

☆ 单位

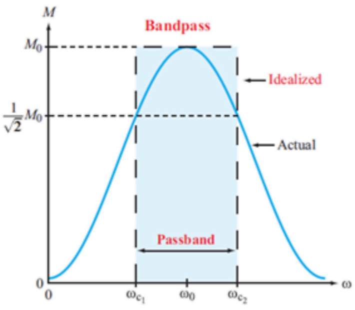
2. General Passive Filter (15 + 15 + 15 = 45 points) ②

① Determine the type of the following filters. Find the bandwidth and the ③ center frequency of the following filters.

(2a) The transfer function of an example filter is

$$H(\omega) = \frac{j\omega K_1}{(j\omega)^2 + j\omega K_1 + K_2^{\,2}}$$

where $K_1 > 0$ and $K_2 > 0$.



$H(0)=0, \quad H(\infty)=0 \Rightarrow$ Band-pass filter ①

$$H(\omega) = \frac{\omega K_1}{j\omega^2 + \omega K_1 - jK_2^2}$$

center frequency: $\omega - \frac{k_2^2}{\omega} = 0 \Rightarrow \omega_0 = k_2$ rad/s ③

$$|H(\omega)| = \frac{\omega K_1}{\sqrt{(k_2^2-\omega^2)^2 + \omega^2 k_1^2}} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow (k_2^2 - \omega^2)^2 = \omega^2 k_1^2$$

① when $\omega < k_2$

$$\omega k_1 = k_2^2 - \omega^2$$

$$\omega^2 + \omega k_1 - k_2^2 = 0$$

$$\Rightarrow \omega_1 = \frac{-k_1 - \sqrt{k_1^2 + 4k_2^2}}{2} < 0$$

$$\omega_2 = \frac{-k_1 + \sqrt{k_1^2 + 4k_2^2}}{2} > 0$$

② $\omega > k_2$

$$\omega k_1 = \omega^2 - k_2^2$$

$$\omega^2 - k_1\omega - k_2^2 = 0$$

$$\omega_1 = \frac{k_1 + \sqrt{k_1^2 + 4k_2^2}}{2} > 0$$

$$\omega_2 = \frac{k_1 - \sqrt{k_1^2 + 4k_2^2}}{2} < 0$$

取大于0

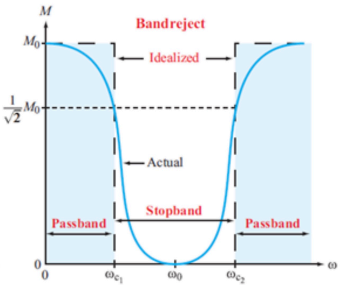
$$\therefore B = \frac{k_1 + \sqrt{k_1^2 + 4k_2^2}}{2} - \frac{-k_1 + \sqrt{k_1^2 + 4k_2^2}}{2}$$

$$= k_1 \text{ rad/s}$$ ②

(2b) The transfer function of an example filter is

$$H(\omega) = \frac{(j\omega)^2 + K_2^{\,2}}{(j\omega)^2 + j\omega K_1 + K_2^{\,2}}$$

where $K_1 > 0$ and $K_2 > 0$.



$H(0) = 1,\quad H(\infty) = 1 \Rightarrow$ band reject filter ①

$$H(\omega) = \frac{-\omega^2 + k_2^2}{-\omega^2 + j\omega k_1 + k_2^2}$$

$\because\ H(\omega = k_2) = 0 \Rightarrow \omega_0 = k_2$ rad/s ③

$$|H(\omega)| = \frac{k_2^2 - \omega^2}{\sqrt{\omega^2 k_1^2 + (k_2^2 - \omega^2)}} = \frac{1}{\sqrt{2}}$$

$$(k_2^2 - \omega^2) = \omega^2 k_1^2$$

与 2(a) 类似
舍去负的

① $\omega > k_2$

$$\omega_2 = \frac{k_1 + \sqrt{k_1^2 + 4k_2^2}}{2} > 0$$

$$\omega_1 = \frac{k_1 - \sqrt{k_1^2 + 4k_2^2}}{2} < 0$$

② $\omega < k_2$

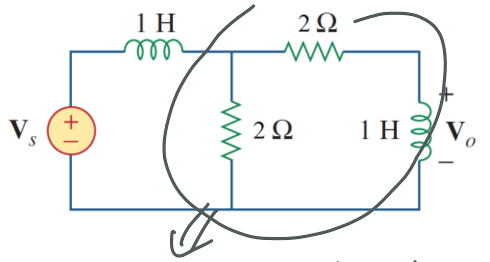
$$\omega_2 = \frac{-k_1 + \sqrt{k_1^2 + 4k_2^2}}{2} > 0$$

$$\omega_1 = \frac{-k_1 - \sqrt{k_1^2 + 4k_2^2}}{2} < 0$$

$\therefore\ B = \bigcirc - \bigcirc$

$= k_1$ rad/s ②

(2c) An example filter in the following figure has the output of $\mathbf{V}_o$ and the input of $\mathbf{V}_i$.



$$Z = 2 \,||\, (2+j\omega) = \frac{2(2+j\omega)}{4+j\omega}$$

$$V_o = \frac{\frac{4+2j\omega}{4+j\omega}}{j\omega + \frac{4+2j\omega}{4+j\omega}} \cdot \frac{j\omega}{2+j\omega} V_s$$

$$H(\omega) = \frac{4+2j\omega}{(4+j\omega)\cdot j\omega + 4+2j\omega} \cdot \frac{j\omega}{2+j\omega}$$

$$= \frac{2j\omega}{4-\omega^2+6j\omega} = \frac{2\omega}{-4j+j\omega^2+6\omega}$$

$H(0)=0$, $H(\infty)=0$, $\Rightarrow$ band-pass filter ①

$\omega^2 = 4 \Rightarrow \omega_0 = 2$ rad/s ③

$$\max H(\omega) = \frac{1}{3}$$

$$\therefore |H(\omega)| = \frac{2\omega}{\sqrt{(4-\omega^2)^2+36\omega^2}} = \frac{1}{3\sqrt{2}}$$

$$\omega_1 = \sqrt{22+12\sqrt{13}} = 3+\sqrt{13}$$

$$\omega_2 = \sqrt{13}-3$$

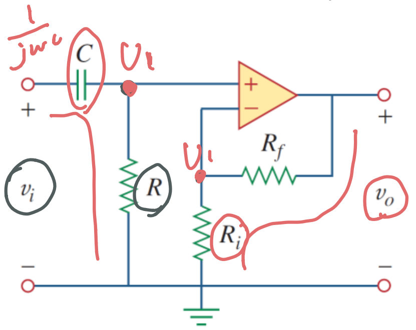
} 同前(a)(b)，舍负

$\therefore B = \omega_1 - \omega_2 = \sqrt{13}+3-\sqrt{13}+3 = 6$ rad/s ②

1 **3. General Active Filters (15 + 15 + 15 = 45 points)** 2

Find the transfer function of the filter and determine the type of the filter.

(3a) An example filter in the following figure has the output of v_o and the input of v_i.



$$V_1 = \frac{R}{R + \frac{1}{j\omega C}} V_i$$

KCL

node1: $\dfrac{V_i - V_1}{\frac{1}{j\omega C}} = \dfrac{V_1}{R}$

node2: $\dfrac{V_1}{R_i} = \dfrac{V_o - V_1}{R_f}$

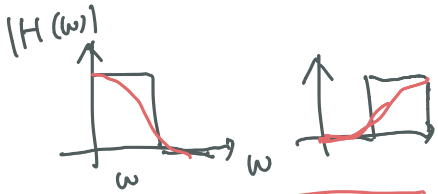
分压

$$\frac{R}{R + \frac{1}{j\omega C}} V_i = \frac{R_i}{R_i + R_f} V_o$$

$$\Rightarrow \quad H(\omega) = \frac{R_i + R_f}{R_i} \cdot \frac{j\omega RC}{1 + j\omega RC} \quad ①$$

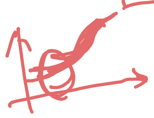
$H(0) = 0, \quad H(\infty) = 1 \quad \Rightarrow$ highpass ②

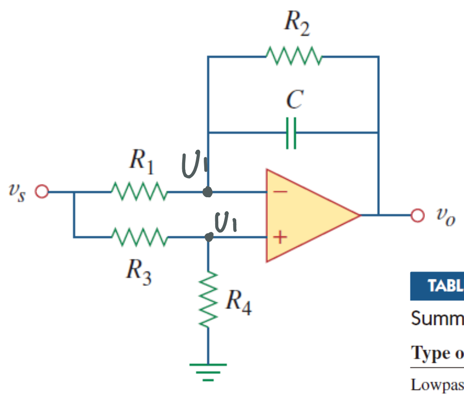
(3b) An example filter in the following figure has the output of v_o and the input of v_S. (Hint: the filter can be either highpass or lowpass filter based on the actual parameters of the circuit. Determine the parameter condition for the filter to be highpass and lowpass filter, respectively.)



$\boxed{H(0) < H(\infty)}$

$H(0)=0$ $\quad H(\infty)\neq 0$





几个错误:

① $H(0) < H(\infty)$

② $H(\infty) = 1$

TABLE 14.5

Summary of the characteristics of ideal filters.

Type of Filter	$H(0)$	$H(\infty)$	$H(\omega_c)$ or $H(\omega_0)$
Lowpass	1	0	$1/\sqrt{2}$
Highpass	0	1	$1/\sqrt{2}$
Bandpass	0	0	1
Bandstop	1	1	0

$$\begin{cases} \dfrac{U_1 - U_s}{R_1} + \dfrac{U_1 - U_o}{R_2 \parallel \frac{1}{j\omega C}} = 0 \\ U_1 = \dfrac{R_4}{R_3 + R_4} U_s \end{cases}$$

$$9' \Rightarrow H(\omega) = \frac{R_1 R_4 - R_2 R_3 + j\omega C R_1 R_2 R_3 R_4}{(R_4 + R_3)\cdot R_1 \cdot (1 + j\omega C R_2)}$$

$$\text{OR } H(\omega) = \frac{R_4}{R_3 + R_4} - \frac{R_2 R_3}{R_1 (R_3 + R_4)} \frac{1}{1 + R_2 j\omega C}$$

$$\text{OR } H(\omega) = \frac{R_4}{R_3 + R_4} \cdot \left(1 - \frac{R_3}{R_1 R_4} \cdot \frac{R_2}{j\omega C R_2 + 1}\right)$$

$$\text{OR } H(\omega) = \frac{R_4}{R_3 + R_4} \cdot \frac{j\omega C + \frac{1}{R_1} + \frac{1}{R_2} - \frac{R_3 + R_4}{R_1 R_4}}{j\omega C + \frac{1}{R_2}}$$

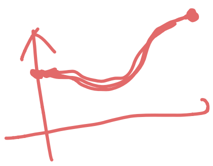
$$\text{OR } H(\omega) = \frac{R_4}{R_3 + R_4} \cdot \frac{j\omega + \frac{\frac{R_1}{R_2} - \frac{R_3}{R_4}}{C R_1}}{j\omega + \frac{1}{C R_2}}$$

$$\text{OR } H(\omega) = \frac{\frac{1}{R_2 R_3} - \frac{1}{R_1 R_4} + \frac{j\omega C}{R_3}}{\frac{1}{R_2 R_3} + \frac{1}{R_2 R_4} + j\omega C \left(\frac{1}{R_3} + \frac{1}{R_4}\right)}$$

令 $a = \frac{1}{R_1}$

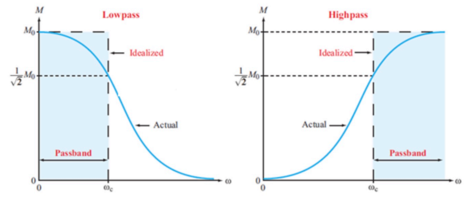
$b = \frac{1}{R_2}$

$s = j\omega$



$$H(0) = \frac{R_1 R_4 - R_2 R_3}{R_1 (R_4 + R_3)}$$

$$H(\infty) = \frac{R_4}{R_4 + R_3}$$

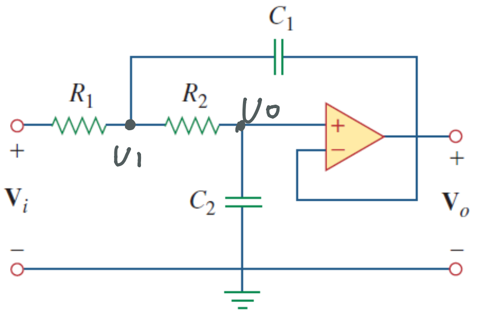


(a) Lowpass filter (b) Highpass filter

if highpass: $H(0) = 0$, $H(\infty) \neq 0 \Rightarrow R_4 \neq 0$, $R_1 R_4 = R_2 R_3$

if lowpass: $H(0) \neq 0$, $H(\infty) = 0 \Rightarrow R_4 = 0$, $R_2 R_3 \neq 0$

(3c) An example filter in the following figure has the output of $\mathbf{V}_o$ and the input of $\mathbf{V}_i$.



$$\begin{cases} \dfrac{V_1-V_i}{R_1} + \dfrac{V_1-V_0}{R_2} + \dfrac{V_1-V_0}{\frac{1}{j\omega C_1}} = 0 \\[2ex] \dfrac{V_1-V_0}{R_2} = \dfrac{V_0}{\frac{1}{j\omega C_2}} \end{cases}$$

$\Downarrow$ 自个儿算

$$\Rightarrow H(\omega) = \frac{V_0}{V_i} = \frac{1}{1-\omega^2 C_1 C_2 R_1 R_2 + j\omega C_2 (R_1+R_2)} \quad 1$$

$H(0) = 1$, $H(\infty) = 0$ $\Rightarrow$ lowpass 2