CS243: Homework #2

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Problem 1: Game Playing

(a) 1 credit

Alice and Bob are choosing a movie to watch together at home. They can only choose either a tragedy, a comedy or a documentary. Alice is preparing/choosing a movie and Bob going out to buy some popcorn or coke. Since the popcorn shop and coke shop are in different directions, Bob can only choose to buy one of them. Bob prefers coke to popcorn and he hates tragedy very much. Alice prefers popcorn to coke and she loves tragedy. Although she doesn't like comedy when having some coke, she enjoys watching comedy while having some popcorn very much. The payoff of the game is modelled as follows:

Alice \ Bob	Popcorn	Coke
Comedy	4,4	0,2
Documentary	2,0	2,2
Tragedy	3,0	1,0

Is there any pure strategy Nash equilibria? Is there any dominant strategy? If yes, list all of them.

Solution

(Comedy, Popcorn) and (Documentary, Coke) are pure strategy Nash equilibria. no dominant

(b) 1 credit

One hour later, Alice's mum came home and brought ten chicken wings. Alice and Bob will share the ten chicken wings and they both want to have as many as possible. Therefore, they played a game to decide: Each of them tells Alice's mum privately how many chicken wings they want. Suppose Alice wants a wings and Bob wants b wings. If $a + b \leq 10$, they receive all what they want. If $a + b > 10$, then the person who wants fewer wings gets what he/she wants while the other receives the rest of the wings. Is (5,5) a nash equilibrium? Is it unique? Prove your answer.

Solution

Neither Alice or Bob can get more chicken wings if the other doesn't change the strategy. So (5, 5) is a nash equilibrium. (5, 6) , (6, 5) and (6, 6) are also nash equilibrium. Not unique.

(c) 1 credit

The neighbourhood committee wants to show a great film in an opening area but needs a volunteer to arrange it with no payment. Suppose the volunteer have a utility of 3 while the other people who watch the film will have a utility of 5. Suppose the neighbourhood has a very large number of population. Is there any Nash equilibrium (including mixed strategies)? Explain your answer.
Assumption: we assume that all people apply the same strategy.

Solution
Suppose the probability that the person doesn't want to be a volunteer is p.
$5(1 - p^{n-1}) = 3$
$p = 1$ if n is very large.
People won't be volunteer.

Problem 2: Auctions

(a)1 credit

Tom wants to hire a temporary worker to do some work. Each worker has a private cost for doing the work. Help Tom to design a Vickrey-like auction in which workers report their costs and the auction chooses a worker and a payment. Truthful reporting should be a dominant strategy in the auction and the auction should also minimizes the cost for the hiring.

Solution
Each worker reports her valuation.
Choose the worker with the smallest private cost.
Charge the winner the second smallest private cost.

(b)1 credit

Tom is selling k identical copies of a digital good (e.g. a software). Suppose that there are $n > k$ bidders and each bidder wants at most one copy. Can you help Tom to design a second-price-like auction? Prove that your auction design is truthful.

Solution
Each buyer reports her valuation to Tom.
Tom sells the items to the buyers with the top k highest valuation report.
Charge the winners the $(k+1)^{th}$ highest valuation report.
Proof:
1. The payment p_i does not depend on v_i. 2. The mechanism optimizes for each player. For those who win the items, if he gives a higher valuation, his utility is still the same. If he gives a lower valuation, his utility may decrease to 0. For those who lose the items, if he gives a higher valuation and gets an item, his payment will be higher than his true valuation and thus his utility will be negative. So everyone will report his true valuation.

(c)1 credit

Consider an auction for selling one item: the item is allocated to the highest bidders and all bidders have to pay what they have bid. Is this auction truthful, efficient and individually rational? Give the proof or counter examples.

Solution
It is not truthful. For example, if person i report v_i truthfully and lose the auction, it could be better for him to report 0.
It is efficient. It maximises social welfare for all valuation reports (though the social welfare might be less than zero).
It is not individually rational because $u_i \leq 0$ if person i lose the auction.

Problem 3: VCG

An auctioneer is selling five items $M = a, b, c, d, e$. There are seven buyers $\{1, 2, 3, 4, 5, 6, 7\}$ with the following seven bids.
$B_1 = (a, b, 9)$,
$B_2 = (b, e, 12)$,
$B_3 = (c, d, e, 10)$,

$B_4 = (c, 6)$,
$B_5 = (a, c, 13)$,
$B_6 = (b, c, e, 16)$,
$B_7 = (b, d, e, 18)$.

(a)1 credit

What is the allocation of applying VCG?

Solution
$B_5 : \{a, c\}$
$B_7 : \{b, d, e\}$

(b)1 credit

What are their payments under VCG (each buyer pays the harm he caused to the others due to his participation)?

Solution
Without B_5, B_4 and B_7 will win. The social welfare will be 24. So B_5 should pay $24 - 18 = 6$. // Without B_7, B_2 and B_5 will win. The social welfare will be 25. So B_7 should pay $25 - 13 = 12$.

Problem 4: Social choice

Consider there are 5 candidates a, b, c, d, e for the president of a student union in ShanghaiTech with the following five preferences from five voters:

$$a \succ_1 b \succ_1 c \succ_1 d \succ_1 e$$

$$c \succ_2 a \succ_2 d \succ_2 e \succ_2 b$$

$$b \succ_3 c \succ_3 d \succ_3 a \succ_3 e$$

$$c \succ_4 b \succ_4 d \succ_4 a \succ_4 e$$

$$d \succ_5 b \succ_5 c \succ_5 a \succ_5 e$$

(a)0.5 credit

Consider a social choice function that assigns 1 score to each candidate ranked at top-3 in each preference and 0 score to the rest, and choose the winner with the highest total score (with random tie-breaking). Who will be the winner?

Solution

$$a = 2 \quad b = 4 \quad c = 5 \quad d = 4 \quad e = 0$$

Thus c would be chosen.

(b)1.5 credit

Is the above social choice function truthful? Give the proof or counter examples.

Solution
No.

$$b \succ_3 e \succ_3 a \succ_3 d \succ_3 c$$

$a \rightarrow 3 \quad b \rightarrow 4 \quad c \rightarrow 4 \quad d \rightarrow 3 \quad e \rightarrow 1$
After this misreport, b will have the probability to be chosen.