Homework 6

Due date: Dec.9th, 2021

Turn in your homework in class

Rules:

- Work on your own. Discussion is permissible, but extremely similar submissions will be judged as plagiarism.
- Please show all intermediate steps: a correct solution without an explanation will get zero credit.
- Please submit on time. No late submission will be accepted.
- Please prepare your submission in English only. No Chinese submission will be accepted.
- If needed, round the number to the nearest hundredths, i.e., rounding it to 2 decimal places.

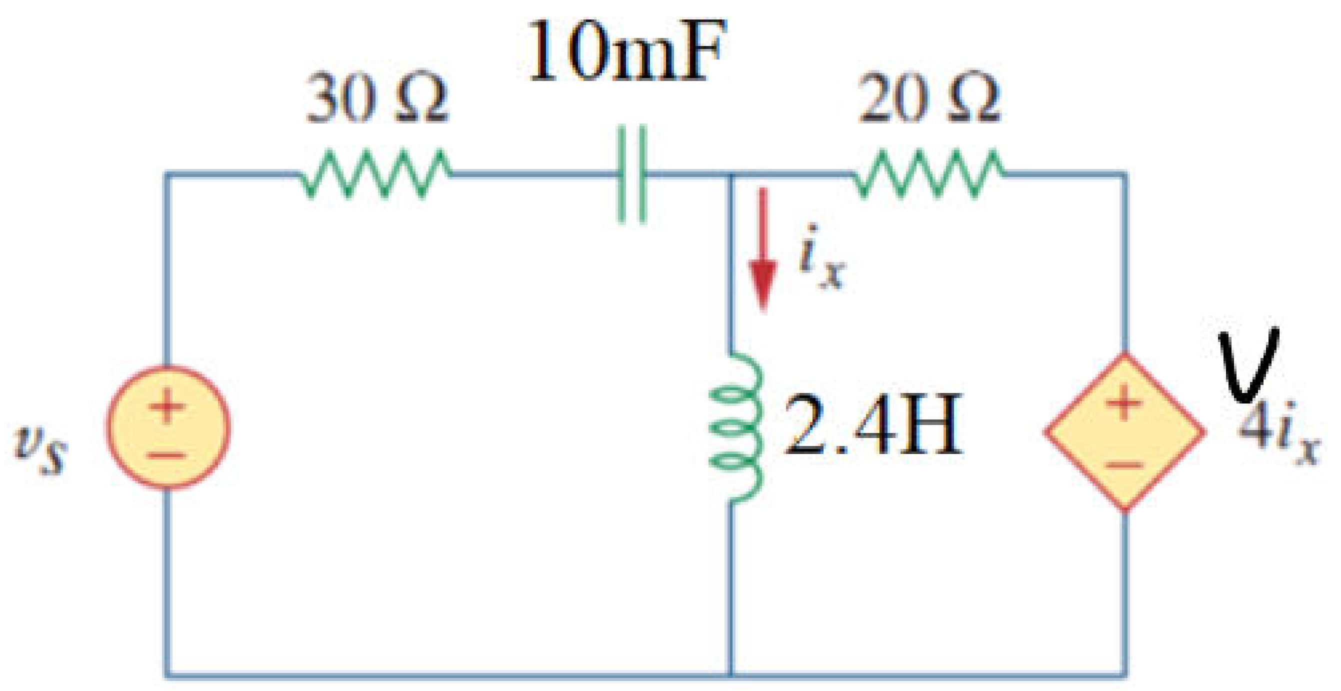
23

1. The circuit below is in steady state and $V_S = 40\sin(5t + 15°)V$ in time domain

a) Calculate $i_x(t)$ in S.S.

b) Calculate the **apparent power** on the 10mF capacitor.

c) Calculate the **complex power** absorbed by the **controlled source**.



$$Z_C = \frac{1}{j\omega C} = -20j\ \Omega$$

$$Z_L = j\omega L = 12j\ \Omega$$

$$V_s = 40\angle -75°\ V$$

(a)

$$KCL: \frac{V_s - V_x}{30-20j} + \frac{4i_x - V_x}{20} = i_x$$

$$V_x = 12j\ i_x$$

$$\frac{V_s}{30-20j} = \left[12j \cdot \left(\frac{1}{20} + \frac{1}{30-20j}\right) + 1 - \frac{1}{5}\right] i_x$$

$$i_x = -0.113 - 1.029j\ A = 1.034\angle -96.25°A$$
$$= 1.034\cos(5t - 96.25°)A$$

(b) $$i_C = \frac{V_s - 12j \cdot i_x}{30-20j} = 1.034\angle -59.38°A$$

$$V_C = i_C \cdot -20j = -17.796 - 10.533j\ V = 20.68\angle -149.38°V$$

$$S_{apparent} = \frac{1}{2} V_m I_m = 10.692\ VA$$

(c) $$i_{cs} = i_C - i_x = 0.654\angle 12.185°\ A$$

$$V_{cs} = 4i_x = 4.136\angle -96.25°V$$

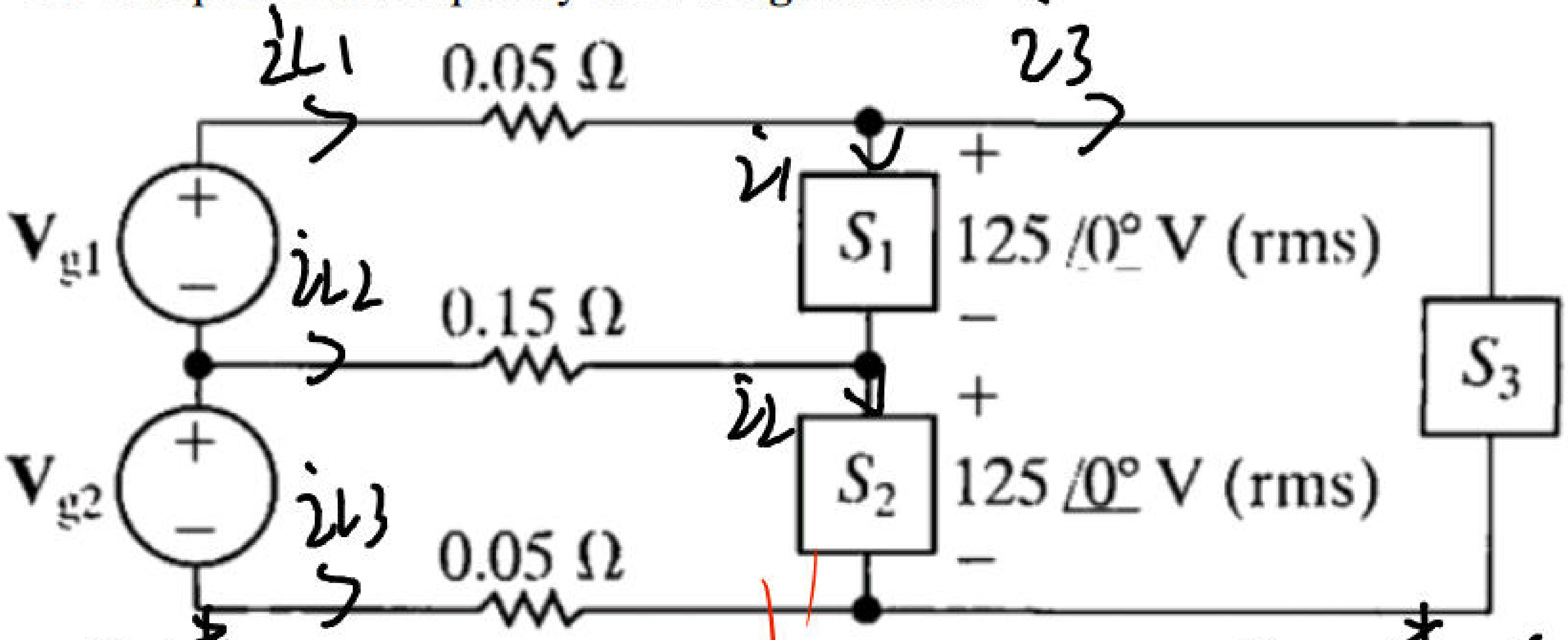
$$S = \frac{1}{2} V_{cs} \cdot i_{cs}^{*} = -0.428 - 1.283j\ VA = 1.352\angle -108.44°VA$$

31

2. Consider the following circuit with three elements S_1 S_2 and S_3. The voltages on the S_1 and S_2 are given. The complex power absorbed by S_1 S_2 and S_3 are

$S_1 = (10 + j2.5)kVA$, $S_2 = (12.5 + j5)kVA$, $S_3 = 4kVA$, respectively

a) Find $\mathbf{V}_{g1}$ (rms) and $\mathbf{V}_{g2}$ (rms).
b) Calculate the **complex power** developed by the voltage sources $\mathbf{V}_{g1}$ and $\mathbf{V}_{g2}$.
c) Prove that the total power dissipated by S_1 S_2 and S_3 **and the resistors** is equal to the total power developed by **the voltage sources**.



a) $i_1 = \left(\frac{S_1}{V_1}\right)^* = (80 - 20j)\,A_{rms}$ $\quad i_2 = \left(\frac{S_2}{V_2}\right)^* = (100 - 40j)\,A_{rms}$

$i_3 = \left(\frac{S_3}{V_3}\right)^* = 16\,A_{rms}$

$i_{L1} = i_1 + i_3 = (96 - 20j)\,A_{rms}$

$i_{L2} = -i_1 + i_2 = (20 - 20j)\,A_{rms}$

$i_{L3} = -i_2 - i_3 = (-116 + 40j)\,A_{rms}$

$\begin{cases} V_{g1} = 126.8\angle 0.9^\circ\,V_{rms} \\ V_{g2} = 133.9\angle -2.14^\circ\,V_{rms} \end{cases}$

KVL: $\begin{cases} V_{g1} = i_{L1} \cdot 0.05 + V_1 - i_{L2} \cdot 0.15 \\ V_{g2} = i_{L2} \cdot 0.15 + V_2 - i_{L3} \cdot 0.05 \end{cases} \Rightarrow \begin{cases} V_{g1} = 126.8 + 2j\,V_{rms} \\ V_{g2} = 133.8 - 5j\,V_{rms} \end{cases}$

b) $S_{g1} = -V_{g1} \cdot i_{L1}^* = -12132.8 - 2728j\ VA = 12435.7\angle -167^\circ\,VA$

$S_{g2} = V_{g2} \cdot i_{L3}^* = -15720.8 - 4772j\ VA = 16429.1\angle -163^\circ\,VA$

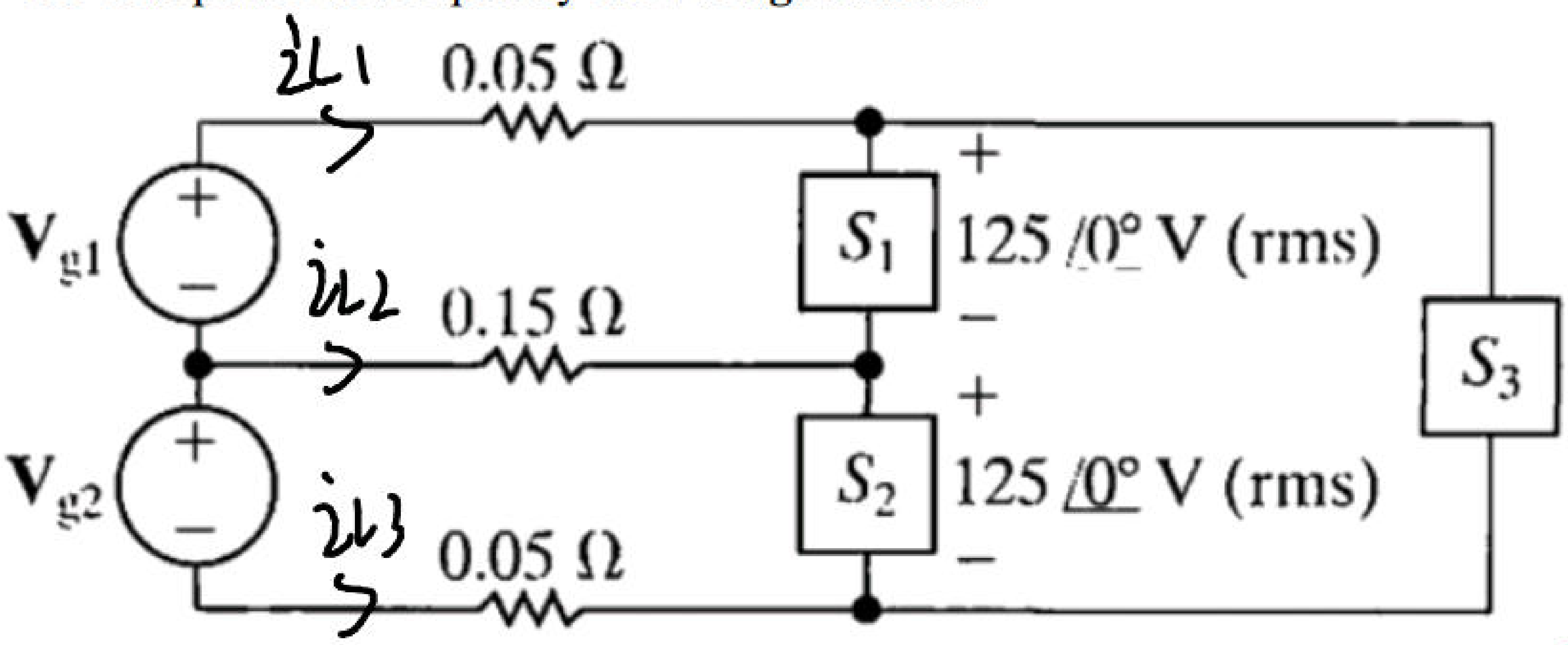
2. Consider the following circuit with three elements S_1 S_2 and S_3. The voltages on the S_1 and S_2 are given. The complex power absorbed by S_1 S_2 and S_3 are

$S_1 = (10 + j2.5)kVA, \quad S_2 = (12.5 + j5)kVA, \quad S_3 = 4kVA$, respectively

a) Find $\mathbf{V}_{g1}$ (rms) and $\mathbf{V}_{g2}$ (rms).

b) Calculate the **complex power** developed by the voltage sources $\mathbf{V}_{g1}$ and $\mathbf{V}_{g2}$.

c) Prove that the total power dissipated by S_1 S_2 and S_3 **and the resistors** is equal to the total power developed by **the voltage sources**.



c) $S_{R1} = \|i_{L1}\|^2 R_1 = 480.8\,VA$ 2'

$S_{R2} = \|i_{L2}\|^2 \cdot R_2 = 120\,VA$ 2'

$S_{R3} = \|i_{L3}\|^2 \cdot R_3 = 752.8\,VA$ 2'

$S_{release} = -27853.6 - 7500j\,VA = S_{g1} + S_{g2}$ 2'

$S_{dissipate} = \sum_{i=1}^{3} S_i + \sum_{i=1}^{3} S_{Ri}$ 2'

$= 27853.6 + 7500j\,VA$

$S_{release} + S_{dissipate} = 0$ QED 2'

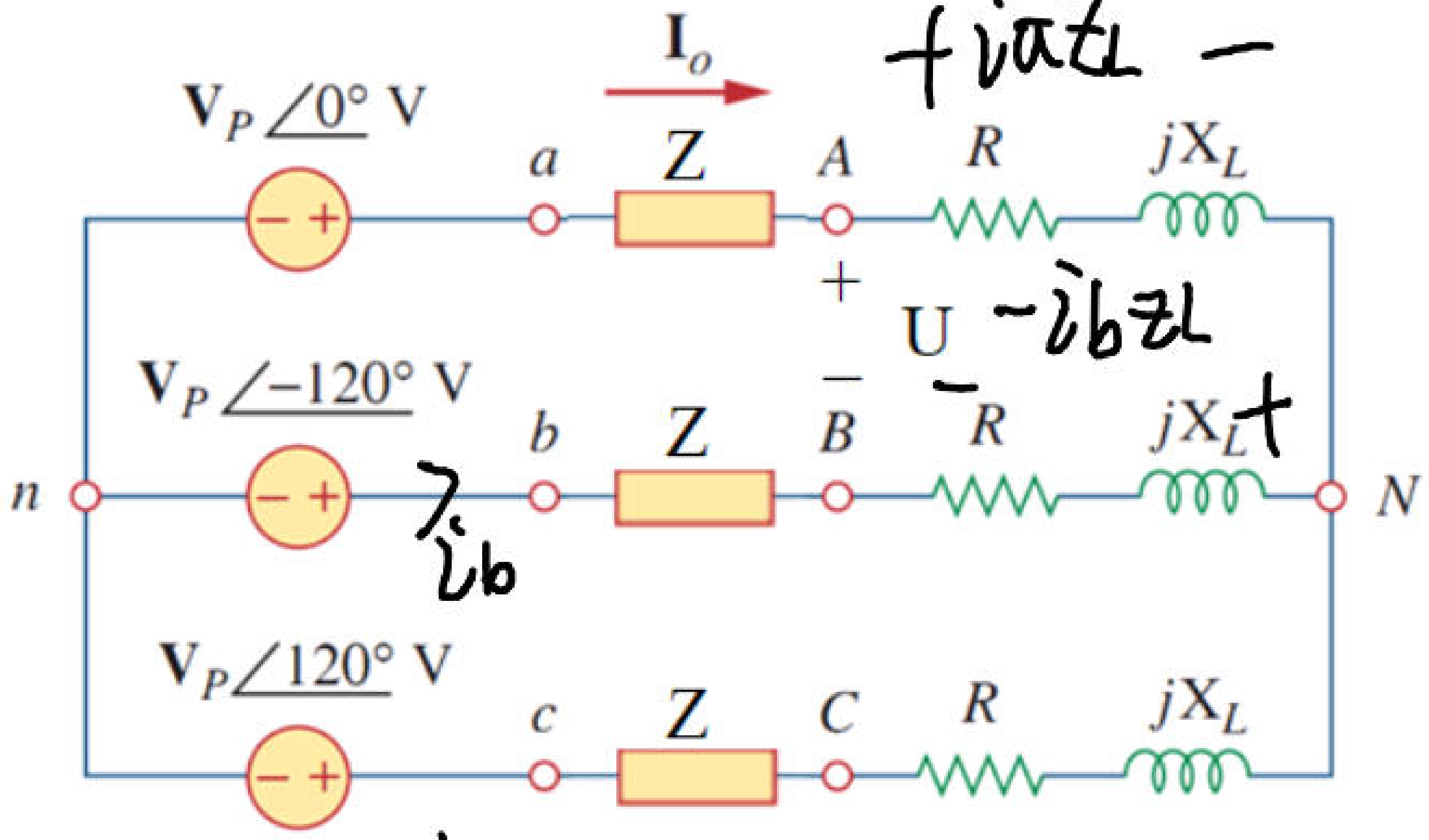
12'

3. Consider the following three-phase circuit. R=20Ω, X_L=5, V_p=50 (*rms*). Line impedance Z=1+j0.5Ω.

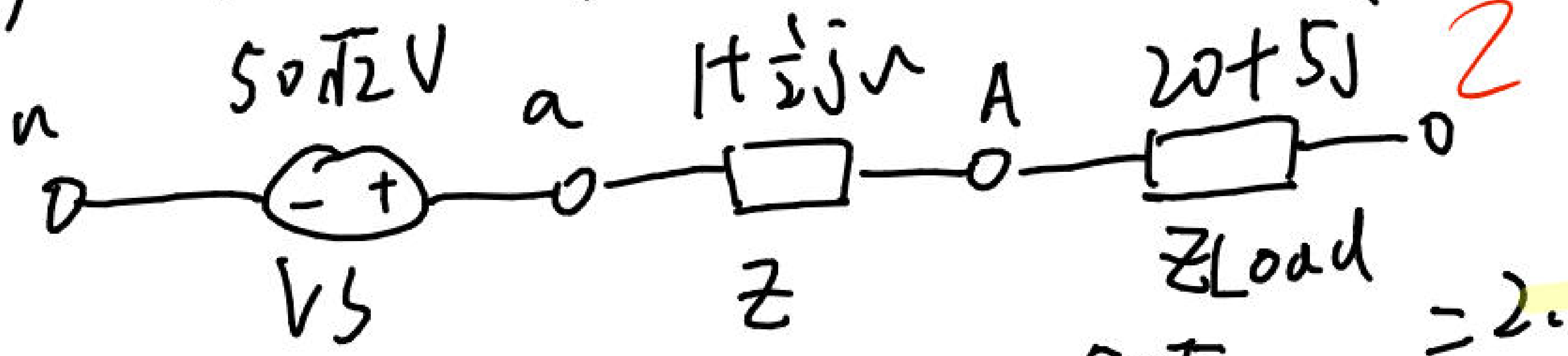
a) Calculate the **line current I_0**.

b) Calculate **the voltage U** shown in the circuit.

c) Calculate the total complex power of **all the loads**.



a) For one phase:



$$I_0 = i_a = \frac{V_s}{Z + Z_{load}} = \frac{50\angle 0}{21+5.5j} = 3.257\angle -14.676° A$$

$= 2.3\angle -14.676° A_{rms}$

2'

b) $i_b = i_a \angle -120° = 3.257 \angle -134.676° A$ 2'

$U = i_a Z_{Load} - i_b Z_{load} = 116.298\angle 29.360° V$

2' $= 82.235\angle 29.360° V_{rms}$

c)

2'

$S = \frac{1}{2} \cdot i_a \cdot i_a^* \cdot Z_{load} \cdot 3 = 318.24 + 79.56j$ VA

2'

If All Passive Elements

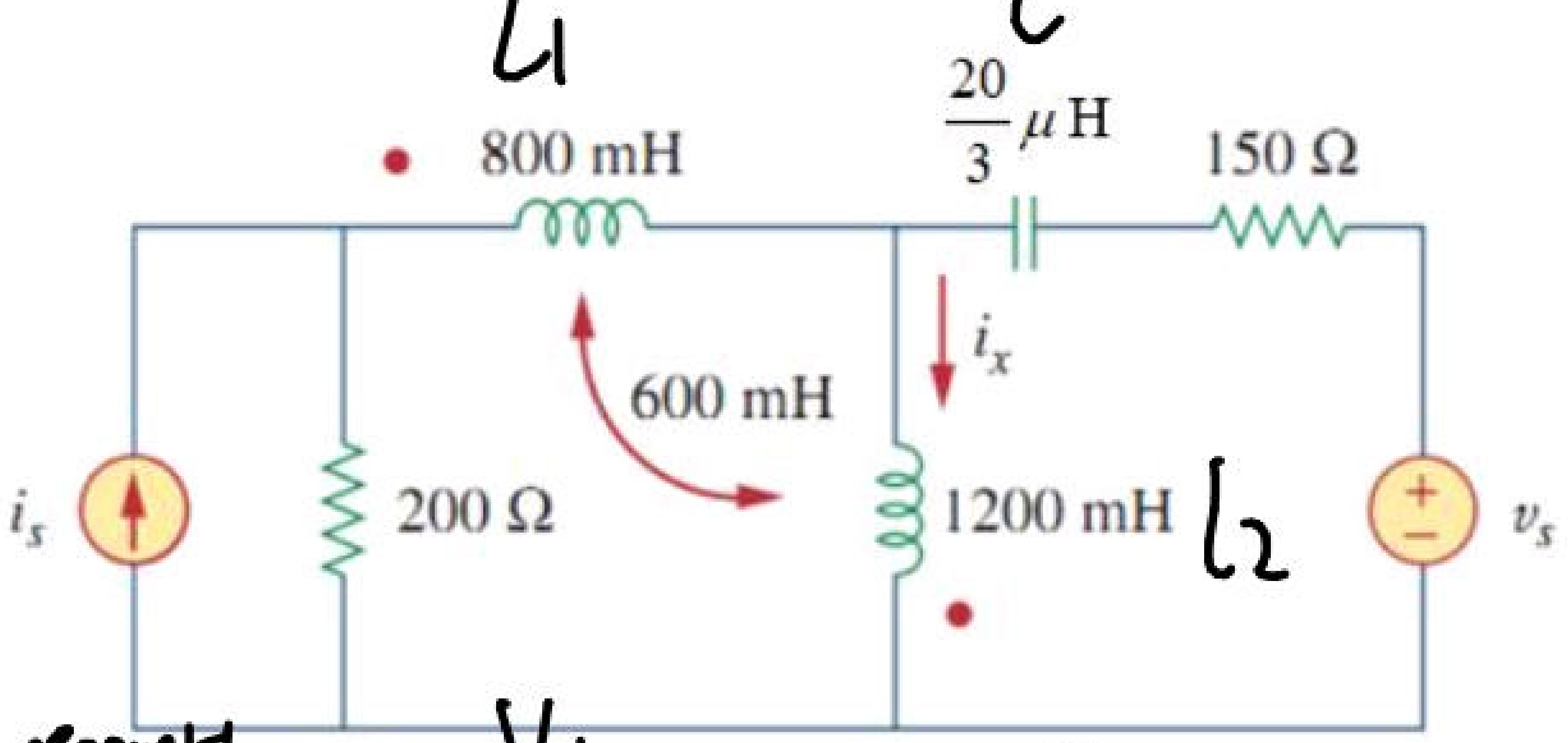
or

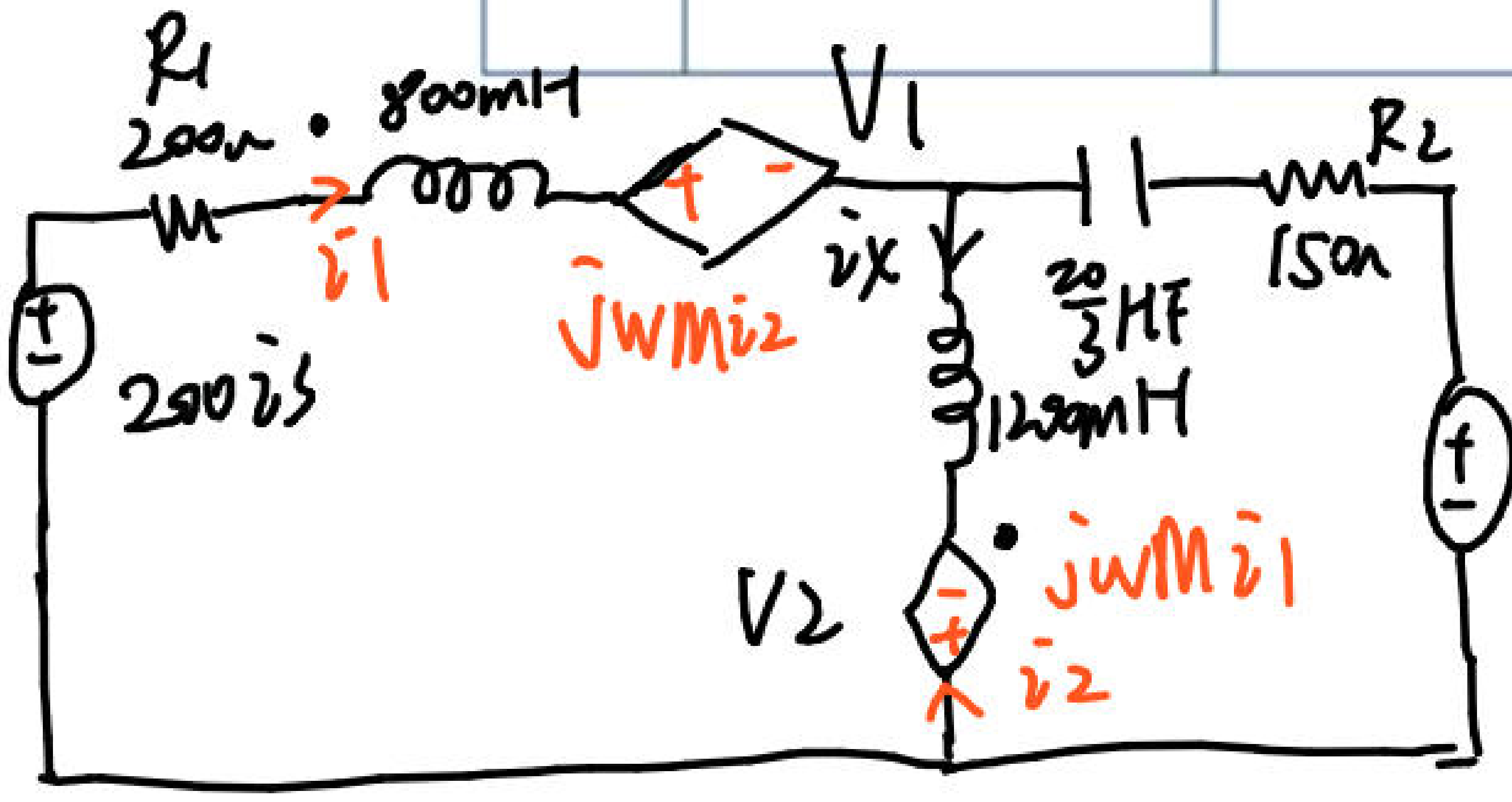
$S = \frac{1}{2}\, i_a \cdot i_a^* \cdot (Z_L + Z_{Line}) \cdot 3 = 345.42\angle 14.676°$ VA

Both Answers Give Full Points

34'

4. The following circuit includes a mutual inductance M=600mH, and is in steady state. Given $i_s = 4\sin(600t)A$, $v_s = 100\cos(300t + 60°)V$, calculate $i_x(t)$ in *S.S.*



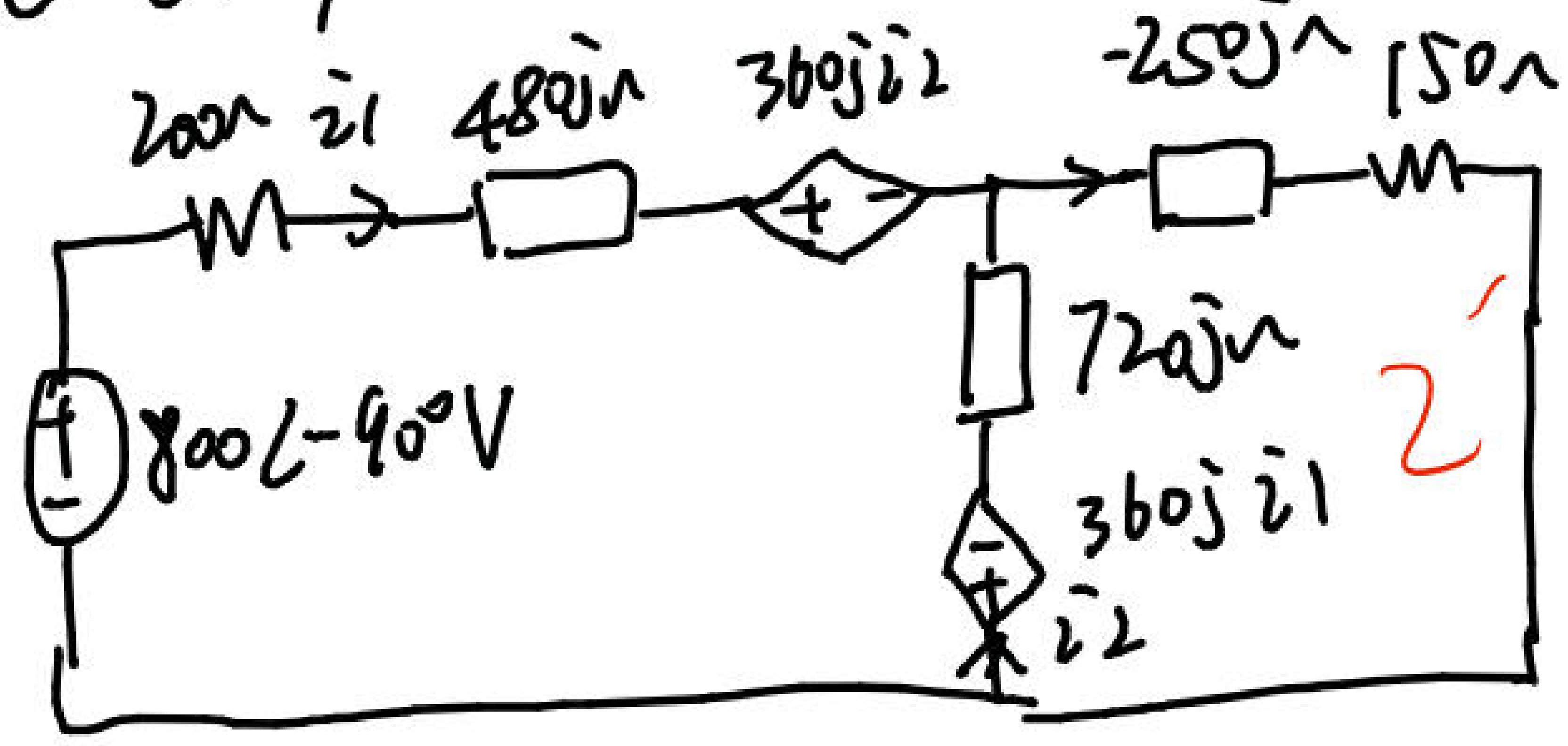


Equivalent Circuit

$\dot{i}_2 = -\dot{i}_x$

2

① Only is is on $w = 600$ rad/s



2'

$Z_{L1} = jwL_1 = 480j\,\Omega$

$Z_{L2} = jwL_2 = 720j\,\Omega$

$Z_C = \frac{1}{jwC} = -250j\,\Omega$

$V_1 = jwM\dot{i}_2 = 360j\,\dot{i}_2\ V$

$V_2 = jwM\dot{i}_1 = 360j\,\dot{i}_1$

KVL:

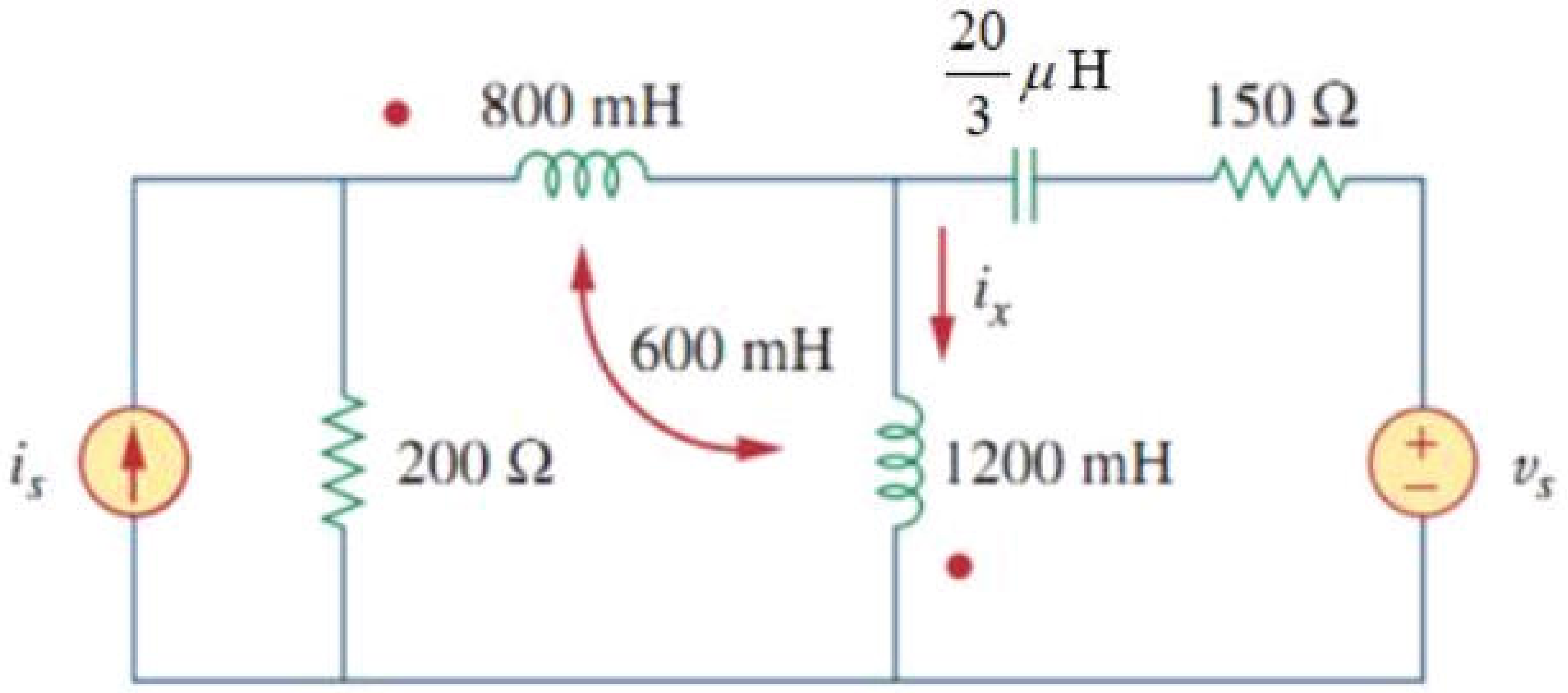
$$\begin{cases} R_1 \dot{i}_s = \dot{i}_1 \cdot Z_{L1} + V_1 + (\dot{i}_1 + \dot{i}_2)(Z_C + R_2) \\ V_2 + \dot{i}_2 \cdot Z_{L2} + (\dot{i}_1 + \dot{i}_2)(Z_C + R_2) = 0 \end{cases}$$ 2' 2'

$$800\angle -90° = \dot{i}_1 \cdot (200 + 480j) + 360j\,\dot{i}_2 + (150 - 250j)(\dot{i}_1 + \dot{i}_2)$$

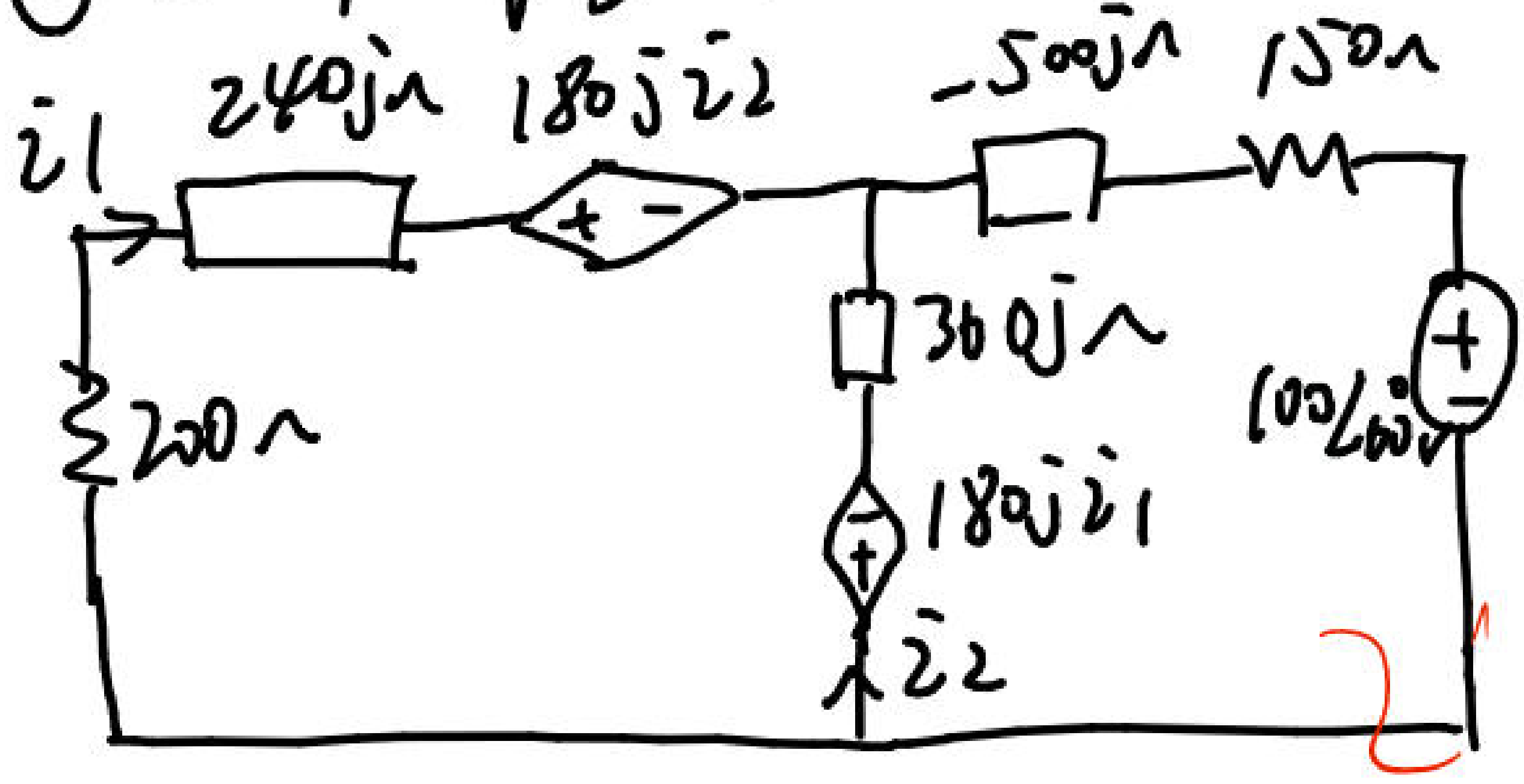
$$360j\,\dot{i}_1 + \dot{i}_2 \cdot 720j + (\dot{i}_1 + \dot{i}_2) \cdot (150 - 250j) = 0$$

$$\begin{cases} \dot{i}_2 = 0.833\angle 14.572°A \\ \dot{i}_1 = 2.209\angle -129.38°A \end{cases}$$ 2' 2'

4. The following circuit includes a mutual inductance M=600mH, and is in steady state. Given $i_s = 4\sin(600t)A$, $v_s = 100\cos(300t + 60°)V$, calculate $i_x(t)$ in ***S.S.***



② Only Vs is on $\quad W = 300 \text{rad/s}$



$Z_{L1} = 240j\,\Omega$

$Z_{L2} = 360j\,\Omega$

$Z_C = -500j\,\Omega$

$V_1 = 180j\,i_2$

$V_2 = 180j\,i_1$

$$\text{KVL:}\begin{cases} i_1\cdot(R_1 + Z_{L1}) + V_1 + (i_1+i_2)(Z_C + R_2) + V_S = 0 \\ i_2\cdot Z_{L2} + V_2 + (i_1+i_2)(Z_C + R_2) + V_S = 0 \end{cases}$$

$$\begin{cases} i_1\cdot(200+240j) + 180j\,i_2 + (i_1+i_2)(-500j+150) + 100\angle 60° = 0 \\ i_2\cdot 360j + 180j\,i_1 + (i_1+i_2)\cdot(-500j+150) + 100\angle 60° = 0 \end{cases}$$

$$\Rightarrow \begin{cases} i_1 = 0.187\angle -34.76°A \\ i_2 = 0.217\angle -108.06°A \end{cases} \quad i_x = -i_2 = 0.217\angle 71.94°A$$

Above all: $i_x(t) = 0.217\cos(300t + 71.94°) + 0.833\cos(600t + \cancel{14.52}^{\,-165.43}°)\,A$