Homework 7

Due date: 18:30 of Dec.16th, 2021

Turn in your homework in Class or to Tutorial Course Classroom 1B-110

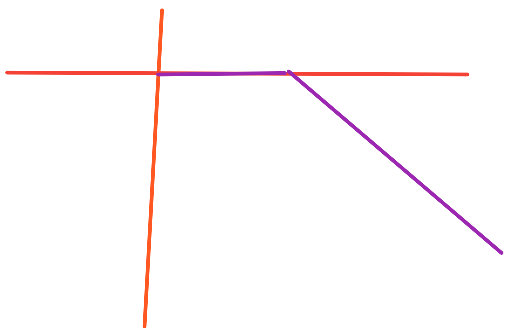
Rules:

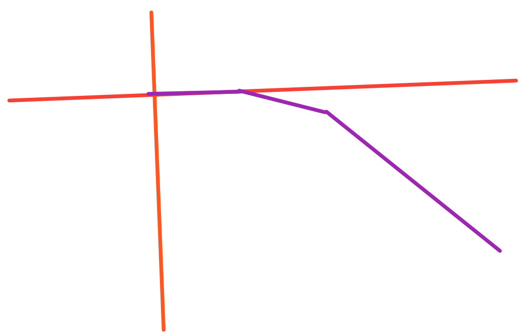
- Work on your own. Discussion is permissible, but extremely similar submissions will be judged as plagiarism.
- Please show all intermediate steps: a correct solution without an explanation will get zero credit.
- Please submit on time. No late submission will be accepted.
- Please prepare your submission in English only. No Chinese submission will be accepted.
- If needed, round the number to the nearest hundredths, i.e., rounding it to 2 decimal places.

Notes: $\frac{1}{Aj^2w^2+Bjw+C}$

or $\frac{1}{\left(\frac{jw}{w1}+1\right)\left(\frac{jw}{w2}+1\right)}$

These plots are all correct.





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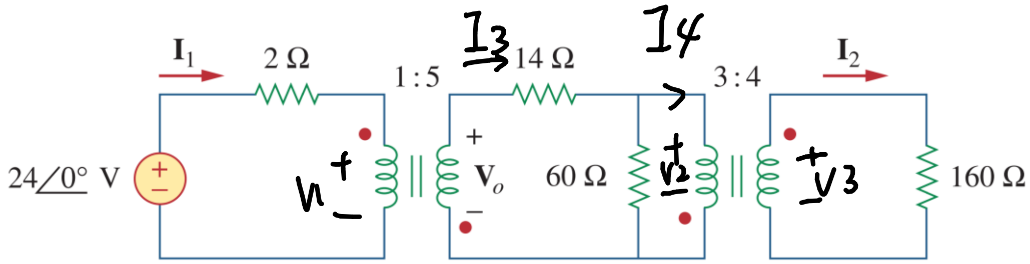
1. For the following ideal transformer circuit, calculate **I_1, I_2, and V_o**.



$$\begin{cases} \frac{V_o}{V_1} = -5 \quad 1 \quad \frac{I_3}{I_1} = -\frac{1}{5} \quad 1 \\ \frac{V_3}{V_2} = -\frac{4}{3} \quad 1 \quad \frac{I_2}{I_4} = -\frac{3}{4} \quad 1 \end{cases}$$

Suppose $\begin{cases} V_1 = X \\ I_1 = Y \\ I_4 = Z \end{cases} \Rightarrow \begin{cases} V_o = -5X \\ I_3 = -\frac{1}{5}Y \\ I_2 = -\frac{3}{4}Z \end{cases}$

KVL: $\begin{cases} V_s = I_1 \cdot 2\Omega + V_1 \Rightarrow 24\angle 0^\circ = 2Y + X \quad (1) \quad 3 \\ V_o = I_3 \cdot 14\Omega + V_2 \Rightarrow -5X = -\frac{14}{5}Y + V_2 \quad 3 \end{cases}$

KCL: $I_3 = I_4 + \frac{V_2}{60\Omega} \Rightarrow -\frac{1}{5}Y = Z + \frac{V_2}{60}$ 3

$V_2 = -5X + \frac{14}{5}Y \quad Z = -\frac{37}{150}Y + \frac{1}{12}X$ 1

$V_3 = -\frac{4}{3}V_2 = \frac{20}{3}X - \frac{56}{15}Y \quad I_2 = \frac{37}{200}Y - \frac{1}{16}X$ 1

KVL: $V_3 = I_2 \cdot 160\Omega \Rightarrow X = 2Y$ 3

Back to (1): $Y = 6\angle 0^\circ \quad X = 12\angle 0^\circ$ 1

$$\begin{cases} I_1 = Y = 6\angle 0^\circ A \quad 2, \quad I_2 = \frac{37}{200}Y - \frac{1}{16}X = 0.36\angle 0^\circ A \quad 2 \\ V_o = -5X = 60\angle -180^\circ V \quad 2 \end{cases}$$

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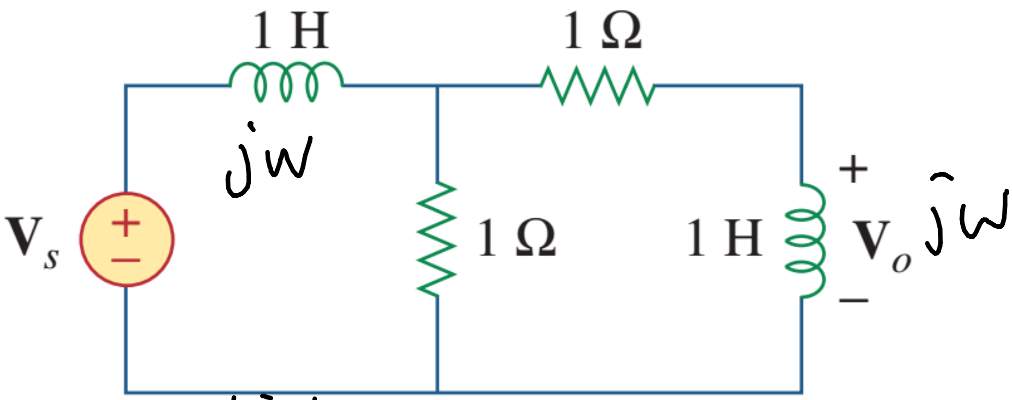
2. Derive the transfer function Y(ω), the voltage gain between $\mathbf{V_o}$ and the voltage source $\mathbf{V_s}$. Draw the corresponding bode plot.

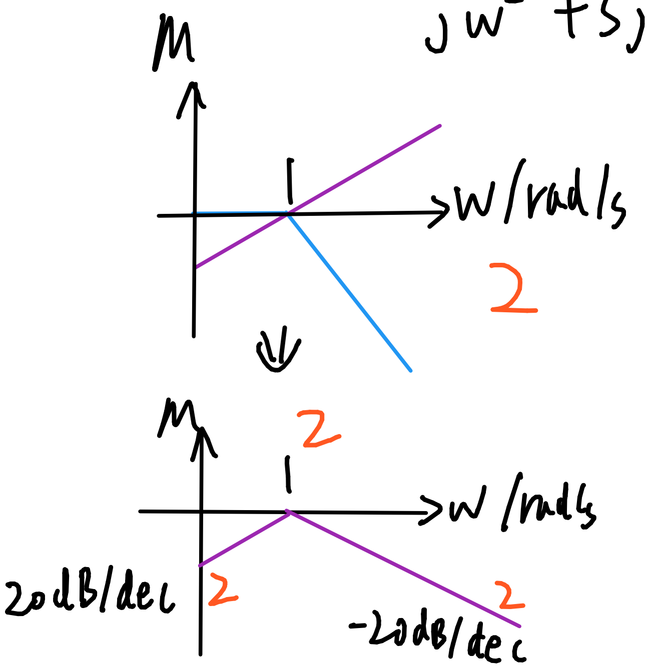


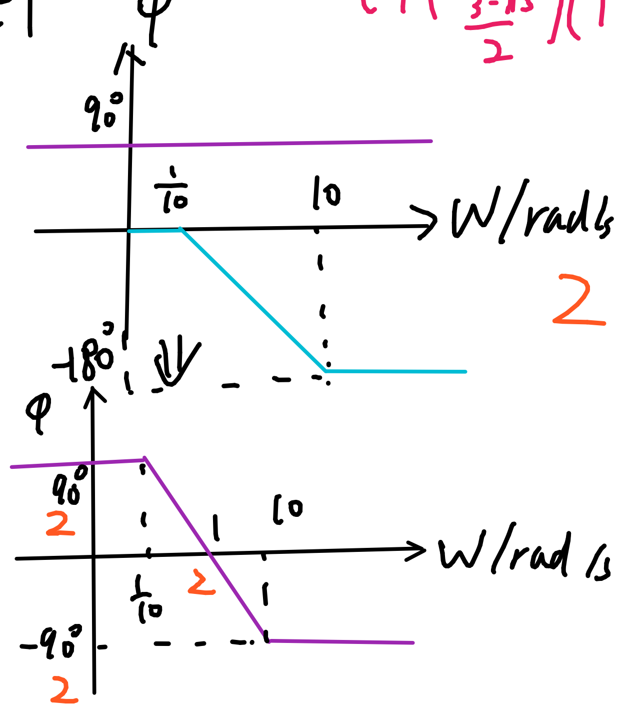
$$V_o = V_s \cdot \frac{\frac{1+jw}{2+jw}}{\frac{1+jw}{2+jw}+jw} \cdot \left(\frac{jw}{1+jw}\right)$$ 3

$$= V_s \frac{1+jw}{1+jw+2jw-w^2} \cdot \frac{jw}{1+jw}$$

$$= V_s \cdot \frac{jw}{(j^2w^2+3jw+1)}$$ 2

$$Y(w) = \frac{jw}{j^2w^2+3jw+1} = \frac{jw}{\left(1+\frac{jw}{\frac{3-\sqrt{5}}{2}}\right)\left(1+\frac{jw}{\frac{3+\sqrt{5}}{2}}\right)}$$





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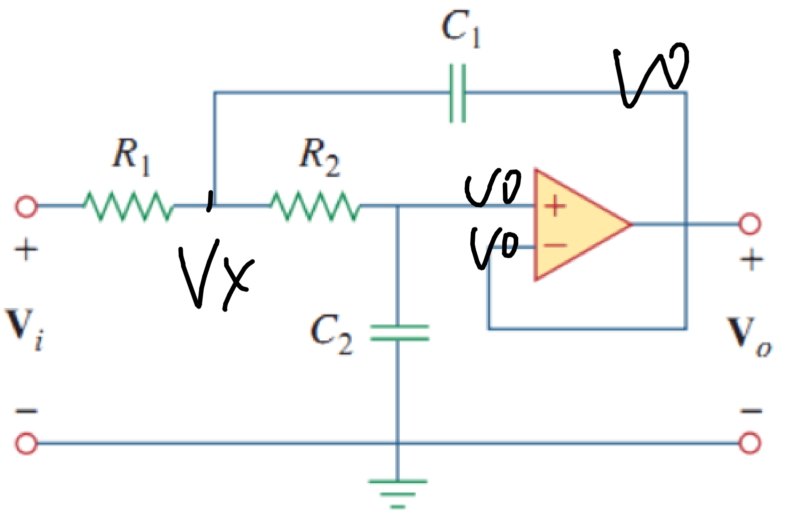
3. Consider the following circuit with an operational amplifier working in the linear region. The input/output voltage are **V**i, **V**o **, respectively**. The circuit is operating at the angular frequency **ω** rad/s.

1) Find the transfer function of the circuit **Y(ω)** = **V**o / **V**i.
2) Sketch the magnitude-frequency relation of bode plot wof **Y(ω).**
3) Determine what kind of filter it is from the bode plot.



1) $$\frac{V_i - V_x}{R_1} + \frac{V_o - V_x}{R_2} + j\omega C_1 \cdot (V_o - V_x) = 0$$ 4

$$\frac{V_x - V_o}{R_2} = j\omega C_2 \cdot V_o$$ 4

$$V_x = V_o \cdot (R_2 j\omega C_2 + 1)$$ 2

$$R_2 V_i - R_2 V_x + R_1 V_o - R_1 V_x = j\omega C_1 R_1 R_2 (V_x - V_o)$$

$$R_2 V_i + R_1 V_o - (R_1 + R_2)(R_2 j\omega C_2 + 1) V_o = j^2\omega^2 C_1 R_1 R_2^2 C_2 V_o$$ 2

$$\frac{V_o}{V_i} = \frac{R_2}{R_2 + (R_1 + R_2)(R_2 j\omega C_2) + j^2\omega^2 R_1 R_2^2 C_1 C_2}$$ 2

$$= \frac{1}{R_1 R_2 C_1 C_2 j^2 \omega^2 + (R_1 + R_2) C_2 j\omega + 1}$$

3. Consider the following circuit with an operational amplifier working in the linear region. The input/output voltage are V_i, V_o, **respectively**. The circuit is operating at the angular frequency ω rad/s.

1) Find the transfer function of the circuit $\mathbf{Y(\omega)} = V_o / V_i$.
2) Sketch the magnitude-frequency relation of bode plot wof $\mathbf{Y(\omega)}$.
3) Determine what kind of filter it is from the bode plot.

3) Low Pass Filter

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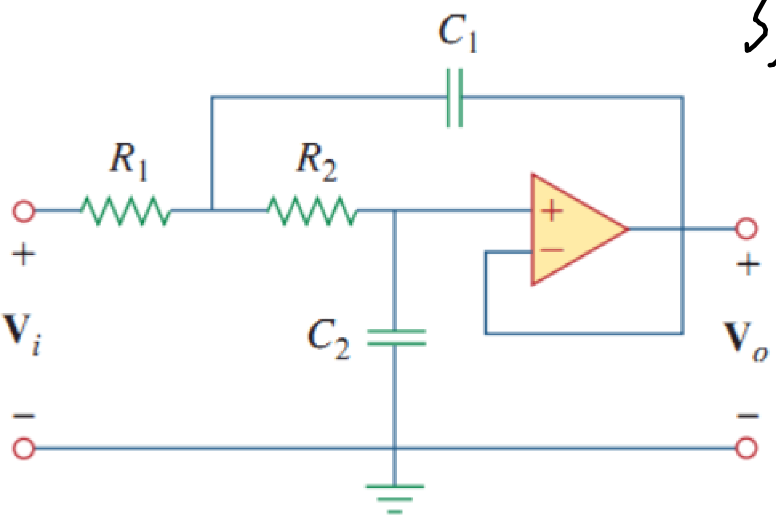


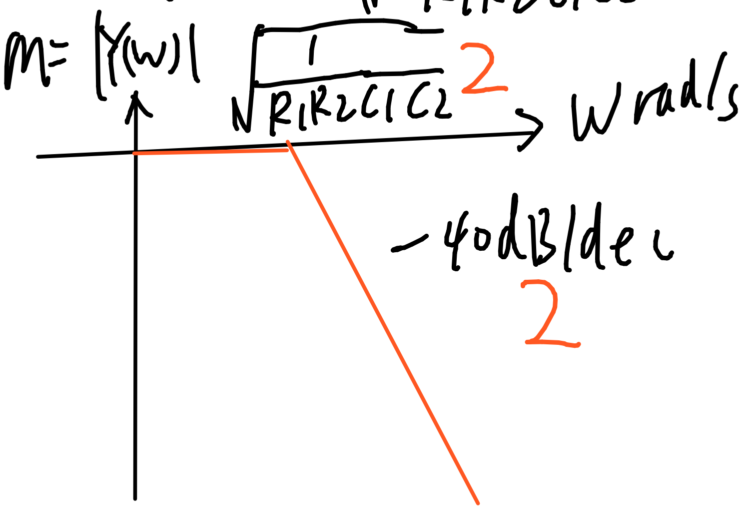
$$Y(w) = \frac{1}{R_1R_2C_1C_2\, j^2 w^2 + (R_1+R_2)C_2\, jw + 1}$$

3

$$= \frac{1}{\frac{j^2 w^2}{\frac{1}{R_1R_2C_1C_2}} + (R_1+R_2)C_2\, jw + 1}$$

$$w_k = \sqrt{\frac{1}{R_1R_2C_1C_2}}$$

2



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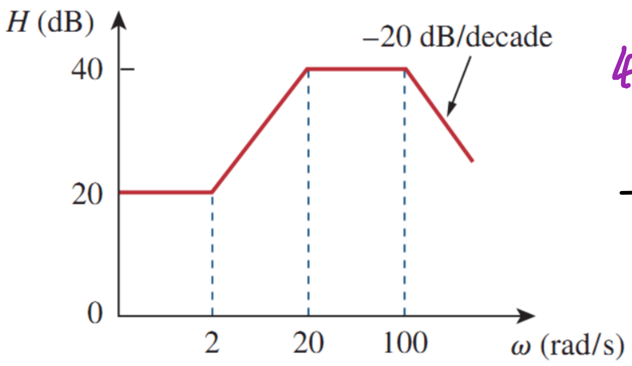
4. Determine a possible **transfer function** for the following magnitude graph; Draw the **phase-frequency plot** according to the transfer function that you obtained.



Constant: $20\log_{10}K = 20$ 2 $K = 10$ 2

Slope 1 = 20 dB/decade 2

$H_1(w) = (1 + \frac{jw}{2})$ 2

Slope 2 = 0 2 $\Rightarrow H_2(w) = \frac{1}{(1+\frac{jw}{20})}$ 2

Slope 3 = -20 dB 2 $\Rightarrow H_3(w) = \frac{1}{1+\frac{jw}{100}}$ 2

$$H(w) = \frac{10(1+\frac{jw}{2})}{(1+\frac{jw}{20})\cdot(1+\frac{jw}{100})}$$ 4