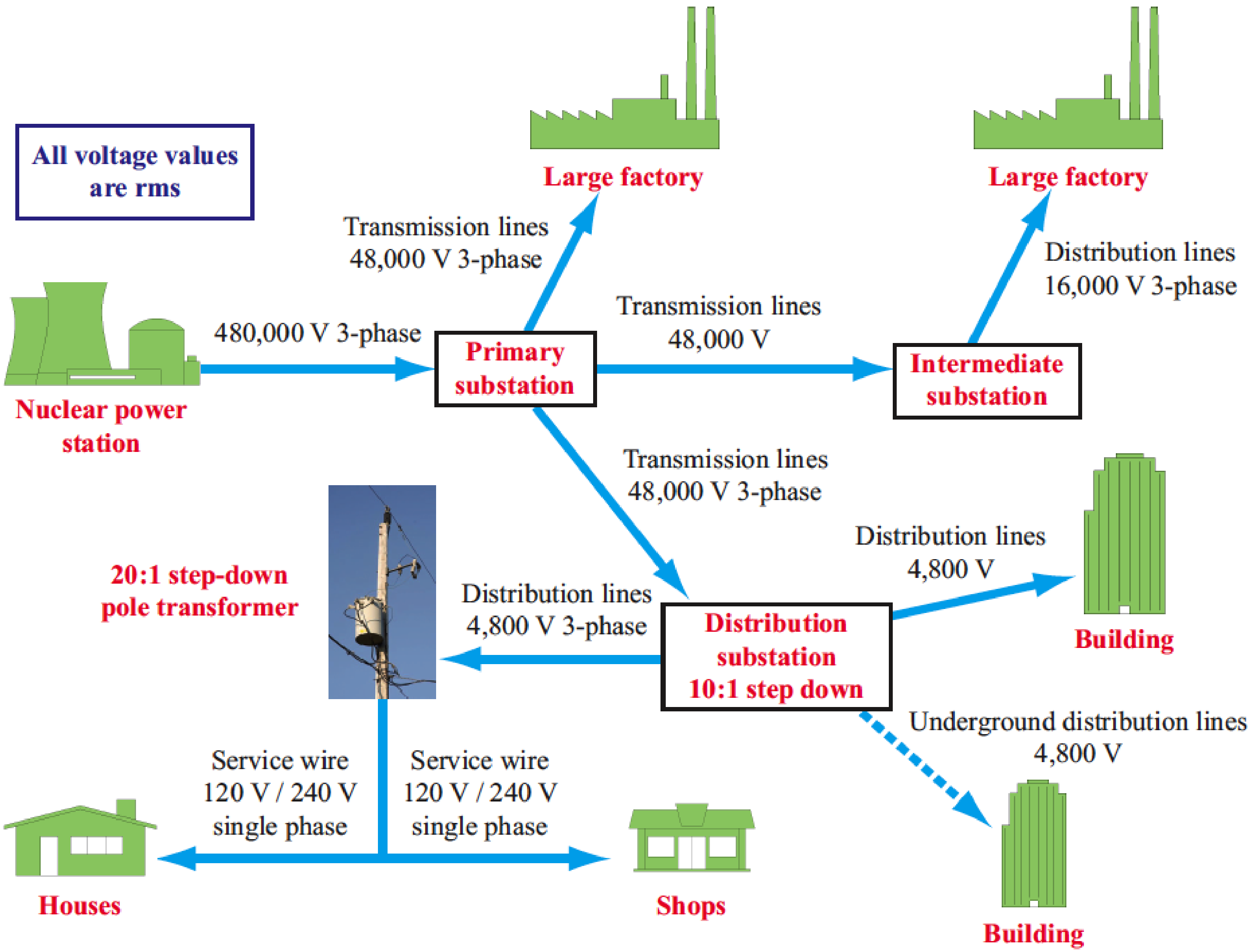
Lecture 10

- Three-Phase Circuits

Outline--Three-Phase Circuits

- Balanced Three-Phase System
  - Balanced sources
  - Balanced loads
- Circuit analysis
  - Phase voltage/current
  - Line voltage/current

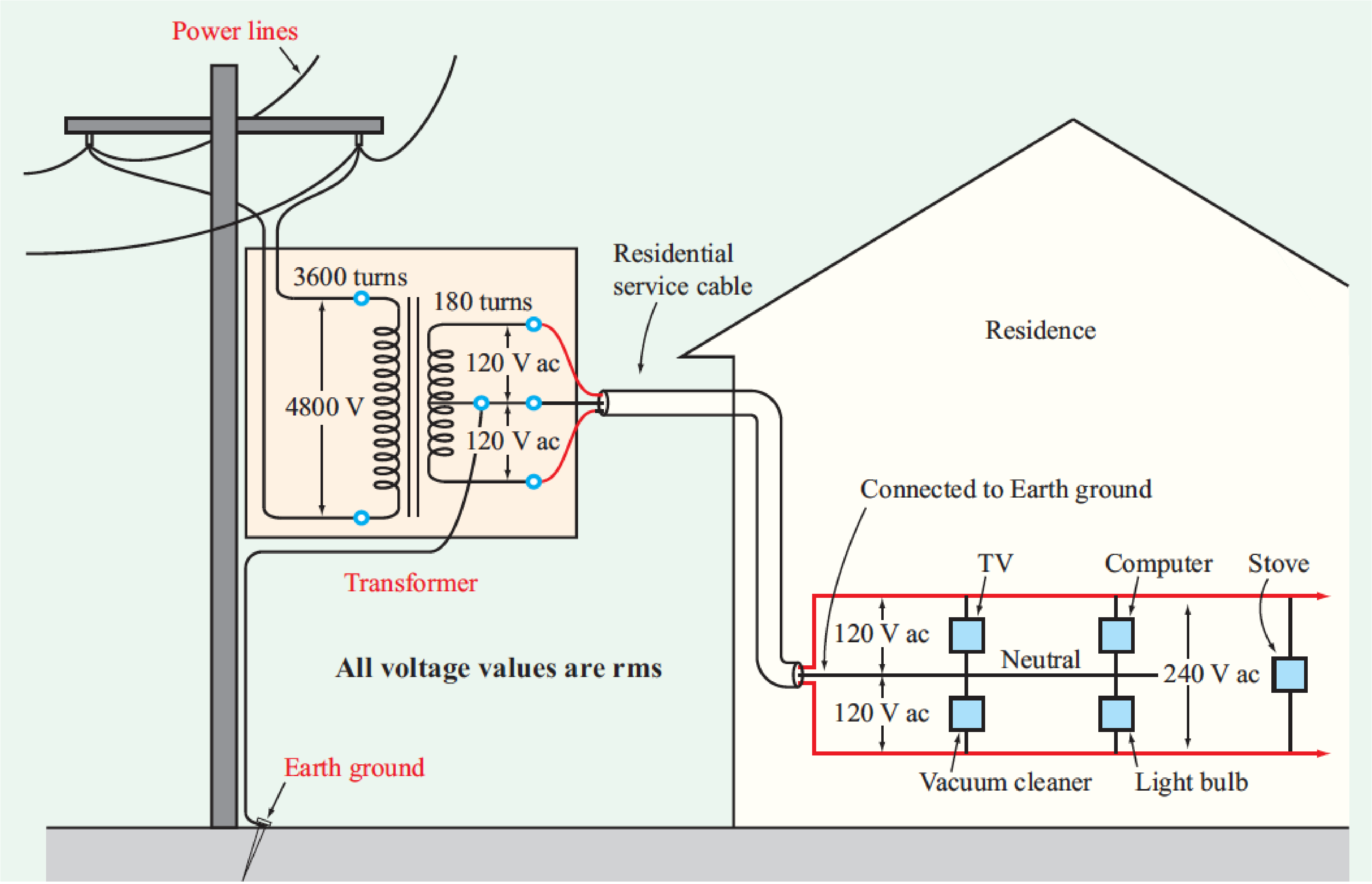
Three-Phase System (in USA)



[Source: Berkeley]

Figure 10-1: Typical electrical power grid.

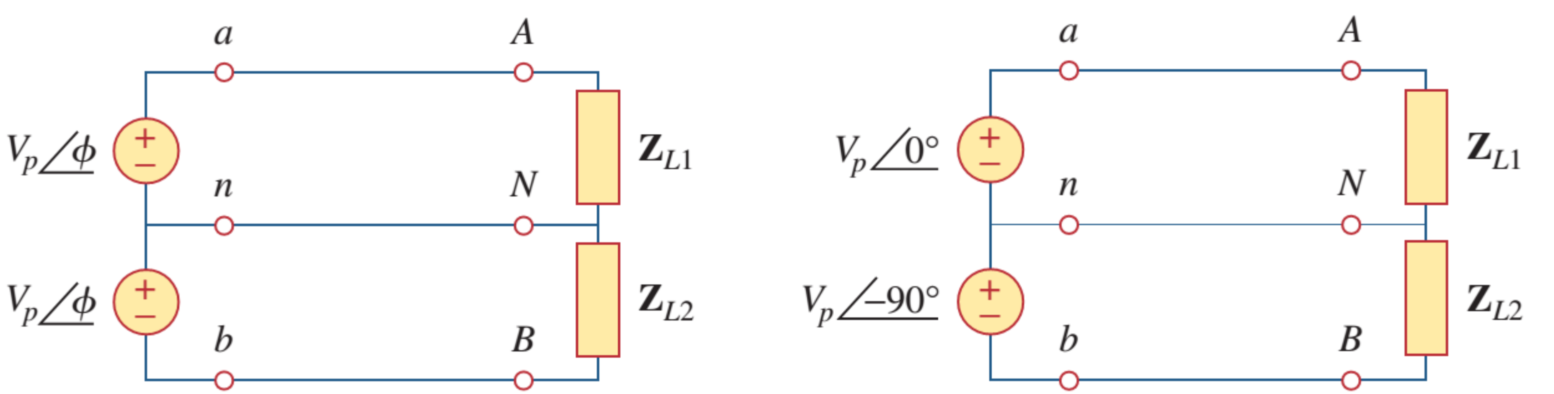
A 4800-V rms single-phase connected to residential user through a 20 : 1 step-down transformer



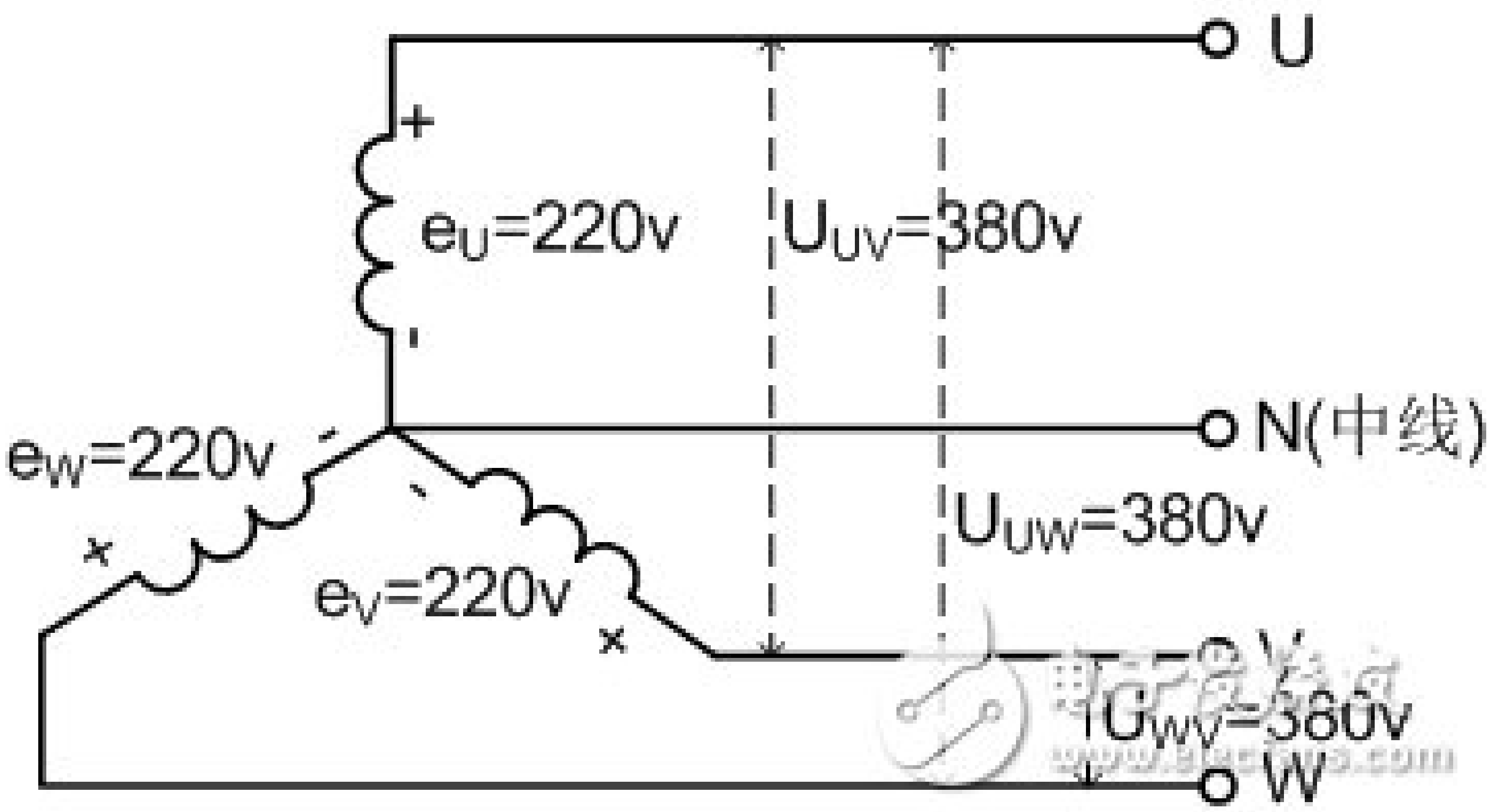
[Source: Berkeley]

Single Phase vs. Polyphase

- Households have single-phase power supply
  - This typically in a three wire form, where two 120V sources with the same phase are connected in series.
  - This allows for appliances to use either 120 or 240V
- Circuits that operate at the same frequency but with multiple sources at different phases are called <u>polyphase</u>.

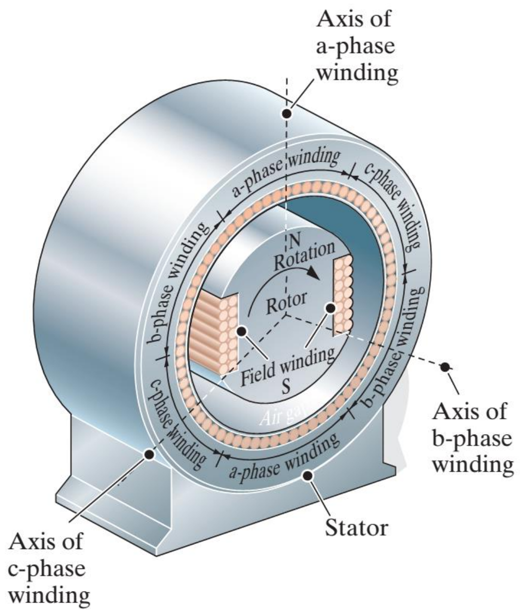


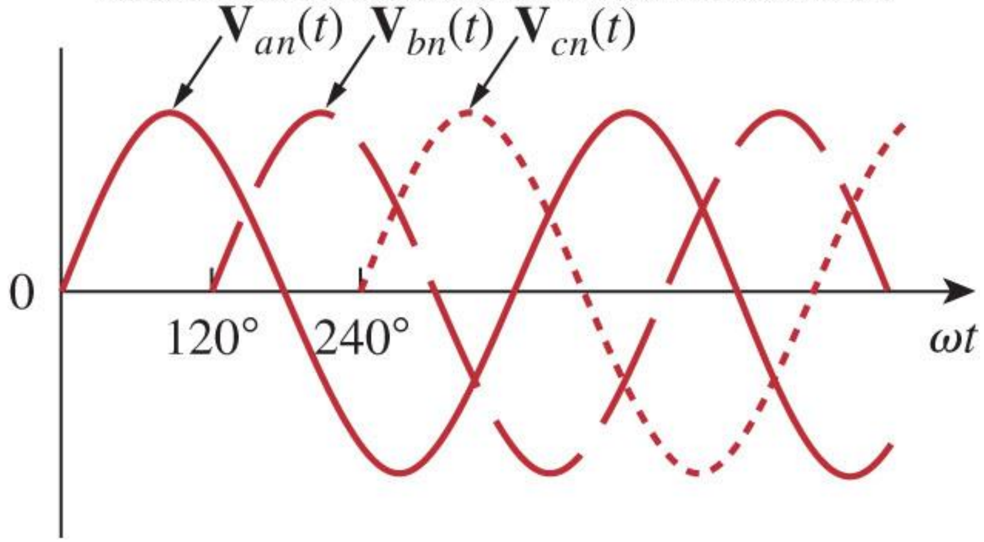
Three-phase four-wire system in China

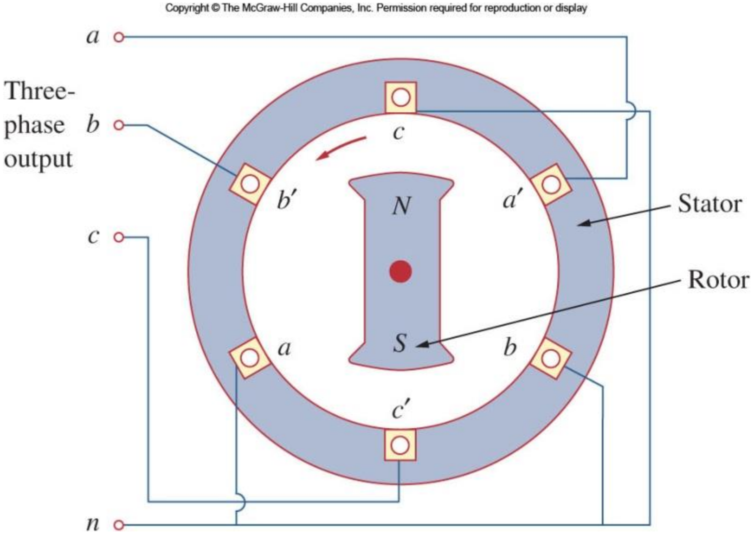


Three-Phase Sources

- Three phase voltages are typically produced by a three-phase AC generator.
- The output voltages look like below.

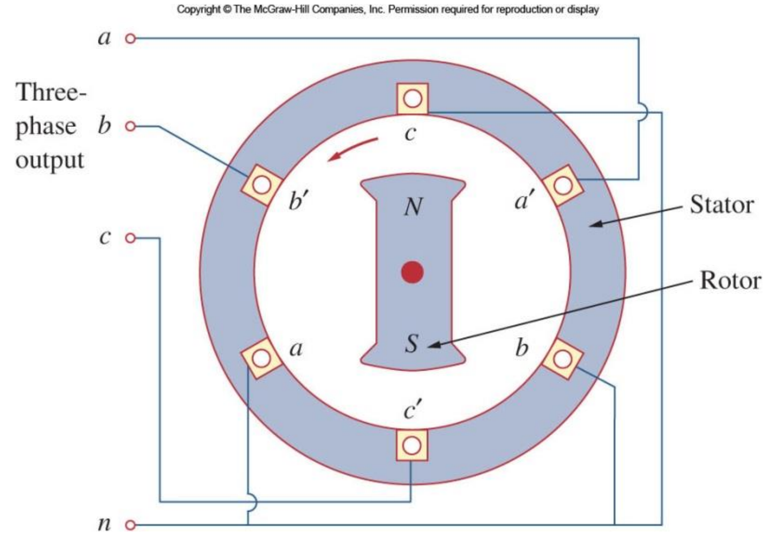




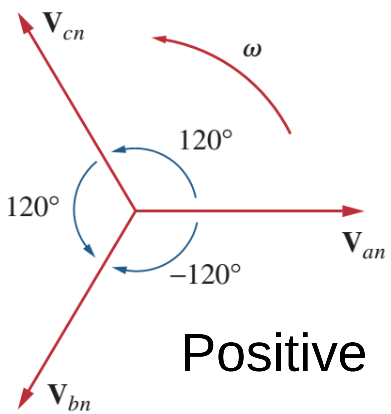


Balanced Three-Phase Sources

- Balanced phase voltage are equal in magnitude and are out of phase with each other by 120deg
- It's easy to know $V_{an} + V_{bn} + V_{cn} = 0$
- Two sequences for the phases:

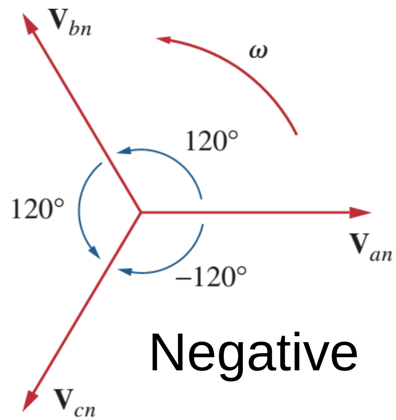


$$\left|V_{an}\right| = \left|V_{bn}\right| = \left|V_{cn}\right|$$



Positive

$$V_{an} = V_p \angle 0^\circ$$
$$V_{bn} = V_p \angle -120^\circ$$
$$V_{cn} = V_p \angle -240^\circ = V_p \angle +120^\circ$$

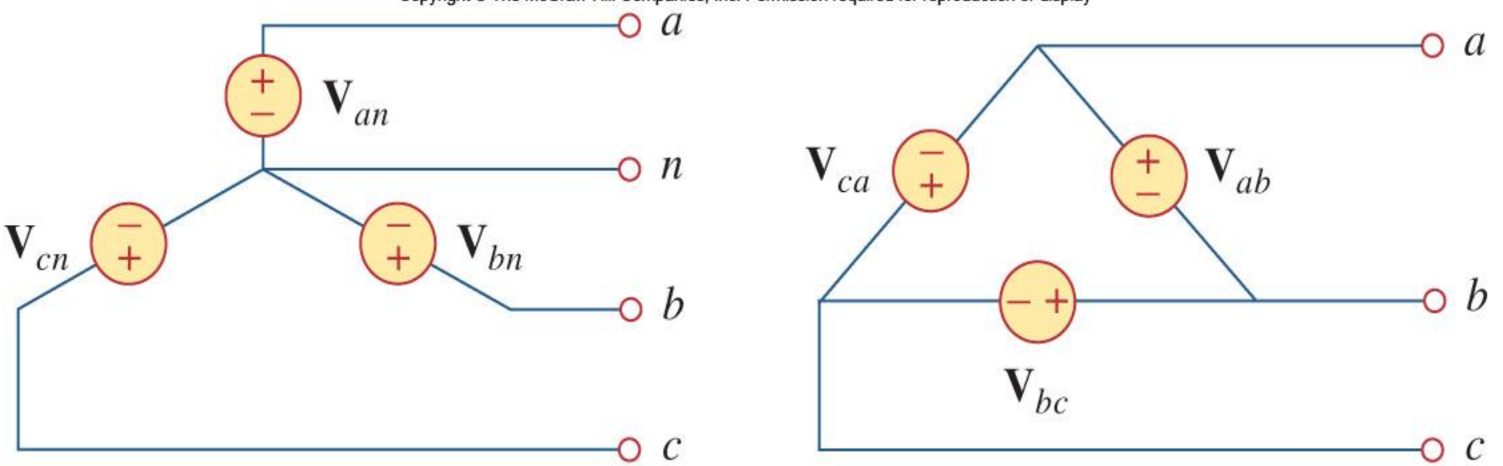


Negative

$$V_{an} = V_p \angle 0^\circ$$
$$V_{cn} = V_p \angle -120^\circ$$
$$V_{bn} = V_p \angle -240^\circ = V_p \angle +120^\circ$$

Connecting the Sources

- Three phase voltage sources can be connected by either three or four wire configurations.
  - Four-wire system accomplished using a Y(Wye) connected source.
  - Three-wire configuration accomplished by Delta connected source.



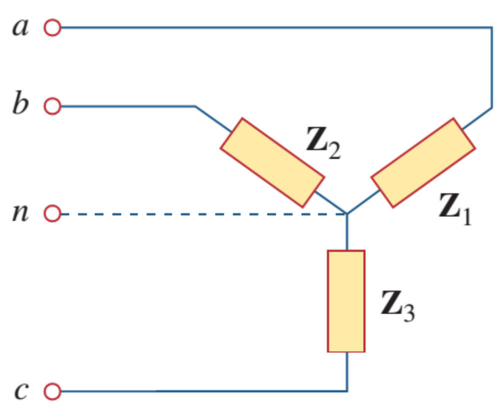
(a) (b)

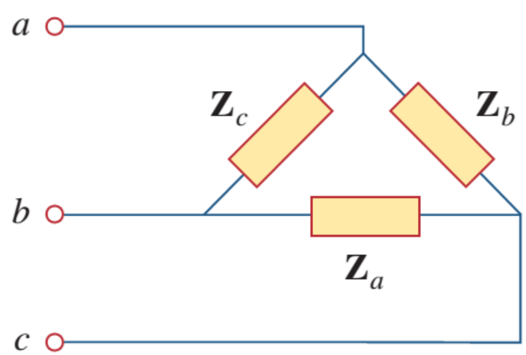
Balanced Loads

- A <u>balanced</u> load is one that has the same impedance presented to all three voltage sources.

-- Impedance are equal in magnitude and in phase

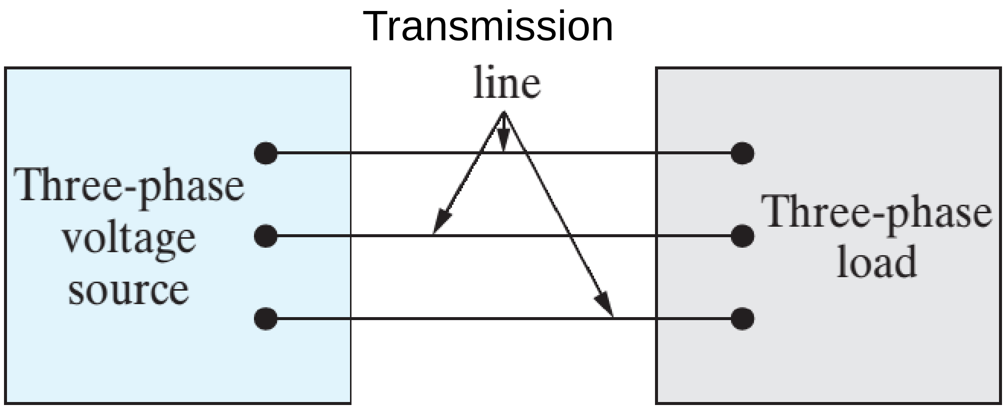
- They may also be connected in either Delta or wye
  - For a balanced wye connected load: $Z_1 = Z_2 = Z_3 = Z_Y$
  - For a balanced delta connected load: $Z_a = Z_b = Z_c = Z_\Delta$





- The load impedance per phase for the two load configurations can be interchanged.

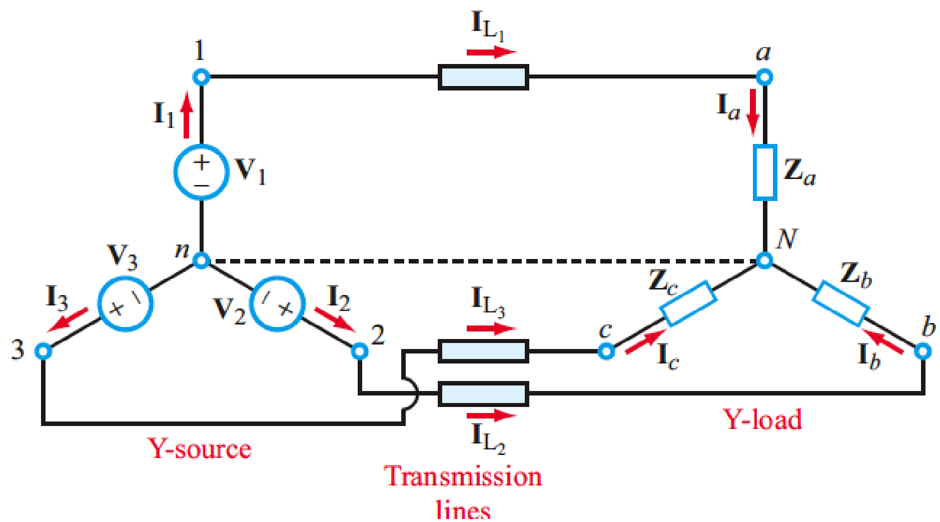
Source-Load configurations



Source	Load
Y	Y
Y	Δ
Δ	Y
Δ	Δ

Source-Load Configurations

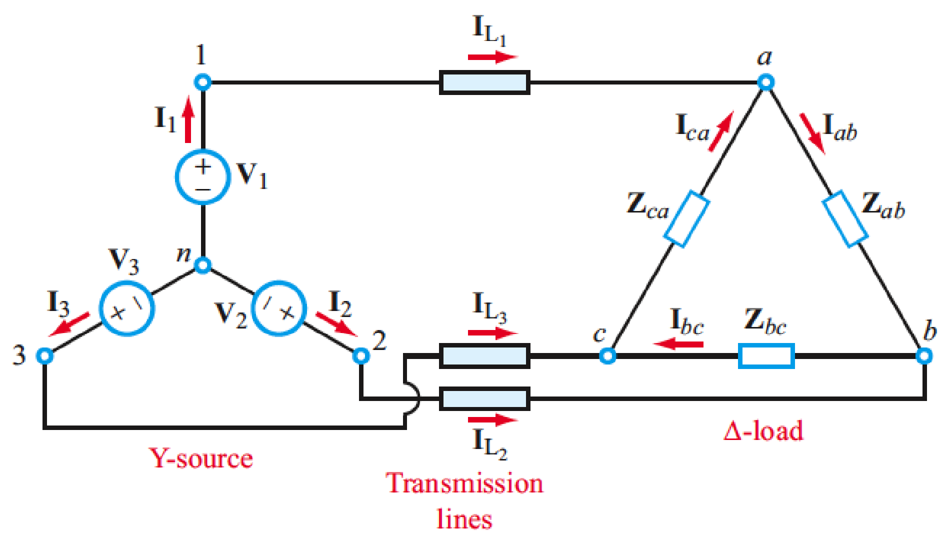
Y-Y



Load Phase Currents
$\mathbf{I}_a$, $\mathbf{I}_b$, $\mathbf{I}_c$
(same as line currents
$\mathbf{I}_{L_1}$, $\mathbf{I}_{L_2}$, and $\mathbf{I}_{L_3}$)

Load Phase Voltages
$\mathbf{V}_{aN}$, $\mathbf{V}_{bN}$, $\mathbf{V}_{cN}$

Y-Delta

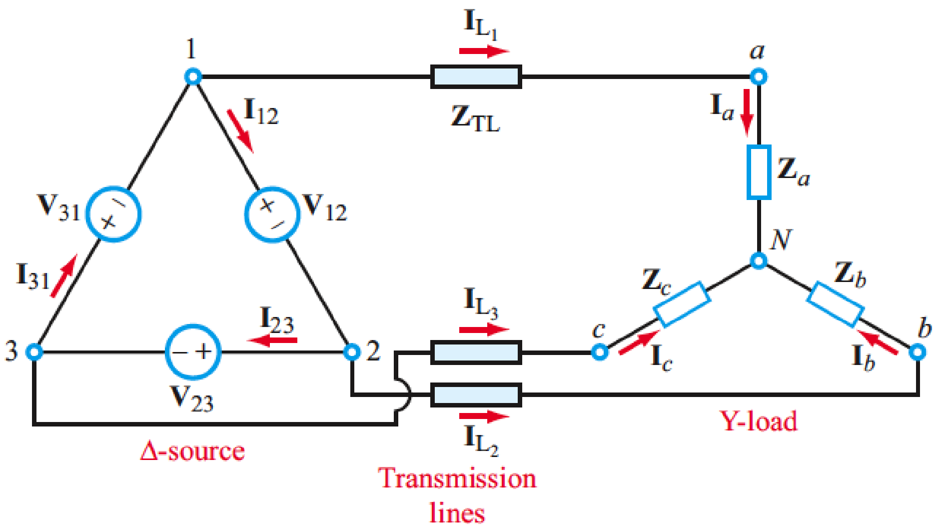


Load Phase Currents
$\mathbf{I}_{ab}$, $\mathbf{I}_{bc}$, $\mathbf{I}_{ca}$

Load Phase Voltages
$\mathbf{V}_{ab}$, $\mathbf{V}_{bc}$, $\mathbf{V}_{ca}$

Source-Load Configurations

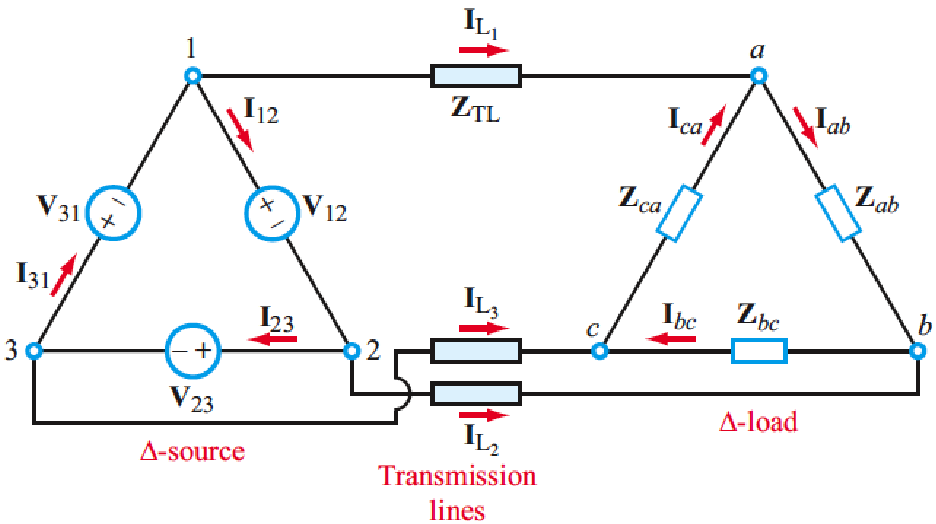
Delta-Y



Load Phase Currents
$\mathbf{I}_a$, $\mathbf{I}_b$, $\mathbf{I}_c$
(same as line currents
$\mathbf{I}_{L_1}$, $\mathbf{I}_{L_2}$, and $\mathbf{I}_{L_3}$)

Load Phase Voltages
$\mathbf{V}_{aN}$, $\mathbf{V}_{bN}$, $\mathbf{V}_{cN}$

Delta-Delta



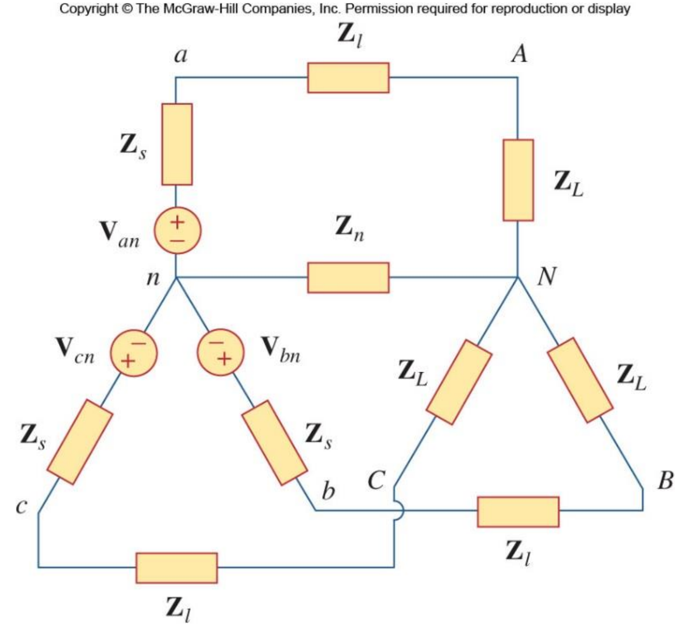
Load Phase Currents
$\mathbf{I}_{ab}$, $\mathbf{I}_{bc}$, $\mathbf{I}_{ca}$

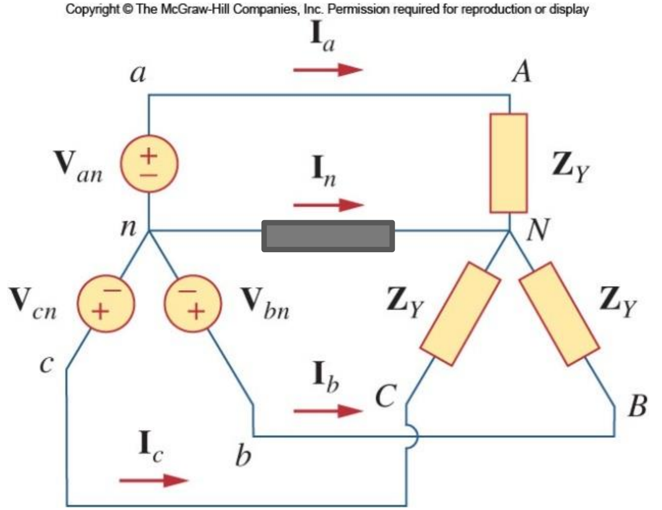
Load Phase Voltages
$\mathbf{V}_{ab}$, $\mathbf{V}_{bc}$, $\mathbf{V}_{ca}$
(same as source voltages
if $\mathbf{Z}_{TL}$ is negligible)

Balanced Y-Y connection

- Any three-phase system can be reduced to an equivalent Y-Y system.
- The load impedances Z_Y will be assumed to be balanced.
  - This can be the source Z_s, line Z_l and load Z_L together.

$$\mathbf{Z}_Y = \mathbf{Z}_s + \mathbf{Z}_\ell + \mathbf{Z}_L$$





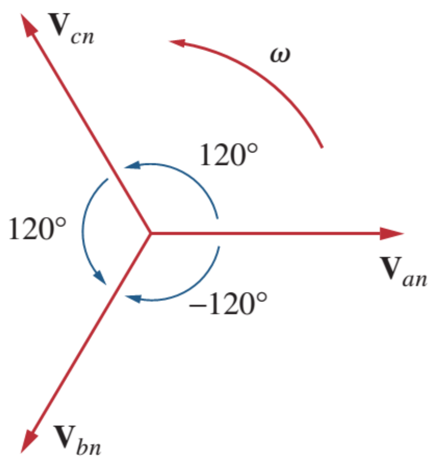
Phase Voltage & Line-to-Line Voltage

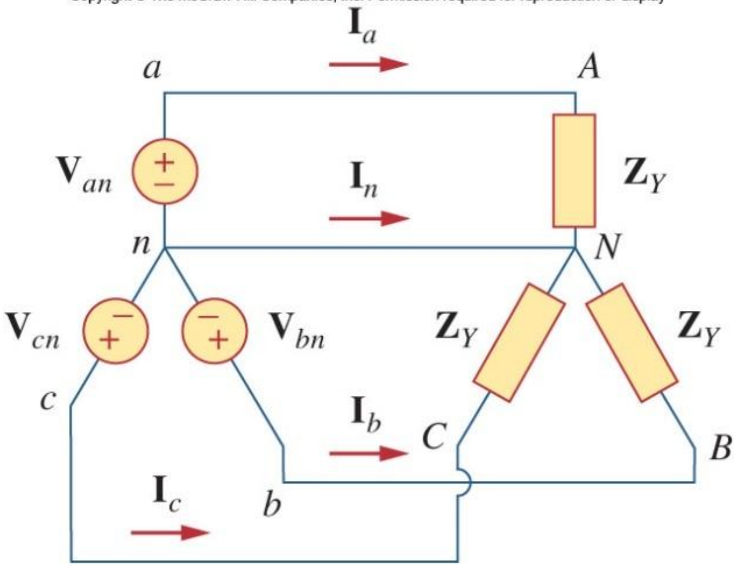
- Use the positive sequence:

Phase Voltage $V_{an} = V_p \angle 0^\circ$

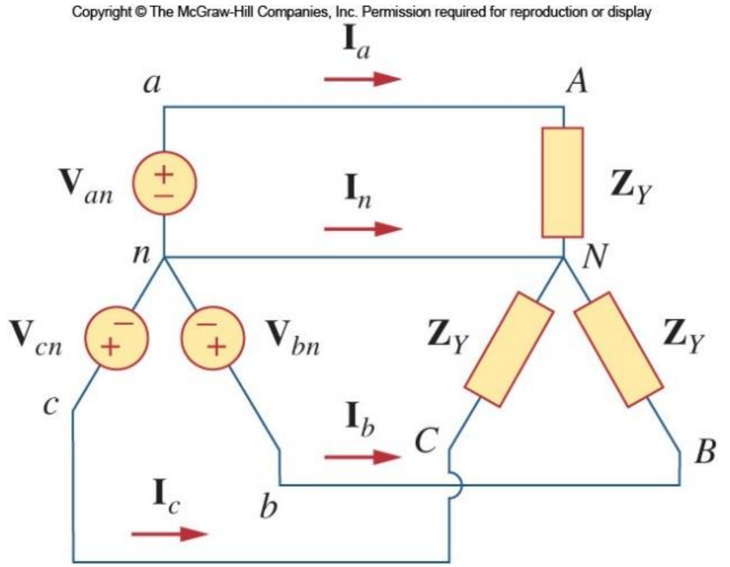
$$V_{bn} = V_p \angle -120^\circ \quad V_{cn} = V_p \angle +120^\circ$$

- The line to line (or line in short) voltages:



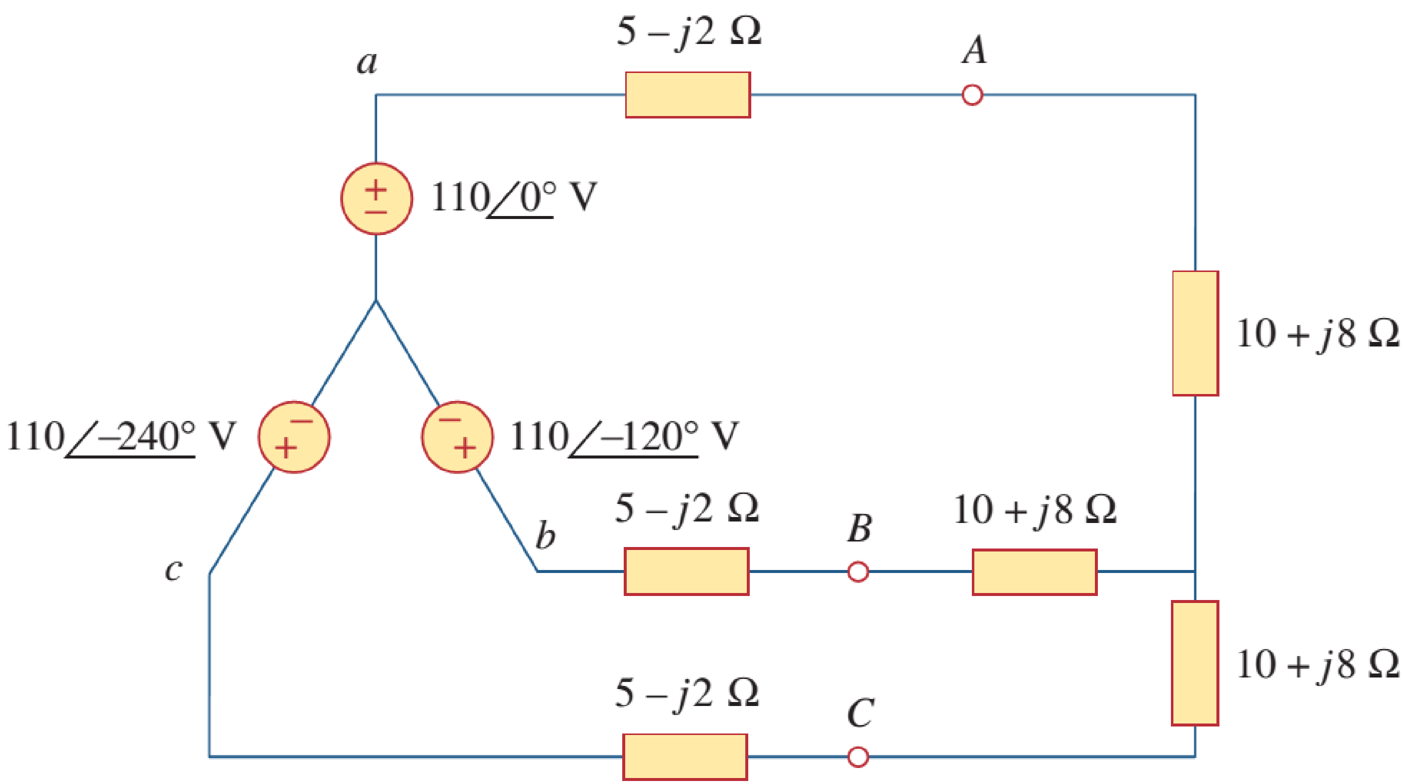


Line Currents

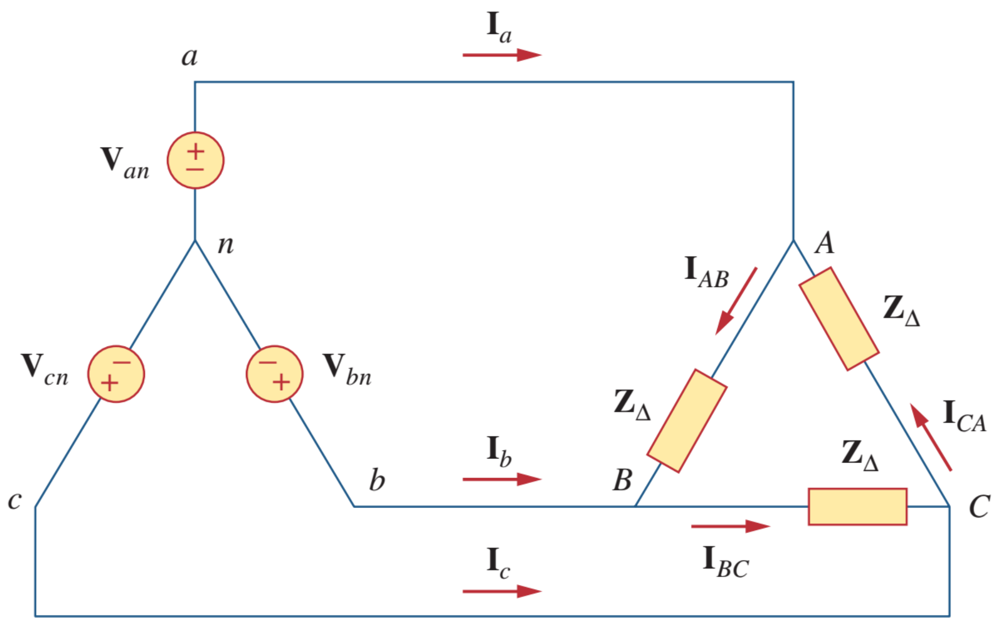


Example

- Calculate the line currents.



Wye-Δ



$$\mathbf{V}_{an} = V_p\angle 0^\circ$$

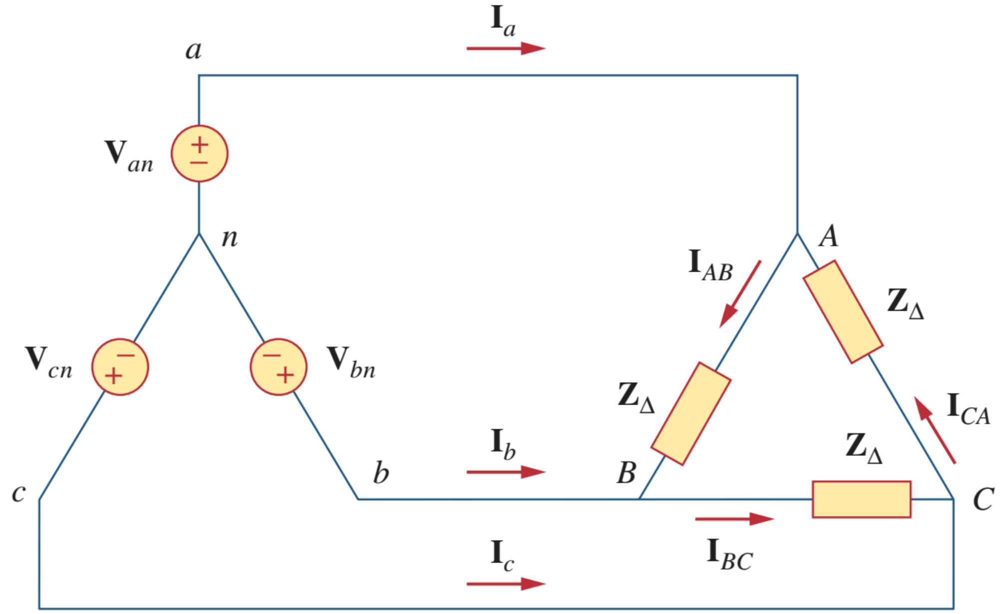
$$\mathbf{V}_{bn} = V_p\angle -120^\circ, \qquad \mathbf{V}_{cn} = V_p\angle +120^\circ$$

$$\mathbf{V}_{ab} = \sqrt{3}V_p\angle 30^\circ = \mathbf{V}_{AB}, \qquad \mathbf{V}_{bc} = \sqrt{3}V_p\angle -90^\circ = \mathbf{V}_{BC}$$

$$\mathbf{V}_{ca} = \sqrt{3}V_p\angle -150^\circ = \mathbf{V}_{CA}$$

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_\Delta}, \qquad \mathbf{I}_{BC} = \frac{\mathbf{V}_{BC}}{\mathbf{Z}_\Delta}, \qquad \mathbf{I}_{CA} = \frac{\mathbf{V}_{CA}}{\mathbf{Z}_\Delta}$$

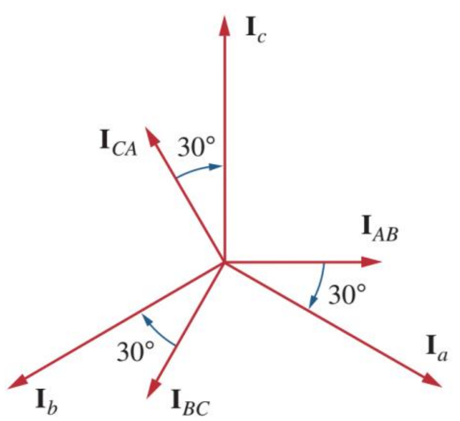
Wye-Δ



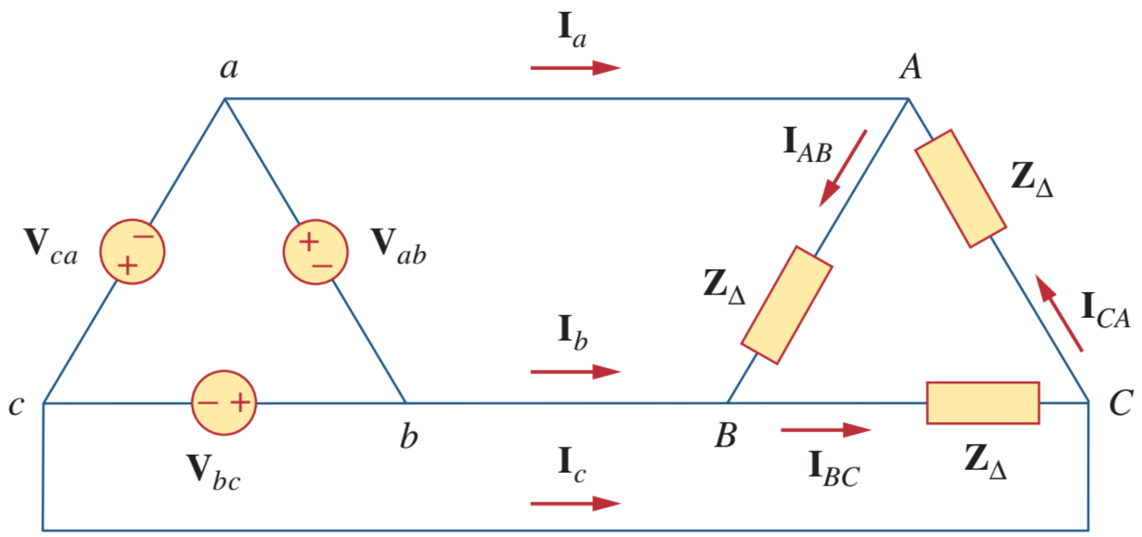
$$\mathbf{I}_a = \mathbf{I}_{AB} - \mathbf{I}_{CA}, \qquad \mathbf{I}_b = \mathbf{I}_{BC} - \mathbf{I}_{AB}, \qquad \mathbf{I}_c = \mathbf{I}_{CA} - \mathbf{I}_{BC}$$

Since $\mathbf{I}_{CA} = \mathbf{I}_{AB}\angle{-240^\circ}$,

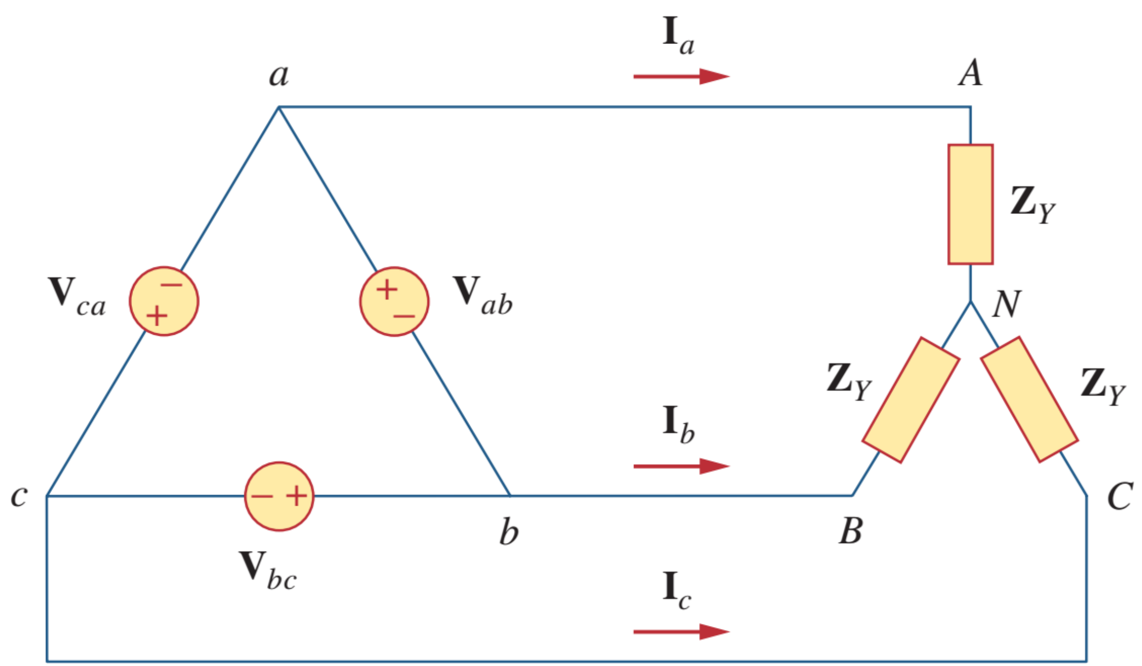
$$\begin{aligned}\mathbf{I}_a = \mathbf{I}_{AB} - \mathbf{I}_{CA} &= \mathbf{I}_{AB}(1 - 1\angle{-240^\circ}) \\ &= \mathbf{I}_{AB}(1 + 0.5 - j0.866) = \mathbf{I}_{AB}\sqrt{3}\angle{-30^\circ}\end{aligned}$$



- Δ-Δ


I$_a$
a
A
I$_{AB}$
Z$_Δ$
V$_{ca}$
V$_{ab}$
Z$_Δ$
I$_b$
I$_{CA}$
c
C
b
B
V$_{bc}$
I$_c$
I$_{BC}$
Z$_Δ$

Δ-Wye


I$_a$
a
A
Z$_Y$
V$_{ca}$
V$_{ab}$
N
Z$_Y$
Z$_Y$
I$_b$
c
C
b
B
V$_{bc}$
I$_c$

Connection	Phase voltages/currents	Line voltages/currents
Y-Y	$\mathbf{V}_{an} = V_p\angle 0^\circ$ $\mathbf{V}_{bn} = V_p\angle -120^\circ$ $\mathbf{V}_{cn} = V_p\angle +120^\circ$ Same as line currents	$\mathbf{V}_{ab} = \sqrt{3}V_p\angle 30^\circ$ $\mathbf{V}_{bc} = \mathbf{V}_{ab}\angle -120^\circ$ $\mathbf{V}_{ca} = \mathbf{V}_{ab}\angle +120^\circ$ $\mathbf{I}_a = \mathbf{V}_{an}/\mathbf{Z}_Y$ $\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ$ $\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ$
Y-Δ	$\mathbf{V}_{an} = V_p\angle 0^\circ$ $\mathbf{V}_{bn} = V_p\angle -120^\circ$ $\mathbf{V}_{cn} = V_p\angle +120^\circ$ $\mathbf{I}_{AB} = \mathbf{V}_{AB}/\mathbf{Z}_\Delta$ $\mathbf{I}_{BC} = \mathbf{V}_{BC}/\mathbf{Z}_\Delta$ $\mathbf{I}_{CA} = \mathbf{V}_{CA}/\mathbf{Z}_\Delta$	$\mathbf{V}_{ab} = \mathbf{V}_{AB} = \sqrt{3}V_p\angle 30^\circ$ $\mathbf{V}_{bc} = \mathbf{V}_{BC} = \mathbf{V}_{ab}\angle -120^\circ$ $\mathbf{V}_{ca} = \mathbf{V}_{CA} = \mathbf{V}_{ab}\angle +120^\circ$ $\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle -30^\circ$ $\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ$ $\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ$
Δ-Δ	$\mathbf{V}_{ab} = V_p\angle 0^\circ$ $\mathbf{V}_{bc} = V_p\angle -120^\circ$ $\mathbf{V}_{ca} = V_p\angle +120^\circ$ $\mathbf{I}_{AB} = \mathbf{V}_{ab}/\mathbf{Z}_\Delta$ $\mathbf{I}_{BC} = \mathbf{V}_{bc}/\mathbf{Z}_\Delta$ $\mathbf{I}_{CA} = \mathbf{V}_{ca}/\mathbf{Z}_\Delta$	Same as phase voltages $\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle -30^\circ$ $\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ$ $\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ$
Δ-Y	$\mathbf{V}_{ab} = V_p\angle 0^\circ$ $\mathbf{V}_{bc} = V_p\angle -120^\circ$ $\mathbf{V}_{ca} = V_p\angle +120^\circ$ Same as line currents	Same as phase voltages $\mathbf{I}_a = \dfrac{V_p\angle -30^\circ}{\sqrt{3}\mathbf{Z}_Y}$ $\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ$ $\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ$

Unbalanced 3-phase circuits

Power calculations