Lecture 21 Representation & Description (chapter 11)

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Course piazza link：
piazza.com/shanghaitech.edu.cn/spring2021/cs270spring2021

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Image analysis fundamental steps

image acquisition

preprocessing,
enhancement

segmentation

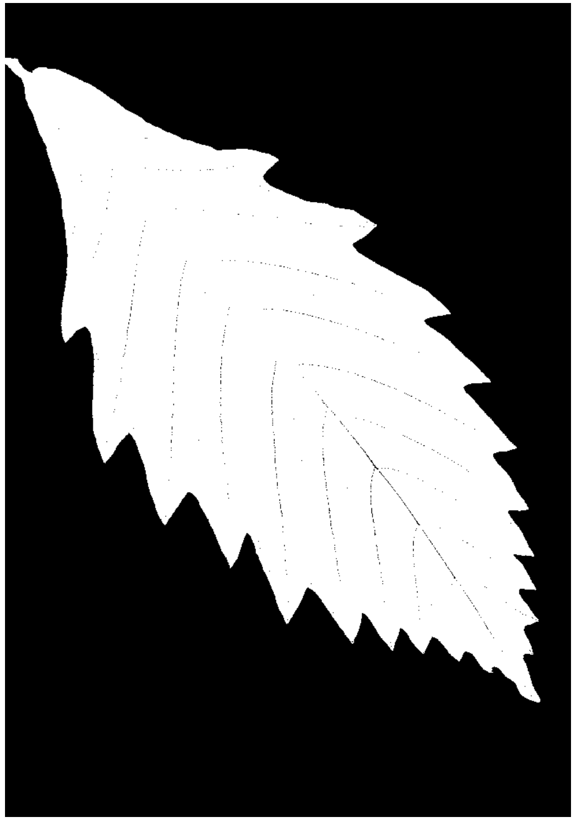
Representation, description, feature
extraction

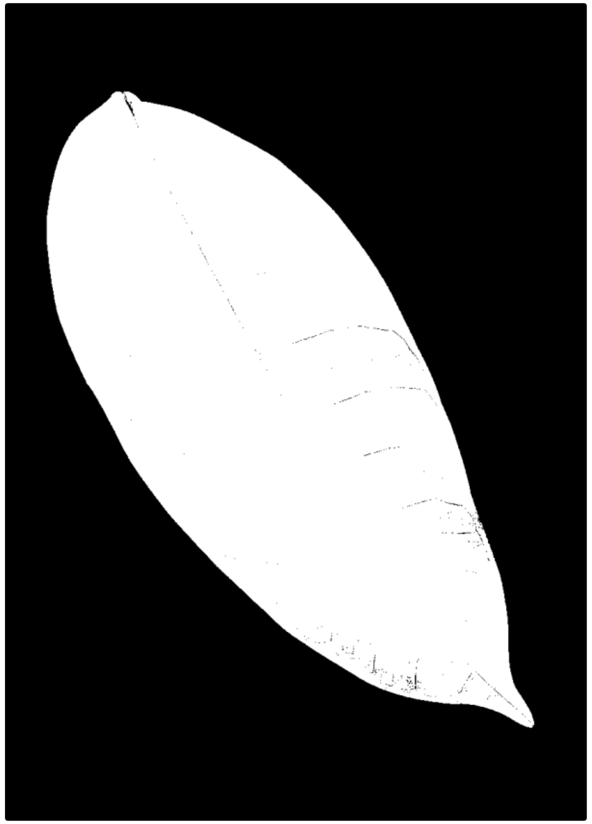
classification,
interpretatio,
recognition

result

Boundary and region description.

- Normally, after segmentation one needs the representation of the objects to describe them





- External (boundary):
  - Representation: Polygon of the boundary
  - Description: The circumference
- Internal (regional)
  - Representation: Pixels inside the object
  - Description: The average color

Outline

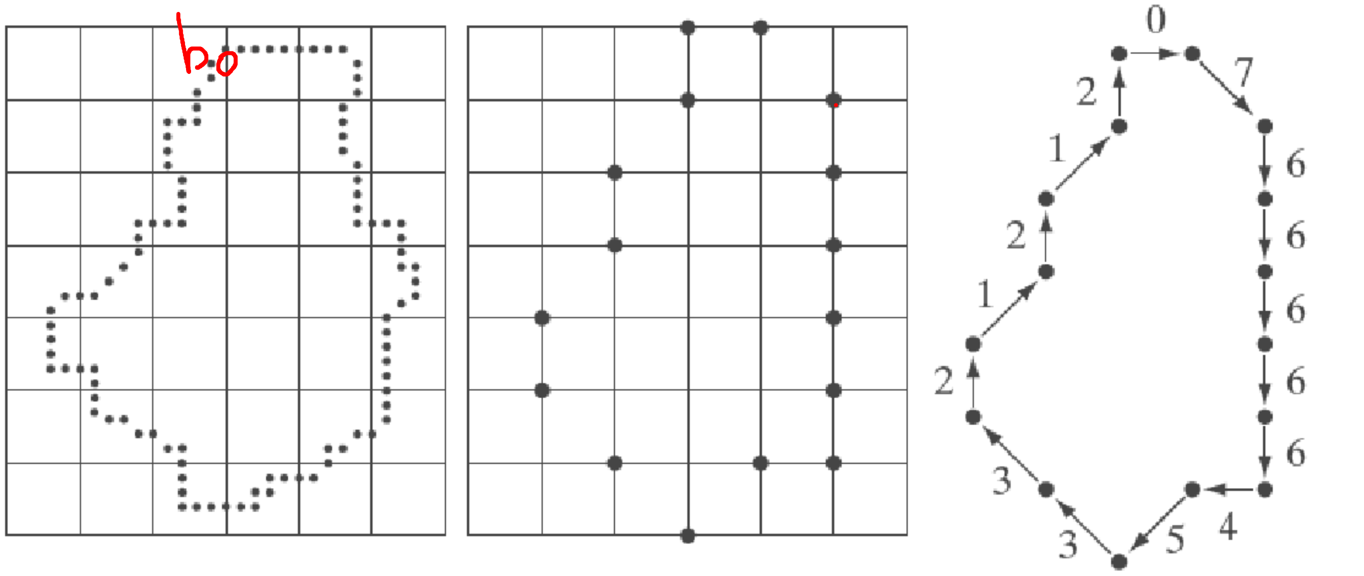
This lecture will cover:

- Boundary and region description.
- Topology (Euler number).
- Skeleton.
- Statistic on histogram of intensity.
- Gray-level co-occurrence matrix (GLCM).

External Descriptor

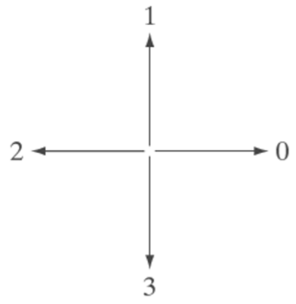
Boundary representation: Chain code

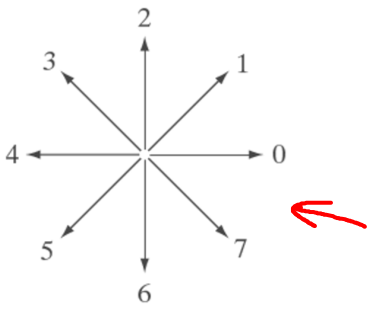
Boundary representation = 0766666453321212



Original boundary　　Sub-sampled boundary　　Chain code of boundary

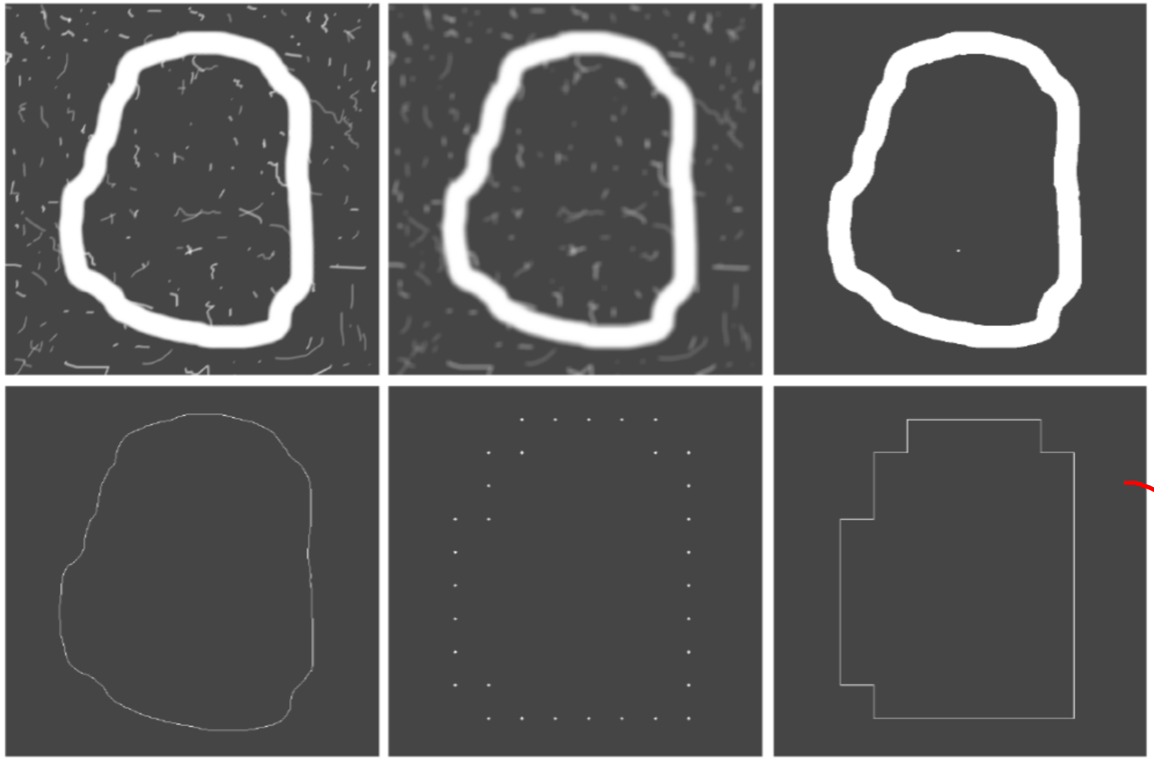
Chain code for
4-neighborhood





Chain code for
8-neighborhood

Chain code: example

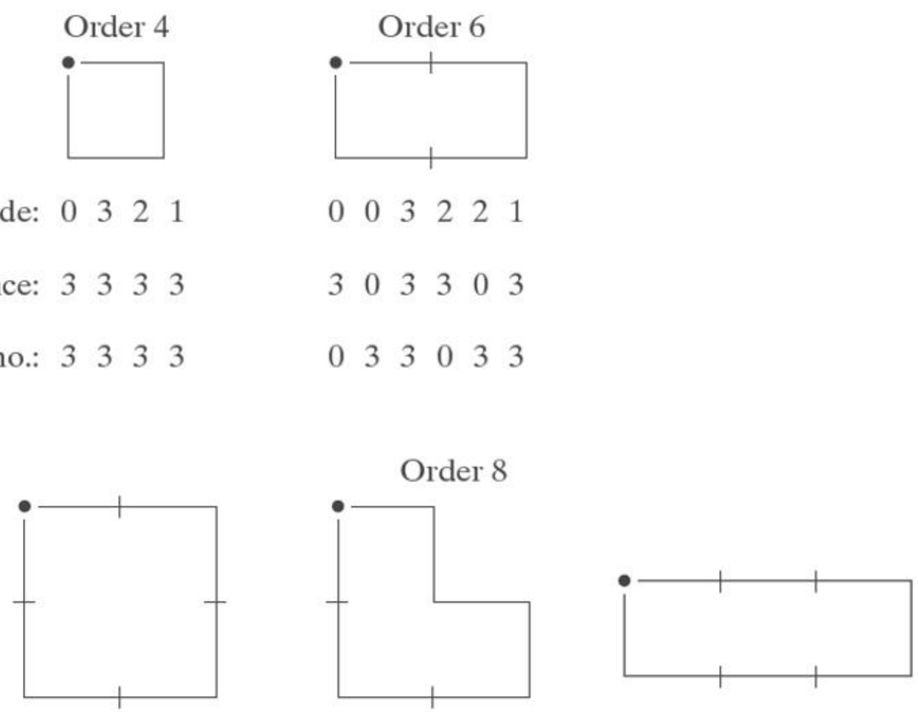


8-directional chain code → 00006066666666444444242222202202

Starting point normalized chain code → 00006066666666444444242222202202

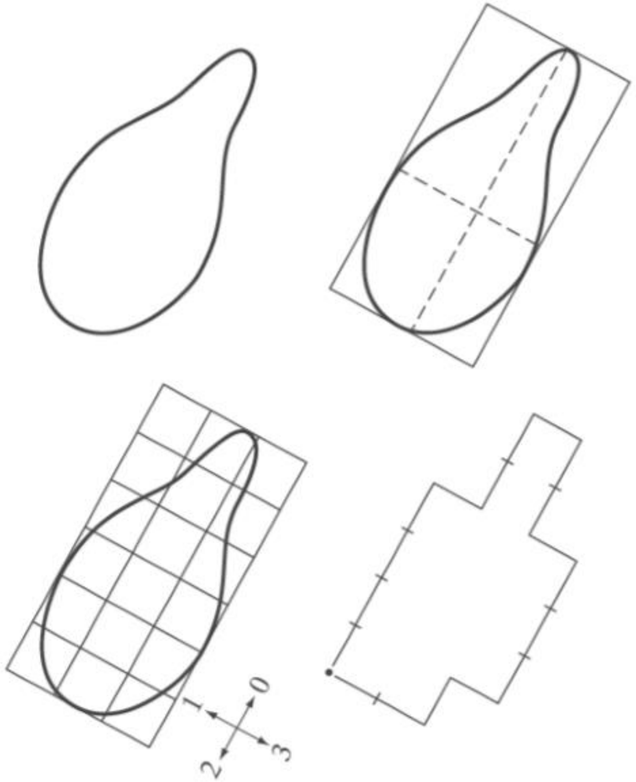
Rotation normalized chain code → 0006200000006000006260000620626

Shape number: A boundary descriptor



	Order 4	Order 6
Chain code:	0 3 2 1	0 0 3 2 2 1
Difference:	3 3 3 3	3 0 3 3 0 3
Shape no.:	3 3 3 3	0 3 3 0 3 3

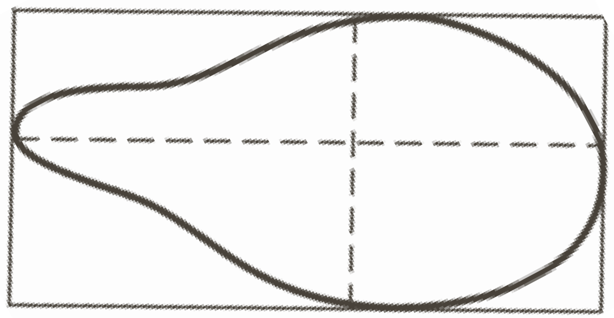
	Order 8		
Chain code:	0 0 3 3 2 2 1 1	0 3 0 3 2 2 1 1	0 0 0 3 2 2 2 1
Difference:	3 0 3 0 3 0 3 0	3 3 1 3 3 0 3 0	3 0 0 3 3 0 0 3
Shape no.:	0 3 0 3 0 3 0 3	0 3 0 3 3 1 3 3	0 0 3 3 0 0 3 3



Chain code:	0 0 0 0 3 0 0 3 2 2 3 2 2 2 1 2 1 1
Difference:	3 0 0 0 3 1 0 3 3 0 1 3 0 0 3 1 3 0
Shape no.:	0 0 0 3 1 0 3 3 0 1 3 0 0 3 1 3 0 3

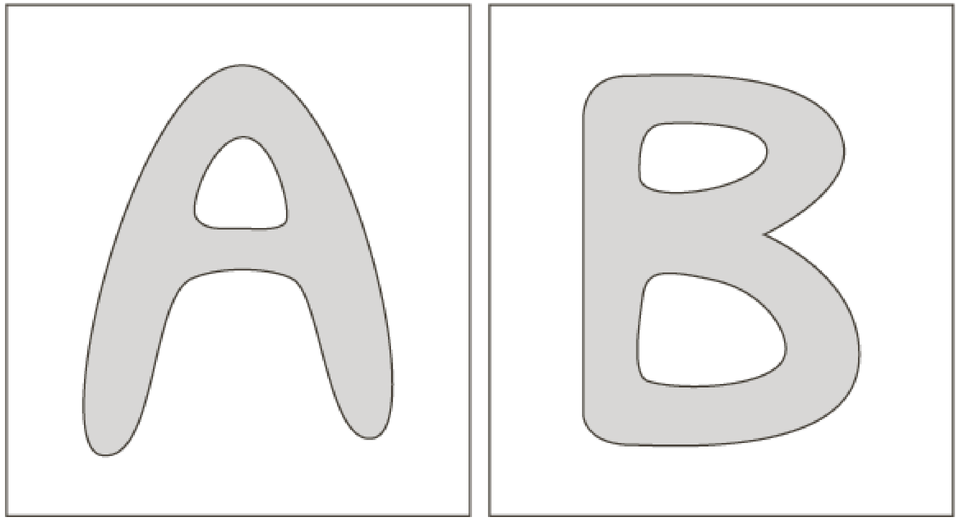
Simple Boundary Descriptors

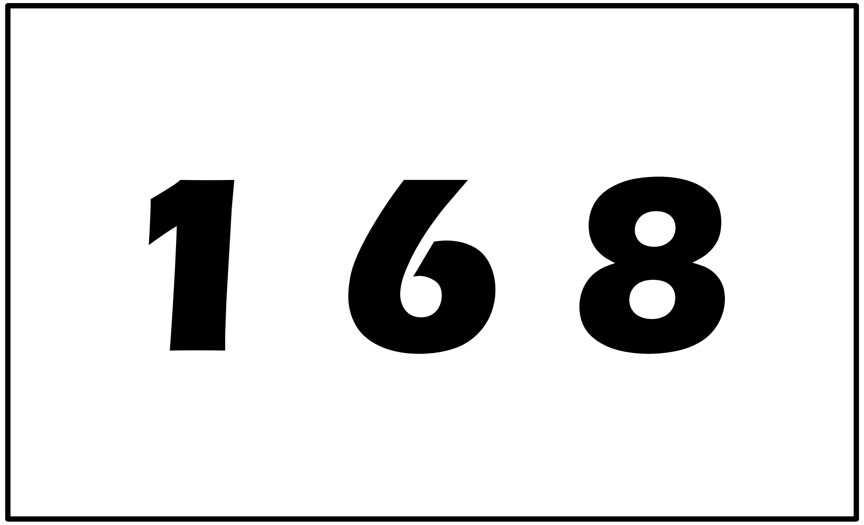
- Perimeter (周长)
- Area (面积)
- Bounding Box.
- Diameter (直径): longest path between two edge pixels.
- Compactness : $\frac{(Perimeter)^2}{(area)}$
- Circularity: $\frac{4\pi(area)}{(Perimeter)^2}$
- Centroid (形心): $c(x,y) = \frac{1}{k}\sum_{p\in Object} p(x,y)$
- Major Axis（长轴）,Minor Axis（短轴）
- Eccentricity (偏心率).



Topological Descriptors（拓扑描绘子）

- Euler Number (欧拉数): $E = C - H$
- C stands for # of components and H stands for # of Holes.





Fourier Descriptors（傅里叶描绘子）

- Represent the boundary by a sequence of points (assume clockwise order)

$$\{(x_0, y_0), (x_1, y_1), \cdots, (x_N, y_N)\}$$

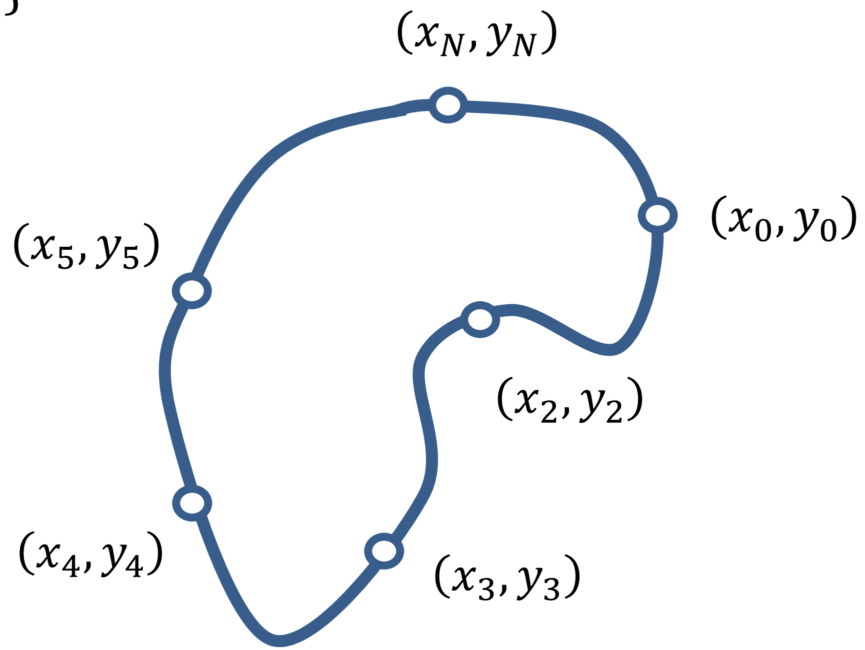
- Write each point (x_n, y_n) as a complex number

$$s(n) = x(n) + jy(n)$$

- Take 1D Fourier series of $s(n)$ to get coefficient $a(u)$

$$a(u) = \sum_{n=1}^{N} s(n) e^{-j2\pi un/N}$$

- Fourier descriptors are a concise description of (object) contours
- Can be used for
    - Contour processing (filtering, interpolation, morphing)
    - Image analysis (characterizing and recognizing shapes)



Fourier Descriptors（傅里叶描绘子）

- We have Fourier transform coefficients $a(u)$

$$a(u) = \sum_{n=1}^{N} s(n) e^{-j2\pi un/N}$$

What is $a(0)$?

- Given coefficients, we can reconstruct boundary

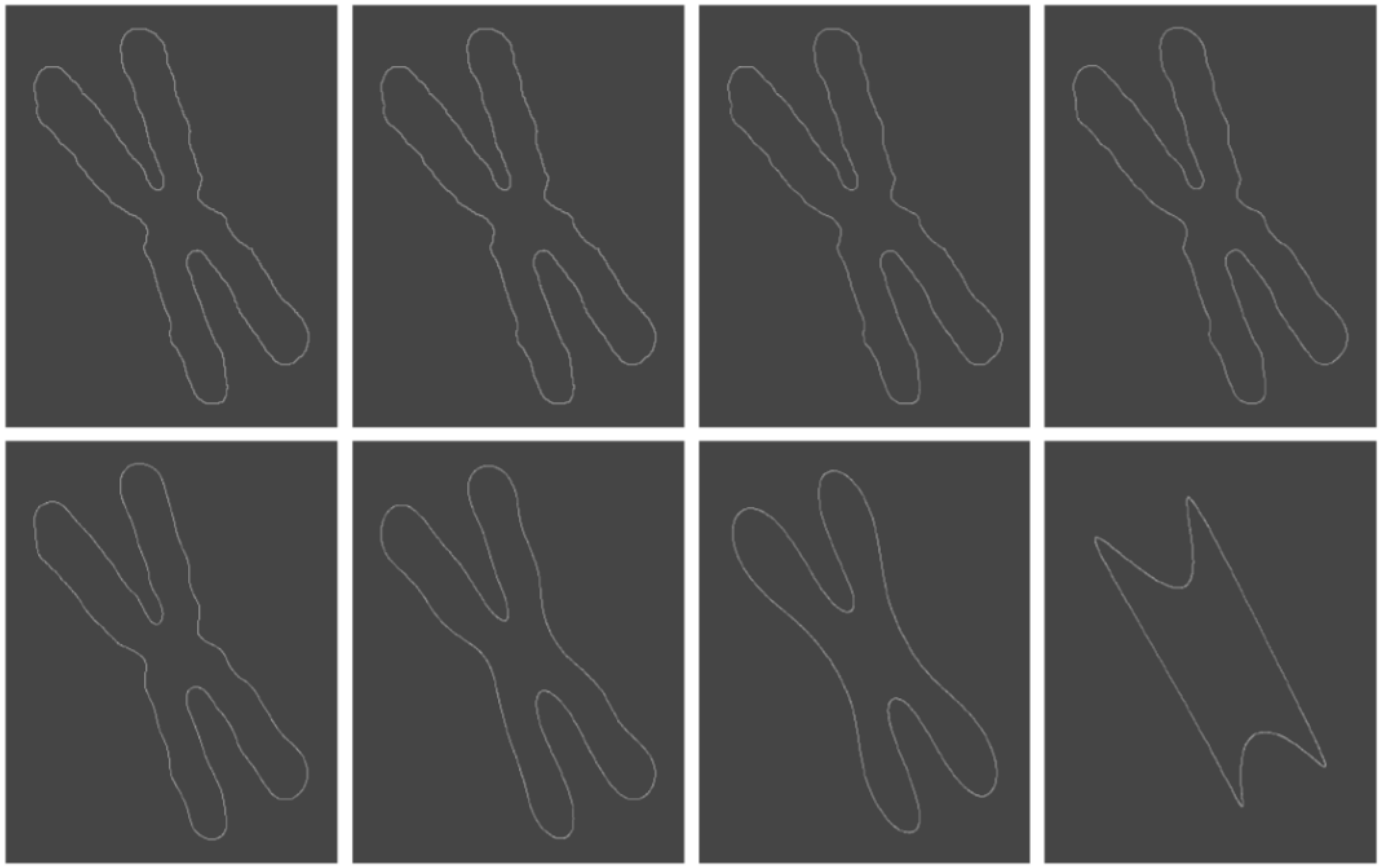
$$s(n) = \frac{1}{N} \sum_{u=1}^{N} a(u) e^{j2\pi un/N}$$

- Higher order coefficients can be truncated for a more concise representation (e.g. low pass filter)
- *Other filters: Sharpening, edge extraction……*

Boundary Reconstruction using Fourier Descriptors

100% 50% 10% 5%

2868 descriptors



2.5% 1.25% 0.63% 0.28%

Fourier Descriptors

Transformation	Boundary	Fourier Descriptor
Identity	$s(k)$	$a(u)$
Rotation	$s_r(k) = s(k)e^{j\theta}$	$a_r(u) = a(u)e^{j\theta}$
Translation	$s_t(k) = s(k) + \Delta_{xy}$	$a_t(u) = a(u) + \Delta_{xy}\delta(u)$
Scaling	$s_s(k) = \alpha s(k)$	$a_s(u) = \alpha a(u)$
Starting point	$s_p(k) = s(k - k_0)$	$a_p(u) = a(u)e^{-j2\pi k_0 u/K}$

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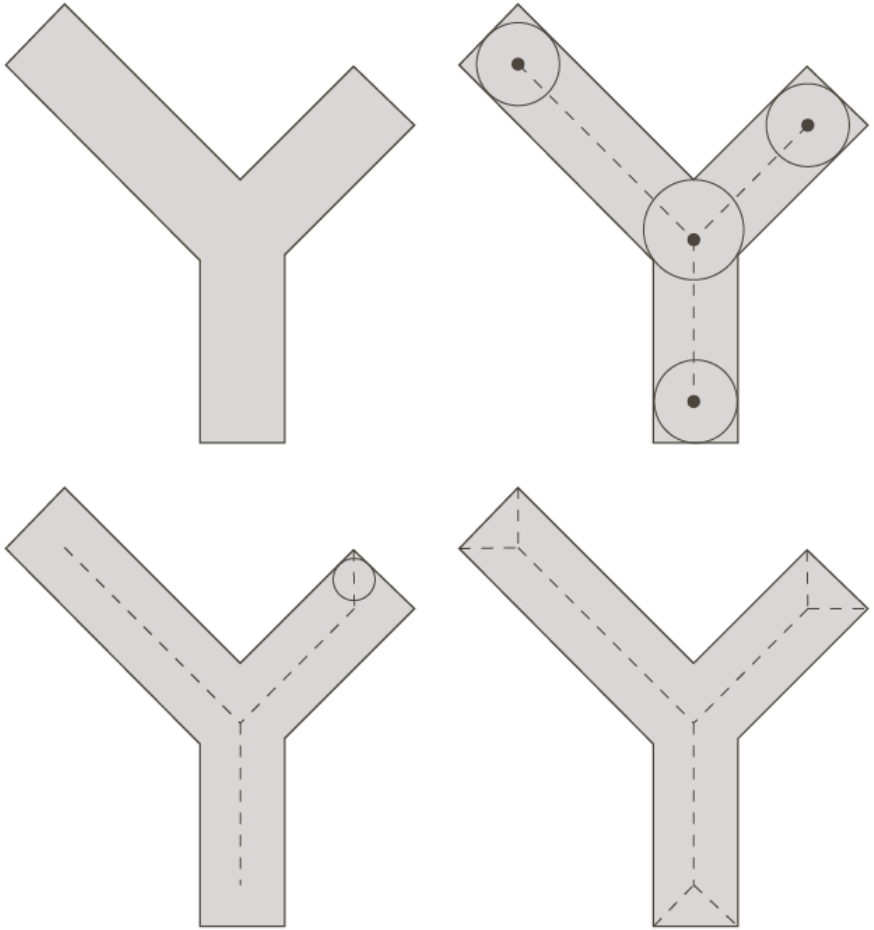
Skeletons（骨架）



- Estimation:
  - Successive erosions
  - Distance transform
  - Points that have more than one nearest neighbor.
- Bw = bwmorph(im,'skel',Inf);

Internal Descriptor

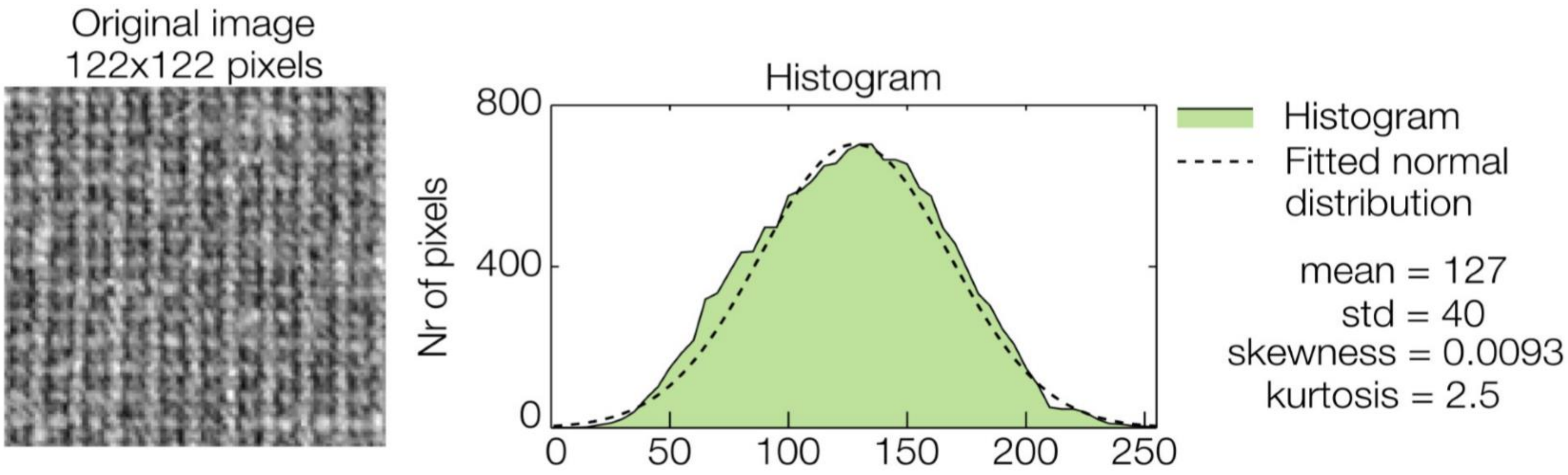
Statistic on histogram of intensity in a region

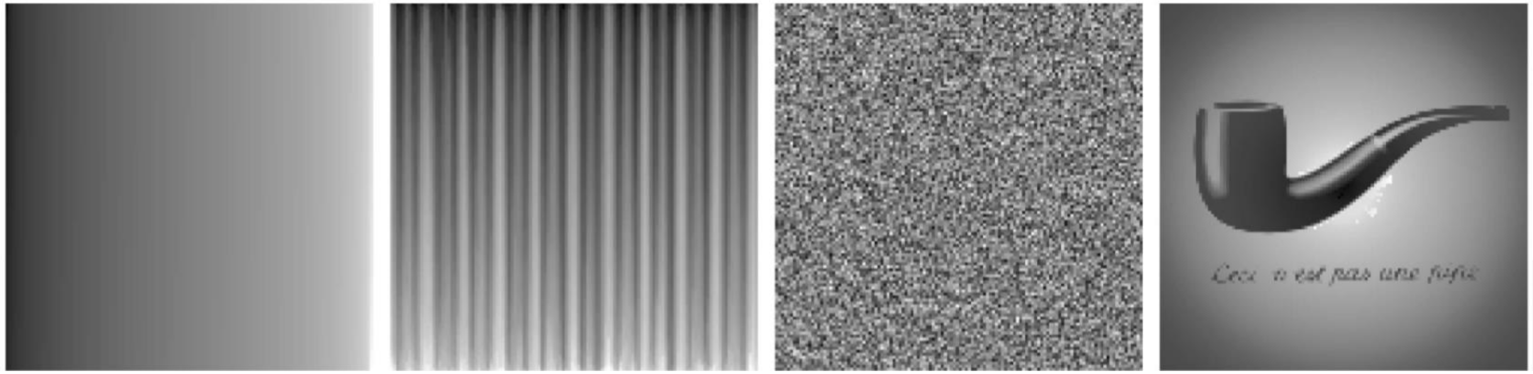
- There is also underlying intensities/ colors inside each region we found.
- Texture can also be filtered.
  - Flat
  - Noisy
  - Stripy

Statistic on histogram of intensity in a region

- Statistics on histogram of intensity in the region:
  - Mean & variance (contrast)
  - Flat -- var=0; Noisy -- var = high;
  - Skewness (locally bright or dark)
  - Entropy (how random)

Intensity histogram says nothing about the spatial distribution of the pixel intensities





Gray-level co-occurence matrix (GLCM)

- How pixels intensity correlate to each other.
- 1) Specify an operation *Q* (spatial relationship between 2 pixels).
  - e.g. *Q* = "1 pixel to the right".
  - If N gray levels, this makes NxN matrix.
- 2) $P\big((x_0, y_0), (x_1, y_1)\big) = [intensity1, intsnsity2]$, the pair of $(x_0, y_0), (x_1, y_1)$ depends on the operation *Q*.
  - *Where P stands for possibility. e.g. How often do I see (1,1) in the given pixel pairs.*
  - *Matlab commend: graycomatrix ()*
- 3) *In practice, # of gray levels is quantized, the quantization depends on the area of the region of interest. (e.g. 8 or 16)*

Gray-level co-occurence matrix (GLCM)

1	1	7	5	3	2
5	1	6	1	2	5
8	8	6	8	1	2
4	3	4	5	5	1
8	7	8	7	6	2
7	8	6	2	6	2

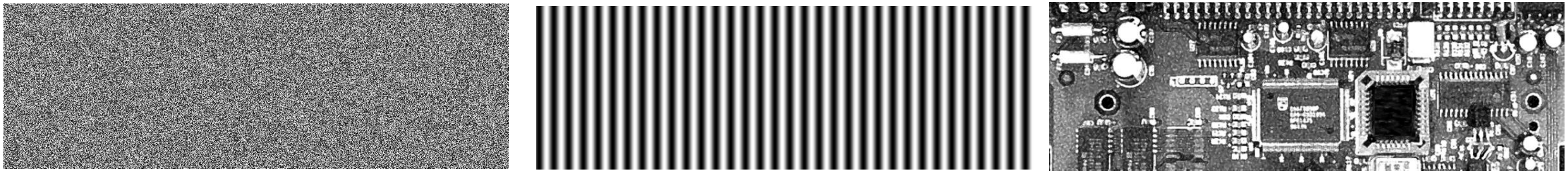
Image *f*

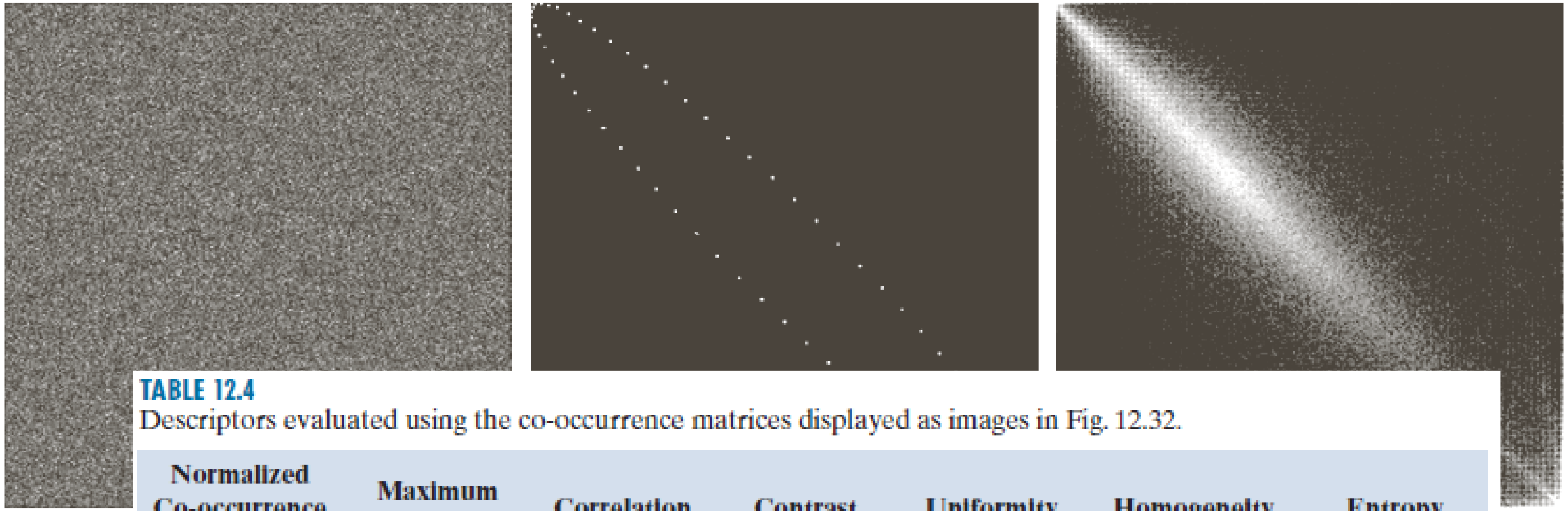
	1	2	3	4	5	6	7	8
1	1	2	0	0	0	1	1	0
2	0	0	0	0	1	1	0	0
3	0	1	0	1	0	0	0	0
4	0	0	1	0	1	0	0	0
5	2	0	1	0	1	0	0	0
6	1	3	0	0	0	0	0	1
7	0	0	0	0	1	1	0	2
8	1	0	0	0	0	2	2	1

Co-occurrence matrix **G**

- 1) Specify an operation *Q* (spatial relationship between 2 pixels).
  - e.g. *Q* = “1 pixel to the right”.
  - If N gray levels, this makes NxN matrix.
- 2) $P\big((x_0, y_0), (x_1, y_1)\big) = [intensity1, intsnsity2]$, the pair of $(x_0, y_0), (x_1, y_1)$ depends on the operation *Q*.

Gray-level co-occurence matrix (GLCM)





a b c

FIGURE 11.31 256 × 256 co-occurrence matrices, $\mathbf{G}_1$, $\mathbf{G}_2$, and $\mathbf{G}_3$, corresponding from left to right to the images in Fig. 11.30.

TABLE 12.4
Descriptors evaluated using the co-occurrence matrices displayed as images in Fig. 12.32.

Normalized Co-occurrence Matrix	Maximum Probability	Correlation	Contrast	Uniformity	Homogeneity	Entropy
$\mathbf{G}_1/n_1$	0.00006	−0.0005	10838	0.00002	0.0366	15.75
$\mathbf{G}_2/n_2$	0.01500	0.9650	00570	0.01230	0.0824	06.43
$\mathbf{G}_3/n_3$	0.06860	0.8798	01356	0.00480	0.2048	13.58

Take home message

- The Representation of the Object

  An encoding of the object

  Truthful but possible approximation

- A Descriptor of the Object:

  Only an aspect of the object

  Suitable for classification

  Consider invariance to e.g. noise, translation