Lecture 10: Model-Free Control

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Outline

Outline

Model-Free Reinforcement Learning

- Last lecture:
  - Model-free prediction
  - *Estimate* the value function of an *unknown* MDP
- This lecture:
  - Model-free control
  - *Optimize* the value function of an *unknown* MDP

Uses of Model-Free Control

Some example problems that can be modeled as MDPs

- Elevator
- Parallel Parking
- Ship Steering
- Bioreactor
- Helicopter
- Aeroplane Logistics
- Robocup Soccer
- Quake
- Portfolio management
- Protein Folding
- Robot walking
- Game of Go

For most of these problems, either:

- MDP model is unknown, but experience can be sampled
- MDP model is known, but is too big to use, except by samples

Model-free control can solve these problems

On & Off-Policy Learning

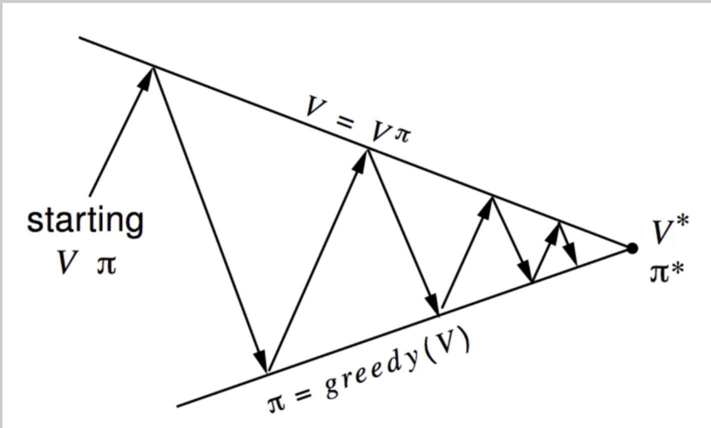
- Behavior-policy: determines which action to take, from which we determine the next state to visit (also called "sampling policy")
- Target-policy: determines the action that appears to be best (also called "learning policy")
- Goal in reinforcement learning:
  - improve the target policy
  - while using a behavior policy to ensure that we visit states often enough (exploration schemes such as ϵ-greedy)
- On-policy learning: when the learning policy and the sampling policy are the same
- Off-policy learning: when the learning policy and the sampling policy are different

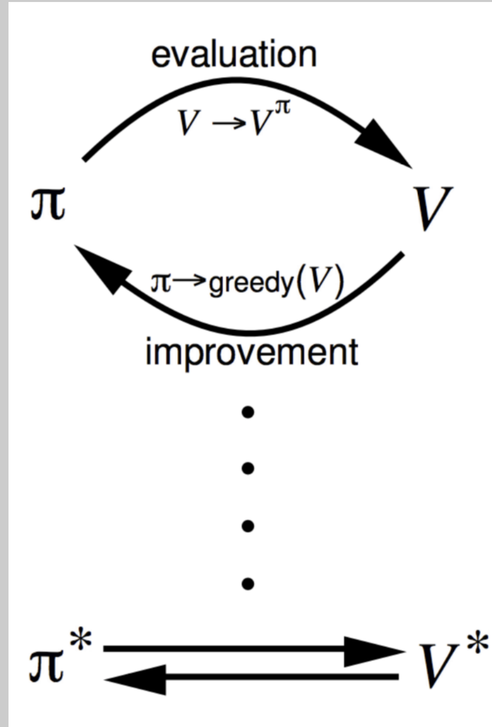
On & Off-Policy Learning

- On-policy learning
  - "Learn on the job"
  - Learn the value of the target policy π from experience sampled from behavior policy π
  - May not ensure the enough exploration of state space
- Off-policy learning
  - "Look over someone's shoulder"
  - Learn the value of the target policy π from experience sampled from behavior policy μ
  - Learning is from experience(data) "off" the target policy
  - Compared to on-policy learning, off-policy learning is
    - ★ more powerful & general
    - ★ often of greater variance & slower to converge

Outline

Generalized Policy Iteration (Refresher)

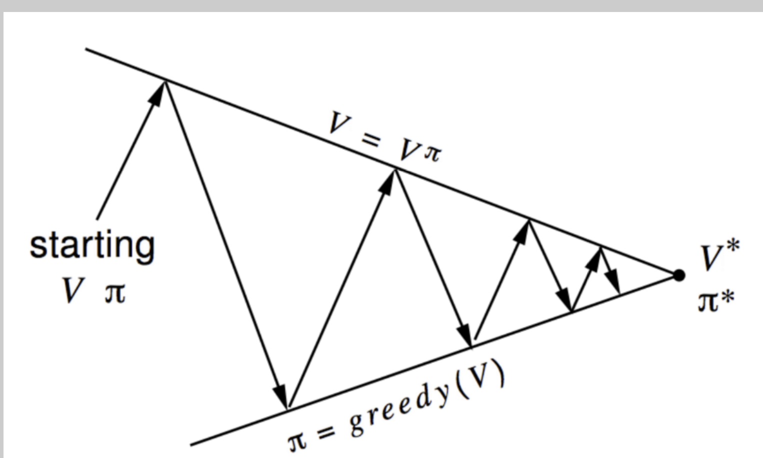




Policy evaluation Estimate v_π
e.g. Iterative policy evaluation

Policy improvement Generate $\pi' \geq \pi$
e.g. Greedy policy improvement

Generalized Policy Iteration With Monte-Carlo Evaluation



Policy evaluation Monte-Carlo policy evaluation, $V = v_\pi$?
Policy improvement Greedy policy improvement?

Model-Free Policy Iteration Using Action-Value Function

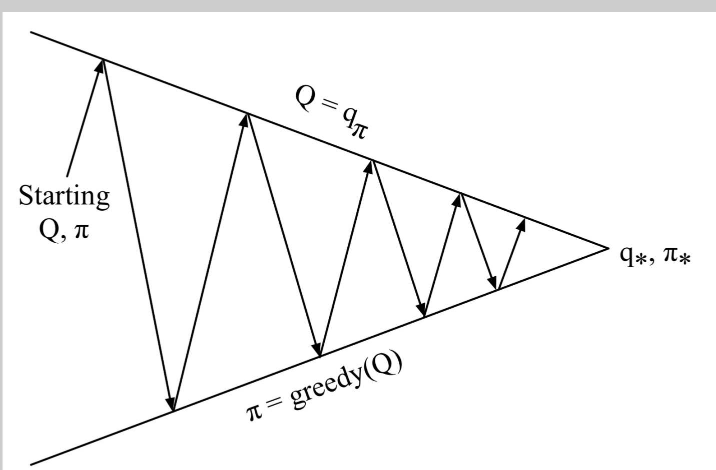
- Greedy policy improvement over $V(s)$ requires model of MDP

$$\pi'(s) = \underset{a\in\mathcal{A}}{\arg\max} \left\{\mathcal{R}_s^a + \mathcal{P}_{ss'}^a V(s')\right\}$$

- Greedy policy improvement over $Q(s,a)$ is model-free

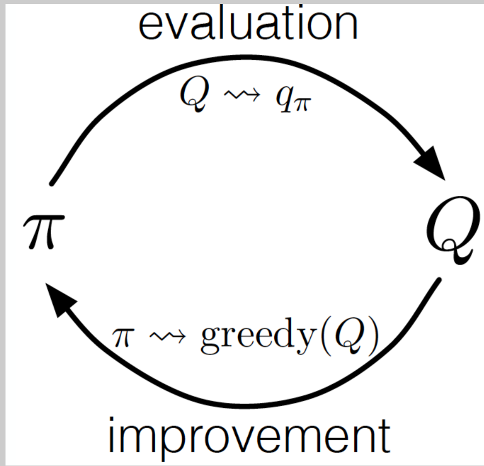
$$\pi'(s) = \underset{a\in\mathcal{A}}{\arg\max}\ Q(s,a)$$

Generalized Policy Iteration with Action-Value Function



Policy evaluation Monte-Carlo policy evaluation, $Q = q_\pi$
Policy improvement Greedy policy improvement?

Monte-Carlo Control



- MC policy iteration: policy evaluation using MC methods followed by policy improvement
- Policy improvement: greedify with respect to value (or action-value) function

Example of Greedy Action Selection


"Behind one door is tenure - behind the other is flipping burgers at McDonald's."

- There are two doors in front of you.
- You open the left door and get reward 0
  $V(left) = 0$
- You open the right door and get reward +1
  $V(right) = +1$
- You open the right door and get reward +3
  $V(right) = +3$
- You open the right door and get reward +2
  $V(right) = +2$

$\vdots$

- Are you sure you've chosen the best door?

ϵ-Greedy Exploration

- Tradeoff between exploitation and exploration
- All m actions are tried with non-zero probability
- With probability $1-\epsilon$ choose the greedy action
- With probability ϵ choose an action at random

$$\pi(a|s) = \begin{cases} \epsilon/m + 1 - \epsilon & \text{if } a^* = \arg\max_{a \in \mathcal{A}} Q(s,a) \\ \epsilon/m & \text{otherwise} \end{cases}$$

ϵ-Greedy Policy Improvement

Theorem

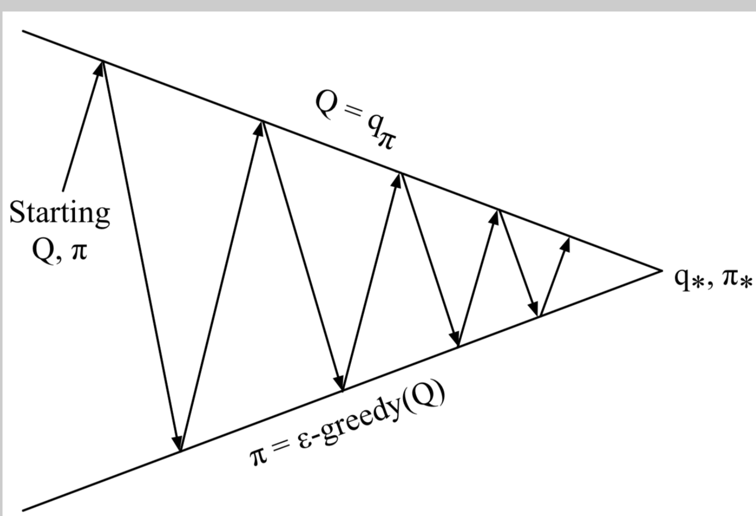
For any ϵ-greedy policy π, the ϵ-greedy policy π' with respect to q_π is an improvement, $v_{\pi'}(s) \geq v_\pi(s)$

$$\begin{aligned} q_\pi(s, \pi'(s)) &= \sum_{a \in \mathcal{A}} \pi'(a|s) q_\pi(s,a) \\ &= \epsilon/m \sum_{a \in \mathcal{A}} q_\pi(s,a) + (1-\epsilon) \max_{a \in \mathcal{A}} q_\pi(s,a) \\ &\geq \epsilon/m \sum_{a \in \mathcal{A}} q_\pi(s,a) + (1-\epsilon) \sum_{a \in \mathcal{A}} \frac{\pi(a|s) - \epsilon/m}{1-\epsilon} q_\pi(s,a) \\ &= \sum_{a \in \mathcal{A}} \pi(a|s) q_\pi(s,a) = v_\pi(s) \end{aligned}$$

Therefore from policy improvement theorem, $v_{\pi'}(s) \geq v_\pi(s)$

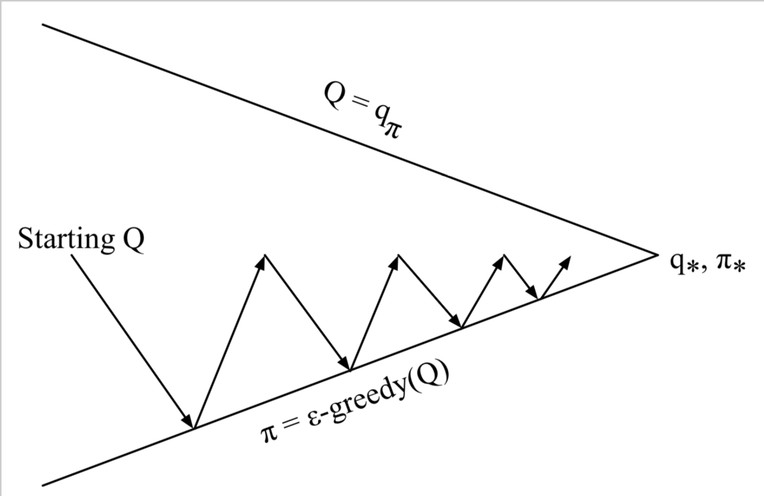
Remark

Monte-Carlo Policy Iteration



Policy evaluation Monte-Carlo policy evaluation, $Q = q_\pi$
Policy improvement ϵ-greedy policy improvement

Monte-Carlo Control



Every episode:

Policy evaluation Monte-Carlo policy evaluation, $Q \approx q_\pi$

Policy improvement ϵ-greedy policy improvement

GLIE

Definition

Greedy in the Limit with Infinite Exploration (GLIE)

- All state-action pairs are explored infinitely many times,

$$\lim_{k \to \infty} N_k(s, a) = \infty$$

- The policy converges on a greedy policy,

$$\lim_{k \to \infty} \pi_k(a|s) = 1(a = \arg\max_{a' \in \mathcal{A}} Q_k(s, a'))$$

GLIE

- For example, ϵ-greedy is GLIE if ϵ reduces to zero at $\epsilon_k = \frac{1}{k}$
- Keep a declining exploration probability at a sufficiently slow rate that try all actions infinitely ofen
- In practice, $\epsilon_k = \frac{1}{k^\beta}$ with $\beta \in (0.5, 1]$

GLIE Monte-Carlo Control

- Sample kth episode using π: $\{S_1, A_1, R_2, \ldots, S_T\} \sim \pi$
- For each state S_t and action A_t in the episode,

$$N(S_t, A_t) \leftarrow N(S_t, A_t) + 1$$
$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \frac{1}{N(S_t, A_t)}(G_t - Q(S_t, A_t))$$

- Improve policy based on new action-value function

$$\epsilon \leftarrow 1/k$$
$$\pi \leftarrow \epsilon\text{-greedy}(Q)$$

Theorem

GLIE Monte-Carlo control converges to the optimal action-value function, $Q(s, a) \rightarrow q_*(s, a)$

On-Policy First-Visit MC Control

On-policy first-visit MC control (for ε-soft policies), estimates $\pi \approx \pi_*$

Algorithm parameter: small $\varepsilon > 0$
Initialize:
  $\pi \leftarrow$ an arbitrary ε-soft policy
  $Q(s,a) \in \mathbb{R}$ (arbitrarily), for all $s \in \mathcal{S}$, $a \in \mathcal{A}(s)$
  $Returns(s,a) \leftarrow$ empty list, for all $s \in \mathcal{S}$, $a \in \mathcal{A}(s)$

Repeat forever (for each episode):
  Generate an episode following π: $S_0, A_0, R_1, \ldots, S_{T-1}, A_{T-1}, R_T$
  $G \leftarrow 0$
  Loop for each step of episode, $t = T-1, T-2, \ldots, 0$:
    $G \leftarrow \gamma G + R_{t+1}$
    Unless the pair S_t, A_t appears in $S_0, A_0, S_1, A_1 \ldots, S_{t-1}, A_{t-1}$:
      Append G to $Returns(S_t, A_t)$
      $Q(S_t, A_t) \leftarrow \text{average}(Returns(S_t, A_t))$
      $A^* \leftarrow \arg\max_a Q(S_t, a)$ (with ties broken arbitrarily)
      For all $a \in \mathcal{A}(S_t)$:

$$\pi(a|S_t) \leftarrow \begin{cases} 1 - \varepsilon + \varepsilon/|\mathcal{A}(S_t)| & \text{if } a = A^* \\ \varepsilon/|\mathcal{A}(S_t)| & \text{if } a \neq A^* \end{cases}$$

Partial Summary of MC

- MC has several advantages over DP
  - can learn directly from interaction with environment
  - No need for full models
  - No need to learn about ALL states (no bootstrapping)
  - Less harmed by violating Markov property
- MC methods provide an alternate policy evaluation process
- One issue to watch for: maintaining sufficient exploration
  - exploring starts, soft policies

Boltzmann Exploration

- A limitation of ϵ-greedy: when we explore, we choose actions at random without regard to their estimated values
- For larger action spaces, this can mean that we are spending a lot of time evaluating actions that are quite poor
- Boltzmann Exploration (also known as "Gibbs sampling" & "soft-max"): choosing an action based on its estimated value.

$$\pi(a|s) = \frac{e^{\beta \hat{Q}(s,a)}}{\sum_{a'} e^{\beta \hat{Q}(s,a')}}$$

- $\hat{Q}(s, a)$ is an estimate of the value of being in state s and taking action a.
- β is a tunable parameter
  - $\beta = 0$ produces a pure exploration policy
  - $\beta \to \infty$ produces a greedy policy

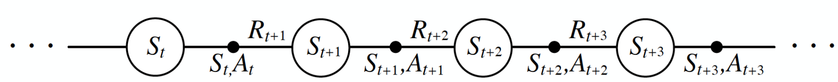
Outline

MC vs. TD Control

- Temporal-difference (TD) learning has several advantages over Monte-Carlo (MC)
  - Lower variance
  - Online
  - Incomplete sequences
- Natural idea: use TD instead of MC in our control loop
  - Apply TD to $Q(S, A)$
  - Use ϵ-greedy policy improvement
  - Update every time-step

Learning An Action-Value Function

Estimate q_π for the current policy π

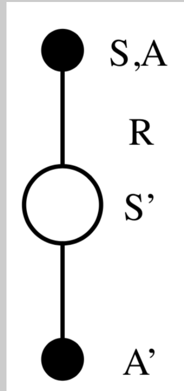


After every transition from a nonterminal state, S_t, do this:

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha\left[R_{t+1} + \gamma Q(S_{t+1}, A_{t+1}) - Q(S_t, A_t)\right]$$

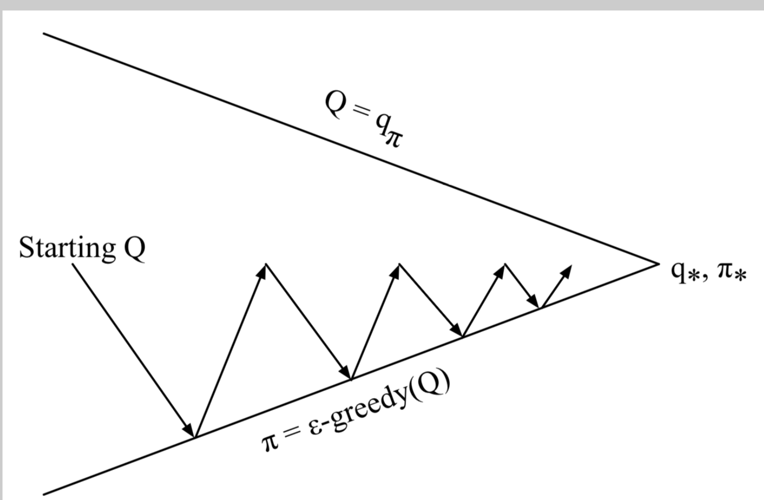
If S_{t+1} is terminal, then define $Q(S_{t+1}, A_{t+1}) = 0$

Updating Action-Value Functions with Sarsa



$$Q(S,A) \leftarrow Q(S,A) + \alpha(R + \gamma Q(S',A') - Q(S,A))$$

On-Policy Control With Sarsa



Every time-step:

Policy evaluation Sarsa, $Q \approx q_\pi$

Policy improvement ϵ-greedy policy improvement

Sarsa Algorithm for On-Policy Control

Sarsa (on-policy TD control) for estimating $Q \approx q_*$

Algorithm parameters: step size $\alpha \in (0, 1]$, small $\varepsilon > 0$
Initialize $Q(s, a)$, for all $s \in \mathcal{S}^+, a \in \mathcal{A}(s)$, arbitrarily except that $Q(\mathit{terminal}, \cdot) = 0$

Loop for each episode:
  Initialize S
  Choose A from S using policy derived from Q (e.g., ε-greedy)
  Loop for each step of episode:
    Take action A, observe R, S'
    Choose A' from S' using policy derived from Q (e.g., ε-greedy)
    $Q(S, A) \leftarrow Q(S, A) + \alpha\big[R + \gamma Q(S', A') - Q(S, A)\big]$
    $S \leftarrow S'$; $A \leftarrow A'$;
  until S is terminal

Convergence of Sarsa

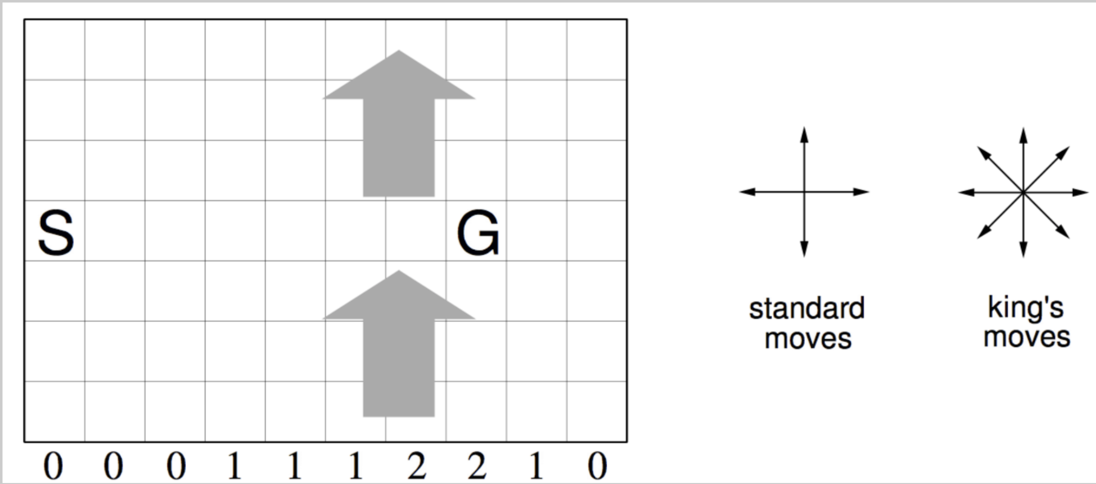
Theorem

Sarsa converges to the optimal action-value function, $Q(s, a) \rightarrow q_*(s, a)$, under the following conditions:

- *GLIE sequence of policies $\pi_t(a|s)$*
- *Robbins-Monro sequence of step-sizes α_t*

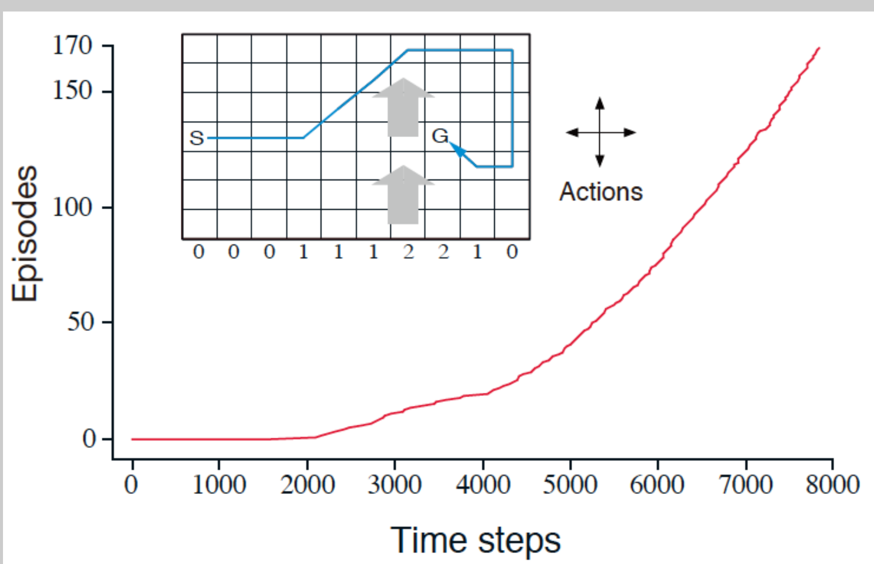
$$\sum_{t=1}^{\infty} \alpha_t = \infty$$
$$\sum_{t=1}^{\infty} \alpha_t^2 < \infty$$

Windy Gridworld Example



- Reward = -1 per time-step until reaching goal
- Undiscounted

Sarsa on the Windy Gridworld



n-Step Sarsa

- Consider the following n-step returns for $n = 1, 2, \infty$:

$$\begin{array}{lll} n = 1 & (Sarsa) & q_t^{(1)} = R_{t+1} + \gamma Q(S_{t+1}) \\ n = 2 & & q_t^{(2)} = R_{t+1} + \gamma R_{t+2} + \gamma^2 Q(S_{t+2}) \\ \vdots & & \vdots \\ n = \infty & (MC) & q_t^{(\infty)} = R_{t+1} + \gamma R_{t+2} + \ldots + \gamma^{T-1} R_T \end{array}$$

- Define the n-step Q-return

$$q_t^{(n)} = R_{t+1} + \gamma R_{t+2} + \ldots + \gamma^{n-1} R_{t+n} + \gamma^n Q(S_{t+n})$$

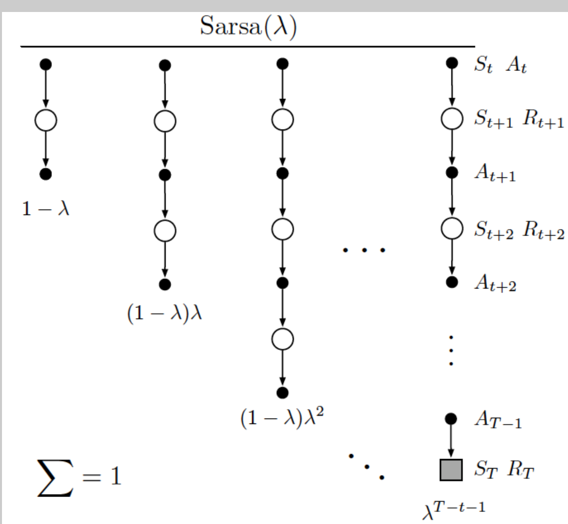
- n-step Sarsa updates $Q(s, a)$ towards the n-step Q-return

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha \left(q_t^{(n)} - Q(S_t, A_t) \right)$$

Forward View Sarsa(λ)

- Forward-view Sarsa(λ)

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha \left(q_t^{\lambda} - Q(S_t, A_t) \right)$$



- The q^{λ} *return* combines all n-step Q-returns $q_t^{(n)}$
- Using weight $(1 - \lambda)\lambda^{n-1}$

$$q_t^{\lambda} = (1 - \lambda) \sum_{n=1}^{\infty} \lambda^{n-1} q_t^{(n)}$$

Outline

Off-Policy Learning

- Learn the value of the target policy π from experience due to behavior policy μ
- In this sense, learning is from experience(data) "off" the target policy, and the overall process is termed off-policy learning.
- For example, π is the greedy policy (and ultimately the optimal policy) while μ is exploratory (e.g.,ϵ-soft)
- In general, we only require coverage, i.e., that μ generates behavior that covers, or includes, π

$$\mu(a|s) > 0 \text{ for every } s, a \text{ at which } \pi(a|s) > 0$$

Off-Policy Learning

- Evaluate target policy $\pi(a|s)$ to compute $v_\pi(s)$ or $q_\pi(s,a)$
- While following behaviour policy $\mu(a|s)$

$$\{S_1, A_1, R_2, \ldots, S_T\} \sim \mu$$

- Why is this important?
  - Learn from observing humans or other agents
  - Re-use experience generated from old policies $\pi_1, \pi_2, \ldots, \pi_{t-1}$
  - Learn about *optimal* policy while following *exploratory* policy
  - Learn about *multiple* policies while following *one* policy

Importance Sampling

- Estimate the expectation of a function

$$\mathbb{E}_{X\sim\pi}[g(X)] = \sum \pi(x)f(x) \approx \frac{1}{n}\sum_{k=1}^{n} g(x_k), x_k \sim \pi$$

- But sometimes it is difficult to sample x from π, then we can sample x from another distribution μ, then correct the weight

$$\mathbb{E}_{X\sim\pi}[g(X)] = \sum \pi(x)g(x) = \sum \mu(x)\frac{\pi(x)}{\mu(x)}g(x)$$

$$= \mathbb{E}_{X\sim\mu}\left[\frac{\pi(X)}{\mu(X)}g(X)\right] \approx \frac{1}{n}\sum_{k=1}^{n}\frac{\pi(x_k)}{\mu(X_k)}g(x_k), x_k \sim \mu$$

- For off-policy learning: weight each return by the ratio of the probabilities of the trajectory under the two policies

Importance Sampling for Off-Policy Monte-Carlo

- Use returns generated from μ to evaluate π
- Weight return G_t according to similarity between policies
- Multiply importance sampling corrections along whole episode

$$G_t^{\pi/\mu} = \frac{\pi(A_t|S_t)}{\mu(A_t|S_t)} \frac{\pi(A_{t+1}|S_{t+1})}{\mu(A_{t+1}|S_{t+1})} \dots \frac{\pi(A_T|S_T)}{\mu(A_T|S_T)} G_t$$

- Update value towards *corrected* return

$$V(S_t) \leftarrow V(S_t) + \alpha \left(G_t^{\pi/\mu} - V(S_t) \right)$$

- Cannot use if μ is zero when π is non-zero
- Importance sampling can dramatically increase variance

Importance Sampling Ratio

- Probability of the rest of the trajectory, after S_t, under π

$$\begin{aligned}
&\Pr\{A_t, S_{t+1}, A_{t+1}, \dots, S_T | S_t, A_{t:T-1} \sim \pi\} \\
&\quad = \pi(A_t|S_t)\, p(S_{t+1}|S_t, A_t)\, \pi(A_{t+1}|S_{t+1}) \cdots p(S_T|S_{T-1}, A_{T-1}) \\
&\quad = \prod_{k=t}^{T-1} \pi(A_k|S_k)\, p(S_{k+1}|S_k, A_k),
\end{aligned}$$

- In importance sampling, each arm is weighted by the relative probability of the trajectory under the two policies

$$\rho_t^T = \frac{\prod_{k=t}^{T-1} \pi(A_k|S_k)\, p(S_{k+1}|S_k, A_k)}{\prod_{k=t}^{T-1} \mu(A_k|S_k)\, p(S_{k+1}|S_k, A_k)} = \prod_{k=t}^{T-1} \frac{\pi(A_k|S_k)}{\mu(A_k|S_k)}$$

- This is called the *importance sampling ratio*
- All importance sampling ratios have expected value 1

$$\mathbb{E}_{A_k \sim \mu}\left[\frac{\pi(A_k|S_k)}{\mu(A_k|S_k)}\right] = \sum_a \mu(a|S_k) \frac{\pi(a|S_k)}{\mu(a|S_k)} = \sum_a \pi(a|S_k) = 1.$$

Off-Policy MC Policy Evaluation

Incremental off-policy every-visit MC policy evaluation (returns $Q \approx q_\pi$

Input: an arbitrary target policy π

Initialize, for all $s \in \mathcal{S}$, $a \in \mathcal{A}(s)$:
  $Q(s,a) \leftarrow$ arbitrary
  $C(s,a) \leftarrow 0$

Repeat forever:
  $\mu \leftarrow$ any policy with coverage of π
  Generate an episode using μ:
    $S_0, A_0, R_1, \ldots, S_{T-1}, A_{T-1}, R_T, S_T$
  $G \leftarrow 0$
  $W \leftarrow 1$
  For $t = T-1, T-2, \ldots$ downto 0:
    $G \leftarrow \gamma G + R_{t+1}$
    $C(S_t, A_t) \leftarrow C(S_t, A_t) + W$
    $Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \frac{W}{C(S_t,A_t)}[G - Q(S_t, A_t)]$
    $W \leftarrow W \frac{\pi(A_t|S_t)}{\mu(A_t|S_t)}$
    If $W = 0$ then ExitForLoop

Off-Policy MC Control

Off-policy every-visit MC control (returns $\pi \approx \pi_*$)

Initialize, for all $s \in \mathcal{S}$, $a \in \mathcal{A}(s)$:
  $Q(s,a) \leftarrow$ arbitrary
  $C(s,a) \leftarrow 0$
  $\pi(s) \leftarrow \arg\max_a Q(S_t, a)$ (with ties broken consistently)

Repeat forever:
  $\mu \leftarrow$ any soft policy
  Generate an episode using μ:
    $S_0, A_0, R_1, \ldots, S_{T-1}, A_{T-1}, R_T, S_T$
  $G \leftarrow 0$
  $W \leftarrow 1$
  For $t = T-1, T-2, \ldots$ downto 0:
    $G \leftarrow \gamma G + R_{t+1}$
    $C(S_t, A_t) \leftarrow C(S_t, A_t) + W$
    $Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \frac{W}{C(S_t,A_t)}[G - Q(S_t, A_t)]$
    $\pi(S_t) \leftarrow \arg\max_a Q(S_t, a)$ (with ties broken consistently)
    If $A_t \neq \pi(S_t)$ then ExitForLoop
    $W \leftarrow W \frac{1}{\mu(A_t|S_t)}$

Target policy is greedy and deterministic

Behavior policy is soft, typically ϵ-greedy

Importance Sampling for Off-Policy TD

- Use TD targets generated from μ to evaluate π
- Weight TD target $R + \gamma V(S')$ by importance sampling
- Only need a single importance sampling correction

$$V(S_t) \leftarrow V(S_t) + \alpha \left(\frac{\pi(A_t|S_t)}{\mu(A_t|S_t)} (R_{t+1} + \gamma V(S_{t+1})) - V(S_t) \right)$$

- Much lower variance than Monte-Carlo importance sampling
- Policies only need to be similar over a single step

Outline

1. Introduction
2. On-Policy Monte-Carlo Control
3. On-Policy Temporal-Difference Learning
4. Off-Policy Learning: Importance Sampling
5. **Off-policy Learning: Q-learning**
6. Summary
7. References

Q-Learning

- We now consider off-policy learning of action-values $Q(s, a)$
- No importance sampling is required
- Next action to evaluate is chosen using behavior policy $A_{t+1} \sim \mu(\cdot|S_t)$
- But we consider alternative successor action $A' \sim \pi(\cdot|S_t)$
- Which means a separate policy is used to choose the alternative action in the future
- And update $Q(S_t, A_t)$ towards value of alternative action

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha(R_{t+1} + \gamma Q(S_{t+1}, A') - Q(S_t, A_t))$$

Off-Policy Control with Q-Learning

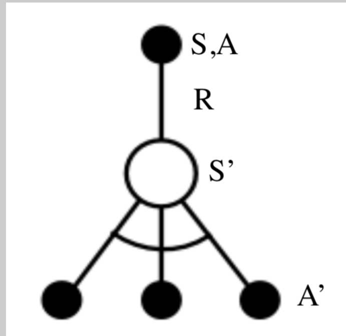
- We now allow both behaviour and target policies to improve
- The target policy π is greedy w.r.t. $Q(s, a)$

$$\pi(S_{t+1}) = \arg\max_{a'} Q(S_{t+1}, a')$$

- The behaviour policy μ is e.g. ϵ-greedy w.r.t. $Q(s, a)$
- The Q-learning target then simplifies:

$$\begin{aligned}
&R_{t+1} + \gamma Q(S_{t+1}, A') \\
=&R_{t+1} + \gamma Q(S_{t+1}, \arg\max_{a'} Q(S_{t+1}, a')) \\
=&R_{t+1} + \max_{a'} \gamma Q(S_{t+1}, a')
\end{aligned}$$

Q-Learning Control Algorithm



$$Q(S,A) \leftarrow Q(S,A) + \alpha \left(R + \gamma \max_{a'} Q(S', a') - Q(S,A) \right)$$

Theorem

Q-learning control converges to the optimal action-value function, $Q(s,a) \to q_*(s,a)$

Q-Learning Algorithm for Off-Policy Control

Q-learning (off-policy TD control) for estimating $\pi \approx \pi_*$

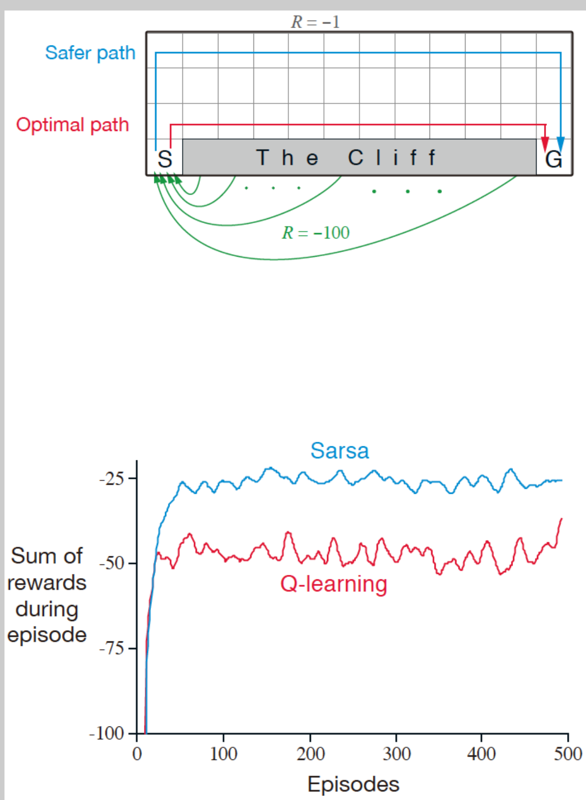
Algorithm parameters: step size $\alpha \in (0,1]$, small $\varepsilon > 0$
Initialize $Q(s,a)$, for all $s \in \mathcal{S}^+, a \in \mathcal{A}(s)$, arbitrarily except that $Q(terminal, \cdot) = 0$

Loop for each episode:
  Initialize S
  Loop for each step of episode:
    Choose A from S using policy derived from Q (e.g., ε-greedy)
    Take action A, observe R, S'
    $Q(S,A) \leftarrow Q(S,A) + \alpha\big[R + \gamma \max_a Q(S',a) - Q(S,A)\big]$
    $S \leftarrow S'$
  until S is terminal

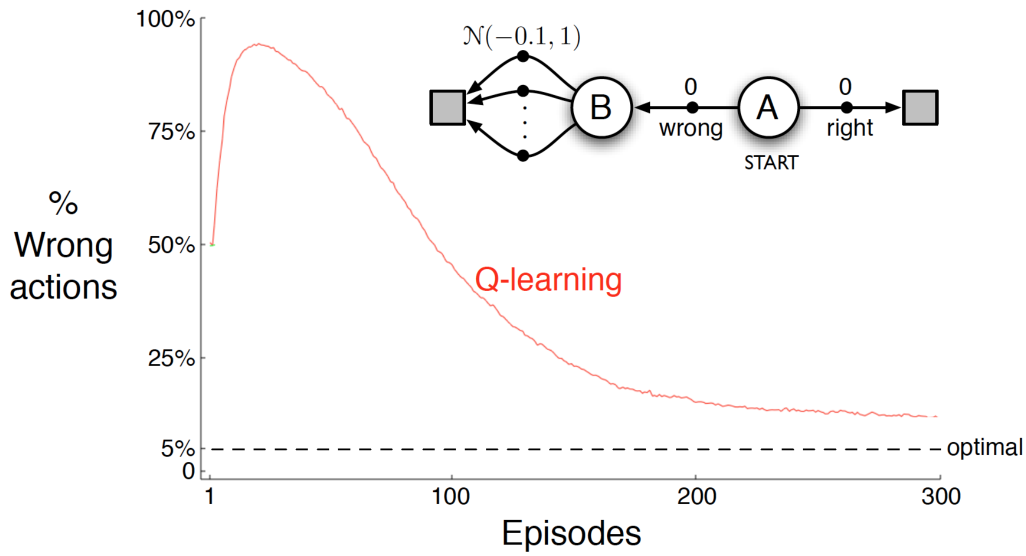
Q-Learning Demo

`https://www.cs.ubc.ca/~poole/demos/rl/q.html`

Cliff Walking Example



Maximization Bias Example



Tabular Q-learning: $Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha \Big[R_{t+1} + \gamma \max_a Q(S_{t+1}, a) - Q(S_t, A_t) \Big]$

Solution: Double Q-Learning

- Train 2 action-value functions Q_1 & Q_2
- Do Q-learning on both, but
  - never on the same time steps (Q_1 & Q_2 are independent)
  - pick Q_1 or Q_2 at random to be updated on each step
- If updating Q_1, use Q_2 for the value of the next state:

$$Q_1(S_t, A_t) \leftarrow Q_1(S_t, A_t) + \alpha \Big(R_{t+1} + Q_2\big(S_{t+1}, \arg\max_a Q_1(S_{t+1}, a)\big) - Q_1(S_t, A_t) \Big)$$

- Action selections are (say) ϵ-greedy with respect to the sum of Q_1 and Q_2

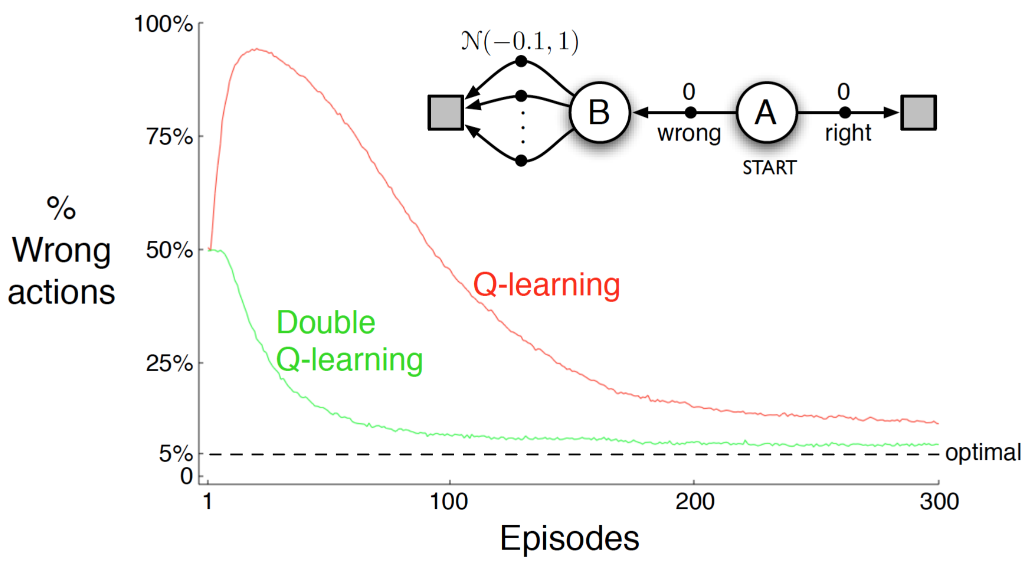
Solution: Double Q-Learning

Double Q-learning, for estimating $Q_1 \approx Q_2 \approx q_*$

Algorithm parameters: step size $\alpha \in (0, 1]$, small $\varepsilon > 0$
Initialize $Q_1(s, a)$ and $Q_2(s, a)$, for all $s \in \mathcal{S}^+, a \in \mathcal{A}(s)$, such that $Q(terminal, \cdot) = 0$

Loop for each episode:
  Initialize S
  Loop for each step of episode:
    Choose A from S using the policy ε-greedy in $Q_1 + Q_2$
    Take action A, observe R, S'
    With 0.5 probabilility:
      $Q_1(S, A) \leftarrow Q_1(S, A) + \alpha\Big(R + \gamma Q_2\big(S', \arg\max_a Q_1(S', a)\big) - Q_1(S, A)\Big)$
    else:
      $Q_2(S, A) \leftarrow Q_2(S, A) + \alpha\Big(R + \gamma Q_1\big(S', \arg\max_a Q_2(S', a)\big) - Q_2(S, A)\Big)$
    $S \leftarrow S'$
  until S is terminal

Maximization Bias Example

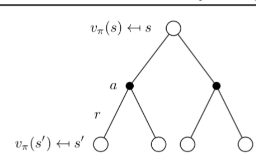
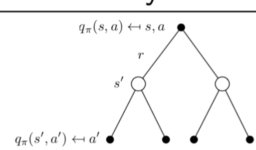
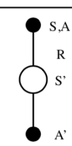
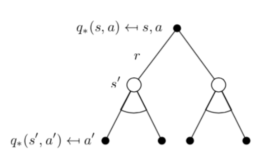
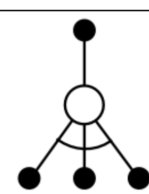


Double Q-learning:

$$Q_1(S_t, A_t) \leftarrow Q_1(S_t, A_t) + \alpha\Big(R_{t+1} + Q_2\big(S_{t+1}, \arg\max_a Q_1(S_{t+1}, a)\big) - Q_1(S_t, A_t)\Big)$$

Outline

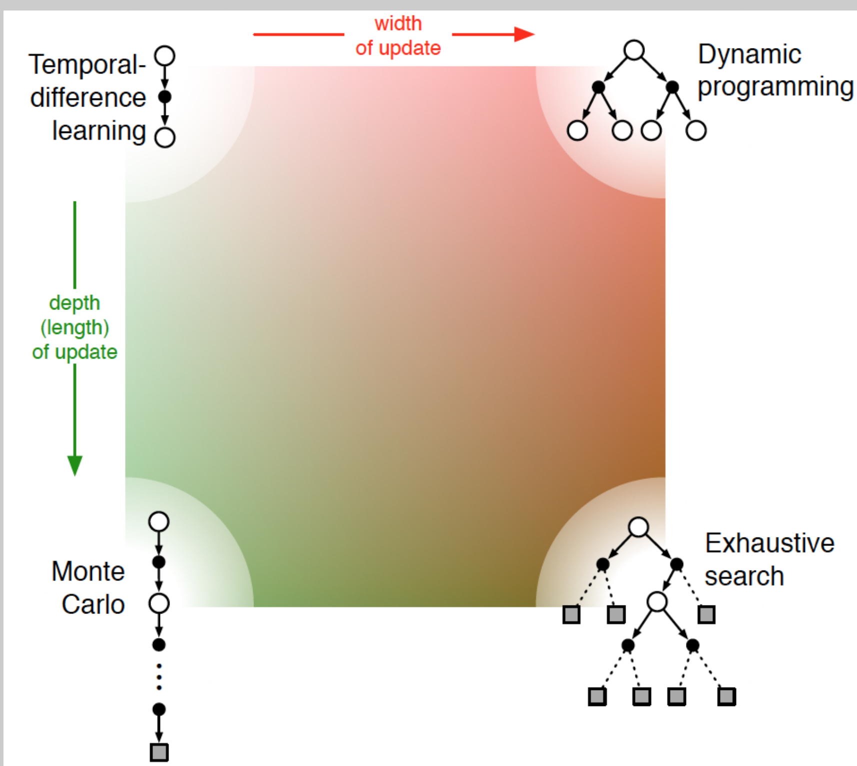
Relationship Between DP and TD

	Full Backup (DP)	*Sample Backup (TD)*
Bellman Expectation Equation for $v_\pi(s)$	 Iterative Policy Evaluation	 TD Learning
Bellman Expectation Equation for $q_\pi(s,a)$	 Q-Policy Iteration	 Sarsa
Bellman Optimality Equation for $q_*(s,a)$	 Q-Value Iteration	 Q-Learning

Relationship Between DP and TD(2)

Full Backup (DP)	*Sample Backup (TD)*
Iterative Policy Evaluation $V(s) \leftarrow \mathbb{E}\left[R + \gamma V(S') \mid s\right]$	TD Learning $V(S) \xleftarrow{\alpha} R + \gamma V(S')$
Q-Policy Iteration $Q(s,a) \leftarrow \mathbb{E}\left[R + \gamma Q(S',A') \mid s,a\right]$	Sarsa $Q(S,A) \xleftarrow{\alpha} R + \gamma Q(S',A')$
Q-Value Iteration $Q(s,a) \leftarrow \mathbb{E}\left[R + \gamma \max_{a' \in \mathcal{A}} Q(S',a') \mid s,a\right]$	Q-Learning $Q(S,A) \xleftarrow{\alpha} R + \gamma \max_{a' \in \mathcal{A}} Q(S',a')$

Unified View of Reinforcement Learning



Outline

Main References

- Reinforcement Learning: An Introduction (second edition), R. Sutton & A. Barto, 2018.
- RL course slides from Richard Sutton, University of Alberta.
- RL course slides from David Silver, University College London.