EE 111 Homework 7

Due date: May. 17th,2019

Turn in your homework in class

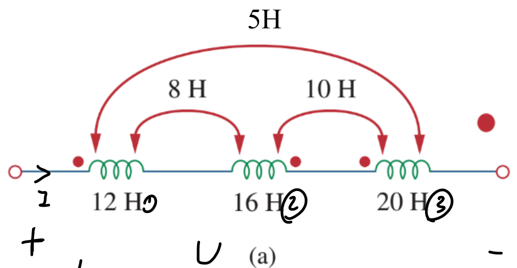
Rules:

- Work on your own. Discussion is permissible, but similar submissions will be judged as plagiarism.
- Please show all intermediate steps: a correct solution without an explanation will get zero credit.
- Please submit on time. No late submission will be accepted.
- Please prepare your submission in English only. No Chinese submission will be accepted.

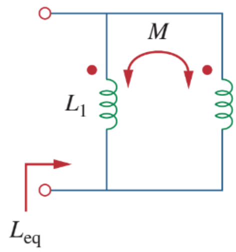
1. (a) For the three coupled coils in Fig. (a), calculate the total inductance. 6'

(b) For the coupled coils in Fig. (b), show that 6'

$$L_{eq} = \frac{L_1L_2 - M^2}{L_1 + L_2 - 2M}.$$

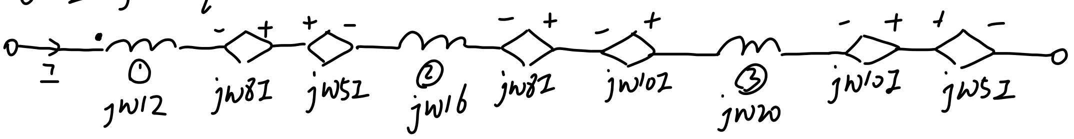


(a)



(b)

(a) $U = I\, j\omega L_{eq}$



3'

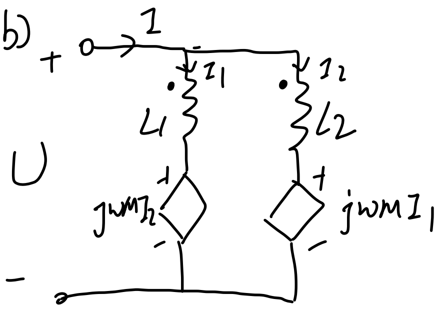
$\therefore U = j\omega 12I - j\omega 8I + j\omega 5I + j\omega 16I - j\omega 8I - j\omega 10I + j\omega 20I - j\omega 10I + j\omega 5I$

$= j\omega 22 I$

$\Rightarrow L_{eq} = 22H$ 3'

Or (a) $L_{eq} = L_1 + L_2 + L_3 + 2M_{13} - 2M_{12} - 2M_{23} = 22H$ (6')

(b)



$$\begin{cases} j\omega L_1 I_1 + j\omega M I_2 = j\omega L_2 I_2 + j\omega M I_1 \\ I_1 + I_2 = I \end{cases}$$

$$\Rightarrow \begin{cases} I_1 = \dfrac{L_2 - M}{L_1 + L_2 - 2M} I \\ I_2 = \dfrac{L_1 - M}{L_1 + L_2 - 2M} I \end{cases}$$

4'

$\therefore U = j\omega L_1 I_1 + j\omega M I_2$

$$= j\omega \left(\frac{L_1(L_2 - M)}{L_1 + L_2 - 2M} + \frac{M(L_1 - M)}{L_1 + L_2 - 2M} \right) I = j\omega \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M} I$$

$$\therefore L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$$

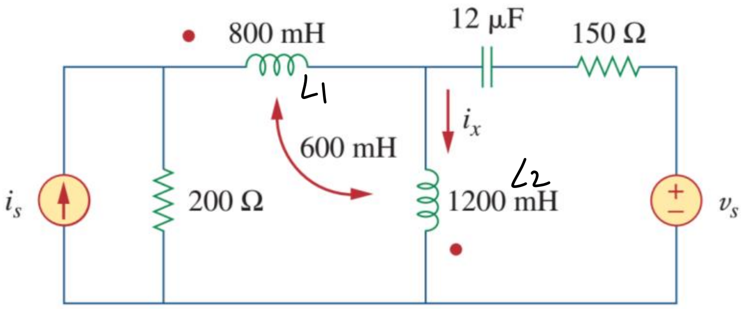
2'

2、With $i_s = 8\cos(600t)$ A and $v_s = 100\cos(600t + 60°)$V,

(a) find the coupling coefficient, 2'

(b) use mesh analysis to find i_x, 6'

(c) determine the energy stored in the coupled inductors at $t = 2$ s. 4'



(a) $k = \dfrac{M}{\sqrt{L_1 L_2}} = \dfrac{\sqrt{6}}{4} = 0.6124 \simeq 0.61$ 2'

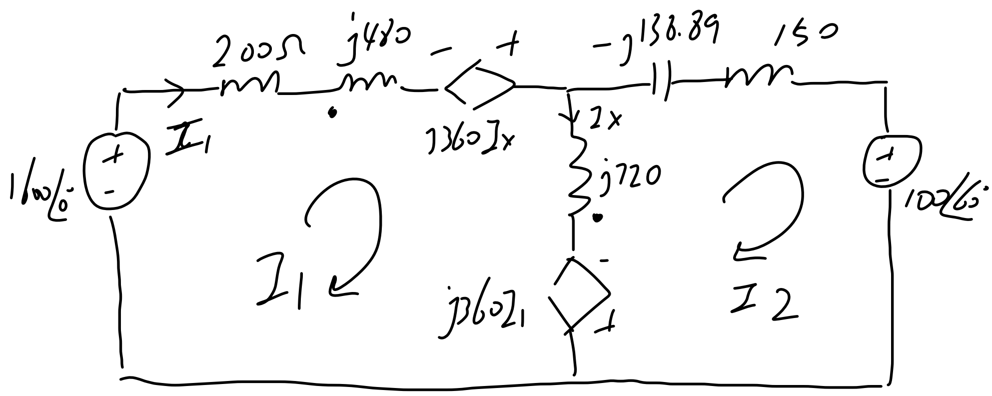
(b) $\omega = 600$ rad/s

$\therefore$ 800mH $\leftrightarrow$ j480Ω

600mH $\leftrightarrow$ j360Ω

1200mH $\leftrightarrow$ j720Ω

12uF $\leftrightarrow$ -j138.89Ω



$I_x = I_1 - I_2$

Mesh①: $-1600 + 200 I_1 + j480 I_1 - j360(I_1 - I_2) + j720(I_1 - I_2) - j360 I_1 = 0$ 2'

Mesh②: $(-j138.89 + 150) I_2 + 100\angle 60° + j360 I_1 + j720(I_2 - I_1) = 0$ 2'

$\Rightarrow \begin{cases} -1600 + (200 + j480) I_1 - j360 I_2 = 0 \\ (j720 - j138.89 + 150) I_2 - j360 I_1 + 100\angle 60° = 0 \end{cases} \Rightarrow \begin{cases} I_1 = 2.80 - 3.22j = 4.27\angle -48.99° \\ I_2 = 1.95 - 1.40j = 2.40\angle -35.68° \end{cases}$

$\therefore I_x = I_1 - I_2 = 0.85 - 1.82j = 2.01\angle -64.97°$ A

$\therefore i_x = 2.01\cos(600t - 64.97°)$ A 2'

(c) $t = 2s$. $i_x = 2.01(\cos 600\cdot 2 - 64.97°) = 0.69$A 1', $i_1 = 4.27\cos(600\cdot 2 - 48.99°) = 2.51$A 1'

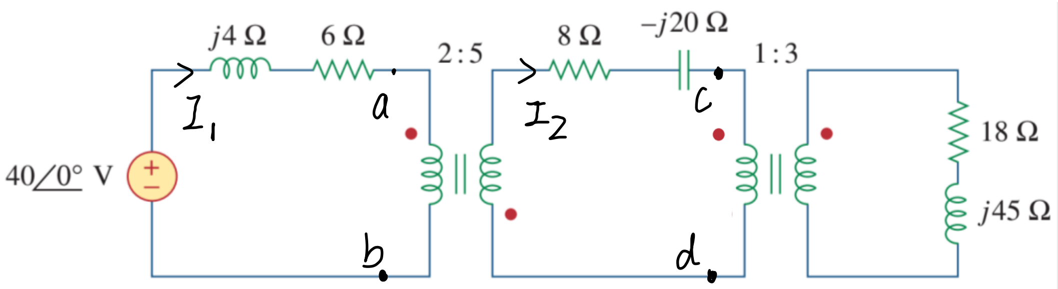
$\therefore W_{L_1 \leftrightarrow L_2} = \frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_x^2 - M i_1 \cdot i_x = \frac{1}{2}\, 0.8\, 2.51^2 + \frac{1}{2}\, 1.2\, 0.69^2 - 0.6\, 2.51\, 0.69$

$= 1.77$J 2'

3、For the network in the figure, find

(a) the complex power supplied by the source, 8'

(b) the average power delivered to the 8-Ω resistor. 4'



(a) $Z_{cd} = \dfrac{18+j45}{\left(\frac{3}{1}\right)^2} = 2+j5$ 2'

$Z_{ab} = \dfrac{8-j20+Z_{cd}}{\left(\frac{5}{2}\right)^2} = 1.6-2.4j$ 2'

$\therefore I_1 = \dfrac{40\angle 0^\circ}{j4+6+Z_{ab}} = \dfrac{40\angle 0^\circ}{7.6+1.6j} = 5.04-1.06j = 5.15\angle -11.89^\circ \text{ A}$ 2'

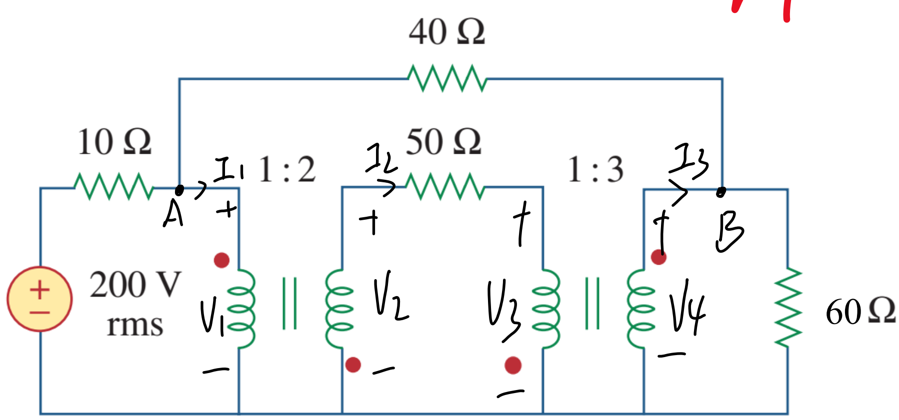
$\therefore S = \frac{1}{2}\, 40\angle 0^\circ \cdot I_1^* = \frac{1}{2} 40\angle 0^\circ\, 5.15\angle 11.89^\circ = 103\angle 11.89^\circ \text{ VA}$ 2'

(b) $\because \dfrac{I_2}{I_1} = -\dfrac{2}{5}$

$\therefore I_2 = -0.4 I_1 = -0.4 \cdot 5.15\angle -11.89^\circ = -2.06\angle -11.89^\circ \text{ A}$ 2'

$\therefore P_{8\Omega} = \frac{1}{2}|I_2|^2 \cdot 8 = 16.97 W.$ 2'

4. Calculate the average power dissipated by the 40Ω resistor. 14'



Nodal analysis.

Node A: $\frac{V_1-200}{10}+\frac{V_1-V_4}{40}+I_1=0$

Node B: $\frac{V_4-V_1}{40}+\frac{V_4}{60}-I_3=0$

$V_2 \Leftrightarrow V_3$: $I_2=\frac{V_2-V_3}{50}$

transformers: $\frac{V_2}{V_1}=-\frac{2}{1}$ $\quad \frac{I_2}{I_1}=-\frac{1}{2}$

$\frac{V_4}{V_3}=-\frac{3}{1}$ $\quad \frac{I_3}{I_2}=-\frac{1}{3}$

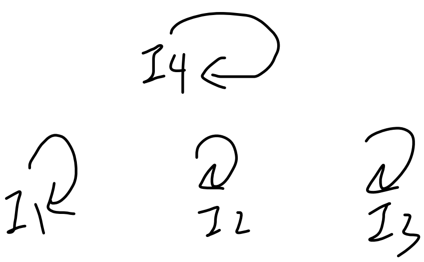
4' 4'

$\Rightarrow$
$I_1=7.97A$
$I_2=-3.99A$
$I_3=1.33A$
$V_1=116.61V$
$V_2=-233.21V$
$V_3=-33.95V$
$V_4=101.85V$

4'

$\therefore P_{40\Omega}=\frac{(V_1-V_4)^2}{40}=\frac{14.76^2}{60}=5.45W$ 2'

Mesh Analysis.



Mesh1: $-200+10I_1+V_1=0$

Mesh2: $-V_2+50(I_2-I_4)+V_3=0$

Mesh3: $-V_4+60I_3=0$

Mesh4: $40I_4+V_4-V_3+50(I_4-I_2)+V_2-V_1=0$

4'

Transformers: $\frac{V_2}{V_1}=-\frac{2}{1}$ $\quad \frac{I_2-I_4}{I_1-I_4}=-\frac{1}{2}$

$\frac{V_4}{V_3}=-\frac{3}{1}$ $\quad \frac{I_3-I_4}{I_2-I_4}=-\frac{1}{3}$

4'

$\Rightarrow$
$I_1=8.34A$
$I_2=-3.62A$
$I_3=1.70A$
$I_4=0.37A$
$V_1=116.61V$
$V_2=-233.21V$
$V_3=-33.95V$
$V_4=101.85V$

4'

$P_{40\Omega}=I_4^2\,40=0.37^2\,40=5.48W$

or $P_{40\Omega}=I_4^2\,40=0.369^2\,40=5.45W$ 2'

5. Sketch the Bode plots for

$$G(s) = \frac{s}{(s+2)^2(s+1)}, \qquad s = j\omega$$

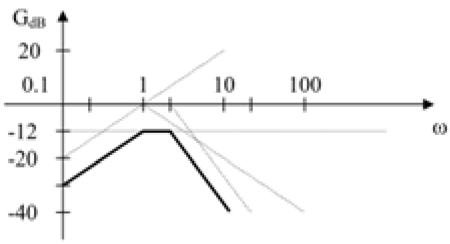
5. Solution:

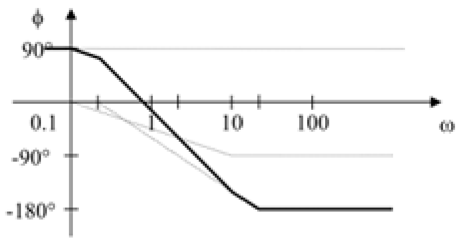
$$\mathbf{G}(\omega) = \frac{(1/4)\, j\omega}{(1+j\omega)(1+j\omega/2)^2}$$

$$G_{dB} = -20\log_{10}4 + 20\log_{10}|j\omega| - 20\log_{10}|1+j\omega| - 40\log_{10}|1+j\omega/2|$$

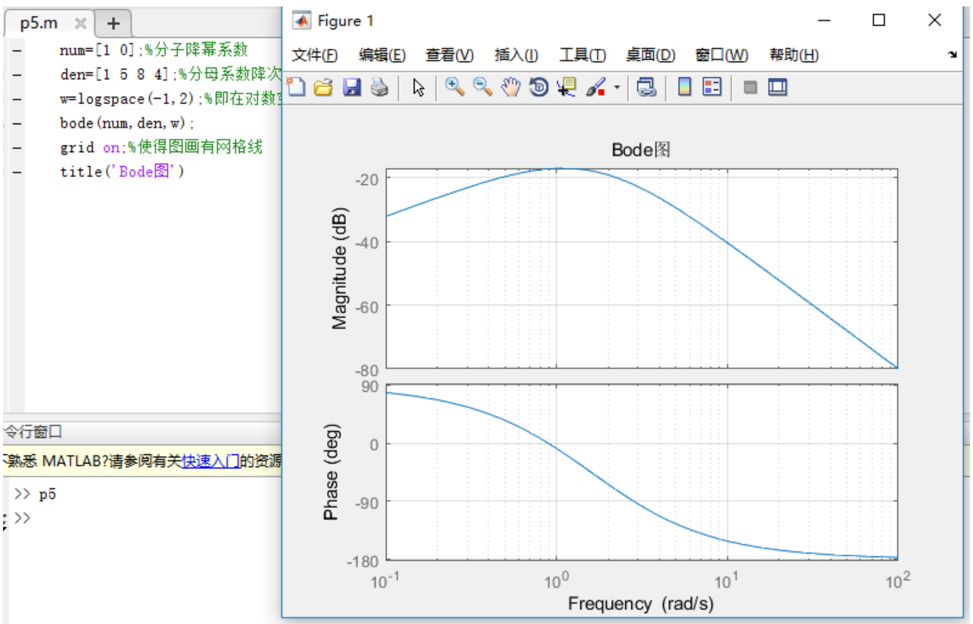
$$\phi = 90° - \tan^{-1}\omega - 2\tan^{-1}\omega/2$$

The magnitude and phase plots are shown below.

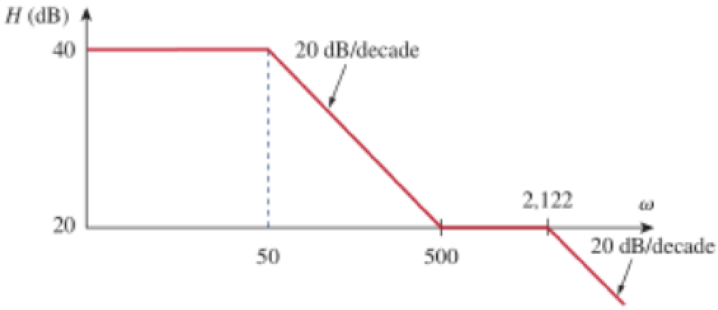




MATLAB:



6. The magnitude plot below represents the transfer function of a preamplifier. Find H(s).



6.Solution

$$40 = 20\log_{10} K \longrightarrow K = 100$$

There is a pole at ω=50 giving 1/(1+jω/50)
There is a zero at ω=500 giving (1 + jω/500).
There is another pole at ω=2122 giving 1/(1 + jω/2122).
Thus,

$$H(\omega) = \frac{100(1+j\omega/500)}{(1+j\omega/50)(1+j\omega/2122)} = \frac{100x\frac{1}{500}(s+500)}{\frac{1}{50}x\frac{1}{2122}(s+50)(s+2122)}$$

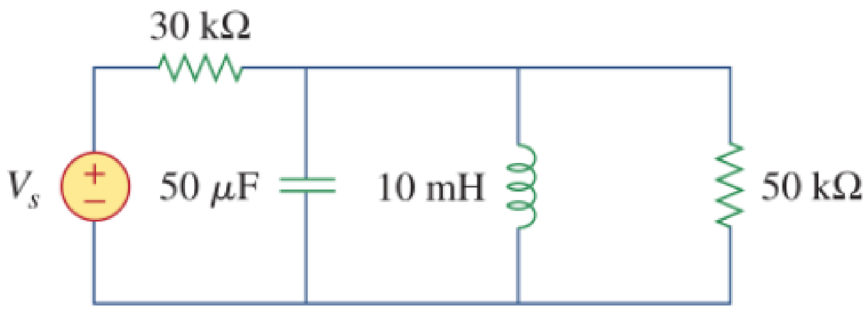
or

$$H(s) = \frac{21220(s+500)}{(s+50)(s+2122)}$$

➢ Hint:

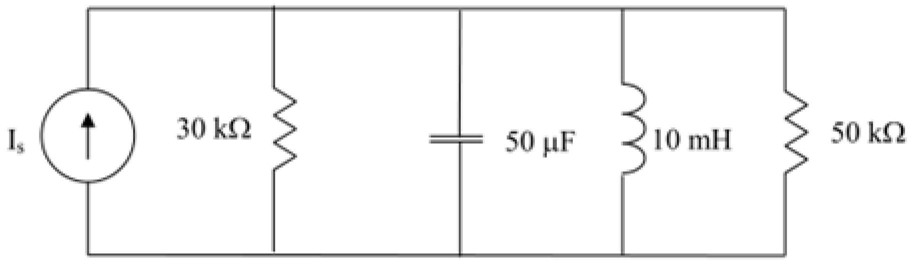
- $H(S) = \frac{-21220(S+500)}{(S+50)(S+2122)}$ is also correct.
- One solution gets full marks.

7. For the circuit shown, find ω_0, B, and Q, as seen by the voltage across the inductor.



7.Solution:

Convert the voltage source to a current source as shown below.



R = 30//50 = 30x50/80 = 18.75 kΩ

This is a parallel resonant circuit.

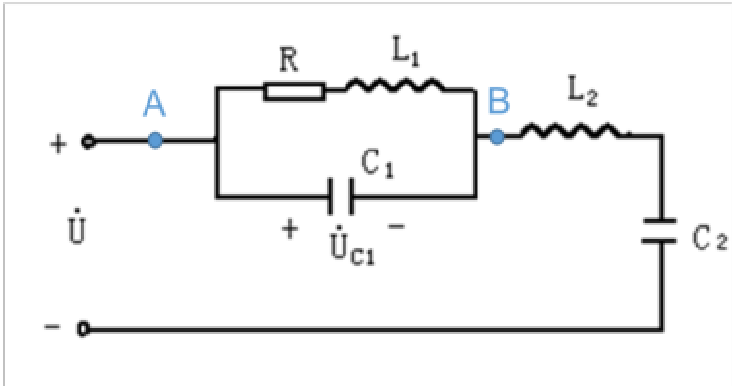
$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10x10^{-3}x50x10^{-6}}} = \underline{1414.21\text{ rad/s}}$$

$$B = \frac{1}{RC} = \frac{1}{18.75x10^{3}x50x10^{-6}} = \underline{1.067\text{ rad/s}}$$

$$Q = \frac{\omega_o}{B} = \frac{447.21}{1.067} = \underline{1325.41}$$

8. For the circuit below, R=50Ω, L_1=5mH, L_2=20mH, C_2=1μF, when the frequency of voltage source $f=\frac{10^4}{2\pi}$ Hz, R、L_1、C_1 is **in resonance as observed between Points A and B**.

At this moment, the voltage U_{C1} of capacitor C_1 is 10 V (U_{C1}=10 V). Please find C_1 and U(rms).



8.Solution: when R、L_1、C_1 is resonance, the resonance angular frequency is: $\omega_1 = 2\pi f = 2\pi \times \frac{10^4}{2\pi} = 10^4$ Hz

$$Y_1 = \frac{1}{R + j\omega_1 L_1} + j\omega_1 C_1 = \left[\frac{R}{R^2 + (\omega_1 L_1)^2}\right] + j\left[\omega_1 C_1 - \frac{\omega_1 L_1}{R^2 + (\omega_1 L_1)^2}\right]$$

$$I_m[Y_1] = 0, \qquad \omega_1 C_1 - \frac{\omega_1 L_1}{R^2 + (\omega_1 L_1)^2} = 0$$

$$\therefore \qquad C_1 = \frac{L_1}{R^2 + (\omega_1 L_1)^2} = 10^{-6}\,F = 1\mu\,F$$

$$Z = \frac{1}{\mathrm{Re}[Y_1]} + j\omega_1 L_2 + \frac{1}{j\omega_1 C_2} = \frac{R^2 + (\omega_1 L_1)^2}{R} + j(\omega_1 L_2 - \frac{1}{\omega_1 C_2}) = 100(1+j)$$

$$R_1 = \frac{1}{\mathrm{Re}[Y_1]} = 100\Omega \qquad I = \frac{U_{C1}}{R_1} = \frac{10\,V}{100\Omega} = 0.1\,A$$

$$U = I \cdot Z = 0.1 \times 100(1+j) = 10(1+j)\ V = 10\sqrt{2}\angle 45^\circ\ V$$

$$\dot{U} = 20\cos(\omega t + 45^\circ)\ V \qquad \therefore \quad U = \frac{20}{\sqrt{2}} = \sqrt{2} \times 10 = 14.14\ V$$