Problem 1

Determine the energy E_∞ and power P_∞ of those signals. Which are energy signals? Which are power signals?

a. $x_1(t) = \cos(t)$

b. $x_2[n] = e^{j(\frac{\pi}{2n}+\frac{\pi}{8})}$

Solution:

a. $E_\infty = \lim_{T\to\infty} \int_{-T}^{T} |\cos(t)|^2\, dt = \int_{-\infty}^{\infty} \frac{1+\cos(2t)}{2} dt = \infty$ (2')

$P_\infty = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |\cos(t)|^2\, dt = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \frac{1+\cos(2t)}{2} dt = \lim_{T\to\infty} \left(\frac{1}{2} + \frac{\sin(2T)}{4T}\right) = \frac{1}{2}$ (2')

It is a power signal. (1')

b. $|e^{j\theta}| = \sqrt{cos^2(\theta) + sin^2(\theta)} = 1$ (1')

$E_\infty = \lim_{N\to\infty} \sum_{n=-N}^{N} 1 = \sum_{-\infty}^{\infty} 1 = \infty$ (1')

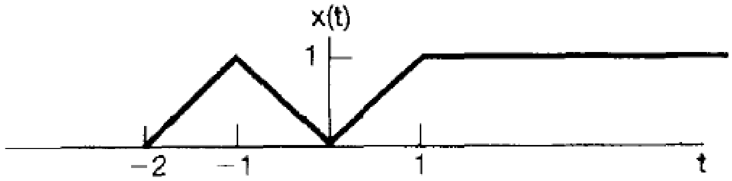
$P_\infty = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} 1 = \lim_{N\to\infty} \frac{2N+1}{2N+1} = 1$ (2')

It is a power signal. (1')

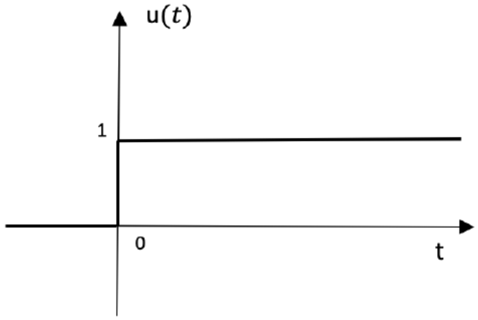
Problem 2

Sketch the signals according to the requirement.

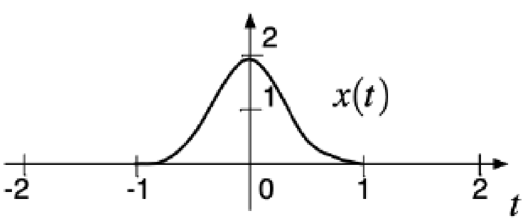
a. Given the signal x(t) shown below, determine and sketch the even part of the signal.



b. Given the signal u(t) shown below, determine and sketch $f(t) = (t-1)u(t-1)$

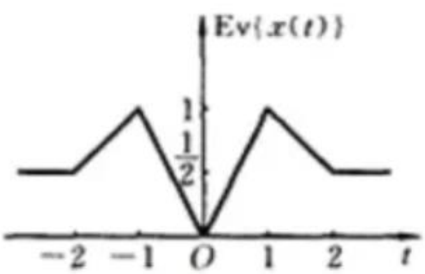


c. Given the signal x(t) shown below, determine and sketch x(2(t-1)) and x(2t-1)

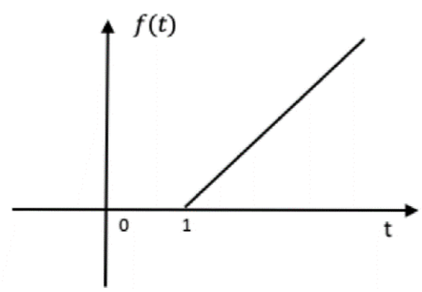


Solution:

a(5')



b(5')



c(10')

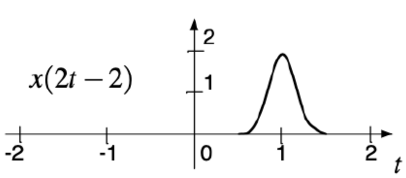


Figure 1: $x(2(t-1))$

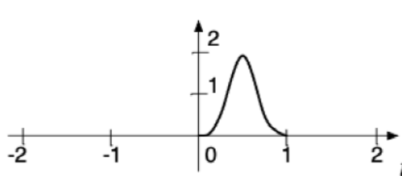


Figure 2: $x(2t-1)$

Problem 3

Determine whether or not each of the following continuous-time signals is periodic. If the signal is periodic, determine its fundamental period. If not, explain why.

a. $x_1(t) = \mathrm{j}e^{j10t}$

b. $x_2[n] = e^{j\frac{2\pi}{3}n} + e^{j\frac{3\pi}{4}n}$

c. $x_3(t) = \mathrm{Ev}\{\sin(4\pi t)u(t)\}$

Solution:

a. $w_0 = 10, T_0 = \frac{2\pi}{10} = \frac{\pi}{5}$ (5’)

b. $w_1 = \frac{2\pi}{3}, N_1 = 3;\ w_2 = \frac{3\pi}{4}, N_2 = 8$→N=24 (5’)

c. $x_3(t) = \frac{1}{2}\sin(4\pi t)\,u(t) + \frac{1}{2}\sin(-4\pi t)\,u(-t) = \frac{1}{2}\sin(4\pi t)\,u(t) - \frac{1}{2}\sin(4\pi t)\,u(-t)$

--> Aperiodic (5’)

Problem 4

In this chapter, we introduced a number of general properties of systems. In particular, a system may or may not be

(1) Memoryless
(2) Time invariant
(3) Linear
(4) Causal
(5) Stable
(6) Invertible

Determine which of the properties hold for each of the following continuous-time systems. Justify your answers. In each example, y(t) denotes the system output and x(t) is the system input.

a. $y(t) = \frac{dx(t)}{dt}$
b. $y[n] = nx[n]$

Solution: (每个性质证明 2'，结论 1')

a. (1) $y(t) = \lim_{\Delta t \to 0} \frac{x(t) - x(t-\Delta t)}{\Delta t}$ ⇒ y(t) is assoaited with $x(t-\Delta t)$ ⇒ The system is memorable.

(2). Given $x_1(t) = x(t-t_0)$. s.t.

$$x_1(t) \to y_1(t) = \frac{dx_1(t)}{dt} = \frac{dx(t-t_0)}{dt} = \frac{dx(t-t_0)}{d(t-t_0)} = y(t-t_0).$$

⇒ The system is time-invariant.

(3). Suppose that $x_1(t) \to y_1(t) = \frac{dx_1(t)}{dt}$, $x_2(t) \to y_2(t) = \frac{dx_2(t)}{dt}$

Give $x_3(t) = ax_1(t) + bx_2(t)$, then

$$x_3(t) \to y_3(t) = \frac{dx_3(t)}{dt} = a\frac{dx_1(t)}{dt} + b\frac{dx_2(t)}{dt} = ay_1(t) + by_2(t)$$

⇒ The system is linear.

(4). $y(t) = \frac{dx(t)}{dt} = \lim_{\Delta t \to 0} \frac{x(t) - x(t-\Delta t)}{\Delta t}$

For that the sign of Δt is undefined. $(t-\Delta t)$ could be before or after t.

⇒ The system is non causal.

(5). When $x(t) = u(t)$ is bounded, $y(t) = \delta(t)$ is unbound.

⇒ The system is unstable.

(6). Suppose $x_1(t) = A(t) + C_1$, $x_2(t) = A(t) + C_2$, $C_1 \neq C_2$

Then $y_1(t) = \frac{dx_1(t)}{dt} = \frac{dA(t)}{dt} = \frac{dx_2(t)}{dt} = y_2(t)$.

⇒ Different x(t) lead to the same y(t).

⇒ The system is non invertible.

b. (1). The system is memoryless and causal.

(2). Suppose that $x_1[n] = x[n-n_0]$, then

$$x_1[n] \to y_1[n] = nx_1[n] = nx[n-n_0]$$
$$\neq (n-n_0)x[n-n_0] = y[n-n_0]$$

$\Rightarrow$ The system is T.V.

(3). Suppose $x_3[n] = ax_1[n] + bx_2[n]$, then

$$x_3[n] \to y_3[n] = nx_3[n] = anx_1[n] + bnx_2[n] = ay_1[n] + by_2[n]$$

$\Rightarrow$ The system is linear.

(4). When $x[n] = u[n]$ is bounded, $y[n] = nx[n]$ is unbounded.

$\Rightarrow$ The system is unstable.

(5) The system is non-invertible.

e.p. $x[n] = \delta[n]$, $y[n] = 0$

$x[n] = 2\delta[n]$, $y[n] = 0$

Problem 5

a. Is the following statement true or false? Justify your answer.
   The series interconnection of two linear time-invariant systems is itself a linear, time-invariant system.

b. Is the following statement true or false? Justify your answer.
   The series interconnection of two nonlinear systems is itself nonlinear.

c. Consider three systems with the following input-output relationships:

$$\text{System 1:} \mathrm{y[n]} = \begin{cases} \mathrm{x}\left[\frac{\mathrm{n}}{2}\right], & \mathrm{n\ even} \\ 0, & \mathrm{n\ odd} \end{cases}$$

$$\text{System 2:} \mathrm{y[n]} = \mathrm{x[n]} + \frac{1}{2}x[n-1] + \frac{1}{4}x[n-2]$$

$$\text{System 3:} \mathrm{y[n]} = x[2n]$$

Suppose that these systems are connected in series as depicted in Figure below. Find the input-output relationship for the overall interconnected system. Is this system linear? Is it time invariant?

Solution:

(a). True. ↗ series interconnection

Suppose two LTI system S_1, S_2 is cascaded:

Suppose that $x_1(t), x_2(t)$ is the input of S_1, $y_1(t), y_2(t)$ be the output of S_1;

$y_1(t), y_2(t)$ is the input of S_2, $z_1(t), z_2(t)$ be the output of S_2;

Then $a x_1(t) + b x_2(t) \xrightarrow{S_1} a y_1(t) + b y_2(t)$

$a y_1(t) + b y_2(t) \xrightarrow{S_2} a z_1(t) + b z_2(t)$ $(a, b \in \mathbb{C})$.

S.t. $a x_1(t) + b x_2(t) \xrightarrow{S_1 S_2} a z_1(t) + b z_2(t)$.

$\Rightarrow$ The cascade of two LTI system is linear. (5')

For S_1, S_2 be time-invariant, then

$x_1(t-T_0) \xrightarrow{S_1} y_1(t-T_0)$.

$y_1(t-T_0) \xrightarrow{S_2} z_1(t-T_0)$.

Then $x_1(t-T_0) \xrightarrow{S_1 S_2} z_1(t-T_0)$.

$\Rightarrow$ The cascade of ~ is time-invariant. (5')

(b). Wrong. (1')

Suppose $y(t) = x(t) + 1$, $z(t) = y(t) - 1 \xRightarrow{\text{cascade}} z(t) = x(t)$, linear. (3')

(C). True. (1')

$$x[n] \rightarrow \boxed{S_1} \xrightarrow{z[n]} \boxed{S_2} \xrightarrow{u[n]} \boxed{S_3} \xrightarrow{y[n]}$$

$$y[n] = u[2n] = z[2n] + \frac{1}{2} z[2n-1] + \frac{1}{4} z[2n-2]$$

$$= x[n] + \frac{1}{4} x[n-1] \qquad (4')$$

$\Rightarrow$ LTI system.