扩展 Kalman 滤波器（EKF）

设系统模型具有非线性关系：

$$x(k)=f(x(k-1)\ ,w(k-1)) \tag{7-1}$$

$$z(k)=h(x(k)\ ,v(k)) \tag{7-2}$$

其中 $x(k)$ 为系统待估计状态， $w(k-1)$ 为系统过程噪声， $z(k)$ 为系统测量， $v(k)$ 为测量噪声， $f(\cdot)$ 和 $h(\cdot)$ 是非线性的过程方程与测量方程。

$$Z(k)-C(k)\hat{x}(k|k-1) \longrightarrow \mathrm{z(k)}-h(\hat{x}(k|k-1),\ 0)$$

即估计可修改为：

$$\hat{\mathrm{x}}(k|k)=\hat{x}(k|k-1)+K(k)(z(k)-h(\hat{x}(k|k-1),\ 0) \tag{7-3}$$

向前一步预测状态可修改为：

$$\hat{x}(k|k-1)=f(\hat{x}(k-1|k-1),\ 0) \tag{7-4}$$

表 7.1 标准 Kalman 滤波器和 EKF 的差别

<table>
<tr><td colspan="2"></td><td>标准 Kalman 滤波器</td><td colspan="2">EKF</td></tr>
<tr><td colspan="2" rowspan="2">系统模型描述</td><td rowspan="2">$x(k+1)=A(k)x(k)+w(k)$
$z(k)=C(k)x(k)+v(k)$</td><td colspan="2">$x(k)=f(x(k-1),w(k-1))$
$z(k)=h(x(k),v(k))$</td></tr>
<tr><td>线性化</td><td>$\frac{\partial f}{\partial x}\vert_{\hat{x}(k-1\vert k-1)}=F(k)$ $\frac{\partial h}{\partial x}\vert_{x=\hat{x}(k\vert k-1)}=H(k)$
$\frac{\partial f}{\partial w}\vert_{\hat{x}(k-1\vert k-1)}=L(k)$ $\frac{\partial h}{\partial v}\vert_{x=\hat{x}(k\vert k-1)}=M(k)$</td></tr>
<tr><td rowspan="5">Kalman 滤波器</td><td>预测</td><td>$\hat{x}(k\vert k-1)=$
$A(k-1)\hat{x}(k-1\vert k-1)$</td><td colspan="2">$\hat{x}(k\vert k-1)=f(\hat{x}(k-1\vert k-1),\ 0)$</td></tr>
<tr><td>更新</td><td>$\hat{x}(k\vert k)=\hat{x}(k\vert k-1)+$
$K(k)\big(z(k)-C(k)\hat{x}(k\vert k-1)\big)$</td><td colspan="2">$\hat{x}(k\vert k)=\hat{x}(k\vert k-1)$
$+K(k)(z(k)-h(\hat{x}(k\vert k-1),\ 0)$</td></tr>
<tr><td>滤波器增益</td><td>$K(k)=P(k\vert k-1)C^T(k)$
$\times(C(k)P(k\vert k-1)C^T(k)+R(k))^{-1}$</td><td colspan="2">$K(k)=P(k\vert k-1)H^T(k)$
$\times(H(k)P(k\vert k-1)H^T(k)$
$+M(k)R(k)M^T(k))^{-1}$</td></tr>
<tr><td>向前一步预测方差</td><td>$P(k\vert k-1)=$
$A(k-1)P(k-1\vert k-1)A^T(k-1)$
$+Q(k-1)$</td><td colspan="2">$P(k\vert k-1)=$
$F(k-1)P(k-1\vert k-1)F^T(k-1)$
$+L(k-1)Q(k-1)L^T(k-1)$</td></tr>
<tr><td>状态估计方差</td><td>$P(k\vert k)=\big(I-K(k)C(k)\big)$
$\times P(k\vert k-1)$</td><td colspan="2">$P(k\vert k)=\big(I-K(k)H(k)\big)$
$\times P(k\vert k-1)$</td></tr>
</table>

不敏 Kalman 滤波器

a. 一般型不敏变换

选取 $2n+1$ 个 sigma 点，并且取值的选择和前面也有所不同。

$$
\begin{aligned}
&x^{(0)} = \bar{x} \\
&x^{(i)} = \bar{x} + \tilde{x}^{(i)} && i = 1,2\cdots 2n \\
&\tilde{x}^{(i)} = (\sqrt{(n+k)P})_i^T && i = 1,2\cdots n \\
&\tilde{x}^{(i)} = -(\sqrt{(n+k)P})_i^T && i = 1,2\cdots n
\end{aligned}
\tag{7-47}
$$

2n+1 个加权系数为：

$$
\begin{aligned}
&w^{(0)} = \frac{k}{n+k} \\
&w^{(i)} = \frac{1}{2(n+k)} && i = 1,2\cdots 2n
\end{aligned}
\tag{7-48}
$$

经过非线性变换：

$$
z^{(i)} = h(x^{(i)}) \tag{7-49}
$$

均值和协方差分别为：

$$\bar{z} = \sum_{i=0}^{2n} w^{(i)} z^{(i)}$$
$$P_z = \sum_{i=0}^{2n} w^{(i)} (z^{(i)} - \bar{z})(z^{(i)} - \bar{z})^T \tag{7-50}$$

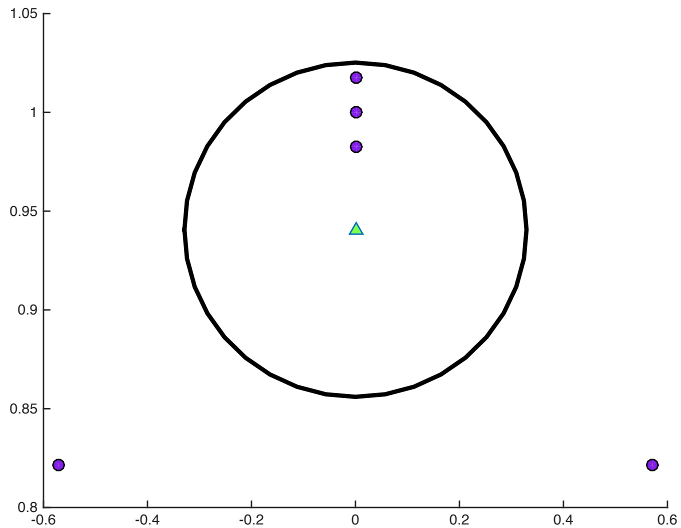

图 7.4 一般型不敏变换的均值与协方差

表 7.2 标准 Kalman 滤波器和 UKF 的差别

<table>
<tr><td colspan="2"></td><td>标准 Kalman 滤波器</td><td colspan="2">UKF</td></tr>
<tr><td colspan="2" rowspan="2">系统模型描述</td><td rowspan="2">$x(k+1)=A(k)x(k)+w(k)$
$z(k)=C(k)x(k)+v(k)$</td><td colspan="2">$x(k)=f(x(k-1),w(k-1))$
$z(k)=h(x(k),v(k))$</td></tr>
<tr><td>设置状态初值</td><td>$x_a(k)=\begin{bmatrix} x(k) \\ w(k) \\ v(k) \end{bmatrix}$, $\hat{x}_a=\begin{bmatrix} E(x_0) \\ 0 \\ 0 \end{bmatrix}=\begin{bmatrix} \hat{x}(0\mid 0) \\ 0 \\ 0 \end{bmatrix}$
$P_a(0\mid 0)=\begin{bmatrix} E\left[(x-\hat{x}(0\mid 0))(x-\hat{x}(0\mid 0))^T\right] & 0 & 0 \\ 0 & Q_0 & 0 \\ 0 & 0 & R_0 \end{bmatrix}$</td></tr>
<tr><td>Kalman 滤波器</td><td>预测</td><td>$\hat{x}(k\mid k-1)=$
$A(k-1)\hat{x}(k-1\mid k-1)$</td><td colspan="2">$\hat{x}^{(i)}(k-1\mid k-1)=\hat{x}(k-1\mid k-1)+\tilde{x}^{(i)} \qquad i=1,\cdots,2n$
$\tilde{x}^{(i)}=(\sqrt{nP(k-1\mid k-1)})_i^T, \qquad i=1,\cdots,n$
$\tilde{x}^{(n+i)}=-(\sqrt{nP(k-1\mid k-1)})_i^T, \qquad i=1,\cdots,n$
$\hat{x}^{(i)}(k\mid k-1)=f\left(\hat{x}^{(i)}(k-1\mid k-1)\right)$
$\hat{x}(k\mid k-1)=\frac{1}{2n}\sum_{i=1}^{2n}\hat{x}^{(i)}(k\mid k-1)$</td></tr>
</table>

	更新	$\hat{x}(k\|k)=\hat{x}(k\|k-1)+$ $K(k)\big(z(k)-C(k)\hat{x}(k\|k-1)\big)$	$\hat{x}^{(i)}(k\|k-1)=\hat{x}(k\|k-1)+\widetilde{x}^{(i)},\quad i=1,\cdots,2n$ $\widetilde{x}^{(i)}=\left(\sqrt{nP(k\|k-1)}\right)_i^T,\quad i=1,\cdots,n$ $\widetilde{x}^{(n+i)}=-\left(\sqrt{nP(k\|k-1)}\right)_i^T,\quad i=1,\cdots,n$ $\hat{z}^{(i)}(k\|k-1)=h\left(\hat{x}^{(i)}(k\|k-1)\right)$ $\hat{z}(k\|k-1)=\frac{1}{2n}\sum_{i=1}^{2n}\hat{z}^{(i)}(k\|k-1)$ $\hat{x}(k\|k)=\hat{x}(k\|k-1)+K(k)\big(z(k)-\hat{z}(k\|k-1)\big)$
	滤波器增益	$K(k)=P(k\|k-1)C^T(k)$ $\times(C(k)P(k\|k-1)C^T(k)+R(k))^{-1}$	$P_z=\frac{1}{2n}\sum_{i=1}^{2n}\left(\hat{z}^{(i)}(k\|k-1)-\hat{z}(k\|k-1)\right)\left(\hat{z}^{(i)}(k\|k-1)-\hat{z}(k\|k-1)\right)^T$ $P_{xz}=\frac{1}{2n}\sum_{i=1}^{2n}\left(\hat{x}^{(i)}(k\|k-1)-\hat{x}(k\|k-1)\right)\left(\hat{z}^{(i)}(k\|k-1)-\hat{z}(k\|k-1)\right)^T$ $K(k)=P_{xz}P_z^{-1}$
	向前一步预测方差	$P(k\|k-1)=$ $A(k-1)P(k-1\|k-1)A^T(k-1)$ $+Q(k-1)$	$P(k\|k-1)=\frac{1}{2n}\sum_{i=1}^{2n}\left(\hat{x}^{(i)}(k\|k-1)-\hat{x}(k\|k-1)\right)\left(\hat{x}^{(i)}(k\|k-1)-\hat{x}(k\|k-1)\right)^T$
	状态估计方差	$P(k\|k)=\big(I-K(k)C(k)\big)$ $\times P(k\|k-1)$	$P(k+1\|k+1)=P(k\|k+1)-K(k)P_zK^T(k)$