9.6.16

EE24BTECH11004 - ANKIT JAINAR

Question: Find the equation of a curve passing through the origin, given that the slope of the tangent to the curve at any point (x, y) is equal to the sum of the coordinates of the point.
Solution:

Variable	**Description**
C	Integration constant
n	Order of given differential equation
$\mathbf{y}(t)$	$\begin{pmatrix} c \\ y(t) \\ y'(t) \\ \vdots \\ y^{n-1}(t) \end{pmatrix}$
h	step size, taken to be 0.001

Theoretical Solution:
Given:

$$\frac{dy}{dx} = x + y \tag{0.1}$$

This is a first-order linear differential equation. Rewriting:

$$\frac{dy}{dx} - y = x \tag{0.2}$$

Using the integrating factor:

$$\text{IF} = e^{\int -1 dx} = e^{-x} \tag{0.3}$$

Multiplying through by the integrating factor:

$$e^{-x}\frac{dy}{dx} - e^{-x}y = xe^{-x} \tag{0.4}$$

$$\frac{d}{dx}\left(e^{-x}y\right) = xe^{-x} \tag{0.5}$$

Integrating both sides:

$$e^{-x}y = \int xe^{-x}dx \tag{0.6}$$

Using integration by parts:

$$\int xe^{-x}dx = -xe^{-x} + \int e^{-x}dx \tag{0.7}$$

$$= -xe^{-x} - e^{-x} \tag{0.8}$$

Thus:

$$e^{-x}y = -xe^{-x} - e^{-x} + C \tag{0.9}$$

Multiply through by e^x:

$$y = -x - 1 + Ce^x \tag{0.10}$$

Applying the initial condition $y(0) = 0$:

$$0 = -0 - 1 + C \cdot e^0 \implies C = 1 \tag{0.11}$$

Therefore, the solution is:

$$y = -x - 1 + e^x \tag{0.12}$$

Computational Solution:

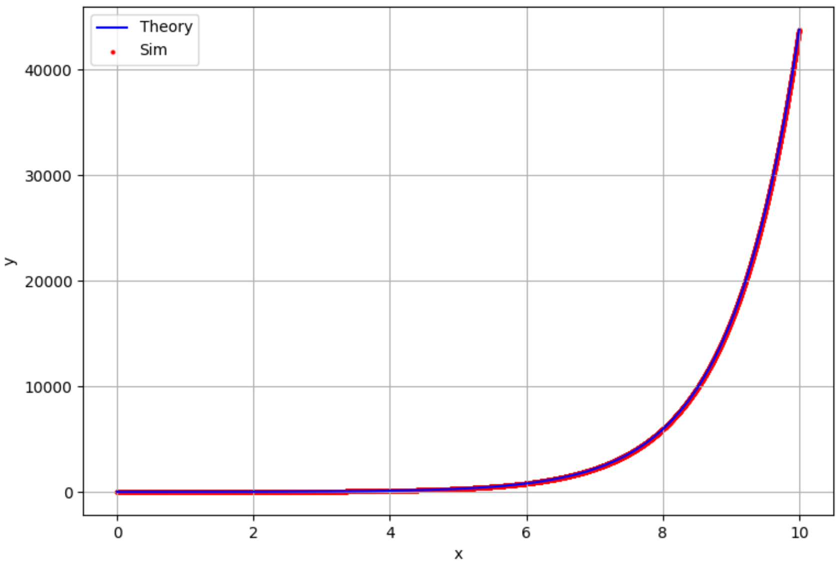
Given:

$$\frac{dy}{dx} = x + y \tag{0.13}$$

Using the discretized form:

$$y_{n+1} = y_n + h(x_n + y_n) \tag{0.14}$$

Where h is the step size. Starting with $x_0 = 0$, $y_0 = 0$, compute y at successive points. By iterating over the time steps, the computational solution is computed. A comparison of theoretical and computational results is shown below.


Theory
Sim
40000
30000
20000
10000
0
y
0
2
4
6
8
10
x