

# 11.16.3.7

EE24BTECH11004 - Ankit Jainar

**Question:** Three coins are tossed once. Find the probability of getting exactly two tails.

## THEORETICAL SOLUTION

The sample space for tossing three coins is:

$$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\} \quad (1)$$

The total number of outcomes is:

$$|S| = 8 \quad (2)$$

The favorable outcomes (exactly two tails) are:

$$A = \{HTT, THT, TTH\} \quad (3)$$

The number of favorable outcomes is:

$$|A| = 3 \quad (4)$$

The probability of getting exactly two tails is:

$$P(A) = \frac{|A|}{|S|} = \frac{3}{8} = 0.375 \quad (5)$$

## INTRODUCTION

This task involves simulating the random tossing of three coins using a C program, compiling it into a shared object (.so) file, and using Python to process the results and generate a probability distribution plot.

## C CODE DESCRIPTION

The C program generates random samples for the coin tosses, where the outcomes are categorized based on the number of tails. The program uses the `rand()` function to simulate the random tosses and increments a counter for each outcome with exactly two tails.

## PYTHON CODE DESCRIPTION

The Python code performs the following:

- 1) Loads the shared object file generated from the C program using the `ctypes` library.
- 2) Simulates a specified number of random coin tosses (e.g., 1,000,000 trials).
- 3) Calculates the probability of getting exactly two tails using the formula:

$$P(\text{exactly two tails}) = \frac{\text{frequency of exactly two tails}}{\text{total trials}} \quad (6)$$

- 4) Plots the probability distribution using `matplotlib`.

## GRAPHICAL OUTPUT

The Python code generates a bar chart where:

- The x-axis represents the outcomes: "0 tails", "1 tail", "2 tails", and "3 tails".
- The y-axis represents the probabilities, ranging from 0 to 1.
- The bar height for "2 tails" corresponds to the probability  $P(A) = 0.375$ .

*Probability Mass Function (PMF):* The PMF represents the probability of each individual outcome in the sample space  $S$ . For the coin toss:

$$S = \{0 \text{ tails}, 1 \text{ tail}, 2 \text{ tails}, 3 \text{ tails}\}, \quad (7)$$

the PMF is given as:

$$P(X = x) = \begin{cases} \frac{1}{8}, & x = 0 \text{ tails}, \\ \frac{3}{8}, & x = 1 \text{ tail}, \\ \frac{3}{8}, & x = 2 \text{ tails}, \\ \frac{1}{8}, & x = 3 \text{ tails}, \\ 0, & x \notin S. \end{cases} \quad (8)$$

*Cumulative Distribution Function (CDF):* The CDF represents the cumulative probability of outcomes up to a given value  $x$ , defined as:

$$F(x) = P(X \leq x) = \sum_{k \in S, k \leq x} P(X = k). \quad (9)$$

For the coin toss:

$$F(x) = \begin{cases} 0, & x < 0 \text{ tails}, \\ \frac{1}{8}, & x = 0 \text{ tails}, \\ \frac{4}{8}, & x = 1 \text{ tail}, \\ \frac{7}{8}, & x = 2 \text{ tails}, \\ 1, & x \geq 3 \text{ tails}. \end{cases} \quad (10)$$

### Simulation Process

We simulate the tossing of three coins using the following steps:

- 1) The sample space consists of outcomes in the set:

$$S = \{0 \text{ tails}, 1 \text{ tail}, 2 \text{ tails}, 3 \text{ tails}\}. \quad (11)$$

- 2) For each simulated toss, a random integer  $X$  is generated such that:

$$X \in \{0, 1, 2, 3\}, \quad (12)$$

using a random number generator function based on binomial trials.

- 3) The number of occurrences of each outcome is tracked over  $N$  trials, where  $N$  is the total number of simulations.
- 4) Both the PMF and CDF are computed:
  - **\*\*PMF\*\***: The frequency of each outcome is divided by the total number of trials to compute the probabilities.
  - **\*\*CDF\*\***: The cumulative probabilities are calculated as the running total of the PMF values.

### Calculation of Probabilities

*Probability of Exactly Two Tails (PMF):* The probability of exactly two tails is computed as:

$$P(\text{exactly two tails}) = \frac{3}{8} = 0.375. \quad (13)$$

*Cumulative Probability (CDF):* The cumulative probability of outcomes up to a given value is:

$$F(x) = \begin{cases} P(0 \text{ tails}), & x = 0 \text{ tails}, \\ P(0 \text{ tails}) + P(1 \text{ tail}), & x = 1 \text{ tail}, \\ P(0 \text{ tails}) + P(1 \text{ tail}) + P(2 \text{ tails}), & x = 2 \text{ tails}, \\ 1, & x = 3 \text{ tails}. \end{cases} \quad (14)$$

For the coin toss:

$$F(0 \text{ tails}) = 0.125, \quad F(1 \text{ tail}) = 0.5, \quad F(2 \text{ tails}) = 0.875, \quad F(3 \text{ tails}) = 1. \quad (15)$$

*Probability of Selecting  $X \notin S$ :* Since all outcomes belong to the set  $S = \{0, 1, 2, 3\}$ , the probability of selecting  $X \notin S$  is:

$$P(X \notin S) = 0. \quad (16)$$

### Output Representation

The computed probabilities are represented in two forms:

- PMF: The probabilities of each outcome (0 tails, 1 tail, 2 tails, 3 tails).
- CDF: The cumulative probabilities up to each outcome.

### CONCLUSION

This task demonstrates the integration of C and Python for simulating and visualizing a probabilistic experiment. The probability of getting exactly two tails from tossing three coins is calculated as **0.375**, matching the theoretical value.



