11.16.3.7

EE24BTECH11004 - Ankit Jainar

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Question

Three coins are tossed once. Find the probability of getting exactly two tails.

Theoretical Solution

The sample space for tossing three coins is:

$$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}.$$

The total number of outcomes is:

$$|S| = 8.$$

The favorable outcomes for exactly two tails are:

$$A = \{HTT, THT, TTH\}.$$

The number of favorable outcomes is:

$$|A| = 3.$$

The probability of getting exactly two tails is:

$$P(A) = \frac{|A|}{|S|} = \frac{3}{8} = 0.375.$$

Introduction

This task involves simulating the random tossing of three coins using a C program, compiling it into a shared object (.so)file and using Python to process the results and generate a Probability Distribution plot, Probability Mass Function, Cumulative Distribution Function.

C Code Description

The C program generates random samples for the coin tosses, where the outcomes are categorized based on the number of tails. The program uses the `rand()` function to simulate the random tosses and increments a counter for each outcome with exactly two tails.

Python Code Description

The Python code performs the following:

1. Loads the shared object file generated from the C program using the `ctypes` library.
2. Simulates a specified number of random coin tosses (e.g., 1,000,000 trials).
3. Calculates the probability of getting exactly two tails using the formula:

$$P(\text{exactly two tails}) = \frac{\text{frequency of exactly two tails}}{\text{total trials}} \tag{1}$$

4. Plots the probability distribution,mass and cumulative distribution functions using `matplotlib`.

Graphical Output

The Python code generates a bar chart where:

- The x-axis represents the outcomes: "0 tails", "1 tail", "2 tails", and "3 tails".
- The y-axis represents the probabilities, ranging from 0 to 1.
- The bar height for "2 tails" corresponds to the probability $P(A) = 0.375$.

Probability Mass Function (PMF)

The PMF represents the probability of each individual outcome in the sample space S. For the coin toss:

$$S = \{0 \text{ tails}, 1 \text{ tail}, 2 \text{ tails}, 3 \text{ tails}\}.$$

The PMF is given as:

$$P(X = x) = \begin{cases} \frac{1}{8}, & x = 0 \text{ tails}, \\ \frac{3}{8}, & x = 1 \text{ tail}, \\ \frac{3}{8}, & x = 2 \text{ tails}, \\ \frac{1}{8}, & x = 3 \text{ tails}. \end{cases}$$

Cumulative Distribution Function (CDF)

The CDF represents the cumulative probability of outcomes up to a given value x, defined as:

$$F(x) = P(X \leq x) = \sum_{k \in S, k \leq x} P(X = k).$$

For the coin toss:

$$F(x) = \begin{cases} 0, & x < 0, \\ \frac{1}{8}, & x = 0, \\ \frac{4}{8}, & x = 1, \\ \frac{7}{8}, & x = 2, \\ 1, & x \geq 3. \end{cases}$$

Simulation Process

We simulate the tossing of three coins using the following steps:

1. The sample space consists of outcomes in the set:

$$S = \{0 \text{ tails}, 1 \text{ tail}, 2 \text{ tails}, 3 \text{ tails}\}.$$

2. For each simulated toss, a random integer X is generated such that:

$$X \in \{0, 1, 2, 3\},$$

   using a random number generator function based on binomial trials.

3. The number of occurrences of each outcome is tracked over N trials, where N is the total number of simulations.
4. Both the PMF and CDF are computed:
   - **PMF**: The frquency of each outcome is divided by the total number of trials to compute the probabilities.
   - **CDF**: The cumulative probabilities are calculated as the running total of the PMF values.

Conclusion

This task demonstrates the integration of C and Python for simulating and visualizing a probabilistic experiment. The probability of getting exactly two tails from tossing three coins is calculated as **0.375**, matching the theoretical value.

Graphical Results

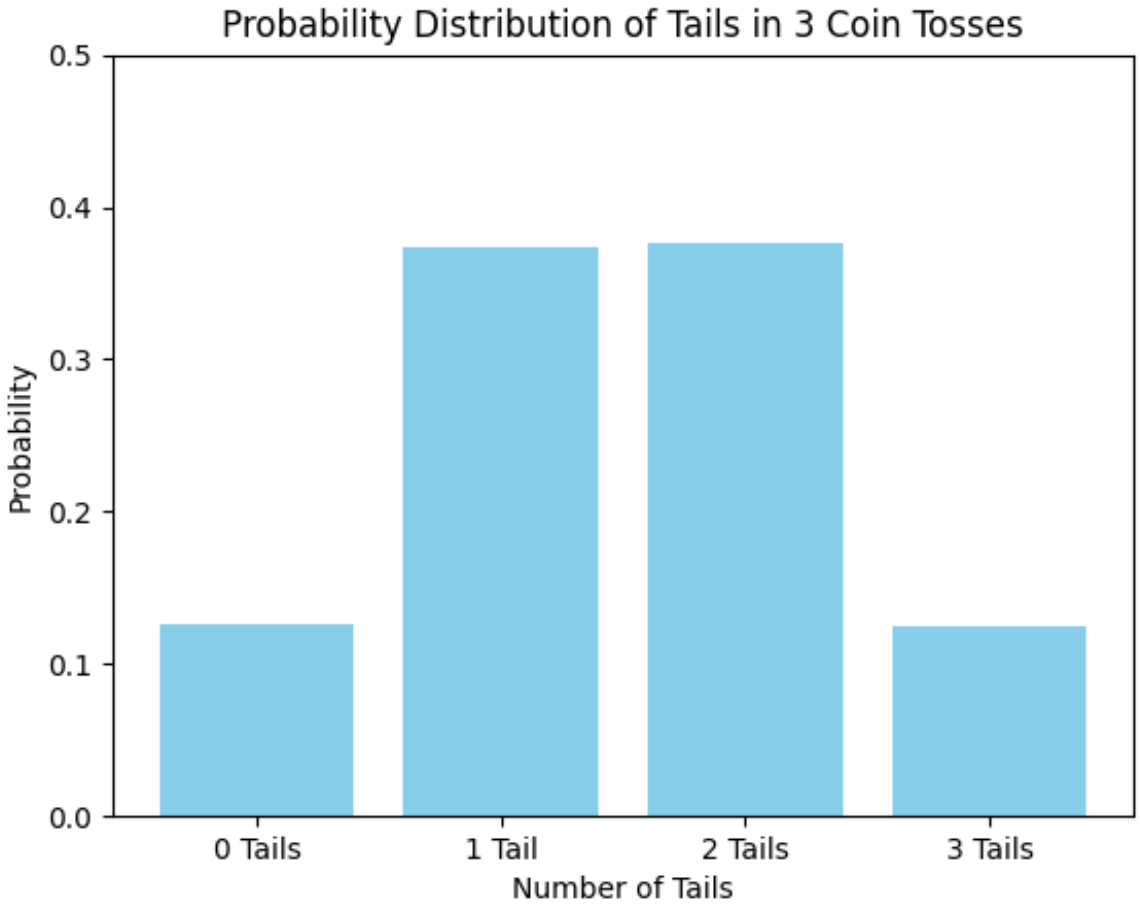

Figure: Probability Distributive Function (PDF)

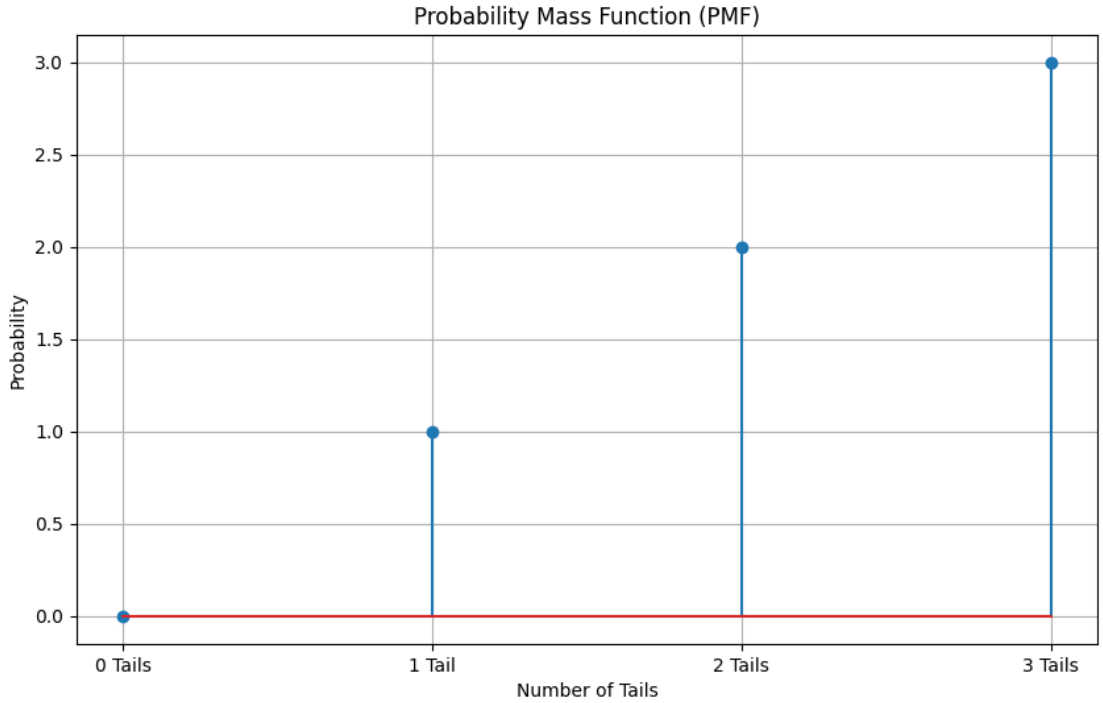

Figure: Probability Mass Function (PMF)

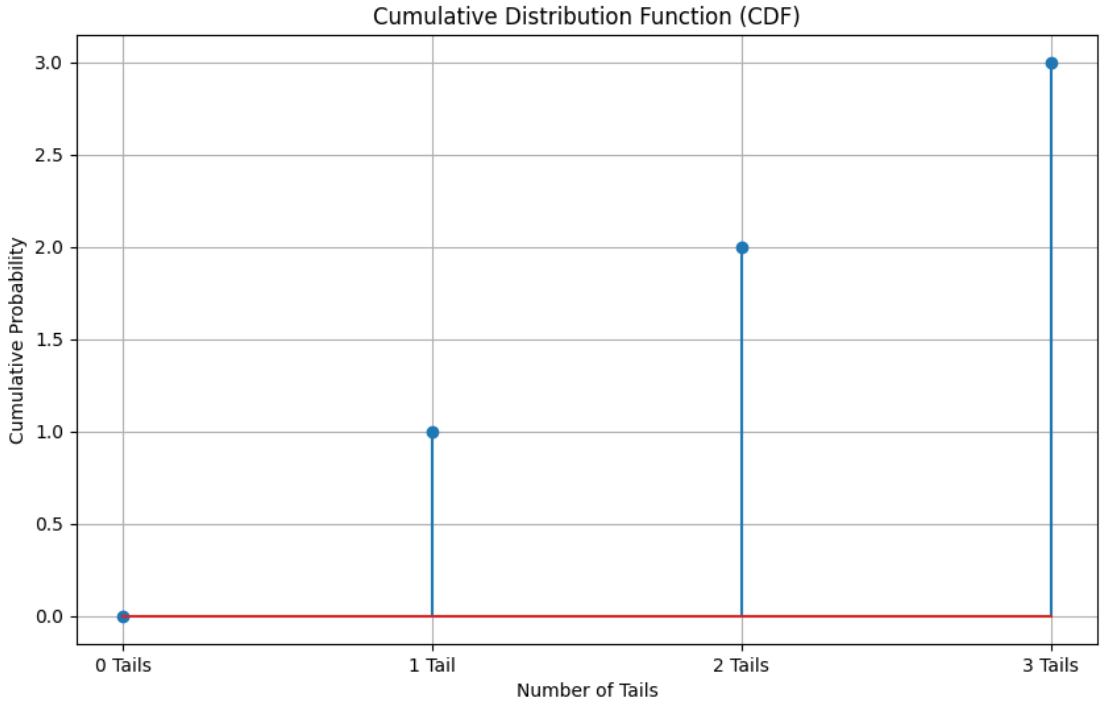

Figure: Cumulative Distribution Function (CDF)