9.1.5

EE24BTECH11004 - ANKIT JAINAR

Question: Solve the differential equation $\frac{d^2y}{dx^2} = \cos(3x) + \sin(3x)$.
Solution:

Variable	Description
C_1	First Integration constant
C_2	Second Integration constant
n	Order of differential equation
$\mathbf{y}(t)$	$\begin{pmatrix} c \\ y(t) \\ y'(t) \\ \vdots \\ y^{n-1}(t) \end{pmatrix}$
h	step size, taken to as 0.001

Theoretical Solution:

$$\frac{d^2y}{dx^2} = \cos(3x) + \sin(3x) \tag{0.1}$$

$$\frac{dy}{dx} = \int \cos(3x)dx + \int \sin(3x)dx + c_1 \tag{0.2}$$

$$= \frac{\sin(3x)}{3} - \frac{\cos(3x)}{3} + c_1 \tag{0.3}$$

$$y = \int \left(\frac{\sin(3x)}{3} - \frac{\cos(3x)}{3} + c_1\right) dx + c_2 \tag{0.4}$$

$$= -\frac{\cos(3x)}{9} - \frac{\sin(3x)}{9} + c_1 x + c_2 \tag{0.5}$$

Assuming the initial conditions $y(0) = 0$ and $y'(0) = 0$

substituting the initial conditions:

$$y(0) = 0 \implies -\frac{\cos(0)}{9} - \frac{\sin(0)}{9} + c_1 \cdot 0 + c_2 = 0 \tag{0.6}$$

$$c_2 = \frac{1}{9} \tag{0.7}$$

$$y'(0) = 0 \implies \frac{\sin(0)}{3} - \frac{\cos(0)}{3} + c_1 = 0 \tag{0.8}$$

$$c_1 = \frac{1}{3} \tag{0.9}$$

Thus, the theoretical solution is:

$$y(x) = -\frac{\cos(3x)}{9} - \frac{\sin(3x)}{9} + \frac{x}{3} + \frac{1}{9} \tag{0.10}$$

Computational Solution:

Consider the given linear differential equation:

$$\frac{d^2y}{dx^2} = \cos(3x) + \sin(3x) \tag{0.11}$$

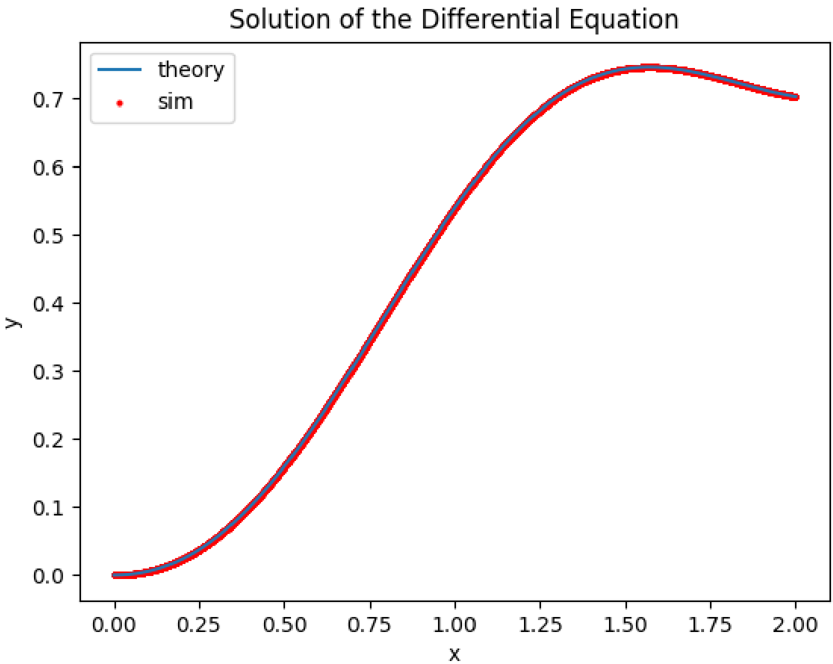
Using discretization, the second-order derivative can be approximated as:

$$y''(t+h) = y''(t) + h \cdot (\cos(3t) + \sin(3t)) \tag{0.12}$$

To express this as a system of equations:

$$\mathbf{y}_{k+1} = \begin{pmatrix} 1 & h & \frac{h^2}{2} \\ 0 & 1 & h \\ 0 & 0 & 1 \end{pmatrix} \cdot \mathbf{y}_k + \begin{pmatrix} 0 \\ 0 \\ h^2 (\cos(3x_k) + \sin(3x_k)) \end{pmatrix} \tag{0.13}$$

By iterating over the time steps, the computational solution is computed. A comparison of theoretical and computational result is shown in the figure below.


Solution of the Differential Equation
theory
sim
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
y
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
x