11.16.3.6

EE24BTECH11004 - Ankit Jainar

Question: There are four men and six women on the city council. If one council member is selected for a committee at random, how likely is it that it is a woman?

THEORETICAL SOLUTION

The total number of council members is:

$$|S| = 4 + 6 = 10 \tag{1}$$

The favorable outcomes (selecting a woman) are:

$$|A| = 6 \tag{2}$$

The probability of selecting a woman is:

$$P(A) = \frac{|A|}{|S|} = \frac{6}{10} = 0.6 \tag{3}$$

INTRODUCTION

This task involves simulating the random selection of council members using a C program, compiling it into a shared object (.so) file, and using Python to process the results and generate a probability distribution plot.

C CODE DESCRIPTION

The C program generates random samples for the selection process, where the outcomes are categorized as either man or woman. The program uses the `rand()` function to simulate the random selection and increments a counter for each outcome.

PYTHON CODE DESCRIPTION

The Python code performs the following:
1) Loads the shared object file generated from the C program using the `ctypes` library.
2) Simulates a specified number of random selections (e.g., 1,000,000 trials).
3) Calculates the probability of selecting a woman using the formula:

$$P(\text{woman}) = \frac{\text{frequency of selecting a woman}}{\text{total trials}} \tag{4}$$

4) Plots the probability distribution using `matplotlib`.

GRAPHICAL OUTPUT

The Python code generates a bar chart where:

- The x-axis represents the outcomes: Man and Woman.
- The y-axis represents the probabilities, ranging from 0 to 1.
- The bar height for Woman corresponds to the probability $P(A) = 0.6$.

STEMPLOT DISTRIBUTION

The stemplot shows a single vertical line at Woman on the x-axis with a height corresponding to its probability (0.6).

Probability Mass Function (PMF): The PMF represents the probability of each individual outcome in the sample space S. For the city council:

$$S = \{\text{Man, Woman}\},$$

the PMF is given as:

$$P(X = x) = \begin{cases} \frac{6}{10}, & x = \text{Woman}, \\ \frac{4}{10}, & x = \text{Man}, \\ 0, & x \notin S. \end{cases}$$

Cumulative Distribution Function (CDF): The CDF represents the cumulative probability of outcomes up to a given value x, defined as:

$$F(x) = P(X \leq x) = \sum_{k \in S, k \leq x} P(X = k).$$

For the city council:

$$F(x) = \begin{cases} 0, & x < \text{Man}, \\ \frac{4}{10}, & x = \text{Man}, \\ 1, & x \geq \text{Woman}. \end{cases}$$

Simulation Process

We simulate the selection of a council member using the following steps:

1) The city council consists of members in the set:

$$S = \{\text{Man, Woman}\},$$

   with 4 men and 6 women.
2) For each simulated selection, a random integer X is generated such that:

$$X \in \{1, 2, 3, \dots, 10\},$$

   using a random number generator function:

$$X = (\text{rand()} \mod 10) + 1.$$

3) If $X \leq 6$, it corresponds to a woman; otherwise, it corresponds to a man.
4) The number of occurrences of each outcome is tracked over N trials, where N is the total number of simulations.

5) Both the PMF and CDF are computed:
- **PMF**: The frequency of selecting a woman or a man is divided by the total number of trials to compute the probabilities.
- **CDF**: The cumulative probabilities are calculated as the running total of the PMF values.

Calculation of Probabilities

Probability of Selecting a Woman (PMF): The probability of selecting a woman is computed as:

$$P(\text{Woman}) = \frac{\text{Number of women}}{\text{Total council members}} = \frac{6}{10} = 0.6.$$

Cumulative Probability (CDF): The cumulative probability of selecting up to a given member type is:

$$F(x) = \begin{cases} P(\text{Man}), & x = \text{Man}, \\ P(\text{Man}) + P(\text{Woman}), & x = \text{Woman}. \end{cases}$$

For the city council:

$$F(\text{Man}) = 0.4, \quad F(\text{Woman}) = 1.$$

Probability of Selecting $X \notin S$: Since all members of the council belong to the set $S = \{\text{Man, Woman}\}$, the probability of selecting $X \notin S$ is:

$$P(X \notin S) = 0.$$

Output Representation

The computed probabilities are represented in two forms:
- PMF: The probabilities of selecting each type of council member (Man, Woman).
- CDF: The cumulative probabilities up to each member type (Man, Woman).

CONCLUSION

This task demonstrates the integration of C and Python for simulating and visualizing a probabilistic experiment. The probability of selecting a woman from the council is calculated as **0.6**, matching the theoretical value.

Probability of Selecting a Council Member

Probability of Selecting a Woman from the Council

CDF of Selecting a Council Member