4.2.1.1

EE24BTECH11004 - Ankit Jainar

Question Find the roots of quadratic equation:

$$x^2 - 3x - 10 = 0 \tag{0.1}$$

Solution

The given equation can be solved using analytical and numerical methods. Let us explore both approaches.

Quadratic Formula

The standard quadratic equation is:

$$ax^2 + bx + c = 0 \tag{0.2}$$

Here, $a = 1, b = -3, c = -10$. The roots are given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{0.3}$$

Substitute the values of a, b, and c:

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-10)}}{2(1)} \tag{0.4}$$

$$x = \frac{3 \pm \sqrt{9 + 40}}{2} \tag{0.5}$$

$$x = \frac{3 \pm \sqrt{49}}{2} \tag{0.6}$$

Simplify further:

$$x_1 = \frac{3 + 7}{2} = 5, \quad x_2 = \frac{3 - 7}{2} = -2 \tag{0.7}$$

Thus, the roots of the equation are:

$$x_1 = 5, \quad x_2 = -2 \tag{0.8}$$

Solution using Fixed Point Iteration

We rewrite the equation as:

$$x = g(x) \tag{0.9}$$

A possible choice for $g(x)$ is:

$$g(x) = \sqrt{3x + 10} \tag{0.10}$$

The iterative update becomes:

$$x_{n+1} = \sqrt{3x_n + 10} \tag{0.11}$$

Starting with an initial guess $x_0 = 2$, the iterations are as follows:

$$x_1 = \sqrt{3(2) + 10} = \sqrt{16} = 4 \tag{0.12}$$

$$x_2 = \sqrt{3(4) + 10} = \sqrt{22} \approx 4.69 \tag{0.13}$$

$$x_3 = \sqrt{3(4.69) + 10} \approx 5.14 \tag{0.14}$$

$$\vdots \tag{0.15}$$

Observation: The iterations converge to $x = 5$, one of the roots of the equation. For $x_2 = -2$, a similar setup with $g(x) = -\sqrt{3x + 10}$ would be used.

Newton-Raphson Method

The Newton-Raphson method is defined as:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \tag{0.16}$$

Here:

$$f(x) = x^2 - 3x - 10, \quad f'(x) = 2x - 3 \tag{0.17}$$

Substitute into the formula:

$$x_{n+1} = x_n - \frac{x_n^2 - 3x_n - 10}{2x_n - 3} \tag{0.18}$$

Example: Starting with an initial guess $x_0 = 3$:

$$x_1 = 3 - \frac{3^2 - 3(3) - 10}{2(3) - 3} = 3 - \frac{9 - 9 - 10}{6 - 3} = 3 + \frac{10}{3} \approx 6.33 \tag{0.19}$$

$$x_2 = 6.33 - \frac{6.33^2 - 3(6.33) - 10}{2(6.33) - 3} \approx 5.02 \tag{0.20}$$

Observation: The iterations quickly converge to $x = 5$. Similarly, starting with $x_0 = -1$ converges to $x = -2$.

Computational Approach

The following results were obtained using a computational method:

$$\begin{aligned}\text{Running Fixed Point Iterations Method:}&\text{Root 1: }5\\&\text{Root 2: }-2\\\text{Running Newton-Raphson Method:}&\\&\text{Root 1: }5\\&\text{Root 2: }-2\end{aligned}$$

Conclusion

The roots of the quadratic equation $x^2 - 3x - 10 = 0$ are:

$$x_1 = 5, \quad x_2 = -2 \tag{0.21}$$

Both numerical and analytical methods confirm these results.