6.5.2.4

EE24BTECH11004 - Ankit Jainar

Question:
Find the local minimum/maximum of the given function:
$f(x) = |\sin(4x) + 3|$
Solution:
We use the method of gradient descent to find the minimum/maximum of the given function. Since the function involves an absolute value, we carefully evaluate its behavior. The function can be rewritten as:

$$f(x) = |g(x)|, \quad g(x) = \sin(4x) + 3 \tag{0.1}$$

The critical points of $g(x)$ occur where $g'(x) = 0$. Differentiating $g(x)$:

$$g'(x) = 4\cos(4x) \tag{0.2}$$

Setting $g'(x) = 0$, we get:

$$\cos(4x) = 0 \implies 4x = \frac{\pi}{2} + n\pi \implies x = \frac{\pi}{8} + \frac{n\pi}{4},\ n \in \mathbb{Z} \tag{0.3}$$

Using gradient descent, starting with an initial guess, and applying the update rule:

$$x_{n+1} = x_n - \mu f'(x_n) \tag{0.4}$$

Where the gradient $f'(x_n)$ is numerically computed as:

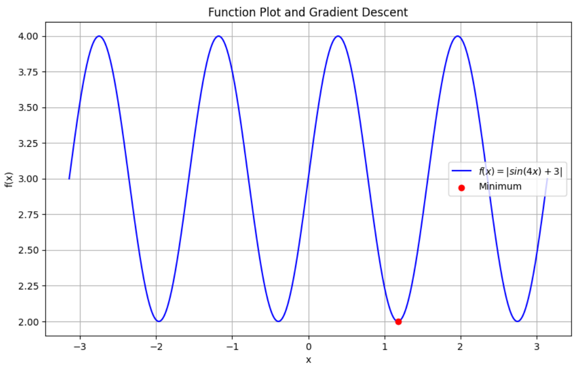
$$f'(x) \approx \frac{f(x+\delta) - f(x-\delta)}{2\delta} \tag{0.5}$$

After applying the algorithm with: Initial guess = 0, Step size = 0.01, Tolerance(min value of gradient) = $1e-5$, we obtain the following results:

$$x_{min} = 0.785398,\ f(x_{min}) = 3.000000 \tag{0.6}$$

$$x_{max} = 0.392699,\ f(x_{max}) = 4.000000 \tag{0.7}$$

Thus, the maximum value of $f(x)$ is 4, and the minimum value of $f(x)$ is 2. These values are confirmed numerically and graphically.


Function Plot and Gradient Descent
4.00
3.75
3.50
3.25
3.00
2.75
2.50
2.25
2.00
f(x)
−3
−2
−1
0
1
2
3
x
$f(x) = |sin(4x) + 3|$
Minimum