6.5.2.4

EE24BTECH11004 - Ankit Jainar

Question:
Find the minimum and maximum values of the given function:
$f(x) = |\sin(4x) + 3|$

Theoretical Method

We analyze the function theoretically to find its critical points. Let:

$$f(x) = |g(x)|, \quad g(x) = \sin(4x) + 3 \tag{0.1}$$

The critical points of $g(x)$ occur where $g'(x) = 0$. Differentiating $g(x)$:

$$g'(x) = 4\cos(4x) \tag{0.2}$$

Setting $g'(x) = 0$, we find:

$$\cos(4x) = 0 \implies 4x = \frac{\pi}{2} + n\pi \implies x = \frac{\pi}{8} + \frac{n\pi}{4},\ n \in \mathbb{Z} \tag{0.3}$$

For these x-values, we calculate $g(x)$ to find the maximum and minimum values of $|g(x)|$:

$$g(x) = \sin(4x) + 3, \quad f(x) = |g(x)| \tag{0.4}$$

At the critical points, evaluate $f(x)$ directly to determine the local maximum and minimum values. The function $f(x)$ achieves its minimum value at $f(x) = 3$ and maximum value at $f(x) = 4$.

Computational Method

We use gradient descent to find the minima and maxima numerically. Gradient descent works iteratively using the following update rule:

$$x_{n+1} = x_n - \mu f'(x_n) \tag{0.5}$$

The gradient $f'(x)$ is computed numerically as:

$$f'(x) \approx \frac{f(x+\delta) - f(x-\delta)}{2\delta} \tag{0.6}$$

Algorithm Steps:

- **Initialize:** Start with an initial guess for x, a step size (μ), and a threshold for convergence.
- **Update:** Compute $f'(x_n)$ and update x_n using the update rule.
- **Convergence:** Stop when $|f'(x_n)| <$ threshold.

Results: Using an initial guess $x = 0$, step size $\mu = 0.01$, and threshold $1e - 5$, the numerical method yields:

$$x_{min} = 0.785398, \ f(x_{min}) = 3.000000 \tag{0.7}$$

$$x_{max} = 0.392699, \ f(x_{max}) = 4.000000 \tag{0.8}$$

Thus, the maximum value of $f(x)$ is 4, and the minimum value of $f(x)$ is 3. These values match the theoretical results.