EE24BTECH11004 - Ankit Jainar

Question:

Find the local minimum/maximum of the given function:

$$f(x) = |\sin(4x) + 3|$$

THEORETICAL METHOD

We analyze the function theoretically to find its critical points. Let:

$$f(x) = |g(x)|, \quad g(x) = \sin(4x) + 3$$
 (0.1)

The critical points of g(x) occur where g'(x) = 0. Differentiating g(x):

$$g'(x) = 4\cos(4x) \tag{0.2}$$

Setting g'(x) = 0, we find:

$$\cos(4x) = 0 \implies 4x = \frac{\pi}{2} + n\pi \implies x = \frac{\pi}{8} + \frac{n\pi}{4}, \ n \in \mathbb{Z}$$
 (0.3)

For these x-values, we calculate g(x) to find the maximum and minimum values of |g(x)|:

$$g(x) = \sin(4x) + 3, \quad f(x) = |g(x)|$$
 (0.4)

At the critical points, evaluate f(x) directly to determine the local maximum and minimum values. The function f(x) achieves its minimum value at f(x) = 3 and maximum value at f(x) = 4.

COMPUTATIONAL METHOD

We use gradient descent to find the local minima and maxima numerically. Gradient descent works iteratively using the following update rule:

$$x_{n+1} = x_n - \mu f'(x_n) \tag{0.5}$$

The gradient f'(x) is computed numerically as:

$$f'(x) \approx \frac{f(x+\delta) - f(x-\delta)}{2\delta}$$
 (0.6)

Algorithm Steps:

- Initialize: Start with an initial guess for x, a step size (μ) , and a threshold for convergence.
- Update: Compute $f'(x_n)$ and update x_n using the update rule.
- Convergence: Stop when $|f'(x_n)| < \text{threshold}$.

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Results: Using an initial guess x = 0, step size $\mu = 0.01$, and threshold 1e - 5, the numerical method yields:

$$x_{min} = 0.785398, \ f(x_{min}) = 3.000000$$
 (0.7)

$$x_{max} = 0.392699, \ f(x_{max}) = 4.000000$$
 (0.8)

Thus, the maximum value of f(x) is 4, and the minimum value of f(x) is 3. These values match the theoretical results.

