

## 3.2.1.2

EE24BTECH11004 - Ankit Jainar

**Question:** 5 pencils and 7 pens together cost Rs.50, whereas 7 pencils and 5 pens together cost Rs.46. Find the cost of one pencil and that of one pen.

SOLUTION:

Let the cost of one pencil be denoted by  $x$  and the cost of one pen by  $y$ . The situation can be described using the following system of linear equations:

$$5x + 7y = 50, \quad (1)$$

$$7x + 5y = 46. \quad (2)$$

### I. THEORETICAL SOLUTION

We solve the above equations using elimination:

- Multiply equation (1) by 5 and equation (2) by 7.
- Subtract the resulting equations to eliminate  $y$  and solve for  $x$ .
- Substitute the value of  $x$  back into either equation to find  $y$ .

Performing these steps:

$$x = 3, \quad y = 5.$$

### II. NUMERICAL METHOD:

### III. LU DECOMPOSITION TO SOLVE THE SYSTEM

We now solve the system of equations using LU decomposition.

#### A. Matrix Form

The system of equations can be expressed in matrix form as:

$$\begin{bmatrix} 5 & 7 \\ 7 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 50 \\ 46 \end{bmatrix}. \quad (1)$$

Here, the coefficient matrix is:

$$A = \begin{bmatrix} 5 & 7 \\ 7 & 5 \end{bmatrix}, \quad \vec{b} = \begin{bmatrix} 50 \\ 46 \end{bmatrix}. \quad (2)$$

*B. Step 1: Decomposing A into L and U*

The matrix  $A$  can be decomposed into:

$$A = L \cdot U, \quad (3)$$

where:

$$L = \begin{bmatrix} 1 & 0 \\ \frac{7}{5} & 1 \end{bmatrix}, \quad (4)$$

$$U = \begin{bmatrix} 5 & 7 \\ 0 & -\frac{14}{5} \end{bmatrix}. \quad (5)$$

*C. Step 2: Forward Substitution*

The system  $A\vec{x} = \vec{b}$  is transformed into  $L \cdot U \cdot \vec{x} = \vec{b}$ . Let  $\vec{y}$  satisfy  $L\vec{y} = \vec{b}$ :

$$\begin{bmatrix} 1 & 0 \\ \frac{7}{5} & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 50 \\ 46 \end{bmatrix}. \quad (6)$$

Using forward substitution:

$$y_1 = 50, \quad (7)$$

$$\frac{7}{5}y_1 + y_2 = 46 \implies y_2 = 46 - \frac{7}{5}(50) = -24. \quad (8)$$

Thus:

$$\vec{y} = \begin{bmatrix} 50 \\ -24 \end{bmatrix}. \quad (9)$$

*D. Step 3: Back Substitution*

Next, solve  $U\vec{x} = \vec{y}$ :

$$\begin{bmatrix} 5 & 7 \\ 0 & -\frac{14}{5} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 50 \\ -24 \end{bmatrix}. \quad (10)$$

Using back substitution:

$$-\frac{14}{5}y = -24 \implies y = 5, \quad (11)$$

$$5x + 7(5) = 50 \implies x = 3. \quad (12)$$

*E. Updated Equation:*

$$A\vec{x} = L \cdot U \cdot \vec{x} = \vec{b}, \quad (13)$$

$$A = L \cdot U, \quad (14)$$

$$L \cdot U \cdot \vec{x} = \vec{b}, \quad (15)$$

$$U \cdot \vec{x} = \vec{y}, \quad (16)$$

$$L \cdot \vec{y} = \vec{b}. \quad (17)$$

*F. Final Answer*

The cost of one pencil is Rs.3, and the cost of one pen is Rs.5.

