11.16.3.6

EE24BTECH11004 - Ankit Jainar

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Question

There are four men and six women on the city council. If one council member is selected for a committee at random, how likely is it that it is a woman?

Theoretical Solution

The total number of council members is:

$$|S| = 4 + 6 = 10$$

The favorable outcomes (selecting a woman) are:

$$|A| = 6$$

The probability of selecting a woman is:

$$P(A) = \frac{|A|}{|S|} = \frac{6}{10} = 0.6$$

Introduction

This task involves simulating the random selection of council members using:

- A C program to generate random samples.
- Compiling the program into a shared object (.so) file.
- Using Python to process the results and generate a probability distribution plot.

C Code Description

The C program performs the following:

- Generates random samples for the selection process.
- Uses the `rand()` function to simulate the random selection.
- Tracks the number of outcomes categorized as either "man" or "woman."

Python Code Description

The Python code performs the following:

1. Loads the shared object file generated from the C program using the `ctypes` library.
2. Simulates a specified number of random selections (e.g., 1,000,000 trials).
3. Calculates the probability of selecting a woman using the formula:

$$P(\text{woman}) = \frac{\text{frequency of selecting a woman}}{\text{total trials}}$$

4. Plots the probability distribution using `matplotlib`.

Graphical Output

The Python code generates a bar chart where:

- The x-axis represents the outcomes: Man and Woman.
- The y-axis represents the probabilities, ranging from 0 to 1.
- The bar height for Woman corresponds to the probability $P(A) = 0.6$.

Probability Mass Function (PMF)

The PMF represents the probability of each individual outcome in the sample space S. For the city council:

$$S = \{\text{Man, Woman}\},$$

the PMF is given as:

$$P(X = x) = \begin{cases} \frac{6}{10}, & x = \text{Woman}, \\ \frac{4}{10}, & x = \text{Man}, \\ 0, & x \notin S. \end{cases}$$

Cumulative Distribution Function (CDF)

The CDF represents the cumulative probability of outcomes up to a given value x, defined as:

$$F(x) = P(X \leq x) = \sum_{k \in S, k \leq x} P(X = k).$$

For the city council:

$$F(x) = \begin{cases} 0, & x < \text{Man}, \\ \frac{4}{10}, & x = \text{Man}, \\ 1, & x \geq \text{Woman}. \end{cases}$$

Simulation Process

Steps to simulate the selection of a council member:

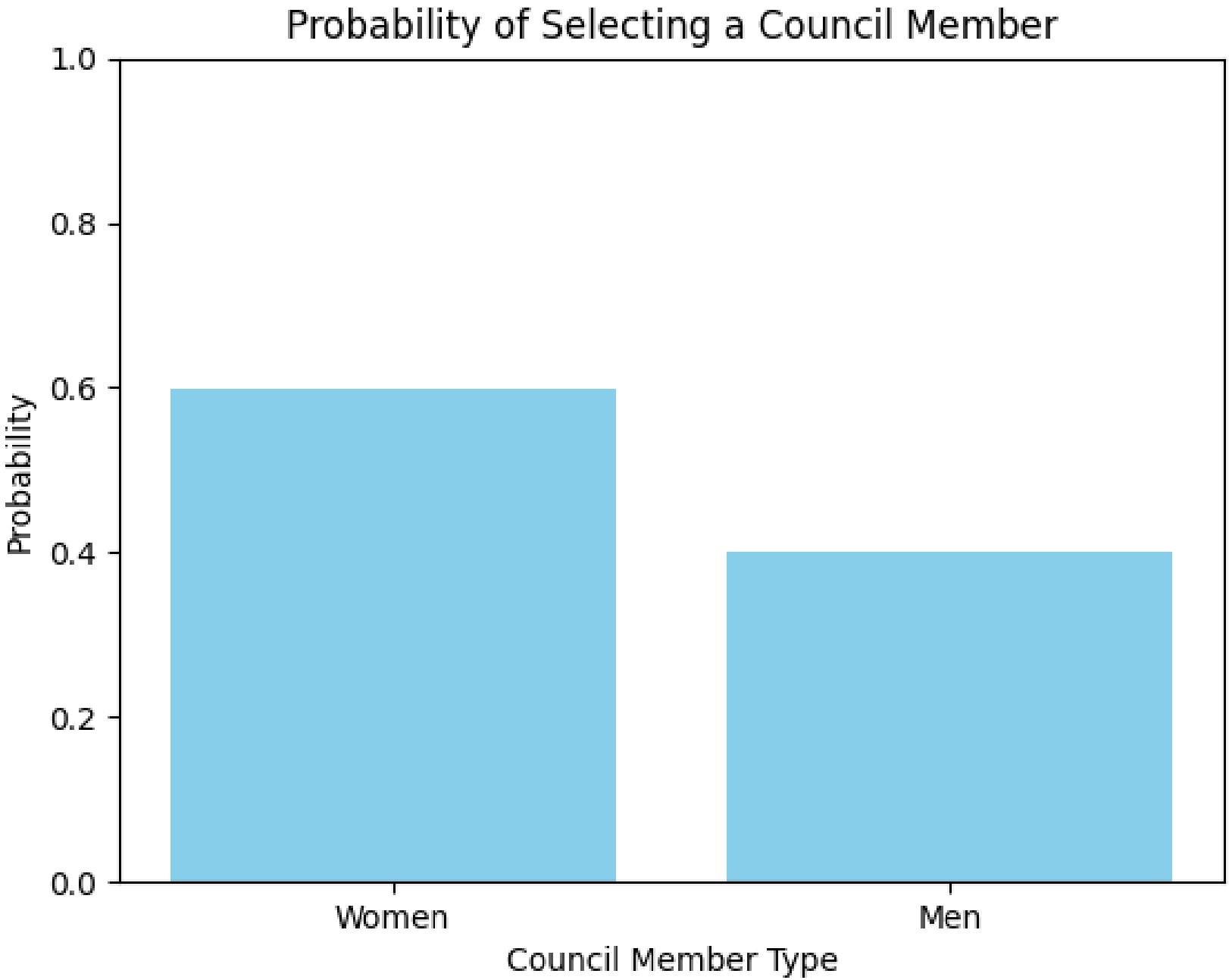
1. The council consists of members in the set:

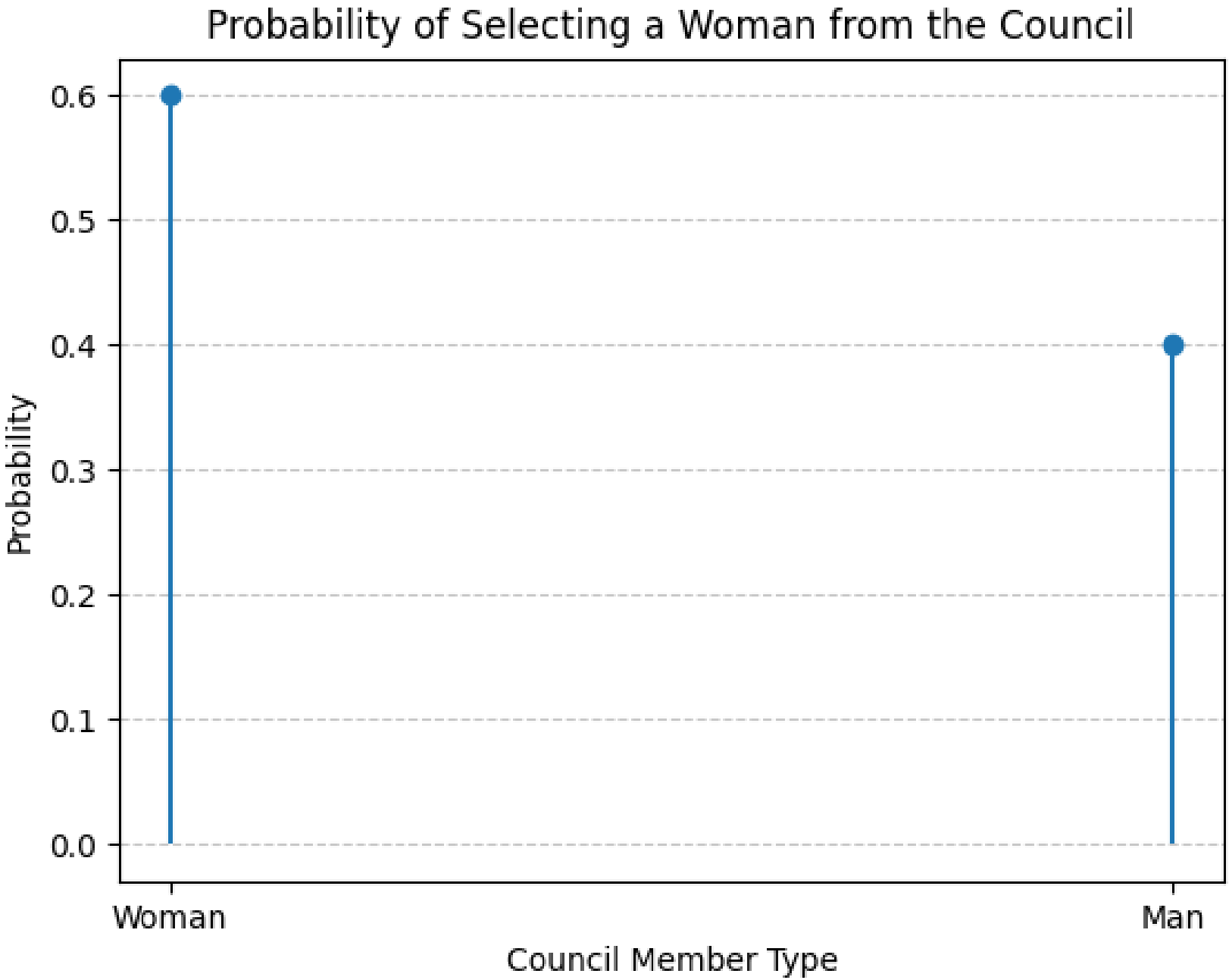
$$S = \{\text{Man, Woman}\}.$$

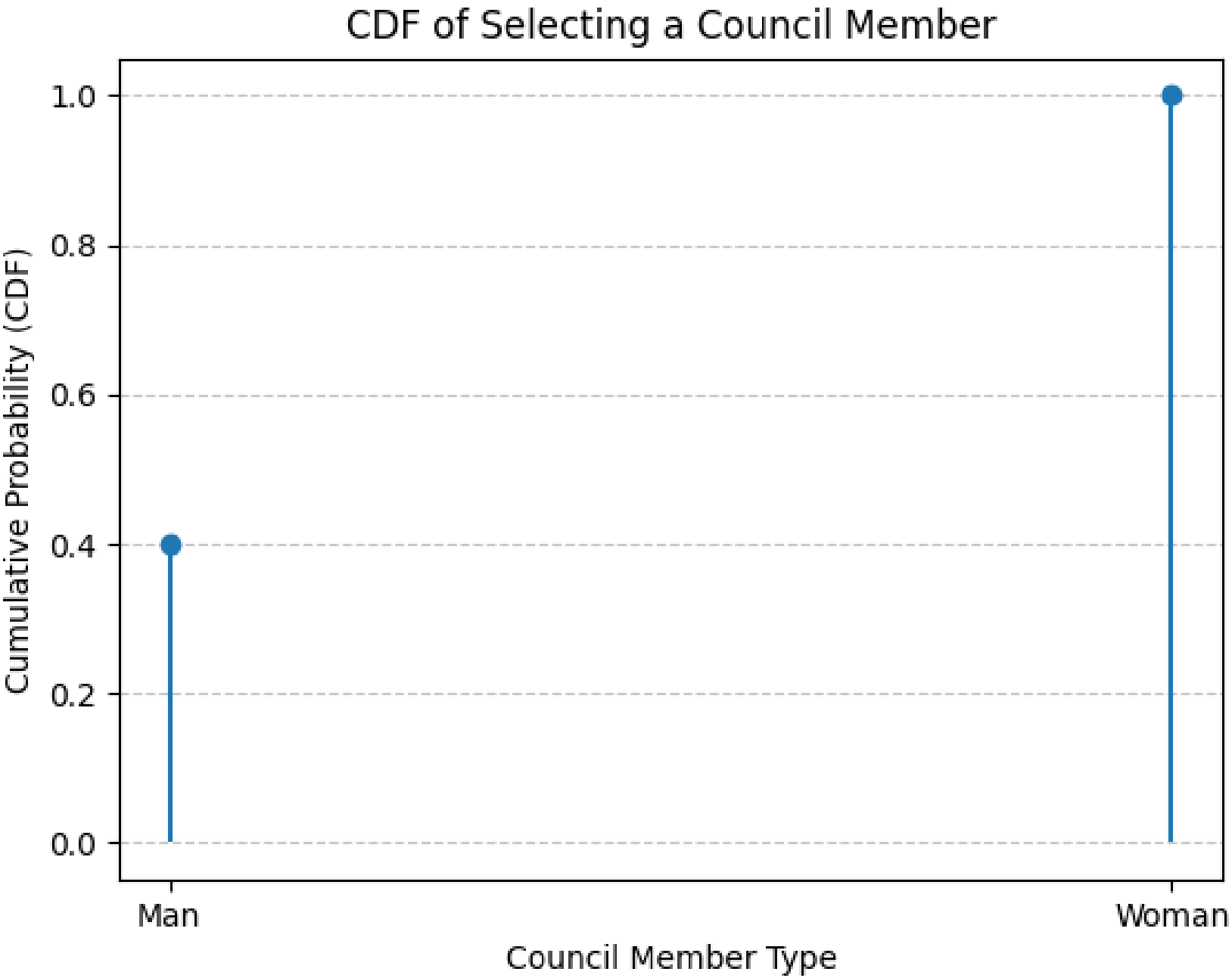
2. Generate a random integer X such that:

$$X \in \{1, 2, \ldots, 10\}.$$

3. If $X \leq 6$, it corresponds to a woman; otherwise, it corresponds to a man.
4. Track the number of occurrences of each outcome over N trials.


Probability of Selecting a Council Member
1.0
0.8
0.6
0.4
0.2
0.0
Probability
Women
Men
Council Member Type


Probability of Selecting a Woman from the Council
Probability
0.0
0.1
0.2
0.3
0.4
0.5
0.6
Woman
Man
Council Member Type


CDF of Selecting a Council Member
Cumulative Probability (CDF)
1.0
0.8
0.6
0.4
0.2
0.0
Man
Woman
Council Member Type

Conclusion

This task demonstrates the integration of C and Python for simulating and visualizing a probabilistic experiment. The probability of selecting a woman from the council is calculated as:

$$P(\text{Woman}) = 0.6,$$

matching the theoretical value.