8.2.2

EE24BTECH11004 - Ankit Jainar

Question: Find the area bounded by the curves:

$$(x-1)^2 + y^2 = 1 \quad \text{and} \quad x^2 + y^2 = 1$$

Solution:

Theoretical Solution:

The curves are two circles:

- Circle 1: $(x-1)^2 + y^2 = 1$, centered at $(1, 0)$ with radius 1.
- Circle 2: $x^2 + y^2 = 1$, centered at $(0, 0)$ with radius 1.

To find the area of the region bounded by these curves, we determine the points of intersection by solving:

$$(x-1)^2 + y^2 = x^2 + y^2$$
$$\Rightarrow x^2 - 2x + 1 + y^2 = x^2 + y^2$$
$$\Rightarrow -2x + 1 = 0 \quad \Longrightarrow \quad x = \frac{1}{2}$$

The points of intersection are $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ and $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$.

The area between the curves is computed by subtracting the integrals of the two curves in the interval where they overlap.

Computational Solution (Trapezoidal Rule):

Using the trapezoidal rule to approximate the area, we write:

$$A \approx \int_{x_1}^{x_2} [f(x) - g(x)]\, dx$$

Here, $f(x)$ and $g(x)$ represent the upper and lower curves, respectively, and $[x_1, x_2]$ is the interval of integration.

Divide the interval into n subintervals of width $h = \frac{x_2 - x_1}{n}$. The trapezoidal rule states:

$$A \approx \frac{h}{2}\left[(f(x_1) - g(x_1)) + 2\sum_{i=1}^{n-1}(f(x_i) - g(x_i)) + (f(x_n) - g(x_n))\right]$$

Substituting the functions:

$$f(x) = \sqrt{1 - (x-1)^2}, \quad g(x) = \sqrt{1 - x^2}$$

Calculate x_i, $f(x_i)$, and $g(x_i)$ at each step i:

$$x_i = x_1 + i \cdot h, \quad f(x_i) = \sqrt{1 - (x_i - 1)^2}, \quad g(x_i) = \sqrt{1 - x_i^2}$$

By iterating over n intervals, approximate the area. A comparison of the theoretical and computational results is shown below.

Comparison of Results: