## EE24BTECH11004 - ANKIT JAINAR

**Question:** Find the area of the region  $\{(x, y) : x^2 \le y \le x\}$ 

**Solution:** The problem is to find the area of the region:  $\{(x,y): x^2 \le y \le x\}$  This can

| Variable                          | Description                              |
|-----------------------------------|--|
| g(x)                              | Equation of the Conic                    |
| L                                 | Equation of the line                     |
| h                                 | A point on the line $L$                  |
| m                                 | Direction vector of line L               |
| $\mathbf{x_1}$ and $\mathbf{x_2}$ | Points of intersection of $L$ and $g(x)$ |

TABLE 0: Variables are

be solved using the following steps:

$$g(x, y) = \mathbf{x}^{\mathsf{T}} V \mathbf{x} + 2 \mathbf{u}^{\mathsf{T}} \mathbf{x} + f = 0 \tag{0.1}$$

$$V = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \tag{0.2}$$

$$\mathbf{u} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \tag{0.3}$$

$$f = 0 \tag{0.4}$$

The line equation is:  $L : \mathbf{x} = \mathbf{h} + k\mathbf{m}$  where:

$$\mathbf{h} = \begin{pmatrix} -2\\0 \end{pmatrix} \tag{0.5}$$

$$\mathbf{m} = \begin{pmatrix} 1 \\ \frac{1}{4} \end{pmatrix} \tag{0.6}$$

The points  $\mathbf{x}_1$  and  $\mathbf{x}_2$  are given by:

$$\mathbf{x}_i = \mathbf{h} + k_i \mathbf{m} \tag{0.7}$$

The values of  $k_1$  and  $k_2$  are obtained by solving the quadratic equation:

$$k_1 = \frac{1}{\mathbf{m}^{\top} V \mathbf{m}} \left( -\mathbf{m}^{\top} (V \mathbf{h} + \mathbf{u}) + \sqrt{[\mathbf{m}^{\top} (V \mathbf{h} + \mathbf{u})]^2 - g(\mathbf{h})(\mathbf{m}^{\top} V \mathbf{m})} \right)$$
(0.8)

$$k_2 = \frac{1}{\mathbf{m}^{\top} V \mathbf{m}} \left( -\mathbf{m}^{\top} (V \mathbf{h} + \mathbf{u}) - \sqrt{[\mathbf{m}^{\top} (V \mathbf{h} + \mathbf{u})]^2 - g(\mathbf{h})(\mathbf{m}^{\top} V \mathbf{m})} \right)$$
(0.9)

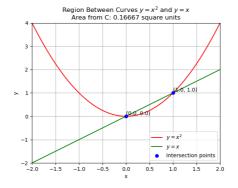


Fig. 0.1: Stem Plot of y(n)

Area bounded by the curves  $y = x^2$  and y = x is given by:

Area = 
$$\int_0^1 (x - x^2) dx = \frac{1}{6} squnits.$$
 (0.10)

(0.11)

Hence, the area bounded by the region is  $\frac{1}{6}$  sq units.