6-04-2024 Shift-1

EE24BTECH11004 - ANKIT JAINAR

1) $I = \int_0^{\frac{\pi}{4}} \frac{\cos^2 x \sin^2 x}{\left(\cos^3 x + \sin^3 x\right)^2} dx$

2) An equilateral triangle of side 12. A circle is embedded inside the triangle, and a square is embedded inside the circle. If the area and perimeter of the square are m and n, respectively, then find $m + n^2$.

3) Solve: $\left(1 + x^2\right)\frac{dy}{dx} + y = e^{\tan^{-1} x}$, $y(1) = 0$. Then $y(0) =$

4) Find the range of x for which $f(x) = x^x \, (x > 0)$ is strictly increasing.

5) Let $A = \{100, 101, 102, \ldots, 700\}$. Find the number of numbers in set A which are neither divisible by 3 nor by 4.

6) Given that $\frac{dy}{dx} + 2x \ln x \cdot y = 3 \ln x$, and $y(1) = 0$, find y.

7) Let $A_r = \begin{vmatrix} r & 1 & \frac{n^2}{2} + \alpha \\ 2r & 2 & n^2 - \beta \\ 3r - 2 & 3 & n(n-1) \end{vmatrix}$ Find $2A_{10} - A_8$.

8) In an octagon how many triangles are possible so that no side of triangle is side of octagon?

9) A variable line is passing through $(4, -9)$, slope of line is positive and it make intercepts on x and y-axis on point A and B. Find the minimum area of triangle OAB.

10) If mean of 20 observation is10, $SD = 2$. One of the observation which is 12 is replaced by8. Find the value of new SD?

11) Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = \frac{x^2 - 2x - 15}{x^2 - 4x + 9}$, then f is:
    a) one-one onto
    b) many-one onto
    c) many-one into
    d) one-one into

12) A company has two branches A and B. Branch A produces 60% of the total production and the remaining by branch B. Branch A produces 80% good quality products, and branch B produces 90% good quality products. A product is randomly selected, and it is found to be of good quality. Let P be the probability that the selected product is from branch B. Find the value of $126P$.

13) Find the shortest distance between two lines: $\frac{x-3}{2} = \frac{y+15}{-7} = \frac{z-9}{5}$ $\quad and \quad$ $\frac{x+1}{2} = \frac{y-1}{1} = frac z - 9 - 3$.

14) If in the expansion of $(x + y)^n$, the terms are: $T_2 = 15$, $\quad T_3 = 10$, $\quad T_4 = \frac{10}{3}$ For $n = 5$, find the value of $n^3 + x^5 + 243y^5$.

15) Let $S = \{1, 2, 3, \ldots, 20\}$ be a given set. Relation R_1 is defined as $R_1 = \{(x, y) : 2x - 3y = 2\}$ and R_2 as $R_2 = \{(x, y) : 4x = 5y\}$, where $x, y \in S$. If m denotes the number of elements required to make R_1 symmetric, and n denotes the number of elements to make R_2 symmetric,find $m + n$.