JEE ADVANCED

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I. MCQs with One or More than One Correct

1) The minimum value of expression $\sin\alpha+\sin\beta+\sin\gamma$, where (α,β,γ) are real numbers satisfying $(\alpha+\beta+\gamma)=\pi$ is (1995)
   a) positive
   b) 0
   c) negative
   d) -3

2) The number of values of x in the interval $[0,5\pi]$ satisfying equation $3\sin(x^2) - 7\sin x + 2 = 0$ (1998-2 Marks)
   a) 0
   b) 5
   c) 6
   d) 10

3) Which of the following number(s) is/are rational? (1998-2 Marks)
   a) $\sin 15^\circ$
   b) $\cos 15^\circ$
   c) $\sin 15^\circ \cos 15^\circ$
   d) $\sin 15^\circ \cos 75^\circ$

4) For a positive integer n, let $f_n(\theta) = (\tan\frac{\theta}{2})(1+\sec\theta)(1+\sec 2\theta)(1+\sec 4\theta)\ldots(1+\sec 2^n\theta)$. Then (1999 - 3Marks)
   a) $f_2(\frac{\pi}{16}) = 1$
   b) $f_3(\frac{\pi}{32}) = 1$
   c) $f_4(\frac{\pi}{64}) = 1$
   d) $f_5(\frac{\pi}{128}) = 1$

5) If $\frac{\sin^4 x}{2} + \frac{\cos^4 x}{3} = \frac{1}{5}$, Then (2009)
   a) $\tan^2 x = \frac{2}{3}$
   b) $\frac{\sin^8 x}{8} + \frac{\cos^8 x}{27} = \frac{1}{125}$
   c) $\tan^2 x = \frac{1}{3}$
   d) $\frac{\sin^8 x}{8} + \frac{\cos^8 x}{27} = \frac{2}{125}$

6) For $0 < \theta < \frac{\pi}{2}$, the solution(s) of $\sum_{m=1}^{6} \operatorname{cosec}(\theta + \frac{(m-1)\pi}{4})\operatorname{cosec}(\theta + \frac{m\pi}{4}) = 4\sqrt{2}$ is(are) (2009)
   a) $\frac{\pi}{4}$
   b) $\frac{\pi}{6}$
   c) $\frac{\pi}{12}$
   d) $\frac{5\pi}{12}$

7) Let $\theta,\varphi \in [0,2\pi]$ be such that $2\cos(\theta(1-\sin\varphi) = \sin^2(\theta(\tan\frac{\theta}{2}) + \cot\frac{\theta}{2})\cos\varphi - 1$,$\tan(2\pi-\theta) > 0$ and $-1 < \sin\theta < -\frac{\sqrt{3}}{2}$, then φ cannot satisfy (2012)
   a) $0 < \varphi < \frac{\pi}{2}$
   b) $\frac{\pi}{2} < \varphi < \frac{4\pi}{3}$
   c) $\frac{4\pi}{3} < \varphi < \frac{3\pi}{2}$
   d) $\frac{3\pi}{2} < \varphi < 2\pi$

8) The number of points in $(-\infty,\infty)$, for which $x - x\sin x - \cos x = 0$, is (JEE Adv.2013)
   a) 6
   b) 4
   c) 2
   d) 0

9) Let $f(x) = x\sin\pi x$, $x > 0$. Then for all natural numbers n, f'(x) vanishes at (JEE Adv. 2013)
   a) A unique point in the interval$(n, n+\frac{1}{2})$
   b) A unique point in the interval$(n+\frac{1}{2}, n+1)$
   c) A unique point in the interval$(n, n+1)$
   d) Two points in the interval$(n, n+1)$

10) Let α and β be non-zero real numbers such that $2(\cos\beta-\cos\alpha)+\cos\alpha\cos\beta = 1$.Then which of the following is/are true? (JEE Adv.2017)
   a) $\tan(\frac{\alpha}{2}) + \sqrt{3}\tan(\frac{\beta}{2}) = 0$
   b) $\sqrt{3}(\tan\frac{\alpha}{2}) + \tan(\frac{\beta}{2}) = 0$
   c) $\tan(\frac{\alpha}{2}) - \tan(\frac{\beta}{2}) = 0$
   d) $\sqrt{3}\tan(\frac{\alpha}{2}) - \tan(\frac{\beta}{2}) = 0$

II. Subjective Problems

11) If $\tan\alpha = \frac{m}{m+1}$ and $\tan\beta = \frac{1}{2m+1}$, find the possible values of $(\alpha+\beta)$ (1978)

12) a) Draw the graph of $y = \frac{1}{\sqrt{2}}(\sin x+\cos x)$ from $x = -\frac{\pi}{2}$ to $x = \frac{\pi}{2}$
   b) If $\cos(\alpha+\beta) = \frac{4}{5}$,$\sin(\alpha-\beta) = \frac{5}{13}$, and α,β lies between 0 and $\frac{\pi}{4}$, find $\tan 2\alpha$ (1979)

13) Given $\alpha+\beta-\gamma = \pi$, prove that $\sin^2\alpha + \sin^2\beta - \sin^2\gamma = 2\sin\alpha\sin\beta\cos\gamma$ (1980)

14) Given A=$\{x : \frac{\pi}{6} \le x \le \frac{\pi}{3}\}$ and f(x)=$\cos x - x(1+x)$; find f(A) (1980)

15) For all θ in $\left[0, \frac{\pi}{2}\right]$ show that, $\cos(\sin\theta) \geq \sin(\cos\theta)$. (1981-4 Marks)