25-01-2023 Shift-1

EE24BTECH11004 - ANKIT JAINAR

1) Let M be the maximum value of the product of two positive integers when their sum is 66. Let the sample space $S = \{x \in \mathbb{Z} : x(66 - x) \geq \frac{5}{9}M\}$ and the event $A = \{x \in S : x \text{ is a multiple of } 3\}$. Then $P(A)$ is equal to:
   a) $\frac{15}{44}$
   b) $\frac{1}{3}$
   c) $\frac{1}{5}$
   d) $\frac{7}{22}$
2) Let a, b, and c be three non-zero vectors such that $b \cdot c = 0$ and $a = \frac{b \times c}{2} - b$. If d is a vector such that $b \cdot d = a \cdot b$, then $(a \times b) \cdot (c \times d)$ is equal to:
   a) $\frac{3}{4}$
   b) $\frac{1}{2}$
   c) $-\frac{1}{4}$
   d) $\frac{1}{4}$
3) Let $y = y(x)$ be the solution curve of the differential equation $e^{2y}\frac{dy}{dx} = \frac{1+xy+(1+\log x)}{x}$ with $x > 0$, and $y(1) = 3$. Then $\frac{2y(x)}{9}$ is equal to:
   a) $\frac{2}{3}e^x(5 - 2x + 2\log x)$
   b) $\frac{2}{3}e^x(2x + 2\log x - 3)$
   c) $\frac{2}{3}e^x(3x - (1 + \log x) + 2)$
   d) $\frac{2}{3}e^x(7 + 3x - 2\log x)$
4) The value of $\lim_{n\to\infty} \frac{1+2-3+4+5-6+\cdots+(3n-2)+(3n-1)-3n}{\sqrt{2n^4+4n+3}-\sqrt{n^4+5n+4}}$ is:
   a) $\frac{\sqrt{2}+1}{2}$
   b) $3(\sqrt{2} + 1)$
   c) $\frac{3}{2}(\sqrt{2} + 1)$
   d) $\frac{3}{2\sqrt{2}}$
5) The points of intersection of the line $ax + by = 0$, $(a \neq b)$ and the circle $x^2 + y^2 - 2x = 0$ are $A(\alpha, 0)$ and $B(1, \beta)$. The image of the circle with AB as diameter in the line $x + y + 2 = 0$ is:
   a) $x^2 + y^2 + 5x + 5y + 12 = 0$
   b) $x^2 + y^2 + 3x + 5y + 8 = 0$
   c) $x^2 + y^2 + 3x + 3y + 4 = 0$
   d) $x^2 + y^2 - 5x - 5y + 12 = 0$
6) The mean and variance of the marks obtained by the students n a test are 10and 4, respectively. Later, the marks of one of the students is increased from 8 to 12. If the new mean of the marks is 10.2, then their new variance is equal to:
   a) 4.04
   b) 4.08
   c) 3.96
   d) 3.92
7) Let $y(x) = (1 + x)(1 + x^2)(1 + x^4)(1 + x^8)(1 + x^{16})$. Then $y' - y''$ at $x = -1$ is equal to:
   a) 976
   b) 464

c) 496
d) 944

8) The vector $\mathbf{a} = -\hat{i} + 2\hat{j} + \hat{k}$ is rotated through a right angle, passing through the y-axis in its way, and the resulting vector is $\mathbf{b}$. Then the projection of $3\mathbf{a} + \sqrt{2}\mathbf{b}$ on $\mathbf{c} = 5\hat{i} + 4\hat{j} + 3\hat{k}$ is:
a) $3\sqrt{2}$
b) 1
c) $\sqrt{6}$
d) $2\sqrt{3}$

9) The minimum value of the function $f(x) = \int_0^2 e^{|x-t|}\,dt$ is:
a) $2(e-1)$
b) $2e-1$
c) 2
d) $e(e-1)$

10) Consider the lines L_1 and L_2 given by

$$L_1 : \frac{x-1}{2} = \frac{y-3}{1} = \frac{z-2}{2}$$

$$L_2 : \frac{x-2}{1} = \frac{y-2}{2} = \frac{z-3}{3}$$

A line L_3 having direction ratios $1, -1, -2$, intersects L_1 and L_2 at the points P and Q respectively. Then the length of line segment PQ is:
a) $2\sqrt{6}$
b) $3\sqrt{2}$
c) $4\sqrt{3}$
d) 4

11) Let $x = 2$ be a local minima of the function $f(x) = 2x^4 - 18x^2 + 8x + 12$, $x \in (-4, 4)$. If M is the local maximum value of the function f in $(-4, 4)$, then M is:
a) $12\sqrt{6} - \frac{33}{2}$
b) $12\sqrt{6} - \frac{31}{2}$
c) $18\sqrt{6} - \frac{33}{2}$
d) $18\sqrt{6} - \frac{31}{2}$

12) Let $z_1 = 2 + 3i$ and $z_2 = 3 + 4i$. The set $S = \{z \in \mathbb{C} : |z - z_1| - |z - z_2| = |z_1 - z_2|\}$ represents a:
a) straight line with sum of its intercepts on the coordinate axes equals 14
b) hyperbola with the length of the transverse axis 7
c) straight line with the sum of its intercepts on the coordinate axes equals -18
d) hyperbola with eccentricity 2

13) The distance of the point $\left(6, -2\sqrt{2}\right)$ from the common tangent $y = mx + c$, $m > 0$, of the curves $x = 2y^2$ and $x = 1 + y^2$ is:
a) $\frac{1}{3}$
b) 5
c) $\frac{14}{3}$
d) $5\sqrt{3}$

14) Let S_1 and S_2 be respectively the sets of all $a \in \mathbb{R} \setminus \{0\}$ for which the system of linear equations:

$$ax + 2ay - 3az = 1$$

$$(2a+1)x + (2a+3)y + (a+1)z = 2$$

$$(3a+5)x + (a+5)y + (a+2)z = 3$$

has a unique solution and infinitely many solutions. Then:

a) $n(S_1) = 2$ and S_2 is an infinite set
b) S_1 is an infinite set and $n(S_2) = 2$
c) $S_1 = \emptyset$ and $S_2 = \mathbb{R} \setminus \{0\}$
d) $S_1 = \mathbb{R} \setminus \{0\}$ and $S_2 = \emptyset$

15) Let $f(x) = \int \frac{2x}{(x^2+1)(x^2+3)}\,dx$. If $f(3) = \frac{1}{2}(\log_e 5 - \log_e 6)$, then $f(4)$ is equal to:

a) $\frac{1}{2}(\log_e 17 - \log_e 19)$
b) $\log_e 17 - \log_e 18$
c) $\frac{1}{2}(\log_e 19 - \log_e 17)$
d) $\log_e 19 - \log_e 20$