# 29-06-2022 Shift-2

EE24BTECH11004 - ANKIT JAINAR

1) The probability that a relation $R$ from $\{x, y\}$ to $\{x, y\}$ is both symmetric and transitive, is equal to
   a) $\frac{5}{16}$
   b) $\frac{9}{16}$
   c) $\frac{11}{16}$
   d) $\frac{13}{16}$
2) The number of values of $a \in \mathbb{N}$ such that the variance of $3, 7, 12, a, 43 - a$ is a natural number is
   a) 0
   b) 2
   c) 5
   d) infinite
3) From the base of a pole of height 20 meters, the angle of elevation of the top of a tower is $60°$. The pole subtends an angle $30°$ at the top of the tower. Then the height of the tower is
   a) $15\sqrt{3}$
   b) $20\sqrt{3}$
   c) $20 + 10\sqrt{3}$
   d) 30
4) Negation of the Boolean statement $(p \vee q) \Rightarrow ((\neg r) \vee p)$ is equivalent to
   a) $p \wedge (\neg q) \wedge r$
   b) $(\neg p) \wedge (\neg q) \wedge r$
   c) $(\neg p) \wedge q \wedge r$
   d) $p \wedge q \wedge (\neg r)$
5) Let $n \geq 5$ be an integer. If $9^n - 8n - 1 = 64\alpha$ and $6^n - 5n - 1 = 25\beta$, then $\alpha - \beta$ is equal to
   a) $1 + {}^nC_2(8 - 5) + {}^nC_3(8^2 - 5^2) + \ldots + {}^nC_n(8^{n-1} - 5^{n-1})$
   b) $1 + {}^nC_3(8 - 5) + {}^nC_4(8^2 - 5^2) + \ldots + {}^nC_n(8^{n-2} - 5^{n-2})$
   c) ${}^nC_3(8 - 5) + {}^nC_4(8^2 - 5^2) + \ldots + {}^nC_n(8^{n-2} - 5^{n-2})$
   d) ${}^nC_4(8 - 5) + {}^nC_5(8^2 - 5^2) + \ldots + {}^nC_n(8^{n-3} - 5^{n-3})$
6) Let $\mathbf{a} = \hat{i} - 2\hat{j} + 3\hat{k}$, $\mathbf{b} = \hat{i} + \hat{j} + \hat{k}$, and $\mathbf{c}$ be a vector such that $\mathbf{a} + (\mathbf{b} \times \mathbf{c}) = 0$ and $\mathbf{b} \cdot \mathbf{c} = 5$. Then, the value of $3(\mathbf{c} \cdot \mathbf{a})$ is
   a) 10
   b) 15
   c) 20
   d) 25
7) Let $y = y(x)$, $x > 1$, be the solution of the differential equation $(x - 1)\frac{dy}{dx} + 2xy = frac1x - 1$ with $y(2) = \frac{1+e^4}{2e^4}$. If $y(3) = \frac{e^\alpha+1}{\beta e^\alpha}$, then the value of $\alpha + \beta$ is equal to
8) Let $3, 6, 9, 12, \ldots$ up to 78 terms and $5, 9, 13, 17, \ldots$ up to 59 terms be two series. The sum of the terms common to both series is
   a) 378
   b) 405
   c) 450
   d) 495

9) The number of solutions of the equation $\sin x = \cos^2 x$ in the interval $(0, 10)$ is
   a) 1
   b) 2
   c) 3
   d) 4
10) The total number of four-digit numbers such that each of the first three digits is divisible by the last digit, is equal to
11) For real numbers $a, b\ (a > b > 0)$, let Area$\left\{(x, y) : x^2 + y^2 \leq a^2 \text{ and } \frac{x^2}{a^2} + \frac{y^2}{b^2} \geq 1\right\} = 30\pi$ and Area$\left\{(x, y) : x^2 + y^2\right.$ $18\pi$ Then the value of $(a - b)^2$ is equal to
12) Let $f$ and $g$ be twice differentiable even functions on $(-2, 2)$ such that $f\left(\frac{1}{4}\right) = 0, f\left(\frac{1}{2}\right) = 0, f(1) = 1,$ and $g\left(\frac{3}{4}\right) = 0, g(1) = 2$. Then, the minimum number of solutions of $f(x)g''(x) + f'(x)g'(x) = 0$ in $(-2, 2)$ is equal to
13) Let $M = \begin{bmatrix} 0 & -\alpha \\ \alpha & 0 \end{bmatrix}$, where $\alpha$ is a non-zero real number, and $N = \sum_{k=1}^{49} M^{2k}$. If $(I - M^2)N = -2I$, then the positive integral value of $\alpha$ is
   a) 1
   b) 2
   c) 3
   d) 4
14) Let the coefficients of $x^{-1}$ and $x^{-3}$ in the expansion of $\left(2x^5 - \frac{1}{x^5}\right)^{15}, x > 0$, be $m$ and $n$ respectively. If $r$ is a positive integer such that $mn^2 =^{15} C_r, 2^r$, then the value of $r$ is equal to
15) Let $f(x)$ and $g(x)$ be two real polynomials of degree 2 and 1, respectively. If $f(g(x)) = 8x^2 - 2x$, and $g(f(x)) = 4x^2 + 6x + 1$, then the value of $f(2) + g(2)$ is