

# 6-04-2024 Shift-1

EE24BTECH11004 - ANKIT JAINAR

- 1)  $I = \int_0^{\frac{\pi}{4}} \frac{\cos^2 x \sin^2 x}{(\cos^3 x + \sin^3 x)^2} dx$ 
  - a)  $\frac{1}{6}$
  - b)  $\frac{1}{3}$
  - c)  $\frac{1}{2}$
  - d) 1
- 2) Find the range of  $x$  for which  $f(x) = x^x$  ( $x > 0$ ) is strictly increasing.
  - a)  $(0, \infty)$
  - b)  $\left(0, \frac{1}{e}\right]$
  - c)  $\left[\frac{1}{e^2}, \infty\right)$
  - d)  $\left[\frac{1}{e}, \infty\right)$
- 3) Let  $A = \{100, 101, 102, \dots, 700\}$ . Find the number of numbers in set  $A$  which are neither divisible by 3 nor by 4.
- 4) Given that  $\frac{dy}{dx} + 2x \ln x \cdot y = 3 \ln x$ , and  $y(1) = 0$ , find  $y$ .
- 5) Let  $A_r = \begin{vmatrix} r & 1 & \frac{n^2}{2} + \alpha \\ 2r & 2 & n^2 - \beta \\ 3r - 2 & 3 & n(n-1) \end{vmatrix}$  Find  $2A_{10} - A_8$ .
  - a)  $4\alpha + 2\beta$
  - b)  $2n$
  - c) 0
  - d)  $2\alpha + 4\beta$
- 6) If mean of 20 observation is 10,  $SD = 2$ . One of the observation which is 12 is replaced by 8. Find the value of new  $SD$ ?
  - a)  $\sqrt{3.96}$
  - b) 1.8
  - c)  $\sqrt{3.8}$
  - d) 1.93
- 7) Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be defined by  $f(x) = \frac{x^2 - 2x - 15}{x^2 - 4x + 9}$ , then  $f$  is:
  - a) one-one onto
  - b) many-one onto
  - c) many-one into
  - d) one-one into
- 8) A company has two branches  $A$  and  $B$ . Branch  $A$  produces 60% of the total production and the remaining by branch  $B$ . Branch  $A$  produces 80% good quality products, and branch  $B$  produces 90% good quality products. A product is randomly selected, and it is found to be of good quality. Let  $P$  be the probability that the selected product is from branch  $B$ . Find the value of  $126P$ .
  - a) 54
  - b) 52
  - c) 48
  - d) 27
- 9) Find the shortest distance between two lines:  $\frac{x-3}{2} = \frac{y+15}{-7} = \frac{z-9}{5}$  and  $\frac{x+1}{2} = \frac{y-1}{1} = \frac{z-9}{-3}$ .

- a)  $4\sqrt{3}$   
 b)  $8\sqrt{3}$   
 c)  $6\sqrt{3}$   
 d)  $2\sqrt{3}$
- 10) If in the expansion of  $(x+y)^n$ , the terms are:  $T_2 = 15$ ,  $T_3 = 10$ ,  $T_4 = \frac{10}{3}$  For  $n = 5$ , find the value of  $n^3 + x^5 + 243y^5$ .  
 a)  $\frac{e^2+1}{e}$   
 b)  $\frac{e^2-1}{e}$   
 c)  $\frac{e^2+2}{e}$   
 d)  $\frac{e^2-2}{e}$
- 11) Let  $S = \{1, 2, 3, \dots, 20\}$  be a given set. Relation  $R_1$  is defined as  $R_1 = \{(x, y) : 2x - 3y = 2\}$  and  $R_2$  as  $R_2 = \{(x, y) : 4x = 5y\}$ , where  $x, y \in S$ . If  $m$  denotes the number of elements required to make  $R_1$  symmetric, and  $n$  denotes the number of elements to make  $R_2$  symmetric, find  $m + n$ .  
 a) 10  
 b) 12  
 c) 8  
 d) 20
- 12) An equilateral triangle of side 12. A circle is embedded inside the triangle, and a square is embedded inside the circle. If the area and perimeter of the square are  $m$  and  $n$ , respectively, then find  $m + n^2$ .
- 13) In an octagon how many triangles are possible so that no side of triangle is side of octagon?
- 14) A variable line is passing through  $(4, -9)$ , slope of line is positive and it make intercepts on x and y-axis on point A and B. Find the minimum area of triangle OAB.
- 15) Solve:  $(1 + x^2) \frac{dy}{dx} + y = e^{\tan^{-1} x}$ ,  $y(1) = 0$ . Then  $y(0) =$