JEE ADVANCED

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I. MCQs with One or More than One Correct

- 1) The minimum value of expression $\sin \alpha + \sin \beta + \sin \gamma$, where (α, β, γ) are real numbers satisfying $(\alpha + \beta + \gamma) = \pi$ is
 - a) positive
 - b) 0
 - c) negative
 - d) -3
- 2) The number of values of x in the interval $[0, 5\pi]$ satisfying equation

$$3\sin(x^2) - 7\sin x + 2 = 0$$

(1998-2 Marks)

- a) 0
- b) 5
- c) 6
- d) 10
- 3) Which of the following number(s) is/are rational?

(1998-2 Marks)

- a) sin 15°
- b) cos 15°
- c) $\sin 15^{\circ} \cos 15^{\circ}$
- d) $\sin 15^{\circ} \cos 75^{\circ}$
- 4) For a positive integer n, let $f_n(\theta) = \left(\tan\frac{\theta}{2}\right)(1+\sec\theta)(1+\sec2\theta)(1+\sec4\theta)\dots(1+\sec2^n\theta)$.

 - a) $f_2\left(\frac{\pi}{16}\right) = 1$ b) $f_3\left(\frac{\pi}{32}\right) = 1$ c) $f_4\left(\frac{\pi}{64}\right) = 1$

 - d) $f_5(\frac{\pi}{128}) = 1$

5) If
$$\frac{\sin^4 x}{2} + \frac{\cos^4 x}{3} = \frac{1}{5}$$
, Then

a) $\tan^2 x = \frac{2}{3}$

b) $\frac{\sin^8 x}{8} + \frac{\cos^8 x}{27} = \frac{1}{125}$

c) $\tan^2 x = \frac{1}{3}$

- d) $\frac{\sin^8 x}{8} + \frac{\cos^8 x}{27} = \frac{2}{125}$
- 6) For $0 < \theta < \frac{\pi}{2}$, the solution(s) of $\sum_{m=1}^{6} \csc\left(\theta + \frac{(m-1)\pi}{4}\right) \csc\left(\theta\right) + \frac{m\pi}{4} = 4\sqrt{2}$ is(are) (2009)

 - a) $\frac{\pi}{4}$ b) $\frac{\pi}{6}$ c) $\frac{\pi}{12}$ d) $\frac{5\pi}{12}$
- 7) Let $\theta, \varphi \in [0, 2\pi]$ be such that $2\cos(\theta(1-\sin\varphi)) = \sin^2(\theta(\tan\frac{\theta}{2}) + \cot\frac{\theta}{2})\cos\varphi 1, \tan(2\pi-\theta) > 0$ and $-1 < \sin \theta < -\frac{\sqrt{3}}{2}$, then φ cannot satisfy (2012)
 - a) $0 < \varphi < \frac{\pi}{2}$

- b) $\frac{\pi}{2} < \varphi < \frac{4\pi}{3}$ c) $\frac{4\pi}{3} < \varphi < \frac{3\pi}{2}$ d) $\frac{3\pi}{2} < \varphi < 2\pi$
- 8) The number of points in $(-\infty, \infty)$, for which $x x \sin x \cos x = 0$, is (JEE Adv.2013)
 - a) 6
 - b) 4
 - c) 2
 - d) 0
- 9) Let $f(x) = x \sin \pi x$, x > 0. Then for all natural numbers n, f'(x) vanishes at (JEE Adv. 2013)
 - a) A unique point in the interval $(n, n + \frac{1}{2})$
 - b) A unique point in the interval $\left(n + \frac{1}{2}, n + 1\right)$
 - c) A unique point in the interval (n, n + 1)
 - d) Two points in the interval (n, n + 1)
- 10) Let α and β be non-zero real numbers such that $2(\cos \beta \cos \alpha) + \cos \alpha \cos \beta = 1$. Then which of the following is/are true? (JEE Adv.2017)

 - a) $\tan\left(\frac{\alpha}{2}\right) + \sqrt{3}\tan\left(\frac{\beta}{2}\right) = 0$ b) $\sqrt{3}\left(\tan\frac{\alpha}{2}\right) + \tan\left(\frac{\beta}{2}\right) = 0$
 - c) $\tan\left(\frac{\alpha}{2}\right) \tan\left(\frac{\beta}{2}\right) = 0$
 - d) $\sqrt{3} \tan \left(\frac{\alpha}{2}\right) \tan \left(\frac{\beta}{2}\right) = 0$

II. Subjective Problems

- 1) If $\tan \alpha = \frac{m}{m+1}$ and $\tan \beta = \frac{1}{2m+1}$, find the possible values of $(\alpha + \beta)$ 2) Draw the graph of $y = \frac{1}{\sqrt{2}} (\sin x + \cos x)$ from $x = -\frac{\pi}{2}$ to $x = \frac{\pi}{2}$ (1978)
- 3) If $\cos(\alpha + \beta) = \frac{4}{5}$, $\sin(\alpha \beta) = \frac{5}{13}$, and α, β lies between 0 and $\frac{\pi}{4}$, find $\tan 2\alpha$ (1979)
- 4) Given $\alpha + \beta \gamma = \pi$, prove that $\sin^2 \alpha + \sin^2 \beta \sin^2 \gamma = 2 \sin \alpha \sin \beta \cos \gamma$ (1980)
- 5) Given $A = \left\{ x : \frac{\pi}{6} \le x \le \frac{\pi}{3} \right\}$ and $f(x) = \cos x x(1+x)$; find f(A)(1980)
- 6) For all θ in $\left(0, \frac{\pi}{2}\right)$ show that, $\cos(\sin \theta) \ge \sin(\cos \theta)$. (1981-4 Marks)