JEE ADVANCED

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I. MCQs with One or More than One Correct

1) The minimum value of expression $\sin\alpha + \sin\beta + \sin\gamma$, where (α, β, γ) are real numbers satisfying $(\alpha + \beta + \gamma) = \pi$ is (1995)
   a) positive
   b) 0
   c) negative
   d) -3

2) The number of values of x in the interval $[0, 5\pi]$ satisfying equation $3\sin\left(x^2\right) - 7\sin x + 2 = 0$ $(1998 - 2Marks)$
   a) 0
   b) 5
   c) 6
   d) 10

3) Which of the following number(s) is/are rational? $(1998 - 2Marks)$
   a) $\sin 15^\circ$
   b) $\cos 15^\circ$
   c) $\sin 15^\circ \cos 15^\circ$
   d) $\sin 15^\circ \cos 75^\circ$

4) For a positive integer n, let $f_n(\theta) = \left(\tan\frac{\theta}{2}\right)(1 + \sec\theta)(1 + \sec 2\theta)(1 + \sec 4\theta)\ldots(1 + \sec 2^n\theta)$. Then $(1999 - 3Marks)$
   a) $f_2\left(\frac{\pi}{16}\right) = 1$
   b) $f_3\left(\frac{\pi}{32}\right) = 1$
   c) $f_4\left(\frac{\pi}{64}\right) = 1$
   d) $f_5\left(\frac{\pi}{128}\right) = 1$

5) If $\frac{\sin^4 x}{2} + \frac{\cos^4 x}{3} = \frac{1}{5}$, Then (2009)
   a) $\tan^2 x = \frac{2}{3}$
   b) $\frac{\sin^8 x}{8} + \frac{\cos^8 x}{27} = \frac{1}{125}$
   c) $\tan^2 x = \frac{1}{3}$
   d) $\frac{\sin^8 x}{8} + \frac{\cos^8 x}{27} = \frac{2}{125}$

6) For $0 < \theta < \frac{\pi}{2}$, the solution($s$) of $\sum_{m=1}^{6} \operatorname{cosec}\left(\theta + \frac{(m-1)\pi}{4}\right)\operatorname{cosec}(\theta) + \frac{m\pi}{4} = 4\sqrt{2}$ is(are) (2009)
   a) $\frac{\pi}{4}$
   b) $\frac{\pi}{6}$
   c) $\frac{\pi}{12}$
   d) $\frac{5\pi}{12}$

7) Let $\theta, \varphi \in [0, 2\pi]$ be such that $2\cos(\theta(1 - \sin\varphi)) = \sin^2\left(\theta\left(\tan\frac{\theta}{2}\right) + \cot\frac{\theta}{2}\right)\cos\varphi - 1$,$\tan(2\pi - \theta) > 0$ and $-1 < \sin\theta < -\frac{\sqrt{3}}{2}$, then φ cannot satisfy (2012)
   a) $0 < \varphi < \frac{\pi}{2}$

b) $\frac{\pi}{2} < \varphi < \frac{4\pi}{3}$
c) $\frac{4\pi}{3} < \varphi < \frac{3\pi}{2}$
d) $\frac{3\pi}{2} < \varphi < 2\pi$

8) The number of points in $(-\infty, \infty)$, for which $x - x\sin x - \cos x = 0$, is (*JEEAdv*.2013)
a) 6
b) 4
c) 2
d) 0

9) Let $f(x) = x\sin\pi x$, $x > 0$. Then for all natural numbers n, $(f'(x))$vanishes at (*JEEAdv*.2013)
a) A unique point in the interval $\left(n, n + \frac{1}{2}\right)$
b) A unique point in the interval $\left(n + \frac{1}{2}, n + 1\right)$
c) A unique point in the interval $(n, n + 1)$
d) Two points in the interval $(n, n + 1)$

10) Let α and β be non-zero real numbers such that $2(\cos\beta - \cos\alpha)+\cos\alpha\cos\beta = 1$.Then which of the following is/are true? (*JEEAdv*.2017)
a) $\tan\left(\frac{\alpha}{2}\right) + \sqrt{3}\tan\left(\frac{\beta}{2}\right) = 0$
b) $\sqrt{3}\left(\tan\frac{\alpha}{2}\right) + \tan\left(\frac{\beta}{2}\right) = 0$
c) $\tan\left(\frac{\alpha}{2}\right) - \tan\left(\frac{\beta}{2}\right) = 0$
d) $\sqrt{3}\tan\left(\frac{\alpha}{2}\right) - \tan\left(\frac{\beta}{2}\right) = 0$

II. Subjective Problems

1) If $\tan\alpha = \frac{m}{m+1}$ and $\tan\beta = \frac{1}{2m+1}$, find the possible values of $(\alpha + \beta)$ (1978)
2) Draw the graph of $y = \frac{1}{\sqrt{2}}(\sin x + \cos x)$ from $x = -\frac{\pi}{2}$ to $x = \frac{\pi}{2}$
3) If $\cos(\alpha + \beta) = \frac{4}{5}$,$\sin(\alpha - \beta) = \frac{5}{13}$, and α, β lies between 0 and $\frac{\pi}{4}$, find $\tan 2\alpha$ (1979)
4) Given $\alpha + \beta - \gamma = \pi$, prove that $\sin^2\alpha + \sin^2\beta - \sin^2\gamma = 2\sin\alpha\sin\beta\cos\gamma$ (1980)
5) Given $A = \left\{x : \frac{\pi}{6} \le x \le \frac{\pi}{3}\right\}$ and f(x)=$\cos x - x(1 + x)$; find $f(A)$ (1980)
6) For all θ in $\left(0, \frac{\pi}{2}\right)$ show that, $\cos(\sin\theta) \ge \sin(\cos\theta)$. (1981 – 4*Marks*)