

# 9-9.3-8

EE24BTECH11004 - ANKIT JAINAR

**Question:** Find the area of the region  $\{(x, y) : x^2 \leq y \leq x\}$

**Solution:** : The problem is to find the area of the region:  $\{(x, y) : x^2 \leq y \leq x\}$  This can

| Variable                          | Description                              |
|-----------------------------------|--|
| $g(x)$                            | Equation of the Conic                    |
| $L$                               | Equation of the line                     |
| $\mathbf{h}$                      | A point on the line $L$                  |
| $\mathbf{m}$                      | Direction vector of line $L$             |
| $\mathbf{x}_1$ and $\mathbf{x}_2$ | Points of intersection of $L$ and $g(x)$ |

TABLE 0: Variables are

be solved using the following steps:

$$g(x, y) = \mathbf{x}^T V \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (0.1)$$

$$V = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad (0.2)$$

$$\mathbf{u} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \quad (0.3)$$

$$f = 0 \quad (0.4)$$

The line equation is:  $L : \mathbf{x} = \mathbf{h} + k\mathbf{m}$  where:

$$\mathbf{h} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} \quad (0.5)$$

$$\mathbf{m} = \begin{pmatrix} 1 \\ \frac{1}{4} \end{pmatrix} \quad (0.6)$$

The points  $\mathbf{x}_1$  and  $\mathbf{x}_2$  are given by:

$$\mathbf{x}_i = \mathbf{h} + k_i \mathbf{m} \quad (0.7)$$

The values of  $k_1$  and  $k_2$  are obtained by solving the quadratic equation:

$$k_1 = \frac{1}{\mathbf{m}^T V \mathbf{m}} \left( -\mathbf{m}^T (V \mathbf{h} + \mathbf{u}) + \sqrt{[\mathbf{m}^T (V \mathbf{h} + \mathbf{u})]^2 - g(\mathbf{h})(\mathbf{m}^T V \mathbf{m})} \right) \quad (0.8)$$

$$k_2 = \frac{1}{\mathbf{m}^T V \mathbf{m}} \left( -\mathbf{m}^T (V \mathbf{h} + \mathbf{u}) - \sqrt{[\mathbf{m}^T (V \mathbf{h} + \mathbf{u})]^2 - g(\mathbf{h})(\mathbf{m}^T V \mathbf{m})} \right) \quad (0.9)$$

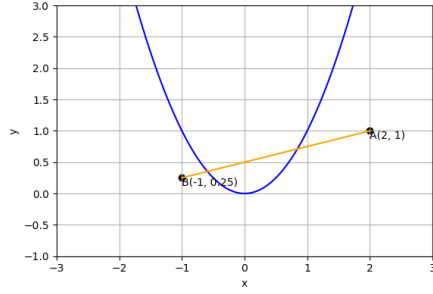


Fig. 0.1: Stem Plot of  $y(n)$

Area bounded by the curves  $y = x^2$  and  $y = x$  is given by:

$$Area = \int_0^1 (x - x^2) dx = \frac{1}{6} squnits. \quad (0.10)$$

(0.11)

Hence, the area bounded by the region is  $\frac{1}{6}$  sq units.