9-9.3-8

EE24BTECH11004 - ANKIT JAINAR

Question: Find the area of the region $\{(x, y) : x^2 \leq y \leq x\}$
Solution: : The problem is to find the area of the region: $\{(x, y) : x^2 \leq y \leq x\}$ This can be solved using the following steps:

Variable	Description
$g(x)$	Equation of the Conic
L	Equation of the line
$\mathbf{h}$	A point on the line L
$\mathbf{m}$	Direction vector of line L
$\mathbf{x_1}$ and $\mathbf{x_2}$	Points of intersection of L and $g(x)$

TABLE 0: Variables are

$$g(x, y) = \mathbf{x}^\top V\mathbf{x} + 2\mathbf{u}^\top\mathbf{x} + f = 0 \tag{0.1}$$

$$V = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \tag{0.2}$$

$$\mathbf{u} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \tag{0.3}$$

$$f = 0 \tag{0.4}$$

The line equation is: $L : \mathbf{x} = \mathbf{h} + k\mathbf{m}$ where:

$$\mathbf{h} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} \tag{0.5}$$

$$\mathbf{m} = \begin{pmatrix} 1 \\ \frac{1}{4} \end{pmatrix} \tag{0.6}$$

The points $\mathbf{x}_1$ and $\mathbf{x}_2$ are given by:

$$\mathbf{x}_i = \mathbf{h} + k_i\mathbf{m} \tag{0.7}$$

The values of k_1 and k_2 are obtained by solving the quadratic equation:

$$k_1 = \frac{1}{\mathbf{m}^\top V\mathbf{m}} \left(-\mathbf{m}^\top(V\mathbf{h} + \mathbf{u}) + \sqrt{[\mathbf{m}^\top(V\mathbf{h} + \mathbf{u})]^2 - g(\mathbf{h})(\mathbf{m}^\top V\mathbf{m})} \right) \tag{0.8}$$

$$k_2 = \frac{1}{\mathbf{m}^\top V\mathbf{m}} \left(-\mathbf{m}^\top(V\mathbf{h} + \mathbf{u}) - \sqrt{[\mathbf{m}^\top(V\mathbf{h} + \mathbf{u})]^2 - g(\mathbf{h})(\mathbf{m}^\top V\mathbf{m})} \right) \tag{0.9}$$

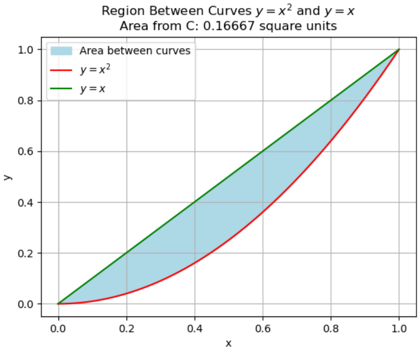

Fig. 0.1: Stem Plot of y(n)

Area bounded by the curves $y = x^2$ and $y = x$ is given by:

$$Area = \int_0^1 (x - x^2)dx = \frac{1}{6} squnits. \tag{0.10}$$

$$\tag{0.11}$$

Hence, the area bounded by the region is $\frac{1}{6}$ sq units.