JEE ADVANCED

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- I. MCQs with One or More than One Correct
- 1) The minimum value of expression $\sin \alpha + \sin \beta +$ $\sin \gamma$, where (α, β, γ) are real numbers satisfying $(\alpha + \beta + \gamma) = \pi$ is (1995)
 - a) positive
 - b) 0
 - c) negative
 - d) -3
- 2) The number of values of x in the interval $[0, 5\pi]$ satisfying equation $3\sin(x^2)$ – $7\sin x + 2 = 0$ (1998-2 Marks)
 - a) 0
 - b) 5
 - c) 6
 - d) 10
- 3) Which of the following number(s) is/are rational? (1998-2)
 - Marks)
 - a) $\sin 15^{\circ}$
 - b) cos 15°
 - c) $\sin 15^{\circ} \cos 15^{\circ}$
 - d) $\sin 15^{\circ} \cos 75^{\circ}$
- 4) For a positive integer n, let $f_n(\theta) = (\tan \frac{\theta}{2})(1 + \frac{\theta}{2})$ $\sec \theta$)(1 + $\sec 2\theta$)(1 + $\sec 4\theta$)...(1 + $\sec 2^n \theta$). (1999 - 3Marks) Then
 - a) $f_2(\frac{\pi}{16}) = 1$
 - b) $f_3(\frac{\pi}{32}) = 1$

 - c) $f_4(\frac{\pi}{64}) = 1$ d) $f_5(\frac{\pi}{128}) = 1$
- 5) If $\frac{\sin^4 x}{2} + \frac{\cos^4 x}{3} = \frac{1}{5}$, Then a) $\tan^2 x = \frac{2}{3}$ (2009)

 - b) $\frac{\sin^8 x}{8} + \frac{\cos^8 x}{27} = \frac{1}{125}$ c) $\tan^2 x = \frac{1}{3}$

 - d) $\frac{\sin^8 x}{8} + \frac{\cos^8 x}{27} = \frac{2}{125}$
- 6) For $\theta < \theta < \frac{\pi}{2}$, the solution(s) of $\sum_{m=1}^{6} \operatorname{cosec} \left(\theta + \frac{(m-1)\pi}{4}\right) \operatorname{cosec} \left(\theta + \frac{m\pi}{4}\right) = 4\sqrt{2}$ (2009)is(are)
 - a) $\frac{\pi}{4}$
 - b) $\frac{\pi}{6}$
 - c) $\frac{\pi}{12}$

- d) $\frac{5\pi}{12}$
- 7) Let $\theta, \varphi \in [0, 2\pi]$ be such that $2\cos(\theta(1 \theta))$ $\sin \varphi$) = $\sin^2(\theta(\tan \frac{\theta}{2}) + \cot \frac{\theta}{2})\cos \varphi - 1, \tan(2\pi - \frac{\theta}{2})\cos \varphi$ θ) > 0 and -1 < $\sin \theta$ < $-\frac{\sqrt{3}}{2}$, then φ cannot satisfy

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- a) $0 < \varphi < \frac{\pi}{2}$ b) $\frac{\pi}{2} < \varphi < \frac{4\pi}{3} \frac{4\pi}{3} < \varphi < \frac{3\pi}{2}$ c) $\frac{3\pi}{2} < \varphi < 2\pi$
- 8) The number of points in $(-\infty, \infty)$, for which $x - x \sin x - \cos x = 0$, is (JEE Adv.2013)
 - a) 6
 - b) 4
 - c) 2
 - d) 0
- 9) Let $f(x) = x \sin \pi x$, x > 0. Then for all natural numbers n, f'(x) vanishes at (JEE Adv. 2013)
 - a) A unique point in the interval $(n, n + \frac{1}{2})$
 - b) A unique point in the interval $(n + \frac{1}{2}, n + 1)$
 - c) A unique point in the interval(n, n + 1)
 - d) Two points in the interval(n, n + 1)
- 10) Let α and β be non-zero real numbers such that $2(\cos\beta - \cos\alpha) + \cos\alpha\cos\beta = 1$. Then which of the following is/are true? (JEE Adv.2017)
 - a) $\tan\left(\frac{\alpha}{2}\right) + \sqrt{3}\tan\left(\frac{\beta}{2}\right) = 0$
 - b) $\sqrt{3}(\tan\frac{\alpha}{2}) + \tan(\frac{\beta}{2}) = 0$
 - c) $\tan\left(\frac{\alpha}{2}\right) \tan\left(\frac{\beta}{2}\right) = 0$
 - d) $\sqrt{3} \tan \left(\frac{\alpha}{2}\right) \tan \left(\frac{\beta}{2}\right) = 0$

II. Subjective Problems

- 1) If $\tan \alpha = \frac{m}{m+1}$ and $\tan \beta = \frac{1}{2m+1}$, find the possible values of $(\alpha + \beta)$
- 2) Draw the graph of $y = \frac{1}{\sqrt{2}} (\sin x + \cos x)$ from $x = -\frac{\pi}{2}$ to $x = \frac{\pi}{2}$
- 3) If $\cos(\alpha + \beta) = \frac{4}{5}$, $\sin(\alpha \beta) = \frac{5}{13}$, and α, β lies between 0 and $\frac{\pi}{4}$, find $\tan 2\alpha$ (1979)
- 4) Given $\alpha + \beta \gamma = \pi$, prove that $\sin^2 \alpha + \sin^2 \beta \sin^2 \gamma = 2 \sin \alpha \sin \beta \cos \gamma$ (1980)
- 5) Given $A=\{x : \frac{\pi}{6} \le x\}$ $\frac{\pi}{3}$ and $f(x) = \cos x - x(1+x)$; find f(A)(1980)
- 6) For all θ in $\left(0,\frac{\pi}{2}\right)$ show that, $\cos(\sin\theta) \ge$ $\sin(\cos\theta)$. (1981-4 Marks)