

9-9.3-8

EE24BTECH11004 - ANKIT JAINAR

Question: Find the area of the region $\{(x, y) : x^2 \leq y \leq x\}$

Solution: : The problem is to find the area of the region:

$$\{(x, y) : x^2 \leq y \leq x\}$$

This can be solved using the following steps:

Variable	Description
$g(x)$	Equation of the Conic
L	Equation of the line
\mathbf{h}	A point on the line L
\mathbf{m}	Direction vector of line L
\mathbf{x}_1 and \mathbf{x}_2	Points of intersection of L and $g(x)$

TABLE 0: Variables are

$$g(x, y) = \mathbf{x}^\top V \mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \quad (0.1)$$

$$V = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad (0.2)$$

$$\mathbf{u} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \quad (0.3)$$

$$f = 0 \quad (0.4)$$

The line equation is:

$$L : \mathbf{x} = \mathbf{h} + k\mathbf{m}$$

where:

$$\mathbf{h} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} \quad (0.5)$$

$$\mathbf{m} = \begin{pmatrix} 1 \\ \frac{1}{4} \end{pmatrix} \quad (0.6)$$

The points \mathbf{x}_1 and \mathbf{x}_2 are given by:

$$\mathbf{x}_i = \mathbf{h} + k_i \mathbf{m} \quad (0.7)$$

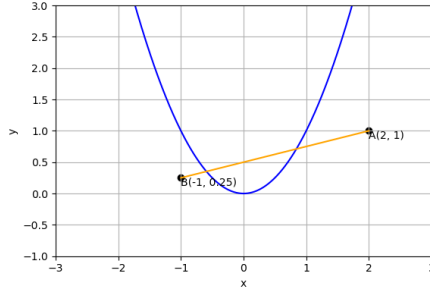


Fig. 0.1: Stem Plot of $y(n)$

The values of k_1 and k_2 are obtained by solving the quadratic equation:

$$k_1 = \frac{1}{\mathbf{m}^\top V \mathbf{m}} \left(-\mathbf{m}^\top (V \mathbf{h} + \mathbf{u}) + \sqrt{[\mathbf{m}^\top (V \mathbf{h} + \mathbf{u})]^2 - g(\mathbf{h})(\mathbf{m}^\top V \mathbf{m})} \right) \quad (0.8)$$

$$k_2 = \frac{1}{\mathbf{m}^\top V \mathbf{m}} \left(-\mathbf{m}^\top (V \mathbf{h} + \mathbf{u}) - \sqrt{[\mathbf{m}^\top (V \mathbf{h} + \mathbf{u})]^2 - g(\mathbf{h})(\mathbf{m}^\top V \mathbf{m})} \right) \quad (0.9)$$

Area bounded by the curves $y = x^2$ and $y = x$ is given by:

$$Area = \int_0^1 (x - x^2) dx = \frac{1}{6} \text{sq units}. \quad (0.10)$$

$$(0.11)$$

Hence, the area bounded by the region is $\frac{1}{6}$ sq units.