MOD-1

MATH (LINEAR)

① $A = \begin{pmatrix} a_{11} & a_{12} \dots a_{1n} \\ a_{21} & a_{22} \dots a_{2n} \\ \vdots & \\ a_{m1} & a_{m2} \dots a_{mn} \end{pmatrix}$ ← coefficient matrix

$\left(\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & b_1 \\ a_{21} & a_{22} & a_{23} & b_2 \\ a_{31} & a_{32} & a_{33} & b_3 \end{array}\right)$ ← Agumented matrix

(*)

② $0x_1 + 0x_2 + \dots + 0x_n = b$ [Degenerate eqn in N variable]

Condition

→ $b \neq 0$ ⟶ No solution (*)

→ $b = 0$ ⟶ Every solution $(a_1, a_2 \dots a_n) \in \mathbb{R}^n$ is Solⁿ of (*)

$0m + 0x_2 + \dots + 0x_m = 0$

LHS = RHS

Result

→ $B \neq 0$, System has No Solⁿ

→ $B = 0$, Degenerate eqn related from System of eqn if Solⁿ set of system is not changed

Note:- if two system of Linear eqn are co-equidistant And has same set of solⁿ

if No. of eqn = No. of unknown then that sys of eqn has unique Solⁿ.

Elimination Algorithm → Gauss Algorithm → Row echelon
Elimination Algorithm → Gauss Jordan → Row Reducing echelon

(1) $2y + 4z = 2$
$x + 2y + 2z = 3$
$3x + 4y + 6z = -1$

$A = \begin{pmatrix} 0 & 2 & 4 \\ 1 & 2 & 2 \\ 3 & 4 & 6 \end{pmatrix}$ $B = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$ $X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

$\boxed{Ax = b}$

$$M = \left[\begin{array}{ccc|c} 0 & 2 & 4 & 2 \\ 1 & 2 & 2 & 3 \\ 3 & 4 & 6 & -1 \end{array}\right] \xrightarrow[R_3 \to R_3 + R_2]{R_1 \leftrightarrow R_2,\ R_3 \to R_3 - 3R_1} \left[\begin{array}{ccc|c} 1 & 2 & 2 & 3 \\ 0 & 2 & 4 & 2 \\ 0 & 0 & 4 & -8 \end{array}\right]$$

← echlon form (final)

Now, Back - substitution

$x + 2y + 2z = 3$
$2y + 4z = 2$
$4z = -8$

$x = -3$
$y = 5$
$z = -2$

$(-3, 5, -2)$ (Ans)

Row echelon form

$$\left[\begin{array}{ccc|c} 1 & * & * & * \\ 0 & 1 & * & * \\ 0 & 0 & 1 & * \end{array}\right]$$

$*$ → Any number (may or may not be zero)

Row Reducing echelon (Gauss Jordan)

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & * \\ 0 & 1 & 0 & * \\ 0 & 0 & 1 & * \end{array}\right]$$

Homogenous

$$\left[\begin{array}{ccc|c} -1 & 1 & 2 & 0 \\ 0 & 7 & 7 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$-x + y + 2z = 0$
~~$7x + 7y = 0$~~
Let $z = t$

$-x - t + 2t = 0$
$x = t$

$7y + 7z = 0$
$y + z = 0$
$\boxed{y = -t}$

$(t, -t, t)$
$(1, -1, 1)$ (Ans)

Non-homogenous

$$\begin{array}{c} \left[\begin{array}{cccc|c} 1 & 1 & 1 & 0 & 3 \\ 0 & -1 & 4 & 2 & 0 \\ 0 & 0 & 1 & 4 & 3 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right] \\ \begin{array}{cccc} w & x & y & z \end{array} \end{array}$$

$w+x+y=3$ $-x+4y+2z=10$

$y+4z=13$ $\boxed{x=10}$ $\boxed{w=2}$

$\boxed{z=3}$ $\boxed{y=1}$ $(2,0,1,3)$

Sys of eqn in echelon form, same with R no. of eqn in n unknown

(I) if $R=n$, There are many eqns as unknowns (triangular form) then system has unique solⁿ.

(II) if $R<n$, N no. of eqn < No. of unknown.
Then we can arbitary assign to values of $n-R$, 3 variables and solve uniquely for R pivot variable obtaining a solⁿ.

Homogenous sys

$S=0$

- More unknown than eqn has non-zero solⁿ.
- if no. of ~~sol~~ eqn = No of unknown then it has 3 solⁿ.

Rank of matrix [R(A), r(A), Rank(A)]

↳ equal to No. of pivots in an echelon form of A matrix.

- → first pivot [first row first column which will be needed to eliminate below it]
- → Second pivot (first Non-zero element in second zero)
- → Third pivot (Non-zero element in third row [may or may not present])

Note:-

① $R(A)=R(A^T)$

② $R(A)$ unchanged by elevatory transformation.

③ $R(AB) \le R(A)$ & $R(B)$

④ $R(A)=0$ [A is keornal matrix]

⑤ $R(A)=$ order of A [A → Non sequence matrix]

⑥ for $m\times n$ matrix
Rank $\le \min(m,n)$

Q) $\begin{pmatrix} 1 & 3 & 5 \\ 2 & -1 & 4 \\ -2 & 3 & 2 \end{pmatrix}$ $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 5 \\ 1 & 5 & 5 & 7 \\ 8 & 1 & 14 & 17 \end{pmatrix}$ $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$

$\begin{pmatrix} 1 & 2 & 5 \\ 0 & -5 & -6 \\ 0 & 0 & 0 \end{pmatrix}$ $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & -3 & -2 & -3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \cancel{0} & -1 \end{pmatrix}$ $\begin{pmatrix} 1 & 2 \\ 0 & -2 \end{pmatrix}$

$R = 2$ $\begin{bmatrix} \text{Non-zero} \\ \text{Non-zero} \\ \text{zero} \end{bmatrix}$

$R = 3$ $\begin{bmatrix} \text{Non-zero} \\ \text{Non-zero} \\ \text{zero} \\ \text{Non-zero} \end{bmatrix}$

$R = 2$ $\begin{bmatrix} \text{Non-zero} \\ \text{Non-zero} \end{bmatrix}$

Q) find

$x + 2y + z = 8$

$2x + 2y + 2z = 13$

$3x + 4y + \lambda z = \mu$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 8 \\ 2 & 2 & 2 & 13 \\ 2 & 4 & \lambda & \mu \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & 2 & 1 & 8 \\ 0 & -2 & 0 & -3 \\ 0 & 0 & \lambda-3 & \mu-21 \end{array}\right]$$

<u>Rank = 3</u>

$\lambda - 3 = \mu - 21$ [Degenerate solⁿ]

for Non-zero

$\lambda = 0$ when $\mu = 21$ $\mu = 0$ when $\lambda = 3$

if $\lambda \neq 3$, $\mu \neq 21$ $\boxed{R(A) = r(M) = 3}$

<u>Inverse Matrix</u>

$$[A : I] \longrightarrow [I : A^{-1}]$$

Eigen value & Eigen vector

$|A - \lambda I| = 0$ $\dfrac{n \times n}{\text{characterial Polynomial}}$ = eigen value.

Characterial matrix of A → A is identical matrix of order n.
$\det(A - \lambda I) = 0$

Rules

- $tr(A)$ = Sum of its diagonal element & sum of its eigen value.
  $\lambda_1 \lambda_2 \lambda_3 = \det(A)$
- if $\det(A) = 0$, Matrix is singular
  One of the eigen value of A must be zero
  vice versa.
- if A is diagonal or upper or lower triangular matrix, then diagonal matrix of eigen value is A.
- zero can be Eigen value But A Eigen vector must be a Non-zero vector
- if $\lambda = \alpha + i\beta$ is an eigenvalue of any matrix
  $\lambda = \alpha + i\beta$
  $\lambda = \alpha - i\beta$

Properties

- $\lambda \rightarrow$ eigen value of A $\quad \alpha \rightarrow$ corresponding eigenvector
  A $\quad \lambda \quad\quad \lambda \rightarrow$ non-zero scalar
  $\alpha A \quad \alpha\lambda$
- ~~A~~ $A \pm kI \quad \lambda \pm k$ [for any positive integer m]
- A, A^T same eigen value
- A^{-1} has eigen value $\frac{1}{\lambda}$ [A^{-1} exist]
- $x \rightarrow$ eigen vector of matrix A, x can't correspond to more eigen value of A.
- Eigen value of ~~Id~~ Idempontent matrix is either 1 or 0.
- eigen vector correspond distinct eigen value of matrix A. is linearly INdependent.

Q) $A = \begin{pmatrix} 0 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 0 & 3 \end{pmatrix}$ $\quad |A - \lambda I| = 0$

$$|A - \lambda I| = \begin{vmatrix} -\lambda & 0 & 2 \\ 0 & 2-\lambda & 0 \\ 2 & 0 & 3-\lambda \end{vmatrix} = 0$$

determinant

$$(2-\lambda)(\lambda-4)(-\lambda+1) = 0 \quad | \lambda = 1, 2, 4$$

eigen value

for $\lambda = 2$

$[A - 2I]x = 0$

$$\begin{bmatrix} 0 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 2 \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\left\{ \begin{bmatrix} 0 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \right\} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \longrightarrow$$

$y - z = 0$

$y = 1$

$z = 0$

$(0, 1, 0)$

$\lambda = 4 \rightarrow (1, 0, 2)$ $\leftarrow$ eigen vectors

$\lambda = -1 \rightarrow (2, 0, -1)$

if we take $A = \begin{bmatrix} 2 & 0 \\ 1 & -1 \end{bmatrix} \Rightarrow$ LDU

we want to write LDU decomposition

$$\underbrace{\begin{bmatrix} 1 & 0 \\ l_{21} & 1 \end{bmatrix}}_{L} \underbrace{\begin{bmatrix} d_{11} & 0 \\ 0 & d_{22} \end{bmatrix}}_{D} \underbrace{\begin{bmatrix} 1 & u_{12} \\ 0 & 1 \end{bmatrix}}_{U}$$

$$= \begin{bmatrix} d_{11} & 0 \\ d_{21} d_{11} & d_{22} \end{bmatrix} \begin{bmatrix} 1 & u_{12} \\ 0 & 1 \end{bmatrix}$$

$$= \cancel{\begin{bmatrix} d_{11} & d_{11}u_{12} \\ d_{11} & d_{22}u \end{bmatrix}} = \begin{bmatrix} d_{11} & d_{11}u_{12} \\ d_{11}d_{21} & d_{11}u_{12} + d_{22} \end{bmatrix}$$

$d_{11} = 0$

$d_{11}d_{12} = 0$

$d_{11}d_{21} = 1$

$d_{11}u_{12} + d_{22} = -1$ ~~$d_{11}u_{12} = 0$~~

$\boxed{d_{11} = 2 \quad d_{22} = -1 \quad l_{21} = \frac{1}{2} \quad u_{12} = 0}$

$$\begin{bmatrix} 2 & 0 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1/2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

if equal then ans correct

(i) $\begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} =$

$$= \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} d_{11} & 0 & 0 \\ 0 & d_{12} & 0 \\ 0 & 0 & d_{33} \end{bmatrix} \begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} d_{11} + 0 + 0 & 0+0+0 & 0+0+0 \\ d_{11} l_{21} + 0 & 0 + d_{12} & 0 \\ d_{11} l_{31} & d_{12} l_{32} & d_{33} \end{bmatrix}$$

$$= \begin{bmatrix} d_{11} & 0 & 0 \\ d_{11} l_{21} & d_{12} & 0 \\ d_{11} l_{31} & d_{12} l_{32} & d_{33} \end{bmatrix} \begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} d_{11} + 0 + 0 & d_{11} u_{12} + 0 & d_{11} u_{13} + 0 \\ d_{11} l_{21} + 0 & d_{11} l_{21} u_{12} + d_{12} & d_{11} l_{21} u_{13} + u_{23} d_{12} \\ d_{11} l_{31} & d_{11} l_{31} u_{12} + d_{12} l_{32} & d_{11} l_{31} u_{13} + d_{12} l_{32} u_{23} + d \end{bmatrix}$$

$$\begin{bmatrix} d_{11} & d_{11}u_{12} & d_{11}u_{13} \\ d_{11}l_{21} & d_{11}l_{21}u_{12} + d_{12} & d_{11}l_{21}u_{13} + u_{23}d_{12} \\ d_{11}l_{31} & d_{11}l_{31}u_{12} + d_{12}l_{32} & d_{11}l_{31}u_{13} + d_{12}l_{32}u_{23} + d_{33} \end{bmatrix}$$

$d_{11} = 1$

$d_{11}u_{12} = -1$

$d_{11}u_{13} = 0$

$d_{11}l_{21} = -1$

$d_{11}l_{21}u_{12} + d_{12} = 2$

$d_{11}l_{21}u_{13} + u_{23}d_{12} = -1$

$d_{11}l_{31} = 0$

$d_{11}l_{31}u_{12} + d_{12}l_{32} = -1$

$d_{11}l_{31}u_{13} + d_{12}l_{32}u_{23} + d_{33} = 2$

$$\boxed{\begin{array}{lll} l_{21} = -1 & l_{31} = 0 & l_{32} = -1 \\ d_{11} = 1 & d_{12} = 1 & d_{33} = 1 \\ u_{12} = -1 & u_{13} = 0 & u_{23} = -1 \end{array}}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

(ii) $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 4 & 5 \\ 1 & 4 & 7 \end{bmatrix}$

(iii) $A = \begin{bmatrix} 1 & -3 & 5 \\ 2 & -4 & 7 \\ -1 & -2 & 1 \end{bmatrix}$

Cryptography

$$\begin{array}{cccccccccccccc} A & B & C & D & E & F & G & H & \dots & Z & \text{Blank} & ? & \\ \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & & & & \updownarrow & \updownarrow & \updownarrow & \updownarrow \\ 0 & 1 & 2 & 3 & 4 & 5 & & & & 25 & 26 & 27 & 28 \end{array}$$

G O O D L U C K

$\underbrace{6, 14, 14}, \underbrace{3, 26, 11}, \underbrace{20, 2, 10}$

$X_1 = \begin{bmatrix} 6 \\ 14 \\ 14 \end{bmatrix}$ $X_2 = \begin{bmatrix} 3 \\ 26 \\ 11 \end{bmatrix}$ $X_3 = \begin{bmatrix} 20 \\ 2 \\ 10 \end{bmatrix}$

$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

$$AX_1 = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 6 \\ 14 \\ 14 \end{bmatrix}$$

$$= \begin{bmatrix} 6 \\ 12 + 14 \\ 6 + 14 + 14 \end{bmatrix} = \begin{bmatrix} 6 \\ 26 \\ 34 \end{bmatrix}$$

$$AX_2 = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 26 \\ 11 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ 6 + 26 \\ 40 \end{bmatrix} = \begin{bmatrix} 3 \\ 32 \\ 40 \end{bmatrix}$$

$$AX_3 = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 20 \\ 2 \\ 10 \end{bmatrix}$$

$$= \begin{bmatrix} 20 \\ 40 + 2 \\ 20 + 10 + 2 \end{bmatrix} = \begin{bmatrix} 20 \\ 42 \\ 32 \end{bmatrix}$$

Receive

6, 26, 34, 3, 32, 40, 20, 42, 32

Q) $\underbrace{19, 45, 26}, \underbrace{13, 36, 41}$

$$A X_1 = \begin{bmatrix} 19 \\ 45 \\ 26 \end{bmatrix} \qquad A x_2 = \begin{bmatrix} 13 \\ 36 \\ 41 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

Make this identity matrix

Then this will become inverse

$$A^{-1} = \left[\begin{array}{ccc:ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 2 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1 & -1 & 1 \end{bmatrix}$$

$$X_1 = A^{-1} \begin{bmatrix} 19 \\ 45 \\ 26 \end{bmatrix} \qquad x_2 = A^{-1} \begin{bmatrix} 13 \\ 36 \\ 41 \end{bmatrix}$$

$$x_1 = \begin{bmatrix} 19 \\ 7 \\ 0 \end{bmatrix} \qquad x_2 = \begin{bmatrix} 13 \\ 10 \\ 48 \end{bmatrix}$$

$$
\begin{array}{cccccc}
T & H & A & N & K & S \\
\uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow \\
19 & 7 & 0 & 13 & 10 & 18
\end{array}
$$

TAKE UFO (DIY)

$$
\begin{array}{c}
T \\
\uparrow \\
8
\end{array}
$$

we are making a place matrix

So we will assume

$$
\begin{array}{ccccccccc}
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
T & A & K & E & \_ & U & F & O & \_
\end{array}
$$

space

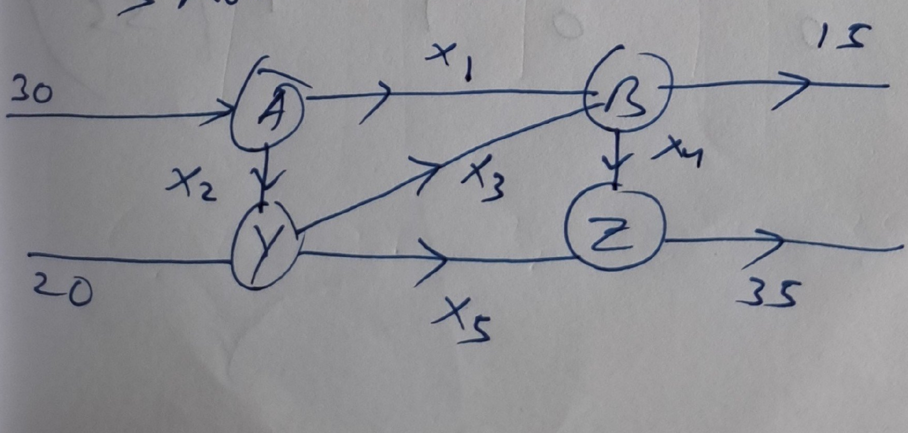
Key

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

Traffic/Network flow

Input = output

① The flow of a traffic through a network of telephone towers is shown in the figure



Q) Solve this system for 5 unknown x_1, x_2, x_3, x_4, x_5 when ~~$x_3 = 0$ and~~ ~~$x_5 = 20$~~

$x_3 = 5$

$x_5 = 20$

Find traffic flow when $x_3 = 0$, $x_5 = 15$

$$x_1 + x_2 = 30$$

$$x_1 + x_3 - x_4 = 15$$

$$x_2 - x_3 - x_5 = -20$$

$$x_4 + x_5 = 35$$

5 unknown
4 equation.

Augmented and simplify ~~simplify~~ with echlon form

$$\left(\begin{array}{ccccc|c} 1 & 1 & 0 & 0 & 0 & 30 \\ 1 & 0 & 1 & -1 & 0 & 15 \\ 0 & 1 & -1 & 0 & -1 & -20 \\ 0 & 0 & 0 & 1 & 1 & 35 \end{array}\right)$$

$R_2 \longrightarrow R_2 - R_1$

$$\left(\begin{array}{ccccc|c} 1 & 1 & 0 & 0 & 0 & 30 \\ 0 & -1 & 1 & -1 & 0 & -15 \\ 0 & 1 & -1 & 0 & -1 & -20 \\ 0 & 0 & 0 & 1 & 1 & 35 \end{array}\right)$$

$R_3 \longrightarrow R_3 + R_2$

$$\left(\begin{array}{ccccc|c} 1 & 1 & 0 & 0 & 0 & 30 \\ 0 & -1 & 1 & -1 & 0 & -15 \\ 0 & 0 & 0 & -1 & -1 & -35 \\ 0 & 0 & 0 & 1 & 1 & 35 \end{array}\right)$$

$R_4 \longrightarrow R_4 + R_3$

$R_2 \longrightarrow -1 \times R_2$

$R_3 \longrightarrow -1 \times R_3$

$$\left(\begin{array}{ccccc|c} 1 & 1 & 0 & 0 & 0 & 30 \\ 0 & 1 & -1 & 1 & 0 & 15 \\ 0 & 0 & 0 & 1 & 1 & 35 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$x_3 = t$ $\qquad x_5 = 5$

$x_1 + x_2 = 30$

$x_2 - x_3 \cancel{\#} + x_4 = 15$

$x_4 + x_5 = 35$

$x_2 = 15 + x_3 - x_4$

$x_4 = \boxed{35 - 5}$

$x_2 = 15 + x_3 - x_4$

$= 15 + t - (35 - 5)$

$= 15$

$\boxed{= 5 + t - 20}$

$x_1 = 30 - x_2$

$x_1 = 30 - (x_3 - x_4 + 15)$

$x_1 = \boxed{50 - 5 - t}$

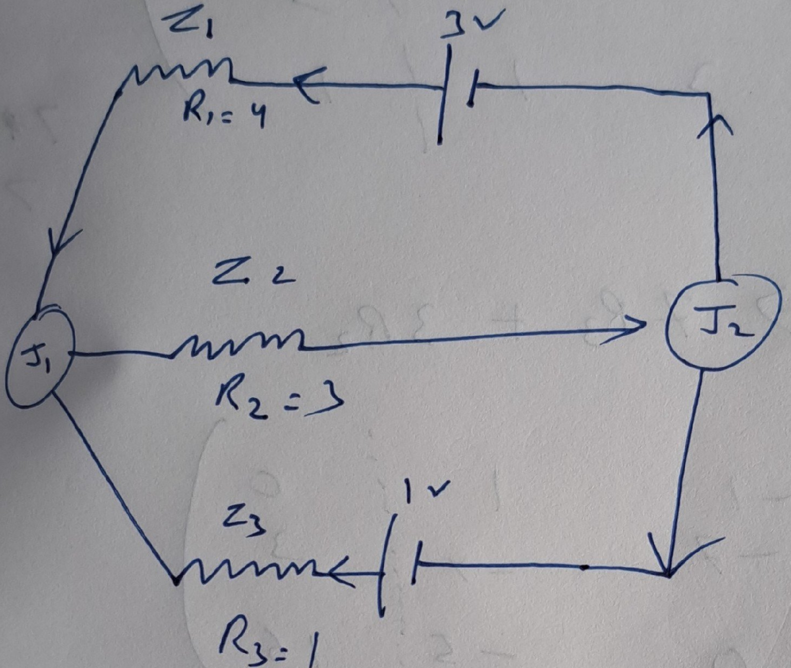
$x_1 = 25$ $\qquad x_2 = 5$ $\qquad x_3 = 5$

$x_4 = 15$ $\qquad x_5 = 20$

Electric Circuit Problem

KCL, KVL, ohms Law

Q) Consider the electrical circuit



$$I_1 + I_3 = I_2$$

$$R_1 I_1 + R_2 I_2 = 3$$

$$R_2 I_2 + R_3 I_3 = 1$$

$$4I_1 + 3I_2 = 3$$

$$3I_2 + I_3 = 1$$

$$\left(\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 4 & 3 & 0 & 3 \\ 0 & 3 & 1 & 1 \end{array}\right)$$

$R_2 \longrightarrow R_2 - 4R_1$ 3 - -4

$$\left(\begin{array}{ccc|c} 1 & -1 & 1 & \\ 0 & -7 & -4 & 0 \\ & & & 3 \\ 0 & 3 & 1 & 1 \end{array}\right)$$

47

7 - 12

7 + 9

$R_3 \longrightarrow 7R_3 + 3R_2$

$$\left(\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & -7 & -4 & 3 \\ 0 & 0 & -5 & 16 \end{array}\right)$$

$x_1 - x_2 + x_3 = 0$

$-7x_2 - 7x_3 = 3$

$-5x_3 = 16$

$x_3 = -\frac{16}{5}$

4.0
3.2
80
120
~~1280~~

$5 \overline{)16} \; 3.2$

15

10

x

$\boxed{x_3 = 3.2}$

$-7x_2 - 12.8 = 3$

$x_2 = -\frac{15.8}{70} =$

$$\boxed{\begin{aligned} x_1 &= 9/19 \\ x_2 &= 7/19 \\ x_3 &= -2/19 \end{aligned}}$$