TUTORIAL-04

Answer-01:- $T(n) = 3T(n/2) + n^2$

$a = 3 \quad b = 2 \quad f(n) = n^2$

$\because$ a & b are constant and f(n) is a +ve function.

$\therefore$ Master's theorem is applicable

$c = \log_b a$

$= \log_2 3 = 1.58$

$\Rightarrow n^c = n^{1.58}$

which is $n^2 > n^{1.58}$

$\therefore$ case 3 is applied here

$$\boxed{T(n) = \Theta(n^2)}$$

Answer-02:- $T(n) = 4T(n/2) + n^2$

$a = 4, \; b = 2, \; f(n) = n^2$

$\because$ a & b are constant and f(n) is a positive function.

$\therefore$ Master's theorem is applicable

$c = \log_b a$

$= \log_2 4 = \log_2 2^2 = 2\log_2 2 = 2$

$\therefore n^c = n^2$

$\because n^c = f(n)$

$\therefore$ case 2 is applied here

$$\Rightarrow \boxed{T(n) = \Theta(n^2 \log n)}$$

Answer-03:- $T(n) = T(n/2) + 2^n$

$a = 1, \; b = 2, \; f(n) = 2^n$

$\because$ a & b are constant and f(n) is a +ve function

$\therefore$ Master's theorem is applicable.

$c = \log_b a = \log_2 1$
$= 0$

$\Rightarrow n^c = n^0 = 1$

$\because f(n) = n^2$

$\therefore$ case 3 is applied here

$\boxed{T(n) = O(2^n)}$

Answer-04:- $T(n) = 2^n T(n/2) + n^n$

$a = 2^n \quad b = 2 \quad f(n) = n^2$

$\because$ a is not constant, its value depends on n.

$\therefore$ Master's theorem is not applicable here.

Answer-05:- $T(n) = 16T(n/4) + n$

$a = 16, \; b = 4, \; f(n) = n$

$\because$ a & b are constant and f(n) is a +ve function

$c = \log_b a$

$= \log_4 16 = \log_4 4^2 = 2\log_4 4 = 2$

$\Rightarrow n^c = n^2$

$\because f(n) < n^2$

$\therefore$ case 1 is applied here.

$\boxed{T(n) = \Theta(n^2)}$

Answer-6:- $T(n) = 2T(n/2) + n\log n$

$a = 2 \quad b = 2 \quad f(n) = n\log n$

$\because$ a, b are constant and f(n) is a +ve function

$c = \log_b a$
$= \log_2 2 = 1$

$n^c = n^1 = n$

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$\because f(n) > n^c$

$\therefore$ case 3 is applied

$\boxed{T(n) = \Theta(n \log n)}$

Answer-07:- $T(n) = 2T(n/2) + n/\log n$

$a = 2 \quad b = 2 \quad f(n) = n/\log n.$

$\therefore$ a & b are constant & f(n) is a +ve function

$c = \log_b a$

$= \log_2 2 = 1$

$n^c = n^1 = n$

non-polynomial difference b/w f(n) & n^c.

$\therefore$ Master's theorem is not applicable.

Answer-08:- $T(n) = 2T(n/4) + n^{0.51}$

$a = 2, \; b = 4, \; f(n) = n^{0.51}$

$\because$ a & b are constant & f(n) is a +ve function.

$\therefore$ Master's theorem is applicable.

$c = \log_b a = \log_4 2 = 0.50$

$n^c = n^{0.50}$

$\because f(n) > n^c$

$\therefore$ case 3 is applicable.

$\boxed{T(n) = \Theta(n^{0.51})}$

-wer-09:- $T(n) = 0.5 * T(n/2) + 1/n$

$a = 0.5 \quad b = 2 \quad f(n) = 1/n$

$\because a < 1$

$\therefore$ Master's theorem is not applicable.

Answer-10:- $T(n) = 16T(n/4) + n!$

$a = 16 \quad b = 4 \quad f(n) = n!$

$\because$ a & b are const. & f(n) is a +ve function.

$\therefore$ Master's theorem is applicable.

$c = \log_b a$

$= \log_4 16 = 2$

$n^c = n^2$

$\because f(n) > n^c$

$\therefore$ case 3 is applied here

$\boxed{T(n) = \Theta(n!)}$

Answer-11:- $T(n) = 4T(n/2) + \log n$

$a = 4 \quad b = 2 \quad f(n) = \log n$

$\because$ a & b are constant & f(n) is a +ve function.

$\therefore$ Master's theorem is applicable.

$c = \log_b a = \log_2 4 = \log_2 2^2 = 2\log_2 2 = 2$

$n^c = n^2$

$\because f(n) < n^2$

$\therefore$ case 1 is applied

$\boxed{T(n) = \Theta(n^2)}$

Answer-12:- $\sqrt{n}\, T(n/2) + \log n$

$a = \sqrt{n} \quad b = 2 \quad f(n) = \log n$

$\therefore$ a is not constant

$\therefore$ Master's theorem is not applicable.

Answer-13:- $T(n) = 3T(n/2) + n$

$a = 3 \quad b = 2 \quad f(n) = n$

$\because$ a & b are constant & f(n) is a +ve function.

$\therefore$ Master's theorem is applicable.

$c = \log_b a = \log_2 3 = 1.58$

$n^c = n^{1.58}$

$f(n) < n^c$

$\therefore$ case 1 is applied here.

$\boxed{T(n) = \theta(n^{1.58})}$

Answer-14:- $T(n) = 3T(n/3) + \sqrt{n}$

$a = 3 \quad b = 3 \quad f(n) = \sqrt{n}$

$\because$ a & b are constant & f(n) is a +ve function.

$\therefore$ Master's theorem is applicable.

$c = \log_b a = \log_3 3 = 1$

$n^c = n^1 = n$

$\because f(n) < n^c$

$\therefore$ case 1 is applicable.

$\boxed{T(n) = \theta(n)}$

Answer-15:- $T(n) = 4T(n/2) + c \cdot n$

$a = 4 \quad b = 2 \quad f(n) = c \cdot n$

$\because$ a & b are constant & f(n) is a +ve function.

$\therefore$ Master's theorem is applicable here.

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

$\therefore f(n) < n^2$

$\therefore$ case 1 is applicable here

$\boxed{T(n) = \theta(n^2)}$

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Answer-16:- $T(n) = 3T(n/4) + n\log n$

$a=3 \quad b=4 \quad f(n) = n\log n$

$c = \log_b a = \log_4 3 = 0.79$

$n^c = n^{0.79}$

$\because f(n) > n^c$

$\therefore$ case c is applicable here.

$\boxed{T(n) = \theta(n\log n)}$

Answer-17:- $T(n) = 3T(n/3) + n/2$

$a=3, \; b=3 \; ; f(n) = n/2$

$\therefore$ Master's theorem is applicable here.

$c = \log_b a = \log_3 3 = 1$

$n^c = n^1 = n$

$\because f(n) = n^c$

$\therefore$ case 2 is applied here

$\boxed{T(n) = n\log n}$

Answer-18:- $T(n) = 6T(n/3) + n^2\log n$

$a=6 \quad b=3 \quad f(n) = n^2\log n$

$\therefore$ Master's theorem is applicable here.

$c = \log_b a = \log_3 6 = 1.63$

$n^c = n^{1.63}$

$\because f(n) > n^c$

$\Rightarrow$ case 3 is applied here

$\boxed{T(n) = \theta(n^2\log n)}$

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Answer-19:- $T(n) = 4T(n/2) + n/\log n$

$a = 4 \quad b = 2 \quad f(n) = n/\log n$

∴ Master's theorem is applicable here.

$c = \log_b a = \log_2 4 = \log_2 2^2 = 2\log_2 2 = 2$

∵ $f(n) < n^c$

∴ case 1 is applied here.

$\Rightarrow \boxed{T(n) = \Theta(n^2)}$

Answer:-20:- $T(n) = 64T(n/8) + n^2 \log n$

∴ a & b are constant but f(n) is a +ve function.

Master's theorem is not applicable here.

Answer-21:- $T(n) = 7T(n/3) + n^2$

$a = 7 \quad b = 3 \quad f(n) = n^2$

∴ Master's theorem applied here

$c = \log_b a = \log_3 7 = 1.77$

$n^c = n^{1.77}$

∵ $f(n) > n^c$

∴ case 3 is applied here.

$\Rightarrow \boxed{T(n) = \Theta(n^2)}$

Answer-22:- $T(n) = T(n/2) + n(2 - \cos n)$

∵ f(n) is not regular function

∴ Master's theorem does not applied here.