Assignment 8

AI1110: Probability and Random Variables

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CBSE Probability Grade 12

Exercise 13.1.6 A coin is tossed three times. Determine $\Pr(E|F)$ for the following three cases:

(i) E: head on third toss
F: heads on first two tosses
(ii) E: at least two heads
F: at most two heads
(iii) E: at most two tails
F: at least one tail

Solution. Let a Bernoulli random variable $X \in \{0, 1\}$ denote the possible outcomes of a coin toss.

X	**Outcome**	**Probability**
0	Tail	$q = \frac{1}{2}$
1	Head	$p = \frac{1}{2}$

TABLE 1: Bernoulli distribution

Consider an experiment consisting of 3 Bernoulli trials X_1, X_2, X_3 and denote the number of heads obtained by a binomial random variable $Y \in \{0, 1, 2, 3\}$. This can be expressed as a binomial distribution with probability mass function given by:

$$p_Y(k) = \binom{n}{k}(1-p)^{n-k}p^k, \; 0 \le k \le n \tag{1}$$

where $n = 3$ and $p = \frac{1}{2}$

Y	**Probability**
0	$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$
1	$3 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{3}{8}$
2	$3 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{3}{8}$
3	$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

TABLE 2: Binomial distribution

(i) Since the outcome of every coin toss is independent of the outcomes of all preceding coin tosses, E and F are independent events, i.e.,

$$\Pr(E|F) = \Pr(E) \tag{2}$$
$$= \Pr(X_3 = 1) \tag{3}$$
$$= p \tag{4}$$
$$= \frac{1}{2} = 0.5 \tag{5}$$

(ii)

$$\Pr(E|F) = \Pr(Y \ge 2 | Y \le 2) \tag{6}$$
$$= \frac{\Pr(Y \ge 2, Y \le 2)}{\Pr(Y \le 2)} \tag{7}$$
$$= \frac{\Pr(Y = 2)}{\sum_{i=0}^{2} \Pr(Y = i)} \tag{8}$$
$$= \frac{\frac{3}{8}}{\frac{1}{8} + \frac{3}{8} + \frac{3}{8}} \tag{9}$$
$$= \frac{3}{7} \approx 0.429 \tag{10}$$

(iii)

$$\Pr(E|F) = \Pr(Y \ge 1 | Y \le 2) \tag{11}$$
$$= \frac{\Pr(Y \ge 1, Y \le 2)}{\Pr(Y \le 2)} \tag{12}$$
$$= \frac{\sum_{i=1}^{2} \Pr(Y = i)}{\sum_{i=0}^{2} \Pr(Y = i)} \tag{13}$$
$$= \frac{\frac{3}{8} + \frac{3}{8}}{\frac{1}{8} + \frac{3}{8} + \frac{3}{8}} \tag{14}$$
$$= \frac{6}{7} \approx 0.857 \tag{15}$$

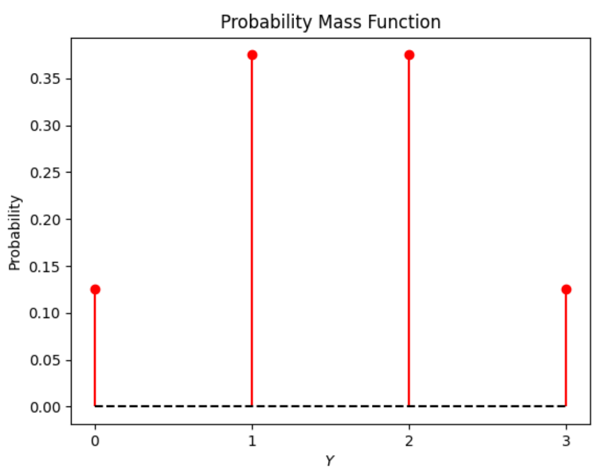

Fig. 1: Plot of the probability mass function