

Assignment 8

AI1110: Probability and Random Variables

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CBSE Probability Grade 12

Exercise 13.1.6 A coin is tossed three times. Determine $\Pr(E|F)$ for the following three cases:

- (i) E : head on third toss
 F : heads on first two tosses
- (ii) E : at least two heads
 F : at most two heads
- (iii) E : at most two tails
 F : at least one tail

Solution. Let a Bernoulli random variable $X \in \{0, 1\}$ denote the possible outcomes of a coin toss.

X	Outcome	Probability
0	Tail	$q = \frac{1}{2}$
1	Head	$p = \frac{1}{2}$

TABLE 1: Bernoulli distribution

Consider an experiment consisting of 3 Bernoulli trials X_1, X_2, X_3 and denote the number of heads obtained by a binomial random variable $Y \in \{0, 1, 2, 3\}$. This can be expressed as a binomial distribution with probability mass function given by:

$$p_Y(k) = \binom{n}{k} (1-p)^{n-k} p^k, \quad 0 \leq k \leq n \quad (1)$$

where $n = 3$ and $p = \frac{1}{2}$

Y	Probability
0	$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$
1	$3 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{3}{8}$
2	$3 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{3}{8}$
3	$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

TABLE 2: Binomial distribution

- (i) Since the outcome of every coin toss is independent of the outcomes of all preceding coin tosses, E and F are independent events, i.e.,

$$\Pr(E|F) = \Pr(E) \quad (2)$$

$$= \Pr(X_3 = 1) \quad (3)$$

$$= p \quad (4)$$

$$= \frac{1}{2} = 0.5 \quad (5)$$

(ii)

$$\Pr(E|F) = \Pr(Y \geq 2|Y \leq 2) \quad (6)$$

$$= \frac{\Pr(Y \geq 2, Y \leq 2)}{\Pr(Y \leq 2)} \quad (7)$$

$$= \frac{\Pr(Y = 2)}{\sum_{i=0}^2 \Pr(Y = i)} \quad (8)$$

$$= \frac{\frac{3}{8}}{\frac{1}{8} + \frac{3}{8} + \frac{3}{8}} \quad (9)$$

$$= \frac{3}{7} \approx 0.429 \quad (10)$$

(iii)

$$\Pr(E|F) = \Pr(Y \geq 1|Y \leq 2) \quad (11)$$

$$= \frac{\Pr(Y \geq 1, Y \leq 2)}{\Pr(Y \leq 2)} \quad (12)$$

$$= \frac{\Pr(1 \leq Y \leq 2)}{\Pr(Y \leq 2)} \quad (13)$$

$$= \frac{\sum_{i=1}^2 \Pr(Y = i)}{\sum_{i=0}^2 \Pr(Y = i)} \quad (14)$$

$$= \frac{\frac{3}{8} + \frac{3}{8}}{\frac{1}{8} + \frac{3}{8} + \frac{3}{8}} \quad (15)$$

$$= \frac{6}{7} \approx 0.857 \quad (16)$$

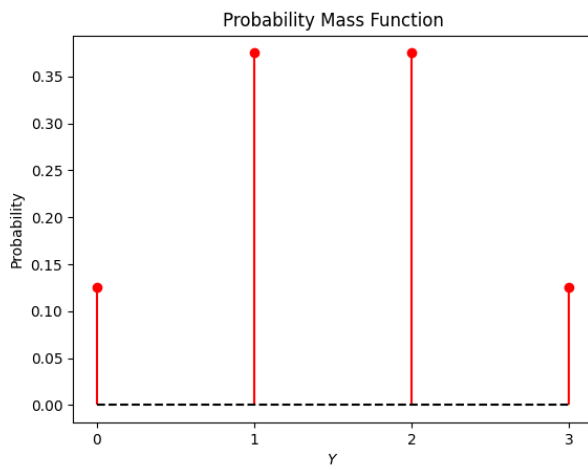


Fig. 1: Plot of the probability mass function