Assignment 10

AI1110: Probability and Random Variables

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CBSE Probability Grade 12

Exercise 13.3.4 In answering a question on a multiple choice test, a student either knows the answer or guesses. Let $\frac{3}{4}$ be the probability that he knows the answer and $\frac{1}{4}$ be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with a probability $\frac{1}{4}$. What is the probability that the student knows the answer given that he answered it correctly?

Solution. Let random variables $X, Y \in \{0, 1\}$ denote the following events in Table (1)

Event	Description
$X = 0$	Student answered wrongly
$X = 1$	Student answered correctly
$Y = 0$	Student knows the answer
$Y = 1$	Student guesses

TABLE 1: Description of events

Probability	Value
$\Pr(Y = 0)$	$\frac{3}{4}$
$\Pr(Y = 1)$	$\frac{1}{4}$
$\Pr(X = 1\|Y = 1)$	$\frac{1}{4}$
$\Pr(X = 1\|Y = 0)$	1
$\Pr(Y = 0\|X = 1)$	?

TABLE 2: Input probabilities

The desired probability is given by:

$$\Pr(Y = 0|X = 1) \tag{1}$$

$$= \frac{\Pr(Y = 0, X = 1)}{\Pr(X = 1)} \tag{2}$$

$$= \frac{\Pr(X = 1|Y = 0)\Pr(Y = 0)}{\sum_{i=0}^{1}\Pr(X = 1, Y = i)} \tag{3}$$

$$= \frac{\Pr(X = 1|Y = 0)\Pr(Y = 0)}{\sum_{i=0}^{1}\Pr(X = 1|Y = i)\Pr(Y = i)} \tag{4}$$

On substituting the values from Table (2) we get:

$$\Pr(Y = 0|X = 1) = \frac{1 \times \frac{3}{4}}{1 \times \frac{3}{4} + \frac{1}{4} \times \frac{1}{4}} \tag{5}$$

$$= \frac{12}{13} \approx 0.923 \tag{6}$$