Assignment 2

AI1110: Probability and Random Variables

Indian Institute of Technology Hyderabad

Ankit Saha
AI21BTECH11004

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Question 1(ii) Solve: $\sin(2\tan^{-1}x) = 1$
Solution.

$$\sin(2\tan^{-1}x) = 1 \tag{1}$$

The general solution to this equation is given by:

$$2\tan^{-1}x = (4n+1)\frac{\pi}{2},\ n \in \mathbb{Z} \tag{2}$$

$$\implies \tan^{-1}x = (4n+1)\frac{\pi}{4},\ n \in \mathbb{Z} \tag{3}$$

But we know that the range of $\tan^{-1}x$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$$\implies -\frac{\pi}{2} < (4n+1)\frac{\pi}{4} < \frac{\pi}{2} \tag{4}$$

$$\implies -2 < 4n+1 < 2 \tag{5}$$

$$\implies -3 < 4n < 1 \tag{6}$$

$$\implies -\frac{3}{4} < n < \frac{1}{4} \tag{7}$$

As $n \in \mathbb{Z}$, we have $n = 0$

$$\implies \tan^{-1}x = \frac{\pi}{4} \tag{8}$$

$$\implies \tan(\tan^{-1}x) = \tan\left(\frac{\pi}{4}\right) \tag{9}$$

$$\implies x = 1 \tag{10}$$

$\therefore$ The solution to the equation is $x = 1$

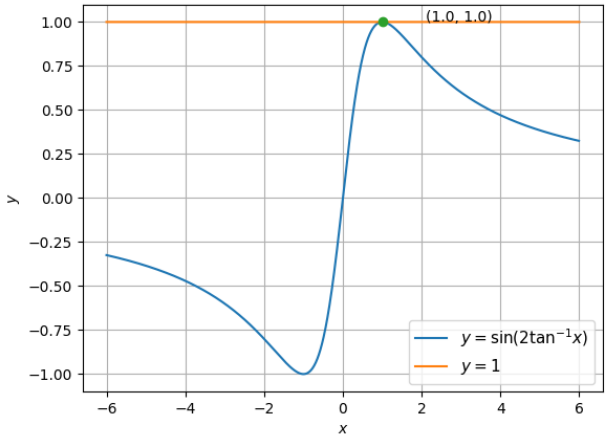

Fig. 1. Graph showing the intersection of $y = \sin(2\tan^{-1}x)$ and $y = 1$