AI1110
Assignment 10

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Outline

Exercise 13.3.4

In answering a question on a multiple choice test, a student either knows the answer or guesses. Let $\frac{3}{4}$ be the probability that he knows the answer and $\frac{1}{4}$ be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with a probability $\frac{1}{4}$. What is the probability that the student knows the answer given that he answered it correctly?

Solution

Let random variables $X, Y \in \{0, 1\}$ denote the following events in Table (1)

Event	Description
$X = 0$	Student answered wrongly
$X = 1$	Student answered correctly
$Y = 0$	Student knows the answer
$Y = 1$	Student guesses

Table 1: Description of events

Input probabilities

The following are the input probabilities as given in the question:

Probability	Value
$\Pr(Y=0)$	$\frac{3}{4}$
$\Pr(Y=1)$	$\frac{1}{4}$
$\Pr(X=1\|Y=1)$	$\frac{1}{4}$
$\Pr(X=1\|Y=0)$	1
$\Pr(Y=0\|X=1)$	?

Table 2: Input probabilities

Computation

The desired probability is given by:

$$\Pr\left(Y=0|X=1\right) \tag{1}$$

$$= \frac{\Pr\left(Y=0,X=1\right)}{\Pr\left(X=1\right)} \tag{2}$$

$$= \frac{\Pr\left(X=1|Y=0\right)\Pr\left(Y=0\right)}{\sum_{i=0}^{1}\Pr\left(X=1,Y=i\right)} \tag{3}$$

$$= \frac{\Pr\left(X=1|Y=0\right)\Pr\left(Y=0\right)}{\sum_{i=0}^{1}\Pr\left(X=1|Y=i\right)\Pr\left(Y=i\right)} \tag{4}$$

Answer

On substituting the values from Table (2) we get:

$$\Pr\left(Y=0|X=1\right)=\frac{1\times\frac{3}{4}}{1\times\frac{3}{4}+\frac{1}{4}\times\frac{1}{4}} \tag{5}$$

$$=\frac{12}{13} \tag{6}$$

$$\approx 0.923 \tag{7}$$