AI1110
Assignment 14

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Outline

1. Question

2. Proof

Papoulis Exercise 6-74

We have a pile of m coins. The probability of heads of the i^{th} coin equals p_i. We select at random one of the coins, we toss it n times and heads shows k times. Show that the probability that we selected the r^{th} coin equals

$$\frac{p_r^k(1-p_r)^{n-k}}{p_1^k(1-p_1)^{n-k}+\cdots+p_m^k(1-p_m)^{n-k}} \tag{1}$$

Proof

Let a random variable $X \in \{1, 2, \ldots, m\}$ denote the coin that has been picked.
Let binomial random variables $Y_i \in \{0, 1, \ldots, n\}, i \in \{1, 2, \ldots, m\}$ denote the number of heads obtained from n tosses of the i^{th} coin. The probability distribution of each of these random variables is given by

$$\Pr\left(Y_i = k\right) = \binom{n}{k} p_i^k \left(1 - p_i\right)^{n-k} \tag{2}$$

Let Y denote the number of heads obtained from n tosses of any coin.

The desired probability is given by

$$\Pr\left(X = r|Y = k\right) \tag{3}$$
$$= \frac{\Pr\left(X = r, Y = k\right)}{\Pr\left(Y = k\right)} \tag{4}$$
$$= \frac{\Pr\left(X = r, Y = k\right)}{\sum_{i=1}^{m} \Pr\left(X = i, Y = k\right)} \tag{5}$$
$$= \frac{\Pr\left(Y_r = k\right)}{\sum_{i=1}^{m} \Pr\left(Y_i = k\right)} \tag{6}$$
$$= \frac{\binom{n}{k} p_r^k (1 - p_r)^{n-k}}{\sum_{i=1}^{m} \binom{n}{k} p_i^k (1 - p_i)^{n-k}} \tag{7}$$
$$= \frac{p_r^k (1 - p_r)^{n-k}}{\sum_{i=1}^{m} p_i^k (1 - p_i)^{n-k}} \tag{8}$$