

Assignment 2

EE3900: Linear Systems and Signal Processing

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Discrete-time Signal Processing

Oppenheim and Schafer

Problem 2.3 By direct evaluation of the convolution sum, determine the step response of a linear time-invariant system whose impulse response is

$$h(n) = a^{-n}u(-n) \quad 0 < a < 1 \quad (1)$$

Solution: We have

$$x(n) = u(n) = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases} \quad (2)$$

and

$$h(n) = a^{-n}u(-n) = \begin{cases} 0 & n > 0 \\ a^{-n} & n \leq 0 \end{cases} \quad (3)$$

Their convolution is given by

$$y(n) = x(n) * h(n) \quad (4)$$

$$= \sum_{k=-\infty}^{\infty} h(k)x(n-k) \quad (5)$$

$$= \sum_{k=-\infty}^0 a^{-k}u(n-k) \quad (6)$$

$$= \sum_{k=0}^{\infty} a^k u(n+k) \quad (7)$$

If $n \geq 0$ then $n+k \geq 0 \forall k \geq 0$

$$y(n) = \sum_{k=0}^{\infty} a^k \quad (8)$$

$$= \frac{1}{1-a} \quad \because 0 < a < 1 \quad (9)$$

Else, if $n < 0$ then $n+k \geq 0 \forall k \geq -n > 0$

$$y(n) = \sum_{k=-n}^{\infty} a^k \quad (10)$$

$$= \frac{a^{-n}}{1-a} \quad \because 0 < a < 1 \quad (11)$$

Therefore, for $0 < a < 1$

$$y(n) = \begin{cases} \frac{1}{1-a} & n \geq 0 \\ \frac{a^{-n}}{1-a} & n < 0 \end{cases} \quad (12)$$

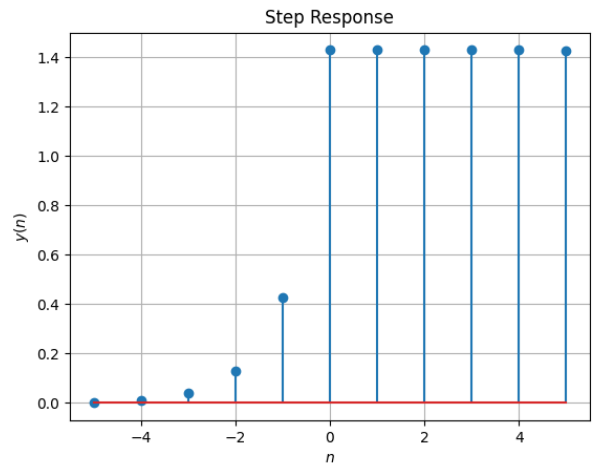


Fig. 1. Plot of the step response for the given impulse response for $a = 0.3$