Circuits and Transforms

EE3900: Linear Systems and Signal Processing

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1. Definitions

1.1 The unit step function is defined as

$$u(t) = \begin{cases} 1 & t > 0 \\ \frac{1}{2} & t = 0 \\ 0 & t < 0 \end{cases} \tag{1.1}$$

1.2 The Laplace transform of $g(t)$ is defined as

$$G(s) = \int_{-\infty}^{\infty} g(t) e^{-st} \, \mathrm{d}t \tag{1.2}$$

2. Laplace Transform

2.1. In the circuit, the switch S is connected to position P for a long time so that the charge on the capacitor becomes q_1 μC. Then S is switched to position Q. After a long time, the charge on the capacitor is q_2 μC

2.2. Draw the circuit using latex-tikz
Solution:

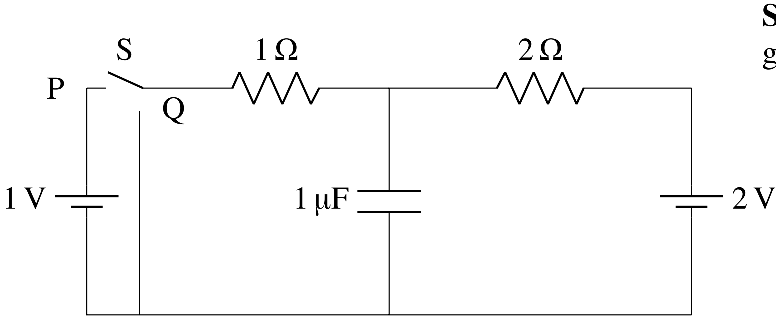
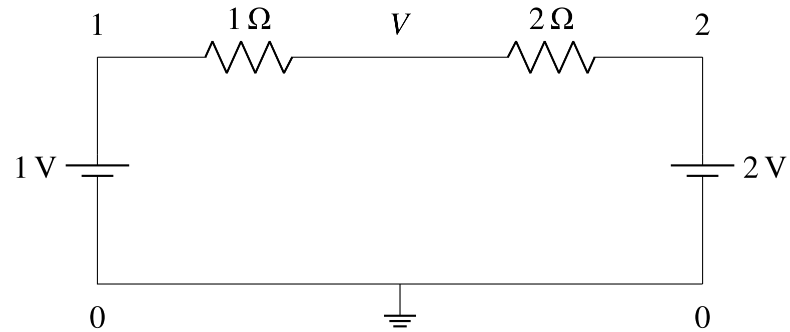


Fig. 2.2. Circuit diagram of the circuit in question

2.3. Find q_1
Solution: After a long time, when steady state is achieved, a capacitor behaves like an open circuit, i.e., current passing through it is zero



By Kirchoff's junction law, we get

$$\frac{V-1}{1} + \frac{V-2}{2} = 0 \tag{2.1}$$

$$\implies V = \frac{4}{3} \,\text{V} \tag{2.2}$$

$$\implies q_1 = CV = \frac{4}{3} \,\mu\text{C} \tag{2.3}$$

2.4. Show that the Laplace transform of $u(t)$ is $\frac{1}{s}$ and find the ROC
Solution: The Laplace transform of $u(t)$ is given by

$$\mathcal{L}\{u(t)\} = \int_{-\infty}^{\infty} u(t) e^{-st} \, \mathrm{d}t \tag{2.4}$$

$$= \int_{0}^{\infty} e^{-st} \, \mathrm{d}t \tag{2.5}$$

$$= \lim_{R \to \infty} \frac{1 - e^{-sR}}{s} \tag{2.6}$$

This limit is finite only if $\Re(s) > 0$, which is going to be its ROC
Therefore

$$u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s} \qquad \Re(s) > 0 \tag{2.7}$$

2.5. Show that

$$e^{-at}u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s+a} \qquad a > 0 \tag{2.8}$$

and find the ROC

Solution: The Laplace transform of $e^{-at}u(t)$ for $a > 0$ is given by

$$\mathcal{L}\{u(t)\} = \int_{-\infty}^{\infty} e^{-at}u(t)e^{-st}\,\mathrm{d}t \tag{2.9}$$

$$= \int_{0}^{\infty} e^{-(s+a)t}\,\mathrm{d}t \tag{2.10}$$

$$= \lim_{R\to\infty} \frac{1-e^{-(s+a)R}}{s+a} \tag{2.11}$$

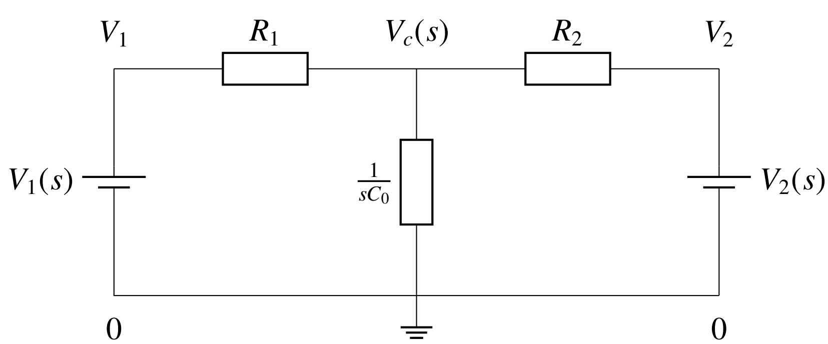
This limit is finite only if $\Re(s+a) > 0$, which is going to be its ROC

Therefore

$$e^{-at}u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s+a} \qquad \Re(s) > -a \tag{2.12}$$

since a is real

2.6. Now consider the following resistive circuit transformed from Fig. 2.2



where

$$u(t) \overset{\mathcal{L}}{\longleftrightarrow} V_1(s) \tag{2.13}$$

$$2u(t) \overset{\mathcal{L}}{\longleftrightarrow} V_2(s) \tag{2.14}$$

Find the voltage across the capacitor $V_{C_0}(s)$

Solution:

$$V_1(s) = \frac{1}{s} \qquad \Re(s) > 0 \tag{2.15}$$

$$V_2(s) = \frac{2}{s} \qquad \Re(s) > 0 \tag{2.16}$$

By Kirchoff's junction law, we get

$$\frac{V_c - V_1}{R_1} + \frac{V_c - V_2}{R_2} + \frac{V_c - 0}{\frac{1}{sC_0}} = 0 \tag{2.17}$$

$$\implies V_c\left(\frac{1}{R_1} + \frac{1}{R_2} + sC_0\right) = \frac{V_1}{R_1} + \frac{V_2}{R_2} \tag{2.18}$$

$$\implies V_c(s) = \frac{\frac{1}{sR_1} + \frac{2}{sR_2}}{\frac{1}{R_1} + \frac{1}{R_2} + sC_0} \tag{2.19}$$

$$= \frac{\frac{1}{R_1C_0} + \frac{2}{R_2C_0}}{s\left(s + \frac{1}{R_1C_0} + \frac{1}{R_2C_0}\right)} \tag{2.20}$$