Assignment 2

EE3900: Linear Systems and Signal Processing

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Discrete-time Signal Processing

Oppenheim and Schafer

Problem 2.3 By direct evaluation of the convolution sum, determine the step response of a linear time-invariant system whose impulse response is

$$h(n) = a^{-n}u(-n) \qquad 0 < a < 1 \tag{1}$$

Solution: We have

$$x(n) = u(n) = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases} \tag{2}$$

and

$$h(n) = a^{-n}u(-n) = \begin{cases} 0 & n > 0 \\ a^{-n} & n \leq 0 \end{cases} \tag{3}$$

Their convolution is given by

$$y(n) = x(n) * h(n) \tag{4}$$

$$= \sum_{k=-\infty}^{\infty} h(k)x(n-k) \tag{5}$$

$$= \sum_{k=-\infty}^{0} a^{-k}u(n-k) \tag{6}$$

$$= \sum_{k=0}^{\infty} a^{k}u(n+k) \tag{7}$$

If $n \geq 0$ then $n + k \geq 0 \ \forall k \geq 0$

$$y(n) = \sum_{k=0}^{\infty} a^k \tag{8}$$

$$= \frac{1}{1-a} \qquad \because 0 < a < 1 \tag{9}$$

Else, if $n < 0$ then $n + k \geq 0 \ \forall k \geq -n > 0$

$$y(n) = \sum_{k=-n}^{\infty} a^k \tag{10}$$

$$= \frac{a^{-n}}{1-a} \qquad \because 0 < a < 1 \tag{11}$$

Therefore, for $0 < a < 1$

$$y(n) = \begin{cases} \dfrac{1}{1-a} & n \geq 0 \\ \dfrac{a^{-n}}{1-a} & n < 0 \end{cases} \tag{12}$$

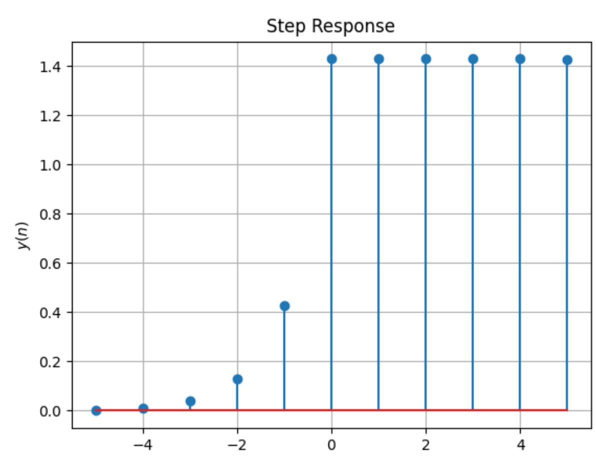

Fig. 1. Plot of the step response for the given impulse response