Indian Institute of Technology Patna
Department of Mathematics
MA225: Probability Theory and Random Process
B.Tech. 2nd year

Tutorial Sheet-4

1. A continuous random variable has probability density function (PDF) defined as $f_X(x) = cxe^{-\frac{x}{2}}$, $x \geq 0$ and $f_X(x) = 0$, $x < 0$.(i) Find the constant c, (ii) What is the CDF of X. (iii) Find the mean, variance and standard deviation of X.(iv) Where is the median of X located.

2. In a dart game the player wins at a circular target having a radius of 25 centimeters. Let X be the distance (in centimeters) between the dart's impact point and the center of the target. Suppose that $P(X \leq x) = c\pi x^2$, $0 \leq x \leq 25$ and $= 1$, $x > 25$, where c is a constant.Evaluate (i) the constant c (ii) the PDF of X (iii) the mean of X (iv) the probability $P(X \leq 10|X \geq 5)$ (v) It costs 1$ to throw a dart and the player wins 10$ if $X \leq r$, 1$ if $r < X \leq 2r$, 0$ if $2r < X \leq 25$. For what values of r is the average gain of the player equal to 0.25$.

3. Show that the function defined as $f_X(x) = \frac{x(6+x)}{3(3+x)^2}$, $0 < x \leq 3$ and $= \frac{9(3+2x)}{x^2(3+x)^2}$, $x > 3$ is a probability density function (PDF).

4. Does the function $\theta^2 x e^{-\theta x}$, $x > 0$, and $= 0$, $x \leq 0$, $\theta > 0$ defines a probability density function? If yes, find the corresponding distribution function and also evaluate $P(X \geq 1)$.

5. Are the following functions distribution functions. If so, find the corresponding PDF/PMF.
(i) $F(x) = 0,\ x \leq 0, = x/2,\ 0 \leq x < 1, = 1/2,\ 1 \leq x < 2,\ \ = x/4,\ 2 \leq x < 4,\ \ = 1,\ x \geq 4$.
(ii) $F(x) = 0,\ x < -\theta, = \frac{1}{2}(x/\theta + 1),\ |x| \leq \theta,\ \ = 1, x > \theta$
(iii) $F(x) = 0,\ x < 1, = \frac{(x-1)^2}{8},\ 1 \leq x < 3, = 1,\ x \geq 3$.

6. Let X be an RV with pdf $f(x) = \frac{\Gamma(m)}{\Gamma(1/2)\Gamma(m-\frac{1}{2})(1+x^2)^m},\quad -\infty < x < \infty,\ \ m \geq 1$. Evaluate $E(X^{2r})$ whenever it exists.

7. Let $f(x)$ be the density function of the RV X. Suppose that X has symmetric distribution about a. Show that the mean of X is a itself.

8. (i) Let X be a continuous random variable with density function function $f(x)$ and distribution function $F(x)$. Then Show that $E(X) = \int_0^{\infty}[1 - F(x)]dx - \int_{-\infty}^{0} F(x)dx$ provided $x\{1 - F(x) - F(-x)\} \to 0$ as $x \to \infty$.
(ii) When X is a nonnegative RV, then $E(X) = \int_0^{\infty}[1 - F(x)]dx$.

9. Let X be a RV with Distribution Function $F(x) = 1 - 0.8e^{-x},\ x \geq 0$ and $F(x) = 0,\ x < 0$. Find EX.

10. Let X be an RV with density function $f(x) = 1/2,\ \ -1 \leq x \leq 1$, and $= 0$ otherwise. Find the distribution function of $\max(X, 0)$.

11. Find the moment generating function for the density function $\frac{1}{2a}e^{-\frac{|x-\mu|}{a}},\ \ -\infty < x < \infty, a > 0,\ \ -\infty < \mu < \infty$. Check whether or not it is a density function.

12. Let $f_X(x) = \frac{1}{2}[1 - \frac{|x-3|}{2}]$, $1 < x < 5$. Check that $f_X(x)$ is a PDF. Find mean, median, variance and p^{th} quantile of X.

13. Let $f_X(x) = \frac{k}{\beta}[1 - \frac{(x-\alpha)^2}{\beta^2}]$, $(\alpha - \beta) < x < (\alpha + \beta)$ where $-\infty < \alpha < \infty$,$\beta > 0$. Find the value of k so that $f_X(x)$ is a PDF. Find mean, median, variance and p^{th} quantile of X. Also evaluate $E(|X - \alpha|)$