

Normalization 3NF and BCNF

**CS 4750
Database Systems**

[A. Silberschatz, H. F. Korth, S. Sudarshan, Database System Concepts, Ch.7]
[Ricardo and Urban, Database Illuminated, Ch.6]
[<https://www.w3schools.in/dbms/database-normalization/>]

Recap: Normalization

Goals:

- Reduce redundancy
- Improve data integrity
- Queries are logical and efficient

Decomposition: Three properties that must be satisfied

- **Lossless join decomposition** – avoid data corruption
 - No gain/no loss
- **Dependency preserving** – improve performance
 - No joins needed to check a dependency
- **Remove duplication** – keep size and structure of DB stable
 - Minimize redundant data in a table

3NF and Decomposition

- **Lossless-join**
- Always **dependency preserving**
- Possible to have extra data (there may be redundancy)

Questions:
Is the relation in 3NF?
Is any refinement needed?

To calculate 3NF

- Identify PK of the original table
- Take **Canonical Cover (Fc)**
- Turn **(minimal set of) FDs** into tables

Canonical Cover (Fc)

- A **minimal set** of functional dependencies that has the same closure as the original set F
- Extraneous attributes = attribute of FDs that we can removed without changing the closure of FDs
- F logically implies all dependencies in Fc
- Fc logically implies all dependencies in F
- No FD in Fc contains an extraneous attribute

F and F+ are logically equivalent

Minimal basis for a set of FDs: For any set of FDs, there is at least one minimal basis, which is a set of FDs **equivalent to the original** (each set implies the other set), with singleton right sides, **no FD that can be eliminated while preserving equivalence**, and **no attribute in a left side that can be eliminated while preserving equivalence**

Canonical Cover (Fc)

Compute the canonical cover of a set of functional dependencies F

Always start with F and use rules to minimize

$$Fc = F$$

repeat

 apply union rule to replace any dependencies $f: X_1 \rightarrow Y_1$
 and $f: X_1 \rightarrow Y_2$ with $f: X_1 \rightarrow Y_1Y_2$

for each functional dependency f_i

if f_i contains an extraneous attribute either in X or in Y
 then remove an extraneous attribute

until Fc does not change any further

Example 1: 3NF and Fc

Given $R(A,B,C,D,E)$

Let's do this together

FDs = { $A \rightarrow B$, $AB \rightarrow D$, $B \rightarrow BDE$, $C \rightarrow D$, $D \rightarrow D$ }

Compute Fc and convert the relation into 3NF

Observation: AC is a minimal super key of the given R

(1) write all LHS

$A \rightarrow$
 $B \rightarrow$
 $AB \rightarrow$
 $C \rightarrow$
 $D \rightarrow$

(2) copy FDs as is

$A \rightarrow B$
 $B \rightarrow B DE$
 $AB \rightarrow D$
 $C \rightarrow D$
 $D \rightarrow D$

(3) remove reflexivity

$A \rightarrow B$
 $B \rightarrow DE$
 $AB \rightarrow D$
 $C \rightarrow D$
 $D \rightarrow D$

(4) remove extraneous attr

$A \rightarrow B$
 $B \rightarrow DE$
 ~~$AB \rightarrow D$~~
 $C \rightarrow D$

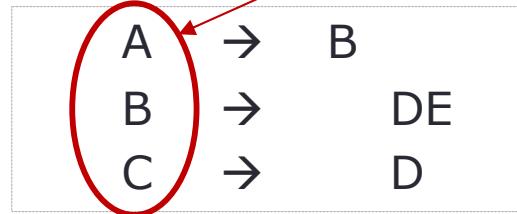
A \rightarrow B and B \rightarrow D.
Thus, remove AB \rightarrow D

Example 1: 3NF and Fc

Let's do this together

(from previous page)

(4) Remove
extraneous attr



Super key

Turn Fc into tables

AC (a minimal super key of the original R)
AB
BDE
CD

A relation $R(A, B, C, D, E)$ is converted into 3NF by putting LHS and RHS of each FD in Fc together in **one** relation.

Dependency preserving

$R(A, B, C, D, E)$ becomes $R_1(A, C)$, $R_2(A, B)$, $R_3(B, D, E)$, $R_4(C, D)$

Or write it in another format: AC // AB // BDE // CD

Example 2: 3NF and Fc

Given $R(A,B,C)$

Let's do this together

FDs = { $A \rightarrow BC$, $B \rightarrow C$, $A \rightarrow B$, $AB \rightarrow C$ }

Compute Fc and convert the relation into 3NF

(1) write all LHS

$A \rightarrow$
 $B \rightarrow$
 $AB \rightarrow$

(2) copy FDs as is

$A \rightarrow BC$
 $B \rightarrow C$
 $AB \rightarrow C$

(3) remove
extraneous attr

$A \rightarrow BC$
 $B \rightarrow C$
 ~~$AB \rightarrow C$~~

(4) remove
extraneous attr

$A \rightarrow BC$
 $B \rightarrow C$

Combine the given
FDs $A \rightarrow BC$ and
 $A \rightarrow B$

No reflexivity
to remove

Consider $AB \rightarrow C$ and
 $B \rightarrow C$,
A is an extraneous attr,
remove A from
 $AB \rightarrow C$ (resulting in
 $B \rightarrow C$)

Apply decomposition to
 $A \rightarrow BC$, thus, $A \rightarrow B$
and $A \rightarrow C$.

$A \rightarrow C$ is logically
equivalent to $A \rightarrow B$
and $B \rightarrow C$ (transitivity).

Thus, C is an
extraneous attr, remove
C from $A \rightarrow BC$

Example 2: 3NF and Fc

Let's do this together

(from previous page)

(4) Remove
extraneous attr



Super key

Turn Fc into tables

AB
BC

$R(A, B, C)$ becomes $R_1(A, B)$ and $R_2(B, C)$

Or write it in another format: AB // BC

BCNF and Decomposition

- **Lossless-join**
- Guarantee **redundancy free**
- May involve dependency across relations

Given a relation R,

for every **nontrivial** FD $X \rightarrow Y$ in R, X is a **super key**

- For all FDs, “key \rightarrow everything”

Questions:
Is the relation in BCNF?
Is any refinement needed?

BCNF and Decomposition

To calculate BCNF

Compute F^+

repeat given a relation R (or a decomposed R) and FDs F
for each functional dependency f_i in a relation R

if f_i violates $X \rightarrow Y$

then decompose R into two relations:

one with $X \cup Y$ as its attributes (i.e., everything f)

one with $X \cup (\text{attrs}(R) - X - Y)$ as its attributes

until no violation

Example: BCNF and F+

Given $R(A,B,C,D,E)$

Let's do this together

FDs = { $A \rightarrow B$, $AB \rightarrow D$, $B \rightarrow BDE$, $C \rightarrow D$, $D \rightarrow D$ }

Compute F+ and convert the relation into BCNF

Compute F+

(1) write all LHS & remaining

$A \rightarrow$
 $B \rightarrow$
 $AB \rightarrow$
 $C \rightarrow$
 $D \rightarrow$
 $E \rightarrow$

(2) copy FDs as is

$A \rightarrow B$
 $B \rightarrow B DE$
 $AB \rightarrow D$
 $C \rightarrow D$
 $D \rightarrow D$
 $E \rightarrow$

(3) apply reflexivity

$A \rightarrow AB$
 $B \rightarrow B DE$
 $AB \rightarrow AB D$
 $C \rightarrow CD$
 $D \rightarrow D$
 $E \rightarrow E$

(4) apply transitivity

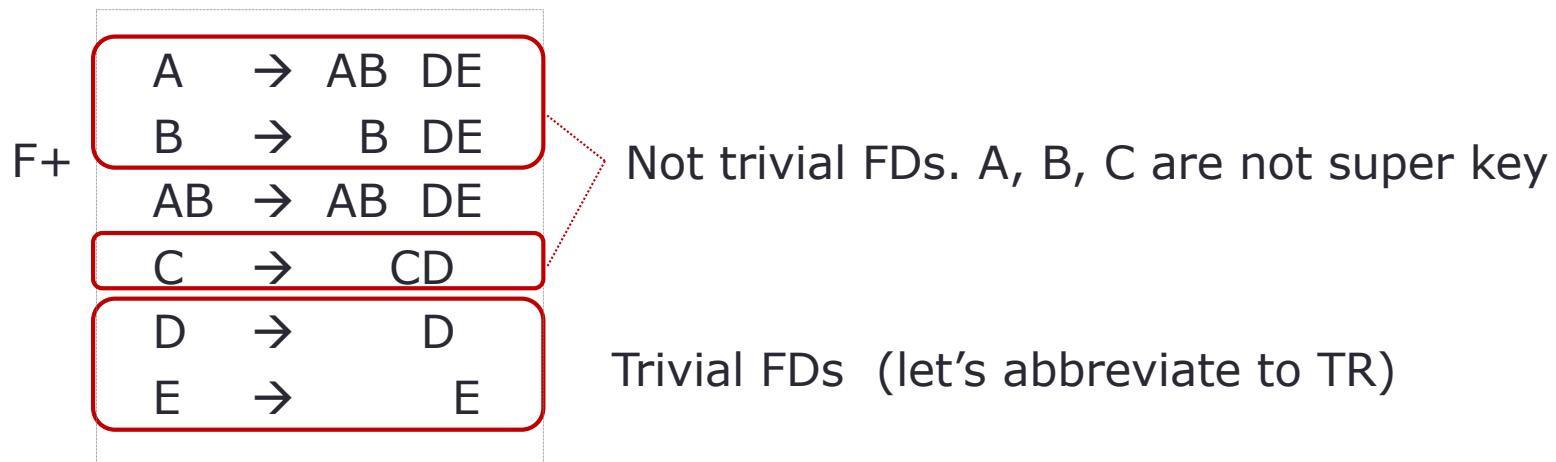
$A \rightarrow AB DE$
 $B \rightarrow B DE$
 $AB \rightarrow AB DE$
 $C \rightarrow CD$
 $D \rightarrow D$
 $E \rightarrow E$

Example: BCNF and F+

(from previous page)

(4) apply transitivity

Let's do this together



Based on F+, let's rewrite using the following format to help us calculate

R (A B C D E)
! ! ! TR TR

TR	- trivial
SK	- super key
X	- neither trivial nor super key
!	- (possibly) need to work on

Example: BCNF and F+

R (A B C D E)
! ! ! TR TR

Let's do this together

F+	A → AB DE
B → B DE	
AB → AB DE	
C → CD	
D → D	
E → E	

To choose which FD to work on, two ways:

- Choose the first FD, or
- Choose the longest FD (yield better solution)

Let's consider A:

A is not a super key, not trivial, thus $A \rightarrow ABDE$ violates BCNF,
break a relation on A

Example: BCNF and F+

R (A **B** C ~~D~~ ~~E~~)
! ! ! TR TR

$A \rightarrow ABDE$

Take *RHS*, make a table: ABDE

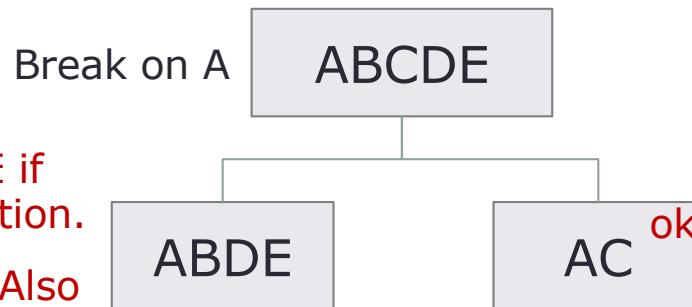
Take *LHS*, make a table where A is a key

A plus (original - (RHS)) --- thus, AC

Let's do this together

F+	A	\rightarrow	AB	DE
	B	\rightarrow	B	DE
	AB	\rightarrow	AB	DE
	C	\rightarrow		CD
	D	\rightarrow		D
	E	\rightarrow		E

There are only 2 attrs,
this relation is ok



Restriction: Cannot break on A 2 times in a row

Next: consider B. B is neither trivial nor super key, break on B

Example: Calculate BCNF

$R (A \boxed{B} D E)$
 $X ! TR TR$

$B \rightarrow BDE$

Take *RHS*, make a table: BDE

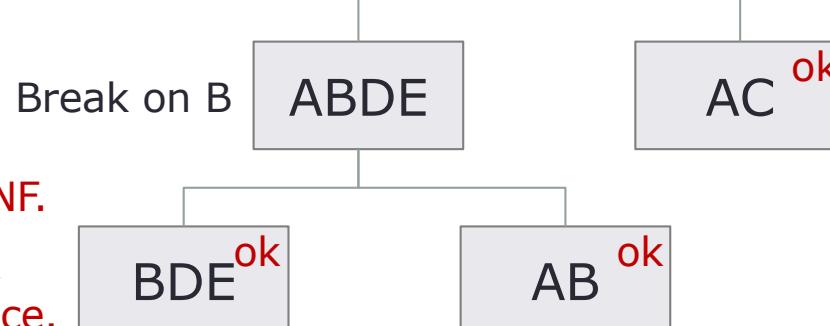
Take *LHS*, make a table where B is a key

B plus (original – (RHS)) --- thus, AB

Let's do this together

F+	A	\rightarrow	AB	DE
	B	\rightarrow	B	DE
	AB	\rightarrow	AB	DE
	C	\rightarrow		CD
	D	\rightarrow		D
	E	\rightarrow		E

There are only 2 attrs,
this relation is ok



Verify that $B \rightarrow BDE$ does not violate BCNF.

B is super key. Also, can't break on B twice.

D and E are trivial.

$R(ABCDE)$ becomes
AC // AB // BDE

(notice: results are different 3NF)

Wrap-Up

- Compute F+ and Fc
- 3NF and decomposition
- BCNF and decomposition

What's next?

- SQL

Additional practice

Practice 1: Decomposition

Given $R(A, B, C)$

FDs = { $A \rightarrow B$, $B \rightarrow C$ }

Supposed R is decomposed in two different ways :

1. $R_1(A, B)$, $R_2(B, C)$

- Does this satisfy lossless-join decomposition?

Yes: $R_1 \cap R_2 = \{B\}$ and $B \rightarrow BC$ (B is super key of R_2)

- Does this satisfy dependency preserving?

Yes: dependencies can be checked without joining tables

2. $R_1(A, B)$, $R_2(A, C)$

- Does this satisfy lossless-join decomposition?

Yes: $R_1 \cap R_2 = \{A\}$ and $A \rightarrow AB$ (A is super key of R_1)

- Does this satisfy dependency preserving?

No: cannot check $B \rightarrow C$ without joining R_1 and R_2

Practice 2: 3NF and BCNF

Given $R(A, B, C, D, E)$

FDs = { $A \rightarrow C$, $C \rightarrow DE$, $D \rightarrow B$, $A \rightarrow D$ }

Decompose table R using 3NF

F_c :

$A \rightarrow C$
 $C \rightarrow DE$
 $D \rightarrow B$

AC // BD // CDE

Decompose table R using BCNF

F^+ :

$A \rightarrow ABCDE$
 $B \rightarrow B$
 $C \rightarrow BCDE$
 $D \rightarrow BD$
 $E \rightarrow E$

AC // BD // CDE

Practice 3: 3NF

Does the Customer_order table satisfy 3NF requirements?
If not, convert the table into 3NF

Customer_order

OrderId	CustomerID	Date	Store	Address
1	2	10/1/2019	South	11 Sorth Str
2	1	9/25/2019	West	22 West Str
3	3	8/12/2019	East	33 East Str
4	4	10/23/2019	West	22 West Str
5	8	5/11/2019	North	44 North Str
6	6	5/11/2019	South	11 Sorth Str
7	5	7/31/2019	East	33 East Str
8	7	10/17/2019	West	22 West Str
9	6	9/19/2019	North	44 North Str
10	4	10/23/2019	North	44 North Str
...

Practice 3: 3NF - solution

No. OrderId → Store → Address
"Transitive dependency"

Customer_order

OrderId	CustomerID	Date	Store
1	2	10/1/2019	South
2	1	9/25/2019	West
3	3	8/12/2019	East
4	4	10/23/2019	West
5	8	5/11/2019	North
6	6	5/11/2019	South
7	5	7/31/2019	East
8	7	10/17/2019	West
9	6	9/19/2019	North
10	4	10/23/2019	North
...

Store

Store	Address
South	11 Sorth Str
West	22 West Str
East	33 East Str
North	44 North Str

Practice 4: BCNF

Does the Student_Major_Advisor table satisfy BCNF requirements?
If not, convert the table into BCNF

Student_Major_Advisor

ComputingID	Major	Advisor
ht1y	Computer Science	someone1
dt2y	Physics	someone2
dt2y	Engineering	somoone3
md3y	Computer Science	someone4
mn4e	Math	someone5
md3y	Computer Science	someone1

computingID, Major → Advisor
Advisor → Major

Assume:
(semantic/business rules)

- Each Student may major in several subjects.
- For each Major, a given Student has only one Advisor.
- Each Major has several Advisors.
- Each Advisor advises only one Major.
- Each Advisor advises several Students in one Major.

Practice 4: BCNF - solution

No. Contain non-key attribute(s)

Student_Advisor

ComputingID	Advisor
ht1y	someone1
dt2y	someone2
dt2y	somoone3
md3y	someone4
mn4e	someone5
md3y	someone1

Advisor_Major

Advisor	Major
someone1	Computer Science
someone2	Physics
somoone3	Engineering
someone4	Computer Science
someone5	Math