Chapter Sums: Numerical Problems

Dr. Mayank Agarwal

Department of CSE
IIT Patna

CS6206 Selected Topics in Wireless Networks

Problem 1: Transmission Time Calculation I

Problem: An 802.11ac AP transmits a 2000-byte data frame to a client using a PHY rate of 867 Mbps. Ignoring MAC overhead, preambles, and waiting times, calculate the transmission time of the data portion over the air.

Solution Steps:

1. Convert packet size to bits: 2000 bytes $\times$ 8 bits/byte $= 16,000$ bits
2. Time = Size / Rate
3. Rate = 867 Mbps = 867×10^6 bps
4. Time = $\dfrac{16,000}{867 \times 10^6} = 1.845 \times 10^{-5}$ seconds
5. Convert to microseconds: $1.845 \times 10^{-5} \times 10^6 = 18.45\mu s$

Model Answer: The raw data transmission takes approximately $\mathbf{18.45}$ μs.

Problem 2: Protocol Efficiency Calculation I

Problem: An 802.11n transmission has the following timing parameters:

- DIFS: 34 μs
- PHY/MAC headers + preamble: 40 μs
- Data transmission time: 27.7 μs
- SIFS: 16 μs
- ACK transmission time: 20 μs

Calculate the total airtime and protocol efficiency.

Solution Steps:

1. Total time = DIFS + Headers + Data + SIFS + ACK
2. Total $= 34 + 40 + 27.7 + 16 + 20 = 137.7\ \mu s$
3. Efficiency = Data time / Total time $= 27.7/137.7 = 0.201 = 20.1\%$

Model Answer: Total airtime = $\mathbf{137.7}\ \mu$s, Efficiency = $\mathbf{20.1}\%$.

Problem 3: Capacity vs. Coverage Design I

Problem: You are designing Wi-Fi for a 60m × 40m exhibition hall. Option A uses 2 APs with 867 Mbps each. Option B uses 8 APs with 433 Mbps each. Which design provides better aggregate throughput for 100 simultaneous clients? Why?

Solution Steps:

1. Option A aggregate capacity: $2 \times 867 = 1734$ Mbps (theoretical)
2. Option B aggregate capacity: $8 \times 433 = 3464$ Mbps (theoretical)
3. Client density: 100 clients / 2 APs = 50 clients/AP vs. 100/8 = 12.5 clients/AP
4. With CSMA/CA, more clients per AP = more collisions and backoff

Model Answer: Option B provides $\mathbf{2}\times$ theoretical capacity and $4\times$ better client distribution, resulting in significantly higher real-world throughput.

Problem 4: Overhead Dominance Analysis I

Problem: Compare the efficiency of transmitting a 64-byte VoIP packet vs a 1500-byte data frame at 54 Mbps. Assume:

- DIFS: 34 μs, SIFS: 16 μs
- PHY preamble: 20 μs
- ACK: 14 bytes at 6 Mbps
- MAC header: 30 bytes
- Average backoff: 67.5 μs

Solution Steps:

(VoIP Size + MAC Header Size) * 8 / transmission speed

1. VoIP: Data $= (64 + 30) \times 8/54 = 13.93\ \mu s$, ACK $= 14 \times 8/6 + 20 = 38.67\ \mu s$
2. Total VoIP $= 34 + 67.5 + 20 + 13.93 + 16 + 38.67 = 190.1\ \mu s$, Efficiency $= 13.93/190.1 = 7.3\%$

Problem 4: Overhead Dominance Analysis II

3. Data: Data $= (1500 + 30) \times 8/54 = 226.67\ \mu s$, Total $= 34 + 67.5 + 20 + 226.67 + 16 + 38.67 = 402.84\ \mu s$, Efficiency $= 226.67/402.84 = 56.3\%$

Model Answer: VoIP efficiency $= \mathbf{7.3}\%$, Data efficiency $= \mathbf{56.3}\%$. Small packets are $\mathbf{7.7}\times$ less efficient.

Problem 5: Channel Overlap Calculation I

Problem: In 2.4 GHz band, channel 6 uses 2.426-2.448 GHz (center 2.437 GHz). Channel 7 uses 2.431-2.453 GHz (center 2.442 GHz). Calculate the overlap in MHz and as a percentage.

Solution Steps:

1. Channel 6 range: 2.426 to 2.448 GHz (22 MHz wide)
2. Channel 7 range: 2.431 to 2.453 GHz (22 MHz wide)
3. Overlap: from 2.431 to 2.448 GHz = 17 MHz
4. Percentage: $17/22 \times 100\% = 77.3\%$

Model Answer: Overlap = $\mathbf{17\ MHz}$ or $\mathbf{77.3}\%$ of channel bandwidth.

Problem 6: MCS and Airtime Inequality I

Problem: Client A uses MCS 9 (650 Mbps) and Client B uses MCS 0 (6.5 Mbps). Both download 50 MB files. How much longer does Client B take? If they share airtime equally, what effective throughput does each get?

Solution Steps:

1. File size: 50 MB $= 50 \times 8 = 400$ Mb
2. Time A $= 400/650 = 0.615$ seconds
3. Time B $= 400/6.5 = 61.54$ seconds
4. Ratio $= 61.54/0.615 = 100\times$ longer
5. With equal airtime sharing: A gets 50% airtime $= 0.5 \times 650 = 325$ Mbps, B gets $0.5 \times 6.5 = 3.25$ Mbps

Model Answer: Client B takes $\mathbf{100}\times$ longer. With equal airtime sharing, A gets $\mathbf{325}$ **Mbps**, B gets $\mathbf{3.25}$ **Mbps**.

Problem 7: Beacon Overhead Calculation I

Problem: An AP transmits beacons every 100 TU (102.4 ms) at 1 Mbps. Beacon size is 200 bytes. Calculate:

1. Percentage of airtime consumed by beacons
2. If 5 APs share the same channel, what's the combined overhead?

Solution Steps:

1. Beacon time $= 200 \times 8/1 = 1600\ \mu s = 1.6$ ms
2. Beacon interval $= 102.4$ ms
3. Percentage $= (1.6/102.4) \times 100\% = 1.56\%$
4. 5 APs $= 5 \times 1.56\% = 7.8\%$

Model Answer: Single AP overhead $= \mathbf{1.56}\%$, 5 APs $= \mathbf{7.8}\%$ (ideal non-colliding case).

Problem 8: TCP Throughput Estimation I

Problem: An 802.11ac link has a PHY rate of 1.3 Gbps. Protocol efficiency is 65% and contention factor reduces throughput by another 20% in a moderately loaded network. What is the expected TCP throughput?

Solution Steps:

1. Throughput = PHY rate × Protocol efficiency × (1 - Contention factor)
2. $= 1300 \times 0.65 \times 0.8 = 1300 \times 0.52 = 676$ Mbps

Model Answer: Expected TCP throughput = $\mathbf{676\ Mbps}$.

Problem 9: IFS Timing Calculation I

Problem: In 802.11ac (5 GHz), SIFS = 16 μs, Slot Time = 9 μs.
Calculate:

1. DIFS duration
2. Total wait time with backoff of 4 slots

Solution Steps:

1. DIFS = SIFS + (2 × Slot Time) $= 16 + (2 \times 9) = 16 + 18 = 34\ \mu s$
2. Backoff time $= 4 \times 9 = 36\ \mu s$
3. Total wait = DIFS + Backoff $= 34 + 36 = 70\ \mu s$

Model Answer: DIFS = $\mathbf{34}$ μs, Total wait = $\mathbf{70}$ μs.

Problem 10: Backoff Window Calculation I

Problem: 802.11ac has CWmin = 15, CWmax = 1023, Slot Time = 9 μs.

1. Initial backoff range (slots and time)?
2. After 3 collisions, what's the new CW and backoff range?

Solution Steps:

1. Initial: CW = 15, slots = 0-15, time = 0 to $15 \times 9 = 0 - 135\ \mu s$
2. After 1st: CW $= 2 \times (15 + 1) - 1 = 31$
3. After 2nd: CW $= 2 \times (31 + 1) - 1 = 63$
4. After 3rd: CW $= 2 \times (63 + 1) - 1 = 127$
5. Slots = 0-127, time = 0 to $127 \times 9 = 0 - 1143\ \mu s$

Model Answer: Initial: $\mathbf{0 - 15}$ **slots** $(\mathbf{0 - 135}\ \mu\mathbf{s})$, After 3 collisions: $\mathbf{0 - 127}$ **slots** $(\mathbf{0 - 1143}\ \mu\mathbf{s})$.

Problem 11: Collision Probability Calculation I

Problem: Two stations use 802.11ac with CWmin = 15. What is the probability they select the same backoff slot? What is this probability after one collision?

Solution Steps:

1. Initial: CW = 15, so 16 possible slots (0-15)
2. Probability both pick same slot = $1/16 = 0.0625 = 6.25\%$
3. After collision: CW = 31, 32 possible slots
4. Probability = $1/32 = 0.03125 = 3.125\%$

Model Answer: Initial collision probability = $\mathbf{6.25}\%$, after collision = $\mathbf{3.125}\%$.

Problem 12: Complete Frame Transmission Time I

Problem: Calculate total transmission time for a 1000-byte frame at 130 Mbps using 802.11ac parameters:

- DIFS: 34 μs, SIFS: 16 μs, Slot: 9 μs
- PHY preamble: 20 μs, MAC header: 36 bytes
- ACK: 14 bytes at 6 Mbps (basic rate)
- Average backoff: 7.5 slots

Solution Steps:

1. Data: $(1000 + 36) \times 8 = 8288$ bits, time = $8288/130 = 63.75\ \mu s$
2. Data with preamble: $63.75 + 20 = 83.75\ \mu s$
3. ACK: $14 \times 8/6 = 18.67\ \mu s$, with preamble: $18.67 + 20 = 38.67\ \mu s$
4. Backoff: $7.5 \times 9 = 67.5\ \mu s$
5. Total $= 34 + 67.5 + 83.75 + 16 + 38.67 = 239.92\ \mu s$

Model Answer: Total transmission time = $\mathbf{239.92}$ μs.

Problem 13: Rate Anomaly Impact I

Problem: A network has 1 client at 6 Mbps and 4 clients at 54 Mbps. Each gets equal airtime. Calculate:

1. Throughput per client
2. Total network throughput
3. Compare to ideal case (all at 54 Mbps)

Solution Steps:

1. Total clients = 5, each gets 20% airtime
2. Slow client: $0.2 \times 6 = 1.2$ Mbps
3. Fast clients: $0.2 \times 54 = 10.8$ Mbps each
4. Total $= 1.2 + 4 \times 10.8 = 1.2 + 43.2 = 44.4$ Mbps
5. Ideal (all 54 Mbps): $5 \times 0.2 \times 54 = 54$ Mbps
6. Loss $= 54 - 44.4 = 9.6$ Mbps (18% reduction)

Model Answer: Slow client = $\mathbf{1.2\ Mbps}$, Fast clients = $\mathbf{10.8\ Mbps}$ each, Total = $\mathbf{44.4\ Mbps}$, $\mathbf{18}\%$ reduction from ideal.

Problem 14: RTS/CTS Efficiency Analysis I

Problem: Compare efficiency for 1500-byte frames at 130 Mbps:

- Without RTS/CTS: Total time = 240 μs (from Problem 12)
- With RTS/CTS: Add RTS (44 μs), CTS (38 μs), 2 extra SIFS (32 μs)
- Collision probability without RTS/CTS: 10%, collision causes retransmission

Solution Steps:

1. Without RTS/CTS: 10% retransmissions = $240/0.9 = 266.7\ \mu s$ effective Effective Time = Time per attempt / Success Probability
2. With RTS/CTS: Add $44 + 38 + 32 = 114\ \mu s$, total = $240 + 114 = 354\ \mu s$
3. Without RTS: $266.7\ \mu s$, With RTS: $354\ \mu s$, RTS adds 33% overhead

Model Answer: Without RTS/CTS (with collisions): $\mathbf{266.7}\ \mu$s, With RTS/CTS: $\mathbf{354}\ \mu$s. RTS/CTS is beneficial when collision probability exceeds $\sim \mathbf{32}\%$.

Problem 15: Slot Time and Network Scale I

Problem: 802.11b uses 20 μs slot time. What is the maximum one-way propagation delay this accommodates? If speed of light in air is 3×10^8 m/s, what's the maximum network diameter?

Solution Steps:

1. Slot time includes 2$\times$ propagation delay + PHY switching
2. For 20 μs slot, propagation delay budget 10 μs (round trip 20 μs)
3. One-way propagation = $10\ \mu s = 10 \times 10^{-6}$ s
4. Distance = speed $\times$ time = $3 \times 10^8 \times 10 \times 10^{-6} = 3000$ m = 3 km

Model Answer: Maximum one-way propagation = $\mathbf{10}$ μs, maximum network diameter = $\mathbf{3\ km}$.

Problem 16: CWmin Optimization I

Problem: For a network with 25 active stations, which CWmin value minimizes collisions? Calculate collision probability for CWmin=15 vs CWmin=31.

Solution Steps:

1. For N stations, approximate collision probability = $1 - (1 - 1/(CW + 1))^{N-1}$
2. CWmin=15 (16 slots): $P_c = 1 - (1 - 1/16)^{24} = 1 - (0.9375)^{24} = 1 - 0.22 = 0.78 = 78\%$
3. CWmin=31 (32 slots): $P_c = 1 - (1 - 1/32)^{24} = 1 - (0.96875)^{24} = 1 - 0.47 = 0.53 = 53\%$

Model Answer: CWmin=31 gives $\mathbf{53}\%$ collision probability vs $\mathbf{78}\%$ for CWmin=15. Larger CWmin recommended for dense networks.

Problem 17: NAV Duration Calculation I

Problem: A station sends a DATA frame of 1500 bytes at 130 Mbps. Calculate the Duration field value that should be set in the DATA frame header.

Given: SIFS = 16 μs, ACK at 6 Mbps = 38.67 μs

Solution Steps:

1. Duration field covers the time until end of ACK
2. Duration = SIFS + ACK transmission time
3. $= 16 + 38.67 = 54.67\ \mu s$

Model Answer: Duration = $\mathbf{54.67}$ μs (rounded to integer in actual implementation).

Problem 18: Retry Limit Impact I

Problem: A VoIP packet (160 bytes payload + 40 bytes IP/UDP + 30 bytes MAC) is transmitted at 54 Mbps. Short Retry Limit = 7. If packet error rate is 20% per transmission, what's the probability the packet is discarded?

Solution Steps:

1. Probability of success on any attempt $= 0.8$
2. Probability of failure on all 7 attempts $= (0.2)^7 = 0.0000128 = 0.00128\%$
3. Probability of eventual success $= 1 - 0.0000128 = 0.9999872 = 99.99872\%$

Model Answer: Discard probability = $\mathbf{0.00128}\%$ (extremely low).

Problem 19: Hidden Node Throughput Impact I

Problem: Two hidden stations each attempt to transmit with probability 0.3 per slot. What is the collision probability? What is the throughput relative to collision-free case?

Solution Steps:

1. Collision probability = $P(A) \times P(B) = 0.3 \times 0.3 = 0.09 = 9\%$
2. Successful transmission probability = $P(A)(1 - P(B)) + P(B)(1 - P(A)) = 0.3 \times 0.7 + 0.3 \times 0.7 = 0.42 = 42\%$
3. Relative throughput = $0.42/0.3 = 1.4$? Wait, that's not right.
4. Actually, with no collisions, one station would get 0.3 throughput.
5. With collisions, total successful transmissions = 0.42
6. Throughput ratio = $0.42/0.6 = 0.7 = 70\%$ of ideal two-station case

Model Answer: Collision probability = $\mathbf{9}\%$, Throughput = $\mathbf{70}\%$ of collision-free.

Problem 20: Deauthentication Attack Impact I

Problem: A deauthentication frame is 30 bytes, transmitted at 1 Mbps. An attacker sends 100 deauth frames per second. What percentage of airtime is consumed? If each deauth forces a client to reauthenticate (taking 5 ms), what's the client availability reduction?

Solution Steps:

1. Deauth frame time = $30 \times 8/1 = 240\ \mu s$ (ignoring preamble)
2. With 802.11b preamble (192 μs): total = $240 + 192 = 432\ \mu s$
3. 100 frames/second = $100 \times 432 = 43,200\ \mu s = 43.2$ ms
4. Percentage = $(43.2/1000) \times 100\% = 4.32\%$
5. Each deauth forces 5 ms reauthentication
6. 100 deauth/sec $\times$ 5 ms = 500 ms reauth time per second
7. Client availability = $(1000 - 500)/1000 = 50\%$

Model Answer: Attack consumes $\mathbf{4.32}\%$ airtime, reduces client availability to $\mathbf{50}\%$.

Problem 21: Fragmentation Overhead Analysis I

Problem: A 2304-byte MSDU is fragmented into 3 fragments of 768 bytes each. Each fragment adds 30-byte MAC header and 20 μs PHY preamble. Compare total overhead with unfragmented transmission (same header/preamble).

Solution Steps:

1. Unfragmented: 1 header (30 bytes) + 1 preamble (20 μs)
2. Overhead $= 30 \times 8/130 + 20 = 1.85 + 20 = 21.85\ \mu s$
3. Fragmented: 3 headers + 3 preambles $= 3 \times 21.85 = 65.55\ \mu s$
4. Additional overhead $= 65.55 - 21.85 = 43.7\ \mu s$ (3× increase)

Model Answer: Fragmentation adds $\mathbf{43.7}$ μs (200%) overhead, but reduces retransmission cost from 2304 to 768 bytes on error.

Problem 22: Sequence Number Wrap Time I

Problem: 802.11 uses 12-bit sequence numbers (0-4095). If a station transmits 1500-byte frames at 130 Mbps continuously, how long until the sequence number wraps?

Solution Steps:

1. Frame transmission time = $83.38\ \mu s$ (from Problem 22)
2. 4096 frames take $4096 \times 83.38\ \mu s = 341,524\ \mu s = 341.5\ ms$
3. Approximately 0.34 seconds

Model Answer: Sequence number wraps after $\mathbf{0.34\ seconds}$ of continuous transmission at max rate.

Problem 23: Beacon Interval in TUs I

Problem: An AP is configured with Beacon Interval = 75 TU. How many milliseconds between beacons? If beacon size is 180 bytes at 1 Mbps, what's the airtime percentage?

Solution Steps:

1. 1 TU = 1024 μs = 1.024 ms
2. Beacon interval = $75 \times 1.024 = 76.8$ ms
3. Beacon time = $180 \times 8/1 = 1440\ \mu s = 1.44$ ms
4. Percentage = $(1.44/76.8) \times 100\% = 1.875\%$

Model Answer: Beacon interval = $\mathbf{76.8\ ms}$, overhead = $\mathbf{1.875}\%$.

Problem 24: Listen Interval and Sleep Time I

Problem: A station sets Listen Interval = 10. Beacon Interval = 100 TU. How long does the station sleep between wake-ups? If it sleeps 99.9% of the time, what's the power savings factor? [awake current 100 mA, sleep 1 mA]

Solution Steps:

1. Beacon interval = $100 \times 1024\ \mu s = 102.4$ ms
2. Listen Interval = $10 \times 102.4 = 1024$ ms = 1.024 seconds
3. Sleep time = 1.024 s - wake time
4. If 99.9% sleep, awake = 0.1% = 1.024 ms
5. Power consumption ratio = sleep current / awake current
6. If awake current 100 mA, sleep 1 mA, average = $(0.001 \times 1024 + 100 \times 1.024)/1024 \approx 1.1$ mA
7. Savings factor vs always awake = $100/1.1 \approx 90\times$

Model Answer: Sleep interval = $\mathbf{1.024}$ s, power savings $\sim \mathbf{90}\times$.

Problem 25: PS-Poll Efficiency I

Problem: Legacy power save uses one PS-Poll per buffered frame. PS-Poll is 20 bytes at 1 Mbps (basic rate) plus PHY preamble 192 μs (802.11b). Compare to retrieving 5 buffered frames with individual PS-Polls vs block transfer.

Solution Steps:

1. PS-Poll time $= 20 \times 8/1 + 192 = 160 + 192 = 352\ \mu s$
2. 5 PS-Polls $= 5 \times 352 = 1760\ \mu s$
3. Data frames: 5 $\times$ (data + ACK) time
4. If AP could send all 5 in one TXOP, save 4 PS-Polls = 1408 μs

Model Answer: PS-Poll overhead $= \mathbf{352}$ μs each. Block transfer saves $\mathbf{1.408\ ms}$ for 5 frames.

Problem 26: Maximum MSDU Size Calculation I

Problem: 802.11 MSDU maximum is 2304 bytes. With 30-byte MAC header, 8-byte LLC/SNAP header, and 4-byte FCS, what's the maximum PHY payload size? What's the airtime at 130 Mbps including 20 μs preamble?

Solution Steps:

1. MSDU = 2304 bytes
2. MAC header = 30 bytes
3. FCS = 4 bytes
4. Total MPDU $= 2304 + 30 + 4 + 8 = 2346$ bytes
5. Bits $= 2346 \times 8 = 18,768$ bits
6. Data time $= 18768/130 = 144.37\ \mu s$
7. With preamble $= 144.37 + 20 = 164.37\ \mu s$

Model Answer: Maximum MPDU = $\mathbf{2346}$ **bytes**, airtime = $\mathbf{164.37}\ \mu$s.

Problem 27: A-MSDU Aggregation Efficiency I

Problem: An A-MSDU aggregates 5 $\times$ 1500-byte frames into one MPDU. MAC header is 30 bytes, A-MSDU subframe headers add 14 bytes per frame. Calculate efficiency gain vs sending 5 separate frames. 20 μs preamble.

Solution Steps:

1. Separate: 5 $\times$ (1500 + 30) = 7650 bytes data + headers, plus 5 preambles
2. Separate airtime = $5\times(7650\times8/130+20) = 5\times(470.77+20) = 5\times490.77 = 2453.85\ \mu s$
3. Aggregated: 5$\times$1500 = 7500 bytes payload, 5$\times$14 = 70 bytes subheaders, + 30 MAC header
4. Total $= 7500 + 70 + 30 = 7600$ bytes = 60,800 bits
5. Airtime $= 60800/130 + 20 = 467.69 + 20 = 487.69\ \mu s$

Problem 27: A-MSDU Aggregation Efficiency II

6. Gain $= 2453.85/487.69 = 5.03\times$ faster!

Model Answer: A-MSDU aggregation provides $\mathbf{5.03}\times$ throughput improvement for 5 frames.

Key Formulas Reference I

- **Transmission time:** $T = \dfrac{\text{bits}}{\text{PHY rate}}$
- **Protocol efficiency:** $\eta = \dfrac{T_{\text{data}}}{T_{\text{total}}}$
- **DIFS**: $\text{DIFS} = \text{SIFS} + 2 \times \text{SlotTime}$
- **Backoff time:** $T_{\text{backoff}} = \text{random}(0, \text{CW}) \times \text{SlotTime}$
- **CW after collision:** $\text{CW}_{\text{new}} = \min(2 \times (\text{CW}_{\text{old}} + 1) - 1, \text{CW}_{\text{max}})$
- **Collision probability:** $P_c \approx 1 - (1 - \frac{1}{\text{CW}+1})^{N-1}$
- **RTS Duration:** $T_{\text{RTS}} = T_{\text{CTS}} + T_{\text{DATA}} + T_{\text{ACK}} + 3 \times \text{SIFS}$
- **Throughput:**
  $\text{TCP Throughput} \approx \text{PHY Rate} \times \text{Efficiency} \times (1 - \text{Contention})$