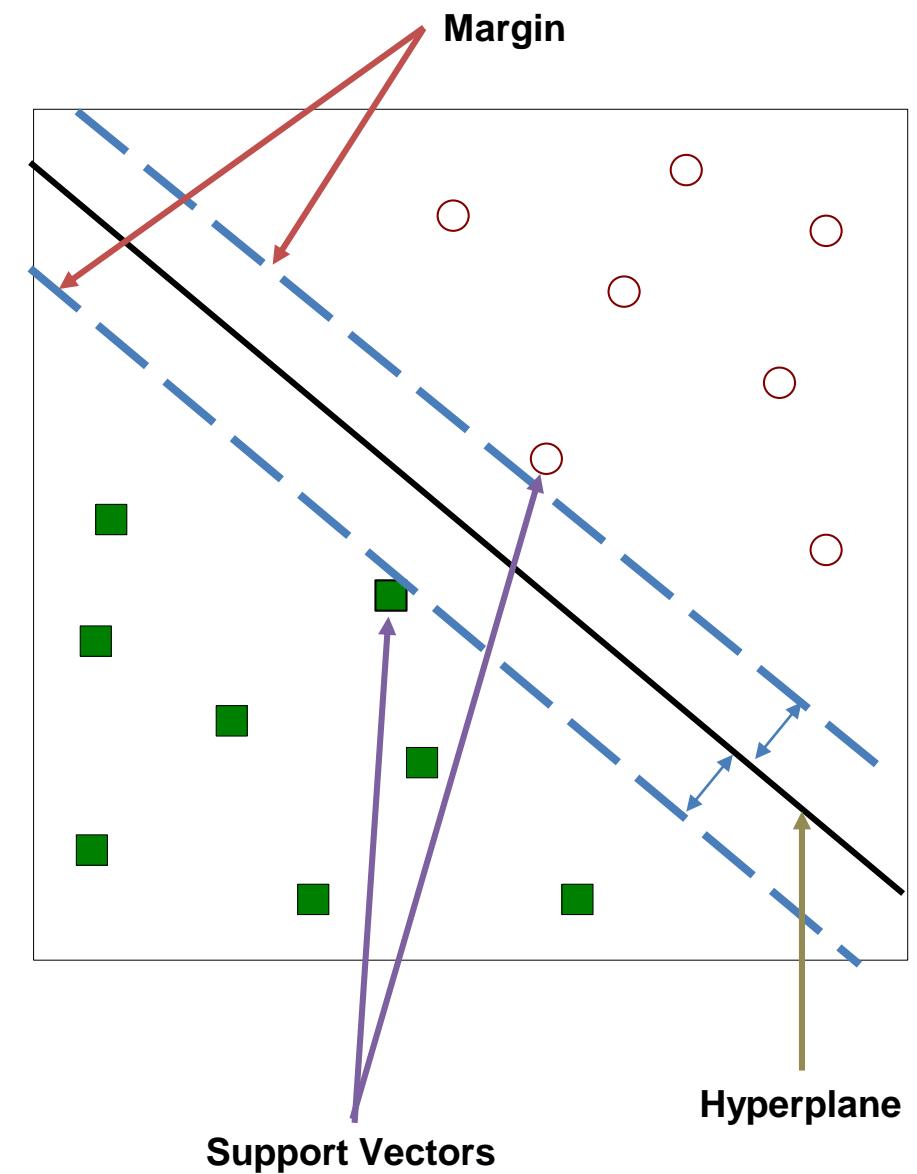


Support Vector Machine

Support Vector Machine(SVM)

- Decision Boundary
- Which Solution is Better?
- **Hyperplane**
 - Best divides the dataset into classes
 - Line for 2D space, Plane or a hyperplane for higher dimensions.
- **Margin**
 - The margin is the distance between the hyperplane and the nearest points from either class
 - A larger margin implies a better generalization for the classifier
 - **Hard Margin SVM:** Assumes data is perfectly separable by a hyperplane without any errors
 - **Soft Margin SVM:** Allows some misclassifications or errors
- **Support Vectors**
 - Points closest to the hyperplane
 - Decide the position and orientation of hyperplane
 - “Supporting” points that define the boundary between classes.



SVM

$$y = \beta_0 + \beta_1 x$$

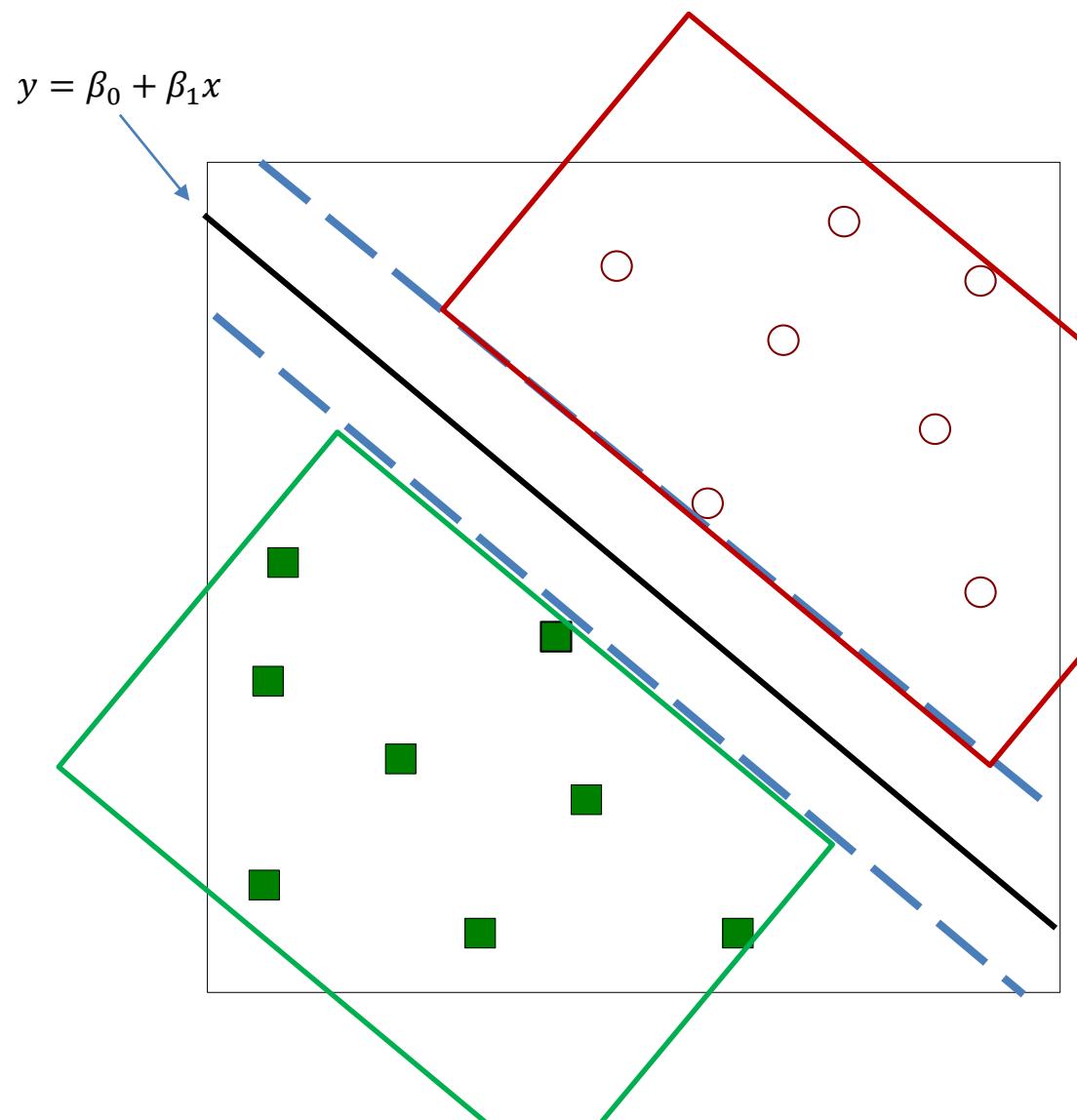
$$\beta_0 + \beta_1 x - y = 0$$

What if

$$\beta_0 + \beta_1 x - y > 0$$

Or

$$\beta_0 + \beta_1 x - y < 0$$



SVM Example 1

Positive Labels (+1)

$$\left\{ \begin{pmatrix} 3 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ -1 \end{pmatrix}, \begin{pmatrix} 6 \\ 1 \end{pmatrix}, \begin{pmatrix} 6 \\ -1 \end{pmatrix} \right\}$$

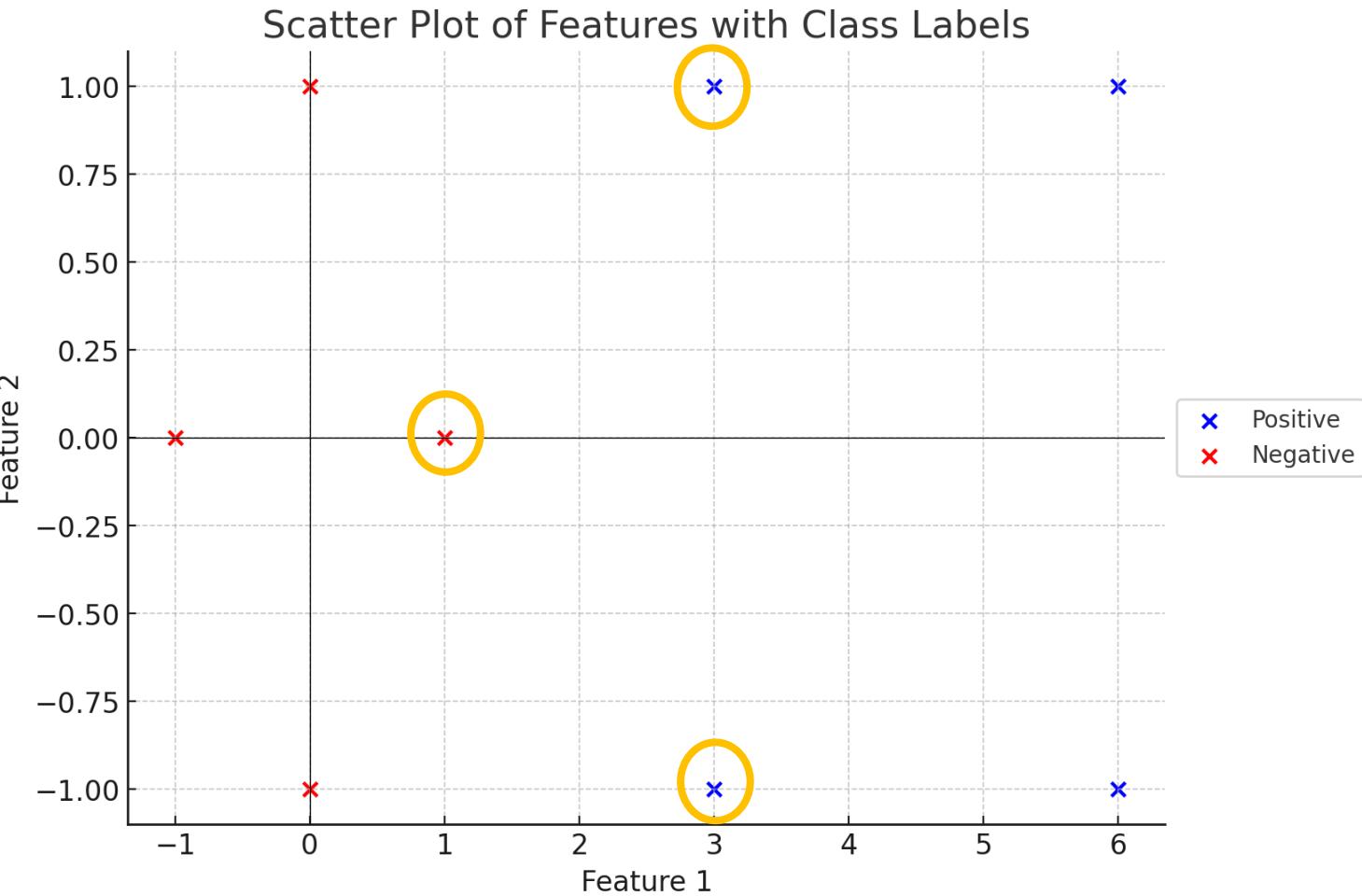
Negative Labels (-1)

$$\left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \end{pmatrix} \right\}$$

Support Vectors

$$\{s_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, s_2 = \begin{pmatrix} 3 \\ 1 \end{pmatrix}, s_3 = \begin{pmatrix} 3 \\ -1 \end{pmatrix}\}$$

Sl. No.	Feature 1	Feature 2	Class/Labels
1	3	1	Positive
2	3	-1	Positive
3	6	1	Positive
4	6	-1	Positive
5	1	0	Negative
6	0	1	Negative
7	0	-1	Negative
8	-1	0	Negative



SVM Example 1

Support Vectors

$$\{s_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, s_2 = \begin{pmatrix} 3 \\ 1 \end{pmatrix}, s_3 = \begin{pmatrix} 3 \\ -1 \end{pmatrix}\}$$

Vector Augmentation

$$s_1 = (10), \tilde{s}_1 = (101) \text{ Negative}$$

$$s_2 = (31), \tilde{s}_2 = (311) \text{ Positive}$$

$$s_3 = (3 (-1)), \tilde{s}_3 = (3 (-1)1) \text{ Positive}$$

Augmented Support Vectors

$$\{\tilde{s}_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \tilde{s}_2 = \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}, \tilde{s}_3 = \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix}\}$$

Find the values of α_i

$$\alpha_1 \phi(s_1) \cdot \phi(s_1) + \alpha_2 \phi(s_2) \cdot \phi(s_1) + \alpha_3 \phi(s_3) \cdot \phi(s_1) = -1$$

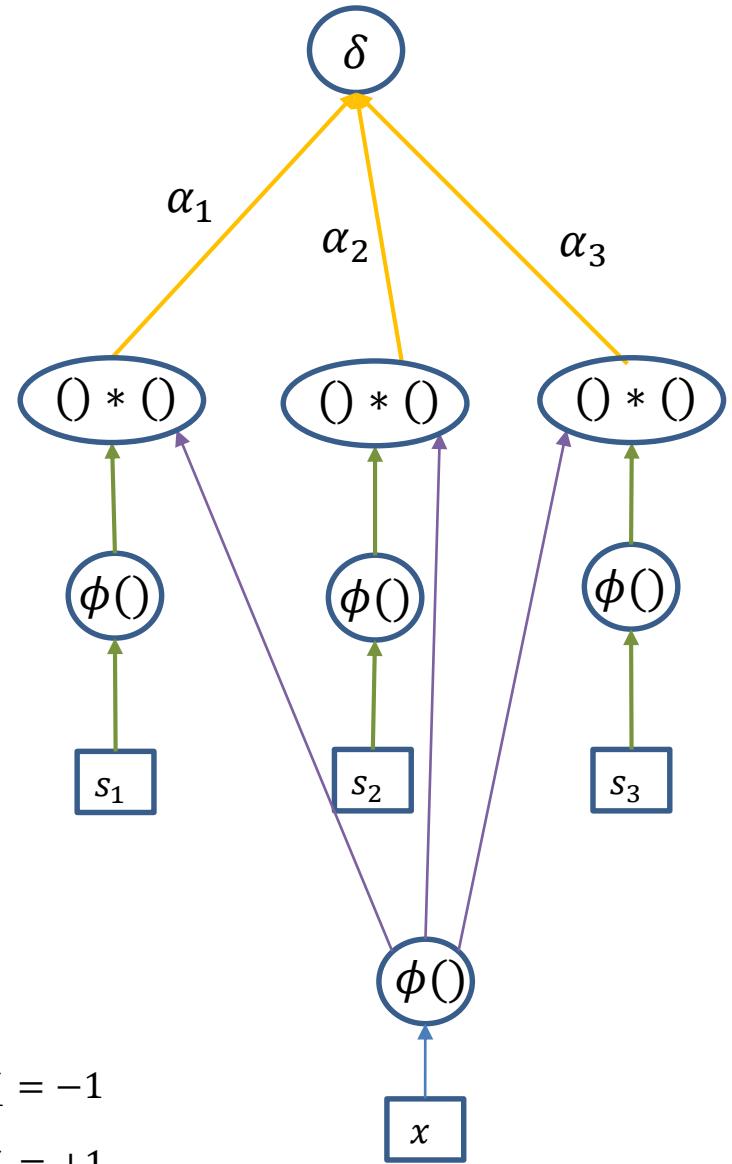
$$\alpha_1 \phi(s_1) \cdot \phi(s_2) + \alpha_2 \phi(s_2) \cdot \phi(s_2) + \alpha_3 \phi(s_3) \cdot \phi(s_2) = +1 \quad \rightarrow$$

$$\alpha_1 \phi(s_1) \cdot \phi(s_3) + \alpha_2 \phi(s_2) \cdot \phi(s_3) + \alpha_3 \phi(s_3) \cdot \phi(s_3) = +1$$

$$\alpha_1 \tilde{s}_1 \cdot \tilde{s}_1 + \alpha_2 \tilde{s}_2 \cdot \tilde{s}_1 + \alpha_3 \tilde{s}_3 \cdot \tilde{s}_1 = -1$$

$$\alpha_1 \tilde{s}_1 \cdot \tilde{s}_2 + \alpha_2 \tilde{s}_2 \cdot \tilde{s}_2 + \alpha_3 \tilde{s}_3 \cdot \tilde{s}_2 = +1$$

$$\alpha_1 \tilde{s}_1 \cdot \tilde{s}_3 + \alpha_2 \tilde{s}_2 \cdot \tilde{s}_3 + \alpha_3 \tilde{s}_3 \cdot \tilde{s}_3 = +1$$



SVM Example 1

$$\alpha_1 \tilde{s}_1 \cdot \tilde{s}_1 + \alpha_2 \tilde{s}_2 \cdot \tilde{s}_1 + \alpha_3 \tilde{s}_3 \cdot \tilde{s}_1 = -1$$

$$\alpha_1 \tilde{s}_1 \cdot \tilde{s}_2 + \alpha_2 \tilde{s}_2 \cdot \tilde{s}_2 + \alpha_3 \tilde{s}_3 \cdot \tilde{s}_2 = +1$$

$$\alpha_1 \tilde{s}_1 \cdot \tilde{s}_3 + \alpha_2 \tilde{s}_2 \cdot \tilde{s}_3 + \alpha_3 \tilde{s}_3 \cdot \tilde{s}_3 = +1$$

$$\alpha_1(101) \cdot (101) + \alpha_2(311) \cdot (101) + \alpha_3(3(-1)1) \cdot (101) = -1$$

$$\alpha_1(101) \cdot (311) + \alpha_2(311) \cdot (311) + \alpha_3(3(-1)1) \cdot (311) = +1$$

$$\alpha_1(101) \cdot (3(-1)1) + \alpha_2(311) \cdot (3(-1)1) + \alpha_3(3(-1)1) \cdot (3(-1)1) = +1$$

Compute Dot Product of $(101) \cdot (3(-1)1)$

$$(101) \cdot (3(-1)1) = 1 * 3 + 0 * (-1) + 1 * 1 = 4$$

$$2\alpha_1 + 4\alpha_2 + 4\alpha_3 = -1 \quad \text{Eq. 1}$$

$$4\alpha_1 + 11\alpha_2 + 9\alpha_3 = +1 \quad \text{Eq. 2}$$

$$4\alpha_1 + 9\alpha_2 + 11\alpha_3 = +1 \quad \text{Eq. 3}$$

Divide Eq 1 by 2

$$2\alpha_1 + 4\alpha_2 + 4\alpha_3 = -1 \rightarrow \alpha_1 + 2\alpha_2 + 2\alpha_3 = -\frac{1}{2} \quad \text{Eq. 4}$$

Subtract (Eq 2 - Eq 4)

$$(4\alpha_1 + 11\alpha_2 + 9\alpha_3) - (\alpha_1 + 2\alpha_2 + 2\alpha_3) = +1 - \left(-\frac{1}{2}\right)$$

$$3\alpha_1 + 9\alpha_2 + 7\alpha_3 = \left(\frac{3}{2}\right) \quad \text{Eq. 5}$$

Subtract (Eq 3 - Eq 4)

$$(4\alpha_1 + 9\alpha_2 + 11\alpha_3) - (\alpha_1 + 2\alpha_2 + 2\alpha_3) = +1 - \left(-\frac{1}{2}\right)$$

$$3\alpha_1 + 7\alpha_2 + 9\alpha_3 = \left(\frac{3}{2}\right) \quad \text{Eq. 6}$$

Subtract (Eq 6 - Eq 5)

$$(3\alpha_1 + 7\alpha_2 + 9\alpha_3) - (3\alpha_1 + 9\alpha_2 + 7\alpha_3) = \left(\frac{3}{2}\right) - \left(\frac{3}{2}\right)$$

$$-2\alpha_2 + 2\alpha_3 = 0 \rightarrow \alpha_3 = \alpha_2 \quad \text{Eq. 7}$$

Substitute Eq 7 in Eq 4

$$\alpha_1 + 2\alpha_2 + 2\alpha_3 = -\frac{1}{2} \rightarrow \alpha_1 + 2\alpha_3 + 2\alpha_3 = -\frac{1}{2}$$

$$\alpha_1 = -\frac{1}{2} - 4\alpha_3 \quad \text{Eq. 8}$$

Substitute Eq 7 & Eq 8 in Eq 5

$$3\alpha_1 + 9\alpha_2 + 7\alpha_3 = \left(\frac{3}{2}\right) \rightarrow 3\left(-\frac{1}{2} - 4\alpha_3\right) + 9\alpha_3 + 7\alpha_3 = \left(\frac{3}{2}\right)$$

$$\alpha_3 = \frac{3}{4} \quad \text{Eq. 7}$$

$$\alpha_2 = \frac{3}{4} \quad \text{Eq. 8}$$

$$\alpha_1 = -\frac{7}{2} \quad \text{Eq. 8}$$

SVM Example 1

$$\alpha_1 = -\frac{7}{2} \quad \alpha_2 = \frac{3}{4} \quad \alpha_3 = \frac{3}{4}$$

Augmented Support Vectors

$$\left\{ \tilde{s}_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \tilde{s}_2 = \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}, \tilde{s}_3 = \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix} \right\}$$

How to get the hyperplane that separates the positive and negative class

$$\tilde{w} = \sum_{i=1}^n \alpha_i \tilde{s}_i$$

$$\tilde{w} = -\frac{7}{2} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + \frac{3}{4} \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} + \frac{3}{4} \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}$$

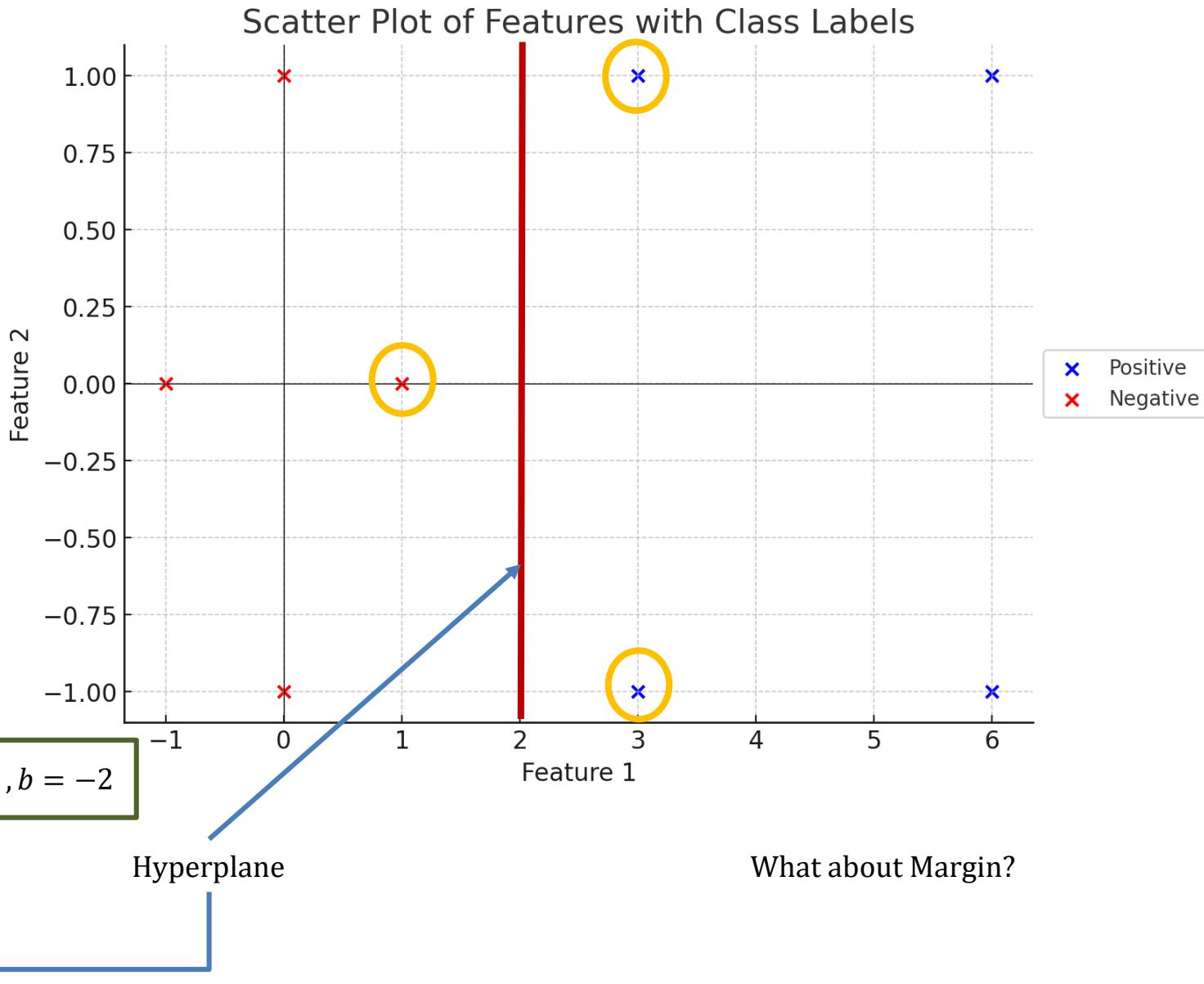
$\begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}$

- ← x coefficient
- ← Y Coefficient
- ← Bias (Intercept)

$$\beta_0 + \beta_1 x - y = 0$$

$$-2 + 1x - 0y = 0 \rightarrow x = 2$$

(Weight Vector(w) = $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $b = -2$)



SVM Example 1

$\begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}$

x coefficient
Y Coefficient
Bias (Intercept)

$$(\text{Weight Vector}(w) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, b = -2)$$

Margin Calculation

Step 1: Compute the Norm of w

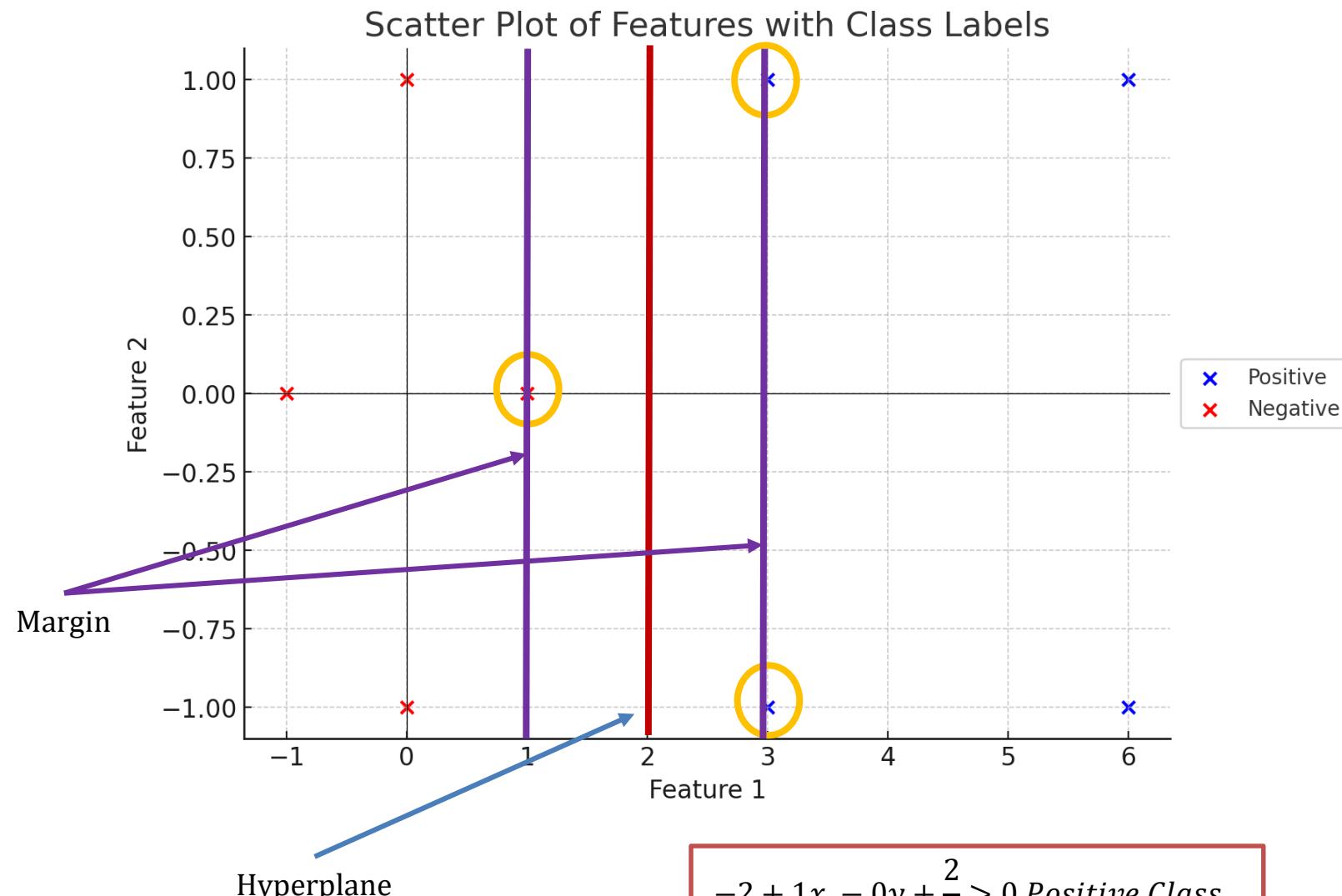
$$\|w\| = \sqrt{1^2 + 0^2} = 1$$

Step 2: Compute the Margin

$$M = \frac{2}{\|w\|}$$

$$M = \frac{2}{1} = 2$$

Hyperplane is exactly in the middle of Margin



$$-2 + 1x - 0y + \frac{2}{2} > 0 \text{ Positive Class}$$
$$-2 + 1x - 0y - \frac{2}{2} < 0 \text{ Negative Class}$$

SVM Characteristics

- The learning problem is formulated as a convex optimization problem
 - Efficient algorithms are available to find the global minima
 - Many of the other methods use greedy approaches and find locally optimal solutions
 - High computational complexity for building the model
- Robust to noise
- Overfitting is handled by maximizing the margin of the decision boundary,
- SVM can handle irrelevant and redundant attributes better than many other techniques
- The user needs to provide the type of kernel function and cost function
- Difficult to handle missing values

SVM – RBF Kernel

- Radial Basis Function(RBF) also known as **Gaussian Kernel**

$$K(x_1, x_2) = e^{(\gamma ||x_1 - x_2||^2)}$$

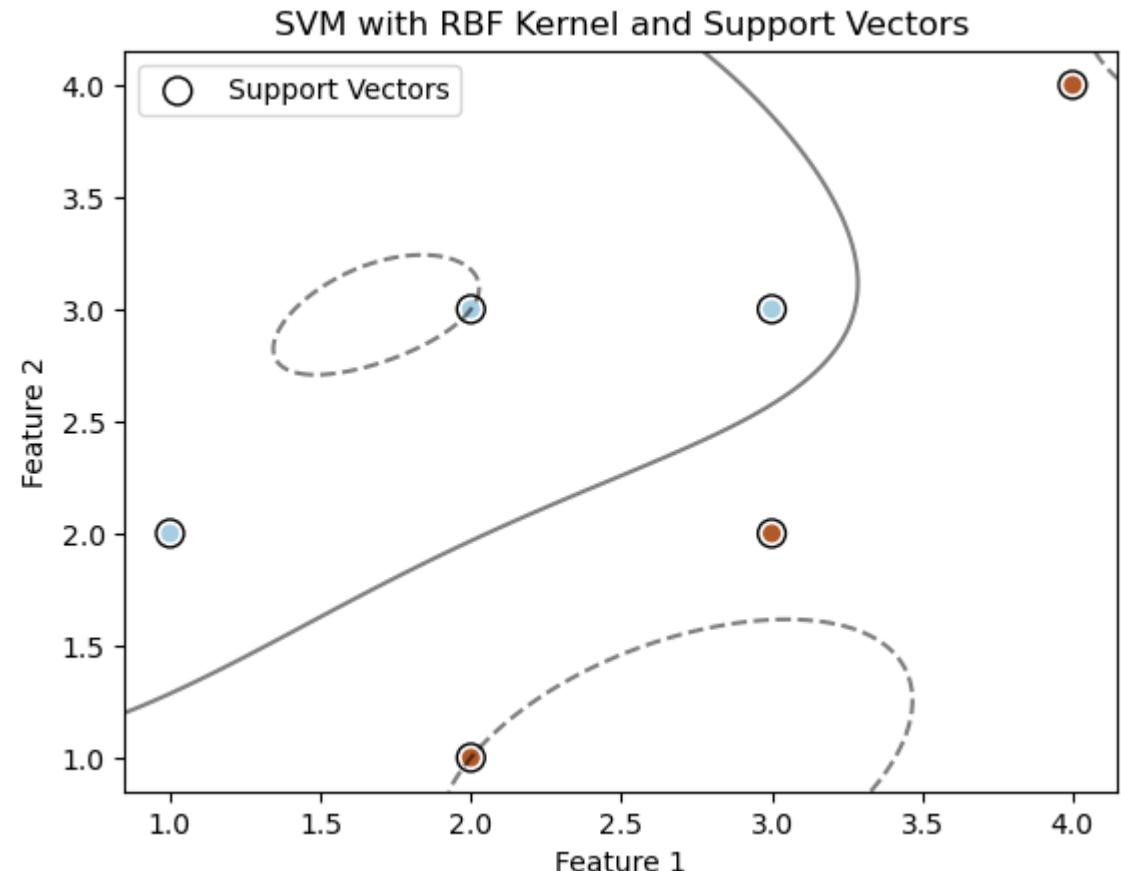
x_1, x_2 are two data points

$||x_1 - x_2||^2$ is the squared euclidean distance between x_1 & x_2

γ defines the spread of kernel

- Commonly used Kernels

- Linear
- Polynomial
- RBF
- Sigmoid
- Laplacian
- Rational Quadratic
- Anova (Analysis of Variance)



SVM Example 2(Non-Linear)

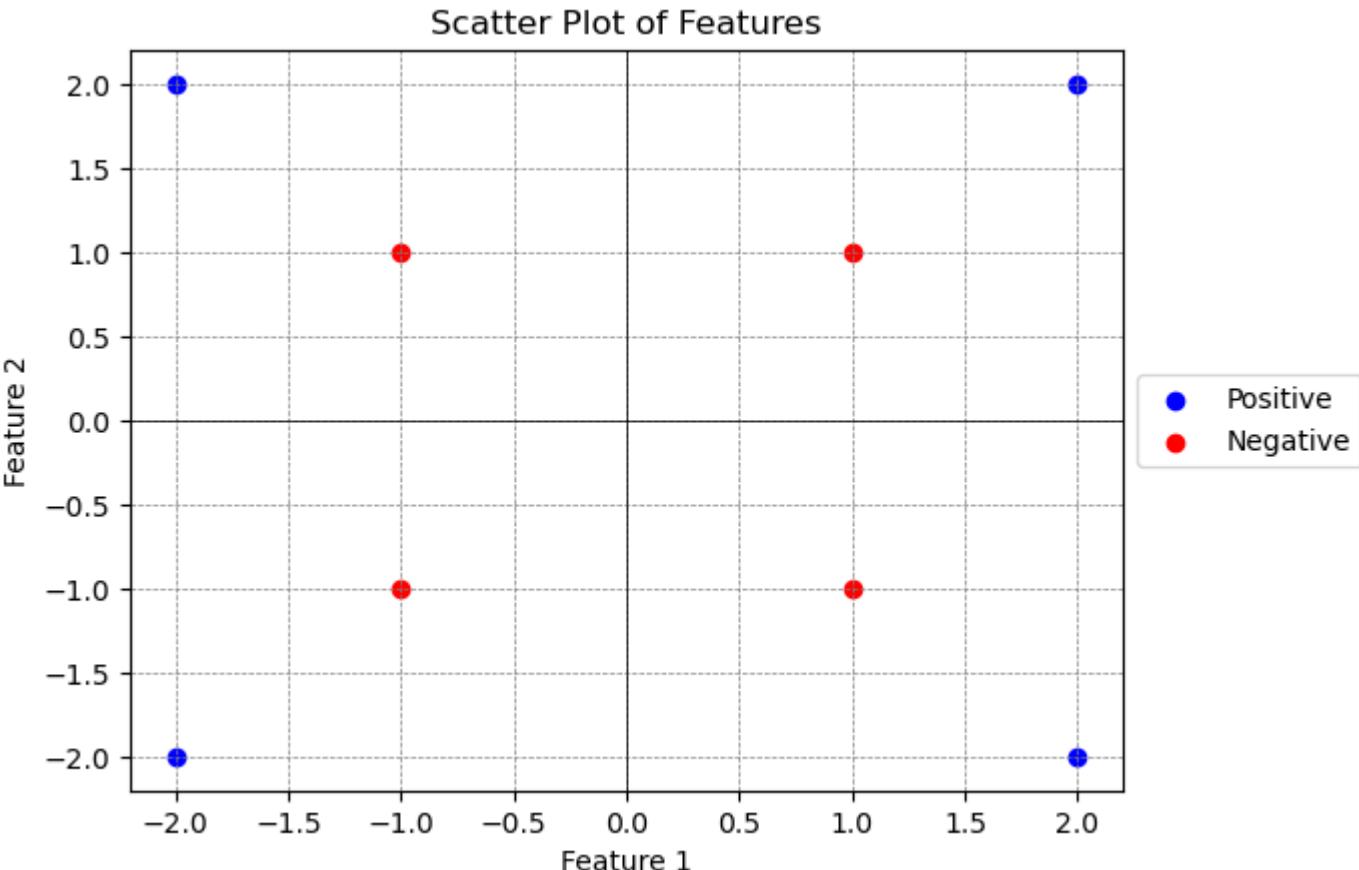
Positive Labels (+1)

$$\left\{ \begin{pmatrix} 2 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ -2 \end{pmatrix}, \begin{pmatrix} -2 \\ -2 \end{pmatrix}, \begin{pmatrix} -2 \\ 2 \end{pmatrix} \right\}$$

Negative Labels (-1)

$$\left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \begin{pmatrix} -1 \\ -1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \end{pmatrix} \right\}$$

Sl. No.	Feature 1	Feature 2	Class/Labels
1	2	2	Positive
2	2	-2	Positive
3	-2	-2	Positive
4	-2	2	Positive
5	1	1	Negative
6	1	-1	Negative
7	-1	-1	Negative
8	-1	1	Negative



SVM Example 2(Non-Linear)

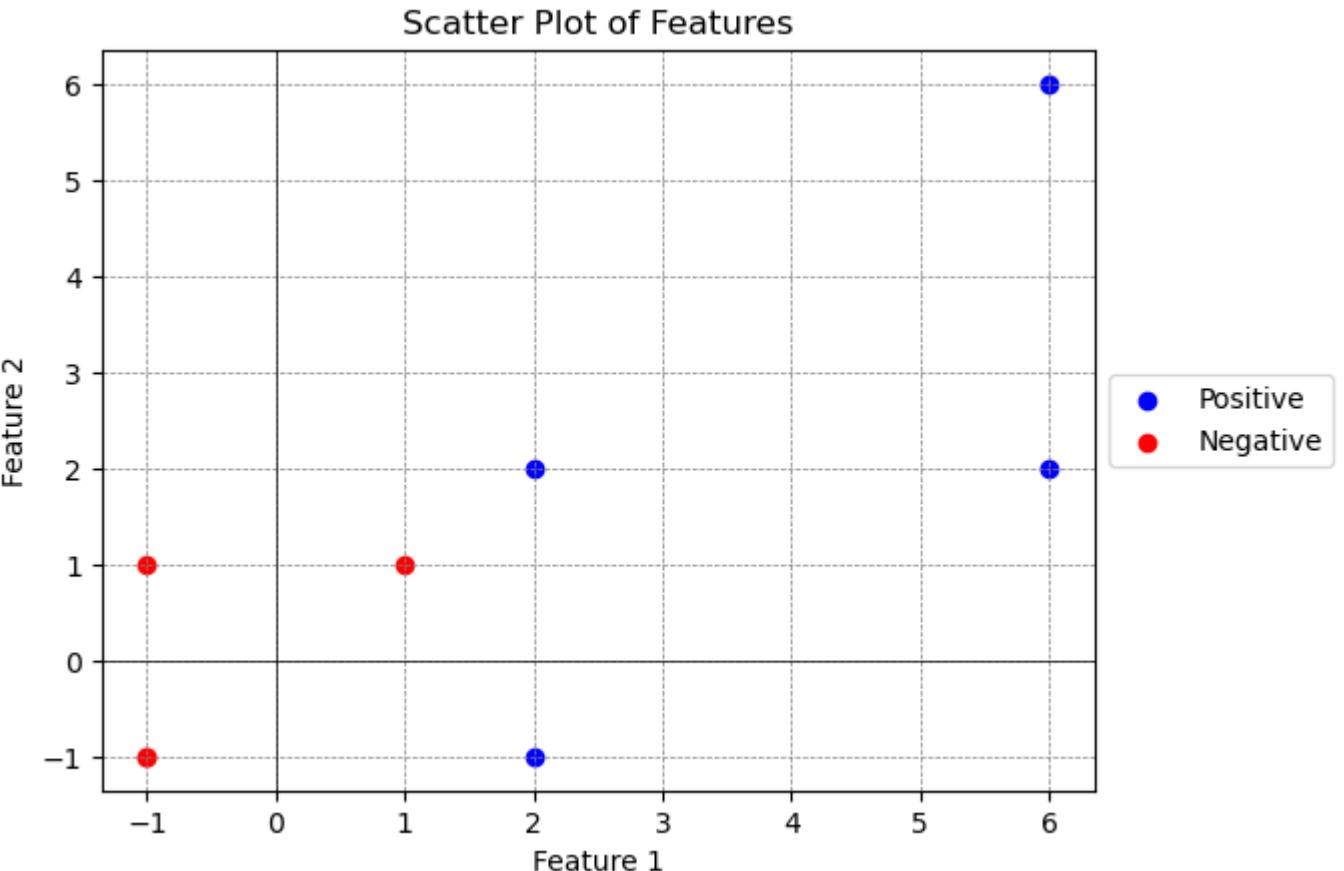
Positive Labels (+1)

$$\left\{ \begin{pmatrix} 2 \\ 2 \end{pmatrix} \begin{pmatrix} 6 \\ 2 \end{pmatrix} \begin{pmatrix} 6 \\ 6 \end{pmatrix} \begin{pmatrix} 2 \\ 6 \end{pmatrix} \right\}$$

Negative Labels (-1)

$$\left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \begin{pmatrix} -1 \\ -1 \end{pmatrix} \begin{pmatrix} -1 \\ 1 \end{pmatrix} \right\}$$

Sl. No.	Feature 1	Feature 2	Class/Labels
1	2	2	Positive
2	6	2	Positive
3	6	6	Positive
4	2	6	Positive
5	1	1	Negative
6	1	-1	Negative
7	-1	-1	Negative
8	-1	1	Negative



Thank You