INDIAN INSTITUTE OF TECHNOLOGY PATNA

CS372: Computer Graphics
Class Test – I — Spring 2025— Solution Sheet

Time: 1 Hour **Maximum Marks: 20 marks**

All questions are compulsory. Without proper derivation and explanation, marks will not be given.

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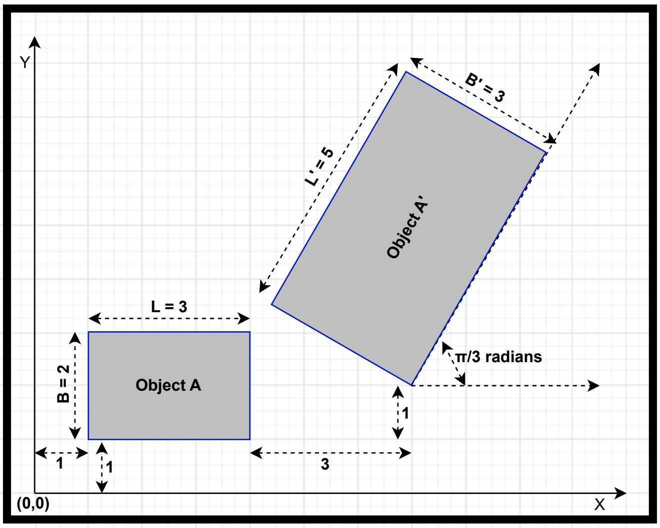
1. Consider the following display: Object A has been transformed to object A' using composite transform.



(a) Write the transformation names and transformation matrix involved, in their correct order of operation (label as Step 1,2,3,....), to transform object A to transformed object A'. **[10]**

Solution:

To transform object A into object A', the following transformations are applied:

Step 1: Translation 1: Before complex transformation, the pivotal point need to be translated to origin, i.e. a translation by vector $(-1, -1)$. The first translation matrix:

$$T_1 = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

Step 2: Scaling: Object A has dimensions $L = 3$, $B = 2$, which transform to $L' = 5$, $B' = 3$. The scaling factors are:

$$S_x = \frac{5}{3}, \quad S_y = \frac{3}{2}$$

The scaling matrix is:

$$S = \begin{bmatrix} \frac{5}{3} & 0 & 0 \\ 0 & \frac{3}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1.67 & 0 & 0 \\ 0 & 1.5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Step 3: Rotation: The object is rotated by $\theta = \frac{\pi}{3}$ radians (or 60°). The rotation matrix is:

$$R = \begin{bmatrix} \cos\frac{\pi}{3} & -\sin\frac{\pi}{3} & 0 \\ \sin\frac{\pi}{3} & \cos\frac{\pi}{3} & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.5 & -0.87 & 0 \\ 0.87 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Step 4: Translation 2: The final position of the transformed object requires a translation by vector $(7, 2)$. The translation matrix is:

$$T_2 = \begin{bmatrix} 1 & 0 & 7 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

(b) What is the final transformation matrix. **[3]**

Solution:

Final Transformation Matrix:

$$T = T_2 * R * S * T_1$$

$$T = \begin{bmatrix} 1 & 0 & 7 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 0.5 & -0.87 & 0 \\ 0.87 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1.67 & 0 & 0 \\ 0 & 1.5 & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T = \begin{bmatrix} 0.84 & -1.31 & 7.47 \\ 1.45 & 0.75 & -0.20 \\ 0 & 0 & 1 \end{bmatrix}$$

(c) Write the co-ordinate of the vertices of the transformed object A'. **[2]**

Solution:

$$[A'] = [T][A] = \begin{bmatrix} 0.84 & -1.31 & 7.47 \\ 1.45 & 0.75 & -0.20 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 1 & 4 & 4 \\ 1 & 3 & 3 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 4.38 & 6.90 & 9.52 \\ 2 & 3.50 & 7.85 & 6.35 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

2. A unit square is transformed by 2×2 transformation matrix. The resulting position vector are:

$$\begin{bmatrix} 0 & 2 & 8 & 6 \\ 0 & 3 & 4 & 1 \end{bmatrix}$$

What is the transformation matrix? **[5]**

Solution:

Let the transformation matrix be:

$$T = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

The original unit square has vertices:

$$A = (0,0), \quad B = (1,0), \quad C = (1,1), \quad D = (0,1)$$

Applying the transformation T:

$$T \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \Rightarrow \begin{cases} a = 2 \\ c = 3 \end{cases}$$

$$T \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 8 \\ 4 \end{bmatrix} \Rightarrow \begin{cases} a + b = 8 \\ c + d = 4 \end{cases}$$

$$T \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 1 \end{bmatrix} \Rightarrow \begin{cases} b = 6 \\ d = 1 \end{cases}$$

Solving the equations:

$$a + b = 8 \Rightarrow 2 + 6 = 8 \quad \text{(Correct)}$$
$$c + d = 4 \Rightarrow 3 + 1 = 4 \quad \text{(Correct)}$$

Thus, the transformation matrix is:

$$T = \begin{bmatrix} 2 & 6 \\ 3 & 1 \end{bmatrix}$$

Correct solution in any sequence would be provided full marks.

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