

Clustering

Lecture 4: Density-based Methods

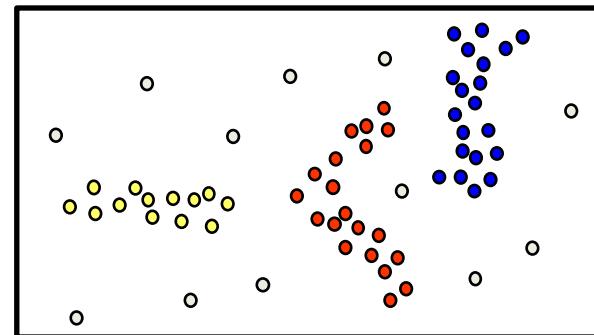
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Outline

- **Basics**
 - Motivation, definition, evaluation
- **Methods**
 - Partitional
 - Hierarchical
 - Density-based
 - Mixture model
 - Spectral methods
- **Advanced topics**
 - Clustering ensemble
 - Clustering in MapReduce
 - Semi-supervised clustering, subspace clustering, co-clustering, etc.

Density-based Clustering

- **Basic idea**
 - Clusters are dense regions in the data space, separated by regions of lower object density
 - A cluster is defined as a maximal set of density-connected points
 - Discovers clusters of arbitrary shape
- **Method**
 - DBSCAN

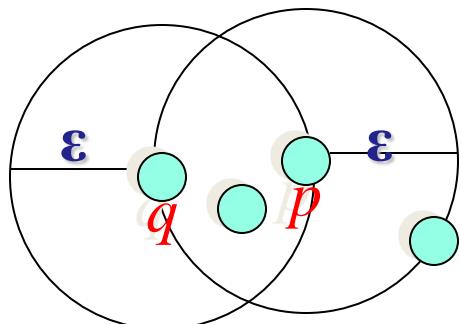


Density Definition

- ε -Neighborhood – Objects within a radius of ε from an object.

$$N_\varepsilon(p) : \{q \mid d(p, q) \leq \varepsilon\}$$

- “High density” - ε -Neighborhood of an object contains at least $MinPts$ of objects.



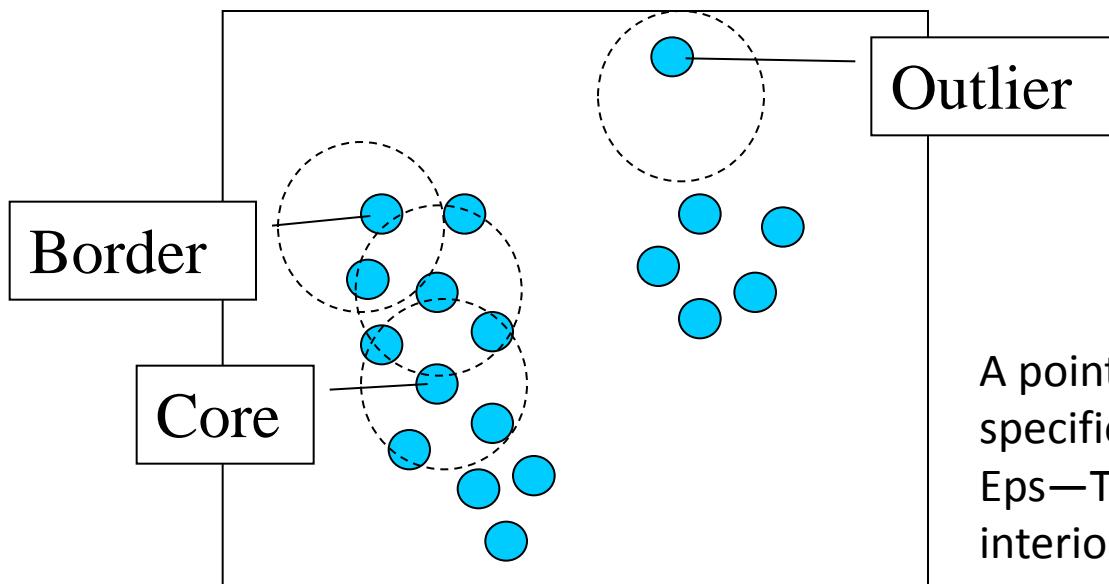
ε -Neighborhood of p

ε -Neighborhood of q

Density of p is “high” ($MinPts = 4$)

Density of q is “low” ($MinPts = 4$)

Core, Border & Outlier



Given ϵ and $MinPts$, categorize the objects into three exclusive groups.

A point is a **core point** if it has more than a specified number of points ($MinPts$) within Eps —These are points that are at the interior of a cluster.

$$\epsilon = 1\text{unit}, MinPts = 5$$

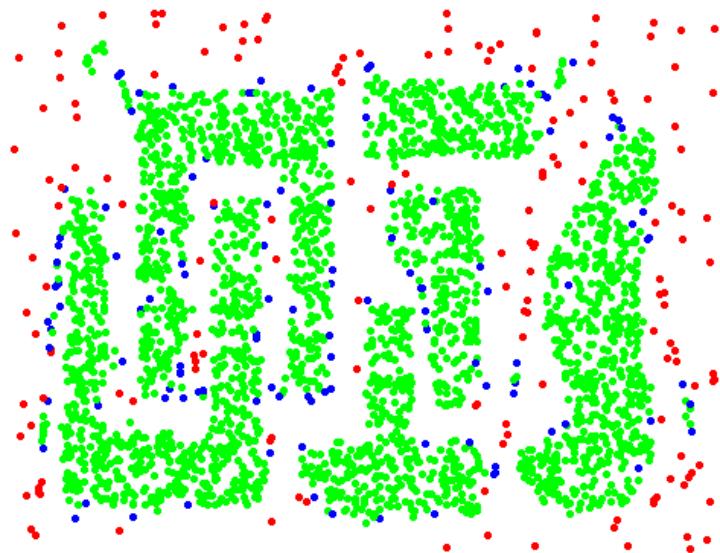
A **border point** has fewer than $MinPts$ within Eps , but is in the neighborhood of a core point.

A **noise point** is any point that is not a core point nor a border point.

Example



Original Points

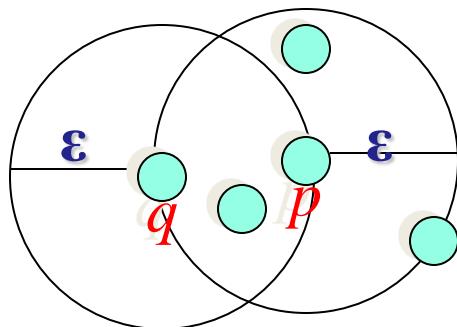


Point types: **core**,
border and **outliers**

$\varepsilon = 10$, MinPts = 4

Density-reachability

- Directly density-reachable
 - An object q is directly density-reachable from object p if p is a core object and q is in p 's ε -neighborhood.

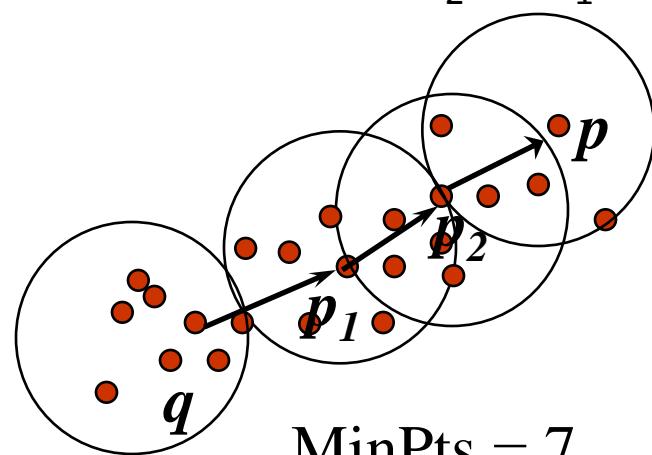


- q is directly density-reachable from p
- p is not directly density-reachable from q
- Density-reachability is asymmetric

MinPts = 4

Density-reachability

- Density-Reachable (directly and indirectly):
 - A point p is directly density-reachable from p_2
 - p_2 is directly density-reachable from p_1
 - p_1 is directly density-reachable from q
 - $p \leftarrow p_2 \leftarrow p_1 \leftarrow q$ form a chain

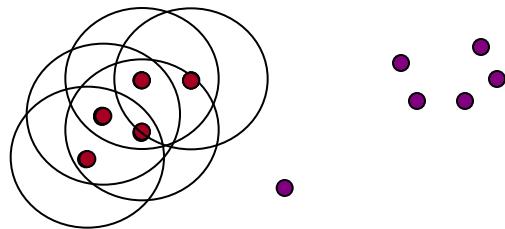


- p is (indirectly) density-reachable from q
- q is not density-reachable from p

DBSCAN Algorithm: Example

- **Parameter**

- $\varepsilon = 2 \text{ cm}$
- $MinPts = 3$

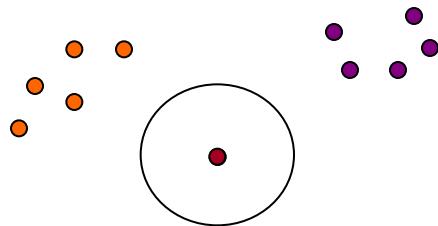


```
for each  $o \in D$  do
    if  $o$  is not yet classified then
        if  $o$  is a core-object then
            collect all objects density-reachable from  $o$ 
            and assign them to a new cluster.
        else
            assign  $o$  to NOISE
```

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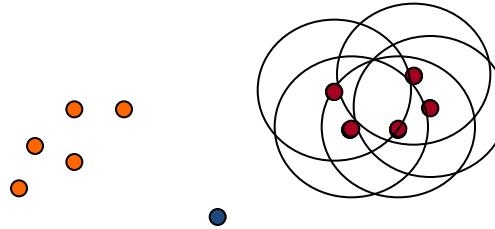


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DBSCAN: Sensitive to Parameters

Figure 8. DBScan results for DS1 with MinPts at 4 and Eps at (a) 0.5 and (b) 0.4.

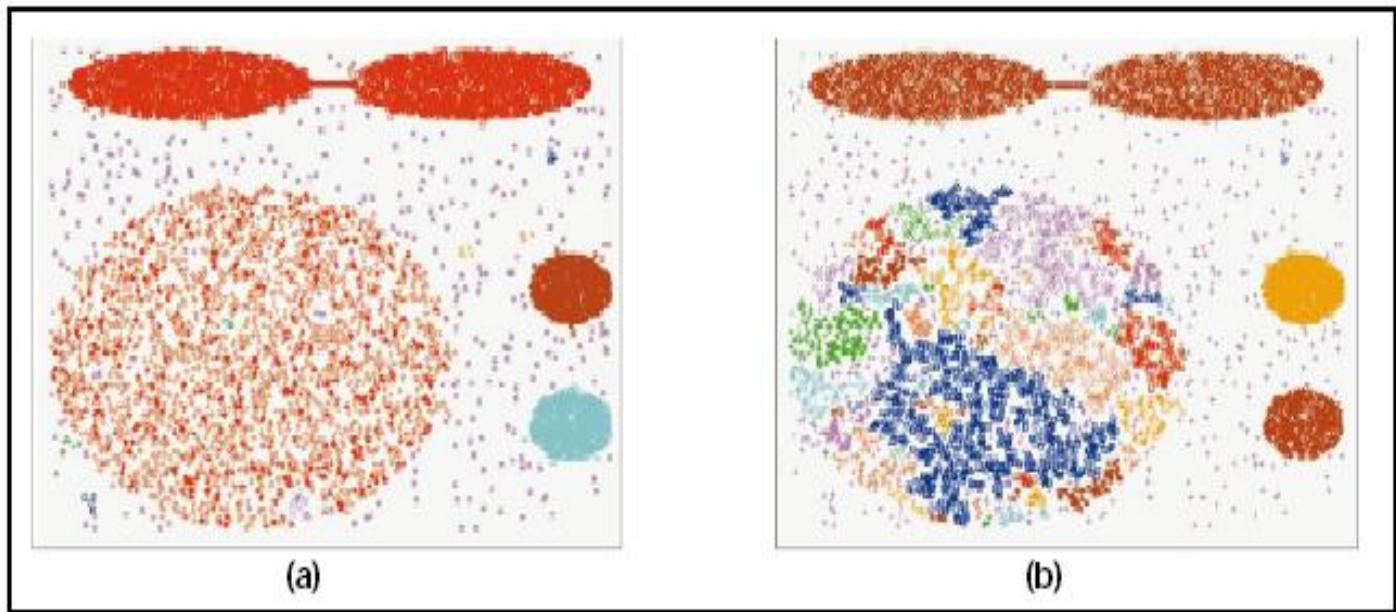
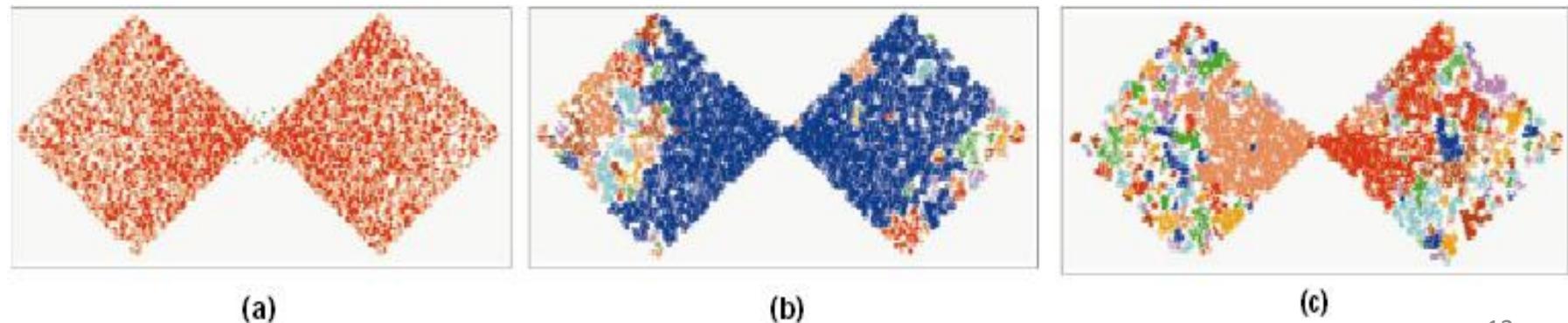
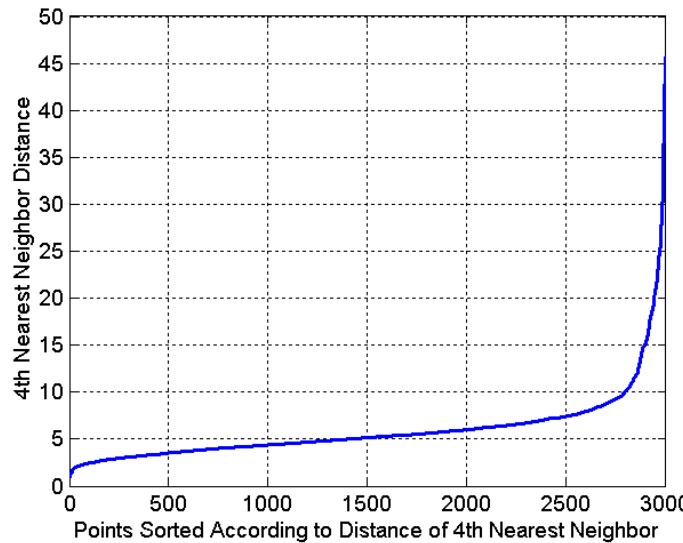


Figure 9. DBScan results for DS2 with MinPts at 4 and Eps at (a) 5.0, (b) 3.5, and (c) 3.0.



DBSCAN: Determining EPS and MinPts

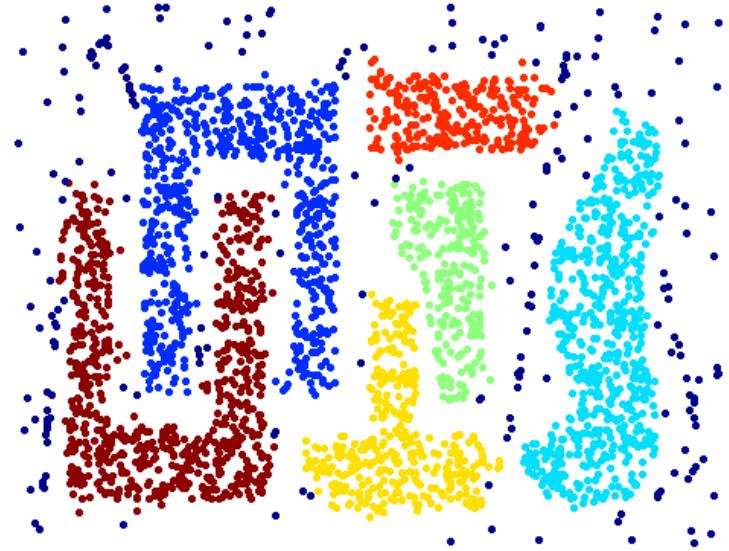
- Idea is that for points in a cluster, their k^{th} nearest neighbors are at roughly the same distance
- Noise points have the k^{th} nearest neighbor at farther distance
- So, plot sorted distance of every point to its k^{th} nearest neighbor



When DBSCAN Works Well



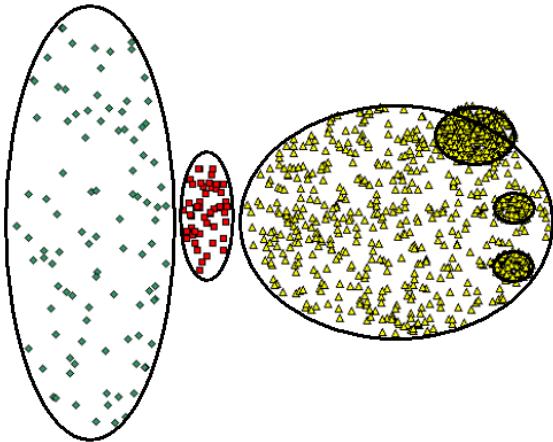
Original Points



Clusters

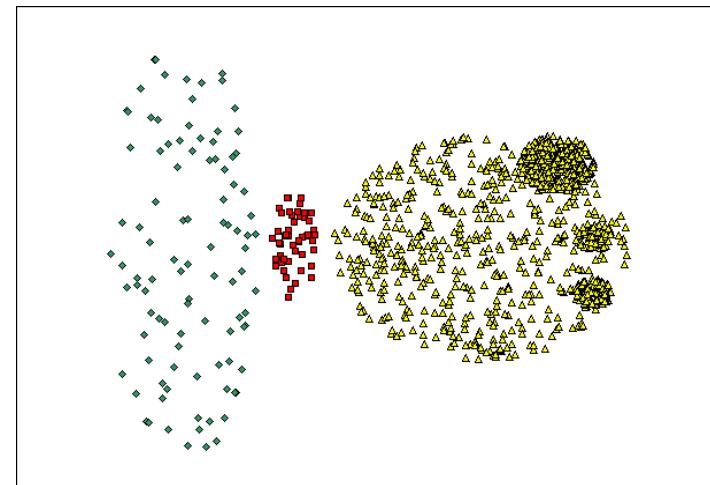
- Resistant to Noise
- Can handle clusters of different shapes and sizes

When DBSCAN Does NOT Work Well

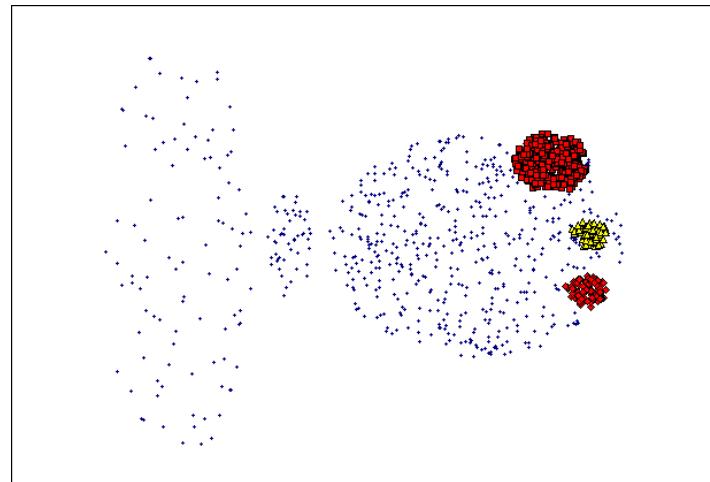


Original Points

- Cannot handle varying densities
- sensitive to parameters—hard to determine the correct set of parameters



($\text{MinPts}=4$, $\text{Eps}=9.92$).



($\text{MinPts}=4$, $\text{Eps}=9.75$)

Take-away Message

- The basic idea of density-based clustering
- The two important parameters and the definitions of neighborhood and density in DBSCAN
- Core, border and outlier points
- DBSCAN algorithm
- DBSCAN's pros and cons