Indian Institute of Technology Patna
Department of Mathematics
MA225: Probability and Statistics
B.Tech. 2nd year

Tutorial Sheet-7

1. In a normal distribution 31% of the items lie below 45 and 8% are over 64. Find the mean and standard deviation of the distribution.

2. If the RV X is distributed as $N(\mu, \sigma^2)$, identify the constant c, in terms of μ and σ, for which: $P(X < c) = 2 - 9P(X > c)$.

3. Suppose the temperature T during June is normally distributed with mean 68° and standard deviation 6°. Find the probability p that the temperature is between 70° and 80°.

4. The lifetime X of a radio has an exponential distribution with mean equal to ten years. What is the probability that a ten year old radio will still work after ten additional years?

5. Suppose that the length X (in meters) of an arbitrary parking place follows a $N(\mu, 0.01\mu^2)$ distribution. (i) A man owns a car whose length is 15% greater than the average length of a parking place. What proportion of free parking places can he uses? (ii) Suppose that $\mu = 4$. What should be the length of a car if we want its owner to be able to use 90% of the free parking places?

6. A company pays its employees an average wage of \$15.9 an hour with a standard deviation of \$1.5. If the wages are normally distributed and paid to the nearest cent, (i) what percentage of the workers receive wages between \$13.75 and \$16.22 an hour inclusive, (ii) the highest 5% of the employee hourly wages is greater than what amount.

7. For any RV X with expectation μ and variance σ^2, use Chebyshev inequality to determine a lower bound for the probabilities: $P(|X - \mu| < k\sigma)$, for $k = 1, 2, 3$. Compare these bounds with the respective probabilities when $X \sim N(\mu, \sigma^2)$.

8. A random variable X has mean 10 and variance 4 and an unknown probability distribution. Find the value of C such that $P(|X - 10| \geq c)| \leq 0.04$.

9. The length of time for one individual to be served at a cafeteria is an exponential random variable with a mean of 4 minutes. What is the probability that a person is served in less than 3 minutes on at least 4 of the next 6 days?

10. Let $X \sim Exp(\lambda)$. Using the Markov's inequality find an upper bound for $P(X \geq a)$ where $a > 0$. Compare the upper bound with the actual value of $P(X \geq a)$.

11. Suppose that the average grade on the upcoming Statistics exam is 70%. Give an upper bound on the proportion of Students who score at least 90%.

12. Let us flip a fair coin n-times. Let X_i be the indicator random variable for the event that the ith coin flip is head. Find the probability to obtain 80% or more heads in such a sequence of coin flips?

13. A perfect coin is tossed twice. Find the MGF of the number of heads. Also, find mean and variance.

14. The MGF of a random variable X is given by $M_X(t) = e^{3(e^t - 1)}$. Find $P(X = 1)$.

15. Find the MGF of Exp(λ) and hence calculate mean and variance using MGF.

16. Let X is a discrete random variable with PMF $P_X(k) = 0.2,\ k = 0;\ = 0.2,\ k = 1;\ = 0.3,\ k = 2;\ = 0.3,\ k = 3;\ = 0,\ otherwise.$ Define $Y = X(X - 1)(X - 2)$. Find the MGF of Y.