

CS349

Artificial

Intelligence - 2

UNCERTAINTY:

→ Quantify Uncertainty → for every event in a series of events we need to quantify.

(Partial Observation)

→ Agent may need to handle whether due to P.O., non-determinism,

or combining these the agent may never know for certain what state it is in or where it will end up after a sequence of actions

→ Problem Solving Agents } AI-1
 → LOGICAL Agents } ↓

designed to handle uncertainty by keeping track of a BELIEF STATE (A representation of all possible world states)

HANDLING UNCERTAIN KNOWLEDGE:

→ ~~H P~~ Symptom (p, toothache)

Problem → Disease (p, cavity)

not correct : T.A can be caused by many other causes
 ↴
 toothache

→ ~~H P~~ Symptom (p, toothache) → D(p, cavity) ∨ D(p, gum disease) ∨ D(p, wisdom teeth)
 ↴
 Disease ∨ --- . . .

→ ~~H P~~ Disease (p, cavity) → Symptom (p, Toothache)

not correct : all cavities don't cause tooth ache

REASONS TO USE PROBABILITIES:

- ① specifications come too large: It is too much work to list the complete set of antecedents or consequents needed to assume an exceptionless rule. (Laziness)
- ② THEORETICAL IGNORANCE : Complete set of antecedents are not known
- ③ Practical Ignorance

$$P(A|B) = \frac{P(B|A) P(A)}{P(B)}$$

→ Probability that x is fat ≈ 0.2

if x is fat then x has coronary heart disease (CHD) ≈ 0.7
with prob

$$P(x \text{ has CHD}) \approx (0.2 \times 0.7) + \text{other terms}$$

↑ Data ↑ → Knowledge ↑ → certainty ↑

AXIOMS OF PROBABILITY:

1) All probability btw. 0 and 1.

$$0 \leq P(A) \leq 1$$

2) $P(\text{True}) = 1$, $P(\text{False}) = 0$

$$3) P(A \vee B) = P(A) + P(B) - P(A \wedge B)$$

$$4) \text{ Bayes : } P(B|A) = \frac{P(A|B) P(B)}{P(A)} \quad [P(A) \neq 0]$$

$P(A \wedge B)$

$$P(A \wedge B) = P(B|A) P(A)$$

$$P(A \wedge B) = P(A|B) P(B)$$

→ we use probability to quantify the uncertainty.

→ Decision making function agent, returns all actions

Persistent : belief state (Prob. beliefs about the current state of)
the world

update belief state based on Action and percept

calculate outcome probability for Actions

↳ given action description and belief state

Select action with highest expected Utility

given prob. of outcome and utility info

Return Action

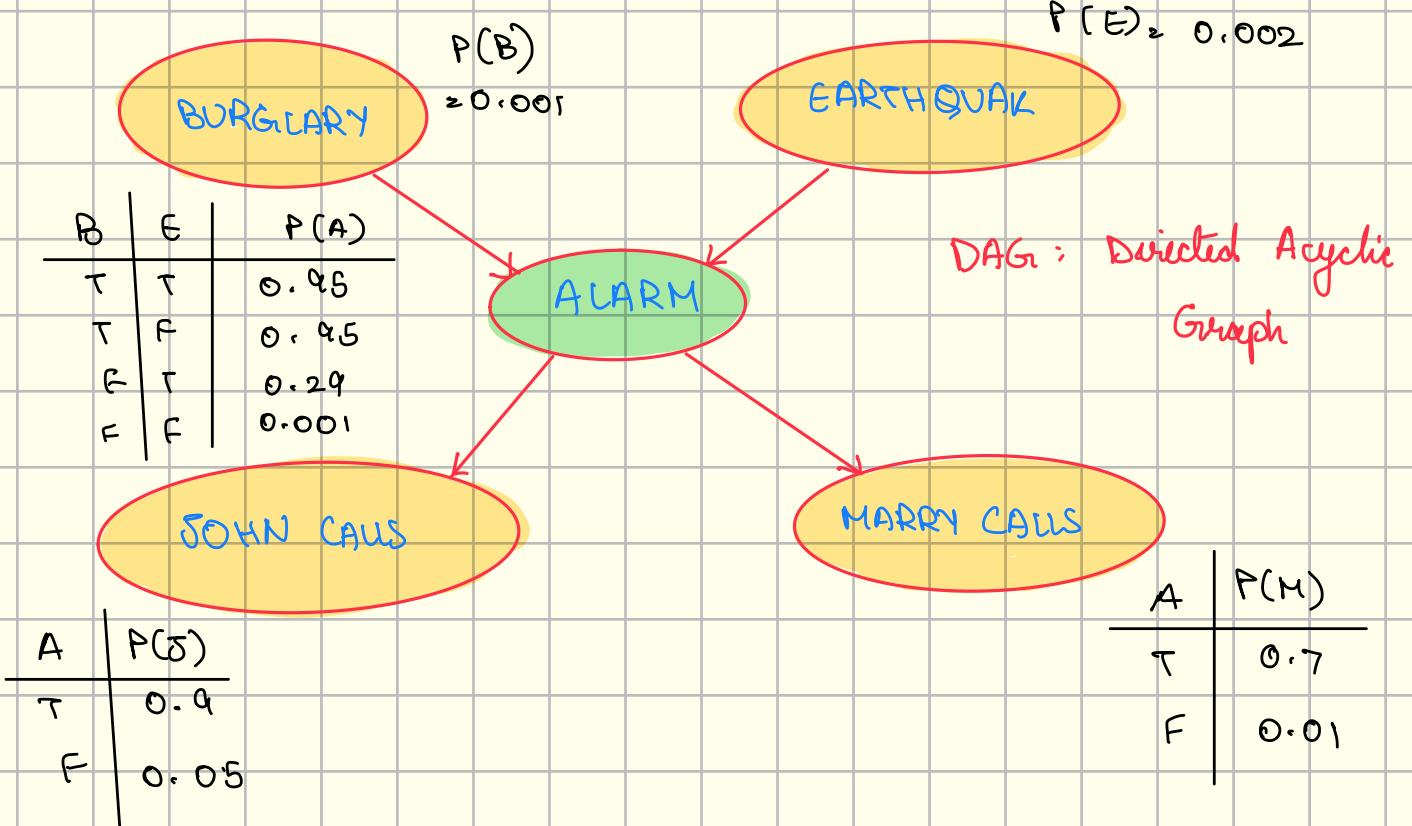
→ Decision Theory = Prob. theory + Utility Theory

→ The fundamental idea of DT is that an Agent is Rational iff

it chooses the actions that yields the highest expected utility, averaged over all possible outcomes of Action

↳ Maximum Expected Utility (MEU)

BELIEF NETWORK EX. :



→ A Belief Network is a graph with following:

- ① Nodes: set of Random variables
- ② Directed lines: The Intuitive meaning of an edge from node X to node Y is that X has direct influence on node Y
- ③ Each node has a conditional Probability table that quantifies the effect that the Parent have on the node.

(ii) DAG (no Cycles)

→ BUGLAR ALARM at home:

- ↳ fairly reliable on detecting BUGLAR
- ↳ Responds at times to minor EO

→ Two neighbours on hearing ALARM calls police

- ↳ John always calls when he hears the alarm
- ↳ But sometimes confuses the telephone ringing with the ALARM & calls them too.
- ↳ MARRY likes LOUD MUSIC and sometimes misses the ALARM

The Joint Prob. dist.:

→ a generic entry in the Joint Prob. distribution is given by

$$\hookrightarrow P(x_1, x_2, \dots, x_n)$$

$$P(x_1, \dots, x_n) = \prod_{i=1}^n P(x_i | \text{Parents}(x_i))$$

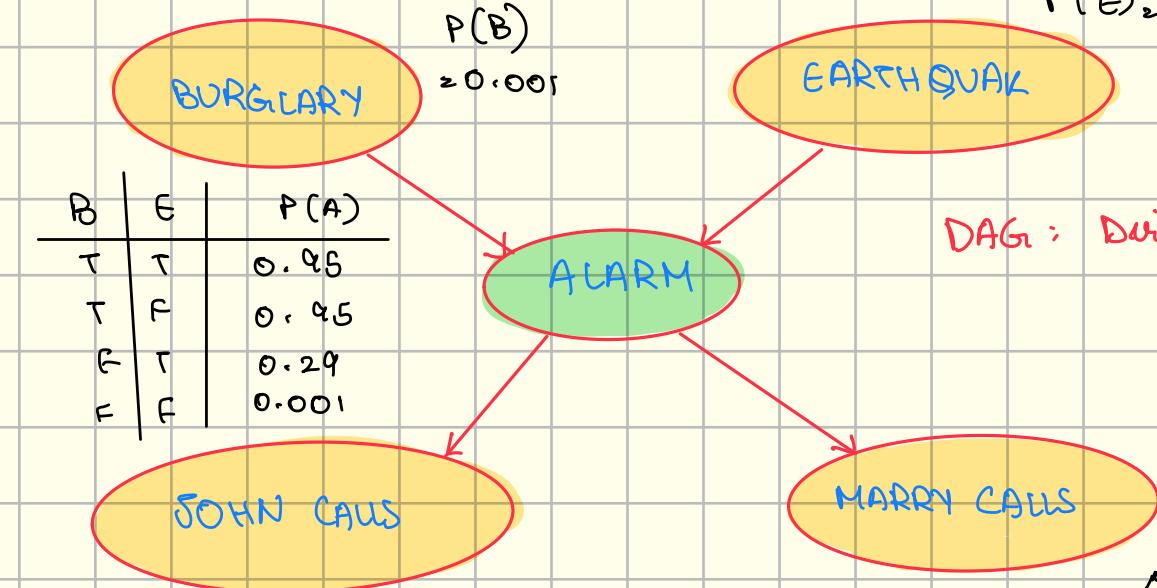
→ Prob. of the event that the ALARM has sounded but neither a Buglar nor a EO has occurred, but both John & Marry call.

$$\begin{aligned} \Rightarrow P(\bar{J} \wedge \bar{M} \wedge A \wedge \bar{B} \wedge \bar{E}) &= P(\bar{J}|A) P(\bar{M}|A) P(A|\bar{B} \wedge \bar{E}) P(\bar{B}) P(\bar{E}) \\ &\geq 0.9 \times 0.7 \times 0.001 \times 0.999 \times 0.998 \end{aligned}$$

Joint Prob. Distribution :

→ A generic entry in the joint prob. dist. is given by,

$$P(x_1, x_2, \dots, x_n) = \prod_{i=1}^n P(x_i | \text{Parents}(x_i))$$



DAG : Directed Acyclic Graph

A	$P(M)$
T	0.7
F	0.01

$$\begin{aligned} P(J \wedge M \wedge \bar{B} \wedge \bar{E}) &= P(J|A) P(M|A) P(A|\bar{B} \wedge \bar{E}) P(\bar{B}) P(\bar{E}) \\ &\approx 0.00062 \end{aligned}$$

$$\begin{aligned} P(A) &= P(AB'G') + P(AB'E') + P(AB'E) + P(ABG) \\ &= P(A|B'G') P(B'G') + P(A|B'E') P(B'E') + P(A|BE) P(BE) + P(A|BG) P(BG) \\ &= 0.001 \times 0.999 \times 0.998 + 0.29 \times 0.999 \times 0.002 + \\ &\quad 0.95 \times 0.001 \times 0.998 + 0.95 \times 0.001 \times 0.002 \\ &\approx 0.0025 \end{aligned}$$

$$P(\bar{S}) = P(SA) + P(SA') = P(S|A)P(A) + P(S|A')P(A)$$

$$= 0.9 \times 0.0025 + 0.05 \times (1 - 0.0025)$$

$$= 0.052125$$

$$P(AB) = P(ABE) + P(ABE') = P(A|BE)P(BE) + P(A|BE')P(BE')$$

$$= 0.95 \times 0.001 \times 0.002 + 0.95 \times 0.001 \times 0.998$$

$$= 0.00095$$

$$P(A'B) = 0.00005$$

$$P(A'E') = 0.996$$

$$P(AE) = 0.00058$$

$$P(SB) = 0.00086$$

$$P(AE') = 0.001945$$

$$P(B) = 0.001$$

$$P(S|B) = \frac{P(SB)}{P(B)} = \frac{0.00086}{0.001} =$$

$$P(MB) = 0.00067$$

$$P(M|B) = \frac{P(MB)}{P(B)}$$

$$P(MB) = 0.00067$$

$$P(M|B) = 0.67$$

$$P(B|S) = 0.016$$

$$P(B|A) = 0.3$$

$$P(B|AE) = P(ABE) / P(AE) = \frac{P(A|BE)P(BE)}{P(AE)} = \frac{0.95 \times 0.001 \times 0.002}{0.00058}$$

$$P(\delta E) = P(A \delta E) + P(A' \delta E)$$

$$P(A' \delta E)$$

$$P(B \delta E)$$

$$P(B | \delta E) = \frac{P(B \delta E)}{P(\delta E)}$$

INFERENCE USING BELIEF NETWORK:

→ DIAGNOSTIC INFERENCES (from effect to cause)

given that John calls, infer that

$$P(\text{Burglary} | \text{John calls}) = 0.016$$

→ CAUSAL INFERENCE

given Burglary, infer that

$$P(\text{John calls} | \text{Burglary}) = 0.86$$

$$P(\text{Mary calls} | \text{Burglary}) = 0.67$$

→ INTER-CAUSAL INFERENCES (btw. causes a common effect)

Given A, we have $P(B | A) = 0.376$

if we add evidence that E is true, then $P(B | A \wedge E) = 0.003$

→ MIXED INFERENCE:

Setting that John calls to TRUE and the cause G.Q. to false.

$$P(A \cap \bar{E}) = 0.003$$

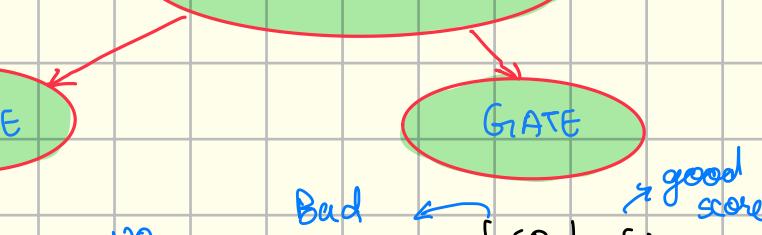
Not intelligent

I0	I1
0.7	0.3

do	di
0.6	0.4
↓	
not difficult paper	difficult paper



	g1	g2	g3
i0, do	0.3	0.4	0.3
i0, di	0.5	0.25	0.7
i1, do	0.9	0.08	0.02
i1, di	0.5	0.3	0.2



	SO	SI
i0	0.95	0.05
i1	0.2	0.8

	l0	l1
g1	0.1	0.9
g2	0.4	0.6
g3	0.99	0.01

- Q1) what is the prob. of getting excellent recommendation letter given the question very easy.
- Q2) what is the Prob. of the student being very intelligent gets a poor Recommendation letter?
- Q3) what is the Probability of getting poor grade score given grade good?

Conditional Independence:

$$P(x_1, \dots, x_n) = P(x_n | x_{n-1}, \dots, x_1) P(x_{n-1}, \dots, x_1)$$

$$= P(x_n | x_{n-1}, \dots, x_1) P(x_{n-1} | x_{n-2}, \dots, x_1) \dots P(x_2 | x_1) P(x_1)$$

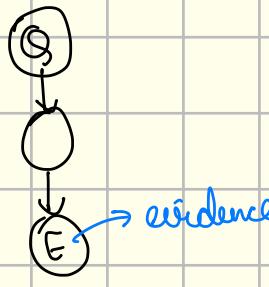
$$= \prod_{i=1}^n P(x_i | x_{i-1}, \dots, x_1)$$

→ The belief Network represents conditional Prob.

$$P(x_i | x_{i-1}, \dots, x_1) = P(x_i | \text{Parents}(x_i))$$

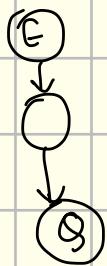
Graph Patterns

Diagnostic

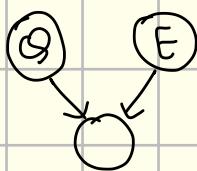


$$P(Q|E)$$

Causal



Inter Causal



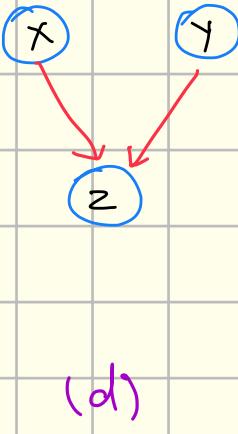
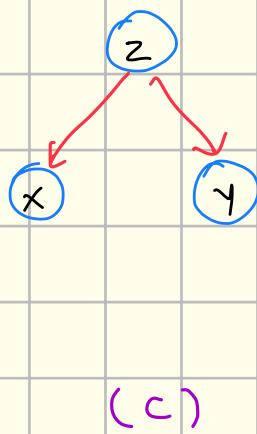
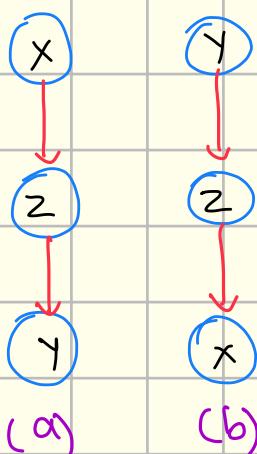
Mixed



Incremental Network Construction:

- 1) Choose the set of relevant x_i that describes the domain
- 2) Choose an ORDERING for the variables
- 3) while there are variables left
 - a) Pick variable x and add a node for it
 - b) Set $\text{Parent}(x)$ to some minimal set of existing nodes such that conditional independence property is satisfied
 - c) Define cond" Prob. tables for x .

Conditional Independence Relations

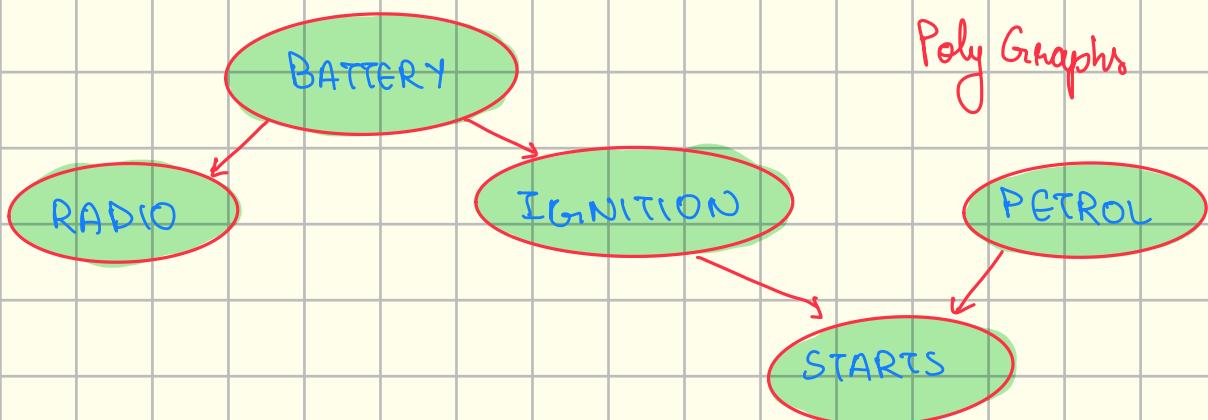


→ A path is blocked given a set of nodes E if there is a node z on the path for which one of 3 conditions holds.

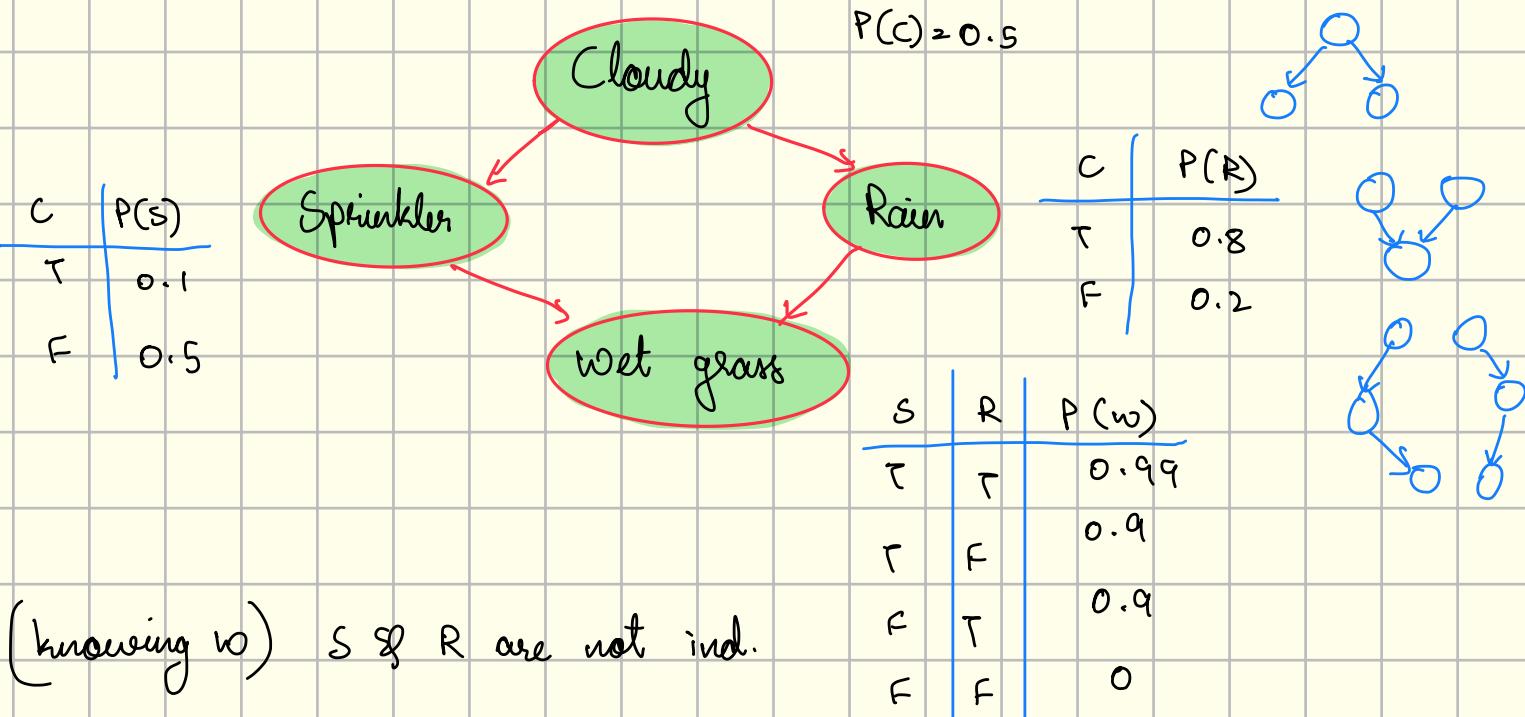
- 1) z is in E and z has one arrow on the path leading in and 1 arrow out (a,b) $x \not\perp\!\!\!\perp y$ are independent
given z , $x \not\perp\!\!\!\perp y$ are independent
- 2) z is in E and z has both path arrows leading out (c)
- 3) Neither z nor any decendent of z is in E , and both path arrows lead to z (d) if z and its successors are not given, $x \not\perp\!\!\!\perp y$ are independent

→ if every undirected path from a node in X to a node in Y is desperation by a given set of evidence nodes E then $x \not\perp\!\!\!\perp y$ are conditionally independent given E

→ A set of nodes E desperation two sets of nodes $X \times Y$
if every undirected path from a node in X to a node
in Y is blocked given E .

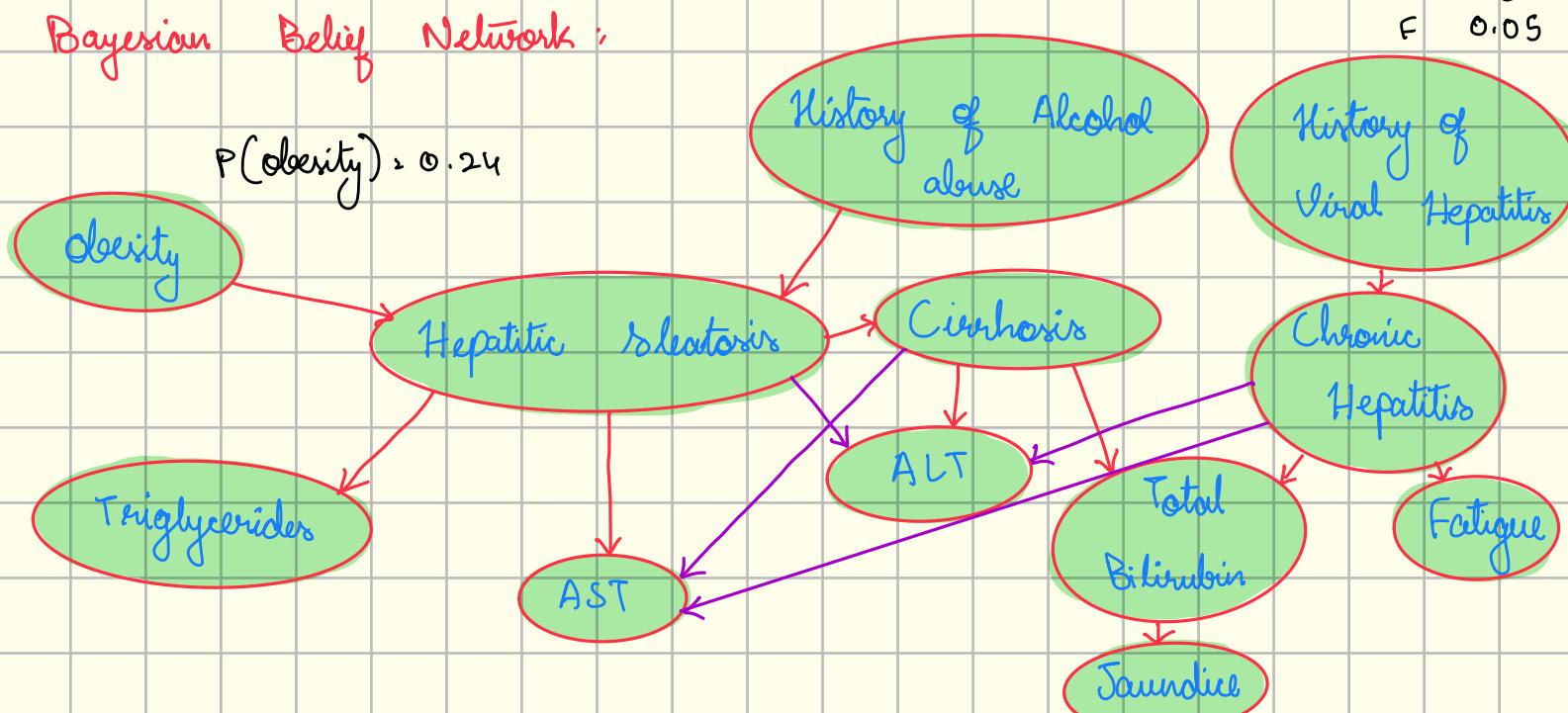


- (1) → whether there is petrol and whether the radio plays are independent given evidence about whether ignition takes place
- (2) → R and P are independent if B works
it is known
a $\xrightarrow{\text{evidence}}$
- (3) → Petrol and R are independent given no evidence at all
- (4) → P and R are dependent given Evidence whether car starts
- (5) → (If car does not start, then R playing is \uparrow evidence
that out of petrol.)
- Inference is multiply connected Belief Network



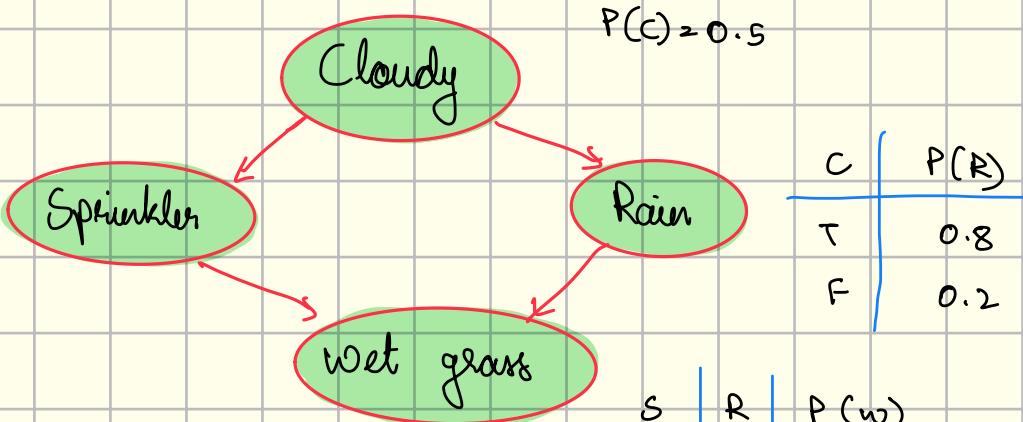
→ (knowing w) S & R are not ind.

→ (knowing c) S & R are ind.



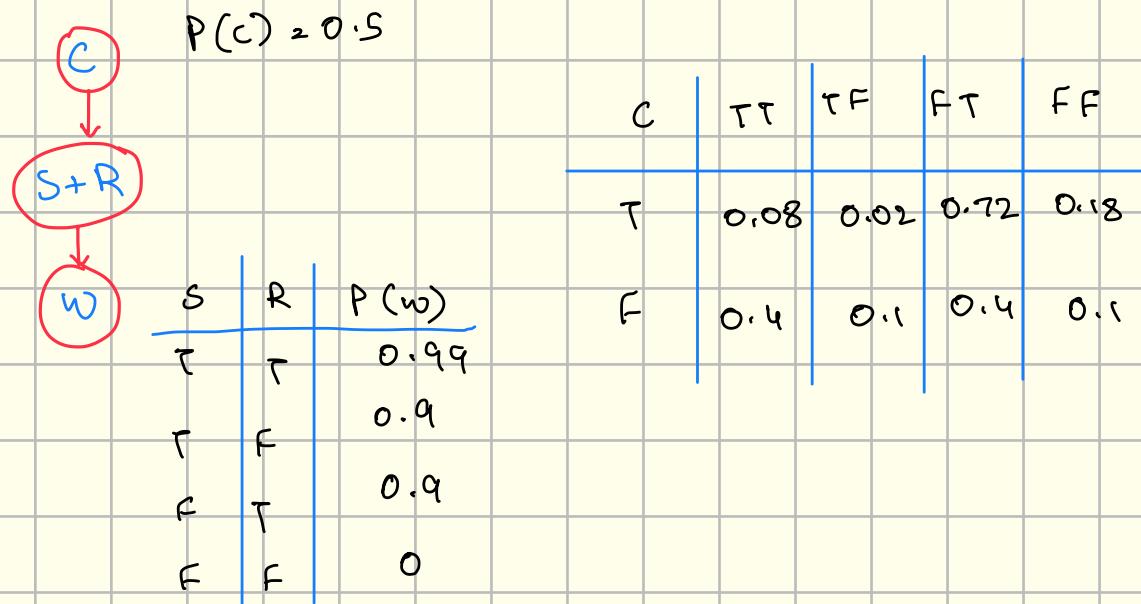
AST : Aspartate Amino Transference

ALT : Alanine n "



Clustering Method :

→ transform the network into a probabilistically equivalent (but topologically different) Poly tree by merging ordering nodes



Cut Set Conditioning Method:



- A set of variables that can be instantiated to yield a poly-tree is called cutset
- Instantiated the cutset variable to define values then evaluate a polytree for each possible instantiation.

Inference in Multiple connected Belief Network

- Stochastic Simulation Method
- Use the NW to generate a large no. of concrete models of the domain that are consistent with the Network distribution
- They give an approx. of exact evaluation

Simpson's Paradox :

Male	Recovered	Not Recover	Recovery Rate
Given drug	18	12	60 %
Not given drug	7	13	35 %
Female	Recovered	Not Recover	Recovery Rate
Given drug	2	8	20 %
Not given drug	9	21	30 %

Combined	Recovery	Not Recover	Recovery Rate
given drug	20	20	50%
Not given drug	16	24	40%

→ Should the drug be administered or not?

$$P(\text{Rec} | M \wedge D) = 0.6$$

$$P(R | F \wedge D) = 0.2$$

$$\begin{aligned} P(R | \text{given drug}) &= P(R | M \wedge D) P(D|M) + P(R | F \wedge D) P(F|D) \\ &= 0.6 \times \frac{30}{40} + 0.2 \times \frac{10}{40} = 0.5 \end{aligned}$$

Dempster - Shafer Theory

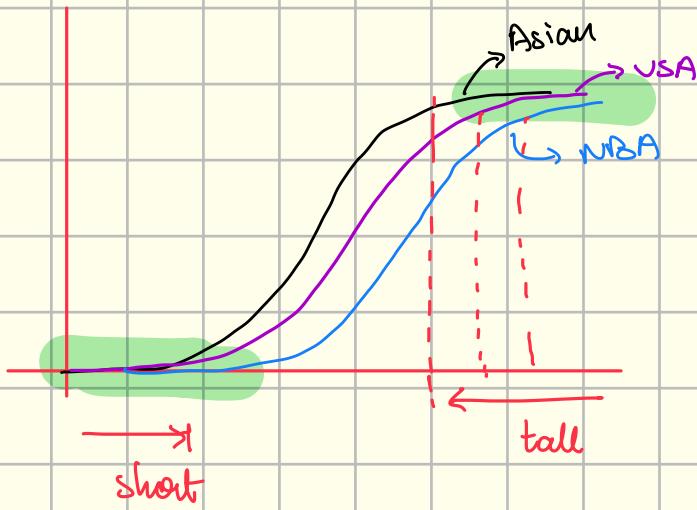
- Designed to deal with the distribution b/w UNCERTAINTY & IGNORANCE
- we use a belief function $B(x)$: Probability that the evidence supports the proposition
- when we don't have any evidence about x we assign $B(x) = 0$ as well as $B(\bar{x}) = 0$
- if we are given that the coin is fair with 90% certainty then

$$B(\text{head}) = 0.9 \times 0.5 = 0.45$$

$$B(\bar{\text{head}}) = 0.9 \times 0.5 = 0.45$$
- we will have a gap of 0.1 that is not accounted by the given evidence

FUZZY LOGIC :

→ Paradigm Shift from Probability to Possibility



→ Probability → Possibility
 of an event (degree of truth of an event)

either fat or
 non-fat
 (Boolean)

If is not Boolean, it is a set
 {very fat, medium fat, ...}

if very fat more chances of cardiac problem
 " less " less " " " "
 " NO " NO " " " "

(Possibility of cardiac problem depends on how
 fat she is -)

→ $\tau(A \wedge B) = \min(\tau(A), \tau(B))$. A & B are 2 events

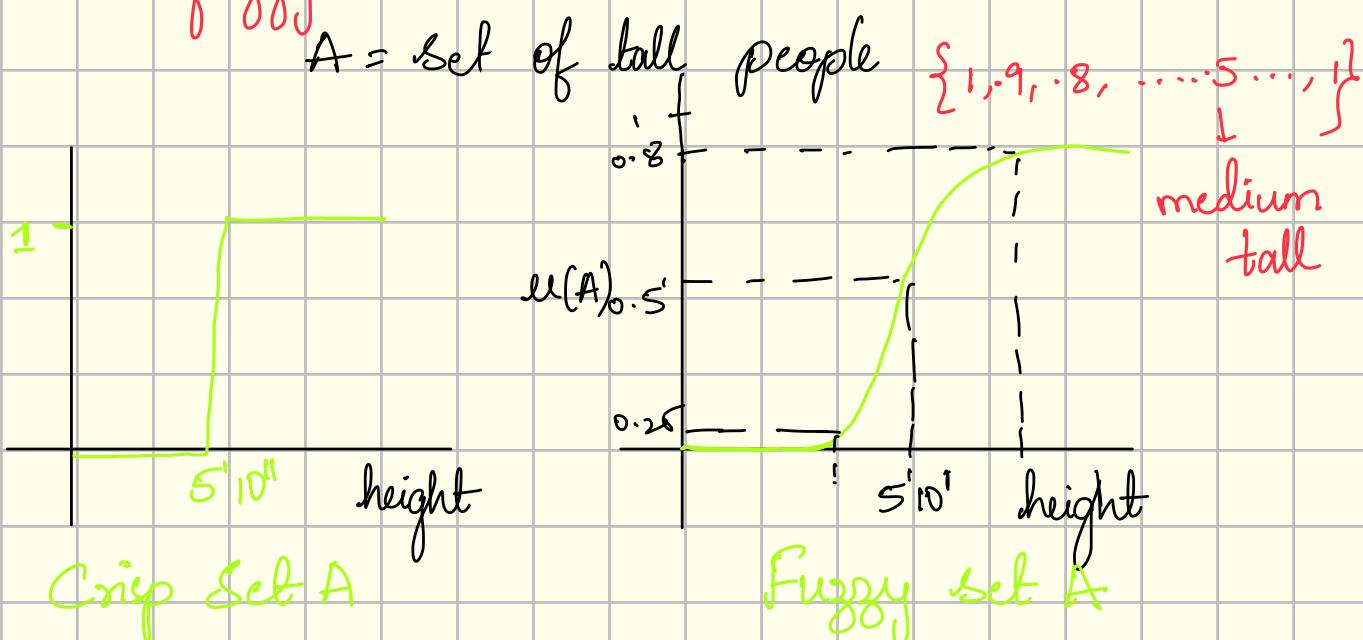
$$\tau(A \vee B) = \max(\tau(A), \tau(B))$$

Fuzzy logic

Fuzzy Set theory is a mean of specifying how well an object satisfies a vague description.

- Truth is a value between 0 and 1
- Uncertainty means from lack of evidence. but given the dimensions of a man whether he is fat the no uncertainty involves

Set with fuzzy Boundaries.



Membership function (MF):

Subjective measure not probability function



A Fuzzy set A in X is expression as a set of ordered pair.

$$A = \{(x, \mu_A(x)) \mid x \in X\}$$

↑ ↑
universe or

↓ Uni of
Membership discourse
functⁿ
(MF)

A F. Set is totally characterized by a MF

* Fuzzy Set with discrete Universe

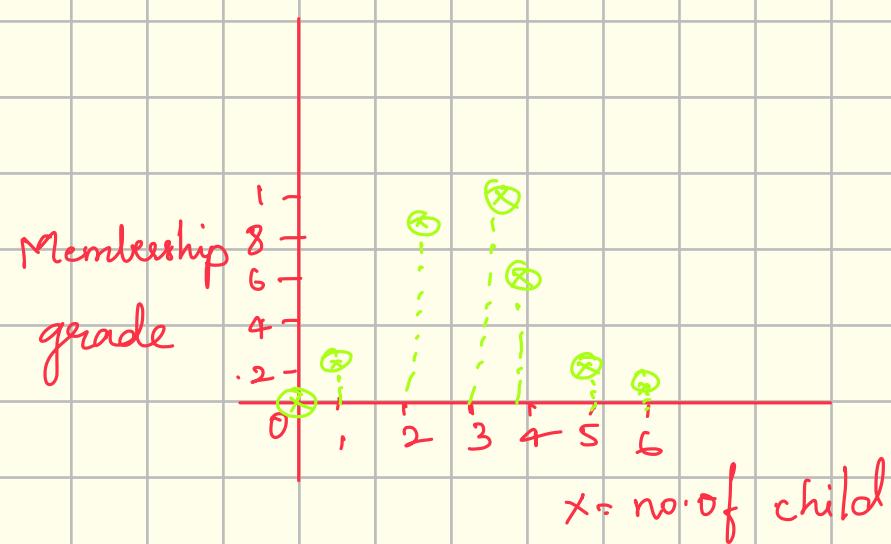
Fuzzy Set C = "desirable city to live in"
 $X = \{ \text{Bhuba, MP, SM} \}$ (discrete and non-ordered)

$$C = \{ (\text{Bhuba}, 0.9), (\text{MP}, 0.8), (\text{SM}, 0.6) \}$$

Fuzzy set A = "Sensible no. of children"

$$X = \{ 0, 1, 2, 3, 4, 5, 6 \} \text{ (discrete Uni.)}$$

$$A = \{ (0, 0.1), (1, 0.3), (2, 0.7), (3, 1), (4, 0.6), (5, 0.2), (6, 0.1) \}$$



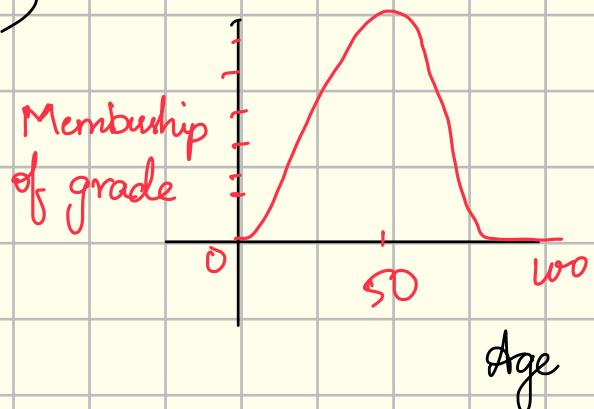
Fuzzy Set with continuous Universe

fuzzy Set B = "about 50 years old"

X , Set of positive real values \mathbb{R}^+ (continuous)

$$B = \{(x, \mu_B(x)) \mid x \in X\}$$

$$\mu_B(x) = \frac{1}{1 + \left(\frac{x - 50}{10}\right)^2}$$



Fuzzy Rules:

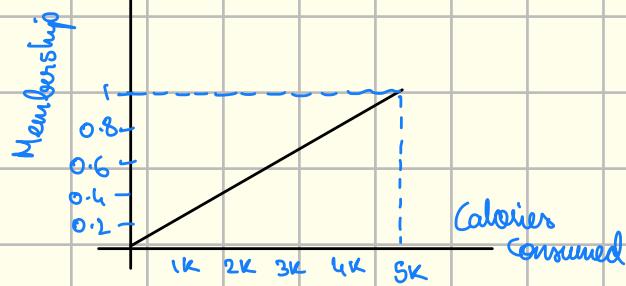
- Diet is low and exercise is high \Rightarrow Balanced
- Diet is high or exercise is low \Rightarrow Unbalanced
- Balanced \Rightarrow Risk is low
- Unbalanced \Rightarrow Risk is high
- For a person it is given that
exercise = Burning 1000 calories/day

What is the risk of heart disease?

Membership Function (MF)

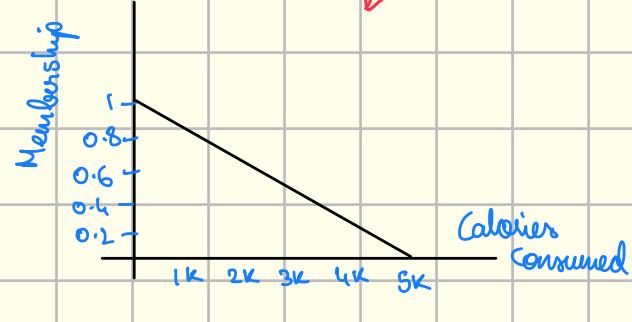
$$f_{DH}(x) = \frac{x}{5000}$$

(Diet high)



$$f_{DL}(x) = 1 - \frac{x}{5000}$$

(Diet low)



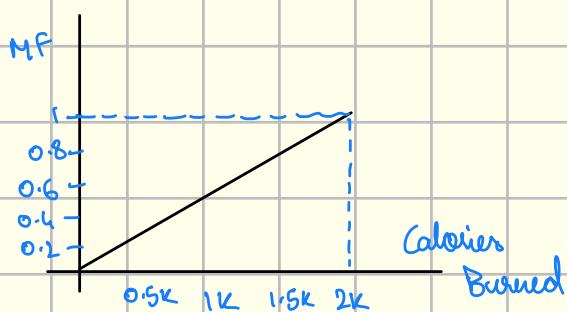
→ for daily calorie intake of 3000

$$\text{Membership of DH} = \frac{3000}{5000} = 0.6$$

$$\text{“ “ DL } = 0.4$$

$$\rightarrow f_{EH} = \frac{x}{2000}$$

(Exercise
high)



$$f_{EL} = 1 - \frac{x}{2000}$$

(Exercise
low)



→ for daily burnout of 1K Calories:

$$\text{Membership of EH} = 0.5$$

$$\text{“ “ EL } = 0.5$$

Rule Evaluation:

$$\rightarrow \text{truth (Diet high)} = \tau(DH) = 0.6 \quad | \quad \tau(DL) = 0.4$$

$$\tau(EH) = 0.5 \quad | \quad \tau(EL) = 0.5$$

→ $DL \wedge EH \Rightarrow \text{Balanced}$

$$\tau(\text{Balanced}) = \min \{ \tau(DL), \tau(EH) \}$$

$$= \min (0.4, 0.5)$$

$$= 0.4$$

→ $DH \vee EL \Rightarrow \text{Unbalanced}$

$$\tau(\text{Unbalanced}) = \max (\tau(DH), \tau(EL))$$

$$= \max(0.6, 0.5)$$

$$\approx 0.6$$

→ Balance \Rightarrow Risk \downarrow

Unbalance \Rightarrow Risk \uparrow

$$\tau(R\downarrow) = \tau(Bal) = 0.4$$

$$\tau(R\uparrow) = \tau(UnBal) = 0.6$$

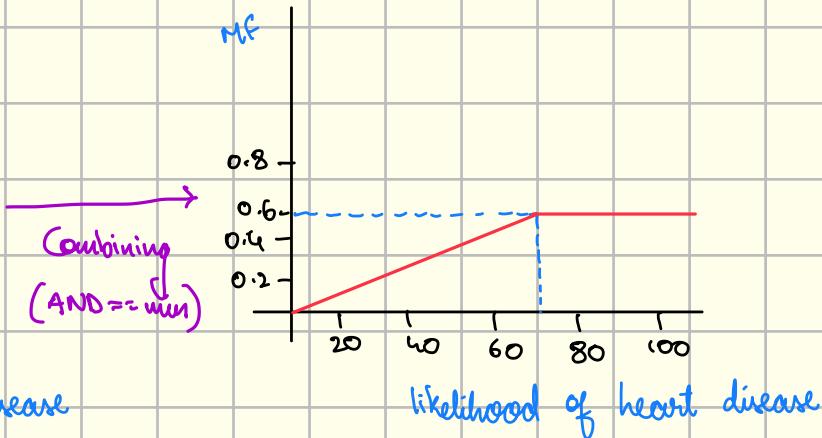
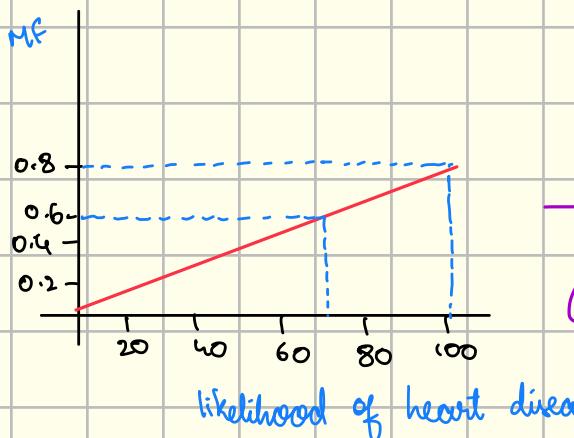
$$\rightarrow f_{RH} = f_{R\uparrow} = \frac{x}{125}$$

(Risk high)

$$\tau(R\uparrow) = \tau(RH) = 0.6$$

$$0.6 = \frac{x}{125}$$

$$\Rightarrow x = 75$$

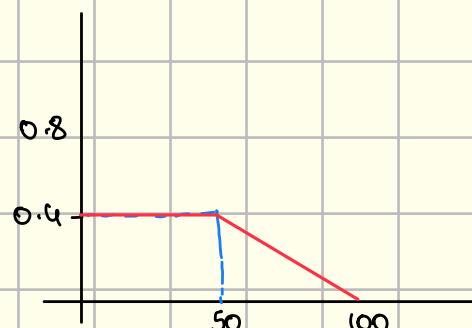
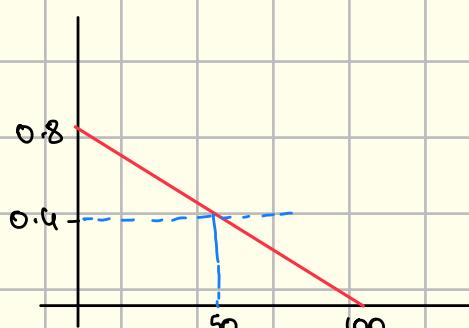


$$\rightarrow f_{RL} = f_{R\downarrow} = 0.8 - \frac{x}{125}$$

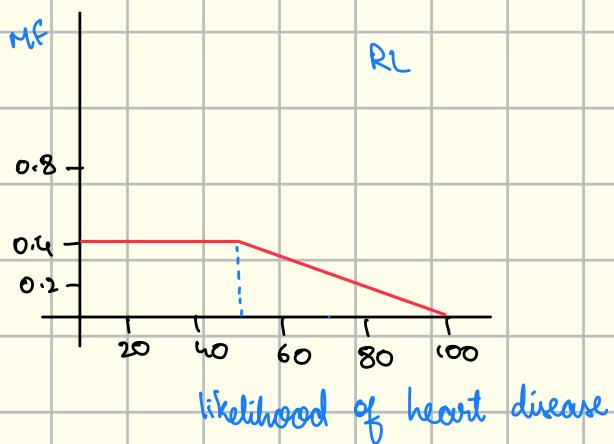
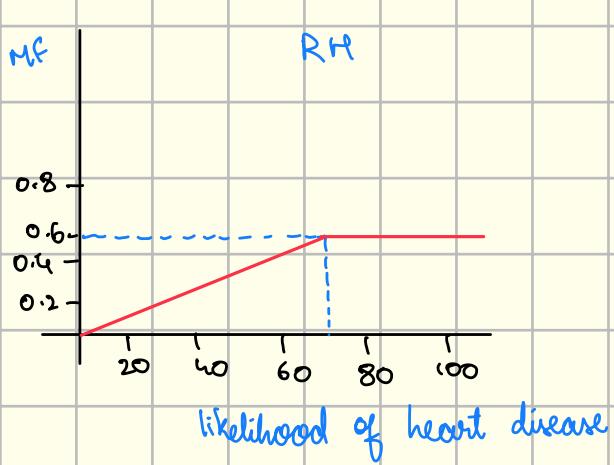
$$\tau(R\downarrow) = 0.4$$

$$0.4 = 0.8 - \frac{x}{125}$$

$$\Rightarrow x = 50$$

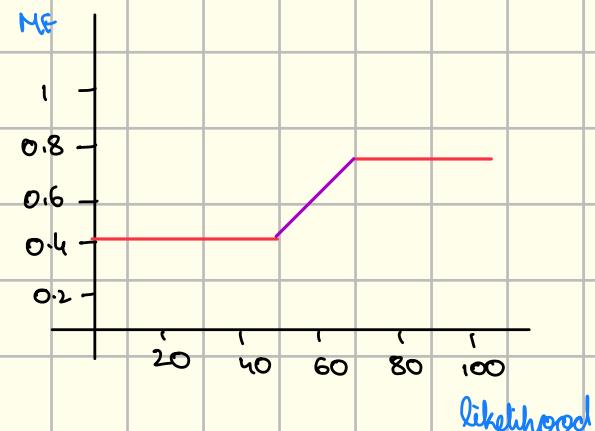


$$\rightarrow f_{\text{aggregated Risk}}(x) = \begin{cases} 0.4, & \text{if } x \in [0, 50) \\ 3x+1, & \text{if } x \in [50, 75) \\ 0.6, & \text{if } x \in [75, 100] \end{cases}$$



$$\tau(A \wedge B) = \min(\tau(A), \tau(B))$$

$$\tau(A \vee B) = \max(\tau(A), \tau(B))$$



→ Defuzzification

$$\begin{aligned} \int_0^{100} f_{A_H} dx &= \int_0^{50} 0.4 dx + \int_{50}^{75} \frac{x}{125} dx + \int_{75}^{100} 0.8 dx \\ &= 50 \times 0.4 + \frac{1}{125} \left[\frac{x^2}{2} \right]_{50}^{75} + 25 \times 0.8 \\ &= 47.5 \end{aligned}$$

→ the likelihood of HD for the person is 47.5 %
(Heart Disease)

A Lumerier's Notion of Fuzzy sets in Continuous & discrete

→ x is discrete $\Rightarrow A = \sum_{x_i \in x} \mu_A(x_i) | x_i$

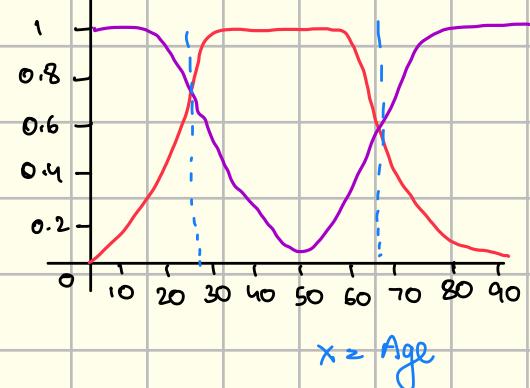
$$\rightarrow x \text{ is continuous} \Rightarrow A = \int_x u_A(x) dx$$

"/" stands for a marker, not decision

FUZZY PARTITIONS:

\rightarrow formed by the linguistic values, "Young", "Middle Age", "old"

Membership grades



BASIC DEFINITIONS & TERMINOLOGIES

\rightarrow support $(A) = \{x \in X \mid u_A(x) > 0\}$

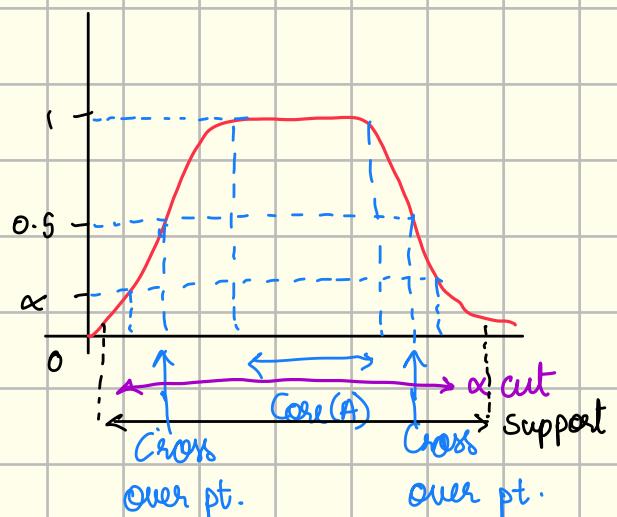
Core $(A) = \{x \in X \mid u_A(x) = 1\}$

NORMALITY: $\text{Core}(A) \neq \emptyset$ [A is a normal fuzzy set]

Crossover $(A) = \{x \in X \mid u_A(x) = 0.5\}$

α -cut: $A_\alpha = \{x \in X \mid u_A(x) \geq \alpha\}$

strong α -cut: $A'_\alpha = \{x \in X \mid u_A(x) > \alpha\}$

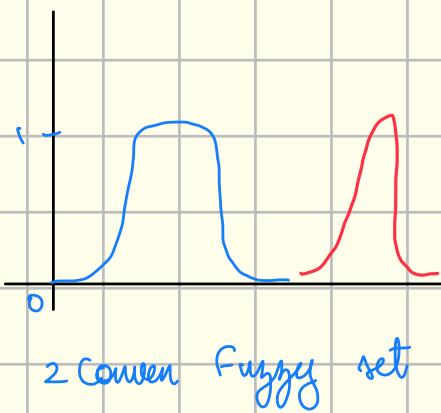


Convexity of Fuzzy Set:

→ A fuzzy set A is convex if for any λ in $[0, 1]$

$$\mu_A(\lambda x_1 + (1-\lambda)x_2) \geq \min(\mu_A(x_1), \mu_A(x_2))$$

→ Alternatively A is convex, if all α -cuts are convex



Fuzzy Numbers:

→ a fuzzy no. A is a fuzzy set in \mathbb{R} that satisfies normality & convexity

Bandwidth (Bw):

→ for a normal and convex set the bandwidth is the dist. b/w 2 unique crossover pts.

$$Bw(A) = |x_2 - x_1|, \text{ with } \mu_A(x_1) = \mu_A(x_2) = 0.5$$

Symmetry:

→ fuzzy set A is symmetric if its MF is symmetric around a

certain pt. $x = c$,

$$\mu_A(x+c) = \mu_A(x-c) \quad \forall x \in X$$

Open left, Open Right, Closed fuzzy set A

OL OR CL

$$A_{OL} : \lim_{x \rightarrow -\infty} \mu_A(x) = 1$$
$$A_{OR} : \lim_{x \rightarrow \infty} \mu_A(x) = 0$$

$$K_1$$

$$K_2$$

$$A_{OR} : K_1 = 0 \quad K_2 = 1$$

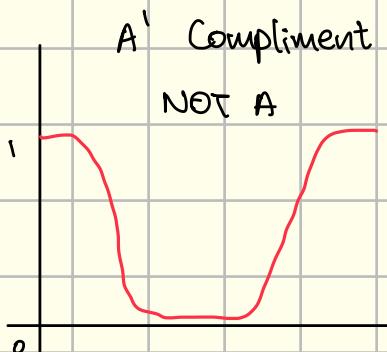
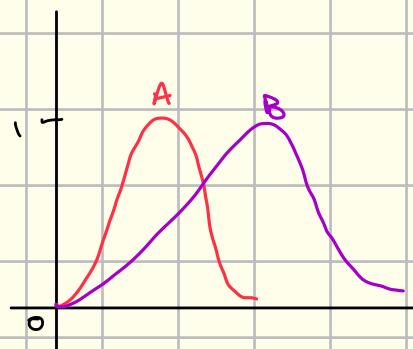
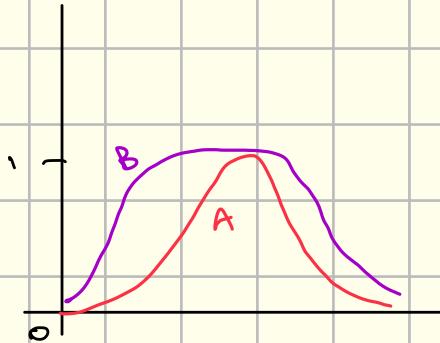
$$A_{CL} : K_1 = 0 \quad K_2 = 0$$

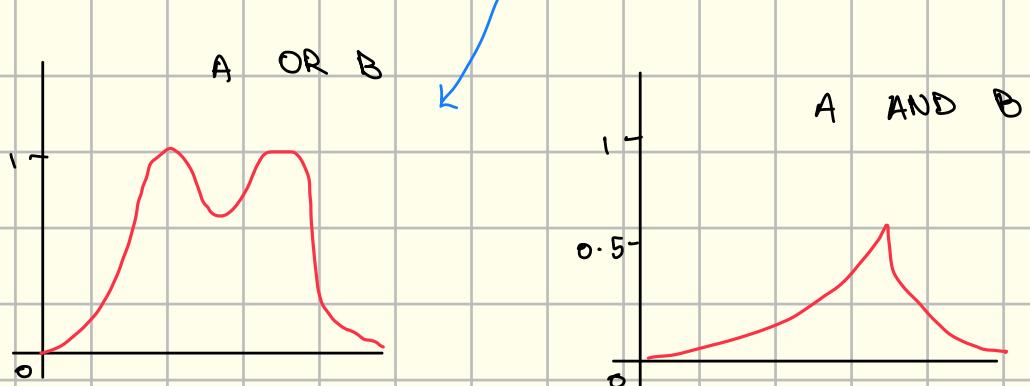
Subset : $A \subseteq B : \mu_A \leq \mu_B$

Complement : $\bar{A} = X - A : \mu_{\bar{A}}(x) = 1 - \mu_A(x)$

Union : $C = A \cup B : \mu_C(x) = \max(\mu_A(x), \mu_B(x)) = \mu_A(x) \vee \mu_B(x)$

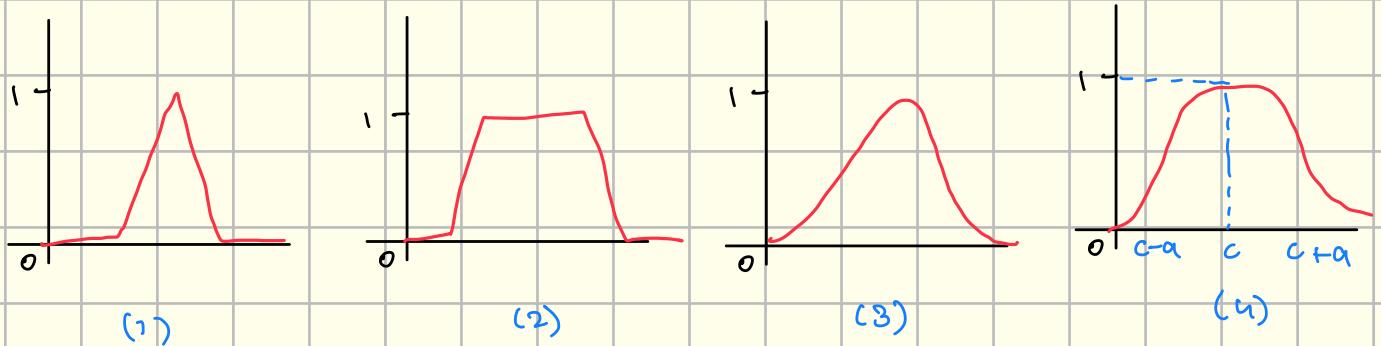
Intersection : $C = A \cap B : \mu_C(x) = \min(\mu_A(x), \mu_B(x)) = \mu_A(x) \wedge \mu_B(x)$





MF in one dimension:

- Triangular MF: $\text{trimf}(x, a, b, c) = \max \left\{ \min \left(\frac{x-a}{b-a}, \frac{c-x}{c-b} \right), 0 \right\}$
- Trapezoidal MF: $\text{trapmf}(x, a, b, c, d) = \max \left\{ \min \left(\frac{x-a}{b-a}, 1, \frac{d-x}{d-c} \right), 0 \right\}$
- Gaussian MF: $\text{gaussmf}(x, c, \sigma) = e^{-\frac{1}{2} \left(\frac{x-c}{\sigma} \right)^2}$
- Generalized bell MF: $\text{gbellmf}(x, a, b, c) = \frac{1}{1 + | \frac{x-c}{a} |^b}$

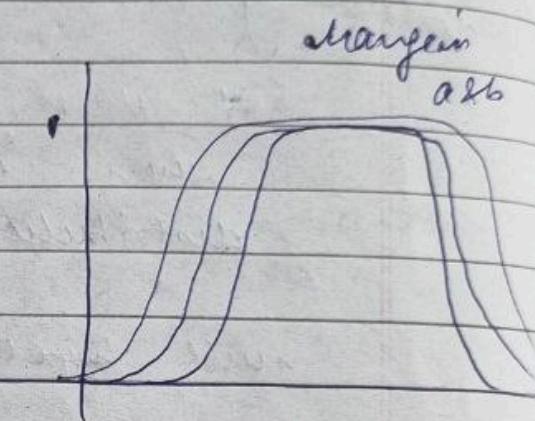
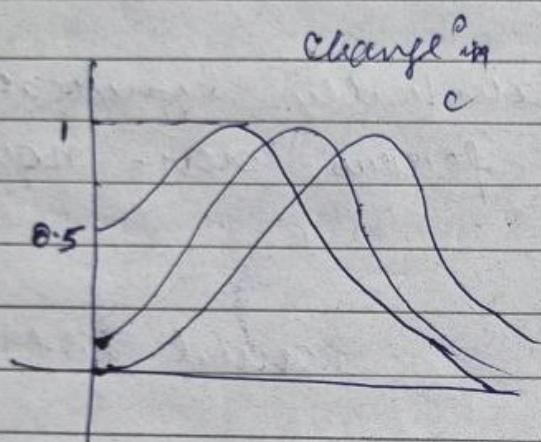
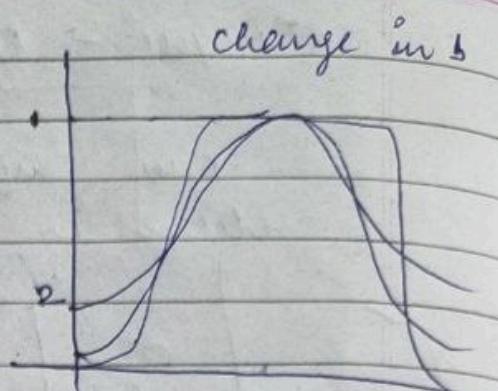
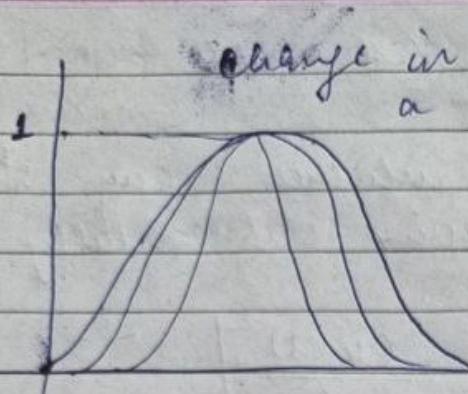


Artificial Intelligence - I

Generalized Bell MF (n, a, b, c)

$$= \frac{1}{1 + \left| \frac{n-c}{a} \right|^{2b}}$$

change of parameters in Bell MF :-



Gaussian MF and Bell MF achieve smoothness, they are unable to specify Asymmetric MFs which are important in many applications

Asymmetric and closed-loop MFs can be synthesized using either the absolute difference or the product of 2 sigmoid functions.

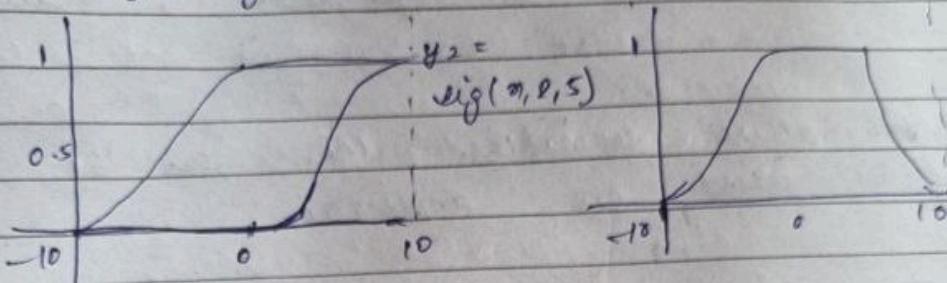
Sigmoidal MFs: $\text{sigmf}(\alpha, a, c) = \frac{1}{1 + e^{(\frac{a}{\alpha - c})}}$

MF + Matrix factorization

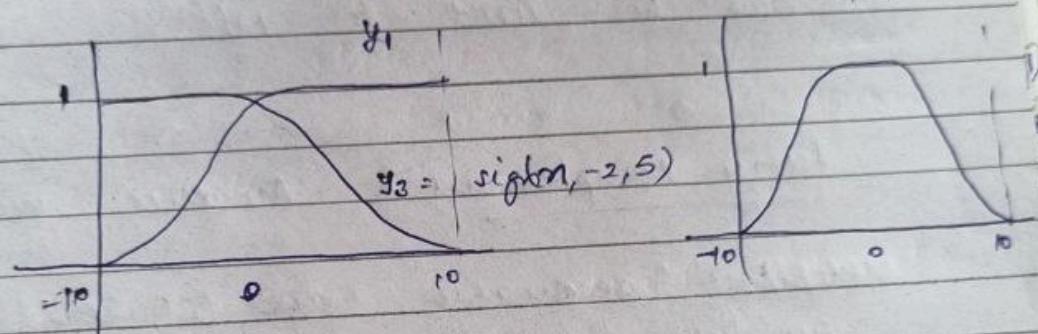
Datta
Page

$$|y_1 - y_2|$$

$$y_1 = \text{sig}(x, 1, -5)$$



$$y_1 \times y_2$$



* A sigmoid MF is inherently open right or left and thus it is appropriate for representing concepts of very large or very +ve / very -ve.

* Sigmoid MF mostly used as activation func' in NN

* A NN should ~~use~~ synthesis a closed MF to simulate the behaviour of a fuzzy sys.

- * Triangular, Trapezoidal, Gaus, Bell
→ not exhaustive
- * other specialized MFs can be created based on application
- * Any type of cont. prob- dist. fun' can be used as MF.

fuzzy complement : Another way to

define reasonable and consistent operation on fuzzy sets.

- General requirement

Boundary : $N(0) = 1$, $N(1) = 0$

monotonicity : $N(a) \geq N(b)$, if $a \leq b$

Inversion : $N(N(a)) = a$

2 Types of fuzzy complement -

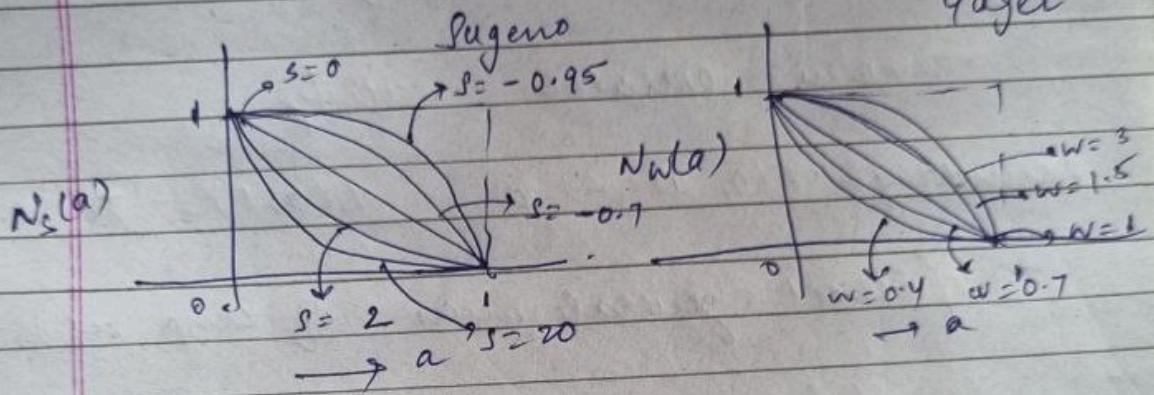
→ SUGENO's complement

$$N_s(a) = \frac{1-a}{1+s a} \quad [s > -1]$$

family of fuzzy complement operator

→ YAGER's complement

$$N_w(a) = (1 - a^w)^{1/w} \quad (w > 0)$$



fuzzy intersection and union, complement

The intersection of 2 fuzzy sets A and B is specified in general by a func".

$T: [0, 1] \times [0, 1] \rightarrow [0, 1]$ with

$$\mu_{A \cap B}(x) = T(\mu_A(x) = \mu_B(x)) = \mu_A(x)$$

$\approx \mu_B(x)$

↑
binary operator for
func" T

crisp

This class of fuzzy intersection operators is called T-norm
(Triangular operators)

T-norm operator satisfy

boundary: $T(0, a) = 0, T(a, 1) = T(1, a) = a$

(correct generalization of crisp set)

Monotonicity & $T(a, b) \leq T(c, d)$
if $a \leq c$ and $b \leq d$

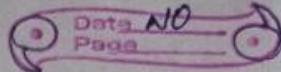
A decrease of membership in A & B
cannot increase a membership in
 $A \cap B$

Commutativity $T(a, b) = T(b, a)$

T is indifferent to the order of
fuzzy sets to be combined.

Associativity $T(a, T(b, c)) = T(T(a, b), c)$

Imp
Kuchi
Singh
Mital
aa
who



Intersection is independent of the order of pairwise grouping.

Example of T-norm

$$\text{minimum } T_m(a, b) = \min(a, b) = a \wedge b$$

$$\text{Algebraic product : } T_a(a, b) = ab$$

$$\text{Bounded product : } T_b(a, b) = \begin{cases} 0 & \text{if } a+b=1 \\ ab & \text{if } 0 < a, b < 1 \\ 1 & \text{if } a+b=1 \end{cases}$$

$$\text{Drastic product : } T_d(a, b) = \begin{cases} a & \text{if } b=1 \\ b & \text{if } a=1 \\ 0 & \text{if } a, b < 1 \end{cases}$$

T-conorm or S-norm

$$S: [0, 1] \times [0, 1] \rightarrow [0, 1]$$

$$M_A \cup M_B = S(M_A(n), M_B(n)) = M_A(n) \approx M_B(n)$$

$$\text{Boundary } S(1, 1) = 1, S(a, 0) = S(0, a) = a$$

$$\text{Monotonicity } S(a, b) < S(c, d) \text{ if } a < c, b < d$$

commutativity $s(a, b) = s(b, a)$

associativity $s(a, s(b, c)) = s(s(a, b), c)$

Example:

Max : $s_m(a, b) = \max(a, b) = \text{and}$

Algebraic sum : $s_a(a, b) = a + b - ab$

Bounded sum : $s_b(a, b) = 1 \wedge (a+b)$

Plastic sum : $s_d(a, b) = \begin{cases} a, & \text{if } b \geq 0 \\ b, & \text{if } a = 0 \\ 1, & \text{if } a, b > 0 \end{cases}$

Generalized De Morgan's law

$$T(a, b) = \neg(N(s(N(a)), N(b)))$$

$$N(s(N(a), N(b)))$$

$$S(a, b) = N(T(N(a), N(b)))$$

T-neums and S-neums are duals
 which support the generalization of
 De Morgan's laws.