Indian Institute of Technology Patna
Department of Mathematics
MA - 225: B.Tech. II year

Tutorial Sheet-1

1. (i) Let $P(A) = 1/3,\ P(B) = 1/4$, can events A and B be disjoint? Explain.
   (ii) Show that if $A \cap B = \{\phi\}$, then $P(A) \leq P(\bar{B})$.

2. Let two events A and B be such that $B \subset A$. Then show that (i) $P(A \cap \bar{B}) = P(A) - P(B)$ (ii) $P(B) \leq P(A)$.

3. Show that (a) $\overline{\bar{A} \cup \bar{B}} \cup \overline{\bar{A} \cup B} = A$
   (b) $(A \cup B) \cap (\overline{A \cap B}) = (A \cap \bar{B}) \cup (B \cap \bar{A})$.

4. Show that If $A = \{2 \leq x \leq 5\}$ and $B = \{3 \leq x \leq 6\}$, find $(A \cup B), (A \cap B)$ and $(A \cup B) \cap (\overline{A \cap B})$

5. Show that (a) If $P(A) = P(B) = P(A \cap B)$, then $P((A \cap \bar{B}) \cup (B \cap \bar{A})) = 0$; (b) $P(A) = P(B) = 1$, then $P(A \cap B) = 1$.

6. For any three events A, B and C defined on the sample space S such that $B \subset C$ and $P(A) > 0$ then $P(B \mid A) \leq P(C \mid A)$.

7. Event A and B are such that $P(A \cup B) = \frac{3}{4}$, $P(A \cap B) = \frac{1}{4}$ and $P(\bar{A}) = \frac{2}{3}$, show that $P(B) = \frac{2}{3}$ and $P(A \cap \bar{B}) = \frac{1}{12}$.

8. Each coefficient in equation $ax^2 + bx + c = 0$ is determined by throwing an ordinary die. Find the probability that the equation will have (a) Real Root (b) Complex Root.

9. Prove the Bonferroni inequality:
   For some arbitrary events $A_1, A_2, \ldots, A_n$ we have $P(A_1 \cap A_2 \cap \cdots \cap A_n) \geq \sum_{i=1}^{n} P(A_i) - (n-1)$.

10. Suppose that there are n students in a class room and assume that $n \leq 365$. Also let no student has birthday on 29th February. What is the probability that at least two students share the same birthday.

11. Suppose that the population of a certain city is 40% mail and 60% female. Suppose also that 50% of the males and 30% of the females smoke. Find the probability that a smoker is male.

12. Let two fair coin are tossed once. (i) Find the probability that both coins show head given that the first shows a head. (ii) What is the probability that the both are heads given that at least on of them is a head.

13. Find the minimum number of times a die has to be thrown such that the probability of no six is less than $1/2$.

14. Why does it pay to bet consistently on seeing 6 at least once in 4 throws of a die, but not seeing a double six at least once in 24 throws with two die?

15. A problem is given to three students A, B and C whose chance of solving it are $1/2, 3/4$ and $1/4$ respectively. What is the probability that the problem is solved if all of them try independently?

16. Consider two boxes, one containing 1 black and 1 white marble, the other, 2 black and 1 white marble. A box is selected at random and a marble is drawn at random from the selected box. What is the probability that the marble is black?

17. Suppose that each of N men at a party throws his hat into the center of the room. The hats are first mixed up and then each man randomly selects a hat. What is the probability that: (i) none of the men selects his own hat (ii) exactly k of the men select their own hats? (iii) Evaluate part (i) when $N = 3$ and $N = 4$ (iv) Discuss the case when N approaches infinity.

18. A box contains m white balls and n black balls. Balls are drawn at random one at time without replacement. Find the probability of encountering a white ball by the kth draw.

19. Two players A and B draw balls one at time alternatively from a box containing m white and n black balls. Suppose the player who picks the first white ball wins the game. What is the probability that the player who starts the game will win?

20. A box contain n identical balls numbered 1 through n. Suppose k balls are drawn in succession. (i) What is the probability that m is the largest number drawn ? (ii) What is the probability that the largest number drawn is less than or equal to m ?

21. A box contains m white and n black balls. Suppose k balls are drawn. Find the probability of drawing at least one white ball ?

22. An urn A contains 5 black and 6 white balls and urn B contains 8 black and 4 white balls. Two balls are transferred from B to A and then a ball is drawn from A. (i) What is the probability that this ball is white? (ii) Given that the ball drawn is white what is the probability that at least one white ball was transferred to A?