Indian Institute of Technology Patna
Department of Mathematics
MA225: Probability and Statistics
B.Tech. 2nd year

Tutorial Sheet-6

1. The distribution of I.Q.s of the people in a given group is approximated well by the Normal distribution with $\mu = 105$ and $\sigma = 20$. What proportion of the individuals in the group in question has an I.Q. : (i) At least 50? (ii) At most 80? (iii) Between 95 and 125?

2. Consider a one meter long string which is cut into two unequal pieces at a random point along its length. Find the probability that the longer piece is at least twice the length of the shorter.

3. Let the roots of the equation $x^2 - ax + b = 0$ be real and b is a positive random variable uniformly distributed in a appropriately permissible range. Find the expected values of the roots of the equation.

4. Define $f(x) = 0.5ae^{-ax}, x \geq 0$ and $f(x) = 0.5ae^{ax}, x < 0$, where $a > 0$. Verify that $f(x)$ is a PDF. Let X be a RV having the PDF $f(x)$. Find $P(X < x)$ and $P(|X| < x)$ for all x.

5. Let X be a RV with PDF $f(x) = 1/3, -1 < x < 2$ and $f(x) = 0$, otherwise. Find the PDF of $Y = |X|$.

6. Let X be a $N(0, 1)$ random variable. Find PDF of X^2 , $|X|$, e^X , $2X^2 + 1$.

7. Suppose that $F_X(x) = 0, x < 0, F_X(x) = x/2, 0 \leq x < 1, F_X(x) = (x/6) + (1/3), 1 < x < 4$ and $F_X(x) = 1, x \geq 4$ be the CDF of a random variable X. Find the PDF, 75th percentile and mean value of X. Also let $Y = -1$, if $X \leq 1$ and $Y = 1$, if $X > 1$ then evaluate $F_Y(0)$ and variance of Y. $(2.5, 1.5, 0.5, 1)$

8. The PDF of a random variable X is given by $f_X(x) = 6x(1 - x), 0 \leq x \leq 1$ and $f_X(x) = 0$, elsewhere. Find mean value of $1/X$, CDF of X and PDF of X^2. Let $Y = 2$, if $X \geq (1/4)$ and $Y = 0$, if $X < (1/4)$ then evaluate mean value of Y^k, k a natural number.$(3, (27/32)2^k)$

9. (i) Let X be distributed as $Exp(2)$ and consider the transformation $Y = (X - 2)^2$. Find the PDF of Y . (ii) Let X be distributed as $Exp(2)$ and consider the transformation $Y = [X] + 1$. Find the probability distribution of Y.

10. In each of the following case find the PDF of Y. (i) $f_X(x) = 42x^5(1 - x), 0 < x < 1, Y = X^3$ (ii) $f_X(x) = 7e^{-7x}, x > 0, Y = 4X + 3$ (iii) $f_X(x) = 30x^2(1 - x)^2, 0 < x < 1, Y = X^2$.

11. Let X be an RV with PDF $f(x) = \theta e^{-\theta x}$, $x > 0$, $\theta > 0$ and $f(x) = 0$, otherwise. Find the PDF of $(X - (1/\theta))^2$.

12. Let X be a RV with PDF $f(x) = 1/3$, $-1 < x < 2$ and $f(x) = 0$, otherwise. Find the PDF of $Y = |X|$.

13. Let X be a RV with PDF $f(x) = 1/2\theta$,-$\theta < x < \theta$ and $f(x) = 0$, otherwise. Find the PDF of $Y = 1/X^2$.