1/12/2023

Tutorial 3

(1)

1. (i) $\sum_{k=0}^{7} P(X=k) = 1$

$\Rightarrow (10k^2 + 9k - 1) = 0$

$\Rightarrow k = \frac{1}{10}, (-1) \times$

$k \neq -1$ otherwise $P(X=1) < 0$ which is not possible.

$\Rightarrow k = 10$

(ii) $P(X<6) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5)$

$= k(+8) = \frac{81}{100}$

$P(X \geq 6) = 1 - P(X<6) = 1 - \frac{81}{100} = \frac{19}{100}$

(iii) $P(0 < X < 5) = P(X=1) + \cdots + P(X=4)$

$= 8k = \frac{8}{10}$

(iv) CDF is -

$F(x) = 0, \quad x < 0$

$= 0, \quad 0 \leq x < 1$

$= 0 + k, \quad 1 \leq x < 2$

$= 0 + k + 2k, \quad 2 \leq x < 3$

$= 0 + k + 2k + 2k, \quad 3 \leq x < 4$

$= \underbrace{0 + k + 2k + 2k + 3k}_{8k}, \quad 4 \leq x < 5$

$= 8k + k^2, \quad 5 \leq x < 6$

$= 8k + k^2 + 2k^2, \quad 6 \leq x < 7$

$= 1, \quad x \geq 7$

$\lfloor \ \rfloor$ floor func^n → greatest integer

2. $X \sim$ RV $\rightarrow E|X| < \infty$

Median: A real no. M is said to be median of r.v X (discrete), if $P(X \leq M) \geq \frac{1}{2}$ & $P(X \geq M) \geq \frac{1}{2}$

Since $P(X \leq M) \geq \frac{1}{2}$

$\Rightarrow \boxed{F(M) \geq \frac{1}{2}}$ [$\because F(\cdot)$ is CDF of X]

If $c > M$ then

$E(|X-c|) = \sum_{x=-\infty}^{\infty} |x-c| p(x)$ where $p(x)$ is pmf of X

$= \sum_{-\infty}^{c} (c-x) p(x) + \sum_{x=c}^{\infty} (x-c) p(x)$

$= \sum_{-\infty}^{M} (c-x) p(x) + \sum_{x=M}^{c} (c-x) p(x) + \sum_{x=M}^{\infty} (x-c) p(x) - \sum_{x=M}^{c} (x-M) p(x)$

$= \sum_{x=-\infty}^{M} [(c-M) p(x) + (M-x) p(x)] + 2 \sum_{x=M}^{c} (c-x) p(x) + \sum_{x=M}^{\infty} [(x-M) p(x) + (M-c) \cdot p(x)]$

$= (c-M) \sum_{x=-\infty}^{M} p(x) + \sum_{x=-\infty}^{M} (M-x) p(x) + \sum_{x=M}^{\infty} (x-M) p(x) + (M-c) \sum_{x=M}^{\infty} p(x) + 2 \sum_{x=M}^{c} (c-x) p(x)$

$= (c-M) \sum_{x=-\infty}^{M} p(x) + \sum_{x=-\infty}^{\infty} (M-x) p(x) + \sum_{x=M}^{\infty} (x-M) p(x) + (M-c) \sum_{x=M}^{\infty} p(x) + 2 \sum_{x=M}^{c} (c-x) p(x)$

$= (c-M) \sum_{x=-\infty}^{M} p(x) + \sum_{x=-\infty}^{\infty} |x-M| p(x) + (M-c) \sum_{x=M}^{\infty} p(x) + 2 \sum_{x=M}^{c} (c-M) p(x)$

$= (c-M) \sum_{x=-\infty}^{\infty} p(x) + E(|x-M|) + (M-c) \sum_{x=M}^{\infty} p(x) + 2 \sum_{x=M}^{c} (c-x) p(x)$

$= E(|X-M|) + (c-M) \sum_{x=-\infty}^{M} p(x) + (M-c) \sum_{x=M}^{\infty} p(x) + 2 \sum_{x=M}^{c} (c-x) p(x)$

$= E(|X-M|) + (c-M) \sum_{x=-\infty}^{M} p(x) + (M-c) \left[1 - \sum_{x=-\infty}^{M} p(x)\right] + 2 \sum_{x=M}^{c} (c-x) p(x)$

[$\because \sum_{x=-\infty}^{\infty} p(x) = 1$

$\Rightarrow \sum_{x=M}^{\infty} p(x) = 1 - \sum_{x=-\infty}^{M} p(x)$]

$= E(|x-M|) + (c-M) \sum_{x=-\infty}^{M} p(x) + (c-M) \left[\sum_{x=-\infty}^{M} p(x) - 1\right] + 2 \sum_{x=M}^{c} (c-x) p(x)$

$= E(|X-M|) + (c-M) \left[2 \sum_{x=-\infty}^{M} p(x) - 1\right] + 2 \sum_{x=M}^{c} (c-x) p(x)$

$= E(|X-M|) + (c-M) [2F(M) - 1] + 2 \sum_{x=M}^{c} (c-x) p(x)$ — (1)

$\because F(M) = P(X \leq M) = \sum_{x=-\infty}^{M} p(x)$

Since M is median then

$F(M) \geq \frac{1}{2} \Rightarrow 2F(M) - 1 \geq 0$

From (1), we have

$E|X-c| \geq E(|X-M|) + 2 \sum_{x=M}^{c} (c-x) p(x)$

Since, $c > M$, then

$\sum_{x=M}^{c} (c-x) p(x) \geq 0$

$\therefore E(|X-c|) \geq E(|X-M|)$

Sl:

Similarly for $c < M$,

$$E(|x-c|) \geq E(|x-M|) + 2\sum_{x=c}^{M}(x-c)p(x)$$

$$\therefore \quad E(|x-c|) \geq E(|x-M|)$$

Thus, $E(|x-c|) \geq E(|x-M|)$

(3) X: No. of right answers

Then X can take value 0, 1 & 2

$P(X=0) = \frac{2}{3} \times \frac{4}{5}$, $P(X=1) = \frac{2}{3}\times\frac{1}{5} + \frac{1}{3}\times\frac{4}{5} = \frac{6}{15}$

$P(X=2) = \frac{1}{3} \times \frac{1}{5} = 1/15$

$E(X) = 0 \cdot P(X=0) + 1 \cdot P(X=1) + 2 \cdot P(X=2)$
$= 8/15$

$E(X^2) = 0\,P(X=0) + 1 \cdot P(X=1) + 4P(X=2)$
$= 10/15$

$V(X) = E(X^2) - \{E(X)\}^2 = \frac{86}{225}$

(4)? $F(x) = 0$, $x < -2$
$= 0 + \frac{1}{4}$, $-2 \leq x < 0$
$= \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$; $0 \leq x < 1$
$= \frac{1}{2} + \frac{1}{3} = \frac{5}{6}$, $1 \leq x < 2$
$= 1$; $x \geq 2$

Now, $P(0) \geq 1/2$ [discrete case

& $P(X \geq 0) = 1 - P(X<0) = 1 - F(0) + P(X=0)$
$= 1 - \frac{1}{2} + \frac{1}{4} = \frac{3}{4} \geq 1/2$

$\Rightarrow 0$ is a median

x is a quantile of order 0.2 if

$P(X \leq x) \geq 0.2$ — Quantile

$\Rightarrow P(X \geq x) \geq 0.8$. / position based

$x = -2$?

(5) X: No. of tosses required for first head appear

$R_X = \{1, 2, \ldots\}$

$P(X=1) = p$ where p is probability for getting head

$P(X=2) = q \cdot p$ where $q = 1-p$

$P(X=3) = q^2 p$

$\vdots$

$P(X=k) = q^{k-1}p$

$$E(X) = \sum_{k=-\infty}^{\infty} k\,P(X=k) = \sum_{k=1}^{\infty} k\,P(X=k)$$
$$= \sum_{k=1}^{\infty} k\,q^{k-1}p = p\sum_{k=1}^{\infty}\frac{d}{dq}q^k$$
$$= p\frac{d}{dq}\left[\sum_{k=1}^{\infty} q^k\right]$$
$$= p\,\frac{d}{dq}\left[\frac{1}{1-q}\right] \quad (\because q<1)$$
$$= p \cdot \frac{1}{(1-q)^2} = \frac{1}{1-q} = \frac{1}{p}$$

(6)? X: No. of trials required to open the door

(i) with replacement

$R_X = \{1, 2, \ldots n\}$

$P(X=1) = \frac{1}{n}$

$P(X=2) = \frac{n-1}{n}\cdot\frac{1}{n}$ [n-1 key jo back open nhi kr sktे

$\vdots$

$P(X=k) = \left(\frac{n-1}{n}\right)^{k-1}\cdot\frac{1}{n}$

$$E(X) = \sum_{k=1}^{\infty} k\,P(X=k) = \sum_{k=1}^{\infty} k\cdot\left(\frac{n-1}{n}\right)^{k-1}\cdot\frac{1}{n}$$
$$= \frac{1}{n}\cdot\frac{1}{\left(1-\frac{(n-1)}{n}\right)^2} = n$$

$$E(X^2) = \sum_{k=1}^{\infty} k^2\,P(X=k) = \sum_{k=1}^{\infty} k^2\left(\frac{n-1}{n}\right)^{k-1}\frac{1}{n}$$
$$= \frac{1}{n}\sum_{k=1}^{\infty} k^2\left(\frac{n-1}{n}\right)^{k-1} = \frac{1}{n}\cdot\frac{1+\frac{n-1}{n}}{\left(1-\frac{n-1}{n}\right)^3}$$
$$= (2n-1)\cdot n$$

$$= \frac{d}{dx}\left(\sum_{k=1}^{\infty} k x^k\right) = \frac{d}{dx}\left(x\sum_{k=1}^{\infty} k x^{k-1}\right) = \frac{d}{dx}\left(x\frac{1}{(1-x)^2}\right)$$
$$= \frac{1+x}{(1-x)^3}$$

$$V(X) = E(X^2) - (E(X))^2$$
$$= n(2n-1) - n^2 = n(n-1)$$

(ii) If the keys are eliminated then the key we select in trials will be excluded.

$R_X = \{1, 2, \ldots n\}$

$P(X=1) = 1/n$; $P(X=2) = \frac{n-1}{n}\cdot\frac{1}{n-1} = \frac{1}{n}$

$P(X=3) = \frac{(n-1)}{n}\cdot\frac{n-2}{n-1}\cdot\frac{1}{n-2} = \frac{1}{n}$

$P(X=n) = \frac{n-1}{n}\cdot\frac{n-2}{n-1}\cdots\frac{1}{2}\cdot 1 = \frac{1}{n}$

$$E(X) = \sum_{k=1}^{n} k\cdot P(X=k) = \sum_{k=1}^{n} k\cdot\frac{1}{n} = \frac{1}{n}\sum_{k=1}^{n} k$$
$$= \frac{1}{n}\cdot\frac{n(n+1)}{2} = \frac{(n+1)}{2}$$

$$E(X^2) = \sum_{k=1}^{n} k^2\,P(X=k) = \frac{1}{n}\sum_{k=1}^{n} k^2 = \frac{1}{n}\cdot\frac{n(n+1)(2n+1)}{6} = \frac{(n+1)(2n+1)}{6}$$

$$V(X) = \frac{(2n+1)(n+1)}{6} - \left(\frac{n+1}{2}\right)^2$$

③

(i) $X \sim B(n,p)$

$E(X) = np$ & $V(X) = npq$

Now $q \le 1 \Rightarrow V(X) \le E(X)$

So variance cannot be greater than mean.

(ii) The most likely outcome is corresponding to the mode of X, since $X \sim Bin(n,p)$

the mode of $X = (n+1)p = ((6+1) \times 0.5$
$= [7 \times 0.5] = 3$

(iii) X: no. of defective articles.
Probability that a article is defective is $= \frac{10}{100} = 0.1$

$X \sim B(10, 0.1)$

$P(X=2) = \binom{10}{2} \cdot (0.1)^2 \cdot (0.9)^8$

(iv) X: No of defective article in the sample

$X \sim B(20, 10)$

So, pmf of $X = \binom{20}{x} p^x (1-p)^{(20-x)}$

For $p = 0.25$ & s: $P(X=10) = \binom{20}{10} (0.25)^{10} \cdot (1-0.25)^{10}$

$P(X \ge 2) = 1 - P(X < 2) = 1 - P(X=0) - P(X=1)$
$= 1 - \binom{20}{0}(0.25)^0 (0.75)^{20} - \binom{20}{1} (0.25)^1 (.75)^{19}$

For poisson approximate $\lambda = 20 \times 0.25 = 5$

$P(X=10) = \frac{e^{-5} 5^{10}}{10!}$

$P(X \ge 2) = 1 - P(X=0) - P(X=1)$
$= 1 - \frac{e^{-5} 5^0}{0!} - e^{-5} \cdot \frac{5^1}{1!}$

(v) X: No. of active components

$X \sim B(5, 0.95)$

$P(X \ge 4) = P(X=4) + P(X=5)$
$= \binom{5}{4} (0.95)^4 \cdot (0.05) + \binom{5}{5} \cdot (.95)^5 \cdot (0.05)^0$

(vi) Prob. that the ship will arrive safely is $p = \frac{8}{9}$

X: no. of ships arrive safely

$X \sim B(6, \frac{8}{9})$

$P(X=3) = \binom{6}{3} \left(\frac{8}{9}\right)^3 \left(\frac{1}{9}\right)^3$

(vii) prob. that the vessel will arrive safely is $p = \frac{97}{100} = 0.97$

$X \sim B(10, 0.97)$

$P(X=6) = \binom{10}{6} (0.97)^6 (0.03)^4$

$P(X \ge 6) = P(X=6) + P(X=7) + \cdots + P(X=10)$

(viii) $X \sim P(5)$

$\therefore$ pmf of $X = p(x) = \frac{e^{-5} 5^x}{x!}$

Now, $P(X \ge 1 \mid X \le 1) = \frac{P(X=1)}{P(X \le 1)}$
$= e^{-5} \frac{5^1}{1!} \Big/ \frac{e^{-5} 5^0}{0!} + e^{-5} \cdot \frac{5^1}{1!}$
$= 5/6$

⑧ prob. that a candidate will pass $= \frac{60}{100} = 0.6$

X: No. of candidate passed the examination

$X \sim B(6, 0.6)$

$P(X \ge 4) = P(X=4) + P(X=5) + P(X=6)$

⑨ X: No. of correct guesses.

$X \sim B(10, 0.5)$

The prob. that a guess is correct is $= 0.5$.

(i) $P(X \ge 5) = 1 - P(X < 5)$
$= 1 - \sum_{k=0}^{4} P(X=k)$
$= 1 - \sum_{k=0}^{4} \binom{10}{k} (0.5)^k (0.5)^{10-k}$

(ii) $P(X=9) = \binom{10}{9} \cdot (0.5)^9 (0.5)$

(iii) $P(X \ge n) < \frac{1}{2}$

$\Rightarrow 1 - P(X \le n) < 1/2$

$\Rightarrow P(X < n) > 1/2$

For $n = 6$, we have $P(X < 6) = \sum_{k=0}^{5} P(X=k) \ge \frac{1}{2}$

$\therefore n = 6$ is the smallest.

⑩ prob. that a product is defective is $10/100 = 0.1$

X: No. of defective sample

$X \sim B(10, 0.1)$

$P(X=3) = \binom{10}{3} (0.1)^3 (0.9)^7$

Now $\lambda = np = 10 \cdot 0.1 = 1$

$P(X=3) = \frac{e^{-1} \cdot 1^3}{3!} =$

(11) Prob. of getting a TV set = 0.5

x: No. of request for TV set

$x \sim B(5, 0.5)$

(i) $P(x \geq 4) = P(x=4) + P(x=5)$
$= 3/16$

(ii) $P(x \leq 3) = 1 - P(x > 3)$
$= 1 - 3/16 = \frac{13}{16}$

(iii) $3C = R\left[1 \cdot P(x=1) + 2 \cdot P(x=2) + 3 \cdot P(x=3)\right]$

$= 1R\binom{5}{1}(0.5)^5 + 2R\binom{5}{2}(0.5)^5 + 3R\binom{5}{3}(0.5)^5$

$R = \frac{96C}{73}$

12 : To evaluate k-th central moment, we use moment generating funcn (MGF)

MGF of r.v. x is -

$$M_x(t) = E(e^{tx})$$

provided that the expectation exists for some t satisfying $|t| < h$, $h > 0$

Now,

$$M_x(t) = \sum_x e^{tx} p(x)$$

$$\frac{d}{dt} M_x(t) = \sum_x t e^{tx} p(x) \quad \text{— (1)}$$

$$\frac{d}{dt} M_x(t)\Big|_{t=0} = \sum_x x p(x) = E(x)$$

From (1)

$$\frac{d^2}{dt^2}(M_x(t))\Big|_{t=0} = E(x^2)$$

Thus we can determine the k-th moment by diff. MGF k times.

$$\frac{d^k}{dt^k}(M_x(t)) = E(x^k)$$

Now,

$$E\left((x - E(x))^k\right) = \sum_{i=0}^{k} \binom{k}{i} E(x^i) \cdot (E(x))^{k-i}$$

$$= \sum_{i=0}^{k} \binom{k}{i} \frac{d^i}{dt^i}(M_x(t))\Big|_{t=0} [E(x)]^{k-i} \quad \text{— (2)}$$

For Binomial Distribution

$$M_x(t) = \sum_x e^{xt}\binom{n}{x} p^x q^{n-x} = (pe^t + q)^n$$

using the MGF & (2) we can determine the k-th central moment

Similarly for Poisson Dist^n case.

(13) $x \sim B(4, p)$

$P(x=1) = 2/3$, $P(x=2) = 1/3$

$\Rightarrow \binom{4}{1} p(1-p)^3 = 2/3$ — (1) | $\binom{4}{2} p^2(1-p)^2 = \frac{1}{3}$ — (2)

(2)/(1) gives

$$\frac{6p^2(1-p)^2}{4p(1-p)^3} = \frac{1}{2} \Rightarrow 3p = 1-p$$
$$\Rightarrow p = 1/4$$

$E(x) = 4 \times \frac{1}{4} = 1$

$V(x) = 4 \times \frac{1}{4} \times \frac{3}{4} = \frac{3}{4}$

(14) x: No. of heads appeared in five tosses.

prob. that the head will appear $= p$

——— tail ——— $= p/3$

$p + \frac{p}{3} = 1 \Rightarrow p = 3/4$

$x \sim B(5, 3/4)$

(i) $P(x \geq 3) = 1 - P(x < 3)$

$= 1 - P(x=0) - P(x=1) - P(x=2)$

$= 1 - \binom{5}{0}\left(\frac{3}{4}\right)^0\left(\frac{1}{4}\right)^5 - \ldots$

(ii) $P(x \leq 3) = P(x=0) + \cdots + P(x=3)$

$= \binom{5}{0}\left(\frac{3}{4}\right)\left(\frac{1}{4}\right)^5 + \cdots +$

(iii) $P(x=3) = \binom{5}{3}\left(\frac{3}{4}\right)^3\left(\frac{1}{4}\right)^2$

(15) Prob. of success = P(getting 4) + P(getting 5)

$= \frac{1}{6} + \frac{1}{6} = 1/3$

x: # of success in 9 throws

$x \sim B(9, 1/3)$

(i) $E(x) = 9 \cdot \frac{1}{3} = 3$; $V(x) = 9 \cdot \frac{1}{3} \cdot \frac{2}{3} = 2$

(ii) $P(x=2) = \binom{9}{2}(1/3)^2(2/3)^7$

(iii) $P(x \leq 2) = P(x=0) + P(x=1) + P(x=2)$

(iv) $P(x \geq 2) = 1 - P(x < 2)$
$= 1 - P(x=0) - P(x=1)$

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14 Prob. that the blade is defective = 0.01

X : # of defective blades in packet of 10

$X \sim B(10, 0.01)$

(i) $P(X=0) = \binom{10}{0} \cdot (0.01)^0 (0.99)^{10}$

# of packet containing no defective blade $= 1000 \times P(X=0)$

(ii) # of ——— one defective blade $= 1000 \times P(X=1)$

(iii) ——— at most two defective

$= 1000 \times P(X \le 2)$

$P(X=0) + P(X=1) + P(X=2)$

(iv) ——— atleast two defective

$= 1000 \times P(X \ge 2)$

$1 - P(X < 2)$

$= 1 - P(X=0) - P(X=1)$

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X : No. of trial before first target is shot

$X \sim Geo(0.8)$

pmf of X $= P(X=k) = q^{k-1} \cdot p$, $q = 0.2$, $p = 0.8$

$P(X = odd) = P(X=1) + P(X=3) + \dots$

$= \frac{1}{2-p}$

$P(X = even) = 1 - P(X = odd)$

$= 1 - \frac{1}{2-p} = \frac{1-p}{2-p}$

18

NB Prob. that a product is defective $= \frac{3}{100} = 0.03$

X : # of components to be examined to get 3 defectives

# of trials for k^{th} success use negative binomial

$X \sim NB(3, 0.03)$

$P(X=x) = \binom{x-1}{2} (0.03)^2 (0.97)^{x-3} (0.03)$

$P(X \ge 6) = 1 - P(X < 6)$

$= 1 - P(X=3) - P(X=4) - P(X=5)$

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X : # of shots for fourth hit

Prob. of hitting the target = 0.7.

marksman 4, 7

$X \sim NB(4, 0.7)$

prob of 7^{th} shot of 4 hit $= P(X=7)$

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X : # of defect

Prob. of item is defective $= \frac{10}{100} = 0.1$

$X \sim B(10, 0.1)$

The machine will not stop when there is no defective product in sample $= P(X=0)$

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Prob. of a person getting into the accident $= \frac{1}{1000}$

# of people insured = 5000

using poisson approximation

$\lambda = np = 5000 \times \frac{1}{1000} = 5$

X : # of people gotten into the accident

$X \sim P(5)$

$\therefore P(X=x) = \frac{e^{-5} \cdot 5^x}{x!}$

$\Rightarrow P(X \le 2) = P(X=0) + P(X=1) + P(X=2)$

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18 prob. of the person making reservation on flight donot show up is $\frac{5}{100} = 0.05$

$\therefore$ prob. of the person making reservation on flight show up is $= 1 - 0.05 = 0.95$

X : # of people show up for the flight

$X \sim B(100, 0.95)$

$\therefore P(X=x) = \binom{100}{x} (0.95)^x (0.05)^{100-x}$

Everyone shows up for flight will get a seat if no. of people show up for flight is less or equal to 95

$P(X \le 95) = 1 - P(X > 95)$

$= 1 - P(X=96) - \dots - P(X=100)$

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19

Prob. of getting double six by rolling a pair of die $= \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$

X : No. of times double six occur

The pair of die is rolled 50 times

$X \sim B(50, \frac{1}{36})$

$\therefore P(X=x) = \binom{50}{x} (\frac{1}{36})^x (\frac{35}{36})^{50-x}$

Prob. for getting a double six at least three times

$= P(X \ge 3) = 1 - P(X < 3)$

$= 1 - P(X=0) - P(X=1) - P(X=2)$

(24) X: # of accidents occurring on a highway each day

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$X \sim P(3)$

$P(X=3) = e^{-3}\frac{3^x}{x!}$

(i) prob. of 3 or more accidents occur today

$P(X \geq 3) = 1 - P(X < 3)$
$= 1 - P(X=0) - P(X=1) - P(X=2)$

(ii) $P(X \geq 3 / X \geq 1) = \frac{P(X \geq 3)}{P(X \geq 1)}$

(25) Prob. of a engine will fail $= 1-p$

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——— operate $= p$

X: # of engines are operative for four engine plane,

Y: ——— two ———

$X \sim B(4, p)$

$Y \sim B(2, p)$

Now, for a successfull flight at least 50% of the engines remains operative

∴ For a four engine plane we need at least 2 engine remain operative & for two engine plane we need at least 1 engine remain operating

ie $P(X \geq 2)$ → for four engine

$P(Y \geq 1)$ → for two eng

Therefor $P(X \geq 2) > P(Y \geq 1)$

$\Rightarrow 1 - P(X < 2) > 1 - P(Y < 1)$

$\Rightarrow 1 - P(X=0) - P(X=1) > 1 - P(Y=0)$

$\Rightarrow \binom{4}{0} p^0 (1-p)^4 + \binom{4}{1} p(1-p)^3 < \binom{2}{0} p^0 (1-p)^2$

$\Rightarrow 1 - 2p + p^2 + 4p - 4p^2 < 1$

$\Rightarrow p > \frac{2}{3}$

(28) $X \sim Geo(p)$

$P(X=k) = (1-p)^{k-1} p = q^{k-1} p \quad (\because q = 1-p)$

$P(X = even) = P(X=2) + P(X=4) + P(X=6) + \cdots$
$= qp + q^3 p + q^5 p + \cdots$
$= p \times \frac{q}{1-q^2} = p \cdot \frac{q}{(1-q)(1+q)} = \frac{q}{1+q}$

& $P(X = odd)$
$= 1 - \frac{q}{1+q}$
$= \frac{1}{1+q}$

(6)

→ n - sample large & prob less — poisson

→ Binom → x no. of success

→ Geo → 1st success

→ Neg. Binom → x^{th} success

(12) $\mu_r = E[X - E(X)]^r = E[X - np]^r$

$= \sum_{x=0}^{n} (x-np)^r \, {}^nC_x \, p^x q^{n-x}$

Diff. w.r.t p we get ↓ Romanovsky formula

$\frac{d\mu_r}{dp} = -nr\sum_x$

(20) $X \sim HG(8, 5, 6)$

$Y \sim HG(12, 2, 6)$

Binomial Dist^n :- It used when there are two possible outcomes (success or failure) in a fixed no. of independent trial.

Poission Dist^n :- It is used to model the no. of events occuring within a fixed interval of times or space, given that these event occur independently of the time since the last event & at a constant rate

Geometric Dist^n :- The no. of trials needed to achieve the first success in a sequence of independent Bernoulli trials, where each trial has the same probability of success.

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