Problem given:-

1. Concentration profile in the gel:-

We have a 1-D gel slab of thickness L.

At $x=0$, the concentration is held at:-

$$C(0,t) = C_0$$

At $x=L$, since the drug is instantaneously consumed, so:-

$$C(L,t) = 0.$$

Initially the gel is drug-free,

$$C(x,0) = 0, \quad 0 < x \leq L.$$

Fick's second law:-

$$\frac{\partial c}{\partial t} = D\frac{\partial^2 c}{\partial x^2}$$

By standard solving, we get the expression:-

$$C(x,t) = C_0\left[1 - \frac{2}{\pi}\sum_{n=0}^{\infty}\frac{1}{(2n+1)}\sin\left(\frac{(2n+1)\pi x}{2L}\right)x \exp\left(\frac{-(2n+1)^2\pi^2 Dt}{4L^2}\right)\right]$$

Time to reach the concentration at the
wound-

$$C(L, t_{eff}) = 0.005 C_0$$

Evaluating the series at $x = L$, $\sin\left(\frac{(2n+1)\pi}{2}\right) = (-1)^n$;

$$C(L, t) = C_0 \frac{4}{\pi} \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} \exp\left(-\frac{(2n+1)^2 \pi^2 D t}{4L^2}\right)$$

~~to~~

$$\frac{4}{\pi} \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} \exp\left(-\frac{(2n+1)^2 \pi^2 D t_{eff}}{4L^2}\right) = 0.005$$

Approximating for $n = 0$, we got—

$$C(L, t) \simeq C_0 \times \frac{4}{\pi} \exp\left(-\frac{\pi^2 D t}{4L^2}\right)$$

$$0.005 = \frac{4}{\pi} \exp\left(\frac{-\pi^2 D t_{eff}}{4L^2}\right)$$

$$t_{eff} = \frac{4L^2}{\pi^2 D} \ln\left(\frac{4}{0.005\pi}\right)$$

$$\Rightarrow \frac{4L^2}{\pi^2 D} \ln\left(800/\pi\right)$$

Let the patch hold a total active mass M.
The instantaneous flux into the would be—

$$J(t) = -D \frac{\partial C}{\partial x}\Big|_{x = L^*}$$

The total mass deposited upto time t is:

$$m(t) = A \int_0^t J(\tau) \, d\tau$$

where A is the cross-sectional area of the gel.

$$m(t_{final}) = M$$

After a short initial transient, the concentration profile becomes almost linear.

$$c(x) \approx c_0 \left(1 - \frac{x}{L}\right) \Rightarrow \frac{\partial c}{\partial x} \approx -\frac{c_0}{L}$$

$$\therefore J_{ss} \approx D \frac{c_0}{L}, \text{ constant in time.}$$

Thus,

$$M = A J_{ss} \, t_{final}$$

$$= A \frac{D c_0}{L} \, t_{final}$$

$$\Rightarrow t_{final} \approx \frac{M L}{A D c_0}$$