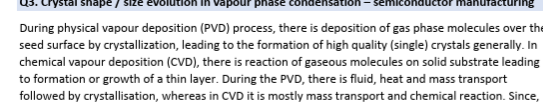
# Mathematical Model for Skin Patch Drug Release

## 1. Introduction

This report presents a mathematical model for a two-layer drug patch designed for constant-rate release of an active ingredient into a skin wound. The patch consists of an active ingredient layer and a gel layer adjacent to the skin, where the active ingredient diffuses through the gel and is instantaneously consumed by the wound.

## Patch Structure

Figure 1: Schematic of the two-layer patch structure.



## 2. Model Formulation

We model diffusion in two one-dimensional layers:  
- Layer 1 (0 ≤ x ≤ L1): Active ingredient reservoir with diffusion coefficient D1.  
- Layer 2 (L1 ≤ x ≤ L1+L2): Gel layer with diffusion coefficient D2.  
The skin at x = L1 + L2 acts as a perfect sink (C = 0). The backing at x = 0 is impermeable.

## 3. Governing Equations

### 3.1 Diffusion in Layers

Layer 1: ∂C₁/∂t = D₁ ∂²C₁/∂x², 0 < x < L₁  
Layer 2: ∂C₂/∂t = D₂ ∂²C₂/∂x², L₁ < x < L₁ + L₂

### 3.2 Boundary Conditions

- x = 0: ∂C₁/∂x = 0 (no flux at backing)  
- x = L₁: Continuity of concentration and flux:  
 • C₁(L₁, t) = C₂(L₁, t)  
 • D₁ ∂C₁/∂x = D₂ ∂C₂/∂x  
- x = L₁ + L₂: C₂ = 0 (perfect sink at skin)

### 3.3 Initial Conditions

C₁(x,0) = C₀ for 0 ≤ x ≤ L₁,  
C₂(x,0) = 0 for L₁ ≤ x ≤ L₁ + L₂.

## 4. Performance Metrics

1. \*\*Lag Time (t\_lag)\*\*: Time until concentration at the skin reaches 0.5% of C₀, solved from the series solution for C₂(L₁+L₂, t).  
2. \*\*Replacement Time (t\_rep)\*\*: Time until total mass M is delivered, given by ∫₀ᵗ J\_s(t) dt = M/A.

## 5. Conclusion

The above formulation provides a complete mathematical framework to predict the release dynamics of the active ingredient from the patch. Numerical evaluation of the eigenvalues and coefficients in the series solutions is required to compute lag and replacement times for specific parameter values.