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B.Tech 1st Semester Special Exam., 2020

(New Course)

MATHEMATICS-I

(Calculus and Linear Algebra)

Time: 3 hours

Full Marks: 70

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Instructions:

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- (i) The marks are indicated in the right-hand margin.
- (ii) There are NINE questions in this paper.
- (iii) Attempt FIVE questions in all.
- (iv) Question No. 1 is compulsory.
- 1. Choose the correct answer of the following (any seven): $2 \times 7 = 14$
 - (a) If

$$Y = \int_0^\infty \frac{x^a}{a^x} dx, \ a > 1$$

then the value of Y is

(i)
$$\frac{\Gamma(a)}{(\log_e a)^a}$$
 (ii) $\frac{\Gamma(a+1)}{(\log_e a)^a}$

(ii)
$$\frac{\Gamma(a+1)}{(\log_e a)^a}$$

(iii)
$$\frac{\Gamma(a+1)}{(\log_e a)^{a+1}}$$
 (iv)
$$\frac{\Gamma(a)}{(\log_e a)^{a+1}}$$

$$(\log_e a)^{a+1}$$

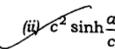
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The area bounded by the axis of x, and the curve and ordinates $y = \cosh \frac{x}{x}$ from x = 0 to x = a is

(i)
$$\cosh \frac{a}{c}$$



- (iii) csinh a
- (iv) None of the above
- Consider the following functions:

1.
$$y = x \sin \frac{1}{x}$$
, $x \ne 0$; and $y = 0$ if $x = 0$

2.
$$y = x^2 \sin \frac{1}{x}$$
, $x \ne 0$; and $y = 0$ if $x = 0$

3.
$$y = x^2 \cos \frac{1}{x}$$
, $x \ne 0$; and $y = 0$ if $x = 0$

4.
$$y = x \cos \frac{1}{x}$$
, $x \ne 0$; and $y = 0$ if $x = 0$

The functions, differentiable at x = 0, are

- (i) 1 and 2
- (ii) 2 and 3
- (iii) 3 and 4
- (iv) 1 and 4

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- For a positive term series $\sum a_n$, the ratio test states that
 - (i) the series converges, if

$$\lim_{n\to\infty}\frac{a_{n+1}}{a_n}>1$$

the series converges, if

$$\lim_{n\to\infty}\frac{a_{n+1}}{a_n}<1$$

(iii) the series converges, if

$$\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=1$$

- (iv) None of the above
- (e) Ιf

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$$\lim_{x\to\infty}\frac{\sin 2x + a\sin x}{x^3} = b$$

where b is finite, then the values of a and b respectively will be

- (ii) (2, 1)
- (iii) (-2, 1)
- (iv) (2, -1)

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The expansion of $\tan x$ in powers of x by Maclaurin's theorem is valid in the interval

(ii)
$$\left(-\frac{3\pi}{2}, \frac{3\pi}{2}\right)$$

(iii)
$$(-\pi, \pi)$$
(iv) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

The value of

$$\lim_{(x, y)\to(k, 0)} \left(1 + \frac{x}{y}\right)^y$$

is

- (i) 1

- (iv) Does not exist
- gradient of the function $f(x, y, z) = \sin(xyz)$, at $(1, -1, \pi)$, is
 - $(\hat{i}\hat{j}, \pi(\hat{i}-\hat{j}+\hat{k})$
 - (ii) $\pi(\hat{i} + \hat{j} + \hat{k})$
 - (iii) $(\hat{i} + \hat{j} + \hat{k})$
 - (iv) $(\pi \hat{i} \rightarrow \pi \hat{j} + \hat{k})$

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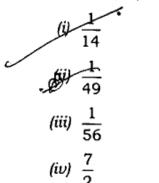
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(i) If det(A) = 7, where

$$A = \begin{bmatrix} a & b & c \\ 1 & 1 & g \\ g & \omega & 1 \end{bmatrix}$$

then $det(2A)^{-1}$ is equal to



- (j) If 3x+2y+z=0, x+4y+z=0 and 2x+y+4z=0 be a system of equations, then https://www.akubihar.com
 - it is inconsistent
 - (ii) it has only the trivial solution (0, 0, 0)
 - (iii) it can be reduced to a single equation and so a solution does not exist
 - (iv) the determinant of the matrix of coefficients is zero

2. (a) Evaluate

$$\int_0^\infty \log \left(x + \frac{1}{x} \right) \frac{dx}{1 + x^2}$$

(b) Find the volume of the solid generated by rotating completely about the x-axis where the area enclosed between $y^2 = x^3 + 5x$ and the line x = 2 and x = 4 about its major axis.

3. (a) Find the maximum value of the function

$$f(x) = \frac{x}{1 + x \tan x}$$

(b) It is given that Rolle's theorem holds for the function $f(x) = x^3 + bx^2 + cx$, $1 \le x \le 2$ at the point $x = \frac{4}{3}$. Find the values of b and c.

4. (a) Discuss the convergence of the sequence whose n-th term is

$$a_n = \frac{(-1)^n}{n} + 1$$

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(b) Test the convergence of the following series:

$$x^2 + \frac{2^2x^4}{3.4} + \frac{2^24^2x^6}{3.4.5.6} + \frac{2^24^26^2x^8}{3.4.5.6.7.8} \dots$$

5. (a) Find the Fourier series expansion of the function $f(x) = \{x^2, -2 \le x \le 2\}$. Hence deduce that

$$\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots$$

(b) Find the Fourier cosine series and Fourier sine series of the following function in given interval:

$$f(x) = \begin{cases} x, & 0 < x < 2 \\ 2, & 2 \le x < 4 \end{cases}$$

6 (a) Discuss continuity of the following function at the point (0, 0):

function at the point (0, 0):
$$f(x, y) = \begin{cases} \frac{x^2y^2}{(x^3 + y^3)}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

(b) Find the maximum value of xyz under the constraints $x^2 + z^2 = 1$ and y - x = 0.

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7. (a) Find the value of

$$\lim_{x\to\infty}\left(\frac{x+4}{x+2}\right)^{x+3}$$

(b) Find the equation of the tangent plane to the surface $x^2 - 3y^2 - z^2 = 2$, at the point (3, 1, 2).

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8. Find the eigenvalues and eigenvectors of the following matrix :

$$\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

9 (a) Verify Cayley-Hamilton theorem for the matrix

$$\begin{bmatrix} 0 & 0 & 1 \\ 3 & 1 & 0 \\ -2 & 1 & 4 \end{bmatrix}$$

(b) Determine the range of the following linear: transformation. Also find the rank of T, where it exists. $T: V_2 \rightarrow V_3$ defined by

$$T(x_1, x_2) = (x_1, x_1 + x_2, x_2)$$
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