# Code : 103102

## B.Tech 1st Semester Exam., 2021
## (New Course)

## MATHEMATICS—I

**( Calculus and Differential Equation )**

*Time* : 3 hours

*Full Marks* : 70

*Instructions* :

*(i) The marks are indicated in the right-hand margin.*

*(ii) There are **NINE** questions in this paper.*

*(iii) Attempt **FIVE** questions in all.*

*(iv) Question No. **1** is compulsory.*

**1.** Choose the correct answer (any *seven*) : 2×7=14

(a) Let $f(x) = \sin 2x$, $0 \le x \le \frac{\pi}{2}$ and $f'(c) = 0$ for $c \in \left(0, \frac{\pi}{2}\right)$. Then $c$ is equal to

(i) $\frac{\pi}{4}$

(ii) $\frac{\pi}{3}$

(iii) $\frac{\pi}{6}$

(iv) None of the above





(b) $\tan\left(\frac{\pi}{4} + x\right)$ when expanded in Taylor's series gives

(i) $1 + x + x^2 + \frac{4}{3}x^3 + \ldots$

(ii) $1 + 2x + 2x^2 + \frac{8}{3}x^3 + \ldots$

(iii) $1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \ldots$

(iv) None of the above

(c) The differential equation whose linearly independent solutions are $\cos 2x$, $\sin 2x$ and $e^{-x}$ is

(i) $(D^3 + D^2 + 4D + 4)y = 0$

(ii) $(D^3 - D^2 + 4D - 4)y = 0$

(iii) $(D^3 + D^2 - 4D - 4)y = 0$

(iv) $(D^3 - D^2 - 4D + 4)y = 0$

(d) For what value of $a$

$$\lim_{x \to 1} \left( \frac{-ax + \sin(x-1) + a}{x + \sin(x-1) - 1} \right)^{\frac{1-x}{1-\sqrt{x}}} = \frac{1}{4}$$

(i) 1 (ii) 2

(iii) 3 (iv) 4

(e) The evolute of a cycloid is

(i) circle

(ii) another cycloid

(iii) an ellipse

(iv) None of the above

(f) The value of improper integral $\int_0^\infty \sqrt{x}\, e^{-x^2} dx$ is

(i) $\sqrt{\pi}$

(ii) $\frac{\sqrt{\pi}}{2}$

(iii) $\Gamma\left(\frac{3}{8}\right)$

(iv) $\frac{1}{2}\Gamma\left(\frac{3}{4}\right)$

(g) Radius of convergence of $\sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}}$ is

(i) 1

(ii) −1

(iii) 0

(iv) $\infty$

( Turn Over )



(h) If $J_n$ is the Bessel's function of first kind, then the value of $J_{\frac{3}{2}}$ is

(i) $\sqrt{\frac{2}{\pi x}}\left(\frac{\cos x}{x} - \sin x\right)$

(ii) $\sqrt{\frac{2}{\pi x}}\left(\frac{\sin x}{x} - \cos x\right)$

(iii) $\sqrt{\frac{2}{\pi x}} \sin x$

(iv) $\sqrt{\frac{2}{\pi x}} \cos x$

(i) The Fourier series of the periodic function $f(x) = x + x^2$, $-\pi < x \le \pi$ at $x = \pi$ converges to

(i) $\pi$

(ii) $2\pi$

(iii) $\pi^2$

(iv) $\pi + \pi^2$

(j) The singular solution of $p = \log(px - y)$ is

(i) $y = x(\log x - 1)$

(ii) $y = x \log x - 1$

(iii) $y = \log x - 1$

(iv) $y = x \log x$



( Continued )

**2.** *(a)* Show that the equation to the evolutes of the curve $x^{2/3} + y^{2/3} = a^{2/3}$ is

$$(x+y)^{2/3} + (x-y)^{2/3} = 2a^{2/3}$$ 7

*(b)* Show that

$$\Gamma(2n) = \frac{2^{2n-1}}{\sqrt{\pi}}\Gamma\left(n+\frac{1}{2}\right)\Gamma(n) \text{ and }$$

$$\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right) = \pi\sqrt{2}$$ 7

**3.** *(a)* Obtain the Taylor's polynomial expression of the function $f(x) = \sin(x)$ about the point $x = \frac{\pi}{4}$. Show that the error term tends to zero as $n \to \infty$ for any real $x$. Hence, write the Taylor's series expansion of $f(x)$. 7

*(b)* Find the Fourier series expansion of the function $f(x) = \pi + x$, $-\pi \le x \le \pi$. Hence, show that

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$$ 7

**4.** *(a)* Find the power series solution about $x = 0$ of the differential equation

$$(1-x^2)y'' - 2xy' + 2y = 0$$ 7



( Turn Over



*(b)* Find the directional derivative of $d(x, y, z) = 2x^2 + y^2 + z^2$ at $(1, 2, 3)$ in the direction of the line $\frac{x}{3} = \frac{y}{4} = \frac{z}{4}$. 7

**5.** *(a)* Find the smallest distance and largest distance between the points $P$ and $Q$, such that $P$ lies on the plane $x + y + z = 2a$ and $Q$ lies on the sphere $x^2 + y^2 + z^2 = a^2$, where $a$ is any constant. 7

*(b)* Find the volume of ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$ 7

**6.** *(a)* Evaluate the integral $\iiint_T z\,dx\,dy\,dz$, where $T$ is the region bounded by the cone $z^2 = x^2 \tan^2\alpha + y^2 \tan^2\beta$ and the plane $z = 0$ to $z = h$ in the first octant. 7

*(b)* Verify Stokes' theorem for the vector field $\mathrm{v} = (3x - y)\hat{i} - 2yz^2\hat{j} - 2y^2z\hat{k}$, where $S$ is the surface of the sphere $x^2 + y^2 + z^2 = 16$, $z > 0$. 7

**7.** *(a)* Solve the differential equation

$$(2xy + x^2)y' = 3y^2 + 2xy$$ 7

*(b)* Find the general solution and singular solution of the Clairaut's equation $y = xy' - (1/y')$. 7

**8.** *(a)* State and prove the orthogonal property of Legendre polynomials. 7

*(b)* Show that

$$J_0^2 + 2(J_1^2 + J_2^2 + J_3^2 + \dots) = 1$$ 7

**9.** *(a)* Find the general solution of the partial differential equation

$$(3 - 2yz)\,p + x(2z - 1)q = 2x(y - 3)$$

*(b)* Find the complete integral of the partial differential equation

$$2\sqrt{p} + 3\sqrt{q} = 6x + 2y$$

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