Doubt :- Q.8

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CSE-10

AFL ASSIGNMENT-2 (Due 8th April)

1) Convert the grammar $S \to aSS|a|b$ into Chomsky normal form.

Ans: Here;

$$S \to aSS|a|b$$

Since S appears in RHS; we add a ~~new~~ state S' and S' → S is added to production.

$$S' \to S$$
$$S \to aSS|a|b$$

We remove unit production $S' \to S$.

$$S' \to aSS|a|b$$
$$S \to aSS|a|b$$

We introduce new variables.

$$S' \to A(a)B|a|b$$

$$S \to A(a)B|a|b$$

$$A(a) \to a$$

$$B \to SS$$

2) Convert the grammar $S \rightarrow aSB \mid Sab \mid ab$ into Chomsky normal form.

Ans: Here;

$$S \rightarrow aSb \mid Sab \mid ab$$

Since; S appears in RHS; we add a new state S'; such that :-

$$S' \rightarrow S$$
$$S \rightarrow aSb \mid Sab \mid ab$$

We need to remove unit production.

$$S' \rightarrow aSb \mid Sab \mid ab$$

We introduce new variables.

$$S' \rightarrow A(a)\ SB(b) \mid SA(a)B(b) \mid A(a)\ B(b)$$

$$S \rightarrow A(a)\ SB(b) \mid SA(a)B(b) \mid A(a)B(b)$$

$$A(a) \rightarrow a$$

$$B(b) \rightarrow b$$

Introducing additional variables to get in normal form.

$$S' \rightarrow A(a)\ D_1 \mid S\ D_2 \mid A(a)\ B(b)$$

$S \to A(a)\, D_1 \mid S D_2 \mid A(a)\, B(b)$

$A(a) \to a$

$B(b) \to b$

$D_1 \to S\, B(b)$

$D_2 \to A(a)\, B(b)$

3) Transform the grammar

$S \to aSaaA \mid A \;,\; A \to abA \mid bb$

into Chomsky normal form.

Ans: Here;

$S \to aSaaA \mid A$
$A \to abA \mid bb$

Since; S appears in RHS; we add a new state S'; such that:-

$S' \to S$
$S \to aSaaA \mid A$
$A \to abA \mid bb$

We remove unit production

$S' \rightarrow aSaaA \mid abA \mid bb$
$S \rightarrow aSaaA \mid abA \mid bb$
$A \rightarrow abA \mid bb$

We add new variables.

$S' \rightarrow A'SA'A'A \mid A'B'A \mid B'B'$

$S \rightarrow A'SA'A'A \mid A'B'A \mid B'B'$

$A \rightarrow A'B'A \mid B'B'$

$A' \rightarrow a$

$B' \rightarrow b$

Introducing new variables to get normal form.

$S' \rightarrow A'SA'D_1 \mid A'D_2 \mid B'B'$

$S \rightarrow A'SA'D_1 \mid A'D_2 \mid B'B'$

$D_1 \rightarrow A'A$

$A \rightarrow A'D_2 \mid B'B'$

$D_2 \rightarrow B'A$

$A' \to a$

$B' \to b$

Further;

$S' \to A'S D_3 \mid A'D_2 \mid B'B'$

$S \to A'S D_3 \mid A'D_2 \mid B'B'$

$D_3 \to A'D_1$

$D_1 \to A'A$

$A \to A'D_2 \mid B'B'$

$A' \to a$

$B' \to b$

Finally;

$S' \to A'D_4 \mid A'D_2 \mid B'B'$

$S \to A'D_4 \mid A'D_2 \mid B'B'$

$D_4 \to S D_3$

$D_3 \to A'D_1$

$D_1 \to A'A$

$A \to A'D_2 \mid B'B'$

$A' \to a$

$B' \to b$

4) Transform the grammar with productions

$S \to baAB,$

$A \to baB \mid \lambda$

$B \to BAa \mid A \mid \lambda$

into Chomsky normal form.

Ans: Here;

We remove null production

$S \to baAB \mid baB \mid baA \mid ba$

$A \to bAB \mid bB \mid bA \mid b$

$b \to BAa \mid A \mid Ba \mid Aa \mid a$

We remove unit production;

$S \to baAB \mid baB \mid baA \mid ba \mid baAA \mid baA$

$A \to bAB \mid bB \mid bA \mid b \mid bAA \mid bB$

$B \to BAa \mid Ba \mid Aa \mid a \mid AAa$

We introduce new ~~variab~~ variable.

$S \to XYAB \mid YXB \mid YXA \mid bYX \mid YXAA \mid YXA$

$A \to YAB \mid YB \mid YA \mid b \mid YAA \mid YB$

$B \to BAX \mid BY \mid BX \mid a \mid AAX$

$X \to a$

$Y \to b$

Then;

$S \to XYD \mid YD_1 \mid YD_2 \mid YX \mid YXD_3 \mid YD_4$

$A \to YD \mid YB \mid YA \mid b \mid YD_3 \mid YB$

$D \to AB$

$D_1 \to XB$

$D_3 \to AA$

$D_2 \to XA$

$B \rightarrow BD_4 \mid BY \mid BX \mid a \mid AD_4$

$D_4 \rightarrow AX$

$X \rightarrow a$

$Y \rightarrow b$

<u>Finally</u>;

$S \rightarrow XE \mid YD_1 \mid YD_2 \mid YX \mid YE_1 \mid YD_4$

$E \rightarrow YD$

$E_1 \rightarrow XD_3$

$A \rightarrow YD \mid YB \mid YA \mid b \mid YD_3 \mid YB$

$D \rightarrow AB$

$D_1 \rightarrow XB$

$D_3 \rightarrow AA$

$D_2 \rightarrow XA$

$B \rightarrow BD_4 \mid BY \mid BX \mid a \mid AD_5$

$D_4 \rightarrow AX$

$X \to a$
$Y \to b$

5) Convert the grammar

$S \to AB | aB,$
$A \to abb | \lambda,$
$B \to bbA$

into Chomsky normal form.

Ans: Here;

We remove λ production,

$S \to AB | aB | B$
$A \to abb$
$B \to bbA | bb$

Now, we remove unit production

$S \to AB | aB | bbA | bb$

$A \to abb$

$B \to bbA | bb$

We introduce new variable;

$$S \to AB \mid XB \mid YYA \mid YY$$
$$A \to XYY$$
$$B \to YYA \mid YY$$
$$X \to a$$
$$Y \to b$$

Further;

$$S \to AB \mid XB \mid YD_1 \mid YY$$
$$A \to XD_2$$
$$B \to YD_1 \mid YY$$
$$D_2 \to YY$$
$$D_1 \to YA$$
$$X \to a$$
$$Y \to b$$

6) Let $G = (V, T, S, P)$ be any context-free grammar without any λ-productions or unit-productions. Let R be the maximum number of symbols on the right of any ~~produc~~ production in P. Show that there is an equivalent grammar in Chomsky normal form with no more than $(k-1)|P| + |T|$ production rules.

Ans: Here; $G = (V, T, S, P)$ is context-free grammar without any λ-productions or unit-productions. Let (k) be the maximum number of symbols on the right of any prediction in P.

Here; we are using the definition of Chomsky normal form; the grammar with initial symbol S, terminal symbols a and b, and productions $S \to Sa \mid bb$ would be a counter example to the theorem.

Clearly; $|P| = |T| = k = 2$;

so; $(k-1)|P| + |T| = 4$.

However; the smallest equivalent grammar in (this version of) Chomsky normal form has six productions

$S \to BB \mid XA$
$X \to BB \mid XA$
$A \to a$
$B \to b$

If we allow S to appear on the right hand side; we can reduce this to four; as in the theorem:-

$S \to BB \mid SA$
$A \to a$
$B \to a$

Hence; proved.

Hence; that there is an equivalent grammar in Chomsky normal form with no more than $(k-1)|P| + |T|$ production rules.

7) Draw the dependency graph for the grammar in Exercise 5.

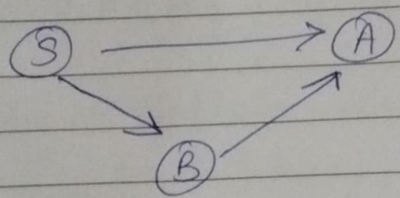
Soln:-

The grammar is:-

$S \rightarrow AB \mid aB$

$A \rightarrow abb \mid \lambda$

$B \rightarrow bbA$



10) Convert the grammar:-

$S \rightarrow aSb \mid bSa \mid a \mid b \mid ab$

into ~~th~~ Greibach normal form.

Ans: Here;

$S \rightarrow aSb \mid bSa \mid a \mid b \mid ab$

Since; they are not in CNF; we convert them to CNF. S occurs in RHS; so we use new production $S' \rightarrow S$.

Sol; $S' \to aSb \mid bSa \mid a \mid b \mid ab$

$S \to aSb \mid bSa \mid a \mid b \mid ab$

We introduce new variable;

$S' \to XSY \mid YSX \mid a \mid b \mid XY$

$S \to XSY \mid YSX \mid a \mid b \mid XY$

$X \to a$

$Y \to b$

We introduce additional variable.

$S' \to XE_1 \mid YE_2 \mid a \mid b \mid XY$

$S \to XE_1 \mid YE_2 \mid a \mid b \mid XY$

$X \to a$

$Y \to b$

$E_1 \to SY$

$E_2 \to SX$

This is in CNF. Now; we rename variables.

$S' \to A_1$

$X \to A_2$

$E_1 \to A_3$

$Y \to A_4$

$E_2 \to A_5$

$S \to A_6$

So; the grammar becomes

$A_1 \to A_2A_3 \mid A_4A_5 \mid a \mid b \mid A_2A_4$

$A_6 \to A_2A_3 \mid A_4A_5 \mid a \mid b \mid A_2A_4$

This doesn't follow rule $i < j$ where i is in LHS & j in RHS.

$A_2 \to a$

$A_4 \to b$

$A_3 \to A_6A_4$

$A_5 \to A_6A_2$

Now;

$A_1 \to A_2A_3 \mid A_4A_5 \mid a \mid b \mid A_2A_4$

$A_6 \to aA_3 \mid bA_5 \mid a \mid b \mid aA_4$

$A_2 \to a$

$A_4 \to b$

$A_3 \rightarrow A_6 A_4$

$A_5 \rightarrow A_6 A_2$

This is the required Greibach normal form.

11) Convert the following grammar into GNF.

$S \rightarrow aSB \mid ab \mid bb$

Here;

$S \rightarrow aSb \mid ab \mid bb$

It is not in CNF; so we first convert it to CNF.

S occurs in RHS also; ~~so~~ we introduce new production $S' \rightarrow S$.

So; $S' \rightarrow aSb \mid ab \mid bb$

$S \rightarrow aSb \mid ab \mid bb$

We introduce new variable.

$S' \rightarrow XSY \mid XY \mid YY$

$S \rightarrow XSY \mid XY \mid YY$

$X \to a$
$Y \to b$

We introduce additional variables.

$S' \to XE_1 \mid XY \mid YY$

$S \to XE_1 \mid XY \mid YY$

$X \to a$

$Y \to b$

$E_1 \to SY$

It is in CNF. Now; we rename variables in order.

$S' \to A_1$
$X \to A_2$
$E_1 \to A_3$
$Y \to A_4$
$S \to A_5$

So; the grammar becomes;

$A_1 \to A_2 A_3 \mid A_2 A_4 \mid A_4 A_4$

$A_5 \to A_2 A_3 \mid A_2 A_4 \mid A_4 A_4$ (This doesnt follow rule.)

$A_2 \to a$

$A_4 \to b$

$A_3 \to A_5 A_4$

Then ;

$A_1 \to A_2 A_3 \mid A_2 A_4 \mid A_4 A_4$

$A_5 \to aA_3 \mid aA_4 \mid bA_4$

$A_2 \to a$
$A_4 \to b$
$A_3 \to A_5 A_4$

This is the required greibach normal form.

12) Convert the grammar $S \to ab \mid aS \mid aaS \mid aSS$ in GNF.

Ans: Here ;

$S \to ab \mid aS \mid aaS \mid aSS$

It is not in CNF. We introduce new production S'; such that :-

$S' \to S$

$S \to ab \mid aS \mid aaS \mid aSS$

We remove unit production.

$S' \to ab \mid aS \mid aaS \mid aSS$

$S \to ab \mid aS \mid aaS \mid aSS$

We introduce new variable;

$S' \to XY \mid XS \mid XXS \mid XSS$
$S \to XY \mid XS \mid XXS \mid XSS$

$X \to a$
$Y \to b$

Then;

$S' \to XY \mid XS \mid XE_1 \mid XE_2$
$S \to XY \mid XS \mid XE_1 \mid XE_2$

$X \to a$
$Y \to b$
$E_1 \to XS$
$E_2 \to SS$

This is in CNF.

We rename the variables;

$S' \to A_1$
$X \to A_2$
$Y \to A_3$

$S \rightarrow A_4$

$E_1 \rightarrow A_5$

$E_2 \rightarrow A_6$

The production becomes;

$A_1 \rightarrow A_2A_3 \mid A_2A_4 \mid A_2A_5 \mid A_2A_6$

$A_4 \rightarrow A_2A_3 \mid A_2A_4 \mid A_2A_5 \mid A_2A_6$

$A_2 \rightarrow a$

$A_3 \rightarrow b$

$A_5 \rightarrow A_2A_4$

$A_6 \rightarrow A_4A_4$

Then;

$A_1 \rightarrow A_2A_3 \mid A_2A_4 \mid A_2A_5 \mid A_2A_6$

$A_4 \rightarrow aA_3 \mid aA_4 \mid aA_5 \mid aA_6$

$A_2 \rightarrow a$

$A_3 \rightarrow b$

$A_5 \rightarrow aA_4$

$A_6 \rightarrow aA_3A_4 \mid aA_4A_4 \mid aA_5A_4 \mid aA_6A_4$

This is the required GNF.

13) Convert the grammar

$S \to ABb \mid a \mid b$

$A \to aaA \mid B$

$B \to bAb$ into Greibach Normal form.

Ans: Here;

$S \to ABb \mid a \mid b$
$A \to aaA \mid B$
$B \to bAa$

It is not in GNF.

We remove unit production.

$S \to ABb \mid a \mid b$
$A \to aaA \mid bAb$
$B \to bAb$

Introducing new variables;

$S \to ABY \mid X \mid b$

$A \to XXA \mid YAY$

$B \to YAY$
$X \to a$
$Y \to b$

Then; $S \to AE_1 \mid a \mid b$
$A \to XE_2 \mid YE_3$
$B \to YE_3$
$E_1 \to BY$
$E_2 \to XA$
$E_3 \to AY$
$X \to a$
$Y \to b$

This is in CNF.

We rename variables;

$S \to F_1$
$A \to F_2$
$E \to F_3$
$X \to F_4$
$E_2 \to F_5$
$Y \to F_6$
$E_3 \to F_7$
$B \to F_8$

Then;

$F_1 \to F_2F_3 \mid a \mid b$

$f_2 \to f_4 f_5 \mid f_6 f_7$

$f_8 \to f_6 f_7$

$f_3 \to f_8 f_6$

$f_5 \to f_4 f_2$

$f_7 \to f_2 f_6$

$f_4 \to a$

$f_6 \to 6$

<u>Then</u>;

$f_1 \to f_2 f_3 \mid a \mid b \Rightarrow f_1 \to a f_5 f_3 \mid b f_7 f_3 \mid a \mid b$

$f_2 \to a f_5 \mid b f_7$

$f_8 \to b f_7$

$f_3 \to b f_7 f_6$

$f_5 \to a f_2$

$f_7 \to a f_5 f_6 \mid b f_7 f_6$

$f_4 \to a$

$f_6 \to b$

This is the required GNF.

14) Can every linear grammar be converted to a form in which all productions look like $A \to ax$; where $a \in T$ and $x \in V \cup \{\lambda\}$

Ans) Since; a linear grammar is a special case of context-free grammar ; the answer is yes.

Yes; every linear grammar can be converted to a form in which all productions look like $A \to ax$; where $a \in T$ and $x \in V \cup \{\lambda\}$.

Here; T is the set of terminals;

V is the set of non-terminals;

λ is empty sequence.

A simpler transformation:—

- Any rule $X \to a_1 a_2 \ldots a_k Y$ you transform them in k rules :—

1) $X \to a_1 X_1 Y$

2) ....

3) $X_{i-1} \to a_i X_i$

4) $X_{k-1} \to a_k Y$

- Any rule $X \to aYb$ should be transformed in two rules :-

1) $X \to aYY_1$

2) $Y_1 \to b$

; where the capital letters belong to V and the small letters to the alphabet (terminals).

9) Show that for every context-free grammar $G = (V, T, S, P)$; there is an equivalent one in which all productions have the form:-

$$A \to aBC;$$

or

$$A \to \lambda;$$

where $a \in \Sigma \cup \{\lambda\}$, $A, B, C \in V$

Ans: For every context-free grammar; $G = \{V, T, S, P\}$; there is an equivalent grammar in which productions are of form:-

$A \rightarrow aBC$
$A \rightarrow d$

Here; $d \rightarrow a$ are terminals
A, B, C are vertices.

Let us take an example $S \rightarrow ABCD$
$A \rightarrow a$
$B \rightarrow bB / c$
$D \rightarrow f$

Apply 3 steps to convert into the following form:-

(i) Substitution

(ii) Left Recursion Elimination

(iii) Renaming

★ Substitution, A in S

$S \rightarrow ABCD$
$A \rightarrow a$
$B \rightarrow bB$

$D \rightarrow f$
$B \rightarrow e$

★ Left Recursion Elimination

$S \rightarrow aBX_eD$
$A \rightarrow a$
$B \rightarrow bB$
$B \rightarrow e$
$D \rightarrow f$
$X_e \rightarrow e$

We have eliminated the $B \rightarrow bB$ which is a left recursion using X_e vertices & we have taken another production $X_e \rightarrow e$.

Using these steps; we can convert any grammar into the required form.