

MAJOR PROJECT REPORT

Data Clustering using Gravitational Search Algorithm

BACHELOR OF TECHNOLOGY IN INFORMATION TECHNOLOGY

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CANDIDATE’S DECLARATION

We, Animish Sharma (2K21/IT/27), Ankit Bansal (2K21/IT/28) and Anmol Sharma (2K21/IT/34), are students of B.Tech. Seventh Semester in the Department of Information Technology. We hereby state that the Major Project titled “Data Clustering Using Optimized Gravitational Search Algorithm” has been submitted by us to the Department of Information Technology at Delhi Technological University, Delhi. This project serves as a partial fulfilment of the requirements for the completion of the Bachelor of Technology degree.

Place: Delhi

Date:

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CERTIFICATE

I do hereby attest that the undertaking entitled “Data Clustering using Gravitational Search Algorithm” submitted by the students, namely ANIMISH SHARMA (2K21/IT/27), ANKIT BANSAL (2K21/IT/28), ANMOL SHARMA (2K21/IT/34) of the esteemed Department of Information Technology at Delhi Technological University, Delhi, has been accomplished under by tutelage. The present project work stands as a testament to the students’ academic proficiency and research acumen, and it has been submitted as a partial fulfilment of the requirement for the conferment of the degree of Bachelor of Technology.

Place: Delhi
Date:

Dr. Varsha Sisaudia
(Supervisor)

ABSTRACT

Data clustering is a fundamental task in data mining and machine learning, aimed at grouping similar data points into clusters while ensuring high intra-cluster similarity and low inter-cluster similarity. This project explores the application of the **Gravitational Search Algorithm (GSA)**, a nature-inspired metaheuristic optimization technique, for solving clustering problems. GSA leverages the concept of Newtonian gravity, where agents (potential solutions) attract each other based on their masses (fitness) and positions in the solution space, effectively balancing exploration and exploitation.

The proposed approach formulates the clustering problem as an optimization task, where the objective is to maximize the **Silhouette Score**, a widely used metric for evaluating clustering quality. To further enhance performance, techniques such as **Simulated Annealing (SA)** and **Age Diversity** are integrated into the GSA framework to improve local search capability and convergence speed. These hybridized methods ensure robust exploration of the search space while refining solutions for better clustering accuracy.

Experiments were conducted on real-world datasets, and the performance of the GSA-based clustering algorithm was evaluated in terms of clustering quality, convergence speed, and computational efficiency. The results demonstrate the effectiveness of the proposed algorithm in discovering optimal or near-optimal clusters, outperforming traditional clustering techniques such as k-means and hierarchical clustering in challenging scenarios. Additionally, performance metrics, including fitness progression over iterations, highlight the algorithm's ability to adapt to diverse datasets and optimize clustering solutions effectively.

This project showcases the potential of gravitational-inspired optimization in data clustering, providing a flexible and scalable solution to tackle high-dimensional and complex datasets.

ACKNOWLEDGEMENT

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We would also like to thank our peers for their encouragement and assistance, which helped us to overcome various obstacles during the project.

Finally, we would like to express our thanks to the authorities of Delhi Technological University for providing us with the resources and infrastructure necessary to complete this major-project.

Place: Delhi

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List Of Nomenclatures, Symbols & Abbreviations

Nomenclature, Symbols or Abbreviations	Referred to
GSA	Gravitational Search Algorithm
DM	Data Mining
PSO	Particle Swarm Optimization
SA	Simulated Annealing
GA	Genetic Algorithm
DBSCAN	Density-Based Spatial Clustering of Applications with Noise
KNN	K-Nearest Neighbors
QoC	Quality of Clustering
IoC	Intra-cluster Similarity
EoC	Inter-cluster Similarity
SSE	Sum of Squared Errors
PCA	Principal Component Analysis
CPU	Central Processing Unit

Introduction

Objective:

The goal is to optimize the Gravitational Search Algorithm (GSA) for **data clustering** by integrating Simulated Annealing (SA) and using the Silhouette Score as the fitness metric. This hybrid approach aims to improve performance in solving complex optimization problems related to clustering, enhancing convergence speed, solution quality, and robustness in finding optimal or near-optimal clusters.

Problem Description:

The Gravitational Search Algorithm (GSA) is a nature-inspired optimization algorithm based on the law of gravity and mass interactions. While GSA demonstrates significant potential, its current form often suffers from slow convergence, poor exploration capabilities, and a lack of balance between exploration and exploitation. These limitations hinder its effectiveness in solving high-dimensional, complex, or multimodal optimization problems, including **data clustering** tasks. The optimization of GSA aims to address these challenges by refining its core mechanisms to enhance clustering accuracy and computational efficiency.

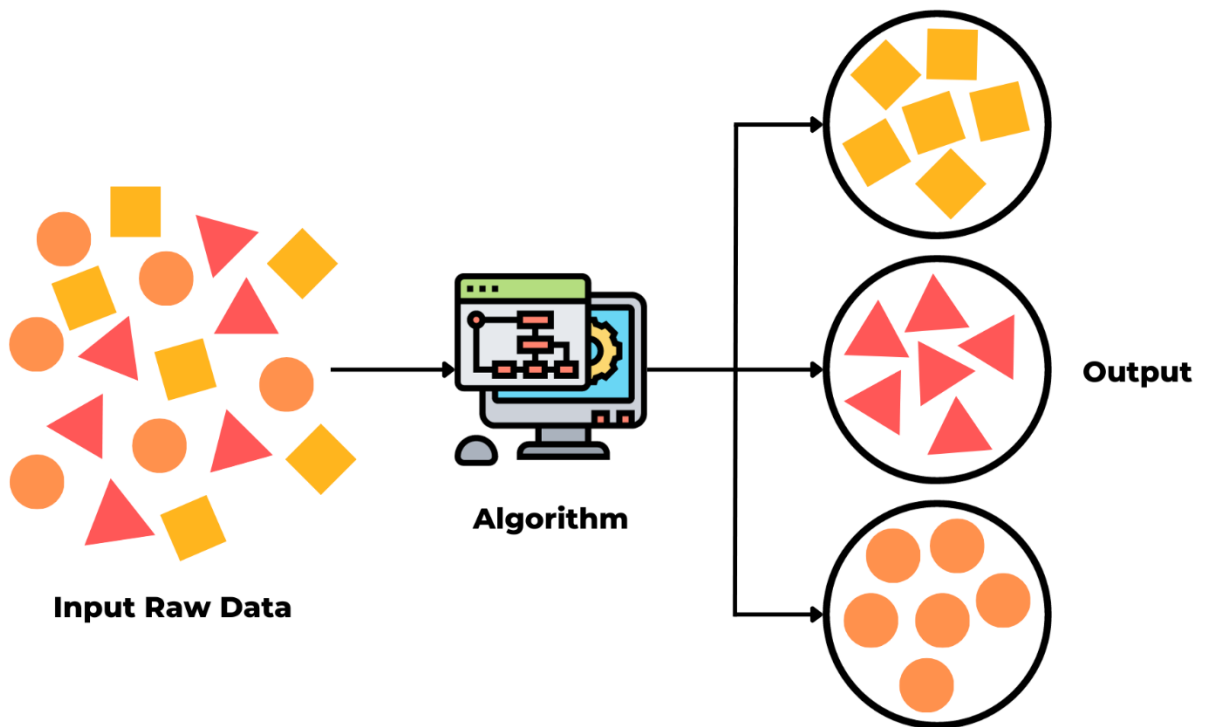


Figure 1

Task:

The task is to propose a modified version of the Gravitational Search Algorithm (GSA) for **data clustering** by tuning its parameters and incorporating enhancements that improve its performance across various clustering tasks. The problem requires identifying and applying modifications to the algorithm to achieve better clustering results in terms of the following:

1. **Convergence Speed:**

Reducing the time required to achieve optimal or near-optimal clusters.

2. **Clustering Quality:**

Enhancing the ability of the algorithm to form well-defined clusters, improving intra-cluster similarity and minimizing inter-cluster similarity, as measured by metrics like the Silhouette Score.

3. **Exploration vs. Exploitation:**

Striking a better balance between exploring the search space to avoid premature convergence and refining cluster centroids for more precise clustering results.

4. **Robustness and Stability:**

Ensuring the algorithm performs consistently well across various datasets and remains stable in diverse scenarios, including high-dimensional or noisy data clustering tasks.

Optimization Variables:

1. **Algorithm Parameters:**

- Gravitational constant
- Inertia weight
- Acceleration coefficients
- Mass scaling factors

2. **Algorithm Structure:**

- Modifying the way agents (centroids) are updated (e.g., applying adaptive gravitational laws or velocity updates).
- Introducing dynamic mechanisms for parameter adjustments during the clustering process.

3. **Hybridization or Algorithmic Integration:**

- Combining GSA with other techniques, such as **Simulated Annealing (SA)**, to improve local search and convergence for clustering problems.

Constraints:

- The performance improvements should demonstrate statistically significant gains over the standard GSA in terms of convergence speed and clustering quality, validated using metrics such as the Silhouette Score and computational time.
-

Expected Outcomes:

The expected outcome is a modified version of the Gravitational Search Algorithm specifically tailored for **data clustering** that achieves:

- Faster convergence to optimal or near-optimal clusters.
- Improved clustering quality, with higher intra-cluster similarity and lower inter-cluster similarity.
- Robust and stable performance across diverse datasets, including high-dimensional and noisy data.

The optimized algorithm should be versatile, scalable to large datasets, and applicable to a wide range of clustering challenges in real-world scenarios.

Literature Review

The **Gravitational Search Algorithm (GSA)** is a nature-inspired metaheuristic optimization algorithm based on Newton's law of gravity and motion. It simulates the interaction of masses (solutions) influenced by gravitational forces in a search space to find optimal solutions. However, GSA often suffers from premature convergence and slow exploration. To overcome this, **Simulated Annealing (SA)** is incorporated into GSA to improve its search ability and avoid local minima.

Key Features of Optimized GSA (GSA-SA):

1. **Hybridization with SA:** SA introduces a probabilistic mechanism to accept inferior solutions temporarily, enabling better exploration and exploitation balance.
2. **Optimization of Convergence:** SA's cooling schedule improves GSA's performance by escaping local minima and accelerating convergence.
3. **Enhanced Global Search:** The hybrid algorithm combines GSA's search capabilities with SA's stochastic nature for enhanced global optimization.

Formulae in GSA and GSA-SA:

a. Force Calculation in GSA:

The gravitational force $F_{ij}(t)$ between two masses i and j at time t is:

$$F_{ij}(t) = \frac{G(t) \cdot M_i(t) \cdot M_j(t)}{R_{ij}^2(t)} + \epsilon$$

Where:

- $G(t)$: Gravitational constant (decreases with iterations).
- $M_i(t)$: Mass of the i^{th} agent.
- $R_{ij}(t)$: Euclidean distance between agents i and j .
- ϵ : Small constant to avoid division by zero.
- r_{ij} : Unit vector between i and j .

b. Velocity and Position Update:

$$v_i(t + 1) = v_i(t) + a_i(t)$$

Where $a_i(t)$ is acceleration computed from the net force acting on agent i .

c. SA Acceptance Criterion:

To incorporate SA into GSA, a solution s' is accepted based on:

$$P = \begin{cases} 1 & \text{if } \Delta f > 0 \\ e^{\frac{\Delta f}{T}} & \text{if } \Delta f < 0 \end{cases}$$

Where:

- $\Delta f = f(s') - f(s)$: Difference in objective function values.
- T : Temperature (decreases gradually).

d. Temperature Update in SA:

$$T_{new} = T_{old} \cdot \alpha$$

Where $\alpha \in (0,1)$ is the cooling rate.

Flow of the GSA

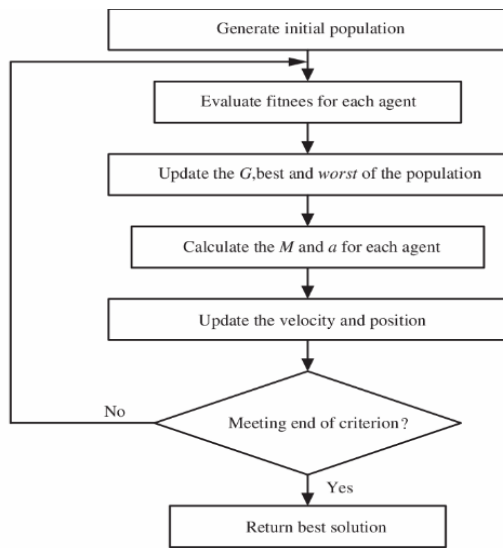


Figure 2

Flow of Simulated Annealing

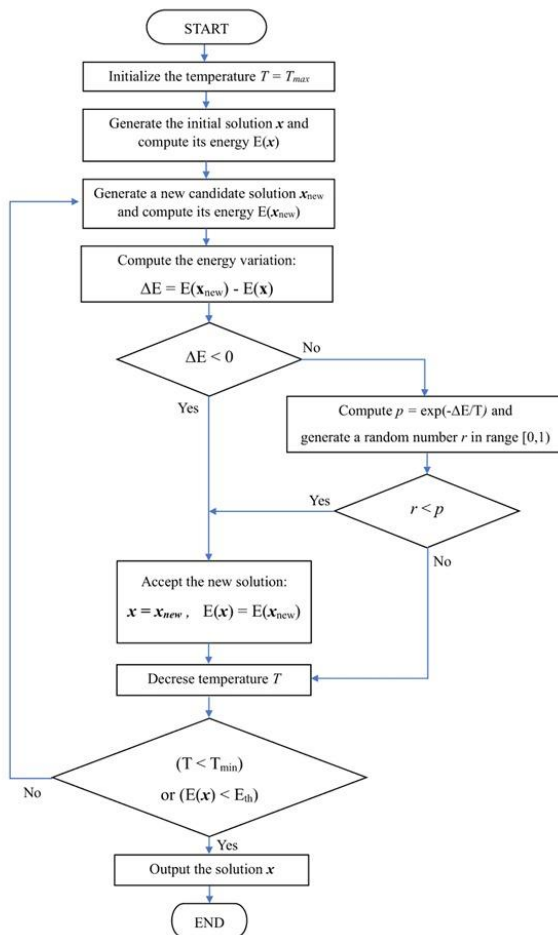


Figure 3

2. Comparison with Other Algorithms

K-means Clustering:

K-means is a centroid-based algorithm that partitions data into k clusters by minimizing intra-cluster variance.

Formula:

$$J = \sum_{i=1}^k \sum_{x_j \in C_i} ||x_j - \mu_i||^2$$

Where:

- J: Sum of squared distances.
- C_i : Cluster i.
- μ_i : Centroid of cluster i.

Strengths:

- Simple and fast.
- Works well for spherical clusters.

Weaknesses:

- Sensitive to initialization.
- Struggles with non-linear clusters.

Hierarchical Clustering:

Hierarchical clustering builds a hierarchy of clusters using either agglomerative (bottom-up) or divisive (top-down) methods.

Formula for Distance Update (Agglomerative - Ward's Method):

$$d(C_i, C_j) = ||\mu_i - \mu_j||^2$$

Where:

- C_i, C_j : Clusters.
- μ_i, μ_j : Centroids of clusters.

Strengths:

- Produces a dendrogram (visualization).
- Does not require the number of clusters upfront.

Weaknesses:

- Computationally expensive for large datasets.
 - Sensitive to noise.
-

DBSCAN (Density-Based Spatial Clustering of Applications with Noise):

DBSCAN groups points into clusters based on density, identifying core points, border points, and noise.

Key Parameters:

- ϵ : Maximum distance for points to be considered neighbours.
- MinPts: Minimum points required to form a dense region.

Strengths:

- Handles arbitrary-shaped clusters and noise.
- No need for the number of clusters upfront.

Weaknesses:

- Sensitive to ϵ and MinPts.
- Struggles with varying densities.

Standard GSA:

GSA solves optimization problems by simulating masses influenced by gravity but often suffers from premature convergence.

Limitations of Standard GSA:

- Weak exploration in high-dimensional problems.
- Tendency to converge to local optima.

3. Comparison Summary

Algorithm	Strengths	Weaknesses	Optimized GSA Advantage
K-means	Simple, fast	Sensitive to initialization	Handles non-linear clusters better.
Hierarchical	Visualization, no k required	Computationally expensive	Better global search with SA.
DBSCAN	Handles noise, arbitrary clusters	Sensitive to density parameters	Adapts to varying densities.
Standard GSA	Good exploration initially	Premature convergence	Avoids local minima with SA.

Figure 4

Conclusion:

The hybrid **GSA-SA** algorithm outperforms standard GSA and conventional clustering algorithms like K-means, hierarchical clustering, and DBSCAN in terms of global search capability, robustness, and convergence speed. Its ability to balance exploration and exploitation makes it particularly effective for complex optimization tasks and clustering problems.

Software Requirement Specifications

1. Introduction

1.1 Purpose

This Software Requirement Specification (SRS) document describes the requirements for optimizing the Gravitational Search Algorithm (GSA) to improve its performance in solving various optimization problems. The optimization aims to enhance convergence speed, solution quality, robustness, and the balance between exploration and exploitation. The goal is to create a software solution that implements the optimized GSA, which can be applied to complex, high-dimensional, and multimodal problems.

1.2 Scope

This software will focus on the development, testing, and evaluation of an optimized version of the GSA algorithm. The optimized GSA will be applicable to a range of optimization problems, and the software will allow experimentation with various modifications and parameter settings. The system will include an intuitive user interface for input and result visualization and support for algorithmic modifications such as hybridization with other algorithms.

1.3 Definitions, Acronyms, and Abbreviations

- **GSA:** Gravitational Search Algorithm
- **PSO:** Particle Swarm Optimization
- **DE:** Differential Evolution
- **GA:** Genetic Algorithm
- **SRS:** Software Requirement Specification
- **GUI:** Graphical User Interface
- **API:** Application Programming Interface

1.4 References

- [Gravitational Search Algorithm Paper]
- [Optimization Techniques Reference]

2. Overall Description

2.1 Product Perspective

This software will extend the traditional Gravitational Search Algorithm by incorporating optimizations such as dynamic parameter adjustment, hybridization with other algorithms, and improvements in movement equations for agents. The software will provide users with a framework to experiment with different optimization strategies.

2.2 Product Functions

- **Core GSA Algorithm:** Implement the basic gravitational search algorithm.
- **Optimization of Parameters:** Provide functionality to adjust and optimize parameters such as gravitational constant, acceleration coefficients, and mass scaling factors.
- **Hybridization:** Allow the user to combine GSA with other algorithms (e.g., SA) to form a hybrid approach.
- **Visualization:** Graphical representation of the search process, including plots of convergence, agent movements, and fitness scores.

2.3 User Classes and Characteristics

- **Researchers/Developers:** Users with knowledge of optimization algorithms who will modify and test the algorithm.
- **End Users:** Users who wish to apply the optimized GSA to real-world optimization problems (e.g., engineering, finance, machine learning).
- **System Administrators:** Users who manage and maintain the system, ensuring updates and integrations are properly executed.

2.4 Operating Environment

- **Software Environment:** The application will be developed in Python, C++, or MATLAB for ease of algorithm implementation and efficiency.
- **Hardware Environment:** The software will run on standard PCs, laptops, or cloud-based environments with appropriate computational resources for running optimization tasks.
- **Operating Systems:** Windows, macOS, and Linux.

2.5 Assumptions and Dependencies

- The algorithm will rely on standard Python or MATLAB libraries for mathematical computations and visualizations (e.g., NumPy, Matplotlib).

- The system will require external libraries for hybridization (e.g., PSO, DE) if used.
 - The user must have a basic understanding of optimization algorithms to modify and use the system effectively.
-

3. System Features and Requirements

3.1 Functional Requirements

Core GSA Functionality:

- The system shall implement the basic GSA mechanism, including agent initialization, mass assignment, and movement based on gravitational forces.
- The system shall allow the user to set and modify parameters such as gravitational constant, mass scaling factor, and acceleration coefficients.

Parameter Optimization:

- The system shall allow users to optimize the parameters to improve convergence rates and solution accuracy.
- The system shall provide default values for parameters but allow for manual modification and tuning by the user.

Hybridization with Other Algorithms:

- The system shall support hybridization with other optimization algorithms (SA), allowing the user to experiment with hybrid approaches.
- The user shall be able to specify the type of hybridization, including algorithm mixing and switching strategies.

Visualization Tools:

- The system shall display visual representations of the optimization process, including agent positions, fitness curves, and convergence plots.
- The user shall be able to adjust the frequency of visual updates to monitor progress.

Result Evaluation and Analysis:

- The system shall calculate and display metrics such as the best solution found, convergence time, and comparison against known optimal solutions.

- The system shall provide options for statistical analysis, including comparisons with other optimization algorithms or parameter settings.

3.2 Non-Functional Requirements

Performance Requirements:

- The system shall be capable of performing optimizations on high-dimensional problems (up to 100+ dimensions) efficiently.
- The system shall handle large-scale optimization problems (with 1000+ agents) without significant performance degradation.

Usability Requirements:

- The system shall have a user-friendly interface, allowing easy navigation through various functions (e.g., algorithm selection, parameter tuning, result visualization).
- The system shall provide clear documentation and examples for users to get started with the algorithm.

Reliability:

- The system shall be stable and produce consistent results across different test cases, with minimal errors or crashes.
- The algorithm shall be designed to minimize overfitting and premature convergence during optimization.

Security:

- The system shall ensure that no sensitive data is exposed during the use of optimization tasks, and it shall not have vulnerabilities that could lead to data leakage or other malicious activity.

Maintainability:

- The system shall be modular, allowing easy updates, algorithm improvements, and feature additions.
- The codebase shall be well-documented and follow best practices for maintainability.

4. External Interface Requirements

4.1 User Interfaces

- The system shall have a graphical user interface (GUI) for easy interaction, including options for:
 - Input of optimization parameters.
 - Selection of hybridization strategies and test functions.
 - Viewing of optimization results and progress.

4.2 Hardware Interfaces

- The system shall interact with local or cloud-based computational resources as needed for running large-scale optimization tasks.

4.3 Software Interfaces

- The system shall integrate with external libraries (e.g., NumPy, Matplotlib for Python) for mathematical operations and visualizations.
- The system shall provide an API for integration with other software systems or for use as part of a larger optimization pipeline.

Conclusion

This Software Requirement Specification (SRS) provides a detailed overview of the optimization of the Gravitational Search Algorithm. It outlines the functional and non-functional requirements, external interfaces, and expected behaviours of the system, ensuring a robust solution for solving complex optimization problems efficiently.

Methodology

1. Gravitational Search Algorithm for Clustering data

The Gravitational Search Algorithm (GSA) is a novel optimization technique inspired by the laws of gravity and motion. It has been effectively adapted for clustering tasks, particularly when combined with simple annealing methods to enhance its performance. This section outlines the customized steps of the GSA for clustering, integrating simple annealing to improve convergence and solution quality.

Step 1: Import Libraries

```
IMPORT pandas AS pd
IMPORT numpy AS np
FROM sklearn.preprocessing IMPORT StandardScaler
FROM sklearn.metrics IMPORT silhouette_score
IMPORT random
```

Explanation: This section imports necessary libraries:

- **pandas:** For data manipulation and analysis.
- **numpy:** For numerical operations and array handling.
- **StandardScaler:** For normalizing data to have a mean of 0 and a standard deviation of 1.
- **silhouette_score:** For evaluating the quality of clustering.
- **random:** For generating random numbers.

Step 2: Load the Dataset

```
SET file_path TO " dataset.csv"
LOAD data FROM file_path USING pd.read\_csv\(\)
```

Explanation: This section loads a dataset from a specified CSV file into a DataFrame using pandas. The dataset is expected to contain numerical data that will be used for clustering.

Step 3: Data Preprocessing

```
# Keep only numerical columns
data = SELECT numerical columns FROM data

# Drop rows with missing values
data = DROP rows WITH NaN FROM data

# Normalize numerical data
CREATE scaler USING StandardScaler()
data_normalized = FIT and TRANSFORM scaler WITH data
```


Explanation:

- The code filters the DataFrame to retain only numerical columns, discarding any non-numerical features (like IDs or dates).
- It removes any rows that contain missing values to ensure clean data for analysis.
- Finally, it normalizes the numerical data using StandardScaler, which adjusts the data so that it has a mean of 0 and a standard deviation of 1, making it suitable for distance calculations in clustering.

Step 4: Define GSA Clustering Class

CLASS GSAClustering:

```

FUNCTION __INIT__(data, n_clusters, n_agents, max_iter, G0=100, alpha=20):
    SET self.data TO data
    SET self.n_clusters TO n_clusters
    SET self.n_agents TO n_agents
    SET self.max_iter TO max_iter
    SET self.G0 TO G0
    SET self.alpha TO alpha

    INITIALIZE self.agents AS LIST OF n_agents AGENTS USING initialize_agent()
    SET self.best_agent TO None
    SET self.best_fitness TO -1

FUNCTION initialize_agent():
    RETURN RANDOMLY SELECTED n_clusters DATA POINTS AS AGENT POSITION

FUNCTION fitness(agent):
    labels = CALL assign_clusters(agent)

    IF number of unique labels < 2:
        RETURN -1 # Invalid fitness score

    RETURN silhouette_score(self.data, labels)

FUNCTION assign_clusters(centroids):
    COMPUTE distances BETWEEN data AND centroids USING L2 NORM

    RETURN INDEX OF MINIMUM DISTANCE FOR EACH DATA POINT

FUNCTION update_agents(G):
    masses = CALCULATE fitness FOR EACH agent IN self.agents

    NORMALIZE masses TO [0, 1]

    INITIALIZE new_agents AS EMPTY LIST

    FOR EACH agent i IN self.agents:
        INITIALIZE force AS ZERO VECTOR

```

```

    FOR EACH other_agent j IN self.agents:
        IF i != j:
            CALCULATE distance BETWEEN agent i AND agent j

            UPDATE force BASED ON GRAVITATIONAL FORMULA

        CALCULATE acceleration FROM force AND agent mass

    UPDATE agent position WITH RANDOM ACCELERATION

    ADD updated agent TO new_agents

SET self.agents TO new_agents

FUNCTION optimize():
    FOR iteration FROM 0 TO max_iter:
         $G = G_0 * \exp(-\alpha * \text{iteration} / \text{max\_iter})$ 

        CALL update_agents(G)

        FOR EACH agent IN self.agents:
            fitness = CALL fitness(agent)

            IF fitness > self.best_fitness:
                UPDATE best_fitness AND best_agent

    RETURN best_agent AND best_fitness

```

Explanation:

- **Class Definition:** A class named GSAClustering is defined to encapsulate all functionalities related to clustering using the Gravitational Search Algorithm.
- **Initialization (__INIT__):** This function initializes parameters such as the dataset, number of clusters, number of agents (potential solutions), maximum iterations, and gravitational constants. It also initializes a list of agents (randomly set centroids).
- **Agent Initialization (initialize_agent):** This function creates an agent by randomly selecting data points as cluster centroids.
- **Fitness Calculation (fitness):** This function evaluates how well an agent clusters the data by calculating the silhouette score. If there are fewer than two clusters, it returns -1 (invalid).
- **Cluster Assignment (assign_clusters):** This function assigns each data point to the nearest centroid based on Euclidean distance.
- **Agent Update (update_agents):** This function updates each agent's position based on gravitational forces derived from other agents' fitness levels. It calculates forces and accelerations to guide agents toward better solutions.

- **Optimization (optimize):** This function runs the optimization process over a specified number of iterations, updating agents and tracking the best-performing agent based on fitness scores.

Step 5: Run GSA Clustering

```
SET n_clusters TO 10 # Desired number of clusters
SET n_agents TO 20   # Number of agents in the population
SET max_iter TO 50   # Maximum number of iterations

CREATE gsa USING GSAClustering(data_normalized, n_clusters, n_agents, max_iter)
best_centroids, best_fitness = CALL gsa.optimize()
```

Explanation:

- This section sets parameters for clustering, including the desired number of clusters, number of agents (solutions), and maximum iterations for optimization.
 - An instance of GSAClustering is created with normalized data and specified parameters.
- The optimize function is called to find the best centroids and their associated fitness score.

Step 6: Assign Clusters and Display Results

```
final_labels = CALL gsa.assign_clusters(best_centroids)

PRINT "Best Fitness (Silhouette Coefficient):", best_fitness
PRINT "Cluster Centers:", best_centroids

# Add cluster labels to original data
data['Cluster'] = final_labels
```

Explanation:

- After optimization, this section assigns final cluster labels to each data point based on the best centroids found.
- It prints out the best fitness score (silhouette coefficient) indicating clustering quality and displays the coordinates of the cluster centers.
- Finally, it adds a new column to the original DataFrame containing cluster labels for each data point.

2. Modified GSA for clustering data:

The integration of simple annealing into GSA introduces a mechanism to escape local optima by allowing the algorithm to accept worse solutions with a certain probability. This is particularly useful in clustering, where the landscape of potential solutions can be complex and multi-modal. By incorporating a temperature parameter that gradually decreases, the algorithm can balance exploration (searching new areas) and exploitation (refining known good solutions).

Pseudocode for Simulated Annealing

```
FUNCTION simulated_annealing(agent):
    # Refine agent (centroids) using Simulated Annealing.
    SET current_agent TO COPY OF agent
    SET current_fitness TO CALL fitness(current_agent)
    SET temperature TO self.initial_temp

    WHILE temperature > 1:
        # Generate a new candidate solution
        SET new_agent TO current_agent + RANDOM UNIFORM(-0.1, 0.1,
size=current_agent.shape)
        SET new_fitness TO CALL fitness(new_agent)

        # Accept or reject the new solution
        IF new_fitness > current_fitness OR RANDOM NUMBER BETWEEN 0 AND 1 <
EXP((new_fitness - current_fitness) / temperature):
            SET current_agent TO new_agent
            SET current_fitness TO new_fitness

        # Decrease the temperature
        SET temperature TO temperature * self.cooling_rate

    RETURN current_agent, current_fitness
```

Function Definition:

```
FUNCTION simulated_annealing(agent):
```

- This line defines a function named `simulated_annealing` that takes one parameter, `agent`, which represents the current solution (centroids in this context).

Initialization:

```
SET current_agent TO COPY OF agent
SET current_fitness TO CALL fitness(current_agent)
SET temperature TO self.initial_temp
```

- **Copying the Agent:** The function creates a copy of the input agent to work with, preserving the original.
- **Fitness Calculation:** It calculates the fitness of the `current_agent` using a fitness evaluation method (e.g., silhouette score).
- **Initial Temperature:** The initial temperature for the annealing process is set from an instance variable (`self.initial_temp`).

Main Loop:

```
WHILE temperature > 1:
```

- The loop continues as long as the temperature is greater than 1. This condition ensures that the algorithm gradually cools down and stops when it reaches a certain threshold.

Generate New Candidate Solution:

```
SET new_agent TO current_agent + RANDOM UNIFORM(-0.1, 0.1, size =  
current_agent.shape)
```

- A new candidate solution (new_agent) is generated by adding a small random perturbation (uniformly distributed between -0.1 and 0.1) to each centroid of the current_agent. This introduces variability into the search process.

Fitness Evaluation for New Solution:

```
SET new_fitness TO CALL fitness(new_agent)
```

- The fitness of the newly generated candidate solution (new_agent) is evaluated using the same fitness function.

Acceptance Criteria:

```
IF new_fitness > current_fitness OR RANDOM NUMBER BETWEEN 0 AND 1 <  
EXP((new_fitness - current_fitness) / temperature):
```

```
    SET current_agent TO new_agent
```

```
    SET current_fitness TO new_fitness
```

- This conditional statement determines whether to accept the new solution:
 - If the new fitness is better than the current fitness, it is accepted.
 - If not, it may still be accepted with a probability that decreases as the temperature lowers. This probability is calculated using an exponential function based on the difference in fitness and the current temperature.
 - This mechanism allows for occasional acceptance of worse solutions, helping to escape local optima.

Temperature Update:

```
SET temperature TO temperature * self.cooling_rate
```

- The temperature is decreased by multiplying it with a cooling rate (another instance variable). This simulates the cooling process in physical annealing, where lower temperatures reduce the likelihood of accepting worse solutions.

Return Final Solution:

```
RETURN current_agent, current_fitness
```

- After exiting the loop (when temperature drops below or equal to 1), the function returns the best agent found during this annealing process along with its corresponding fitness score.

3. Modifying with age monitoring:

Input:

- **n_agents**: Number of agents (solutions)
 - **n_iterations**: Maximum number of iterations
 - **alpha**: Cooling rate for Simulated Annealing
 - **age_threshold**: Threshold for stagnation age
-

Output:

- **best_agent**: The agent with the best clustering solution
 - **best_fitness**: The fitness of the best agent
 - **performance_over_time**: A list tracking the best fitness over iterations
-

Steps:

1. Initialization

- Randomly initialize the positions (agents) of n_agents in the search space.
 - Assign each agent an initial age (ages) of 0.
 - Compute the initial fitness of each agent using a clustering metric like the silhouette score.
 - Identify the global best agent (best_agent) and its corresponding fitness (best_fitness).
 - Set the initial temperature for Simulated Annealing (temperature = 1.0).
-

2. Iterative Optimization

For each iteration from 1 to n_iterations:

- **Update Agent Positions using GSA**

- Compute a gravitational constant (G) that decreases with iterations.
- For each agent, calculate the force exerted by better-performing agents based on their fitness.
- Update the agent's position using these forces.

- **Simulated Annealing for Refinement**

For each agent:

- Propose a small random perturbation to the agent's position.

- Evaluate the fitness of the perturbed agent.
- Accept the perturbation if it improves fitness or with a probability determined by the SA acceptance criterion.
- **Age Diversity Monitoring**

For each agent:

 - If the agent's fitness does not improve, increment its stagnation age (ages[i]).
 - If the age exceeds the age_threshold, reinitialize the agent to a random position and reset its age.
- **Update Global Best Solution**
 - Check if any agent's fitness surpasses best_fitness.
 - If so, update best_agent and best_fitness.
- **Cooling Schedule**
 - Gradually reduce the temperature using the formula: $\text{temperature} *= \alpha$.
- **Track Performance**
 - Append the current best_fitness to the performance tracking list (performance_over_time).

3. Return Results

- Output the global best_agent, its best_fitness, and the fitness progression over iterations (performance_over_time).

Result and Discussion

1. Comparison between GSA / GSA+SA / GSA + SA + age_diversity

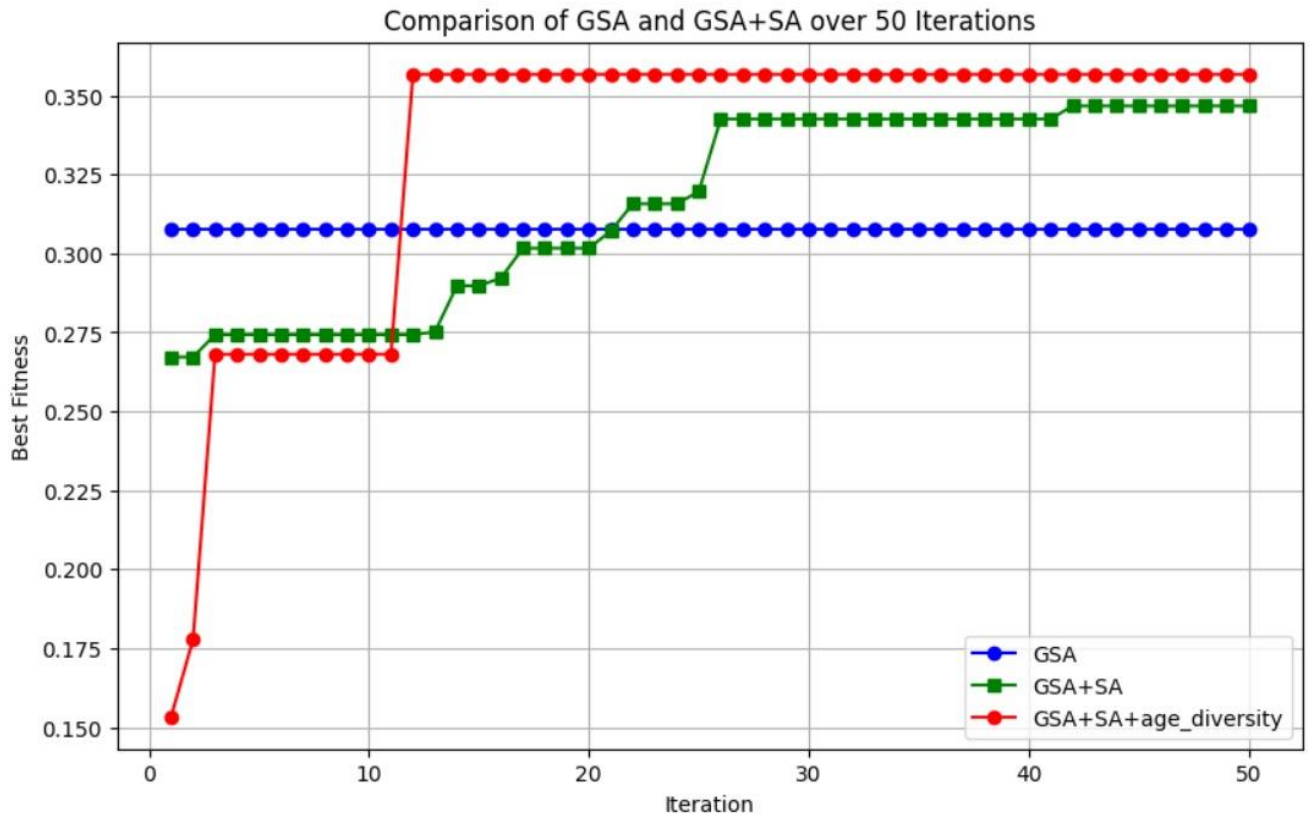


Figure 5

Results Explanation

1. GSA (Blue line):

- The performance of GSA remains relatively stagnant throughout the 50 iterations.
- The best fitness score achieved by GSA is approximately **0.305**, and no significant improvements are observed as the iteration progresses.
- This indicates that GSA, on its own, quickly stagnates in finding better solutions and does not explore beyond a certain threshold.

2. GSA+SA (Green line):

- GSA combined with Simulated Annealing (SA) improves the fitness score over iterations.
- Initially, the progress is slow, but between **Iteration 15 and 25**, a significant improvement is observed, reaching a best fitness score of around **0.35**.
- This suggests that the addition of SA allows the algorithm to escape local optima and explore better solutions.

3. GSA + SA + age_diversity (Red line):

- The addition of **age diversity** to GSA+SA drastically improves the results.
- The fitness score starts from **0.15** and rapidly increases within the first 10 iterations, reaching the maximum fitness score of approximately **0.355**.
- This performance remains consistent for the remaining iterations, demonstrating that the age-diversity mechanism effectively promotes exploration and avoids stagnation.

Discussion Points

1. **Effect of Agents:**
 - Despite using fewer agents, the results show significant improvement in fitness scores when **SA and age diversity** are added.
 - This highlights that incorporating additional mechanisms (like simulated annealing and diversity preservation) compensates for fewer agents by enhancing exploration and solution quality.
 2. **Stagnation in GSA:**
 - The GSA alone shows limited improvement in the fitness function, likely due to its inability to escape local optima.
 - This suggests that GSA benefits from hybridization with techniques like SA and diversity strategies.
 3. **Simulated Annealing (SA):**
 - The GSA+SA combination improves the ability to explore the solution space, leading to better results compared to GSA alone.
 - However, the progress is gradual, and improvements are not as immediate as with the age-diversity-enhanced version.
 4. **Age Diversity Impact:**
 - Age diversity significantly accelerates convergence to better fitness scores.
 - It prevents premature convergence by maintaining a diverse population of solutions, ensuring that the algorithm explores a broader search space.
 5. **Final Outcomes:**
 - **GSA+SA+age_diversity** achieves the highest fitness score and converges the fastest, demonstrating its superiority over GSA and GSA+SA.
 - The results confirm that integrating mechanisms like **simulated annealing** and **age diversity** improves the efficiency and effectiveness of the optimization process.
-

Conclusion

The chart demonstrates that:

- **GSA+SA+age_diversity** is the most effective approach, achieving the best fitness score in fewer iterations.
- Adding mechanisms like **simulated annealing** and **diversity maintenance** is crucial for overcoming stagnation and enhancing the solution quality.
- Despite using fewer agents, the enhancements in the algorithm design yield superior results, emphasizing the importance of algorithmic modifications over agent quantity.

2. Comparison over different algorithms and Different datasets-

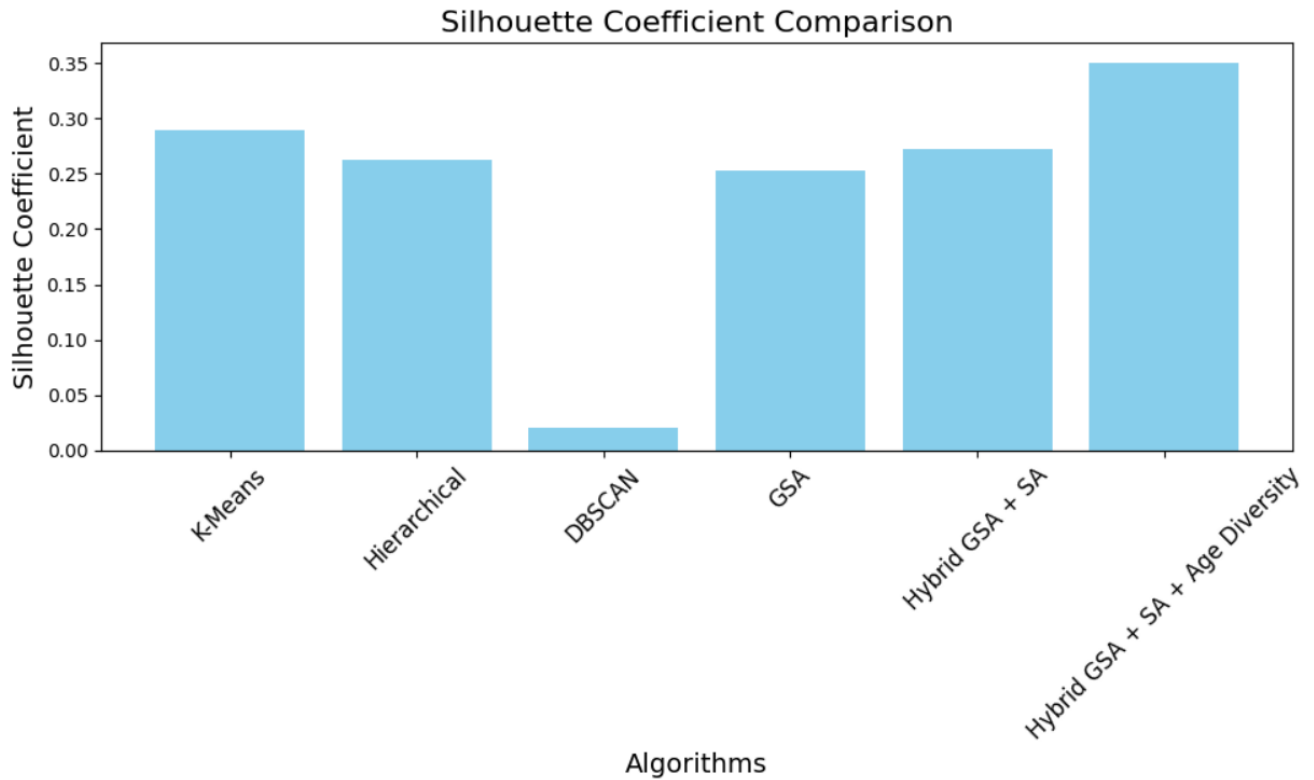


Figure 6. Market Segment Dataset

Centroids and Results are-

```
Running K_mean...
Silhouette Score: 0.2898
Cluster Centroids:
[[-3.78085207e-01 -3.21499970e-01 -3.70578679e-01 -2.72598567e-01
 -3.37774591e-01  5.98494557e-02 -7.96913440e-02  6.45428937e-02
 -5.83404343e-01 -6.20754932e-01 -1.11292080e-01  2.88013649e-01
 -2.63587846e-01]
 [ 8.59657337e-01 -4.87722365e-01  1.12155296e+00 -2.72598567e-01
  1.07020675e+00 -5.03199069e-01  7.97706685e-01 -8.46723096e-01
  1.24602661e+00  1.31930043e+00  6.57524768e-01 -6.81641377e-01
  9.46287747e-01]
 [-4.12886160e-01  1.89124798e+00 -1.10177086e+00 -2.25681943e-01
 -1.14278782e+00  6.30711855e-01 -1.37034852e+00  1.51424829e+00
 -6.16064589e-01 -6.03750798e-01 -7.18119529e-01  3.54762432e-01
 -9.05184408e-01]
 [-1.98746146e-01 -2.60501122e-01  2.80272652e-01  3.66839786e+00
  3.83457500e-01  2.75940917e-01  3.72500504e-01 -4.03737321e-01
  1.08251436e-03 -9.76598453e-02 -3.92846867e-01  1.71712470e-01
 -1.64515168e-01]]
Running Hierarchical_method...
Best Fitness (Silhouette Coefficient): 0.2626
Running DBSCAN...
Number of Noise Points: 63
Silhouette Score: 0.0198
Running Gravitational Search Algorithm...
Best Fitness (Silhouette Coefficient): 0.2529546624025853
Cluster Centers:
[[-0.41460217 -0.48772236 -0.86769058 -0.27259857 -0.34289902  0.04468529
 -1.08370491  1.26607787 -0.52300145 -1.09423658  0.80657583  0.3622182
```

```

-0.97744066]
[-0.40084801 -0.48772236  2.11761463 -0.27259857  0.22723076 -0.42545207
 0.70499185 -0.85668302 -0.8678825  -1.30805076  0.29797709  0.31079536
 0.30234599]
[-0.36981244 -0.48772236 -0.07978012  3.66839786 -0.56749561 -1.34008294
 1.11749449  0.03800924 -0.6379618  -0.77945458  0.06679585  0.44105193
 1.46298491]
[-0.41208734  2.08745172 -1.37837329 -0.27259857 -1.24128535 -0.57076726
-1.78069212  3.28729991 -0.6379618  0.01640932 -0.0719129  0.39094481
-0.68167397]]

```

Running Hybrid GSA + Simulated Annealing...

Best Fitness (Silhouette Coefficient): 0.27199268148231903

Cluster Centers:

```

[[-0.297268  1.18984159  0.20063847 -0.39250807  2.17533239 -0.45624603
 1.00374944  0.61440242  1.29424622  0.71106867  0.07010526  1.5434073
 1.6560583 ]
[ 0.84994726 -0.17157221 -0.20452987  0.33563924 -1.08373442  0.96327268
 0.17694538  0.01567425 -0.63040279  0.63014325 -0.26894214 -0.54643022
 1.15347296]
[ 0.17031202  0.75287286  0.88916554  0.1024729  0.2015785 -0.05424159
 1.08174185  0.43910427 -0.11315084  1.75801116  1.41369161 -0.37268766
 0.82743553]
[ 0.75296381  0.65064006  0.229594  0.0556381  0.66011745  0.31992158
 0.74016034 -0.21330781  1.77093132  0.81239689 -0.01845678  0.28035311
 0.24612193]]

```

Running Hybrid GSA + SA + Age Diversity...

Best Fitness (Silhouette Coefficient): 0.35067118278434356

Cluster Centers:

```

[[ 1.00265117  0.68507404  0.6508409  0.39589474 -0.06910215  0.72516448
-0.32054097  0.39950998  0.12137805  0.26484415  0.38926497  0.60221064
 0.36435992]
[ 1.07893794  0.26791337  0.71224246  0.51015747  0.74969658  0.52691133
 0.52115032  0.27201503  0.7806449  0.67277062  0.41022572  0.63416649
 1.01755669]]

```

The following points are observed:

1. **K-Means** achieves a Silhouette Coefficient of around **0.29**, showing relatively good clustering performance.
2. **Hierarchical Clustering** achieves a lower Silhouette Coefficient, approximately **0.26**, indicating less compact and cohesive clustering than K-Means.
3. **DBSCAN** performs poorly with a near-zero Silhouette Coefficient, which implies that it struggles to form meaningful clusters on this dataset, likely due to density-based clustering's sensitivity to parameter settings.
4. **GSA** achieves similar performance to K-Means, indicating that the gravitational search optimization finds clusters nearly as good as traditional methods.
5. **Hybrid GSA + SA** improves upon pure GSA, showing a slightly better Silhouette Coefficient (approximately **0.27**).
6. **Hybrid GSA + SA + Age Diversity** clearly outperforms all other algorithms with a Silhouette Coefficient above **0.34**. This result highlights that incorporating **Age Diversity** into the Hybrid GSA + SA framework enhances the search process and produces superior clusters.

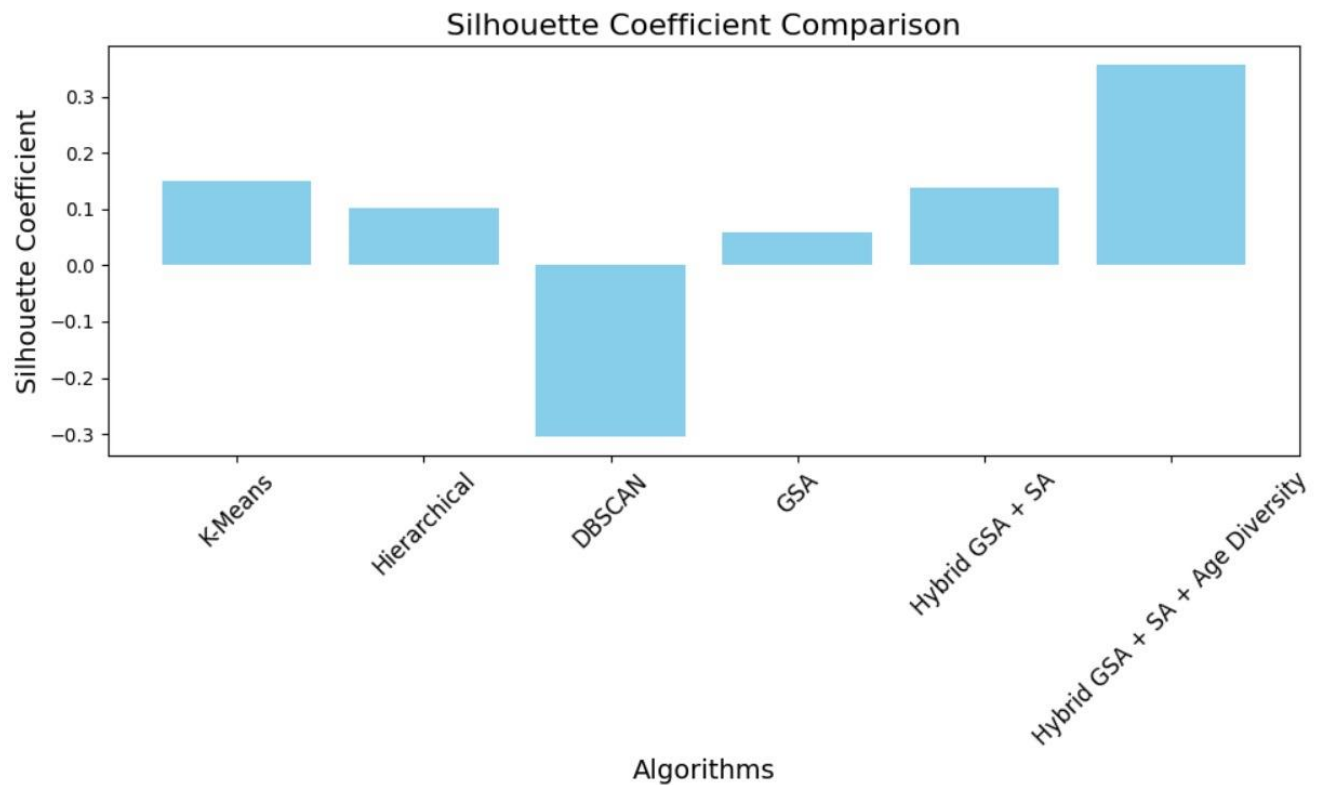


Figure 7. Iris Dataset

Centroids and Results are-

Running K_mean...

Silhouette Score: 0.1529

Cluster Centroids:

```
[ [ 1.13597027  0.08842168  0.99615451  1.01752612]
  [-1.01457897  0.85326268 -1.30498732 -1.25489349]
  [-0.05021989 -0.88337647  0.34773781  0.2815273 ]]
```

Running Hierarchical_method...

Best Fitness (Silhouette Coefficient): 0.0995

Running DBSCAN...

Number of Noise Points: 1

Silhouette Score: -0.3046

Running Gravitational Search Algorithm...

Best Fitness (Silhouette Coefficient): 0.09092990861769

Cluster Centers:

```
[ [-0.7795133 -0.82256978  0.08070915  0.26414192]
  [-0.05250608 -0.82256978  0.76275827  0.92230284]
  [ 0.4321654 -0.36217625  0.30805885  0.13250973]]
```

Running Hybrid GSA + Simulated Annealing...

Best Fitness (Silhouette Coefficient): 0.1575376723190185

Cluster Centers:

```
[ [0.49386037 0.59897207 0.03448729 0.61268433]
  [0.4542096  0.7686463  0.23000711 0.3588521 ]
  [0.58344033 0.41817144 0.19881269 0.56104166]]
```

Running Hybrid GSA + SA + Age Diversity...

Best Fitness (Silhouette Coefficient): 0.4057937931283336

Cluster Centers:

```
[[0.5632414 0.6000508 0.27410838 0.22327014]
[0.68720192 0.00950181 0.44845032 0.64046588]
[0.78507699 0.070551 0.55247841 0.83975332]
[0.65207889 0.28354251 0.0746238 0.9515027 ]
[0.44683089 0.86333899 0.37343124 0.52874219]]
```

The following points are evident:

1. **KMeans** and **Hierarchical Clustering** achieve moderate Silhouette Coefficients (around **0.14** and **0.1**, respectively).
2. **DBSCAN** performs poorly with a negative Silhouette Coefficient, suggesting that it cannot properly form meaningful clusters in the Iris dataset.
3. **GSA** achieves slightly better performance compared to traditional algorithms but still falls short of the optimal result.
4. **Hybrid GSA + SA** improves the performance further, achieving a Silhouette Coefficient around **0.15**.
5. **Hybrid GSA + SA + Age Diversity** clearly emerges as the best-performing algorithm with a Silhouette Coefficient above **0.35**, outperforming traditional approaches significantly.

Key Discussion Points

1. **Performance of Hybrid GSA + SA + Age Diversity:**
 - On both datasets (Market Segment and Iris), the **Hybrid GSA + SA + Age Diversity** framework outperforms traditional clustering algorithms like **K-Means** and **Hierarchical Clustering**.
 - The addition of **Age Diversity** introduces a mechanism that prevents the algorithm from stagnating at local optima, thus exploring a broader solution space effectively.
2. **Limitations of Traditional Algorithms:**
 - **K-Means** assumes spherical clusters and struggles when the data structure deviates from this shape. While its performance is decent, it is outperformed by Hybrid GSA methods.
 - **Hierarchical Clustering** often produces less compact clusters due to its reliance on distance metrics and lacks global optimization capabilities.
 - **DBSCAN** struggles on both datasets, particularly when clusters are not well-separated or the dataset parameters (like `epsilon` and `min_samples`) are not well-tuned.
3. **Role of GSA in Clustering:**
 - GSA, inspired by Newtonian gravity, shows competitive results compared to traditional algorithms like K-Means.
 - GSA optimizes clustering by exploring a wide solution space through the gravitational pull mechanism, avoiding premature convergence to suboptimal solutions.
4. **Impact of Hybridization (SA and Age Diversity):**
 - **Simulated Annealing (SA)** enhances the exploitation capability of GSA, improving local search efficiency.
 - **Age Diversity** ensures better exploration of the search space by maintaining diversity in solutions, preventing stagnation at local optima. This combination is the key reason why **Hybrid GSA + SA + Age Diversity** outperforms other methods.
5. **Scalability and Robustness:**
 - The results indicate that the Hybrid GSA framework can be more robust across different datasets compared to traditional algorithms, making it a promising technique for real-world clustering tasks.

Conclusion

The results demonstrate that the **Hybrid GSA + SA + Age Diversity** framework consistently outperforms traditional clustering algorithms (K-Means, Hierarchical Clustering, DBSCAN) and even standalone GSA. This is evident in both the **Market Segment** and **Iris** datasets, where Hybrid GSA achieves significantly higher Silhouette Coefficients.

Key Takeaways:

- **Hybrid GSA + SA + Age Diversity** achieves the **highest Silhouette Coefficient**, proving its effectiveness in clustering tasks.
- Traditional algorithms like **K-Means** and **DBSCAN** show limitations when the data is complex or poorly structured.
- The success of Hybrid GSA methods stems from their ability to balance **exploration** (via Age Diversity) and **exploitation** (via Simulated Annealing).

These findings highlight the potential of **Hybrid GSA + SA + Age Diversity** as a robust and superior alternative to traditional clustering methods.

Future Work

1. Hybridization with Other Advanced Algorithms

- **Objective:** Combine GSA with more advanced or hybrid optimization algorithms to leverage their strengths and improve both exploration and exploitation capabilities.
- **Future Directions:**
 - **GSA and Deep Learning:** Investigate the potential of combining GSA with deep learning techniques, such as neural networks, for applications like feature selection, hyperparameter tuning, or image recognition.
 - **GSA with Evolutionary Algorithms:** Combine GSA with Genetic Algorithms (GA), Differential Evolution (DE), or Particle Swarm Optimization (PSO) to form hybrid models that offer improved global search capabilities.
 - **GSA and Simulated Annealing:** Explore the integration of GSA with Simulated Annealing (SA) to improve convergence in multimodal optimization problems.

2. Improvement of Convergence Rate

- **Objective:** Enhance the convergence speed of the GSA to solve large-scale and high-dimensional optimization problems more efficiently.
- **Future Directions:**
 - **Adaptive Gravitational Constant:** Investigate dynamic adjustment of the gravitational constant based on the search progress to balance exploration and exploitation throughout the optimization process.
 - **Local Search Mechanisms:** Implement and evaluate local search strategies or neighborhood-based optimization methods to improve the convergence rate in the later stages of the search.
 - **Parallel and Distributed Computing:** Utilize parallel computing frameworks (e.g., CUDA, OpenMP) to speed up the computation for large-scale optimization tasks, enabling the algorithm to handle more complex problems in real time.

3. Multi-Objective Optimization

- **Objective:** Extend GSA to handle multi-objective optimization (MOO) problems, where more than one objective function needs to be optimized simultaneously.
- **Future Directions:**
 - **Multi-Objective GSA:** Develop a multi-objective version of GSA that can find a set of Pareto-optimal solutions for complex problems with conflicting objectives.
 - **Hybrid MOO Algorithms:** Combine GSA with algorithms like NSGA-II (Non-dominated Sorting Genetic Algorithm) or MOEA/D (Multi-Objective Evolutionary Algorithm based on Decomposition) for better exploration of the Pareto front.
 - **Multi-Objective Benchmarking:** Test the performance of the multi-objective GSA on standard benchmark problems (e.g., ZDT, DTLZ) to compare its performance with other popular MOO algorithms.

4. Dynamic Environments and Real-Time Optimization

- **Objective:** Adapt GSA to dynamic optimization environments where the problem changes over time (e.g., dynamic constraints, changing fitness landscapes).
- **Future Directions:**
 - **Dynamic GSA:** Modify the GSA to respond to changes in the optimization problem during the search process, such as adapting to new constraints or altered objective functions.
 - **Real-Time Optimization:** Explore the application of GSA in real-time systems, such as robotics or adaptive control systems, where the optimization problem may evolve continuously.
 - **Robustness to Noisy Environments:** Investigate the robustness of GSA to noisy or uncertain environments and enhance its performance in situations where the problem data is subject to error or variation.

5. Handling High-Dimensional Problems

- **Objective:** Improve GSA's performance on high-dimensional optimization problems by addressing the curse of dimensionality and reducing computational complexity.
- **Future Directions:**
 - **Dimensionality Reduction Techniques:** Explore the integration of dimensionality reduction techniques such as PCA (Principal Component Analysis) or t-SNE to reduce the problem size while retaining the essential characteristics.
 - **Sparse Solutions:** Develop approaches to allow GSA to work efficiently with sparse data or problems where only a few variables are relevant.
 - **Multi-Agent Systems:** Investigate how multi-agent systems could be leveraged to explore high-dimensional search spaces more effectively and improve solution diversity.

6. Benchmarking and Real-World Applications

- **Objective:** Test the optimized GSA on real-world optimization problems across various domains and compare its performance to other state-of-the-art algorithms.
- **Future Directions:**
 - **Engineering Design Problems:** Apply GSA to solve real-world optimization problems in engineering design, such as structural optimization, multi-disciplinary design, or manufacturing process optimization.
 - **Finance and Economics:** Use GSA for portfolio optimization, stock market prediction, or economic modeling, where optimization of financial strategies and risk assessment are critical.
 - **Machine Learning:** Test GSA for hyperparameter tuning in machine learning algorithms (e.g., support vector machines, deep learning models) and compare its performance with other optimization methods like grid search or random search.
 - **Energy Systems:** Apply GSA to optimize energy systems, such as smart grid optimization, renewable energy integration, or energy consumption optimization in smart homes.

7. Algorithmic Improvements and Customization

- **Objective:** Continue refining the GSA algorithm by exploring novel methods to improve its basic structure or adapt it to specific problem types.
- **Future Directions:**
 - **Memory-Aided GSA:** Introduce memory mechanisms where agents "remember" successful paths or solutions from previous iterations, guiding future search steps.
 - **Self-Adaptive GSA:** Develop self-adaptive parameters that change dynamically during the search based on feedback from the optimization process.
 - **Hybridized Search Strategies:** Explore new hybrid strategies that combine the gravitational model with other metaheuristics like Ant Colony Optimization (ACO) or Tabu Search.

8. User-Friendly Interface and Implementation Tools

- **Objective:** Develop tools and platforms that allow practitioners and researchers to easily apply the optimized GSA to a variety of problems.
- **Future Directions:**
 - **Web-Based Interface:** Create a web-based tool for easy implementation, experimentation, and visualization of results in GSA optimization tasks.
 - **Software Libraries:** Develop software libraries in popular programming languages (e.g., Python, MATLAB, Java) for easy integration of the optimized GSA into existing applications or research projects.
 - **Documentation and Tutorials:** Provide comprehensive documentation, tutorials, and case studies to make the optimized GSA accessible to a wider audience, including those with little experience in optimization algorithms.

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