

# Beam Search

# Hill Climbing Vs. Beam Search

- Hill climbing just explores all nodes in one branch until goal found or not being able to explore more nodes.
- Beam search explores more than one path together. A factor **k** is used to determine the number of branches explored at a time.
- If k=2, then two branches are explored at a time. For k=4, four branches are explored simultaneously.
- The branches selected are the **best branches** based on the used **heuristic evaluation function**.

# Beam Search, $k=2$

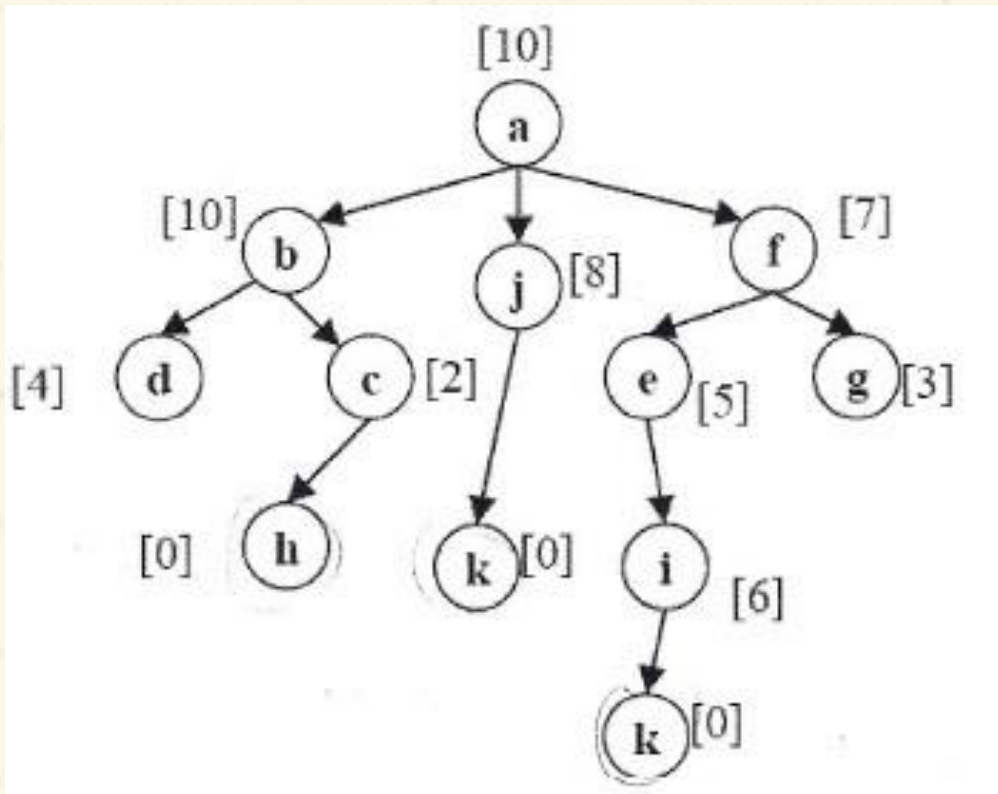
## Goal – Node K

Current

a

Children

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# Beam Search

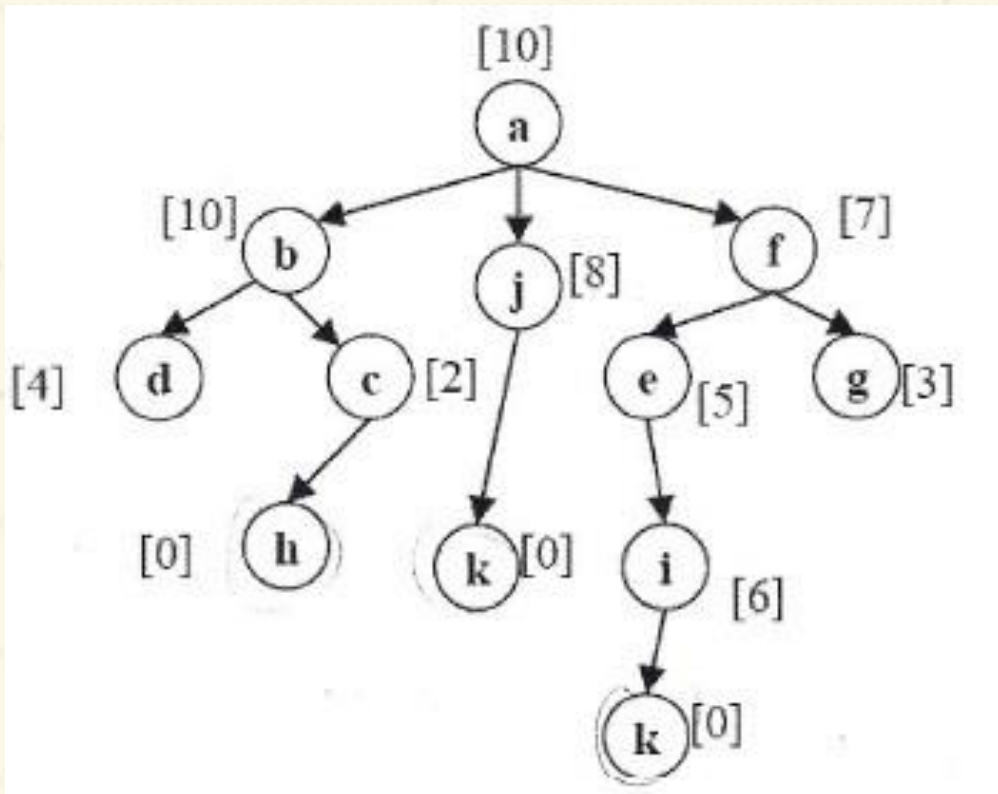
Goal – Node K

Current

a

Children

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# Beam Search

## Goal – Node K

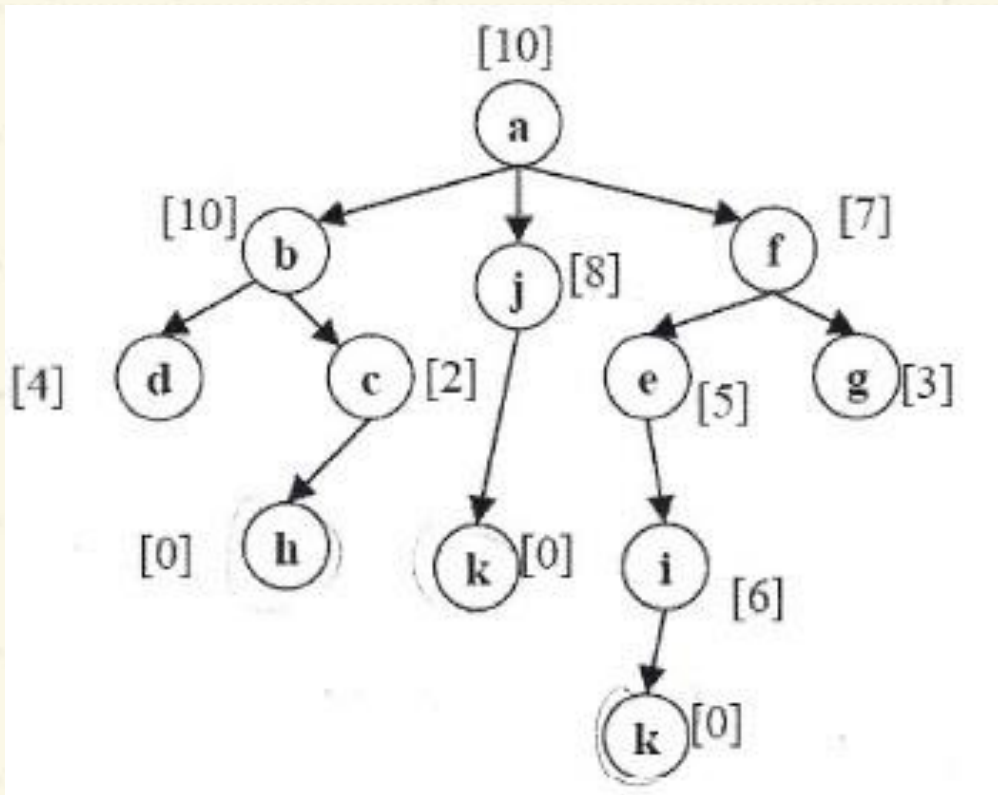
Current

a

a

Children

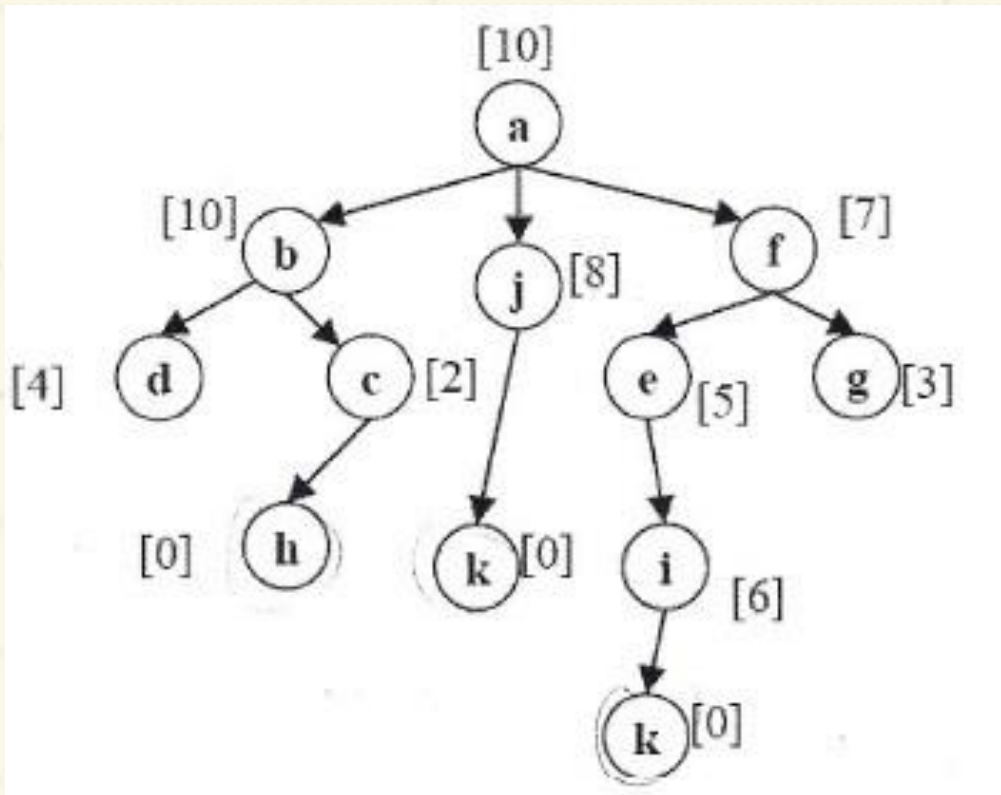
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# Beam Search

## Goal – Node K

Current	Children
a	---
a	$f_7, j_8, b_{10}$



# Beam Search

Goal – Node K

Best k  
Successors

Current

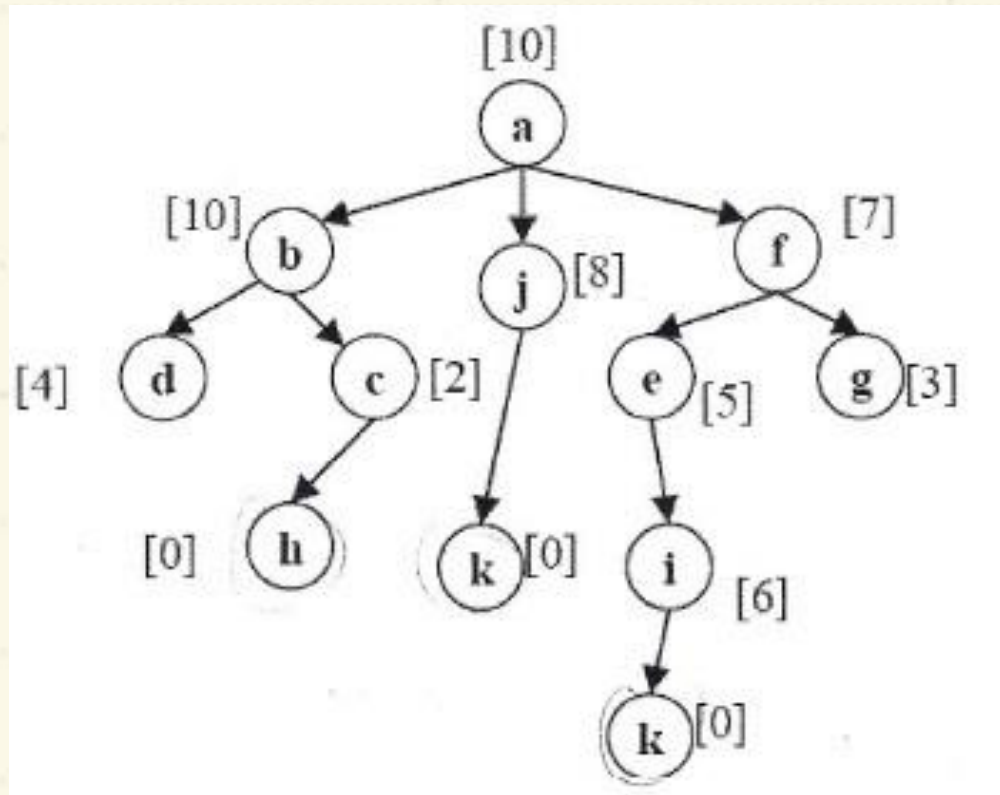
a

a

Children

---

$f_7, j_8, b_{10}$





# Beam Search

## Goal – Node K

Best k  
Successors

Current

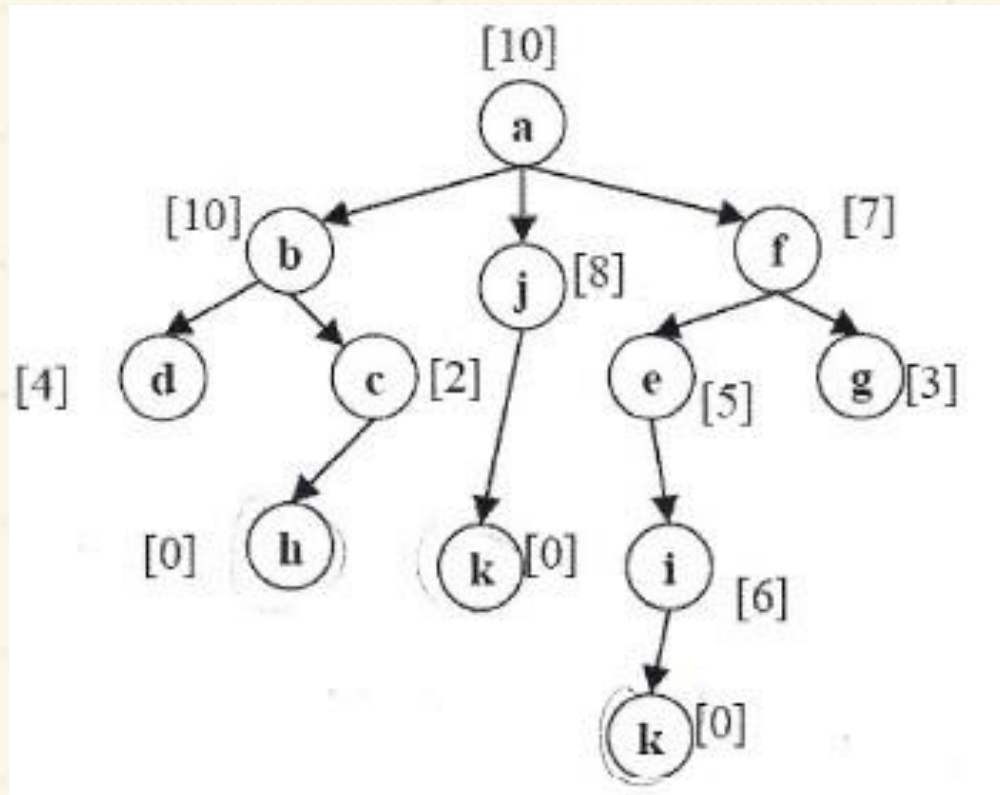
a

a

Children

---

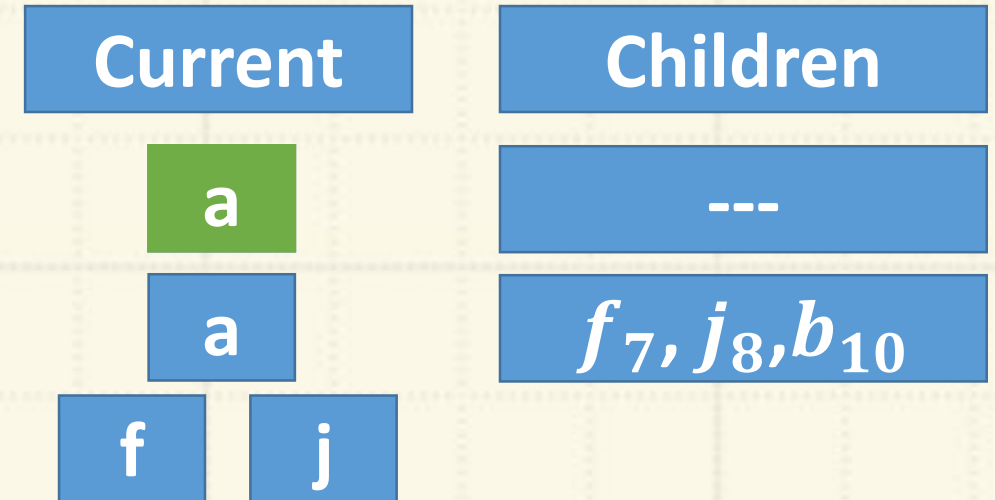
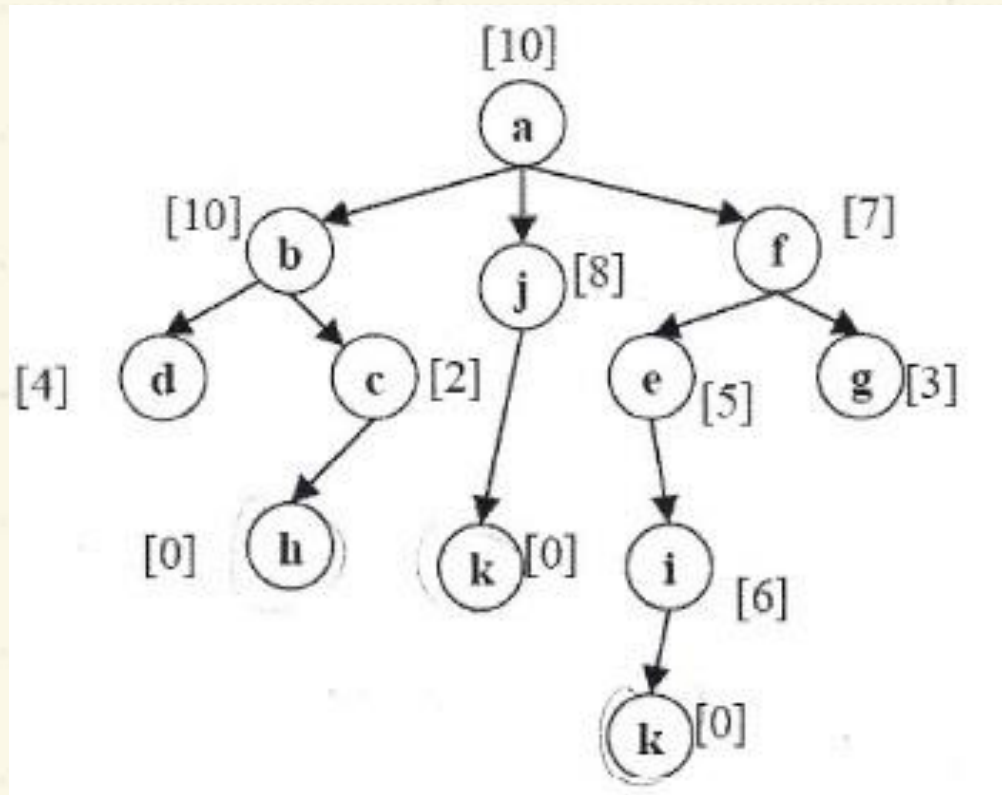
*f*<sub>7</sub>, *j*<sub>8</sub>, *b*<sub>10</sub>





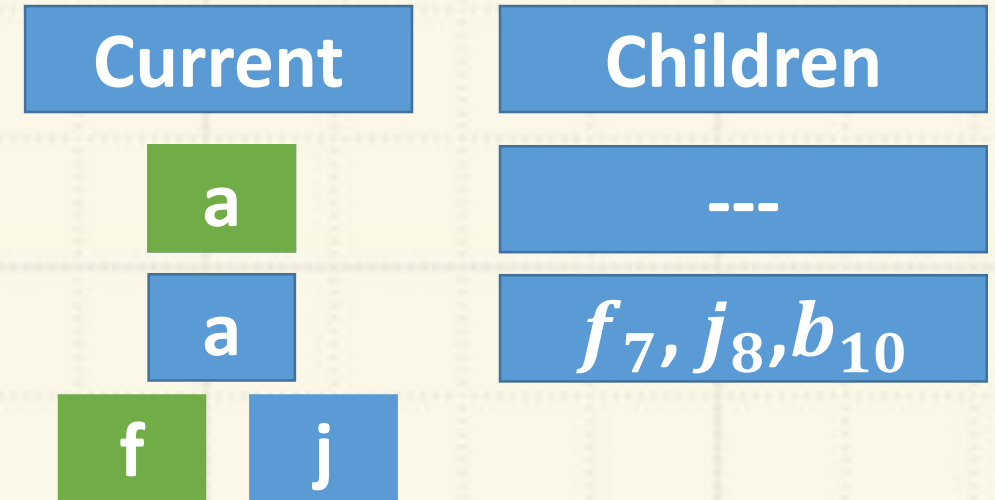
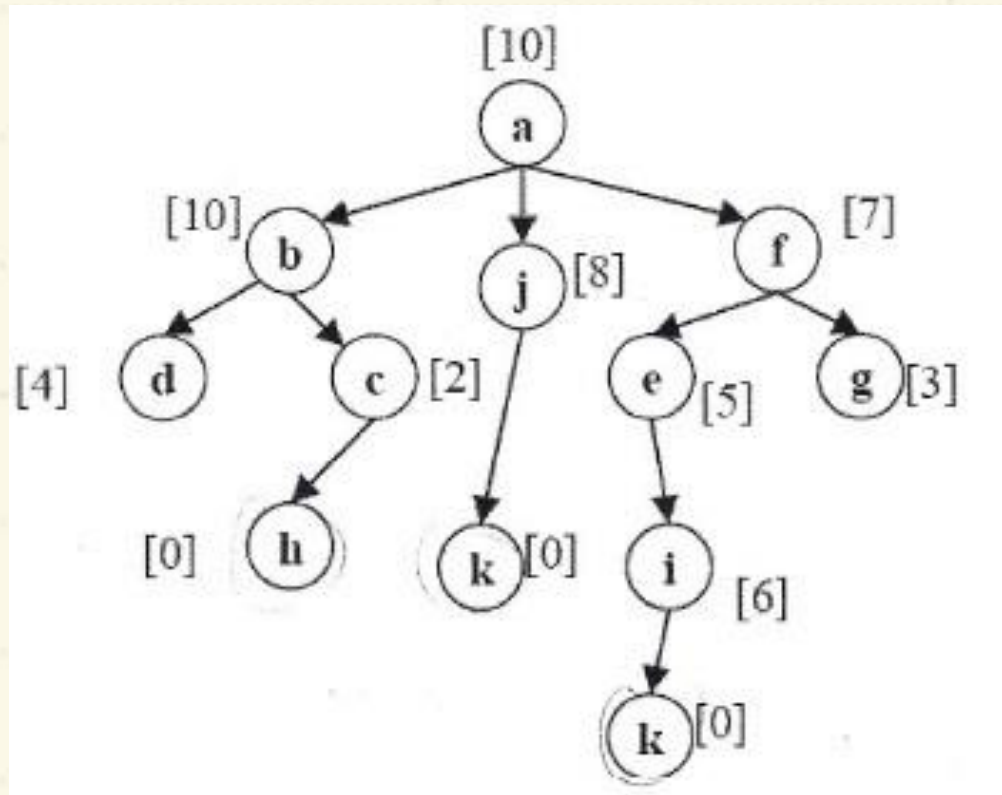
# Beam Search

## Goal – Node K



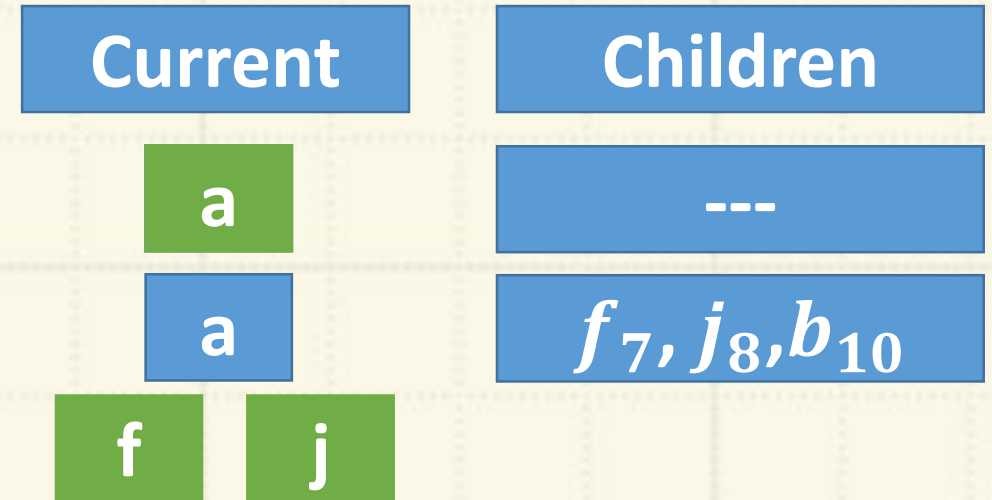
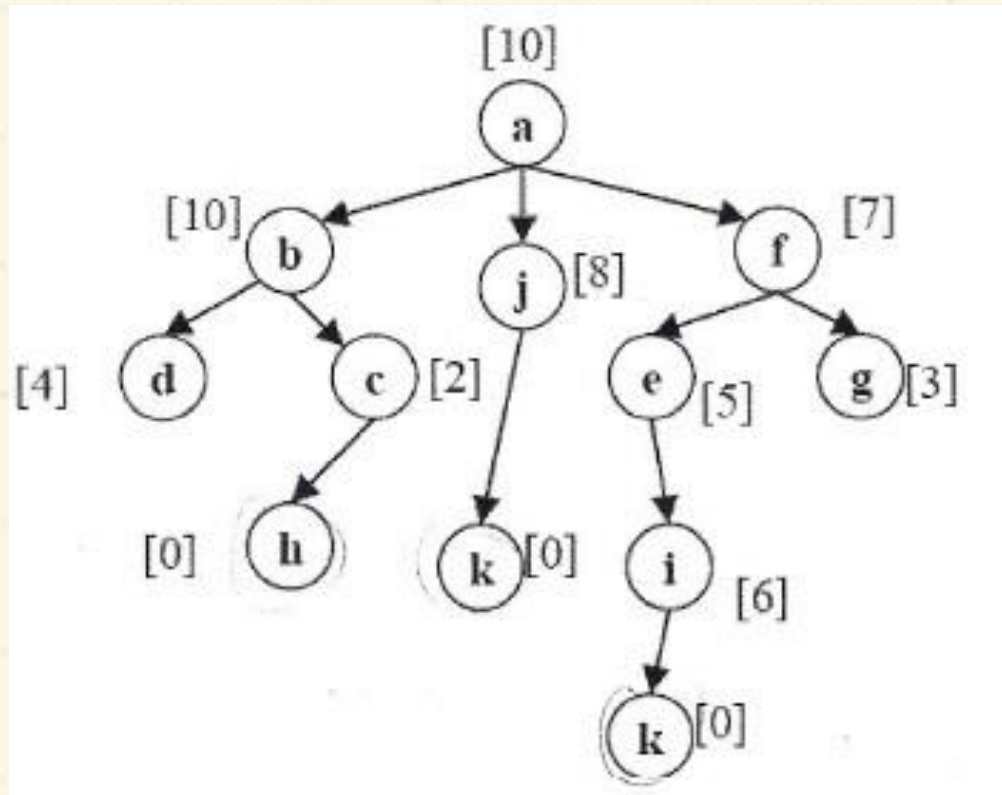
# Beam Search

## Goal – Node K



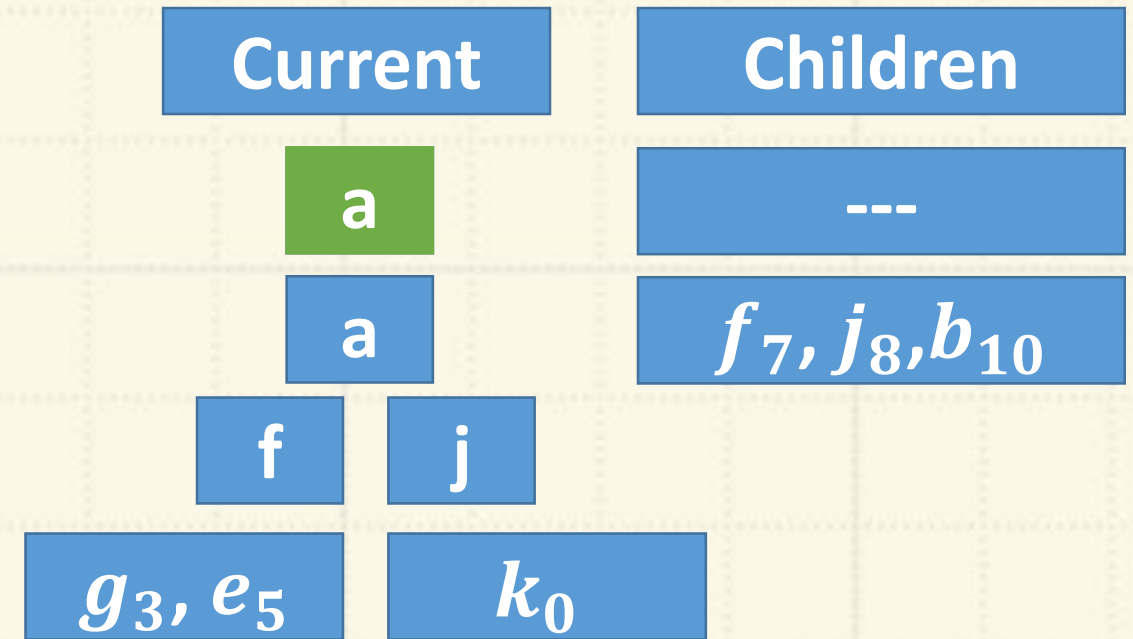
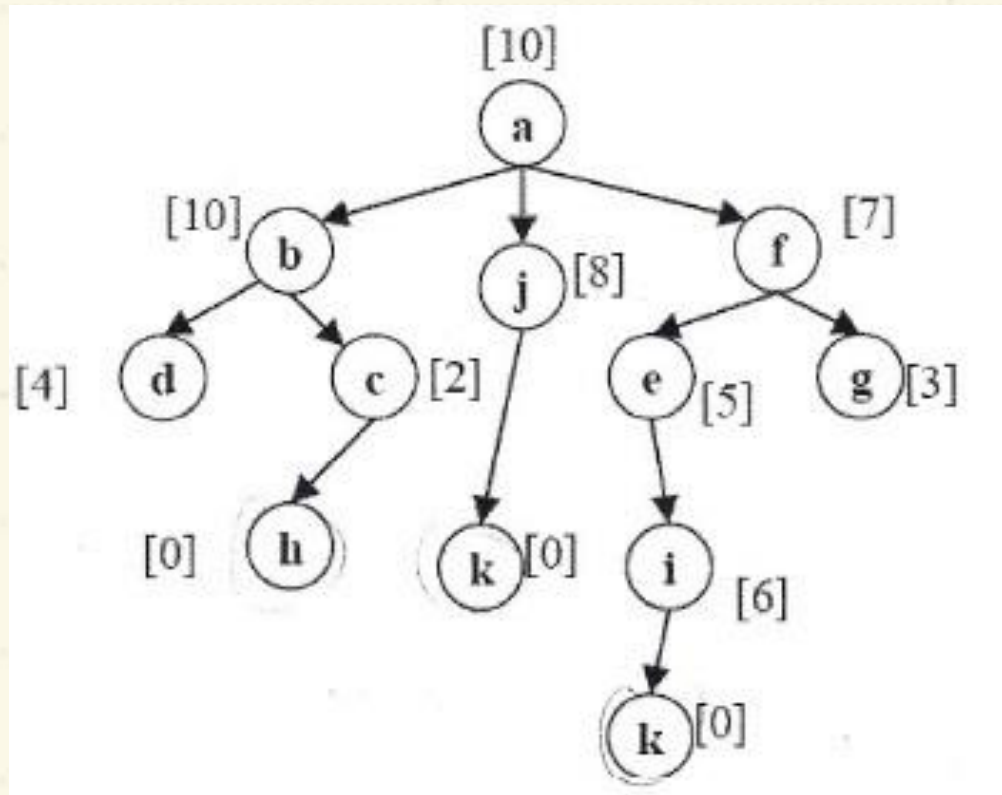
# Beam Search

## Goal – Node K



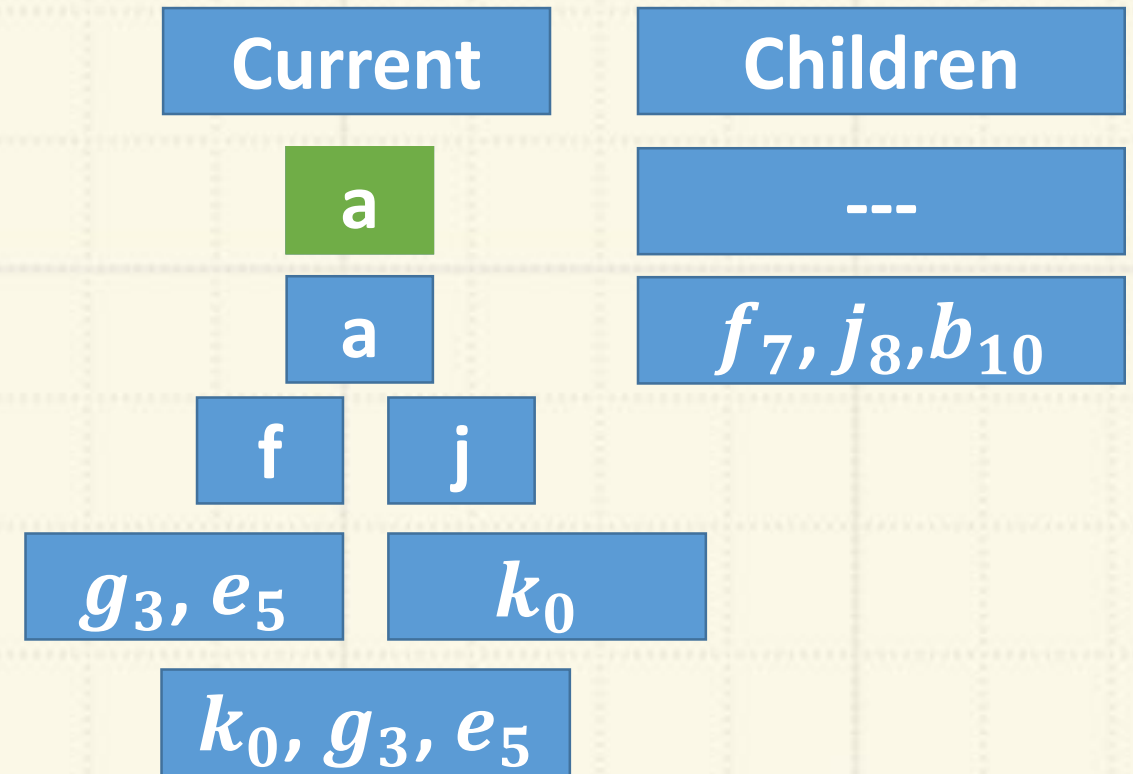
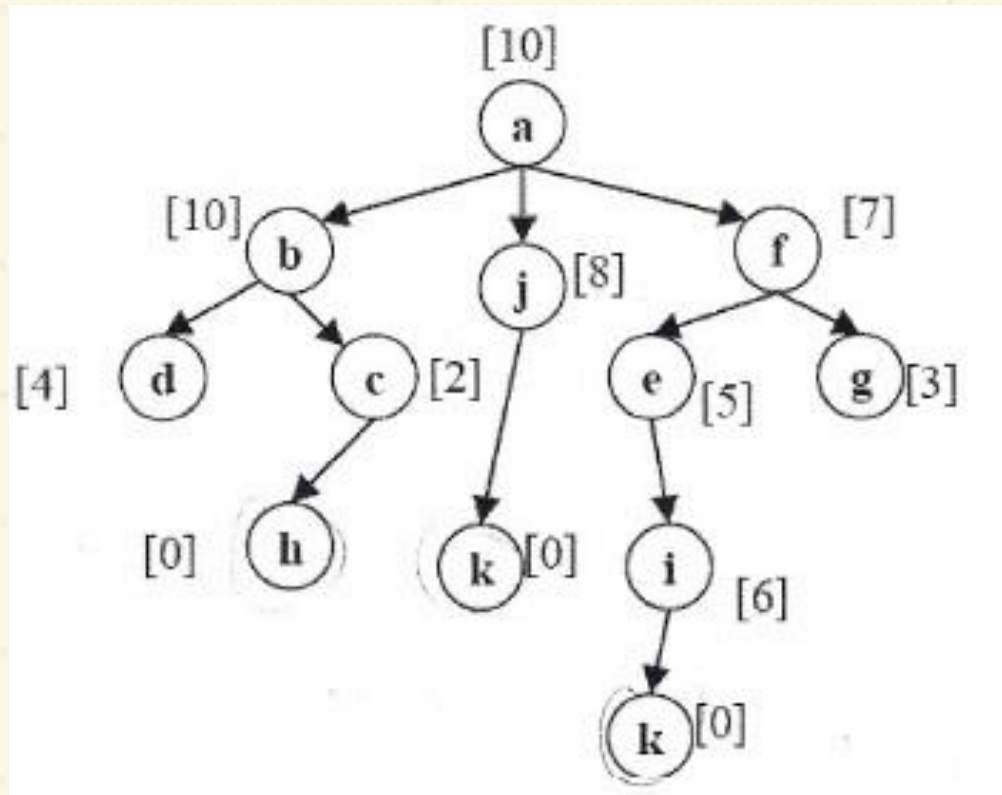
# Beam Search

## Goal – Node K



# Beam Search

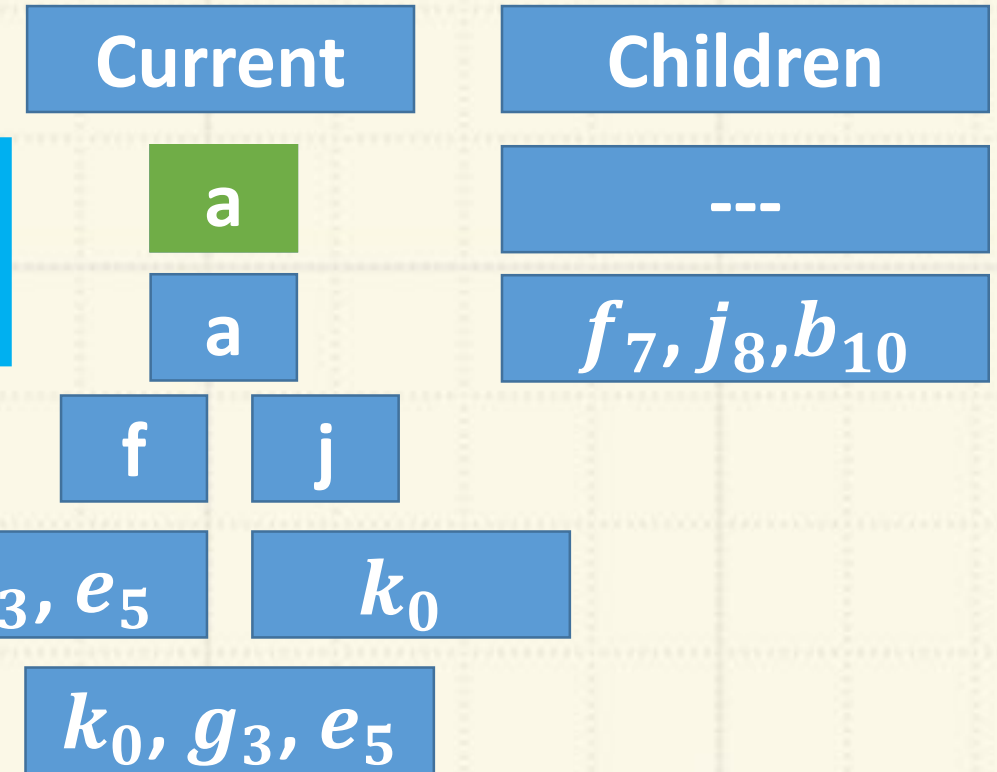
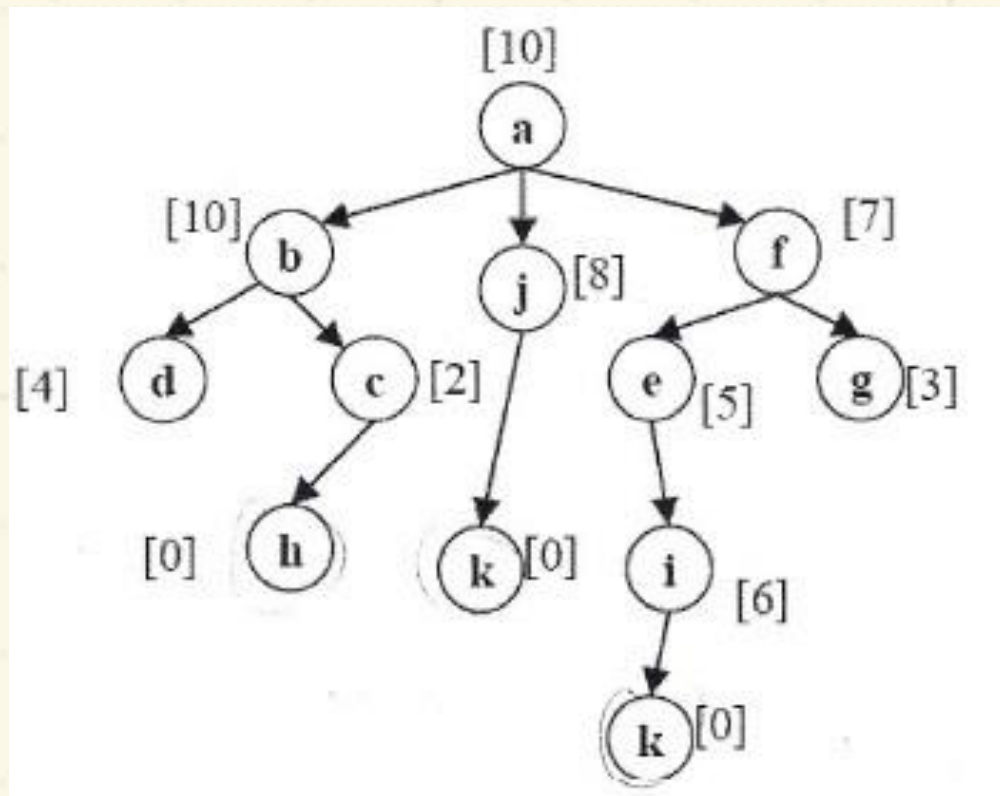
## Goal – Node K



# Beam Search

## Goal – Node K

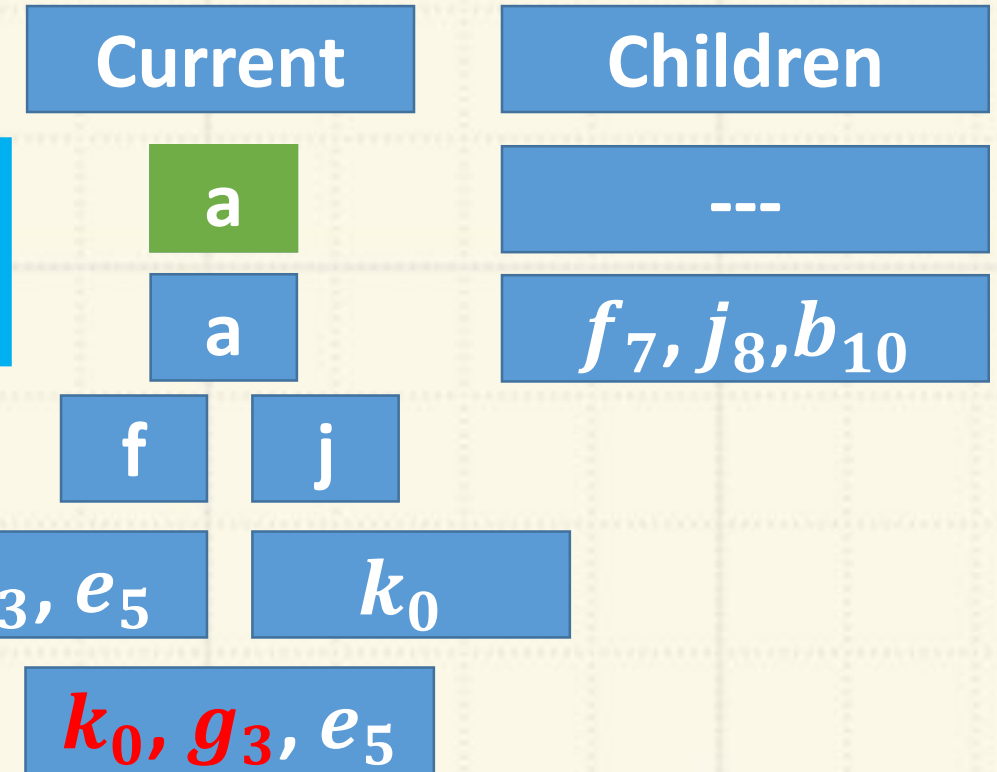
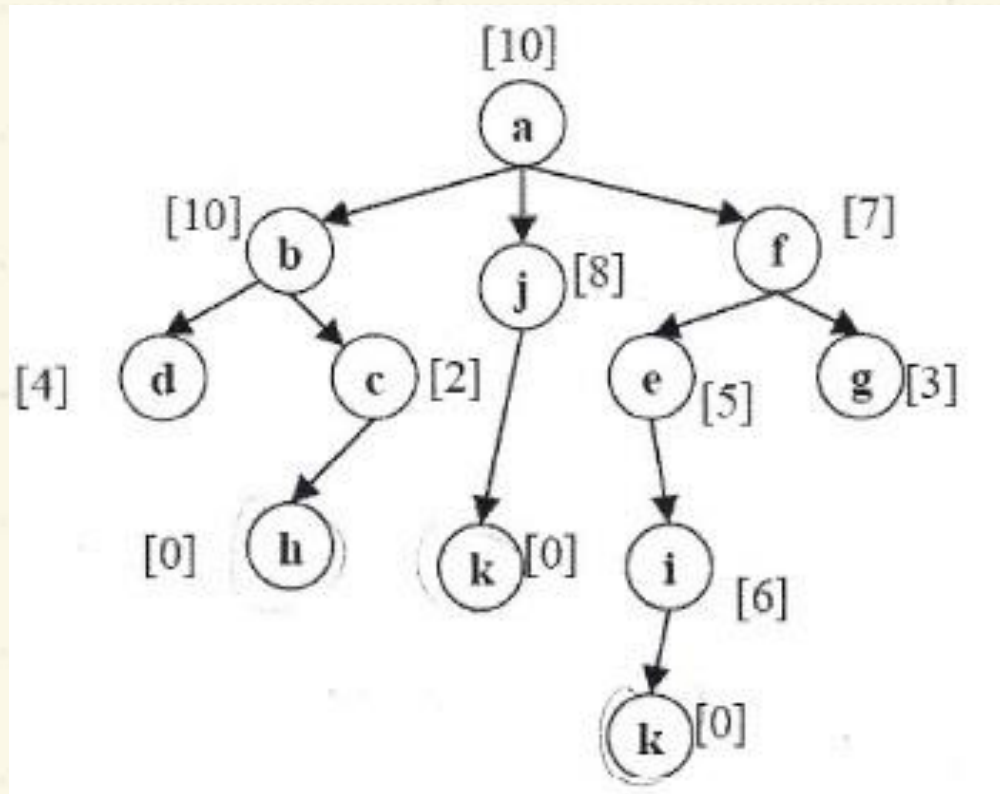
Best k  
Successors



# Beam Search

## Goal – Node K

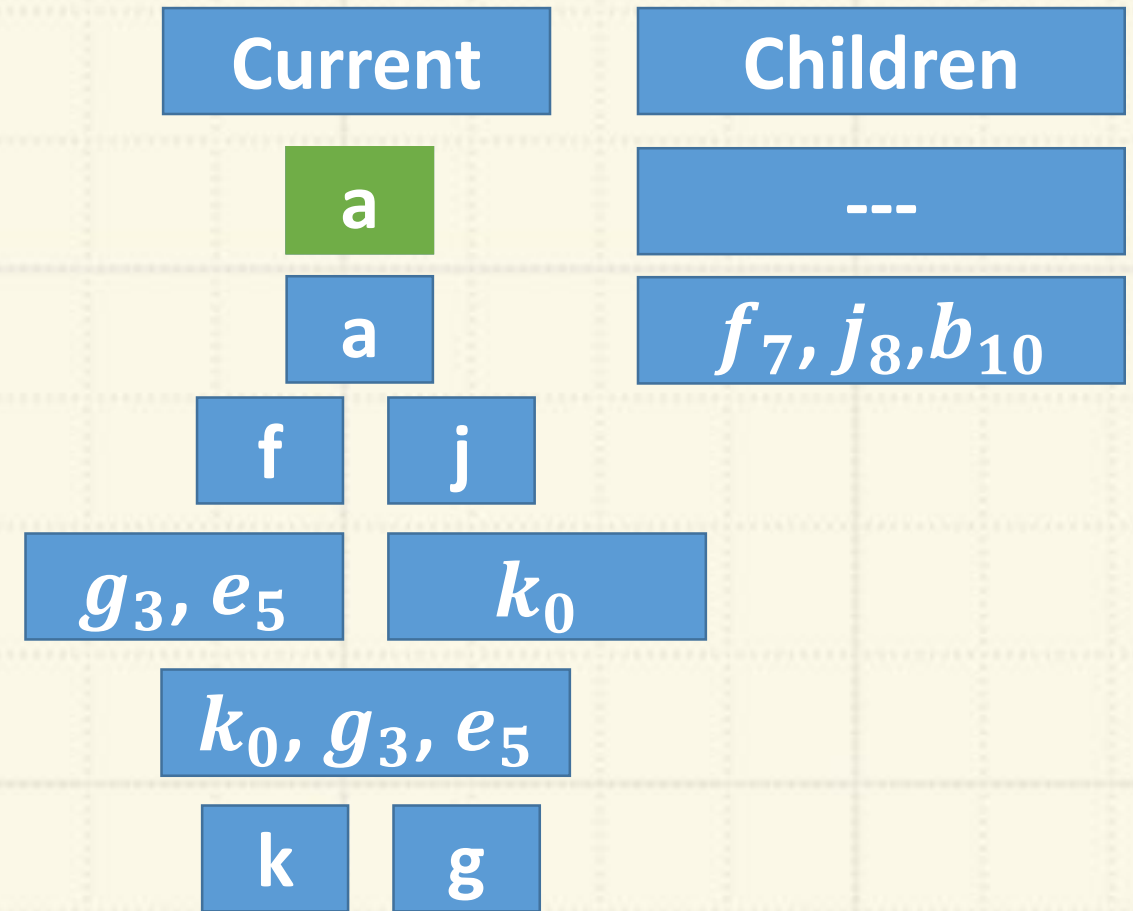
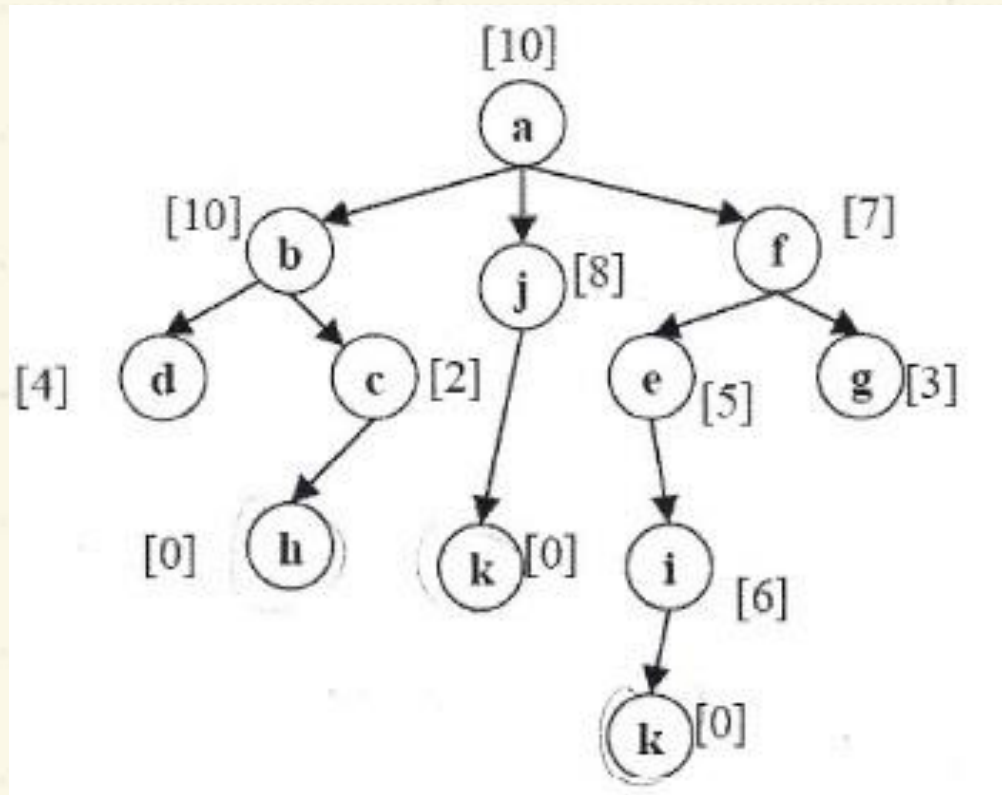
Best k  
Successors





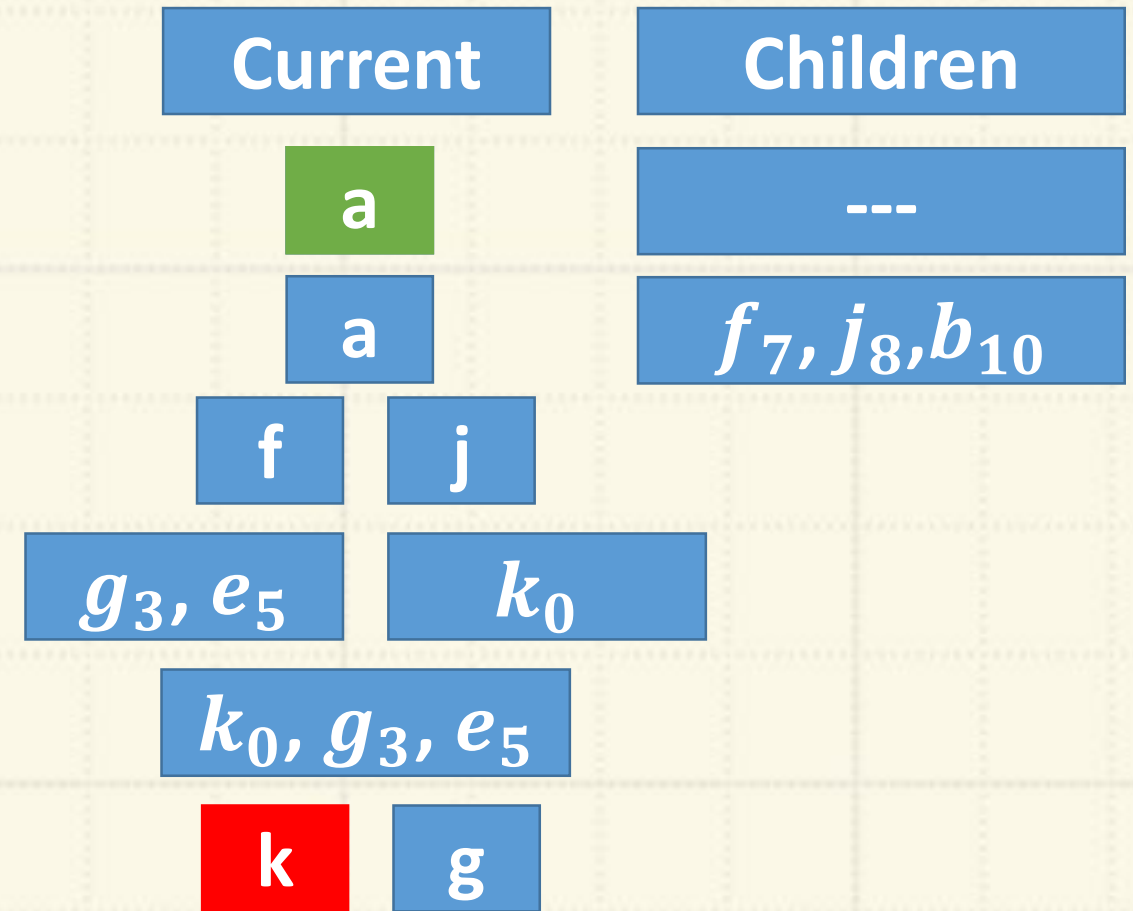
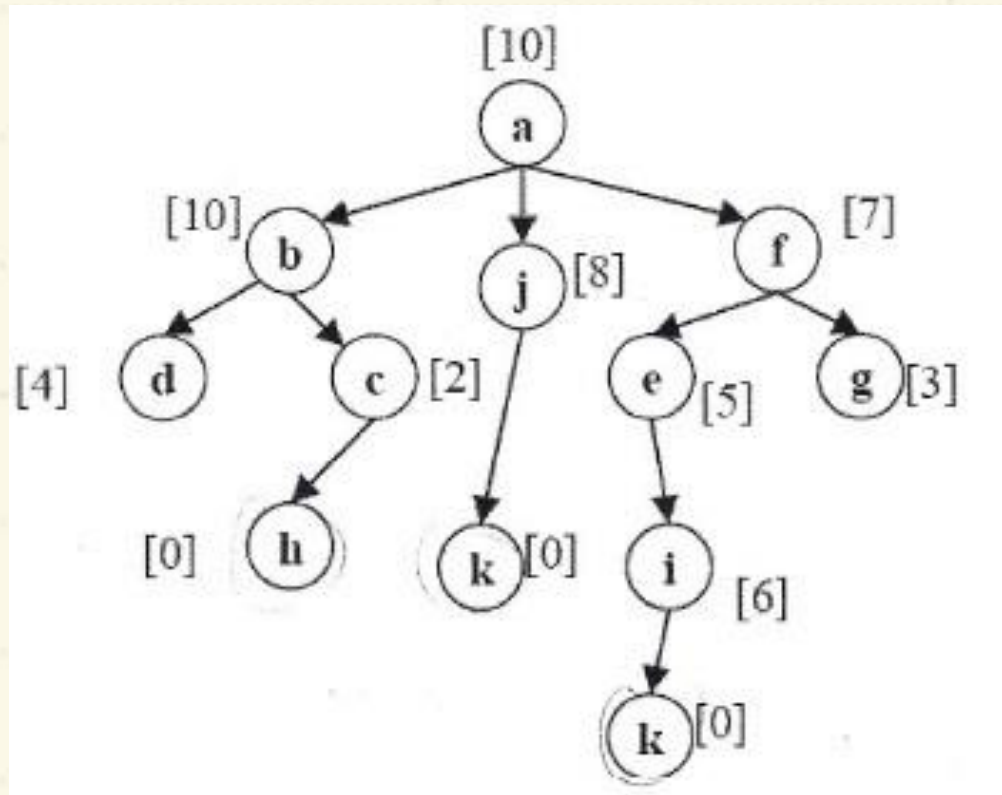
# Beam Search

## Goal – Node K



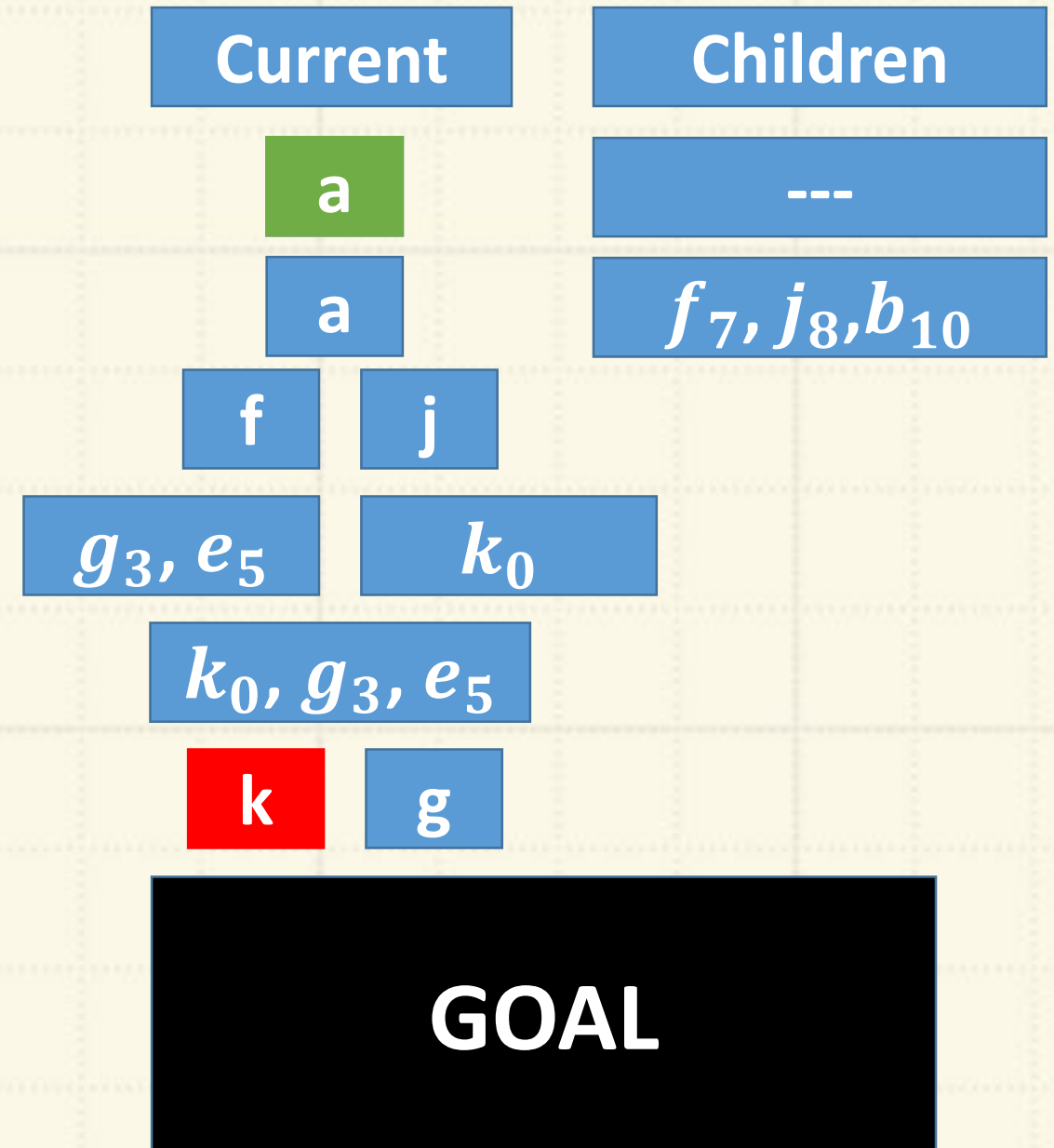
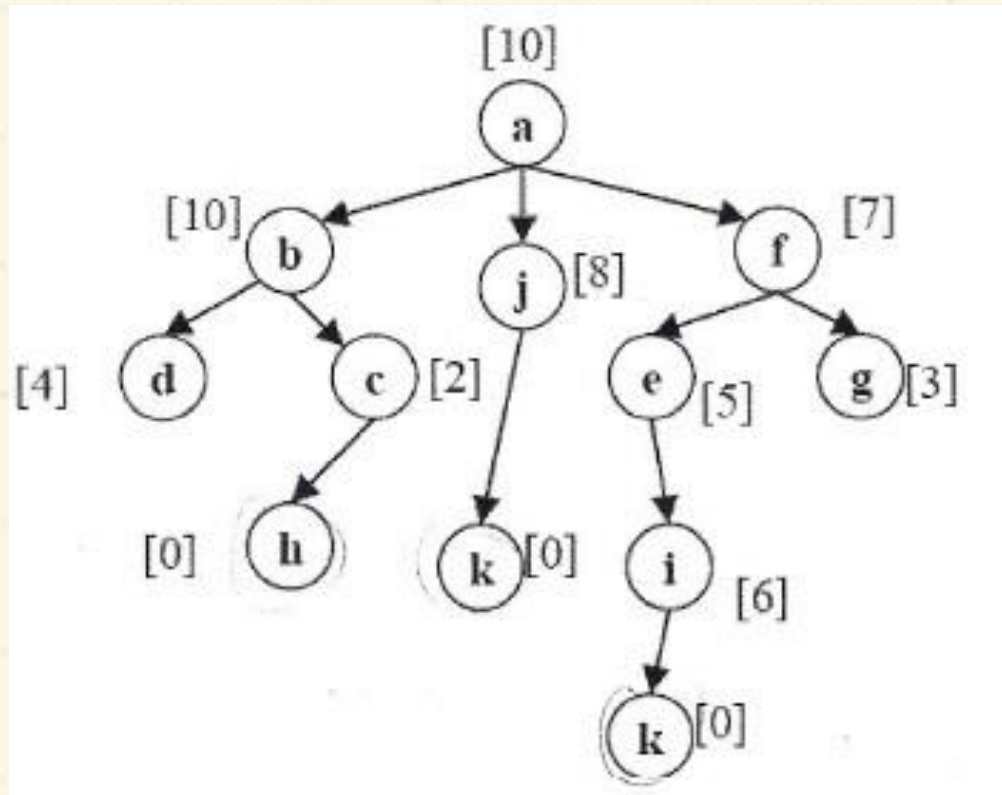
# Beam Search

## Goal – Node K



# Beam Search

## Goal – Node K



# Simulated Annealing

# Simulated Annealing Steps

1. Select a start node (root node).
2. Randomly select a child of the current node, calculate a value reflecting how good such child is like  $\text{value}(\text{node}) = -\text{heuristic}(\text{node})$ .
3. Select the child if it is better than the current node. Else try another child.

A node is better than the current node if  $\Delta E = \text{value}[\text{next}] - \text{value}[\text{current}] > 0$ .

Else if  $\Delta E < 0$ , then try to find another child.

4. If the child was not better than the current node then it will be selected with probability equal to  $p = e^{\frac{\Delta E}{T}}$

where  $\Delta E = \text{value}[\text{next}] - \text{value}[\text{current}]$

$T$  is a temperature.

5. Stop if no improvement can be found or after a fixed time.

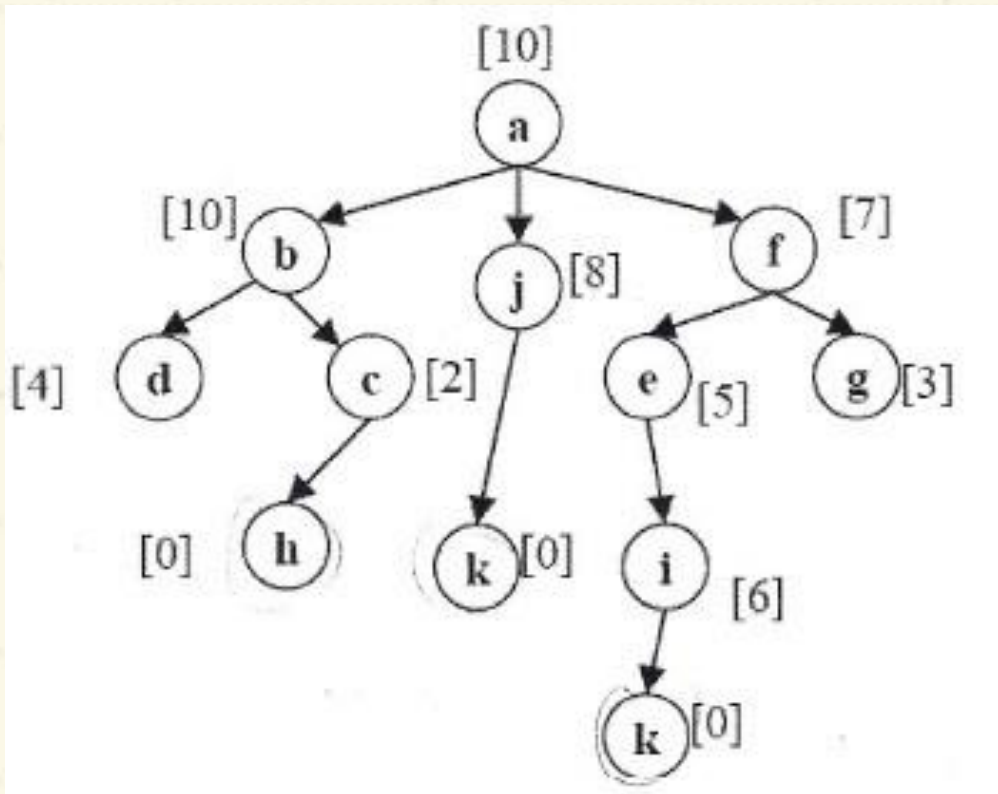
# Simulated Annealing

## Example – $T=10$

Current

a

Children

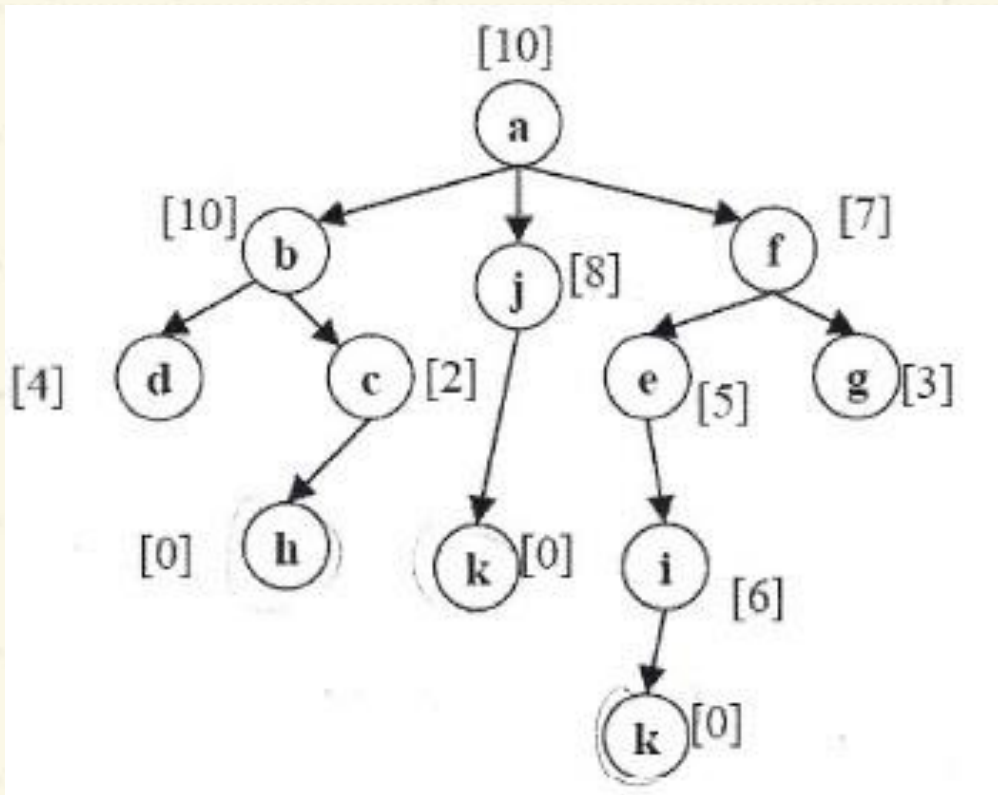


# Simulated Annealing Starting from Node a

Current

a

Children



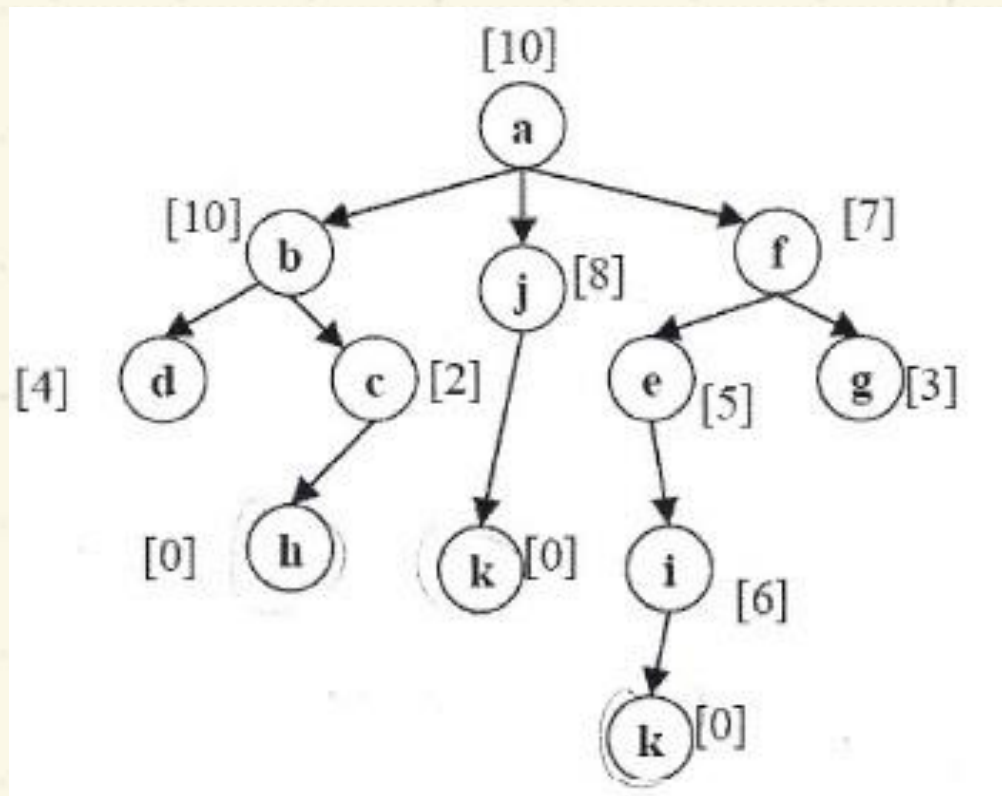


# Simulated Annealing Starting from Node a

Current

a

Children



# Simulated Annealing

Current

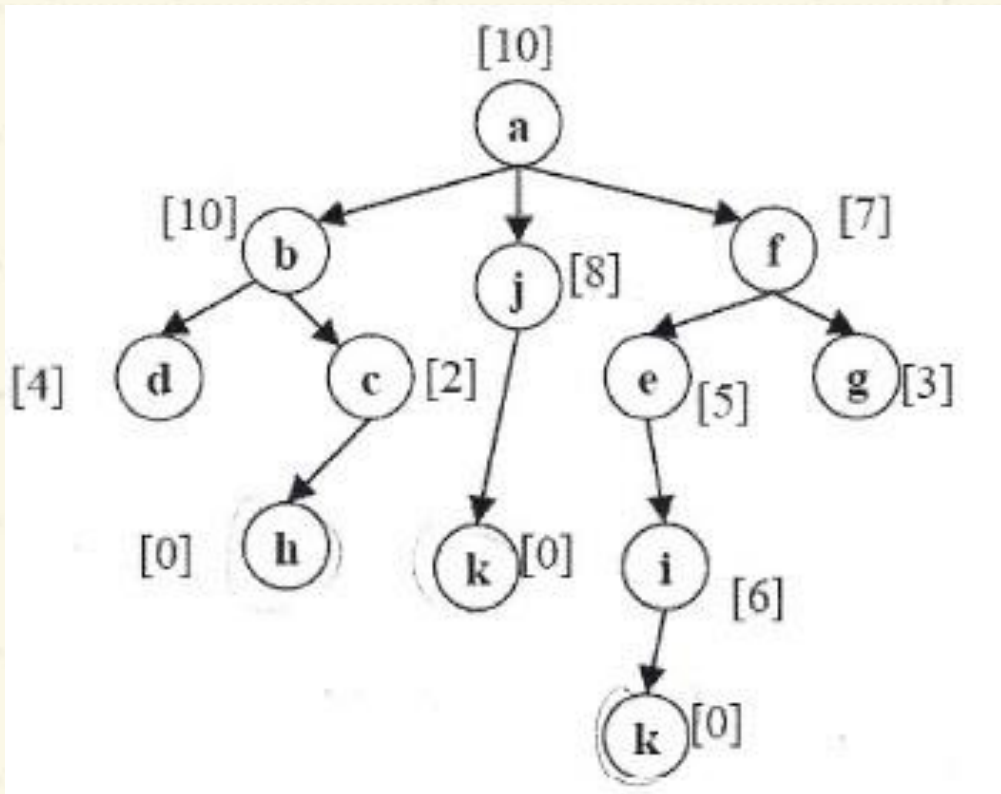
a

a

Children

---

$f_7, j_8, b_{10}$



# Simulated Annealing

Current

a

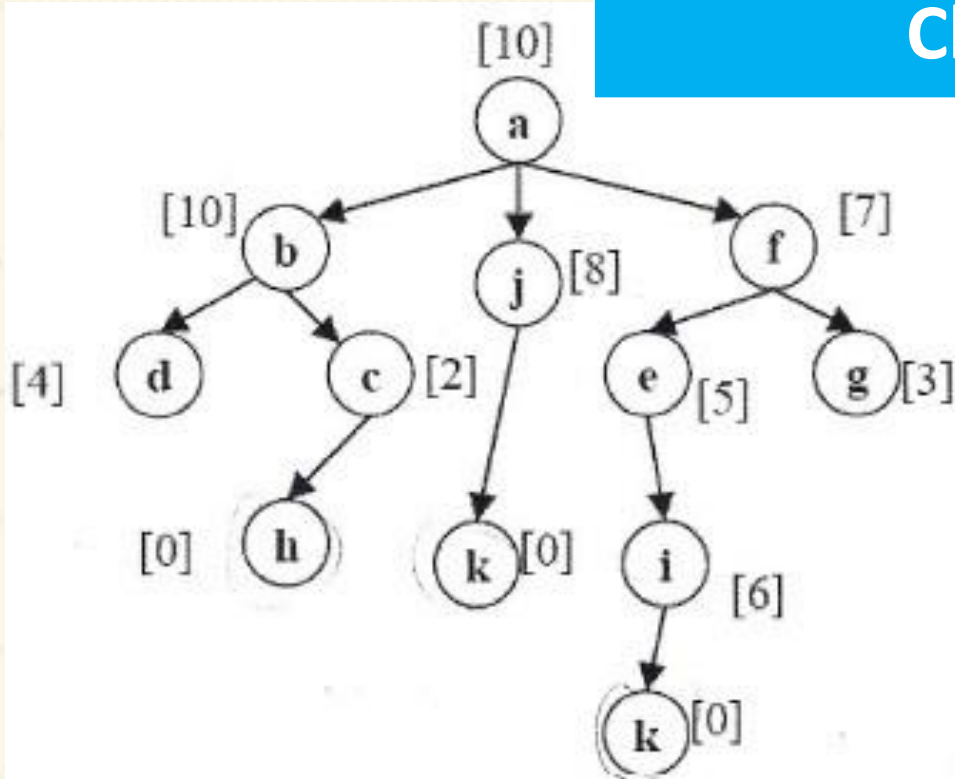
a

Children

---

$f_7, j_8, b_{10}$

Randomly Select a  
Child



# Simulated Annealing

Current

a

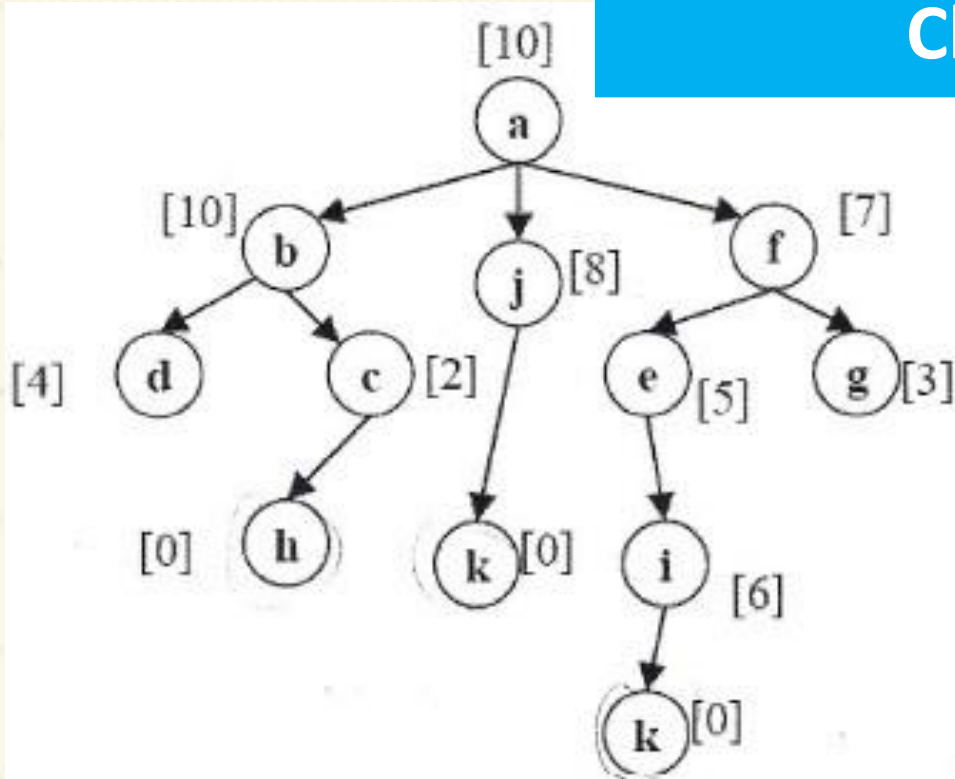
a

Children

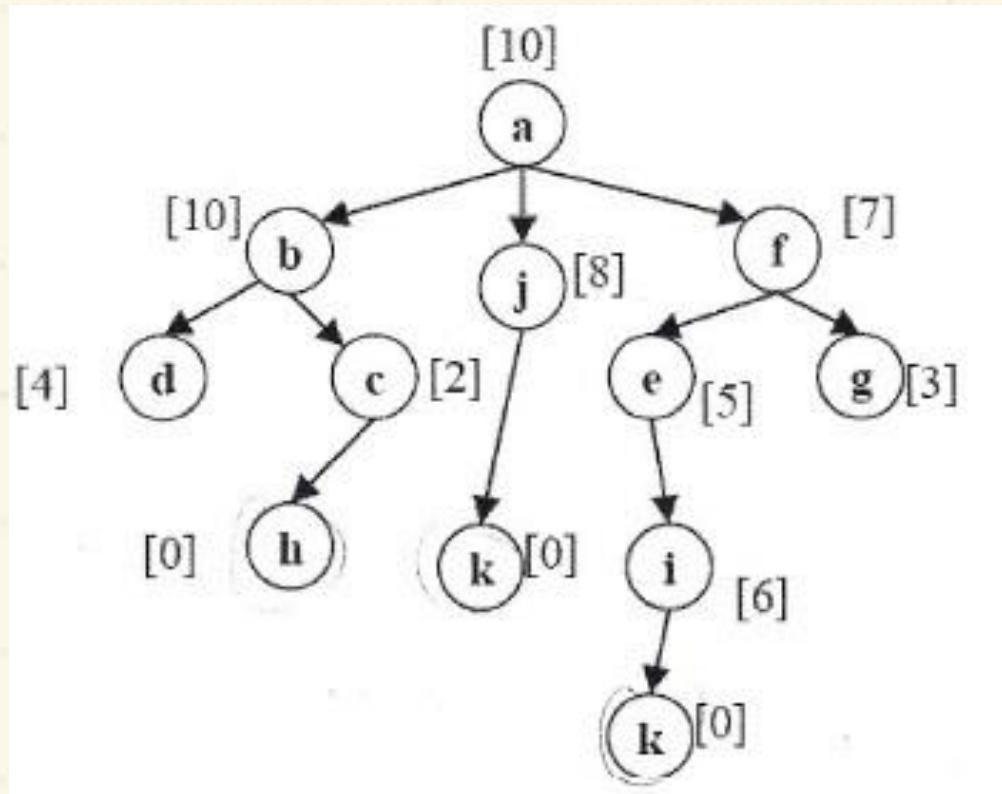
---

$f_7, j_8, b_{10}$

Randomly Select a  
Child



# Simulated Annealing



Current

a

a

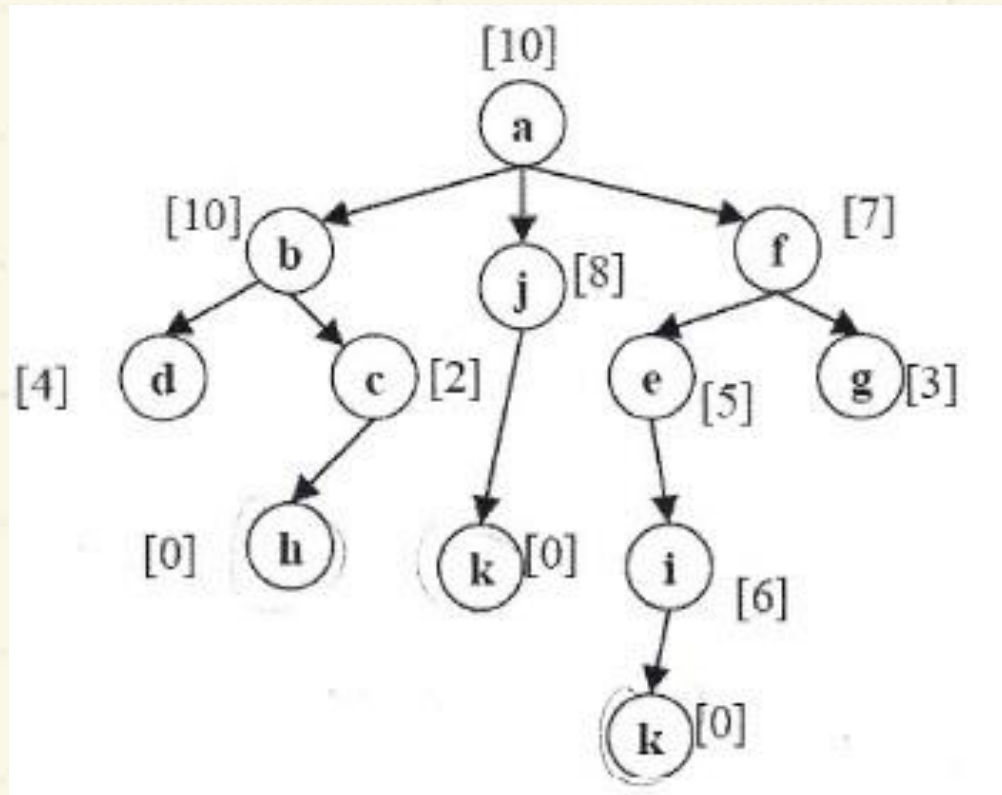
Children

---

$f_7, j_8, b_{10}$

Check if next node  $f_7$  is better than current node

# Simulated Annealing



Current

a

a

Children

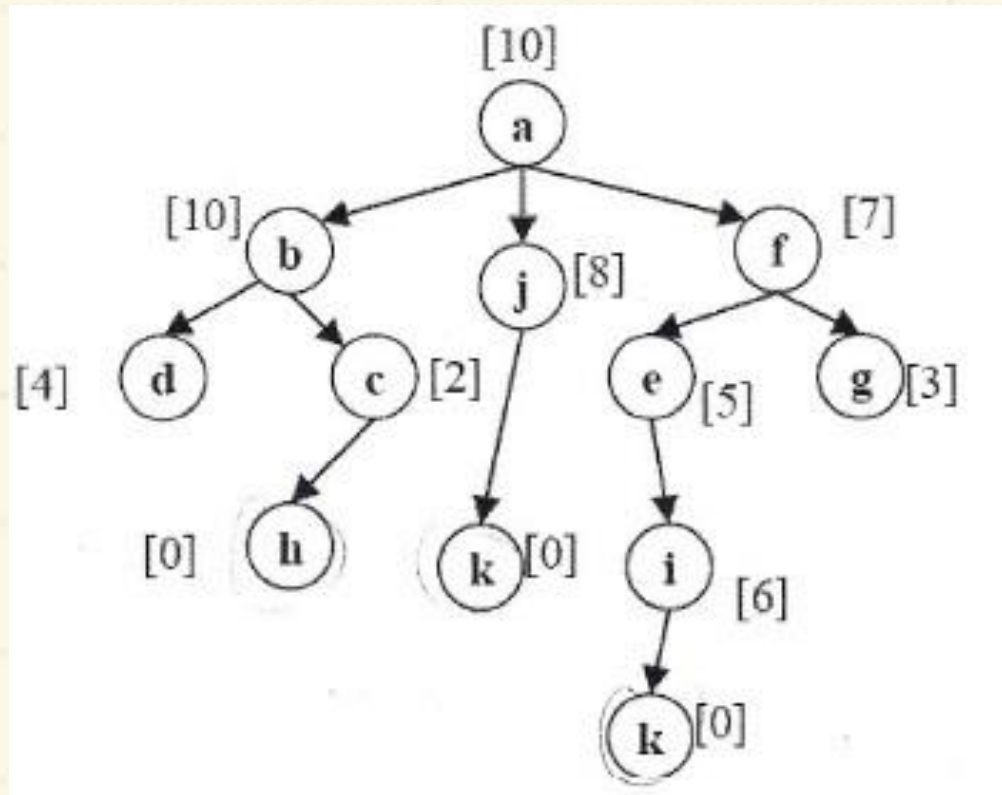
---

$f_7, j_8, b_{10}$

Check if next node  $f_7$  is better than current node

$$\Delta E > 0$$

# Simulated Annealing



Current

a

a

Children

---

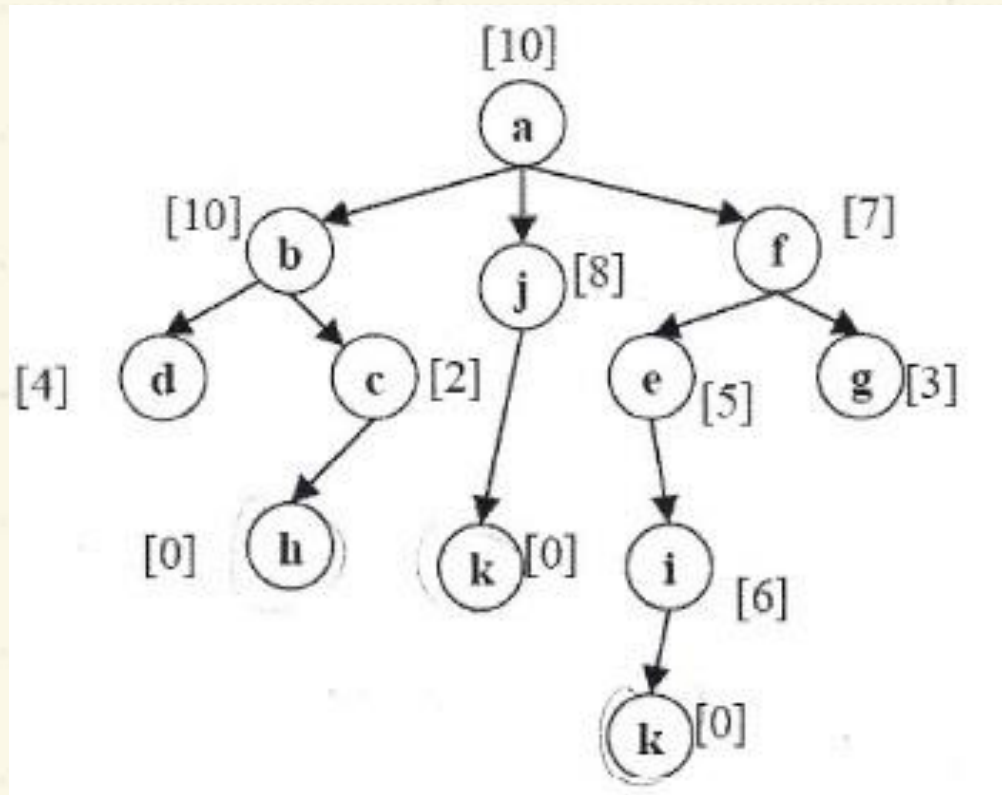
$f_7, j_8, b_{10}$

Check if next node  $f_7$  is better than current node

$$\Delta E = \text{value}(\text{next}) - \text{value}(\text{current})$$



# Simulated Annealing



Current

a

a

Children

---

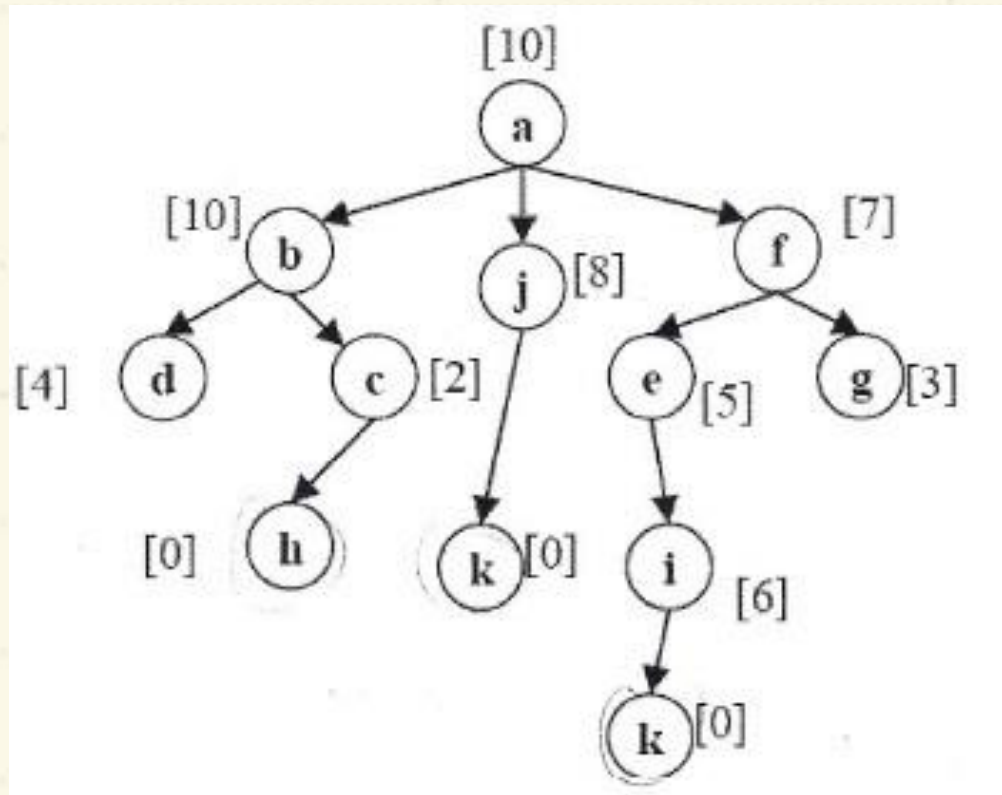
$f_7, j_8, b_{10}$

Check if next node  $f_7$  is better than current node

$$\Delta E = \text{value}(\text{next}) - \text{value}(\text{current})$$

$$\Delta E = \text{value}(f_7) - \text{value}(a_{10})$$

# Simulated Annealing



Current

a

a

Children

---

$f_7, j_8, b_{10}$

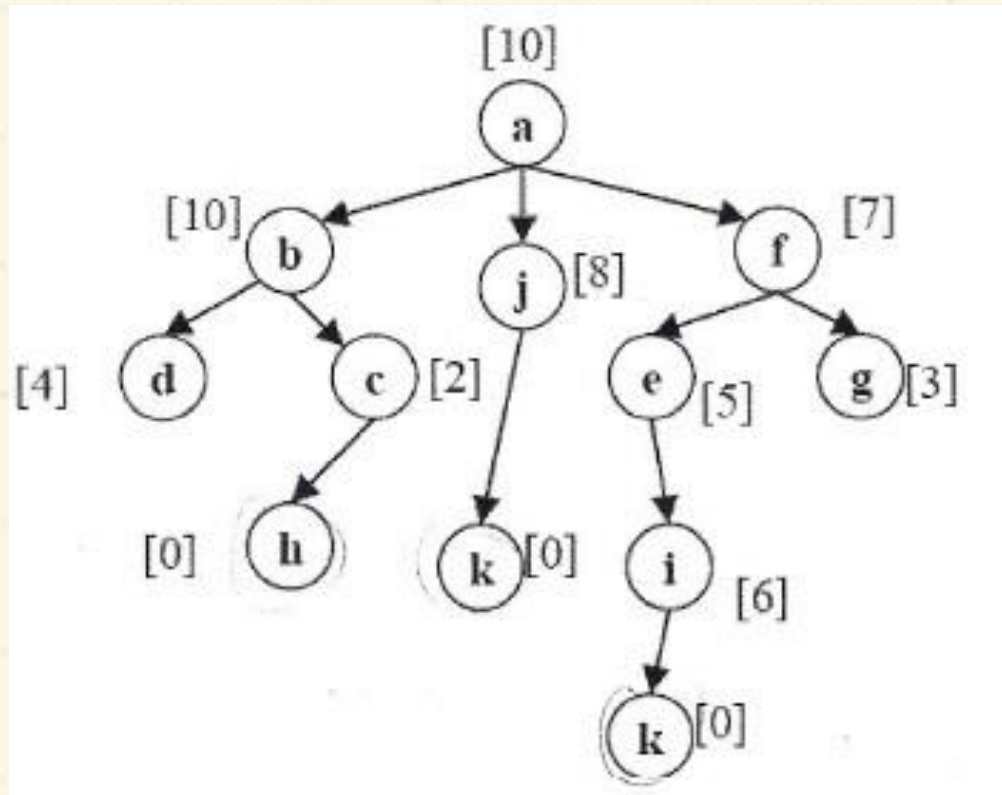
Check if next node  $f_7$  is better than current node

$$\Delta E = \text{value}(\text{next}) - \text{value}(\text{current})$$

$$\Delta E = \text{value}(f_7) - \text{value}(a_{10})$$

$$\text{value}(f_7) = -\text{heuristic}(f_7) = -7$$

# Simulated Annealing



Current

a

a

Children

---

$f_7, j_8, b_{10}$

Check if next node  $f_7$  is better than current node

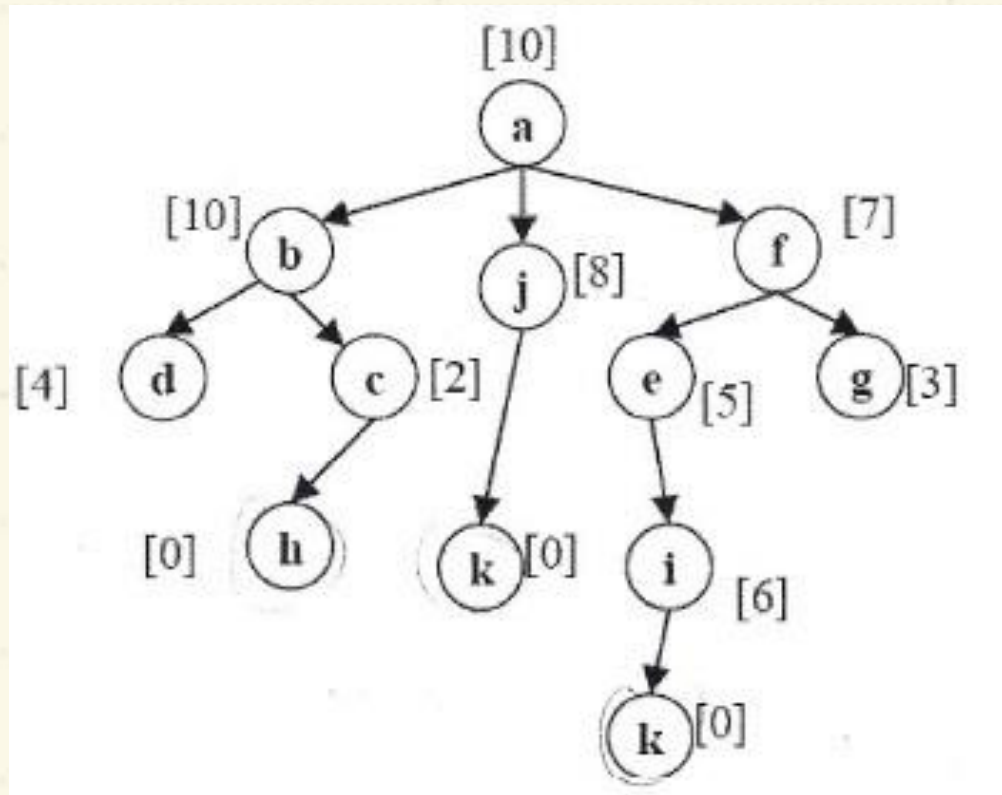
$$\Delta E = \text{value}(\text{next}) - \text{value}(\text{current})$$

$$\Delta E = \text{value}(f_7) - \text{value}(a_{10})$$

$$\text{value}(f_7) = -\text{heuristic}(f_7) = -7$$

$$\text{value}(a_{10}) = -\text{heuristic}(a_{10}) = -10$$

# Simulated Annealing



Current

a

a

Children

---

***f***<sub>7</sub>, *j*<sub>8</sub>, *b*<sub>10</sub>

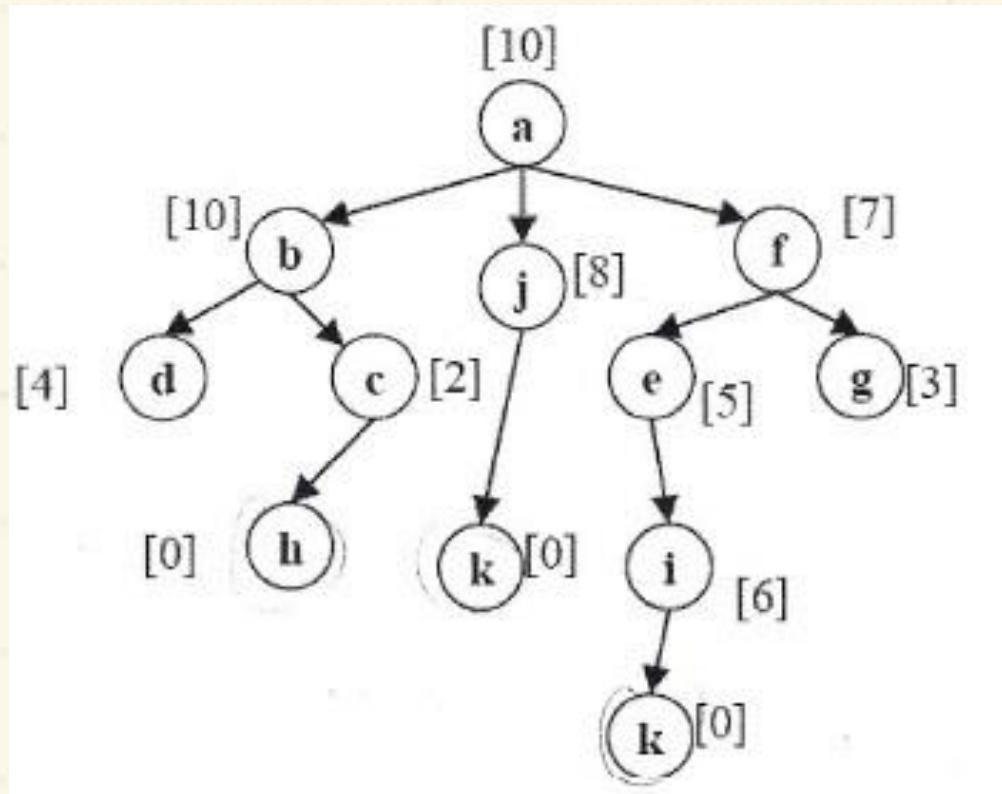
Check if next node *f*<sub>7</sub> is better than current node

$$\Delta E = \text{value}(\text{next}) - \text{value}(\text{current})$$

$$\Delta E = \text{value}(f_7) - \text{value}(a_{10})$$

$$\Delta E = -7 - (-10) = +3$$

# Simulated Annealing



Current

a

a

Children

---

$f_7, j_8, b_{10}$

Check if next node  $f_7$  is better than current node

$$\Delta E = \text{value}(\text{next}) - \text{value}(\text{current})$$

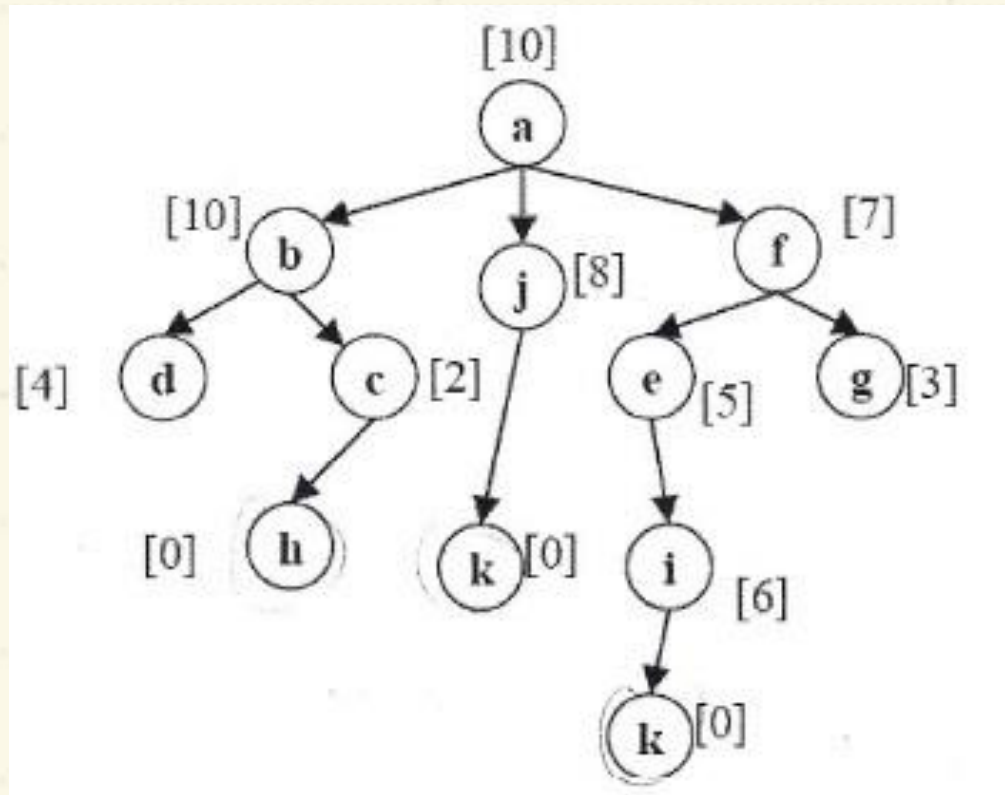
$$\Delta E = \text{value}(f_7) - \text{value}(a_{10})$$

$$\Delta E = -7 - (-10) = +3$$

$$\because \Delta E > 0$$

$\therefore f_7$  will be selected with probability 1

# Simulated Annealing



Current

a

a

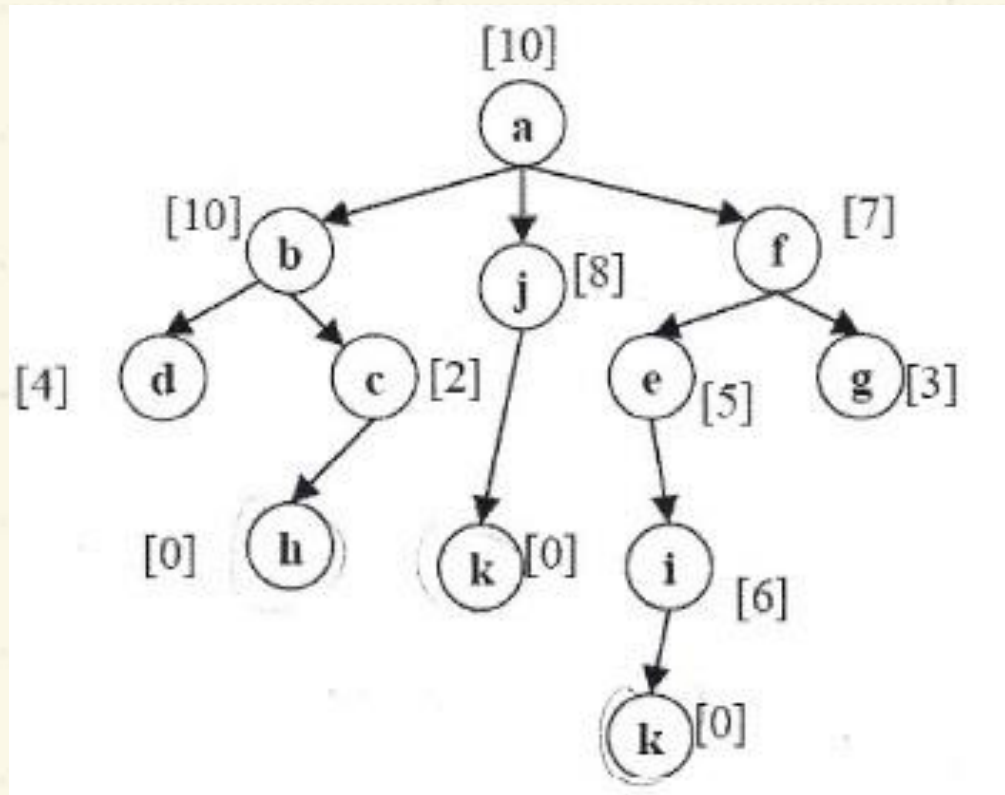
f

Children

---

$f_7, j_8, b_{10}$

# Simulated Annealing



Current

a

a

f

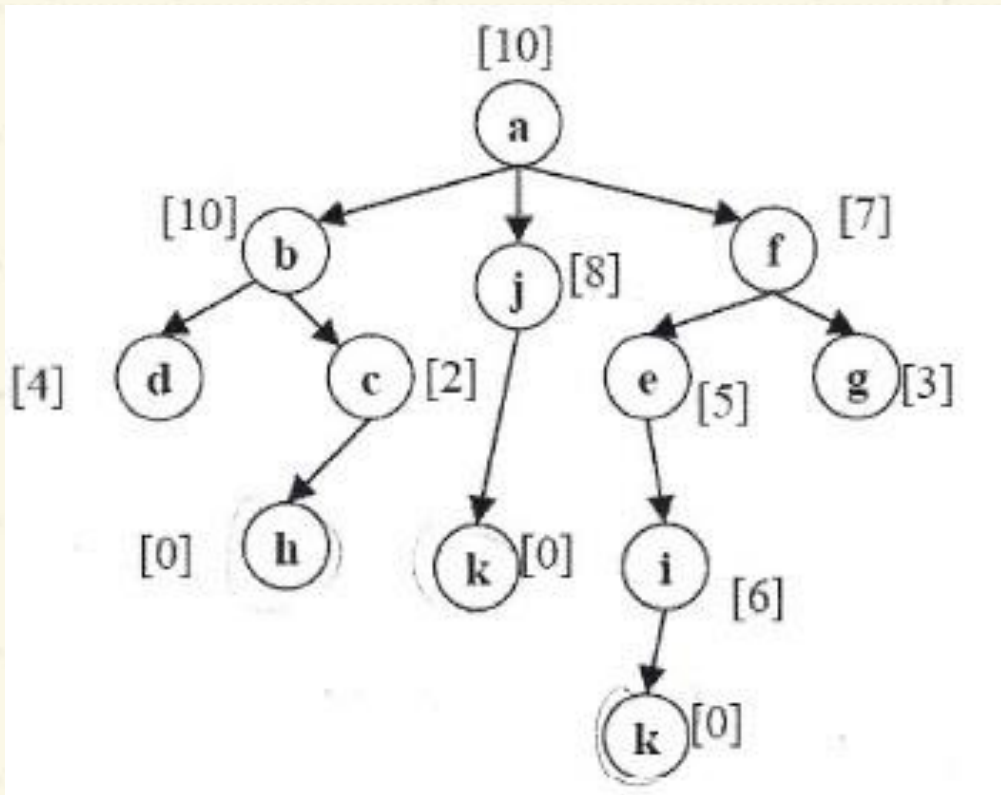
Children

---

$f_7, j_8, b_{10}$



# Simulated Annealing



Current

a

a

f

Children

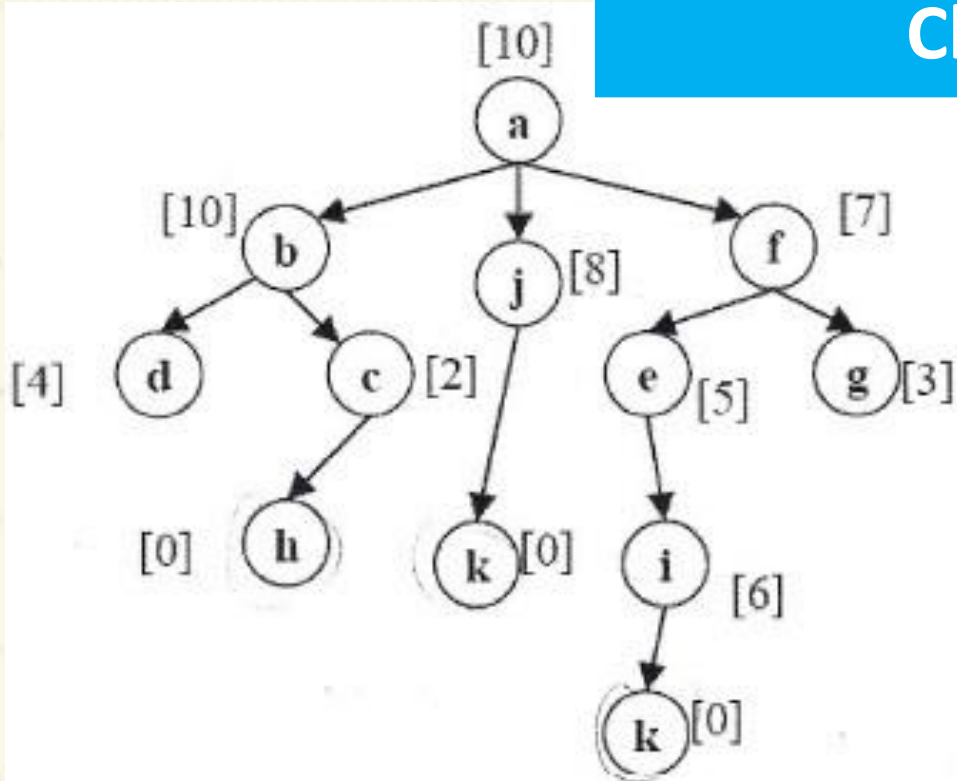
---

$f_7, j_8, b_{10}$

$e_5, g_3$

# Simulated Annealing

Randomly Select a  
Child



Current

a

a

f

Children

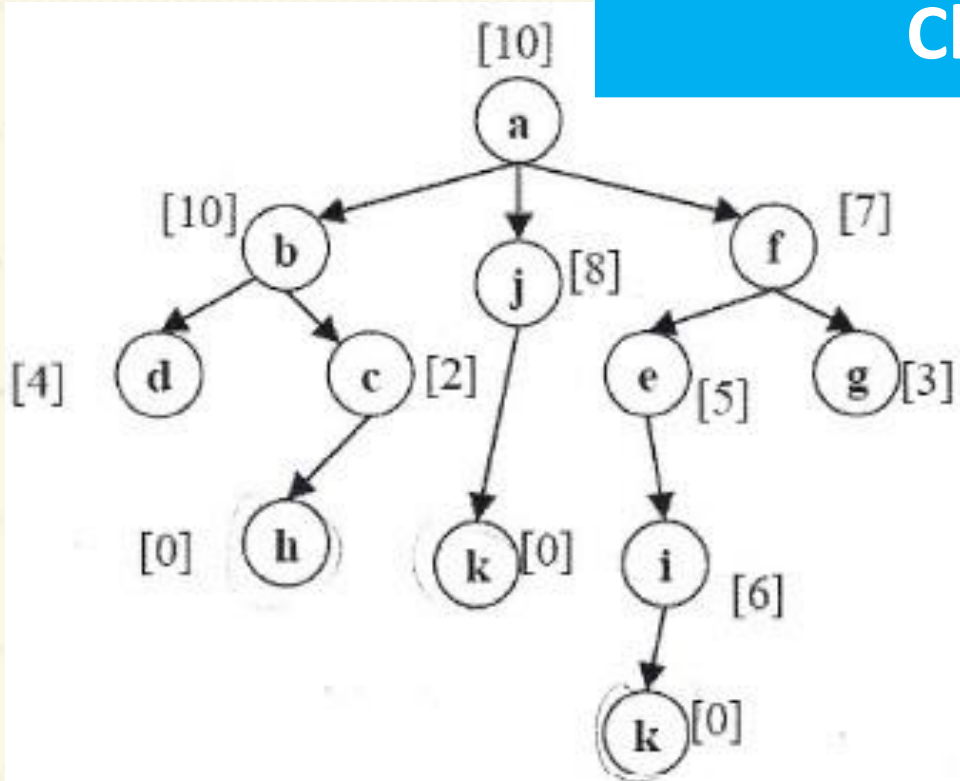
---

$f_7, j_8, b_{10}$

$e_5, g_3$

# Simulated Annealing

Randomly Select a  
Child



Current

a

a

f

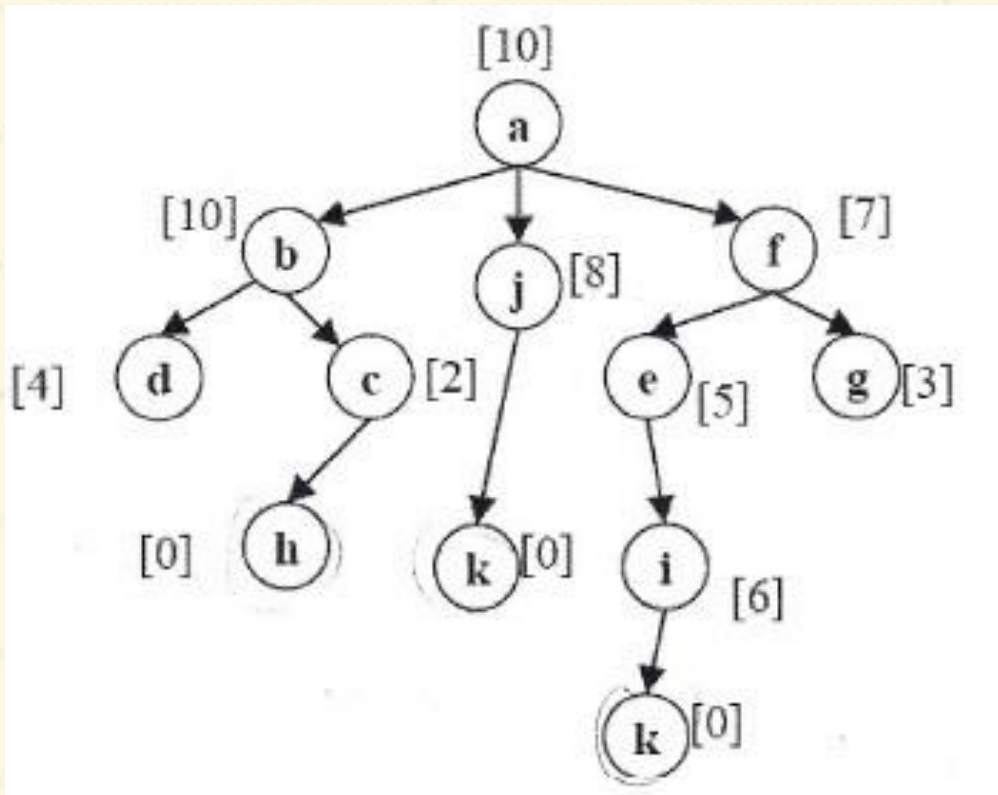
Children

---

$f_7, j_8, b_{10}$

$e_5, g_3$

# Simulated Annealing



Current

a

a

f

Children

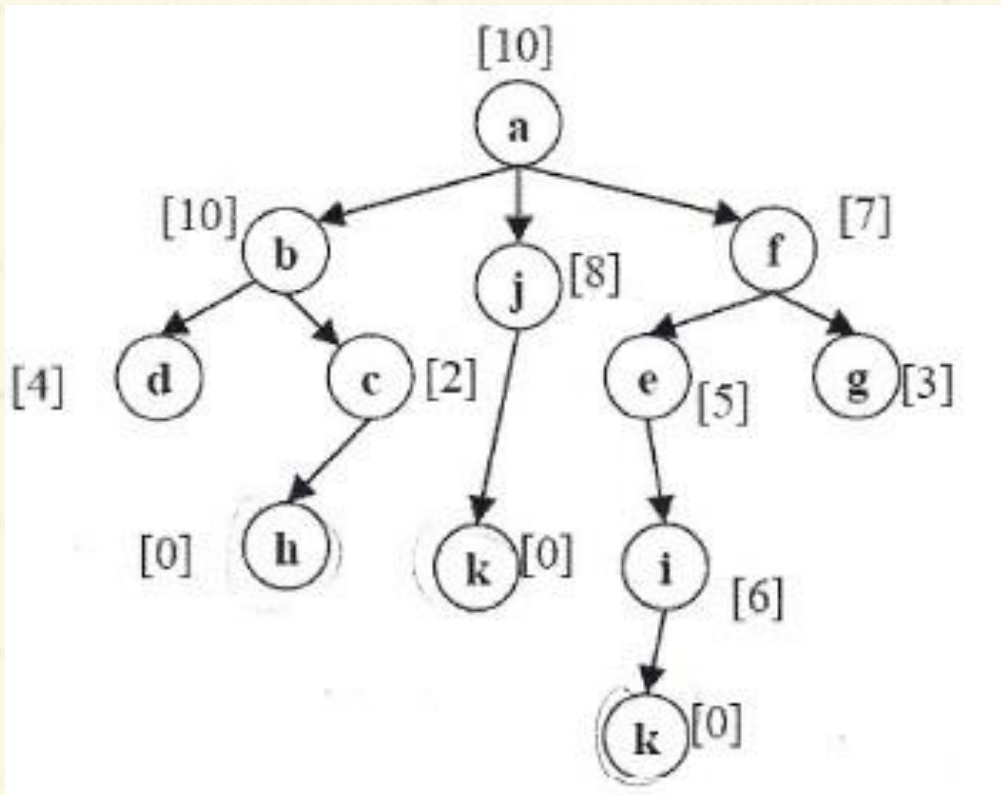
---

$f_7, j_8, b_{10}$

$e_5, g_3$

Check if next node  $e_5$  is better than current node

# Simulated Annealing



Current

a

a

f

Children

---

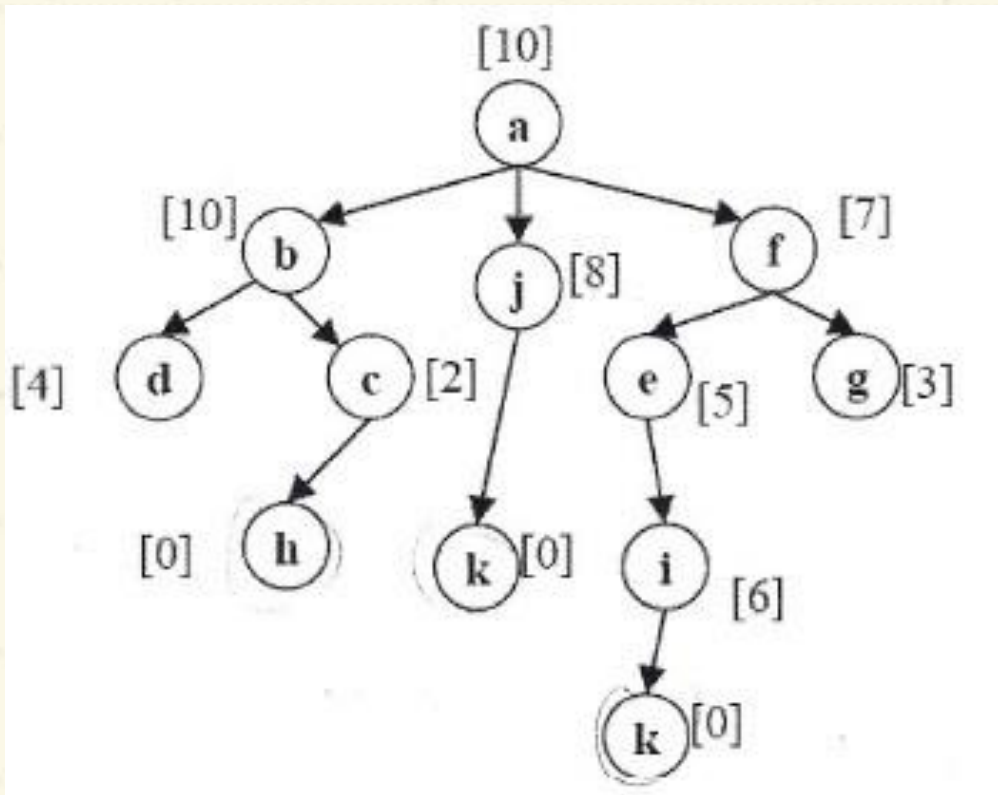
$f_7, j_8, b_{10}$

$e_5, g_3$

Check if next node  $e_5$  is better than current node

$$\Delta E > 0$$

# Simulated Annealing



Current

a

a

f

Children

---

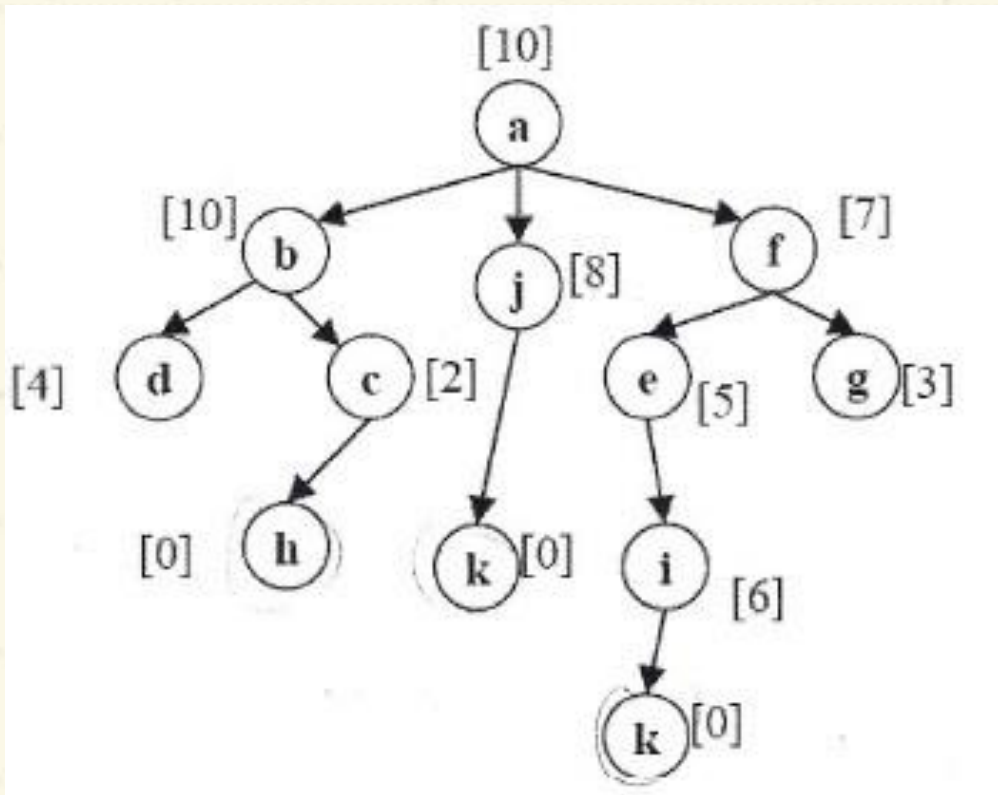
$f_7, j_8, b_{10}$

$e_5, g_3$

Check if next node  $e_5$  is better than current node

$$\Delta E = \text{value}(\text{next}) - \text{value}(\text{current})$$

# Simulated Annealing



Current

a

a

f

Children

---

$f_7, j_8, b_{10}$

$e_5, g_3$

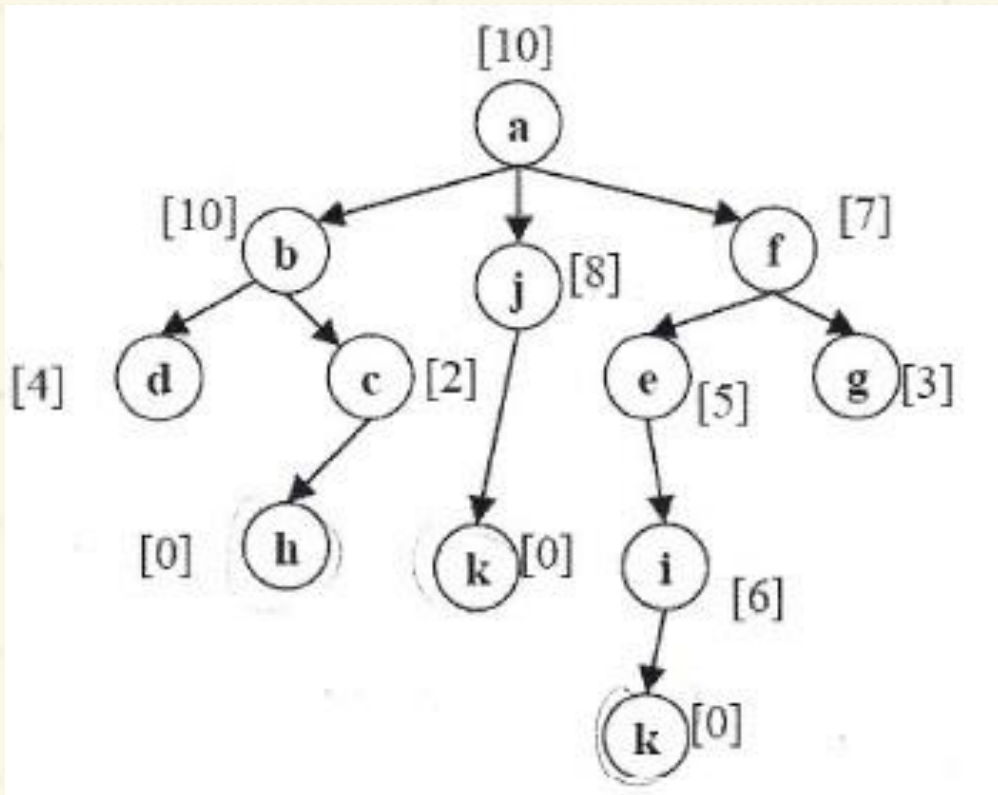
Check if next node  $e_5$  is better than current node

$$\Delta E = \text{value}(\text{next}) - \text{value}(\text{current})$$

$$\Delta E = \text{value}(e_5) - \text{value}(f_7)$$



# Simulated Annealing



Current

Children

a

---

a

$f_7, j_8, b_{10}$

f

$e_5, g_3$

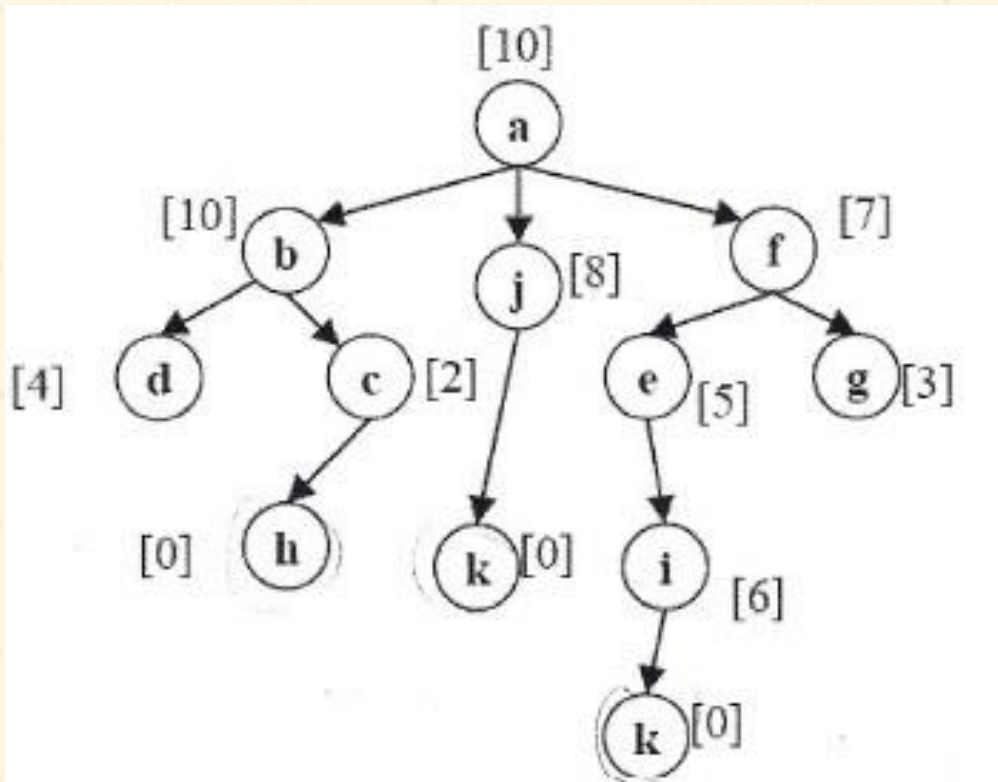
Check if next node  $e_5$  is better than current node

$$\Delta E = \text{value}(\text{next}) - \text{value}(\text{current})$$

$$\Delta E = \text{value}(e_5) - \text{value}(f_7)$$

$$\text{value}(e_5) = -\text{heuristic}(e_5) = -5$$

# Simulated Annealing



$$\text{value}(f_7) = -\text{heuristic}(f_7) = -7$$

Current

a

a

f

Children

---

$f_7, j_8, b_{10}$

$e_5, g_3$

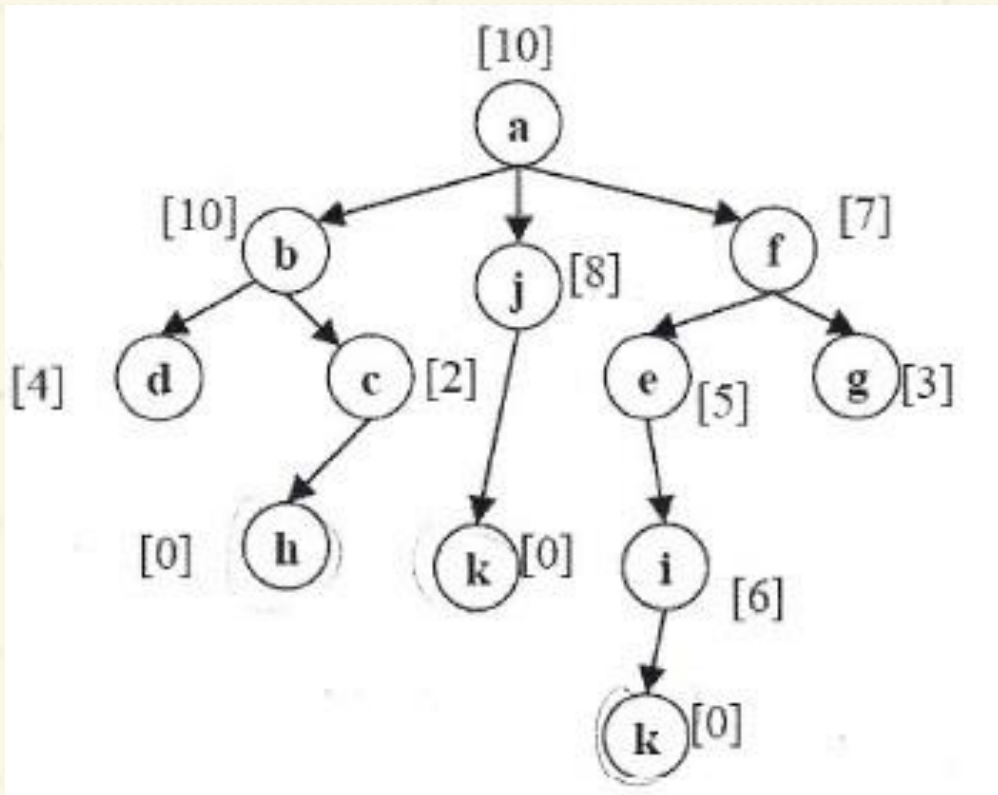
Check if next node  $e_5$  is better than current node

$$\Delta E = \text{value}(\text{next}) - \text{value}(\text{current})$$

$$\Delta E = \text{value}(e_5) - \text{value}(f_7)$$

$$\text{value}(e_5) = -\text{heuristic}(e_5) = -5$$

# Simulated Annealing



Current

Children

a

---

a

$f_7, j_8, b_{10}$

f

$e_5, g_3$

Check if next node  $e_5$  is better than current node

$$\Delta E = \text{value}(\text{next}) - \text{value}(\text{current})$$

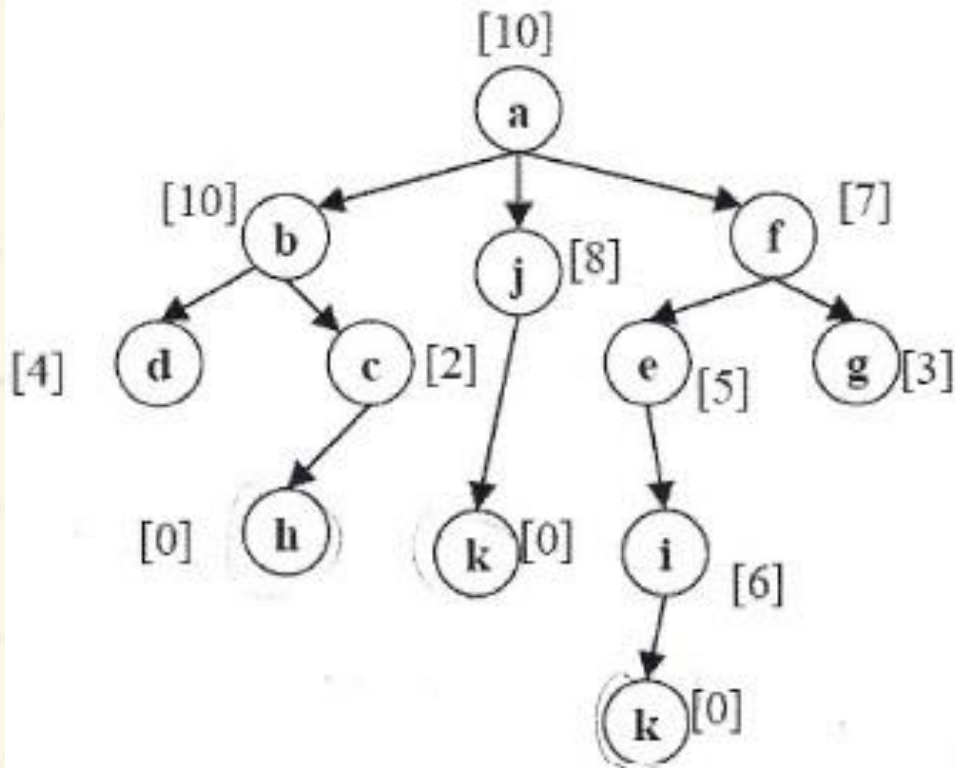
$$\Delta E = \text{value}(e_5) - \text{value}(f_7)$$

$$\Delta E = -5 - (-7) = +2$$

# Simulated Annealing

$\because \Delta E > 0$

$\therefore e_5$  will be selected with probability 1



Current

a

a

f

Children

---

$f_7, j_8, b_{10}$

$e_5, g_3$

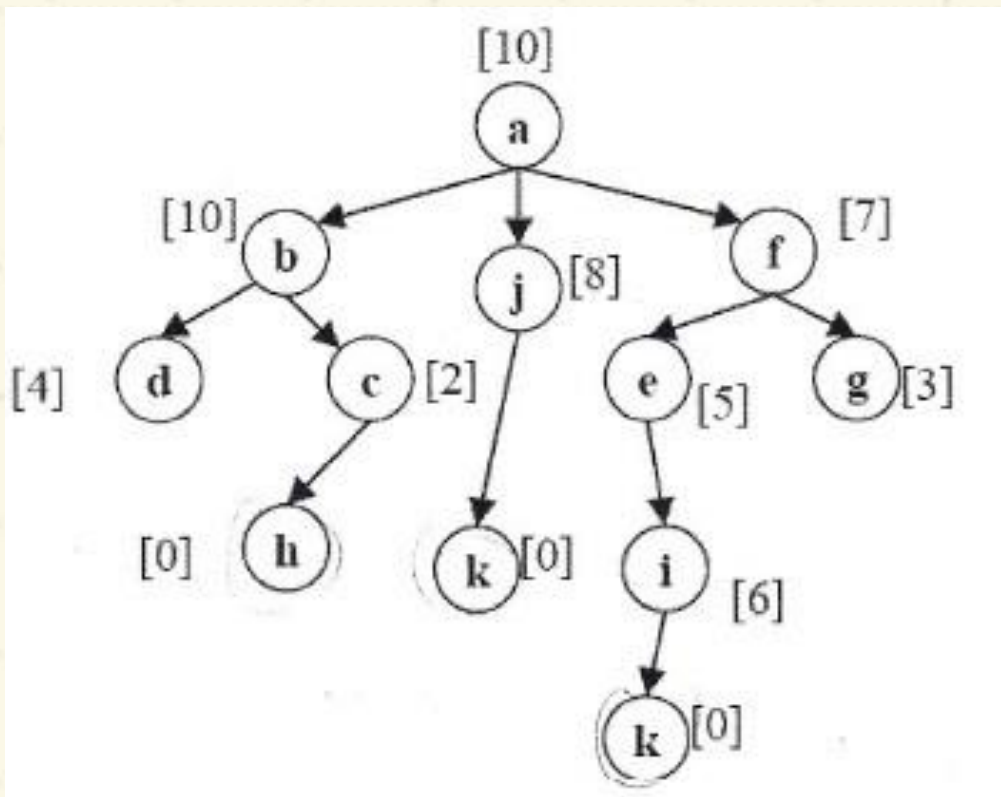
Check if next node  $e_5$  is better than current node

$\Delta E = \text{value}(\text{next}) - \text{value}(\text{current})$

$\Delta E = \text{value}(e_5) - \text{value}(f_7)$

$\Delta E = -5 - (-7) = +2$

# Simulated Annealing



Current

a

a

f

e

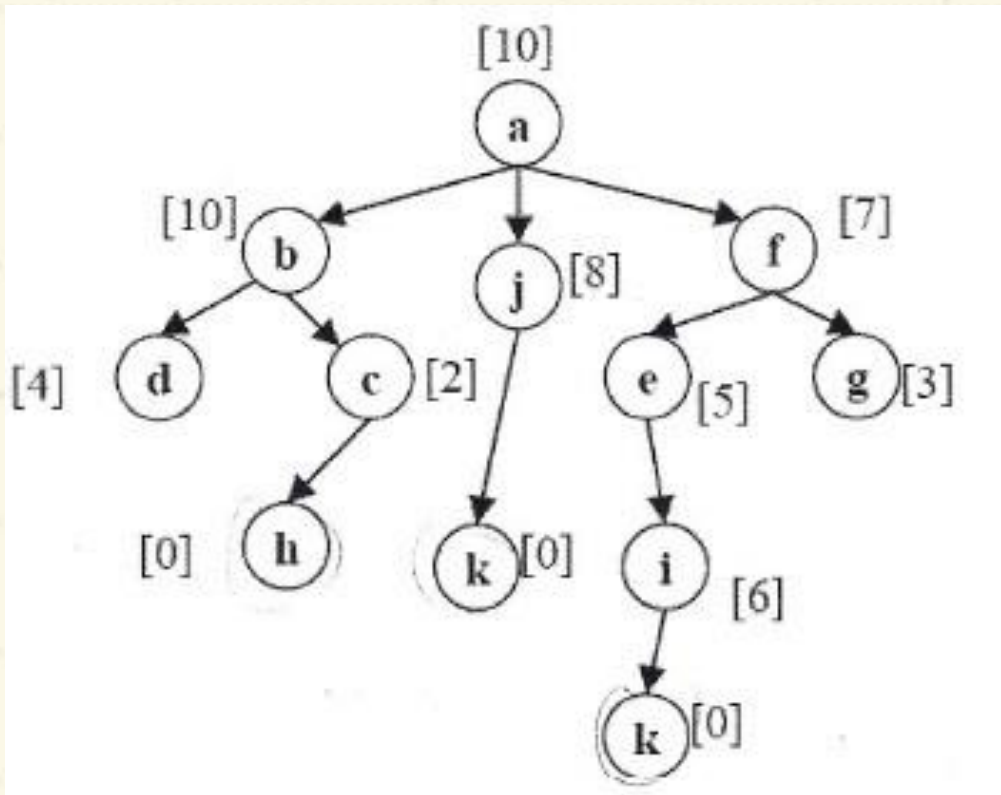
Children

---

$f_7, j_8, b_{10}$

$e_5, g_3$

# Simulated Annealing



Current

a

a

f

e

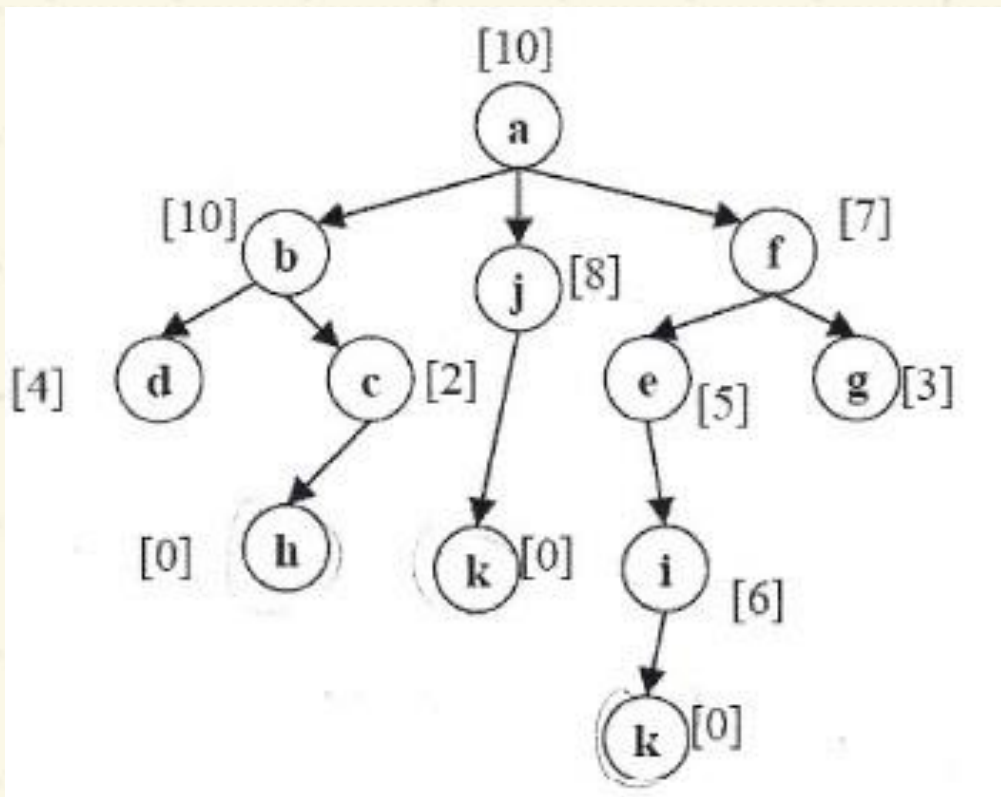
Children

---

$f_7, j_8, b_{10}$

$e_5, g_3$

# Simulated Annealing



Current

a

a

f

e

Children

---

$f_7, j_8, b_{10}$

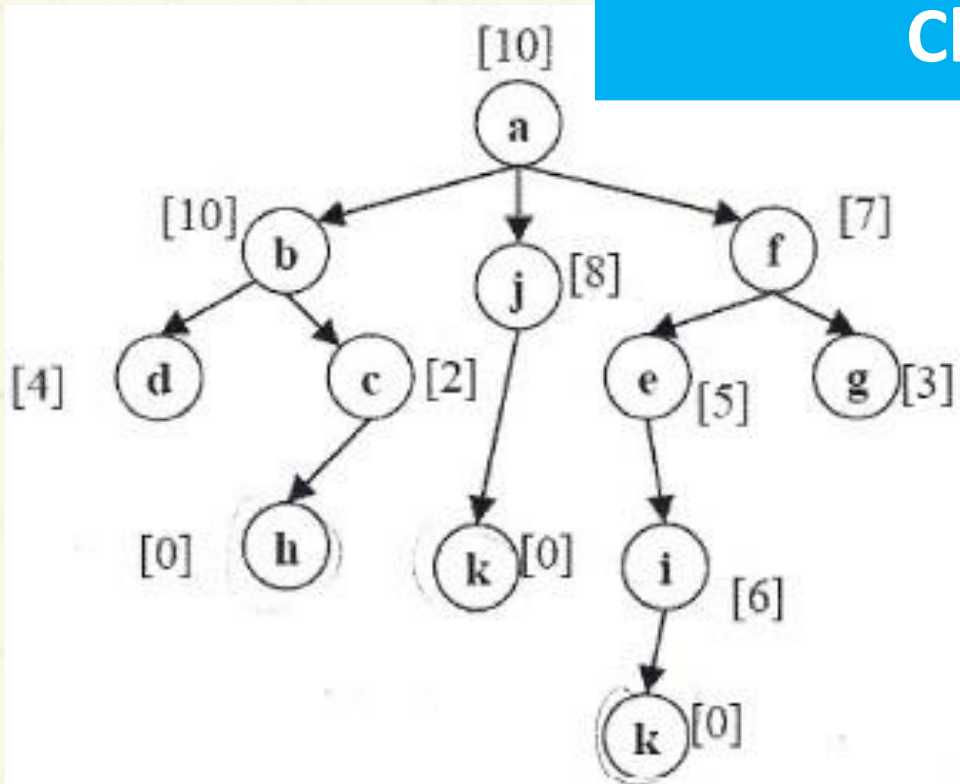
$e_5, g_3$

$i_6$



# Simulated Annealing

Randomly Select a  
Child



Current

a

a

f

e

Children

---

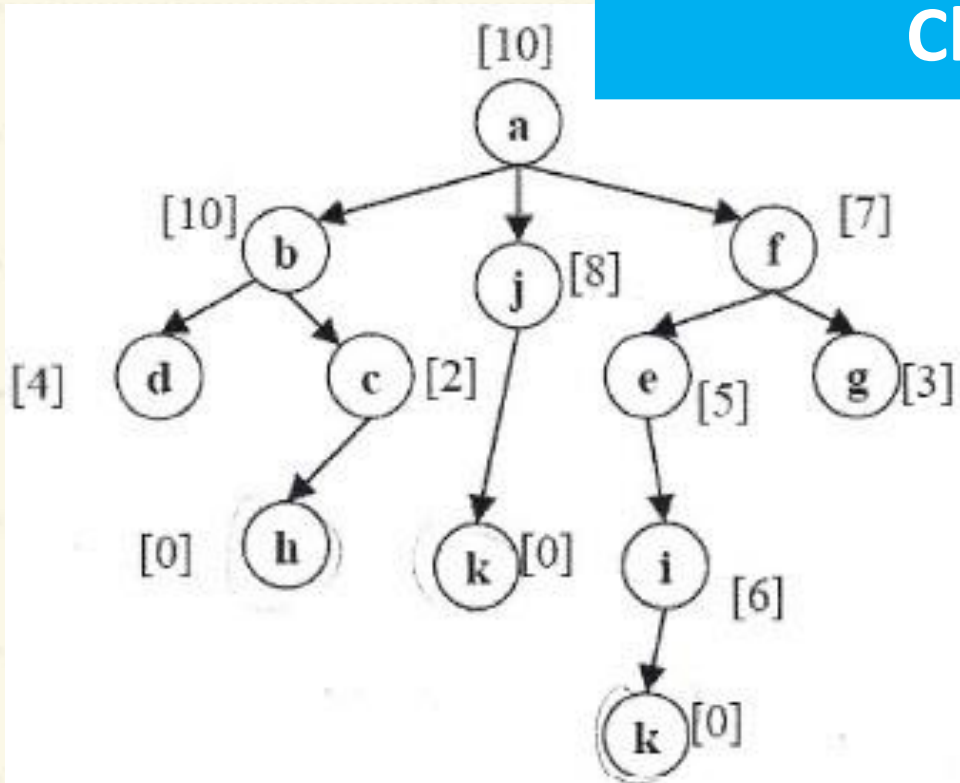
$f_7, j_8, b_{10}$

$e_5, g_3$

$i_6$

# Simulated Annealing

Randomly Select a  
Child



Current

a

a

f

e

Children

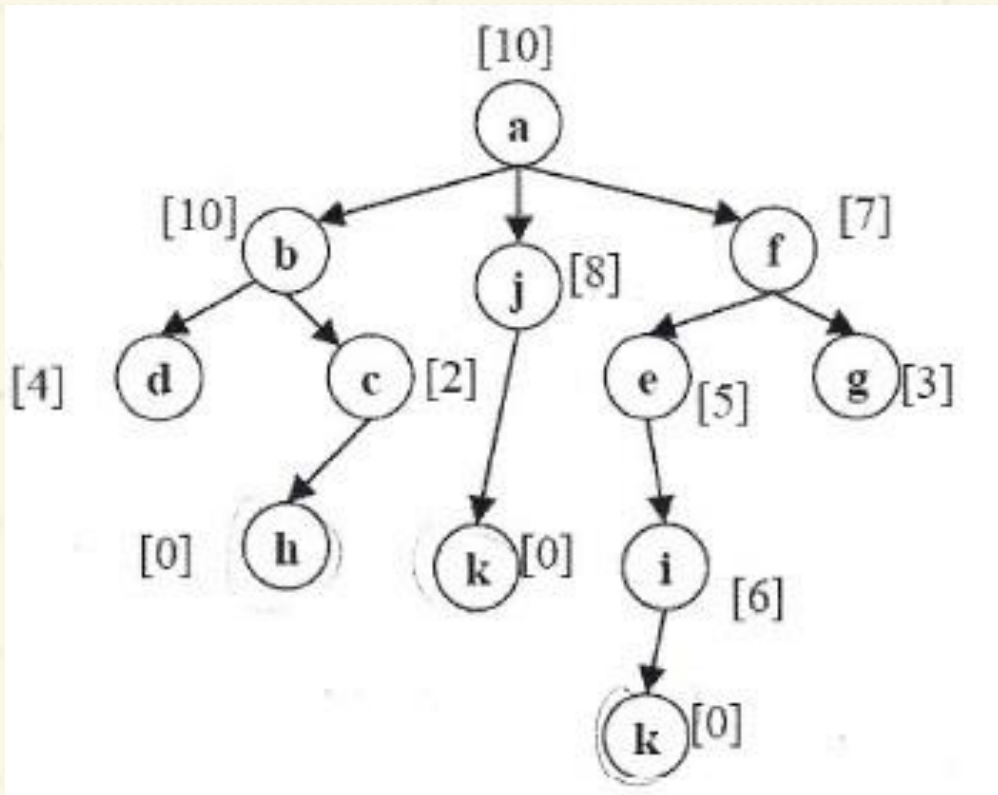
---

$f_7, j_8, b_{10}$

$e_5, g_3$

$i_6$

# Simulated Annealing



Current

a

a

f

e

Children

---

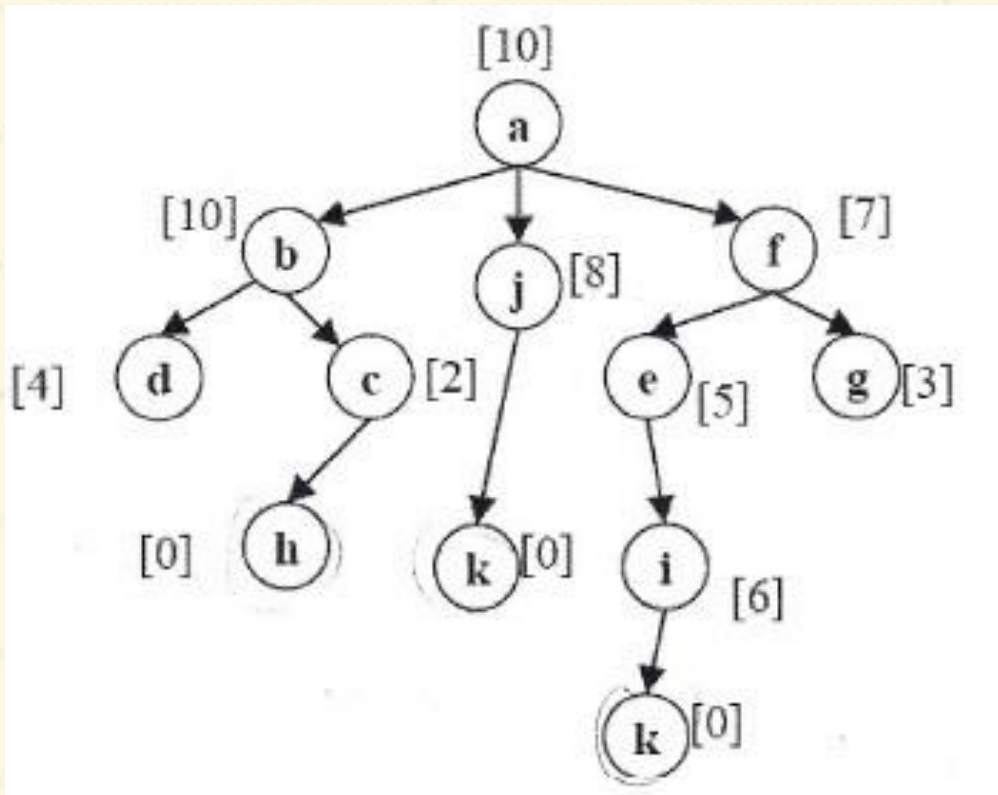
$f_7, j_8, b_{10}$

$e_5, g_3$

$i_6$

Check if next node  $i_6$  is better than current node

# Simulated Annealing



Current

a

a

f

e

Children

---

$f_7, j_8, b_{10}$

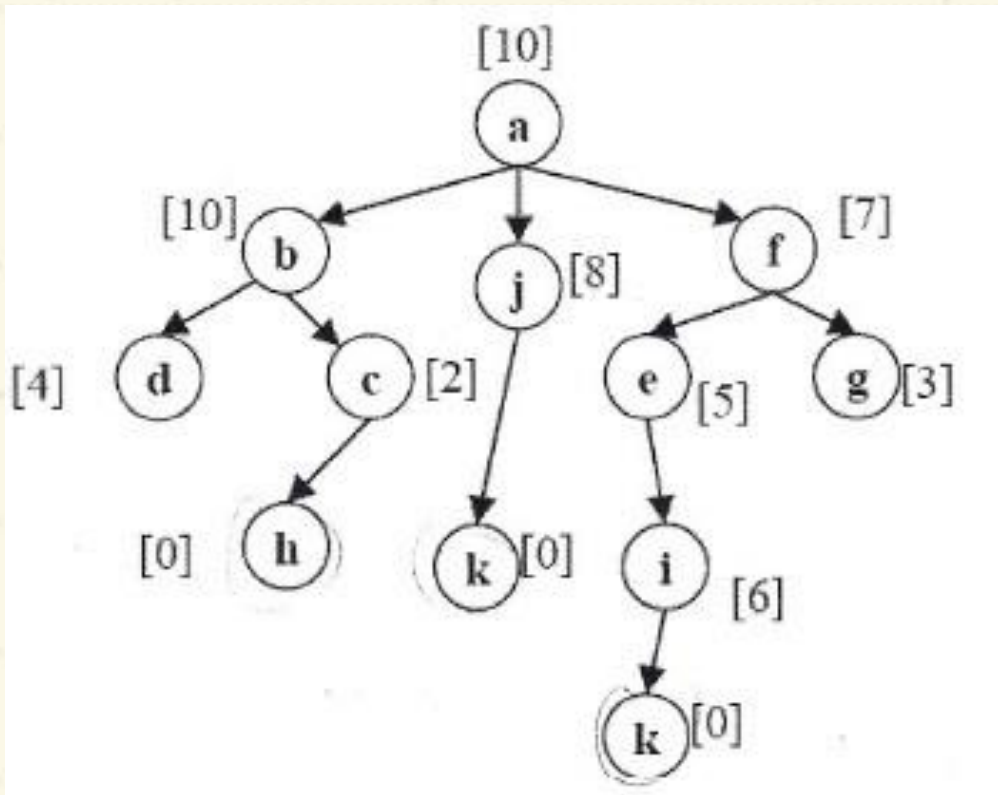
$e_5, g_3$

$i_6$

Check if next node  $i_6$  is better than current node

$$\Delta E > 0$$

# Simulated Annealing



Current

Children

a

---

a

$f_7, j_8, b_{10}$

f

$e_5, g_3$

e

$i_6$

Check if next node  $i_6$  is better than current node

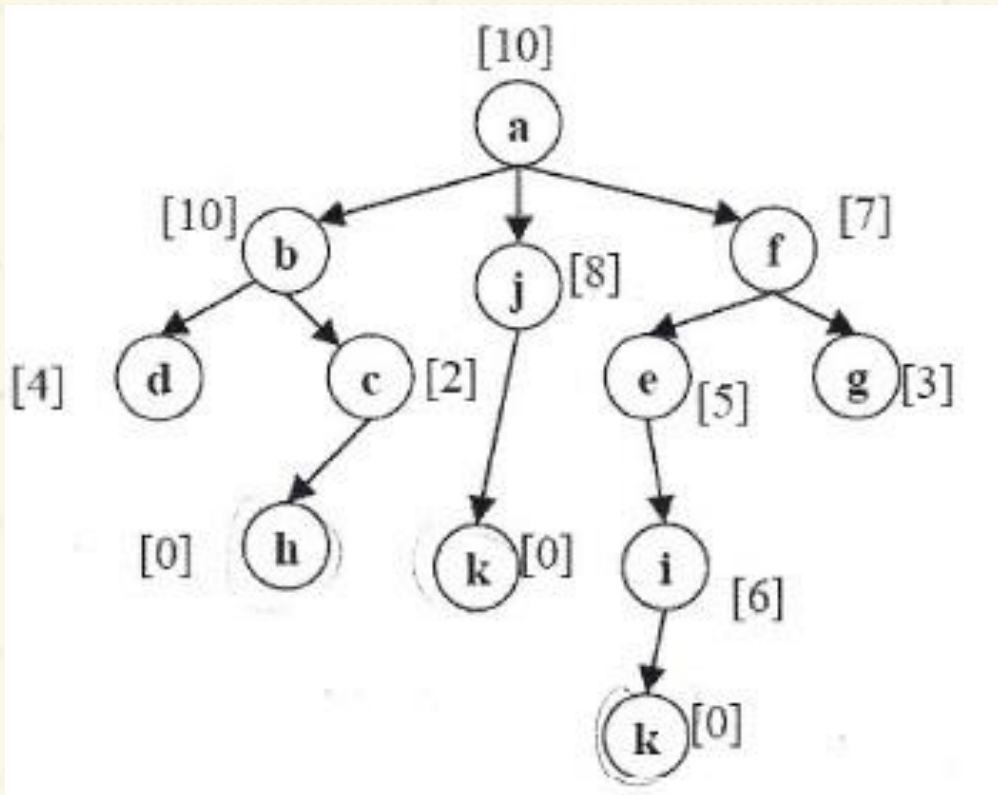
$\Delta E > 0$

$\Delta E = \text{value}(\text{next}) - \text{value}(\text{current})$

$\Delta E = \text{value}(i_6) - \text{value}(e_5)$

# Simulated Annealing

$$\text{value}(i_6) = -\text{heuristic}(i_6) = -6$$



Current

a

a

f

e

Children

---

$f_7, j_8, b_{10}$

$e_5, g_3$

$i_6$

Check if next node  $i_6$  is better than current node

$$\Delta E > 0$$

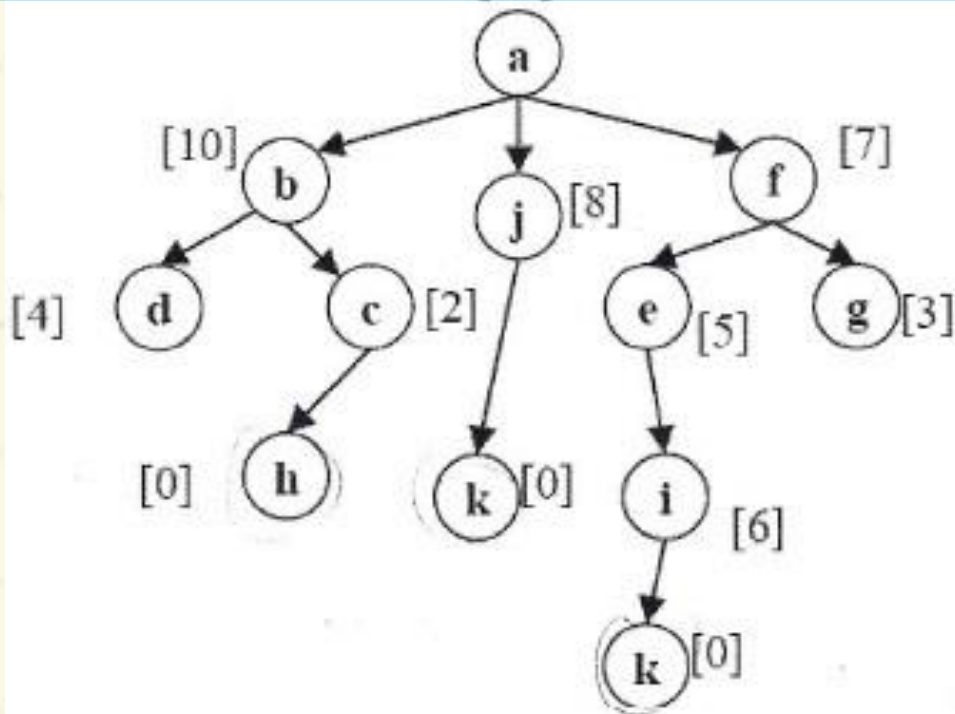
$$\Delta E = \text{value}(\text{next}) - \text{value}(\text{current})$$

$$\Delta E = \text{value}(i_6) - \text{value}(e_5)$$

# Simulated Annealing

$$\text{value}(i_6) = -\text{heuristic}(i_6) = -6$$

$$\text{value}(e_5) = -\text{heuristic}(e_5) = -5$$



Current

Children

a

---

a

$f_7, j_8, b_{10}$

f

$e_5, g_3$

e

$i_6$

Check if next node  $i_6$  is better than current node

$$\Delta E > 0$$

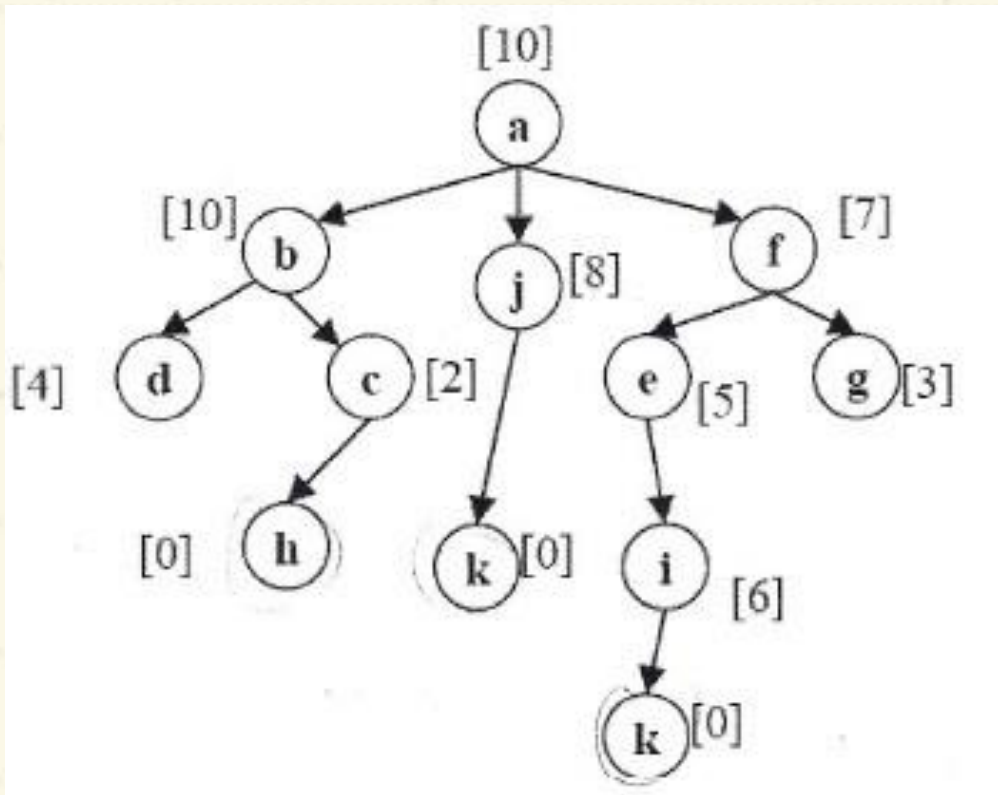
$$\Delta E = \text{value}(\text{next}) - \text{value}(\text{current})$$

$$\Delta E = \text{value}(i_6) - \text{value}(e_5)$$



# Simulated Annealing

$$\Delta E = -6 - (-5) = -1$$



Current

Children

a

---

a

$f_7, j_8, b_{10}$

f

$e_5, g_3$

e

$i_6$

Check if next node  $i_6$  is better than current node

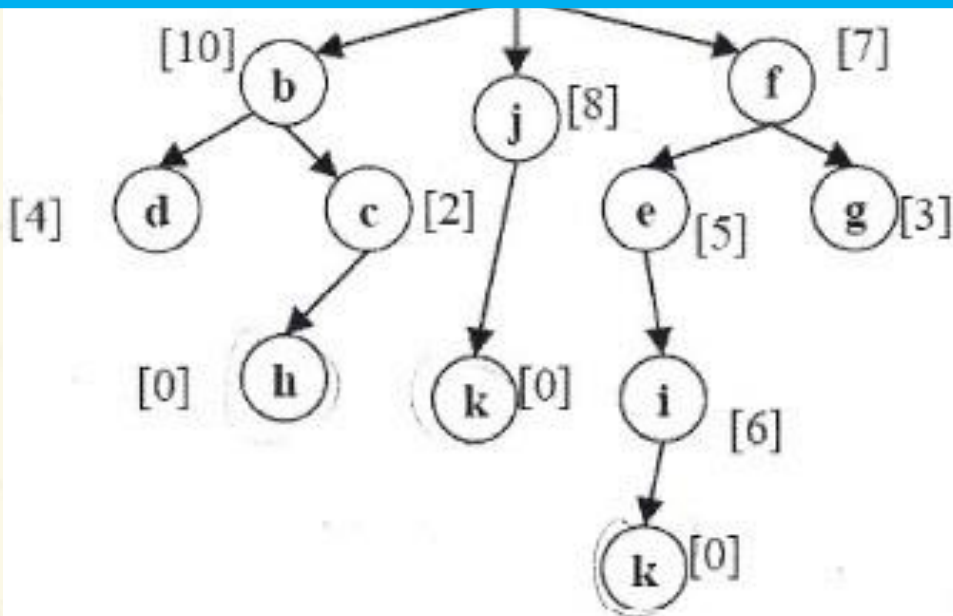
$$\Delta E > 0$$

$$\Delta E = \text{value}(\text{next}) - \text{value}(\text{current})$$

$$\Delta E = \text{value}(i_6) - \text{value}(e_5)$$

# Simulated Annealing

$\because \Delta E < 0$   
 $\therefore i_6$  can be selected with  
probability  $p = e^{\frac{\Delta E}{T}}$



Current

Children

a

---

a

$f_7, j_8, b_{10}$

f

$e_5, g_3$

e

$i_6$

Check if next node  $e_5$  is  
better than current node

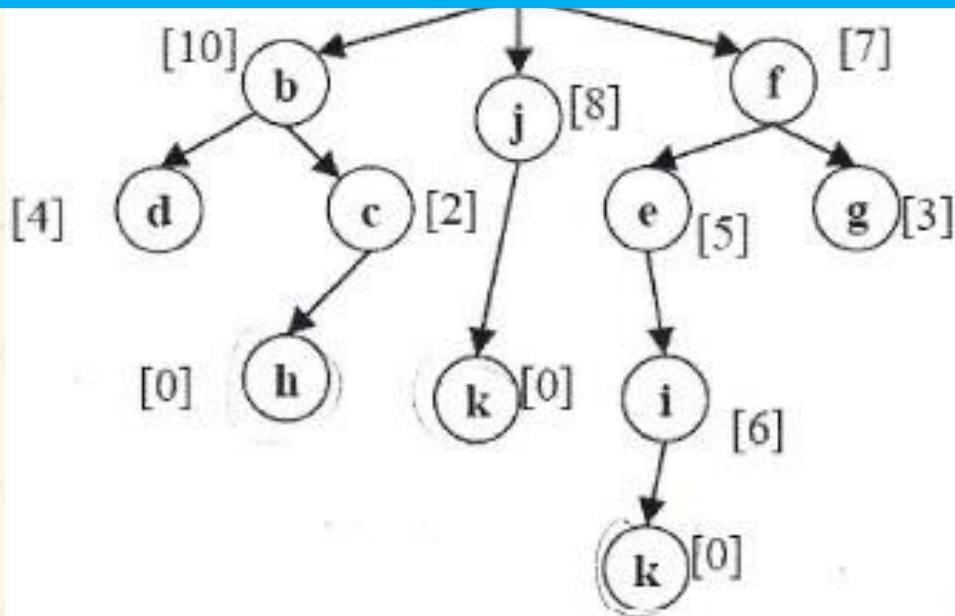
$$\Delta E = \text{value}(\text{next}) - \text{value}(\text{current})$$

$$\Delta E = \text{value}(e_5) - \text{value}(f_7)$$

$$\Delta E = -5 - (-7) = +2$$

# Simulated Annealing

$\because \Delta E < 0$   
 $\therefore i_6$  can be selected with  
 probability  $p = e^{\frac{\Delta E}{T}}$



Current

a

a

f

e

Children

---

$f_7, j_8, b_{10}$

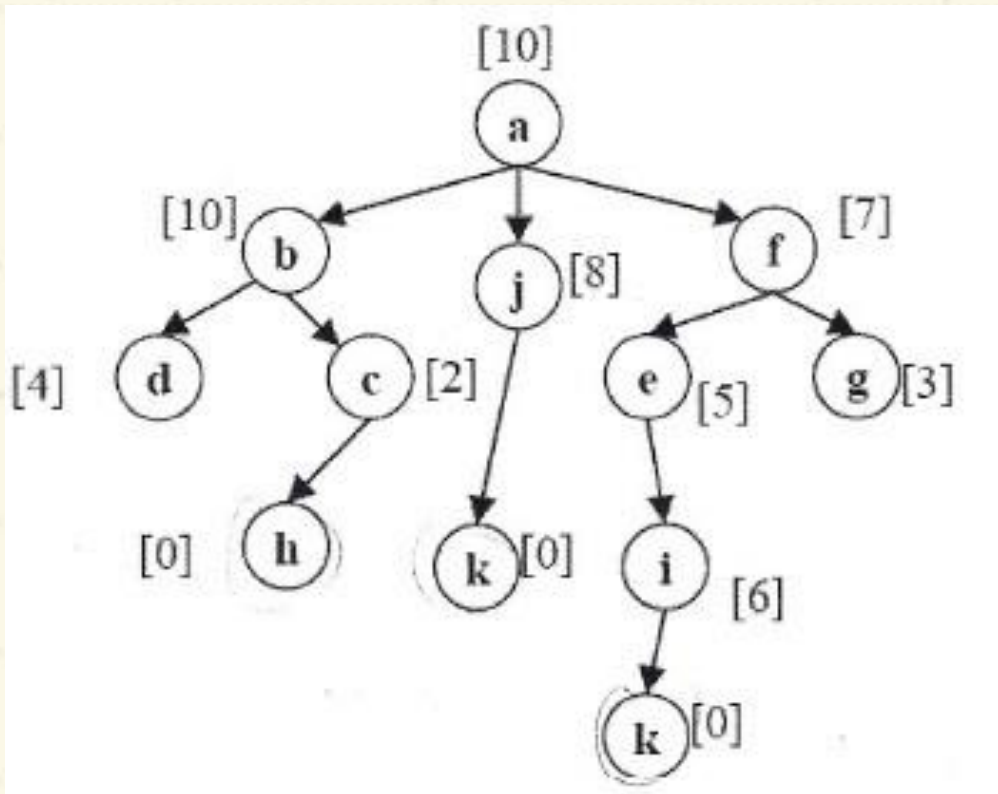
$e_5, g_3$

$i_6$

Check if next node  $e_5$  is better than current node

$$p = e^{\frac{-1}{10}} = e^{\frac{-1}{10}} = .905$$

# Simulated Annealing



Current

a

a

f

e

Children

---

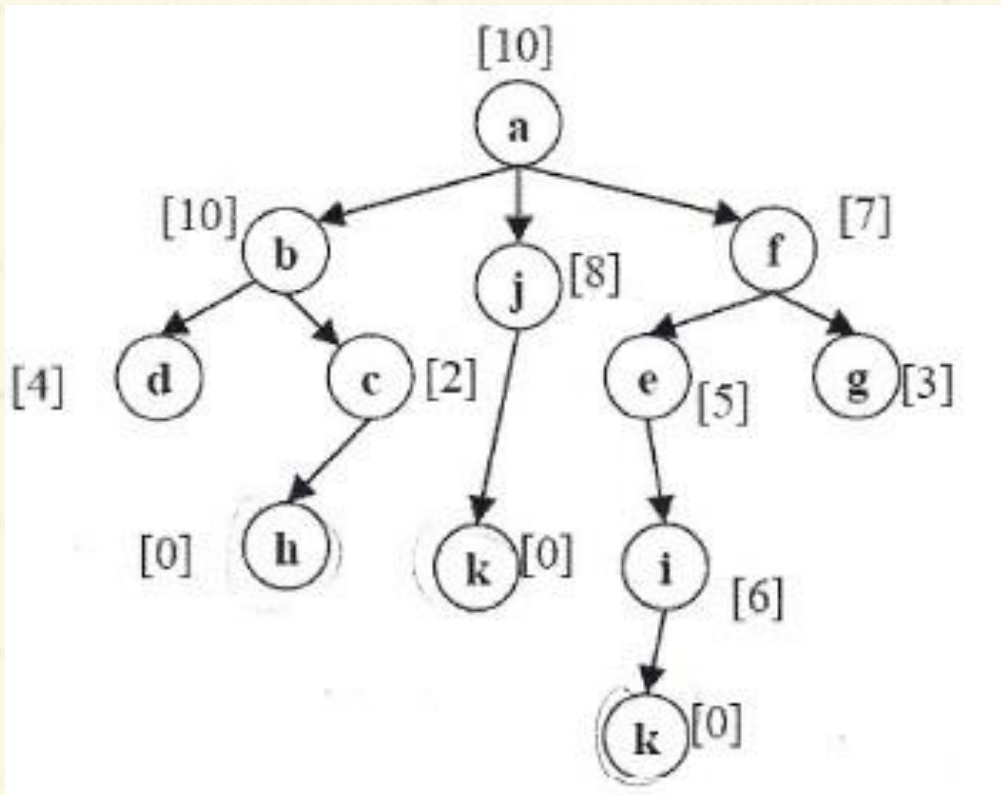
$f_7, j_8, b_{10}$

$e_5, g_3$

$i_6$

Because the only child of  $e_5$  is  $i_6$  then it will be selected even if its probability is not 1.

# Simulated Annealing



Current

a

a

f

e

i

Children

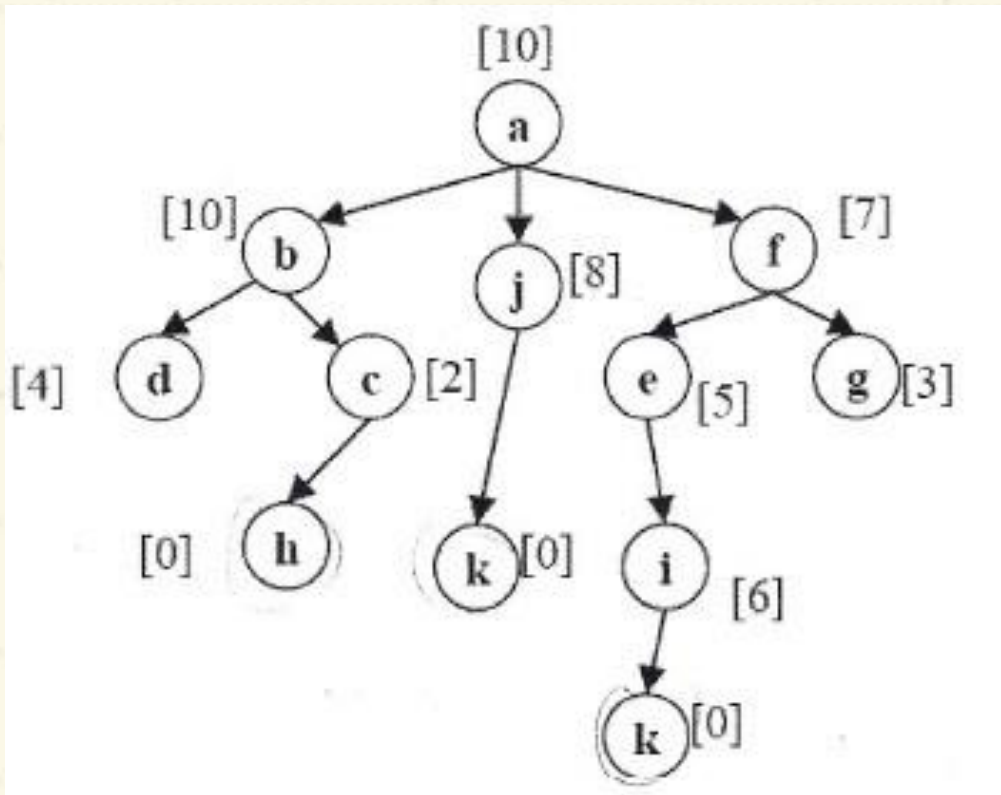
---

$f_7, j_8, b_{10}$

$e_5, g_3$

$i_6$

# Simulated Annealing



Current

a

a

f

e

i

Children

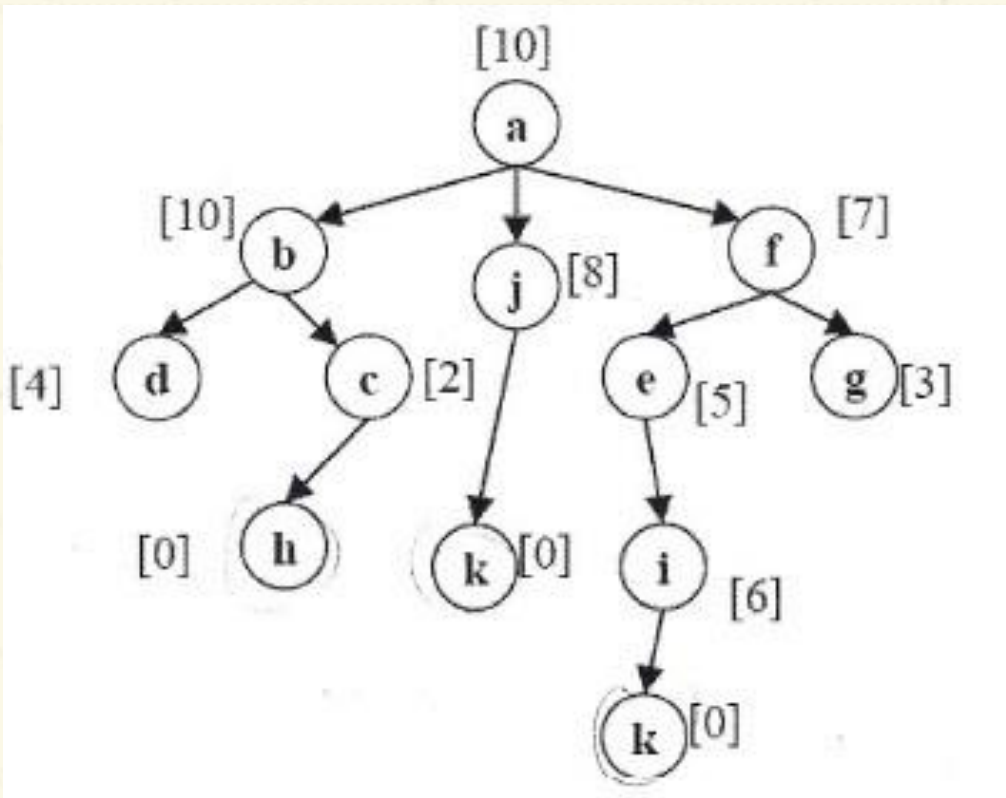
---

$f_7, j_8, b_{10}$

$e_5, g_3$

$i_6$

# Simulated Annealing

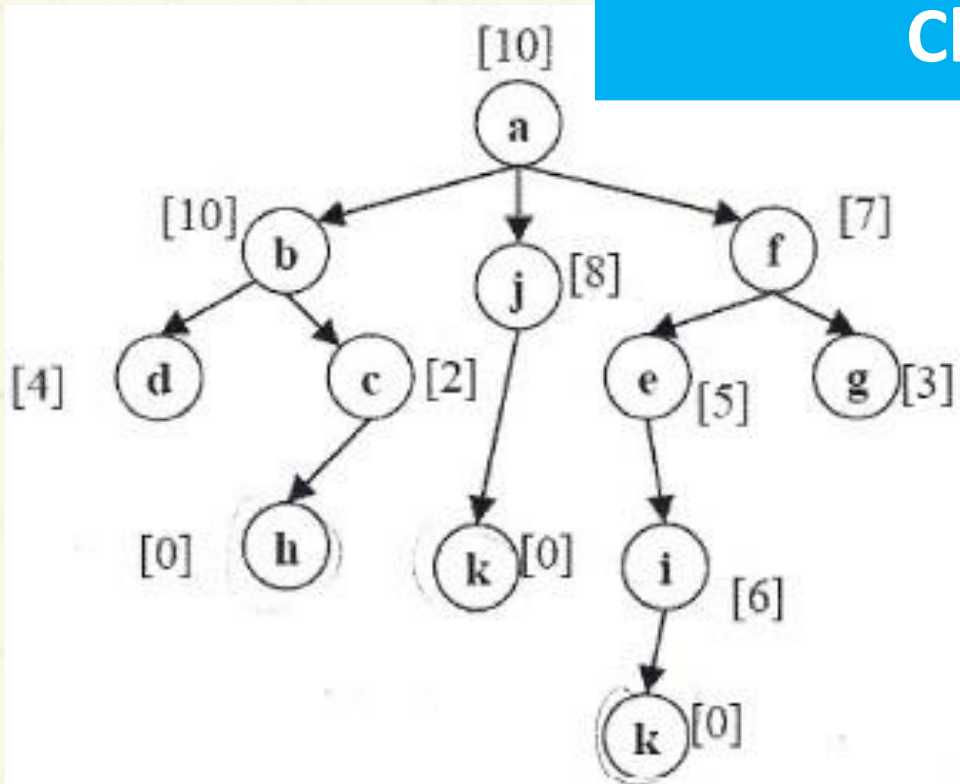


Current	Children
a	---
a	$f_7, j_8, b_{10}$
f	$e_5, g_3$
e	$i_6$
i	$k_0$



# Simulated Annealing

Randomly Select a  
Child



Current

a

a

f

e

i

Children

---

$f_7, j_8, b_{10}$

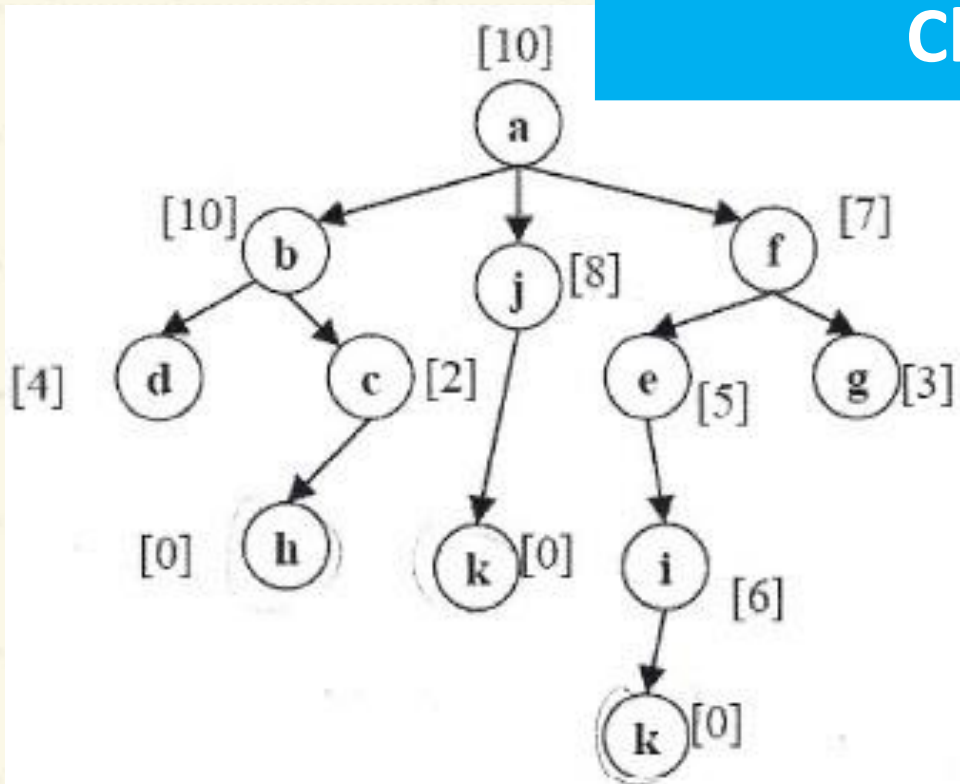
$e_5, g_3$

$i_6$

$k_0$

# Simulated Annealing

Randomly Select a  
Child



Current

a

a

f

e

i

Children

---

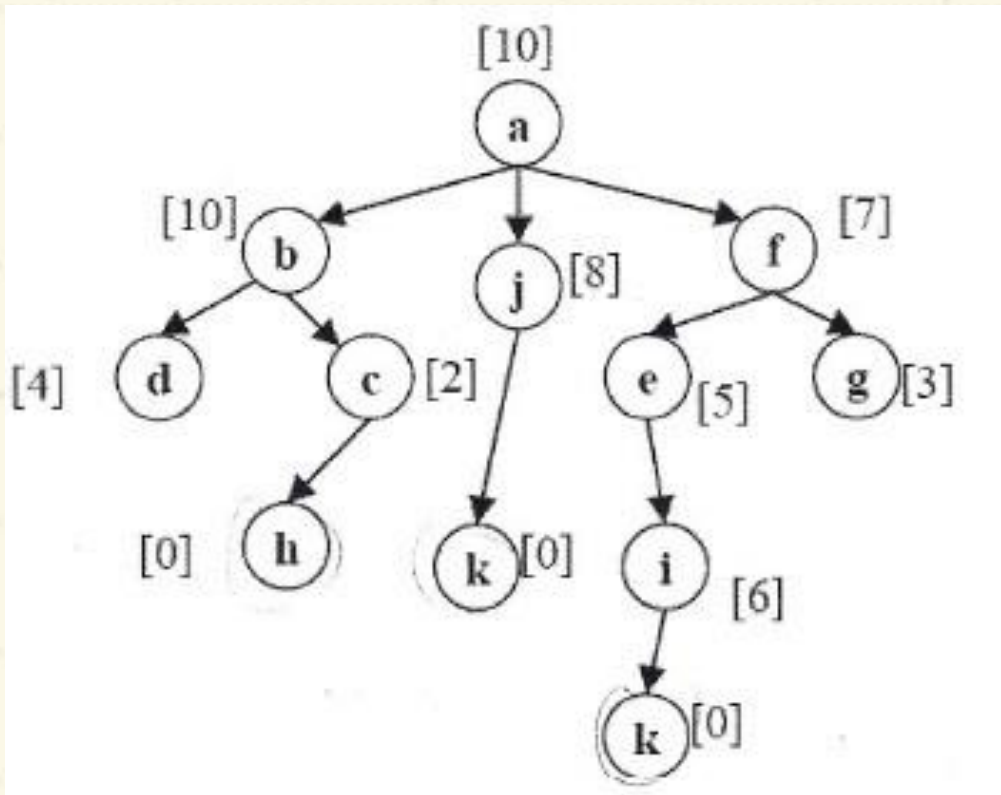
$f_7, j_8, b_{10}$

$e_5, g_3$

$i_6$

$k_0$

# Simulated Annealing



Current

a

a

f

e

i

Children

---

$f_7, j_8, b_{10}$

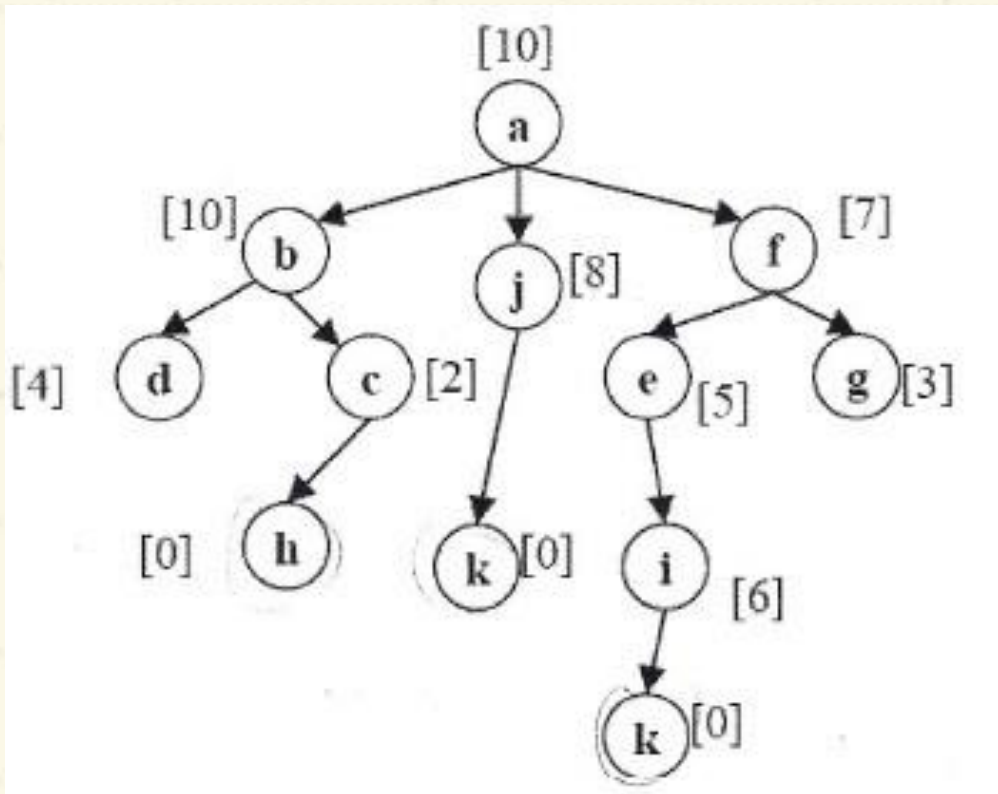
$e_5, g_3$

$i_6$

$k_0$

Check if next node  $k_0$  is better than current node

# Simulated Annealing



Current

a

a

f

e

i

Children

---

$f_7, j_8, b_{10}$

$e_5, g_3$

$i_6$

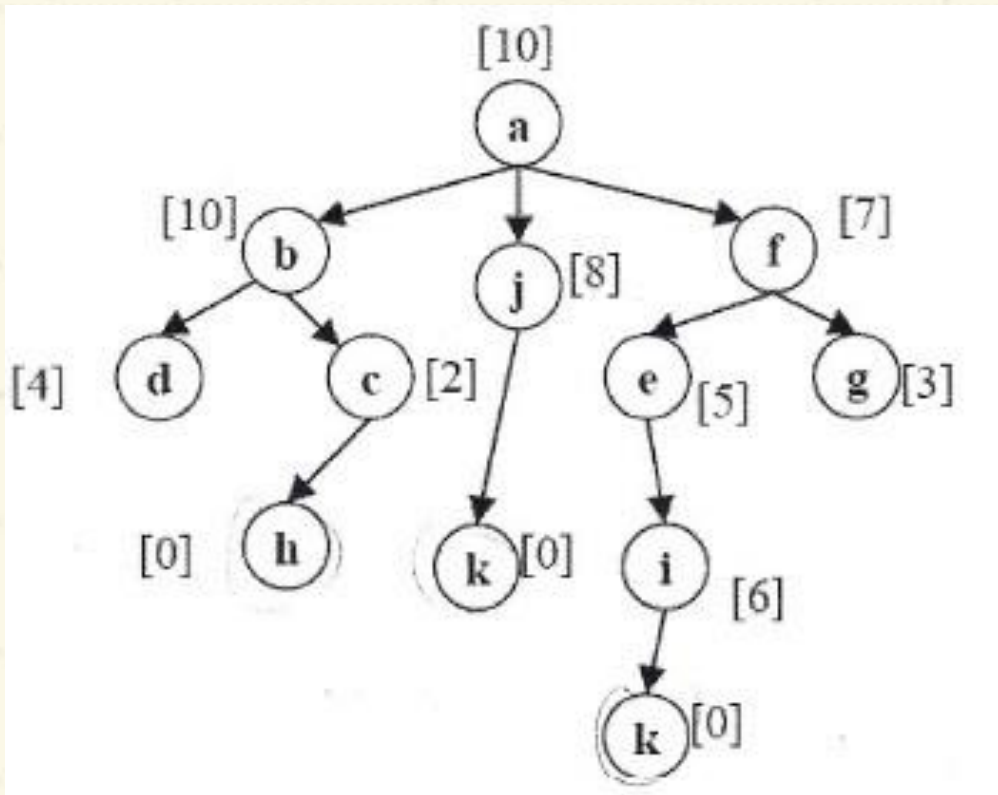
$k_0$

Check if next node  $k_0$  is better than current node

$\Delta E > 0$

# Simulated Annealing

$$\Delta E = \text{value}(\text{next}) - \text{value}(\text{current})$$



Current

a

a

f

e

i

Children

---

$f_7, j_8, b_{10}$

$e_5, g_3$

$i_6$

$k_0$

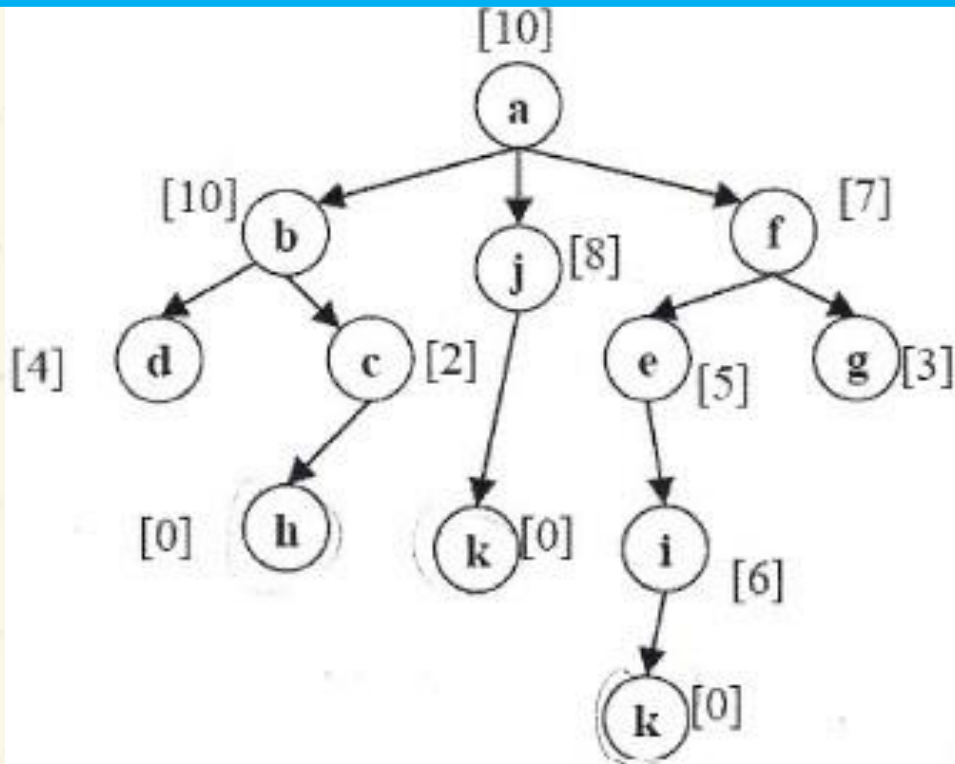
Check if next node  $k_0$  is better than current node

$\Delta E > 0$

# Simulated Annealing

$$\Delta E = \text{value}(\text{next}) - \text{value}(\text{current})$$

$$\Delta E = \text{value}(k_0) - \text{value}(i_6)$$



Current

a

a

f

e

i

Children

---

$f_7, j_8, b_{10}$

$e_5, g_3$

$i_6$

$k_0$

Check if next node  $k_0$  is better than current node

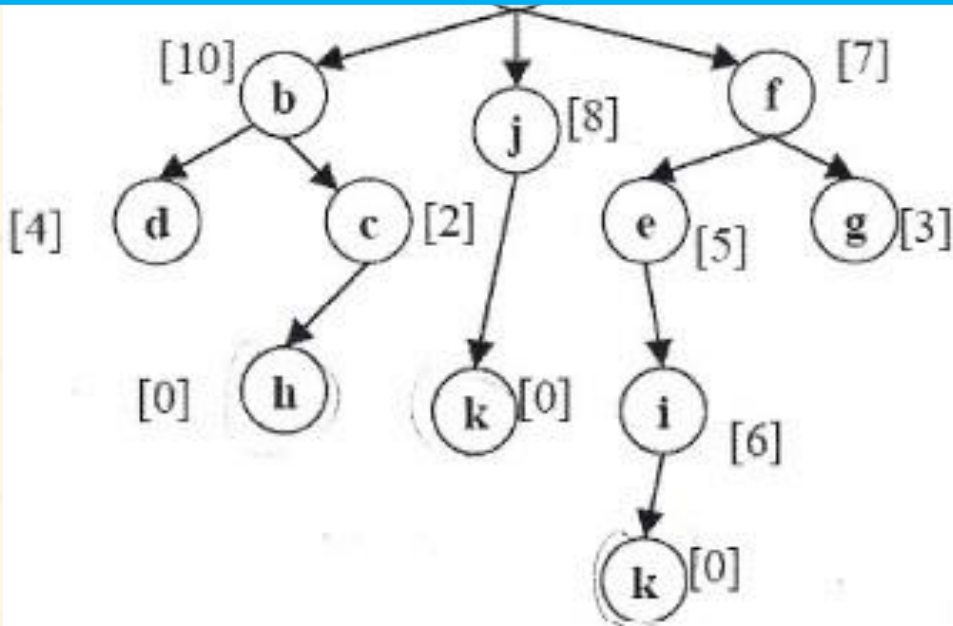
$$\Delta E > 0$$

# Simulated Annealing

$$\Delta E = \text{value}(\text{next}) - \text{value}(\text{current})$$

$$\Delta E = \text{value}(k_0) - \text{value}(i_6)$$

$$\text{value}(k_0) = -\text{heuristic}(k_0) = 0$$



Current

a

a

f

e

i

Children

---

$f_7, j_8, b_{10}$

$e_5, g_3$

$i_6$

$k_0$

Check if next node  $k_0$  is better than current node

$$\Delta E > 0$$



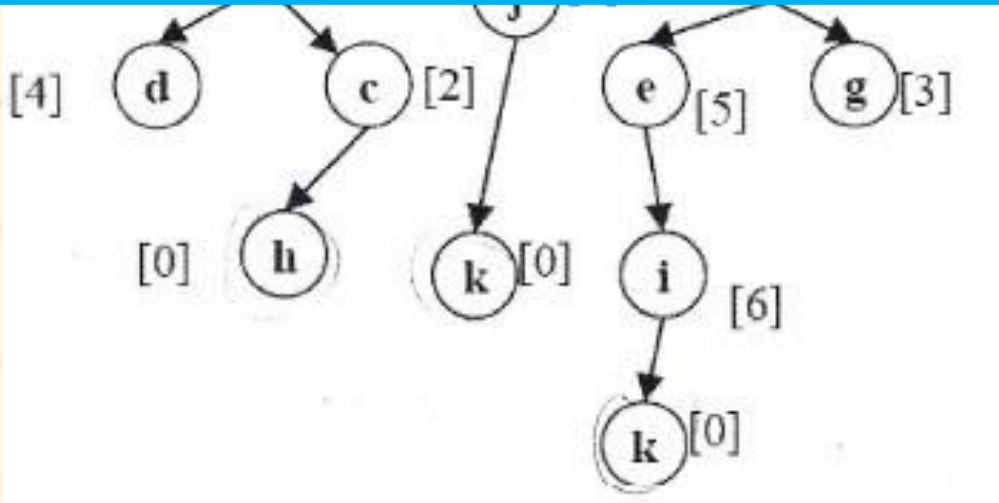
# Simulated Annealing

$$\Delta E = \text{value}(\text{next}) - \text{value}(\text{current})$$

$$\Delta E = \text{value}(k_0) - \text{value}(i_6)$$

$$\text{value}(k_0) = -\text{heuristic}(k_0) = 0$$

$$\text{value}(i_6) = -\text{heuristic}(i_6) = -6$$



Current

a

a

f

e

i

Children

---

$f_7, j_8, b_{10}$

$e_5, g_3$

$i_6$

$k_0$

Check if next node  $k_0$  is better than current node

$$\Delta E > 0$$

# Simulated Annealing

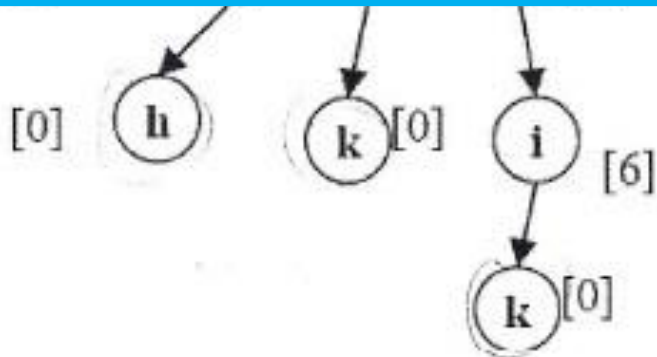
$$\Delta E = \text{value}(\text{next}) - \text{value}(\text{current})$$

$$\Delta E = \text{value}(k_0) - \text{value}(i_6)$$

$$\text{value}(k_0) = -\text{heuristic}(k_0) = 0$$

$$\text{value}(i_6) = -\text{heuristic}(i_6) = -6$$

$$\Delta E = 0 - (-6) = +6$$



Current

a

a

f

e

i

Children

---

$f_7, j_8, b_{10}$

$e_5, g_3$

$i_6$

$k_0$

Check if next node  $k_0$  is better than current node

$$\Delta E > 0$$

# Simulated Annealing

$$\Delta E = \text{value}(\text{next}) - \text{value}(\text{current})$$

$$\Delta E = \text{value}(k_0) - \text{value}(i_6)$$

$$\text{value}(k_0) = -\text{heuristic}(k_0) = 0$$

$$\text{value}(i_6) = -\text{heuristic}(i_6) = -6$$

$$\Delta E = 0 - (-6) = +6$$

$$\because \Delta E > 0$$

$\therefore k_0$  will be selected with  
probability 1

Current

a

a

f

e

i

Children

---

$f_7, j_8, b_{10}$

$e_5, g_3$

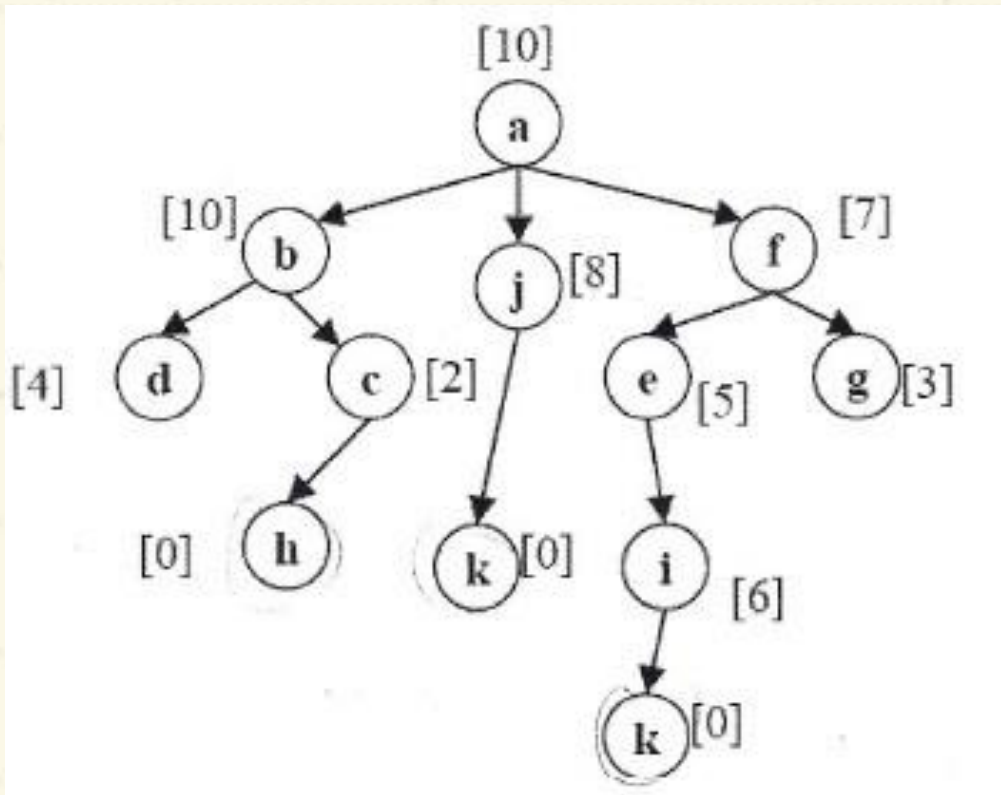
$i_6$

$k_0$

Check if next node  $k_0$  is  
better than current node

$$\Delta E > 0$$

# Simulated Annealing



Current

a

a

f

e

i

k

Children

---

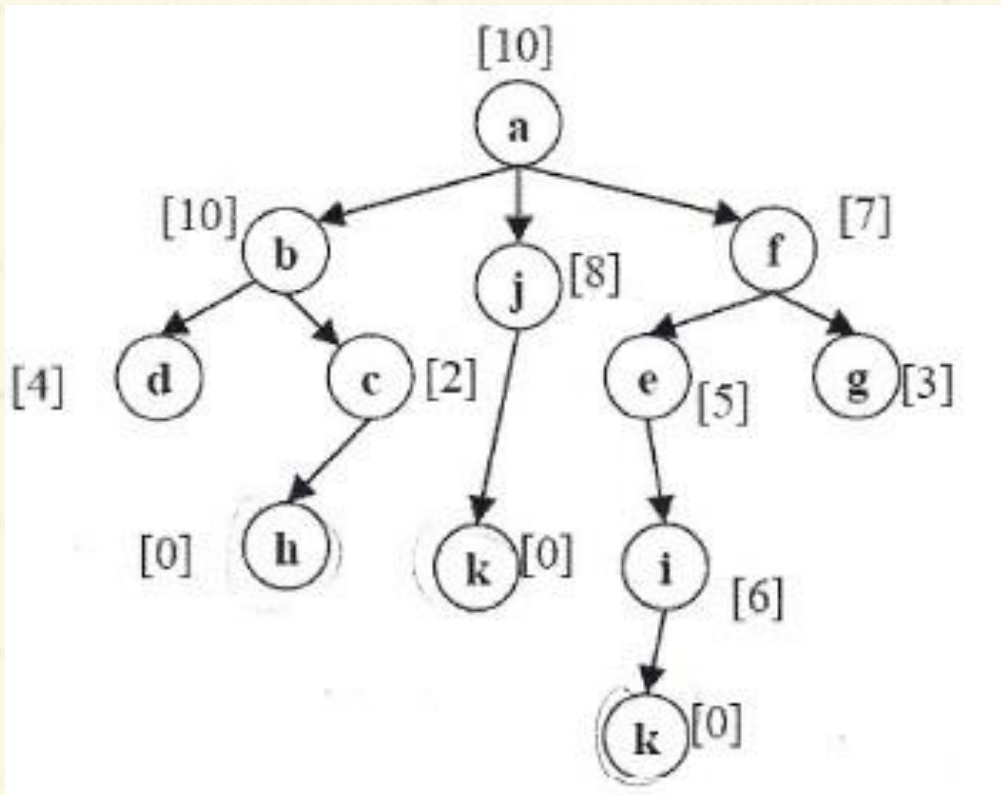
$f_7, j_8, b_{10}$

$e_5, g_3$

$i_6$

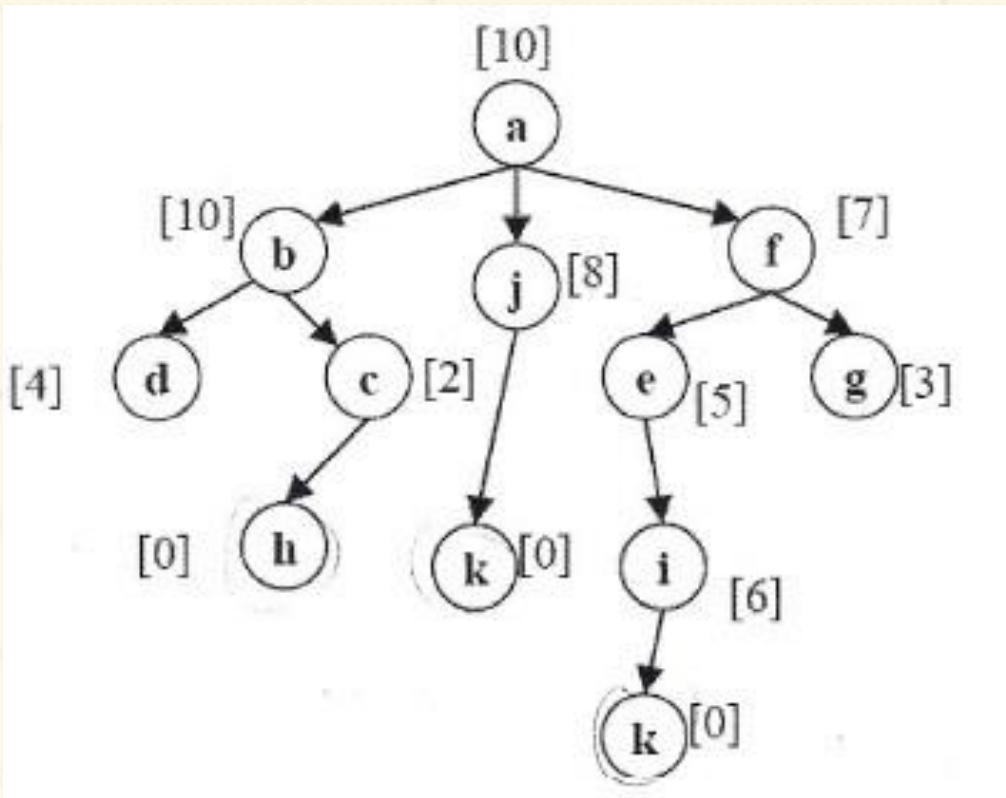
$k_0$

# Simulated Annealing



Current	Children
a	---
a	$f_7, j_8, b_{10}$
f	$e_5, g_3$
e	$i_6$
i	$k_0$
k	

# Simulated Annealing



Current	Children
a	---
a	$f_7, j_8, b_{10}$
f	$e_5, g_3$
e	$i_6$
i	$k_0$
k	
GOAL	