

# Beam Search

# Hill Climbing Vs. Beam Search

- Hill climbing just explores all nodes in one branch until goal found or not being able to explore more nodes.
- Beam search explores more than one path together. A factor  $k$  is used to determine the number of branches explored at a time.
- If  $k=2$ , then two branches are explored at a time. For  $k=4$ , four branches are explored simultaneously.
- The branches selected are the **best branches** based on the used **heuristic evaluation function**.

# Beam Search, k=2

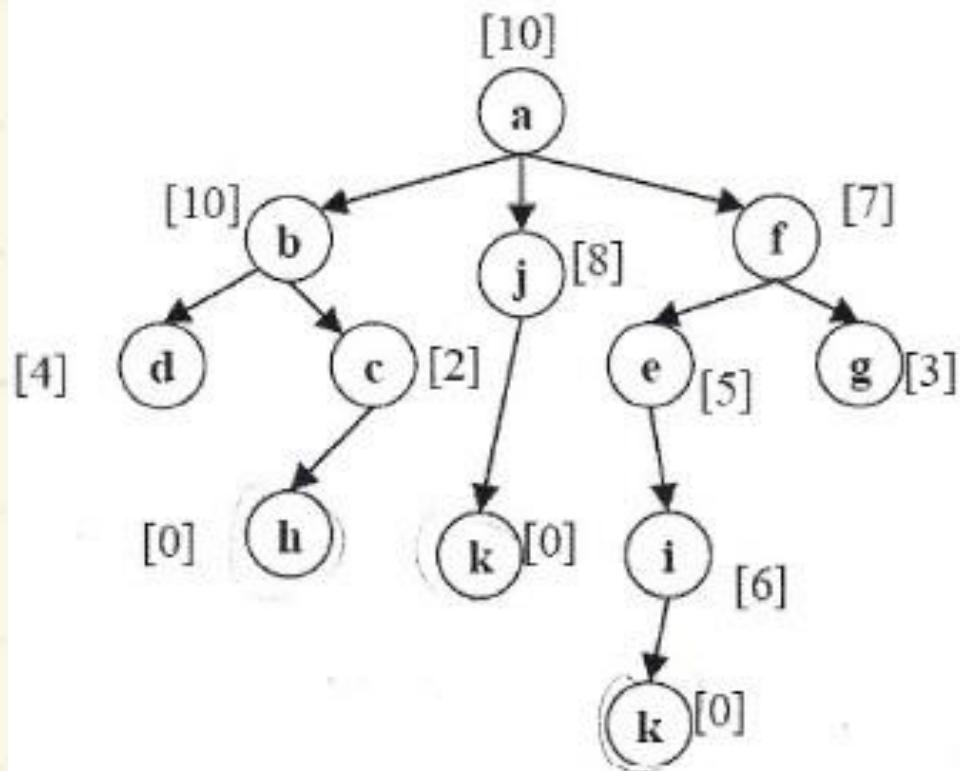
## Goal – Node K

Current

a

Children

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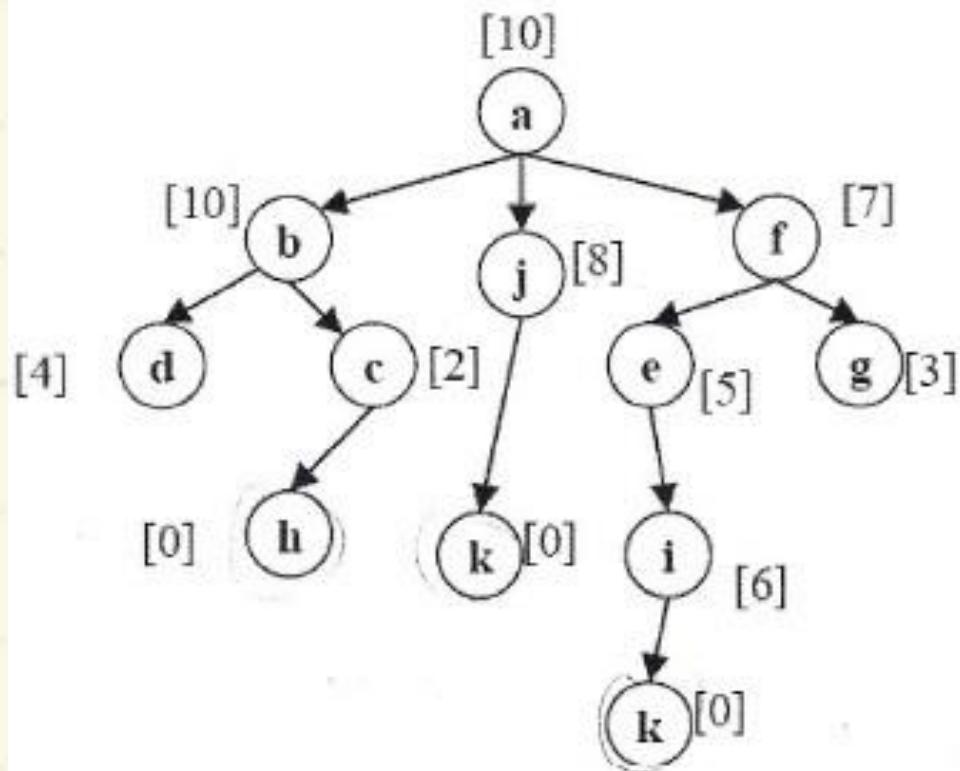
# Beam Search Goal – Node K

Current

a

Children

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# Beam Search Goal – Node K

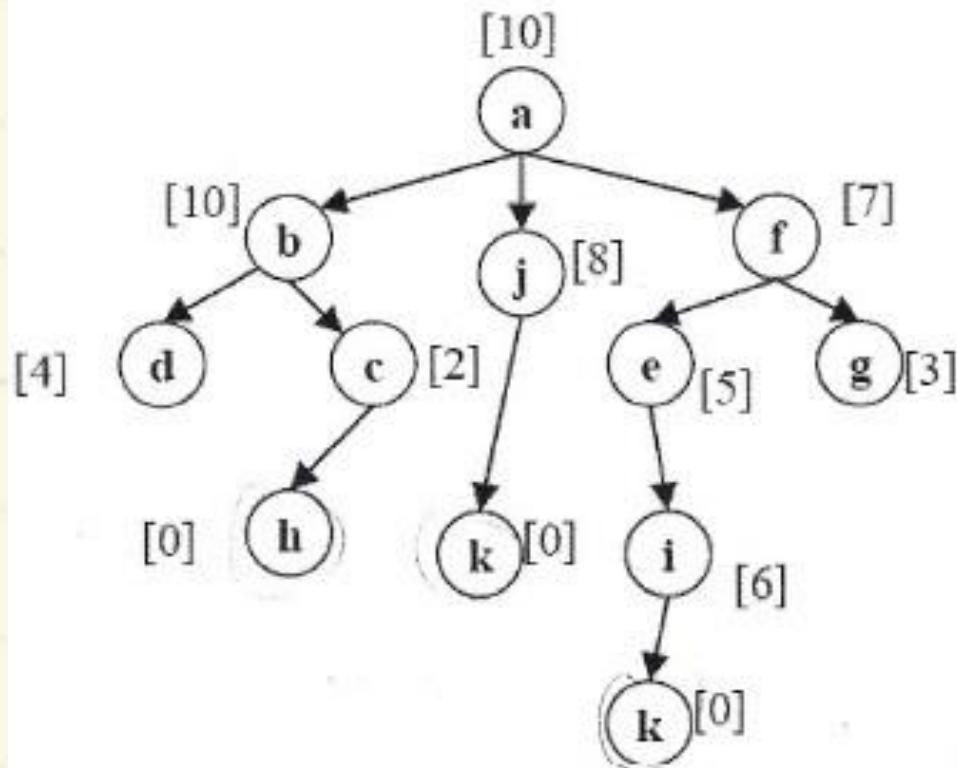
Current

a

a

Children

---



# Beam Search Goal – Node K

Current

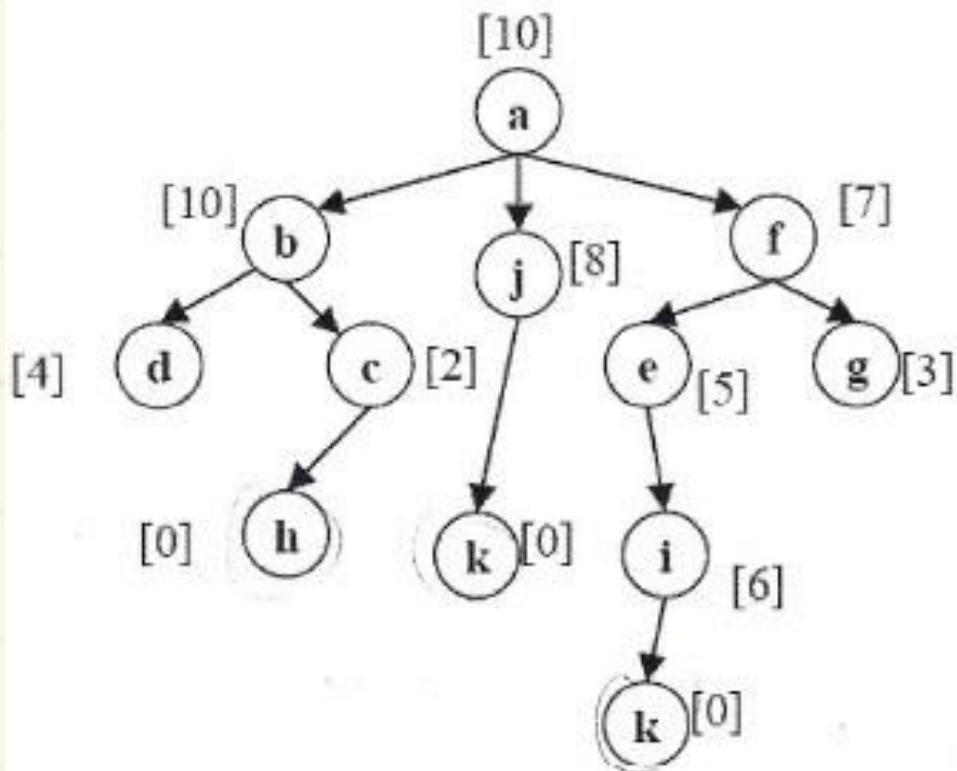
a

a

Children

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$f_7, j_8, b_{10}$



# Beam Search Goal – Node K

Best k  
Successors

Current

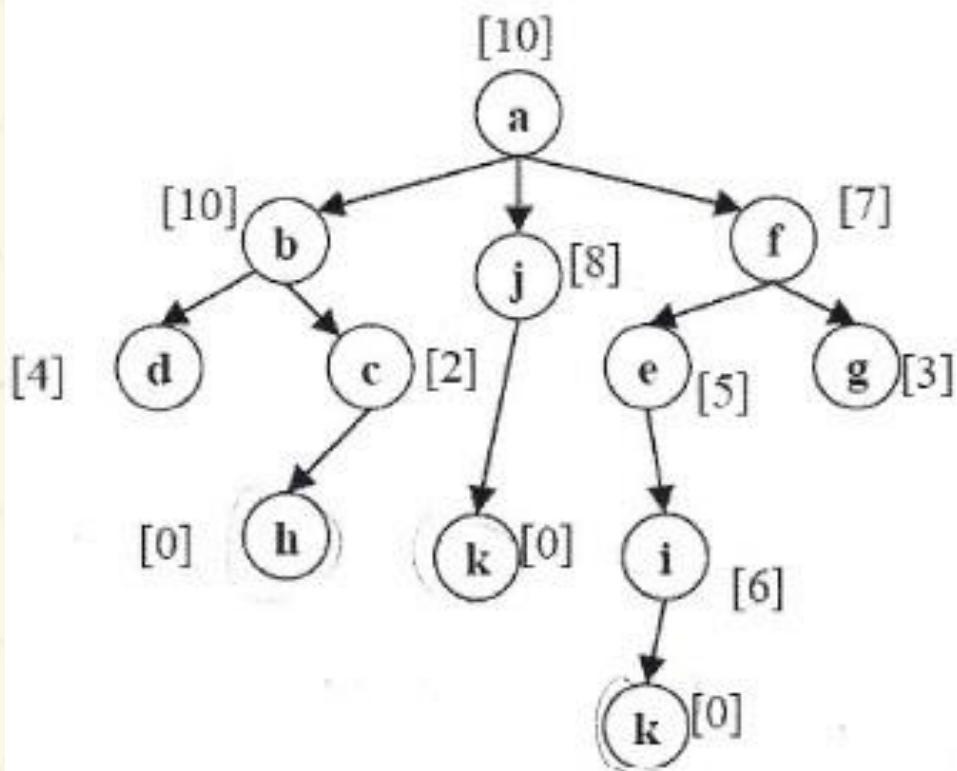
a

a

Children

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$f_7, j_8, b_{10}$



# Beam Search Goal – Node K

Best k  
Successors

Current

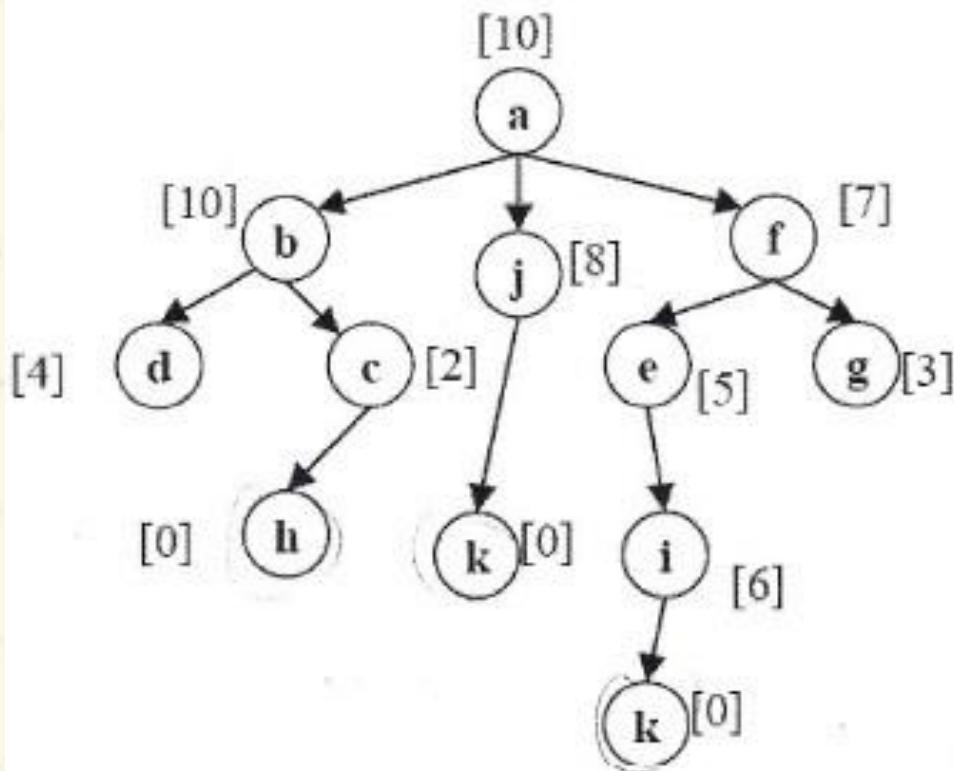
a

a

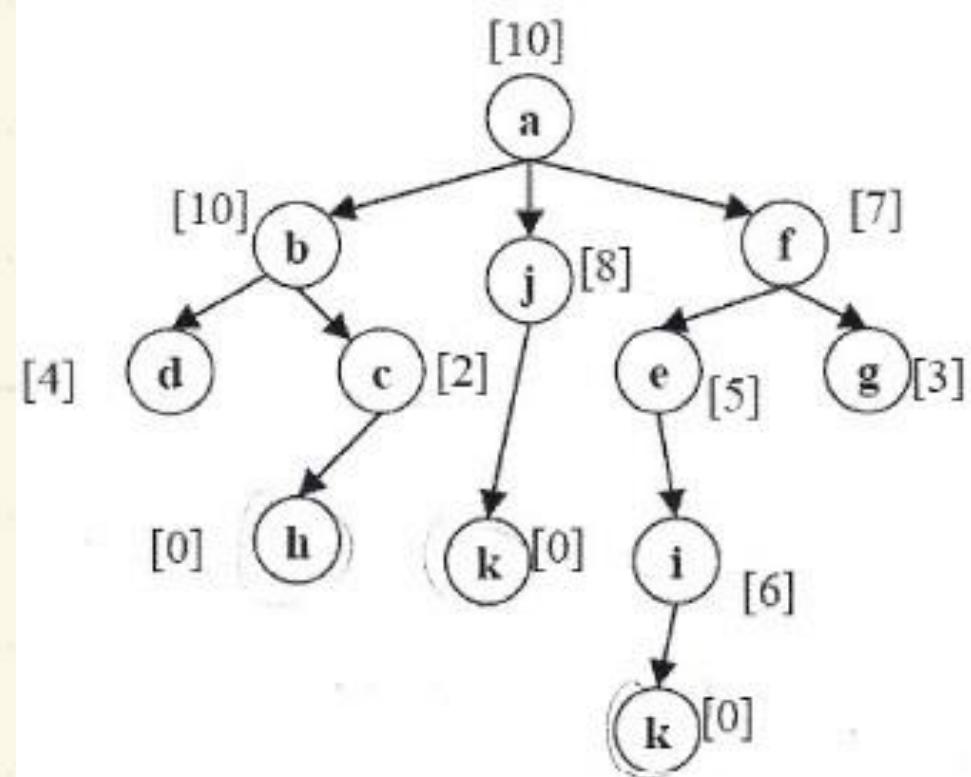
Children

---

$f_7, j_8, b_{10}$

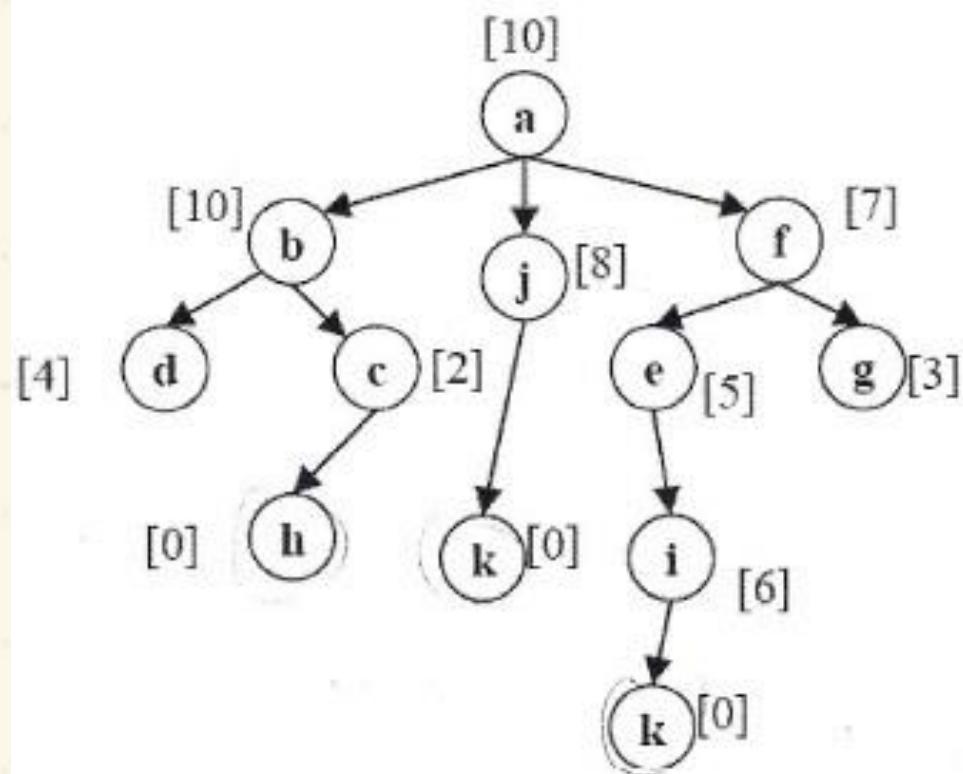


# Beam Search Goal – Node K



| Current | Children           |
|---------|--------------------|
| a       | ---                |
| a       | $f_7, j_8, b_{10}$ |
| f       | j                  |

# Beam Search Goal – Node K



Current

a

a

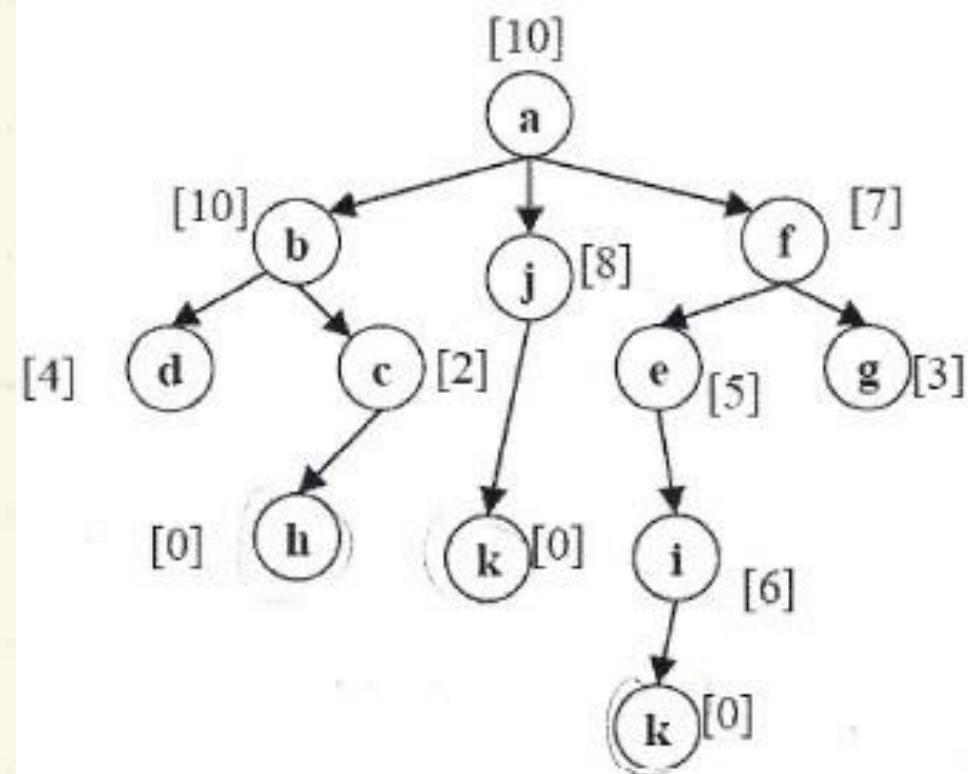
f j

Children

...

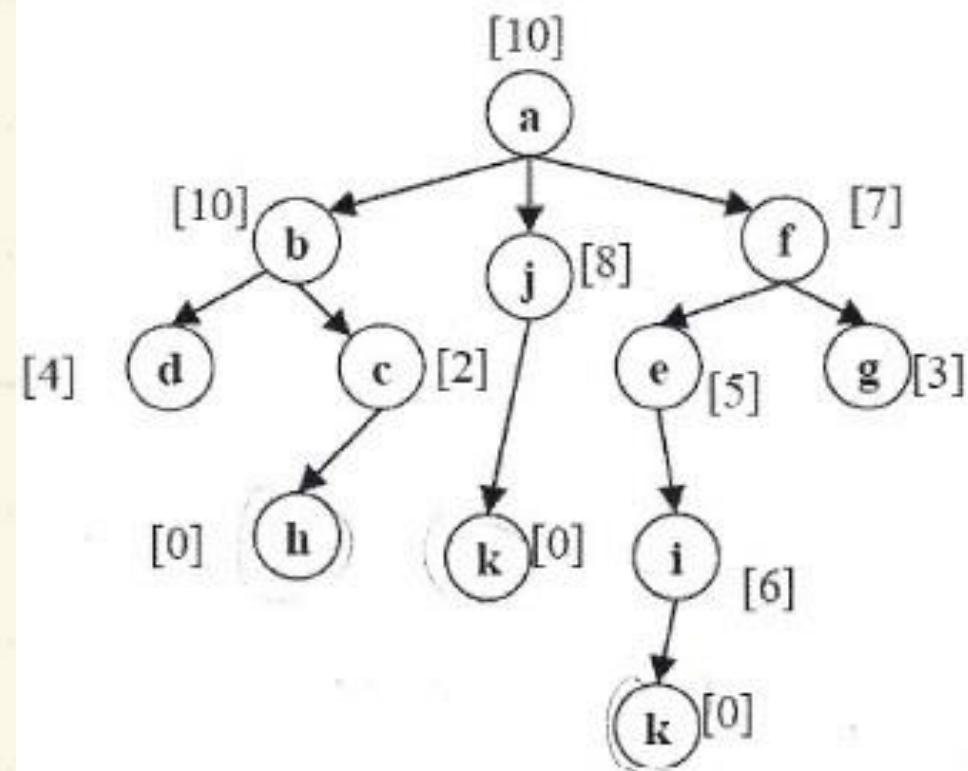
$f_7, j_8, b_{10}$

# Beam Search Goal – Node K



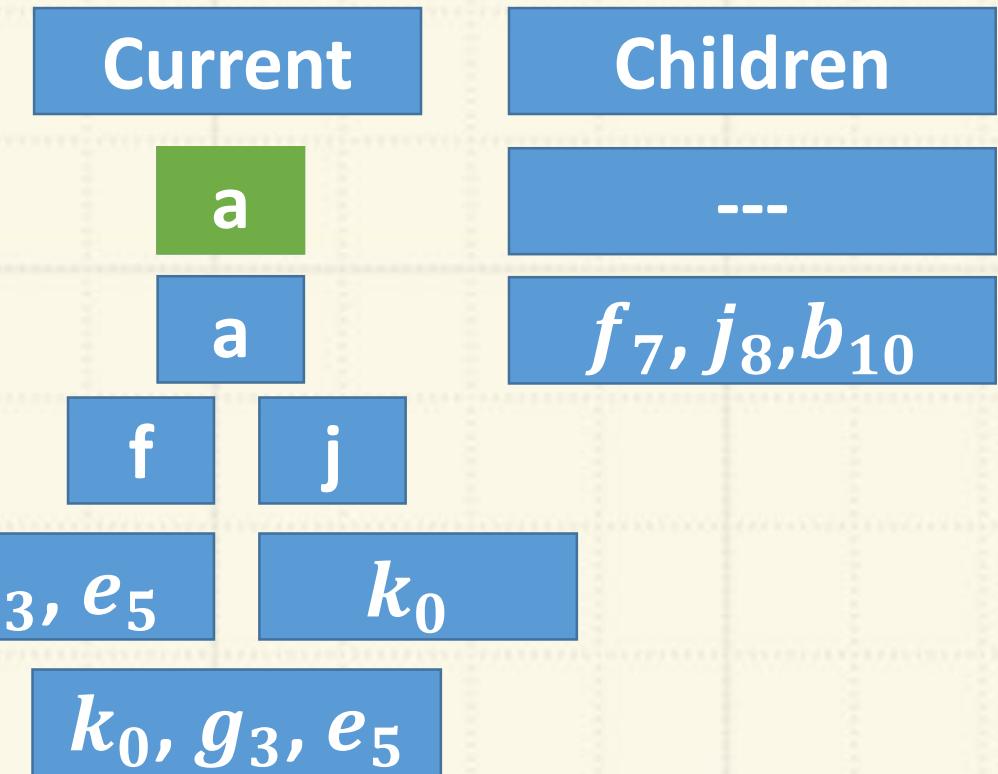
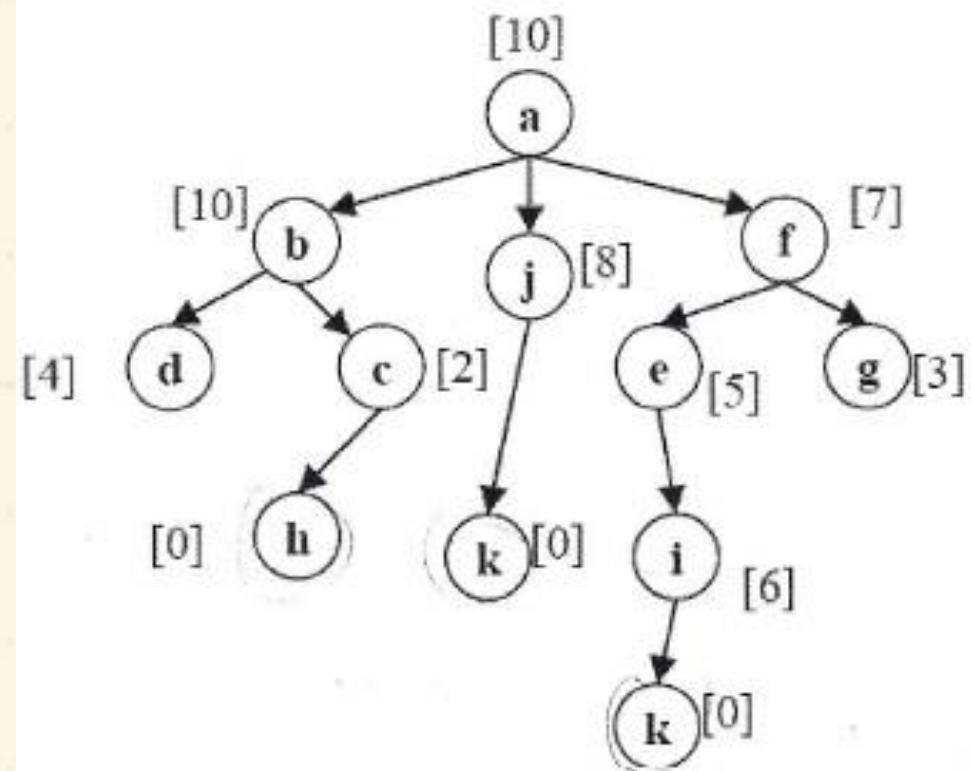
| Current | Children           |
|---------|--------------------|
| a       | ...                |
| a       | $f_7, j_8, b_{10}$ |
| f       | j                  |

# Beam Search Goal – Node K



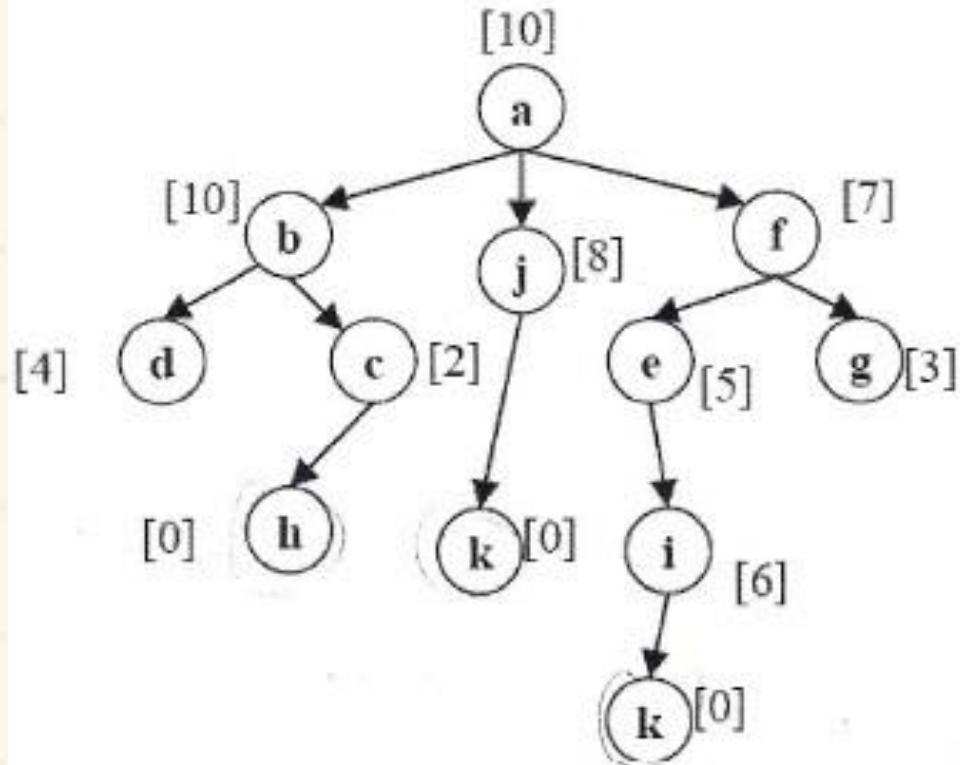
| Current    | Children           |
|------------|--------------------|
| a          | ---                |
| a          | $f_7, j_8, b_{10}$ |
| f          | j                  |
| $g_3, e_5$ | $k_0$              |

# Beam Search Goal – Node K



# Beam Search Goal – Node K

Best k  
Successors



Current

a

a

f j

$g_3, e_5$

$k_0$

$k_0, g_3, e_5$

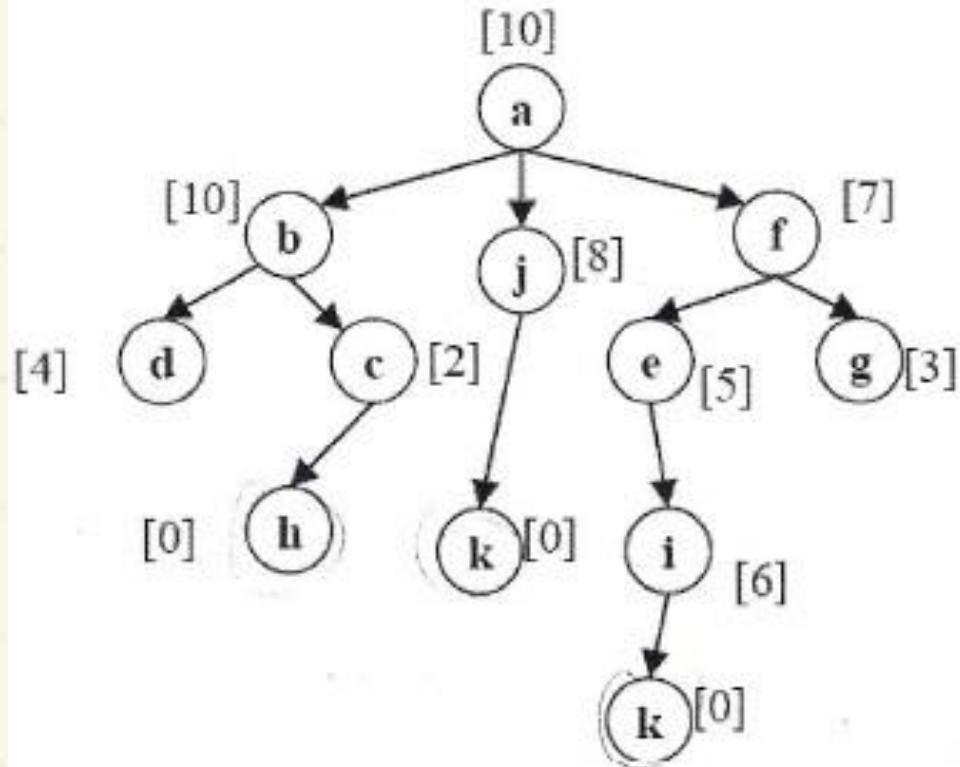
Children

---

$f_7, j_8, b_{10}$

# Beam Search Goal – Node K

Best k  
Successors



Current

a

a

f

j

$g_3, e_5$

$k_0$

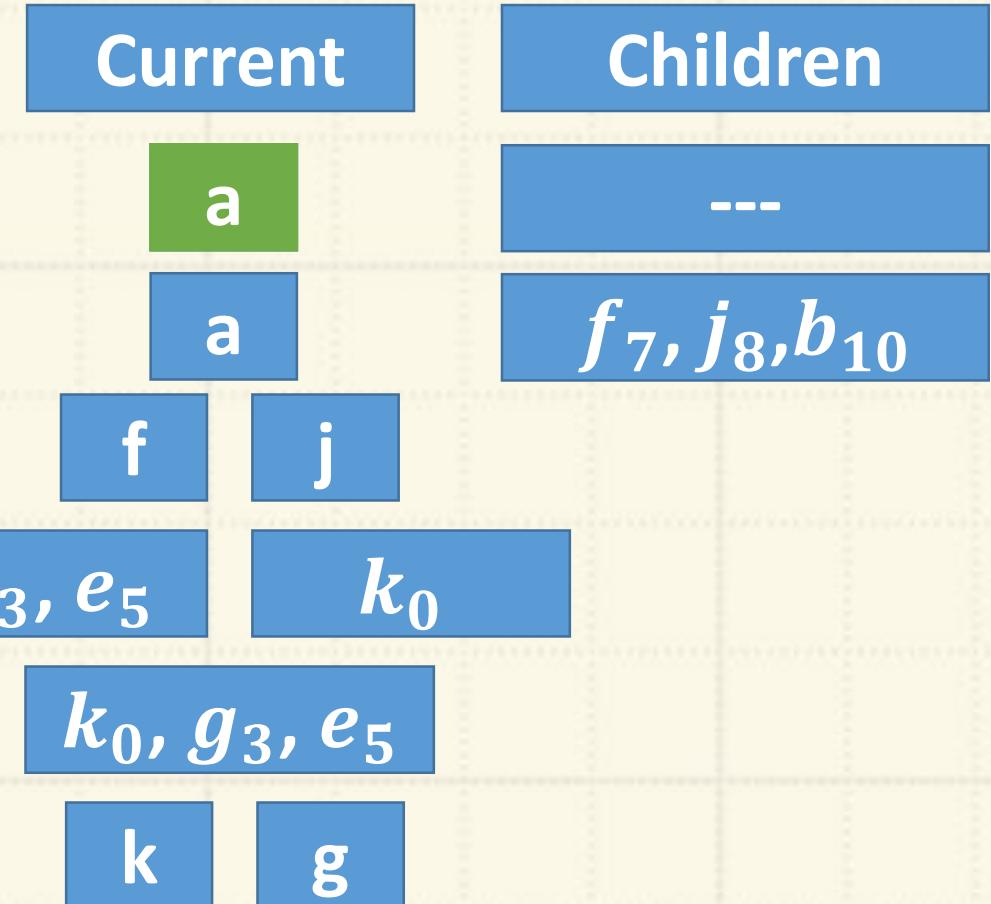
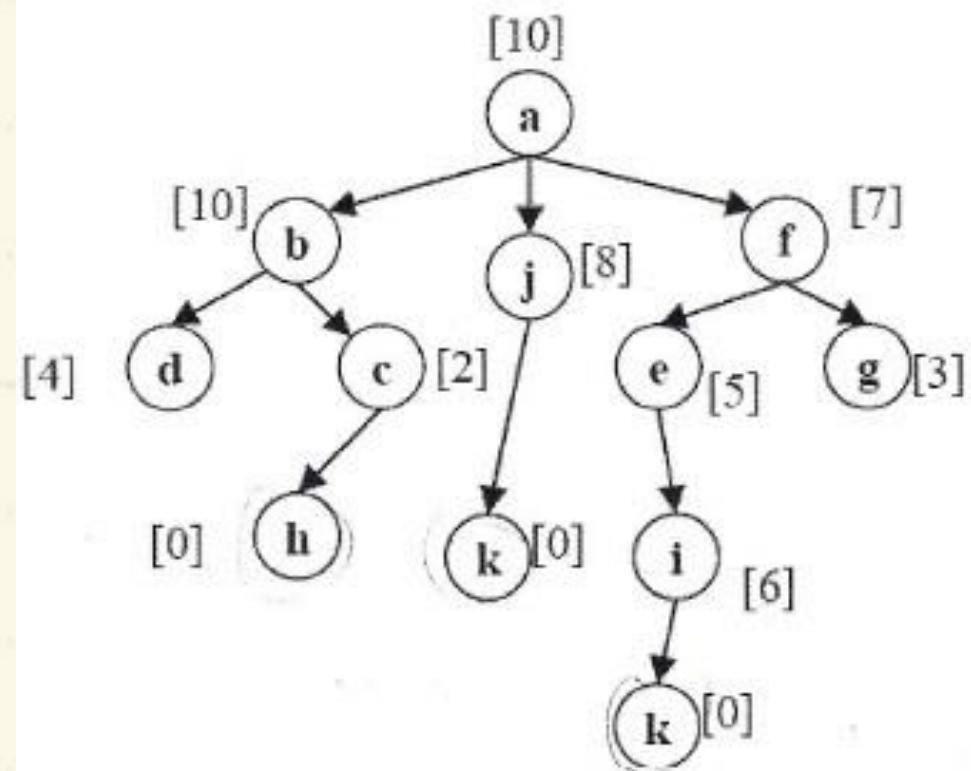
$k_0, g_3, e_5$

Children

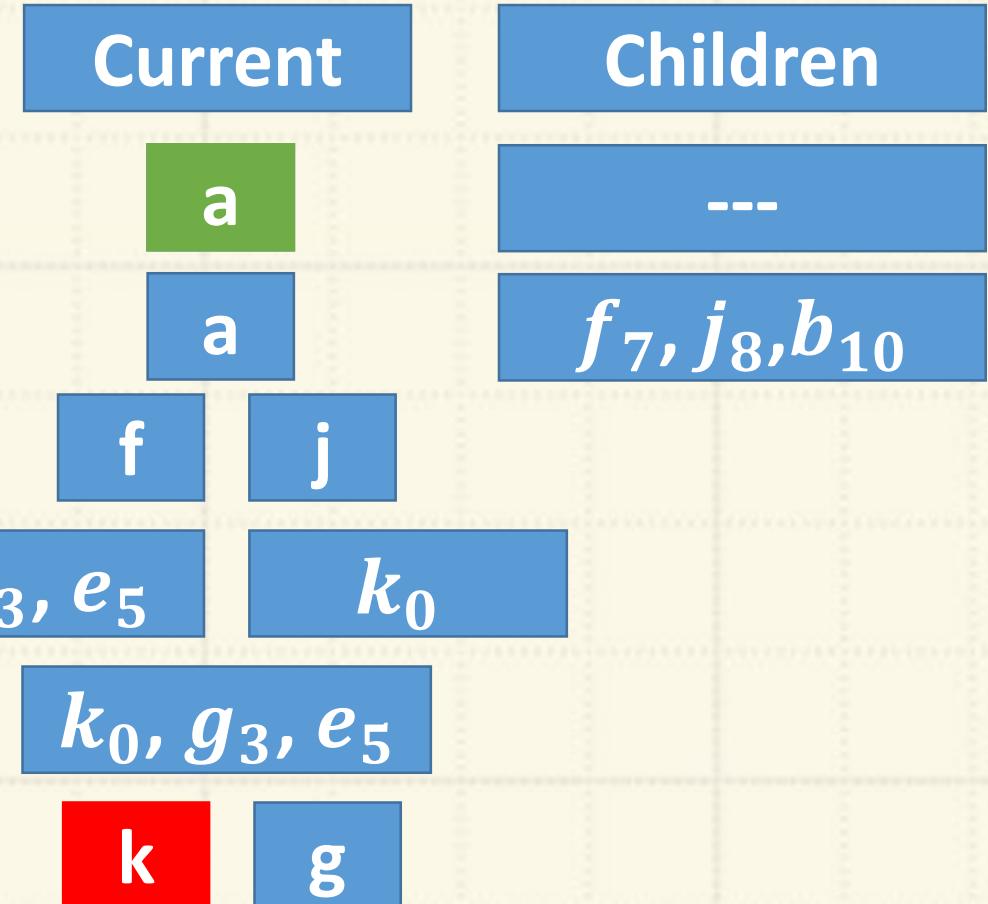
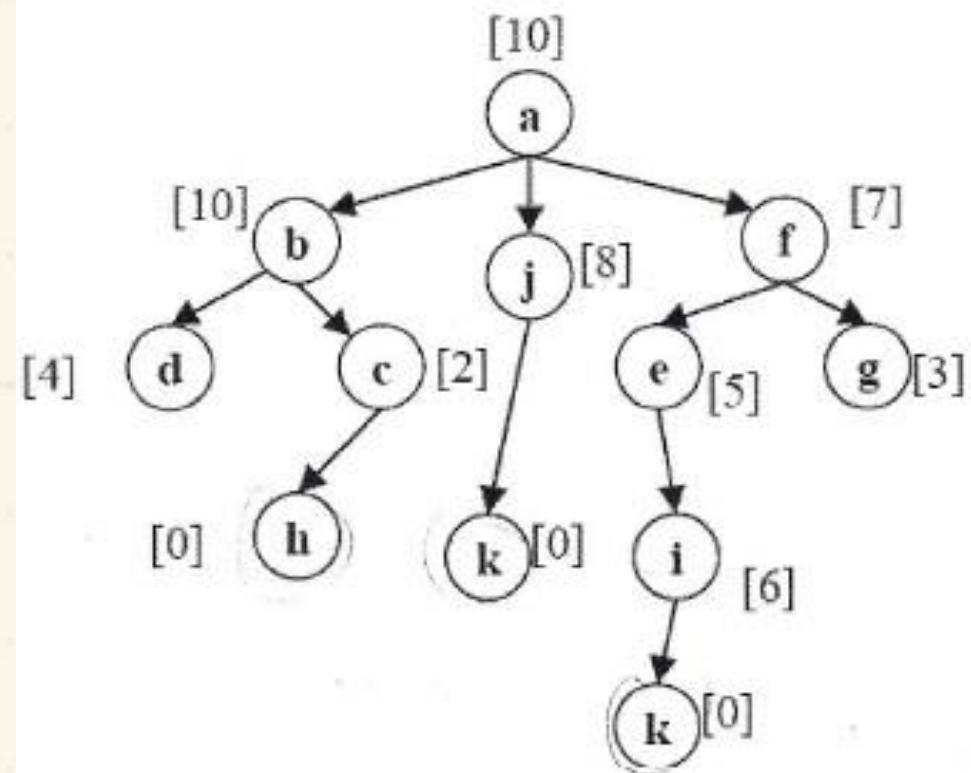
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$f_7, j_8, b_{10}$

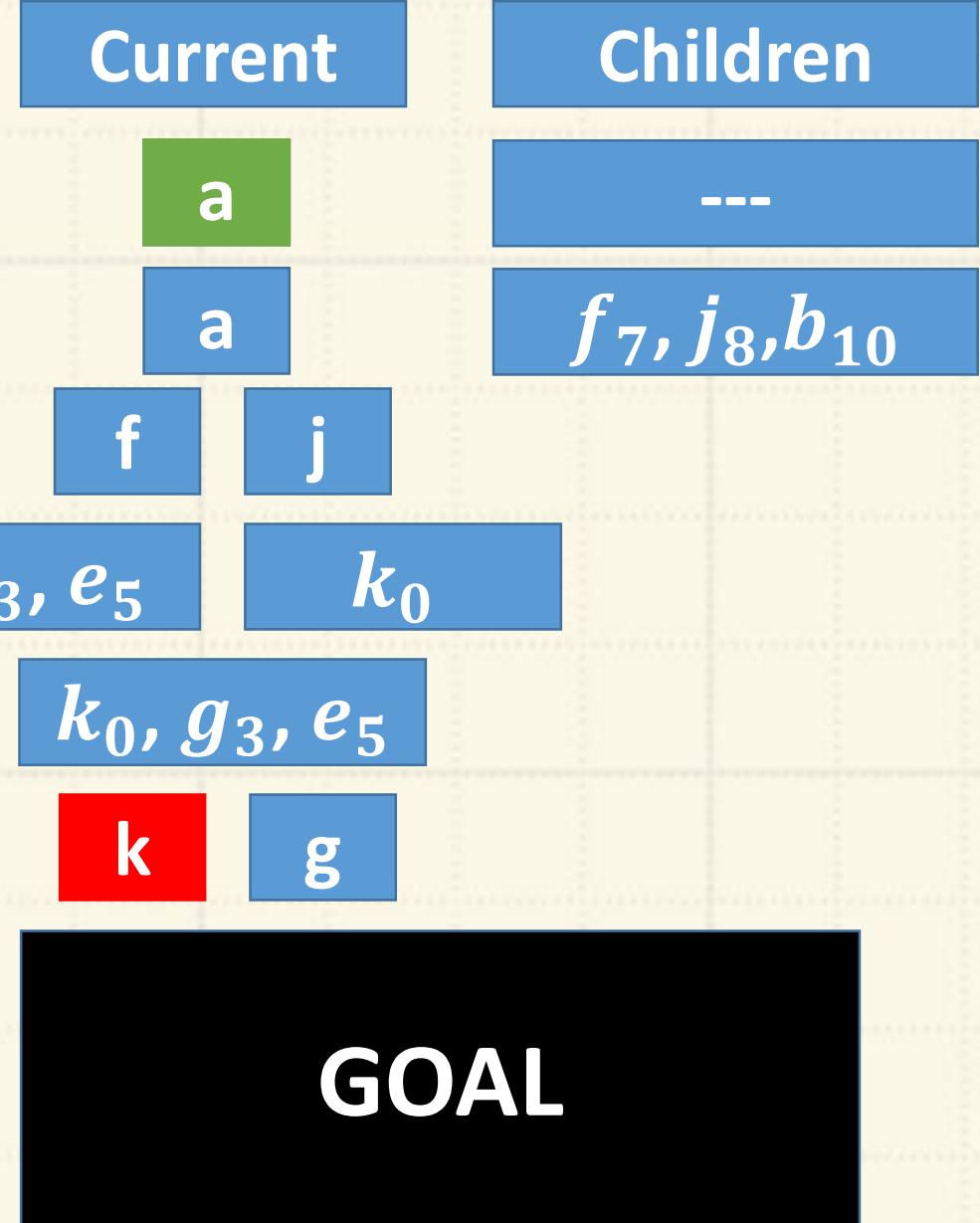
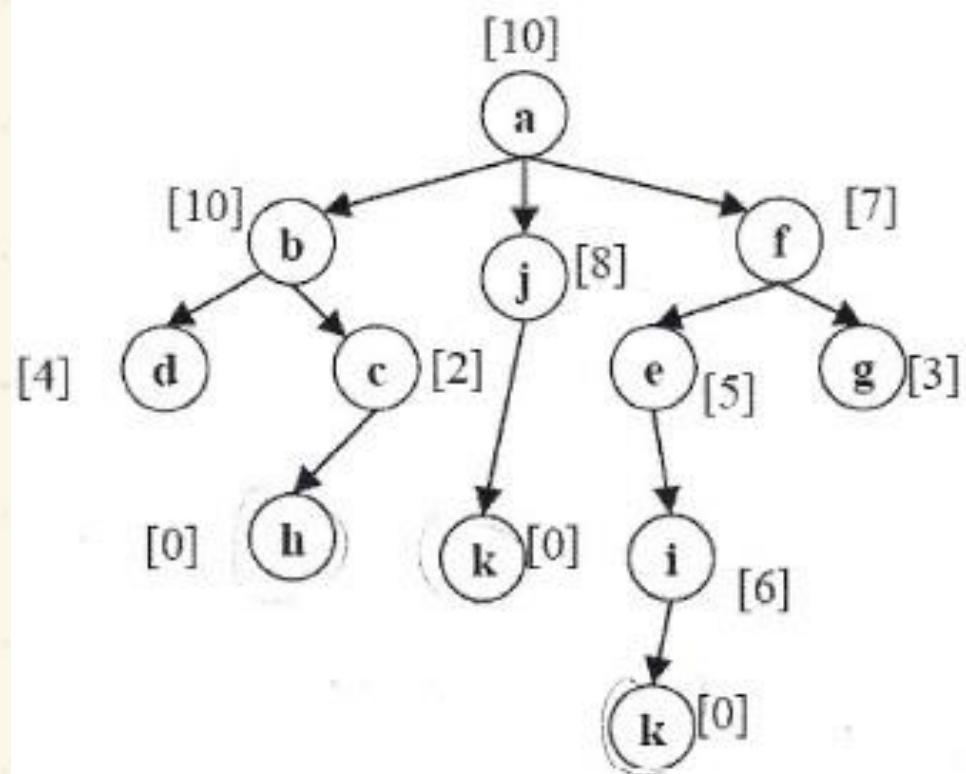
# Beam Search Goal – Node K



# Beam Search Goal – Node K



# Beam Search Goal – Node K



# Simulated Annealing

# Simulated Annealing

## Steps

1. Select a start node (root node).
2. Randomly select a child of the current node, calculate a value reflecting how good such child is like  $\text{value}(\text{node}) = -\text{heuristic}(\text{node})$ .
3. Select the child if it is better than the current node. Else try another child.

A node is better than the current node if  $\Delta E = \text{value}[\text{next}] - \text{value}[\text{current}] > 0$ .

Else if  $\Delta E < 0$ , then try to find another child.

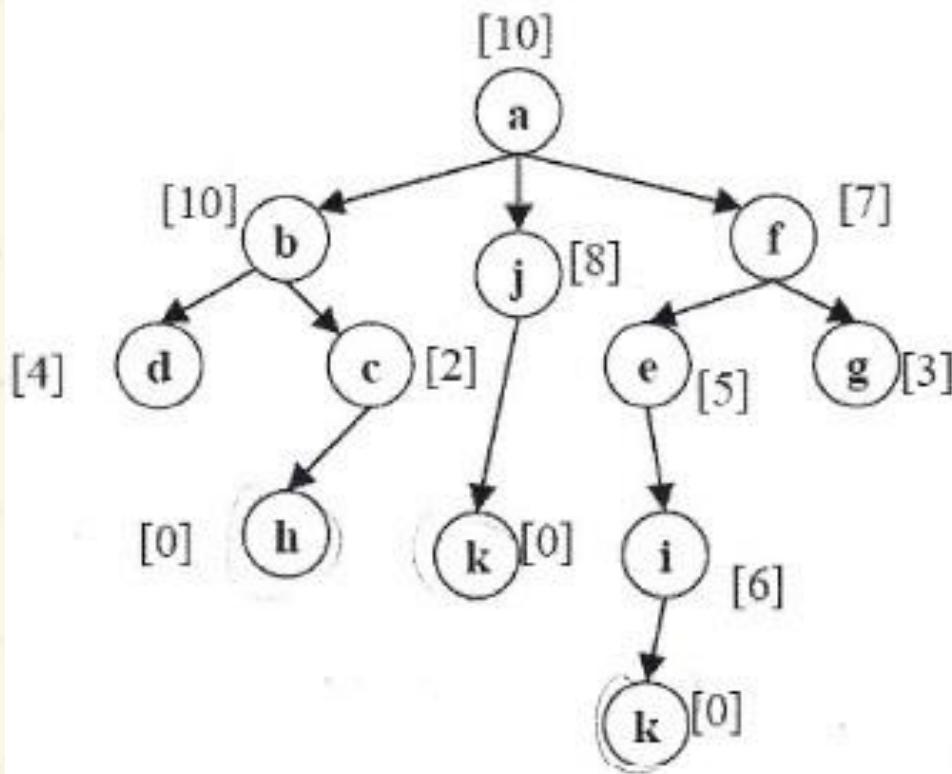
4. If the child was not better than the current node then it will be selected with probability equal to  $p = e^{\frac{\Delta E}{T}}$   
where  $\Delta E = \text{value}[\text{next}] - \text{value}[\text{current}]$   
 $T$  is a temperature.
5. Stop if no improvement can be found or after a fixed time.

# Simulated Annealing Example – $T=10$

Current

Children

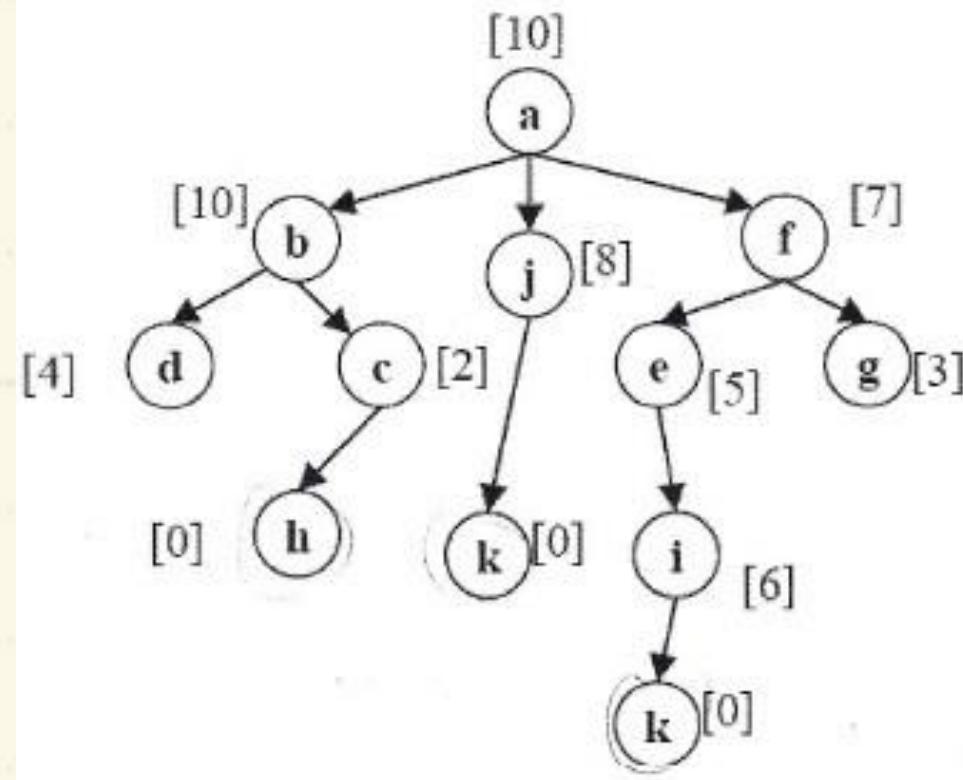
a



# Simulated Annealing Starting from Node a

Current

Children

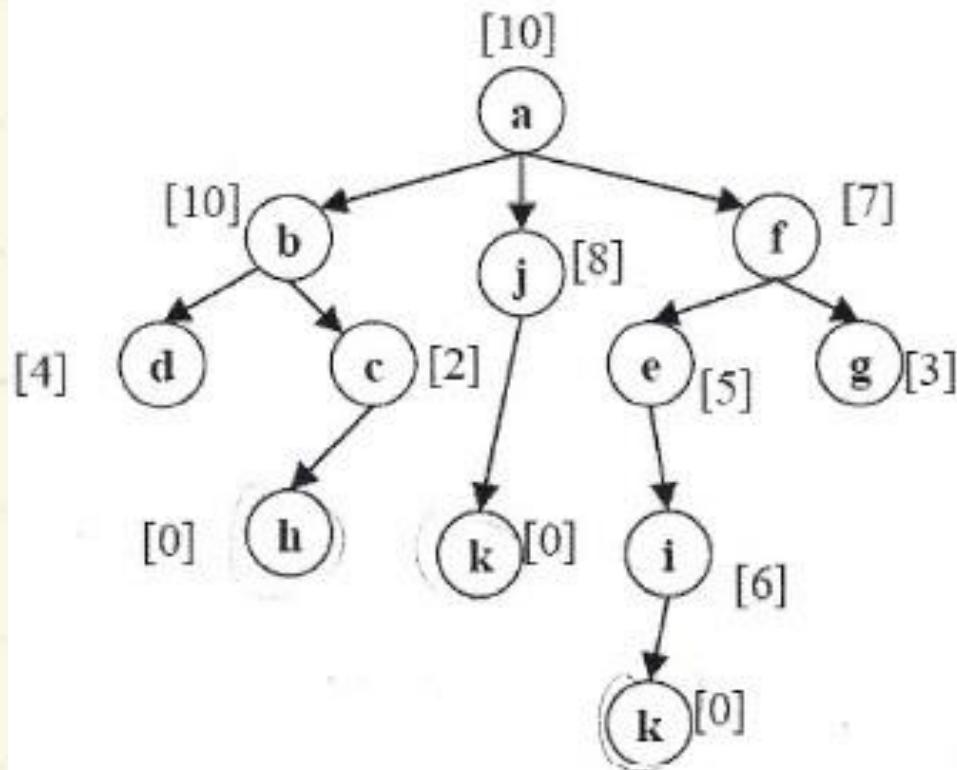


# Simulated Annealing Starting from Node a

Current

Children

a



# Simulated Annealing

Current

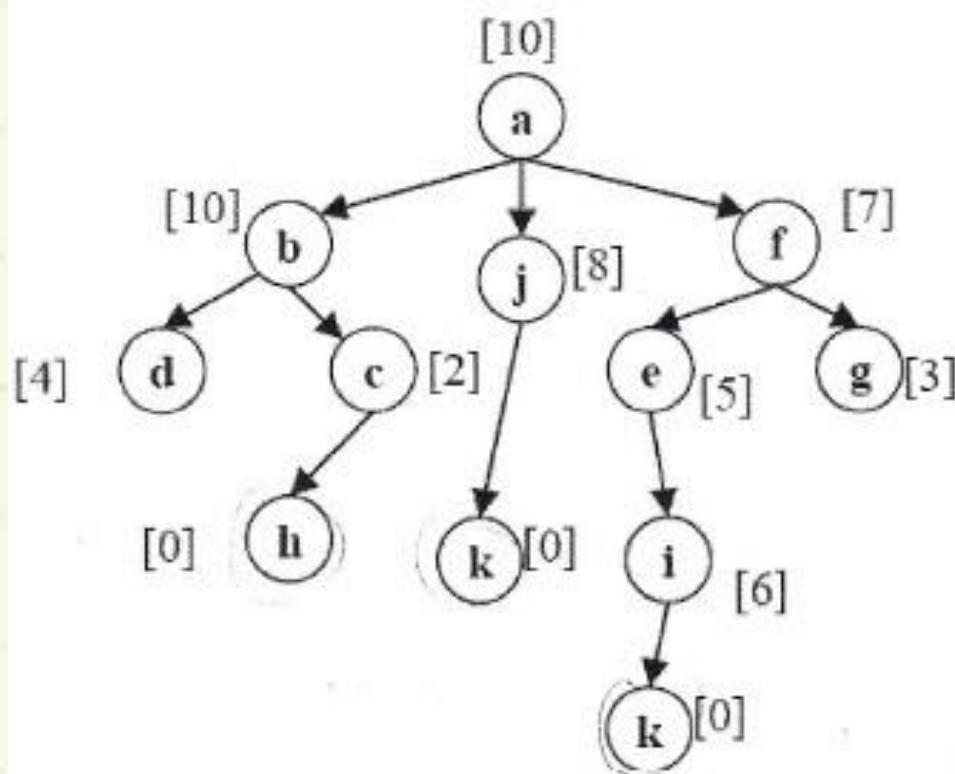
a

a

Children

---

$f_7, j_8, b_{10}$



# Simulated Annealing

Current

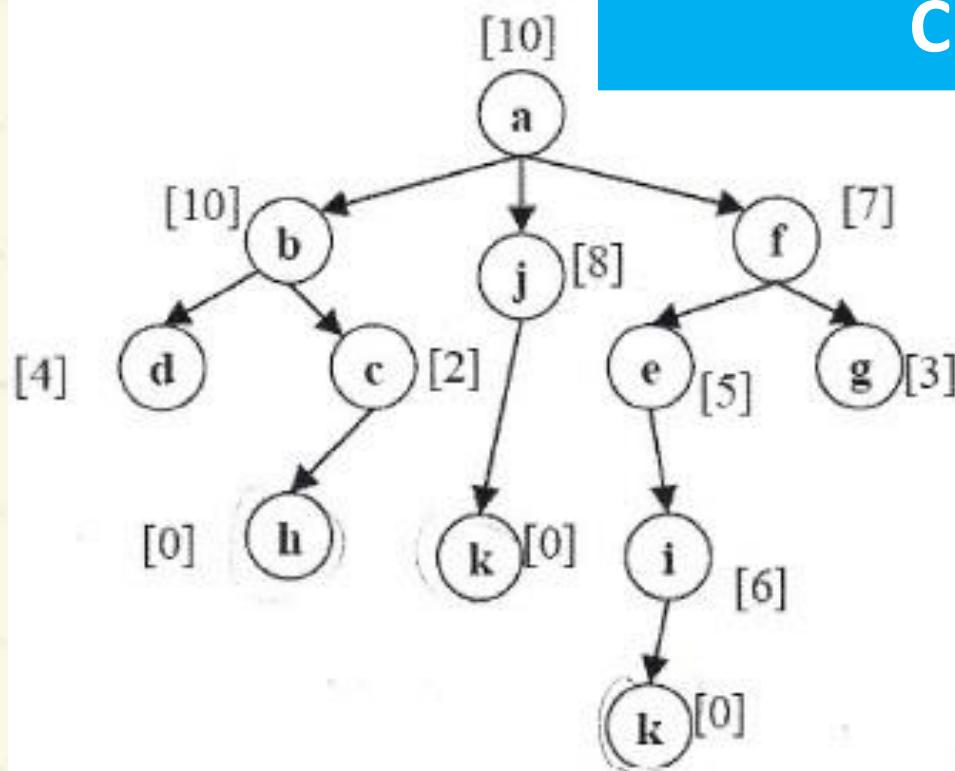
a

Children

---

$f_7, j_8, b_{10}$

Randomly Select a  
Child



# Simulated Annealing

Current

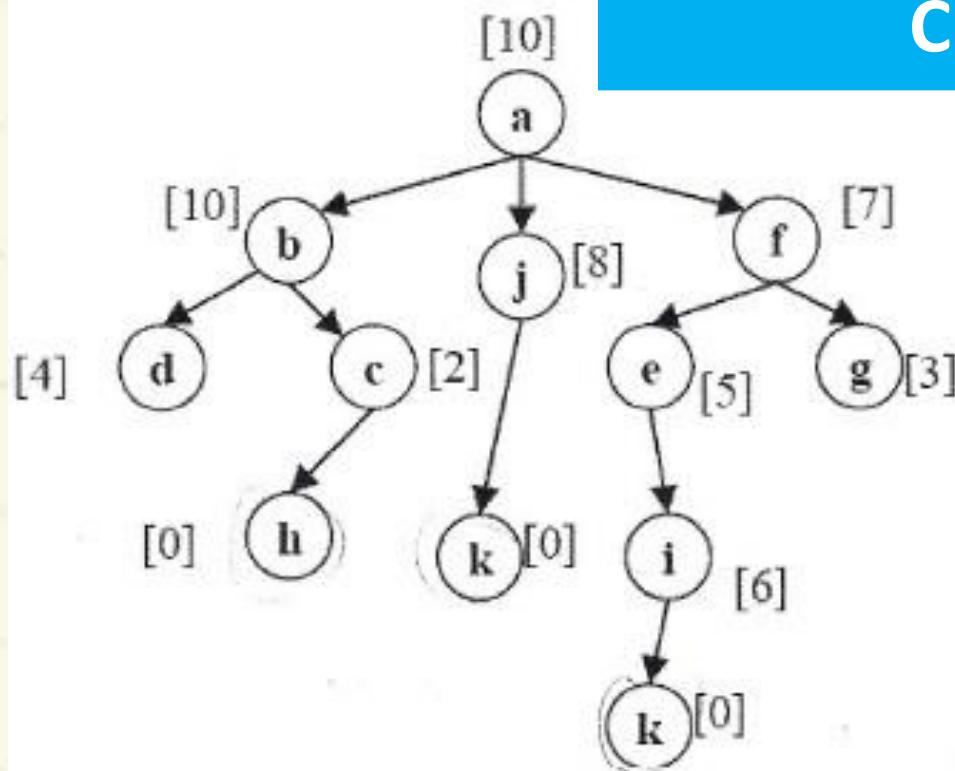
a

Children

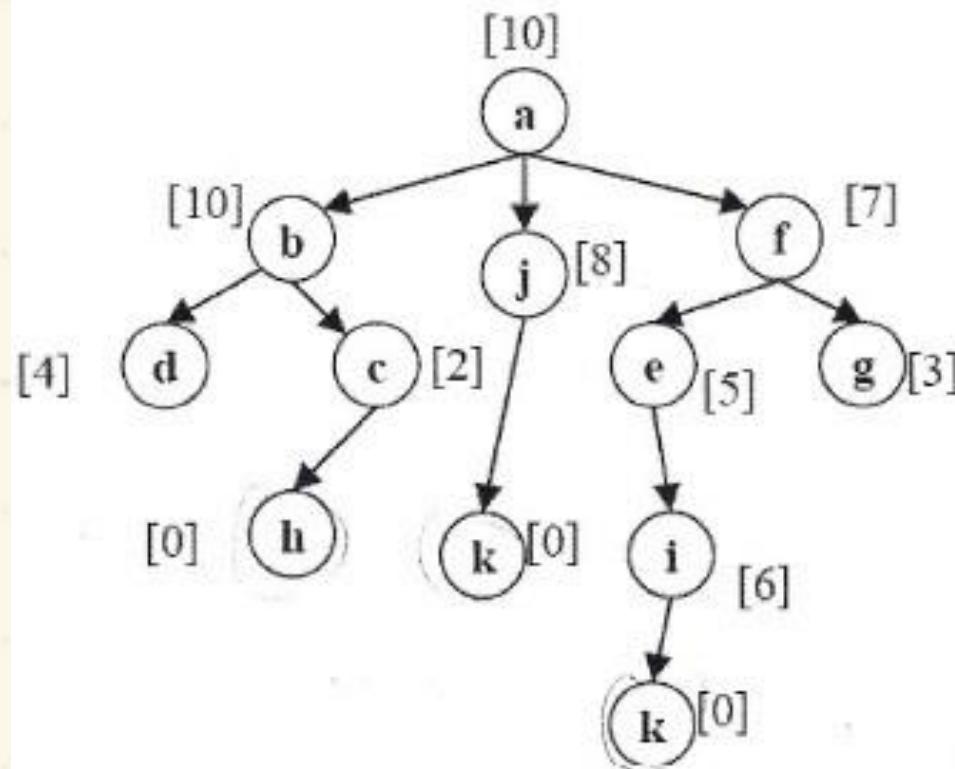
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$f_7, j_8, b_{10}$

Randomly Select a  
Child



# Simulated Annealing



Current

a

a

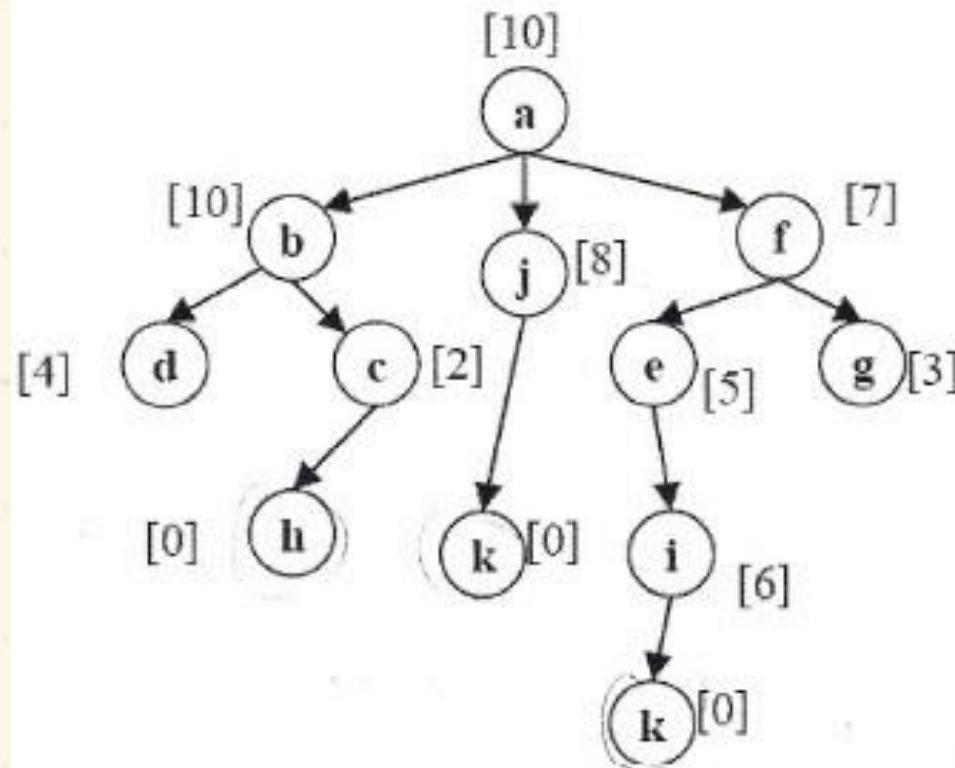
Children

---

$f_7, j_8, b_{10}$

Check if next node  $f_7$  is better than current node

# Simulated Annealing



Current

a

a

Children

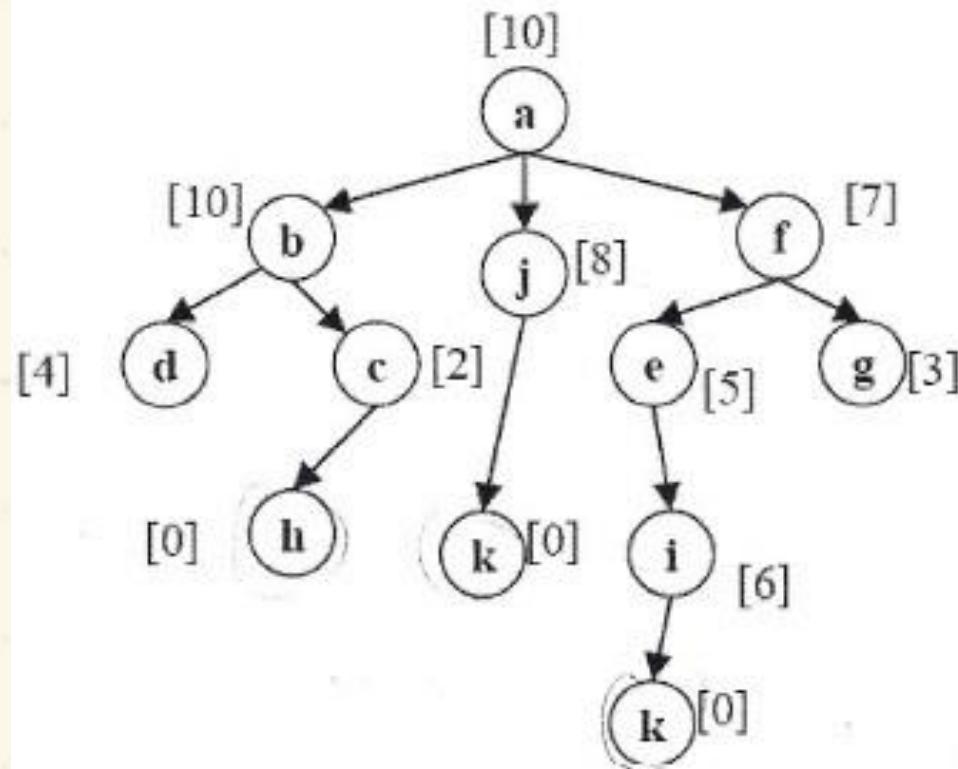
---

$f_7, j_8, b_{10}$

Check if next node  $f_7$  is better than current node

$\Delta E > 0$

# Simulated Annealing



Current

a

a

Children

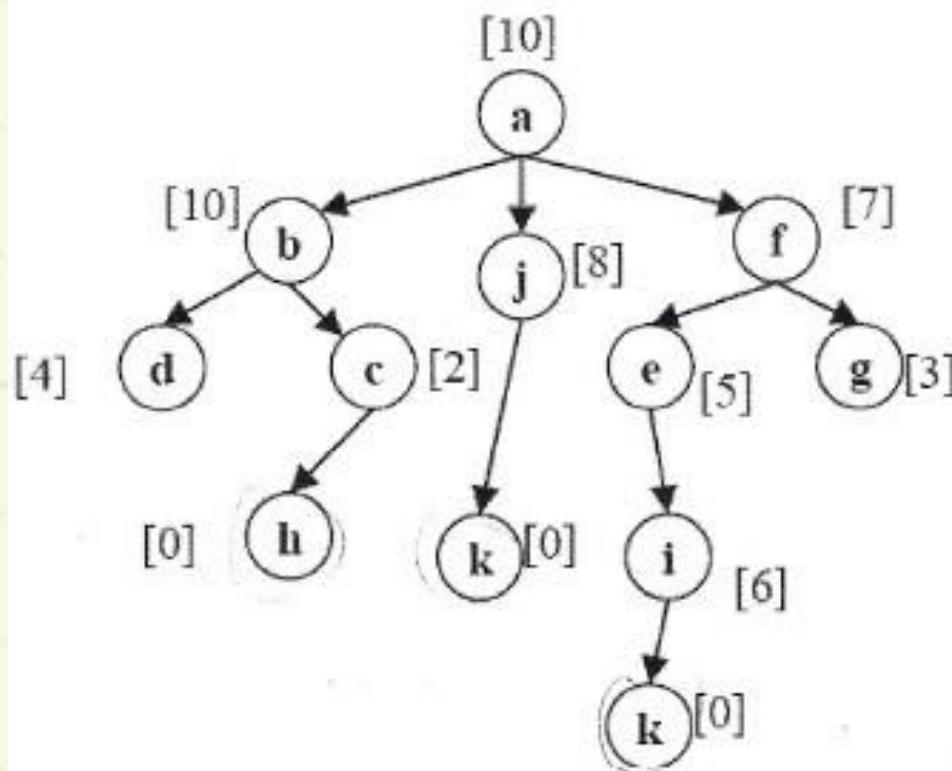
---

$f_7, j_8, b_{10}$

Check if next node  $f_7$  is better than current node

$$\Delta E = \text{value(next)} - \text{value(current)}$$

# Simulated Annealing



Current

a

a

Children

---

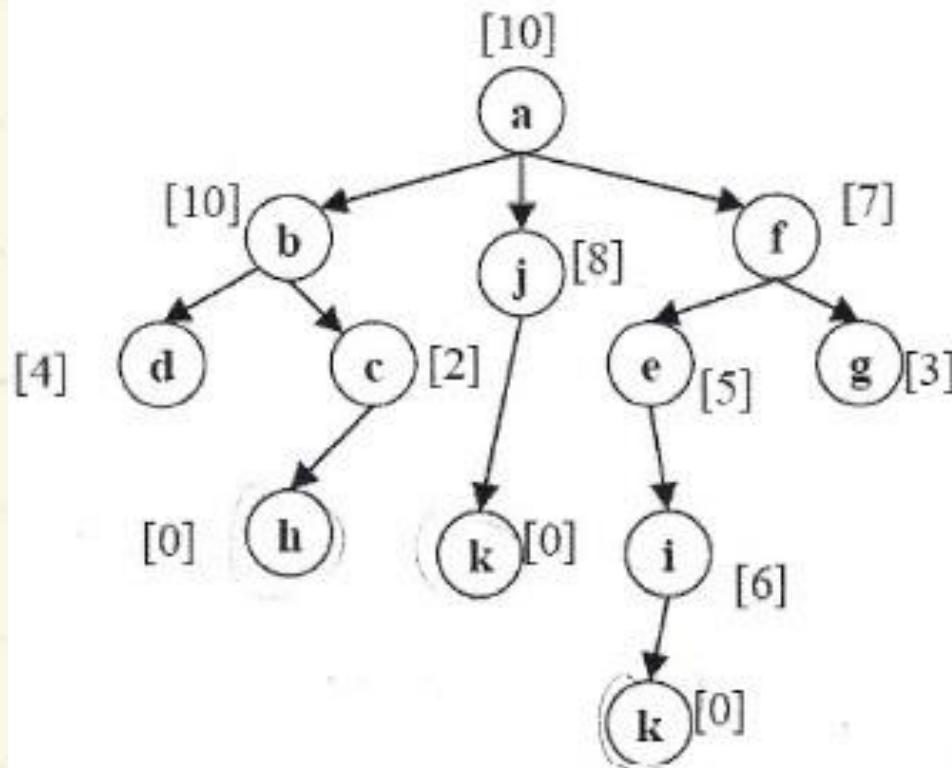
$f_7, j_8, b_{10}$

Check if next node  $f_7$  is better than current node

$$\Delta E = \text{value(next)} - \text{value(current)}$$

$$\Delta E = \text{value}(f_7) - \text{value}(a_{10})$$

# Simulated Annealing



Current

a

a

Children

---

$f_7, j_8, b_{10}$

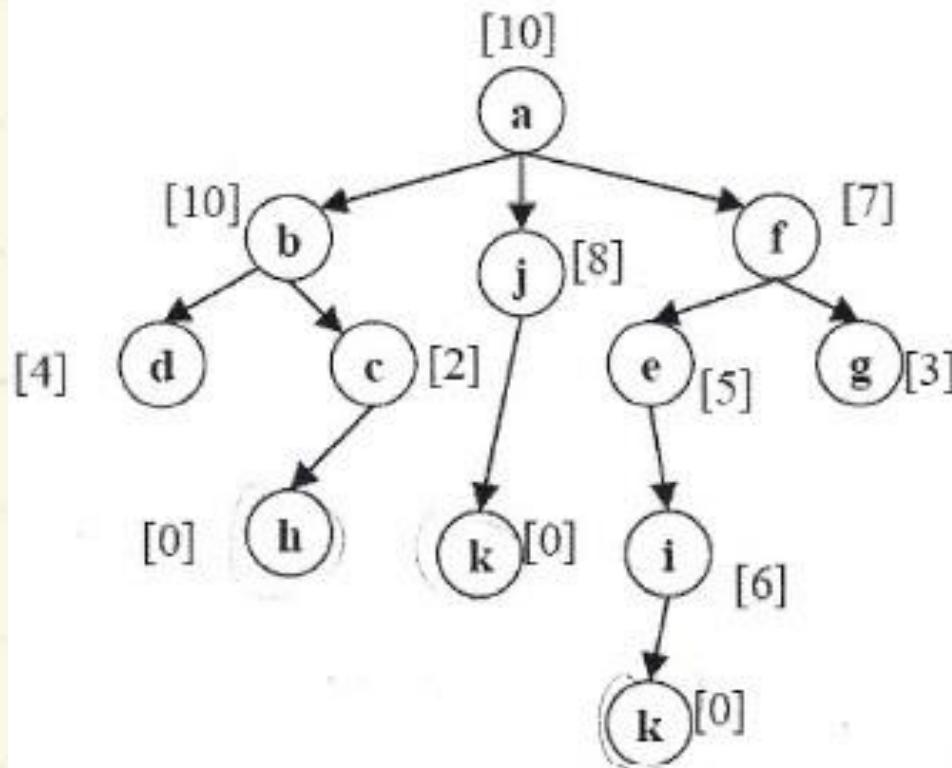
Check if next node  $f_7$  is better than current node

$$\Delta E = \text{value(next)} - \text{value(current)}$$

$$\Delta E = \text{value}(f_7) - \text{value}(a_{10})$$

$$\text{value}(f_7) = -\text{heuristic}(f_7) = -7$$

# Simulated Annealing



Current

a

a

Children

---

$f_7, j_8, b_{10}$

Check if next node  $f_7$  is better than current node

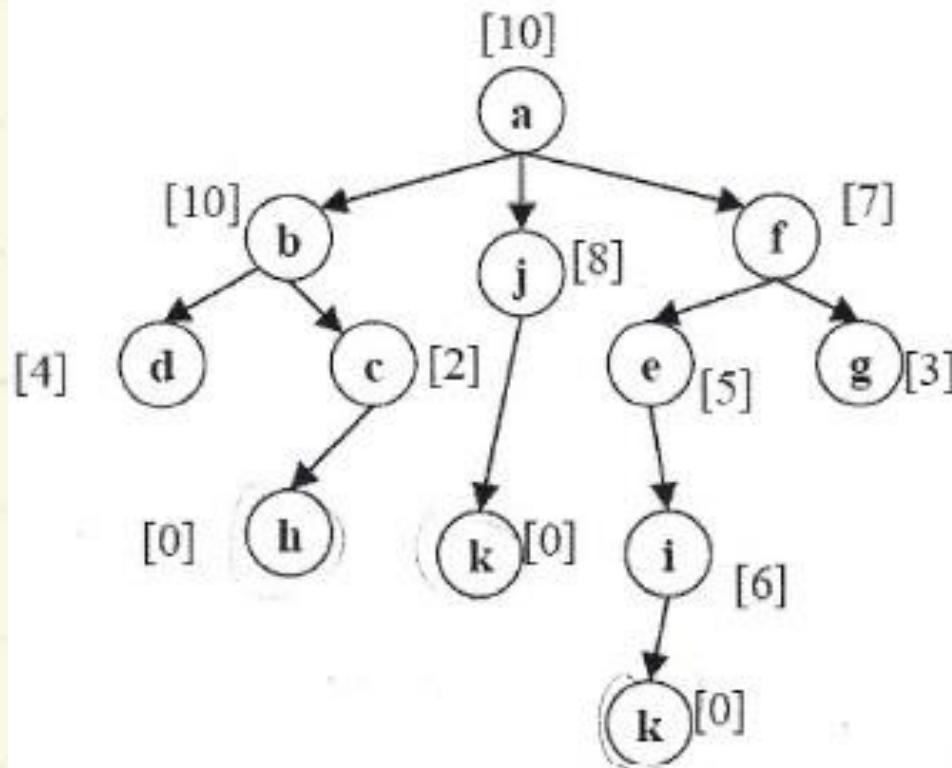
$$\Delta E = \text{value(next)} - \text{value(current)}$$

$$\Delta E = \text{value}(f_7) - \text{value}(a_{10})$$

$$\text{value}(f_7) = -\text{heuristic}(f_7) = -7$$

$$\text{value}(a_{10}) = -\text{heuristic}(a_{10}) = -10$$

# Simulated Annealing



Current

a

a

Children

---

$f_7, j_8, b_{10}$

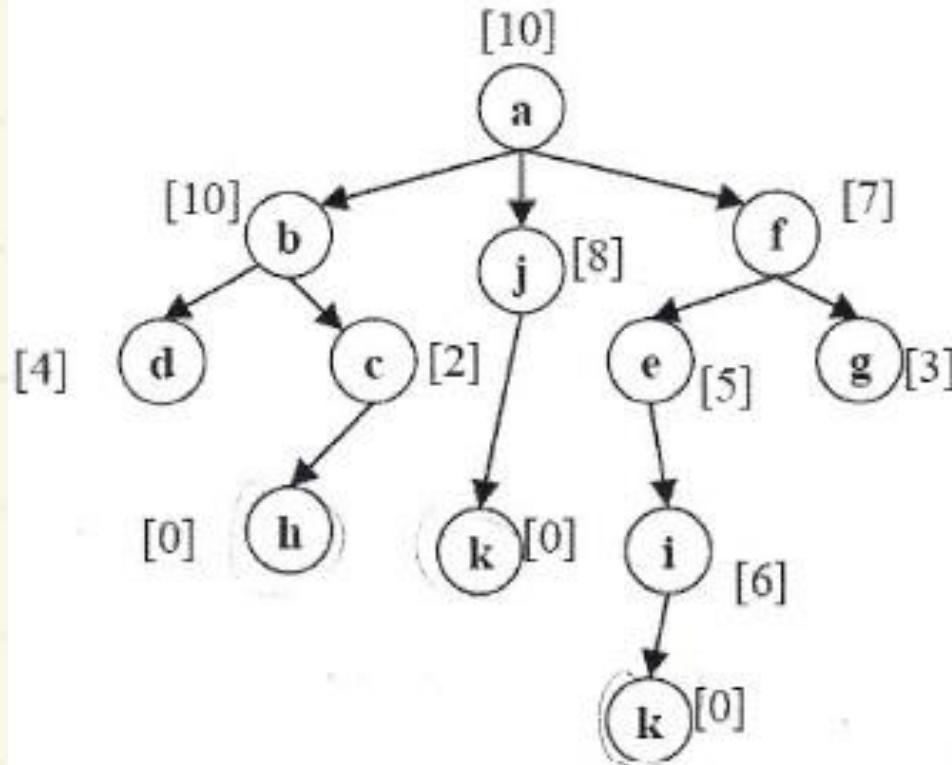
Check if next node  $f_7$  is better than current node

$$\Delta E = \text{value(next)} - \text{value(current)}$$

$$\Delta E = \text{value}(f_7) - \text{value}(a_{10})$$

$$\Delta E = -7 - (-10) = +3$$

# Simulated Annealing



Current

a

a

Children

---

$f_7, j_8, b_{10}$

Check if next node  $f_7$  is better than current node

$$\Delta E = \text{value(next)} - \text{value(current)}$$

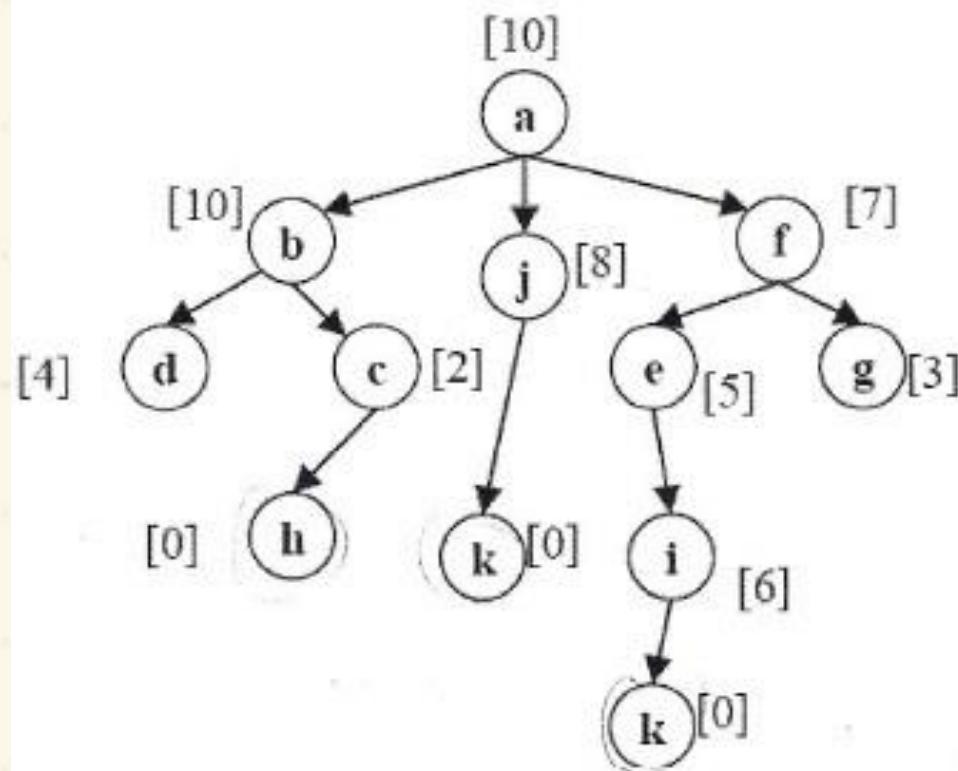
$$\Delta E = \text{value}(f_7) - \text{value}(a_{10})$$

$$\Delta E = -7 - (-10) = +3$$

$$\therefore \Delta E > 0$$

$\therefore f_7$  will be selected with probability 1

# Simulated Annealing



Current

a

a

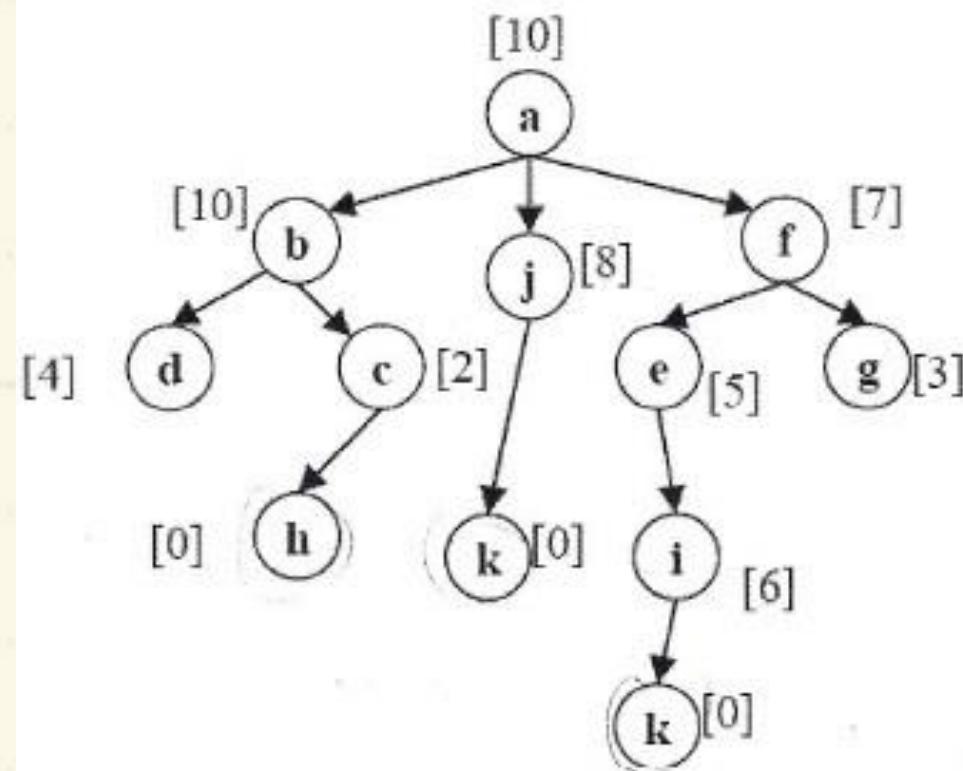
f

Children

---

$f_7, j_8, b_{10}$

# Simulated Annealing



Current

a

a

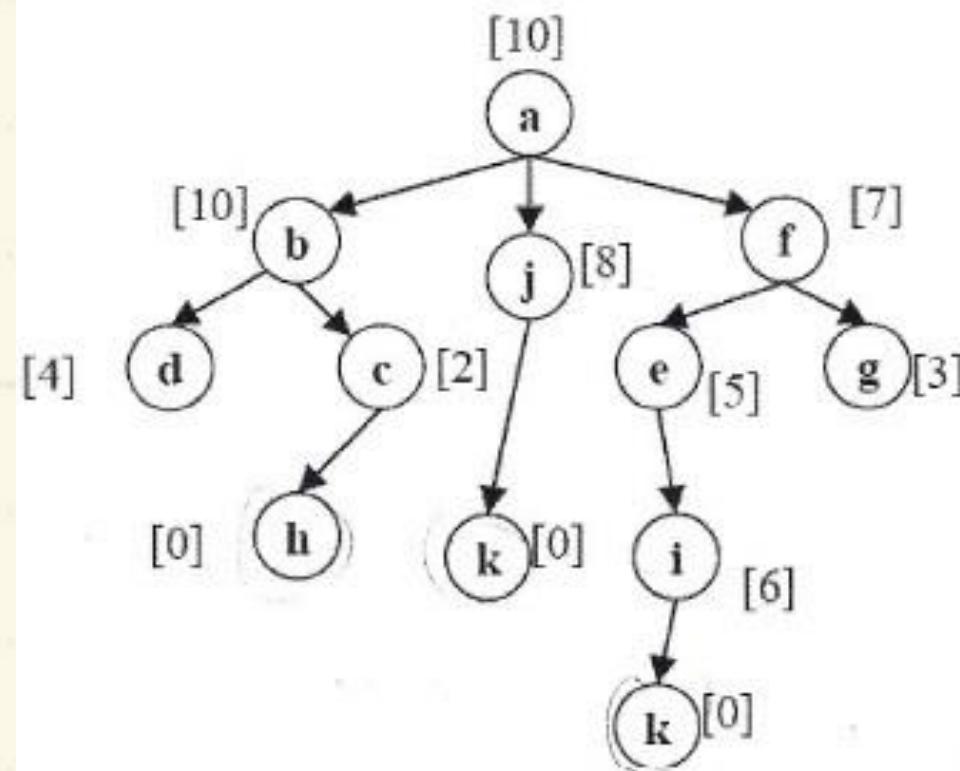
f

Children

---

$f_7, j_8, b_{10}$

# Simulated Annealing



Current

a

a

f

Children

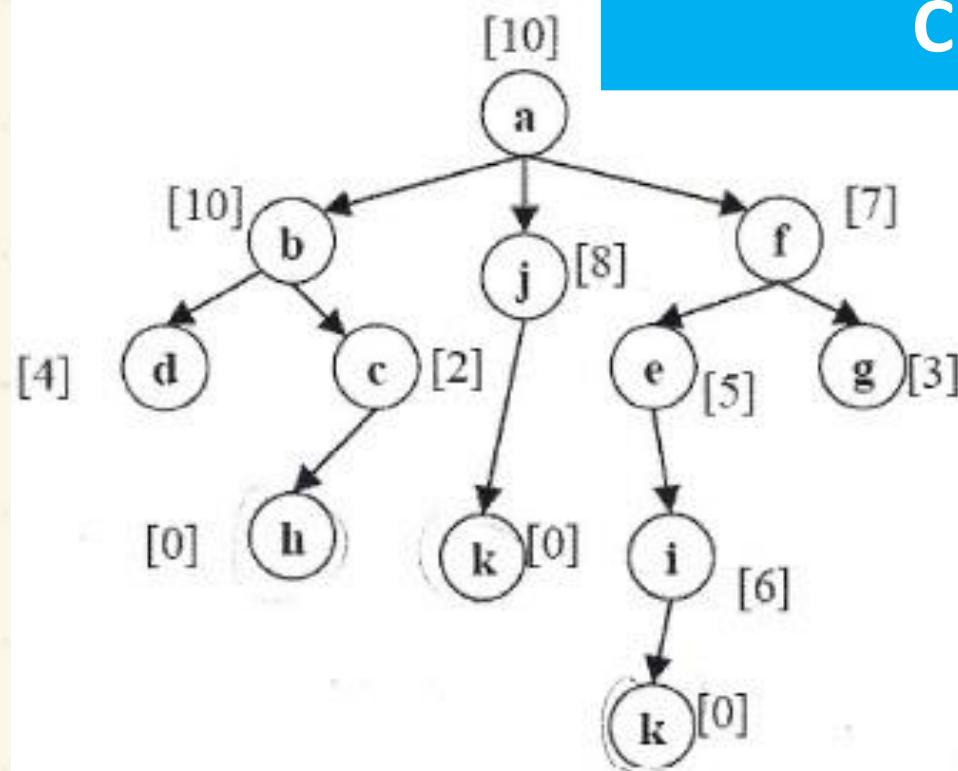
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$f_7, j_8, b_{10}$

$e_5, g_3$

# Simulated Annealing

Randomly Select a  
Child



Current

a

a

f

Children

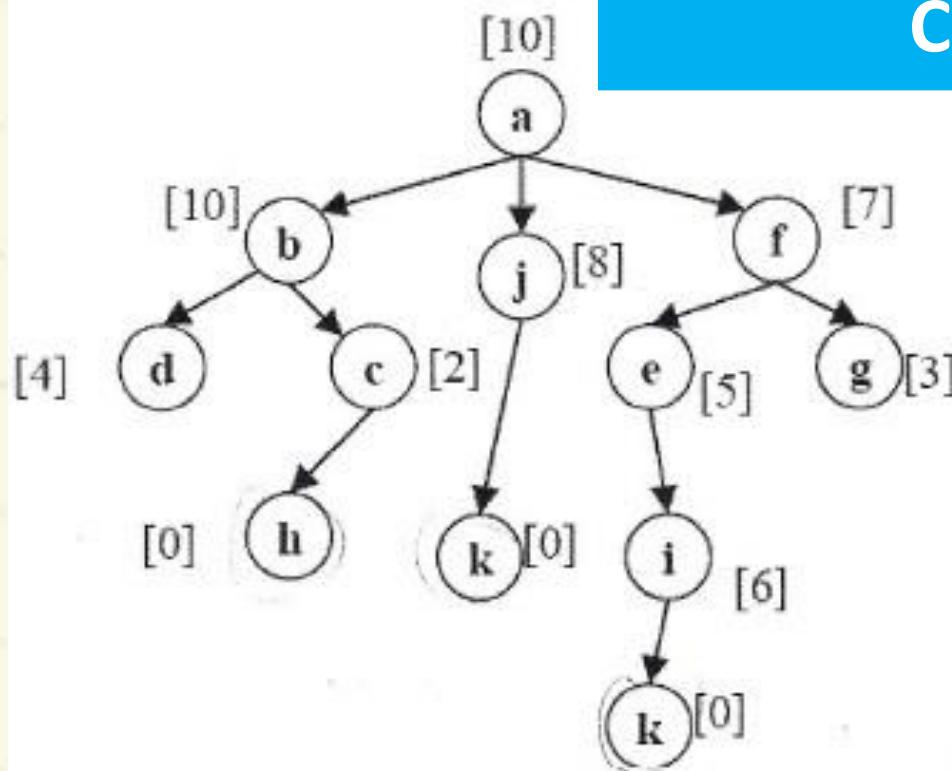
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$f_7, j_8, b_{10}$

$e_5, g_3$

# Simulated Annealing

Randomly Select a  
Child



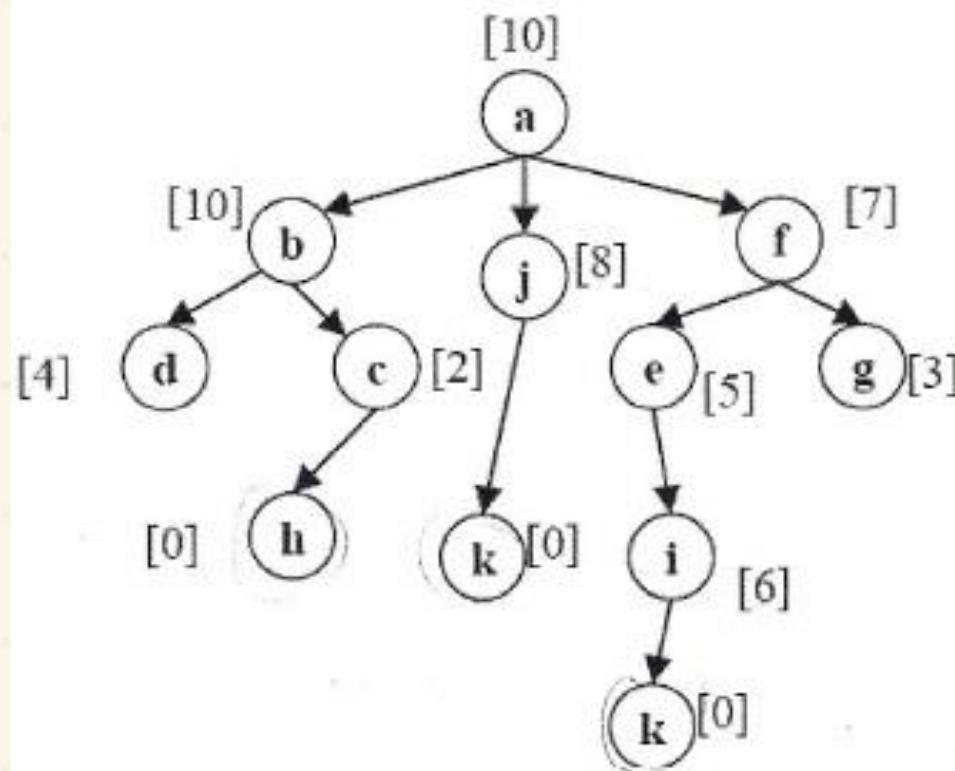
Current

a  
a  
f

Children

...  
 $f_7, j_8, b_{10}$   
 $e_5, g_3$

# Simulated Annealing



Current

a

a

f

Children

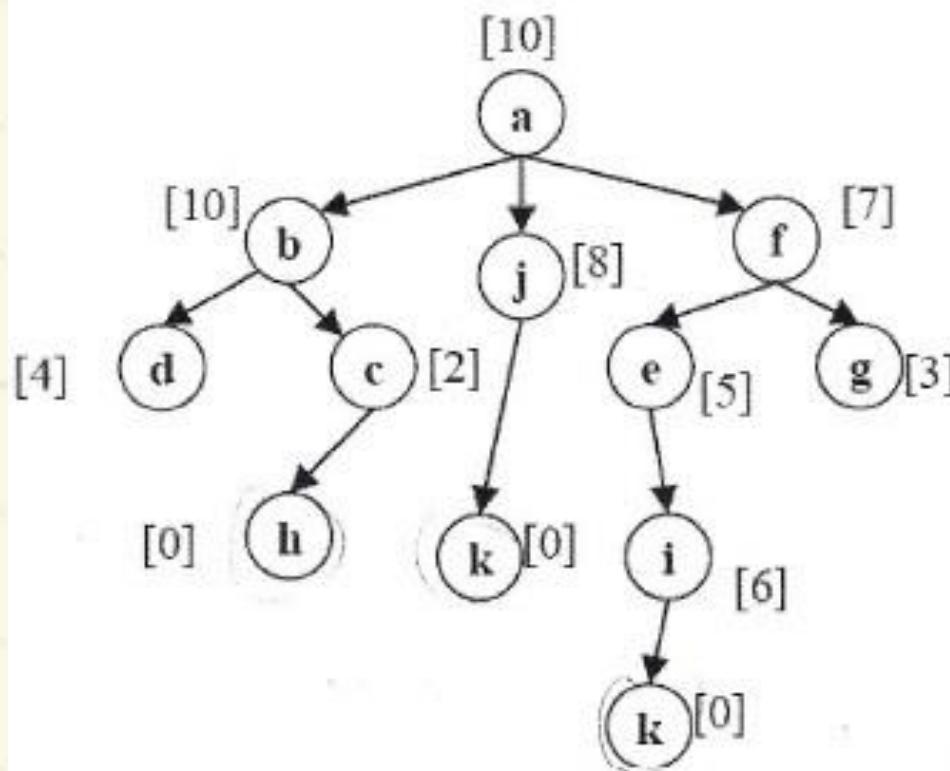
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$f_7, j_8, b_{10}$

$e_5, g_3$

Check if next node  $e_5$  is better than current node

# Simulated Annealing



Current

a

a

f

Children

---

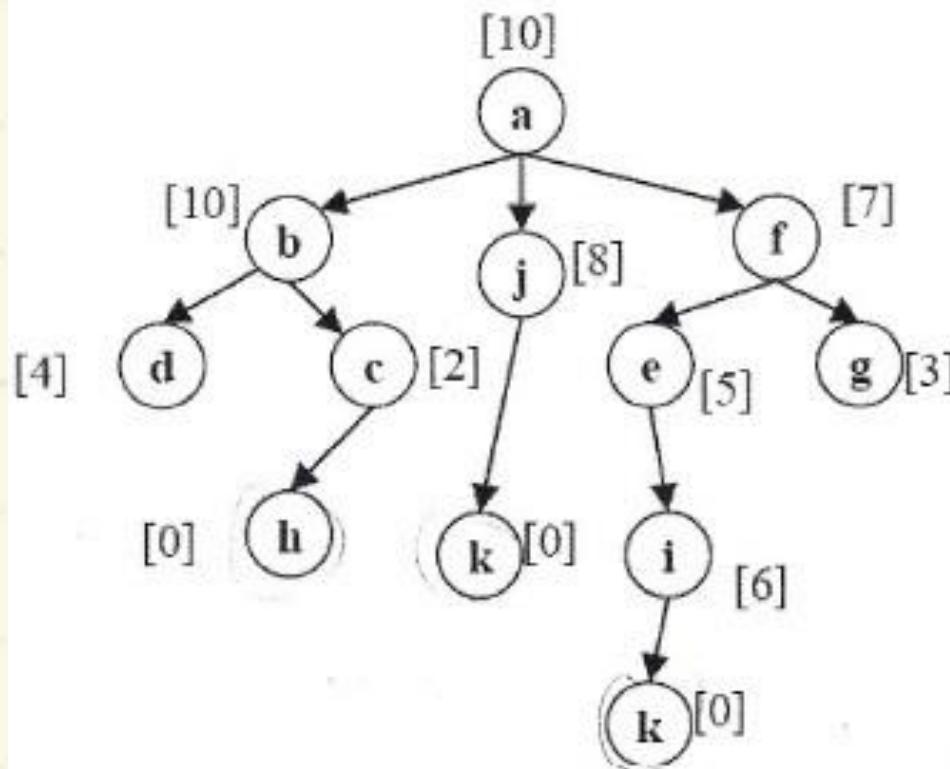
$f_7, j_8, b_{10}$

$e_5, g_3$

Check if next node  $e_5$  is better than current node

$\Delta E > 0$

# Simulated Annealing



Current

a

a

f

Children

---

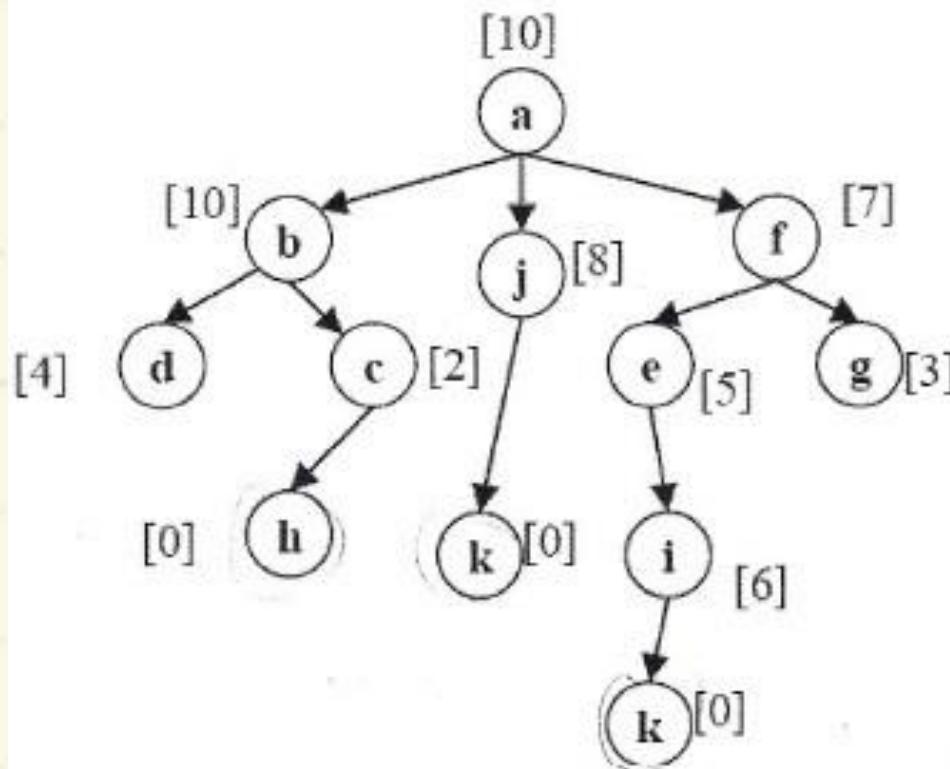
$f_7, j_8, b_{10}$

$e_5, g_3$

Check if next node  $e_5$  is better than current node

$$\Delta E = \text{value(next)} - \text{value(current)}$$

# Simulated Annealing



Current

a

a

f

Children

---

$f_7, j_8, b_{10}$

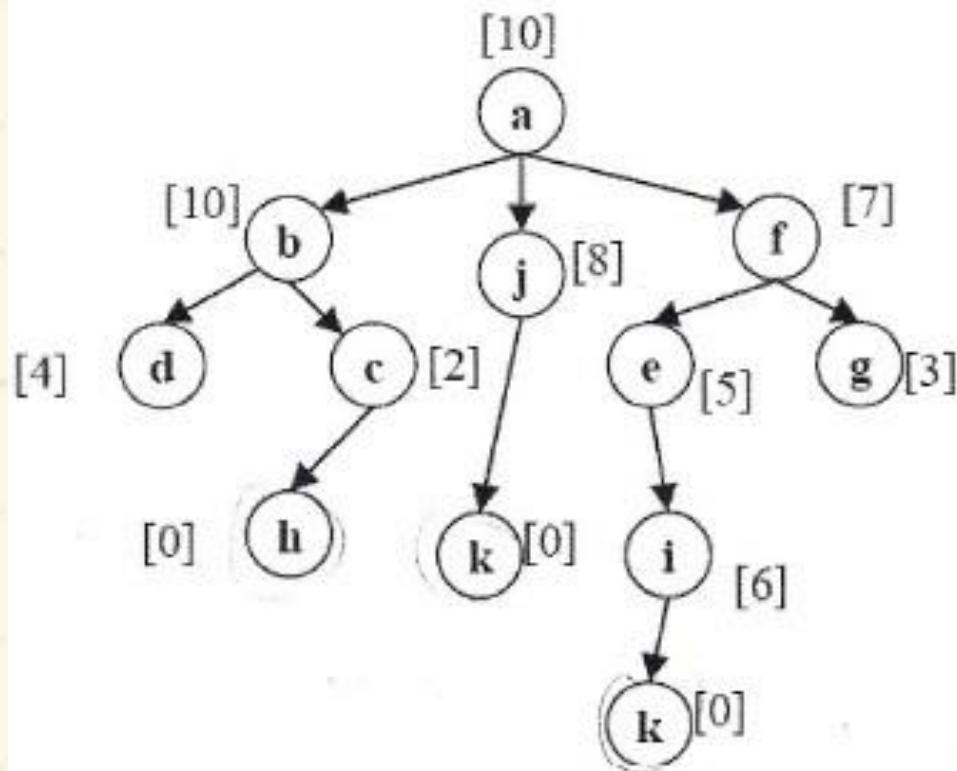
$e_5, g_3$

Check if next node  $e_5$  is better than current node

$$\Delta E = \text{value(next)} - \text{value(current)}$$

$$\Delta E = \text{value}(e_5) - \text{value}(f_7)$$

# Simulated Annealing



Current

a

a

f

Children

---

$f_7, j_8, b_{10}$

$e_5, g_3$

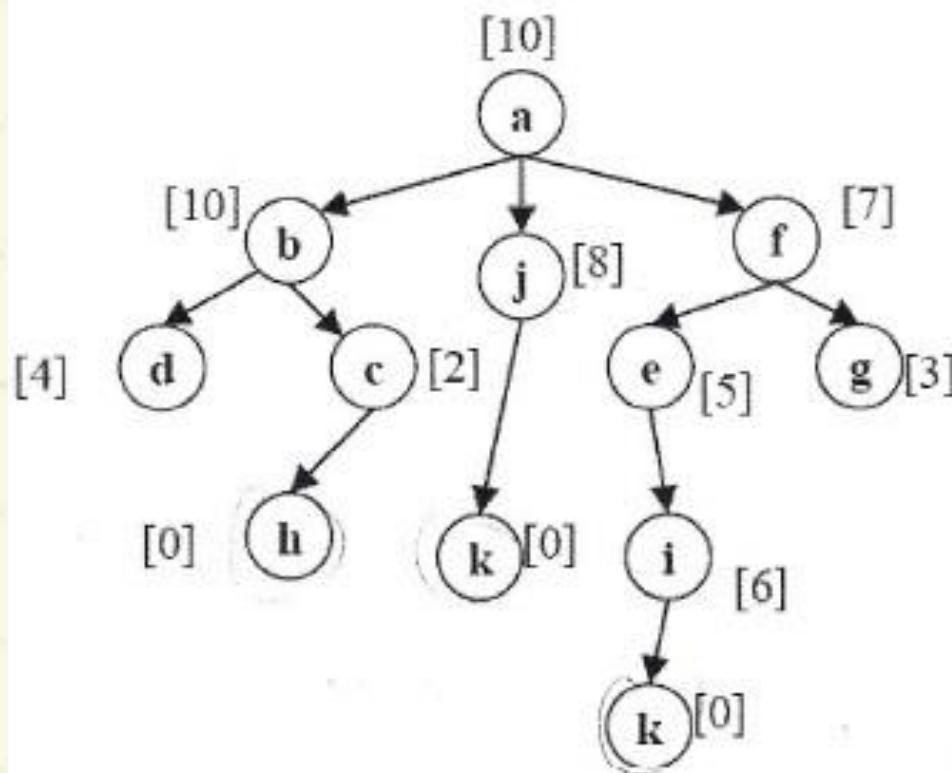
Check if next node  $e_5$  is better than current node

$$\Delta E = \text{value(next)} - \text{value(current)}$$

$$\Delta E = \text{value}(e_5) - \text{value}(f_7)$$

$$\text{value}(e_5) = -\text{heuristic}(e_5) = -5$$

# Simulated Annealing



**value( $f_7$ ) = -heuristic( $f_7$ ) = -7**

| Current | Children           |
|---------|--------------------|
| a       | ---                |
| a       | $f_7, j_8, b_{10}$ |
| f       | $e_5, g_3$         |

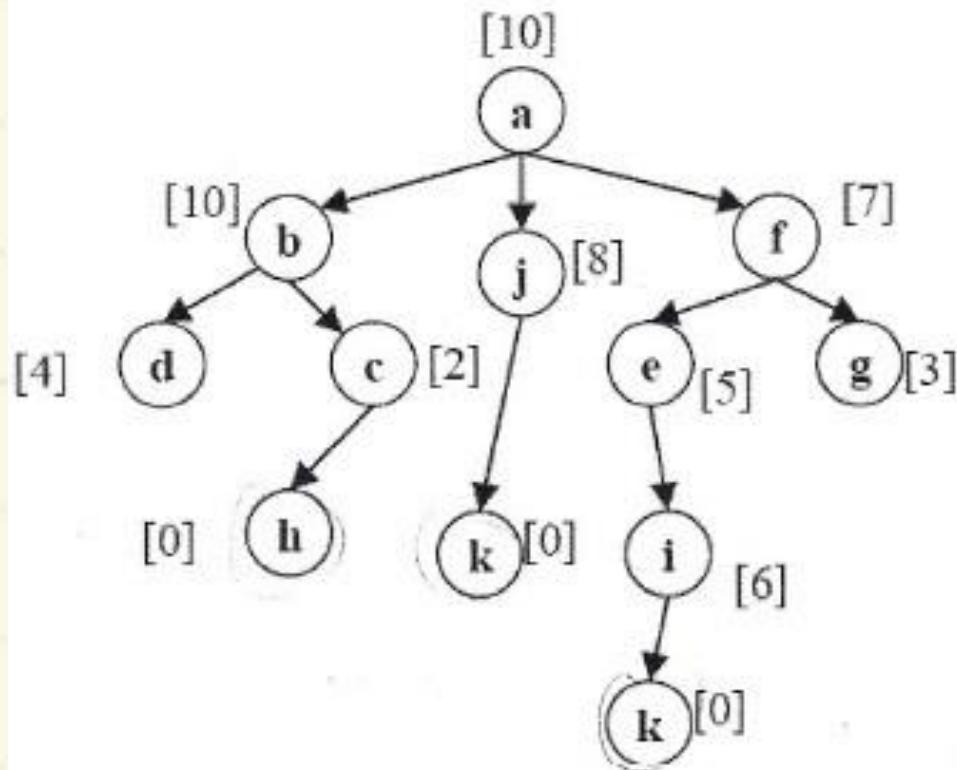
**Check if next node  $e_5$  is better than current node**

$$\Delta E = \text{value(next)} - \text{value(current)}$$

$$\Delta E = \text{value}(e_5) - \text{value}(f_7)$$

$$\text{value}(e_5) = -\text{heuristic}(e_5) = -5$$

# Simulated Annealing



Current

a

a

f

Children

---

$f_7, j_8, b_{10}$

$e_5, g_3$

Check if next node  $e_5$  is better than current node

$$\Delta E = \text{value(next)} - \text{value(current)}$$

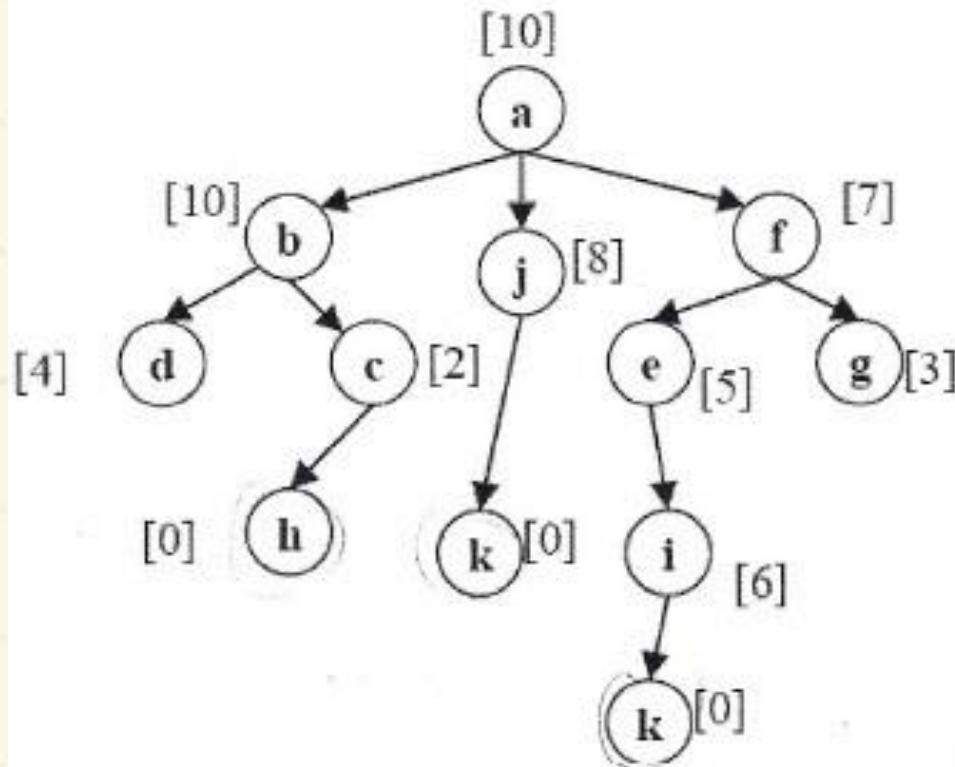
$$\Delta E = \text{value}(e_5) - \text{value}(f_7)$$

$$\Delta E = -5 - (-7) = +2$$

# Simulated Annealing

$$\because \Delta E > 0$$

$\therefore e_5$  will be selected with probability 1



Current

a

a

f

Children

---

$f_7, j_8, b_{10}$

$e_5, g_3$

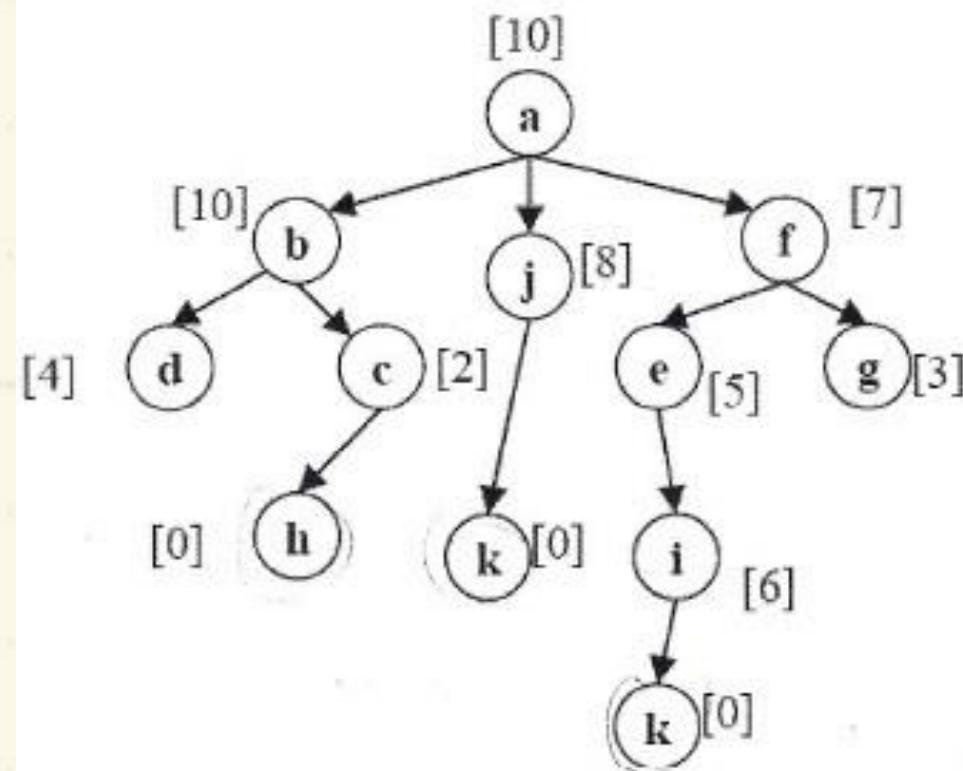
Check if next node  $e_5$  is better than current node

$$\Delta E = \text{value(next)} - \text{value(current)}$$

$$\Delta E = \text{value}(e_5) - \text{value}(f_7)$$

$$\Delta E = -5 - (-7) = +2$$

# Simulated Annealing



Current

a

a

f

e

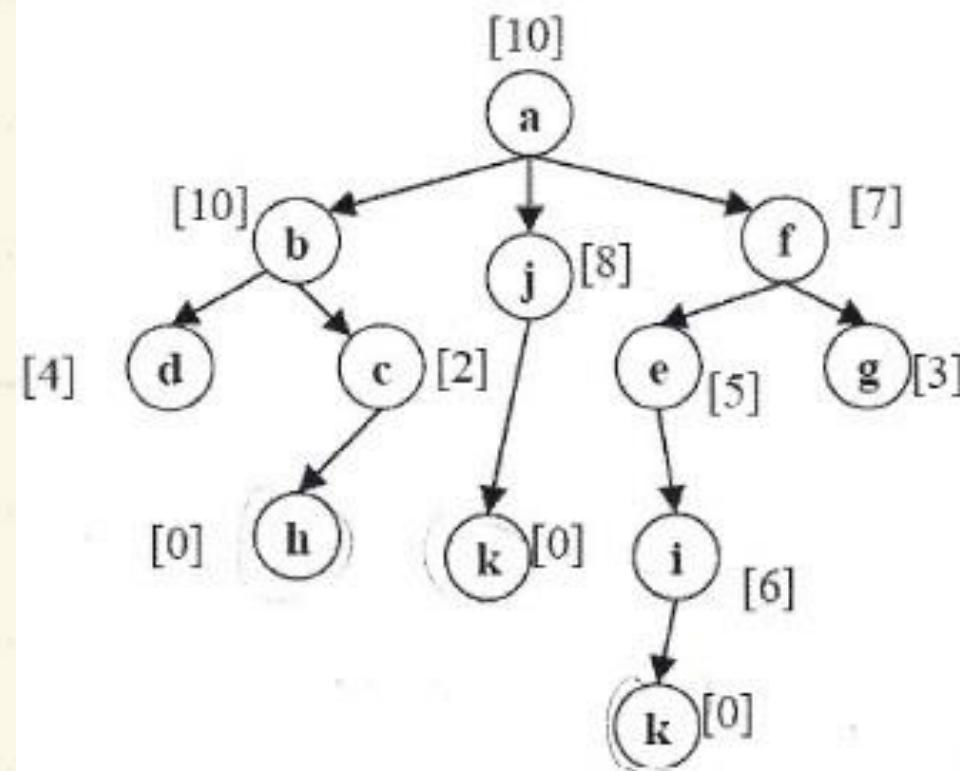
Children

---

$f_7, j_8, b_{10}$

$e_5, g_3$

# Simulated Annealing



Current

a

a

f

e

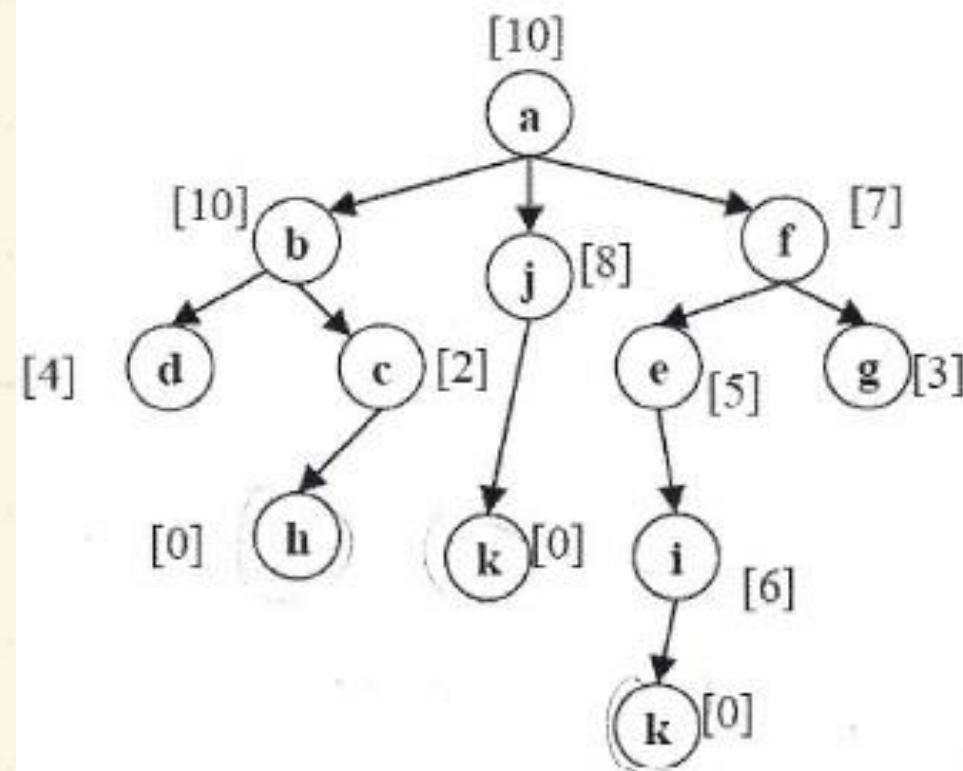
Children

---

$f_7, j_8, b_{10}$

$e_5, g_3$

# Simulated Annealing



Current

a

a

f

e

Children

---

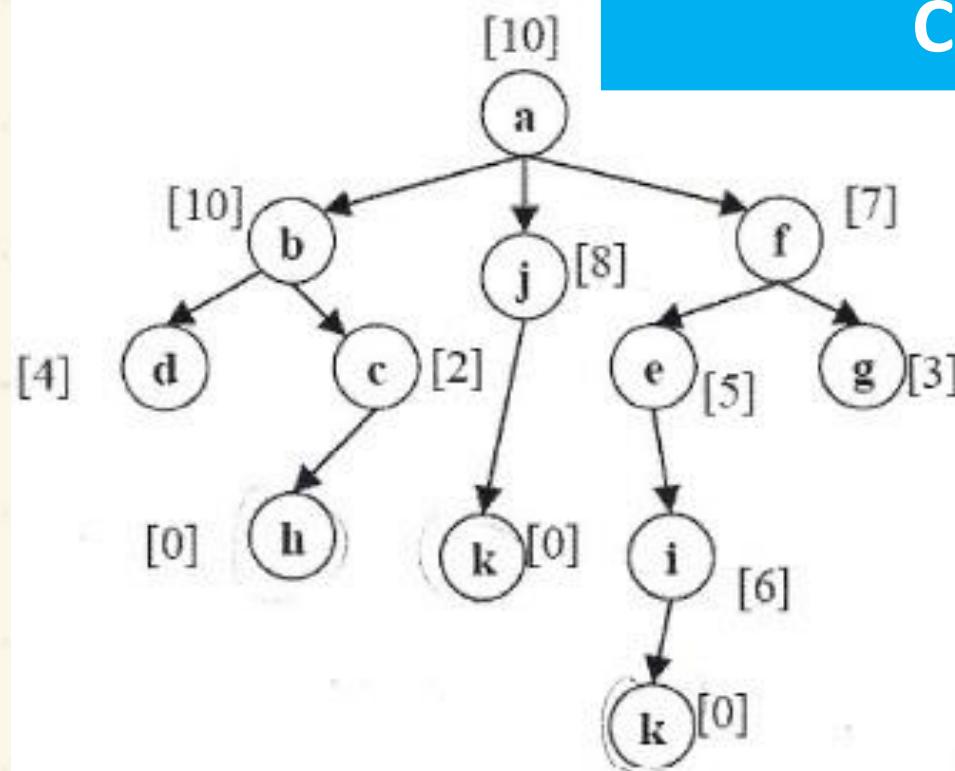
$f_7, j_8, b_{10}$

$e_5, g_3$

$i_6$

# Simulated Annealing

Randomly Select a  
Child



Current

a

a

f

e

Children

---

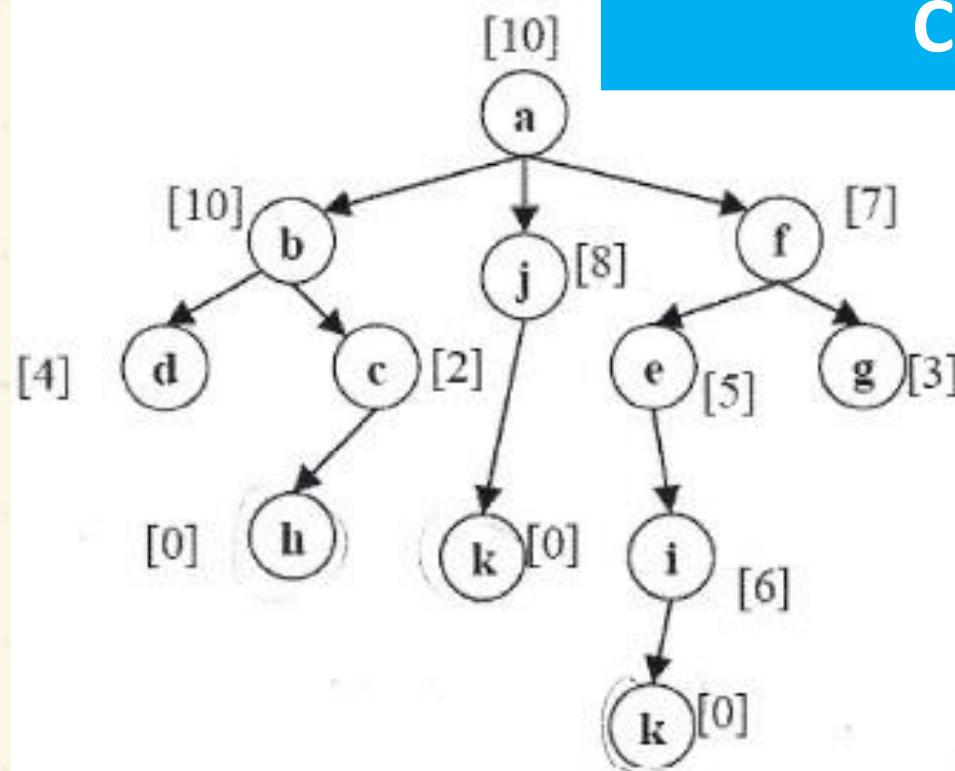
$f_7, j_8, b_{10}$

$e_5, g_3$

$i_6$

# Simulated Annealing

Randomly Select a  
Child



Current

a

a

f

e

Children

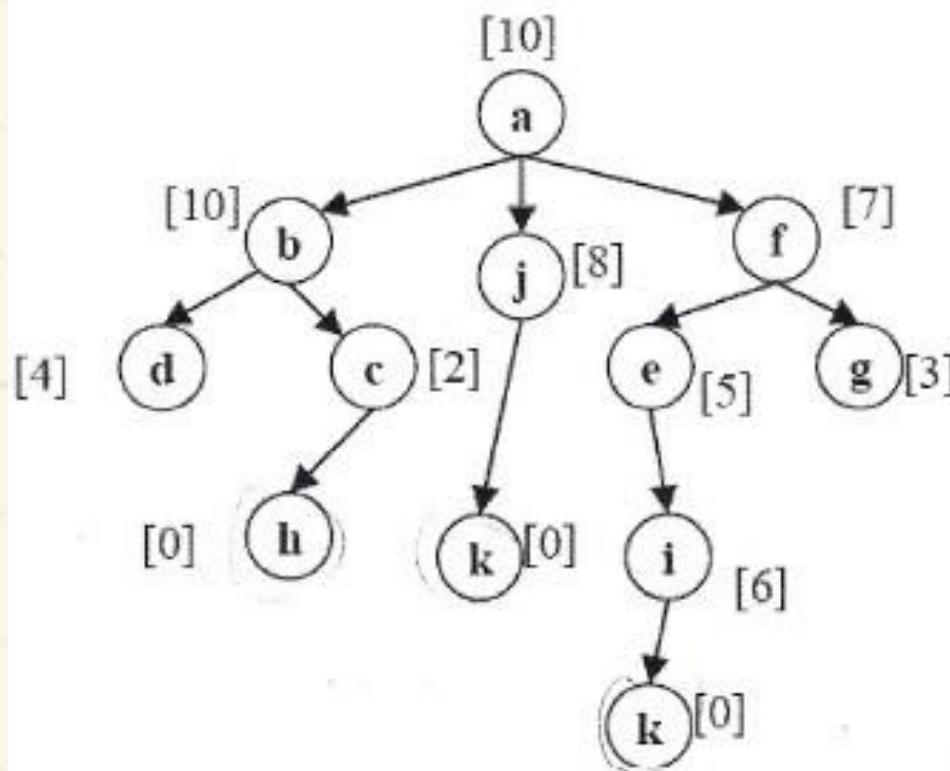
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$f_7, j_8, b_{10}$

$e_5, g_3$

$i_6$

# Simulated Annealing



Current

a

a

f

e

Children

---

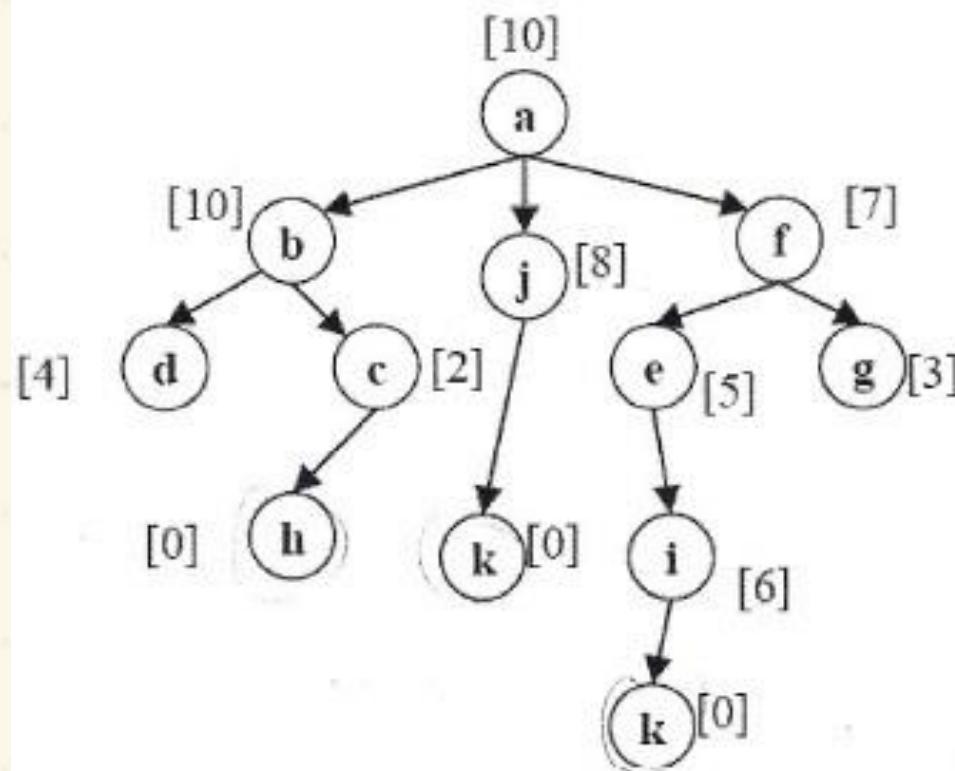
$f_7, j_8, b_{10}$

$e_5, g_3$

$i_6$

Check if next node  $i_6$  is better than current node

# Simulated Annealing



Current

a

a

f

e

Children

---

$f_7, j_8, b_{10}$

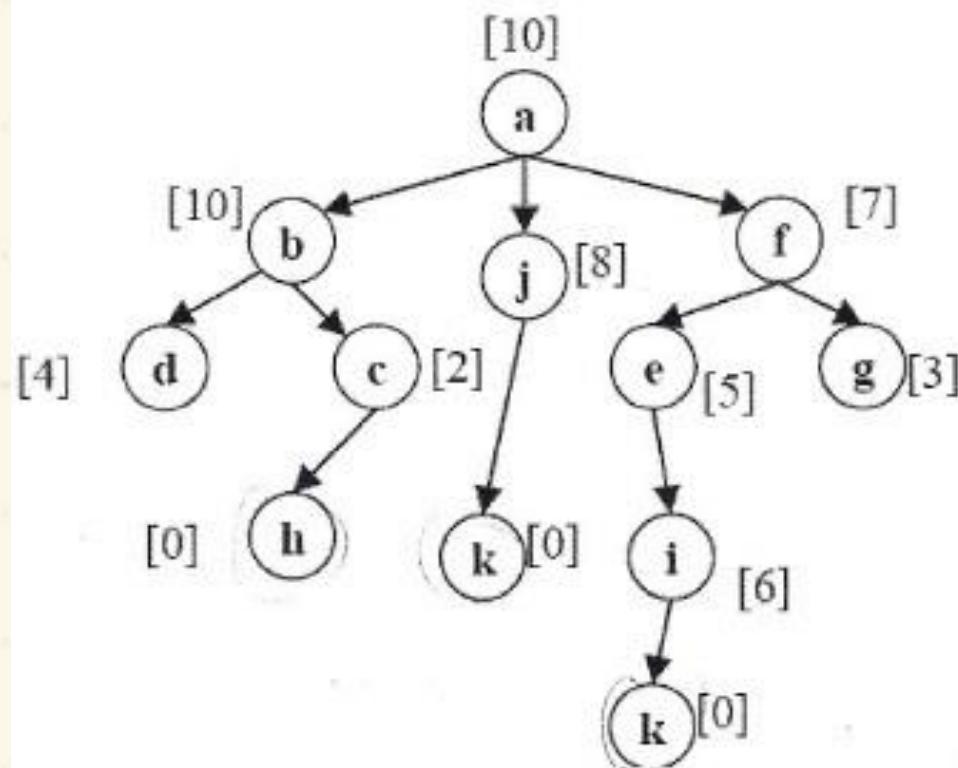
$e_5, g_3$

$i_6$

Check if next node  $i_6$  is better than current node

$\Delta E > 0$

# Simulated Annealing



Current

a

a

f

e

...

$f_7, j_8, b_{10}$

$e_5, g_3$

$i_6$

Check if next node  $i_6$  is better than current node

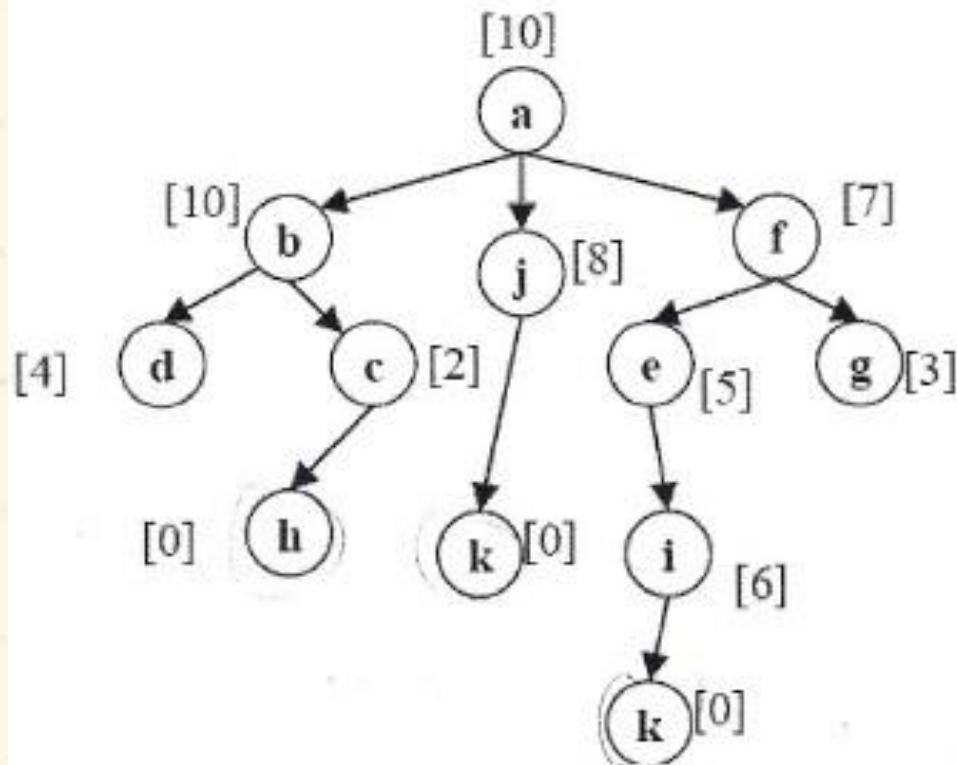
$$\Delta E > 0$$

$$\Delta E = \text{value(next)} - \text{value(current)}$$

$$\Delta E = \text{value}(i_6) - \text{value}(e_5)$$

# Simulated Annealing

$$\text{value}(i_6) = -\text{heuristic}(i_6) = -6$$



Current

a

a

f

e

Children

---

$f_7, j_8, b_{10}$

$e_5, g_3$

$i_6$

Check if next node  $i_6$  is better than current node

$$\Delta E > 0$$

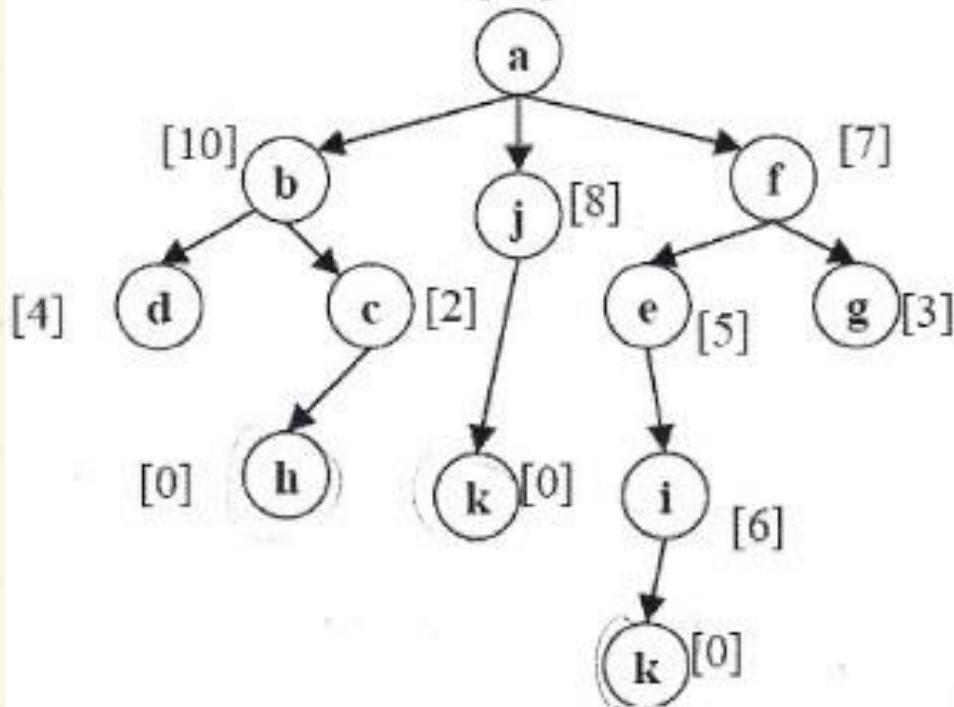
$$\Delta E = \text{value}(\text{next}) - \text{value}(\text{current})$$

$$\Delta E = \text{value}(i_6) - \text{value}(e_5)$$

# Simulated Annealing

$\text{value}(i_6) = -\text{heuristic}(i_6) = -6$

$\text{value}(e_5) = -\text{heuristic}(e_5) = -5$



Current

a

a

f

e

---

$f_7, j_8, b_{10}$

$e_5, g_3$

$i_6$

Check if next node  $i_6$  is better than current node

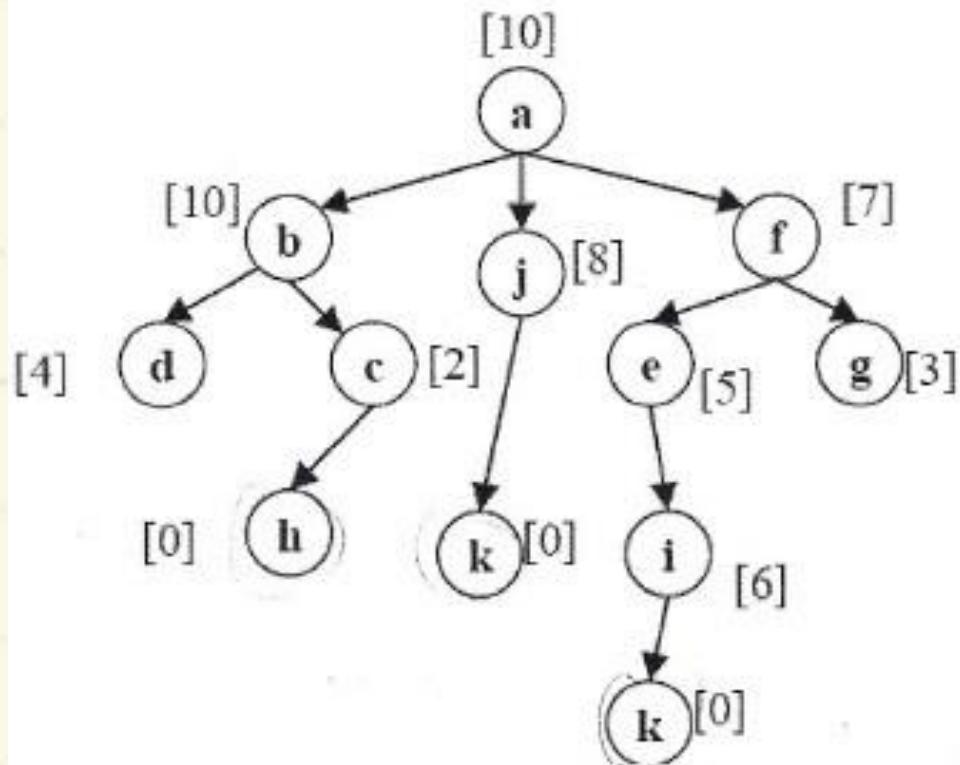
$\Delta E > 0$

$\Delta E = \text{value}(\text{next}) - \text{value}(\text{current})$

$\Delta E = \text{value}(i_6) - \text{value}(e_5)$

# Simulated Annealing

$$\Delta E = -6 - (-5) = -1$$



Current

a

a

f

e

...

$f_7, j_8, b_{10}$

$e_5, g_3$

$i_6$

Check if next node  $i_6$  is better than current node

$\Delta E > 0$

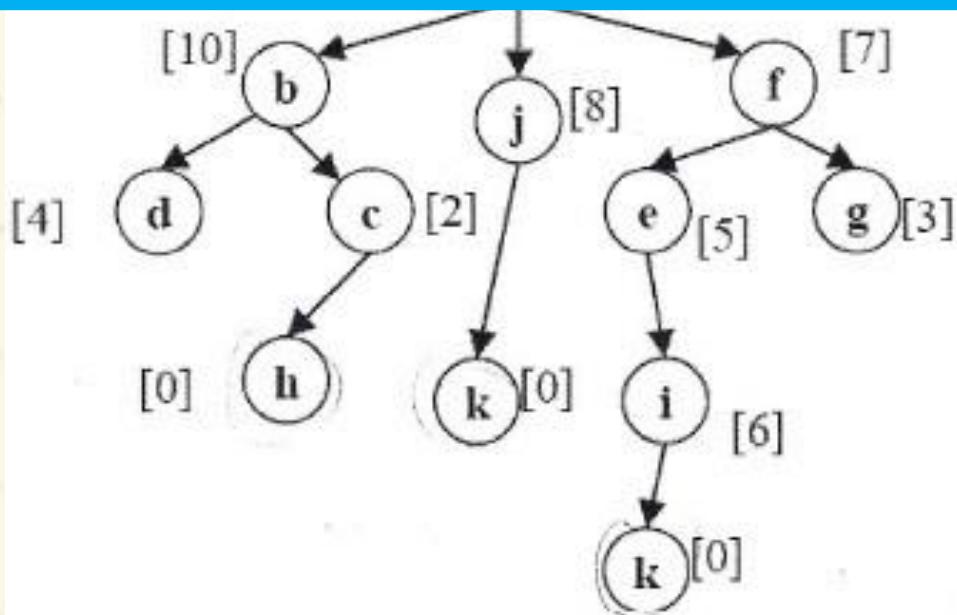
$\Delta E = \text{value}(\text{next}) - \text{value}(\text{current})$

$\Delta E = \text{value}(i_6) - \text{value}(e_5)$

# Simulated Annealing

$$\because \Delta E < 0$$

$\therefore i_6$  can be selected with  
probability  $p = e^{\frac{\Delta E}{T}}$



Current

a

a

f

e

Children

---

$f_7, j_8, b_{10}$

$e_5, g_3$

$i_6$

Check if next node  $e_5$  is better than current node

$$\Delta E = \text{value(next)} - \text{value(current)}$$

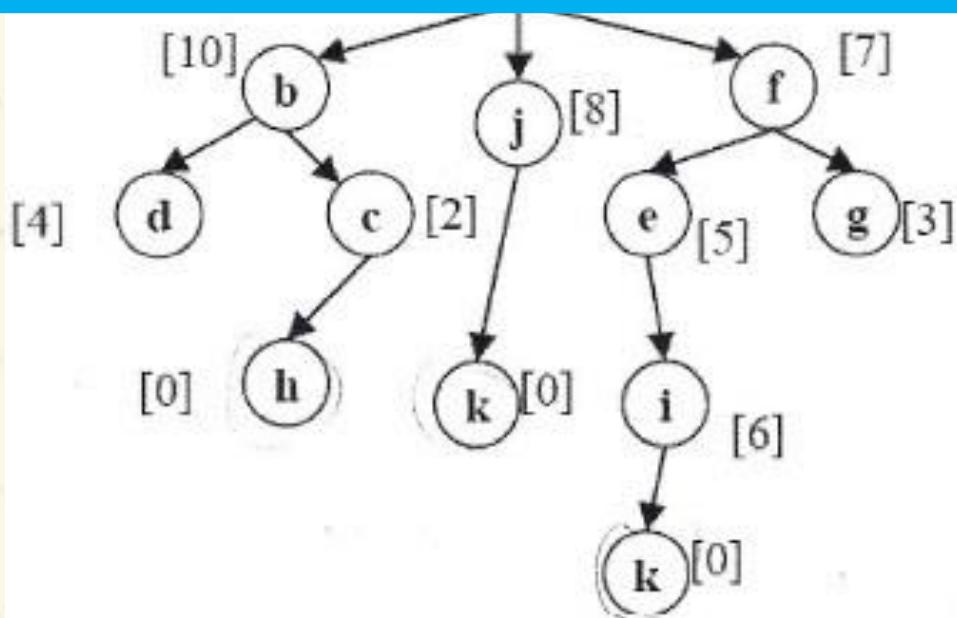
$$\Delta E = \text{value}(e_5) - \text{value}(f_7)$$

$$\Delta E = -5 - (-7) = +2$$

# Simulated Annealing

$$\because \Delta E < 0$$

$\therefore i_6$  can be selected with  
probability  $p = e^{\frac{\Delta E}{T}}$



Current

a

a

f

e

Children

...

$f_7, j_8, b_{10}$

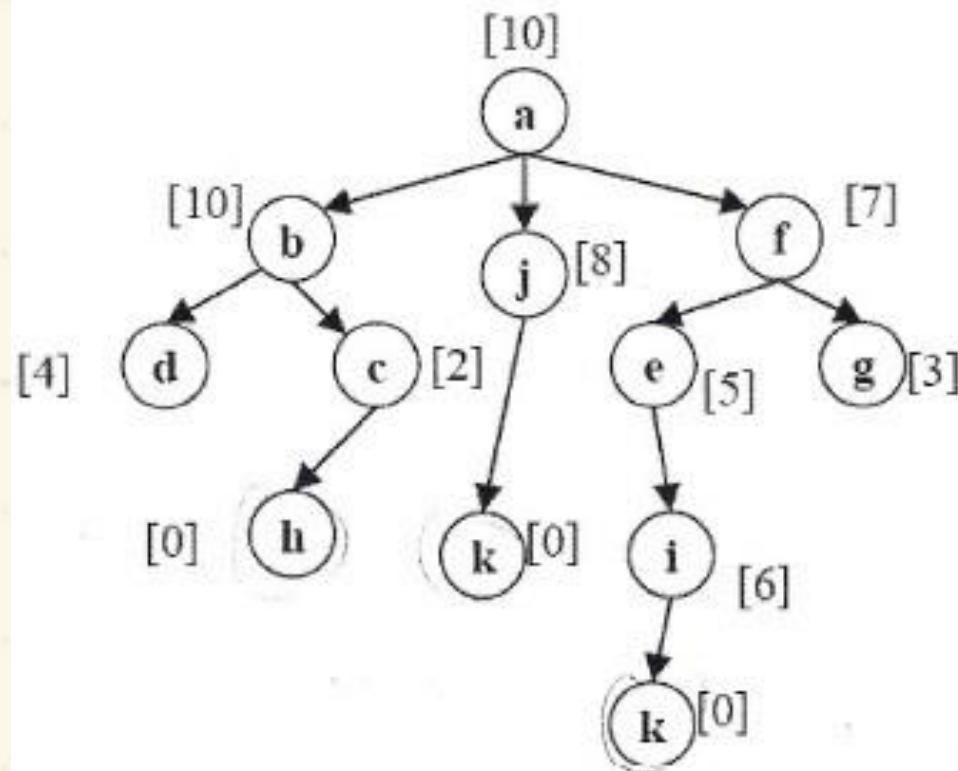
$e_5, g_3$

$i_6$

Check if next node  $e_5$  is better than current node

$$p = e^{\frac{-1}{10}} = e^{\frac{-1}{10}} = .905$$

# Simulated Annealing



Current

a

a

f

e

Children

---

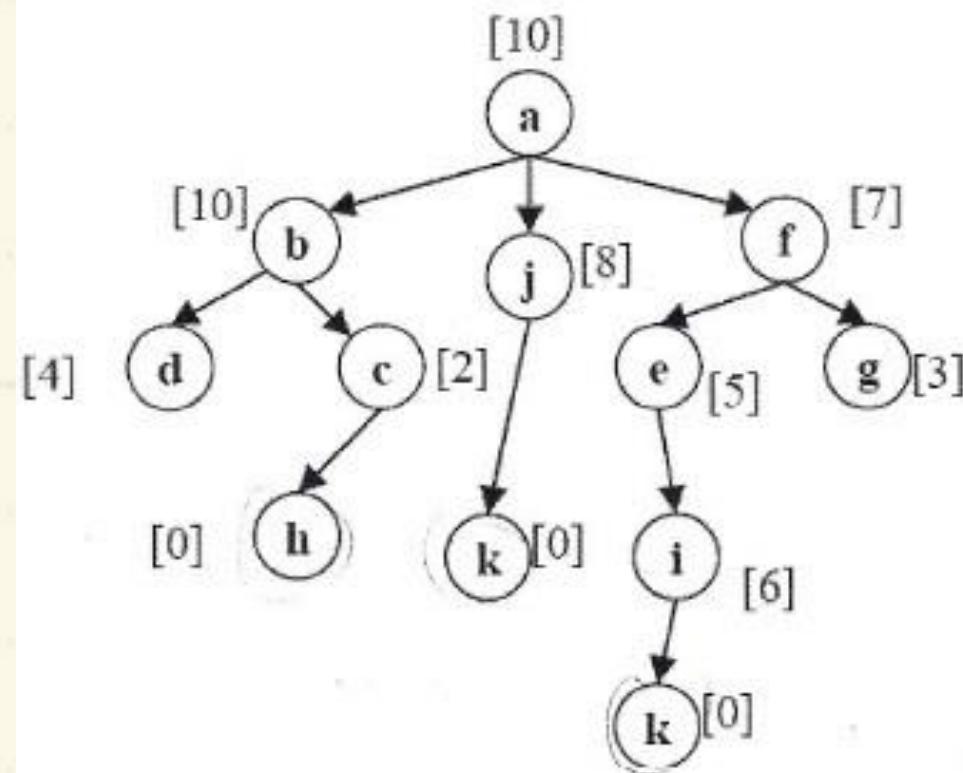
$f_7, j_8, b_{10}$

$e_5, g_3$

$i_6$

Because the only child of  $e_5$  is  $i_6$  then it will be selected even if its probability is not 1.

# Simulated Annealing



Current

a

a

f

e

i

Children

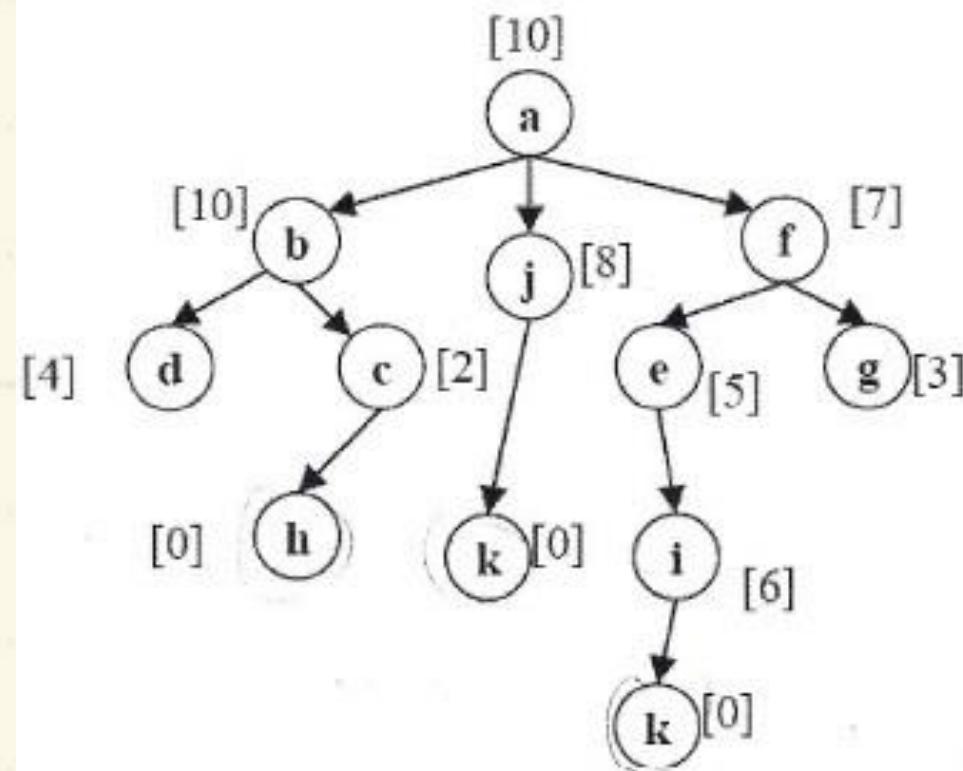
---

$f_7, j_8, b_{10}$

$e_5, g_3$

$i_6$

# Simulated Annealing



Current

a

a

f

e

i

Children

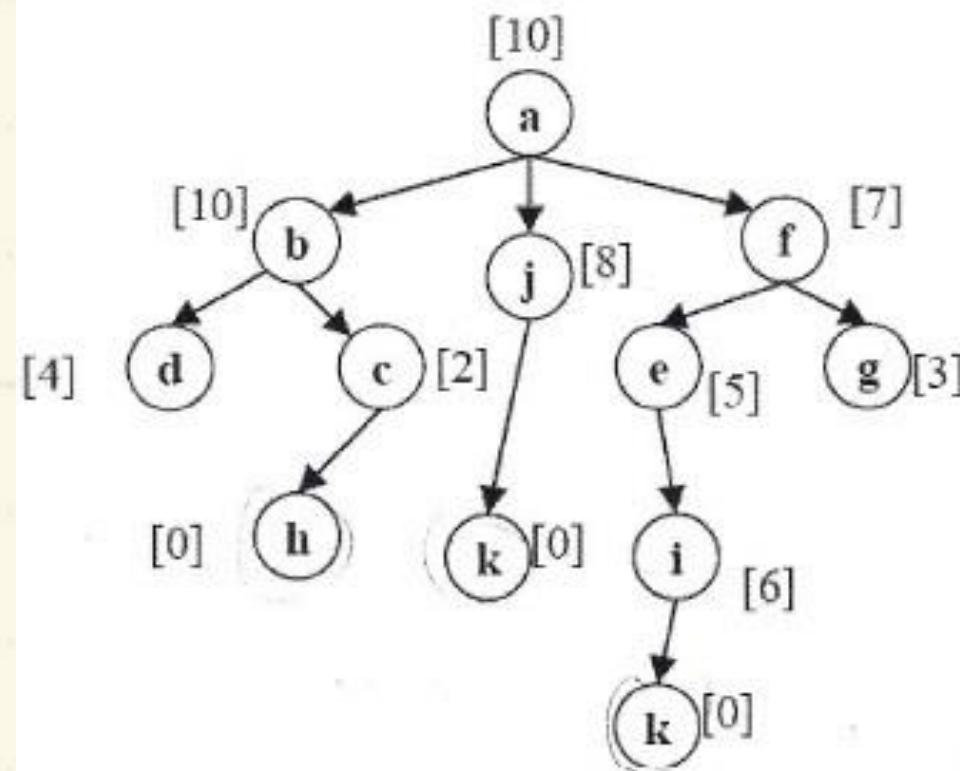
---

$f_7, j_8, b_{10}$

$e_5, g_3$

$i_6$

# Simulated Annealing



Current

a

a

f

e

i

Children

---

$f_7, j_8, b_{10}$

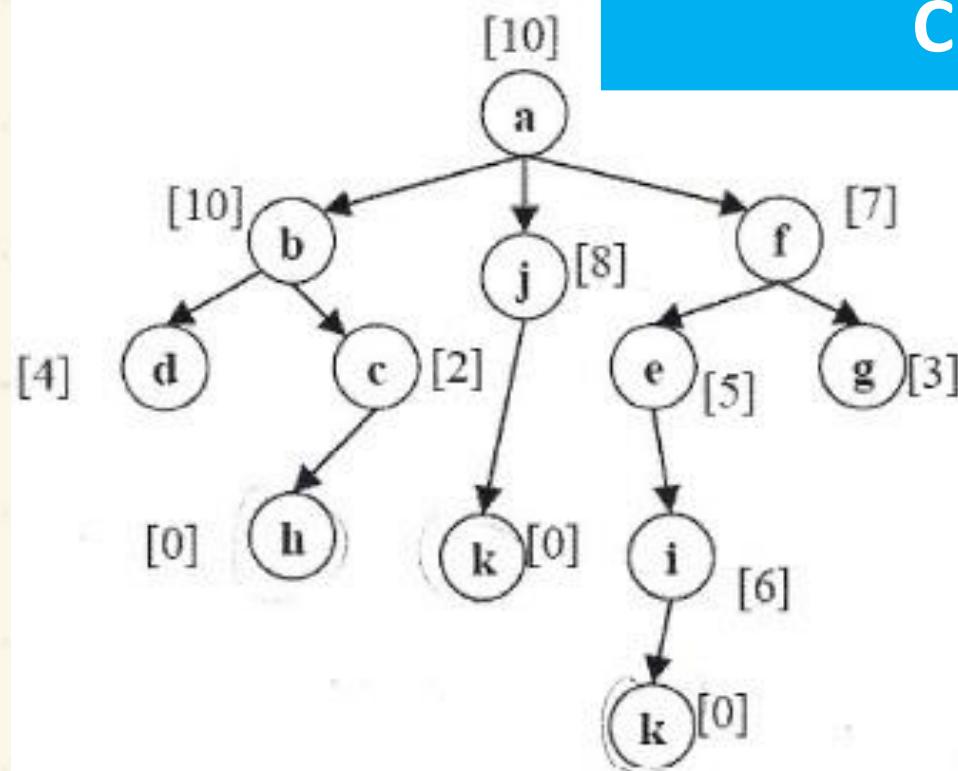
$e_5, g_3$

$i_6$

$k_0$

# Simulated Annealing

Randomly Select a Child



Current

a

a

f

e

i

Children

---

$f_7, j_8, b_{10}$

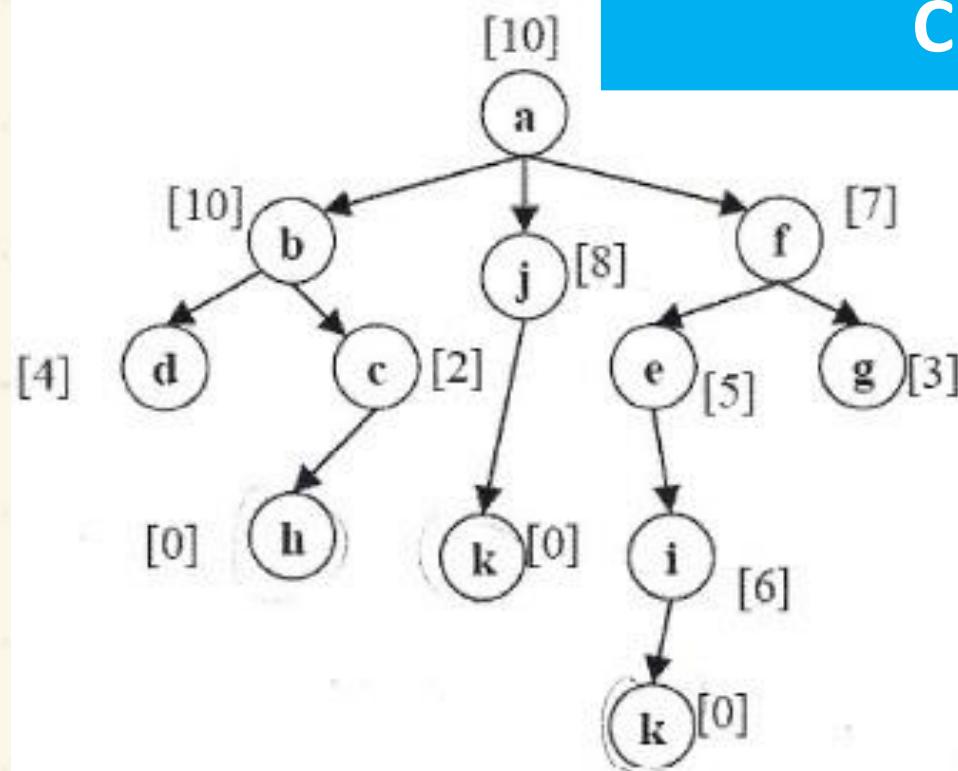
$e_5, g_3$

$i_6$

$k_0$

# Simulated Annealing

Randomly Select a Child



Current

a

a

f

e

i

Children

---

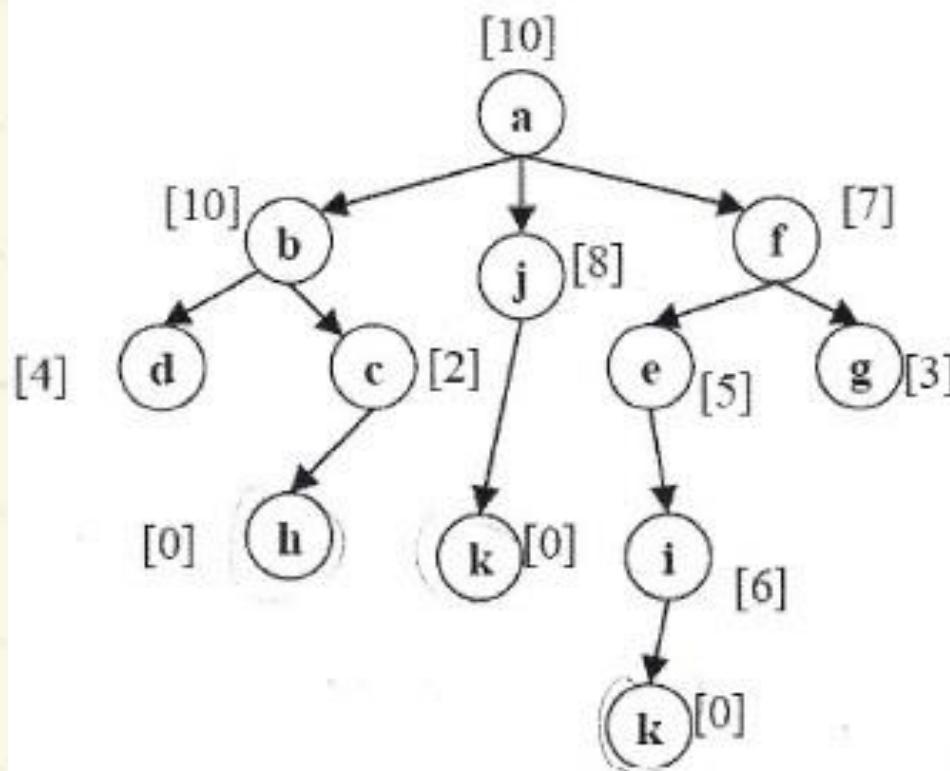
$f_7, j_8, b_{10}$

$e_5, g_3$

$i_6$

$k_0$

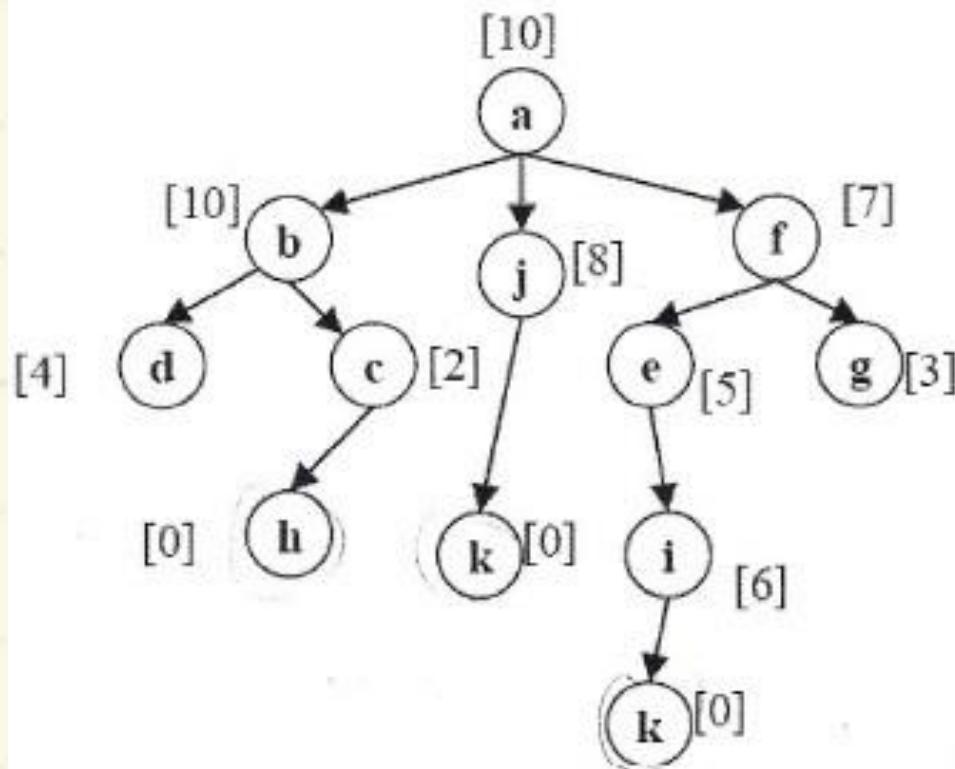
# Simulated Annealing



| Current | Children           |
|---------|--------------------|
| a       | ---                |
| a       | $f_7, j_8, b_{10}$ |
| f       | $e_5, g_3$         |
| e       | $i_6$              |
| i       | $k_0$              |

Check if next node  $k_0$  is better than current node

# Simulated Annealing



Current

a

a

f

e

i

Children

---

$f_7, j_8, b_{10}$

$e_5, g_3$

$i_6$

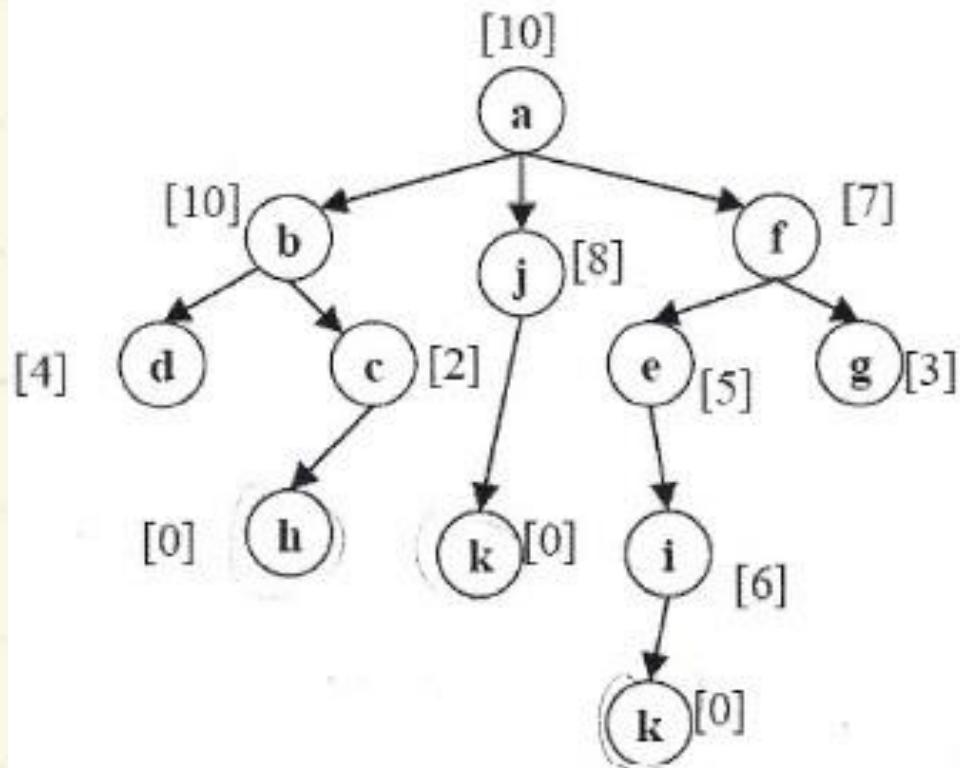
$k_0$

Check if next node  $k_0$  is better than current node

$\Delta E > 0$

# Simulated Annealing

$$\Delta E = \text{value(next)} - \text{value(current)}$$



Current

a

a

f

e

i

Children

---

$f_7, j_8, b_{10}$

$e_5, g_3$

$i_6$

$k_0$

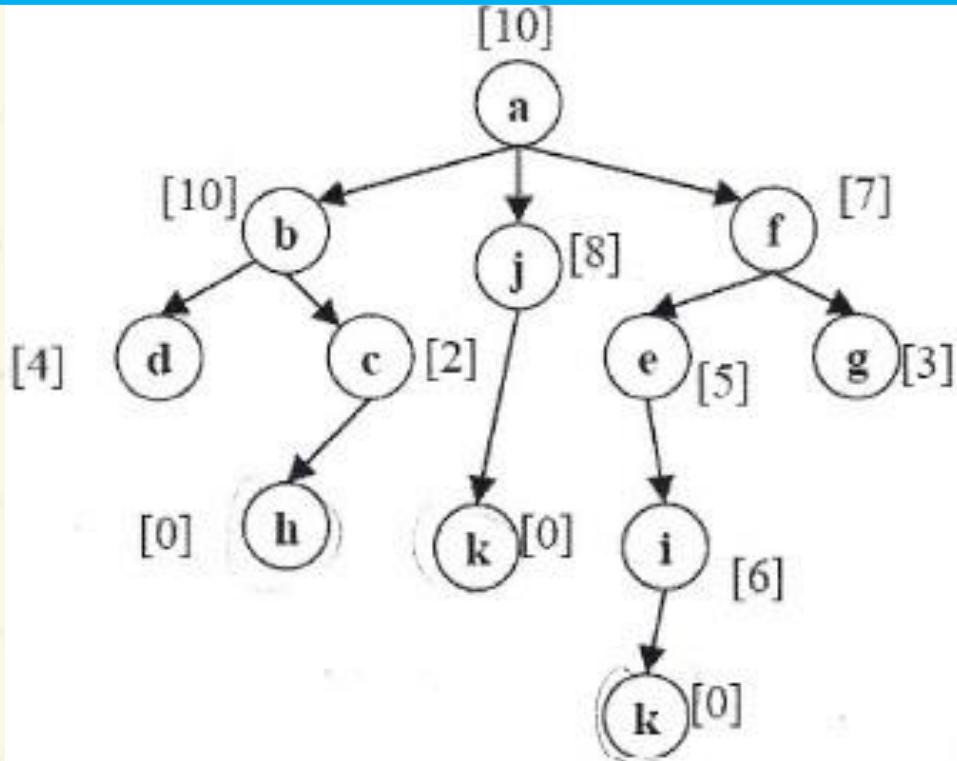
Check if next node  $k_0$  is better than current node

$$\Delta E > 0$$

# Simulated Annealing

$$\Delta E = \text{value(next)} - \text{value(current)}$$

$$\Delta E = \text{value}(k_0) - \text{value}(i_6)$$



Current

a

a

f

e

i

Children

---

$f_7, j_8, b_{10}$

$e_5, g_3$

$i_6$

$k_0$

Check if next node  $k_0$  is better than current node

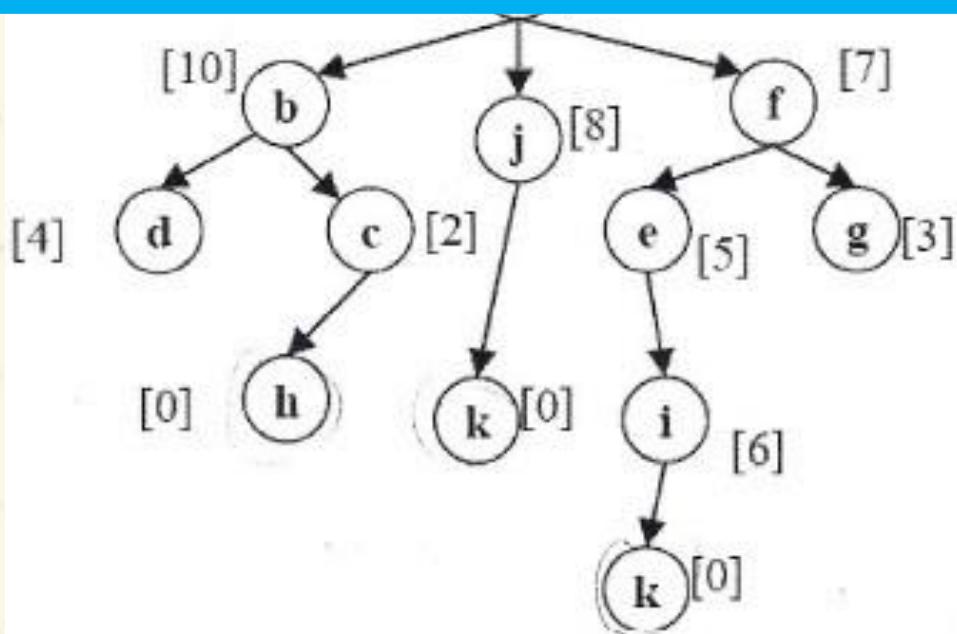
$$\Delta E > 0$$

# Simulated Annealing

$$\Delta E = \text{value(next)} - \text{value(current)}$$

$$\Delta E = \text{value}(k_0) - \text{value}(i_6)$$

$$\text{value}(k_0) = -\text{heuristic}(k_0) = 0$$



Current

a

a

f

e

i

Children

---

$f_7, j_8, b_{10}$

$e_5, g_3$

$i_6$

$k_0$

Check if next node  $k_0$  is better than current node

$$\Delta E > 0$$

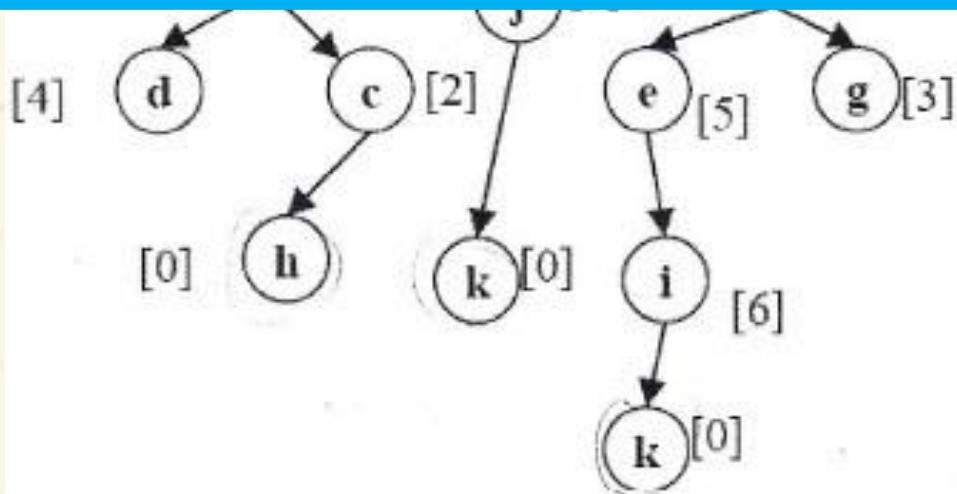
# Simulated Annealing

$$\Delta E = \text{value(next)} - \text{value(current)}$$

$$\Delta E = \text{value}(k_0) - \text{value}(i_6)$$

$$\text{value}(k_0) = -\text{heuristic}(k_0) = 0$$

$$\text{value}(i_6) = -\text{heuristic}(i_6) = -6$$



Current

a

a

f

e

i

Children

---

$f_7, j_8, b_{10}$

$e_5, g_3$

$i_6$

$k_0$

Check if next node  $k_0$  is better than current node

$$\Delta E > 0$$

# Simulated Annealing

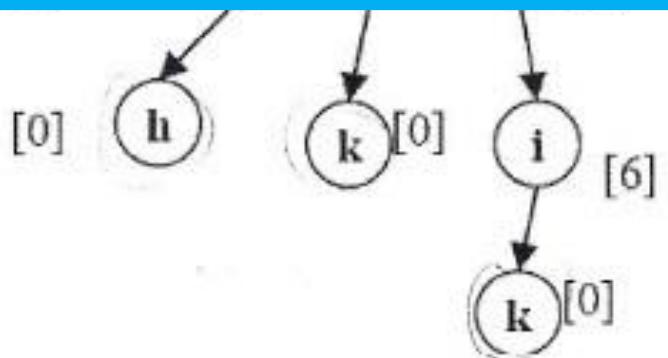
$$\Delta E = \text{value(next)} - \text{value(current)}$$

$$\Delta E = \text{value}(k_0) - \text{value}(i_6)$$

$$\text{value}(k_0) = -\text{heuristic}(k_0) = 0$$

$$\text{value}(i_6) = -\text{heuristic}(i_6) = -6$$

$$\Delta E = 0 - (-6) = +6$$



Current

a

a

f

e

i

Children

---

$f_7, j_8, b_{10}$

$e_5, g_3$

$i_6$

$k_0$

Check if next node  $k_0$  is better than current node

$$\Delta E > 0$$

# Simulated Annealing

$$\Delta E = \text{value(next)} - \text{value(current)}$$

$$\Delta E = \text{value}(k_0) - \text{value}(i_6)$$

$$\text{value}(k_0) = -\text{heuristic}(k_0) = 0$$

$$\text{value}(i_6) = -\text{heuristic}(i_6) = -6$$

$$\Delta E = 0 - (-6) = +6$$

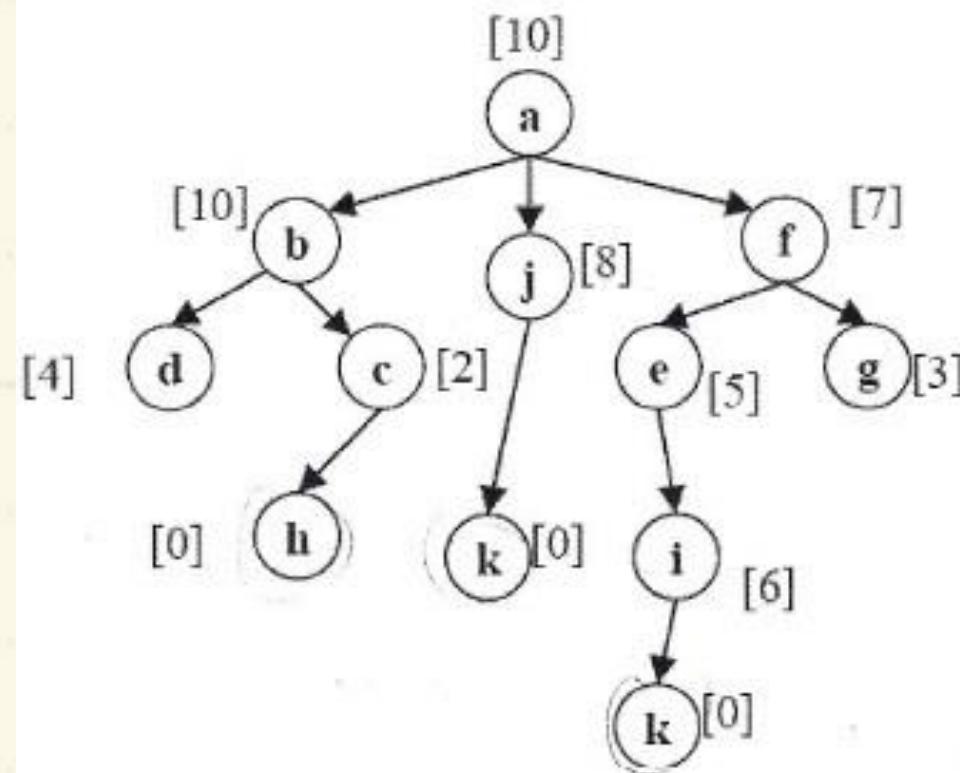
$\therefore \Delta E > 0$   
 $\therefore k_0$  will be selected with probability 1

| Current | Children           |
|---------|--------------------|
| a       | ---                |
| a       | $f_7, j_8, b_{10}$ |
| f       | $e_5, g_3$         |
| e       | $i_6$              |
| i       | $k_0$              |

Check if next node  $k_0$  is better than current node

$$\Delta E > 0$$

# Simulated Annealing



Current

a

a

f

e

i

k

Children

---

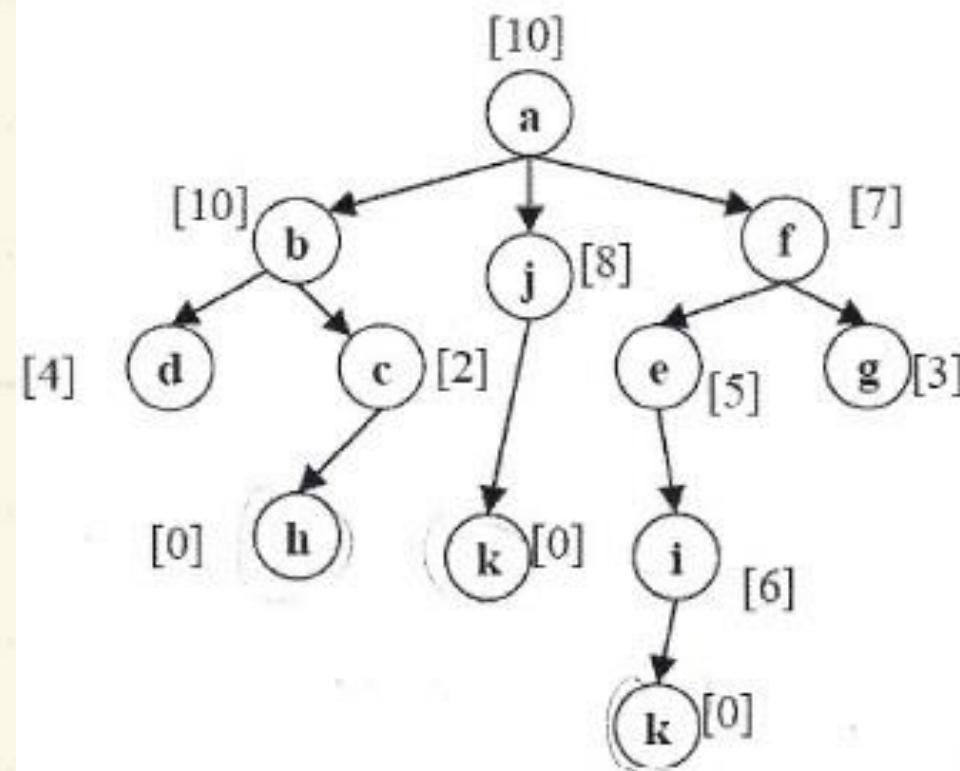
$f_7, j_8, b_{10}$

$e_5, g_3$

$i_6$

$k_0$

# Simulated Annealing



Current

a

a

f

e

i

k

Children

---

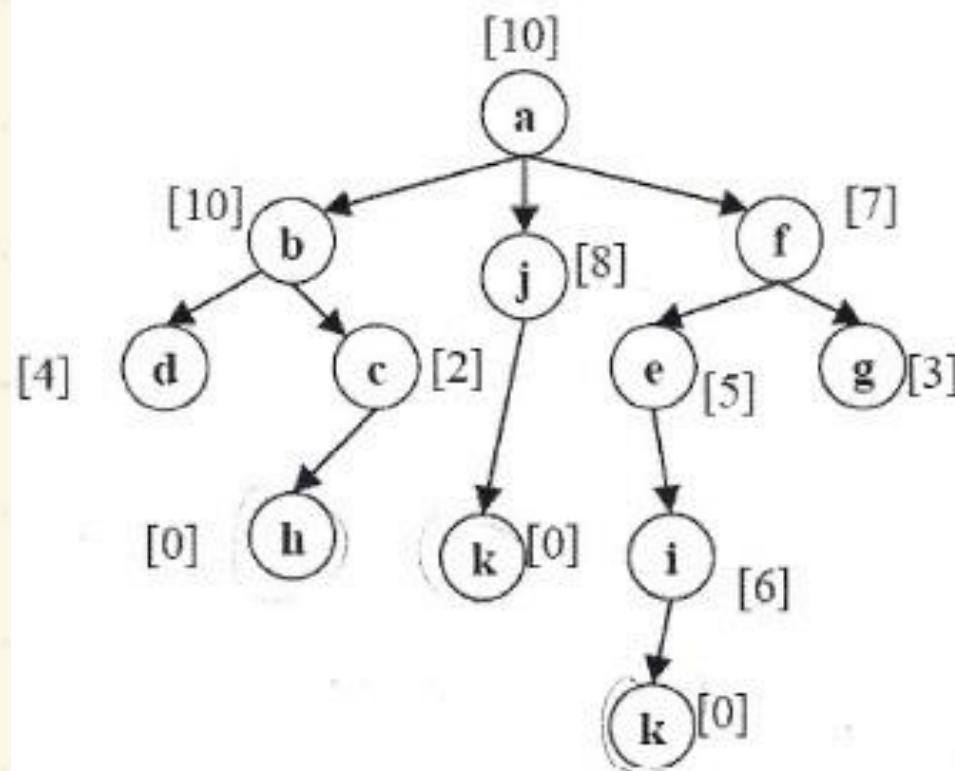
$f_7, j_8, b_{10}$

$e_5, g_3$

$i_6$

$k_0$

# Simulated Annealing



| Current | Children           |
|---------|--------------------|
| a       | ---                |
| a       | $f_7, j_8, b_{10}$ |
| f       | $e_5, g_3$         |
| e       | $i_6$              |
| i       | $k_0$              |
| k       |                    |
| GOAL    |                    |