CS & IT ENGINEERING

COMPUTER NETWORKS

Error Control

Lecture No-6

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TOPICS TO BE COVERED

- Hamming code
- Checksum

1. If the generator has more than one term and coefficient of x° is 1, all single bit error can be detected.
2. If a generator cannot divide $\boxed{x^t + 2}$ (t between 0 and $n - 1$) then all isolated Double error can be detected
3. A generator that contains a Factor of $\boxed{x + 1}$ can detect all odd numbered errors.

① Data = 1011

CRC generator = 1 (Generator is not valid)

② Data = 1011

CRC generator = $x = 1 \cdot x^1 + 0 \cdot x^0 = 10$

Sender

```
10 ) 10110
     10
     00110
       10
       010
        10
        00  — CRC or Remainder
```

Transmitted code word: $\boxed{10110}$ — 1 bit error → Received code word: $\boxed{10010}$

Receiver

```
10 ) 10010
     10
     00010
        10
        00   syndrom = 0
```

No error

Data word Accepted

so It is not able to detect the error

Note: Generator should not contain 'x'

③ data = 1011

CRC generator $= x+1 = 1\cdot x^1 + 1\cdot x^0 = 11$

Sender

```
11 ) 10110 (
     11
     01110
      11
      0010
        11
        01  -> CRC or Remainder
```

Sent code word

(i) $\boxed{10111}$

1 bit error

Received code word

$\boxed{10101}$

Receiver

```
11 ) 10101 (
     11
     01101
      11
      0001  => syndrm ≠ 0 (error)
```

one bit error detected

sent codeword

(ii) $\boxed{10111}$

Received code word

$\boxed{10100}$

2 bit error

Receiver

$$
\begin{array}{r|l}
11 & 10100 \\
 & 11 \\
\hline
 & 01100 \\
 & \ \ 11 \\
\hline
 & \ \ 0000 \\
\hline
\end{array}
$$

⇒ syndrom = 0 (No error)

Dataword Accepted

two bit error not detected

A good polynomial generator needs to have the following characteristics:

1. It should have at least two terms.
2. The coefficient of the term x^0 should be 1.
3. It should not divide $\boxed{x^t + 1}$, for t between 2 and $n - 1$.
4. It should have the factor $x + 1$.

Q

Let G(x) be the generator polynomial used for CRC checking. What is the condition that should be satisfied by G(x) to detect odd number of bits in error?

X **(A)** G(x) contains more than two terms

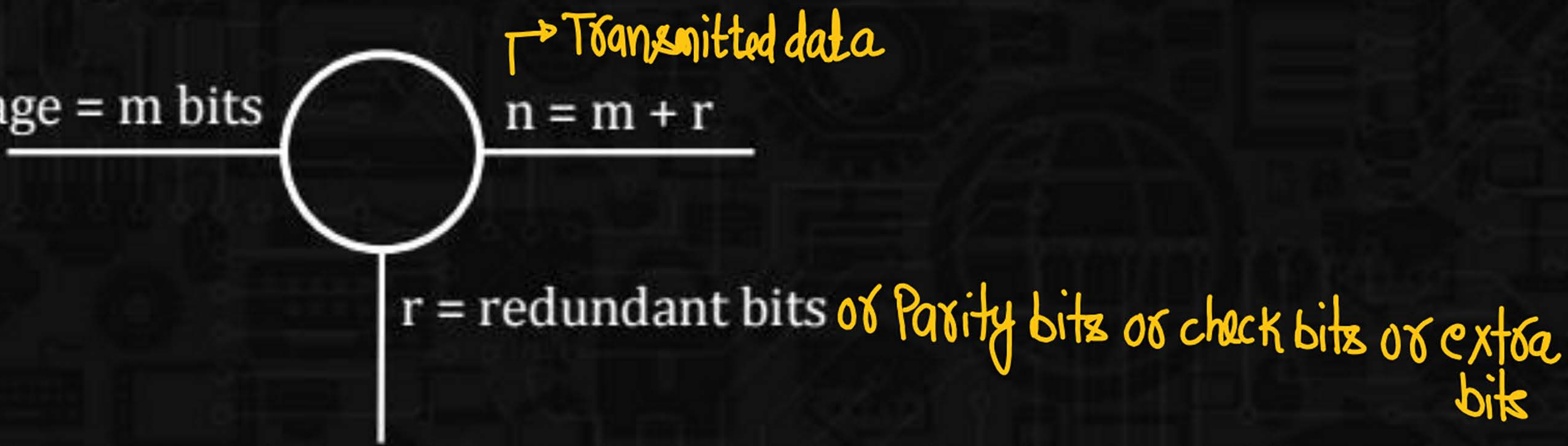
(B) G(x) does not divide 1+x^k, for any k not exceeding the frame length

✓ **(C)** 1+x is a factor of G(x)

(D) G(x) has an odd number of terms.

Hamming Code

- Hamming code can correct 1 bit error only.
- Hamming code can detect upto 2 bit error.
- Hamming code is used for error correction.



According to the hamming code, number of redundant bits

$\boxed{m + r + 1 \le 2^r}$ where r = lower limitation ex:

data or msg $= 1010111$, $m = 7$, $r = 4$, $n = m + r$

$n = 7 + 4 = 11$ bit

Transmitted bits

By using even Parity

$$\boxed{m + r + 1 \le 2^r}$$

$r = 1 \rightarrow 7 + 1 + 1 \le 2^1$, $9 \le 2$ (No)

$r = 2 \rightarrow 7 + 2 + 1 \le 2^2$, $10 \le 4$ (No)

$r = 3 \rightarrow 7 + 3 + 1 \le 2^3$, $11 \le 8$ (No)

✓ $r = 4 \rightarrow 7 + 4 + 1 \le 2^4$, $12 \le 16$ (yes) ✓

$r = 5 \rightarrow 7 + 5 + 1 \le 2^5$, $13 \le 32$ (yes)

$r = 6 \rightarrow 7 + 6 + 1 \le 2^6$, $14 \le 64$ (yes)

⋮

Redundant bit Position $= 2^i$ (when $i \ge 0$)
or
Parity bits Position
or
check bits Position
or
extra bits Position

$2^0, 2^1, 2^2, 2^3, 2^4, \ldots$

$1, 2, 4, 8, 16, \ldots$

P_1	P_2	3	P_4	5	6	7	P_8	9	10	11
1	0	1	1	0	1	0	1	1	1	1
2^0	2^1		2^2				2^3			

$1 = 2^0$
$2 = 2^1$
$3 = 2^1 + 2^0$
$4 = 2^2$
$5 = 2^2 + 2^0$
$6 = 2^2 + 2^1$
$7 = 2^2 + 2^1 + 2^0$
✓ $8 = 2^3$
✓ $9 = 2^3 + 2^0$
✓ $10 = 2^3 + 2^1$
✓ $11 = 2^3 + 2^1 + 2^0$

$\underline{P_1}$

1	3	5	7	9	11
$\boxed{1}$	1	0	0	1	1

$\underline{P_4}$

4	5	6	7
$\boxed{1}$	0	1	0

$\underline{P_2}$

2	3	6	7	10	11
$\boxed{0}$	1	1	0	1	1

$\underline{P_8}$

8	9	10	11
$\boxed{1}$	1	1	1

Transmitted data =

1	2	3	4	5	6	7	8	9	10	11
1	0	1	1	0	1	0	1	1	1	1

IF Receiver Received uncorrupted data

Received data =

1	2	3	4	5	6	7	8	9	10	11
1	0	1	1	0	1	0	1	1	1	1

P_1

1, 3, 5, 7, 9, 11

1 1 0 0 1 1 → even ($P_1 = 0$)

P_2

2, 3, 6, 7, 10 11

0 1 1 0 1 1 → even ($P_2 = 0$)

P_4:

4, 5, 6, 7

1 0 1 0 → even ($P_4 = 0$)

P_8

8, 9, 10, 11

1 1 1 1 → even ($P_8 = 0$)

P_8	P_4	P_2	P_1
0	0	0	0

↓

No error

(i) If Receiver Received corrupted data (1 bit error)

Received data = 1 0 1 1 0 1 0 1 0 1 1 (bit positions 1 2 3 4 5 6 7 8 9 10 11)

1	2	3	4	5	6	7	8	9	10	11
1	0	1	1	0	1	0	1	0	1	1

P_1

1, 3, 5, 7, 9, 11

1 1 0 0 0 1 → odd ($P_1 = 1$)

P_2

2, 3, 6, 7, 10, 11

0 1 1 0 1 1 → even ($P_2 = 0$)

P_4

4, 5, 6, 7

1 0 1 0 → even ($P_4 = 0$)

P_8

8, 9, 10, 11

1 0 1 1 → odd ($P_8 = 1$)

P_8	P_4	P_2	P_1
1	0	0	1

Nonzero means Error

↓

decimal val = 9

↓

9^{th} bit got corrupted

(ii) If Receiver received corrupted data (2 bit error)

Received data = 1 0 1 1 1 1 0 1 0 1 1
(positions 1 2 3 4 5 6 7 8 9 10 11)

P_1

1, 3, 5, 7, 9, 11

1 1 1 0 0 1 → even ($P_1 = 0$)

P_2

2, 3, 6, 7, 10, 11

0 1 1 0 1 1 → even ($P_2 = 0$)

P_4

4, 5, 6, 7

1 1 1 0 → odd ($P_4 = 1$)

P_8

8, 9, 10, 11

1 0 1 1 → odd ($P_8 = 1$)

P_8	P_4	P_2	P_1
1	1	0	0

Non zero means error

2 bit error detected

decimal value = 12

~~12th bit got corrupted~~

It can't correct 2 bit error

9:05 PM

Problem Solving on Hamming Code

NAT

Q.1 If a 7 bit hamming code word received by receiver is 1011011. assume even parity state whether the received code word is correct or not ?if it is incorrect then locate the bit having error.____

P_1	P_2	3	P_4	5	6	7
1	0	1	1	0	1	1

P_1

1, 3, 5, 7

1 1 0 1 → odd (P_1 = 1)

P_2

2 3 6 7

0 1 1 1 → odd (P_2 = 1)

P_4

4, 5, 6, 7

1 0 1 1 → odd (P_4 = 1)

P_4	P_2	P_1
1	1	1

Non zero means error

⇓

decimal value = 7

↓

7th bit got corrupted

Q.2 Assume that a 12-bit Hamming codeword consisting of 8-bit data and 4 check bits is $d_8d_7d_6d_5c_8d_4d_3d_2c_4d_1c_2c_1$, where the data bits and check bits are given in

Gate-२०२१ (२m)

Data bits							
d_8	d_7	d_6	d_5	d_4	d_3	d_2	d_1
1	1	0	x	0	1	0	1

Check bits			
c_8	c_4	c_2	c_1
y	0	1	0

Which one of the following choices gives the correct values of x and y?

A. x is 1 and y is 0

B. x is 1 and y is 1

C. x is 0 and y is 0

D. x is 0 and y is 1

c_1	c_2	3	c_4	5	6	7	c_8	9	10	11	12
0	1	1	0	0	1	0	y	x	0	1	1

c_1

1, 3, 5, 7, 9, 11

0 1 0 0 x 1

x = 0 (For even parity)

c_2

2, 3, 6, 7, 10, 11

1 1 1 0 0 1 → even (even parity)

c_4

4, 5, 6, 7, 12

0 0 1 0 1 → even

c_8

8, 9, 10, 11, 12

y x 0 1 1

0 0 0 1 1

y = 0 (For even parity)

Note

"Odd Parity is Preferable over even Parity"

Q.3 Consider hamming code (Signal bit error detection and correction technique), the minimum parity bits needed for 60 data bits is 7.

$m = 60$

$m + r + 1 \leq 2^r$

$r = 5 \rightarrow 60 + 5 + 1 \leq 2^5$ (No)

$r = 6 \rightarrow 60 + 6 + 1 \leq 2^6$ (No)

✓ $r = 7 \rightarrow 60 + 7 + 1 \leq 2^7$ (Yes)

Q.4 For single bit error correcting hamming code, the code length for 12 data bit is 17

$m = 12$

$$\boxed{m + r + 1 \leq 2^r}$$

$r = 4 \rightarrow 12 + 4 + 1 \leq 2^4$, $17 \leq 16$ (No)

✓ $r = 5 \rightarrow 12 + 5 + 1 \leq 2^5$, $18 \leq 32$ (yes)

Code length $(n) = m + r$
$= 12 + 5 = 17$

Q.5 After encoding using Hamming method, the pattern for $\boxed{1010111}$ is (consider Even Parity)

1 2 3 4 5 6 7 8 9 10 11

A. 10110101111

B. 10111101111

C. 11110101111

D. 10011101111

$m = 7$, $r = 4$, $n = m + r$

$= 7 + 4 = 11$ bit

$$\boxed{m + r + 1 \leq 2^r}$$

$r = 3 \rightarrow 7 + 3 + 1 \leq 2^3$, $11 \leq 8$ (No)

$r = 4 \rightarrow 7 + 4 + 1 \leq 2^4$, $12 \leq 16$ (yes)

P_1	P_2	3	P_4	5	6	7	P_8	9	10	11
		1		0	1	0		1	1	1

$\underline{P_1}$

1, 3, 5, 7, 9, 11

$\underline{P_2}$

2, 3, 6, 7, 10, 11

$\underline{P_4}$

4, 5, 6, 7

$\underline{P_8}$

8, 9, 10, 11

Q.6 Identify valid 7 bit hamming code. (By using even parity)

A. 0110011

B. 1011011

C. Both A & B

D. None of these


THANK YOU
GW
SOLDIERS !