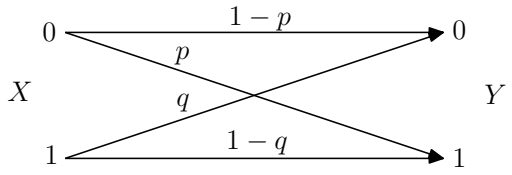
IC252 - Lab 5

Preamble: In practice, we use channels (e.g., mobile communication via a wireless channel) to communicate information. But in the physical world, the channels are usually not "perfect". That is, due to the noise in the channels, a transmitted message may be received as some other message. In this lab session, we shall study a model for such channel and understand marginal, conditional and joint distributions and their interrelation via the model.

Problem statement: The input messages to a binary channel are chosen from the set $\{0, 1\}$ with probability $p(X = 0)$ and $p(X = 1)$. Output of the channel is a stream of messages from the set $\{0, 1\}$ with probability $P(Y = 0)$ and $P(Y = 1)$. In a binary channel, an input message 0 (1) is altered to 1 (0) with probability p (q). Hence,

$$P(Y = 0|X = 0) = 1 - p, \tag{1}$$
$$P(Y = 1|X = 0) = p, \tag{2}$$
$$P(Y = 1|X = 1) = 1 - q, \tag{3}$$
$$P(Y = 0|X = 1) = q. \tag{4}$$

A binary channel is depicted in the following figure.



Let $p = .25, q = .35$. Let the length of the input stream be 10000, in which the messages are drawn according to the uniform distribution, i.e., $P(X = 0) = .5$ and $P(X = 1) = .5$.

1. (a) Find via simulation the distribution of Y and plot it. (b) Verify that the distribution $P(Y = y), y \in \{0, 1\}$ obtained by simulation approximately matches the values analytically given by

$$P(Y = 0) = P(Y = 0|X = 0)P(X = 0) + P(Y = 0|X = 1)P(X = 1),$$
$$P(Y = 1) = P(Y = 1|X = 0)P(X = 0) + P(Y = 1|X = 1)P(X = 1).$$

2. (a) Find via simulation the joint distribution for (X, Y) and plot it. (b) Verify that the conditional distribution $P(Y = y|X = x), x, y \in \{0, 1\}$ obtained by simulation approximately matches the conditional probabilities (1)-(4) via the equality

$$P(Y = y|X = x) = \frac{P(X = x, Y = y)}{P(X = x)}.$$

Hint: To obtain an input sequence of size 10000, you need to use a random number generator which generates 0 with probability .5 and 1 with probability .5. To implement the channel, you again need to use two random number generators as follows. If $X = 0$ then Generator 1 should generate (i.e., output Y) the number 0 with probability .75 and 1 with probability .25. If $X = 1$ then Generator 2 should generate (i.e., output Y) the number 0 with probability .35 and 1 with probability .65.