Input: $V_p \cos \omega t$

Output: $\dfrac{k V_p}{\sqrt{1+\dfrac{\omega^2 k^2}{\omega_u^2}}} \cdot \cos\left[\omega t - \tan^{-1}\left(\dfrac{\omega k}{\omega_u}\right)\right]$

① For low freq:-

$\hookrightarrow \omega << \dfrac{\omega_u}{k}$ } $\boxed{\text{Output:- } k V_p.}$

So we neglected $\dfrac{\omega^2 k^2}{\omega_u^2}$.

$\hookrightarrow k V_p \cos\left(\omega t - \dfrac{\omega k}{\omega_u}\right)$

$= \underbrace{k V_p}_{\substack{\text{Amp} \\ (\text{gain}=k)}} \cos\underbrace{\left[\omega\left(t - \dfrac{k}{\omega_u}\right)\right]}$

$\hookrightarrow$ delay $\Rightarrow \dfrac{k}{\omega_u}$.

② For high freq:-

$\omega >> \dfrac{\omega_u}{k}$ } Output $= \dfrac{k V_p}{\sqrt{\dfrac{k^2 \omega^2}{\omega_u^2}}} \cos\left[\omega t - \tan^{-1}\left(\dfrac{\omega k}{\omega_u}\right)\right]$

neglecting '1' here

Very large

So, $\tan^{-1}\left(\dfrac{\omega k}{\omega_u}\right) \simeq \pi/2$

So, output $= \dfrac{k V_p}{\sqrt{\dfrac{k^2 \omega^2}{\omega_u^2}}} \cdot \cos(\omega t - \pi/2)$

$$\boxed{\text{Output} = V_p \frac{\omega_u}{\omega} \cos(\omega t - \pi/2)}$$

$\rightarrow$ The output is independent of 'k'.

$\rightarrow$ The gain is arbitrary & phase lag of $\frac{\pi}{2}$.

Summary:-

(1) At low frequencies $\left(\omega << \frac{\omega_u}{k}\right)$ → Amp ~~woorking~~ working properly

→ Output $\simeq kV_p \cos\left[\omega\left(t - \frac{k}{\omega_u}\right)\right]$

→ Ideal gain :→ Independent of ω_u, the parameter of the integrator

Delay '$\frac{k}{\omega_u}$'

(2) ~~At~~ At High frequencies $\left(\omega >> \frac{\omega_u}{k}\right)$ → Amp. not working properly.

→ Output $\simeq \frac{\omega_u}{\omega} V_p \cos\left(\omega t - \pi/2\right)$

→ Arbitrary gain :- independent of k, the gain of amp.

Delay '$\frac{\pi}{2}$'

thats why ω_u/k → Bandwidth → Usable range of freq.

Frequency domain analysis

Bandwidth of system

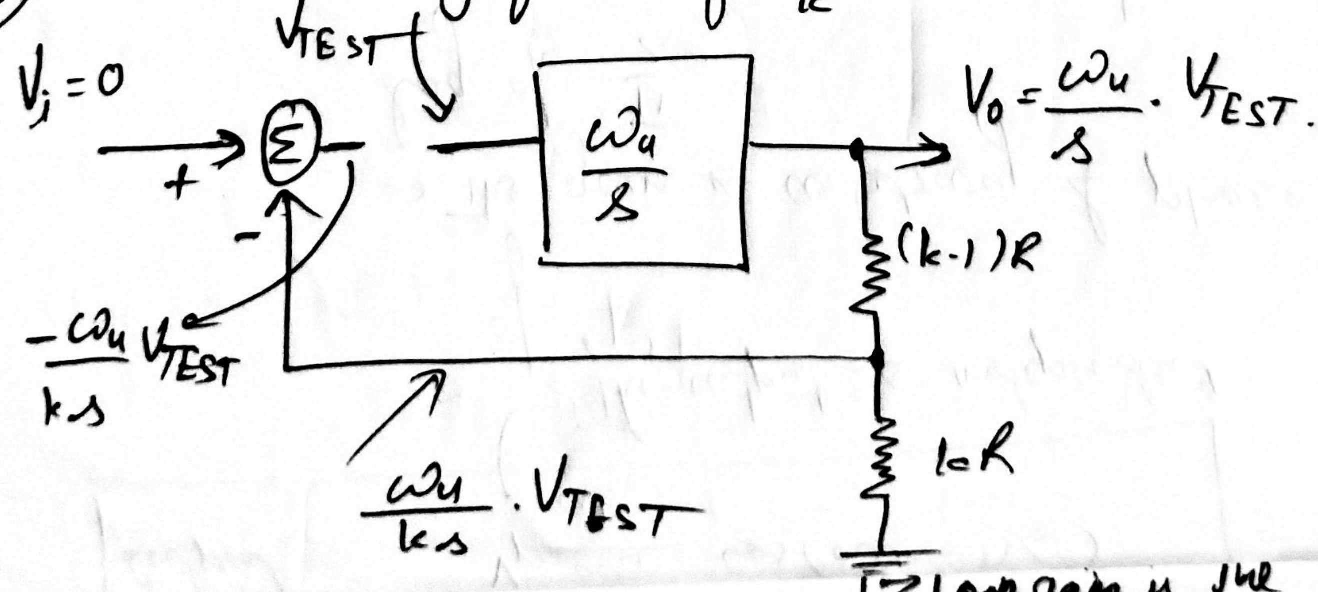
The amp. is usable for frequencies below $\left(\frac{\omega_u}{k}\right)$

For higher bandwidth, use an integrator with a higher ω_u.

Time domain	Frequency domain
Step Response: $V_o(t) = kV_x\left[1-\exp\left(-\frac{\omega_u t}{k}\right)\right]$	Pole: $-\omega_u/k$ → Bandwidth $= \frac{\omega_u}{k}$.

Time constant $= k/\omega_u$

Q) What is the significance of $\frac{\omega_u}{k}$?



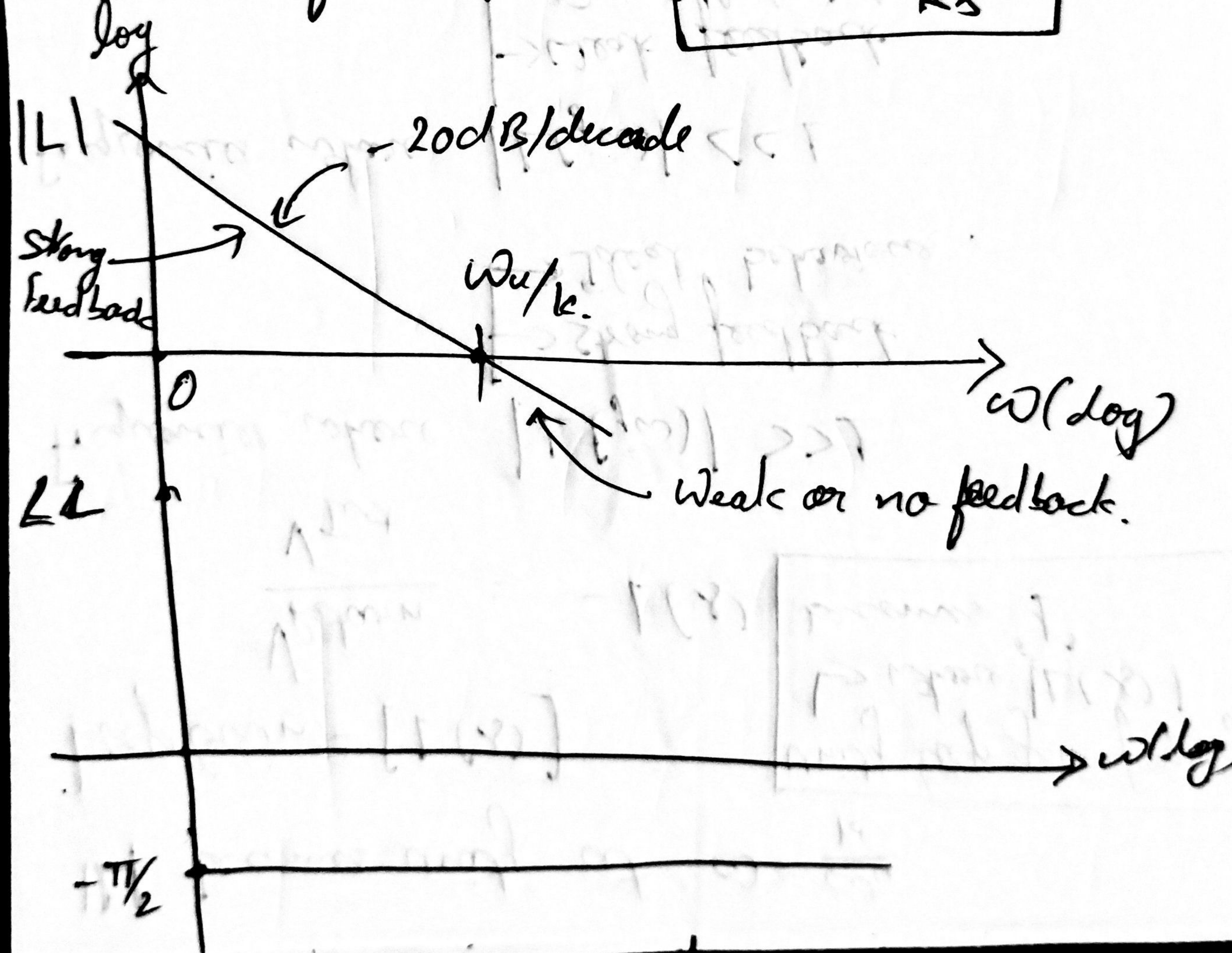
$$V_{Return} = -\frac{\omega_u}{ks} V_{TEST}$$

→ Loop gain is the quantity that quantifies the amount of feedback.

$$\cancel{\text{Loop Gain}} = \frac{V_{Return}}{V_{TEST}} = -\frac{\omega_u}{k s} = -L(s)$$

$$\boxed{L(s) \rightarrow \text{Loop gain}}$$

Now, for our case $\boxed{L(s) = \dfrac{\omega_u}{k s}}$



$|L|$ becomes unity at $\omega = \frac{\omega_u}{k}$

Loop Gain:- $[L(s)]$

$$\frac{V_{Return}}{V_{Test}} = -L(s)$$

$\boxed{\text{Unity loop gain frequency } \hookrightarrow \text{ where } |L(s)| \text{ becomes '1'}}$

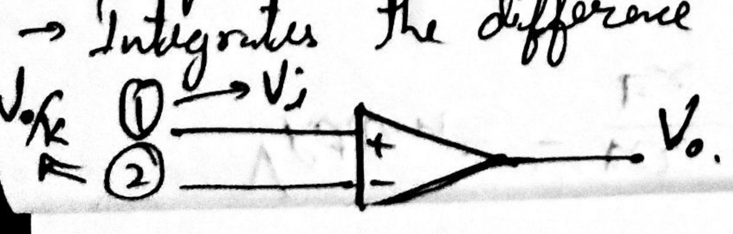
Frequencies where $|L(j\omega)| >> 1$
- → Strong feedback
- → Ideal behaviour.

frequencies where $|L(j\omega)| << 1$
- → Weak feedback
- → Non-Ideal behaviour.

→ Now combining the Σ & $\boxed{\frac{\omega_a}{s}}$ and making one block.

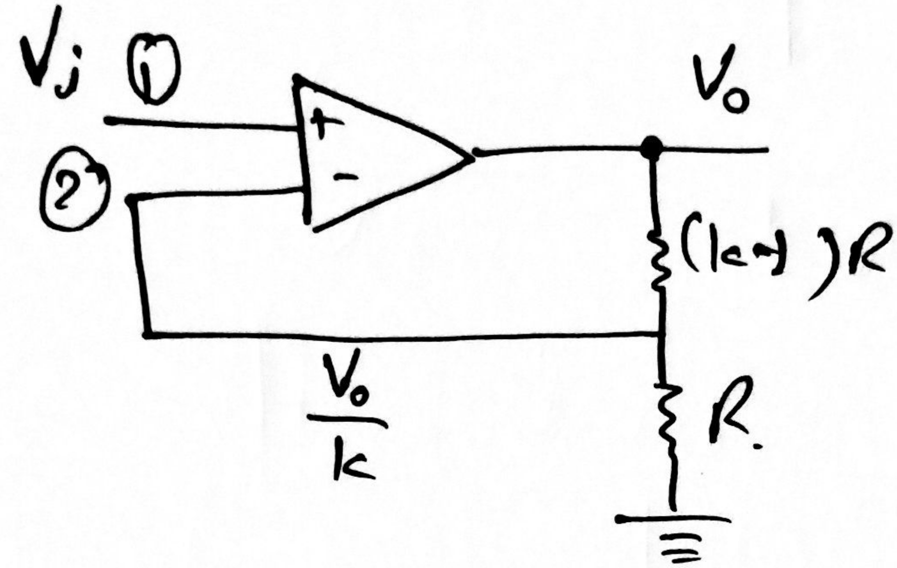
→ Diffrence between input & feedback Σ

→ Integrates the difference



Known as Op-amp.

Now,



A classical non-inverting op-amp.

DC gain > 0

We know that

DC gain = k

→ And we define the op-amp using a single parameter 'ω_u'

→ Unity gain frequency of the op-amp.

$$\boxed{\frac{V_o(s)}{V_i(s)} = \frac{k}{1 + \frac{s k}{\omega_u}}}$$

→ BW

Bandwidth = $\frac{\omega_u}{k}$

Ideal op-amp:-

Now, if $\omega_u = \infty \rightarrow BW = \infty$

So, $\boxed{\frac{V_o(s)}{V_i(s)} = k}$