Problems with negative feedback amp.

↳ Delay

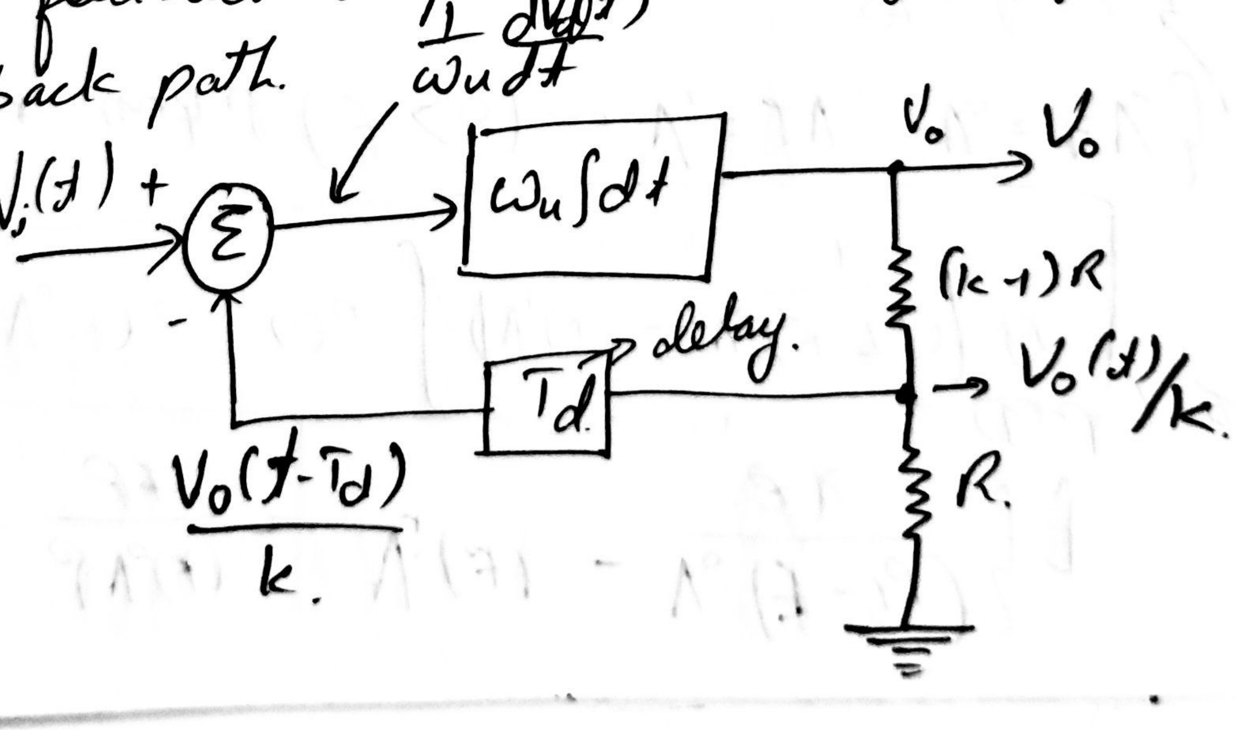
→ Feedback is delayed

→ Comparison is with the actual output some time ago.

→ Don't know if actual output has reached the target → Overshoot [Go past the target]

→ Go below the target [Ringing]

Negative feedback amp. after having delay in feedback path.



So, $\frac{1}{\omega_u}\frac{dV_o(t)}{dt} = V_i(t) - \frac{V_o(t-T_d)}{k}$ (2B)

Check page (4)

$$\boxed{V_o(t) = \omega_u \int \left[V_i(t) - \frac{V_o(t-T_d)}{k}\right] dt}$$

Assume initial $(t<0) \rightarrow V_i = 1V,\ V_o = kV_i$ Steady state.

At $t = 0 \rightarrow V_i = 0$

$$\boxed{\frac{1}{\omega_u}\frac{dV_o(t)}{dt} = -\frac{V_o(t-T_d)}{k}}$$

Without delay $\rightarrow \frac{1}{\omega_u}\frac{dV_o(t)}{dt} = -\frac{V_o(t)}{k}$

Scaling factor $-\frac{\omega_u}{k}$

$$\frac{dV_o(t)}{dt} = -\left(\frac{\omega_u}{k}\right) V_o(t)$$

$$\frac{dV_o(t)}{dt} = V_o(t) \rightarrow \text{(K)}$$

$$\boxed{\frac{dV_o(t)}{dt} = -\left(\frac{\omega_u}{k}\right) \cdot V_o(t-T_d)} \text{ (T)}$$

Without the scaling factor: $\frac{dV_o(t)}{dt} = V_o(t-T_d) \rightarrow$ (A)

(29)

→ Eqⁿ (A) & (B) is saying that derivative of the function is same as the delayed function.

Assum an exponential form: $V_o(t) = V_p \exp(\sigma t)$

By Eqⁿ (J)

$$\frac{1}{\omega_u} \cdot \cancel{V_p}\, \sigma \exp(\sigma t) = -\frac{\cancel{V_p} \exp[\sigma(t - T_d)]}{k}$$

$$\frac{1}{\omega_u} \sigma \cancel{\exp(\sigma t)} = -\frac{\cancel{\exp(\sigma t)} \cdot \exp(-\sigma T_d)}{k}$$

$$\boxed{\frac{\sigma}{\omega_u} = -\frac{\exp(-\sigma T_d)}{k}}$$

$$\exp(-\sigma T_d) = \frac{-k\sigma}{\omega_u}$$

$$-\sigma T_d = \ln\left(\frac{-k\sigma}{\omega_u}\right)$$

$$T_d = -\frac{1}{\sigma} \ln\left(\frac{-k\sigma}{\omega_u}\right)$$

$$\frac{\sigma}{\omega_u} k + \exp(-\sigma T_d) = 0$$

$$\sigma\left(\frac{k}{\omega_u}\right) + \exp\left[\sigma\left(\frac{k}{\omega_u}\right)\cdot T_d \cdot \left(\frac{\omega_u}{k}\right)\right] = 0$$ (30)

Now, $\sigma\left(\frac{k}{\omega_u}\right) = \sigma'$

$\& \ \tau = T_d \cdot \left(\frac{\omega_u}{k}\right) = \frac{T_d}{(k/\omega_u)}$

$$\boxed{\sigma' + \exp(\sigma' \cdot \tau) = 0.}$$

Now plotting the functions

$$f(\sigma') = \sigma' + \exp(\sigma' \cdot \tau) = 0$$

Without delay:- $\tau = 0 \Rightarrow \cancel{f(\sigma') = \sigma'}$ $\boxed{\sigma' = -1}$

$\sigma' = 0 \rightarrow f(0) = 0$ $\boxed{\sigma = -\frac{\omega_u}{k}.}$

$\tau \to -ve \to \frac{1}{e^{'\tau}}$

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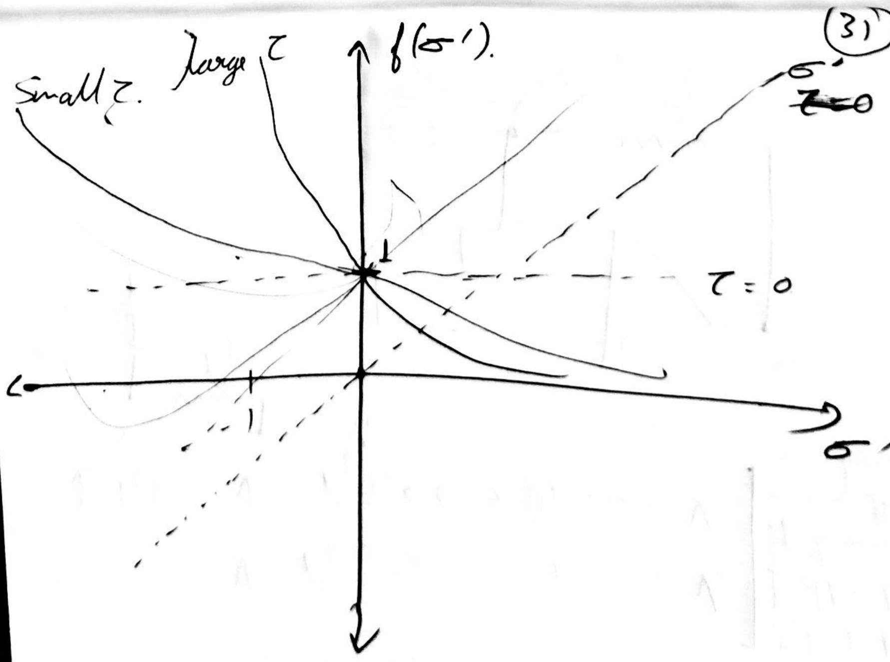


Delays

$\rightarrow$ Solution. (32)

* Negative feedback with delay : $V_p \exp(\sigma t)$

$$\sigma' + \exp(\sigma' \tau) = 0. \quad \Bigg| \quad \sigma' = \frac{\sigma}{\omega_u/k} \qquad \tau = \frac{T_d}{k/\omega_u}.$$

$\rightarrow$ Now has 2 solutions for small τ

$\sigma_1' \,\&\, \sigma_2' \Rightarrow \sigma_1 \,\&\, \sigma_2$

$\rightarrow$ It has no solutions for large τ.

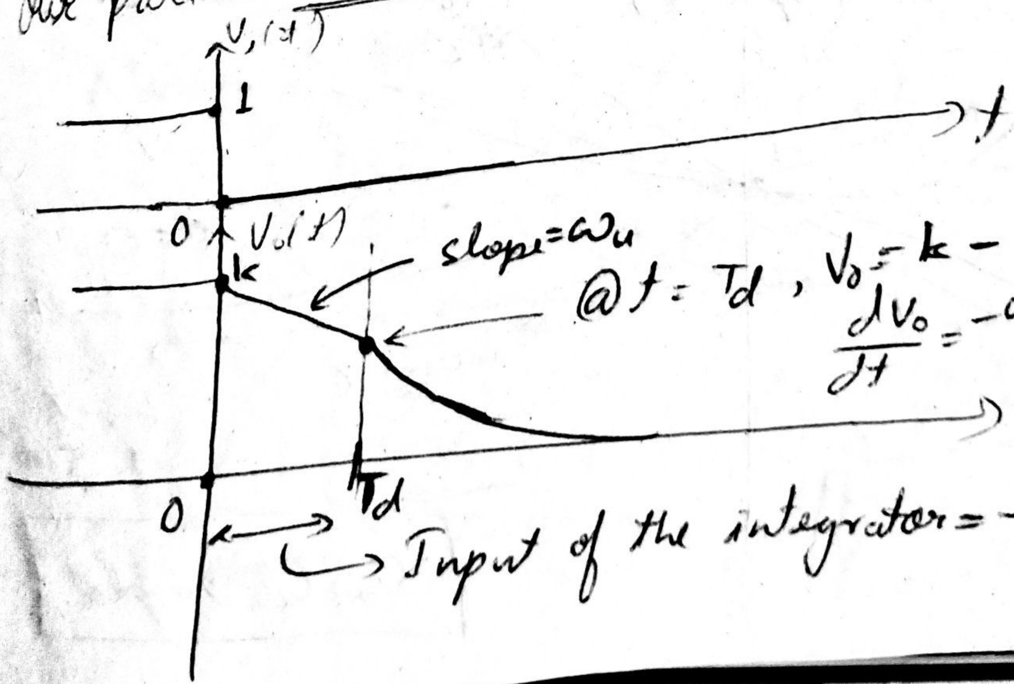
Since $\sigma_1 \,\&\, \sigma_2$ are the 2 solutions

So, $\exp(\sigma_1 t)$, $\exp(\sigma_2 t)$ are solutions

to $\dfrac{1}{\omega_u}\dfrac{dV_0}{dt} = -\dfrac{V_0(t - T_d)}{k}$

Complete solution : $A_1 \exp(\sigma_1 t) + A_2 \exp(\sigma_2 t) = V_0(t)$

Our particular case



@ $t = T_d$, $V_0 = k - \omega_u T_d$

$\dfrac{dV_0}{dt} = -\omega_u$.

$\hookrightarrow$ Input of the integrator $= -1V$

Solution - $V_0(t) = A_1 \exp(\sigma_1 t) + A_2 \exp(\sigma_2 t)$ (33)

$$V_0(T_d) = k - \omega_u T_d = k\left[1 - \frac{T_d}{k/\omega_u}\right]$$

$$= \cancel{k\left[1 - \frac{T_d}{\omega_u/k}\right]}$$

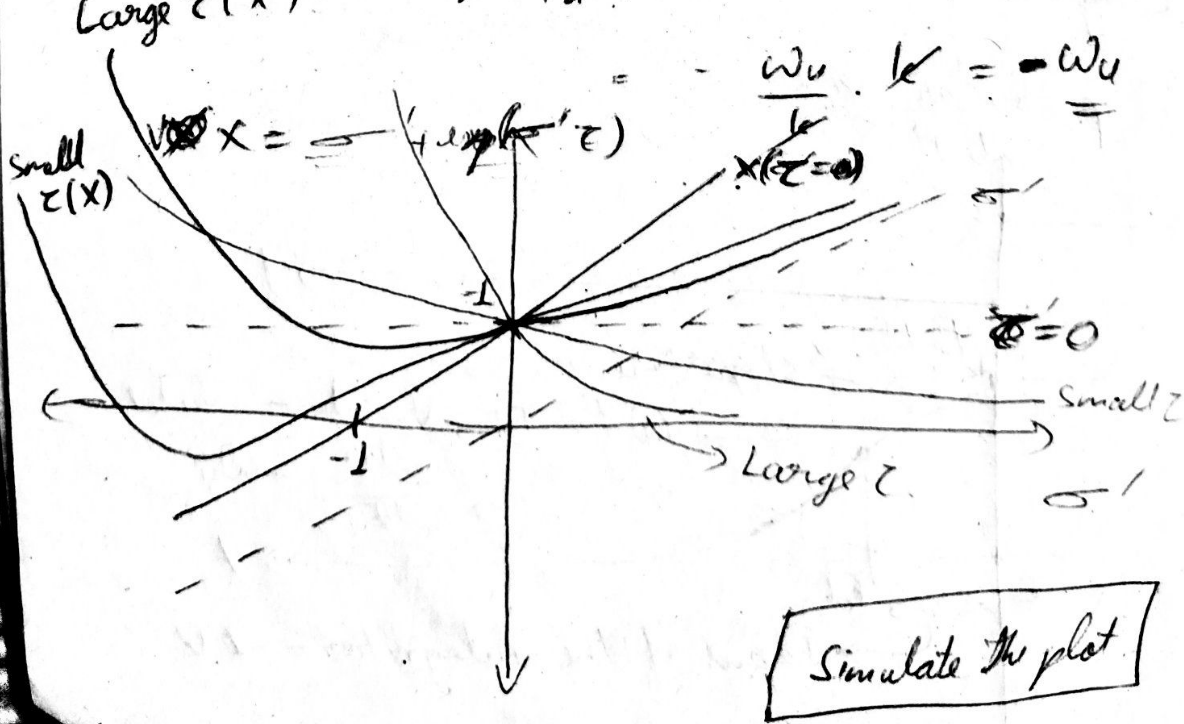
$$V_0(T_d) = k(1 - \tau)$$

So,
$$\left.\frac{dV_0}{dt}\right|_{t=T_d} = -\underline{\omega_u}.$$

$$\frac{dV_0}{dt} = -\left(\frac{\omega_u}{k}\right) V_0(t - T_d) \quad \Bigg| \quad V_0(0) = k[1-0] = k.$$

So,
$$\left.\frac{dV_0}{dt}\right|_{t=T_d} = -\left(\frac{\omega_u}{k}\right) V_0(0)$$

$$= -\frac{\omega_u}{k} \cdot k = -\underline{\omega_u}$$



Simulate the plot

$\sigma' + \exp(-\sigma'\tau) = 0$

Find the ~~minimum~~ max value of τ where the curve touches the x-axis & below that we have 2 roots.

$$= \frac{d\left[\sigma' + \exp(-\sigma'\tau)\right]}{d\sigma'}$$

$$= 1 - \tau\exp(-\sigma'\tau) = 0$$

$$\tau e^{-\sigma'\tau} = \frac{1}{1}$$

$$e^{-\sigma'\tau} = \frac{1}{\tau}$$

$$-\sigma'\tau = \ln\left(\frac{1}{\tau}\right)$$

$$\boxed{\sigma' = \frac{\ln(\tau)}{\tau}}$$

Minimum value of σ' where curve touches x-axis once.

Now,

$$\frac{\ln\tau}{\tau} + \exp\left(-\frac{\ln\tau}{\tau}\tau\right) = 0$$

$$\frac{\ln\tau}{\tau} + e^{\ln(1/\tau)} = 0$$

$$\frac{\ln\tau}{\tau} + \frac{1}{\tau} = 0$$

$$\ln\tau = -1$$

Maximum value of τ where curve touches x-axis

$$\boxed{\tau = \frac{1}{e} = 0.3675}$$

(for $\tau > \frac{1}{e}$) No real solutions

For $\tau > \frac{1}{e}$.

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$$\frac{dV_o}{dt} = -\frac{\omega_u}{k} V_o(t - T_d)$$

Previous assumption $\Rightarrow V_o(t) = \exp(\sigma t) \rightarrow \sigma$: real

$$V_o(t) = \exp((\sigma + j\omega)t)$$

Now.

So, $$\frac{dV_o}{dt} = (\sigma + j\omega) \cdot \exp[(\sigma + j\omega)t]$$

$$= -\frac{\omega_u}{k} V_o(t - T_d)$$

$$(\sigma + j\omega)\, \cancel{\exp[(\sigma + j\omega)t]} = -\frac{\omega_u}{k} \left[\cancel{\exp[(\sigma + j\omega)t]}\right] \cdot \left[\exp[-(\sigma + j\omega)T_d]\right]$$

$$\sigma + j\omega = -\left(\frac{\omega_u}{k}\right) \exp\left[-(\sigma + j\omega)T_d\right]$$

$$\sigma + j\omega = -\left(\frac{\omega_u}{k}\right)\left[\exp(-\sigma T_d) \cdot \exp(-j\omega T_d)\right]$$

$$\sigma + j\omega = -\left(\frac{\omega_u}{k}\right)\left[\exp(-\sigma T_d)\right]\left[\cos(\omega T_d) - j\sin(\omega T_d)\right]$$

$$\sigma = -\cos(\omega T_d)\left(\frac{\omega_u}{k}\right)\exp(-\sigma T_d)$$
$$\omega = +\sin(\omega T_d)\left(\frac{\omega_u}{k}\right) \cdot \exp(-\sigma T_d)$$
solve for σ & ω

$$\boxed{\omega' = \frac{\omega_u}{k}} \quad \boxed{\sigma' = \frac{\sigma}{\omega_u/k}} \quad \boxed{\tau = \frac{T_d}{k/\omega_u}}$$

$$\boxed{\begin{aligned} \sigma' &= -\cos(\omega'\tau)\cdot\exp(-\sigma'\tau) \rightarrow (1) \\ \omega' &= \sin(\omega'\tau)\cdot\exp(-\sigma'\tau) \rightarrow (2) \end{aligned}}$$

$(1)^2 + (2)^2 : (\sigma')^2 + (\omega')^2 = \exp(-2\sigma'\tau) \rightarrow (3)$

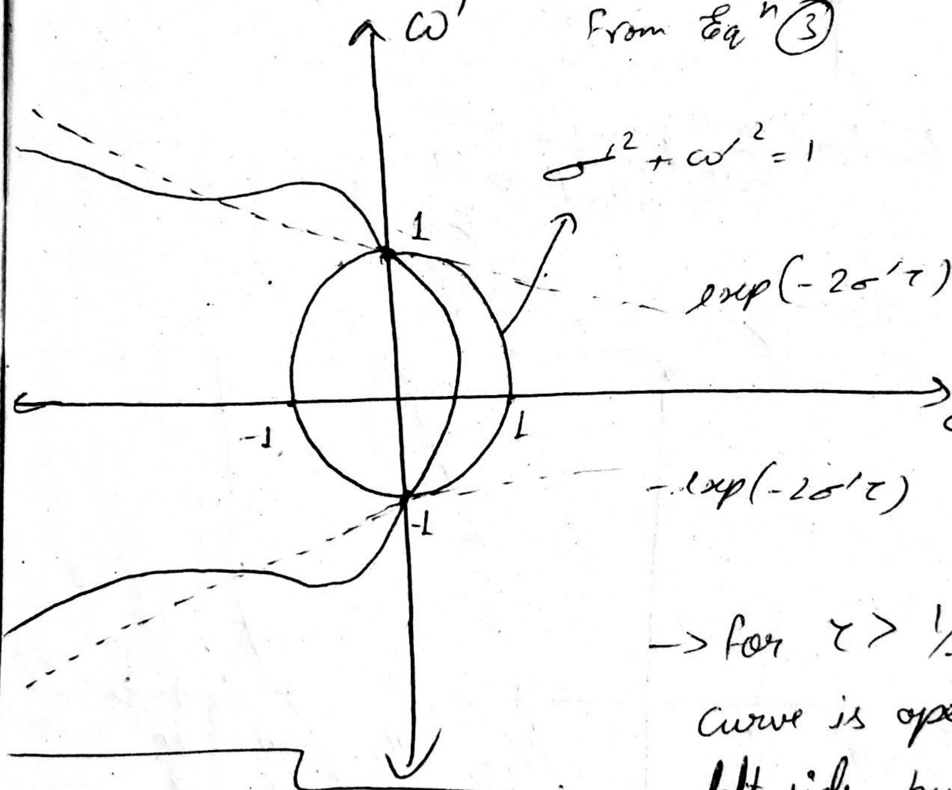
$$\boxed{\cos^2\theta + \sin^2\theta = 1}$$

Now,

$$\frac{\sigma'}{\omega'} = \frac{(1)}{(2)} = -\frac{\cos\omega'\tau}{\sin\omega'\tau} \rightarrow (4)$$

From Eq^n (3)

$x^2+y^2=1$
↳ circle



→ for $\tau > 1/e$, the curve is open on the left side, but passes through $(0,1)$ & $(0,-1)$

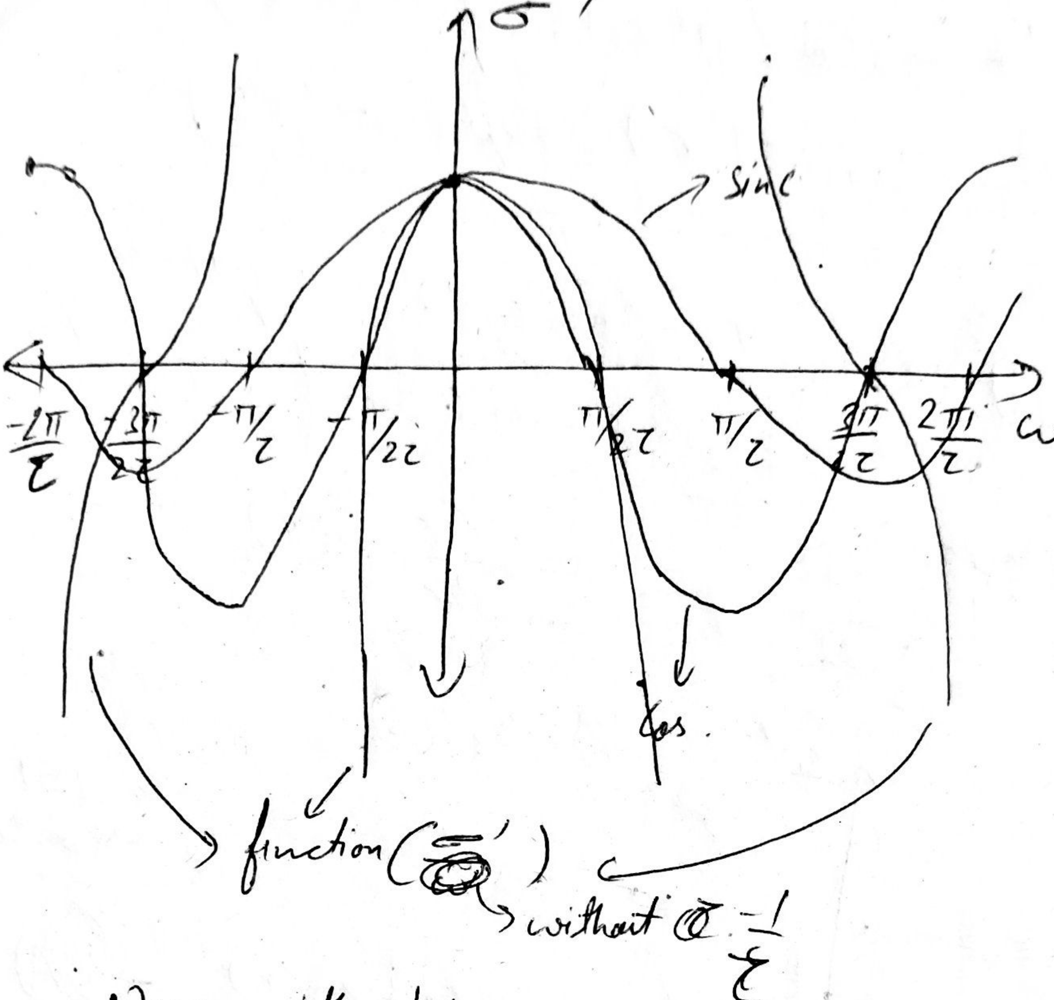
From Eq^n (4)

$$\frac{\sigma'}{\omega'} = \frac{-\cos(\omega'\tau)}{\sin(\omega'\tau)}$$

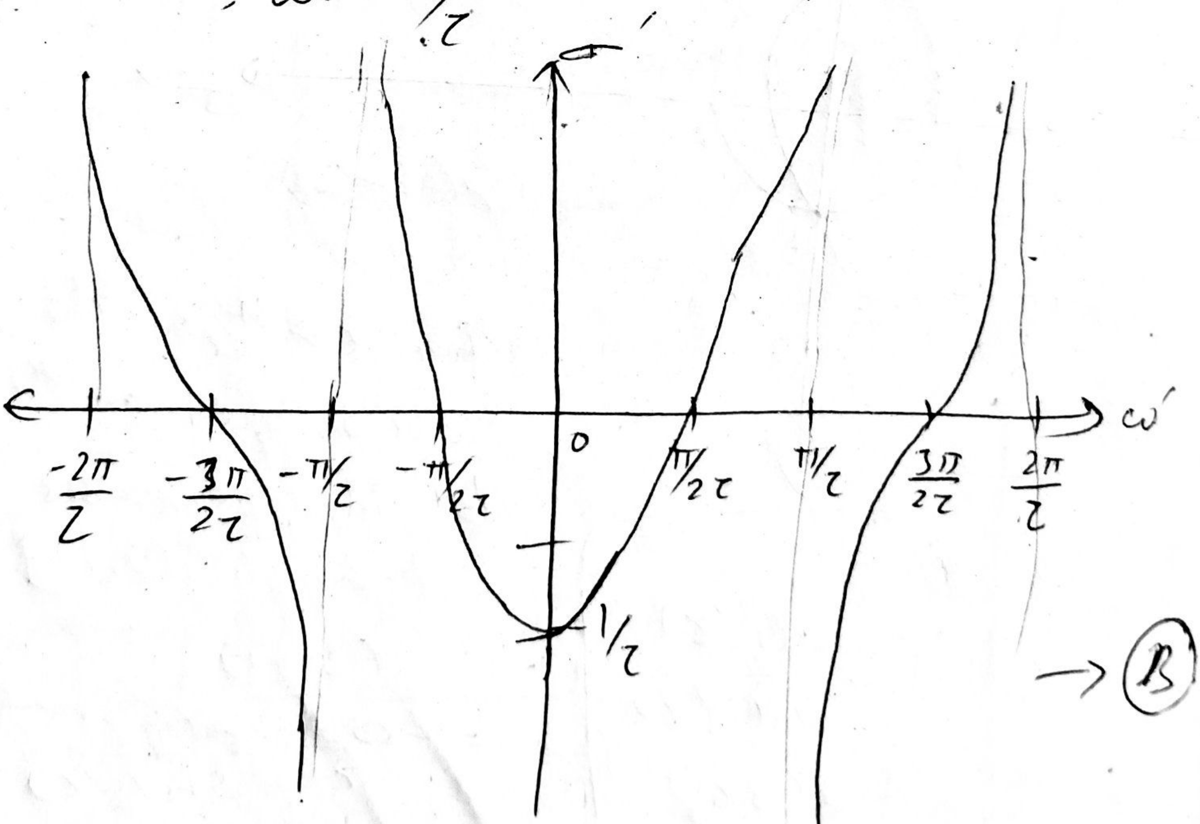
$$\sigma' = -\omega'\frac{\cos(\omega'\tau)}{\sin(\omega'\tau)}$$

$$= -\frac{\omega'\tau}{\tau}\frac{\cos(\omega'\tau)}{\sin(\omega'\tau)}$$

$$= -\frac{\omega'1}{\tau}\frac{\cos(\omega'\tau)}{\mathrm{Sinc}(\omega'\tau)}$$



Now, with $-1/\tau$

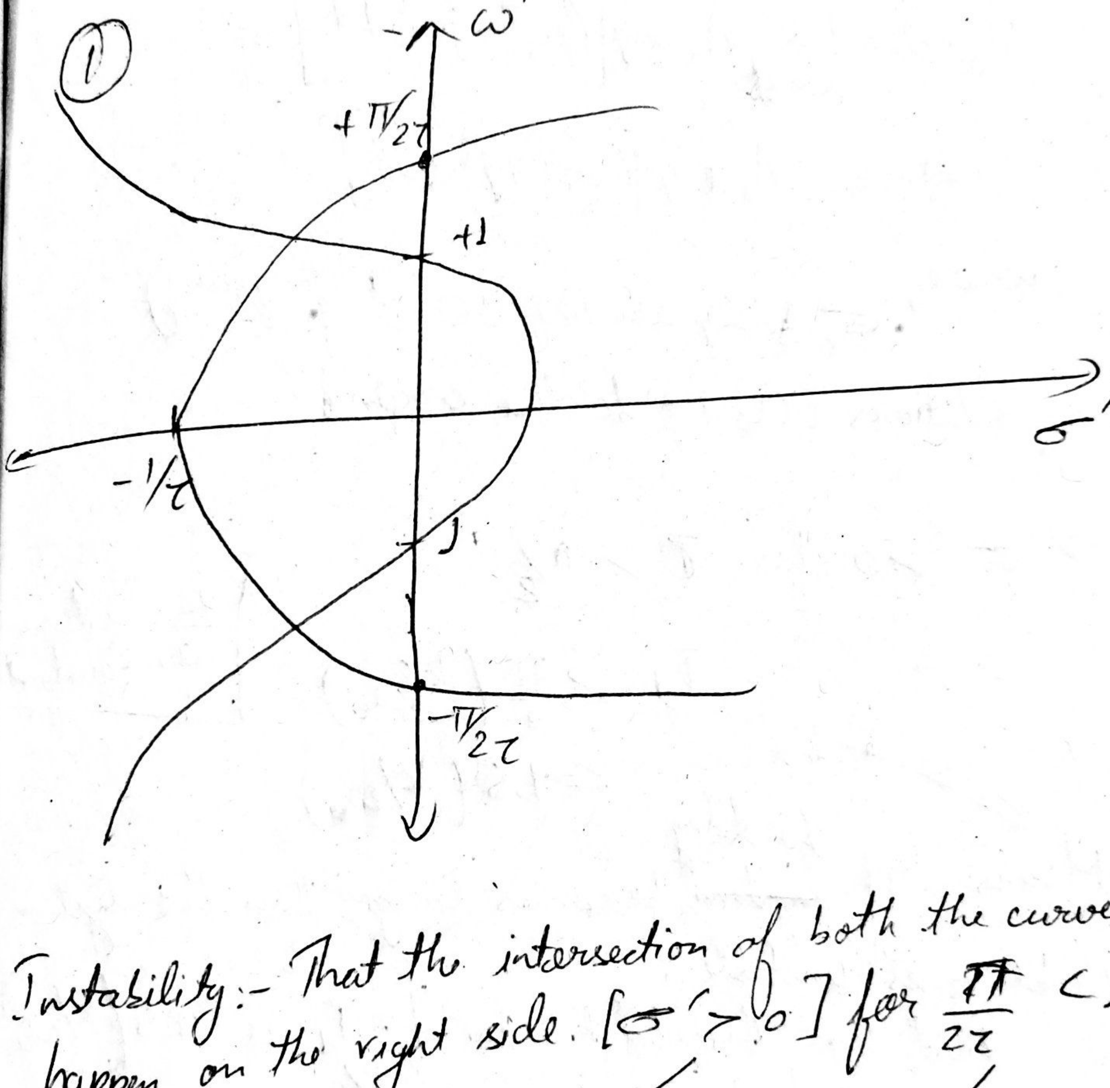


→ B

Now, combining the plots A + B

$$\sigma'^2 + \omega'^2 = \exp(-2\sigma'^2 \tau) \rightarrow (1)$$

$$\frac{\sigma'}{\omega'}\sigma' = -\frac{1}{\tau}\frac{\cos\omega'\tau}{\sin\omega'\tau} \rightarrow (2)$$



Instability:- That the intersection of both the curves happen on the right side. $[\sigma' > 0]$ for $\frac{\pi}{2\tau} < 1$

$$\tau > \pi/2$$

We are using only 1 middle curve from B because the ~~the~~ other curves are at higher frequencies (ω is higher) so the exponential of it will die off.

Summary for $\tau > \frac{1}{e}$ $\simeq 1$

→ Infinite number of solutions :- $\sigma_1', \sigma_2', \ldots$
$\omega_1', \omega_2' \ldots$

$$V_0(t) = \sum_{k=1}^{\infty} A_k \exp\left[(\sigma_k^{*} + j\omega_k^{*})t\right]$$

$$V_0(t) \simeq A_1 \exp\left[(\sigma_1^{\beta} + j\omega_1)t\right]$$

where, σ_1 & ω_1 are the lowest frequency solutions [closest to the origin]

→ $\sigma' > 0$ for $\tau > \pi/2$

$$T_d > \frac{\pi}{2}(k/\omega_u)$$
$$\simeq 1.5(k/\omega_u)$$

$\frac{k}{\omega_u}$ → time Constant

Means → If ~~system~~ the delay becomes large then the system becomes unstable.
And the threshold value of T_d is $1.5(k/\omega_u)$
i.e, 1.5 times the time constant

~~Sed~~ Solutions :-

$$A_1 \exp((\sigma_1 + j\omega_1)t) + A_2 \exp((\sigma_1 - j\omega_1)t)$$

$$\hookrightarrow a_1 \exp(\sigma_1 t)\cos(\omega_1 t) + a_2 \exp(\sigma_1 t)\sin(\omega_1 t)$$

→ The sol. will be in terms of sinusoids which are modulated by an exponential. | Simulate it

Imp Points :- [Negative feedback amp. with delay] (40)

(1) Small delays speeds up the response

(2) $\tau = \frac{1}{e} \left[T_d = 2.718 \frac{k}{\omega_u} \right]$, fastest response without overshoot.

(3) $\frac{1}{e} < \tau < \pi/2$: stable response, with ringing

(4) $\tau > \pi/2$: Unstable.

(5) In practice : $\tau \leq 0.5$.