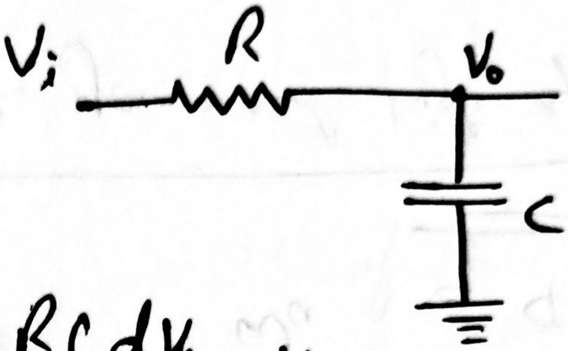
Negative feedback amp	RC filter.
$V_i \rightarrow DC$	

$V_o = k V_i$ in steady state.

$$\frac{1}{\omega_u}\frac{dV_o}{dt} = V_i - \frac{V_o}{k}$$



$$RC\frac{dV_o}{dt} = V_i - V_o$$

1st order DE.

Solution:-

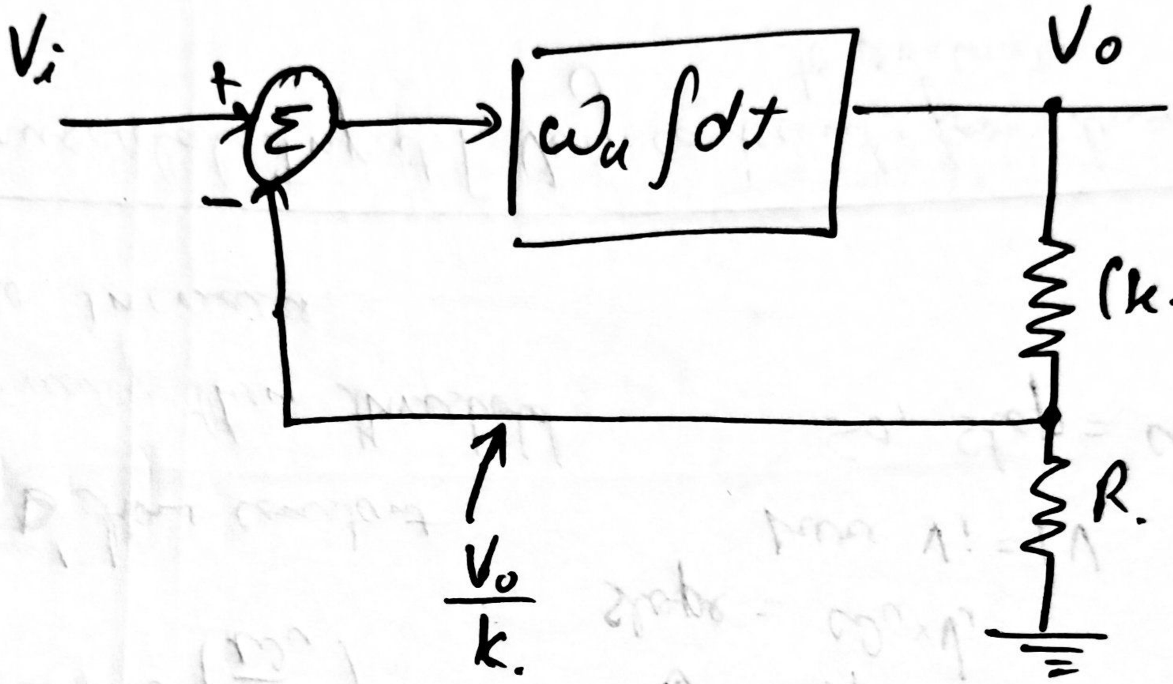
$$V_o(t) = V_o(0).\exp\left(\frac{-\omega_u}{k}.t\right) + kV_i\left[1 - \exp\left(-\frac{\omega_u}{k}.t\right)\right]$$

→ Same behaviour as 1st order RC filter.

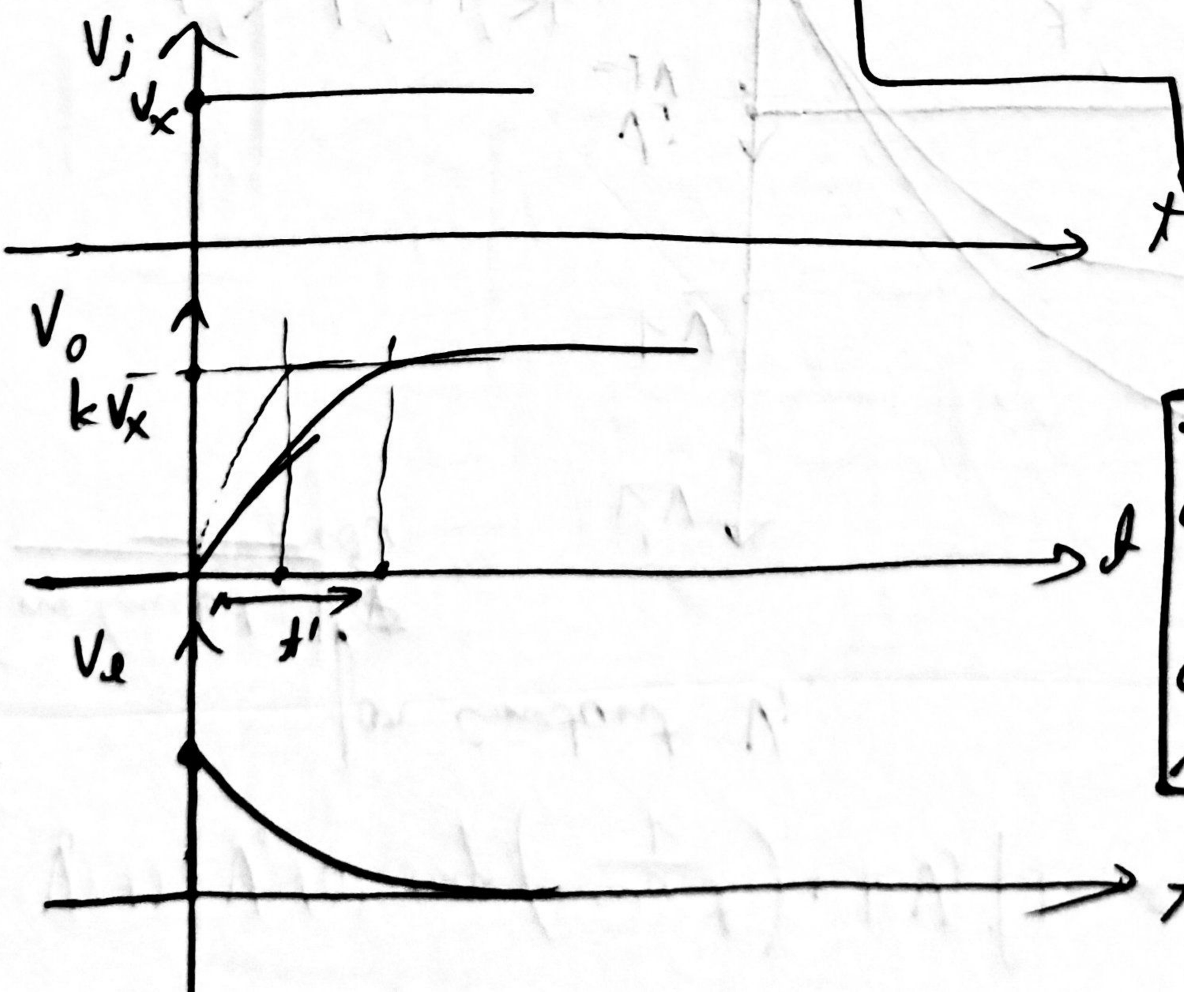
→ Till now we worked with constant input.

→ Now we'll work on step input & sinusoidal input [variable input].

* Response of negative feedback amp. to a step input.



Waveforms



Threshold ← $t' = 4.6\left(\frac{k}{\omega_u}\right)$ to reach 99% of steady state

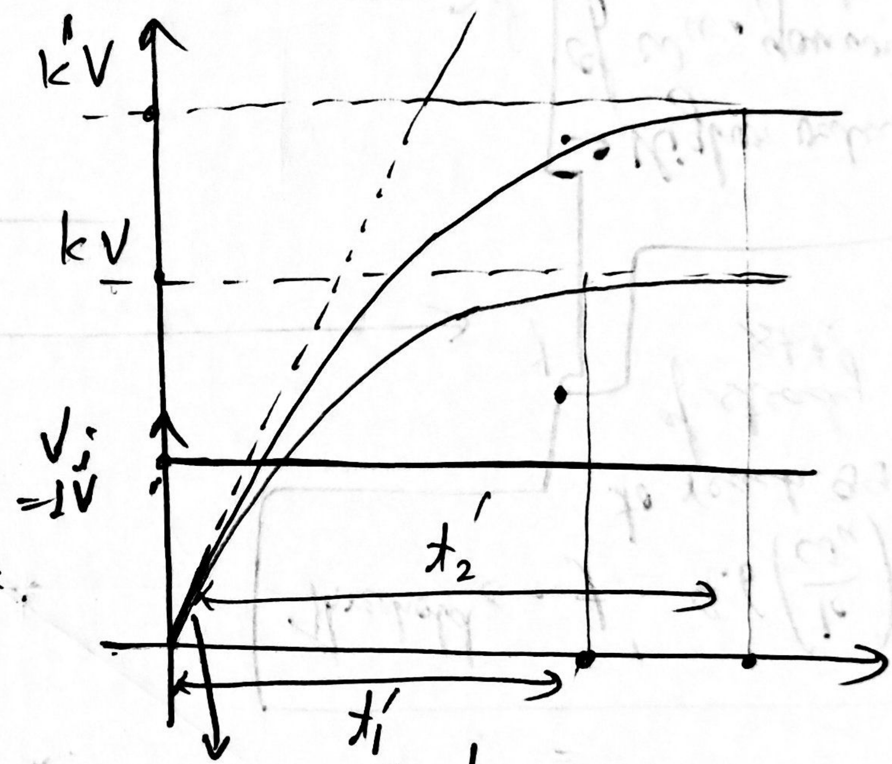
→ Higher value of ω_u, you will be having faster approach to steady state

→ Smaller energy (area) of the error $V_i - V_o/k$

$$V_o(t) = V_o(0)\exp\left(-\frac{\omega_u t}{k}\right) + kV_i\left[1 - \exp\left(-\frac{\omega_u t}{k}\right)\right]$$

for Constant V_i.

Time Constant $= \frac{k}{\omega_u}$



Since $t_2' > t_1' \rightarrow k' > k$.

$$t' = 4.6\left(\frac{k}{\omega_u}\right)$$

Slope $= \omega_u \times V_i$

here $V_i = 1V$

So, slop $= \omega_u$.

→ If time constant increases then threshold also increases.

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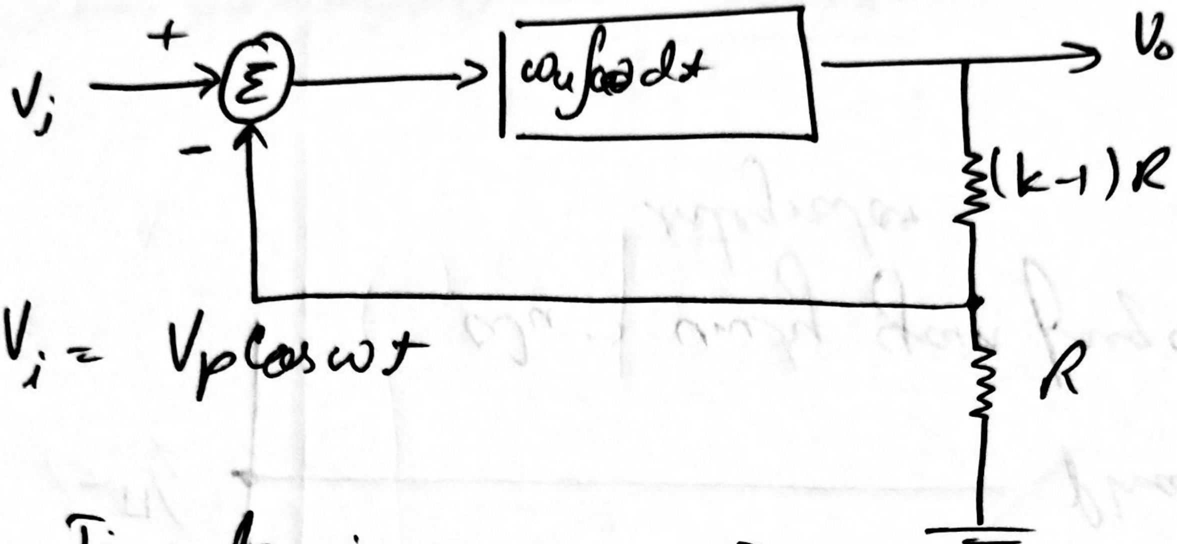
* Sinusoidal Input [Very conbient for linear system to evaluate]



$V_i = V_p \cos \omega t$

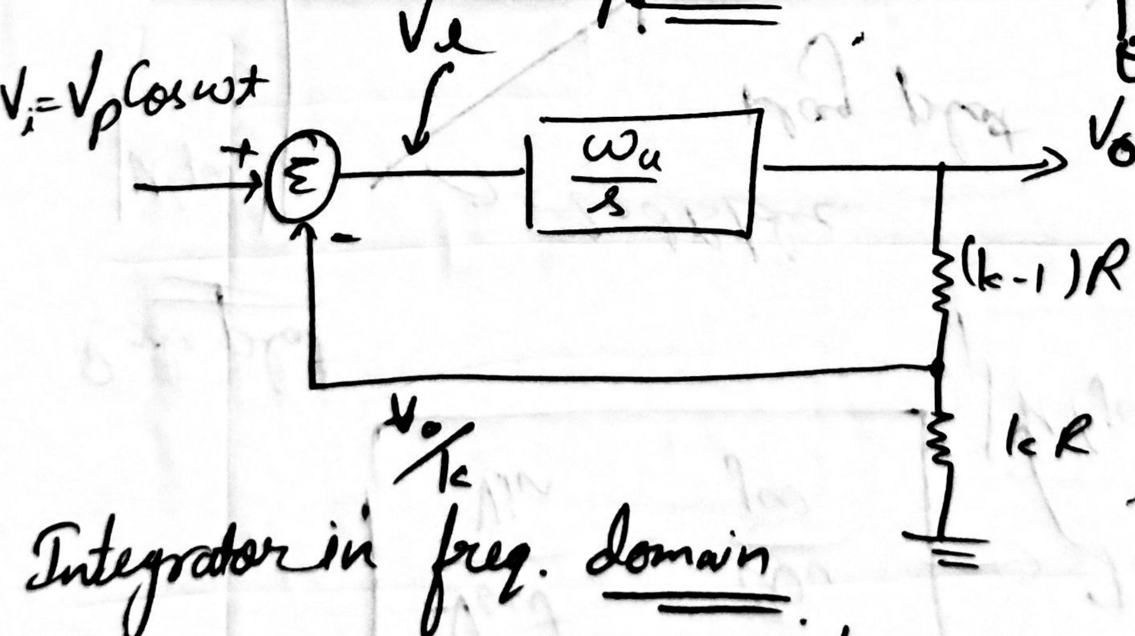
Time domain

Freq. domain

$$\omega_u \int dt \equiv \frac{\omega_u}{s}$$

Integrator

Updated



$V_e \, \omega_u/s$

$$V_o = \left(V_i - \frac{V_o}{k}\right)\frac{\omega_u}{s}$$

$$\frac{V_i \, \omega_u}{s} = V_o\left[\frac{\omega_u}{ks} + 1\right]$$

$$\frac{V_e}{V_i} = \frac{\omega_u/s}{\frac{\omega_u}{ks} + 1}$$

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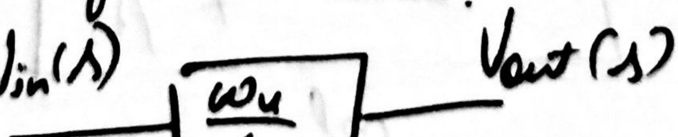
Integrator in freq. domain



$$V_{out}(s) = \frac{\omega_u}{s} \cdot V_{in}(s)$$

$$V_{out}(t) = \omega_u \int V_{in}(t) \cdot dt$$

Substitute $s = j\omega$

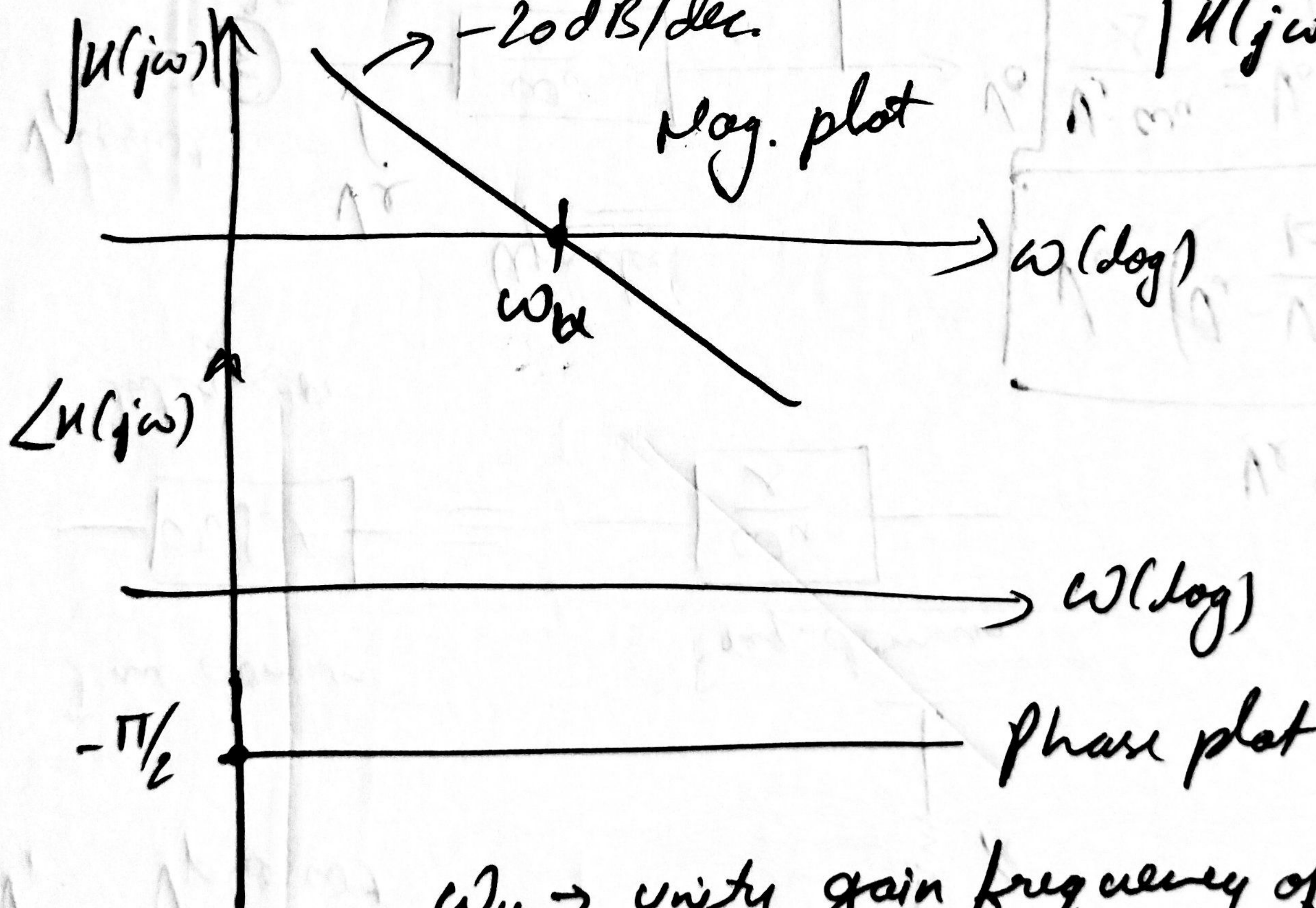
$$V_{out}(j\omega) = \frac{\omega_u}{j\omega} \cdot V_{in}(j\omega)$$

$$\boxed{\frac{V_{out}}{V_{in}} = \frac{\omega_u}{j\omega}} \rightarrow \text{Transfer function}$$

$$\boxed{H(j\omega) = \frac{\omega_u}{j\omega}}$$

<u>Bode plot</u>

$$|H(j\omega)| = \frac{\omega_u}{\omega}$$



$\omega_u \rightarrow$ unity gain frequency of the integrator.

$$H(s) = \frac{V_o}{V_i} = \frac{\omega_u/s}{\left(\frac{\omega_u}{ks} + 1\right)}$$

$\times$ by $\underline{\underline{ks/\omega_u}}$

$$H(s) = \frac{V_o}{V_i} = \frac{k}{\left(1 + \frac{ks}{\omega_u}\right)}$$

Transfer function.

Now, $\boxed{H(0) = k}$ → Input is constant

DC gain of amp. is 'k' [Already known]

$$\boxed{\frac{V_o}{V_i} = \frac{k}{1 + \frac{ks}{\omega_u}}}$$

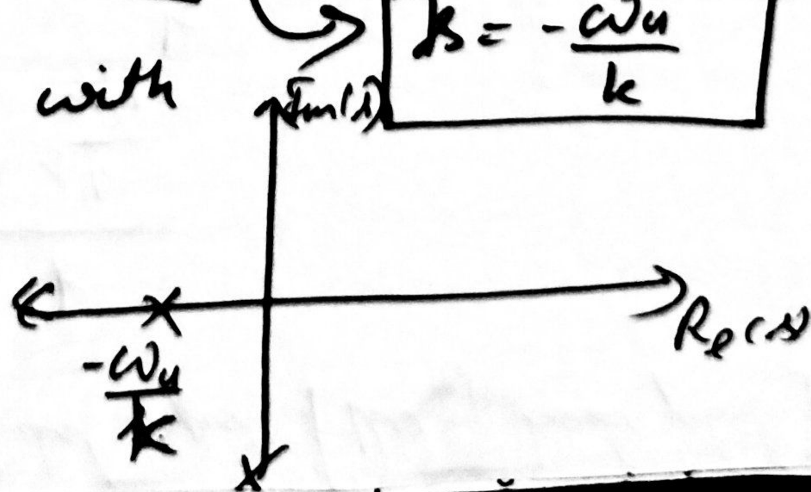
Pole at $-\frac{\omega_u}{k}$

$$1 + \frac{ks}{\omega_u} = 0$$

$$\boxed{s = -\frac{\omega_u}{k}}$$

1st order system with single pole.

→ For the system to be stable the pole should be in left side.



Analysis of Sinusoidal input [Using bode plot]

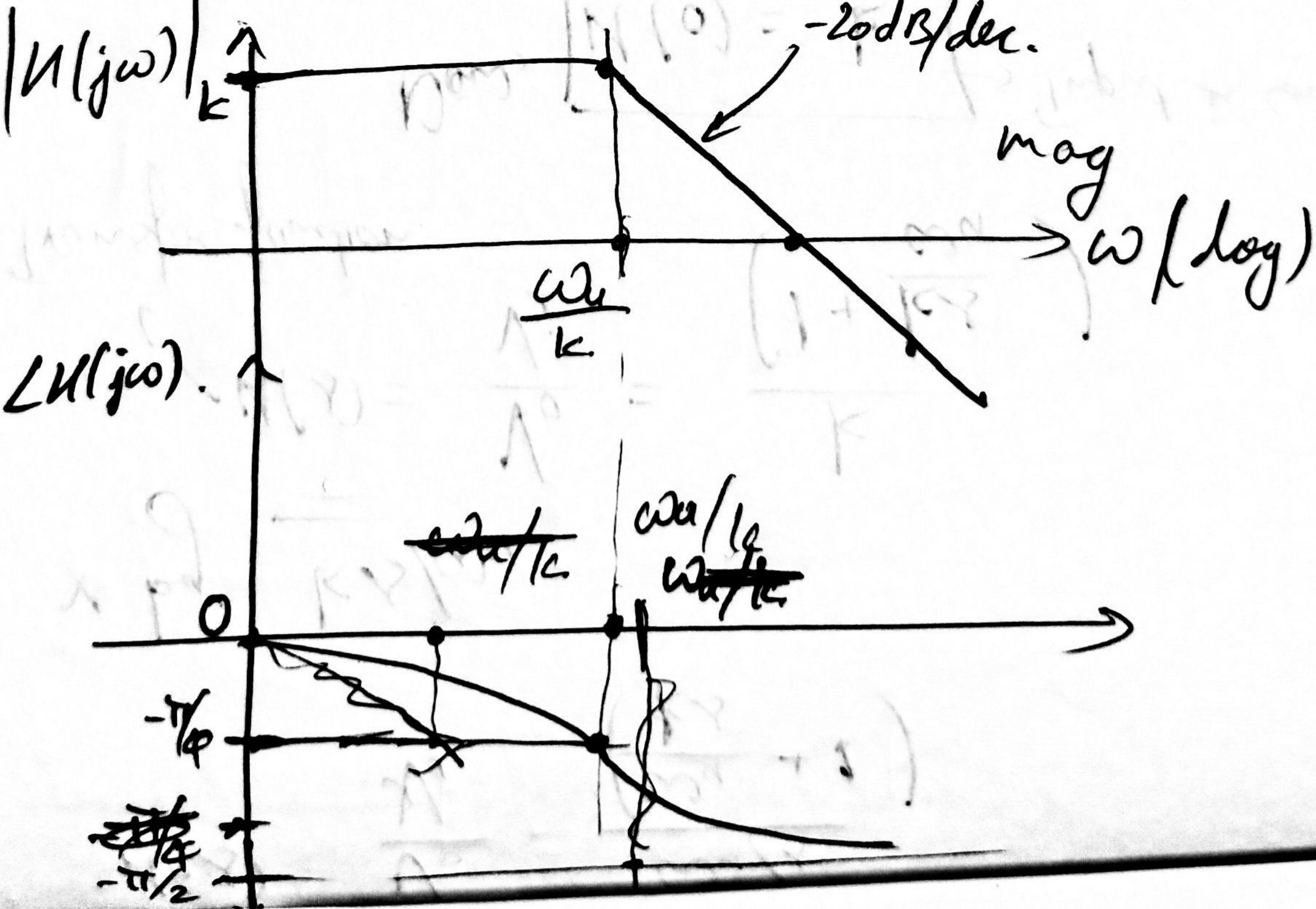
$$U(s) = \frac{k}{1+s\frac{k}{\omega_u}}$$

$$U(j\omega) = \frac{k}{1+j\omega\frac{k}{\omega_u}}$$

$$|U(j\omega)| = \frac{k}{\sqrt{1+\frac{k^2\omega^2}{\omega_u^2}}}$$

$$\angle U(j\omega) = -\tan^{-1}\left(\frac{\omega k}{\omega_u}\right)$$

Revise



Approximation

$$|H(j\omega)| \begin{cases} k. & \omega = 0 \\ \dfrac{k}{\sqrt{\dfrac{k^2\omega^2}{\omega_u^2}}} = \dfrac{\omega_u}{\omega}. & \omega \to \text{very large} \end{cases}$$

neglect 1

or

$$|H(j\omega)| \simeq \begin{cases} \dfrac{\omega_u}{\omega} \to \omega \gg \dfrac{\omega_u}{k} \to \text{High freq.} \\ k \to \omega \ll \dfrac{\omega_u}{k} \to \text{low freq.} \end{cases}$$

Summary

→ The dC gain of The transfer function is 'k'
[Expected]

→ It has a single pole at $\dfrac{'\omega_u'}{k}$ → pole freq.

→ Mag. plot :- $\omega \ll \dfrac{\omega_u}{k} \Rightarrow |H(j\omega)| = k$
Low freq

$\omega \gg \dfrac{\omega_u}{k} \Rightarrow |H(j\omega)| = \dfrac{\omega_u}{\omega}$ → Rolls off at 1st order [20 dB/dec]
High freq.

→ Phase plot :-
Phase lag $= -\pi/4$ at pole freq. | phase lag $= -\pi/2$ at very high freq.