Unit 2
Part I Electrostatics

Electric Field: The electric field space which is around a stable or moving charge in a charged particle form or between the two voltages. The other charged particles in this space undergo force that is exerted by this field. The intensity and force type that is exerted is going to depend on the charge a particle carries.

Electric Charge: Electric Charge can be defined as a physical property of matter which causes it to undergo a force when within an electromagnetic field. It helps the particle to hold an electric field of itself. An electron comes with a charge of -1, and a proton with +1. Neutrons are typically neutral with charge equivalent to 0.

Electrostatics can be defined as the study of electric charges at rest. Electric fields have their sources in electric charges. Two fundamental laws governing the electrostatic fields are (1) Coulomb's Law and (2) Gauss's Law. Coulomb's law is applicable in finding electric field due to any charge distribution, Gauss's law is easier to use when the distribution is symmetrical.

Coulomb's Law states that the force between two-point charges Q1and Q2 is directly proportional to the product of the charges and inversely proportional to the square of the distance between them. Point charge is a hypothetical charge located at a single point in space. It is an idealized model of a particle having an electric charge. $F=kQ_1Q_2/R^2$

Gauss law states that "the total flux linked within a closed surface is equal to the $1/\varepsilon_0$ times the total charge enclosed by that surface."

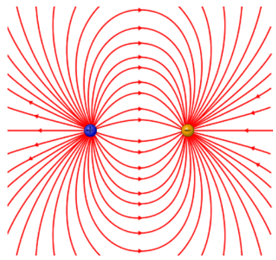
It is defined as $\phi=q/\varepsilon_0$.

Where, q is the charge enclosed in the surface

ε_0 is the permittivity of the vacuum.

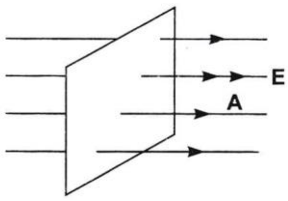
The electric flux through an area is the electric field multiplied by the area of the surface projected in a plane perpendicular to the field using magnetic effect of electric current.

1. Electrostatics field, electric flux density, electric field strength



Like magnetic flux, electric flux lines are not always closed loop. This is because, an isolated magnetic north pole or an isolated magnetic south pole do not exist practically, but an isolated positively charged body and an isolated negatively charged body can exist. We generally denote electric flux with Ψ. We take the measuring unit of flux as the amount flux emanated from one coulomb positive electric charge. We know that number of lines of force emanated from a positive charge body is numerically equal to the charge of the body measured in coulomb. As the flux is total number of lines of force emanated from the charge body, the unit of flux is also taken as Coulomb. So, if Q is the charge of a body and Ψ is the electric flux emanated from the body, then we can write, Q = Ψ. Electric Flux is the rate of flow of an electric field through an area. Electric flux is proportional to the number of electric field lines passing through a virtual surface.

- Flux refers to the presence of a force field in a physical medium.
- flux refers to an electrostatic field.
- Flux is shown as "lines" in a plane that contains or intersects electric charge poles or magnetic poles.
- The total number of electric field lines passing a given area in a unit of time is defined as the electric flux.
- The S.I. unit of electric flux is Volt Metres (V m).



Φ = EA On tilting this plane at an angle of θ, the calculated area is given by Acosθ and the total flux over this surface is given as: $\phi = EA\cos\theta$

Where, E = magnitude of the electric field

A = area of the surface through which the electric flux is to be determined

Θ = angle made by the plane and the axis parallel to the direction of flow of the electric field

Electric Flux Density:

Electric flux is the normal (Perpendicular) flux per unit area. If a flux of ø passes through an area of A m^2 normal to the area then the flux density (Denoted by D) is: D = ø/A

If an electric charge is place in the center of a sphere or virtual sphere then the electric flux on the surface of the sphere is: $D = ø/A = Q/4\pi r^2$, where r =radius of the sphere.

The SI unit of electric flux is Coulomb per meter square.

Relation between Electric flux density and electric field intensity:
If we compare the formula for the Electric flux Density given above with the formula for the Electric Field intensity, we see that: $D = Q/4\pi r^2$ and $E = Q/4\pi\varepsilon_0\varepsilon_r r^2$ Where, ε_0 = Permittivity of vacuum and ε_r = relative Permittivity. Thus, we can conclude that: $D = \varepsilon_0\varepsilon_r.E$ And, $E = D/\varepsilon_0\varepsilon_r$

The number of electric field lines or electric lines of force flowing perpendicularly through a unit surface area is called electric flux density.

- Electric flux density is represented as D, and its formula is $D=\epsilon E$.
- Electric flux is measured in Coulombs C, and surface area is measured in square meters (m^2). Hence, the SI unit of electric flux density is coulomb per square meters (C/m^2).
- The electric flux density depends on the number of electric lines of force passing through the surface area.
- Maximum electric field lines pass through the surface when the surface is perpendicular to the electric field. Therefore, no electric lines of force will pass through the surface when the surface is parallel to the electric field.

2. Permittivity

Permittivity can be explained as the ratio of electric displacement to the electric field intensity. It is the property of a material to measure the opposition generated by the material during the electric current development. The permittivity of a material is represented by the symbol ε. The SI unit of permittivity is Farad per metre. The approximate value for permittivity is 8.85×10^{-12} Faraday/metre, which is found in a vacuum medium. The permittivity measures the number of charges needed for generating a unit of electric flux in a specific channel. Permittivity is expressed in relative terms in engineering applications instead of absolute terms. The permittivity of free space (that is, 8.85×10^{-12} F/m) is represented by ε_o and the permittivity of substance in question (also represented in farads per metre) is represented by ε. Here, the relative permittivity dielectric constant ε_r, is given by: $\varepsilon_r = \varepsilon / \varepsilon_o = e\ (1.13 \times 10^{11})$

The higher the value of the dielectric constant, the greater the opposition the material offers against the formation of an electric field.

Types of permittivity

Although permittivity is a general term represented by ε, there are various types of permittivity. Which term or definition is used depends on the application and environment in which it is being measured.

Absolute permittivity is the measure of permittivity in a vacuum or free space. It measures the resistance encountered when forming an electric field in a vacuum. ε_o is the smallest possible value of permittivity.

Relative permittivity is the permittivity of a material in relation to the permittivity of a vacuum. It is symbolized as ε_r and is always greater than ε_o.

Static permittivity is the permittivity of a material when exposed to a static electric field. The value is usually measured to assess the material's response to the frequency of some applied voltage.

Factors affecting permittivity

- permittivity almost always varies with the frequency of applied voltage. As the frequency increases, permittivity decreases.
- Humidity and the strength of the electric field applied also affect permittivity.
- when temperatures increase, permittivity falls. Permittivity variations often are small and even negligible.

Applications of permittivity

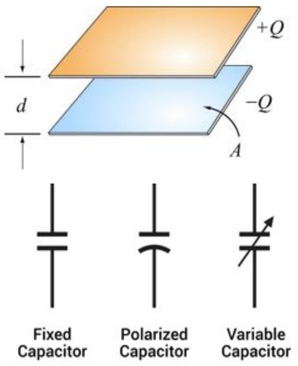
Permittivity plays an important role in capacitor design because the value determines how much electrostatic energy a dielectric material can store per unit of volume.

Permittivity and dielectric constant are also important for applications involving RF transmission lines and the propagation of radio waves.

3. Capacitance and capacitor

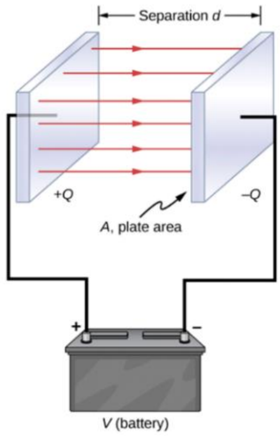
A capacitor is one of several kinds of devices used in the electric circuits of radios, computers and other such equipment. Capacitors provide temporary storage of energy in circuits and can be made to release it when required. The property of a capacitor that characterises its ability to store energy is called its capacitance.

When energy is stored in a capacitor, an electric field exists within the capacitor. The stored energy can be associated with the electric field. Indeed, energy can be associated with the existence of an electric field. A capacitor is a device which stores electric charge. Capacitors vary in shape and size, but the basic configuration is two conductors carrying equal but opposite charges. In the uncharged state, the charge on either one of the conductors in the capacitor is zero. During the charging process, a charge Q is moved from one conductor to the other one, giving one conductor a charge +Q, and the other one a charge −Q. A potential difference ΔV is created, with the positively charged conductor at a higher potential than the negatively charged conductor. Note that whether charged or uncharged, the net charge on the capacitor as a whole is zero. The simplest example of a capacitor consists of two conducting plates of area, which are parallel to each other, and separated by a distance d.



The amount of charge Q stored in a capacitor is linearly proportional to ΔV, the electric potential difference between the plates. Thus, we may write Q = C |ΔV| where C is a positive proportionality constant called capacitance. Physically, capacitance is a measure of the capacity of storing electric charge for a given potential difference ΔV. The SI unit of capacitance is the farad (F): 1 F = 1 farad = 1 coulomb/volt = 1 C/V

A typical capacitance is in the picofarad (1 pF = 10^{-12} F) to millifarad range, (1 mF = 10^{-3} F = 1000μF; 1 μF 10^{-6}F).



Parallel-Plate Capacitor

Consider two metallic plates of equal area A separated by a distance d. The top plate carries a charge +Q while the bottom plate carries a charge –Q. The charging of the plates can be accomplished by means of a battery which produces a potential difference.

C depends only on the geometric factors A and d. The capacitance C increases linearly with the area A since for a given potential difference ΔV, a bigger plate can hold more charge. On the other hand, C is inversely proportional to d, the distance of separation because the smaller the value of d, the smaller the potential difference |ΔV| for a fixed Q.

$$\boxed{C = \frac{Q}{|\Delta V|} = \frac{\varepsilon_0 A}{d} \quad \text{(parallel plate)}}$$

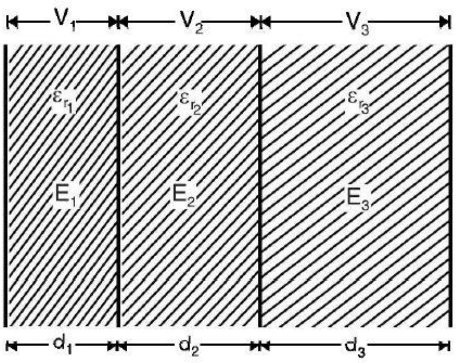
4. Composite dielectric capacitors

When a parallel plate capacitor has two dielectrics or more between the plates, it is said to be composite capacitor. The various types of such composite capacitor exist in practice. Let us study few types of such composite capacitors.

In this type, number of dielectrics having different thicknesses and relative permittivities are filled in between the two parallel plates. The composite capacitor with three different dielectrics with permittivities εr1, εr2 and εr3 and thicknesses d1, d2 and d3

$$E_1 = \frac{D}{\varepsilon_0\, \varepsilon_{r1}} \;;\quad E_2 = \frac{D}{\varepsilon_0\, \varepsilon_{r2}} \;;\quad E_3 = \frac{D}{\varepsilon_0\, \varepsilon_{r3}} \qquad V = V_1 + V_2 + V_3$$

$$= E_1 d_1 + E_2 d_2 + E_3 d_3$$



$$= \frac{D}{\varepsilon_0\, \varepsilon_{r1}} d_1 + \frac{D}{\varepsilon_0\, \varepsilon_{r2}} d_2 + \frac{D}{\varepsilon_0\, \varepsilon_{r3}} d_3$$

$$= \frac{D}{\varepsilon_0}\left[\frac{d_1}{\varepsilon_{r1}} + \frac{d_2}{\varepsilon_{r2}} + \frac{d_3}{\varepsilon_{r3}}\right]$$

$$= \frac{Q}{\varepsilon_0\, A}\left[\frac{d_1}{\varepsilon_{r1}} + \frac{d_2}{\varepsilon_{r2}} + \frac{d_3}{\varepsilon_{r3}}\right] \quad \left(\because D = \frac{Q}{A}\right)$$

$$\text{or} \quad \frac{Q}{V} = \frac{\varepsilon_0\, A}{\left(\frac{d_1}{\varepsilon_{r1}} + \frac{d_2}{\varepsilon_{r2}} + \frac{d_3}{\varepsilon_{r3}}\right)} \qquad C = \frac{\varepsilon_0\, A}{\left(\frac{d_1}{\varepsilon_{r1}} + \frac{d_2}{\varepsilon_{r2}} + \frac{d_3}{\varepsilon_{r3}}\right)} \text{ farad} \qquad C = \frac{\varepsilon_0 A}{\sum \frac{d}{\varepsilon_r}} \text{ farad}$$

5. Capacitors in series and parallel

A capacitor can be charged by connecting the plates to the terminals of a battery, which are maintained at a potential difference ΔV called the terminal voltage. The connection results in sharing the charges between the terminals and the plates.

Parallel Connection

Suppose we have two capacitors C_1 with charge Q_1 and C_2 with charge Q_2 that are connected in parallel, as shown in Figure

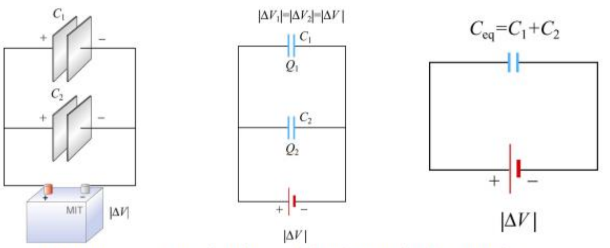


Figure Capacitors in parallel and an equivalent capacitor.

The left plates of both capacitors C_1 and C_2 are connected to the positive terminal of the battery and have the same electric potential as the positive terminal. Similarly, both right plates are negatively charged and have the same potential as the negative terminal. Thus, the potential difference $|\Delta V|$ is the same across each capacitor. This gives

$$C_1=\frac{Q_1}{|\Delta V|},\quad C_2=\frac{Q_2}{|\Delta V|}$$

These two capacitors can be replaced by a single equivalent capacitor C_{eq} with a total charge Q supplied by the battery. However, since Q is shared by the two capacitors, we must have

$$Q=Q_1+Q_2=C_1|\Delta V|+C_2|\Delta V|=(C_1+C_2)|\Delta V|$$

The equivalent capacitance is then seen to be given by

$$C_{eq}=\frac{Q}{|\Delta V|}=C_1+C_2$$

Thus, capacitors that are connected in parallel add. The generalization to any number of capacitors is

$$\boxed{C_{eq}=C_1+C_2+C_3+\cdots+C_N=\sum_{i=1}^{N}C_i \qquad \text{(parallel)}}$$

Series Connection

Suppose two initially uncharged capacitors C_1 and C_2 are connected in series, as shown in Figure 5.3.3. A potential difference $|\Delta V|$ is then applied across both capacitors. The left plate of capacitor 1 is connected to the positive terminal of the battery and becomes positively charged with a charge $+Q$, while the right plate of capacitor 2 is connected to the negative terminal and becomes negatively charged with charge $-Q$ as electrons flow in. What about the inner plates? They were initially uncharged; now the outside plates each attract an equal and opposite charge. So the right plate of capacitor 1 will acquire a charge $-Q$ and the left plate of capacitor $+Q$.

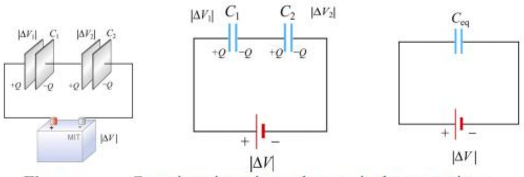


Figure Capacitors in series and an equivalent capacitor

The potential differences across capacitors C_1 and C_2 are

$$|\Delta V_1|=\frac{Q}{C_1},\quad |\Delta V_2|=\frac{Q}{C_2}$$

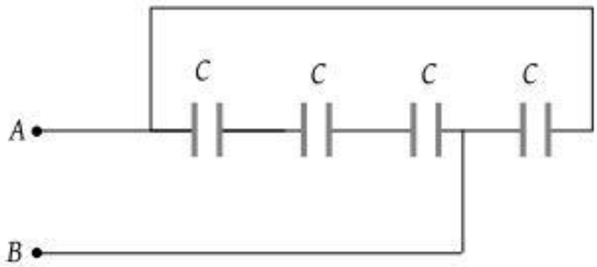
respectively. From Figure , we see that the total potential difference is simply the sum of the two individual potential differences: $|\Delta V| = |\Delta V_1|+|\Delta V_2|$

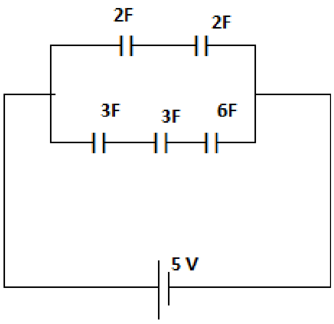
In fact, the total potential difference across any number of capacitors in series connection is equal to the sum of potential differences across the individual capacitors. These two capacitors can be replaced by a single equivalent capacitor $C_{eq}=Q/|\Delta V|$. Using the fact that the potentials add in series, $\frac{Q}{C_{eq}}=\frac{Q}{C_1}+\frac{Q}{C_2}$

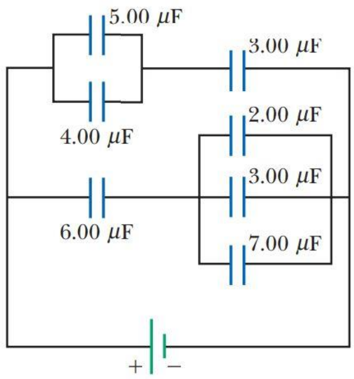
and so the equivalent capacitance for two capacitors in series becomes $\frac{1}{C_{eq}}=\frac{1}{C_1}+\frac{1}{C_2}$

The generalization to any number of capacitors connected in series is

$$\boxed{\frac{1}{C_{eq}}=\frac{1}{C_1}+\frac{1}{C_2}+\cdots+\frac{1}{C_N}=\sum_{i=1}^{N}\frac{1}{C_i} \qquad \text{(series)}}$$







6. Energy stored in capacitors

Capacitors can be used to stored electrical energy. The amount of energy stored is equal to the work done to charge it. During the charging process, the battery does work to remove charges from one plate and deposit them onto the other. Work is done by an external agent in bringing +dq from the negative plate and depositing the charge on the positive plate. Let the capacitor be initially uncharged. In each plate of the capacitor, there are many negative and positive charges, but the number of negative charges balances the number of positive charges, so that there is no net charge, and therefore no electric field between the plates.

Suppose the amount of charge on the top plate at some instant is +q, and the potential difference between the two plates is $|\Delta V|= q/C$. To dump another bucket of charge +dq on the top plate, the amount of work done to overcome electrical repulsion is $dW = |\Delta V|\, dq$. If at the end of the charging process, the charge on the top plate is +Q, then the total amount of work done in this process is

$$W=\int_0^Q dq\,|\Delta V|=\int_0^Q dq\,\frac{q}{C}=\frac{1}{2}\frac{Q^2}{C}$$

This is equal to the electrical potential energy U_E of the system:

$$\boxed{U_E=\frac{1}{2}\frac{Q^2}{C}=\frac{1}{2}Q|\Delta V|=\frac{1}{2}C|\Delta V|^2}$$

Energy Density of the Electric Field

One can think of the energy stored in the capacitor as being stored in the electric field itself. In the case of a parallel-plate capacitor, with $C = \varepsilon_0 A/d$ and $|\Delta V| = Ed$, we have

$$U_E = \frac{1}{2} C |\Delta V|^2 = \frac{1}{2}\frac{\varepsilon_0 A}{d}\left(Ed\right)^2 = \frac{1}{2}\varepsilon_0 E^2 \left(Ad\right)$$

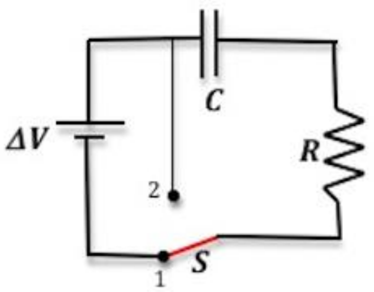
Since the quantity Ad represents the volume between the plates, we can define the electric energy density as

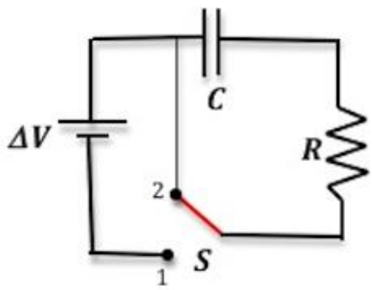
$$\boxed{u_E = \frac{U_E}{\text{Volume}} = \frac{1}{2}\varepsilon_0 E^2}$$

7. Charging and discharging of capacitors and time constant

The Charging and discharging of capacitors hold importance because it is the ability to control as well as predict the rate at which a capacitor charges and discharges that makes capacitors useful in electronic timing circuits. It happens when the voltage is placed across the capacitor and the potential cannot rise to the applied value instantaneously. As the charge on the terminals gets accumulated to its final value, it tends to repel the addition of further charge accumulation. Thus, following are the factors on which rate at which a capacitor can be charged or discharged depends on:

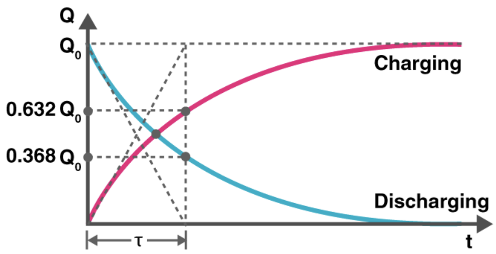
- The capacitance of the capacitor and
- The resistance of the circuit through which it is being charged or is discharged.

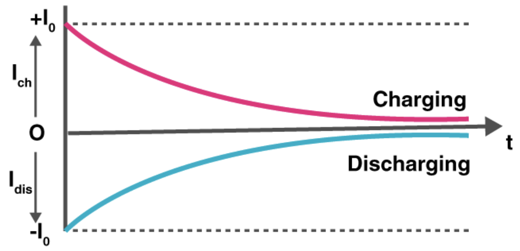


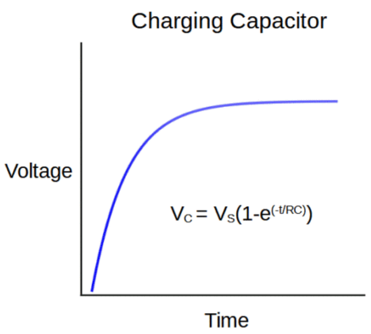


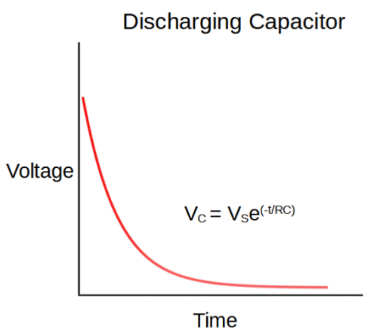
The figure shows a capacitor, (C) in series with a resistor, (R) forming a RC Charging Circuit connected across a DC battery supply (Vs) via a mechanical switch. at time zero, when the switch is first closed, the capacitor gradually charges up through the resistor until the voltage across it reaches the supply voltage of the battery. Let us assume above, that the capacitor, C is fully "discharged" and the switch (S) is fully open.

These are the initial conditions of the circuit, then t = 0, i = 0 and q = 0. When the switch is closed the time begins at t = 0 and current begins to flow into the capacitor via the resistor.









Since the initial voltage across the capacitor is zero, (Vc = 0) at t = 0 the capacitor appears to be a short circuit to the external circuit and the maximum current flows through the circuit restricted only by the resistor R.

Vs-R*i-Vc=0

The capacitor (C), charges up at a rate shown by the graph. The rise in the RC charging curve is much steeper at the beginning because the charging rate is fastest at the start of charge but soon tapers off exponentially as the capacitor takes on additional charge at a slower rate. As the capacitor charges up, the potential difference across its plates begins to increase with the actual time taken for the charge on the capacitor to reach 63% of its maximum possible fully charged voltage, in our curve 0.63Vs, being known as one full Time Constant, (T). This 0.63Vs voltage point is given the abbreviation of 1T, (one time constant). The capacitor continues charging up and the voltage difference between Vs and Vc reduces, so too does the circuit current, i. Then at its final condition greater than five-time constants (5T) when the capacitor is said to be fully charged, t = ∞, i = 0, q = Q = CV. At infinity the charging current finally diminishes to zero and the capacitor acts like an open circuit with the supply voltage value entirely across the capacitor as Vc = Vs.

RC Time Constant, $\tau \equiv R \times C$ **Tau**

This RC time constant only specifies a rate of charge where, R is in Ω and C in Farads. Since voltage V is related to charge on a capacitor given by the equation, Vc = Q/C, the voltage across the capacitor (Vc) at any instant in time during the charging period is given as:

$$V_C = V_S\left(1 - e^{(-t/RC)}\right)$$

Where:

Vc is the voltage across the capacitor

Vs is the supply voltage

e is an irrational number presented by Euler as: 2.7182

t is the elapsed time since the application of the supply voltage

RC is the time constant of the RC charging circuit

After a period, equivalent to 4-time constants, (4T) the capacitor in this RC charging circuit is said to be virtually fully charged as the voltage developed across the capacitors plates has now reached 98% of its maximum value, 0.98Vs. The time period taken for the capacitor to reach this 4T point is known as the Transient Period.

After a time of 5T the capacitor is now said to be fully charged with the voltage across the capacitor, (Vc) being approximately equal to the supply voltage, (Vs). As the capacitor is therefore fully charged, no more charging current flows in the circuit so IC = 0. The time period after this 5T time period is commonly known as the Steady State Period.

Then we can show in the following table the percentage voltage and current values for the capacitor in a RC charging circuit for a given time constant.

discharging of capacitors

When the switch is first closed, the capacitor starts to discharge as shown. The rate of decay of the RC discharging curve is steeper at the beginning because the discharging rate is fastest at the start, but then tapers off exponentially as the capacitor loses charge at a slower rate. As the discharge continues, VC reduces resulting in less discharging current.

For a RC discharging circuit, the voltage across the capacitor (VC) as a function of time during the discharge period is defined as:

$$V_C = V_S \times e^{-t/RC}$$

Where:

VC is the voltage across the capacitor

VS is the supply voltage

t is the elapsed time since the removal of the supply voltage

RC is the time constant of the RC discharging circuit

In a RC Discharging Circuit the time constant (τ) is still equal to the value of 63%. Then for a RC discharging circuit that is initially fully charged, the voltage across the capacitor after one time constant, 1T, has dropped by 63% of its initial value which is 1 – 0.63 = 0.37 or 37% of its final value.

Part II AC Fundamentals

The majority of electrical power in the world is generated, distributed, and consumed in the form of 50- or 60-Hz sinusoidal alternating current (AC) and voltage. It is used for household and industrial applications such as television sets, computers, microwave ovens, electric stoves, to the large motors used in the industry.

AC has several advantages over DC. The major advantage of AC is the fact that it can be transformed, however, direct current (DC) cannot. A transformer permits voltage to be stepped up or down for the purpose of transmission. Transmission of high voltage (in terms of kV) is that less current is required to produce the same amount of power. Less current permits smaller wires to be used for transmission. AC unlike DC flows first in one direction then in the opposite direction. The most common AC waveform is a sine (or sinusoidal) waveform. Sine waves are the signal whose shape neither is nor altered by a linear circuit, therefore, it is ideal as a test signal.

8. Sinusoidal voltages and currents

i) ALTERNATING QUANTITY

An alternating quantity is that which acts in alternate directions and whose magnitude undergoes a definite cycle of changes in definite intervals of time. When a simple loop revolves in a magnetic field, an alternating emf is induced in the loop. If the loop revolves with an uniform angular velocity the induced alternating emf is sinusoidal in nature. The alternating quantity may have various other wave forms like triangular, semicircular, stepped, distorted, etc. The graph repeats after regular intervals. One complete set of positive and negative values of an alternating quantity is called a cycle. The important alternating quantities, $f(t)$ that will be discussed in the chapter are current and voltage.

ii) ALTERNATING VOLTAGE

Alternating voltage may be generated by

(A) By rotating a coil in a stationary magnetic field.

(B) By rotating a magnetic field within a stationary coil.

Sinusoidal waveforms are periodic waveforms whose shape can be plotted using the sine or cosine function from trigonometry. Electrical circuits supplied by sinusoidal waveforms whose polarity changes every cycle and are commonly known as "AC" voltages and current sources. When dealing with sine waves in the time domain and especially current related sine waves the unit of measurement used along the horizontal axis of the waveform can be either time, degrees or radians. In electrical engineering it is more common to use the Radian as the angular measurement of the angle along the horizontal axis rather than degrees. For example, ω = 100 rad/s, or 500 rad/s.

Commercial alternators produce sinusoidal alternating voltage i.e., alternating voltage is a sine wave. The sinusoidal alternating voltage can be expressed by the equation

$v = V_m \sin \omega t$

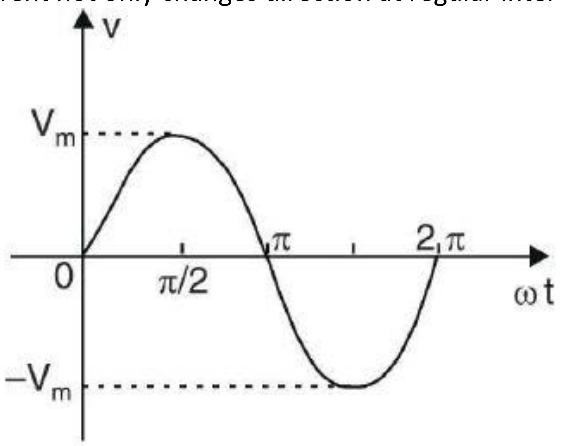
where v = Instantaneous value of alternating voltage

V_m = Max. value of alternating voltage

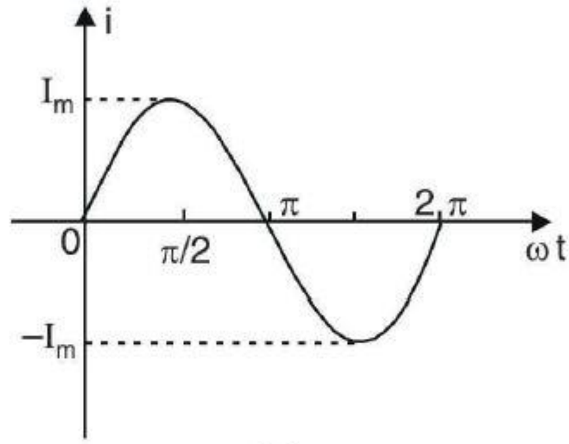
ω = Angular velocity of the coil

Sinusoidal voltages always produce sinusoidal currents, unless the circuit is non-linear.

Therefore, a sinusoidal current can be expressed in the same way as voltage i.e. $i = I_m \sin \omega t$. Note that sinusoidal voltage or current not only changes direction at regular intervals but the magnitude is also changing continuously.



(i)



(ii)

9. Frequency, cycle, period

- The Period, (T) is the length of time in seconds that the waveform takes to repeat itself from start to finish. This can also be called the Periodic Time of the waveform for sine waves, or the Pulse Width for square waves.
- The Frequency, (ƒ) is the number of times the waveform repeats itself within a one second time period. Frequency is the reciprocal of the time period, (ƒ = 1/T) with the unit of frequency being the Hertz, (Hz).
- The Amplitude (A) is the magnitude or intensity of the signal waveform measured in volts or amps.

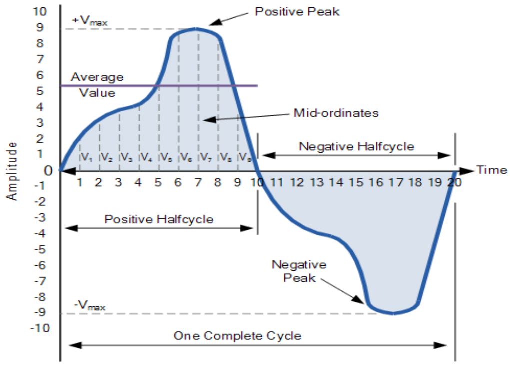
The time taken for an AC Waveform to complete one full pattern from its positive half to its negative half and back to its zero baseline again is called a **Cycle** and one complete cycle contains both a positive half-cycle and a negative half-cycle. The time taken by the waveform to complete one full cycle is called the **Periodic Time** of the waveform, and is given the symbol “T”.

The number of complete cycles that are produced within one second (cycles/second) is called the **Frequency**, symbol ƒ of the alternating waveform. Frequency is measured in Hertz, (Hz) named after the German physicist Heinrich Hertz.

Then we can see that a relationship exists between cycles (oscillations), periodic time and frequency (cycles per second), so if there are ƒ number of cycles in one second, each individual cycle must take 1/ƒ seconds to complete.

$$\text{Frequency}, (f) = \frac{1}{\text{Periodic Time}} = \frac{1}{T} \ \text{Hertz} \qquad \text{Periodic Time}, (T) = \frac{1}{\text{Frequency}} = \frac{1}{f} \ \text{seconds}$$

10. Instantaneous, peak (maximum), average and R.M.S. values



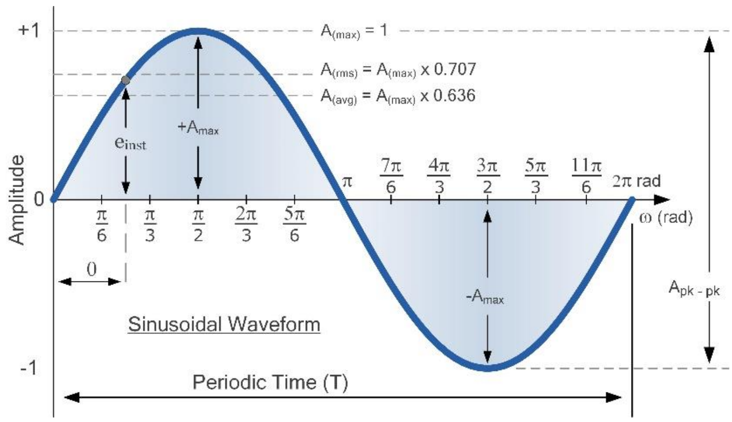
As well as knowing either the periodic time or the frequency of the alternating quantity, another important parameter of the AC waveform is Amplitude, better known as its Maximum or Peak value represented by the terms, Vmax for voltage or Imax for current.

The **peak value** is the greatest value of either voltage or current that the waveform reaches during each half cycle measured from the zero baseline.

For pure sinusoidal waveforms this peak value will always be the same for both half cycles (+Vm = -Vm) but for non-sinusoidal or complex waveforms the maximum peak value can be very different for each half cycle.

Instantaneous value

The instantaneous value is “the value of an alternating quantity (it may ac voltage or ac current or ac power) at a particular instant of time in the cycle”. There are uncountable number of instantaneous values that exist in a cycle.



Average Value of an AC Waveform

$$V_{average} = \frac{V_1 + V_2 + V_3 + V_4 + \ldots + V_n}{n}$$

Where: n equals the actual number of mid-ordinates used. For a pure sinusoidal waveform this average or mean value will always be equal to 0.637*Vmax and this relationship also holds true for average values of current.

Average Value

The arithmetic average of all the values of an alternating quantity over one cycle is called its average value.

$$\text{Average value} = \frac{\text{Area under one cycle}}{\text{Base}}$$

$$v_{avg} = \frac{1}{2\pi}\int_0^{2\pi} v \, d(\omega t)$$

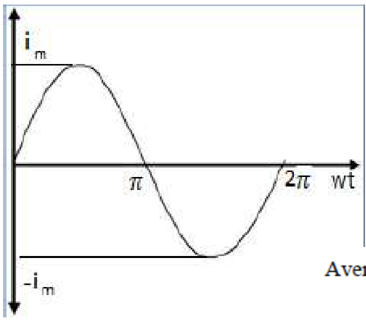
For Symmetrical waveforms, the average value calculated over one cycle becomes equal to zero because the positive area cancels the negative area. Hence for symmetrical waveforms, the average value is calculated for half cycle.

$$Average\ value = \frac{Area\ under\ one\ cycle}{Base}$$

$$v_{avg} = \frac{1}{\pi}\int_0^{\pi} v\, d(\omega t)$$

Average value of a sinusoidal current:

$$i = i_m \sin(\omega t)$$
$$i_{avg} = \frac{1}{\pi}\int_0^{\pi} i\, d(\omega t)$$
$$i_{avg} = \frac{1}{\pi}\int_0^{\pi} i_m \sin(\omega t) d(\omega t)$$
$$i_{avg} = \frac{2i_m}{\pi} = 0.637 i_m$$



Average value of a full wave rectifier output

$$i = i_m \sin(\omega t)$$
$$i_{avg} = \frac{1}{\pi}\int_0^{\pi} i\, d(\omega t)$$
$$i_{avg} = \frac{1}{\pi}\int_0^{\pi} i_m \sin(\omega t) d(\omega t)$$
$$i_{avg} = \frac{2i_m}{\pi} = 0.637 i_m$$

RMS or Effective Value

The effective or RMS value of an alternating quantity is that steady current (dc) which when flowing through a given resistance for a given time produces the same amount of heat produced by the alternating current flowing through the same resistance for the same time.

$$V_{RMS} = \sqrt{\frac{V_1^2 + V_2^2 + V_3^2 + V_4^2 + \ldots + V_n^2}{n}}$$

$$RMS = \sqrt{\frac{Area\ under\ squared\ curve}{Base}}$$

$$V_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} v^2\, d(\omega t)}$$

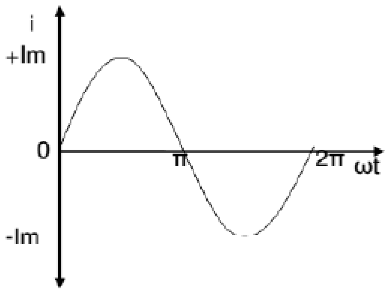
RMS value of a sinusoidal current:

$$i = i_m \sin(\omega t)$$

$$i_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i^2\, d(\omega t)}$$

$$i_{rms} = \sqrt{\frac{1}{\pi}\int_0^{\pi} i_m^2 sin^2(\omega t)\, d(\omega t)}$$

$$i_{rms} = \sqrt{\frac{i_m^2}{\pi}\int_0^{\pi} \frac{(1-\cos(2\omega t))}{2}\, d(\omega t)}$$



$$i_{rms} = \frac{i_m}{\sqrt{2}} = 0.707\, i_m$$

RMS value of a full wave rectifier output:

11. Peak factor and form factor

Both **Form Factor** and **Peak Factor** can be used to give information about the actual shape of the AC waveform. Form Factor is the ratio between the average value and the RMS value and is given as.

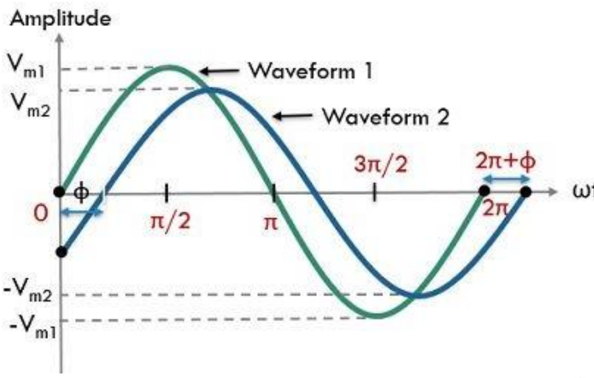
$$\text{Form Factor} = \frac{\text{R.M.S value}}{\text{Average value}} = \frac{0.707 \times \text{Vmax}}{0.637 \times \text{Vmax}} \qquad \text{Crest Factor} = \frac{\text{Peak value}}{\text{R.M.S. value}} = \frac{\text{Vmax}}{0.707 \times \text{Vmax}}$$

For a pure sinusoidal waveform, the Form Factor will always be equal to 1.11. Peak Factor is the ratio between the R.M.S. value and the Peak value of the waveform. For a pure sinusoidal waveform, the Peak Factor will always be equal to 1.414.

12. Phase difference

Phase Difference is defined as the delay between two or more alternating quantities while attaining the maxima or zero-crossings giving rise to the difference in their phases. This difference in two waves is measured in degrees or radians and is also known as phase shift.

It is sometimes defined as the difference between two or more sinusoidal waveforms in consideration with a reference axis. It is denoted by ϕ and corresponds to the shift in the waveform along the horizontal axis from a common reference point. when the peak and zero crossing of the alternating quantities with same frequency do not coincide then these quantities are said to be out of phase with respect to each other and a specific difference in the phase exists between the two.



Equation for Phase Difference

The general equation of the alternating quantities is given as:

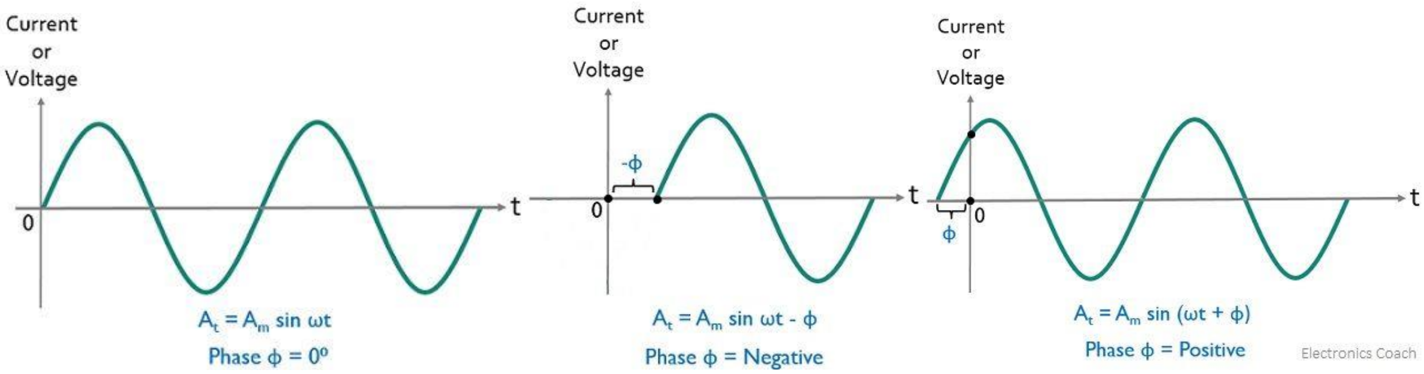
$$A_t = A_m \sin(\omega t \pm \varphi)$$

: ϕ represents the phase of the alternating quantity,

Am is the amplitude of the waveform,

ωt represents the angular frequency of the waveform.

Here the ϕ can be either positive or negative.



The relationship between voltage and current sinusoidal waveforms is very important while dealing with AC circuits as this form the base of AC Circuit analysis.

13. Rectangular and polar representation of phasors

Real numbers are not the only kind of numbers we need to use especially when dealing with frequency dependent sinusoidal sources and vectors. As well as using normal or real numbers, Complex Numbers were introduced to allow complex equations to be solved with numbers that are the square roots of negative numbers, √-1.

In electrical engineering this type of number is called an “imaginary number” and to distinguish an imaginary number from a real number the letter ” j ” known commonly in electrical engineering as the j-operator, is used. Thus, the letter “j” is placed in front of a real number to signify its imaginary number operation. Examples of imaginary numbers are: j3, j12, j100 etc. Then a complex number consists of two distinct but very much related parts, a ” Real Number ” plus an ” Imaginary Number “. Complex Numbers represent points in a two dimensional complex or s-plane that are referenced to two distinct axes. The horizontal axis is called the “real axis” while the vertical axis is called the “imaginary axis”. The real and imaginary parts of a complex number are abbreviated as Re(z) and Im(z), respectively.

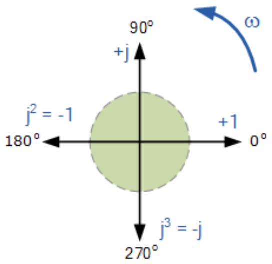
Complex numbers that are made up of real (the active component) and imaginary (the reactive component) numbers can be added, subtracted and used in exactly the same way as elementary algebra is used to analyse DC Circuits.

$$90^\circ \text{ rotation}: \quad j^1 = \sqrt{-1} = +j$$

$$180^\circ \text{ rotation}: \quad j^2 = \left(\sqrt{-1}\right)^2 = -1$$

$$270^\circ \text{ rotation}: \quad j^3 = \left(\sqrt{-1}\right)^3 = -j$$

$$360^\circ \text{ rotation}: \quad j^4 = \left(\sqrt{-1}\right)^4 = +1$$



The rules and laws used in mathematics for the addition or subtraction of imaginary numbers are the same as for real numbers, j2 + j4 = j6 etc. The only difference is in multiplication because two imaginary numbers multiplied together becomes a negative real number. Real numbers can also be thought of as a complex number but with a zero imaginary part labelled j0.
In Electrical Engineering there are different ways to represent a complex number either graphically or mathematically. One such way that uses the cosine and sine rule is called the Cartesian or Rectangular Form.

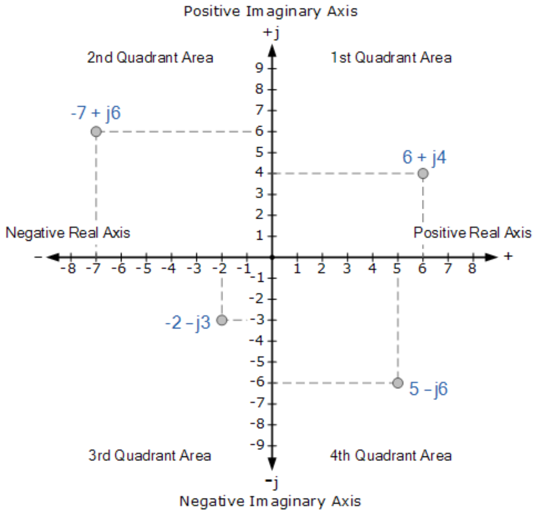
Complex Numbers using the Rectangular Form

A complex number is represented by a real part and an imaginary part that takes the generalised form of:

$$Z = x + jy$$

Where:
Z – is the Complex Number representing the Vector
x – is the Real part or the Active component
y – is the Imaginary part or the Reactive component
j – is defined by √-1

In the rectangular form, a complex number can be represented as a point on a two-dimensional plane called the complex or s-plane. So, for example, Z = 6 + j4 represents a single point whose coordinates represent 6 on the horizontal real axis and 4 on the vertical imaginary axis as shown.



Complex Numbers using the Complex or s-plane

Complex Numbers can also have "zero" real or imaginary parts such as: Z = 6 + j0 or Z = 0 + j4. In this case the points are plotted directly onto the real or imaginary axis. Also, the angle of a complex number can be calculated using simple trigonometry to calculate the angles of right-angled triangles, or measured anti-clockwise around the Argand diagram starting from the positive real axis.
Then angles between 0 and 90° will be in the first quadrant (I), angles (θ) between 90 and 180° in the second quadrant (II). The third quadrant (III) includes angles between 180 and 270° while the fourth and final quadrant (IV) which completes the full circle, includes the angles between 270 and 360° and so on. In all the four quadrants the relevant angles can be found from:
tan-1(imaginary component ÷ real component)

$$0^\circ = \pm 360^\circ = +1 = 1\angle 0^\circ = 1 + j0$$

$$+90^\circ = +\sqrt{-1} = +j = 1\angle +90^\circ = 0 + j1$$

$$-90^\circ = -\sqrt{-1} = -j = 1\angle -90^\circ = 0 - j1$$

$$\pm 180^\circ = \left(\sqrt{-1}\right)^2 = -1 = 1\angle \pm 180^\circ = -1 + j0$$

$$A + B = (4 + j1) + (2 + j3)$$

$$A + B = (4 + 2) + j(1 + 3) = 6 + j4$$

$$A - B = (4 + j1) - (2 + j3)$$

$$A + B = (4 - 2) + j(1 - 3) = 2 - j2$$

$$A \times B = (4 + j1)(2 + j3)$$

$$= 8 + j12 + j2 + j^2 3$$

$$\text{but } j^2 = -1,$$

$$= 8 + j14 - 3$$

$$A \times B = 5 + j14$$

Addition, Subtraction, Multiplication and Division of Complex Numbers

The Addition, Subtraction, Multiplication and Division of complex numbers can be done either mathematically or graphically in rectangular form. For addition, the real parts are firstly added together to form the real part of the sum, and then the imaginary parts to form the imaginary part of the sum and this process is as follows using two complex numbers A and B as examples.

$$\frac{A}{B} = \frac{4 + j1}{2 + j3}$$

Multiply top & bottom by Conjugate of 2 + j3

$$\frac{4 + j1}{2 + j3} \times \frac{2 - j3}{2 - j3} = \frac{8 - j12 + j2 - j^2 3}{4 - j6 + j6 - j^2 9}$$

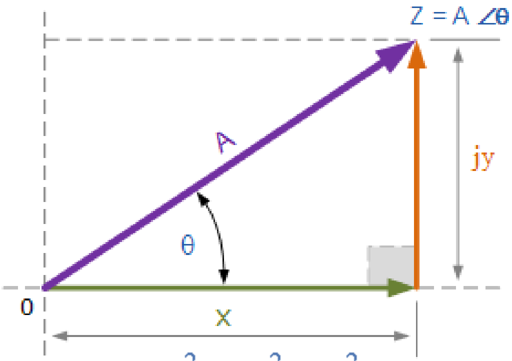
$$= \frac{8 - j10 + 3}{4 + 9} = \frac{11 - j10}{13}$$

$$= \frac{11}{13} + \frac{-j10}{13} = 0.85 - j0.77$$

The Complex Conjugate

The Complex Conjugate, or simply Conjugate of a complex number is found by reversing the algebraic sign of the complex numbers imaginary number only while keeping the algebraic sign of the real number the same and to identify the complex conjugate of z the symbol z is used. For example, the conjugate of z = 6 + j4 is z* = 6 – j4, likewise the conjugate of z = 6 – j4 is z* = 6 + j4.

Complex Numbers using Polar Form



$$A^2 = x^2 + y^2$$

$$A = \sqrt{x^2 + y^2}$$

$$\text{Also,} \quad x = A.\cos\theta, \quad y = A.\sin\theta$$

$$\theta = \tan^{-1}\frac{y}{x}$$

Unlike rectangular form which plots points in the complex plane, the Polar Form of a complex number is written in terms of its magnitude and angle. Thus, a polar form vector is presented as: $Z = A\angle\pm\theta$, where: Z is the complex number in polar form, A is the magnitude or modulo of the vector and θ is its angle or argument of A which can be either positive or negative. The magnitude and angle of the point still remains the same as for the rectangular form above, this time in polar form the location of the point is represented in a "triangular form" as shown below.

As the polar representation of a point is based around the triangular form, we can use simple geometry of the triangle and especially trigonometry and Pythagoras's Theorem on triangles to find both the magnitude and the angle of the complex number.

Then in Polar form the length of A and its angle represents the complex number instead of a point. Also in polar form, the conjugate of the complex number has the same magnitude or modulus it is the sign of the angle that changes, so for example the conjugate of $6\angle 30^o$ would be $6\angle -30^o$.

Converting between Rectangular Form and Polar Form

In the rectangular form we can express a vector in terms of its rectangular coordinates, with the horizontal axis being its real axis and the vertical axis being its imaginary axis or j-component. In polar form these real and imaginary axes are simply represented by "A∠θ".

$$6\angle 30^o = x + jy$$

However,

$$x = A.\cos\theta \quad y = A.\sin\theta$$

Therefore,

$$\begin{aligned} 6\angle 30^0 &= (6\cos\theta) + j(6\sin\theta) \\ &= (6\cos 30^o) + j(6\sin 30^o) \\ &= (6\times 0.866) + j(6\times 0.5) \\ &= 5.2 + j3 \end{aligned}$$

$$(5.2 + j3) = A\angle\theta$$

$$\text{where:} \quad A = \sqrt{5.2^2 + 3^2} = 6$$

$$\text{and} \quad \theta = \tan^{-1}\frac{3}{5.2} = 30^o$$

$$\text{Hence,} \quad (5.2 + j3) = 6\angle 30^0$$

Polar Form Multiplication and Division

Rectangular form is best for adding and subtracting complex numbers as we saw above, but polar form is often better for multiplying and dividing. To multiply together two vectors in polar form, we must first multiply together the two modulus or magnitudes and then add together their angles. Likewise, to divide together two vectors in polar form, we must divide the two modulus and then subtract their angles as shown.

$$Z_1 \times Z_2 = A_1 \times A_2 \angle\theta_1 + \theta_2$$

$$Z_1 \times Z_2 = 6 \times 8 \angle 30^o + (-45^o) = 48\angle -15^o$$

$$\frac{Z_1}{Z_2} = \left(\frac{A_1}{A_2}\right)\angle\theta_1 - \theta_2$$

$$\frac{Z_1}{Z_2} = \left(\frac{6}{8}\right)\angle 30^0 - (-45^o) = 0.75\angle 75^0$$

complex number in exponential form

$$Z = Ae^{j\phi}$$

$$Z = A(\cos\phi + j\sin\phi)$$

a third method for representing a complex number which is similar to the polar form that corresponds to the length (magnitude) and phase angle of the sinusoid but uses the base of the natural logarithm, e = 2.718281.. to find the value of the complex number. This third method is called the Exponential Form.