Unit 1
DC Circuits

Network theory is the study of solving the problems of electric circuits or electric networks.
Basic Terminology

- Electric Circuit
- Electric Network
- Current
- Voltage
- Power

An Electric circuit is an interconnection of various elements in which there is at least one closed path in which current can flow. An Electric circuit is used as a component for any engineering system. The performance of any electrical device or machine is always studied by drawing its electrical equivalent circuit. By simulating an electric circuit, any type of system can be studied for e.g., mechanical, hydraulic thermal, nuclear, traffic flow, weather prediction etc. All control systems are studied by representing them in the form of electric circuits. The analysis, of any system can be learnt by mastering the techniques of circuit theory. The analysis of any system can be learnt by mastering the techniques of circuit theory.

Electric Field

An electric field is the physical field that surrounds electrically charged particles and exerts force on all other charged particles in the field, either attracting or repelling them. It also refers to the physical field for a system of charged particles. Electric fields originate from electric charges and time-varying electric currents. Electric fields and magnetic fields are both manifestations of the electromagnetic field, one of the four fundamental interactions (also called forces) of nature.

Electric field can be considered as an electric property associated with each point in the space where a charge is present in any form. An electric field is also described as the electric force per unit charge.
The formula of electric field is given as;
$E = F / Q$
Where,
E is the electric field.
F is a force.
Q is the charge.
Electric fields are usually caused by varying magnetic fields or electric charges. Electric field strength is measured in the SI unit volt per meter (V/m).

The direction of the field is taken as the direction of the force which is exerted on the positive charge. The electric field is radially outwards from positive charge and radially in towards negative point charge.

Electric Charge

"Electric Charge is the property of subatomic particles that causes it to experience a force when placed in an electric and magnetic field."
Electric charges are of two types: Positive and Negative, commonly carried by charge carrier's protons and electrons.
Examples of the types of charges are subatomic particles or the particles of matter:
Protons are positively charged.
Electrons are negatively charged.
Neutrons have zero charge.

Electric charge is a scalar quantity. Coloumb is the unit of electric charge. "One coulomb is the quantity of charge transferred in one second."

Electric Circuit

An electric circuit contains a closed path for providing a flow of electrons from a voltage source or current source. The elements present in an electric circuit will be in series connection, parallel connection, or in any combination of series and parallel connections.

Electric Network

An electric network need not contain a closed path for providing a flow of electrons from a voltage source or current source. Hence, we can conclude that "all electric circuits are electric networks" but the converse need not be true.

Current

The current "I" flowing through a conductor is nothing but the time rate of flow of charge. Mathematically, it can be written as

$$I = \frac{dQ}{dt}$$

Where,

- Q is the charge and its unit is Coloumb.
- t is the time and its unit is second.

As an analogy, electric current can be thought of as the flow of water through a pipe. Current is measured in terms of Ampere. In general, Electron current flows from negative terminal of source to positive terminal, whereas, Conventional current flows from positive terminal of source to negative terminal. Electron current is obtained due to the movement of free electrons, whereas, Conventional current is obtained due to the movement of free positive charges. Both of these are called as electric current.

Voltage

The voltage "V" is nothing but an electromotive force that causes the charge (electrons) to flow. Mathematically, it can be written as

$$V = \frac{dW}{dQ}$$

Where,

- W is the potential energy and its unit is Joule.
- Q is the charge and its unit is Coloumb.

As an analogy, Voltage can be thought of as the pressure of water that causes the water to flow through a pipe. It is measured in terms of Volt.

Power & Energy

The power "P" is nothing but the time rate of flow of electrical energy. Mathematically, it can be written as

$$P = \frac{dW}{dt}$$

Where,

- W is the electrical energy and it is measured in terms of Joule.
- t is the time and it is measured in seconds.

We can re-write the above equation a

$$P = \frac{dW}{dt} = \frac{dW}{dQ} \times \frac{dQ}{dt} = VI$$

Therefore, power is nothing but the product of voltage V and current I. Its unit is Watt.

2. Types of Network Elements

We can classify the Network elements into various types based on some parameters. Following are the types of Network elements −

- Active Elements and Passive Elements
- Linear Elements and Non-linear Elements
- Bilateral Elements and Unilateral Elements
- Lumped Elements and Distributed Elements

i) Active Elements and Passive Elements

We can classify the Network elements into either active or passive based on the ability of delivering power.

- Active Elements deliver power to other elements, which are present in an electric circuit. Sometimes, they may absorb the power like passive elements. That means active elements have the capability of both delivering and absorbing power. Examples: Voltage sources and current sources.
- Passive Elements can’t deliver power (energy) to other elements; however, they can absorb power. That means these elements either dissipate power in the form of heat or store energy in the form of either magnetic field or electric field. Examples: Resistors, Inductors, and capacitors.

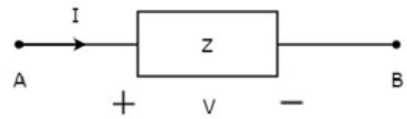
ii) Linear Elements and Non-Linear Elements

We can classify the network elements as linear or non-linear based on their characteristic to obey the property of linearity.

- Linear Elements are the elements that show a linear relationship between voltage and current. Examples: Resistors, Inductors, and capacitors.
- Non-Linear Elements are those that do not show a linear relation between voltage and current. Examples: Voltage sources and current sources.

iii) Bilateral Elements and Unilateral Elements
Network elements can also be classified as either bilateral or unilateral based on the direction of current flows through the network elements.

- Bilateral Elements are the elements that allow the current in both directions and offer the same impedance in either direction of current flow. Examples: Resistors, Inductors and capacitors. The concept of Bilateral elements is illustrated in the following figure.

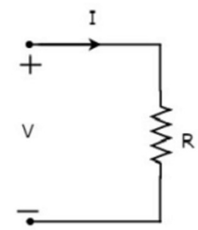


iv) Lumped and Distributed Elements

- Lumped elements are those elements which are very small in size & in which simultaneous actions takes place. Typical lumped elements are capacitors, resistors, inductors.
- Distributed elements are those which are not electrically separable for analytical purposes. For example, a transmission line has distributed parameters along its length and may extend for hundreds of miles.

3. Electrical circuit elements (R, L and C)

Resistor



The main functionality of Resistor is either opposes or restricts the flow of electric current. Hence, the resistors are used in order to limit the amount of current flow and / or dividing (sharing) voltage. Let the current flowing through the resistor is I amperes and the voltage across it is V volts. The symbol of resistor along with current, I and voltage, V are shown in the following figure.
According to **Ohm's law**, the voltage across resistor is the product of current flowing through it and the resistance of that resistor. Mathematically, it can be represented as

$$V = IR \qquad \text{Equation 1}$$

$$\Rightarrow I = \frac{V}{R} \qquad \text{Equation 2}$$

Where, R is the resistance of a resistor.
From Equation 2, we can conclude that the current flowing through the resistor is directly proportional to the applied voltage across resistor and inversely proportional to the resistance of resistor.
Power in an electric circuit element can be represented as

$$P = VI \qquad \text{Equation 3}$$

Substitute, Equation 1 in Equation 3.

$$P = (IR)I$$

$$\Rightarrow P = I^2R \qquad \text{Equation 4}$$

Substitute, Equation 2 in Equation 3.

$$P = V\left(\frac{V}{R}\right)$$

$$\Rightarrow P = \frac{V^2}{R} \qquad \text{Equation 5}$$

So, we can calculate the amount of power dissipated in the resistor by using one of the formulae mentioned in Equations 3 to 5.

Inductor

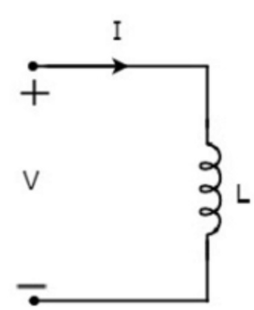
In general, inductors will have number of turns. Hence, they produce magnetic flux when current flows through it. So, the amount of total magnetic flux produced by an inductor depends on the current, I flowing through it and they have linear relationship. Mathematically, it can be written as

$$\Psi \, \alpha \, I$$

$$\Rightarrow \Psi = LI$$

Where,

- Ψ is the total magnetic flux
- L is the inductance of an inductor



Let the current flowing through the inductor is I amperes and the voltage across it is V volts. The symbol of inductor along with current I and voltage V are shown in the following figure.
According to Faraday's law, the voltage across the inductor can be written as

$$V = \frac{d\Psi}{dt}$$

$$V = \frac{d(LI)}{dt}$$

$$\Rightarrow V = L\frac{dI}{dt}$$

$$\Rightarrow I = \frac{1}{L}\int V dt$$

From the above equations, we can conclude that there exists a linear relationship between voltage across inductor and current flowing through it.
We know that power in an electric circuit element can be represented as P= VI
By integrating the above equation, we will get the energy stored in an inductor as $W = \frac{1}{2}LI^2$
So, the inductor stores the energy in the form of magnetic field.

Capacitor

In general, a capacitor has two conducting plates, separated by a dielectric medium. If positive voltage is applied across the capacitor, then it stores positive charge. Similarly, if negative voltage is applied across the capacitor, then it stores negative charge.

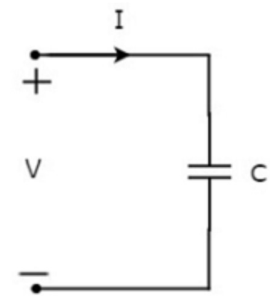
So, the amount of charge stored in the capacitor depends on the applied voltage V across it and they have linear relationship. Mathematically, it can be written as

$$Q \alpha V$$

$$\Rightarrow Q = CV$$

Where,

- Q is the charge stored in the capacitor.
- C is the capacitance of a capacitor.



Let the current flowing through the capacitor is I amperes and the voltage across it is V volts. The symbol of capacitor along with current I and voltage V are shown in the following figure. We know that the current is nothing but the time rate of flow of charge. Mathematically, it can be represented as

$$I = \frac{dQ}{dt}$$

Substitute $Q = CV$ in the above equation.

$$I = \frac{d(CV)}{dt}$$

$$\Rightarrow I = C\frac{dV}{dt}$$

$$\Rightarrow V = \frac{1}{C}\int I dt$$

From the above equations, we can conclude that there exists a linear relationship between voltage across capacitor and current flowing through it.
We know that power in an electric circuit element can be represented as P = VI. By integrating the above equation, we will get the energy stored in the capacitor as $W = \frac{1}{2}CV^2$ So, the capacitor stores the energy in the form of electric field.

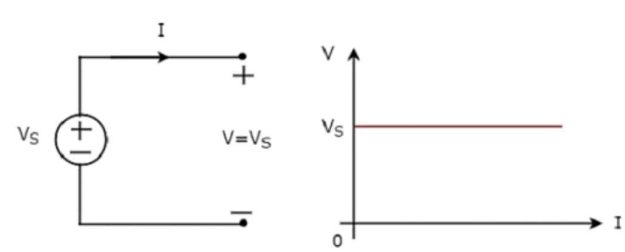
4. Voltage and Current Sources

Types of Sources

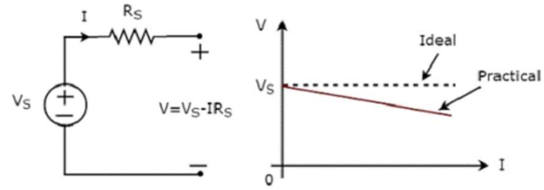
Active Elements are the network elements that deliver power to other elements present in an electric circuit. So, active elements are also called as sources of voltage or current type.

a. Voltage Sources

- An independent voltage source produces a constant voltage across its two terminals. This voltage is independent of the amount of current that is flowing through the two terminals of voltage source. The V-I characteristics of an **independent ideal voltage**



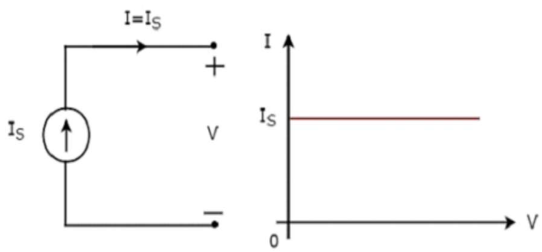
source is a constant line, which is always equal to the source voltage (V_S) irrespective of the current value (I). So, the internal resistance of an independent ideal voltage source is zero Ohms. Hence, the independent ideal voltage sources do not exist practically, because there will be some internal resistance



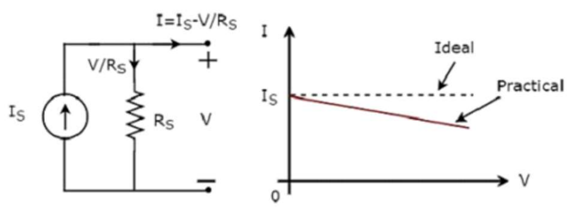
- There is a deviation in the V-I characteristics of an **independent practical voltage source** from the V-I characteristics of an independent ideal voltage source. This is due to the voltage drop across the internal resistance (R_S) of an independent practical voltage source.
- A **Dependent Voltage Source or controlled voltage source**, provides a voltage supply whose magnitude depends on either the voltage across or current flowing through some other circuit element. A dependent voltage source is indicated with a diamond shape and are used as equivalent electrical sources for many electronic devices, such as transistors and operational amplifiers.

(a) Voltage-controlled voltage source (b) Current-controlled voltage source

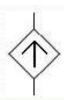
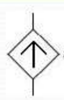
b. Current Sources



- An independent current source produces a constant current. This current is independent of the voltage across its two terminals. The V-I characteristics of an **independent ideal current source** is a constant line, which is always equal to the source current (IS) irrespective of the voltage value (V). So, the internal resistance of an independent ideal current source is infinite ohms. Hence, the independent ideal current sources do not exist practically, because there will be some internal resistance.



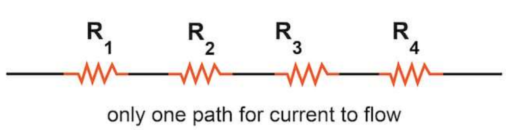
- There is a deviation in the V-I characteristics of an **independent practical current source** from the V-I characteristics of an independent ideal current source. This is due to the amount of current flows through the internal shunt resistance (RS) of an independent practical current source.

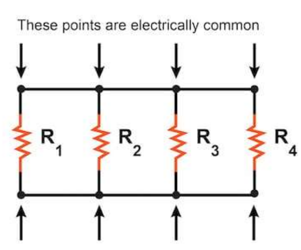
Current Controlled Current Source	CCCS	 aI
Voltage Controlled Current Source	VCCS	 aV

- A controlled or dependent current source on the other hand changes its available current depending upon the voltage across, or the current through, some other element connected to the circuit. In other words, the output of a dependent current source is controlled by another voltage or current.

5. Series and parallel circuit & Network Reduction

The definition of a series circuit is a circuit where the components are connected end-to-end. a parallel circuit is a circuit where all components are connected across each other's leads.



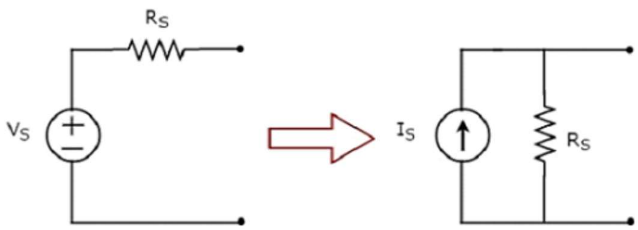


Network reduction is used to transform part of a system or an external system into an equivalent system that is easier to solve and often smaller but has the same overall parameters of the original system. The reduced system may not contain all the elements and modelling of the parent system but it has the same electrical characteristics. By reducing the system size, we can make reduce simulation times which is important for large power systems.

6. Source Transformation Technique

We know that there are two practical sources, namely, voltage source and current source. We can transform (convert) one source into the other based on the requirement, while solving network problems. The technique of transforming one source into the other is called as source transformation technique. Following are the two possible source transformations

- Practical voltage source into a practical current source
- Practical current source into a practical voltage source



Practical voltage source into a practical current source

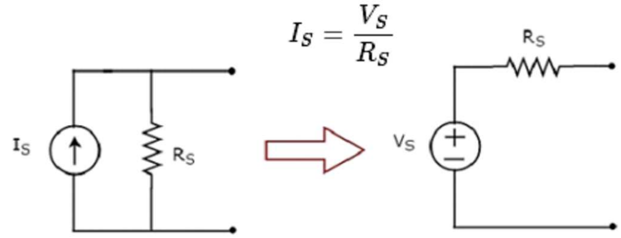
Practical voltage source consists of a voltage source (V_S) in series with a resistor (R_S). This can be converted into a practical current source. It consists of a current source (I_S) in parallel with a resistor (R_S).

The value of I_S will be equal to the ratio of V_S and R_S. Mathematically, it can be represented as

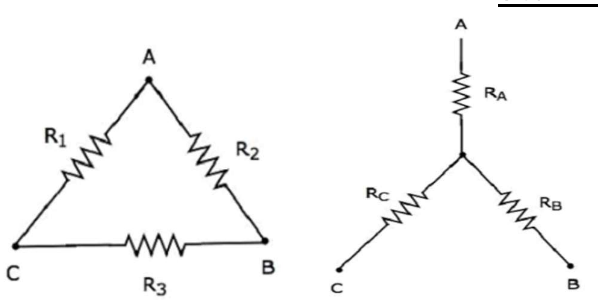
$$I_S = \frac{V_S}{R_S}$$

Practical current source into a practical voltage source

Practical current source consists of a current source (I_S) in parallel with a resistor (R_S). This can be converted into a practical voltage source. It consists of a voltage source (V_S) in series with a resistor (R_S). The value of V_S will be equal to the product of I_S and R_S. Mathematically, it can be represented as $V_s = I_sR_s$



7. Star Delta Transformation



$$R_A = \frac{R_1R_2}{R_1+R_2+R_3}$$

$$R_B = \frac{R_2R_3}{R_1+R_2+R_3}$$

$$R_C = \frac{R_3R_1}{R_1+R_2+R_3}$$

$$\frac{R_AR_B + R_BR_C + R_CR_A}{R_B} = R_1$$

$$\Rightarrow R_1 = R_C + R_A + \frac{R_CR_A}{R_B}$$

$$R_2 = R_A + R_B + \frac{R_AR_B}{R_C}$$

$$R_3 = R_B + R_C + \frac{R_BR_C}{R_A}$$

8. Division Principles

Let us discuss about the following two division principles of electrical quantities.

- Current Division Principle
- Voltage Division Principle

Current Division Principle

When two or more passive elements are connected in parallel, the amount of current that flows through each element gets divided(shared) among themselves from the current that is entering the node.

Voltage Division Principle

When two or more passive elements are connected in series, the amount of voltage present across each element gets divided (shared) among themselves from the voltage that is available across that entire combination.

9. Network Theorems

Electric circuit theorems are always beneficial to help find voltage and currents in multi-loop circuits. These theorems use fundamental rules or formulas and basic equations of mathematics to analyse basic components of electrical or electronics parameters such as voltages, currents, resistance, and so on. These fundamental theorems include the basic theorems like Superposition theorem, Tellegen’s theorem, Norton’s theorem, Maximum power transfer theorem, and Thevenin’s theorems. Another group of network theorems that are mostly used in the circuit analysis process includes the Compensation theorem, Substitution theorem, Reciprocity theorem, Millman’s theorem, and Miller’s theorem.

I. Kirchoff's Law

Network elements can be either of active or passive type. Any electrical circuit or network contains one of these two types of network elements or a combination of both. Now, let us discuss about the following two laws, which are popularly known as Kirchhoff's laws.

- Kirchhoff's Current Law
- Kirchhoff's Voltage Law

a. Kirchoff's Voltage Law

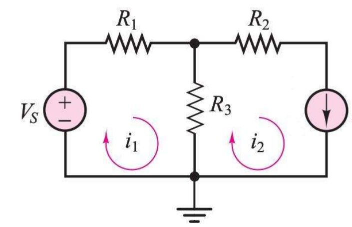
Kirchhoff's Voltage Law (KVL) states that the algebraic sum of voltages around a loop or mesh is equal to zero. A Loop is a path that terminates at the same node where it started from. In contrast, a Mesh is a loop that doesn't contain any other loops inside it.

Mathematically, KVL can be represented as

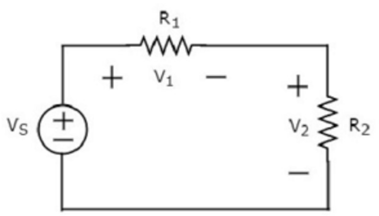
$$\sum_{n=1}^{N} V_n = 0$$

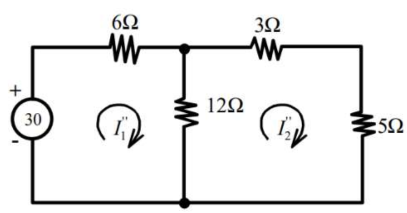
Where,

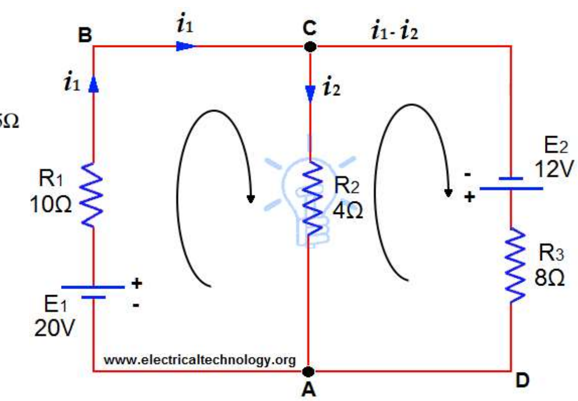
- V_n is the nth element's voltage in a loop (mesh).
- N is the number of network elements in the loop (mesh).

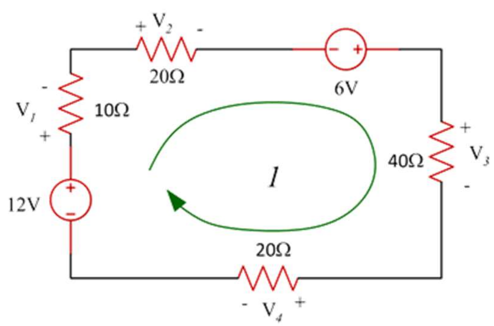


The above statement of KVL can also be expressed as "the algebraic sum of voltage sources is equal to the algebraic sum of voltage drops that are present in a loop."









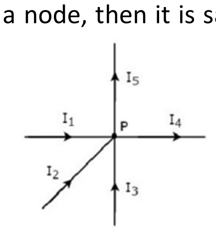
b. Kirchoff's Current Law

Kirchhoff's Current Law (KCL) states that the algebraic sum of currents leaving (or entering) a node is equal to zero.

A Node is a point where two or more circuit elements are connected to it. If only two circuit elements are connected to a node, then it is said to be simple node. If three or more circuit elements are connected to a node, then it is said to be Principal Node.

Mathematically, KCL can be represented as

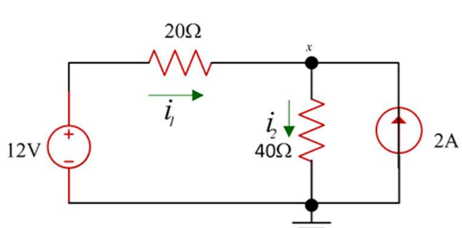
$$\sum_{m=1}^{M} I_m = 0$$

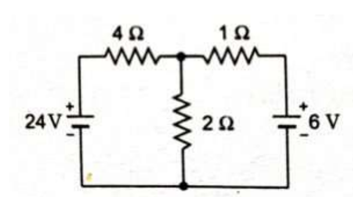


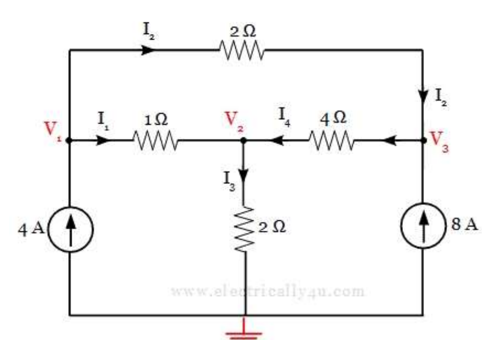
Where,

- I_m is the m^{th} branch current leaving the node.
- M is the number of branches that are connected to a node.

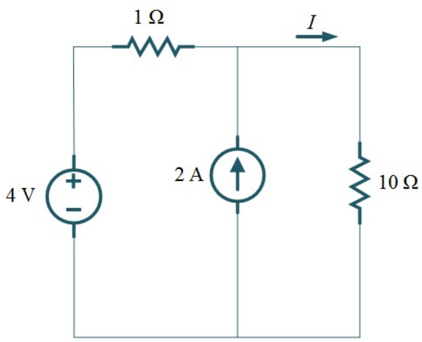
The above statement of KCL can also be expressed as "the algebraic sum of currents entering a node is equal to the algebraic sum of currents leaving a node".

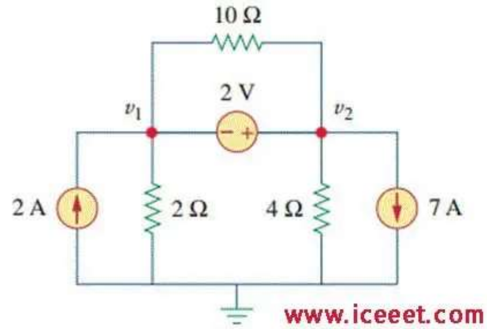


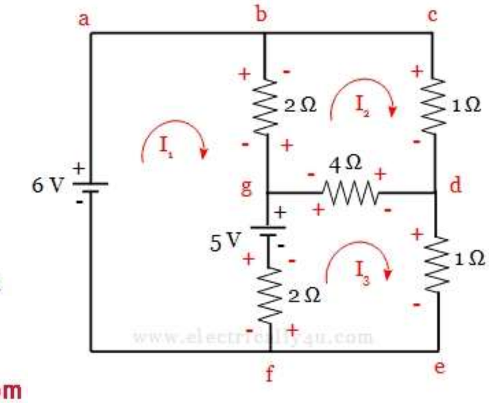




c. Super Mesh and Super node analysis





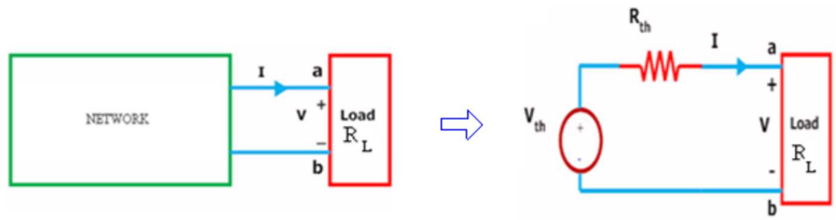


II. Thevenin and Norton's Theorem

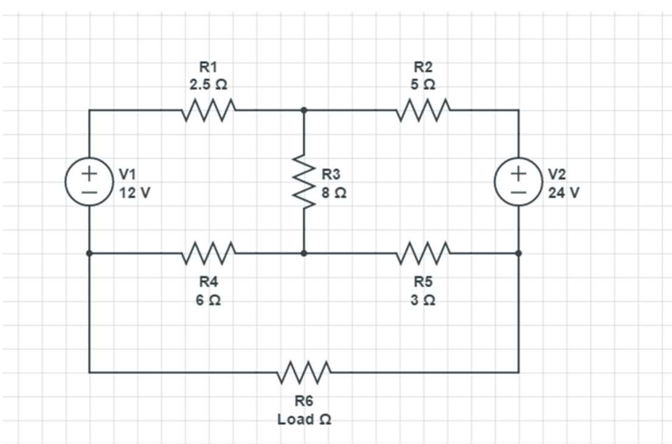
Any complicated network i.e., several sources, multiple resistors are present if the single element response is desired then use the network theorems. Network theorems are also can be termed as network reduction techniques.

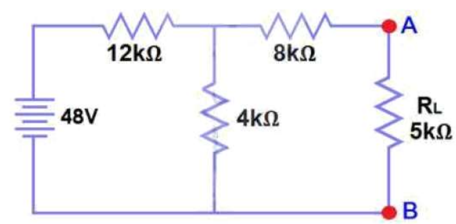
Thevenin's Theorem and Norton's theorem are two important theorems in solving Network problems having many active and passive elements. Using these theorems, the networks can be reduced to simple equivalent circuits with one active source and one element. Thevenin's and Norton's theorems are dual theorems.

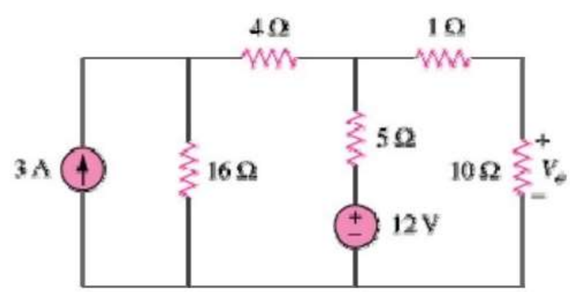
a. Thevenin's theorem



Any linear, bilateral two terminal network consisting of sources and resistors(Impedance),can be replaced by an equivalent circuit consisting of a voltage source in series with a resistance (Impedance).The equivalent voltage source V_{Th} is the open circuit voltage looking into the terminals(with concerned branch element removed) and the equivalent resistance R_{Th} while all sources are replaced by their internal resistors at ideal condition i.e. voltage source is short circuit and current source is open circuit.

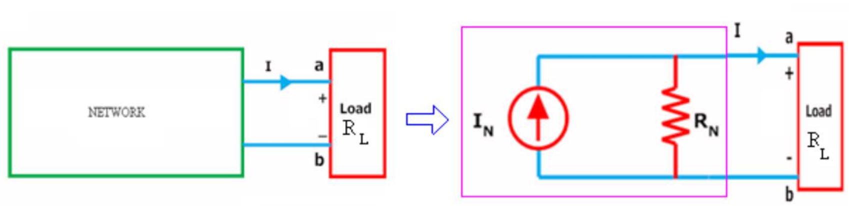






b. Norton's theorem

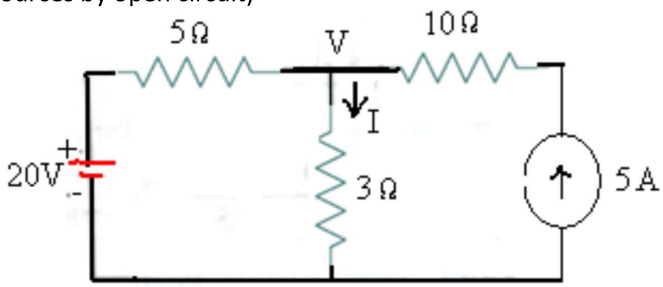
Any linear, bilateral two terminal network consisting of sources and resistors(Impedance),can be replaced by an equivalent circuit consisting of a current source in parallel with a resistance (Impedance),the current source being the short circuited current across the load terminals and the resistance being the internal resistance of the source network looking through the open circuited load terminals.

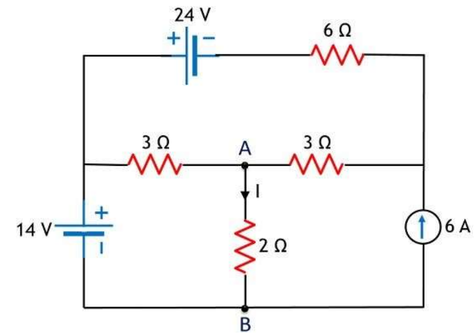


III. Superposition Theorem

The principle of superposition helps us to analyse a linear circuit with more than one current or voltage sources sometimes it is easier to find out the voltage across or current in a branch of the circuit by considering the effect of one source at a time by replacing the other sources with their ideal internal resistances.

Any linear, bilateral two terminal network consisting of more than one sources, the total current or voltage in any part of a network is equal to the algebraic sum of the currents or voltages in the required branch with each source acting individually while other sources are replaced by their ideal internal resistances. (i.e., Voltage sources by a short circuit and current sources by open circuit)



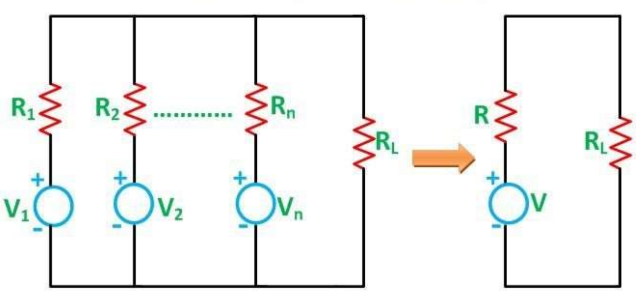


IV. Millman's Theorem

Millman's theorem is one of the most useful methods for particularly electrical circuits which contain many voltage sources. Based on this theorem, each voltage source as well as their particular resistors generates a current & the amount of all currents is equivalent to the whole current generated through the circuit.

In Millman's Theorem the number of parallel voltage sources can be reduced to one equivalent source.

$$R = \frac{1}{G} = \frac{1}{G_1 + G_2 + ,,,,,,, + G_n}$$



And as we know,
I = V/R, and we can also write R = I/G as G = I/R
So, the equation can be written as

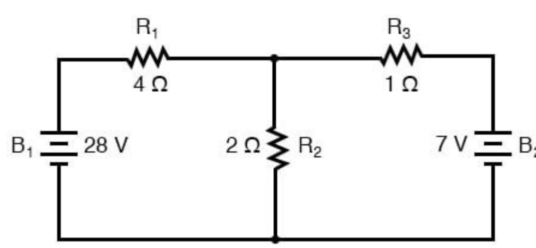
$$V = \frac{\pm \frac{V_1}{R_1} \pm \frac{V_2}{R_2} \pm \dots\dots\dots\dots \pm \frac{V_n}{R_n}}{\frac{1}{R_1} + \frac{1}{R_2} + \dots\dots\dots + \frac{1}{R_n}}$$

Where R is the equivalent resistance connected to the equivalent voltage source in series.

Thus, the final equation becomes

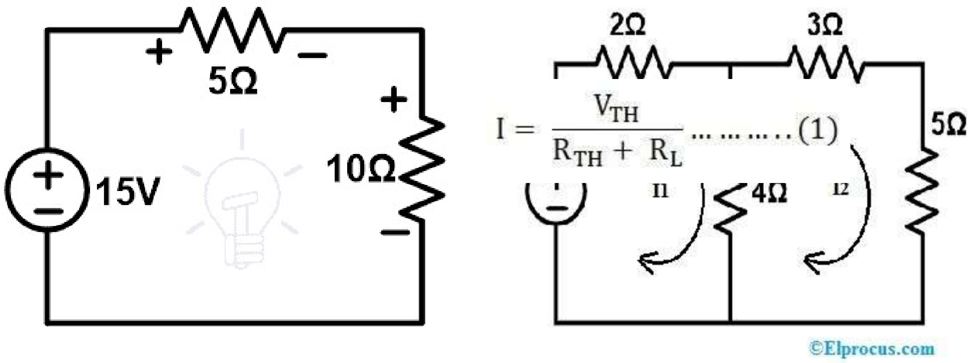
$$V = \frac{\pm V_1 G_1 \pm V_2 G_2 \pm \dots\dots\dots \pm V_n G_n}{G_1 + G_2 + \dots\dots + G_n} \quad \text{or}$$

$$V = \frac{\sum_{K=1}^{n} V_K G_K}{\sum_{K=1}^{n} G_K} \quad \text{and} \quad G_K = \frac{1}{R_K}$$



V. Substitution Theorem

Substitution Theorem states that the voltage across any branch or the current through that branch of a network being known, the branch can be replaced by the combination of various elements that will make the same voltage and current through that branch. In other words, the Substitution Theorem says that for branch equivalence, the terminal voltage and current must be the same.

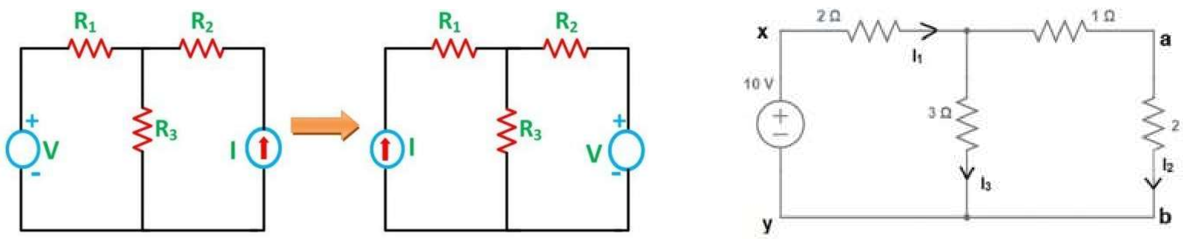


$$I = \frac{V_{TH}}{R_{TH} + R_L} \ldots\ldots\ldots\ldots(1)$$

VI. Reciprocity Theorem

In any branch of a network or circuit, the current due to a single source of voltage (V) in the network is equal to the current through that branch in which the source was originally placed when the source is again put in the branch in which the current was originally obtained. This theorem is used in the bilateral linear network which consists of bilateral components.

The location of the voltage source and the current source may be interchanged without a change in current. However, the polarity of the voltage source should be identical with the direction of the branch current in each position.



VII. Maximum Power Transfer Theorem

Maximum power transfer theorem states that the DC voltage source will deliver maximum power to the variable load resistor only when the load resistance is equal to the source resistance.

The Maximum Power Transfer theorem is used to find the load resistance for which there would be the maximum amount of power transfer from the source to the load.

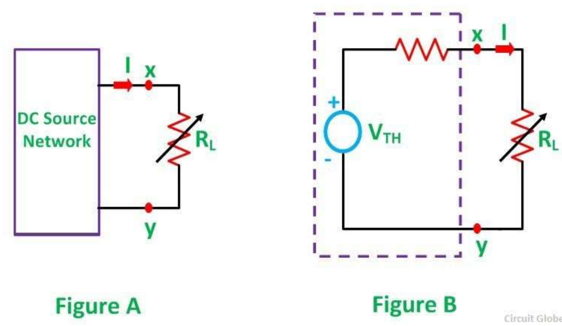


Figure A

Figure B

$$P_L = I^2 R_L \ldots\ldots\ldots\ldots(2)$$

$$P_L = \left(\frac{V_{TH}}{R_{TH} + R_L}\right)^2 x\, R_L$$

$$\frac{dP_L}{dR_L} = \frac{V_{TH}^2\,(R_{TH} + R_L - 2R_L)}{(R_{TH} + R_L)^3} = \frac{V_{TH}^2\,(R_{TH} - R_L)}{(R_{TH} + R_L)^2}$$

$$\frac{V_{TH}^2\,(R_{TH} - R_L)}{(R_{TH} + R_L)^2} = 0 \qquad (R_{TH} - R_L) = 0 \quad \text{or} \quad R_{TH} = R_L$$

$$P_{max} = \frac{V_{TH}^2 R_{TH}}{(R_{TH} + R_{TH})^2} = \frac{V_{TH}^2}{4R_{TH}} \ldots\ldots\ldots\ldots\ldots(3)$$

During Maximum Power Transfer the efficiency η becomes:

$$P = 2\,\frac{V_{TH}^2}{4R_{TH}} = \frac{V_{TH}^2}{2R_{TH}} \qquad\qquad \eta = \left(\frac{P_{max}}{P}\right) x\, 100 = 50\%$$