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ASSIGNMENT - 1

(a) test for consistency and solve.

(i)
$$2x - 3y + 7z = 5$$
$$3x + y - 3z = 13$$
$$2x + 19y - 47z = 32$$

⇒ $AX = B$

$$A = \begin{bmatrix} 2 & -3 & 7 \\ 3 & 1 & -3 \\ 2 & 19 & -47 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} 5 \\ 13 \\ 32 \end{bmatrix}$$

augmented matrix $= [A:B]$

$$\left[\begin{array}{ccc:c} 2 & -3 & 7 & 5 \\ 3 & 1 & -3 & 13 \\ 2 & 19 & -47 & 32 \end{array}\right]$$

$R_2 \to R_2 - 3/2 R_1$, $R_3 \to R_3 - R_1$

$$\left[\begin{array}{ccc:c} 2 & -3 & 7 & 5 \\ 0 & 5.5 & -13.5 & 5.5 \\ 0 & 22 & -54 & 27 \end{array}\right]$$

$R_3 \to R_3 - 22/5.5 R_2$

$$\left[\begin{array}{ccc:c} 2 & -3 & 7 & 5 \\ 0 & 5.5 & -13.5 & 5.5 \\ 0 & 0 & 0 & 5 \end{array}\right]$$

Rank of $[A:B] \neq$ Rank of A

$3 \neq 2$

$\rho[A:B] \neq \rho[A]$ so inconsistency

no solution

(2)

(b) $2x - y + 3z = 8$

$-x + 2y + z = 4$

$3x + y - 4z = 0$

$\Rightarrow$ $AX = B$

$$A = \begin{bmatrix} 2 & -1 & 3 \\ -1 & 2 & 1 \\ 3 & 1 & -4 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} 8 \\ 4 \\ 0 \end{bmatrix}$$

augmented matrix $= [A:B]$

$$= \begin{bmatrix} 2 & -1 & 3 & : & 8 \\ -1 & 2 & 1 & : & 4 \\ 3 & 1 & -4 & : & 0 \end{bmatrix}$$

$R_2 \to R_2 + R_1/2$, $R_3 \to R_3 - 3/2 R_1$

$$= \begin{bmatrix} 2 & -1 & 3 & : & 8 \\ 0 & 1.5 & 2.5 & : & 8 \\ 0 & 2.5 & -8.5 & : & -12 \end{bmatrix}$$

$R_3 \to R_3 - \frac{2.5}{1.5} R_2$

$$= \begin{bmatrix} 2 & -1 & 3 & : & 8 \\ 0 & 1.5 & 2.5 & : & 8 \\ 0 & 0 & -12.667 & : & -25.33 \end{bmatrix}$$

$\rho[A:B] = \rho A = n$

So consistency and one solution

So $-12.66z = -25.33$

$1.5y + 2.5z = 8$

$2x - y + 3z = 8$

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$$-12.66\,z = -25.33$$

$$z = \frac{-25.33}{-12.66}$$

$$\boxed{z = 2.000}$$

$$1.5y + 2.5\times 2 = 8$$

$$1.5y = 8 - 5$$

$$y = \frac{3}{1.5}$$

$$\boxed{y = 2}$$

$$2x - 2 + 3\times 2 = 8$$

$$2x = 8 - 4$$

$$x = \frac{4}{2}$$

$$\boxed{x = 2}$$

(iii) $4x - y = 12$

$-x + 5y - 2z = 0$

$-2x + 4z = -8$

$\Rightarrow$ $AX = B$

$$A = \begin{bmatrix} 4 & -1 & 0 \\ -1 & 5 & -2 \\ -2 & 0 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 12 \\ 0 \\ -8 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

augmented matrix $= [A \vdots B]$

$$= \begin{bmatrix} 4 & -1 & 0 & \vdots & 12 \\ -1 & 5 & -2 & \vdots & 0 \\ -2 & 0 & 4 & \vdots & -8 \end{bmatrix}$$

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$R_2 \to R_2 - 4R_1 \quad , \quad R_3 \to R_3 + 2R_1$

$$= \left[\begin{array}{ccc:c} 1 & -5 & 2 & 0 \\ 0 & 19 & -8 & 12 \\ 0 & -10 & 8 & -8 \end{array}\right]$$

$R_3 \to R_3 + {}^{10}/_{19} R_2$

$$= \left[\begin{array}{ccc:c} 1 & -5 & 2 & 0 \\ 0 & 19 & -8 & 12 \\ 0 & 0 & 3.792 & -1.688 \end{array}\right]$$

$\rho[A:B] = \rho(A) = n = 3$

so consistency and one solution

so $z\,3.79 = -1.68$

$19y - 8z = 12$

$x - 5y + 2z = 0$

$z\,3.79 = -1.68$

$$z = \frac{-1.68}{3.79}$$

$\boxed{Z = -0.44}$

$19y - 8\times(-0.44) = 12$

$19y = 12 - 3.52$

$$y = \frac{8.48}{19}$$

$\boxed{y = 0.44}$

$x - 5(0.44) + 2(-0.44) = 0$

$x - 2.2 - 0.88$

$\boxed{x = 3.08}$

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(b) for what values of λ and μ the given system of equations

$$x + y + z = 6$$
$$x + 2y + 3z = 10$$
$$x + 2y + \lambda z = \mu$$

has (i) no solution (ii) a unique solution (iii) infinite number of solutions?

$\Rightarrow$ $AX = B$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \lambda \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} 6 \\ 10 \\ \mu \end{bmatrix}$$

augmented matrix $= [A:B]$

$$= \left[\begin{array}{ccc:c} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & \lambda & \mu \end{array}\right]$$

$R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$$= \left[\begin{array}{ccc:c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 1 & \lambda - 1 & \mu - 6 \end{array}\right]$$

$R_3 \to R_3 - R_2$

$$= \left[\begin{array}{ccc:c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda - 3 & \mu - 10 \end{array}\right]$$

(i) if $\boxed{\lambda = 3, \mu \neq 10}$ so no solution.

(ii) if $\boxed{\lambda \neq 3}$ a unique solution.

(iii) if $\boxed{\lambda = 3, \mu = 10}$ so infinite solution.

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(C) find for what values of λ the given equation $x+y+z=1$, $x+2y+4z=\lambda$, $x+4y+10z=\lambda^2$ have a solution and solve them completely in each case.

$\Rightarrow$

$$x+y+z=1$$
$$x+2y+4z=\lambda$$
$$x+4y+10z=\lambda^2$$

$AX = B$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 4 & 10 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ \lambda \\ \lambda^2 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

augmented matrix $= [A:B]$

$$= \left[\begin{array}{ccc:c} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & \lambda \\ 1 & 4 & 10 & \lambda^2 \end{array}\right]$$

$$= \left[\begin{array}{ccc:c} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & \lambda-1 \\ 0 & 3 & 9 & \lambda^2-1 \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{array}$$

$R_3 \to R_3 - 3R_2$

$$= \left[\begin{array}{ccc:c} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & \lambda-1 \\ 0 & 0 & 0 & \lambda^2-3\lambda+2 \end{array}\right]$$

for solution $P[A:B] = PA \neq n$

so $\lambda^2 - 3\lambda + 2 = 0$

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$\lambda^2 - 2\lambda - \lambda + 2 = 0$

$\lambda(\lambda-2) - 1(\lambda-2) = 0$

$(\lambda-1)(\lambda-2) = 0$

so at $\lambda = 1, 2$ x, y, z have infinite solution.

(d) find the solution of the system of equations

$x + 3y - 2z = 0$

$2x - y + 4z = 0$

$x - 11y + 14z = 0$

$\Rightarrow$ $AX = B$

$$A = \begin{bmatrix} 1 & 3 & -2 \\ 2 & -1 & 4 \\ 1 & -11 & 14 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

augmented matrix $= [A:B]$

$$= \left[\begin{array}{ccc:c} 1 & 3 & -2 & 0 \\ 2 & -1 & 4 & 0 \\ 1 & -11 & 14 & 0 \end{array}\right]$$

$R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - R_1$

$$= \left[\begin{array}{ccc:c} 1 & 3 & -2 & 0 \\ 0 & -7 & 8 & 0 \\ 0 & -14 & 16 & 0 \end{array}\right]$$

$R_3 \to R_3 - 2R_2$

$$= \left[\begin{array}{ccc:c} 1 & 3 & -2 & 0 \\ 0 & -7 & 8 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\rho[A:B] = \rho(A) \neq n$

so consistancy and infinite solution

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(e) find for what values of λ the given equations $3x+y-\lambda z=0$, $4x-2y-3z=0$, $2\lambda x+4y+\lambda z=0$ may passes non-trival solution and solve them completely in each case.

$\Rightarrow$

$$3x+y-\lambda z=0$$
$$4x-2y-3z=0$$
$$2\lambda x+4y+\lambda z=0$$

$AX = B$

$$A = \begin{bmatrix} 3 & 1 & -\lambda \\ 4 & -2 & -3 \\ 2\lambda & 4 & \lambda \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

for non-trival solution $|A| = 0$

$$\begin{vmatrix} 3 & 1 & -\lambda \\ 4 & -2 & -3 \\ 2\lambda & 4 & \lambda \end{vmatrix} = 0$$

$$3(-2\lambda+12) - 1(4\lambda+6\lambda) - \lambda(\cancel{4\lambda}16+4\lambda) = 0$$
$$-6\lambda+36-4\lambda-6\lambda-16\lambda-4\lambda^2 = 0$$
$$-4\lambda^2-32\lambda+36 = 0$$
$$4\lambda^2+32\lambda-36 = 0$$
$$\lambda^2+8\lambda-9 = 0$$
$$\lambda^2+9\lambda-\lambda-9 = 0$$
$$\lambda(\lambda+9)-1(\lambda+9) = 0$$
$$(\lambda-1)(\lambda+9) = 0$$
$$\lambda = 1, -9$$

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(i) for $\lambda = 1$

$$[A:B] = \left[\begin{array}{ccc:c} 3 & 1 & -1 & 0 \\ 4 & -2 & -3 & 0 \\ 2 & 4 & 1 & 0 \end{array}\right]$$

$R_1 \to R_1 - R_2$

$$= \left[\begin{array}{ccc:c} -1 & 3 & 2 & 0 \\ 4 & -2 & -3 & 0 \\ 2 & 4 & 1 & 0 \end{array}\right]$$

$R_2 \to R_2 + 4R_1$, $R_3 \to R_3 + 2R_1$

$$= \left[\begin{array}{ccc:c} -1 & 3 & 2 & 0 \\ 0 & 10 & 5 & 0 \\ 0 & 10 & 5 & 0 \end{array}\right]$$

$R_3 \to R_3 - R_2$

$$= \left[\begin{array}{ccc:c} -1 & 3 & 2 & 0 \\ 0 & 10 & 5 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

So $\rho[A:B] = \rho(A) \neq n$

So consistancy and infinite solution

(ii) $\lambda = -9$

$$[A:B] = \left[\begin{array}{ccc:c} 3 & 1 & 9 & 0 \\ 4 & -2 & -3 & 0 \\ -18 & 4 & -9 & 0 \end{array}\right]$$

$R_1 \to R_1 - R_2$

$$= \left[\begin{array}{ccc:c} -1 & 3 & 12 & 0 \\ 4 & -2 & -3 & 0 \\ -18 & 4 & -9 & 0 \end{array}\right]$$

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$R_1 \to (-1)R_1$

$$= \left[\begin{array}{ccc|c} 1 & -3 & -12 & 0 \\ 4 & -2 & -3 & 0 \\ -18 & 4 & -9 & 0 \end{array}\right]$$

$R_2 \to R_2 - 4R_1$, $R_3 \to R_3 + 18R_1$

$$= \left[\begin{array}{ccc|c} 1 & -3 & -12 & 0 \\ 0 & 10 & 45 & 0 \\ 0 & -50 & -225 & 0 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 1 & -3 & -12 & 0 \\ 0 & 10 & 45 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad R_3 \to R_3 + R_2$$

$\rho[A:B] = \rho[A] \neq n$

so consistency and infinit solution

so hence in $\lambda = 1, -9$ ~~have~~ equation have non-thival solution.

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ASSIGNMENT-2

Q- Are the following sets of vectors linearly independent or dependent?

1) [1,0,0], [1,1,0], [1,1,1]

⇒ $v_1 = [1,0,0]$
$v_2 = [1,1,0]$
$v_3 = [1,1,1]$

vetor equation $c_1v_1 + c_2v_2 + c_3v_3 = 0$

$c_1(1,0,0) + c_2(1,1,0) + c_3(1,1,1) = (0,0,0)$

$c_1 + c_2 + c_3 = 0$
$c_2 + c_3 = 0$
$c_3 = 0$

$$\begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{vmatrix}$$

$c_1 = c_2 = c_3 = 0$ has a trival solution
so vectares are linearly independent

2) [7 -3 11 -6], [-56 24 -88 48]

eq. $c_1v_1 + c_2v_2 = 0$

$c_1[7\ -3\ 11\ -6] + c_2[-56\ 24\ -88\ 48] = [0\ 0\ 0\ 0]$

$7c_1 - 56c_2 = 0$
$-3c_1 + 24c_2 = 0$
$11c_1 - 88c_2 = 0$
$-6c_1 + 48c_2 = 0$

$$\begin{bmatrix} 7 & -56 \\ -3 & 24 \\ 11 & -88 \\ -6 & 48 \end{bmatrix}$$

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$$= \begin{bmatrix} 7 & -56 \\ -3 & 24 \\ 11 & -88 \\ 6 & 48 \end{bmatrix}$$

$R_2 \to R_2 + \frac{3}{7}R_1$

$$= \begin{bmatrix} 7 & -56 \\ 0 & 0 \\ 11 & -88 \\ 6 & 48 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & -56 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad \begin{matrix} R_3 \to R_3 + \frac{11}{7}R_1 \\ R_4 \to R_4 + \frac{6}{7}R_1 \end{matrix}$$

so c_1 and c_2 are ~~not~~ non-zero
therefore the given vectors are dependent.

3) [-1 5 0] [16 8 -3] [-64 56 9]

$\Rightarrow$
$v_1 = (-1 \ 5 \ 0)$
$v_2 = (16 \ 8 \ -3)$
$v_3 = (-64 \ 56 \ 9)$

so $c_1v_1 + c_2v_2 + c_3v_3 = 0$

$c_1(-1 \ 5 \ 0) + c_2(16 \ 8 \ -3) + c_3(-64 \ 56 \ 9) = 0$

$-c_1 + 16c_2 - 64c_3 = 0$
$5c_1 + 8c_2 + 56c_3 = 0$
$-3c_2 + 9c_3 = 0$

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$$\underbrace{\begin{bmatrix} -1 & 16 & -64 \\ 5 & 8 & 56 \\ 0 & -3 & 9 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}}_{B}$$

$$[A:B] = \left[\begin{array}{ccc:c} -1 & 16 & -64 & 0 \\ 5 & 8 & 56 & 0 \\ 0 & -3 & 9 & 0 \end{array}\right]$$

$R_2 \to R_2 + 5R_1$

$$= \left[\begin{array}{ccc:c} -1 & 16 & +64 & 0 \\ 0 & 88 & -264 & 0 \\ 0 & -3 & 9 & 0 \end{array}\right]$$

$R_3 \to R_3 + \frac{3}{88} R_2$

$$= \left[\begin{array}{ccc:c} -1 & 16 & -64 & 0 \\ 0 & 88 & -264 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

so c_1, c_2 and c_3 are ~~not~~ non-zero therefore the given vectors are linearly dependent

4) [1 -1 1] [1 1 -1] [-1 1 1] [0 1 0]

so vector equation : $c_1 v_1 + c_2 v_2 + c_3 v_3 + c_4 v_4 = 0$

$$c_1(1\ -1\ 1) + c_2(1\ 1\ -1) + c_3(-1\ 1\ 1) + c_4(0\ 1\ 0) = 0$$

$$c_1 + c_2 + c_3 = 0$$
$$-c_1 + c_2 + c_3 + c_4 = 0$$
$$c_1 - c_2 + c_3 = 0$$

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$$\underset{A}{\begin{bmatrix} 1 & 1 & -1 & 0 \\ -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 0 \end{bmatrix}} \underset{X}{\begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix}} = \underset{B}{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}}$$

$$A:B = \left[\begin{array}{cccc|c} 1 & 1 & -1 & 0 & 0 \\ -1 & 1 & 1 & 1 & 0 \\ 1 & -1 & 1 & 0 & 0 \end{array}\right]$$

$R_2 \to R_2 + R_1$
$R_3 \to R_3 - R_1$

$$= \left[\begin{array}{cccc|c} 1 & 1 & -1 & 0 & 0 \\ 0 & 2 & 0 & 1 & 0 \\ 0 & -2 & 2 & 0 & 0 \end{array}\right]$$

$R_3 \to R_3 + R_2$

$$= \left[\begin{array}{cccc|c} 1 & 1 & -1 & 0 & 0 \\ 0 & 2 & 0 & 1 & 0 \\ 0 & 0 & 2 & 1 & 0 \end{array}\right]$$

$c_1 + c_2 - c_3 = 0$
$c_2 + c_4 = 0$
$2c_3 + c_4 = 0$

Let $\boxed{c_4 = k}$ (non zero constant)

$\boxed{c_3 = \frac{-k}{2}}$

$\boxed{c_2 = -k}$

$c_1 = \frac{2k}{2} - \frac{k}{2} \Rightarrow \boxed{c_1 = \frac{k}{2}}$

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Since c_1, c_2, c_3, c_4 are non zero so given vector are linearly dependent.

5 $[2, -4], [1\ 9], [3\ 5]$

vector equation $c_1 v_1 + c_2 v_2 + c_3 v_3 = 0$

$$c_1(2, -4) + c_2(1\ 9) + c_3(3\ 5) = 0$$

$$c_1 + c_2 + 3c_3 = 0$$
$$-4c_1 + 9c_2 + 5c_3 = 0$$

$$\begin{bmatrix} 2 & 1 & 3 \\ -4 & 9 & 5 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$A:B = \left[\begin{array}{ccc|c} 2 & 1 & 3 & 0 \\ -4 & 9 & 5 & 0 \end{array}\right]$$

$$R_2 \to \tfrac{1}{2}R_2 + R_1$$

$$= \left[\begin{array}{ccc|c} 2 & 1 & 3 & 0 \\ 0 & 11/2 & 11/2 & 0 \end{array}\right]$$

$$2c_1 + c_2 + 3c_3 = 0$$
$$\tfrac{11}{2}c_2 + \tfrac{11}{2}c_3 = 0$$

let $c_3 = k$ (non zero constant)

$$\boxed{c_2 = -k}$$

$$2c_1 = k - 3k$$
$$2c_1 = -2k$$
$$\boxed{c_1 = -k}$$

Since c_1, c_2 and c_3 are non - zero so given vector are linearly dependent.

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6. [3 -2 0 4], [5 0 0 1], [-6 1 0 1], [2 0 0 3]

⇒ Let c_1, c_2, c_3 and c_4 be four scalars such that

$$c_1v_1 + c_2v_2 + c_3v_3 + c_4v_4 = 0$$

$$c_1(3\ -2\ 0\ 4) + c_2(5\ 0\ 0\ 1) + c_3(-6, 1, 0, 1)$$
$$c_4(2\ 0\ 0\ 3) = 0$$

$$3c_1 + 5c_2 - 6c_3 + 2c_4 = 0$$
$$-2c_1 + c_3 = 0$$
$$4c_1 + c_2 + c_3 + 3c_4 = 0$$

$$A:B = \begin{bmatrix} 3 & 5 & -6 & 2 \\ -2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 4 & 1 & 1 & 3 \end{bmatrix} \quad X = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$A:B = \left[\begin{array}{cccc|c} 3 & 5 & -6 & 2 & 0 \\ -2 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 4 & 1 & 1 & 3 & 0 \end{array}\right]$$

$R_4 \longleftrightarrow R_3$

$$= \left[\begin{array}{cccc:c} 3 & 5 & -6 & 2 & 0 \\ -2 & 0 & 1 & 0 & 0 \\ 4 & 1 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$|A| = 0\begin{vmatrix} 5 & -6 & 2 \\ 0 & 1 & 0 \\ 1 & 1 & 3 \end{vmatrix} - 0\begin{vmatrix} 3 & -6 & 2 \\ -2 & 1 & 0 \\ 4 & 1 & 3 \end{vmatrix} + 0\begin{vmatrix} 3 & 5 & 2 \\ -2 & 0 & 0 \\ 4 & 1 & 3 \end{vmatrix}$$
$$-0\begin{vmatrix} 3 & 5 & -6 \\ -2 & 0 & 1 \\ 4 & 1 & 1 \end{vmatrix}$$

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$|A| = 0$

so c_1, c_2, c_3 and c_4 are non-zero solution
so given vetor are linearly dependend

7. $[3, 4 7], [2\ 0\ 3], [8\ 2\ 3]\ [5\ 5\ 6]$

→ $c_1 v_1 + c_2 v_2 + c_3 v_3 + c_4 v_4 = 0$

$c_1(3\ 4\ 7) + c_2(2\ 0\ 3) + c_3(8\ 2\ 3) + c_4(5\ 5\ 6) = 0$

$3c_1 + 2c_2 + 8c_3 + 5c_4 = 0$
$4c_1 + 2c_3 + 5c_4 = 0$
$7c_1 + 3c_2 + 3c_3 + 6c_4 = 0$

$$\begin{bmatrix} 3 & 2 & 8 & 5 \\ 4 & 0 & 2 & 5 \\ 7 & 3 & 3 & 6 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$A:B = \left[\begin{array}{cccc:c} 3 & 2 & 8 & 5 & 0 \\ 4 & 0 & 2 & 5 & 0 \\ 7 & 3 & 3 & 6 & 0 \end{array}\right]$$

$R_2 \rightarrow R_2 - \frac{4R_1}{3}$

$R_3 \rightarrow R_3 - \frac{7R_1}{3}$

$$= \left[\begin{array}{cccc:c} 3 & 2 & 8 & 5 & 0 \\ 0 & -8/3 & -26/3 & -5/3 & 0 \\ 0 & -5/3 & -47/3 & -17/3 & 0 \end{array}\right]$$

$R_3 \rightarrow R_3 - \frac{5R_2}{8}$

$$= \left[\begin{array}{cccc:c} 3 & 2 & 8 & 5 & 0 \\ 0 & -8/3 & -26/3 & 5/3 & 0 \\ 0 & 0 & -\frac{82}{8} & & \end{array}\right]$$

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$$= \left[\begin{array}{cccc|c} 3 & 2 & 8 & 5 & 0 \\ 0 & -8/3 & -26/3 & 5/3 & 0 \\ 0 & 0 & -246/24 & -111/24 & 0 \end{array}\right]$$

$$-\frac{246}{24}C_3 - \frac{111}{24}C_4 = 0$$

$$-10.23\,C_3 - 4.6\,C_4 = 0$$

let $C_4 = K$ (non-zero constant)

$$\boxed{C_3 = \frac{4.6K}{10.23}}$$

$$-\frac{8C_2}{3} - \frac{26}{3}\left(\frac{460}{1023}K\right) + \frac{5}{3}K = 0$$

$$C_2 = 2.23K \times \frac{3}{-8}$$

$$\boxed{C_2 = 0.83K}$$

Since C_1, C_2, C_3 and C_4 are non-zero
so given vector are linearly independent

8. $[6, 0\ 3\ 1\ 4\ 2]$ $[0\ -1\ 2\ 7\ 0\ 5]$ $[12\ 3\ 0\ -19\ 8\ -11]$

$\Rightarrow$ $C_1V_1 + C_2V_2 + C_3V_3 = 0$

$C_1(6, 0\ 3\ 1\ 4\ 2) + C_2(0\ -1\ 2\ 7\ 0\ 5)$
$+ C_3(12\ 3\ 0\ -19\ 8\ -11)$

$6C_1 + 12C_3 = 0$
$-C_2 + 3C_3 = 0$
$3C_1 + 2C_2 = 0$
$C_1 + 7C_2 - 19C_3 = 0$
$4C_1 + 8C_3 = 0$
$2C_1 + 5C_2 - 11C_3 = 0$

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$$\begin{bmatrix} 6 & 0 & 12 \\ 0 & -1 & 3 \\ 3 & 2 & 0 \\ 1 & 7 & -19 \\ 4 & 0 & 8 \\ 2 & 5 & -11 \end{bmatrix}$$

$R_3 \leftrightarrow R_4$

$$\begin{bmatrix} 1 & 7 & -19 \\ 0 & -1 & 3 \\ 3 & 2 & 0 \\ 6 & 0 & 12 \\ 4 & 0 & 18 \\ 5 & 5 & -11 \end{bmatrix}$$

$R_3 \to R_3 - 3R_1$
$R_4 \to R_4 - 6R_1$
$R_5 \to R_5 - 4R_1$
$R_6 \to R_6 - 2R_1$

$$\begin{bmatrix} 1 & 7 & -19 \\ 0 & -1 & 3 \\ 0 & -19 & 57 \\ 0 & -42 & 126 \\ 0 & -28 & 54 \\ 0 & -9 & 27 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 7 & -19 \\ 0 & -1 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$R_3 \to R_3 - 19R_2$
$R_4 \to R_4 - 42R_2$
$R_5 \to R_5 - 28R_2$
$R_6 \to R_6 - 9R_6$

Since $c_1 = c_2 = c_3 = 0$ are ~~[illegible]~~ - zero
so given vector are linearly dependent

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Assignment - 3

Q→ find the Eigen Values and Eigen Vectors of following matries.

1. $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$

the characteristec eq is $|A-\lambda I| = 0$

$$\begin{vmatrix} -2-\lambda & 2 & -3 \\ 2 & 1-\lambda & -6 \\ -1 & -2 & -\lambda \end{vmatrix} = 0$$

$= (-2-\lambda)(-\lambda+\lambda^2-12) - 2(-2\lambda-6) - 3(-4+1-\lambda)$

$= (-2-\lambda)(\lambda^2-\lambda-12) + (4\lambda+12) + 9 + 3\lambda$

$= -2\lambda^2 + 2\lambda + 24 - \lambda^3 + \lambda^2 + 12\lambda + (7\lambda+21)$

$= -\lambda^3 - \lambda^2 + 21\lambda + 45 = 0$

$-\lambda^3 - \lambda^2 + 21\lambda + 45 = 0$

$-\lambda^2(\lambda+3) + 2\lambda(\lambda+3) + 15(\lambda+3) = 0$

$(\lambda+3)(-\lambda^2+2\lambda+15) = 0$

$(\lambda+3)(\lambda^2-2\lambda-15) = 0$

$\lambda = \dfrac{-(-2) \pm \sqrt{4+4\times15}}{2}$

$\lambda = \dfrac{2\pm8}{2}$

$\boxed{\lambda=-3}$ $\boxed{\lambda=5}$ $\boxed{\lambda=-3}$

eigen Values are = $-3, 5, -3$

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vectors corresponding to eigen value $\lambda_1 = \lambda_3 = -3$

$[A - \lambda I] X = 0$

$$\begin{bmatrix} 1 & 2 & -3 \\ 2 & -2 & -6 \\ -1 & -2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 + R_1$

$$\begin{bmatrix} 1 & 2 & -3 \\ 0 & -6 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_1 + 2x_2 - 3x_3 = 0$

$-6x_2 = 0$

$x_2 = 0$

$x_1 + 2(0) - 3x_3 = 0$

$x_1 = 3x_3$

Let $x_3 = k$

$x_1 = 3k$

$x_2 = 0$

for $\lambda = -3$, $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3k \\ 0 \\ k \end{bmatrix}$

$= \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$ (for $k = 1$)

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when $\lambda_2 = 5$

$$[A - \lambda I]\, x = 0$$

$$\begin{bmatrix} -7 & 2 & -3 \\ 2 & -4 & -6 \\ -1 & -2 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_1 \to R_1 - 7R_3$

$R_2 \to R_2 + 2R_3$

$$\begin{bmatrix} 0 & 16 & 32 \\ 0 & -8 & -16 \\ -1 & -2 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_1 \leftrightarrow R_3$

$$\begin{bmatrix} -1 & -2 & -5 \\ 0 & -8 & -16 \\ 0 & 16 & 32 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_3 \to R_3 + 2R_2$

$$\begin{bmatrix} -1 & -2 & -5 \\ 0 & -8 & -16 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-x_1 - 2x_2 - 5x_3 = 0$$
$$-8x_2 - 16x_3 = 0$$
$$x_2 = -2x_3$$

let $x_3 = k$

$x_2 = -2k$

$-x_1 = 5k + 2(-2k) = 5k - 4k$

$x_1 = -k$

for $\lambda = 5$, $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -K \\ -2K \\ K \end{bmatrix}$

$= \begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix}$ (for $K = 1$)

2. $\begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$

$\Rightarrow$ the characteristic eq is $|A - \lambda I| = 0$

$$\begin{vmatrix} 4-\lambda & 0 & 1 \\ -2 & 1-\lambda & 0 \\ -2 & 0 & 1-\lambda \end{vmatrix} = 0$$

$\Rightarrow 4-\lambda\,((1-\lambda)^2 - 0) - 0\,(-2(1-\lambda) - 0(-2)) + (-2\times 0 - (1-\lambda)(-2))$

$\Rightarrow (4-\lambda)(1+\lambda^2-2\lambda) + 2 - 2\lambda = 0$

$\Rightarrow 4 + 4\lambda^2 - 8\lambda - \lambda - \lambda^3 + 2\lambda^2 + 2 - 2\lambda = 0$

$\Rightarrow -\lambda^3 + 6\lambda^2 - 11\lambda + 6 = 0$

$\Rightarrow -\lambda^2(\lambda-1) + 5\lambda(\lambda-1) - 6(\lambda-1) = 0$

$\Rightarrow (\lambda-1)(-\lambda^2 + 5\lambda - 6) = 0$

$\Rightarrow (\lambda-1)(-1)(\lambda^2 - 5\lambda + 6) = 0$

$\Rightarrow (\lambda-1)(\lambda-2)(\lambda-3) = 0$

Eigen Values $\lambda_1 = 1$

$\lambda_2 = 2$

$\lambda_3 = 3$

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Vector corresponding to eigen Values

when $\lambda_1 = 1$

$$[A - \lambda I]\, x = 0$$

$$\begin{bmatrix} 3 & 0 & 1 \\ -2 & 0 & 0 \\ -2 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$R_3 \to R_3 + R_2$

$$\begin{bmatrix} 3 & 0 & 1 \\ -2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$-2x_1 = 0 \Rightarrow x_1 = 0$

$3x_1 + x_3 = 0$

$0 + x_3 = 0$ Let $x_2 = K$

$x_3 = 0$

Eigen Vector $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ K \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

when $\lambda_2 = 2$

$[A - \lambda I] = 0$

$$\begin{bmatrix} 2 & 0 & 1 \\ -2 & -1 & 0 \\ -2 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_2 \to R_2 + R_1$

$R_3 \to R_3 + R_1$

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$$\begin{bmatrix} 2 & 0 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$2x_1 + x_3 = 0$

$-x_2 + x_3 = 0$

$x_2 = x_3$

Let $x_1 = K$

$x_3 = -2K = x_2$

eigen vector when $\lambda = 2$ $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} K \\ -2K \\ -2K \end{bmatrix}$

$= \begin{bmatrix} 1 \\ -2 \\ -2 \end{bmatrix}$

for $\lambda = 3$

$$\begin{bmatrix} 1 & 0 & 0 \\ -2 & -2 & 0 \\ -2 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_2 \to R_2 + 2R_1$

$R_3 \to R_3 + 2R_1$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & -2 & 2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_1 + x_3 = 0$

$-2x_2 + x_3 = 0$

$x_2 = x_3$

Let $x_1 = K$

$x_3 = -K$

$x_2 = -K$

eigen vector, $\begin{bmatrix} K \\ -K \\ -K \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$

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3. $\begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 3 \end{bmatrix}$

the characteristec eq is $|A-\lambda I| = 0$

$$\begin{vmatrix} 5-\lambda & 0 & 0 \\ 0 & -\lambda & 0 \\ -1 & 0 & 3-\lambda \end{vmatrix} = 0$$

$(5-\lambda)(-\lambda(3-\lambda)) = 0$

$(5-\lambda)(-3\lambda+\lambda^2) = 0$

$\lambda(5-\lambda)(-3+\lambda) = 0$

$\lambda(\lambda-5)(\lambda-3) = 0$

so eigen Value $\lambda_1 = 0$, $\lambda_2 = 5$, $\lambda_3 = 3$

Vector corresponding to eigen Values

for $\lambda_1 = 0$

$$\begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$5x_1 = 0 \Rightarrow x_1 = 0$

$x_1 + 3x_3 = 0$

$x_3 = 0$

let $x_2 = K$

so eigen Vector $\lambda = 0$, $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ K \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

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for $\lambda_2 = 5$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & -5 & 0 \\ -1 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-5x_2 = 0 \Rightarrow x_2 = 0$$

$$-x_1 - 2x_3 = 0$$

$$x_1 = -2x_3$$

let $x_3 = K$

$$x_1 = -2K$$

eigen vetor $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2K \\ 0 \\ K \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$

when $\lambda_3 = 3$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$2x_1 = 0 \Rightarrow x_1 = 0$$

$$-3x_2 = 0 \Rightarrow x_2 = 0$$

let $x_3 = K$

so eigen Vector $\lambda_3 = 3$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ K \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \rightarrow \text{for } (K = 1)$$

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4. $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & -2 \end{bmatrix}$

$\Rightarrow$ the characteritec eq- $|A - \lambda I| = 0$

$$\begin{vmatrix} -\lambda & 0 & 0 \\ 0 & 3-\lambda & 4 \\ 0 & 0 & -2-\lambda \end{vmatrix} = 0$$

$$-\lambda\,(+3-\lambda)(-2-\lambda) = 0$$
$$\lambda\,(3-\lambda)(\lambda+2) = 0$$

eigen Values $\rightarrow \lambda_1 = 0,\ \lambda_2 = 3,\ \lambda_3 = -2$

Vector corresponding to eigen Values

when $\lambda_1 = 0$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$3x_2 + 4x_3 = 0$$
$$-2x_3 = 0 \quad \Rightarrow \quad x_3 = 0$$
$$x_2 = 0$$

let $x_1 = K$

so eigen vector $= \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} K \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

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when $\lambda_2 = \cancel{3}\ 3$

$$\begin{bmatrix} -3 & 0 & 0 \\ 0 & 0 & 4 \\ 0 & 0 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-3x_1 = 0 \Rightarrow x_1 = 0$

$4x_3 = 0 \Rightarrow x_3 = 0$

Let $x_2 = K$

eigen vector $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ K \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

when $\lambda_3 = -2$

$$\begin{bmatrix} -2 & 0 & 0 \\ 0 & 5 & 4 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-2x_1 = 0 \Rightarrow x_1 = 0$

$5x_2 + 4x_3 = 0$

$x_2 = -4/5\, x_3$

let $x_3 = K$

$x_2 = -4/5\, K$

so eigen vector $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -4/5\, K \\ K \end{bmatrix} = \begin{bmatrix} 0 \\ -4/5 \\ 1 \end{bmatrix}$

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5. for following matrix find one eigen value without calculation and justify your answer

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix}$$

⇒ we know that if two rows or columns are identical then the value of the determinant is zero

so

$$|A| = \begin{vmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{vmatrix}$$

$$|A| = 0$$

we know that the product of the n eigenvalues of A in the same as the $|A|$

so one eigen value of A is zero //

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Assignment - 4

Q-1 find the rank of the matrix A by reducing in Row reduced echelon form

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$$

$R_2 \rightarrow R_2 - 2R_1$
$R_3 \rightarrow R_3 - 3R_1$
$\Rightarrow$ $R_4 \rightarrow R_4 - 6R_1$

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & -4 & -11 & 5 \end{bmatrix}$$

$R_3 \rightarrow R_3 + R_2$

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -11 & 5 \\ 0 & -4 & -11 & 5 \end{bmatrix}$$

$R_3 \rightarrow R_3 + 2R_1$
$R_4 \rightarrow R_4 - R_3$

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & 5 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

so rank of A = $\rho(A) = 3$ (non-zero rows)

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(33) Q.3 ⇒

Q-2 Let W be the vector space of all symmetric 2×2 matrices and let $T: W \to P_2$ the linear transformation defined by
$T\begin{bmatrix} a & b \\ c & d \end{bmatrix} = (a-b) + (b-c)x + (c-a)x^2$. find the rank and nullity of T.

⇒ T defined as $T\begin{pmatrix} a & b \\ c & d \end{pmatrix} = (a-b) + (b-c)x + (c-a)x^2$

so we need to examine the linear independence of the polynomials $1, x, x^2$ after applying T to all possible symmetric 2×2 matrices.

the transformation T maps a symmetric 2×2 matrix to a polynomial of degree at most 2. it maps the entries a, b, c of the matrix to coefficients of the polynomial $(a-b) + (b-c)x + (c-a)x^2$

the polynomials $1, x, x^2$ are linearly independent and since T maps symmetric matrix to polynomials of degree at most 2, it implies that the range of T spans all of P_2. therfor rank of T is 3.

rank + nullity = dimantion

since the dimantion is 3

so

rank + nullity = dimantion

3 + nullity = 3

$\boxed{\text{nullity} = 0}$

then the rank of T = 3

nullity of T = 0

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Q.3 Let $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ find the eigenvalue and eigenvectors of A^{-1} and $A + 4I$

$\Rightarrow$ $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$

$$A^{-1} = \frac{1}{(2\times2)-(-1\times-1)} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$= \frac{1}{4-1} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$= \frac{1}{3} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

the characteristec eq. $[A - \lambda I] = 0$

$$\frac{1}{3} \begin{vmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{vmatrix} = 0$$

$$(2-\lambda)(2-\lambda) - 1 = 0$$
$$4 + \lambda^2 - 4\lambda - 1 = 0$$
$$\lambda^2 - 4\lambda + 3 = 0$$
$$\lambda^2 - 3\lambda - \lambda + 3 = 0$$
$$\lambda(\lambda-3) - 1(\lambda-3) = 0$$
$$(\lambda-1)(\lambda-3) = 0$$

so eigen values $\lambda_1 = 1$, $\lambda_2 = 3$

eigen vectors of corresponding eigen values :

when $\lambda_1 = 1$

$$\frac{1}{3} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$x_1 + x_2 = 0$$

let $x_1 = K$, $x_2 = -K$

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eigen vector $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} k \\ -k \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

when $\lambda_2 = 3$

$\frac{1}{3}\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$

$R_2 \to R_2 + R_1$

$\frac{1}{3}\begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$

$-x_1 + x_2 = 0$

if $x_1 = K$, $x_2 = K$

eigen vector $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} K \\ K \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$A + 4I$

$\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$

$\begin{bmatrix} 6 & -1 \\ -1 & 6 \end{bmatrix}$

the charactectice eq $|A - \lambda I| = 0$

$\begin{vmatrix} 6-\lambda & -1 \\ -1 & 6-\lambda \end{vmatrix} = 0$

$36 + \lambda^2 - 12\lambda - 1 = 0$

$\lambda^2 - 12\lambda + 35 = 0$

$\lambda^2 - 7\lambda - 5\lambda + 35 = 0$

$\lambda(\lambda - 7) - 5(\lambda - 7) = 0$

$(\lambda - 5)(\lambda - 7) = 0$

eigen Values $\lambda_1 = 5$, $\lambda_2 = 7$

when $\lambda_1 = 5$

$$\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$R_2 \to R_2 + R_1$

$$\begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$x_1 - x_2 = 0$

$x_1 = x_2 = k$ (let)

eigen Vector $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} k \\ k \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

when $\lambda_2 = 7$

$$\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$-x_1 - x_2 = 0$

$-x_1 = x_2 = k$ (let)

eigen vector $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -k \\ k \end{bmatrix}$

$= \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

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Q.1 Solve by gauss-seidel method (take three iteration)

$$3x - 0.1y - 0.2z = 7.85$$
$$0.1x + 7y - 0.3z = -19.3$$
$$0.3x - 0.2y + 10z = 71.4$$

with initial values $x(0) = 0$, $y(0) = 0$, $z(0) = 0$

$\Rightarrow$ by gauss-seidel method

$$x_1 = \frac{0.1(0) - 0.2(0) + 7.85}{3}$$

$$= \frac{7.85}{3}$$

$$x_1 = 2.616$$

$$y_1 = \frac{-19.3 - 0.1(2.616) + 0.3(0)}{7}$$

$$= \frac{-19.3 - 0.2616}{7}$$

$$y_1 = -2.7945$$

$$z_1 = \frac{71.4 - 0.3(2.616) + 0.2(-2.7945)}{10}$$

$$z_1 = 7.005$$

$$\Rightarrow (x_1, y_1, z_1) \equiv (2.616, -2.794, 7.005)$$

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2^{nd} iteration

$$x_2 = \frac{7.85 + 0.1(-2.794) + 0.2(7.005)}{3}$$

$$= 2.9905$$

$$y_2 = \frac{-19.3 - 0.1(2.9905) + 0.3(7.005)}{7}$$

$$y_2 = -2.4996$$

$$z_2 = \frac{71.4 - 0.3(2.9905) + 0.2(-2.4996)}{10}$$

$$= 7.0002$$

$$\Rightarrow (x_2, y_2, z_2) = (2.9905, -2.4996, 7.0002)$$

3^{rd} iteration

$$x_3 = \frac{7.85 + 0.1(-2.4996) + 0.2(7.0002)}{3}$$

$$= 3.00002$$

$$y_3 = \frac{-19.3 - 0.1(3.00002) + 0.3(7.0002)}{7}$$

$$y_3 = -2.4999$$

$$z_3 = \frac{71.4 - 0.3(3.00002) + 0.2(-2.4999)}{10}$$

$$= 7.000001$$

So
$(x_1, y_1, z_1) = (2.616, -2.794, 7.005)$
$(x_2, x_2, z_2) = (2.9905, -2.4996, 7.0002)$
$(x_3, y_3, z_3) = (3.00002, -2.4999, 7.000001)$

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Q-5. Define consistent and inconsistent system of eq. Hence solve the following system of equation if consistent

$$x+3y+2z=0$$
$$2x-y+3z=0$$
$$3x-5y+4z=0$$
$$x+17y+4z=0$$

$\Rightarrow$ so $AX=B$

$$\begin{bmatrix} 1 & 3 & 2 \\ 2 & -1 & 3 \\ 3 & -5 & 4 \\ 1 & 17 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

homogeneous

Augmented matrix $[A:B]$

$$= \left[\begin{array}{ccc:c} 1 & 3 & 2 & 0 \\ 2 & -1 & 3 & 0 \\ 3 & -5 & 4 & 0 \\ 1 & 17 & 4 & 0 \end{array}\right]$$

$R_2 \to R_2 - 2R_1$
$R_3 \to R_3 - 3R_1$
$R_4 \to R_4 - R_1$

$$= \left[\begin{array}{ccc:c} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & -14 & -2 & 0 \\ 0 & 14 & 2 & 0 \end{array}\right]$$

$R_3 \to R_3 + R_4$

$$= \left[\begin{array}{ccc:c} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 14 & 2 & 0 \end{array}\right]$$

$R_3 \leftrightarrow R_4$

$$= \left[\begin{array}{ccc:c} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & 14 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc:c} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad R_3 \rightarrow R_3 + 2R_2$$

Rank of $(A:B)$ = Rank of A = 2

$\rho[A:B] = \rho(A) = 2$

so matrix is consistanc

$\because \rho[A:B] = \rho[A] \neq$ no of unnone (varibles)

so infinit solution

$x + 3y + 2z = 0$

$-7y - z = 0$

$z = -7y$

let $y = K$

$z = -7K$

$x = -2(-7K) - 3K$

$= 14K - 3K$

$x = 11K$

so genral so. $\begin{pmatrix} 11K \\ K \\ -7K \end{pmatrix}$

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Q-6 Detemine wheter the fun. $T: P_2 \to P_2$ is linear transformation or not.
where $T(a+bx+cx^2) = (a+1)+(b+1)x+(c+1)x^2$

$\Rightarrow$ for linear transformation we have check 2 - condition

① $T(u+v) = T(u) + T(v)$.
② $cT(u) = T(cu)$

① $u = a+bx+cx^2$
$v = d+ex+fx^2$

$T(u+v) = T((a+bx+cx^2)+(d+ex+fx^2))$
$= T((a+d)+(b+e)x+(c+f)x^2)$
$= (a+d+1) + (b+e+1)x + (c+f+1)x^2$

$T(u)+T(v) = T(a+bx+cx^2) + T(d+ex+fx^2)$
$= (a+1)+(b+1)x+(c+1)x^2 + (d+1)+(e+1)x+(f+1)x^2$
$= (a+d+2)+(b+e+2)x+(c+f+2)x^2$
$\neq T(u+v)$

so function is not linear transformation.

Q-7
$\Rightarrow$

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Q-7 Determine the whether the function set = { (1,2,3), (3,1,0), (-2,1,3) is a basis of $V_3(R)$. in case S is not a basis detemine and the basis of the sub space spanned by S.

⇒ checking spaning

$$c_1\begin{pmatrix}1\\2\\3\end{pmatrix} + c_2\begin{pmatrix}3\\1\\0\end{pmatrix} + c_3\begin{pmatrix}-2\\1\\3\end{pmatrix} = \begin{pmatrix}v_1\\v_2\\v_3\end{pmatrix}$$

Here c_1, c_2, c_3 are scales & $(v_1 v_2 v_3) \in V$

$$\begin{bmatrix}1 & 3 & -2\\2 & 1 & 1\\3 & 0 & 3\end{bmatrix}\begin{pmatrix}c_1\\c_2\\c_3\end{pmatrix} = \begin{pmatrix}v_1\\v_2\\v_3\end{pmatrix}$$

$AX = B$

$$[A:B] = \begin{bmatrix}1 & 3 & -2\\2 & 1 & 1\\3 & 0 & 3\end{bmatrix}$$

$$\begin{bmatrix}1 & 3 & -2\\0 & -5 & 5\\0 & -9 & 9\end{bmatrix} \quad \begin{matrix}R_2 \to R_2 - 2R_1\\R_3 \to R_3 - 3R_1\end{matrix}$$

$$\begin{bmatrix}1 & 3 & -2\\0 & -5 & 5\\0 & 0 & 0\end{bmatrix} \quad R_3 \to R_3 - \frac{9}{5}R_2$$

$\rho(A) = 2 <$ no of variable

so system has infinite solution

there exist a linear dependent

∴ S span.

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Q-8 Using Jacobi's method solve

$$3x - 6y + 2z = 23$$
$$-4x + y - z = -15$$
$$x - 3y + 7z = 16$$

with intial values $x_0 = 1,\ y_0 = 1,\ z_0 = 1$

$\Rightarrow$ $x_1 = \frac{6y - 2z + 23}{3} = \frac{6\times(1) - 2(1) + 23}{3}$

$= \frac{27}{3}$

$x_1 = 9$

$y_1 = 4(1) + (1) - 15$
$= 4 + 1 - 15$
$= -10$

$z_1 = \frac{16 - x_0 + 3y_0}{7} = \frac{16 - 1 + 3(1)}{7} = \frac{18}{7}$

$= 2.5714$

$(x_1, y_1, z_1) \equiv (9, -10, 2.5714)$

$x_2 = \frac{23 + 6y_1 - 2z_1}{3} = \frac{27 + 6\times(-10) - 2\times 2.5714}{3}$

$= -12.7142$

$y_2 = -15 + z_1 + 4x_1 = -15 + 2.5714 + 4\times 9$

$= 23.5714$

$z_2 = \frac{16 - x_1 + 3y_1}{7} = \frac{16 - 9 + 3\times -10}{7} = -3.2557$

$(x_2, y_2, z_2) \equiv (-12.7142, 23.5714, -3.2557)$

$x_3 = \frac{27 + 6y_2 - 2z_2}{3} = \frac{23 + 6(23.5714) - 2(-3.2657)}{3}$

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$= 56.999$

$y_3 = -15 + z_2 + 4x_2 = -15 + (-3.2557) + 4(-12.7142)$

$= -69.1425$

$z_3 = \frac{16x_2 + 3y_2}{7} = \frac{16 - (-12.7142) + 3(23.5714)}{7}$

$= 14.2040$

$(x_3, y_3, z_3) = (56.99, -69.14, 14.20)$

Q-9 Explain one application of matrix operations in image processing with example.

→ One application of matrix opertions in image processing is convolution, where a filter matrix is applied to an image.

for ex →

a 3×3 blur fiter with values 1/9 can smooth an image. By overlaying the filter matrix onto the image and multiplying corresponding pixels, then summing the results, the central pixel is replaced with the sum. this process, repeated across the image, averages pixel values, resulting in a blurred image.

Q-10 Give a brief description of linear transformation for computer vision for rotating 2D image

→ Linear transformations in computer vision, like rotating a 2D image, involve using

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matrices to change how the image looks. to rotate an image, we use a special matrix. this matrix shifts each pixel's position, making it seem ~~as~~ like the image has been turned.
the amount of turning is determined by the rotating angle. these transformations help in tasks like adjusting images to fit better or recognizing objects accurately in computer vision.