Q function sum(list, i) {

if(i== list.length-1)

return list[i]

let res= sum(list, i+1);

return res+ list[i]

}

$$\boxed{T(n)= T(n-1)+c} \rightarrow (1)$$

$$T(n-1)= T(n-2)+c$$

$$T(n) = T(n-2)+c+c \rightarrow (2)$$

$$\boxed{T(n-2) = T(n-3)+c}$$

$$T(n)= T(n-3) +c+c+c \rightarrow (3)$$

$$T(n)=T(n-1) +c$$

$$T(n) =T(n-2)+2c$$

$$T(n)= T(n-3) +3c$$

$$\vdots$$

$$T(n) = T(n-k) +kc$$

$$T(n)= T(n-1) +c$$

$$T(n-1)= T(n-2)+2c$$

$$T(n-2) = T(n-3)+3c$$

$$T(1)=0$$

$$\vdots$$

$$T(n) = T(1)+ nc$$

$$T(n)= T(n-i)+ic$$

$$T(n)= 0+ n\cancel{c}$$

$$\boxed{O(n)}$$

Q $T(n) = \begin{cases} 1 & \text{if } n=1 \\ n*T(n-1) & \text{if } n>1 \end{cases}$

Substitution method

$T(n) = n + T(n-1)$ — (1)

$T(n-1) = (n-1) + T(n-2)$ — (2)

$T(n-2) = (n-2) + T(n-3)$ — (3)

$n(n-1) + T(n-2)$

$T(n) = n + (n-1) + T(n-2)$

$= n + (n-1) + (n-2) + T(n-3)$ $\frac{1}{2}\ \frac{1}{3}\ \frac{1}{4}\ \frac{1}{5}$ — <

$= n + (n-1) + (n-2) + (n-3) + (n-4) \cdots$ $T(n-(n-1))$, $T(n-n+1)$, $T(1)$

$= n + (n-1) + (n-2) + \cdots 3 + 2 + 1$

$= n + n + n + n \cdots$

$= n^n$ $n!$

(a) $O(n)$
(b) $O(n^2)$
(c) $O(n \log n)$
(d) $O(n^n)$

Q.

$$T(n) = \begin{cases} 1 & \text{if } n=1 \\ 2T\left(\frac{n}{2}\right) + n & n \geq 1 \text{ otherwise} \end{cases}$$ — (1)

$T\left(\frac{n}{2}\right) = 2T\left(\frac{n}{4}\right) + \frac{n}{2}$ — (2) $n = n/2$

$T(n/4) = 2T\left(\frac{n}{8}\right) + \frac{n}{4}$ — (3) $n = n/4$

$= 2\left[2T\left(\frac{n}{4}\right) + \frac{n}{2}\right] + n$

$= 2^2 T\left(\frac{n}{2^2}\right) + n + n$

$= 2^2 T\left(\frac{n}{2^2}\right) + 2n$

$= 2^2\left[2T\left(\frac{n}{8}\right) + \frac{n}{4}\right] + 2n$

$= 2^3 T\left(\frac{n}{2^3}\right) + n + 2n$

$= 2^3 T\left(\frac{n}{2^3}\right) + 3n$

$2^4 T\left(\frac{n}{\ }\right) + 4n$

$T(n) = 2T\left(\frac{n}{2}\right) + n$

$= 2\left[2T\left(\frac{n}{4}\right) + \frac{n}{2}\right] + n$

$4T\left(\frac{n}{4}\right) + \frac{2n}{2} + n$

$2^3 T\left(\frac{n}{8}\right) + \frac{4n}{4} + 2n$

$2^5 T\left(\frac{n}{2^5}\right) + 5n$

$\vdots$

k times

$2^k T\left(\frac{n}{2^k}\right) + kn$

$\boxed{T(1) = 1}$

$\log m^n$

$\Downarrow$

$n \log m$

$\frac{n}{2^k} = 1 \Rightarrow n = 2^k$

$\log n = \log 2^k$

$\log n = k \log 2$

$\boxed{\log n = k}$

$n\,T(1) + \log n \cdot n$

$[n + n\log n]$

$\boxed{O(n \log n)}$

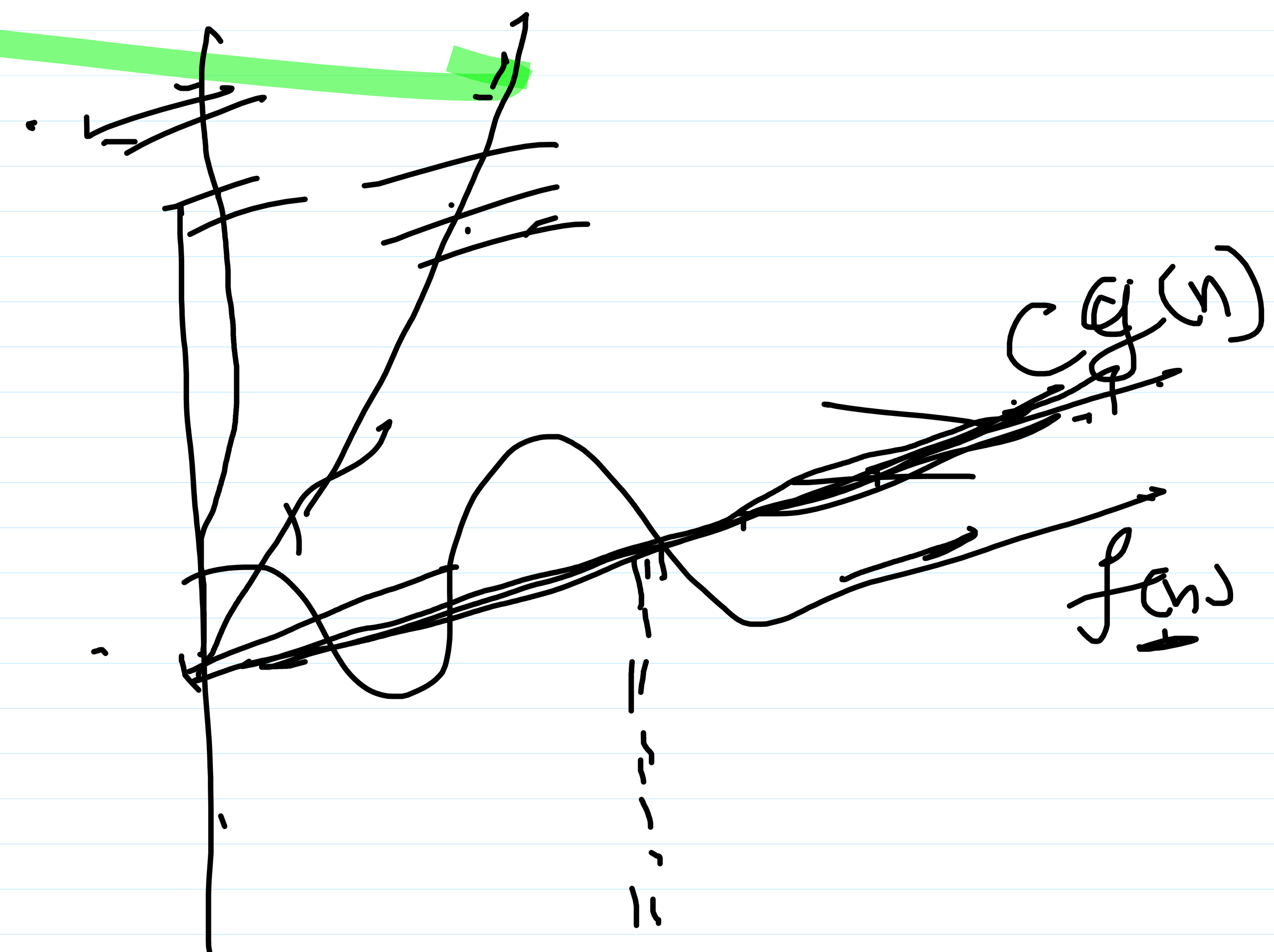
Q.

$f(n) = 2n^2 + 5n + 1$

$g(n) = n^2$

$c = 8$

$n_0 = 1$



$f(n) \leq c\,g(n)$

n_0

$f(n) \leq c\,g(n)$

$n > n_0$

$2n^2+5n+1 \leq 8n^2$

$n=1 \Rightarrow 2+5+1 \leq 8$

$8 \leq 8$

$f(n) = O(g(n))$

$f(n) = O(n^2)$

$\forall n \geq 1$

$f(n) = 2n^2+5n+1$

$g(n) = n^2$

$c = 4$

$f(n) \leq c\,g(n)$

$$2n^2+5n+1 \leq 4n$$

$n=1$ $\quad 2+5+1 \leq 4$

$8 \leq 4$ false

$n=2$ $\quad 8+10+1 \leq 16$

$\underline{19} \leq \underline{16}$

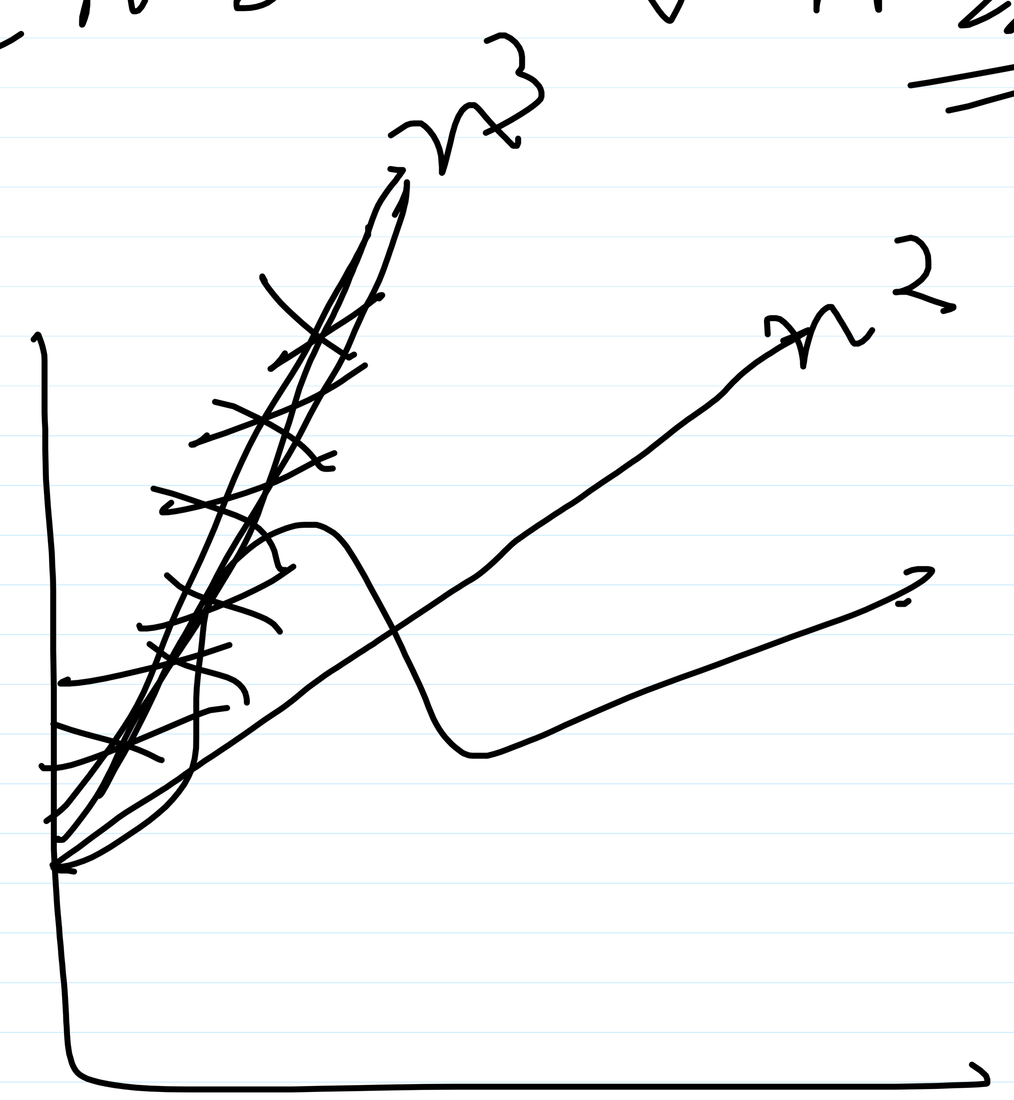
$n=3$ $\quad 2\times 9+15+1 \leq 36$

$18+15+1 \leq 36$

$\underline{34 \leq 36}$

$\boxed{n_0=3}$

$f(n)=O(n^2) \quad \forall n \geq 3$



$f(n)=2\underline{n^2}+5n+1$

$\boxed{g(n)=n^3}$

$C=\underline{4}$