Printed page:02 Subject Code: AAS0103

Roll No:

NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA

(An Autonomous Institute Affiliated to AKTU, Lucknow)

B. Tech.

(SEM: FIRST SEMESTER THEORY EXAMINATION (2020-2021))

Subject Name: Engineering Mathematics-I

Time: 3 Hours **Max. Marks:100**

General Instructions:

- All questions are compulsory. Answers should be brief and to the point.
- This Question paper consists of 02 pages & 8 questions.
- It comprises of three Sections, A, B, and C. You are to attempt all the sections.
- Section A -Question No- 1 is objective type questions carrying 1 mark each, Question No- 2 is very short answer type carrying 2 mark each. You are expected to answer them as directed.
- Section B - Question No-3 is Long answer type -I questions with external choice carrying 6 marks each. You need to attempt any five out of seven questions given.
- Section C -Question No. 4-8 are Long answer type –II (within unit choice) questions carrying 10marks each. You need to attempt any one part *a or b.*
- Students are instructed to cross the blank sheets before handing over the answer sheet to the invigilator.
- No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.

SECTION – A

1. Attempt all the parts. **[10×1=10]** **CO**

a. A is a singular matrix of order 3 with eigen values 2 and 3. The third eigen value is **(1)** **CO1**

(a) 1
(b) 0
(c) 4
(d) -1

b. The rank of the matrix $\begin{bmatrix}1 & 1 & 1\\ 1 & -1 & 0\\ 1 & 1 & 1\end{bmatrix}$ is **(1)** **CO1**

(a) 0
(b) 1
(c) 2
(d) 3

c. If $u=\frac{x^2}{a}+\frac{y^2}{b}-7$ then $\frac{\partial u}{\partial x}$ is .... **(1)** **CO2**

d. If $u=x^2$ and $x=t^3$ then $\frac{du}{dt}$ is .... **(1)** **CO2**

e. If $x=rcos\emptyset$ and $y=rsin\emptyset$ then $\frac{\partial(x,y)}{\partial(r,\emptyset)}$ is .... **(1)** **CO3**

f. The function $z=y^2+x^2y+x^4$ has a minimum at (0,0). (T/F) **(1)** **CO3**

g. The value of the double integral $\int_{x=0}^{3}\int_{y=0}^{1}(x^2+3y^2)dydx$ is 12. (T/F) **(1)** **CO4**

h. The value of $\int_0^\infty e^{-x^2}dx$ is $\sqrt{\pi}$. (T/F) **(1)** **CO4**

i. The value of $81\times 81+68\times 68-2\times 81\times 68$ is .... **(1)** **CO5**

j. Insert the missing number: 11, 13, 17, 19, 23, 29, 31, 37, 41, (......). **(1)** **CO5**

2. Attempt all the parts. **[5×2=10]** **CO**

a. Find a and b such that $A=\begin{bmatrix}a & 4\\ 1 & b\end{bmatrix}$ has 3 and -2 as eigen values. **(2)** **CO1**

b. Find the n^{th} derivative of $y=sin\,(ax+b)$. **(2)** **CO2**

c. The radius of a sphere is found to be 10 meter with a possible error of 0.02 meter. What is the relative error in calculating the volume of sphere? **(2)** **CO3**

d. Prove that Beta function is symmetric. **(2)** **CO4**

e. If 50% of $(x-y)$ is 30% of $(x+y)$ then what percent of x is y? **(2)** **CO5**

SECTION – B

CO

3. Answer any five of the following- [5×6=30]

a. Show that the system of equations (6) CO1

$$\left.\begin{aligned} 3x+4y+5z&=\alpha \\ 4x+5y+6z&=\beta \\ 5x+6y+7z&=\gamma \end{aligned}\right\}$$

is consistent only if α, β and γ are in arithmetic progression.

b. Trace the curve $a^2y^2 = x^2(a^2 - x^2)$. (6) CO2

c. Prove that $\frac{1}{(1-x)} = \frac{1}{3} + \frac{(x+2)}{3^2} + \frac{(x+2)^2}{3^3} + \frac{(x+2)^3}{3^4} + \cdots \ldots$ (6) CO3

d. Change the order of integration and hence evaluate $\int_0^a \int_{x^2/a}^{2a-x} xy\,dydx$. (6) CO4

e. Using the transformation $x + y = u$ and $y = uv$, show that $\int_0^1 \int_0^{1-x} e^{\left(\frac{y}{x+y}\right)}\,dydx = \frac{1}{2}(e-1)$. (6) CO4

f. The selling price of 20 articles is equal to the cost price of 25 articles. Find the profit percent. (6) CO5

g. If the word LEADER is coded as 20-13-9-12-13-26, how would you write LIGHT? (6) CO5

SECTION – C

4 Answer any one of the following- [5×10=50] CO

a. State Cayley-Hamilton theorem. Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and hence find A^{-1}. (10) CO1

b. Find eigen values and corresponding eigen vectors of the matrix $A = \begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$. (10) CO1

5. Answer any one of the following-

a. If $y = x^n logx$, prove that $(i)\ y_{n+1} = \frac{n!}{x}$ $(ii)\ x^2y_{p+2} + (2p-2n+1)xy_{p+1} + (p-n)^2y_p = 0$. (10) CO2

b. State and prove Euler's theorem for homogeneous function. Also prove that if $u = tan^{-1}\left[\frac{x^3+y^3}{x-y}\right]$ then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = sin2u$. (10) CO2

6. Answer any one of the following-

a. Expand x^y in powers of $(x-1)$ and $(y-1)$ upto the third degree terms. (10) CO3

b. Find a point on the paraboloid $z = x^2 + y^2$ nearest to the point (3,-6,4). (10) CO3

7. Answer any one of the following-

a. Prove by the method of double integration that the area lying between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$ is $\frac{16}{3}a^2$. (10) CO4

b. Find the volume of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ by using Dirichlet's theorem. (10) CO4

8. Answer any one of the following-

a. A batsman makes a score of 87 runs in the 17^{th} inning and thus increases his average by 3. Find his average after 17^{th} inning. (10) CO5

b. If three numbers are added in pairs, the sums equal 10, 19 and 21. Find the numbers. (10) CO5