Theory of computation

Assignment-2

1) obtain grammar to generate the language.

$L = \{0^n 1^{n+1} \mid n \geq 0\}$.

Soln:

Recursive definition to generate string $0^n 1^n$ is

$A \rightarrow 0A1 \mid \epsilon$

$L = 1 \quad n = 0$

If $A \rightarrow 0A1$ is applied n times,

$L = 011 \quad n = 1$

$L = 00111 \quad n = 2$

$A \Rightarrow 0A1 \Rightarrow 00A11 \xrightarrow{n \text{ time}} 0^n 1^n$

$L = 0001111 \quad n = 3$

but $L = 0^n 1^{n+1}$

$= 0^n 1^n 1$

$S \rightarrow A1$

$V = \{S, A\}$

$T = \{0, 1\}$

$P \quad \{S \rightarrow A1$

$A \rightarrow 0A1 \mid \epsilon$

$\}$

$S \rightarrow$ is the start symbol.

(or)

$L = \{0^n 1^m \mid n \geq 0 \; m \geq 1\}$

$\because n+1 \geq 1$

$m \geq 1$

$0^n \Rightarrow A \rightarrow 0A \mid \epsilon$

$1^m \Rightarrow B \rightarrow 1B \mid 1$

$S \rightarrow AB$

$V = \{S, A, B\}$

$T = \{0, 1\}$

$P = \{ S \rightarrow AB$

$A \rightarrow 0A \mid \epsilon$

$B \rightarrow 1B \mid 1$

$\}$

$S \rightarrow$ Start symbol.

(2) Obtain a Grammar to generate the language.

$L = \{ww^R \mid w \in \{a,b\}^*\}$ where w^R is reverse of w.

Soln

As it is a even length palindrom, we have to generate all even length palindromes.

Recursive definition of a palindrom is

1. ϵ is a palindrome
2. a & b are palindromes
3. If w is a palindrome, then the strings awa and bwb are also palindromes.

$\therefore$ From 1, 2, 3

$S \rightarrow \epsilon$

$S \rightarrow a \mid b$

$S \rightarrow aSa \mid bSb$

$V = \{S\}$

$T = (a, b\}$

$P \rightarrow \{ S \rightarrow \epsilon \rightarrow (1)$

$S \rightarrow a \mid b \rightarrow (2)$

$S \rightarrow aSa \mid bSb \rightarrow (3)$.

(3) obtain a grammar to generate the language

$L = \{0^n 1^{2n} \mid n \geq 0\}$.

soln)

$L = \{\epsilon, 011, 001111, \ldots\}$

Recursive definition to generate string $0^n, 1^{2n}$

$S \rightarrow 0S11 \mid \epsilon$

$V = \{S\}$

$T = \{0, 1\}$

$P = \{$

$S \rightarrow 0S11 \mid \epsilon$

$\}$

$S \rightarrow$ start symbol.

(4) obtain grammar to generate the language

$L = \{a^{n+2} b^m \mid n \geq 0 \ \& \ m > n\}$.

soln

$L = \{a^{n+2} b^m \mid n \geq 0 \ \& \ m > n\}$

$= \{aaa^n b^m \mid n \geq 0 \ \& \ m > n\}$

$= \{aab, aaabb, aaaabbbbb \ldots\}$

Recursive definition for $a^n b^m$ where $n \geq 0$ & $m > n$

→ For $n = 0$ ~~aa~~b....

→ For $n = 1$ bb....

no of b's should be always greater than a

So For b

$B \to bB | b$ (m is never zero because $m > n$ & $n \geq 0$

For $m > n$

$S \to aSb \mid \cancel{abB \mid bB \mid b}\ B$ $a^n b^m$

$L = \{abb, b, bb$

For $a^n b^m$ $n \geq 0$ & $m > n$

aaabbbb

$S \to aSb \mid \cancel{abB \mid bB \mid b}\ B$

$B \to bB | b$

but we have to find for $a^{n+2} b^m$

$\therefore = aa\, a^n b^m$

$S \to aaaSb \mid a\cancel{abB \mid aabB \mid aab}\ aaB$

$B \to bB | b$

$\therefore$

$V = \{S, B\}$

$T = \{a, b\}$

$P = \{ S \to aaaSb \mid aaB$

$B \to bB \mid b$

$\}$.

(5) Obtain grammar to generate the language L

$L = \{w : |w| \bmod 5 = 0\}$ on $\Sigma = \{a\}$.

Soln

$L = \{w : |w| \bmod 5 = 0\}$ on $\Sigma = \{a\}$

$L = \{\varepsilon, aaaaa, aaaaaaaaaa, \ldots\}$

String generated should have length of multiple of 5

$S \rightarrow aaaaaS \mid \epsilon$

$V = \{S\}$

$T = \{a\}$

$P = \{ S \rightarrow aaaaaS \mid \epsilon$
$\}$

S is the start symbol.

(6) Obtain a grammar to generate the set of all strings with no more than three a's when $\Sigma = \{a, b\}$

Soln

$L = \{a^n b^m \mid n \leq 3 \; m \geq 0\}$

~~For at leas~~

For at most 3 a's

$S \Rightarrow aAb$

$A \Rightarrow aBb$

$B \Rightarrow aCb$

$C \rightarrow bC \mid b \mid \epsilon$

$V = \{S, A, B, C\}$

$T = \{a, b\}$

$P = \{ S \rightarrow aAb,$
$A \rightarrow aBb$
$B \rightarrow aCb$
$C \rightarrow bC \mid b \mid \epsilon \}$