```
In [1]: import Pkg
        Pkg.activate(@__DIR__)
        Pkg.instantiate()
        import MathOptInterface as MOI
        import Ipopt
        import FiniteDiff
        import ForwardDiff
        import Convex as cvx
        import ECOS
        using LinearAlgebra
        using Plots
        using Random
        using JLD2
        using Test
        import MeshCat as mc
        using Statistics
        using Printf
         Activating project at `~/OCRL/HW3_S25`
```

```
In [2]: include(joinpath(@__DIR__, "utils","fmincon.jl"))
  include(joinpath(@__DIR__, "utils","planar_quadrotor.jl"))
```

check_dynamic_feasibility (generic function with 1 method)

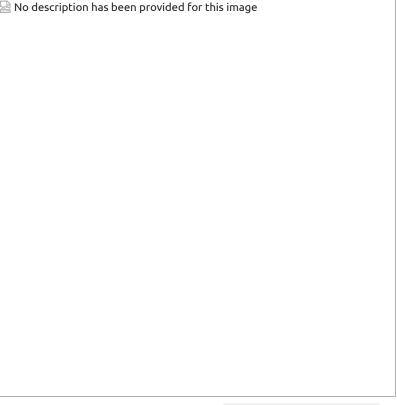
Q3: Quadrotor Reorientation (40 pts)

In this problem, you will use the trajectory optimization tools you have demonstrated in questions one and two to solve for a collision free reorientation of three planar quadrotors. The planar quadrotor (as described in lecture 10) is described with the following state and dynamics:

$$x = egin{bmatrix} p_x \ p_z \ heta \ v_x \ v_z \ \omega \end{bmatrix},$$
 (1) \dot{x} =

where p_x and p_z are the horizontal and vertial positions, v_x and v_z are the corresponding velocities, θ for orientation, ω for angular velocity, ℓ for length of the quadrotor, m for mass, g for gravity acceleration in the -z direction, and a moment of inertia of J.

You are free to use any solver/cost/constraint you would like to solve for three collision free, dynamically feasible trajectories for these quadrotors that looks something like the following:



(if an animation doesn't load here, check out quadrotor_reorient.gif.)

Here are the performance requirements that the resulting trajectories must meet:

- The three quadrotors must start at xlic, x2ic, and x2ic as shown in the code (these are the initial conditions).
- The three quadrotors must finish their trajectories within .2 meters of x1g, x2g, and x2g (these are the goal states).
- ullet The three quadrotors must never be within **0.8** meters of one another (use $[p_x,p_z]$ for this).

There are two main ways of going about this:

- 1. **Cost Shaping**: Design cost functions for each quadrotor that motivates them to take paths that do not result in a collision. You can do something like designing a reference trajectory for each quadrotor to use in the cost. You can use iLQR or DIRCOL for this.
- 2. **Collision Constraints**: You can optimize over all three quadrotors at once by creating a new state $\tilde{x} = [x_1^T, x_2^T, x_3^T]^T$ and control $\tilde{u} = [u_1^T, u_2^T, u_3^T]^T$, and then directly include collision avoidance constraints. In order to use constraints, you must use DIRCOL (at least for now).

Hints

- You should not use norm() >= R in any constraints, instead you should square the constraint to be $norm()^2 >= R^2$. This second constraint is still non-convex, but it is differentiable everywhere.
- If you are using DIRCOL, you can initialize the solver with a "guess" solution by linearly interpolating between the initial and terminal conditions. Julia let's you create a length N linear interpolated vector of vectors between a::Vector and b::Vector like this: range(a, b, length = N) (experiment with this to see how it works).

You can use either RK4 (iLQR or DIRCOL) or Hermite-Simpson (DIRCOL) for your integration. The dt = 0.2, and tf = 5.0 are given for you in the code (you may change these but only if you feel you really have to).

```
In [3]: function single_quad_dynamics(params, x,u)
    # planar quadrotor dynamics for a single quadrotor

# unpack state
    px,pz,0,vx,vz,w = x

xdot = [
    vx,
    vz,
```

```
(1/params.mass)*(u[1] + u[2])*sin(\theta),
        (1/params.mass)*(u[1] + u[2])*cos(\theta) - params.q,
        (params.\ell/(2*params.J))*(u[2]-u[1])
   1
    return xdot
end
function combined_dynamics(params, x,u)
   # dynamics for three planar quadrotors, assuming the state is stacked
    # in the following manner: x = [x1; x2; x3]
   # NOTE: you would only need to use this if you chose option 2 where
   # you optimize over all three trajectories simultaneously
   # quadrotor 1
   x1 = x[1:6]
   u1 = u[1:2]
   xdot1 = single_quad_dynamics(params, x1, u1)
   # quadrotor 2
   x2 = x[(1:6) .+ 6]
   u2 = u[(1:2) .+ 2]
   xdot2 = single_quad_dynamics(params, x2, u2)
   # quadrotor 3
   x3 = x[(1:6) .+ 12]
   u3 = u[(1:2) .+ 4]
   xdot3 = single_quad_dynamics(params, x3, u3)
    # return stacked dynamics
    return [xdot1;xdot2;xdot3]
function hermite_simpson(params::NamedTuple, x1::Vector, x2::Vector, u, dt::Real)::Vector
    # TODO: input hermite simpson implicit integrator residual
   x_{k+h} = 0.5 * (x1 + x2) +
            0.125 * dt * (combined_dynamics(params,x1,u) - combined_dynamics(params,x2,u))
    residual_hs = x1 + (dt/6) * (combined_dynamics(params, x1,u) +
       4 * combined_dynamics(params, xk+h,u) + combined_dynamics(params, x2,u)) - x2
    return residual hs
end
```

hermite_simpson (generic function with 1 method)

```
In [4]: function quadrotor_cost(params::NamedTuple, Z::Vector)::Real
            idx, N, xg = params.idx, params.N, params.xg
            Q, R, Qf = params.Q, params.R, params.Qf
            J = 0
            for i = 1:(N-1)
                xi = Z[idx.x[i]]
                ui = Z[idx.u[i]]
                J += 0.5 * (xi - xg)' * Q * (xi - xg) + 0.5 * ui' * R * ui
            xN = Z[idx.x[N]]
            J += 0.5 * (xN - xg)' * Qf * (xN - xg)
            return J
        end
        function quadrotor_dynamics_constraints(params::NamedTuple, Z::Vector)::Vector
            idx, N, dt = params.idx, params.N, params.dt
            c = zeros(eltype(Z), idx.nc)
            for i = 1:(N-1)
                xi = Z[idx.x[i]]
                ui = Z[idx.u[i]]
                xip1 = Z[idx.x[i+1]]
                c[idx.c[i]] = hermite_simpson(params, xi, xip1, ui, dt)
            end
             return c[:]
        end
        function quadrotor_equality_constraints(params::NamedTuple, Z::Vector)::Vector
            N, idx, xic, xg = params.N, params.idx, params.xic, params.xg
            \# x = [Z[idx.x[i]] \text{ for } i = 1:N]
```

```
return [Z[idx.x[1]] - xic;
            Z[idx.x[N]] - xq;
            quadrotor dynamics constraints(params, Z)]
end
function quadrotor_inequality_constraints(params::NamedTuple, Z::Vector)::Vector
    N, idx, xg = params.N, params.idx,params.xg
    \# c = zeros(eltype(Z), idx.nc)
    # for i = 1:N-1
    \# c[idx.c[i]] = x[i] - xg
    d = zeros(eltype(Z), 3, N-1)
    for i = 1:(N-1)
       xi1 = Z[idx.x[i]][1:2]
       xi2 = Z[idx.x[i]][7:8]
       xi3 = Z[idx.x[i]][13:14]
        d[:,i] = [norm(xi1-xi2)^2;norm(xi2-xi3)^2;norm(xi3-xi1)^2]
    end
    return d[:]
end
```

quadrotor inequality constraints (generic function with 1 method)

```
In [5]: function create idx(nx,nu,N)
            # This function creates some useful indexing tools for Z
            \# x_i = Z[idx.x[i]]
            \# u_i = Z[idx.u[i]]
            # Feel free to use/not use anything here.
            # our Z vector is [x0, u0, x1, u1, ..., xN]
            nz = (N-1) * nu + N * nx # length of Z
            x = [(i - 1) * (nx + nu) .+ (1 : nx) for i = 1:N]
            u = [(i - 1) * (nx + nu) .+ ((nx + 1):(nx + nu)) for i = 1:(N - 1)]
            # constraint indexing for the (N-1) dynamics constraints when stacked up
            c = [(i - 1) * (nx) .+ (1 : nx) for i = 1:(N - 1)]
            nc = (N - 1) * nx # (N-1)*nx
            return (nx=nx,nu=nu,N=N,nz=nz,nc=nc,x=x,u=u,c=c)
        end
            quadrotor_reorient
        Function for returning collision free trajectories for 3 quadrotors.
        Outputs:
            x1::Vector{Vector} # state trajectory for quad 1
            x2::Vector{Vector} # state trajectory for quad 2
x3::Vector{Vector} # state trajectory for quad 3
            u1::Vector{Vector} # control trajectory for quad 1
            u2::Vector{Vector} # control trajectory for quad 2
            u3::Vector{Vector} # control trajectory for quad 3
            t_vec::Vector
            params::NamedTuple
        The resulting trajectories should have dt=0.2, tf = 5.0, N = 26
        where all the x's are length 26, and the u's are length 25.
        Each trajectory for quad k should start at `xkic`, and should finish near
         `xkg`. The distances between each quad should be greater than 0.8 meters at
        every knot point in the trajectory.
        function quadrotor_reorient(;verbose=true)
            # problem size
            nx = 18
            nu = 6
            dt = 0.2
            tf = 5.0
            t_vec = 0:dt:tf
```

```
N = length(t_vec)
# indexing
idx = create_idx(nx,nu,N)
# initial conditions and goal states
lo = 0.5
mid = 2
hi = 3.5
x1ic = [-2, lo, 0, 0, 0, 0] # ic for quad 1
x2ic = [-2, mid, 0, 0, 0, 0] # ic for quad 2
x3ic = [-2,hi,0,0,0,0] # ic for quad 3
xic = [x1ic; x2ic; x3ic]
x1g = [2,mid,0,0,0,0] # goal for quad 1
x2g = [2,hi,0,0,0,0] # goal for quad 2
x3g = [2,lo,0,0,0,0]
                      # goal for quad 3
xg = [x1g; x2g; x3g]
# load all useful things into params
Q = diagm(ones(nx))
R = 0.1*diagm(ones(nu))
Qf = 10*diagm(ones(nx))
# TODO: include anything you would need for a cost function (like a Q, R, Qf if you were doing an
# LQR cost)
params = (xlic=xlic,
          x2ic=x2ic,
          x3ic=x3ic
          x1g = x1g,
          x2g = x2g
          x3g = x3g,
          dt = dt,
          N = N,
          idx = idx,
          mass = 1.0, # quadrotor mass
          g = 9.81, # gravity
                     # quadrotor length
          \ell = 0.3,
          J = .018, # quadrotor moment of inertia
          nx = nx,
          nu = nu,
          xic = xic,
          xg = xg
          Q = Q
          R = R,
          Qf = Qf
x_l = -Inf * ones(idx.nz)
x_u = Inf * ones(idx.nz)
min_distance = 0.8
c_l = min_distance^2 * ones(3 * (idx.N-1))
c_u = Inf * ones(3 * (idx.N-1))
# initial guess
z0 = 0.001*randn(idx.nz)
x_initial_guess = range(xic, xg, length = N)
for i = 1:(N)
    z0[idx.x[i]] = x_initial_guess[i]
diff_type = :auto
Z = fmincon(quadrotor_cost,quadrotor_equality_constraints,quadrotor_inequality_constraints,
            x_l,x_u,c_l,c_u,z0,params, diff_type;
            tol = 1e-6, c_tol = 1e-6, max_iters = 10_000, verbose = verbose)
# return the trajectories
x1 = [Z[idx.x[i]][1:6] for i = 1:N]
x2 = [Z[idx.x[i]][7:12] for i = 1:N]
x3 = [Z[idx.x[i]][13:18] for i = 1:N]
u1 = [Z[idx.u[i]][1:2] for i = 1:(N-1)]
```

```
u2 = [Z[idx.u[i]][3:4] for i = 1:(N-1)]
u3 = [Z[idx.u[i]][5:6] for i = 1:(N-1)]

return x1, x2, x3, u1, u2, u3, t_vec, params
end
```

quadrotor_reorient

```
In [6]: @testset "quadrotor reorient" begin
            X1, X2, X3, U1, U2, U3, t_vec, params = quadrotor_reorient(verbose=true)
            #-----testing-----
            # check lengths of everything
            @test length(X1) == length(X2) == length(X3)
            @test length(U1) == length(U2) == length(U3)
            @test length(X1) == params.N
            @test length(U1) == (params.N-1)
            # check for collisions
            distances = [distance\_between\_quads(x1[1:2],x2[1:2],x3[1:2]) \ \ \textbf{for} \ \ (x1,x2,x3) \ \ \textbf{in} \ \ zip(X1,X2,X3)]
            @test minimum(minimum.(distances)) >= 0.799
            # check initial and final conditions
            @test norm(X1[1] - params.xlic, Inf) <= 1e-3</pre>
            @test norm(X2[1] - params.x2ic, Inf) <= 1e-3</pre>
            (x_3[1] - params.x_3ic, Inf) \le 1e-3
            (x_1] = 0 (test norm(x_1[end] - params.x_1g, Inf) <= 2e-1
            @test norm(X2[end] - params.x2g, Inf) \le 2e-1
            [atest norm(X3[end] - params.x3g, Inf) \le 2e-1
            # check dynamic feasibility
            @test check dynamic feasibility(params,X1,U1)
            @test check_dynamic_feasibility(params,X2,U2)
            @test check_dynamic_feasibility(params,X3,U3)
            #-----plotting/animation-----
            display(animate_planar_quadrotors(X1,X2,X3, params.dt))
            plot(t_vec, 0.8*ones(params.N), ls = :dash, color = :red, label = "collision distance",
                 xlabel = "time (s)", ylabel = "distance (m)", title = "Distance between Quadrotors")
            display(plot!(t_vec, hcat(distances...)', label = ["|r_1 - r_2|" "|r_1 - r_3|" "|r_2 - r_2|"]))
            X1m = hcat(X1...)
            X2m = hcat(X2...)
            X3m = hcat(X3...)
            plot(X1m[1,:], X1m[2,:], color = :red,title = "Quadrotor Trajectories", label = "quad 1")
            plot!(X2m[1,:], X2m[2,:], color = :green, label = "quad 2",xlabel = "p_x", ylabel = "p_z")
            \label{limits} display(plot!(X3m[1,:], X3m[2,:], color = :blue, label = "quad 3"))
            plot(t_vec, X1m[3,:], color = :red,title = "Quadrotor Orientations", label = "quad 1")
            plot!(t_vec, X2m[3,:], color = :green, label = "quad 2", xlabel = "time (s)", ylabel = "0")
            display(plot!(t vec, X3m[3,:], color = :blue, label = "quad 3"))
        end
```

```
-----all dimensions good------
-----diff type set to :auto (ForwardDiff.jl)----
-----testing objective gradient-----
-----testing constraint Jacobian-----
-----successfully compiled both derivatives-----
-----IPOPT beginning solve-----
This program contains Ipopt, a library for large-scale nonlinear optimization.
Ipopt is released as open source code under the Eclipse Public License (EPL).
        For more information visit https://github.com/coin-or/Ipopt
This is Ipopt version 3.14.17, running with linear solver MUMPS 5.7.3.
Number of nonzeros in equality constraint Jacobian...:
                                                     300348
                                                    46350
Number of nonzeros in inequality constraint Jacobian.:
Number of nonzeros in Lagrangian Hessian....:
Total number of variables....:
                                                        618
                   variables with only lower bounds:
               variables with lower and upper bounds:
                                                          0
                   variables with only upper bounds:
                                                          0
Total number of equality constraints....:
                                                         486
Total number of inequality constraints....:
                                                         75
       inequality constraints with only lower bounds:
                                                         75
   inequality constraints with lower and upper bounds:
                                                          0
                                                          0
       inequality constraints with only upper bounds:
                   inf_pr inf_du lg(mu) ||d|| lg(rg) alpha_du alpha_pr ls
iter
       objective
  0 2.7183001e+02 1.96e+00 1.37e+00 0.0 0.00e+00
                                                   - 0.00e+00 0.00e+00
   1 3.8663484e+02 1.96e+00 1.09e+04 -5.8 3.26e+03
                                                     - 4.96e-02 9.59e-04h 5
  2 6.6248921e+02 1.98e+00 1.15e+04 1.1 1.68e+03
                                                    - 8.04e-02 1.58e-03h
     8.6746932e+02 1.98e+00 1.20e+04 2.0 8.83e+02 7.5994192e+02 1.96e+00 1.15e+04 0.2 7.04e+01
                                                    - 8.22e-03 1.54e-03h
                                                    - 1.10e-02 9.40e-03f
     7.8733678e+02 1.95e+00 1.16e+04 2.3 4.08e+01
                                                    - 6.88e-03 4.98e-03f
    7.9058530e+02 1.95e+00 2.33e+04 2.6 4.26e+01
                                                    - 1.04e-02 2.12e-03f
  7 8.4579239e+02 1.90e+00 1.43e+06 2.7 4.59e+01
                                                    - 7.83e-01 3.56e-02f
                                    1.7 2.07e+01
                                                  - 6.48e-01 5.00e-01f
  8 9.1585904e+02 3.60e+00 4.93e+05
  9 1.3674926e+03 4.20e+00 2.48e+05
                                     2.2 7.80e+00
                                                    - 5.33e-01 1.00e+00f
      objective inf_pr inf_du lg(mu) ||d|| lg(rg) alpha_du alpha_pr ls
iter
 10 1.2965671e+03 4.21e+00 5.59e+04 1.5 8.18e+00 - 7.74e-01 1.00e+00f 1
 11 1.4334463e+03 1.97e+00 1.90e+04
                                    1.5 8.71e+00
                                                    - 6.62e-01 1.00e+00f
 12 1.2400552e+03 3.61e-01 4.70e+03 0.5 3.93e+00 - 7.52e-01 1.00e+00f
 13 1.1681081e+03 4.68e-01 9.67e+02 0.4 3.16e+00 14 1.1085875e+03 1.77e-01 6.33e+01 -0.7 8.55e+00
                                                       7.94e-01 1.00e+00f
                                                    - 9.35e-01 1.00e+00f
 15 1.0712880e+03 1.47e-01 1.93e+01 -6.1 1.25e+01
                                                    - 6.95e-01 6.78e-01f 1
 16 1.0579245e+03 1.36e-01 5.32e+00 -1.4 2.93e+00
                                                    - 9.98e-01 7.35e-01f
 17 1.0520382e+03 1.48e-01 5.72e+00 -7.0 4.62e+00
                                                    - 5.13e-01 2.53e-01f 1
                                                    - 1.07e-01 2.15e-01f 1
 18 1.0517410e+03 5.23e-01 7.36e+00 -2.5 1.67e+01
 19 1.0487085e+03 6.79e-01 1.00e+01 -2.5 9.64e+01
                                                    - 8.21e-02 2.69e-02f
      objective inf_pr inf_du lg(mu) ||d|| lg(rg) alpha_du alpha_pr ls
iter
 20 1.0446426e+03 6.62e-01 1.13e+01 -1.7 1.66e+01 - 1.87e-01 3.84e-02f 1
 21 1.0411525e+03 6.42e-01 1.21e+01
                                   -7.3 1.12e+01
                                                    - 1.53e-01 3.55e-02f
 22 1.0402400e+03 8.39e-01 1.86e+01 -2.3 2.46e+01 - 2.20e-01 1.74e-01F
 23 1.0264777e+03 7.32e-01 1.88e+01 -2.0 8.55e+00 
24 1.0215569e+03 6.98e-01 1.88e+01 -1.7 4.86e+00
                                                    - 3.08e-01 1.40e-01f
                                                    - 4.10e-01 4.66e-02f
 25 9.8754196e+02 5.93e-01 1.87e+01 -1.3 8.82e+00
                                                    - 5.69e-02 3.01e-01f
 26 9.6325389e+02 9.40e-01 1.94e+01 -0.5 1.85e+01
                                                    - 3.98e-02 1.38e-01F 1
 27 9.2879519e+02 5.61e-01 1.82e+01 -1.0 5.33e+00
                                                    - 2.72e-01 3.21e-01f 1
 28 8.9538293e+02 4.10e-01 1.49e+01 -1.0 5.13e+00
                                                    - 5.61e-01 2.63e-01f 1
 29 8.6437486e+02 3.43e-01 1.06e+01 -0.3 1.04e+01
                                                    - 2.46e-01 3.24e-01f
      objective inf_pr inf_du lg(mu) ||d|| lg(rg) alpha_du alpha_pr ls
iter
 30 8.3276569e+02 1.93e-01 7.55e+00 -2.7 8.15e+00 - 3.51e-01 5.78e-01f 1
 31 8.2429552e+02 1.14e-01 5.74e+00
                                   -2.1 3.09e+00
                                                    - 7.52e-01 4.14e-01f
 32 8.2137760e+02 7.36e-02 4.04e+00 -1.8 1.41e+00
                                                  - 7.30e-01 4.21e-01f
 33 8.2211315e+02 1.87e-01 4.61e+00 -1.4 2.51e+00 34 8.2542767e+02 2.70e-01 7.46e+00 -1.5 4.74e+00
                                                    - 8.88e-01 1.00e+00f
                                                    - 6.68e-01 1.00e+00H
 35 8.1175414e+02 1.19e-01 8.73e+00 -1.9 2.28e+00
                                                  - 9.98e-01 7.60e-01f 1
 36 8.0535600e+02 2.40e-02 3.64e+00 -2.1 1.27e+00 - 9.99e-01 8.91e-01f
 37 8.0394647e+02 4.45e-02 3.85e+00 -3.8 2.52e+00 - 5.70e-01 4.97e-01f
                                   -1.6 2.03e+01
 38 8.0382762e+02 1.70e-01 7.85e+00
                                                    - 4.09e-01 1.08e-01f
     8.0347841e+02 2.40e-01 1.15e+01 -2.1 4.73e+01
                                                    - 1.14e-01 3.84e-02f
 39
       objective inf_pr inf_du lg(mu) ||d|| lg(rg) alpha_du alpha_pr ls
iter
```

-----checking dimensions of everything-----

```
- 1.73e-01 7.13e-02f
 40 8.0218700e+02 2.52e-01 1.39e+01 -3.3 2.54e+01
 41 8.0230089e+02 3.15e-01 1.60e+01 -1.7 1.90e+01
                                                       4.55e-01 1.21e-01f
     7.9987066e+02 1.74e-01 1.67e+01 -1.9 5.71e+00
                                                       7.75e-01 7.48e-01F
     7.8829402e+02 3.95e-02 1.42e+01 -2.1 1.76e+00
                                                       1.00e+00 1.00e+00f
 44 7.8677311e+02 1.04e-02 6.74e+00 -1.6 1.26e+00
                                                     - 7.86e-01 1.00e+00f
    7.8614383e+02 1.85e-02 5.26e+00 -1.4 1.82e+00
                                                     - 1.00e+00 3.77e-01f
 46
    7.8595019e+02 8.43e-02 5.83e+00 -1.9 9.66e+00
                                                    - 7.52e-01 1.53e-01f
     7.8564018e+02 1.54e-01 7.49e+00
                                    -1.4 1.13e+01
                                                       1.00e+00 1.24e-01f
     7.8487485e+02 2.05e-01 8.50e+00
                                    -7.5 1.42e+01
                                                        2.87e-01 8.56e-02f
 48
 49
    7.8436601e+02 2.36e-01 9.28e+00 -1.8 1.21e+01
                                                     - 5.23e-01 1.07e-01f
iter
      objective
                 inf_pr inf_du lg(mu) ||d|| lg(rg) alpha_du alpha_pr ls
 50 7.8421279e+02 2.83e-01 1.06e+01 -1.3 1.07e+01
                                                   - 1.00e+00 1.25e-01f
                                                       4.60e-01 1.00e+00H
 51 7.9539985e+02 1.23e-01 2.26e+01
                                    -1.3 7.06e+00
     7.7892504e+02 7.74e-02 2.39e+01
                                                       1.00e+00 4.10e-01f
                                    -1.4 2.12e+00
 53 7.7511315e+02 1.56e-01 1.78e+01 -0.6 3.64e+00
                                                    - 4.25e-01 1.00e+00f
    7.6566930e+02 2.71e-01 9.08e+00 -0.8 2.86e+00
                                                     - 7.95e-01 8.62e-01f
    7.5424459e+02 4.04e-01 9.11e+00 -6.8 6.01e+00
                                                     - 5.04e-01 6.78e-01f
    7.5501789e+02 1.36e+00 1.40e+01 -0.9 1.38e+01
                                                     - 5.63e-01 4.70e-01F
     7.5215645e+02 1.54e+00 2.10e+01
                                    -1.1 1.38e+01
                                                       3.42e-01 3.69e-01F
     7.0033459e+02 6.31e-01 2.78e+01 -0.8 4.33e+00
                                                       5.31e-01 1.00e+00f
    6.6684743e+02 1.12e+00 6.41e+01 -0.6 1.15e+01
                                                    - 6.17e-01 9.29e-01f
      objective inf_pr inf_du lg(mu) ||d|| lg(rg) alpha_du alpha_pr ls
 60 6.6433086e+02 1.45e+00 6.83e+01 -0.2 8.95e+02
                                                   - 2.36e-02 8.99e-03f
 61 7.3364823e+02 8.68e-01 2.03e+01
                                    -0.0 2.23e+01
                                                       2.17e-01 1.00e+00f
                                    -0.1 3.31e+01
     6.2571216e+02 4.82e-01 1.54e+01
                                                       8.83e-01 6.13e-01f
    6.0259727e+02 2.26e-01 2.64e+01 -0.7 9.07e+00
                                                    - 7.68e-01 1.00e+00f
    5.9797380e+02 2.08e-01 2.54e+01 -6.5 6.63e+00
                                                     - 8.52e-01 8.29e-02f
    5.9026057e+02 6.20e-02 1.45e+01 -0.8 2.35e+00
                                                     - 6.50e-01 1.00e+00f
                                                     - 1.00e+00 4.12e-01f
    5.8281548e+02 3.97e-02 1.45e+01 -1.7 2.61e+00
     5.7831841e+02 1.16e-02 2.35e+00
                                    -1.4 1.65e+00
                                                        1.00e+00 9.43e-01f
    5.7713838e+02 7.57e-03 8.18e-01 -2.6 5.79e-01
                                                     - 9.96e-01 1.00e+00f
    5.7659948e+02 1.29e-02 1.32e+00 -2.8 7.39e-01
                                                     - 9.98e-01 7.21e-01f 1
 69
      objective inf_pr inf_du lg(mu) ||d|| lg(rg) alpha_du alpha_pr ls
 70 5.7649446e+02 9.20e-02 3.16e+00 -2.3 1.10e+01
                                                   - 1.00e+00 1.25e-01f
     5.7666365e+02 2.13e-01 4.83e+00
                                    -1.4 3.96e+01
                                                       1.00e+00 4.15e-02f
                                    -1.5 2.59e+01
     5.7643781e+02 2.73e-01 6.04e+00
                                                        1.66e-01 5.18e-02f
 73 5.9517974e+02 5.64e-02 9.72e+00 -1.7 7.90e+00
                                                    - 2.48e-01 1.00e+00H
    5.7354451e+02 2.12e-01 7.00e+00
                                    -1.6 2.85e+00
                                                     - 1.00e+00 1.00e+00f
 75 5.7192351e+02 7.94e-02 4.48e+00
                                    -2.0 2.56e+00
                                                     - 1.00e+00 9.91e-01f
                                    -2.2 7.11e-01
                                                     - 1.00e+00 9.25e-01h
    5.7103663e+02 2.78e-02 2.80e+00
     5.7103750e+02 6.95e-02 2.24e+00
                                     -2.9 1.18e+00
                                                        1.00e+00 1.00e+00h
 78
    5.7085347e+02 1.21e-01 3.31e+00
                                    -3.2 4.96e+00
                                                        1.00e+00 2.91e-01f
                                    -3.7 9.55e+00
                                                     - 1.00e+00 1.58e-01f 1
 79
    5.7065094e+02 2.00e-01 5.16e+00
                   inf_pr inf_du lg(mu) ||d|| lg(rg) alpha_du alpha_pr ls
       objective
     5.7023480e+02 1.95e-01 5.68e+00 -4.4 5.83e+00
                                                   - 4.95e-01 1.22e-01f
     5.6864402e+02 1.37e-01 8.40e+00
                                    -2.4 7.98e+00
                                                       1.00e+00 2.49e-01F
     5.7946270e+02 3.51e-02 1.07e+01
                                    -1.6 7.61e+00
                                                       1.24e-01 1.00e+00H
    5.6406397e+02 8.30e-02 8.71e+00 -2.1 7.23e+00
                                                    - 8.80e-01 8.14e-01f
    5.6231629e+02 5.80e-03 1.43e+00
                                    -2.3 8.31e-01
                                                    - 1.00e+00 1.00e+00h
    5.6129101e+02 7.80e-03 2.03e+00
                                    -2.7 9.16e-01
                                                     - 1.00e+00 8.49e-01f
    5.6092617e+02 1.49e-02 2.47e+00
                                    -4.8 2.83e+00
                                                     - 6.22e-01 1.56e-01f
 86
     5.6099619e+02 9.05e-02 3.63e+00
                                     -1.7 7.41e+00
                                                       6.54e-01 1.91e-01f
 88
     5.6070924e+02 1.22e-01 4.52e+00
                                    -2.0 1.60e+01
                                                       1.38e-02 6.25e-02f
                                                     - 1.99e-01 2.15e-02f 4
                                    -1.3 6.71e+01
 89
    5.6060379e+02 1.97e-01 5.50e+00
                 inf_pr inf_du lg(mu) ||d|| lg(rg) alpha_du alpha_pr ls
       objective
 90 5.5927099e+02 1.65e-01 8.27e+00 -7.8 3.04e+01
                                                   - 1.62e-01 4.82e-02F
     5.5897752e+02 1.81e-01 1.02e+01
                                    -1.8 8.65e+00
                                                       2.63e-01 1.59e-01f
     5.5888463e+02 1.17e-01 1.31e+01
                                    -2.2 4.98e+00
                                                     - 8.56e-01 6.31e-01F
    5.5555651e+02 6.57e-02 1.17e+01 -2.2 1.18e+00
                                                    - 1.00e+00 6.00e-01f
    5.5325554e+02 4.06e-03 5.76e+00
                                    -2.0 6.55e-01
                                                    - 1.00e+00 1.00e+00f
    5.5291442e+02 4.22e-03 5.25e+00
                                    -2.2 1.93e+00
                                                     - 1.00e+00 1.75e-01f
    5.5280394e+02 1.56e-01 5.43e+00
                                    -3.1 1.31e+01
                                                     - 6.79e-02 1.52e-01f
                                                        1.52e-01 6.03e-02f
     5.5253686e+02 2.60e-01 6.68e+00
                                     -2.3 3.26e+01
    5.5190730e+02 3.01e-01 8.75e+00
                                    -1.9 4.42e+01
                                                     - 4.84e-01 2.86e-02f
    5.5133254e+02 3.23e-01 1.02e+01 -1.6 1.38e+01
                                                     - 6.71e-01 9.24e-02f 3
       objective
                 inf_pr inf_du lg(mu) ||d|| lg(rg) alpha_du alpha_pr ls
100 5.5106558e+02 3.58e-01 1.27e+01 -2.2 1.86e+01
                                                       5.16e-01 1.14e-01f
     5.5135407e+02 2.96e-01 1.30e+01
                                    -0.9 1.18e+01
                                                       3.05e-01 2.06e-01f
     5.4629490e+02 1.15e-01 9.79e+00
                                    -1.4 1.75e+00
                                                     - 8.09e-01 7.95e-01f
103 5.4606835e+02 2.76e-02 7.71e+00 -0.9 1.38e+00
                                                    - 6.41e-01 1.00e+00f
104 5.4331200e+02 8.54e-02 6.71e+00
                                    -1.9 2.55e+00
                                                    - 1.00e+00 5.24e-01f
105 5.4277616e+02 3.76e-01 1.15e+01
                                    -1.9 2.86e+01
                                                     - 8.19e-01 1.10e-01f
                                                    - 2.01e-01 6.10e-02f
106
     5.3969698e+02 5.50e-01 1.37e+01
                                    -7.5 3.48e+01
                                     -2.8 7.73e+00
                                                        2.50e-01 1.88e-01f
     5.3452734e+02 5.14e-01 1.42e+01
108 5.3095911e+02 4.33e-01 1.38e+01 -1.1 3.05e+00
                                                    - 9.10e-02 1.83e-01f
```

```
- 1.00e+00 1.18e-01f 1
 109 5.2752705e+02 3.90e-01 1.34e+01 -1.1 5.06e+00
iter
        objective inf_pr inf_du lg(mu) ||d|| lg(rg) alpha_du alpha_pr ls
 110 5.2380642e+02 3.76e-01 1.18e+01 -7.2 9.70e+00
                                                           1.82e-01 7.03e-02f
 111 5.1746098e+02 5.09e-01 1.58e+01 -0.8 1.07e+01
                                                        - 3.62e-02 2.50e-01f
 112 1.5177647e+03 2.99e+00 4.15e+01 0.9 3.03e+02
                                                        - 2.73e-02 1.93e-01f 1
 113 1.0306998e+03 2.16e+00 3.49e+01 0.0 1.75e+02 - 3.37e-01 2.92e-01f 1
 114 8.9839912e+02 1.83e+00 2.69e+01 0.0 1.24e+01 - 9.02e-01 1.71e-01f 1
     7.9915226e+02 8.23e+00 6.53e+01 0.0 2.17e+01 5.9546278e+02 3.83e+00 3.31e+01 0.0 1.53e+01
 115
                                                        - 5.63e-01 7.54e-01f
                                                           1.42e-01 8.59e-01f
 116
 117 7.2680501e+02 3.97e+00 2.23e+01 0.0 1.11e+01
                                                        - 3.75e-01 1.00e+00h
 118 5.0565527e+02 2.70e+00 2.04e+01 0.0 1.01e+01
                                                        - 9.34e-01 1.00e+00f
 119 4.8648647e+02 1.53e+00 9.12e+00 -1.9 1.38e+01
                                                      - 9.99e-01 3.99e-01f 1
      objective inf_pr inf_du lg(mu) ||d|| lg(rg) alpha_du alpha_pr ls
iter
                                                      - 9.86e-01 8.30e-01f
 120 4.6457770e+02 1.59e+00 5.67e+00 -1.1 1.16e+01
 121 4.4538885e+02 3.46e-01 5.36e+00 -1.1 2.03e+00
                                                        - 5.46e-01 8.37e-01f
 122 4.3875833e+02 5.38e-02 1.88e+00 -1.8 2.44e+00 - 7.42e-01 9.86e-01f
 123 4.3632826e+02 2.82e-02 7.28e-01 -1.9 2.78e+00 - 6.22e-01 7.79e-01f 1
 124 4.3663076e+02 8.64e-04 8.35e-01 -2.0 1.39e+00 - 8.90e-01 1.00e+00H 1
 125  4.3568660e+02 1.91e-03 5.96e-01 -2.4 1.02e+00 
126  4.3565433e+02 2.98e-04 1.31e-01 -3.0 1.73e-01
                                                        - 9.95e-01 8.08e-01f
                                                           1.00e+00 1.00e+00h
                                                        - 1.00e+00 9.62e-01h
 127 4.3563655e+02 5.25e-05 1.12e-01 -4.6 5.60e-02
                                                        - 1.00e+00 1.00e+00h 1
 128 4.3563232e+02 1.60e-05 8.29e-02 -5.8 5.07e-02
 129 4.3563175e+02 2.05e-05 5.32e-02 -7.1 3.82e-02
                                                        - 1.00e+00 9.94e-01h 1
                    inf_pr inf_du lg(mu) ||d|| lg(rg) alpha_du alpha_pr ls
iter
       obiective
 130 4.3563085e+02 5.78e-06 1.76e-02 -8.6 1.37e-02 131 4.3563078e+02 8.32e-07 7.23e-03 -9.9 9.25e-03
                                                           1.00e+00 1.00e+00h
                                                           1.00e+00 1.00e+00h
 132 4.3563077e+02 5.09e-08 3.35e-03 -11.0 1.98e-03
                                                        - 1.00e+00 1.00e+00h
 133 4.3563076e+02 1.25e-07 5.06e-03 -11.0 5.04e-03
                                                        - 1.00e+00 1.00e+00h 1
 134 4.3563076e+02 4.58e-10 1.02e-03 -11.0 1.55e-03
                                                        - 1.00e+00 1.00e+00H
     4.3563075e+02 4.58e-08 4.13e-04 -11.0 1.67e-03
                                                           1.00e+00 1.00e+00h
 136 4.3563075e+02 3.86e-10 5.21e-05 -11.0 2.03e-04
                                                           1.00e+00 1.00e+00h
                                                        - 1.00e+00 1.00e+00h
 137 4.3563075e+02 4.29e-10 3.29e-05 -11.0 3.75e-04
 138 4.3563075e+02 3.28e-10 6.27e-05 -11.0 1.13e-04
                                                           1.00e+00 1.00e+00h 1
 139 4.3563075e+02 1.44e-10 5.13e-06 -11.0 8.20e-05
                                                           1.00e+00 1.00e+00h 1
iter
       objective
                    inf_pr inf_du lg(mu) ||d|| lg(rg) alpha_du alpha_pr ls
 140 4.3563075e+02 3.27e-12 3.70e-06 -11.0 1.39e-05
                                                           1.00e+00 1.00e+00h 1
 141 4.3563075e+02 4.94e-12 3.03e-06 -11.0 2.24e-05
                                                           1.00e+00 1.00e+00h
 142 4.3563075e+02 1.43e-12 2.84e-06 -11.0 1.09e-05
                                                        - 1.00e+00 1.00e+00h 1
 143 4.3563075e+02 4.45e-13 3.89e-07 -11.0 5.43e-06
                                                        - 1.00e+00 1.00e+00h 1
Number of Iterations....: 143
                                   (scaled)
                                                             (unscaled)
Objective..... 4.3563075204740028e+02
                                                      4.3563075204740028e+02
Dual infeasibility.....: 3.8895207286481111e-07
                                                      3.8895207286481111e-07
Constraint violation...:
Variable bound violation:
                           4.4468595472579864e-13
                                                      4.4468595472579864e-13
                            0.00000000000000000e+00
                                                      0.0000000000000000e+00
Complementarity.....: 1.0000169573587904e-11
                                                      1.0000169573587904e-11
Overall NLP error....: 3.8895207286481111e-07
                                                      3.8895207286481111e-07
Number of objective function evaluations
                                                     = 280
Number of objective gradient evaluations
                                                     = 144
                                                     = 280
Number of equality constraint evaluations
Number of inequality constraint evaluations
                                                     = 280
Number of equality constraint Jacobian evaluations = 144
Number of inequality constraint Jacobian evaluations = 144
Number of Lagrangian Hessian evaluations
                                                     = 0
Total seconds in IPOPT
                                                     = 23.051
EXIT: Optimal Solution Found.
_{\text{\cbsc}} Info: Listening on: 127.0.0.1:8702, thread id: 1 @ HTTP.Servers /home/burger/.julia/packages/HTTP/4AUPl/src/Servers.jl:382
_{\mathsf{\Gamma}} Info: MeshCat server started. You can open the visualizer by visiting the following URL in your browser:
  http://127.0.0.1:8702
L@ MeshCat /home/burger/.julia/packages/MeshCat/9QrxD/src/visualizer.jl:43
```

Open Controls

