In [1]:
```
import Pkg
Pkg.activate(@__DIR__)
Pkg.instantiate()
using LinearAlgebra, Plots
import ForwardDiff as FD
using MeshCat
using Test
using Plots
```

```
Activating project at `~/OCRL/HW1_S25`
```

Q2: Equality Constrained Optimization (25 pts)

In this problem, we are going to use Newton's method to solve some constrained optimization problems. We will start with a smaller problem where we can experiment with Full Newton vs Gauss-Newton, then we will use these methods to solve for the motor torques that make a quadruped balance on one leg.

Part A (10 pts)

Here we are going to solve some equality-constrained optimization problems with Newton's method. We are given a problem

$$\min_x \quad f(x) \tag{1}$$
$$\text{st} \quad c(x) = 0 \tag{2}$$

Which has the following Lagrangian:

$$\mathcal{L}(x, \lambda) = f(x) + \lambda^T c(x),$$

and the following KKT conditions for optimality:

$$\nabla_x \mathcal{L} = \nabla_x f(x) + \left[\frac{\partial c}{\partial x}\right]^T \lambda = 0 \tag{3}$$
$$c(x) = 0 \tag{4}$$

Which is just a root-finding problem. To solve this, we are going to solve for a $z = [x^T, \lambda]^T$ that satisfies these KKT conditions.

Newton's Method with a Linesearch

We use Newton's method to solve for when $r(z) = 0$. To do this, we specify `res_fx(z)` as $r(z)$, and `res_jac_fx(z)` as $\partial r/\partial z$. To calculate a Newton step, we do the following:

$$\Delta z = -\left[\frac{\partial r}{\partial z}\right]^{-1} r(z_k)$$

We then decide the step length with a linesearch that finds the largest $\alpha \leq 1$ such that the following is true:

$$\phi(z_k + \alpha \Delta z) < \phi(z_k)$$

Where ϕ is a "merit function", or `merit_fx(z)` in the code. In this assignment you will use a backtracking linesearch where α is initialized as $\alpha = 1.0$, and is divided by 2 until the above condition is satisfied.

NOTE: YOU DO NOT NEED TO (AND SHOULD NOT) USE A WHILE LOOP ANYWHERE IN THIS ASSIGNMENT.

In [2]:
```
function linesearch(z::Vector, Δz::Vector, merit_fx::Function;
                    max_ls_iters = 20)::Float64 # optional argument with a default

    # TODO: return maximum α≤1 such that merit_fx(z + α*Δz) < merit_fx(z)
    # with a backtracking linesearch (α = α/2 after each iteration)

    # NOTE: DO NOT USE A WHILE LOOP
```

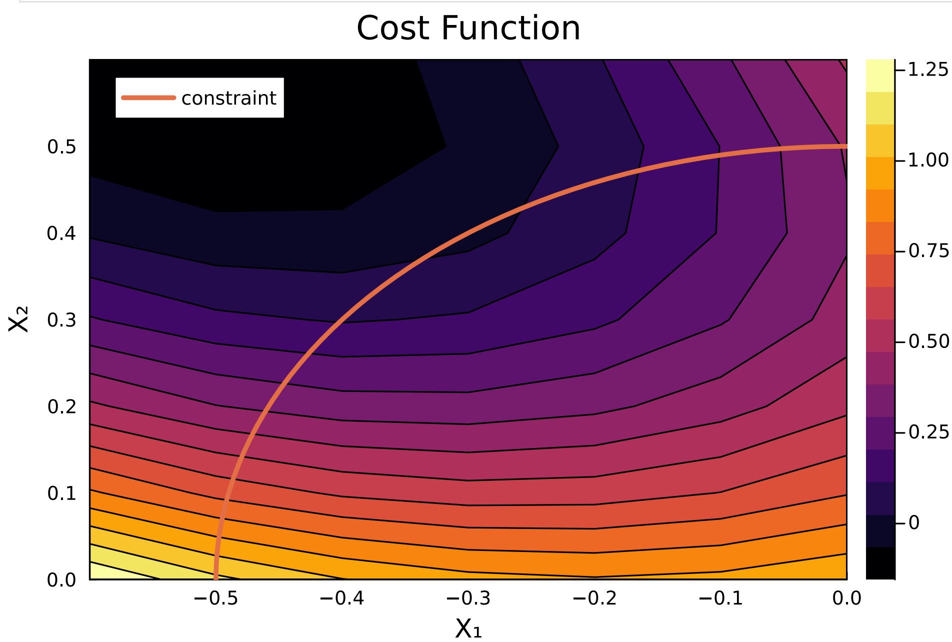
```julia
    b = 1e-4
    for i = 1:max_ls_iters
        alpha = 1.0 / 2^(i-1)

        #if dot(FD.gradient(merit_fx,z),Δz) < 0 #check if the direction is correct
        #    Δz = - Δz
        #end

        if merit_fx(z + alpha*Δz) < merit_fx(z) + b*alpha* dot(FD.gradient(merit_fx,z),Δz)
            return alpha
        end

        # TODO: return α when merit_fx(z + α*Δz) < merit_fx(z)
    end
    error("linesearch failed")
end

function newtons_method(z0::Vector, res_fx::Function, res_jac_fx::Function, merit_fx::Function;
                        tol = 1e-10, max_iters = 50, verbose = false)::Vector{Vector{Float64}}

    # TODO: implement Newton's method given the following inputs:
    # - z0, initial guess
    # - res_fx, residual function
    # - res_jac_fx, Jacobian of residual function wrt z
    # - merit_fx, merit function for use in linesearch

    # optional arguments
    # - tol, tolerance for convergence. Return when norm(residual)<tol
    # - max iter, max # of iterations
    # - verbose, bool telling the function to output information at each iteration

    # return a vector of vectors containing the iterates
    # the last vector in this vector of vectors should be the approx. solution

    # NOTE: DO NOT USE A WHILE LOOP ANYWHERE

    # return the history of guesses as a vector
    Z = [zeros(length(z0)) for i = 1:max_iters]
    Z[1] = z0


    for i = 1:(max_iters - 1)

        # NOTE: everything here is a suggestion, do whatever you want to

        # TODO: evaluate current residual
        r = res_fx(Z[i])
        norm_r = norm(r)
        if verbose
            print("iter: $i    |r|: $norm_r   ")
        end

        # TODO: check convergence with norm of residual < tol

        # if converged, return Z[1:i]
        if norm_r < tol
            return Z[1:i]
        end

        # TODO: caculate Newton step (don't forget the negative sign)
        Δz = - res_jac_fx(Z[i]) \ r

        # TODO: linesearch and update z
        alpha = linesearch(Z[i], Δz, merit_fx)
        Z[i+1] = Z[i] + alpha*Δz


        if verbose
            print("α: $alpha \n")
        end

    end
    error("Newton's method did not converge")
end
```

```
newtons_method (generic function with 1 method)
```

In [3]:
```julia
@testset "check Newton" begin

    f(_x) = [sin(_x[1]), cos(_x[2])]
    df(_x) = FD.jacobian(f, _x)
    merit(_x) = norm(f(_x))

    x0 = [-1.742410372590328, 1.4020334125022704]

    X = newtons_method(x0, f, df, merit; tol = 1e-10, max_iters = 50, verbose = true)

    # check this took the correct number of iterations
    # if your linesearch isn't working, this will fail
    # you should see 1 iteration where α = 0.5
    @test length(X) == 6

    # check we actually converged
    @test norm(f(X[end])) < 1e-10

end
```

```
iter: 1    |r|: 0.9995239729818045   α: 1.0
iter: 2    |r|: 0.9421342427117169   α: 0.5
iter: 3    |r|: 0.1753172908866053   α: 1.0
iter: 4    |r|: 0.0018472215879181287   α: 1.0
iter: 5    |r|: 2.1010529101114843e-9   α: 1.0
iter: 6    |r|: 2.5246740534795566e-16   Test Summary: | Pass  Total  Time
check Newton  |    2      2  1.9s
Test.DefaultTestSet("check Newton", Any[], 2, false, false, true, 1.738902283777001e9, 1.7389022
85656645e9, false, "/home/burger/OCRL/HW1_S25/jl_notebook_cell_df34fa98e69747e1a8f8a730347b8e2f_
W5sZmlsZQ==.jl")
```

In [4]:
```julia
let
    function plotting_cost(x::Vector)
        Q = [1.65539  2.89376; 2.89376  6.51521];
        q = [2;-3]
        return 0.5*x'*Q*x + q'*x + exp(-1.3*x[1] + 0.3*x[2]^2)
    end
    contour(-.6:.1:0,0:.1:.6, (x1,x2)-> plotting_cost([x1;x2]),title = "Cost Function",
            xlabel = "X₁", ylabel = "X₂",fill = true)
    xcirc = [.5*cos(θ) for θ in range(0, 2*pi, length = 200)]
    ycirc = [.5*sin(θ) for θ in range(0, 2*pi, length = 200)]
    plot!(xcirc,ycirc, lw = 3.0, xlim = (-.6, 0), ylim = (0, .6),label = "constraint")
end
```



We will now use Newton's method to solve the following constrained optimization problem. We will write functions for the full Newton Jacobian, as well as the Gauss-Newton Jacobian.

In [5]:
```julia
# we will use Newton's method to solve the constrained optimization problem shown above
```

```julia
function cost(x::Vector)
    Q = [1.65539  2.89376; 2.89376  6.51521];
    q = [2;-3]
    return 0.5*x'*Q*x + q'*x + exp(-1.3*x[1] + 0.3*x[2]^2)
end
function constraint(x::Vector)
    norm(x) - 0.5
end
# HINT: use this if you want to, but you don't have to
function constraint_jacobian(x::Vector)::Matrix
    # since `constraint` returns a scalar value, ForwardDiff
    # will only allow us to compute a gradient of this function
    # (instead of a Jacobian). This means we have two options for
    # computing the Jacobian: Option 1 is to just reshape the gradient
    # into a row vector

    # J = reshape(FD.gradient(constraint, x), 1, 2)

    # or we can just make the output of constraint an array,
    constraint_array(_x) = [constraint(_x)]
    J = FD.jacobian(constraint_array, x)

    # assert the jacobian has # rows = # outputs
    # and # columns = # inputs
    @assert size(J) == (length(constraint(x)), length(x))

    return J'
end
function kkt_conditions(z::Vector)::Vector
    # TODO: return the KKT conditions
    x = z[1:2]
    λ = z[3:end]
    # TODO: return the stationarity condition for the cost function
    # and the primal feasibility

    stationarity_cdn = FD.gradient(cost, x) + constraint_jacobian(x)*λ
    primal_feasibility = constraint(x)
    r = vcat(stationarity_cdn, primal_feasibility)
    #error("kkt not implemented")

    return [stationarity_cdn[1], stationarity_cdn[2], primal_feasibility]
end

function fn_kkt_jac(z::Vector)::Matrix
    # TODO: return full Newton Jacobian of kkt conditions wrt z
    x = z[1:2]
    λ = z[3]


    L_hess = FD.hessian(cost,x) + FD.jacobian(constraint_jacobian,x)

    kkt_jac = [L_hess constraint_jacobian(x);
               constraint_jacobian(x)' zeros(1,1)]
    kkt_jac -= 1e-3 * I(3)
    return kkt_jac

    #error("fn_kkt_jac not implemented")
    #return nothing
end
function gn_kkt_jac(z::Vector)::Matrix
    # TODO: return Gauss-Newton Jacobian of kkt conditions wrt z
    x = z[1:2]
    λ = z[3]

    J = FD.jacobian(x -> FD.gradient(cost,x),x)

    gnkkyt_jac = [FD.hessian(cost,x) constraint_jacobian(x);
               constraint_jacobian(x)' zeros(1,1)]
    # TODO: return Gauss-Newton jacobian with a 1e-3 regularizer
    gnkkyt_jac -= 1e-3 * I(3)
    return gnkkyt_jac
    #error("gn_kkt_jac not implemented")
    #return nothing
end
```

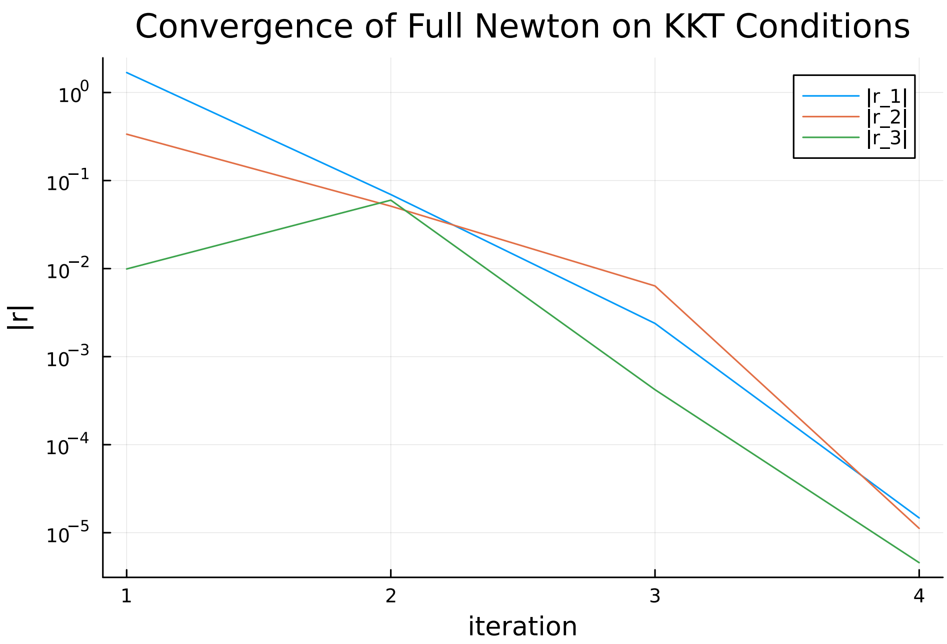
```
gn_kkt_jac (generic function with 1 method)
```

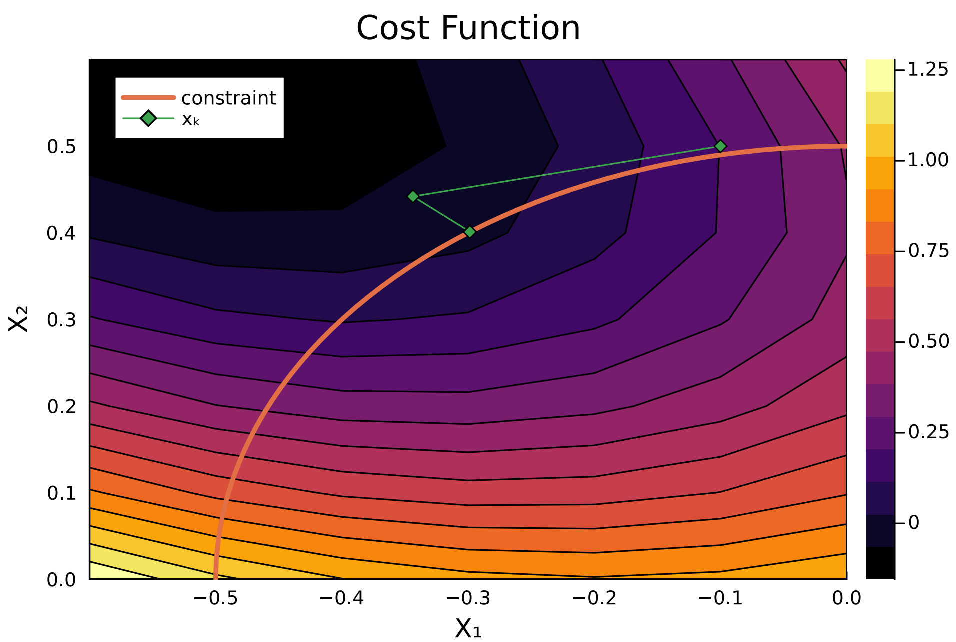
In [6]:
```
@testset "Test Jacobians" begin

    # first we check the regularizer
    z = randn(3)
    J_fn = fn_kkt_jac(z)
    J_gn = gn_kkt_jac(z)

    # check what should/shouldn't be the same between
    @test norm(J_fn[1:2,1:2] - J_gn[1:2,1:2]) > 1e-10
    @test abs(J_fn[3,3] + 1e-3) < 1e-10
    @test abs(J_gn[3,3] + 1e-3) < 1e-10
    @test norm(J_fn[1:2,3] - J_gn[1:2,3]) < 1e-10
    @test norm(J_fn[3,1:2] - J_gn[3,1:2]) < 1e-10
end
```

```
Test Summary:  | Pass  Total  Time
Test Jacobians |    5      5  6.0s
Test.DefaultTestSet("Test Jacobians", Any[], 5, false, false, true, 1.738902288598111e9, 1.73890
2294576447e9, false, "/home/burger/OCRL/HW1_S25/jl_notebook_cell_df34fa98e69747e1a8f8a730347b8e2
f_X12sZmlsZQ==.jl")
```

In [7]:
```
@testset "Full Newton" begin


    z0 = [-.1, .5, 0] # initial guess
    merit_fx(_z) = norm(kkt_conditions(_z)) # simple merit function
    Z = newtons_method(z0, kkt_conditions, fn_kkt_jac, merit_fx; tol = 1e-4,
    max_iters = 100, verbose = true)
    R = kkt_conditions.(Z)

    # make sure we converged on a solution to the KKT conditions
    @test norm(kkt_conditions(Z[end])) < 1e-4
    @test length(R) < 6


    # -----------------------plotting stuff-----------------------
    Rp = [[abs(R[i][ii]) + 1e-15 for i = 1:length(R)] for ii = 1:length(R[1])] # this gets abs (

    plot(Rp[1],yaxis=:log,ylabel = "|r|",xlabel = "iteration",
         yticks= [1.0*10.0^(-x) for x = float(15:-1:-2)],
         title = "Convergence of Full Newton on KKT Conditions",label = "|r_1|")
    plot!(Rp[2],label = "|r_2|")
    display(plot!(Rp[3],label = "|r_3|"))

    contour(-.6:.1:0,0:.1:.6, (x1,x2)-> cost([x1;x2]),title = "Cost Function",
            xlabel = "X₁", ylabel = "X₂",fill = true)
    xcirc = [.5*cos(θ) for θ in range(0, 2*pi, length = 200)]
    ycirc = [.5*sin(θ) for θ in range(0, 2*pi, length = 200)]
    plot!(xcirc,ycirc, lw = 3.0, xlim = (-.6, 0), ylim = (0, .6),label = "constraint")
    z1_hist = [z[1] for z in Z]
    z2_hist = [z[2] for z in Z]
    display(plot!(z1_hist, z2_hist, marker = :d, label = "xₖ"))
    # -----------------------plotting stuff-----------------------
end
```

```
iter: 1    |r|: 1.7188450769812715   α: 1.0
iter: 2    |r|: 0.10496246835869102   α: 1.0
iter: 3    |r|: 0.0067966851555096835   α: 1.0
iter: 4    |r|: 1.9051291860785185e-5   Test Summary: | Pass  Total  Time
Full Newton   |    2      2  3.8s
```





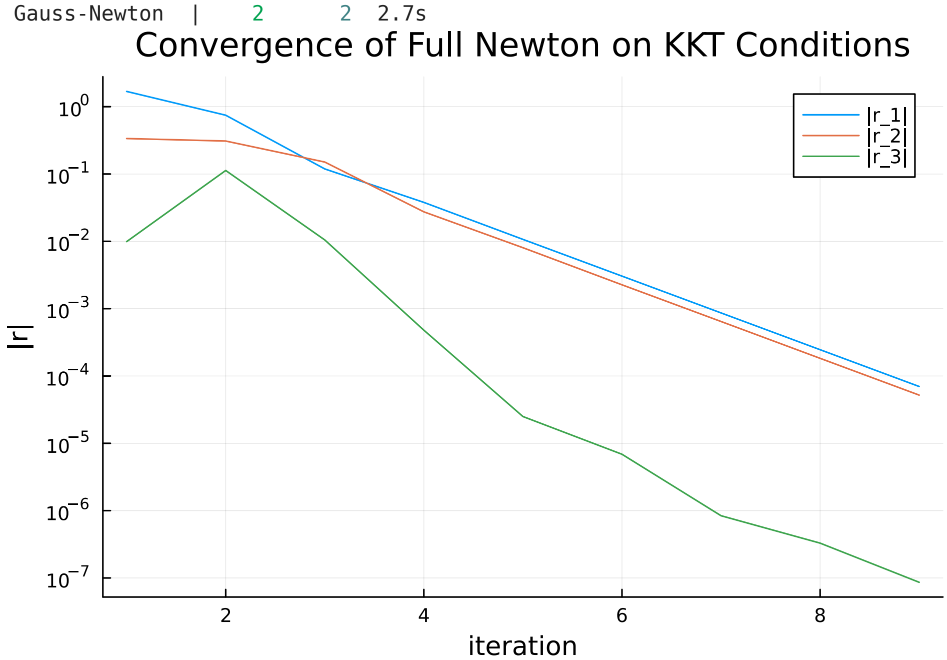

```
Test.DefaultTestSet("Full Newton", Any[], 2, false, false, true, 1.738902294590215e9, 1.73890229
8372363e9, false, "/home/burger/OCRL/HW1_S25/jl_notebook_cell_df34fa98e69747e1a8f8a730347b8e2f_X
13sZmlsZQ==.jl")
```

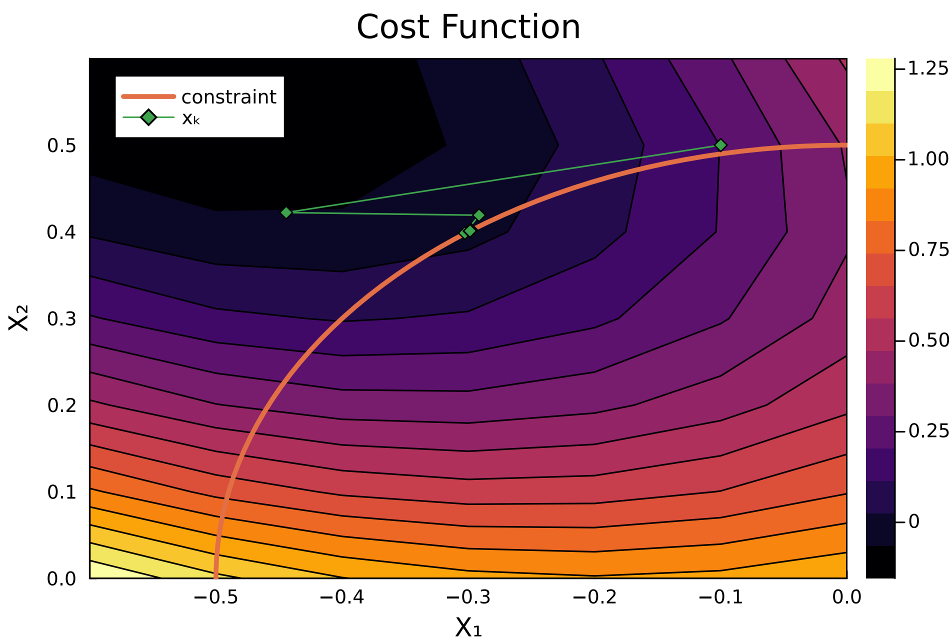
In [8]:

```
@testset "Gauss-Newton" begin


    z0 = [-.1, .5, 0] # initial guess
    merit_fx(_z) = norm(kkt_conditions(_z)) # simple merit function

    # the only difference in this block vs the previous is `gn_kkt_jac` instead of `fn_kkt_jac`
    Z = newtons_method(z0, kkt_conditions, gn_kkt_jac, merit_fx; tol = 1e-4,
     max_iters = 100, verbose = true)
    R = kkt_conditions.(Z)

    # make sure we converged on a solution to the KKT conditions
    @test norm(kkt_conditions(Z[end])) < 1e-4
    @test length(R) < 10


    # ------------------------plotting stuff------------------------
    Rp = [[abs(R[i][ii]) + 1e-15 for i = 1:length(R)] for ii = 1:length(R[1])] # this gets abs o

    plot(Rp[1],yaxis=:log,ylabel = "|r|",xlabel = "iteration",
         yticks= [1.0*10.0^(-x) for x = float(15:-1:-2)],
         title = "Convergence of Full Newton on KKT Conditions",label = "|r_1|")
```

```
    plot!(Rp[2],label = "|r_2|")
    display(plot!(Rp[3],label = "|r_3|"))

    contour(-.6:.1:0,0:.1:.6, (x1,x2)-> cost([x1;x2]),title = "Cost Function",
            xlabel = "X₁", ylabel = "X₂",fill = true)
    xcirc = [.5*cos(θ) for θ in range(0, 2*pi, length = 200)]
    ycirc = [.5*sin(θ) for θ in range(0, 2*pi, length = 200)]
    plot!(xcirc,ycirc, lw = 3.0, xlim = (-.6, 0), ylim = (0, .6),label = "constraint")
    z1_hist = [z[1] for z in Z]
    z2_hist = [z[2] for z in Z]
    display(plot!(z1_hist, z2_hist, marker = :d, label = "xₖ"))
    # -----------------------plotting stuff-----------------------
end
```

```
iter: 1    |r|: 1.7188450769812715   α: 1.0
iter: 2    |r|: 0.8163267519728127   α: 1.0
iter: 3    |r|: 0.1922177677011686   α: 1.0
iter: 4    |r|: 0.04678866823071684   α: 1.0
iter: 5    |r|: 0.0133893914077011   α: 1.0
iter: 6    |r|: 0.003792680422273685   α: 1.0
iter: 7    |r|: 0.0010784701647844727   α: 1.0
iter: 8    |r|: 0.00030635243738783284   α: 1.0
iter: 9    |r|: 8.7049117069183e-5   Test Summary: | Pass  Total  Time
Gauss-Newton  |    2      2  2.7s
```






```
Test.DefaultTestSet("Gauss-Newton", Any[], 2, false, false, true, 1.738902298395068e9, 1.738902301052465e9, false, "/home/burger/OCRL/HW1_S25/jl_notebook_cell_df34fa98e69747e1a8f8a730347b8e2f_X14sZmlsZQ==.jl")
```

Part B (10 pts): Balance a quadruped

Now we are going to solve for the control input $u \in \mathbb{R}^{12}$, and state $x \in \mathbb{R}^{30}$, such that the quadruped is balancing up on one leg at an equilibrium point. First, let's load in a dynamics model from `quadruped.jl`, where

$\dot{x} = f(x, u) =$ `dynamics(model, x, u)`

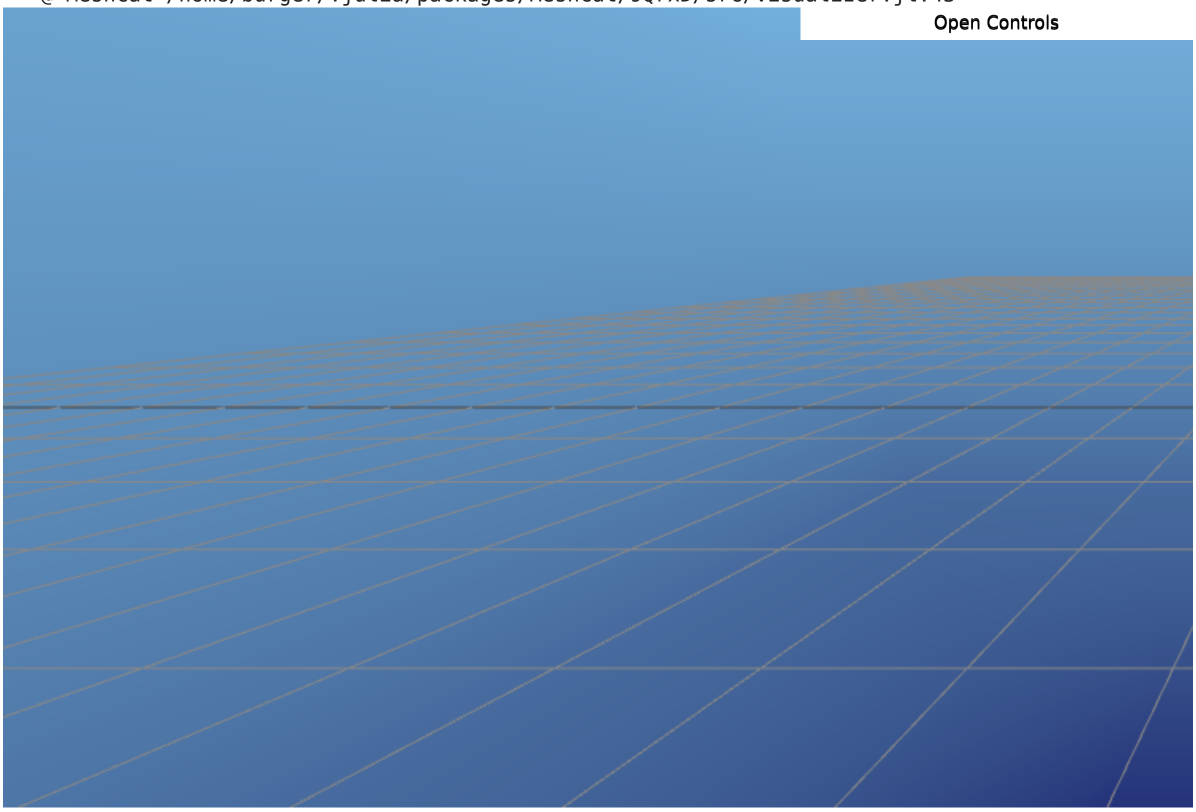
In [9]:
```
# include the functions from quadruped.jl
include(joinpath(@__DIR__, "quadruped.jl"))

# this loads in our continuous time dynamics function xdot = dynamics(model, x, u)
```

```
initialize_visualizer (generic function with 1 method)
```

let's load in a model and display the rough "guess" configuration that we are going for:

In [10]:
```
# --------these three are global variables------------
model = UnitreeA1() # contains all the model properties for the quadruped
mvis = initialize_visualizer(model) # visualizer
const x_guess = initial_state(model) # our guess state for balancing
# ---------------------------------------------------

set_configuration!(mvis, x_guess[1:state_dim(model)÷2])
render(mvis)
```

```
┌ Info: Listening on: 127.0.0.1:8702, thread id: 1
└ @ HTTP.Servers /home/burger/.julia/packages/HTTP/4AUPl/src/Servers.jl:382
┌ Info: MeshCat server started. You can open the visualizer by visiting the following URL in you
r browser:
│ http://127.0.0.1:8702
└ @ MeshCat /home/burger/.julia/packages/MeshCat/9QrxD/src/visualizer.jl:43
```



Now, we are going to solve for the state and control that get us an equilibrium (balancing) on just one leg. We are going to do this by solving the following optimization problem:

$$\min_{x,u} \quad \frac{1}{2}(x - x_{guess})^T(x - x_{guess}) + \frac{1}{2}10^{-3}u^T u \tag{5}$$

$$\text{st} \quad \dot{x} = f(x, u) = 0 \tag{6}$$

Where our primal variables are $x \in \mathbb{R}^{30}$ and $u \in \mathbb{R}^{12}$, that we can stack up in a new variable $y = [x^T, u^T]^T \in \mathbb{R}^{42}$. We have a constraint $\dot{x} = f(x, u) = 0$, which will ensure the resulting configuration is an equilibrium. This constraint is enforced with a dual variable $\lambda \in \mathbb{R}^{30}$. We are now ready to use Newton's method to solve this equality constrained optimization problem, where we will solve for a variable $z = [y^T, \lambda^T]^T \in \mathbb{R}^{72}$.

In this next section, you should fill out `quadruped_kkt(z)` with the KKT conditions for this optimization problem, given the constraint is that `dynamics(model, x, u) = zeros(30)`. When forming the Jacobian of the KKT conditions, use the Gauss-Newton approximation for the hessian of the Lagrangian (see example above if you're having trouble with this).

In [11]:

```
# initial guess
const x_guess = initial_state(model)

# indexing stuff
const idx_x = 1:30
const idx_u = 31:42
const idx_c = 43:72

# I like stacking up all the primal variables in y, where y = [x;u]
# Newton's method will solve for z = [x;u;λ], or z = [y;λ]

function quadruped_cost(y::Vector)
    # cost function
    @assert length(y) == 42
    x = y[idx_x]
    u = y[idx_u]

    # TODO: return cost
    cost_qp = 0.5 * dot(x-x_guess,x-x_guess) + 0.5 * dot(u,u) * 1e-3
    #error("quadruped cost not implemented")
    return cost_qp
end

function quadruped_constraint(y::Vector)::Vector
    # constraint function
    @assert length(y) == 42
    x = y[idx_x]
    u = y[idx_u]

    # TODO: return constraint
    constraint_qp = dynamics(model, x, u)
    #error("quadruped constraint not implemented")

    return constraint_qp
end
function quadruped_kkt(z::Vector)::Vector
    @assert length(z) == 72
    x = z[idx_x]
    u = z[idx_u]
    λ = z[idx_c]

    y = [x;u]

    # TODO: return the KKT conditions
    #error("quadruped kkt not implemented")
    stationarity_cdn = FD.gradient(quadruped_cost, y) +
    FD.jacobian(quadruped_constraint, y)'*λ
    primal_feasibility = quadruped_constraint(y)
    return [stationarity_cdn; primal_feasibility]
end

function quadruped_kkt_jac(z::Vector)::Matrix
    @assert length(z) == 72
    x = z[idx_x]
    u = z[idx_u]
    λ = z[idx_c]

    y = [x;u]

    # TODO: return Gauss-Newton Jacobian with a regularizer (try 1e-3,1e-4,1e-5,1e-6)
    # and use whatever regularizer works for you
    #error("quadruped kkt jac not implemented")
    @show size(FD.hessian(quadruped_cost, y))
    @show size(FD.jacobian(quadruped_constraint, y))
```

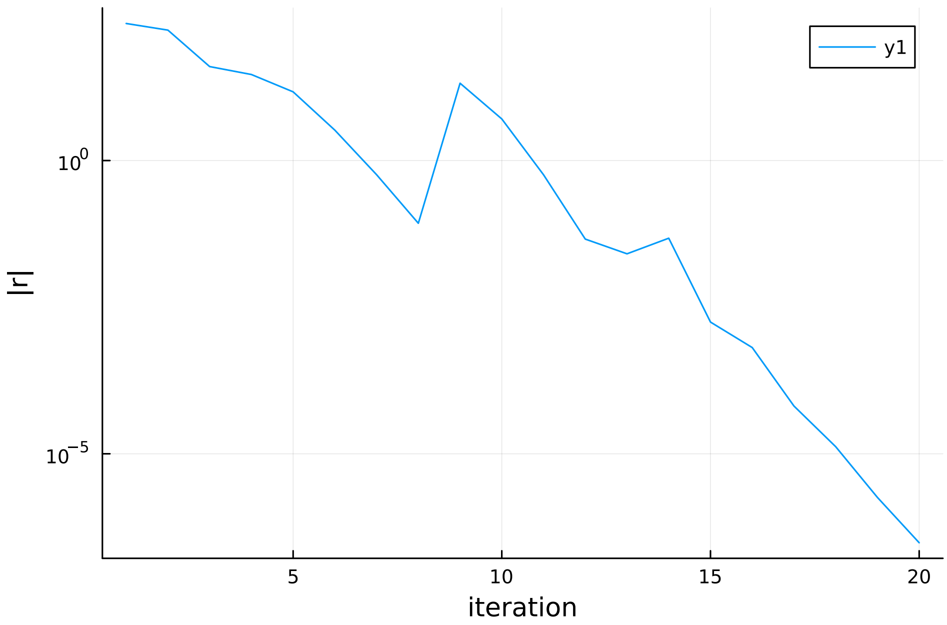
```
    qp_kkt_jac = [FD.hessian(quadruped_cost, y) FD.jacobian(quadruped_constraint, y)';
                  FD.jacobian(quadruped_constraint, y) zeros(30,30)]
    qp_kkt_jac -= 1e-5 * I(length(z))
    return qp_kkt_jac
end
```

```
WARNING: redefinition of constant Main.x_guess. This may fail, cause incorrect answers, or produ
ce other errors.
quadruped_kkt_jac (generic function with 1 method)
```

In [12]:

```
function quadruped_merit(z)
    # merit function for the quadruped problem
    @assert length(z) == 72
    r = quadruped_kkt(z)
    return norm(r[1:42]) + 1e4*norm(r[43:end])
end

@testset "quadruped standing" begin

    z0 = [x_guess; zeros(12); zeros(30)]
    Z = newtons_method(z0, quadruped_kkt, quadruped_kkt_jac, quadruped_merit;
     tol = 1e-6, verbose = true, max_iters = 50)
    set_configuration!(mvis, Z[end][1:state_dim(model)÷2])
    R = norm.(quadruped_kkt.(Z))

    display(plot(1:length(R), R, yaxis=:log,xlabel = "iteration", ylabel = "|r|"))

    @test R[end] < 1e-6
    @test length(Z) < 25

    x,u = Z[end][idx_x], Z[end][idx_u]

    @test norm(dynamics(model, x, u)) < 1e-6

end
```

```
iter: 1    |r|: 217.37236872332167   size(FD.hessian(quadruped_cost, y)) = (42, 42)
size(FD.jacobian(quadruped_constraint, y)) = (30, 42)
α: 1.0
iter: 2    |r|: 167.48123915696675   size(FD.hessian(quadruped_cost, y)) = (42, 42)
size(FD.jacobian(quadruped_constraint, y)) = (30, 42)
α: 1.0
iter: 3    |r|: 40.03191132009891   size(FD.hessian(quadruped_cost, y)) = (42, 42)
size(FD.jacobian(quadruped_constraint, y)) = (30, 42)
α: 0.25
iter: 4    |r|: 29.320216375353176   size(FD.hessian(quadruped_cost, y)) = (42, 42)
size(FD.jacobian(quadruped_constraint, y)) = (30, 42)
α: 1.0
iter: 5    |r|: 14.849304714132273   size(FD.hessian(quadruped_cost, y)) = (42, 42)
size(FD.jacobian(quadruped_constraint, y)) = (30, 42)
α: 1.0
iter: 6    |r|: 3.296340187617205   size(FD.hessian(quadruped_cost, y)) = (42, 42)
size(FD.jacobian(quadruped_constraint, y)) = (30, 42)
α: 1.0
iter: 7    |r|: 0.5739837881065105   size(FD.hessian(quadruped_cost, y)) = (42, 42)
size(FD.jacobian(quadruped_constraint, y)) = (30, 42)
α: 1.0
iter: 8    |r|: 0.08525061806878889   size(FD.hessian(quadruped_cost, y)) = (42, 42)
size(FD.jacobian(quadruped_constraint, y)) = (30, 42)
α: 1.0
iter: 9    |r|: 20.81702110100295   size(FD.hessian(quadruped_cost, y)) = (42, 42)
size(FD.jacobian(quadruped_constraint, y)) = (30, 42)
α: 1.0
iter: 10    |r|: 5.120045496071522   size(FD.hessian(quadruped_cost, y)) = (42, 42)
size(FD.jacobian(quadruped_constraint, y)) = (30, 42)
α: 1.0
iter: 11    |r|: 0.5719239566710616   size(FD.hessian(quadruped_cost, y)) = (42, 42)
size(FD.jacobian(quadruped_constraint, y)) = (30, 42)
α: 1.0
iter: 12    |r|: 0.04577715621455575   size(FD.hessian(quadruped_cost, y)) = (42, 42)
size(FD.jacobian(quadruped_constraint, y)) = (30, 42)
α: 0.5
iter: 13    |r|: 0.02569586712374606   size(FD.hessian(quadruped_cost, y)) = (42, 42)
size(FD.jacobian(quadruped_constraint, y)) = (30, 42)
α: 1.0
iter: 14    |r|: 0.04742035983349077   size(FD.hessian(quadruped_cost, y)) = (42, 42)
size(FD.jacobian(quadruped_constraint, y)) = (30, 42)
α: 1.0
iter: 15    |r|: 0.0017624055595852244   size(FD.hessian(quadruped_cost, y)) = (42, 42)
size(FD.jacobian(quadruped_constraint, y)) = (30, 42)
α: 1.0
iter: 16    |r|: 0.0006476508558171974   size(FD.hessian(quadruped_cost, y)) = (42, 42)
size(FD.jacobian(quadruped_constraint, y)) = (30, 42)
α: 1.0
iter: 17    |r|: 6.522230431670966e-5   size(FD.hessian(quadruped_cost, y)) = (42, 42)
size(FD.jacobian(quadruped_constraint, y)) = (30, 42)
α: 1.0
iter: 18    |r|: 1.3215736684407556e-5   size(FD.hessian(quadruped_cost, y)) = (42, 42)
size(FD.jacobian(quadruped_constraint, y)) = (30, 42)
α: 1.0
iter: 19    |r|: 1.8027817926790468e-6   size(FD.hessian(quadruped_cost, y)) = (42, 42)
size(FD.jacobian(quadruped_constraint, y)) = (30, 42)
α: 1.0
iter: 20    |r|: 3.042359767878103e-7   Test Summary:      | Pass  Total     Time
quadruped standing |    3      3  5m06.2s
```




```
Test.DefaultTestSet("quadruped standing", Any[], 3, false, false, true, 1.738902313851117e9, 1.738902620027923e9, false, "/home/burger/OCRL/HW1_S25/jl_notebook_cell_df34fa98e69747e1a8f8a730347b8e2f_X24sZmlsZQ==.jl")
```

In [13]:

```
let

    # let's visualize the balancing position we found

    z0 = [x_guess; zeros(12); zeros(30)]
    Z = newtons_method(z0, quadruped_kkt, quadruped_kkt_jac, quadruped_merit;
     tol = 1e-6, verbose = false, max_iters = 50)
    # visualizer
    mvis = initialize_visualizer(model)
    set_configuration!(mvis, Z[end][1:state_dim(model)÷2])
    render(mvis)


end
```

```
size(FD.hessian(quadruped_cost, y)) = (42, 42)
size(FD.jacobian(quadruped_constraint, y)) = (30, 42)
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size(FD.jacobian(quadruped_constraint, y)) = (30, 42)
┌ Info: Listening on: 127.0.0.1:8703, thread id: 1
└ @ HTTP.Servers /home/burger/.julia/packages/HTTP/4AUPl/src/Servers.jl:382
┌ Info: MeshCat server started. You can open the visualizer by visiting the following URL in you
r browser:
│ http://127.0.0.1:8703
└ @ MeshCat /home/burger/.julia/packages/MeshCat/9QrxD/src/visualizer.jl:43
```