Smart Derivative Contracts

Detaching Transactions from Counterparty Credit Risk
- Specification, Parametrisation, Valuation -

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Abstract

In this note we describe a *smart derivative contract* with a fully deterministic termination to remove many of the inefficiencies in collateralized OTC transactions. The automatic termination procedure embedded in the smart contracts replaces the counterparty default by an option right of the counterparty.

The application of smart contracts to cure issues in xVAs has been described before, see [7, 8]. However, a direct implementation of an OTC derivative as a smart contract may come with its own issues:

- If the smart contract is implemented on a crypto-currency blockchain it will introduce a currency conversion risk.
- If the smart contract has an automatic termination in case of insufficient wallet amounts, the contract essentially contains a bilateral American option. Both counterparts can willingly terminate the contract by emptying the wallet. This would render the contract useless.

In this note we will fully describe the terms of a smart contract to replace a collateralized OTC transaction. We introduce a penalty payment to modify the American option right in the contract. The penalty and the excess amount in the wallet can be seen as a combination of default fund contribution and initial margin, inducing a per-contract termination probability. Hence, each contract come with its own termination probability (corresponding to the default probability). Based on this, ratings could be assigned on a per-contract basis.

Such smart contracts are also interesting with respect to the mathematical theory of systemic risk, since each contract represents an individual counterparty, increasing the numbers of individual counterparties in the whole system and possibly justifying the application of mean filed theory (compared to a setup with a large central counterpart (CCP)).

Contents

1 Introduction

1.1 Smart Contacts

In this note we consider a special variant of a *smart derivative contract*, i.e., a financial product, which contains an embedded algorithm to handle margining and default risk (failure to pay) in a fully deterministic way. The contract itself then serves as a micro-counterpart.

This smart contract allows to effectively remove counterparty related risks, like inefficiencies in cash-flow netting, determination and processing of the default event, etc.

The contract itself has very limited risk - comparable to a daily settlement. Its maintenance costs are given by funding costs of margin buffers and penalties which are lower and more transparent compared to classical contracts with central or bilateral margining.

A smart contract may be realized using blockchain technologies, but the concepts described here are independent of the effective implementation. The application of smart contracts to cure inefficiencies in settlement and reduce xVAs have been discussed before, see [8]. In direct comparison the smart contract proposed here is different since it has a deterministic termination criteria which simplifies the termination procedure and may detach the contract from counterparty default risk. A termination criteria may introduce an option right. To prevent wilful termination a penalty premium is introduced. The contract specifies a margin buffer amount M, which transparently determines the termination risk and a penalty P which covers the gap risk in case of termination.

Investigating the termination risk it might be possible to assign a rating to the smart derivative contract (like one assigns a rating to a counterparty).

1.2 Layout of the Paper

Before specifying the contract, we shortly review the current status and some of the inefficiencies related to collateralized OTC transactions in Section 2. We then define the exact terms of the smart derivative contract in Section 3.

The following sections will then discuss special topics, like the determination of the termination penalty and the valuation of the contract.

2 Status Quo

2.1 Inefficiencies in bilateral collateralized OTC transactions

To mitigate counterparty credit risk in OTC derivative transactions a daily collateral exchange procedure is established. Changes in market values trigger a variation margin call to adjust the amount of collateral posted. Although the process is more or less standardised some issues exist which may lead to significant unsecured exposures.

First the amount of collateral, which has to be posted, is usually determined with a certain time lag which leads to a mismatch between the actual value and the collateral amount in transit.

Second, cash flows directly arising from the underlying derivative trades trigger a corresponding margin flow in the opposite direction the day after as long as the counterparty is able to pay its obligations. This introduces so-called *Settlement Gap Risk* and is covered in more detail in [6].

In addition, the valuation model determining the variation margin is counterparty specific, which may lead to disputes.

2.2 Central Clearing Hubs, Initial Margin and Settlement Agents

For standardised plain vanilla swap transactions central counterparties (CCP) are seen to be one solution as they enter the bilateral market as an independent third party. The counterparties now trade over (i.e., against) that central hub and do not face each other directly. For all bilateral transactions which are not subject to clearing obligation a so-called *initial margin obligation* has been introduced and is supposed to cover gap risks. Effectively this introduces additional funding costs for both counterparties. A shortcoming of the actual ISDA SIMM initial margin methodology [5] is that initial margin is designed to cover large market moves but not to cover trade flow based settlement risk. A reduction of settlement risk can be performed by specialised agents (swap agent, continuous linked settlement), while leaving counterparty risk an open issue.

Although issues like trade and valuation standardisation as well as transaction netting are solved, new problems get introduced.

2.2.1 Initial Margin

In essence: Due to a lack of efficient transaction netting and determination of the correct market value the best current solution is to over-collateralize that transaction. Furthermore, whether the amount of initial margin determined by standardised models might be enough to cover actual losses is not properly proven yet. That depends especially on the actual time until the default is confirmed and how much the market can move in that period.

2.2.2 Central Clearing Hubs

This approach has several advantages compared with the bilateral world:

1. Standardisation - As the CCP defines the trade details and also their market values there does not exist any dispute process.

2. Cash flow netting can be made much more efficient since one member is not facing several bilateral counterparties. Netting of all derivative cash flows can be done across all trades so settlement risk is reduced massively.

3. Periodic optimization runs try to identify offsetting trades such that the outstanding notional and amount of derivative cash flow transactions can be reduced massively.

4. Concerning possible defaults, all participating clearing members have to participate in a resolution in case of an actual of another member. Significant amounts of initial margin and default contributions have to be posted to a CCP to resolve a possible default event of a member and even prevent the CCP itself going bankrupt.

For CCPs, due to several compression cycles, nobody can really tell its particular risk against other counterparties. In case of a default of a CCP member a complex resolution mechanism - "*the waterfall*" - is introduced to resolve the default of a participating member and to prevent the CCP itself going bankrupt. That mechanism is complex and it should be at least questioned whether it works efficiently in case of a default of a large member party combined with a stressed market scenario - especially because the notional of cleared transactions rose significantly in the recent years without facing a critical crisis scenario.

It is unclear whether bilateral counterparty risk can be effectively handled more efficiently by accumulating large volume on a very few centralized hubs.

For so-called settlement agents handling of settlement-gap risks seems to be solved as cash flow netting is introduced but efficient risk management in case of a bilateral counterparty default further remains an unsolved issue.

3 Smart Contract Mechanics

Given the inefficiencies and complications in the settlement and processing of a standard derivative and the possibilities of smart contracts, we construct a derivative from scratch. Our aim is a derivative which

- is equipped with its own margining process and standardized valuation,
- can handle trade based and collateral cash flows efficiently,
- can manage a possible default of the counterparty.

In the following we consider a smart contract with a single wallet for each counterparty which jointly provides both a source for automatically triggered product cash flows and collateral flow.

Since a netting of product and collateral flows effectively removes any product cash flows, the only flows remaining are variation margin calls which are induced by the market moves. In this sense, the smart contract resembles a derivative with daily settlement.

In addition, the smart contract has an automatic termination feature: if a payment cannot be executed due to insufficient amounts in the corresponding wallet, the contract terminates.

The only interaction with the counterpart is given through counterparties refilling their wallet or withdrawing amounts from their wallet.

3.1 Smart Contracts are American Options

The termination rules imply that a smart contract is a bilateral American option, because a counterparty may intentionally refuse to refill its wallet (or even empty it) and thus intentionally trigger contract termination. Making the smart contract a bilateral American option would render it use-less, because none of the counterparts can effectively rely on the future payments of the contract.

To cover gap risk and handle the American optionality, we introduce a penalty P which is pre-funded and resides in a segregated part of the wallet and which is paid in full to the other counterpart upon termination. In other words: the contract terminates if the total wallet amount falls below P. The penalty corresponds to the strike of the American option.

3.2 Contract Terms

We give the exact terms of the contract. In the following the symbols W (wallet), V (valuation), C (product cash-flow), X (transaction amount), P (penalty) are defined from the perspective of the same counterpart (say, counterparty A), while the corresponding quantities from the perspective of the other counterparty are denoted with an asterix as W^*, V^*, C^*, X^*, P^*.

3.2.1 Event Times

1. The contract is given by a set of event times t_i (settlement times), usually once a day, but possible at higher or lower frequencies. These event times will constitute the times when trade flows (variation margin flows) are executed or an automatic termination is performed.

2. The contract specifies variation margin determination times $t_i^c \leq t_i$ at which the variation margin is determined

3. The contract specifies fixing time $t_i^f \leq t_i^c$ at which the market data required for the valuation of the variation margin is fixed.

4. The contract specifies margin buffer determination times $t_i^m \leq t_i^f$ at which the trade wallet is checked for sufficient margin buffers, i.e. $W \leq M + P$. If the test fails, the contract performs an automatic termination.

5. The contract specifies a rebalancing period, $t_i^{r,s} \leq t_i^{r,e}$ at which the counterparties are allowed to adjust their wallets. The start of the period lies after the previous execution time $t_{i-1} < t_i^{r,s}$. The end of the period lies before $t_i^{r,s} < t_i^m$.

To ensure that no counterparty can profit from additional information, the market data fixing time t_i^f should be close to the settlement t_i and the rebalancing period $[t_i^{r,s}, t_i^{r,e}]$ should be right after the previous settlement time t_{i-1} and kept short. See Figure 3.2.1.

Note that many of these times can be indeed identical.

3.2.2 Net Transaction Amount

1. The contract specifies trade cash-flows C_i fixed on or before the fixing times t_i^f paid at t_i. Such a trade cash-flow may be a coupon. If no trade cash-flow is associated with the time t_i we set $C_i = 0$.[1]

2. The contract specifies a valuation model V and variation margin determination times t_i^c at which the variation margin is determined as

$$X_i \;=\; V(t_i; t_i^c, \mathcal{M}(t_i^f)) - V(t_i; t_i^c, \mathcal{M}(t_{i-1}^f)),$$

   where $V(s; t, \mathcal{M})$ denotes the net present time t-value of the cash flows after s evaluated with the market data $\mathcal{M}$ and $\mathcal{M}(t_i^f$ denotes the market data set as of t_i^f (see below).

[1] The trade cash-flows C_i may be fixed long before t_i^f, e.g. three month before the settlement period. The time t_i^f is the last possible fixing time. The trade cash-flows C_i will only enter into the determination of the value and netted with a hypothetical collateral account.

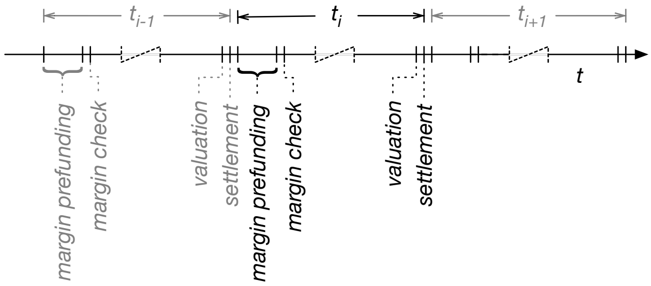

Figure 1: The event times of the pre-funded smart derivative contract: at the begin of a period there are the margin pre-funding ($[t_i^{r,s}, t_i^{r,e}]$) and the margin admissibility check (t_i^m), at the end of the period there are the valuation (t_i^f) and the settlement (t_i).

3. The net cash-flow of the contract at time t_i is determined as the transaction amount X_i.

The transaction amount paid in t_i represents the net cash-flow of a classical collateralized derivative with cash flows C_i. Such a derivative will pays C_i plus the adjustment of the collateral ΔV_i, i.e.,

$$X_i \;=\; C_i + \Delta V_i$$

with

$$\Delta V_i \;=\; V(t_i; t_i^c, \mathcal{M}(t_i^f)) - \left(1 + r^c(t_{i-1}^c)\Delta t_{i-1}^c\right) V(t_{i-1}; t_{i-1}^c, \mathcal{M}(t_{i-1}^f)),$$

where $r^c(t_{i-1}^c)$ denotes the collateral rate for the time period $[t_{i-1}^c, t_i^c]$.

The net present value $V(t_i; t_i^c, \mathcal{M}(t_i^f))$ differs from $V(t_{i-1}; t_i^c, \mathcal{M}(t_i^f))$ by just the removal of the cash flow C_i, that is we have

$$V(t_i; t_i^c, \mathcal{M}(t_i^f)) - V(t_{i-1}; t_i^c, \mathcal{M}(t_i^f)) \;=\; -C_i.$$

The accrued collateral $\left(1 + r^c(t_{i-1}^c)\Delta t_{i-1}^c\right) V(t_{i-1}; t_{i-1}^c, \mathcal{M}(t_{i-1}^f))$ agrees with the time t_i^c-valuation, i.e.,

$$\left(1 + r^c(t_{i-1}^c)\Delta t_{i-1}^c\right) V(t_{i-1}; t_{i-1}^c, \mathcal{M}(t_{i-1}^f)) \;=\; V(t_{i-1}; t_i^c, \mathcal{M}(t_{i-1}^f))$$

Thus we have

$$X_i \;=\; C_i + \Delta V_i \;=\; V(t_i; t_i^c, \mathcal{M}(t_i^f)) - V(t_i; t_{i-1}^c, \mathcal{M}(t_{i-1}^f))$$

Note that the transaction amount X_i "nets" trade based cash flows (e.g. coupons) and cash flows arising from variation margin calls. Hence, if there is a coupon payment the resulting variation margin can be netted against that coupon payment such that no transaction have to take place.

3.2.3 Contract Wallet

1. The contract is endowed with wallets W, W^*, where $W(t)$ represents the amount accessible by counterparty A at time t and $W^*(t)$ represents the amount accessible by counterparty A at time t. The wallets are hosted by a trusted third party (ECB) or on a blockchain, backed by fiat money.

2. The contract specifies minimum margin buffer amounts M_i, M_i^*.

3. To cover gap risk (and remove advantage from the embedded option right), the contract specifies penalty amounts P and P^*.

4. During the rebalancing period $[t_i^{r,s}, t_i^{r,e}]$ counterparties may post or withdraw amounts from their wallets. They may withdraw the amount $W(t) - P$ from the wallet. The amount P is segregated during lifetime of the contract (except for a termination phase).

5. At the end of the rebalancing period the wallet's balance is checked. If $W(t_i^{r,e}) < M_i + P$ the contract transfers the amount P and terminates immediately.

6. Let $W(t_i-)$ denote the value observed on wallet W right before t_i. At trade time t_i the transaction amount X_i is admissible if and only if

$$W(t_i-) + X_i \geq P \qquad \text{and} \qquad W^*(t_i-) + X_i^* \geq P^*,$$

   with $X_i^* = -X_i$.

   (a) If the transaction amount is admissible at trade time t_i, the wallets are adjusted by

$$W(t_i) = W(t_i-) + X_i \qquad \text{and} \qquad W^*(t_i) = W^*(t_i-) + X_i^*.$$

   (b) If the transaction amount is not admissible at trade time t_i, the wallets are adjusted by

$$\begin{aligned} W(t_i) &= W(t_i-) + W^*(t_i-) \quad \text{and} \quad W^*(t_i) = 0 && \text{if } X_i > 0, \\ W(t_i) &= 0 \quad \text{and} \quad W^*(t_i) = W^*(t_i-) + W(t_i-) && \text{if } X_i < 0, \end{aligned}$$

   and the contract is terminated.

3.2.4 Automatic Termination and Introduction of a Penalty Premium

The penalty amount P is introduced to cover gap risk. Upon early termination the amount $\min(W(t), P)$ is transferred to the non-terminating counterparts wallet.

1. Penalty has to be pre-funded and will cover the gap risk in the contract.
2. Penalty will not be touched e.g. from a variation margin call.
3. The trade terminates if the wallet (excluding the penalty amount) cannot provide the transaction amount.
4. In case of contract termination the full penalty amount P of the terminating counterpart will be transferred to the other counter part.

As a guidance, the amount M can be specified to guarantee that the probability of termination is below a certain level α, that is

$$\mathbb{P}\left(X_i \leq M_i\right) \; \geq \; 1 - \alpha$$

and the penalty may be chosen to cover the gap risk in average, that is

$$P_i \; = \; \mathrm{E}\left(X_i - M_i \mid X_i > M_i\right).$$

3.3 Notification, Refill Option and the American Option Problem

After the determination time t_i^f (fixing of trade cash flow) and t_i^c) (determination of variation margin) the trade cash-flow X_i is known. It is expected that the wallets contains a sufficient amount to provide a cash flow induced by a reasonable market move.

One might consider allowing for an adjustment of the wallet, to prevent termination. Note however, that this represents a cancellation option as the counterpart could just empty the wallet (except for the penalty amount P) to enforce termination of the contract. For example, it is optimal to terminate the contract if the transaction amount excesses the penalty, i.e. $P_i + X_i < 0$.

We make the following observations:

- The penalty amount represents a strike for the option to terminate the contract. Increasing P reduces the risk of wilful termination and hence the value of this option. Since both counterparts have that option, P and P^* may be used to reduce the exercise probability and balance the value of the options.
- If rebalancing is only allowed in a short rebalancing period, the values M and P may be used to make the termination payout fair. (The option is worthless).
- One may just ignore the option right and consider it unlikely that the counterpart will exercise it, considering wilful termination as reputation risk.
- The option is a one day option, or with even shorter maturity. Making gap amounts small.

3.4 The Termination Risk (Default Risk) of a Smart Contract

Since the termination of the smart contract is fully deterministic and depends on market moves probabilities only, we can define a per-contract termination risk, which corresponds a per-contract counterparty default risk.

The termination risk is

$$P\left(W(t) + X < P\right).$$

It can be controlled by the wallet $W(t)$ and both counterparties may agree on providing sufficient wallet amounts to achieve that termination risk stays below a certain value. For example, they could agree on ensuring that $W(t) > M + P > P$ with some given $M > 0$. We call M the margin buffer.

Note that the trade termination resembles to the termination of an American option. We will exercise the option if $E(X) < -P$. If a termination is a consequence of a counterparty default, implying that the counterparty cannot provide sufficient fund to the wallet, the option exercise is - in expectation - sub-optimal and termination represents (in expectation) a windfall profit for the other counterpart. In this sense, the contract replaces counterparty risk by an option right.

3.5 Determination of Penalty Premium

The penalty premium is supposed to cover potential losses which occur for counterparty B after termination of a contract. The penalty should cover

1. the possible loss of the last market move, that is the amount $W(t_i) + X - P$
2. possibly additional replacement costs R.

It is up to the counterparties to agree on a suitable penalty. The penalty may be time dependent to ensure a specific rating of the contract or it may be fixed. In this section we will discuss some possible methods for determining P.

3.5.1 Fair Minimum Penalty

We consider the case $W(t_i) = P$, i.e., the worst case, since the penalty cannot be removed from the wallet. We assume that $X < 0$. In this case the contract will terminate.

- If $-X < P$, then $P + X > 0$ is a profit for the counterparty.
- If $-X > P$, then $P + X < 0$ is a loss for the counterparty.

Hence, a fair minimum value for P can be given by assuming that the expected profits agree with the expected losses, conditional to $X < 0$, that is

$$\mathrm{E}\left(P + X \mid -P < X < 0\right) \;=\; -\mathrm{E}\left(P + X \mid X < -P\right).$$

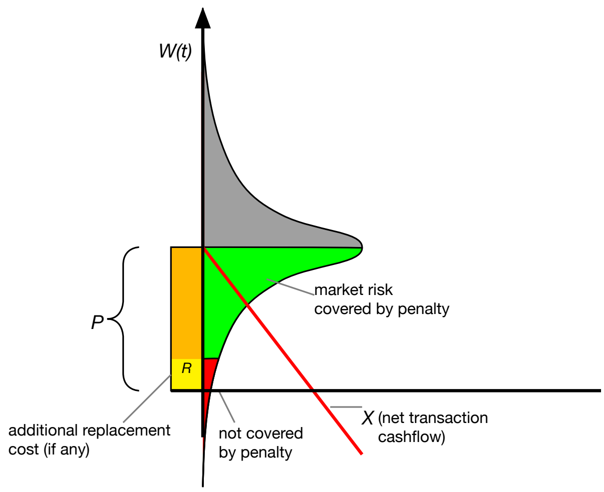

Figure 2: Determination of a penalty P covering the replacement cost R and a predefined portion of the market risk.

An interesting property of the penalty P is that is does not depend on default probability of the counterpart (except for a correlation of the market parameters determining the product cash flows and margin calls X and the counterparty default).

In addition, the penalty is not symmetric. So both counterparts may have different penalties, which cover possibly directional risks in the product (for example, if the product has an unbounded payoff in one direction and a bounded payoff in the other direction).

3.5.2 Comparison of penalty premium with the CCP

The penalty premium might be compared to a contribution to the CCP's default fund. In the CCP's default resolution process a default fund is supposed to be touched only if posted initial margin amounts do not suffice to cover occurring replacement costs for the CCP. From a valuation point of view it is hard to determine at which specific point and to what amount one's posted default fund contribution is at risk as that depends on the behaviour of other CCP members. In case of the bilateral smart derivative contract, the penalty algorithm is fully deterministic and pre-defined in the contract terms and does not depend on the behaviour of our counterpart or any other instance.

3.6 How the Smart Derivative Contract replaces Counterparty Risk by Market Risk

The concept of a pre-funded margin account and a fully deterministic termination in case of un-funded margins allows a complete automatic processes of the contract. While automatic termination may be an undesired behaviour, it should be noted that the margin accounts M can be chosen such that an market induced termination is extremely unlikely and that frequent intra-day settlement even further reduces the probability of automatic termination.

That said, the termination behaviour of the smart derivative contract is rather a feature than an issue: it allows to replace counterparty risk by a transparent market risk. Figure 3.6 illustrates the pay-off V of classical derivative, depending on an index X. In theory it is unbounded, however no counterpart can guarantee a pay-off of arbitrary size. Counterparty default will inevitable lead to unpredictable cut-offs in the payoff.

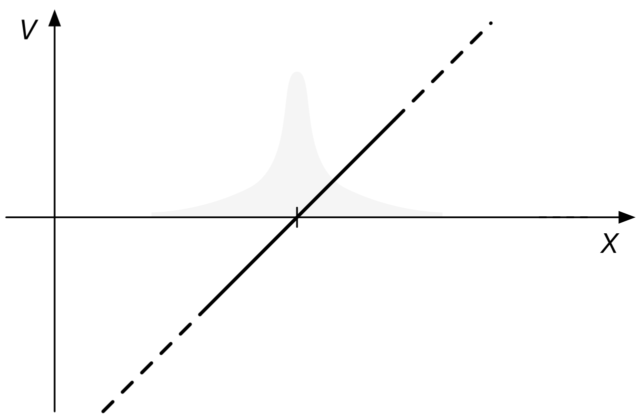

Figure 3: Payoff of a classic product: The payoff is unbounded in the underlying index, leading to theoretically unbounded exposures. A daily settlement keeps the outstanding value distributed around zero.

Figure 3.6 give the corresponding payoff V of a smart derivative contract with margin M and termination fee P. For each settlement period the payoff is bounded by $M + P$. Since the amount $M + P$ is pre-funded (i.e., provided right after the last settlement), we can rely on the contract guaranteeing a settlement of $M + P$. The contract will terminate if $X > M$. Note that the repeated settlement will ensure that larger amounts may be paid.

That a single settlement does not cover payments above $M + P$ is a market risk and the effect of the termination can be investigated in a market-risk model allowing netting across different counterparts. If $P > 0$ the termination event will

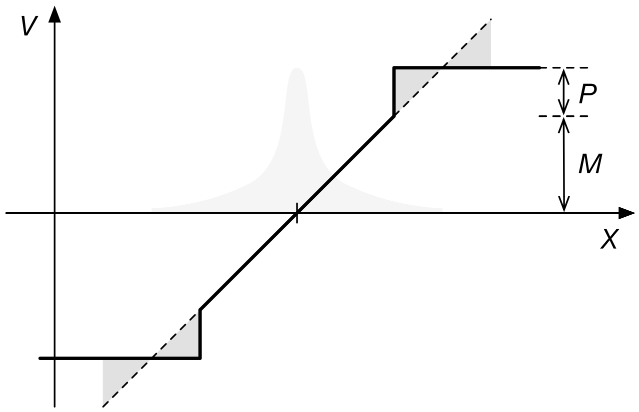

Figure 4: Payoff of a classic product: The payoff is unbounded in the underlying index, leading to theoretically unbounded exposures. A daily settlement keeps the outstanding value distributed around zero.

either result in a profit or a loss (grey area in Figure 3.6), which may be used to cover gap risk and balance the value.

4 Valuation of Smart Contracts

A smart contract is a derivative product which triggers cash-flows though its margining process at event time t_i. That the smart contract has an embedded algorithm determining the variation margin does not imply that the valuation of the contract is trivial. Also note that the valuation model determining V could be a simplified model and - due to symmetry - cannot not include counterparty specific valuation adjustments, like individual funding coasts.

Since the collateral is part of the wallet and can be removed from the contract at will, the contract resembles a classical contract with daily settlement. Hence the net present value is around 0.

The exact net present value of the smart contract consists of the funding costs of the penalty, the valuation of the funding costs of the excess amount $W(t) - P$, the valuation of the mutual American cancellation rights and the valuation of the future margin payment determined by the valuation model V.

The value of the smart contract has a *lower bound* given by

$$W(t) - P. \tag{1}$$

At any time it is possible to withdraw the amount $W(t) - P$ from the contract and trigger cancellation, losing the deposited penalty P which would otherwise be returned at maturity T.

The value of the smart contract is given by the funding of the future margin flows. If we assume a fixed penalty P and a fixed margin buffer M and neglect the termination risk, it is given by

$$N(0)\mathrm{E}\left(\sum_i \frac{X(t_i)}{N(t_i)}\mathrm{d}t + \frac{M+P}{N(T)}\right) \; + \; W(0) - (M(0) + P(0)) \tag{2}$$

where N is the funding numéraire ($N(t) = \exp(r^f(t)\; t)$). If the penalty P and the margin buffer M are time dependent, the valuation is given by

$$N(0)\mathrm{E}\left(\sum_i \frac{X(t_i)}{N(t_i)}\mathrm{d}t + \frac{M(0)+P(0)}{N(0)} - \int_0^\infty r^f(t)\; \frac{P(t)}{N(t)}\mathrm{d}t\right) + W(0) - (M(0) + P(0)).$$

The net present value of the smart contract will likely be positive for both counterparties due to the penalty amount P. This is a similar situation as for total returns swaps, where mutual funding benefits may results in valuations where both counterparties see a positive net present value, see [4].

Assuming that the expectation of the future transaction flows is zero and that these flows are independent of the funding rate (no convexity), then requiring that the value in (1) is smaller than the value in (2) implies that M should be such that

$$M < (M+P)\mathrm{E}\left(N(0)/N(T)\right).$$

That is, the penalty has to be larger than the funding cost of the margin buffer. In this case, it will be disadvantageous for both counterparties to terminate the transaction.

4.1 Maintenance Cost of the Smart Contract - the FVA/MVA

We like to give a rough estimate for the maintenance cost of the smart contract to compare it to classical contracts. If the valuation model is fair, such that the expected future variation margins are zero, the maintenance cost are the funding of the margin buffer M and the penalty P. Excluding the funding benefit from a possible early termination, we assume that M and P have to be funded over the whole maturity.

Introducing a simple model we assume that the product cash flows X follow $\mathrm{d}X = \sigma \mathrm{d}W$ an take first (partly conservative) look at the funding costs of the smart contract: Considering equi-distant transaction times $\Delta t = t_{i+1} - t_i$, given a termination probability q we thus have

$$M \ = \ -\Phi^{-1}\left(q; 0, \sigma/\sqrt{\Delta t})\right),$$

where $q \mapsto \Phi^{-1}(q; \mu, \sigma)$ is the inverse distribution function of the normal distribution with mean 0 and standard deviation σ. The termination probability q here refers to the probability of termination with respect to a single transaction period. If p is the annual termination probability of the contract, then $q = 1 - (1-p)^{\Delta t}$ (given that Δt is measured in fraction of years).

To determine the penalty P we assume that P should cover the expected gap risk on a ΔT horizon. This corresponds to the valuation of a strike 0 option on X with maturity ΔT, such that P is given though the Bachelier formula as (in the special case of an ATM option)

$$P \ = \ \frac{1}{\sqrt{2\pi}} \sigma \sqrt{\Delta T}.$$

While the margin buffer M is determined as a quantile (similar to an initial margin), the penalty is determined in expectation, because it covers the risk to miss the margin requirement M by chance.

Assuming a funding rate of r_f and a maturity of T the total funding costs for M and P are

$$(M+P)\left(1 - \exp(-r_f T)\right)$$

with (approximate) annual funding costs for $r_f(M+P)$.

4.1.1 Example: 10Y Smart Swap and a Comparison to ISDA SIMM™

Let us consider that the variation margin flows X are that of a 10 Y swap. We consider Δt to be one day. The daily (historical) volatility of the 10Y swaprate is assumed to be approximately 0.06% (note: this assumption is in-line with the ISDA SIMM™ specification). Roughly assuming the delta of the swap to be 10 (the time to maturity), we have

$$\sigma\sqrt{\Delta t} = 0.06\% \cdot 10 = 0.6\%.$$

Using $p = 0.01$ we get $q = 1 - (0.99)^{1/365} \approx 0.000028$ and then

$$M \ = \ 2.4\%.$$

That is, the required margin buffer is 2.4% of the notional. Assuming a gap risk period of $\Delta T = 7$ days we find

$$P \ = \ 0.63\%.$$

Reducing the termination risk to $p = 0.0001 = 0.01\%$ would require $M \ = \ 3.0\%$. And covering $\Delta T = 7$ days gap risk (in expectation) would require $P \ = \ 0,90\%$.

So in summary the 10Y swap would require the funding of $M + P = 3.0\%$ (or $M + P = 3.9\%$) of the notional. Assuming a funding rate of 2% this would correspond to 6 bp (p.a.).

These values may be compared to the current initial margin requirements of the ISDA SIMM™ model, [5]. The SIMM™ model specifies a risk weight of 51 for the 10Y swap rate. This risk weights is the 1%-quantile of a normal distribution, assuming a 14-day risk horizon, where the risk factor is given in basis points (factor 10000). This corresponds to a 1-day volatility of 5.9%.

The risk weight 51 directly translates to an initial margin of approximately 5.1% of the notional $(51/10000 \cdot 10)$, where 10 is our rough estimate for the delta of the 10Y swap.

The SIMM margin being a 1%-quantile on 14 days corresponds to a probability for insufficient initial margin of 20% over a period of 1-year. This has to be compared then with the 1% or 0.01% termination risk of the smart contract.

The 2.4% margin buffer M of the smart derivative contract is still lower than then SIMM IM, mainly because the contract does not have an uncertain time period in which default is not confirmed.

In summary the smart contract can be made reliable (termination risk 0.01%), with a coverage of al 14-day gap risk at the fraction of the ISDA SIMM™ model.

4.1.2 Comparison to a possible SIMM™ revisions

In [1] revisions of the SIMM™ are discussed. One aspect under revision is the choice of the 10 business day period in the determination of the initial margin. It is considered as a rough estimate compared with actual processes. representing a conservative estimation of close out risk, leading to unnecessarily high margin buffers.

In [1] the close out risk is split into two stages, where at each stage a value at risk over a specific time horizon is calculated. The initial margin is then the sum of the two VaRs. It is supposed to be less than the initial margin of the current SIMM™ over the assumed time of 10 business days. The first risk quantile is supposed to cover the period in which a default has to be determined. It is assumed that at the end of that period a “macro hedge” has been performed to reduce the open risk on some key risk factors, leaving a residual risk. The second risk quantile is then calculated on the residual portfolio risk and is similar to the current initial margin, but due to the assumption of a macro hedge, smaller. The author stresses that the two risk horizon period should depend on the liquidity of the underlying portfolio and are supposed to be scaled up if a high risk factor concentration may be present.

In the context of the smart derivative contract, there is no uncertainty if termination has occurred and the risk can be reduced further by decreasing the transaction periods. In addition, the argument of the macro-hedge or partial hedge applies to the smart contract as well and may be used to reduce the requirements on the penalty.

4.2 Further possible improvements

4.2.1 Reducing the Margin Buffer M

If the contract provides a notification on the transaction amount X prior to the transaction time and allows for an corresponding adjustment of the wallet, the requirement for an margin buffer M can be reduced / relaxed, further reducing the funding costs.

It remains to value the penalty amount P, used to cover the gap risk. Covering the 14 days gap risk of the 10Y swap under the assumptions used in the previous section would require funding cost less than 3 bp (p.a.).

4.2.2 Reducing the Penalty P

Similar to [1] the gap risk might be reduced by assuming the availability of a partial hedge upon termination, reducing the gap risk. However, it might be hard to determine a ”one-fits-all” replacement cost calculation as macro-hedging and residual hedge errors might depend on portfolio size and complexity in combination with a stressed market environment.

The penalty P can be seen as a measure for the efficiency of a particular type of contracts. If there is a very liquid market for similar or standardize smart derivative contracts, a replacement of a contract could be performed in a very short time period, say even until the next transaction cycle, reducing the requirements for the penalty. Note that if a counterparty drops out in a smart contract the surviving counterparty receives back its own penalty amount (P) and the penalty amount of the other counterparty $P*$. Hence the surviving counterparty can enter a new smart derivative contract with penalty P and the amount $P*$ is solely used to cover addiction frictions in the replacements.

Note that the smart derivative contract is such that there is no outstanding claim against the other counterparty. The only outstanding claim is that of the penalty P against the wallet provider.

4.2.3 Smart contracts and portfolio context

Given several smart contracts with the same counterpart, transaction may take place on each contract standalone. However, variation margin amounts can be netted afterwards as amounts can be re-allocated between wallets to find an optimal allocation as only the penalty restriction has to be fulfilled

In case of more than one smart-swap, resulting penalty will not be netted. Also the daily transaction amounts are not netted. To allow for a netting of individual smart contracts, it is straight forward to define a *smart portfolio contract*. That contract has a reference on all individual swap contracts, performs a netting (or pooling) of all individual transactions amounts X_i^k and provides a single wallet W and a single penalty P for the portfolio.

The penalty P is then determined by the portfolio risk.

5 Future Research

5.1 Legal and Regulatory Implications

Considering P as a reserve, the smart derivative contract may be seen as having zero positive exposure for both counterparties and one may take the view that the smart derivative contract itself requires very few (or even none) regulatory capital. While this is a desirable property, the clarification of the required legal frameworks is maybe the most challenging part in the implementation of such a contract.

5.2 Numerical Analysis and Simulation, Funding of a Smart Derivative Contract

In [3] (work in progress) related tools and analysis are build, e.g. using [2] for the valuation model needed as part of the "oracle" providing $V(t_i)$. While this is work in progress, one may already find spreadsheets with an indicative comparison of a smart derivative contract and the corresponding costs under bi-lateral margining.

5.3 Implications for Systemic Risks

CCPs have been considered problematic since they may be considered as a very large counterparty with systemic relevance. In this context CCPs have developed procedures and models to access and manage the risk of a counterparty default affecting the efficiency of the CCP. However, such rules are complicated and their behaviour under a stress situation is maybe less clear.

The smart contract may be considered as a counterparty by itself, such that a single counterparty is replaced by a multitude of individual contracts, each with fully deterministic termination rules and individual default funds (penalties). Trade repositories can generate transparency, allowing to asses the systemic stability of large portfolios.

The setup may be suitable for future research, e.g. enabling methods like mean field theory to investigate the systemic behaviour of a portfolio of smart contracts.

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Notes

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