(In)Stability for the Blockchain: Deleveraging Spirals and Stablecoin Attacks*

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Abstract

We develop a model of stable assets, including noncustodial stablecoins backed by cryptocurrencies. Such stablecoins are popular methods for bootstrapping price stability within public blockchain settings. We demonstrate fundamental results about dynamics and liquidity in stablecoin markets, demonstrate that these markets face deleveraging spirals that cause illiquidity during crises, and show that these stablecoins have 'stable' and 'unstable' domains. Starting from documented market behaviors, we explain actual stablecoin movements; further our results are robust to a wide range of potential behaviors. In simulations, we show that these systems are susceptible to high tail volatility and failure. Our model builds foundations for stablecoin design. Based on our results, we suggest design improvements that can improve long-term stability and suggest methods for solving pricing problems that arise in existing stablecoins. In addition to the direct risk of instability, our dynamics results suggest a profitable economic attack during extreme events that can induce volatility in the 'stable' asset. This attack additionally suggests ways in which stablecoins can cause perverse incentives for miners, posing risks to blockchain consensus.

1 Introduction

In 2009, Bitcoin [16] introduced a new notion of decentralized cryptocurrency and trustless transaction processing. This is facilitated by blockchain, which introduced a new way for mistrusting agents to cooperate without trusted third parties. This was followed by Ethereum [18], which introduced generalized scripting functionality, allowing 'smart contracts' that execute algorithmically in a verifiable and somewhat trustless manner. Cryptocurrencies promise notions of cryptographic security, privacy (e.g., Zcash), incentive alignment, digital usability, and open accessibility while removing most facets of counterparty risk. However, as these cryptocurrencies are, by their nature, unbacked by governments or physical assets, and the technology is quite new and developing, their prices are subject to wild volatility, which affects their usability.

A stablecoin is a cryptocurrency with an economic structure built on top of blockchain that aims to stabilize the trading price. A true stablecoin, often referred to as the "Holy Grail of crypto" [4], would offer the benefits of cryptocurrencies without the unusable volatility and remains elusive. A more tangible goal is to design a stablecoin that maximizes the probability of remaining stable long-term. If one can find guarantees for the stability regimes of such a

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stablecoin, this would be sufficient to form the basis for a robust decentralized financial system and facilitate economic adoption of cryptocurrencies.

Cryptocurrency volatility Cryptocurrencies face difficult technological, usability, and regulatory challenges to be successful long-term. Many cryptocurrency systems develop different approaches to solving these problems. Even assuming the space is long-term successful, there is large uncertainty about the long-term value of individual systems.

The value of these systems depends on network effects: value changes in a nonlinear way as new participants join. In concrete terms, the more people who use the system, the more likely it can be used to fulfill a given real world transaction. The success of a cryptocurrency relies on a mass of agents—e.g., consumers, businesses, and/or financial institutions—adopting the system for economic transactions and value storage. Which systems will achieve this adoption is highly uncertainty, and so current cryptocurrency positions are very speculative bets on new technology. Further, cryptocurrency markets face limited liquidity and market manipulation. In addition, the decentralized control and privacy features of cryptocurrencies can be at odds with desires of governments, which introduces further uncertainty around attempted interventions in the space.

These uncertainties drive price volatility, which feeds back into fundamental usability problems. It makes cryptocurrencies unusable as short-term stores of value and means of payment, which increases the barriers to adoption. Indeed, today we see that most cryptocurrency transactions represent speculative investment as opposed to typical economic activity.

Stablecoins Stablecoins aim to bootstrap price stability into cryptocurrencies as a stop-gap measure for adoption. Current projects take one of two forms:

- Custodial stablecoins rely on trusted institutions to hold reserve assets off-chain (e.g., \$1 per coin). This introduces counterparty risk that cryptocurrencies otherwise solve.
- Noncustodial (or decentralized) stablecoins create on-chain risk transfer markets via complex systems of algorithmic financial contracts backed by volatile cryptoassets.

We focus on noncustodial stablecoins and, more generally, the stable asset and risk transfer markets that they represent. Noncustodial systems are not well understood whereas custodial stablecoins can be interpreted using the existing well-developed literature on credit risk modeling. Further, noncustodial stablecoins operate in the public/permissionless blockchain setting, in which any agent can participate. In this setting, malicious agents can participate in stablecoin systems. As we will see, this can introduce new economic attacks.

1.1 Noncustodial (decentralized) stablecoins

The noncustodial stablecoins that we consider create systems of contracts on-chain with the following features encoded in the protocol. We refer to these as **DStablecoins**.

- Risk is transferred from stablecoin holders to speculators. Stablecoin holders receive a form of price insurance whereas speculators expect a risky return from a leveraged position.¹
- Collateral is held in the form of cryptoassets, which backs the stable and risky positions.
- An oracle provides pricing information from off-chain markets.
- A dynamic deleveraging process balances positions if collateral value deviates too much.
- Agents can change their positions through some pre-defined process.

These systems are noncustodial (or decentralized) because the contract execution and collateral are all completely on-chain; thus they potentially inherit all of the benefits of cryptocurrencies, such as minimization of counterparty risk. DStablecoins are variants on contracts for difference, which we describe next. The risk transfer typically works by setting up a tranche structure in which losses (or gains) are borne by the speculators and the stablecoin holder holds an instrument

 $^{^{1}}$ 'Leverage' means that the speculator holds $> 1 \times$ their initial assets but faces new liabilities.

like senior debt.² There are also other *non-collateralized* (or *algorithmic*) stablecoins–for a discussion of these, see [4]. We don't consider these directly in this paper; however, we discuss in Section 6 how our model can accommodate these systems as well.

Contract for difference Two parties enter an overcollateralized contract, in which the speculator pays the buyer the difference (possibly negative) between the current value of a risky asset and its value at contract termination.³ For example, a buyer might enter 1 Ether into the contract and a speculator might enter 1 Ether as collateral. At termination, the contract Ether is used to pay the buyer the original dollar value of the 1 Ether at the time of entry. Any excess goes to the speculator. If the contract approaches undercollateralization (if Ether price plummets), the buyer can trigger early settlement or the speculator can add more collateral.

Variants on contracts for difference DStablecoins differ from basic contracts for difference in that (1) the contracts are multi-period and agents can change their positions over time, (2) the positions are dynamically deleveraged according to the protocol, and (3) settlement times are random and dependent on the protocol and agent decisions. The typical mechanics of these contracts are as follows:

- Speculators lock cryptoassets in a smart contract, after which they can create new stablecoins as liabilities against their collateral up to a threshold. These stablecoins are sold to stablecoin holders for additional cryptoassets, thus leveraging their positions.
- At any time, if the collateralization threshold is surpassed, the system attempts to liquidate the speculator's collateral to repurchase stablecoins/reduce leverage.
- The stablecoin price target is provided by an oracle. The target is maintained by a dynamic coin supply based on an 'arbitrage' idea. Notably, this is not true arbitrage as it is based on assumptions about the future value of the collateral.
 - If price is above target, speculators have increased incentive to create new coins and sell them at the 'premium price'.
 - If price is below target, speculators have increased incentive to repurchase coins (reducing supply) to decrease leverage 'at a discount'.
- Stablecoins are redeemable for collateral through some process. This can take the form of global settlement, in which stakeholders can vote to liquidate the entire system, or direct redemption for individual coins. Settlement can take 24 hours-1 week.
- Additionally, the system may be able to sell new ownership/decision-making shares as a last attempt to recapitalize a failing system e.g., the role of MKR in Dai (see [14]).

DStablecoin risks DStablecoins face two substantial risks:

- 1. Risk of market collapse,
- 2. Oracle/governance manipulation.

Our model in this paper focuses on market collapse risk. We further remark on oracle/governance manipulation in Section 6.

Existing DStablecoins Examples of noncustodial stablecoins include Dai and bitUSD (as well as other BitShares Market Pegged Assets). In Steem Dollars, Steem market cap is essentially collateral. Steem dollars can be redeemed for \$1 worth of newly minted Steem, and so redemptions affect all Steem holders via inflation. Notably, unlike custodial stablecoins, Dai is not considered as emoney or payment method subject to the Payment Services Directive in the European Union since there is no single issuer or custodian. Thus it does not have AML/KYC requirements.

²Intuitively, these are like collateralized debt obligations (CDOs) with the important addition of dynamic deleveraging according to the rules of the protocol. As we will see, it is critical to understand deleveraging spirals as they affect the senior tranches.

³Intuitively, this is similar to a forward contract except that the price is only fixed in fiat terms while payout is in the units of the underlying collateral.



- (a) NuBits trades at cents on the dollar.
- (b) BitUSD has broken its USD peg.

Figure 1: Recent decentralized stablecoin failures.

In an academic white paper, [6] proposed a variation on cryptocurrency-collateralized DStable-coin design. It standardizes the speculative positions by restricting leverage to pre-defined bounds using automated resets. A consequence of these leverage resets is that stablecoin holders are partially liquidated from their positions during downward resets—i.e., when leverage rises above the allowed band due to a cryptocurrency price crash. This compares with Dai, in which stablecoin holders are only liquidated in global settlement. An effect of this difference is that, in order to maintain a stablecoin position in the short-term, stablecoin holders need to re-buy into stablecoins (at a possibly inflated price) after downward resets. Of the many designs, it is unclear which deleveraging method would lead to a system that survives longer. This motivates us to study the dynamics of DStablecoin systems.

Noncustodial stablecoins have faced surprising levels of volatility and failure in 2018 [11]. Nubits, which currently trades at cents on the dollar (Figure 1a), is a recent example of extreme failure. BitUSD and Steem Dollars also broke their USD pegs in 2018 (Figure 1b), although both have since recovered following a cryptocurrency recovery. Despite these problems, there is large interest to develop new noncustodial stablecoins. For instance, Basis raised \$133m in 2018 (although it has since closed down), two other projects raised \$32m each, and many other projects raised several million [4].

1.2 Relation to prior work

Stablecoins are active cryptocurrencies, for which no existing models understand how the collateral rule enforces stability. Moreover, existing models completely ignore how the interaction of different agents can affect stability.

With the notable exception of [6], rigorous mathematical work on noncustodial stablecoins is lacking. They applied option pricing theory to valuing tranches in their proposed DStablecoin design using advanced PDE methods. In doing so, they need the simplifying assumption that DStablecoin payouts (e.g., from interest/fee payments and liquidations from leverage resets) are exogenously stable with respect to USD. This may circularly cause stability. In reality, these payouts are made in volatile cryptocurrency (ETH). From these ETH payments, stablecoin holders can

- 1. Hold ETH and so take on ETH exposure,
- 2. Use the ETH to re-buy into stablecoin, likely at an inflated price as it endogenously increases demand after a supply contraction,
- 3. Convert the ETH to fiat, which requires waiting for block confirmations in an exchange (possibly hours) during times when ETH is particularly volatile and paying costs for fiat conversion (fees, potentially taxes). Notably, this is not available in all jurisdictions.

To maintain a DStablecoin position, stablecoin holders need to re-buy into DStablecoins at each reset at endogenously higher price. Stablecoin holders additionally face the risk that the size

of the DStablecoin market collapses such that the position cannot be maintained (and so ends up holding ETH). As no stable asset models exist to understand these endogenous effects, the analysis can't be easily extended using the traditional financial literature.⁴ Our focus in this paper is complementary to understand these endogenous stable asset effects.

[12] studied the evolution of custodial stablecoins. [9] studied whether Tether, a custodial stablecoin, was used to influence Bitcoin prices.

In the context of central counterparty clearinghouses, the default fund contributions, margin requirements and participation incentives have been studied in, e.g., [5], [1], and [8]. The critical question in this area is understanding the effects of a liquidation policy of a member's portfolio in the case of a significant event. The counterpart of this in a decentralized setting is understanding the impact of DStablecoin deleveraging on system stability.

Stablecoin holders bear some resemblance to agents in currency peg and international finance models, e.g., [15] and [10]. In these models, the market maker is essentially the government but is modeled with mechanical behavior and is not a player in the game. For instance, in [10], devaluation is modeled by a simple exogenous threshold rule: the government abandons the peg if the net demand for currency breaches the threshold and is otherwise committed to maintaining the peg. In contrast to currency markets, no agents are committed to maintaining the peg in DStablecoin markets. The best we can hope is that the protocol is well-designed and that the peg is maintained with high probability through the protocol's incentives. The role of government is replaced by decentralized speculators, who issue and withdraw stablecoins in a way to optimize profit. A fully strategic model would be a complicated dynamic game—these tend to be intractable and, indeed, are avoided in the currency peg literature in favor of a sequence of one period games. We enable a more endogenous modeling of speculators' optimization problems under a variety of risk constraints. Our model is a sequence of one-period optimization problems, in which dynamic coupling comes through the risk constraints.

DStablecoin speculators are similar to market makers in market microstructure models (e.g., [17]). Like classical market microstructure, we do have a multi-period system with multiple agents subject to leverage constraints that take recurring actions according to their objectives. In contrast, in the DStablecoin setting, we do not have a truly stable asset that is efficiently and instantaneously available. Instead, agents make decisions that endogenously affect the price of the 'stable' asset and affect the agents' future decisions and incentives to participate in a non-stationary way. In turn, the (in)stability results from the dynamics of these decisions.

1.3 This paper

We build foundations to understand stablecoin design. Our contribution is to provide a dynamic model complex enough to take into account the feedback effects discussed above and yet remains tractable. Our model is easily adapted to consider risks in a wide variety of design choices.

Our model involves agents with different risk profiles; some desire to hold stablecoins and others speculate on the market. These agents solve optimization problems consistent with a wide array of documented market behaviors and well-defined financial objectives. As is common in the literature on market microstructure and currency peg games, these agents' objectives are myopic. These objectives are coupled for non-myopic risk using a flexible class of rules that are widely established in financial markets; these allow us to model the effects of a range of cyclic and counter-cyclic behaviors. The exact form of these rules is selected and self-imposed by speculators to match their desired responses and not part of the stablecoin protocol. Thus well-established manipulation of similar rules as applied to traditional financial regulation is not a problem here. Our model goes largely beyond a one-period model. We introduce this model with supporting rationale for design choices in Section 2.

Our model can be interpreted as a market microstructure model for these new types of markets. It allows a tractable study of dynamics from both analytical (Sections 3, 4) and

⁴A secondary issue with their continuous model is that these systems are inherently discontinuous due to the discrete nature of incorporating blockchain transactions into blocks. Thus resets can occur beyond the set thresholds.

simulation (Section 5) standpoints and allows us to measure failure risks of DStablecoins in a precise way. Our model is comparatively simple, containing very few parameters given the problem complexity.

Using our simple model, we make the following contributions:

- Starting from documented market behaviors, we explain actual complex stablecoin movements. Our results (analytical and simulation) are robust to a wide range of behaviors.
- Our results suggest that the interaction between multiple speculators may be the most interesting to explore from a purely strategic perspective. We see a range of behaviors lead to similar qualitative stablecoin movements; the system is dominated by collateral returns, so long-term viability relies on speculator inflow. Further, we characterize an economic attack between speculators and argue that it can be profitable. This attack additionally suggests ways in which DStablecoins can cause perverse incentives for miners, posing risks to blockchain consensus.
- We demonstrate analytically that DStablecoins have 'stable' and 'unstable' domains, a
 novel result in the literature on financial markets. Further, these results provide a first
 means to quantitatively solve pricing problems that arise in Dai with supporting theory.
- We demonstrate fundamental results about the dynamics and liquidity in DStablecoins, which lead to deleveraging spirals that cause illiquidity during crises.
- Based on our results, we suggest stablecoin design improvements that can improve longterm stability. Our model builds a pillar in the foundations of stablecoin design.

2 Model

Our model couples a number of variables of interest in a risk transfer market between stablecoin holders and speculators. The stablecoin protocol dictates the logic of how agents can interact with the smart contracts that form the system; the design of this influences how the market plays out. Many DStablecoin designs have been proposed. We set up our model to emulate a DStablecoin protocol like Dai with global settlement, but the model is easily adapted to consider different design choices. Note that our model is formulated with very few parameters given the problem complexity.

Our model builds on the model of traditional financial markets in [2] but is new in design by incorporating endogenous stablecoin structure. In the model, we assume that the underlying consensus layer (e.g., blockchain) works well to confirm transactions without censorship or attack and that the system of contracts executes as intended.

Agents Two agents participate in the market.

- The **stablecoin holder** seeks stability and chooses a portfolio to achieve this.
- The **speculator** chooses leverage in a speculative position behind the DStablecoin.

Stablecoin holders are motivated by risk aversion, trade limitations, and budget constraints. They are inherently willing to hold cryptoassets. In the current setting, this means they are likely either traders looking for short-term stability, users from countries with unstable fiat currencies, or users who are using cryptocurrencies to move money across borders. In the future, cryptocurrencies may be more accepted in economic exchange. In this case, stablecoin holders may be ordinary consumers who face risk aversion and budgeting for required consumption.

Speculators are motivated by (1) access to leverage and (2) security lending to borrow against their Ether holdings without triggering tax incidence or giving up Ether ownership. In order to begin participating, speculators need to either have confidence in the future of cryptocurrencies, think they can make money trading the markets, or face unusually high tax rates (or other barriers) that make security lending cheaper than outright selling assets. The simple model in this paper focuses on the first motivation. We propose an extension to the model that considers the second motivation.

Assets There are two assets. For simplicity, we give these assets specific names; however, they could be abstracted to other cryptocurrencies or even outside of a cryptocurrency setting.

- Ether: high risk asset whose USD market prices p_t^E are exogenous
- **DStablecoin**: a 'stable' asset collateralized in Ether whose USD price p_t^D is endogenous Notably, a large DStablecoin system may have endogenous amplification effects on Ether price, similarly to how CDOs affected underlying assets in the 2008 financial crisis. We discuss this further in Section 6 but leave formal modeling of this to future work.

There are several barriers for trading between crypto and fiat, which motivate our choice of assets. Most crypto-fiat pairs are through Bitcoin or Ether, which act as a gateway to other cryptoassets. Trading to fiat can involve moving assets between a number of exchanges and can take considerable time to confirm on the blockchain. Trading to a stablecoin is comparatively simple. Trading to fiat can also trigger more clear tax incidence. Additionally, some countries have imposed strict capital controls on trading between fiat and crypto.

Model outline At t = 0, the agents have endowments and prior beliefs. In each period t:

- 1. New Ether price is revealed
- 2. Ether expectations are updated
- 3. Stablecoin holder decides portfolio weights
- 4. Speculator, seeing demand, decides leverage
- 5. DStablecoin market is cleared

2.1 Stablecoin holder

The stablecoin holder starts with an initial endowment and decides portfolio weights to attain the desired stability. The following table defines the agent's state variables.

Variable	Definition
\bar{n}_t	Ether held at time t
$ar{m}_t$	DStablecoin held at time t
$\mathbf{w_{t}}$	Portfolio weights chosen at time t

The stablecoin holder weights its portfolio by $\mathbf{w_t}$. We denote the components as w_t^E and w_t^D for Ether and Dstablecoin weights respectively. The results in Section 3 hold generally for any $\mathbf{w_t} \geq 0$ (i.e., there is no shorting). In this case, $\mathbf{w_t}$ could be chosen, e.g., from Sharpe ratio optimization, mean-variance optimization, or Kelly criterion (among others). In Sections 4 & 5, we assume that $\mathbf{w_t}$ follows a specific form that leads to constant DStablecoin demand.

The stablecoin holder's portfolio value at time t is

$$\mathcal{A}_{t} = \bar{n}_{t} p_{t}^{E} + \bar{m}_{t} p_{t}^{D} = \bar{n}_{t-1} p_{t}^{E} + \bar{m}_{t-1} p_{t}^{D}.$$

Given weights, \bar{n}_t and \bar{m}_t will be determined based on the stablecoin clearing price p_t^D .

2.2 Speculator

The speculator starts with an endowment of Ether and initial beliefs about Ether's returns and variance and decides leverage to maximize expected returns subject to protocol and self-imposed constraints. The following table defines the speculator's state variables and parameters.

Variable	Definition
n_t	Ether held at time t
r_t	Expected return of Ether at time t
μ_t	Expected log return of Ether at time t
$rac{\mu_t}{\sigma_t^2}$	Expected variance of Ether at time t
\mathcal{L}_t	# outstanding stablecoins at time t
Δ_t	Change to stable coin supply at time t
$ ilde{\lambda}_t$	Leverage bound at time t

Parameter	Definition
γ	Memory parameter for return estimation
δ	Memory parameter for variance estimation
β	Collateral liquidation threshold
α	Inverse measure of riskiness
b	Cyclicality parameter

2.2.1 Ether expectations

The speculator updates expected returns r_t , log-returns μ_t , and variance σ_t^2 based on observed Ether returns as follows:

$$r_{t} = (1 - \gamma)r_{t-1} + \gamma \frac{p_{t}^{E}}{p_{t-1}^{E}},$$

$$\mu_{t} = (1 - \delta)\mu_{t-1} + \delta \log \frac{p_{t}^{E}}{p_{t-1}^{E}},$$

$$\sigma_{t}^{2} = (1 - \delta)\sigma_{t-1}^{2} + \delta \left(\log \frac{p_{t}^{E}}{p_{t-1}^{E}} - \mu_{t}\right)^{2}.$$
(1)

For fixed memory parameters γ , δ (lower memory parameter = longer memory), these are exponential moving averages consistent with the RiskMetrics approach commonly used in finance [13]. For sufficiently stepwise decreasing memory levels and assuming i.i.d. returns, this process will converge to the true values supposing they are well-defined and finite. In reality, speculators don't outright know the Ether return distribution and, as we will see in the simulations, the stablecoin system dynamics occur on timescales shorter than required for convergence of expectations. Thus, we focus on the simpler case of fixed memory parameters.

Note that $\gamma \neq \delta$ may be reasonable. Current cryptocurrency markets are not very price efficient, and so traders might reasonably take into account momentum when estimating returns while using a wider memory for estimating covariance.

We additionally consider the case in which the speculator knows the Ether distribution outright and $\gamma = \delta = 0$. This is consistent with a rational expectations standpoint but ignores how the speculator arrives at that knowledge.

2.2.2 Optimize leverage: choose Δ_t

The speculator is liable for \mathcal{L}_t DStablecoins at time t. At each time t, it decides the number of DStablecoins to create or repurchase. This changes the stablecoin supply $\mathcal{L}_t = \mathcal{L}_{t-1} + \Delta_t$. If $\Delta_t > 0$, the speculator creates and sells new DStablecoin in exchange for Ether at the clearing price. If $\Delta_t < 0$, the speculator repurchases DStablecoin at the clearing price.

Strictly speaking, the speculator will want to maximize its long-term withdrawable value. At time t, the speculator's withdrawable value is the value of its ETH holdings minus collateral required for any issued stablecoins: $n_t p_t^E - \beta \mathcal{L}_t$. Maximizing this is not amenable to a myopic view, however, as maximizing the next step's withdrawable value is only a good choice when the speculator intends to exit in the next step.

Instead, we frame the speculator's objective as maximizing expected equity: $n_t p_t^E - \mathbf{E}[p^D] \mathcal{L}_t$. In this, the speculator expects to be able to settle liabilities at a long-term expected value of $\mathbf{E}[p^D]$. The market price of DStablecoin will fluctuate above and below \$1 naturally depending on prevailing market conditions. The actual expected value is nontrivial to compute as it depends on the stability of the DStablecoin system. For individual speculators with small market power, we argue that $\mathbf{E}[p^D] = 1$ is a an assumption they may reasonably make, as we discuss further below. This is additionally the value realized in the event of global settlement.

Aggregate vs. individual speculators In our model, the single speculative agent, which is not a price-taker, is intended to reflect the aggregate behavior of many individual

speculators, each with small market power.⁵ In a normal liquid market, an individual speculator would be able to repurchase DStablecoins at dollar cost and walk away with the equity. By maximizing equity, the aggregate speculator considers its liabilities to be \$1 per DStablecoin. This may turn out to be untrue during liquidity crises as the repurchase price may be higher. Speculator's don't know the probability of crises and instead account for this in a conservative risk constraint.

Perceived arbitrage This is consistent with the documented perception of 'arbitrage' in these markets. Assuming p_t^D is sufficiently mean-reverting to \$1, a speculator will eventually be able to exit its position in a liquid market. Then the speculator can sell new DStablecoin at a 'premium' if $p_t^D > 1 and repay liabilities at a 'discount' if $p_t^D < 1 . This is not true arbitrage as it depends on the stability of system.

The speculator's optimization The speculator chooses Δ_t by maximizing expected equity in the next period subject to a leverage constraint:

$$\max_{\Delta_t} \quad r_t \Big(n_{t-1} p_t^E + \Delta_t p_t^D(\mathcal{L}_t) \Big) - \mathcal{L}_t$$
s.t.
$$\Delta_t \in \mathcal{F}_t$$

where \mathcal{F}_t is the feasible set for the leverage constraint. This is composed of two separate constraints: (1) a **liquidation constraint** that is fundamental to the protocol, and (2) a **risk constraint** that encodes the speculator's desired behavior. Both are introduced below.

If the leverage constraint is unachievable, we assume the speculator enters a 'recovery mode', in which it tries to maximize its chances of returning to the normal setting. In this case, it solves the optimization using only the liquidation constraint. If the liquidation constraint is unachievable, the DStablecoin system fails with a global settlement.

2.2.3 Liquidation constraint: enforced by the protocol

The liquidation constraint is fundamental to the DStablecoin protocol. A speculator's position undergoes forced liquidation at time t if either (1) after p_t^E is revealed, $n_{t-1}p_t^E < \beta \mathcal{L}_{t-1}$, or (2) after Δ_t is executed, $n_t p_t^E < \beta \mathcal{L}_t$. The speculator aims to control against this as liquidations can occur at unfavorable prices and are associated with fees in existing protocols (we exclude these fees from our simple model, but they can be easily added).

Define the speculator's leverage as the β -weighted ratio of liabilities to assets⁶

$$\lambda_t = \frac{\beta \cdot \text{liabilities}}{\text{assets}}.$$

The liquidation constraint is then $\lambda_t \leq 1$.

2.2.4 Risk constraint: self-imposed speculator behavior

The risk constraint encodes the speculator's desired behavior into the model. We assume no specific type for the risk constraint in our analytical results, which are generic. For our simulations, we explore a variety of speculator behaviors via the risk constraint. We first consider Value-at-Risk (VaR) as an example of a constraint realistically used in markets. This is consistent with narratives shared by Dai speculators about leaving a margin of safety to avoid liquidations. We then construct a generalization that goes well beyond VaR and allows us to explore a spectrum of pro-cyclical and counter-cyclical behaviors encoded in the risk constraint.

⁵We propose to relax this simplification in follow-up work by considering the interaction of many speculators with longer term strategic thinking.

⁶We choose this definition to simplify the model. The alternative definition $\lambda' = \frac{\text{assets}}{\text{assets} - \beta \cdot \text{liabilities}}$ describes the same idea scaled from 0 to ∞. I.e., $\lambda' = \frac{1}{1-\lambda}$ is monotonically increasing in λ for $0 \le \lambda' < 1$.

Manipulation and instability resulting from similar externally-imposed VaR rules is a well-known problem in the risk management and financial regulatory literature (see e.g., [2]). This is of less concern here as the precise parameters of the risk constraint are selected and self-imposed by speculators to approximate their own utility optimization and are not part of the DStablecoin protocol. Further, we consider constraints that go beyond VaR. We instead need to show that our results are robust to a variety of risk constraints that speculators could select.

Example: VaR-based constraint The VaR-based version of the risk constraint is

$$\lambda_t \leq \exp(\mu_t - \alpha \sigma_t),$$

where $\alpha > 0$ is inversely related to riskiness. This is consistent with VaR for normal and maximally heavy-tailed symmetric return distributions with finite variance.

Let $\operatorname{VaR}_{a,t}$ be the a-quantile per-dollar VaR of the speculator's holdings at time t. This is the minimum loss on a dollar in an a-quantile event. With a VaR constraint, the speculator aims to avoid triggering the liquidation constraint in the next period with probability 1-a, i.e., $\mathbf{P}\left(n_t p_{t+1}^E \geq \beta \mathcal{L}_t\right) \geq 1-a$. To achieve this, the speculator chooses Δ_t such that

$$(n_{t-1}p_t^E + \Delta_t p_t^D(\mathcal{L}_t))(1 - \text{VaR}_{a,t}) \ge \beta \mathcal{L}_t.$$

This requires $\lambda_t \leq 1 - \text{VaR}_{a,t}$, which addresses the probability that the liquidation constraint is satisfied next period and implies that it is satisfied this period.

Define $\tilde{\lambda}_t := \exp(\mu_t - \alpha \sigma_t)$. Then $\tilde{\lambda}_t$ is increasing in μ_t and decreasing in σ_t . Further, the fatter the speculator thinks the tails of the return distribution are, the greater α will be, and the lesser $\tilde{\lambda}_t$ will be, as we demonstrate next.

VaR constraint with normal returns If the speculator assumes Ether log returns are (μ_t, σ_t) normal, then $\text{VaR}_{a,t} = 1 - \exp\left(\mu_t + \sqrt{2}\sigma_t \text{erf}^{-1}(2a-1)\right)$. Defining $\alpha = -\sqrt{2}\text{erf}^{-1}(2a-1)$, which is positive for appropriately small a, the VaR constraint is $\lambda_t \leq 1 - \text{VaR}_{a,t} = \exp(\mu_t - \alpha\sigma_t)$.

VaR constraint with heavy tails If Ether log returns X are symmetrically distributed with finite mean μ_t and finite variance σ_t^2 , then for any $\alpha > 1$, Chebyshev's inequality gives us

$$\mathbf{P}(X < \mu_t - \alpha \sigma_t) \le \frac{1}{2\alpha^2}.$$

For the maximally heavy-tailed case, this inequality is tight. Then for VaR quantile a, we can find the corresponding α such that $a = \frac{1}{2\alpha^2}$. The log return VaR is $\mu_t - \alpha \sigma_t$, which gives the per-dollar VaR_{a,t} = $1 - \exp(\mu_t - \alpha \sigma_t)$. Then the VaR constraint is $\lambda_t \leq \exp(\mu_t - \alpha \sigma_t)$.

Generalized risk constraint Similarly to [2], we can generalize the bound to explore a spectrum of different behaviors:

$$\ln \tilde{\lambda} = \mu_t - \alpha \sigma_t^b,$$

where α is an inverse measure of riskiness and b is a cyclicality parameter. A positive b means that $\tilde{\lambda}_t$ decreases with perceived risk (pro-cyclical). A negative b means that $\tilde{\lambda}_t$ increases with perceived risk (counter-cyclical).

2.3 DStablecoin market clearing

The DStablecoin market clears by setting demand = supply in dollar terms:

$$w_t^D \Big(\bar{n}_{t-1} p_t^E + \bar{m}_{t-1} p_t^D(\mathcal{L}_t) \Big) = \mathcal{L}_t p_t^D(\mathcal{L}_t).$$

The demand (left-hand side) comes from the stablecoin holder's portfolio weight and asset value. Notice that while the asset value depends on p_t^D , the portfolio weight w_t^D does not. That is, the stablecoin holder buys with market orders based on weight. This simplification allows for a tractable market clearing; however, it is not a full equilibrium model.

We justify this choice of simplified market clearing with the following observations:

- The clearing is similar to constant product market maker model used in the Uniswap decentralized exchange (DEX) [19].
- Sophisticated agents are known to be able to front-run DEX transactions [7]. As speculators are likely more sophisticated than ordinary stablecoin holders, in many circumstances they can see demand before making supply decisions.⁷
- Evidence from Steem Dollars suggests that demand need not decrease tremendously with price in the unique setting in which stable assets are not efficiently available. Steem Dollars is a stablecoin with a mechanism for price 'floor' but not 'ceiling'. Over significant stretches of time, it has traded at premiums of up to 15× target.

To simplify notation, in a given time t, we define the following and drop subscripts

Definition	\mathbf{Sign}	Interpretation
$x := w_t^D \bar{n}_{t-1} p_t^E$		New DStablecoin demand available
$y := w_t^D \bar{m}_{t-1} - \mathcal{L}_{t-1}$	$y \le 0$	y = 'free supply' in DStablecoin market
$z := n_{t-1} p_t^E$	$z \ge 0$	Speculator value available to maintain market
		<u> </u>
		$\mathcal{L} := \mathcal{L}_{t-1}$
		$\Delta := \Delta_t$
		$ ilde{\lambda} := ilde{\lambda}_t$
		$\mathbf{w} := \mathbf{w_t}$

With $\Delta > y$, which turns out to be always true as discussed later, the clearing price is

$$p_t^D(\Delta) = \frac{x}{\Delta - y}.$$

As the model is defined thus far, stablecoin holders only redeem coins for collateral through global settlement. However, this assumption is easily relaxed to accommodate algorithmic or manual settlements.

3 Stable Asset Market Dynamics

We now present tractable solutions to the proposed interactions and results about liquidity. Note that these results are established in terms of boundaries Δ_{\min} and Δ_{\max} to the feasible set and selected leverage bound $\tilde{\lambda}$. These encode the selection of risk constraint; thus our results are robust to any selection of risk constraint.

3.1 Solution to the speculator's decision

Solving the leverage constraint

Proposition 1. Let $\Delta_{\min} \geq \Delta_{\max}$ be the roots of the polynomial in Δ

$$-\beta\Delta^2 + \Delta\Big(\tilde{\lambda}(z+x) - \beta(\mathcal{L}-y)\Big) - \tilde{\lambda}zy + \beta\mathcal{L}y.$$

Assuming $\Delta > y$,

- If $\Delta_{\min}, \Delta_{\max} \in \mathbb{R}$, then $[\Delta_{\min}, \Delta_{\max}] \cap (y, \infty)$ is the feasible set for the leverage constraint.
- If the roots are not real, then the constraint is unachievable.

⁷This said, DEX mechanics differ slightly from our specific formulation. To make the model more realistic, stablecoin holders could issue buy offers in token units instead of weights at the expense of greater model complexity.

Proof. In each period t, we determine the leverage constraint by setting $\tilde{\lambda} = \lambda$ and solving for Δ . Using the formulation of p_t^D from the market clearing, we have the following equation for Δ :

$$\tilde{\lambda}\left(z + \Delta \frac{x}{\Delta - y}\right) = \beta(\mathcal{L} + \Delta).$$

Given $\Delta > y$, this transforms to the quadratic equation for Δ

$$-\beta\Delta^{2} + \Delta\left(\tilde{\lambda}(z+x) - \beta(\mathcal{L} - y)\right) - \tilde{\lambda}zy + \beta\mathcal{L}y = 0.$$

This is a downward facing parabola. The speculator's leverage constraint is satisfied when the polynomial is positive. The roots, if real, bound the feasible region of the speculator's constraint. Due to the requirement that $\Delta > y$, the feasible set is given by $[\Delta_{\min}, \Delta_{\max}] \cap (y, \infty)$. When there are no real roots, the polynomial is never positive, and so the constraint is unachievable.

Setting $\tilde{\lambda} = 1$ gives the expression for the liquidation constraint alone.

The condition $\Delta > y$ makes sense for two reasons. First, if $\Delta < y$ then $p_t^D < 0$. Second, as we show below, the limit $\lim_{\Delta \to y^+} p_t^D = \infty$. Thus, if we start in the previous step under the condition $\Delta > y$, then the speculator will never be able to pierce this boundary in subsequent steps. We further discuss the implications of this condition later.

Solving the leverage optimization

Proposition 2. Assume that the speculator's constraint is feasible and let $[\Delta_{\min}, \Delta_{\max}] \cap (y, \infty)$ be the feasible region. Define $r := r_t$, let $\Delta^* = y + \sqrt{-yrx}$, and define

$$f(\Delta) = r\Delta \frac{x}{\Delta - y} - \Delta.$$

Then the solution to the speculator's optimization problem is

- Δ^* if $\Delta^* \in [\Delta_{\min}, \Delta_{\max}] \cap (y, \infty)$
- $\bullet \ \Delta_{\min} \ \textit{if} \ \Delta^* < \Delta_{\min}$
- Δ_{\max} if $\Delta^* > \Delta_{\max}$

Proof. By Prop. 1, $[\Delta_{\min}, \Delta_{\max}] \cap (y, \infty)$ is indeed the feasible region. Incorporating the market clearing, the speculator decides Δ in each period t by solving

$$\max \quad r\left(z + \Delta \frac{x}{\Delta - y}\right) - \mathcal{L} - \Delta$$
s.t.
$$\Delta \in [\Delta_{\min}, \Delta_{\max}] \cap (y, \infty)$$

This optimization is solvable in closed form by maximizing over critical points. Maximizing the objective is equivalent to maximizing

$$f(\Delta) = r\Delta \frac{x}{\Delta - y} - \Delta.$$

We first consider the case of Δ approaching y from above and show that this boundary is not relevant in the maximization. The limit is $\lim_{\Delta \to y^+} f(\Delta) = -\infty$. To see this, note that $\mathcal{L}_{t-1} = \bar{m}_{t-1} \geq w_t^D \bar{m}_{t-1}$, and so in order to have $\mathcal{L}_t = w_t^D \bar{m}_{t-1}$, we must have $\Delta < 0$. Thus the sign of the term that tends to infinity is negative. The limit is $-\infty$ because the price for the speculator to buy back DStablecoins goes to ∞ .

To find the critical points of f, we set the derivative equal to zero:

$$\frac{df}{d\Delta} = -\frac{\Delta^2 - 2\Delta y + y(rx + y)}{(\Delta - y)^2} = 0$$

Assuming $\Delta \neq y$, the solutions are the roots to the quadratic $\Delta^2 + -2y\Delta + y(rx+y) = 0$. Notice that the axis of this parabola is at $\Delta = y$. When there are two real solutions, then exactly one

of them will be > y. Given $y \le 0$ and $x \ge 0$ and noting $r \ge 0$, a real solution always exists and the relevant critical point is

$$\Delta^* = y + \sqrt{-yrx}.$$

If it is feasible, Δ^* is the solution to the speculator's optimization problem. If Δ^* is not feasible, then we need to choose along the boundary. The possible cases are as follows.

Suppose $\Delta^* < \Delta_{\min}$. Then Δ_{\min} is feasible since $\Delta^* > y$ implies $\Delta_{\min} > y$. Since f is monotone decreasing to the right of Δ^* , $f(\Delta_{\min}) > f(\Delta_{\max})$, and so Δ_{\min} is the solution.

Suppose $\Delta^* > \Delta_{\text{max}}$. By our assumption that the constraint is feasible, we have that Δ_{max} is feasible. Since f is monotone decreasing to the left of Δ^* on the feasible region, $f(\Delta_{\text{max}}) > f(\Delta_{\text{min}})$, and so Δ_{max} is the solution.

3.2 Maintenance condition for the stable asset market

The next proposition describes when the DStablecoin system is maintainable by the speculator.

Proposition 3. The feasible set for the speculator's liquidation constraint is empty when

$$\left(\tilde{\lambda}(x+z) - \beta \mathcal{L} w^D\right)^2 < 4\beta \tilde{\lambda} \mathcal{L} x w^E$$

Proof. The speculator's leverage constraint is unachievable when the quadratic has no real solutions or when all real solutions are < y. The first case occurs when

$$\left(\tilde{\lambda}(z+x) - \beta(\mathcal{L}-y)\right)^2 + 4\beta(-\tilde{\lambda}zy + \beta\mathcal{L}y) < 0.$$

Noting that $y = -w^D \mathcal{L}$ and $\mathcal{L} - y = \mathcal{L}(2 - w^D)$ and expanding and simplifying terms yields

$$\beta \tilde{\lambda} \mathcal{L} \left(2zw^D + 2x(2 - w^D) \right) - (\beta \mathcal{L} w^D)^2 > \left(\tilde{\lambda} (x + z) \right)^2$$

Completing the square by subtracting $4\beta \tilde{\lambda} \mathcal{L} x (1-w^D)$ from each side then gives the result. \Box

In Prop. 3, $\beta \mathcal{L}w^D \geq 0$ is interpreted as a lower bound on the capital required to maintain the DStablecoin market into the next period (i.e., the collateral required for the minimum size of the DStablecoin market), $\tilde{\lambda} \in [0,1]$, and $x+z \geq 0$ is the capital available to enter the DStablecoin market from both the supply and demand sides. The inequality then states that the difference between the capital available to enter the market and the lower bound maintenance capital must be sufficiently high for the market to be maintainable by the speculator. The constraint $\Delta < y$ implies that the case of the negative difference does not work.

3.3 Deleveraging spirals and limits to market liquidity

Limits to the speculator's ability to decrease leverage Prop. 4 gives fundamental limits on how quickly the speculator can repurchase DStablecoins to lower leverage. These limits are due to the demand in the market. The term $-y = \mathcal{L}(1-w^D)$ represents the 'free supply' of DStablecoin available for exchange, which can be increased by a positive Δ .

Proposition 4. The speculator with asset value z cannot decrease DStablecoin supply at t more than

$$\Delta^- := \frac{z}{z+x}y.$$

Further, even with additional capital, the speculator cannot decrease the DStablecoin supply at t by more than y.

Proof. Setting $z = -\Delta p_t^D = -\Delta \frac{x}{\Delta - y}$ gives the lower bound $\Delta^- := \frac{z}{z + x} y > y$.

Note that $\bar{m}_t = \mathcal{L}_t$, and so $y = \mathcal{L}(w^D - 1) = -w^E \mathcal{L} \leq 0$. The term $w_t^D \bar{m}_{t-1}$ presents a lower bound on the size of the DStablecoin market in the next step from the demand side, and so the speculator can't decrease the size of the market faster than y, even with additional capital beyond z. As shown above, $\Delta \to y^+$ coincides with $p_t^D \to \infty$. The speculator pays increasingly large amounts to buy back more DStablecoins as liquidity dries in the market.

Deleveraging spirals cause liquidity crises The effect on market price of speculator DStablecoin repurchase can lead to deleveraging spirals, which are feedback loops in leverage reduction and drying liquidity. Repurchase tends to push up p_t^D if the outside demand remains the same. The action of the stablecoin holder may actually exacerbate this effect: during extreme Ether price crashes, stablecoin holders will tend to increase their DStablecoin demand in a 'flight to safety' move.

In a higher price/demand environment, it becomes more difficult in subsequent steps for the speculator to lower leverage, whether voluntarily or through forced collateral liquidation. Subsequent repurchases become even more difficult at progressively higher prices as the feedback loop continues. If Ether prices continue to go down,⁸ the deleveraging spiral is only fixed if (1) more money comes into the collateral pool to create more DStablecoins, or (2) people lose faith in the system and no longer want to hold DStablecoins, which can cause the system to fail. There is no guarantee that (1) always happens.

This liquidity effect on DStablecoin price makes sense because the stablecoin (as long as it's working) should be worth more than the same dollar amount of ETH during a downturn because the stablecoin comes with additional protection. If the speculator is forced to buy back a sizeable amount of the coin supply, it will have to do so at a premium price.

One might argue that increase in price is good for stablecoin holders. However, as we will see, the speculator's ability to maintain a stable system deteriorates during these sort of events as it has less control or less willingness to control the coin supply. The deleveraging spiral siphons off collateral value, which can be detrimental to the system in the long-run. The effect is that the price increase can trigger increased volatility in the stablecoin and ultimately design failure.

Algorithmic settlement at \$1, as proposed in [6] can help to alleviate pressure on speculator collateral. As discussed earlier, however, it has the effect of liquidating stablecoin holders into holding risky assets, which should in turn induce volatility into the stablecoin valuation. A spiral effect occurs either way. The dynamic deleveraging process that is used determines who bears the effects of the spiral in the short-term: stablecoin holders or speculators. This said, the effects on long-term stability of the system remain unclear. However, our model is adaptable to study the effects of different deleveraging processes over time.

Results explain real market data Figure 2 shows that our results explain real market data for MakerDAO's Dai stablecoin. Figure 2a shows the Dai price appreciate in Nov-Dec 2018 during multiple large supply decreases. This is consistent with an early phase of a deleveraging spiral. Figure 2b shows trading data from multiple DEXs over Jan-Feb 2019. Price spikes occur in the data from speculator liquidations, which provides empirical evidence that liquidity is limited for lowering leverage in Dai markets. Further, as explained in the next section results, Dai empirically trades below target in many normal circumstances.

4 Stable and Unstable Domains

We now show analytically that DStablecoins have inherent stable and unstable domains. As before, we do not assume any particular form for the risk constraint. We demonstrate that the stable domain results can be used to solve an open problem in the MakerDAO community regarding below target Dai prices. For this section, we make the following simplifications to focus on speculator behavior:

- The market has fixed demand at each t: $w_t^D A_t = \mathcal{D}$. This is consistent with the stablecoin holder having an exogenous constraint to put a fixed amount of wealth in the stable asset.
- Speculator's expected Ether return is constant $r_t = \hat{r} > 1$. This means they always want to fully participate in the market and is consistent with $\gamma = 0$.

⁸Ether price decline can further be facilitated by feedback from large liquidations, as discussed earlier.

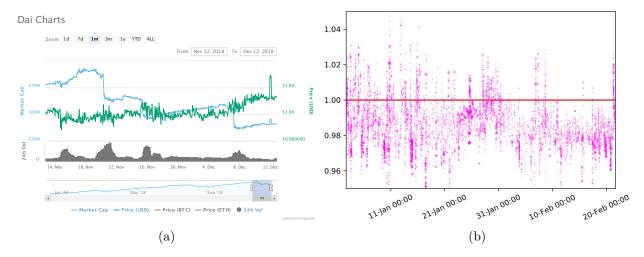


Figure 2: Model Results explain data from Dai market. (a) Dai deleveraging feedback in Nov-Dec 2018 (image from coinmarketcap). (b) Dai normally trades below target with spikes in price due to liquidations (image from Kenny Rowe, Tweet).

This is like setting $x = \mathcal{D}$ and $y = -\mathcal{L}$. Now the DStablecoin market clearing price is $p_t^D = \frac{\mathcal{D}}{\mathcal{L}_t}$. The leverage constraint (assuming $\mathcal{L} + \Delta > 0$) becomes

$$-\beta \Delta^2 + \Delta(\tilde{\lambda}(z+\mathcal{D}) - 2\beta \mathcal{L}) + \mathcal{L}(\tilde{\lambda}z - 2\beta - \beta \mathcal{L}) \ge 0.$$

The speculator's maximization objective becomes $\hat{r}\Delta \frac{\mathcal{D}}{\mathcal{L}+\Delta} - \Delta$, which gives

$$\Delta^* = -\mathcal{L} + \sqrt{\mathcal{L}\mathcal{D}\hat{r}}.$$

While we prove stability results in this simplified setting, we believe the results can be extended beyond the assumption of constant wealth kept in the stable asset.

4.1 Stable domain

Proposition 5. Assume $w_t^D A_t = \mathcal{D}$ (DStablecoin demand) and $r_t = \hat{r}$ (speculator's expected Ether return) remain constant. If the leverage constraint is inactive at time t, then the DStablecoin return is

$$\frac{p_t^D}{p_{t-1}^D} = \sqrt{\frac{\mathcal{L}}{\mathcal{D}\hat{r}}}.$$

Proof. With inactive constraint, $\mathcal{L}_t = \sqrt{\mathcal{L}\mathcal{D}\hat{r}}, \ p_t^D = \frac{\mathcal{D}}{\sqrt{\mathcal{L}\mathcal{D}\hat{r}}} = \sqrt{\frac{\mathcal{D}}{\mathcal{L}\hat{r}}}, \ \text{and} \ \frac{p_t^D}{p_{t-1}^D} = \frac{\sqrt{\frac{\mathcal{D}}{\mathcal{L}\hat{r}}}}{\frac{\mathcal{D}}{\mathcal{L}}} = \sqrt{\frac{\mathcal{L}}{\mathcal{D}\hat{r}}}.$

Supposing that $\mathcal{D} \approx \mathcal{L}$ (i.e., the previous price was close to the \$1 target) and the constraint is inactive, Prop. 5 tells us that the DStablecoin behaves stably like the payment of a coupon on a bond. This is the 'stable domain' of the DStablecoin.

Consider DStablecoin log returns $\bar{\mu}_t$ and volatility $\bar{\sigma}_t$ computed in a similar way to Ether expectations in Eq. 2.2.1. In the stable domain, DStablecoin log returns remain $\bar{\mu}_t \approx 0$, the contribution to volatility at time t is $\ln \frac{p_t^D}{p_{t-1}^D} - \bar{\mu}_t \approx 0$, and the DStablecoin tends toward a steady state. We characterize this formally in the next theorem.

Theorem 1. Assume $w_t^D \mathcal{A}_t = \mathcal{D}$ (DStablecoin demand) and $r_t = \hat{r}$ (speculator's expected Ether return) remain constant. Let $\mathcal{L}_0 = \mathcal{D}$ and $\bar{\mu}_0, \bar{\sigma}_0$ be given. If the leverage constraint remains

inactive through time t, then

$$\mathcal{L}_{t} = \mathcal{D}\hat{r}^{\frac{2^{t}-1}{2^{t}}}, \qquad \bar{\mu}_{t} = \begin{cases} (1-\delta)^{t}\bar{\mu}_{0} - \delta\frac{(1-\delta)^{t}-2^{-t}}{2(1-\delta)-1}\ln\hat{r}, & \text{if } \delta \neq 1/2\\ 2^{-t}\left(\bar{\mu}_{0} - \frac{1}{2}t\ln\hat{r}\right), & \text{if } \delta = 1/2 \end{cases}$$

$$\bar{\sigma}_{t}^{2} = \begin{cases} \sum_{k=1}^{t} (1-\delta)^{t-k}\delta\left((1-\delta)^{k}\bar{\mu}_{0} - \frac{(1-\delta)^{k}-2^{-k+1}(1-\delta)}{2(1-\delta)-1}\ln\hat{r}\right)^{2} + (1-\delta)^{t}\bar{\sigma}_{0}^{2}, & \text{if } \delta \neq 1/2\\ 2^{-t}\sum_{k=1}^{t} 2^{-k-1}\left((k/2-1)\ln\hat{r} - \bar{\mu}_{0}\right)^{2} + 2^{-t}\bar{\sigma}_{0}^{2}, & \text{if } \delta = 1/2 \end{cases}$$

Further, assuming the constraint continues to be inactive and that $\delta \leq \frac{1}{2}$, the system converges exponentially to the steady state $\mathcal{L}_t \to \mathcal{D}\hat{r}$, $\bar{\mu}_t \to 0$, $\bar{\sigma}_t^2 \to 0$.

Proof. It is straightforward to verify $\mathcal{L}_t = \mathcal{D}\hat{r}^{\frac{2^t-1}{2^t}}$ by induction using $\mathcal{L}_t = \sqrt{\mathcal{L}_{t-1}\mathcal{D}\hat{r}}$. Then

$$\frac{p_t^D}{p_{t-1}^D} = \sqrt{\frac{\mathcal{L}_{t-1}}{\mathcal{D}\hat{r}}} = \sqrt{\frac{\mathcal{D}\hat{r}^{\frac{2^{t-1}-1}{2^{t-1}}}}{\mathcal{D}\hat{r}}} = \hat{r}^{\frac{1}{2}\left(\frac{2^{t-1}-1}{2^{t-1}}-1\right)} = \hat{r}^{-2^{-t}}.$$

And so $\ln \frac{p_t^D}{p_{t-1}^D} = -2^{-t} \ln \hat{r}$.

Next, as $\bar{\mu}_t = (1 - \delta)\bar{\mu}_{t-1} + \delta \ln \frac{p_t^D}{p_{t-1}^D}$, it is straightforward to verify by induction that

$$\bar{\mu}_t = (1 - \delta)^t \bar{\mu}_0 - \delta \ln \hat{r} \sum_{k=1}^t 2^{-k} (1 - \delta)^{t-k}.$$

Case I: $\delta = 1/2$. The series in $\bar{\mu}_t$ becomes

$$\sum_{k=1}^{t} 2^{-k} (1-\delta)^{t-k} = \sum_{k=1}^{t} 2^{-k} 2^{-(t-k)} = \sum_{k=1}^{t} 2^{-t} = \frac{t}{2^{t}}.$$

Then we have $\bar{\mu}_t = 2^{-t} \left(\bar{\mu}_0 - \frac{1}{2} t \ln \hat{r} \right)$. The first term $\to 0$ since $0 \le \delta < 1$. The second term $\to 0$ by L'Hopital's rule. Thus $\bar{\mu}_t \to 0$ as $t \to \infty$.

The contributing term to volatility at time t, after substituting and simplifying terms, is

$$\ln \frac{p_t^D}{p_{t-1}^D} - \bar{\mu}_t = \frac{t/2 - 1}{2^t} \ln \hat{r} - 2^{-t} \bar{\mu}_0.$$

Then DStablecoin volatility evolves according to

$$\begin{split} \bar{\sigma}_t^2 &= (1 - \delta)\bar{\sigma}_{t-1}^2 + \delta \left(\ln \frac{p_t^D}{p_{t-1}^D} - \bar{\mu}_t \right)^2 \\ &= \sum_{k=1}^t (1 - \delta)^{t-k} \delta \left(\ln \frac{p_k^D}{p_{k-1}^D} - \bar{\mu}_k \right)^2 + (1 - \delta)^t \bar{\sigma}_0^2 \\ &= \sum_{k=1}^t 2^{-(t-k)} \delta \left(\frac{k/2 - 1}{2^k} \ln \hat{r} - 2^{-k} \bar{\mu}_0 \right)^2 + 2^{-t} \bar{\sigma}_0^2 \\ &= \sum_{k=1}^t 2^{-(t-k)} \delta 2^{-2k} \left((k/2 - 1) \ln \hat{r} - \bar{\mu}_0 \right)^2 + 2^{-t} \bar{\sigma}_0^2 \\ &= 2^{-t} \sum_{k=1}^t 2^{-k-1} \left((k/2 - 1) \ln \hat{r} - \bar{\mu}_0 \right)^2 + 2^{-t} \bar{\sigma}_0^2. \end{split}$$

The second line follows from straightforward induction. As $t \to \infty$, the series converges from exponential decay. Then both terms $\to 0$ because of the factor of 2^{-t} . Thus $\bar{\sigma}_t^2 \to 0$.

Case II: $\delta \neq 1/2$. The series in $\bar{\mu}_t$ is a geometric progression

$$\sum_{k=1}^{t} 2^{-k} (1-\delta)^{t-k} = \sum_{k=1}^{t} (1-\delta)^{t} \left(2(1-\delta) \right)^{-k}$$

$$= \frac{(1-\delta)^{t} \left(2(1-\delta)^{-1} - 2^{-t-1} (1-\delta)^{-t-1} \right)}{1 - 2(1-\delta)^{-1}}$$

$$= \frac{(1-\delta)^{t} - 2^{-t}}{2(1-\delta) - 1}$$

Then we have $\bar{\mu}_t = (1 - \delta)^t \bar{\mu}_0 - \delta \frac{(1 - \delta)^t - 2^{-t}}{2(1 - \delta) - 1} \ln \hat{r}$, which converges to 0 as $t \to \infty$.

The contributing term to volatility at time t, after substituting and simplifying terms, is

$$\ln \frac{p_t^D}{p_{t-1}^D} - \bar{\mu}_t = (1 - \delta)^t \bar{\mu}_0 - \frac{(1 - \delta)^t - 2^{-t+1}(1 - \delta)}{2(1 - \delta) - 1} \ln \hat{r}.$$

The DStablecoin volatility evolves according to

$$\bar{\sigma}_t^2 = \sum_{k=1}^t (1-\delta)^{t-k} \delta \left(\ln \frac{p_k^D}{p_{k-1}^D} - \bar{\mu}_k \right)^2 + (1-\delta)^t \bar{\sigma}_0^2$$

$$= \sum_{k=1}^t (1-\delta)^{t-k} \delta \left((1-\delta)^k \bar{\mu}_0 - \frac{(1-\delta)^k - 2^{-k+1}(1-\delta)}{2(1-\delta) - 1} \ln \hat{r} \right)^2 + (1-\delta)^t \bar{\sigma}_0^2.$$

Note that because $(1 - \delta) \ge 1/2$, we have

$$|(1 - \delta)^t - 2^{-t+1}(1 - \delta)| \le (1 - \delta)^t + 2^{-t+1}(1 - \delta)$$

$$\le 2(1 - \delta)^t.$$

Thus we have

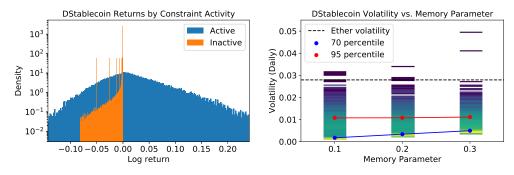
$$\begin{split} \bar{\sigma}_t^2 &\leq (1-\delta)^t \sum_{k=1}^t \frac{\delta}{(1-\delta)^k} \Big((1-\delta)^k \bar{\mu}_0 + \frac{2(1-\delta)^k}{2(1-\delta)-1} \ln \hat{r} \Big)^2 + (1-\delta)^t \bar{\sigma}_0^2 \\ &= (1-\delta)^t \sum_{k=1}^t (1-\delta)^k \delta \Big(\bar{\mu}_0 + \frac{2}{2(1-\delta)-1} \ln \hat{r} \Big)^2 + (1-\delta)^t \bar{\sigma}_0^t. \end{split}$$

As $t \to \infty$, the series converges from exponential decay. Then both terms $\to 0$ because of the factor of $(1 - \delta)^t$. Thus $\bar{\sigma}_t^2 \to 0$.

Notice that if the system has reached the leverage constraint, we can still treat the system as a reset of $\bar{\mu}_0$ and $\bar{\sigma}_0$ when we re-enter the region in which the constraint is inactive. While the system subsequently remains in the inactive region, we again converge to a steady state starting from the new initial conditions.

Solving Dai's below target price problem A consequence of Theorem 1 is that the DStablecoin will trade below target in the stable domain during times in which Ether expectations are high. This explains the behavior in Figure 2b, in which Dai empirically trades below target in Jan-Feb 2019.

Solving this below target price trend is an open problem in the MakerDAO community. There have been several recent governance votes to try to adapt system fees to bring the price back in line with \$1. Thus far, this has been an ad hoc process. Our model can be adapted to solve this quantitatively with supporting theory by adding a fee parameter into the setting of Theorem 1 and solving for the fee that brings the stable domain steady state supply in line with demand.



(a) Histogram of DStablecoin returns in sta- (b) Heat map of volatility under different ble vs. unstable domains with constant \hat{r} . speculator $\gamma = \delta$ memory parameters.

Figure 3: DStablecoin volatility, 10k simulation paths of length 1000.

4.2 Unstable domain

When the speculator's leverage constraint becomes active, DStablecoin returns can be more extreme, leading to high DStablecoin volatility. We conjecture that outside of the above 'stable domain', volatility is bounded away from 0 with high probability. This makes sense because the probability distribution for being outside of the stable domain in the next step has a kink at the boundary of the stable domain. In particular, it becomes increasingly likely that the system is outside of the stable domain in a subsequent step due to the deleveraging spirals described in the previous section. Note that feedback of large liquidations on Ether price, if added to the model, will make the unstable domain even larger.

We show this instability computationally in Figure 3a in simulation results. In this figure, the shape of the inactive histogram reflects the speculator's willingness to sell at a slight discount within the stable domain due to the constant \hat{r} assumption.

We relax this assumption in Figure 3b, which shows the effects on volatility of different speculator memory parameters. This figure is a heat map/2D histogram. A histogram over y-values is depicted in the third dimension (color: light=high density, dark=low density) for each x-value. Each histogram depicts realized volatilities across 10k simulation paths using the simulation setup introduced in the next section and the given memory parameter (x-value). Horizontal lines depict selected percentiles in these histograms. The dotted line depicts the historical level of Ether volatility for comparison.

In Figure 3b, volatility is bounded away from 0 even in the stable domain and the distance increases with the memory parameter. This happens because r updates faster with a higher memory parameter. As the speculator's objective then changes at each step, the steady state itself changes. Thus we expect some nonzero volatility, although it remains low in most cases.

In not-so-rare cases, however, volatility can easily be on the order of magnitude of actual Ether volatility in these simulations. As seen in Figure 4, this result is robust to a wide range of choices for the speculator's risk constraint. This suggests that DStablecoins perform well in median cases, but are subject to heavy tailed volatility.

5 Simulation Results

We now explore simulation results from the model considering a wide range of choices for the speculator's risk constraint. Unless otherwise noted, the simulations use the following parameter set with a simplified constant demand assumption ($\mathcal{D}=100$) and a t-distribution with df=3 to simulate Ether log returns. Cryptocurrency returns are well known for having very heavy tails. This choice gives us these heavy tails with finite variance. Note, however, that this doesn't capture path dependence of Ether returns. We instead assume Ether returns in each period

are independent. We run simulations on 10k paths of 1000 steps (days) each. This is enough time to look at short-term failures and dynamics over time. We plan to release the code for our simulator implementation in the future.

Parameter	Value	Rationale
$\overline{n_0}$	400	4x initial collateralization > typical Dai level
r_0	1.00583	Historical daily Ether mult. return 2017-2018
μ_0	0.00162	Historical daily Ether log return 2017-2018
σ_0	0.027925	Historical daily Ether volatility 2017-2018
$\gamma = \delta$	0.1	\sim Recommended value [13]
β	1.5	Threshold used in MakerDAO's Dai
α	~ 1.28	Value assuming normal distr. $+ a = 0.1$
b	1	Consistent with VaR constraint

Note that our simulations study daily movements. We choose this time step to examine these systems under reasonable computational requirements. More realistic simulations might study intraday movements. One plausible scenario of a Dai freeze is if the price feed moves too far too fast instraday, so that speculators don't have enough time to react before liquidations are triggered and keepers (who perform actual liquidations) are unable to handle the avalanche of liquidations. As the price feed in Dai faces an hourly delay in the price feed, hourly time steps are a natural choice for follow-up simulations. This said, daily time steps can actually be reasonable due to a behavioral trend in Dai data: most Dai speculators realistically don't track their positions with very high frequency as supported by surprisingly high liquidation rates.

5.1 Speculator behavior affects volatility

We compare DStablecoin performance under the following speculator behaviors encoded in the risk constraint.

\mathbf{Name}	Speculator risk constraint
VaRN.1	VaR using $a = 0.1 + \text{normality assumption}$
VaRN.01	VaR using $a = 0.01 + \text{normality assumption}$
VaRM.1	VaR using $a = 0.1 + \text{heavy-tailed assumption}$
VaRM.01	VaR using $a = 0.01 + \text{heavy-tailed assumption}$
AC1	Anti-cyclic constraint, $b = -0.5$, $\alpha = 0.01$
AC2	Anti-cyclic constraint, $b = -0.5$, $\alpha = 0.02$
RN	Risk neutral, only faces liquidation constraint

Figure 4 compares the effects on volatility of these behavioral constraints under various Ether return distributions. These figures are heatmaps/2D histograms similar to that in Figure 3b. The results suggest that DStablecoins face significant tail volatility (on the order of Ether volatility) even under comparatively 'nice' Ether return distributions, such as with significant upward drift (Figure 4b) and a normal distribution (Figure 4c).

Stricter cyclic risk management (e.g., VaR) on the part of the speculator can lead to increased DStablecoin volatility without improving the safety of the system. A risk neutral speculator, who faces no behavioral risk constraint and only the system's forced liquidation constraint, provides the least volatile DStablecoin. This makes sense because the 'stable domain' of the risk neutral speculator is the largest since it faces the widest possible leverage constraints. Stricter risk management serves to reduce the size of the stable domain, and so the system is more likely to enter the unstable domain. Naturally, a risk neutral speculator may be unrealistic in a real world setting.

Figure 3b further suggests that a higher speculator memory parameter (lower memory) tends to increase volatility in typical cases. This makes sense as high memory parameters can lead to noise chasing on the part of the speculator. Note that keeping the speculator's expected Ether returns and variance constant is equivalent to setting a static risk constraint.

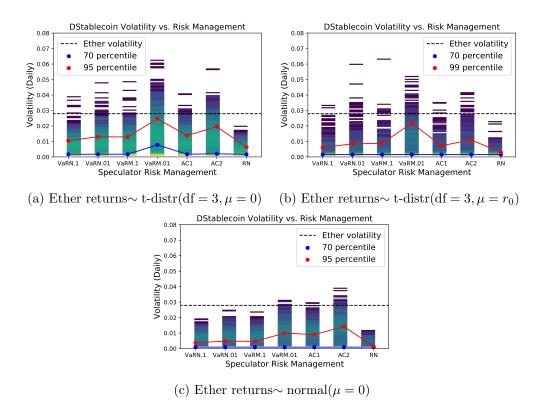


Figure 4: Heatmaps of DStablecoin volatility for different speculator risk management behaviors.

5.2 Stable asset failure is dominated by collateral asset returns

We define the DStablecoin's **failure (or stopping) time** to be either (1) when the speculator's liquidation constraint is unachievable or (2) when the DStablecoin price remains below \$0.5 USD. In these cases, a global settlement would be reasonable, leaving DStablecoin holders with Ether holdings with high volatility in subsequent periods.

Figure 5 compares the effects on failure time of these behavioral risk constraints. The results suggest that DStablecoins are not long-term safe, even under comparatively 'nice' Ether return distributions. They are susceptible to failure within the short-term. To avoid failure, they rely on more speculator capital entering the system during downturns.

DStablecoin failure times appear to be dominated by Ether returns as opposed to speculator behavior. Figure 5 shows that the stopping times on paths are comparable across a wide range of selections for the speculator's risk constraint. They are also comparable across the memory parameters studied above.

6 Discussion

Failure risks DStablecoins are complex systems with substantial failure risks. Our model demonstrates that they work well in their 'stable domain', which includes typical cases, but not outside of these domains. As we explore in this paper, the market can collapse due to feedback effects on liquidity and volatility from deleveraging spirals during cryptocurrency crises. Surviving these events relies ultimately on bringing in increasing amounts of new capital to expand the DStablecoin supply at the right times. In these events speculators may not always be willing and able to take these new risky positions. Indeed, there are may examples of speculative markets drying up during extreme market movements. As we explore below, continued stability during these events additionally relies on new capital entering the system in a well-behaved

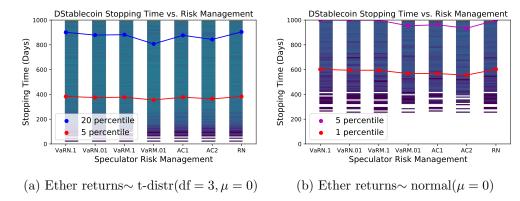


Figure 5: Heatmaps of DStablecoin failure times for different speculator risk management behaviors.

manner as a profitable attack is possible.

As suggested by our simulations, stablecoin holders face the direct tail risk of cryptocurrencies. If the market loses liquidity, there is no guarantee that forced liquidation of speculators' collateral will be possible within reasonable pricing limits. Further, volatile cryptocurrency markets can, in unlikely events, move too fast for speculators to adapt their positions. In these cases, stablecoin holders can only truly rely on the cryptocurrency value from global settlement.

Remark on oracle risks The DStablecoin design also relies on trusted oracles to provide real world price data, which could be subject to manipulation. In MakerDAO's Dai, for instance, oracles are chosen by MKR token holders, who vote on system parameters. This opens a potential 51% attack, in which enough speculators buy up MKR tokens, change the system to use oracles that they manipulate, and trigger global settlement at unfavorable rates to stablecoin holders while pocketing the difference themselves when they recover their excess collateral. A hint of manipulation in oracles or large acquisitions of MKR could potentially trigger market instability issues on its own.

Note that Dai has protections from oracle attacks. First, there is a threshold of maximum price change and an hourly delay on new prices taking effect. This means that emergency oracles have time to react to an attack. Second, at current prices 51% of MKR is substantially more expensive than the ETH collateral supply. However, this second point does not have to be true in general—at least unless Dai holders otherwise bid up the price of MKR for their own security. The value of MKR is linked to expectations around Dai growth as fees paid in the system are used to reduce MKR supply. At some point, the expectation may not be enough to lift MKR value above collateral on its own. This raises the question of whether fees should be used to reduce MKR supply at all. Alternatively, MKR value could be completely based on the potential value of a 51% attack, which may also grow with Dai growth, and the value of fees could be put to different uses, as we discuss further below.

A profitable economic attack on the system We now describe an attack on the system that can trigger DStablecoin volatility while earning a profit for the attacking agent through market manipulation. This attack is made possible by the permissionless cryptocurrency setting, in which agents can enter/exit at any time with a degree of anonymity. In the traditional financial system, market manipulation can be enforced legally. In contrast, in an anonymous cryptocurrency setting, enforcement is only possible to the extent that it can be codified within the protocol and incentive structure, which dictate how interactions take place.

Starting in a declining cryptocurrency environment:

1. The attacking agent bids up the price of DStablecoin.

⁹Though it is notable that most MKR is reputedly held by just a few individuals within the MakerDAO team.

- 2. When Ether price declines enough, speculator liquidations are triggered. 10
- 3. The liquidations trigger a feedback effect: DStablecoin price is pushed up further and the price of Ether can be pushed further down if the size of liquidations is large.
- 4. To exit, the attacking agent has two options
 - (a) Sell DStablecoin position for a profit.
 - (b) Enter as a new speculator at low Ether price and leverage its position.

The effect is akin to a short squeeze on the existing speculators. It takes advantage of the fact that liquidating speculators have to repurchase DStablecoin at the prevailing market rate, which is easily manipulated during these events to cause a deleveraging spiral. It can additionally be facilitated by network congestion, which can result in a fee battle to prevent existing speculators from maintaining their positions. The result is profit for the attacking agent because they facilitate the peaks and bottoms in the market and induced volatility for the DStablecoin. In the second exit, the induced volatility is actually beneficial for the attacking agent. After the event, the DStablecoin may trade at a discount because of realized volatility, which only means the attacking agent can unlock assets at a discount.

We can explore this formally in an expansion to the model by considering the strategic behavior of a rival speculator on the sideline deciding when to enter the market. This attack also suggests that DStablecoins may cause perverse incentives for miners, who are in a position to potentially censor or reorder transactions. We discuss this further below in the context of risks to blockchain consensus.

Stablecoin design insights In general, it is impossible to build a stablecoin system without significant risks. As speculators only want to participate in a DStablecoin system if they believe cryptocurrencies have increasing value, these systems will always have an undiversifiable cryptocurrency risk. However, a stablecoin can aim to be an effective store of value assuming trust in cryptocurrencies as a whole is not undermined. In this case, it is *conceivable* to sustain a dollar peg. Such a stablecoin needs to survive transitory extreme movements.

Our model is adaptable to study the stability of DStablecoins with a wide variety of design characteristics, including methods for dynamic deleveraging, collateral thresholds, and the setting and usage of fees. It will aid in designing stablecoins that perform well in these extreme settings. Such designs will focus on expanding the size of the stable domain and minimizing the severity of the unstable domain.

In the case of Dai, our results suggest some immediate design considerations. The Dai system includes various fees that are imposed on speculators when they are forced to liquidate positions (e.g., liquidation penalty, stability fee, and penalty ratio). These can have the effect of amplifying deleveraging spirals. An alternative design that automatically changes these fees to be counter-cyclical would widen Dai's stable domain. For instance, fees could be collected upon entering or exiting positions while the system is performing well, but these fees could be removed automatically during liquidity crises in order to limit the feedback effects and remove disincentives to bringing new capital into the system.

Speculators in Dai have the freedom to pay back liabilities at any time and come and go from the system, which raises concerns about herd behavior in crises. A herd reducing leverage can trigger a deleveraging spiral, and so the stability of Dai really relies on having a significant mass of speculators who avoid herd behavior. A good mechanism of fees may be able to improve speculator behavior and further widen the stable domain.

In Dai, MKR tokens serve as 'collateral of last resort' in addition to distributing system governance. In particular, if the speculator collateral value in global settlement is insufficient to cover the Dai liabilities, new MKR tokens are created and sold to make up the difference. This may not always be possible as the MKR market can face similar illiquidity in these situations and the market cap may not be high enough to cover collateral losses. Thus this isn't a real 'fix' to DStablecoin problems.

¹⁰This Ether price decline can come naturally or can be facilitated by the attacking agent's sale of Ether for DStablecoin, although this latter market is more liquid and difficult to manipulate than the DStablecoin market.

Further, MKR holders may have an incentive to trigger global settlement early before MKR would be inflated away. This is not ideal for Dai holders, as they will want to hold the stable asset for longer during extreme events. In this way, a global settlement is essentially a failure for stablecoin holders as it liquidates their stablecoin holdings into risky cryptocurrency holdings at a likely volatile time. A global settlement would affect the price of MKR, but the cost may be small if MKR is expected to be able to restart a new version of Dai after the fact. This suggests that other 'last resort' roles for MKR could be more beneficial. For instance, MKR could be used to quell feedback effects from deleveraging spirals by automatically positioning the MKR supply as collateral to expand the Dai supply in an emergency. System fee revenue could also be put to this use.

MakerDAO has also suggested a Dai savings rate paid from fee revenue, suggesting that this could help expand Dai demand to be more in line with supply when Dai price is below target. This is another idea toward solving the problem discussed previously in the context of setting speculator fees. Our model helps provide insight in this discussion as well. There is an inherent trade-off in using valuable fee revenue. A Dai savings rate would put this revenue to work improving stability within the stable domain (as well as paying potentially reasonable interest rates to stablecoin holders on what are meant to be cash equivalent positions). An alternative use, however, is to lessen the severity of deleveraging spirals in the unstable domain. We can incorporate these fees and their uses into our model in order to compare the effects of different design choices.

MakerDAO has also proposed a 'multi-collateral' version of Dai. We discuss this further below in the context of model extensions.

Systemic risks to blockchain consensus While our model abstracts away from the consensus layer to focus on the application layer (i.e., the stablecoin system), the presence of the attacks we uncover suggests scenarios in which DStablecoins can cause perverse incentives that may affect consensus. Many blockchain systems, e.g., proof-of-work, have only probabilistic finality, meaning that there is a nonzero chance that the history can be rewritten to reorder or exclude transactions. Miners may attempt such an attack if there is enough payoff for doing so compared to the mining reward—this is a time-bandit attack introduced in [7]. Miners can also simply censor certain transactions going forward if there is enough payoff for doing so.

The assets locked in a DStablecoin system, if significant enough, can provide a means for miners to manipulate markets and earn a payoff for doing so. In a simple instance, miners can perform the economic attack we present and censor existing speculators from changing their positions, increasing the probability of a mass liquidation that can affect Ether and DStablecoin price. In a more complicated time-bandit attack, miners can re-write the history to reorder and exclude speculator transactions and take advantage of guaranteed Ether price trajectory (via oracle transactions) to trigger such liquidations in the 'new' present in a more risk-free way. If the DStablecoin collateral size is large, the effects of the market manipulation can be large and so the profit from this attack could very well be larger than the mining rewards and transaction fees. Further, the payoff need not be in Ether.

The possibility of such attacks suggests that DStablecoins can pose systemic risks to consensus if not designed well. We leave to future study further exploration to map out how agents' incentives may lead to these attacks in a game theoretic setting based on the foundations set in, e.g., [3].

Managerial insights Our results suggest tools and indicators that can warn about volatility in DStablecoins. We can find proxies for the free supply, estimate the price impact of liquidations, and track the entrance of new capital into speculative positions. We can connect this information with model results to estimate the probability of liquidity problems given the current state. This information is also useful in valuing token positions in these systems (e.g., Dai, MKR, and the speculator's leveraged position).

In a recent trend, several exchanges have begun bundling select stablecoins into a single market that allows users to deposit any stablecoin and in turn withdraw any other stablecoin—

e.g., Huobi. In this case, exchanges are essentially providing insurance to their users against stablecoin failures. These arrangements could lead to a run on exchanges in the event that some stablecoins fail. It is unclear if these exchanges are subject to regulation to protect users against this, and it is further unclear if such regulations would be sufficient to account for risks in stablecoins. Our work provides insight into the risks (to exchanges and users) if such arrangements in the future include noncustodial stablecoins.

Future directions We suggest expansions to our model to explore wider settings.

- Incorporate more speculator decisions, such as locking and unlocking collateral and holding different cryptoassets, accommodating speculators with security lending motivation. This makes the speculator's optimization problem multi-dimensional. In this expanded setting, speculators may make more long-term strategic decisions considering whether tomorrow they would have to buy back stablecoins and at what price.
- Consider multiple speculators with different utility functions who participate in the DStablecoin market. In this expanded setting, we can consider the conditions under which new capital may enter the system and formally study the economic attack described above and the effects of external incentives.
- Incorporate additional assets, such as a custodial stablecoin that faces counterparty risk. This would allow us to study long-term movements between stablecoins in the space and learn about systemic effects that could be triggered by counterparty failures. This is further relevant in evaluating systems like MakerDAO's proposed multi-collateral Dai, which try to capture diversification gains. However, this comes with a trade-off of a new counterparty risk that is very hard to measure. In particular, it's not just custodian default risk, but also risk of targeted interventions on centralized assets. Such interventions (e.g., from a government who wants to shut down Dai) could be highly correlated with cryptocurrency downturns as that is when the system is naturally weakest.
- Incorporate endogenous feedback of liquidations on Ether price, which becomes relevant if the DStablecoin system becomes large relative to the Ether supply.

Additionally, our existing model can be adapted to analyze DStablecoins with different design characteristics. For instance,

- DStablecoins with more general collateral settlement, in which stablecoin holders can individually redeem stablecoins for collateral. This is possible, for instance, in bitUSD and Steem Dollars. In this case, the stablecoin acts as a perpetual option to redeem collateral, and stablecoin volatility will be additionally related to the settlement terms.
- DStablecoins without speculators (e.g., Steem Dollars, in which the whole marketcap of Steem acts as collateral). Our model can directly apply by removing speculator decisions and modeling the growth of collateral from block rewards and growth of stablecoin from other processes.
- Some non-collateralized algorithmic stablecoins. We believe this setting can also be interpreted in our model by thinking of 'synthetic' collateral that ends up describing user faith in the system. The underlying mechanics would be similar, simply recreating 'out of thin air' the value of the underlying blockchain as opposed to building on top of the existing value. The stability of the system ultimately still relies on how people perceive this value over time similarly to how perceived value of Ether changes.

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