PAGE:
DATE: / /

General Properties Of Matter

Elasticity

Elasticity :- The phenomenon of regaining the original shape of the body after external force applied on it is removed is called elasticity and the body is called elastic body but if the body doesn't return to its original shape after the external force applied to it is removed is called elasticity and the body is called Plastic body.

Stress and Strain

Stress :- Applied force per unit cross sectional area is called Stress.

i.e., Stress $= \frac{\text{force}}{\text{Area}} = \frac{F}{A}$

Unit of Stress in S.I system is

Newton / Metre^2.

Strain :– Change in shape per unit original shape of the body is called strain.

i.e., $\text{Strain} = \dfrac{\text{Change in Shape}}{\text{Original Shape}}$

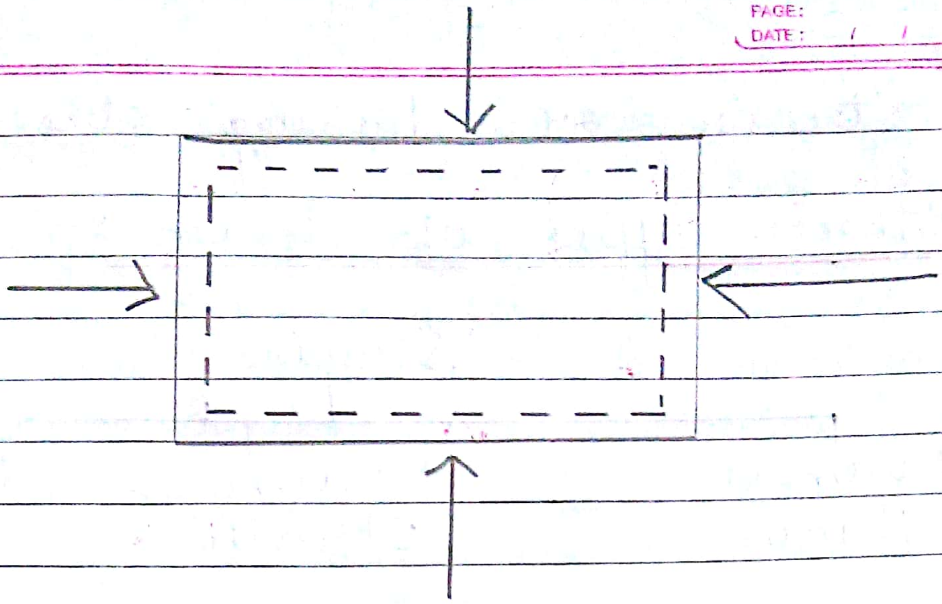
Strain is unitless and Dimensionless.

Since three types of deformation may occur by applying the external force to the body.
therefore there are three types of stress and three types of strain.

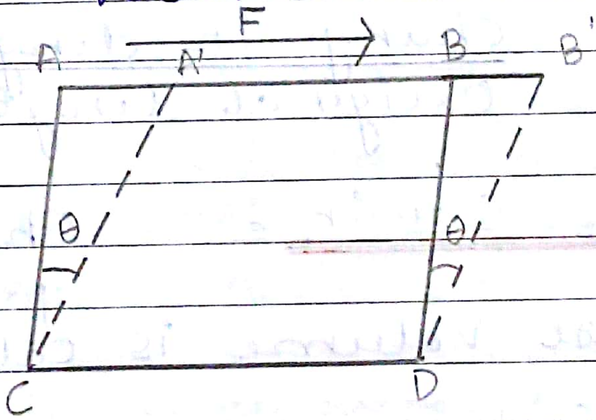
Stress

- Normal Stress
- Volume Stress
- tangential Stress or Shearing Stress

Normal Stress :– When the applied force changes the length of the object then applied force per unit area is called Normal Stress.



Volume Stress :- When the applied force changes the volume of an ... then applied force per unit area is called Volume Stress.



Let ABCD is a side of the cube whose lower face BC is fixed. Let a tangential force F is applied to upper face of the cube then its shape changes to A'B'CD

Sharing stress :- In this case applied tangential force per unit area tangential

PAGE:
DATE: / /

Stress or sharing stress.

Three types of Strain :-

Strain
- Normal Strain
- Volume Strain
- tangential Strain or Shearing strain

Normal Strain :- Change in length per unit original length is called Normal Strain.

i.e., $\frac{\text{Change in length}}{\text{Original length}} \quad \frac{l}{L}$

Volume Strain :- Change in Volume per unit Original Volume is called Volume Strain.

i.e., Volume Strain = $\frac{\text{Change in volume } (v)}{\text{Original Volume } (V)}$

Elastic limit :- The limit up to which the property of elasticity remains in the body is called elastic limit.

PAGE:
DATE: / /

Hooke's law :— Within the elastic limit stress is directly proportional to strain.

i.e., Stress $\propto$ Strain

or Stress = E Strain

$$E = \frac{\text{Stress}}{\text{Strain}}$$

Where E is a constant and is called coefficient of elasticity or Modulus of elasticity.

Since there are three types of stress and three types of strain therefore there are three Coefficient of elasticity.

1) Young's modulus of elasticity (Y)
2) Bulk modulus of elasticity (K)
3) Rigidity modulus of elasticity (η)

1) $$Y = \frac{\text{Normal Stress}}{\text{Normal Strain}} \quad \frac{F/A}{l/L}$$

$$\boxed{Y = \frac{FL}{Al}}$$

2) $$K = \frac{\text{Volume stress}}{\text{Volume strain}}$$

$$K = \frac{F/A}{v/V} = \frac{F \cdot V}{vA}$$

3) $\eta = \dfrac{\text{tangential stress} \quad (T)}{\text{tangential strain} \quad (\theta)}$

= from fig.

In Δ AA'D

$\tan\theta = \dfrac{AA'}{AD} = \dfrac{l}{L}$

$\tan\theta = \dfrac{AA'}{AD} = \dfrac{l}{L}$

$\tan\theta = \theta$

$\theta = \dfrac{l}{L}$

$\eta = \dfrac{F/A}{l/L}$

$$\boxed{\eta = \frac{F \cdot L}{A \cdot l}}$$

Y, K. and η are called elastic constant there is one more elastic constant poisson's ratio denoted by σ.

Hence we have four types of elastic constant.

Y, K, η and σ.

PAGE:
DATE: / /

Poisson's Ratio (σ) :- When a normal force is applied to a wire its length increases but its diameter decreases. Increase in length and decrease in diameter are mutually lar to each other.

Increase in length per unit original length is called Normal Strength (α) and decrease in diameter per unit original diameter is called lateral strength (β).

$$(\beta) \propto (\alpha)$$

$$\beta = \sigma\alpha$$

$$\sigma = \frac{\beta}{\alpha}$$

where α is constant and is called Poisson's ratio.

It is unit less and dimension less.

let L = Original length of wire

l = Increase in length of wire

D = Original diameter of the wire

d = Decrease in diameter of wire

$$\alpha = \frac{l}{L} \quad , \quad \beta = \frac{d}{D}$$

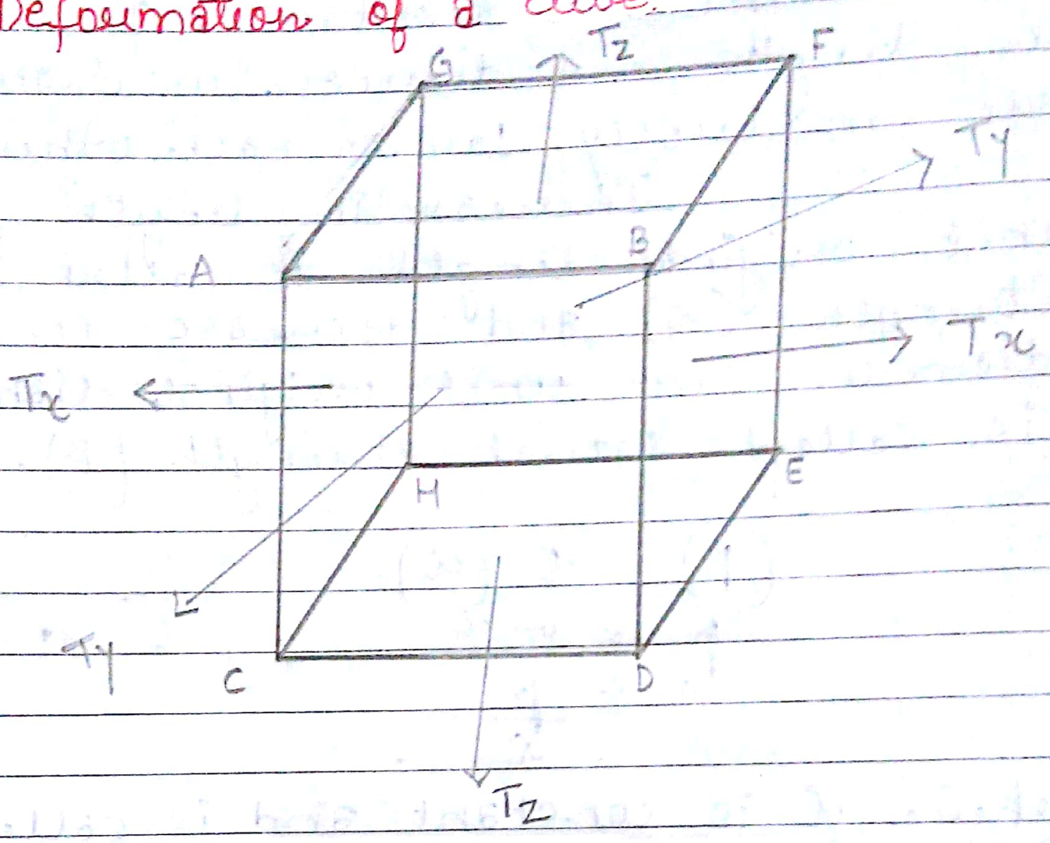
$$\sigma = \frac{dL}{Dl}$$

p.t.o.

PAGE:
DATE: / /

Inter relation b/w elastic constant ($Y K \eta \sigma$)

★ Deformation of a cube.



Let ABCDEFGH is a cube of unit length in which the tension T_x, T_y, and T_z are applied to faces AGHD and BFEC, ABCD and EFGH and DCEH and ABFG respectively. Due to these tension volume of the cube increases.

Let α – Increase in length per unit length per unit tension along the dir^n of applied force.

β – Decrease in length per unit length per unit tension along the perpendicular dir^n of applied force.

Due to these tensions length of AB, BF and BC.

$AB = 1 + T_x\alpha - T_y\beta - T_z\beta$
$BF = 1 + T_y\alpha - T_x\beta - T_z\beta$
$BC = 1 + T_z\alpha - T_x\beta - T_y\beta$

find volume of the cube

$= (1 + T_x\alpha - T_y\beta - T_z\beta)(1 + T_y\alpha - T_x\beta - T_z\beta)$
$(1 + T_z\alpha + T_x\beta - T_y\beta)$

$= 1 + (\alpha - 2\beta)(T_x + T_y + T_z)$

(neglecting the products of α, β and there squares)

Increase in volume of the cube

$1 + (\alpha - 2\beta)(T_x + T_y + T_z)$
$(\alpha - 2\beta)(T_x + T_y + T_z)$

if the same tension are applied on all the faces of the cube

i.e., $T_x = T_y = T_z = T$ (say)

Increase in volume $= 3T(\alpha - 2\beta)$

If instead of apply the tension outward if we apply an invert pressure P on the Q then decrease in volume of the cube

$= 3P(\alpha - 2\beta)$

$\therefore$ Volume strain $= \dfrac{3P(\alpha - 2\beta)}{1}$

$= 3P(\alpha - 2\beta)$

Volume Stress = P

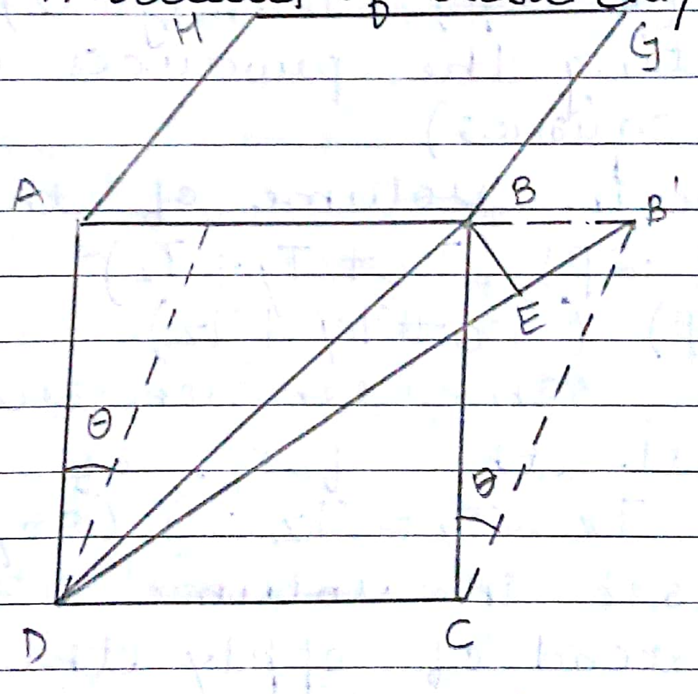
let $K = \frac{\text{Volume stress}}{\text{Volume strain}}$

$$= \frac{P}{3P(\alpha - 2\beta)}$$

$$\boxed{K = \frac{1}{3(\alpha - 2\beta)}}$$

26/08/17

Rigidity modulus of elasticity (η)



Let ABCD is one of the face of the cube of which lower face DC is fixed and a tangential force F is applied to upper face of the cube. due to which cube is sheared by an angle θ. and it becomes A'B'CD.

let L = length of the cube.
tangential stress $(T) = \frac{F}{L^2}$

$$AA' = BB' = l$$

Shearing strain $(\theta) = \frac{l}{L}$

η = Rigidity modulus of elasticity

$$\eta = \frac{T}{\theta} \quad \text{—} \quad (1)$$

we Know that a tangential stress 'T' is equivalent to a linear tensile stress 'T' along DB' and an equal compressive stress 'T' along DA diagonal AC, both mutually $\perp^r$ to each other. tensile stress (T) along DB increases its length. the length of DB will also increases by compressive stress 'T' along AC. thus the length of DB is increased due to two reason.

(i) tensile stress (T) along DB.
(ii) Compressive stress (T) along AC.

let α = Increase in length per unit length per unit tension along the dirn of applied tension.
β = Decrease in length per unit length per unit tension along the perpendicular dirn of the applied force.

PAGE:
DATE: / /

Increase in length of diagonal due to tensile stress T along it

$= DB \cdot T \cdot \alpha$

Increase in length of diagonal DB due to compressive stress T along AC

$= DB \cdot T \cdot \beta$

Total increase in length of diagonal

$DB = DB\,T(\alpha+\beta)$

$= L\sqrt{2}\,T(\alpha+\beta)$ — (2)

Drawn $\perp^r$ 'BE' on 'DB' it is clear that increase in length of DB = EB'

θ is very small

$\angle BB'C \simeq 90^\circ$

$\angle BB'E \simeq 45^\circ$

In $\Delta BB'E$

$\cos 45^\circ = \frac{EB'}{BB'}$ or $EB' = l\frac{1}{\sqrt{2}}$

Therefore $EB' = \frac{l}{\sqrt{2}}$ — (3)

From (2) and (3)

$L\sqrt{2}\,T(\alpha+\beta) = \frac{l}{\sqrt{2}}$

$\frac{L \cdot T}{l} = \frac{1}{2(\alpha+\beta)}$

or $\frac{T}{l} = \frac{1}{2(\alpha+\beta)}$

Or $\frac{T}{\theta} = \frac{1}{2(\alpha+\beta)}$

$$\boxed{\eta = \frac{1}{2(\alpha+\beta)}}$$ from eqn (1)

Interrelation between Y, K, η and σ.

Let us a consider a unit. In which a unit tension is applied to it.

Let α = increase in length per unit length per unit tension along the dirn of applied tension.

β = decrease in length per unit length per unit tension along the ⊥r dirn of applied tension.

Increase in length = α

$\therefore$ Normal strain = $\frac{\alpha}{1} = \alpha$

Normal stress = $\frac{1}{1} = 1$

$Y = \frac{1}{\alpha}$, $\alpha = \frac{1}{Y}$

$K = \frac{1}{3(\alpha - 2\beta)}$

PAGE:
DATE: / /

$$\alpha - 2\beta = \frac{1}{3K} \quad \text{—— (1)}$$

and $\eta = \dfrac{1}{2(\alpha+\beta)}$

$$\alpha + \beta = \frac{1}{2\eta} \quad \text{—— (11)}$$

Multiply eqn (11) by 2 and add it in eqn (1)

$$2\alpha + 2\beta = \frac{1}{\eta}$$

$$\alpha - 2\beta = \frac{1}{3K}$$

$$3\alpha = \frac{1}{\eta} + \frac{1}{3K}$$

or $\dfrac{3}{Y} = \dfrac{3K+\eta}{3K\eta}$

$$Y = \frac{9K\eta}{3K+\eta} \quad \text{—— (111)}$$

eq^n (iii) is the required relation between Y, K and η.

From eq^n (1)

$$\alpha\left(1-\frac{2\beta}{\alpha}\right) = \frac{1}{3K}$$

or $$\frac{1}{Y}\left(1-2\sigma\right) = \frac{1}{3K}$$

or $$Y = 3K\,(1-2\sigma) \quad \text{— (IV)}$$

From eq^n (2)

$$\alpha\left(1+\frac{\beta}{\alpha}\right) = \frac{1}{2\eta}$$

or $$\frac{1}{Y}\left(1+\sigma\right) = \frac{1}{2\eta}$$

$$Y = 2\eta\,(1+\sigma) \quad \text{— (V)}$$

eq^n (IV) is the relation b/w Y, K and σ.
eq^n (V) is the relation b/w Y, η and σ.

from eq^n (IV) and (V)

$$3K\,(1-2\sigma) = 2\eta\,(1+\sigma) \quad \text{— (VI)}$$

or $3K - 6K\sigma = 2\eta + 2\eta\sigma$

or $3K - 2\eta = \sigma(2\eta + 6K)$

$$\sigma = \frac{3K - 2\eta}{2\eta + 6K} \quad \text{— (VII)}$$

eqn (VII) is the relation b/w K, η and σ.

Limiting Value of σ

from eqn (VI)

$$3K(1 - 2\sigma) = 2\eta(1 + \sigma)$$

1) If σ is +ve then write hand expression is +ve. therefore left hand side expression is to be +ve. $2\sigma < 1$ or $\sigma < .5$

2) If σ is -ve then left hand side expression is +ve. therefore right hand side expression to be +ve σ must not be < -1. Hence limiting value of σ lies between -1. to $.5$.

Q) which one is more elastic Steel or rubber and why?

→ Steel is more elastic than rubber.

Let L = Original length of steel and rubber.

F = Applied normal force to both the steel and rubber.

l_s = Increase in length of steel.

l_r = Increase in length of rubber

A = Cross - sectional area of both the steel and rubber.

Y_s = Young's modulus of elasity of steel.

Y_r = Young's modulus of elasticity of rubber

$$Y_s = \frac{F \cdot L}{A \cdot l_s} \qquad Y_r = \frac{F \cdot L}{A \cdot l_r}$$

$$l_s < l_r$$

$$\frac{1}{l_s} > \frac{1}{l_r}$$

$$Y_s > Y_r$$

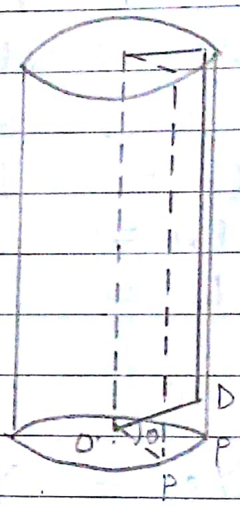
Hence young's modulus of elasticity of steel is greater than young's

PAGE:
DATE: / /

modulus of elasticity of rubber. Therefore steel is more elastic than rubber.

→ Twisting or restoring couple per unit angle twist of the wire cylinder.

→ Let ABDE is a cylinder or wire of radius 'r' and length 'l', upper end AB of the cylinder is fixed and lower end DE is twisted by an angle θ



Let φ = angle of shear
T = Shearing Stress
θ = Angle of twist

fig. 2 is the cross-sectional view of the cylinder. Let the cylinder consist of large no. of imaginary cylinders in contact with each other.

Consider an imaginary cylinder of radius 'x' and thickness dx shown in figure 2.

Let in fig 2 P and P' are assumed to be on the imaginary cylinder.

from figure 1

In Δ CPP'

$$\phi = \frac{PP'}{l}$$

$$PP' = l\phi \quad \text{--- (i)}$$

from fig (2)

In Δ OPP'

$$\theta = \frac{PP'}{x}$$

$$PP' = \theta x \quad \text{--- (ii)}$$

from eqn (i) and (ii)

$$l\phi = x\theta$$

$$\phi = \frac{\theta x}{l} \quad \text{--- (iii)}$$

cross-sectional area of imaginary cylinder.

$$= 2\pi x \, dx$$

$$T = \frac{\text{Shearing force } (F)}{2\pi x \, dx}$$

$$F = T \cdot 2\pi x \, dx \quad \text{--- (iv)}$$

Let η = rigidity modulus of elasticity.

$$\eta = \frac{T}{\phi}$$

$$T = \eta\phi = \frac{\eta \theta x}{l}$$

put the value of T in eqn (4)

$$F = \frac{2\pi \eta \theta x^2 \, dx}{l}$$

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PAGE:
DATE: / /

moment of force or twisting couple

$$= \frac{2\pi\eta\,\theta x^3\,dx}{l} \quad — ⑥$$

Therefore for calculating the total twisting torque, we have to integrate eqn ⑤ from the limit 0—r.

Total twisting couple of the cylinder

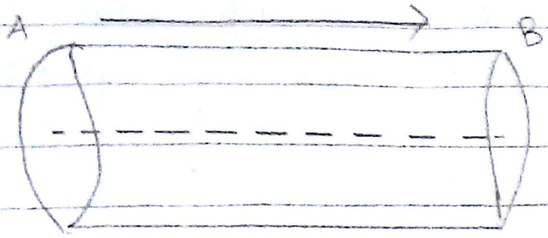
$$= \frac{2\pi\eta\theta}{l}\int_0^r x^3\,dx$$

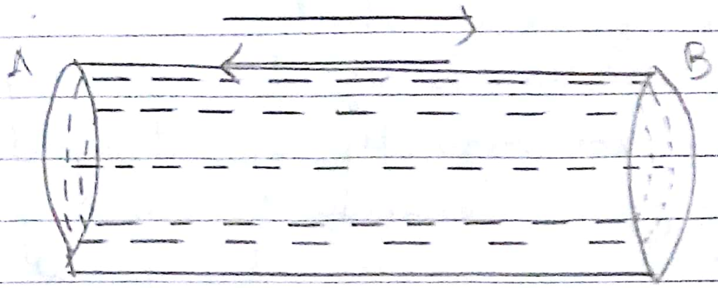
$$= \frac{2\pi\eta\theta}{l}\,\frac{r^4}{4}$$

$$= \frac{\pi\eta r^4}{2l}\,\theta.$$

11/11/17

Viscosity :— Let a liquid flow through a tube AB from left to right. The liquid which flows through the tube can be considered as made of large no. of imaginary layers in contact with each other. The layer which is in contact with the wall of the tube is called fixed layer or stationary layer. The velocity of this layer is assumed to be zero. and the velocity of other layer goes on increased as we move towards the axis of the tube.





Consider two imaginary layer M and N in contact with each other. The velocity of layer N is greater than the velocity of layer M. Therefore an opposite force than the velocity of layer M. Therefore an opposite force then the dirn of flow of the liquid is developed b/w this two Imaginary layer. This will take place for any two consequetive layer of the liquid.

Hence viscosity is the property of liquid and gases by virtue of an opposite force then the dirn of flow of the liquid is developed b/w this two imaginary layer. This will take place for any two consequetive layer of the liquid.

Hence viscosity is the property of liquid and gases by virtue of an opposite force is developed b/w the imaginary

PAGE:
DATE: / /

layer of the liquid which try to maintain the velocity gradient b/w them. Hence viscosity is also called internal friction of the liquid.
Let F= Opposite force due to viscosity
Newton found that
$F \propto A$ (A= area of imaginary layer)
$F \propto V$ (V= Visosity of imaginary layer)
$F \propto \frac{1}{x}$ (x= Distance of imaginary layer from the fixed layer)

$$F \propto \frac{AV}{x}$$

$$F = -\eta \frac{AV}{x}$$

where -ve sign indicate that dirn of F is opp. to the dirn of flow of the liquid.
η is a constant and is called co-efficient of viscosity of the liquid.
In calculas

$$F = -\eta A \frac{dv}{dx}$$

Here, $\frac{dv}{dx}$ is called Velocity gradient.

$$\eta = \frac{-F}{A \frac{dv}{dx}}$$

if $A = 1m^2$

$$\frac{dv}{dx} = 1\ sec^{-1}$$

$$\eta = -1$$

Hence, co-efficient of viscosity of a liquid is defined as the opp. force required to maintain 1 sec⁻¹ velocity gradient in an imaginary layer of $1m^2$ area.

Unit in c.g.s :—

$$\eta = \frac{dyne}{cm^2 \times \frac{cm}{sec} \times \frac{1}{cm}}$$

Unit of η in cgs system is 'dyne sec cm^{-2}'
or
'Poise'

In S.I system → 'N sec m^{-2}'
or 'decapoise'

Dimension $\dfrac{[MLT^{-2}]}{[L^2]\left[\dfrac{LT^{-1}}{L}\right]} = [ML^{-1}T^{-1}]$

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PAGE:
DATE: / /

Poisellceous formula :— for the rate of flow of liquid through a capillarly tube :—

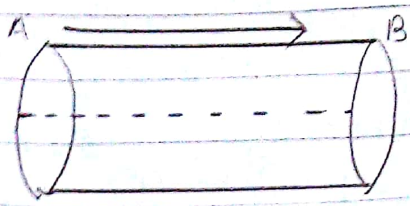

fig. 1

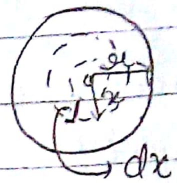

fig 2

Let a liquid is flowing through a tube from A to B.

Let, r = Radius of the tube
l = length of the tube
F = Opposite force from B to A due to viscosity.

fig 2 is the cross-sectional view of the cappilary tube. Let us consider an imaginary layer of radius 'x' and thickness 'dx' as shown in fig 2.

Area of imaginary layer $= 2\pi x l$

v = Velocity of flow of liquid in cylindrical shell between x and $(x+dx)$.

$$F = -\eta \, 2\pi x l \cdot \frac{dv}{dx} \quad — (1)$$

Let P = Pressure difference at the two end of the tube.

$$P = \frac{F}{\pi x^2}$$

$$F = P\pi x^2 \quad \text{—— (ii)}$$

from (i) and (ii)

$$-\eta \cdot 2 \frac{dv}{dx} = P \cdot \pi x^2$$

$$dv = \frac{-P}{2\eta l} \, x dx$$

integrating both side.

$$\int dv = \frac{-P}{2\eta l} \int x \, dx$$

$$v = \frac{-P}{4\eta l} x^2 + C \quad \text{—— (iii)}$$

where C is constant of integration.

when $x = r$, $v = 0$

$0 = \frac{-P}{4\eta l} r^2 + C$, therefore

$$C = \frac{P}{4\eta l} r^2$$

Put the value of C in eq^n (iii)

$$v = \frac{P}{4\eta l} (r^2 - x^2) \quad \text{—— (4)}$$

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PAGE:
DATE: / /

Let v = Volume of liquid flowing per second through the cappillarly tube.

Cross-sectional area of the cylindrical shell of radius x and $(x+dx) = 2\pi x\,dx$

$$v = \frac{P}{4\eta l}(r^2 - x^2) \quad \text{— (IV)}$$

$$dV = 2\pi x\,dx \cdot v$$
$$dV = 2\pi x\,dx \cdot \frac{P}{4\eta l}(r^2 - x^2)$$

$$\boxed{dV = \frac{\pi P}{2\eta l}(r^2 x - x^3)\,dx} \quad \text{— (V)}$$

Hence total volume of the liquid flowing per second through the capilledurly tube can be obtained by integrating eq^n (5) from the limit 0–V and 0–r.

eq^n (5) becomes:—

$$\int_0^V dV = \frac{\pi P}{2\eta l}\int_0^r (r^2 x - x^3)\,dx$$

$$V = \frac{\pi P}{2\eta l}\left(\frac{r^4}{2} - \frac{r^4}{4}\right)$$

$$V = \frac{\pi P}{2\eta l} \cdot \frac{r^4}{4}$$

$$V = \frac{\pi P r^4}{8\eta l} \quad \text{—} (6)$$

Eqn (6) is the required poisellueous formula for the rate of flow of the liquid through a cappilary tube.

co-efficient of viscosity :-

$$\eta = \frac{\pi P r^4}{8 V l}$$

r = radius of tube
l = length of tube
V = Volume of tube
P = rate of flow of tube.