Physics GE-04

Progressive Wave – When a wave propagates along the forward direction, then this type of wave is called progressive wave.

Equation for the displacement of the progressive wave is given by –

$$y = a \sin \frac{2\pi}{\lambda}(vt - x) \quad \text{——— (1)}$$

where,

$y \rightarrow$ displacement at any instant
$a \rightarrow$ amplitude of the wave
$\lambda \rightarrow$ wavelength of the wave
$v \rightarrow$ velocity of the wave
$t \rightarrow$ time at any instant
$x \rightarrow$ distance from the origin at a instant

Equation (1) is used when the wave travells along the +ve direction of x- axis.

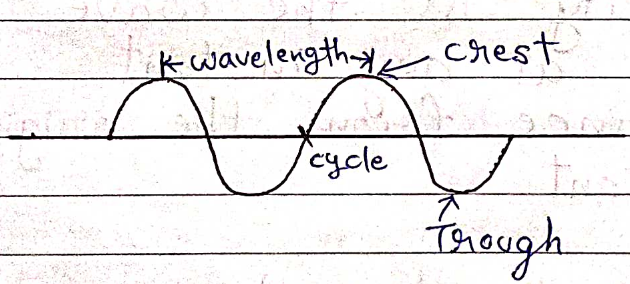
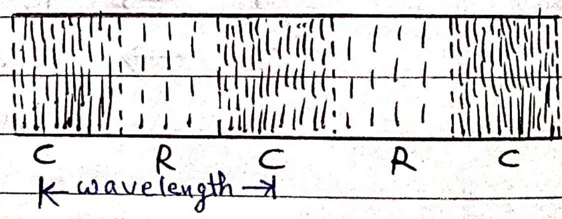
But when ~~tra~~ wave travells along the -ve direction of x-axis then eq^n (1) becomes.

$$y = a \sin \frac{2\pi}{\lambda}(vt + x)$$

There are two types of progressive wave –

1.) Longitudinal wave → Sound wave
2.) Transverse wave ⊥r Light wave

Difference b/w Transverse and Longitudinal wave –

	Transverse Wave	Longitudinal wave
1.)	The distance b/w two consecutive crest or trough gives the wavelength of transverse wave. 	The distance b/w the crest of consecutive ~~comparession~~ compression or refraction gives the wavelength of Longitudinal wave. 
2.)	Particles vibrate in a direction ⊥r to the direction of the propagation of the wave.	Particles vibrate in a direction parallel to the direction of propagation of the wave.
3.)	They can be polarised.	They cannot be polarised.
4.)	They are possible only in Solids.	They are possible in Solids, liquids & gases.

5.)	Crests and Troughs are formed.	Compressions and refractions are formed
6.)	These waves do not involve changes of pressure and density of the medium.	These waves involve changes of pressure and density of the medium.

Differential eqn of progressive wave –

Eqn for the displacement of the progressive wave is given by,

$$y = a \sin \frac{2\pi}{\lambda}(vt - x) \quad \text{—— (I)}$$

on differentiating above eqn w.r.t x,

$$\therefore \frac{dy}{dx} = -\frac{2\pi a}{\lambda} \cos \frac{2\pi}{\lambda}(vt - x)$$

Now, again differentiating w.r.t x

$$\therefore \frac{d^2y}{dx^2} = -\frac{4\pi^2 a}{\lambda^2} \sin \frac{2\pi}{\lambda}(vt - x)$$

$$\frac{d^2y}{dx^2} = -\frac{4\pi^2}{\lambda^2} \cdot y \quad \text{—— (II)} \quad \left[y = a \sin \frac{2\pi}{\lambda}(vt - x)\right]$$

Now, differentiating the eqn (I) w.r.t time 't',

$$\frac{dy}{dt} = \frac{2\pi v}{\lambda} a \cos \frac{2\pi}{\lambda}(vt - x)$$

Again differentiating the above eqⁿ w.r.t 't'

$$\frac{d^2y}{dt^2} = -\frac{4\pi^2 v^2}{\lambda^2} a \sin \frac{2\pi}{\lambda}(vt - x)$$

$$\therefore \frac{d^2y}{dt^2} = -\frac{4\pi^2 v^2}{\lambda^2} \cdot y \quad \text{—— (iii)}$$

from above eqⁿ (ii) and (iii), we get,

$$\boxed{\frac{d^2y}{dt^2} = v^2 \frac{d^2y}{dx^2} \quad \text{—— (iv)}}$$

eqⁿ (iv) represents the differential eqn of progressive wave.

From the above eqⁿ we can write,

Acceleration of the particle = $v^2 \times$ rate of change of the displacement

Electromagnetic Wave – Electromagnetic waves consist of sinusoidaly (sin function of time) time varying electric and magnetic fields acting at right angles of each other as well as $\perp^r$ to the direction of propagation of the waves.

example: Light wave

There are 7 types of electromagnetic wave –

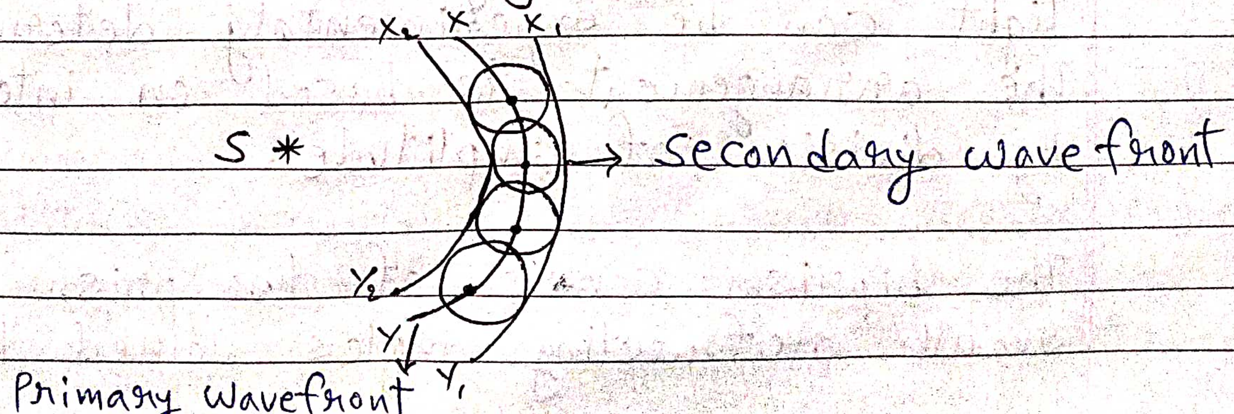
1.) Gamma Rays
2.) X - Rays
3.) Ultraviolet Waves
4.) Visible Light Rays
5.) Infrared Waves
6.) Microwaves
7.) Radio Waves

$\lambda \downarrow$ $\quad f \uparrow$

Wavefront – A wavefront is the locus of all such points which are at the same distance from the source of light.

The assumed initial wavefront from a source of light is called Primary wavefront. All the points on the primary wavefront are in same phase and emits secondary wave at the same time in all direction.

The wavefront after the primary wavefront is called secondary wavefront.



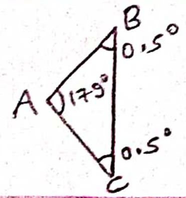
In this figure $XY \rightarrow$ Primary Wavefront
X_1Y_1 & $X_2Y_2 \rightarrow$ secondary wavefront
But X_2Y_2 secondary wavefront is not taken into count because light wave always in the forward direction.

Interference of Light - When two light waves having same amplitude, same frequency, and same phase or constant phase difference move in a medium then in that medium if we place a screen then in that screen we get alternate dark and bright fringes. This phenomenon of light is called Interference of Light.

Two light wave having the same amplitude, same frequency and same phase or constant phase difference are called Coherent Source of Light.

Biprism - Biprism is used to produce the interference pattern, by which the wavelength of monochromatic light can be experimentaly determined. The arrangement is based on interference by division of amplitude.

The biprism consist of two prism of very small refracting angles joined base to



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base. Actually, a biprism is a single prism with an optuse angle of about 179° and the remaining two acute angles of 0.5° as shown in figure. It is made by a thin glass plate whose one of its faces is grounded and polished till biprism is formed.

Newton's formula for the velocity of Sound in air and laplace correction.

Newton's formula for velocity of sound in air is given by,

$$v = \sqrt{\frac{E}{\rho}} \quad \text{———} \quad (1)$$

where $E \rightarrow$ Elasticity of the medium
$\rho \rightarrow$ density of the medium

Let 'P' $\rightarrow$ initial pressure of the gas
'V' $\rightarrow$ initial volume of the gas
$p \rightarrow$ excess pressure due to propagation of sound wave
$v \rightarrow$ decrease in volume

$\because$ Final Pressure $= P + p$
& Final volume $= V + v$

Newton's assumed that when sound wave propagates in gas/air then temperature remains constant i.e. process is isothermal.

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$\therefore PV = (P+p)(V+v)$

$\cancel{PV} = \cancel{PV} - Pv + pV - pv$

$\because$ p & v are very small & so 'pv' can be neglected.

$\therefore pV = Pv$

$\therefore P = \frac{pV}{v}$

or $P = \frac{p}{\frac{v}{V}} = \frac{\text{volume stress}}{\text{volume strain}}$

$\therefore P = K$

$\because \mathcal{E} = K$

$\therefore P = \mathcal{E}$

$\therefore$ Newton's formula becomes,

$$\boxed{v = \sqrt{\frac{P}{\rho}}}$$

At normal temperature & pressure,

$P = h\rho g = 0.76 \times 13.6 \times 9.8 \ N/m^2$

& $\rho = 1.293 \ kg/m^3$

$\therefore v = \sqrt{\frac{0.76 \times 13.6 \times 9.8}{1.293}}$

$v = 280 \ m/sec$

But velocity of sound in air at 0°C is 332 m/sec. It means that there is something wrong with Newton's assumption. which was corrected by Laplace and called "Laplace correction".

Laplace assumed that when sound wave propagates in air then process does not becomes isothermal but it becomes adiabatic. i.e. temperature changes.

$\therefore \quad PV^{\gamma} = (P+p).(V-v)^{\gamma}$

$P\cancel{V^{\gamma}} = (P+p) \cdot \cancel{V^{\gamma}} \left(1 - \frac{v}{V}\right)^{\gamma}$

Expand by ~~Bfo~~ Binomial theorem, we get

$P = (P+p)\left(1 - \frac{\gamma v}{V}\right)$ [neglecting higher power of $\frac{v}{V}$]

or, $\cancel{P} = \cancel{P} - \frac{P\gamma v}{V} + p - \frac{p\gamma v}{V}$

$\because \frac{p\gamma v}{V}$ is very small and hence it can be neglected.

$\therefore \quad \frac{P\gamma v}{V} = p$

or, $P\gamma = \frac{pV}{v}$

or, $P\gamma = \frac{p}{\frac{v}{V}} = K = \varepsilon$

$\therefore$ Newton's formula becomes,

$$\boxed{v = \sqrt{\frac{P.\gamma}{\rho}}}$$

$$= \sqrt{\frac{0.76 \times 13.6 \times 9.8 \times 1.41}{1.293}}$$

$$= 332.5 \ m/sec$$

which is very near to experimental of velocity of Sound in air at $0°c$. Hence Laplace correction is correct.

Expression for energy density of plane progressive wave.

The eqn of simple harmonic progressive wave is given by,

$$y = a \sin \frac{2\pi}{\lambda}(vt - x) \quad \text{——— (I)}$$

we get particle velocity as,

$$v = \frac{dy}{dt} = \frac{2\pi a v}{\lambda} \cdot \cos \frac{2\pi}{\lambda}(vt - x) \quad \text{——— (II)}$$

and the acceleration of the particle is given as,

$$\alpha = \frac{dv}{dt} = -\frac{4\pi^2 a v^2}{\lambda^2} \cdot \sin \frac{2\pi}{\lambda}(vt - x) \quad \text{——— (III)}$$

where the -ve sign shows that acceleration is directed towards the mean position.

Kinetic energy per unit volume is given by,

$$K.E = \frac{1}{2} \rho v^2$$

where ρ is the density of medium.

$$= \frac{1}{2}\rho\left\{\frac{2\pi a v}{\lambda}\cdot\cos\frac{2\pi}{\lambda}(vt-x)\right\}^2$$ from eqn (II)

$$= \frac{2\pi^2 a^2 v^2 \rho}{\lambda^2}\cdot\cos^2\frac{2\pi}{\lambda}(vt-x) \quad \text{—— (IV)}$$

Potential energy per unit volume is given by, for small displacement dy,

$$P\cdot E = W = \rho\, \alpha\, dy$$

$$= \rho\left\{\frac{4\pi^2 a v^2}{\lambda^2}\cdot\sin\frac{2\pi}{\lambda}(vt-x)\right\}dy$$

Total workdone for displacement y is given by,

$$= \int_0^y \rho\left\{\frac{4\pi^2 a v^2}{\lambda^2}\cdot\sin\frac{2\pi}{\lambda}(vt-x)\right\}dy$$

$$\therefore\ W = \left(\frac{4\pi^2 v^2 \rho}{\lambda^2}\right)\int_0^y y\,dy$$

$$= \frac{4\pi^2 v^2 \rho}{2\lambda^2}\cdot y^2$$

$$\therefore\ = \frac{2\pi^2 v^2 \rho}{\lambda^2}\cdot y^2$$

$\therefore$ P.E per unit volume is,

$$P\cdot E = \frac{2\pi^2 v^2 a^2 \rho}{\lambda^2}\sin^2\frac{2\pi}{\lambda}(vt-x) \quad \text{—— (V)}$$

Total energy per unit volume is given by,

$$E = K\cdot E + P\cdot E$$

$$E = \frac{2\pi^2 \rho v^2 a^2}{\lambda^2}\left\{\sin^2\frac{2\pi}{\lambda}(vt-x) + \cos^2\frac{2\pi}{\lambda}(vt-x)\right\}$$

$$\therefore\ E = \frac{2\pi^2 \rho v^2 a^2}{\lambda^2} \quad \text{——— (VI)}$$

we know that,

$$v = n\lambda$$

then from eqn (VI)

$$\boxed{E = 2\pi^2 \rho n^2 a^2}$$

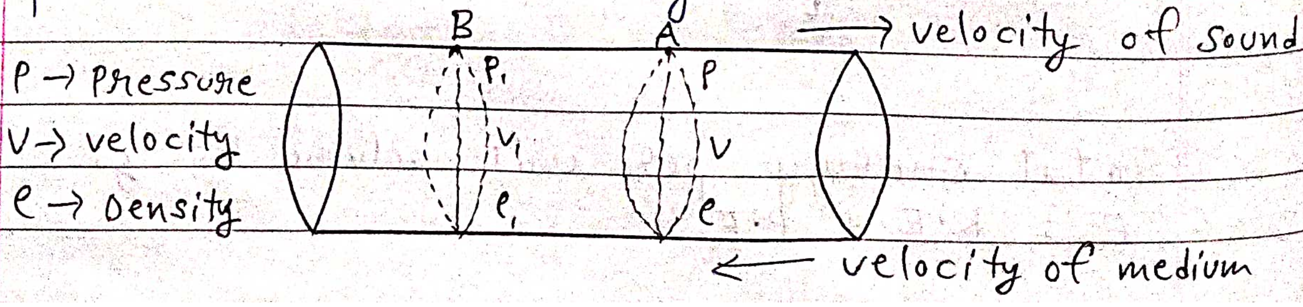
This is expression for energy density.

The average K.E per unit volume and average P.E per unit volume are equal and each is equal to 1/2 the total energy per unit volume.

$\therefore$ Average K.E per unit volume $= \pi^2 \rho n^2 a^2$
and average P.E per unit volume $= \pi^2 \rho n^2 a^2$

Expression for velocity of longitudinal waves in fluid in a pipe.

When a longitudinal progressive wave move in fluids, the particles of the medium vibrate along the direction of propagation. Hence pressure varries on particle inside the gas.



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Consider a long tube. Let a equal to projection area of a tube.
The wave is moving in +ve x direction along the length of the tube. v is the velocity of the wave.

A & B be two projection of the tube & the medium is supposed to move in −ve x ~~direction~~ direction. Then the position of compressions and refractions in the tube will be remain fixed with respect to ground.

Let pressure, density and velocity of the medium at A = P, ρ, v and at B = P_1, ρ_1, v_1.

∴ Mass of the medium entering at A per second = $av\rho$ and mass of the medium leaving at B per second = $av_1\rho_1$

As the average density of the medium is constant, the masses of the medium crossing the sections A & B per second

$(m) = av\rho = av_1\rho_1$

$\therefore \quad v\rho = v_1\rho_1$ ——— (1)

∵ B is the region of compression, so its density will be higher than normal density ρ at A.

$\therefore \quad \rho_1 > \rho$ & $v_1 > v$

$\therefore$ The momentum of the medium entering per second at A equal to mv & the momentum of the medium leaving per second at B equal to mv_1.

$\therefore$ change in momentum per second $= mv - mv_1$

The change in momentum per second is due to difference in pressure $(P_1 - P)$ between A & B.

$\therefore \quad F = a(P_1 - P)$

A/c to Newton's second law of motion, rate of change of momentum at applied force,

$$mv - mv_1 = a(P_1 - P)$$

$$(P_1 - P)a = mv\left(1 - \frac{v_1}{v}\right) \quad \text{——— (ii)}$$

putting the value $m = av\rho$ & $\frac{v_1}{v} = \frac{\rho}{\rho_1}$ in eqn (ii)

$$(P_1 - P)\cancel{a} = \cancel{a}v^2\rho\left(1 - \frac{\rho}{\rho_1}\right)$$

$$(P_1 - P) = v^2\rho\left(1 - \frac{\rho}{\rho_1}\right)$$

$\therefore$ Access pressure $(P_1 - P) = v^2\rho\left(1 - \frac{\rho}{\rho_1}\right)$

$$P_1 - P = v^2\rho\left(\frac{\rho_1 - \rho}{\rho_1}\right)$$

$$v^2\rho = \frac{P_1 - P}{\left(\frac{\rho_1 - \rho}{\rho_1}\right)} \quad \text{——— (iii)}$$

Let E = Co-efficient of elasticity of medium.

$$E = \frac{\text{stress}}{\text{strain}} = \frac{\text{Decrease in Pressure}}{\text{strain}}$$

from eqn (iii)

$$v^2 \rho = E$$

$$v^2 = \frac{E}{\rho}$$

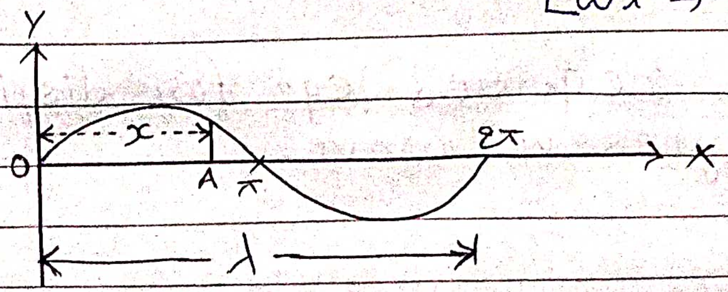
$$\boxed{v = \sqrt{\frac{E}{\rho}}}$$

Thus velocity of longitudinal wave in gas depends upon the elasticity and density of the medium.

Equation for displacement of a progressive wave and difference b/w transverse wave and longitudinal wave.

Let us consider a progressive wave along x-axis. The displacement eqn of the wave at any instant is given by,

$$y = a \sin \omega t \qquad \begin{bmatrix} a \rightarrow \text{amplitude} \\ \omega t \rightarrow \text{phase} \end{bmatrix}$$



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Let us consider a particle at 'p' at a distance x from 0. The particle p is similarly vibrate but due to the distance it will get some phase difference we know that when path difference is λ then the phase difference is 2π.

$\therefore$ when the path difference is x then phase difference is $\frac{2\pi}{\lambda}x$

$$\therefore y = a\sin\left\{\omega t - \frac{2\pi}{\lambda}x\right\} \quad \left[\text{where } \omega = \frac{2\pi}{T}\right]$$

$$= a\sin\left\{\frac{2\pi}{T}t - \frac{2\pi}{\lambda}x\right\}$$

$$= a\sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) \quad \text{——— (1)}$$

we know that,

$\lambda = VT$ ($v \rightarrow$ velocity of wave)

or $\frac{1}{T} = \frac{V}{\lambda}$

putting the value of $\frac{1}{T}$ in eqn (1) we get,

$$y = a\sin 2\pi\left(\frac{Vt}{\lambda} - \frac{x}{\lambda}\right)$$

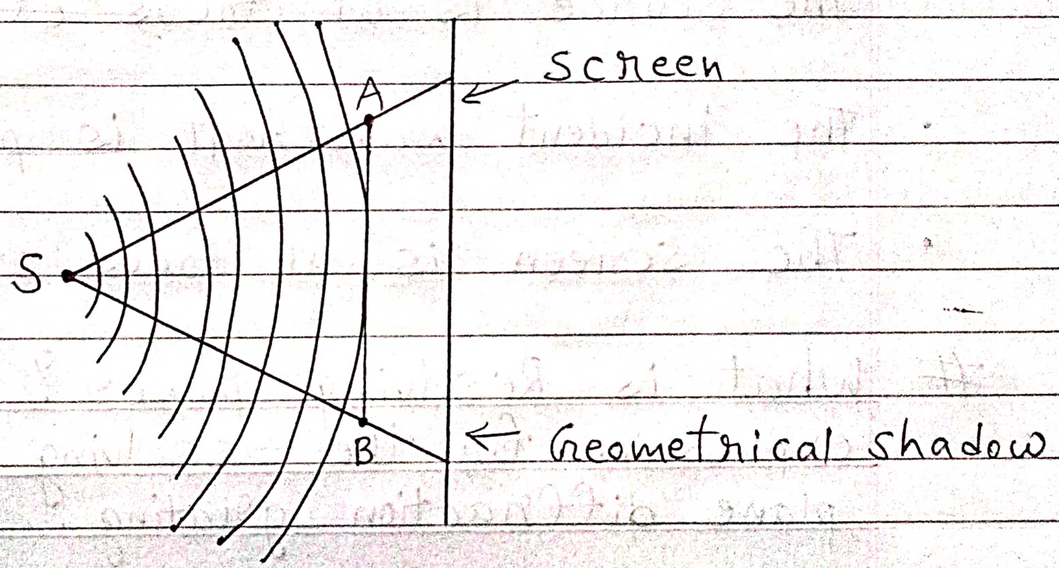
$$\boxed{y = a\sin\frac{2\pi}{\lambda}(vt - x)}$$

This is the required eqn for displacement of a progressive wave.

Diffraction of Light - The phenomenon of bending of light around an obstacle into the region of geometrical shadow is called diffraction of light.

There are two types of diffraction.

1) Fresnel Diffraction
2) Fraunhofer Diffraction



1) Fresnel diffraction -

- The source of light and the screen are at finite distance from the obstacle.
- No lenses are used in this.
- The incident ~~wavelength~~ wavefront is either spherical or cylindrical.

2.) Fraunhoffer diffraction -

- The source of light and screen all at infinite distance from the obstacle.
- one convex lense is used between source and obstacle and other lense is used between obstacle and screen.
- The source is at focus of first lense.
- The incident wavefront is plane.
- The screen is at focus of second lense.

What is Resolving power? Define expression for the resolving power of plane diffraction grating.

Resolving power of an optical instrument is defined as the ratio of wavelength (λ) to the difference in wavelength ($d\lambda$).

$$\therefore \boxed{\text{Resolving power} = \frac{\lambda}{d\lambda}}$$

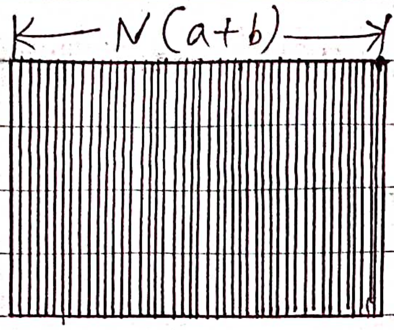
It has no unit.

~~Ref~~ Resolving power of plane diffraction grating -

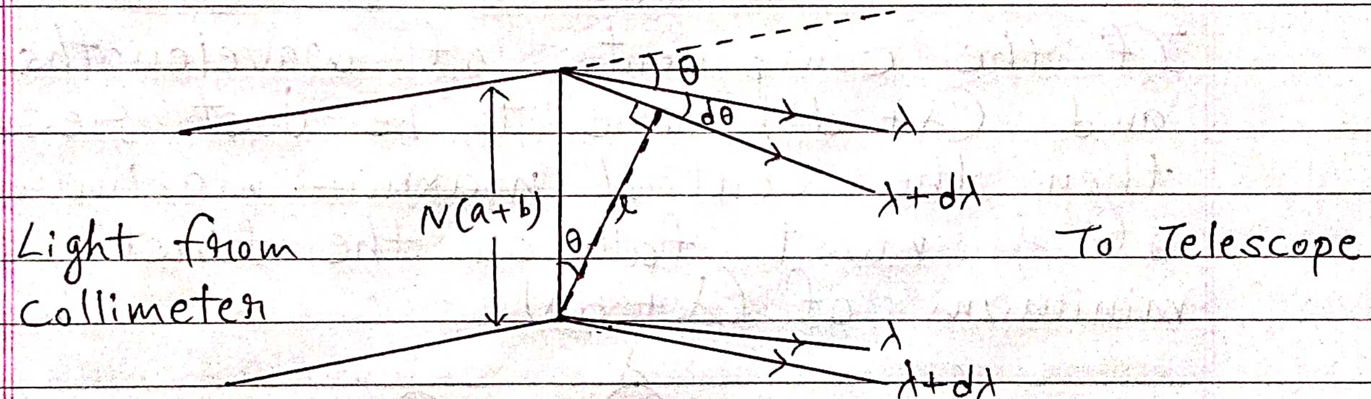
Let N = Numbers of rulings (or lines) in a diffraction grating

$(a+b)$ = separation between the centres of adjacent lines.

Then full width of rulings $= N(a+b)$



1500 lines/inch



The plane diffraction grating is mounted on a spectrometer table. The slit of collimeter is illuminated by the source of light containing wavelengths λ & $\lambda + d\lambda$. The light from the collimeter incident normally on the surface of diffraction grating.

Let θ = angle of diffraction of n^{th} order principle maxima for normally incident light of wavelength λ.

Then by grating eq^n we have,

$$(a+b)\sin\theta = n\lambda \quad \text{--- (I)}$$

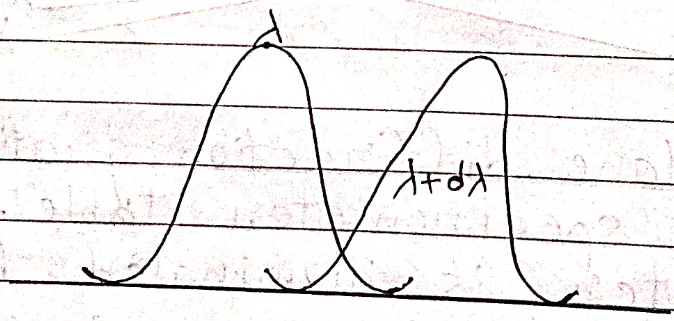
where $n = 1$ for first order
$n = 2$ for second order

differentiate w.r.t λ we get,

$$(a+b)\cos\theta \frac{d\theta}{d\lambda} = n$$

$$\therefore \frac{d\theta}{d\lambda} = \frac{n}{(a+b)\cos\theta} \quad \text{--- (II)}$$

If the components of wavelengths λ and $(\lambda + d\lambda)$ are to be just resolved, then the central ~~maxima~~ maximum of λ must fall on the first minimum of $(\lambda + d\lambda)$.



Let $d\theta$ = angle which separates two images of λ and $(\lambda + d\lambda)$.

Then $d\theta = \frac{\lambda}{l}$ --- (III)

where l = width of wavefront in the parallel light leaving the grating to enter the telescope.

from figure,

$$\cos\theta = \frac{l}{N(a+b)}$$

$$l = N(a+b)\cos\theta$$

putting values of l in eqⁿ (III) we get,

$$d\theta = \frac{\lambda}{N(a+b)\cos\theta} \quad \text{——— (IV)}$$

Now putting this value of $d\theta$ in eqⁿ (II) we get,

$$\frac{1}{d\lambda} \times \frac{\lambda}{N\cancel{(a+b)\cos\theta}} = \frac{n}{\cancel{(a+b)\cos\theta}}$$

or $$\boxed{\frac{\lambda}{d\lambda} = N.n}$$

This is the required eqⁿ for resolving power of grating.

This shows that resolving power of grating increases with —
(i) The total no. of rulings (N) on the grating surface.
(ii) The order no. (n) of spectrum.

What are Fresnel's assumptions in Fresnel's diffraction?

Following are the Fresnel's assumptions —

- Each element of wavefront sends secondary waves continuously.

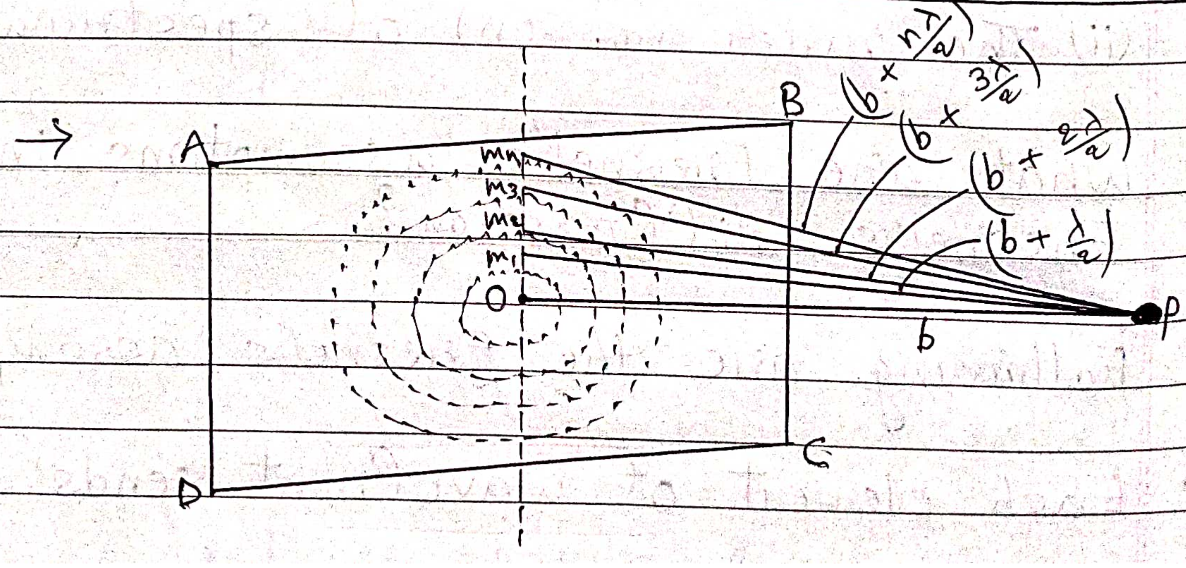
- Each point on the wavefront may be considered as the centre of propagation of secondary wavelets.

- The Wavefront can be divided into a large no of strips or zones called Fresnel's zone.

- The effect at a point due to any particular zone depends on the distance of point from that zone.

- The resultant effect at any point is determined by combining the effects of all the secondary waves reaching there from various zone.

What are Fresnel's Half Period zones? Show that the radii of half period zones are proportional to the square root of the natural number.



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Let ABCD is a plane wavefront ⊥ to the plane of the paper.

Let λ = wavelength of monochromatic light.
Let P is any point at phase the effect of the whole wavefront.

PO is ⊥ drawn from point P to the wavefront ABCD.

Let OP = b

The wavefront ABCD is divided into a large number of concentric half period zones, called Fresnel's zones, with P as centre and radii equal to $(b+\lambda/2)$, $(b+2\lambda/2)$, $(b+3\lambda/2)$, ..., $(b+n\lambda/2)$

We draw a series of spheres on the wavefront. Thus cutting the wavefront into annular zones.

We get concentric circles having radii Om_1, Om_2, Om_3, ..., Om_n.

The secondary wavelets from any two consecutive zones reach point P with a path difference $\lambda/2$. Hence the zones are called "Half Period Zones".

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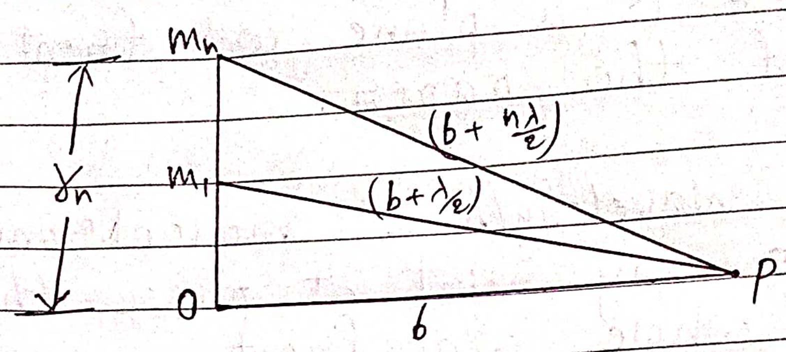


Let $r_1 = OM_1$ = radius of first half period zone.

Then $(M_1P)^2 = (OM_1)^2 + (OP)^2$

$\therefore (OM_1)^2 = (M_1P)^2 - (OP)^2$

$$(r_1)^2 = \left(b+\frac{\lambda}{2}\right)^2 - b^2$$

$$= \cancel{b^2} + \frac{\cancel{2}b\lambda}{\cancel{2}} + \frac{\lambda^2}{4} - \cancel{b^2}$$

$$r_1^2 = b\lambda \qquad \left[\text{neglecting } \frac{\lambda^2}{4}\right]$$

$$r_1 = \sqrt{b\lambda} \quad \text{——— (1)}$$

Let $r_n = OM_n$ = radius of n^{th} half period zone.

Then $(M_1P)^2 = (OM_n)^2 + (OP)^2$

$\therefore (OM_n)^2 = (M_1P)^2 - (OP)^2$

$$(r_n)^2 = \left(b+\frac{n\lambda}{2}\right)^2 - b^2$$

$$= \cancel{b^2} + \frac{\cancel{2}n\lambda b}{\cancel{2}} + \frac{n^2\lambda^2}{4} - \cancel{b^2}$$

$$r_n^2 = nb\lambda$$

$$r_n = \sqrt{nb\lambda} \quad \text{——— (ii)} \quad \left[\text{neglecting } \frac{n^2\lambda^2}{4}\right]$$

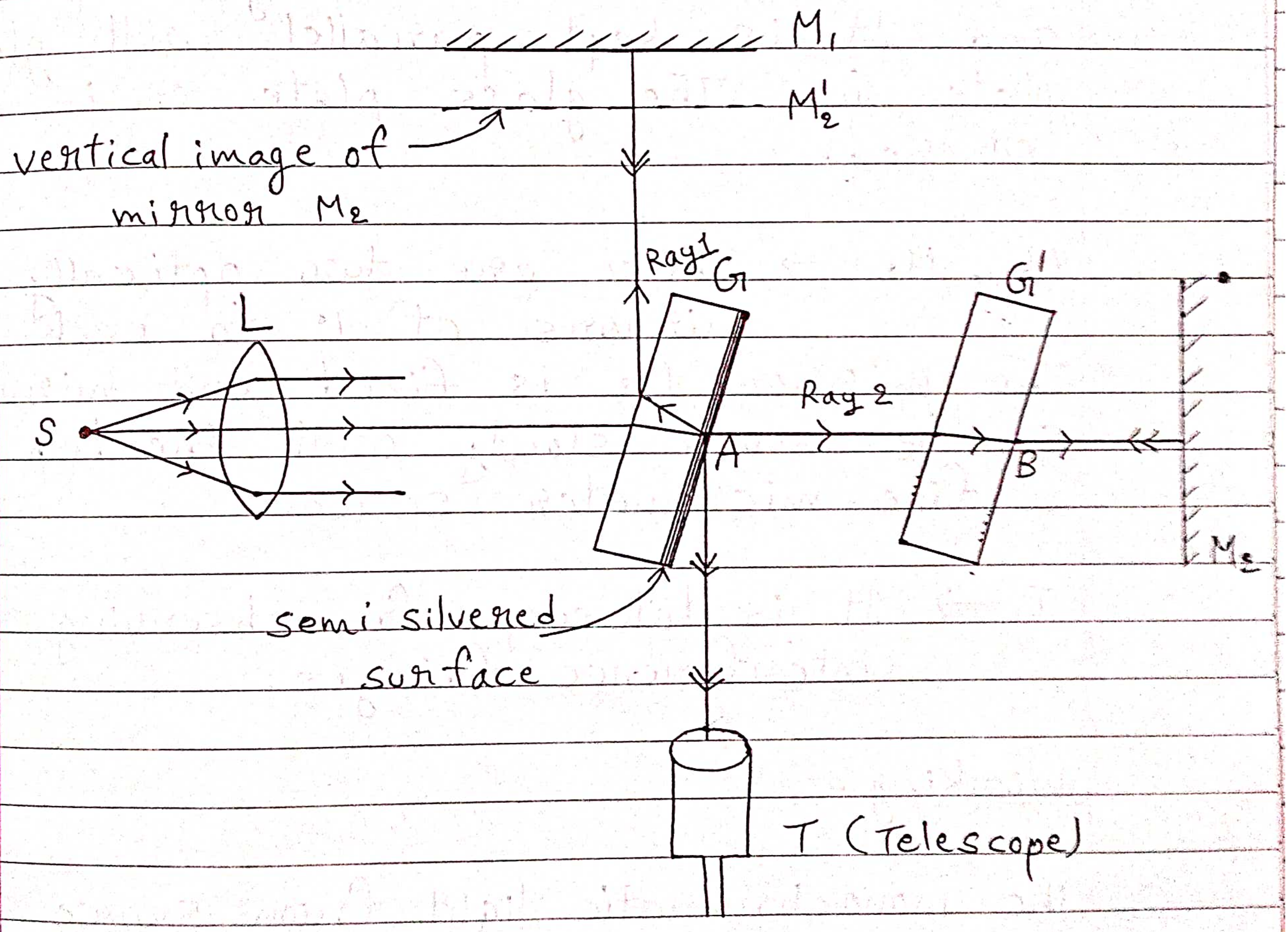
Thus we see that radii of half period zones are proportional to the square root of natural number.

i.e. $\boxed{r_n \propto \sqrt{n}}$

What is interferometer ?

Interferometer is an optical instruement used for obtaining interference fringes.

Describe with a neat diagram the construction and working of Michelson Interferometer ?



The construction of Michelson interferometer is described below –

S → It is a source of monochromatic light placed at the focus of convex lense.

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L → It is a convex lense.

G → It is a plane parallel glass plate inclined at 45° with the axis of incident B. It is semi-silvered on the face nearer to the glass plate G'.

G' → It is a transparent glass plate identical with the glass plate G and It is kept parallel with glass plate G. The glass plate G' is not silvered.

M_1, M_2 → These are two optically plane mirrors at ⊥ to each other. The mirror M_2 is fixed and mirror M_1 Can be moved slowly with the help of a fine micrometer screw.

T → It is telescope for observing interference fringes.

Working :-

The monochromatic light from source is made parallel by the convex lense L. The parallel light is incident on the glass plate G. ~~is partly reflected~~ A ray of light incident on glass plate G is partly reflected and rest transmitted. The reflected ray goes towards mirror M_1 and transmitted ray goes towards mirror M_2. These two ray travel along two mutually $\perp^r$ directions and are reflected back by the mirrors M_1 and M_2 towards the glass plate G. The two rays recombined at the semi-silvered glass plate G at point A. Now, they enter ~~two~~ into the telescope T. Since the rays entering the telescope are derived from the same incident ray. Hence they are coherent. Due to superposition of coherent light wave interference fringes are produced which is observed in the telescope.

Role of glass plate G' :

To compensate the path of rays 1 & 2 in the glass plate G, a similar glass plate G' is kept parallel to G. Hence the glass place G' is called compensating plate.

Formation of fringes :

M_2' is the virtual image of surface of mirror M_2 formed by reflection from the semi-silvered surface of plate G such that

$$GM_2' = GM_2$$

The interference fringes are formed by light reflected from the surface of M_1 and M_2 respectively. Fringes are observed in the air-film enclosed between M_1 & M_2. Depending upon the distance and inclination between M_1 & M_2'. The fringes observed are -

- straight
- circular
- elliptical
- parabolic

Uses :

(i) To determine the wavelength of monochromatic light.

(ii) To determine difference in wavelength.

(iii) To determine refractive index.